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Foreword

VOLUME 62 of the Transactions of The American Society of Mechanical Engineers contains the individual papers published during 1940 under the sponsorship of the Society's professional divisions and technical committees, including the contributions of the Applied Mechanics Division issued originally in a quarterly known as *Journal of Applied Mechanics*, and the 1940 Society Records and Index. The technical papers and reports that make up this volume represent the Society's annual contribution to the permanent record of mechanical-engineering achievement. Most of these papers and reports were presented at meetings of the Society and of its professional divisions and local sections and were published in monthly issues, eight being distributed as the Transactions of The American Society of Mechanical Engineers and four as the *Journal of Applied Mechanics*. Indexes to other A.S.M.E. papers and publications will be found on page RI-75 at the end of this volume. In view of the fact that material of which this volume is composed was originally issued periodically as Transactions, *Journal of Applied Mechanics*, and Society Records, three sets of page numbers will be found. Numbers without letter symbols are those of the eight issues of the Transactions, those with letter symbol A preceding the number refer to the pages of the *Journal of Applied Mechanics*, which follow, and those with letter symbol RI to the Society Records section, which concludes the volume.

All sections of the Transactions are bound together at the end of the year for the convenience of libraries and of engineers who wish all of the papers in permanent form. Copies of the bound Transactions will be found in depositories located in selected engineering, university, and public libraries throughout the world. A complete list of these depositories will be found on pages RI-71 to RI-74 of the Society Records and Index. Copies of the Transactions have also been set aside for sale.

The Society Records of the A.S.M.E. for 1940, which form a part of these Transactions, are the permanent records of the Society's activities for the year, including lists of committees and memorial biographies. The Index provides a means of locating special A.S.M.E. publications and articles in *Mechanical Engineering* as well as those in the Transactions.

THE COMMITTEE ON PUBLICATIONS

Toward Economic Air-Line Equipment

By A. F. BONNALIE,[1] CHICAGO, ILL.

The paper is a discussion of some of the apparent factors determining the worth of the transport airplane. The author treats size, speed, range, and utilization as being among the major considerations for economic operation. Items of interest both in cost of operation and in sales of service are included.

The author finds no immediate limit to the size of airplanes, but does raise the question of crowding when larger numbers of passengers are involved. He also points out that, due to continual improvement, equipment in the future will be more economic.

WHILE the transport airplane has a reasonably long history, it is only recently that some of the economic relationships have become evident. The technical problems of air transportation, in common with similar problems of all other forms of transportation, can be considered under four headings, namely, location, finance, management, and equipment. In the solution of problems of the first two categories, the experience of older transportation can be drawn upon with but little change; management involves much that is familiar, except changed operating technique, but the equipment problem is very different and is of major importance.

To be truly economic, air-line equipment must be not only financially and mechanically efficient but it must have sufficient sales merit to attract a substantial volume of traffic in spite of adverse factors of considerable magnitude. As pointed out by the author previously,[2] the major factors in the development of passenger traffic are speed, comfort, cost, and fear. Other factors of convenience and utility are definitely limiting and reduce the applicability of air transportation to a limited field, auxiliary to surface transportation. Passenger traffic is herein given major consideration for four reasons:

1 It is at present the major source of income for the air lines.

2 It is most demanding and is determinative of much of the design specification for equipment.

3 It offers the largest field for the development of real business volume.

4 Revenue return from other sources of traffic cannot be affected by equipment change to any considerable extent because their rates are determined outside of the control of equipment or management.[3]

The vehicle, with older forms of transportation, affects costs to a limited degree and sales to a maximum degree. The success of the so-called streamline train has proved this. Something of the same relationship exists with air transportation, except that equipment affects costs to a much higher degree. Where the expenses of the railroads are mostly bonded indebtedness, cost of fixed property and its maintenance, none of these appear in the air-line expense column to any marked extent. Instead, equipment depreciation and upkeep, pay of flying personnel, and fuel become the major cost burdens.

Much of this discussion of air-line equipment must be academic in character and while the author believes it to be sound, the problem is so complex that exceptions are bound to exist. It must also be appreciated that the viewpoint is biased and represents the opinions of one student and not necessarily those of any one else. No claim to originality is made and much of the material is well understood throughout the industry, but the author believes that no comprehensive presentation of such data has been made.

Of the many economic factors involved, size, speed, and degree of equipment utilization are receiving a great deal of attention. These three will be considered first.

The Size of Airplanes

Any study of air-transportation equipment must attempt to answer the problem of economic size. That inadequate answers are the usual result is merely indicative of the difficulty of the problem. That this answer will be no more adequate than others is a foregone conclusion, for the final answer can never be written as long as changing economic conditions exist and clever engineers are able to develop new materials, methods, processes, and mechanism.

For the moment it will be accepted that traffic will be sufficient to allow the economic operation of large equipment. The questions then become what is large and how large is economic? How large can airplanes be built? Not all of these questions can be answered in other than general terms. Specific applications as to range and other requirements, together with the need of suitable operation off restricted flying-field areas, nature of the terrain, and weather must be considered.

Fig. 1 shows the effect of increased size—on the costs per airplane-mile and per capacity-ton-mile of pay load. The costs are computed on a basis of average costs for the domestic air lines of the United States and necessarily certain specific design proportions had to be set for the family of airplanes under consideration. This chart was computed some three years ago and it is interesting that the operating costs of equipment built since that time have come well within acceptable estimating margins.

As many of the costs of air-line operation do not vary directly with the increased size of equipment, it is to be expected that the ton-mile costs of transportation will improve as the size of the airplane increases. A radical change in design proportions would change the shape of these curves of costs per ton-mile and costs per airplane-mile, but most of the present-day airplanes are within a reasonable margin of these figures. Some small operations, with less-than-average overhead, have lower costs than those shown, but others will be somewhat higher.

The entire answer to the question of size cannot be covered by cost alone. Fig. 1 is based on the assumption that the proportioning is constant to the installed power. This is not strictly correct. As the size of the power units increases, there is definite decrease in their specific and installed weight. The per-horsepower weight of instrument and other equipment also decreases. Variations in structural weight of the airplane are much more complex, governed somewhat by the law of similitude.

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[1] Assistant to the Director of Flight Operations, United Air Lines Transport Corporation.

[2] "Elementary Principles of Passenger Transportation," by A. F. Bonnalie, *Journal of the Aeronautical Sciences*, vol. 5, August, 1938, p. 397.

[3] Air-mail rates are established by government authority and express rates are largely dictated by competitive services.

Contributed by the Aeronautic Division and presented at the Semi-Annual Meeting, San Francisco, Calif., July 10–15, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

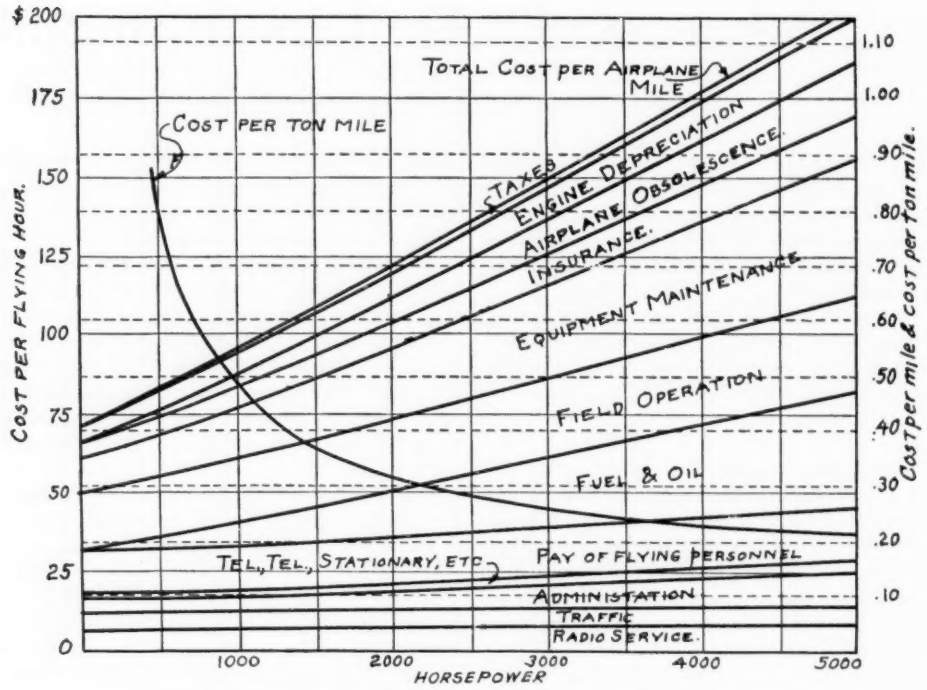


FIG. 1 EFFECT OF AIRPLANE SIZE ON COSTS OF SERVICE
(Based on 200 hr per month, 175 mph, 2¼ lb pay load per hp.)

LAW OF SIMILITUDE

This law, simply stated, is that with a geometric increase in size the weight will vary as the cube of the dimension, and the lifting surface as the square. Accordingly, increasing the dimensions of an airplane would (where d is the ratio between the larger and smaller significant dimension):

(*a*) Multiply gross weight by d^3
(*b*) Multiply wing area by d^2
(*c*) Multiply wing loading by d
(*d*) Multiply power by d^3
(*e*) Multiply power loading by $d^3/d^3 = 1$
(*f*) Multiply load factors by $1/d$.

Increasing the wing loading and reducing the strength would not be practicable except for small changes in dimension. The performance would be affected as follows:

(*a*) Landing speed would be increased approximately as the ratio of $d^{1/2}$.

(*b*) Maximum speed would be improved somewhat better than the ratio of $d^{1/3}$.

(*c*) The take-off run would be lengthened; the relationship is rather involved.

Fortunately, the situation is not as severe as the foregoing would indicate. Many parts of the large airplane would not need to be proportional because smaller airplanes have many members determined by the minimum sizes that can be used, which are accordingly overstrength and overweight. It is also possible to reduce load factors somewhat (or their equivalent) as the size of the airplane is increased. The reducing of load factors cannot be carried too far, however. Better materials are also beneficial. It is to be noted that in the hypothetical case under consideration the useful-load ratio would remain unchanged. It has been argued many times that the law of similitude wrote a definite limit to the size of airplanes. This has always been said whenever the state of the existing art exhausted the available materials, methods, design, and mechanism. Advance has so far invariably pushed the frontier farther, allowing more progress. Unquestionably, some limit exists, but the art has not progressed far enough for it to be apparent at the moment.

Increasing size is accompanied by other problems. It is of course possible that the airplanes now considered huge will, as time passes, be quite as ordinary in the future as the large airplanes of a few years ago seem now. One of the possible limits is the physiological one. This does not seem to be acute however, for the author has seen a test pilot leave a forty-ton transport and, overnight, test-fly a ton-and-a-half airplane, handling it to perfection. In other words, the competent pilot seems to be sufficiently adaptable to the physical size of the aircraft to be capable of making the necessary allowances for the size, thereby insuring equally satisfactory control over a wide range of size.

ABILITY OF CREWS TO HANDLE THE LARGER AIRCRAFT

The heaviest airplane that has been built, the *Dornier DOX* (129,000 lb), or the two versions built for Italy, seemed to be within the capabilities of their crews. Likewise, the *Maxim Gorky*, with the largest span (207 ft 0 in.) that has been flown was controllable and flyable. The use of servo controls makes it practicable to put the control of almost any reasonably sized airplane to within the comfortable strength of one man.

L. C. McCarty, Jr., project engineer of the Glenn L. Martin Company, analyzes the problem of landing large flying boats[4] as follows:

> What is the safe limit for landing speeds? This is a question of .... basic importance to future safety, speed, and efficiency....
>
> With no limiting obstacles the safety in landing and taking off is dependent largely on the impact loads imposed on the structure in the event of bad landings, a bounce, or contact with an advancing wave.
>
> From dimensional analysis, the following important conclusions are indicated:

[4] "Ocean Air Transportation," by L. C. McCarty, Jr., *S.A.E. Journal*, vol. 40, no. 5, May, 1937.

1 The height of free drop increases directly as the linear-scale ratio or as weight to the $^1/_3$ power for the same acceleration.

2 Also for the same acceleration, the landing speed increases as the $^1/_2$ power of the linear scale or as the weight to the $^1/_6$ power.

3 With increasing speeds, for a bounce of the same acceleration, the time interval to the subsequent contact with the water is increasing as the linear scale to the $^1/_2$ power, thus allowing greater time for piloting decision and action.

4 Since the height of free drop varies with the linear scale, the vertical velocity at contact (for the same acceleration) increases as the linear scale to the $^1/_3$ power. Extending this reasoning, greater seaworthiness is indicated with the same design accelerations and with the landing speeds increasing as the $^1/_2$ power of the linear scale.

5 The physiological limitations of the average pilot in judging absolute distance above the water as affected by increased landing speeds, sinking speeds, and his height over the water at the instant of landing are under investigation. Preliminary indications are that, other things being equal, the physiological limitations in this respect have not been approached.

Summarizing it appears that, as the size increases and without "instrument" landings or the use of power in landing, the average pilot will be able to make landings with lower loads on the structure, less discomfort to the occupants, and with greater seaworthiness at increasing landing speeds.

As an example, applying these principles, a landing speed of 75 mph for a 50,000-lb flying boat (Martin 130) would indicate landing speeds of 98 mph for 250,000 lb gross weight and 39 mph for 1000-lb gross weight.

Space Limitations for Passengers

One thing that seems to have definite limits, at least for any long-range aircraft, is the rapidly increasing crowding effect with increased numbers of passengers. Compartmentalization of habitable spaces will help, but even this has limits. The required volume of space per passenger is a variable that needs much consideration. The problem becomes most acute when any more than a relatively small number of passengers are involved for more than a relatively short space of time. Assuming adequate ventilation and heating, apparently the factors to be considered include: number of persons, number of personal contacts, length of trip, whether night or day, the relative diversion of interest, and the gregariousness of those involved.

For instance, a small group of people, well known to each other and making a daylight trip of a few hours in a private automobile, represent the smallest practicable cubature per person. A small car capable of carrying a maximum of six persons will have a total cubature of 114 cu ft, or 19 cu ft per person. It is probable that these six people are either closely related or at least quite well known and friendly to each other. The horizon from the car is limited, the scenery and events are in close proximity, the change of scenery is rapid, and the constant stream of opposing traffic, each unit of which opens a potential hazard of collision, all combine to offer the rider diversion of a high order. Analysis of this situation gives the following:

Number of persons	6
Number of personal contacts, $n\,(n-1)/2$	15
Time of journey	short
Gregarious factor	high
Diversion factor	high

It is probable that this space represents the lower limit per person that could be used. On long journeys the small car becomes rather cramped for capacity loads. The inclusion of strangers lowers the gregarious factor rapidly and any reduction of the diversion factor tends to aggravate conditions. Total strangers with no desire for contact with their fellows and no diversions of any kind would feel comfortable only if the space were proportional to the number of contacts (combinations of two) that are possible. This would be the extreme limit of space demand. Fig. 2 shows the two limiting curves. The lower curve is the straight-line relationship of 19 cu ft per person, the lower limit. The steeper curve is an assumed upper limit of 19 cu ft per possible personal contact or 19 $n\,(n-1)/2$.

This problem of space and diversion is a real one. As the number of passengers increases, the difficulty of providing for the human pay load increases faster than the numbers involved. Even the development of airplanes large enough to have habitable wings may not be more than a minor relief. It is also going to be necessary to take these factors into consideration in the construction of terminal facilities, waiting rooms, and so on.

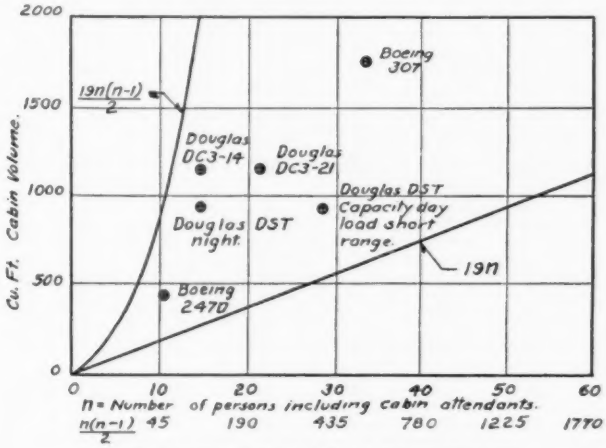


Fig. 2 Limits in Cabin-Space Requirements

The suitability of any design from the standpoint of space for passengers may be expressed as

$$Q = 2\,H_v/[n(n-1)b] \quad \ldots\ldots\ldots\ldots\ldots [1]$$

Where Q = the relative space factor
n = the number of persons in the cabin, including the attendants
H_v = habitable volume of the airplane, cu ft
b = the number of compartments or divisions of space

Q will really be the smallest practicable space per occupant in a rather low-capacity vehicle. Q should vary with the time of uninterrupted occupancy and the available diversions.

The Effects of Speed

Fig. 3 shows the effect of speed on operating costs, assuming that speed is gained by aerodynamic improvement and mechanical development, not by merely increasing power. The reduction in costs per airplane-mile that accompanies increased speed, while not spectacular, is important.

Fig. 4 shows the per-mile costs plotted against speed where change of speed is made by change of power. Much of the increased cost at the higher speeds is due to the decrease in reliability of power plants at the higher outputs. Some assumptions of cost due to power-plant failure were made in this computation, but the decrease in reliability is based on fact. Improvement in power-plant reliability and durability would allow increased speed with accompanying reduced cost. Overpowered equipment is another question, however, that cannot be covered here.

Speed is extremely important to air transportation for it is one of the major reasons for the method. Wright, in his paper[5] expresses the speed-merit relation as

$$F_T = F_0 V_c = V_c^2\,U/(22.7\,W_g + 1000\,P_e + 100\,U) \ldots [2]$$

[5] "The Economics of Speed," by T. P. Wright, read before a meeting of the Institute of the Aeronautical Sciences at Cleveland, Ohio, on Sept. 4, 1937, reported in *Western Flying*, October, 1937.

Where W_g = gross weight, lb
P_c = cruising brake horsepower (usually 60 per cent normal rated power)
U = useful load, lb
F_0 = airplane-operating efficiency factor
F_T = air-transport efficiency factor
V_c = cruising speed, mph.

Wright credits Kendall Perkins with the development of this relation and further says that it is based on the findings that 22

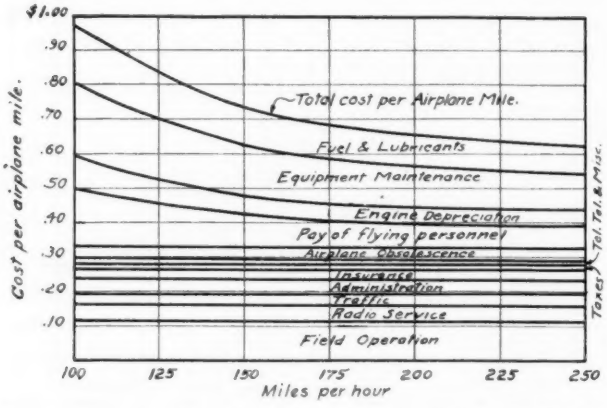


FIG. 3 EFFECT ON PER-MILE COSTS OF INCREASED SPEED DUE TO AERODYNAMIC AND MECHANICAL IMPROVEMENTS
(Typical air-carrier airplane in average U. S. A. domestic operation.)

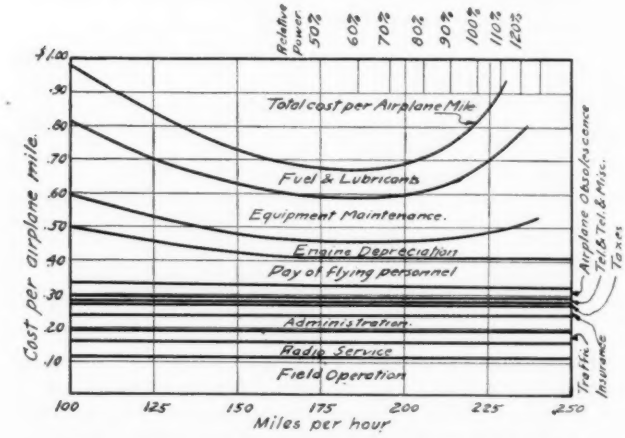


FIG. 4 EFFECT OF INCREASED POWER ON PER-MILE COSTS
(Typical air-carrier airplane in average U. S. A. domestic operation. Speed increased by higher power without improved durability and reliability of power plants.)

per cent of average operating costs can be assigned to gross weight (airplane depreciation and maintenance), 46 per cent can be assigned to power (fuel, oil, engine depreciation, and maintenance), 32 per cent can be assigned to useful load (all types of insurance and crew pay).

According to this analysis, the most important economic factor is speed, followed by useful load, power, and gross weight in that order. Range does not appear in the formula except by the inference that it depends in part on useful load and cruising power.

This rating formula, while it attempts to include the sales value of speed, overlooks other sales factors of relatively great value, particularly comfort. It is a repetition of the old ideal of trying to put into one simple relationship the full merit of the vehicle. Such a hope is obviously a vain one, for no one formula can possibly include the necessary considerations of, say, an operation on the Belgian Congo, across the American continent, and a service into Tibet at one and the same time—to say nothing of including transoceanic services or possibly nonstop over-the-pole operations. Each of these operations and innumerable others would call for widely different characteristics for an ideal piece of equipment. An almost unlimited number of equipment specifications can be drawn and as many or more aircraft built to meet them. Accordingly, only this one figure of merit is considered here, although many have been proposed.

SPEED

The von Kármán formula for speed, somewhat modified by Lesley, expressed in mph, hp units

$$V_{max} = 7.22 \left(\frac{\eta C_{L\max} V_L^2}{C_D P_L \delta} \right)^{1/3} \quad \ldots\ldots [3]$$

Where η = propeller efficiency
C_L = lift coefficient (absolute)
V_L = landing speed, mph
C_D = absolute drag coefficient, whole airplane
P_L = power loading, lb per hp
δ = ratio atmospheric density.

Indirectly this formula takes into account wing loading W/S and wing power hp/S and is directly concerned with the over-all efficiency of the design. From this, it can be seen that the important items for the development of high speed are:

1 Wing load, as high as is practicable considering landing, take-off, and maneuverability.

2 Lowest possible drag for the airplane with the highest possible propeller efficiency.

3 Lowest possible weight per horsepower. In other words, highest possible wing power.

4 Lowest atmospheric-density ratio with maximum thrust (highest propeller efficiency and horsepower output at high altitude).

Putting the above formula into another form, expressed in ft-lb-per-second units

$$V = \left(\frac{C_{L\max} \eta V_L^2}{C_D (W/P) \delta} \right)^{1/3} \quad \ldots\ldots [4]$$

W = weight
P = power = $hp \times 550$
However, $V_L^2 = 2W/C_{L\max} S\rho$
ρ = mass density of the air
S = wing area, sq ft

Substituting $V = (2\eta P/C_D \rho S)^{1/3} \quad \ldots\ldots [5]$

If it is assumed that η (propeller efficiency) is constant, that at any one altitude ρ (mass density of the air) is constant, also that C_D (coefficient of drag) is constant, speed can be seen to vary as the cube root of the wing power. For all practical purposes this relationship has been found to be true. Some authorities, notably Warner and McGinty, have used $(2\eta/C_D\rho)^{1/3}$ as a constant for any one airplane. When English units and constants are used, this figure sometimes called "specific speed," is the speed of the airplane in miles per hour were its engine developing one horsepower for each square foot of wing area. It is a good measure of the relative cleanliness of any design.

DEGREE OF EQUIPMENT UTILIZATION

This factor is of two parts:

(*a*) The actual utilization of pay load in a given vehicle on a given operation called load factor.

(*b*) The amount of hours of flying that can be accomplished by the average airplane on the average operation.

Load Factor (Ratio of Revenue Load to Capacity Pay Load). It is, of course, important that the highest possible load factor be maintained in order that the greatest possible percentage of capacity revenue be developed. Fig. 5 shows a plot of idealized loads showing variations in load factor. This plot is not taken from fact, but shows variations between limits, with even variations being assumed. For instance, when the load is 100 per cent and 0 per cent on two successive trips, the load factor is 50 per cent. When the variation is uniform between 100 and 60 per cent, the load factor is 80 per cent. Actually, in order to develop high load factors it is necessary that full loads be carried oftener than light loads. Whenever the revenue load actually developed approaches the capacity pay load, it is probable that some business has been turned away because of lack of capacity. Furthermore, a great deal of airplane business is emergency in character.

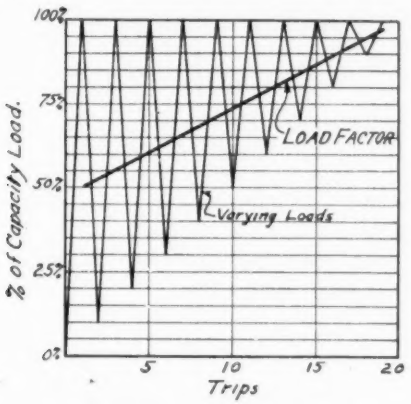


FIG. 5 VARIATION IN LOAD FACTOR

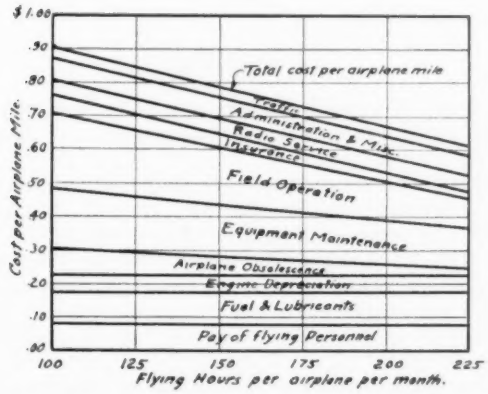


FIG. 6 PER-MILE COSTS VERSUS FLYING HOURS PER MONTH
(Typical air-carrier airplane.)

When the carrier is unable to accommodate an emergency due to lack of capacity, it is failing in its service in the public convenience and necessity.

From Fig. 5 it can be seen that the development of load factors much higher than about 60 per cent allows little reserve capacity for possible demands. It is accordingly essential that the air carrier be able to perform its service with a load factor not much in excess of 60 per cent in order that it may be able to discharge the obligation assumed as a common carrier. Airplanes that can operate on this load factor at a profit on passenger rates are still a matter for future development.

Flight Utilization. Fig. 6 shows the effect of varying the hours of flying per month on the operating costs of the typical air-line airplane previously considered. As it is regular practice to service air-carrier airplanes after not exceeding 20 hr of flight with engine changes at intervals of from 450 to 600 hr of flight, it is difficult to operate at more than about 180 hr per month in regular schedule.

RANGE

The next factor of importance economically is range. Any airplane having a theoretical range of about 10,000 miles will be capable of flying from any habitable portion of the earth to practically any other habitable location. This is still beyond the best that has been done, even with all of the carrying capacity devoted to fuel with the minimum of crew.

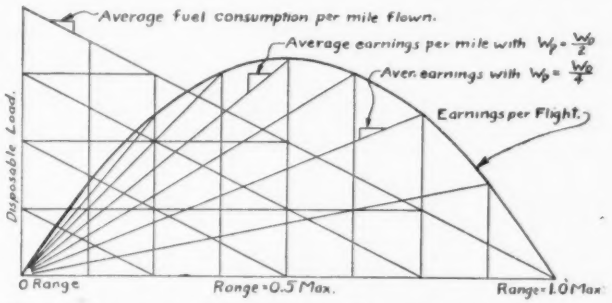


FIG. 7 GRAPHICAL DEVELOPMENT, EARNINGS PER FLIGHT AT VARIOUS RANGES

W_U = useful load = gross weight less empty weight
W_D = disposable load = useful load less crew and supplies
W_P = pay load = disposable load less consumable load (fuel)
Earnings for any one flight are a maximum when
$W_P = W_D/2$ (Consumable load = pay load = ½ disposable load)

Disregarding fuel reserve, other operational and geographical limitations, and assuming that pay load produces revenue on a direct pound-mile basis, a strictly theoretical concept shows the revenue situation in Fig. 7. Fuel is zero when the disposable load is all pay load, so earnings are also zero. Likewise, when the disposable load is all fuel, earnings are again zero for no pay load is carried. Earnings are a maximum when fuel load equals pay load on any one flight. As a transport operation is mostly one flight added to another and so on, the graph shows that earnings will be a maximum for a completed trip when fuel is held to a minimum and frequent stops are made. These relationships have become familiar from many repetitions, and, while academic in character, study will help a true concept to develop.

It is obviously impracticable to use too many operating stops as the operation will be slower and sale of traffic more difficult. The longer nonstop operations assess a penalty in reduction of pay load by the heavier fuel needs, but selling is enhanced and under certain conditions such operations are accordingly justified. The longer the distance between stops the lower is the cost of operation per mile. Additional landings not only slow down the flight speed, but actually increase operating expenses.

If the theory of range be simplified by assuming fixed power output and accordingly fixed fuel consumption per hour, then

$$R = EV_c \quad \ldots\ldots [6]$$

where R = range, miles; E = endurance, hr; and V_c = cruising speed, mph.

But

$$E = (W_0 - W_1)/hp_cC \quad \ldots\ldots [7]$$

where W_0 = initial gross weight, W_1 = final gross weight, C = specific fuel consumption (average), hp_c = average cruising horsepower, and

$$V_c = 7.22\left(\frac{\eta C_{L\,max}\, V_L^2}{C_D\,(W/hp_c)}\right)^{1/3} \quad \ldots\ldots [8]$$

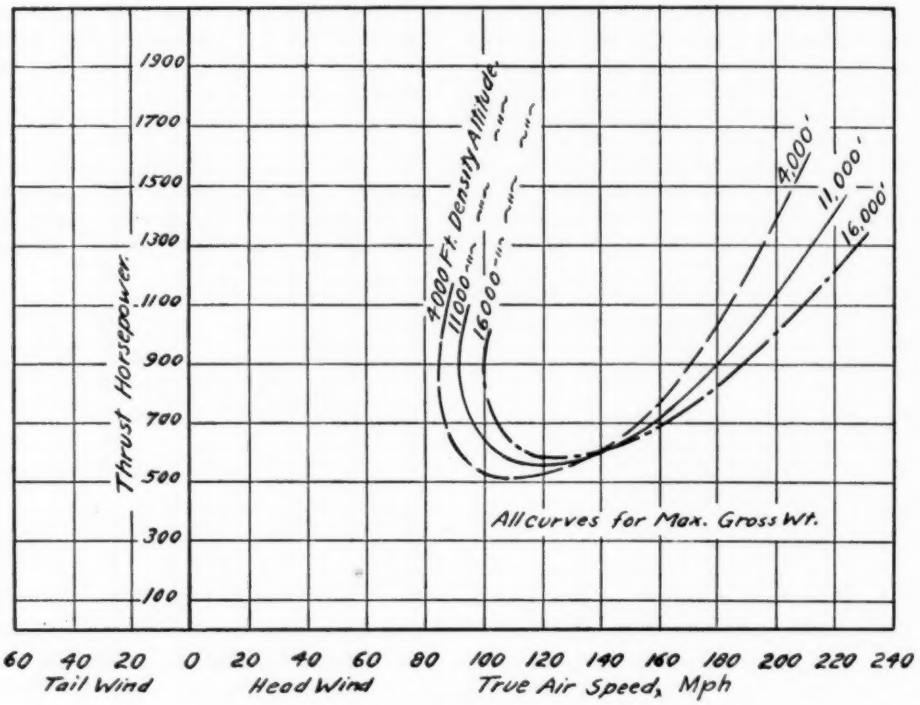


FIG. 8 POWER REQUIREMENTS
(Typical transport airplane.)

substituting $V_L^2 = 2\,W/C_{L\,max}\,S\rho$

$$V_e = 7.22\left(\frac{2\eta hp_e}{C_D\,\rho\,S}\right)^{1/3}$$

then
$$R = 7.22\left(\frac{W_0 - W_1}{hp_e C}\right)\left(\frac{2\,\eta hp_e}{\rho C_D S}\right)^{1/3} \quad \ldots\ldots\ldots [9]$$

This development makes some simplifying assumptions that cannot be applied in practice. The purpose has not been to develop a precise equation, but to establish an expression that will allow an appreciation of the important design factors.

Actually, to develop either the greatest endurance or the greatest range, the power output will be varied in order to maintain the most effective air speed for the purpose. Internal-combustion engines being what they are, the best results will be obtained at the lowest practicable rotational speed of the engine and the highest possible brake mean effective pressure that can be allowed.

Fig. 8 shows the polar curves of power required against true air speed for a typical air-carrier airplane at three altitudes. A tangent drawn from the origin will determine the air speed for most miles per gallon in still air, assuming a flat curve for specific fuel consumption. Moving the point from which the tangent is drawn to the right or left of the origin will allow for head and tail winds, respectively. The actual curve of specific fuel consumption for the engines used will alter the slope of the polar curves somewhat as will the propeller-efficiency curve. Correcting these curves with factors determined from specific fuel consumption and propeller efficiencies will make curves that are quite critical and useful for flight-control purposes.

The foregoing represents a few of the factors in the development of economic air-line equipment. There are many more, most of which are imperfectly understood, but it is felt that more discussion of the problems may lead to a better understanding and appreciation of them.

At the present moment the available air-line equipment is more economic than any that has preceded it. The best has not been developed as yet; there is still enormous room for improvement.

Discussion

WALTER F. MCGINTY.[6] The author is to be congratulated for his very complete presentation of recognized economic factors for air transportation.

A symbol or two seem to be missing, and some minor corrections are required in the equations which have to do with "speed." In Equation [3], for the maximum speed of an airplane in miles per hr, the terms of the various factors indicate that the landing speed is the limiting landing speed at sea level; therefore, in the expressions following Equation [4], the equation for the square of the same landing speed should correspondingly be given in terms of mass density of air at sea level. This will allow a correct transition to Equation [5]. In Equation [8], the atmospheric-density ratio is missing from the denominator. The next equation would be true only if the limiting landing speed were in ft-per-sec units. Due to the fact that the landing speed is defined in this paper as the limiting landing speed in miles per hour, the coefficient, 7.22, in Equation [9] should be eliminated in favor of the value 5.59.

The writer feels that any discussions of air-line economy finally come back to discussions of equipment economy. When reduced to basic essentials, Equation [3] contains every element of economical operating efficiency. An examination of this equation points to the obvious conclusion that high maximum and cruising speeds will be attained with any airplane, provided that the propulsive efficiency, the maximum-lift coefficient, and the landing speed are made as great as possible; and that total-drag coefficient, power loading, and the density ratio be made as small as possible. Landing speeds are dictated by common sense or

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government regulation. Maximum-lift coefficients have been increased greatly these last few years with the development of special wing devices, but practical limits are indicated in this direction already. Power loadings, on the other hand, must be increased considerably over present-day figures in order to insure lowered fuel costs. The choice of a proper operating altitude that will allow intelligent increases in speed, commensurate with power-plant and passenger difficulties with high altitudes, limits the possible decrease in density ratio. What remains then is the ratio of propulsive efficiency to total-drag coefficient. This ratio must be increased if better economies are to be realized. The problem is the same now as it has been for the last 30 years. It is the responsibility, therefore, of airplane and power-plant designers to see that this ratio is made a maximum consistent with structural and maintenance limitations that might well enter.

In the floor discussion that followed the presentation of the paper, the author was asked whether he believed that air-line equipment units were going to increase greatly in size in order to minimize the number of scheduled trips for a given traffic density or whether over-all costs would be lowered if a greater equipment utilization was realized with smaller airplanes operating more frequently. In this regard, the writer feels that we shall see a much greater diversification of airplane equipment types than at present in order to increase over-all traffic by providing transportation to the smaller communities which are not served by air at present. To accomplish this, there will likely be at least three general classes of equipment: Large airplanes with moderate fuel capacity to serve large communities in densely populated areas; slightly smaller transports, designed to operate over long distances for our transcontinental services, stopping only at the larger cities or where refueling may be required; and a much smaller airplane, designed to equip the smaller air lines and to act as "parallel feeders" and "cross feeders" for the main lines.

The Flow of Water in Channels Under Steep Gradients

By W. F. DURAND,[1] STANFORD UNIVERSITY, CALIF.

In the case of water flowing down a steep slope with the upper surface exposed to the air, the usual conditions and equations for hydraulic flow under moderate velocities no longer apply. The changed conditions involve greatly increased velocities, insufflation by air, a surface in contact with the air, uneven, foamy and rolling, with consequent air resistance as an important factor in the problem.

The questions of interest involve the history of the water from its start at a zero or moderate velocity at the top of the slope until it reaches the bottom, or until a limit velocity is reached where the resistance to flow equals the acceleration, and measurably steady conditions continue. Specifically, the questions of chief interest are: With given conditions at the top of the slope, what velocity will be acquired after a given length of run or after a given period of time? The present paper gives a mathematical stepwise solution of these problems, depending on a knowledge of the initial conditions and a known or assumed value of a coefficient β, relating the resistance to flow at a given point in the run or at a given instant of time, to the square of the velocity. Values of the coefficient β await the results of observation on actual cases of such flow. With representative sets of such values, an attempt at generalization could be made. A further important question relates to the details of the way in which the air enters a stream of rapidly flowing water, thus producing the insufflation which is so important a factor in the problem.

THE PROBLEMS arising in connection with the flow of water in channels more commonly relate to moderate gradients, low to moderate velocities, and steady conditions of flow. We are not, as a rule, concerned with the introductory period during which the velocity is increasing from zero, or from some initial velocity, to its final or steady-flow value. Under these latter conditions the forces making for acceleration are balanced by the resistance to flow and steady conditions prevail, that is, over any given cross section of the stream, the distribution of velocity remains sensibly constant in time. Furthermore, the moving medium may be assumed to remain continuously in the liquid phase, that is, without sensible insufflation by air. And again, the resistance opposed to the flow may be considered as that developed between the moving liquid and the solid boundaries which constrain its motion, fluid friction so called.

The final or steady-motion velocity which is acquired under these conditions is, then, the item of principal interest and forms the subject of the well-known classical treatment of the subject.

In the case of water flowing under steep gradients, however, the preceding simple conditions of flow no longer prevail. The velocities acquired may be high and the resistance to the flow can no longer be restricted to the solid boundaries of the stream. The air boundary now begins to play a part, and gradual insufflation with air brings a change in the density of the flowing medium and in the mean hydraulic radius of the section. The surface of the flow becomes broken and irregular due to wave formation (action between air and water) and due also to the marked velocity gradient between the bottom of the flow and the top. The latter is especially marked in case the flow is relatively shallow in depth. As a result, the flow may become transformed from a liquid phase into an extremely turbulent foamy phase with a rough and irregular upper surface involving a greatly increased resistance by the air boundary of the stream.

Under these conditions of flow, we are not only concerned with the ultimate velocity which may be acquired, provided the channel is long enough, but we are also interested in the velocity which may be acquired in a given length of channel or in a given interval of time. We are, therefore, concerned with relations between velocity, distance, and time, during the period of acceleration from an initial velocity v_1 toward a final velocity $\bar{v}$.

The present treatment of this problem differs from the well-known solution by Bakhmeteff in that, with insufflation, the assumption of a continuous liquid phase is no longer tenable. Furthermore, the effect of the fluid pressure within the sheet is neglected and the slope of the flow is taken as that of the bottom surface of the channel. The simplification thus realized by throwing all variable hydraulic conditions into one factor β (as defined later) makes possible the direct solution in terms of hyperbolic functions rather than in terms of a special function, as in the Bakhmeteff solution. The present treatment is presented in no sense as a substitute for the detailed Bakhmeteff solution but rather as a relatively direct mode of approach through well-known tabular functions to problems involving insufflation, always assuming that values of the coefficient β, based on experience, may be available.

The treatment of the problem mathematically will obviously depend on the continuously varying relation between acceleration and resistance or retardation. If we know the angle of slope of the channel at any given point, we can, for a unit of mass, determine the acceleration due to gravity at that point. If the slope is uniform, so also is the acceleration.

The question then turns on the relation between resistance and velocity. For the ordinary case of channel flow we are content, for most practical purposes, to assume the resistance, or fluid friction, to vary as the square of the velocity with a constant coefficient, the value of which will depend on the circumstances of the flow, especially the mean hydraulic radius and the roughness of the surface.

In the present case we may make the same assumption as to velocity, but now with a value of the coefficient varying with the velocity instead of constant. This will give, for the resistance to the flow, a term of the form βv^2 where v is the velocity at any instant of time (or point along the flow) and β is the

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value of the coefficient for that value of v and for the other circumstances of the flow at that time or point. The phrase, "velocity at any instant of time (or point along the flow)," is not precise, since the velocity will vary, perhaps widely, over the cross section of the flow. As usual, however, it may be accepted as meaning the general average velocity of flow over the section.

If then we denote the angle of slope of the channel (not necessarily uniform) at a given point along the line of flow by θ, we shall have, for the general equation of flow the differential form

$$dv/dt = g \sin \theta - \beta v^2 \quad \ldots\ldots [1]$$

With the equation in this form, with θ variable from point to point along the channel and with β variable with the velocity, analytical treatment of the problem appears to be out of the question. The same is true with θ constant, as is more commonly the case, but with β variable. If, however, we know the values of θ from point to point along the channel and if we have some clue to the relation of β to the controlling conditions of the flow and to the velocity, we may then assume these values constant for a brief interval of time—one second or shorter if desired. If then we can solve the equation for constant values of θ and β, we may, knowing the values of velocity or distance at the start of the period, find those for the end of the period, and thus stepwise trace all values through as far as may be desired.

It is seen, of course, that this method of treatment is approximate only. First, the resistance is assumed to vary with the square of the velocity and second, the value of β is assumed to remain constant at some mean value over a short period of time. Justification for the first assumption is found in the fact that it is one of the conditions of a practicable analytical treatment and, further, that it is in line with the general practice in problems involving the relative motion between a fluid and its boundaries. Justification for the second assumption is found in the fact that by decreasing the time interval over which β is taken constant at a mean value, the approximation may be made as close as desired—in any event much closer, presumably, than the observational data on which the values of β may depend.

With this by way of introduction, we may now proceed to the discussion of the equation of motion.

We consider first the simple case where both the slope and β are constant and $v_1 = 0$. We then take the more general case where v_1 is not zero and where, as will be seen, the treatment will provide for a stepwise approximate solution for the case of variable θ or β or both θ and β, from point to point.

Turning then to Equation [1] we put α for $g \sin \theta$ and have

$$dv/dt = \alpha - \beta v^2$$

For $dv/dt = 0$ we have for the limit velocity

$$\bar{v} = (\alpha/\beta)^{1/2} \quad \ldots\ldots [2]$$

Put the equation in the form

$$\frac{dt}{dv} = \frac{1}{\alpha - \beta v^2} \quad \ldots\ldots [3]$$

The fraction on the right is readily separated into two partial fractions giving

$$dt = \frac{1}{2\alpha^{1/2}}\left(\frac{dv}{\alpha^{1/2} - \beta^{1/2}v} + \frac{dv}{\alpha^{1/2} + \beta^{1/2}v}\right) \quad \ldots\ldots [4]$$

This integrates immediately as

$$t\Big]_{t_1}^{t} = \frac{1}{2\alpha^{1/2}\,\beta^{1/2}} \log\left(\frac{\alpha^{1/2} + \beta^{1/2}v}{\alpha^{1/2} - \beta^{1/2}v}\right)\Big]_{v_1}^{v} \quad \ldots\ldots [5][2]$$

[2] Throughout the treatment, log means the Napierian log, or $\log_e$.

With $v_1 = 0$, that is, a start from rest at the top of the slide and with $t_1 = 0$, we have

$$2\alpha^{1/2}\beta^{1/2}t = \log \frac{\alpha^{1/2} + \beta^{1/2}v}{\alpha^{1/2} - \beta^{1/2}v} \quad \ldots\ldots [6]$$

Dividing through numerator and denominator of the fraction by $\beta^{1/2}$, remembering Equation [2], and putting

$$\mu \text{ for } \alpha^{1/2}\beta^{1/2} \text{ and } \lambda^2 \text{ for } \frac{\bar{v} + v}{\bar{v} - v}$$

Equation [6] becomes

$$\mu t = \frac{1}{2} \log \frac{\bar{v} + v}{\bar{v} - v} = \log \lambda \quad \ldots\ldots [7]$$

Here again for $v = \bar{v}$, $t = \infty$ which shows that $\bar{v}$ is reached only after an infinite time. However on the assumption of β constant we find readily the time required to reach any stated fraction of $\bar{v}$. Thus put $v = m\bar{v}$. Then Equation [7] becomes

$$\mu t = \frac{1}{2} \log \left(\frac{1 + m}{1 - m}\right) \quad \ldots\ldots [8]$$

For $m = 0.90$ this gives $t = (1/2\mu) \log 19$
For $m = 0.95$ this gives $t = (1/2\mu) \log 39$
For $m = 0.99$ this gives $t = (1/2\mu) \log 199$

Now taking exponentials and solving Equation [7] for v or Equation [8] for m, we have

$$\frac{v}{\bar{v}} = m = \frac{e^{2\mu t} - 1}{e^{2\mu t} + 1} \quad \ldots\ldots [9]$$

Equation [9] with a series of values of t will give the corresponding progressive values of v.

Equation [9] may, furthermore, be conveniently transformed into an expression in terms of hyperbolic functions as follows:

Since $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \dfrac{e^{2x} - 1}{e^{2x} + 1}$, Equation [9] gives directly

$$\frac{v}{\bar{v}} = m = \tanh (\mu t) \quad \ldots\ldots [10]$$

Thus with a table of hyperbolic tangents and any assumed values of t, the values of m and hence of v may readily be found.

Again in Equation [10] putting $v = ds/dt$ and integrating between limits of 0 and t, we have

$$S = \frac{\bar{v}}{\mu} (\log \cosh \mu t) \quad \ldots\ldots [11]$$

Here again, with tables of hyperbolic cosines and natural logarithms, the value of the distance for any value of t may be directly found.

We return now to Equation [5] and take the more general case where v_1 is not zero. Substitution of the limits t_1, t, and v_1, v, with $\bar{v}$ put for $\alpha^{1/2}/\beta^{1/2}$ gives

$$\mu(t - t_1) = \frac{1}{2} \log \left(\frac{\bar{v} + v}{\bar{v} - v}\right)\left(\frac{\bar{v} - v_1}{\bar{v} + v_1}\right) \quad \ldots\ldots [12]$$

Or putting Δt for $t - t_1$, and remembering that we may put

$$\lambda^2 \text{ for } \frac{\bar{v} + v}{\bar{v} - v} \quad \text{and} \quad \lambda_1^2 \text{ for } \frac{\bar{v} + v_1}{\bar{v} - v_1} \quad \ldots\ldots [13]$$

Equation [12] takes the form

$$\mu \Delta t = \log \left(\frac{\lambda}{\lambda_1}\right) \quad \ldots\ldots [14]$$

This equation gives the time interval Δt required for an increase of the velocity from v_1 to any given value v.

Then going back to Equation [12], taking exponentials, and solving for v, we find

$$\frac{v}{\bar{v}} = \frac{e^{2\mu\Delta t}(\bar{v} + v_1) - (\bar{v} - v_1)}{e^{2\mu\Delta t}(\bar{v} + v_1) + (\bar{v} - v_1)} \text{ or } \ldots\ldots\ldots [15]$$

$$\frac{v}{\bar{v}} = m = \frac{\lambda_1^2 e^{2\mu\Delta t} - 1}{\lambda_1^2 e^{2\mu\Delta t} + 1} \ldots\ldots\ldots\ldots [16]$$

which may be compared with Equation [9].

This equation gives the velocity v acquired after an interval of time Δt, starting with an initial velocity v_1 and knowing $\bar{v}$. See Equations [13].

In a similar manner as for Equation [9], we may express Equation [16] in terms of hyperbolic functions as follows:

Put $e^{2\mu} = \lambda_1^2 e^{2\mu\Delta t}$. Then $2\mu = 2\mu\Delta t + 2 \log \lambda_1$, and, similarly as for Equation [10]

$$\frac{v}{\bar{v}} = \tanh (\mu\Delta t + \log \lambda_1) \ldots\ldots\ldots\ldots [17]$$

With values of $\bar{v}$, v_1, and Δt, the value of v is thus found directly from a table of hyperbolic tangents.

Again putting ds/dt for v, we integrate Equation [17], giving for the space interval ΔS the value

$$\Delta S = \frac{\bar{v}}{\mu} \log \left[\frac{\cosh (\mu\Delta t + \log \lambda_1)}{\cosh (\log \lambda_1)} \right] \ldots\ldots\ldots [18]$$

This gives the space ΔS passed over in the time Δt starting with an initial velocity v_1.

By solving inversely certain of the preceding equations and collecting together, we have explicit expressions for all six possible relations between the variables v, t, and S for initial $v = 0$, and for v, Δt, and ΔS for initial $v = v_1$.

These are as follows:

General Formulas

In the following formulas, note that

$$\mu = \sqrt{(\alpha\beta)}, \quad \lambda^2 = \frac{\bar{v} + v}{\bar{v} - v}, \quad \lambda_1^2 = \frac{\bar{v} + v_1}{\bar{v} - v_1}, \quad \text{and} \quad \gamma = \frac{\mu S}{\bar{v}} \text{ or } \frac{\mu \Delta S}{\bar{v}}.$$

Where $v_1 = 0$

For t in terms of v

$$\mu t = \frac{1}{2} \log \frac{\bar{v} + v}{\bar{v} - v} = \log \lambda \ldots\ldots\ldots\ldots [a]$$

For t in terms of S

$$\mu t = \cosh^{-1} e^{\gamma} \ldots\ldots\ldots\ldots\ldots [b]$$

For v in terms of t

$$v = \bar{v} \frac{e^{2\mu t} - 1}{e^{2\mu t} + 1} = \bar{v} \tanh (\mu t) \ldots\ldots\ldots\ldots [c]$$

For v in terms of S

$$v = \bar{v} \tanh (\cosh^{-1} e^{\gamma}) \ldots\ldots\ldots\ldots [d]$$

For S in terms of t

$$S = \frac{\bar{v}}{\mu} \log \cosh (\mu t) \ldots\ldots\ldots\ldots\ldots [e]$$

For S in terms of v

$$S = \frac{\bar{v}}{\mu} \log \cosh \left(\frac{1}{2} \log \frac{\bar{v} + v}{\bar{v} - v} \right) = \frac{\bar{v}}{\mu} \log \cosh \log \lambda \ldots [f]$$

Where v_1 is not 0

For Δt in terms of v

$$\mu\Delta t = \frac{1}{2} \log \left(\frac{\bar{v} + v}{\bar{v} - v} \right) \left(\frac{\bar{v} - v_1}{\bar{v} + v_1} \right) = \log \frac{\lambda}{\lambda_1} \ldots\ldots [A]$$

For Δt in terms of ΔS

$$\mu\Delta t = \cosh^{-1} (e^{\gamma} \cosh \log \lambda_1) - \log \lambda_1 \ldots\ldots [B]$$

For v in terms of Δt

$$v = \bar{v} \left(\frac{\lambda_1^2 e^{2\mu\Delta t} - 1}{\lambda_1^2 e^{2\mu\Delta t} + 1} \right) = \bar{v} \tanh (\mu\Delta t + \log \lambda_1)$$

$$\text{or } \log \lambda = \mu\Delta t + \log \lambda_1 \ldots\ldots\ldots\ldots [C]$$

For v in terms of ΔS

$$v = \bar{v} \tanh [\cosh^{-1} (e^{\gamma} \cosh \log \lambda_1)]$$

$$\text{or } \log \lambda = \cosh^{-1} (e^{\gamma} \cosh \log \lambda_1) \ldots\ldots\ldots [D]$$

For ΔS in terms of Δt

$$\Delta S = \frac{\bar{v}}{\mu} \log \left[\frac{\cosh (\mu\Delta t + \log \lambda_1)}{\cosh \log \lambda_1} \right] \ldots\ldots\ldots [E]$$

For ΔS in terms of v

$$\Delta S = \frac{\bar{v}}{\mu} \log \frac{\cosh \log \lambda}{\cosh \log \lambda_1} \ldots\ldots\ldots\ldots [F]$$

Thus far the analytical discussion has proceeded on the assumption of constant values for both α and β. While quite commonly α might be found constant, the value of β, as already noted, may be expected to change with the changing physical conditions usually met with in flows of this character. At the start the value of β will involve the coefficient of friction between water and the surface of the conduit, the density of the flow, and the mean hydraulic radius. As the flow follows down the slope, all of these conceivably may change. Air friction will enter as a factor, especially with indraft of air and insufflation of the stream, and in particular if the flow becomes broken and irregular with boiling rolling turbulence, with wave action at the surface raveling out into spray. As the gradient approaches extreme limits, from 80 to 90 deg, there is a tendency, as is well known, for the entire flow to break up into spray and mist, provided the descent be of sufficient length.

These same physical conditions will affect likewise the average density of the flow and the mean hydraulic radius. It would seem not unreasonable to expect a continuous increase in the value of β from point to point down the flow. However, any attempt to introduce β as a function, either of distance or time, into the basic Equation [1], introduces a complexity rendering direct integration impracticable.

The same results follow with variable slope, that is, with α variable. If, however, we assume both α and β constant for any specified time, such as one second, we shall have, for this period, Equations [14] and [16] as the solution of the basic Equation [3] with the known (or assumed) initial velocity v_1. This provides a means for tracing the velocity through from second to second, with any known series of values of α and any assumed values of β. If, as in the more usual case, α is constant, we shall have only to take the assumed values of β from one time interval to another.

Thus for the first second assume a velocity of approach v_1 and for β, some value β_1. This will give v at the end of second 1. This then becomes v_1 for second 2, and with another value β_2, v at the end of second 2 is found. This in turn becomes v_1 for second 3, and so on as far as may be desired.

It now remains to discuss briefly the relation of β in Equation [1] to the more familiar quantities connected with fluid friction.

We may derive immediately the desired relation as follows: From Equation [2] we have

$$(\bar{v})^2 = \frac{\alpha}{\beta} = g\frac{\sin\theta}{\beta}$$

But v is the same v as that given by the familiar formula $v = C\sqrt{(i\rho)}$ where C is the Chezy coefficient, i the hydraulic gradient which for steady flow equals $\sin\theta$, and ρ is the mean hydraulic radius. Equating these values of $(\bar{v})^2$ and v^2 and solving for β, we have

$$\beta = g/C^2\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots [19]$$

However, it will be desirable to develop this relation in such manner as to bring in the two fundamental characteristics of a fluid, viscosity (as evidenced by fluid friction) and density, and to this end we proceed as follows:

In any conduit let f be the coefficient of friction for unit velocity. That is, f is the drag in pounds force per unit area of surface for unit velocity. Assume friction to vary as v^2.

Then let w = the weight of unit volume of the fluid
a = cross-sectional area of channel
p = wetted perimeter of channel.

Then for a small length dx of the channel or conduit, we shall have

Drag for length dx at velocity $v = fv^2pdx$
Weight of contents $= wadx$
Drag per lb, $fv^2pdx \div wadx = fv^2p \div wa$

If then we put $\rho = a/p$ = mean hydraulic radius, this becomes

$$\text{Drag per lb contents of flow} = fv^2/\rho w$$

This then is the drag in pounds force on a pound weight of the contents of the flow and as such it will produce negative acceleration measured by the force divided by the mass. But the mass of 1 lb wt is $1/g$ and hence the negative acceleration produced by the drag $fv^2/\rho w$ will be $gfv^2/\rho w$.

This is represented in Equation [1] by the term βv^2. Hence we have

$$\beta = gf/\rho w \quad \ldots\ldots\ldots\ldots\ldots\ldots [20]$$

Next, if we take the familiar Chezy formula for loss of head, we have

$$h = Lv^2/C^2\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots [21]$$

This is loss of energy per lb wt of fluid in traversing a distance L, and is measured by the work done on 1 lb wt in traversing this distance. Hence the resistance overcome will be measured by dividing the work by L, or, drag $= v^2/C^2\rho$.

But, as we have just seen, this is also measured by $fv^2/\rho w$. Hence equating, we have

$$1/C^2 = f/w \text{ or } f = w/C^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [22]$$

This gives a relation between the coefficient of friction f and the familiar C of the Chezy formula.

Again from Equations [20] and [22] we have, as in [19] $\beta = g/C^2\rho$ giving, as before, a direct relation between β, the Chezy C and the mean hydraulic radius.

Returning to Equation [20], we may write $\beta \sim f/\rho w$.

In comparing the value of β at some point down the slope with its value at the start, we have then to query the changes in f, ρ, and w.

With high velocity, irregular water surface, and insufflation of the stream, air friction will enter as an important factor and the effective perimeter must include the air surface as well as the channel surface. The value of f for the friction between the channel surface and a mixture of air and water will also come into the picture. There seem to be no available data on such a complex value of f. The effect of insufflation, without the air perimeter, will be to increase ρ and decrease w. The inclusion of the air part of the perimeter will decrease ρ. Decrease in the values of either ρ or w, or both, will tend to increase the value of β. On the other hand, the increase in velocity will tend to decrease both a and p, but with a decrease in the value of ρ. That is, increase in v by itself will result in a decrease of ρ, insufflation by itself will increase ρ and decrease w, and including the air part of the perimeter, will decrease ρ. There seem to be available no data on which to base a judgment for any particular case as to the ultimate effect on f, ρ, or w, and still less on the resulting value of β.

Accurate observations on the values of v in any particular case would however make possible, by a cut-and-try method, the determination of a series of values of β which would fit that particular case, but there does not appear to be any theoretical method whereby such values could be generalized to extend to other cases with varying values of the slope, proportions of the flow sheet, and the like.

Such observations would, however, be of value in at least determining the facts and in making possible the determination of ranges of values of β for particular cases. An accumulation of such data would then make possible at least an approximate solution for the usual run of cases which are encountered in practical experience.

A question of fundamental interest and importance, collateral with the main problem is, how, in the course of the flow of water down a steep gradient, that is, with high velocity, does the air get into the water? What are the factors which determine such insufflation and on what conditions of the flow do they depend? In the present paper no attempt is made to deal with these important questions; there seems to be too little to go on in the way of experimental evidence; but it is hoped that the interest and importance of these questions may stimulate, by way of laboratory research, suitable investigating which may throw some light on this apparently obscure phase of the general problem.

It may be noted that the treatment in the present paper implies a continuous discharge, for the time being, of a uniform *quantity* of water down the steep sloping discharge channel, rather than the movement of a discrete gulp or wave of water. The same basic equations would appear likewise to apply even to the case of a sudden wave, but in such case the form of the wave would vary continuously down the slope with resulting increased difficulty in the evaluation, from point to point, of suitable values of the coefficient β.

Furthermore, the discussion thus far has been quite independent of reference to specific applications. One of the principal fields of such application would be found in connection with the design of means for the absorption of the energy of the water flowing down the spillway of a dam, such as the Grand Coulee Dam on the Columbia River, or the Shasta Dam on the Sacramento River. In the former, the water will plunge into the river on the downstream side of the dam, and then under the influence of a submerged curved lip—the so-called "bucket"—it will be deflected upward, with transformation of most of the kinetic energy of flow into the energy of eddies, boils, and mixed turbulence. In the latter the energy is to be absorbed in a special form of hydraulic jump. In both cases the velocities likely to be acquired in the flow down the sloping face of the spillway are of definite interest in connection with the design of the "bucket" in the one case or the hydraulic jump in the other.

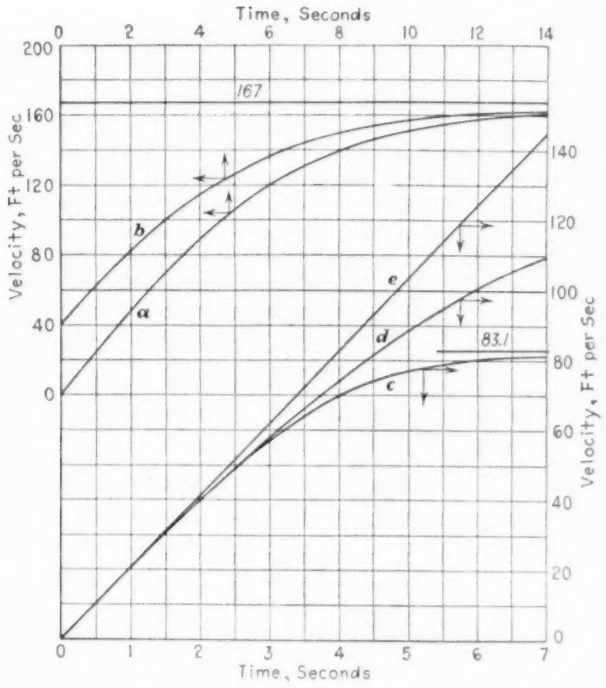


FIG. 1 VELOCITY CURVES OF EXAMPLES GIVEN

Broadly, the methods outlined in the present paper would be applicable to the determination of the velocity which might be anticipated after any given period of time or over any given length of a steep sloping run, once the controlling conditions are known or assumed, and suitable values of the coefficient β available. Such conditions may occur in connection not only with spillway dams, but with many types of hydraulic construction, where waste water is disposed of by discharge down a specially formed channel under a relatively steep gradient.

By way of illustration of the application of these equations, the following problems have been worked out on assumed conditions and values of the coefficient β.

EXAMPLES

In Fig. 1 curve *a* gives the velocity history for the following conditions: $v_1 = 0$ (start from rest), $\alpha = 25(\theta = 50$ deg), and $\beta = 0.0009$ constant. This would correspond to, say, $C = 125$ and $\rho = 2.3$ ft. Then $\bar{v} = 167$ fps and $\mu = 0.15$.

In 14 sec, with a travel of 1583 ft, v has reached a value 162 fps or 97 per cent of $\bar{v}$.

Curve *b* gives the velocity history for the same basic values except that here $v = 40$ fps. In this case, after 14 sec and with a travel of 1813 ft, v has reached a value of 164 fps or 98 per cent of $\bar{v}$.

Curve *c* gives the velocity history for the following conditions: $v_1 = 0$ (start from rest), $\alpha = 20.7$ ($\theta = 40$ deg), $\beta = 0.0009$ as before at the start, with increasing values taken arbitrarily as follows: For the first second, 0.0009; second, 0.0012; third, 0.0015; fourth, 0.0019; fifth, 0.0024; sixth, 0.0029; and for the seventh second, 0.0030.

In this case, after 7 sec, with a run of about 390 ft, v has reached a value of 81.4 fps against an ultimate for $\beta = 0.003$ of 83.1 fps or 97.8 per cent of $\bar{v}$.

Curve *d* gives the corresponding time history with no increase in β, i.e., no increase above ordinary skin friction for water on concrete, for which the limit velocity, with β constant at 0.0009, would be 151 fps.

The line *e* gives the corresponding time history with no retardation at all.

Discussion

G. H. KEULEGAN[3] AND H. N. EATON.[4] The purpose of the following discussion is to show the effect of including the velocity distribution in the equation of flow in steep channels. The author, although recognizing that this will affect the flow, leaves it out of account. However, it is a simple matter to include this effect, and it is far from being negligible in many instances. Because of space limitations, only a bare indication of the results obtained for a wide rectangular channel can be given. The notation used agrees with that of the author, except where otherwise stated.

First introduce the ratios

$$\frac{v_s' - v}{v} = a_2, \text{ and } \frac{v_s'^2 - a_1 v^2}{v^2} = 2a_3 \dots\dots [23]$$

where v_s' is the velocity of the surface filaments, and $a_1 = 1 + \eta$, η being the "Boussinesq coefficient" defined by Equation [31].

Starting with Osborne Reynolds' equation of motion for turbulent flow, we derive the following equation, in which the coefficients a_1, a_2, and a_3 involve the velocity distribution

$$\frac{\partial v}{\partial t} + (a_1 + a_3)\, v \frac{\partial v}{\partial x} + (a_3 - a_2) \frac{v}{h} \frac{\partial h}{\partial t} = - g \cos\theta \frac{\partial h}{\partial x} + g \sin\theta - \frac{\lambda}{2h} v^2 \dots\dots [24]$$

in which h is the depth, and λ is $^1/_4$ the friction coefficient λ in the rational pipe-flow formula.

We assume that the flow is steady and that the term containing $\frac{\partial h}{\partial x}$ is small; whereupon Equation [24] can be written

$$\frac{d(v^2)}{dx} = \frac{2g \sin\theta}{a_1 + a_3} - \frac{\lambda v^2}{h(a_1 + a_3)} = \alpha' - \beta' v^2 \dots\dots [25]$$

Here α' and β' are related to the author's α and β as follows:

$$\alpha' = \alpha/(a_1 + a_3), \text{ and } \beta' = \beta/(a_1 + a_3) \dots\dots [26]$$

Equation [25] can be integrated

$$S = x - x_0 = \frac{1}{\beta'} \ln \frac{\left(\frac{\alpha'}{\beta'} - v_0^2\right)}{\left(\frac{\alpha'}{\beta'} - v^2\right)} \dots\dots [27]$$

with v taken as v_0 at $x = x_0$.

At $$S = \infty, \bar{v} = \sqrt{\frac{\alpha'}{\beta'}} = \sqrt{\frac{\alpha}{\beta}} \dots\dots [28]$$

and we see that the velocity distribution has no effect upon the limiting velocity $\bar{v}$, but only upon the rate at which this velocity is reached. Equation [27] can be written

$$S = \frac{1}{\beta'} \ln \left(\frac{\alpha/\beta - v_0^2}{\alpha/\beta - v_2}\right) = \frac{a_1 + a_3}{\beta} \ln \left(\frac{\bar{v}^2 - v_0^2}{\bar{v}^2 - v^2}\right) \dots [29]$$

and the only effect of allowing for the velocity distribution is to add the factor $a_1 + a_3$ to the equation.

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[4] Senior Hydraulic Engineer, National Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

The values of a_1 and a_3 can be evaluated for any assumed velocity distribution. For example, if

$$v'^2 = Ay, \text{ where } A = v'^2_s/h \quad \ldots\ldots\ldots\ldots [30]$$

where v' is the filament velocity at distance y above the bed, then

$$a_1 = 1 + \eta = \frac{\int_0^{h_2} v'^2\, dy}{h\, v^2} = \frac{\int_0^h y\, dy}{\left(\int_0^h y^{1/2}\, dy\right)^2} = 1.125 \ldots [31]$$

and from Equations [23], [30], and [31]

$$a_3 = \frac{1}{2}\left(\frac{v_s'^2 - a_1 v^2}{v^2}\right) = 0.562$$

Then $a_1 + a_3 = 1.687$.

Thus the effect of a nonuniform velocity distribution is to decrease the mean velocity at a station downstream from the entrance to a channel below what it would be if the velocity were uniform over the cross section. In other words, the computed water-surface profile will be lower for assumed uniform than for nonuniform distribution. Now it has been observed in tests on model channels with steep slopes that the water-surface profiles, computed for assumed uniform velocity distribution lie below the observed surface profile. Computations made for one such channel indicate that, including the effect of velocity distribution as outlined, the foregoing accounts approximately for this discrepancy, although data are not available to test the accuracy of the assumptions made.

Problems Encountered in the Design and Operation of Impulse Turbines

By RAY S. QUICK,[1] SAN FRANCISCO, CALIF.

Because of present-day demands on high-head hydroelectric installations, calling for continuous maximum overload operation of impulse turbines, unusual problems of wear and fatigue of materials have developed. The author discusses such problems frankly and suggests solutions for them. One of the important factors influencing unit efficiency of impulse turbines is the arrangement and design of approach piping due to the effect on the jet of turbulence in the nozzle. Flow-straightening vanes have been developed to correct this turbulence. Various successful forms are illustrated and their characteristics plotted. The form of needle and nozzle tip also has a marked influence upon the appearance and form of the power jet. Another factor affecting the performance of the unit is that of the design, shape, and arrangement of buckets on the water wheel. Investigations have been made on all these elements of the impulse turbine, the results of which have been incorporated in present successful applications. Two outstanding impulse-turbine installations in California are mentioned. A comparison is drawn between field tests on one of these at the Tiger Creek Plant of the Pacific Gas and Electric Company and model tests. The paper concludes with a discussion of the governing problem.

THE DEVELOPMENT and operation of impulse turbines, particularly those for important high-head hydroelectric installations, have introduced many problems both in the selection of the most appropriate materials and in designs which will produce not only high efficiency but a satisfactory working life. In the early days of the industry loads were light and intermittent and it was often customary to shut a plant down over the week end. During the past decade transmission lines have been extended and interconnected. Loads have grown to such an extent that it is customary to operate large impulse turbines continuously at maximum overload for periods of many weeks or months without interruption. Needless to say, unusual problems of wear and fatigue of the materials have developed and, while it is most unusual for a manufacturer to report and discuss such difficulties, it will be the purpose of this paper to mention frankly characteristic problems and the methods followed in solving them.

The impulse turbine or tangential water wheel was developed in the early mining days in California. Early forms were inefficient until Lester A. Pelton about 1880 perfected the water-wheel bucket with the central dividing splitter. Seven years later the company was founded which now bears his name. Much has been written regarding the history of this type of water-turbine prime mover, while textbooks today outline and develop the theory of its operation. Consequently this work will not be reviewed at present.[2]

For many years the company with which the writer is associated has carried on an extensive research program based upon laboratory studies and upon field observations and tests. It has been customary to consider the efficiency as the ratio between the mechanical output of the water-wheel shaft and the theoretical hydraulic-power input at the entrance to the nozzle body.[3] In past years, in considering the requirements for high efficiency, no particular attention was given to the design or arrangement of the approach piping which delivers the power water from the penstock to the nozzle. Recent investigations have shown that the design of this approach piping has a pronounced influence upon the unit efficiency, due to the effect on the jet of turbulence or crossflow and eddies in the nozzle. As an example, refer to the upper jet shown in Fig. 1. This jet is produced by a needle nozzle of standard design having an inlet properly proportioned

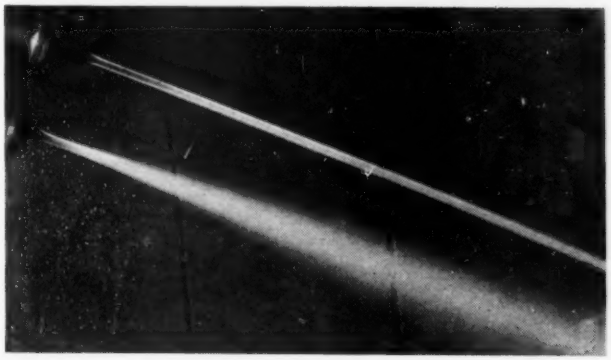


FIG. 1 COMPARISON OF JET PRODUCED BY STANDARD NEEDLE NOZZLE AND THAT PRODUCED BY SIMILAR NEEDLE NOZZLE WITH 90-DEG ELBOW NOZZLE BODY

to reduce turbulence to a minimum. A similar needle nozzle having a body formed with a 90-deg elbow is unable to produce a satisfactory power jet, the form being shown as the lower jet in Fig. 1. The spreading of the jet, due to eddies in the elbow, destroys its power value in a short distance.

Extensive investigations have been carried on in the design of flow-straightening vanes to be inserted in the nozzle body in an effort to correct this turbulence and restore the power of the jet. Fig. 2 illustrates a selected group of such vanes. Some of these vanes are far more effective than others. In general, vanes placed appreciably upstream from the power needle are of little use, while vanes combined with or surrounding the needle are most effective. Of striking interest is the type of vane shown in Fig. 3. It consists of a series of radial plates attached to the needle and stem in such a way as to divide the flow of water into parallel strips which recombine when the jet is formed. The result of

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[1] Chief Engineer, The Pelton Water Wheel Company. Mem. A.S.M.E.

[2] "The Tangential Water Wheel," by W. A. Doble, Transactions of the American Institute of Mining Engineers, vol. 29, 1899, pp. 852–894.

[3] "Test Code for Hydraulic Prime Movers," THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, June, 1938.

Contributed by the Hydraulic Division and presented at the Semi-Annual Meeting, San Francisco, Calif., July 10–15, 1939, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

the installation of vanes of this character is shown in Fig. 4. Comparison with Fig. 1 will show that this is the same needle-nozzle assembly but that the lower jet is now comparable to the upper in appearance and has recovered a substantial portion of its power value and eliminated most of its fringe or sprayed out covering.

A similar form of turbulence with resultant spreading jet may

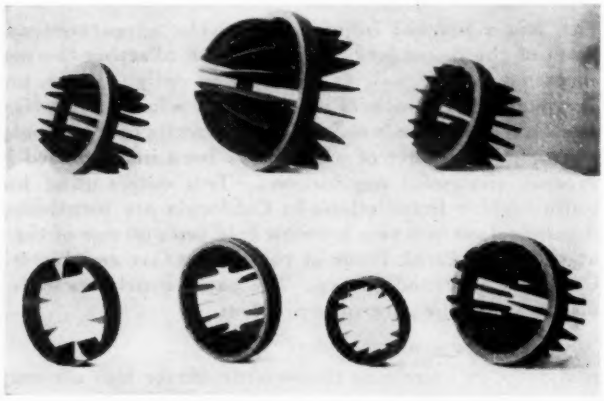


FIG. 2 STRAIGHTENING VANES FOR NOZZLE BODIES



FIG. 3 STRAIGHTENING VANES ON NEEDLE

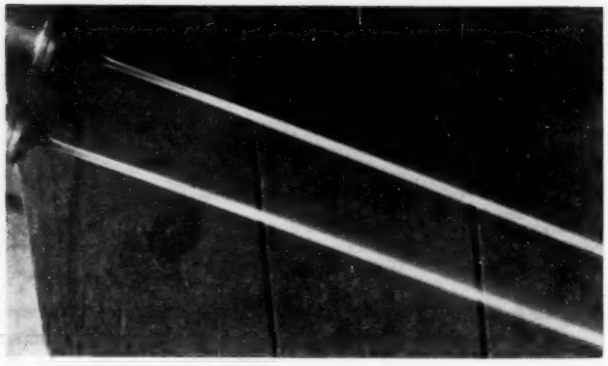


FIG. 4 COMPARISON OF JET PRODUCED BY STANDARD NEEDLE NOZZLE AND THAT PRODUCED BY SIMILAR NEEDLE WITH 90-DEG ELBOW NOZZLE BODY AND STRAIGHTENING VANES

be introduced into a needle nozzle of standard design and arrangement by an unfavorable manifold or system of elbows, wye pipes, and fittings, which otherwise would be considered satisfactory hydraulic practice. In laboratory testing it has been found that an orifice plate, inserted in the pipe line between adjoining flanges a few diameters upstream from the nozzle entrance, may introduce serious turbulence. While the pressure and velocity heads are maintained unchanged in the nozzle body, a serious loss in efficiency results, due to a disturbed stream flow at the point where the jet is formed. Fig. 5 has been taken directly from laboratory tests and shows in a very striking manner the influence of straightening vanes with and without turbulence. The performance curve, designated ① in Fig. 5, is the characteristic of a typical single-nozzle impulse turbine, using a standard needle nozzle, but with the flow in the approach piping disturbed by an orifice plate. For the purpose of the test, bucket loading is considered as the ratio between the diameter of the jet at the vena contracta, as determined by the head and quantity, and the nominal full-load-size rating of the bucket. When straightening vanes are installed in the needle nozzle, the efficiency characteristic is improved to curve ② in Fig. 5. Since the test setup is otherwise unchanged, it is apparent that the improvement has resulted from the addition of the vanes. There is also a visual improvement in the form of jet, the fringing of the outside surface being materially reduced. If the orifice plate is entirely removed and the straightening vanes are left in position, the characteristic is still further improved to curve ③ in Fig. 5. As there is little or no detrimental turbulence in the supply pipe, it is evident that

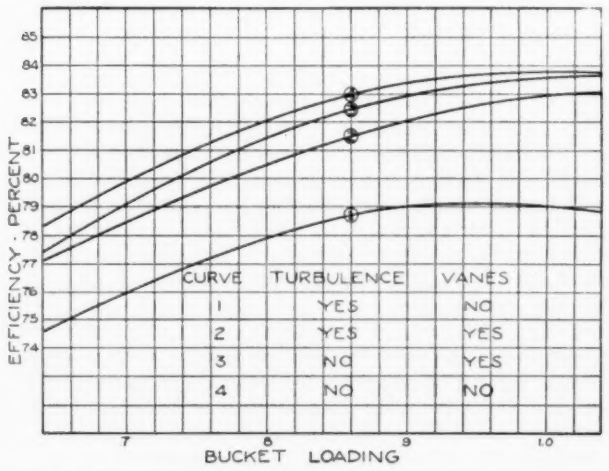


FIG. 5 EFFECT OF TURBULENCE AND STRAIGHTENING VANES IN NOZZLE BODY ON UNIT EFFICIENCY

the efficiency characteristic is approaching its best value. By removing the straightening vanes, thus bringing the assembly to the most favorable arrangement, the efficiency characteristic reaches its optimum value as shown by curve ④ in Fig. 5.

The foregoing characteristics as shown in Fig. 5 are typical and may be repeated qualitatively with other setups. The conclusions which are to be drawn from these tests may be summed up as follows:

1 The best efficiency characteristic is secured when there is neither turbulence nor straightening vanes in the supply system or nozzle.

2 If turbulence exists in the approach piping, it may be overcome or corrected materially by the installation of appropriate straightening vanes.

3 There is a mutual responsibility between all parties concerned with the selection and design of the penstock, approach piping and manifold, and hydraulic prime mover to eliminate the detrimental results of such turbulence.

Water passageways must be smooth, as straight as possible, and free of obstructions or sudden contractions and enlargements. A shutoff valve should have if possible a water passageway, forming a smooth continuation of the conduit walls. The turbulence in a well-designed nozzle is very small.

The form of needle and nozzle tip has a marked influence upon the appearance and efficiency of the power jet. Extensive tests

have been made on many combinations, both in the laboratory and in the field. While tests made under high-head conditions produce results differing slightly from those under low heads, due primarily to the change in relative velocity between the water, wetted surfaces, and the air surrounding the jet, nevertheless, in general, the nozzle producing the best appearing and the most efficient jet, when tested under low-head conditions, will be found to be preferable under high heads.[4] It is interesting to note that even at a high head, such as 2000 ft, a small jet will stay together and show a very fine appearance.

Fig. 6 illustrates a selected group of needles and nozzle tips which have been developed and tested in the laboratory with satisfactory results. The longer needles, having less contraction, require a greater stroke in order to discharge a given water quantity. Therefore, these are most adaptable to governor operation. The jet produced makes a very fine appearance and is relatively smooth and parallel on the surface. The steeper and shorter needles with greater contraction have a much shorter stroke for a given discharge and produce jets which are relatively rougher in external appearance. However, since the water has a much shorter distance to travel at full jet velocity on the wetted surfaces of the steeper needle, the efficiency is higher. This is confirmed by Fig. 7, which shows the efficiency characteristics of a given water-wheel unit, as tested in the laboratory, with needle nozzles having included angles of 45 deg and 90 deg. Other combinations of needles and tips have been tested with comparable results. The appropriate selection for a given case is determined largely by governing requirements, range in load, and general setting.

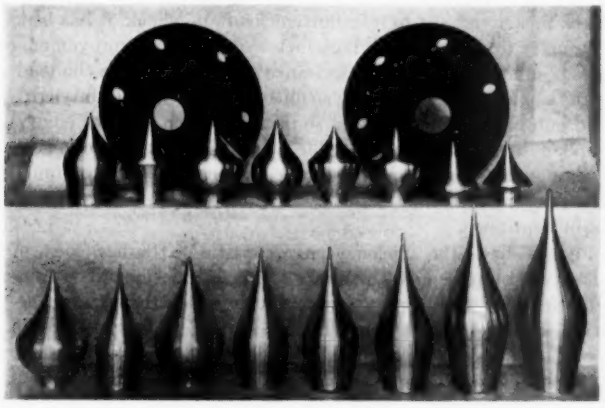


Fig. 6 Needles and Tips Tested in Laboratory

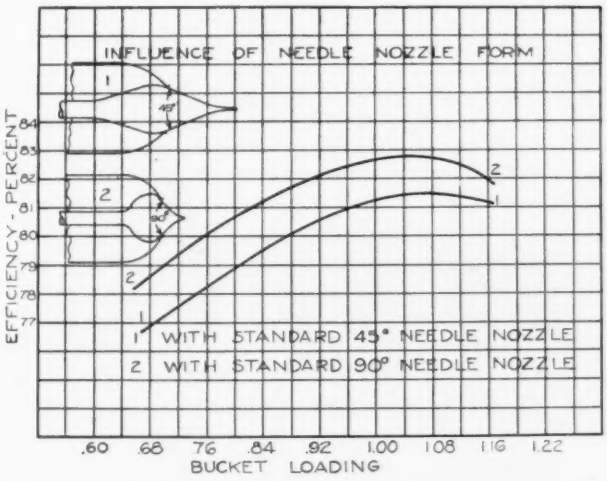


Fig. 7 Influence of Needle-Nozzle Form on Unit Efficiency

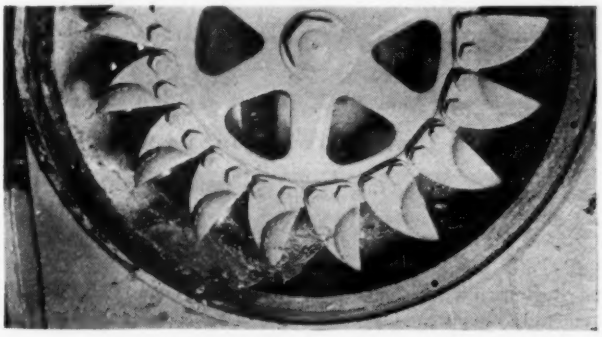


Fig. 8 High-Speed Photograph Illustrating Bucket Discharge

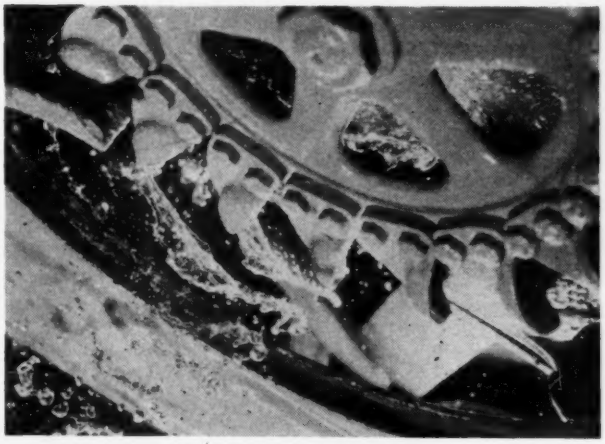


Fig. 9 High-Speed Photograph, Taken From Above Wheel, Showing Bucket Discharge

Problems of cavitation and erosion of needles and nozzle tips have been solved satisfactorily. Cavitation may be avoided almost entirely when the needle nozzle is operated throughout the normal range from say one quarter to full load. Operation for long periods near full shutoff is unfavorable, resulting in erosion and cavitation of the needle in the zone where the thin annular jet is being formed. Needles and nozzle tips made of carbon steel, stainless steel, nitralloy, bronze, and similar materials provide a satisfactory working life under average conditions. Stainless steel of the 18-8 type, used either as a forging or as a layer weld upon a mild-steel base is favored by many for severe conditions.

As the water-wheel runner is the heart of the unit, the design, shape, and arrangement of the buckets upon the wheel center exert a pronounced influence upon the performance of the unit as a whole. While the theory of reversal of the jet in the bowl of a water-wheel bucket has been developed and is fully described in standard textbooks on the subject, it has required the assistance of the stroboscope to arrest the relative motion and show clearly the action of the water with respect to the buckets. The author is indebted to H. E. Edgerton[5] for the perfection of the strobo-

[4] "Tests Conducted at Big Creek No. 1 With Experimental Nozzle and Needle Parts at Penstock Pressures," by J. F. Davenport, *Electrical West*, vol. 68, no. 6, May 15, 1932, p. 378.

[5] "High-Speed Photographic Methods of Measurement," by H. E. Edgerton, J. K. Germeshausen, and H. E. Grier, *Journal of Applied Physics*, vol. 8, no. 1, January, 1937, pp. 2–9.

scope and high-speed photography which has made possible the accompanying illustrations. A Pelton standard laboratory water motor was used by Mr. Edgerton, the glass cover plates on the housing being removed in order to provide a direct view for the camera. The water wheel was driven by a jet of water under relatively low head to avoid heavy mist and spray. The photographs were taken by instantaneous exposures when the wheel

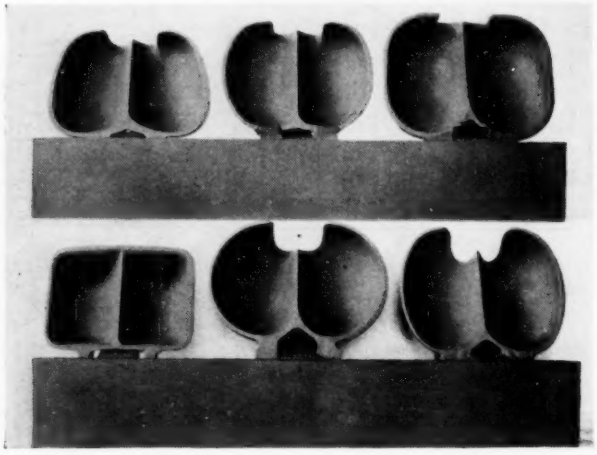


FIG. 10 GROUP OF DIFFERENT TYPES OF IMPULSE-WHEEL BUCKETS

was running approximately at normal speed. Fig. 8 is a side view showing the power jet in action on several buckets simultaneously. The bucket directly below and to the right of the shaft center line has interrupted the jet and the discharge has just started to leave the sides of the bucket bowl. The bucket to the right of this has just entered the jet and is not filled as yet. The bucket to the left is discharging its quota, while the bucket beyond is completing its working cycle. By taking photographs from above the wheel while in operation, as shown in Fig. 9, a definite clearance is indicated between the slugs of water discharging from adjacent buckets. It is also evident that the discharged water does not strike the back of the bowl of the preceding bucket, thus indicating an excellent operating condition.

The form of water-wheel bucket has undergone an evolution not only to improve its efficiency but also to increase its durability and strength, particularly for large high-capacity installations where continuous overload is the rule rather than the exception. Fig. 10 illustrates a representative group of such buckets, from the older square shape to the current ellipsoidal design with filled-in entrance. It has been difficult to secure a design of lip which will enter the jet with minimum shock and splash, while at the same time having adequate strength to resist the heavy loads and intermittent thrust. When loads are severe and the head is high, fatigue cracks may appear in the entering lip of the bucket, progressing rapidly until a portion of the bowl breaks off. Fig. 11 illustrates a bucket of older design which has failed in this manner. Due to the difficulty of making a complete analysis of the stresses existing in the bowl of a water-wheel bucket, subject to intermittent load and shock, it has been customary to design with a high factor of safety. Improvements have been made to stiffen the entrance and to distribute the load. With buckets of recent design, as illustrated in Fig. 10, the sharpened entering lip is retained in so far as possible, but the edge is rounded to minimize stress concentration and the section increases in thickness rapidly, thus providing a section of much greater strength than possessed by the previous thin and sharpened cut-out entrance.

Fig. 12 shows the efficiency characteristic of the new form of bucket in comparison with that of a previous design. It is evident that the modification of the entrance design has been accomplished without detriment. Buckets of this type are in operation under high heads and in large capacities and show great promise of having overcome the fatigue-crack limitations of the previous designs.

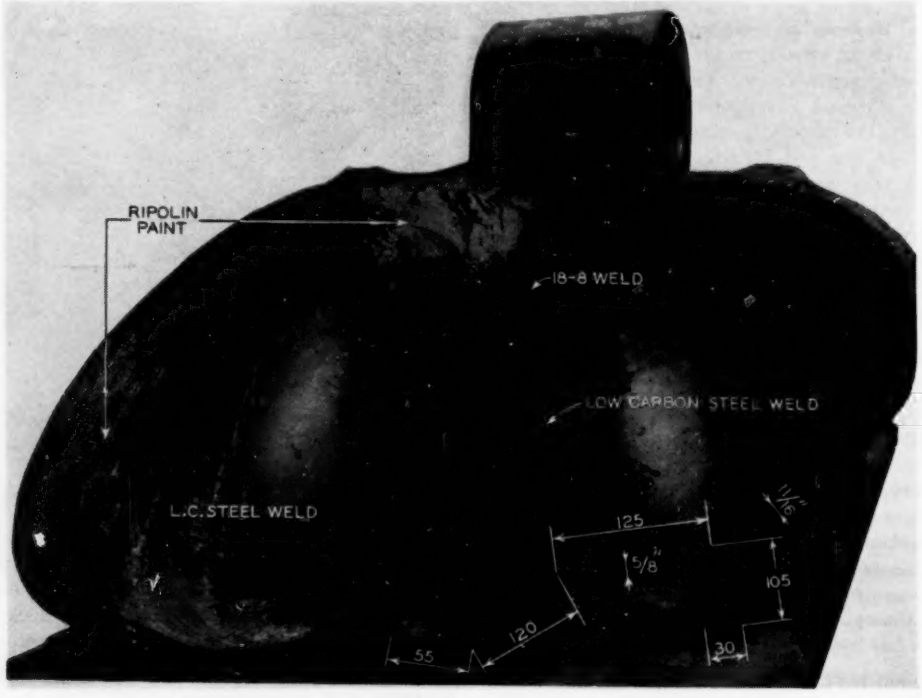


FIG. 11 SERVICE FAILURE OF A BUCKET DUE TO FATIGUE

In order to get a perspective as to the magnitude of the forces and the number of cycles of repeated stress to which a water-wheel bucket is subjected, consider an operating head of 2000 ft, an output of 25,000 hp per wheel, and a speed of 400 rpm. Calculation will show that the number of revolutions per year of the runner is about 211,000,000. Thus, a working life of 10 years requires over 2,000,000,000 working cycles for each bucket. Studies of fatigue would indicate that the material must be stressed appreciably below its fatigue limit if this working life is to be secured without failure of the material. The approximate force on the bucket each time it passes through the jet is 50,000 lb. It is apparent that the attachment of the buckets to the wheel center must be very secure and moderately stressed to avoid failure due to fatigue or to the bucket bolts working loose. The problem of bucket attachment has been solved successfully and reports of failure of bolts and lugs are very rare. The important problem in recent years has been cracking at the entering lip of the bucket.

The material used for high-head buckets is cast steel, the medium grades of carbon steel being preferred. Heat-treated alloy steel has been used to a limited extent but the practical problem in connection with this material is the difficulty in re-heat-treatment following field repairs by welding. In cases where the power water carries abrasive material in suspension, erosion may become an objectionable factor. An increase in bucket life

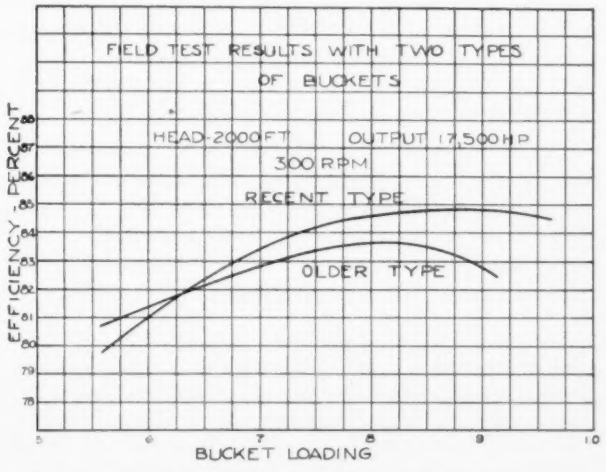


FIG. 12 INFLUENCE OF BUCKET SHAPE ON UNIT EFFICIENCY

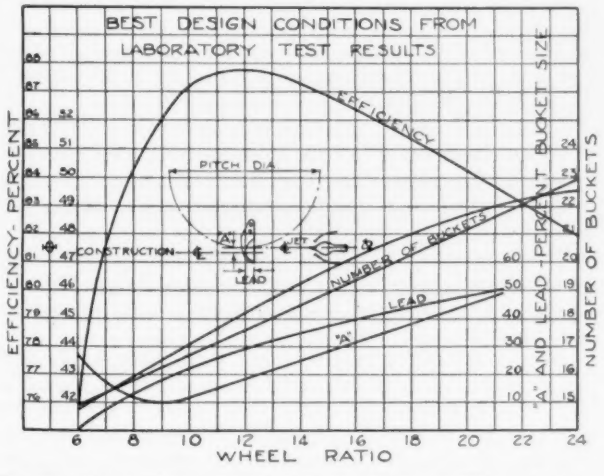


FIG. 13 CHARACTERISTICS OF ONE TYPE OF IMPULSE-WHEEL BUCKET

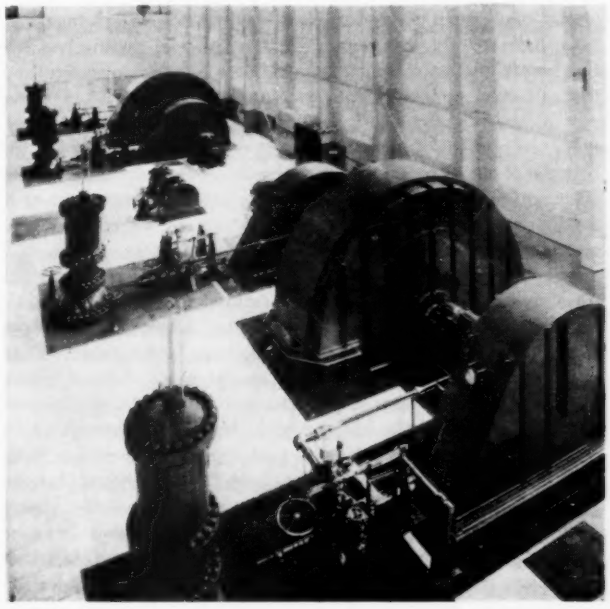


FIG. 14 TWO 40,000-HP DOUBLE-OVERHUNG IMPULSE TURBINES IN TIGER CREEK PLANT OF THE PACIFIC GAS AND ELECTRIC COMPANY

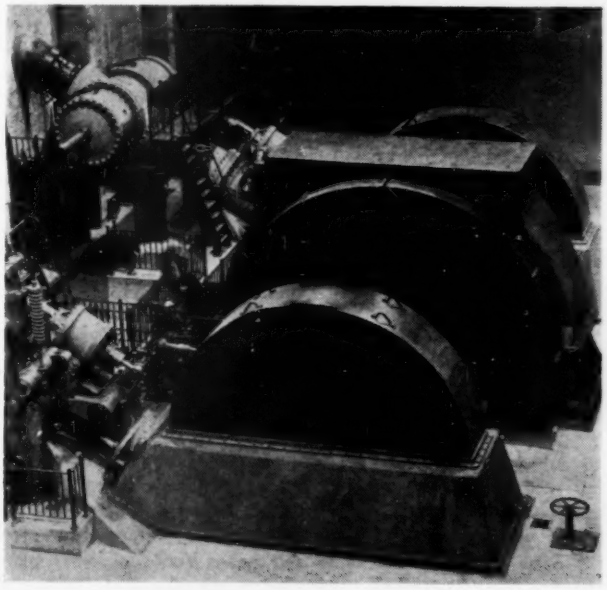


FIG. 15 ONE OF TWO 68,000-HP DOUBLE-OVERHUNG IMPULSE TURBINES IN BIG CREEK 2A PLANT OF THE SOUTHERN CALIFORNIA EDISON COMPANY

has been secured by the layer welding of 18-8 chrome-nickel stainless steel or other hard facings in the bucket bowls which are afterward ground smooth to true shape.

In the past it has been standard practice to construct water-wheel buckets as integral castings using iron for low heads and steel for high heads. Bronze has been favored for small buckets and in some cases stainless steel has been used. The complicated shape of the water-wheel bucket has made it difficult to produce castings of homogeneous physical properties. As a result, extensive study and development work have been done on the design of buckets made either as integral steel forgings or fabricated with

forged-steel bowls welded to cast-steel or plate-steel lugs. To date, however, the first cost of the integral steel casting has been the lowest and as a result, because of economic considerations, the cast-steel bucket is still standard practice. The increased cost of the forgings has been due to the special dies involved. Since the number of buckets of any specific design is very limited, because of their special nature, it has not been possible to standardize on dies. In the case of the fabricated buckets, formed by welding two or more pieces together, the increased costs of labor and heat-treatment have been the deciding factors.

In selecting the most appropriate speed for an impulse turbine for a given head and quantity, the ratio of the bucket size to the pitch diameter of the wheel is a deciding factor. Fig. 13 shows the characteristics of a bucket developed in recent years. These characteristics were established by an extensive series of laboratory tests, using a water-wheel runner so arranged that the number and position of the buckets could be altered through a wide range, while at the same time the jet could be changed in radial position and size. As a result, the characteristics shown illustrate the most favorable combination. It is evident that the highest efficiency can be secured only at the most favorable ratio.

Among the recent outstanding impulse-turbine installations in California may be mentioned the Tiger Creek Plant of the Pacific Gas and Electric Company, shown in Fig. 14, and the Big Creek 2A Plant of the Southern California Edison Company shown in Fig. 15. The units in both plants are of the horizontal, double-overhung, single-straight-flow-nozzle type, using follower-ring gate valves and penstocks carefully laid out to secure favorable approach conditions. Each Tiger Creek unit develops a maximum of 40,000 hp at 225 rpm under 1190 ft effective head, while each Big Creek 2A unit develops 68,000 hp at 250 rpm under 2350 ft effective head. The latter units are the largest-capacity impulse turbines installed in North America, and are exceeded throughout the world only by those in the Serra Plant in Brazil.[6]

Fig. 16 shows a comparison between the model and field tests of the Tiger Creek unit, using single-wheel tests. These characteristics are typical, although the stepup in efficiency between the model and the field units varies to a considerable extent with different designs. The model, which is usually tested under a moderate head in the laboratory, shows its best efficiency at a higher bucket loading than does the high-head field unit. The reason for this change in characteristics as yet has not been explained satisfactorily. Further information on this variation in characteristics, based on additional laboratory and field data, is being studied at the present time.

The systems of governing applied to impulse turbines may be divided into two general classifications, the first being those which are water-saving and the second being those discharging a constant water quantity regardless of load. The water-saving system is of course the most widely used and is best adapted to plants having forebay storage. However, many plants are located on streams where downstream water rights require an uninterrupted flow for irrigation or other use and in such cases the manually operated needle nozzle and the governor-operated jet deflector are preferable.

With water-saving controls, where the governor is connected directly to the power needle, it is necessary to accelerate and retard the water column in the pipe line slowly, in order to avoid objectionable pressure surge, which otherwise would interfere with the safety of the pipe line or with governing. Where this slow operation interferes with satisfactory speed and voltage regulation, it is customary to use a water-saving by-pass control such

[6] "Water Power in Brazil, With Special Reference to the São Paulo Development," by A. W. K. Billings, *Journal of the Institution of Civil Engineers*, vol. 3, 1935–1936 (Supplement, Oct., 1936), pp. 677–698.

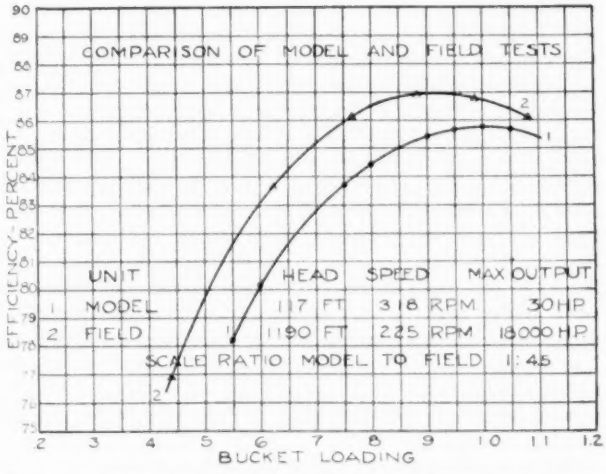


FIG. 16 COMPARISON OF MODEL AND SINGLE-WHEEL FIELD TESTS OF TIGER CREEK UNITS

as the auxiliary relief needle nozzle or a rapidly operated deflector and a slow-moving power needle. When the unit is connected to a large system and the natural stream flow must be maintained, and forebay storage is limited, a very satisfactory method of operation is to use a forebay float-control system, arranged to regulate the opening of the needle nozzle in such a manner as to maintain the forebay elevation within close limits. Governing plants, however, should have adequate forebay storage in order to average up the water supply and the required discharge through the unit.

In conclusion, it should be stated that while the impulse turbine has been developed to a high degree of perfection and reliability, nevertheless, further progress is being made, both in increased efficiency and in obtaining a longer useful working life.

Discussion

GEORGE H. BRAGG.[7] Probably no company has had more experience than the author's or has been more progressive in the development of hydraulic equipment for high-head hydroelectric systems, and so the writer ventures the statement that all the principal steps, by which the needle nozzle and the water-wheel bucket have advanced to reach their present state of efficiency and reliability, have been most ably presented in this paper.

On large interconnected hydroelectric systems, it is essential that some of the plants be designed to regulate the speed. It is not necessary for all of them to be capable of maintaining constant frequency. The needle nozzle, therefore, is indispensable in those plants which are designed to regulate the speed.

In certain plants of comparatively small horsepower rating and low head, needle nozzles are not absolutely necessary and we find in some of them the so-called open nozzle, which may be credited with a higher efficiency than the needle nozzle and a slightly lower cost.

The open nozzle would find a greater field of usefulness if equipped with some simple device designed to increase gradually its discharge from 0 to full jet, without creating disturbances and water hammer in the penstock.

It was suggested at one time that a combination nozzle might be devised which could be opened slowly and at the proper rate and, when completely open, the needle entirely withdrawn from influence to the water discharging at spouting velocity, thus forming a solid jet of maximum efficiency. Needless to say, there

[7] San Francisco, Calif.

are difficulties to overcome in designing such a nozzle, or probably there would be examples in operation today.

The effect of only a slight bend in the approach piping and also the effect of the needle in the center of the discharging jet may be observed in the graph of pressure-gage readings taken by a pitot tube in traversing a jet from the needle nozzle. Fig. 17 illustrates an actual nozzle body having a slight curvature for the needle-stem requirement and also the variation in the velocity head across the diameter of the jet, discharged by the nozzle.

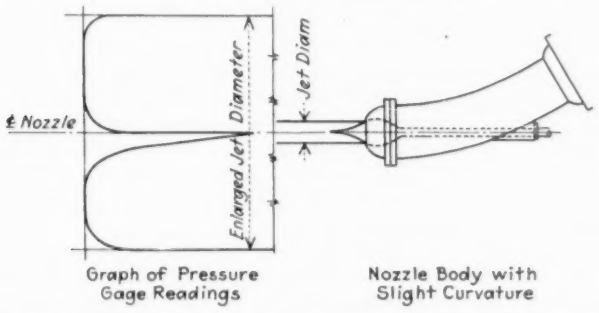


FIG. 17 GRAPH OF PRESSURE-GAGE READINGS TAKEN BY PITOT-TUBE TRAVERSE OF A JET FROM NOZZLE BODY WITH SLIGHT CURVATURE

Notice that the graph is not symmetrical about the center line of the jet and also that the needle causes a depression in the center of the jet, indicating that the power supplied by the jet is not uniformly symmetrical.

Thus far there has been no opportunity to traverse a jet from a straight-flow nozzle which is the latest development, but it is reasonable to suppose that the graph will be symmetrical about the center line of the jet but will have the characteristic depression in the center. The traverse of a solid jet, if uninfluenced by bends in the approach piping, is symmetrical and without the depression. Incidentally, the deformation of the jet has a marked influence on the rate of cavitation of the buckets regardless of the type.

The principal enemies of needle nozzles and water-wheel buckets are cavitation and erosion. Thus far no metal has been found which will resist these forces indefinitely. The hardest and highest-strength metals are the most resistant.

Although the water-wheel bucket, in plan, has been changed from a rectangular shape to the ellipsoidal shape as shown in Fig. 10 of the paper, cavitation has been found to be more rapid on some of the later designs than it is on some having the rectangular shape. There is a case, in which buckets of the rectangular type have been in continuous service for more than 40 years and yet there is little or no cavitation on the surfaces. They are made of ordinary cast steel. Perhaps the author may offer an explanation of this phenomenon.

W. A. DOBLE.[8] The development and operation of high-head hydroelectric generating units has changed merely in degree but not in kind during the last 30 to 40 years. Higher heads have been utilized, but heads of over 1000 ft were developed and placed in commercial use prior to 1897. The power output of the generating units has been increased by installing larger units, but this is also a change merely in degree. Details of design have been refined, based upon the operation of and the experience gained in regular commercial service with large-size units, under heads of 1000 ft or more.

Improved methods of governing or speed control of the units under variable load conditions have been perfected, but there again this work dates back to the years 1900–1913, to the introduction in commercial service of the needle regulating nozzle with manual or direct governor control of the needle, the use of the jet deflector, and/or the combination of the needle nozzle with the jet deflector, the needle being manually controlled, the jet deflector being controlled by the governor.

The author states: "In the early days of the industry loads were light and intermittent and it was often customary to shut a plant down over the week end."

The operating schedule of any hydroelectric public-service plant was the same in the early days as it is at present, depending upon two factors, i.e., the quantity of pressure water available at the plant to drive the water wheels and the load on the system or the power demanded from the generator units. It is the same today. So-called overloads are purely relative and bore the same relationship to the units in 1900 as in 1939. If proper factors were used in the design of the units, continuous service would result without fatigue or failure of elements. So-called overload is really meaningless, as all portions of the wheel unit were or should have been designed to carry successfully and continuously the maximum forces that could be applied. The useful life of the plant or measure of the hours of service was limited by the erosion of the active surfaces of the buckets, nozzle tip, needles, etc., depending upon the quantity and character of sand and grit carried by the water.

In those early days, reliable and continuous service was demanded by the consumer and, as requisite as it is today, the entire system must be kept continuously hot. Current had to be supplied over the week ends, as light and power were as necessary over Saturday, Sunday nights, and holidays as at any other time. The engineer, responsible for the design and successful operation of the system, developed the designs on the basis of apparatus incorporating the necessary factors of safety to insure equipment having the structural stability to develop the full power for which the units were designed and capable of giving continuous service. He could not rely on the units being subjected to intermittent service, with the idea that they could "rest up and recover" over the week ends.

The author's statement gives an incorrect impression because in the earlier days power stations that were built were generally not connected with others to form an interconnected system, but in fact served certain local areas with power. The subject of constancy of load and load factors was well known and it was necessary to maintain continuous service on practically every plant as then built; there could not be any shutdown on Saturday or Sunday.

A large portion of the load of these early plants included supplying current to drive irrigating pumps. During the irrigation season, such pumps must necessarily operate continuously, irrespective of Saturdays or Sundays.

Consider for instance the Mount Whitney power plant completed in June, 1899; this plant was primarily constructed and financed on the basis of furnishing electric current to the agriculturists for driving irrigating pumps. This plant was isolated and independent of any other plants, therefore, it was necessary to provide continuous service to the agriculturists.

As a matter of fact, the contrary situation prevails today where there are a large number of generating plants delivering energy into interconnected systems. In modern practice, it is possible to shift the load from one plant to another and in case of necessity any one of the hydroelectric units or the entire plant may be taken out of service, the load being carried by other plants of the system.

In those early days, the generator units in the large power houses in regular commercial service frequently carried maximum

[8] Doble & Doble, Patent Attorneys and Consulting Engineers, San Francisco, Calif. Mem. A.S.M.E.

full loads over long periods of time without fatigue or excessive stresses. It was absolutely necessary that these characteristics be incorporated in the design. The history of the earlier hydroelectric power plants with their records of many years of continuous service substantiates this statement.

The author also states: "In past years, in considering the requirements for high efficiency, no particular attention was given to the design or arrangement of the approach piping for delivering the power water from the penstock to the nozzle."

This is a serious and unjust charge against the many very able engineers who were responsible for the development, design, construction, and operation of these earlier hydroelectric power plants, and who also blazed the trail and developed the designs which are the basis on which the recent units referred to by the author are founded.

In the design and construction of such early power plants, the design of the pressure pipe line or penstock, including the arrangement of the approach piping or members through which the power water was delivered from the penstock to the nozzles, was most carefully worked out. It was fully appreciated that changes in the direction of flow should be avoided as much as possible and, where necessary, should be by means of curved or angle fittings providing for the least possible change in direction of flow of the power water, all abrupt changes in direction of flow being avoided. This was a fundamental requisite as the success of the project depended upon the development of the highest possible efficiency from the available water which as a rule was of relatively small volume but delivered to the wheels under a high head or pressure. A study of the layout of these earlier plants will demonstrate that these factors were most carefully considered and were provided for in the design and the layout of the plant, penstock, pressure pipe fittings, including careful consideration of the design of the pressure piping in order to insure a uniform and smooth acceleration of velocity of the power water and so designed as to avoid eddy currents, swirling, and loss of head or pressure.

Blue Lakes Water Company

In the power development[9] of the Blue Lakes Water Company, which was placed in commercial service August 25, 1897, under a static head of 1042 ft, the pressure pipe line or penstock was in plan laid on a straight line from the forebay to the steel Y branches or connecting pipe fittings at the powerhouse. The approach-pipe connections that delivered the power water from the penstock to the nozzles were steel castings designed to accelerate the power water and deliver it to the wheels with the least possible disturbance and eddy currents.

Mount Whitney Power Company

The power plant[10] of the Mount Whitney Power Company, completed in June, 1899, operates under 1300 ft static head. In this plant the penstock and special pipe-connecting members, through which the power water was delivered from the penstock to the nozzles, were most carefully designed to avoid abrupt changes in direction of flow and to provide smooth acceleration of the power water, avoiding eddy actions, disturbances, and friction losses.

The gate valves in this plant, which are of the rotatable-plug type, were incorporated in the design and construction of the nozzle body. The cylindrical plug is parallel, the water passageway through the plug was bored on a taper and registered accurately with the passageway through the nozzle body, providing continuous smooth water passageway. The valves were constructed so that the rotatable plug was lubricated and packed with a pressure grease seal.

In this plant radial antiswirling and turbulence vanes were installed in the nozzle and tips to straighten the flow of the water and improve the character of the jet. It developed, however, that regardless of how keen the entrance edges of the radial antiswirling and turbulence vanes were sharpened and, although the head of water at the nozzles exceeded 1000 ft, at times, when the power water carried fine grasses and pine needles, such grasses and needles would accumulate and build up on the radial vanes and thereby plug the nozzles. A further disadvantage was that the radial swirling vanes, being positioned in the flowing water at the point of its highest velocity, substantially increased the wetted-surface area, thereby increasing the friction loss, which more than offset any possible gain due to the improved character of the jet.

The nozzles, plug valves, and radial swirling vanes in this plant were of patented construction.[11]

In this plant, although the character of the jet was improved by the addition of these antiswirling vanes in the tip, and discharged a jet of excellent characteristics, it did not equal the character of the jet from a needle nozzle which projects a parallel jet of water with substantially constant cross-sectional velocity free from eddy action and swirling. The needle nozzle, also presenting the minimum area of wetted surface to the jet, gives a jet of higher power value than when the tip is provided with antiswirling vanes which increase the area of the wetted surface and the friction losses.

In the construction of the Mount Whitney plant, a pipe fitting was installed at the lower end of the pressure pipe line. An exact duplicate of the gate valve, nozzle, and tip was installed at this point, the purpose being to provide an opportunity for testing the character of jet discharged from the tips under this high head.

The De Sabla Power Plant of the Pacific Gas & Electric Company

The De Sabla Plant of the Pacific Gas & Electric Company was constructed by the Valley Counties Power Company. The original installation,[12] including units Nos. 1 and 2 of 2000 kw each at 240 rpm, operating under a static head of 1531 ft, was placed in service October 22, 1903. The gate valves in these two units are of the parallel transverse sliding plug or piston type.

The water passageway through the valve plug was bored to register accurately with the bore through the valve body, forming a uniform smooth water passageway through the over-all length of the valve. The branch Y fitting connecting the end of the penstock with the valves was very carefully designed to avoid eddy action and swirling losses and to insure smooth acceleration of flow of the power water.

For many months after units Nos. 1 and 2 went into commercial service, each unit carried a continuous load of approximately 2200 kw. The original buckets on units Nos. 1 and 2 are still in service, however, these units have not run more than about one quarter of the time during recent years.

In 1904, unit No. 3 was installed including a 4000-kw generator at 400 rpm. This generator was very liberally rated. It has been in service almost continuously since it was installed carrying a load of 5000 kw, during which time worn wheel buckets have been replaced.

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[9] "Sierran Transmission," by George P. Low, *Journal of Electricity*, vol. 5, October, 1897, pp. 1–17.

[10] "The Mt. Whitney Power Transmission," by R. McF. Doble, *Journal of Electricity*, vol. 19, January, 1900, pp. 1–16.

[11] U. S. Letters Patent, No. 659,794; October 16, 1900.

[12] "High Potential Long-Distance Transmission and Control," by F. G. Baum (with an introduction by the editor on the De Sabla power plant), *Journal of Electricity*, vol. 14, no. 11, November, 1904, pp. 439–452.

The San Francisquito Plant No. 1

The San Francisquito plant No. 1 of the Bureau of Power & Light of Los Angeles was designed to develop the highest possible efficiency from the water available at the plant. The design of the elements carrying the power water to the wheels was most carefully considered to avoid friction losses and losses due to eddy currents and swirling in the power water. The main cutoff gate, bolted to the inlet flange of each nozzle, was of the needle type and was designed to avoid unfavorable acceleration, eddy currents, and friction losses in the power water. These needle gates were of patented construction.[13] The plant was installed about 1912 or 1913.

In view of these and many other early hydroelectric plants, it is established that the engineers in past years, considering the requirements for high efficiency, gave very careful study to the design and arrangement of the approach piping which delivered the power water from the penstock to the nozzles, so as to secure the highest possible unit efficiency, by avoiding the losses due to the effect on the jet of turbulence or crossflow and eddies in the nozzles, gate valve, and connecting piping, leading from the penstock to the water-wheel unit.

C. V. Foulds.[14] The author has given an interesting picture of the benefits to users of the routine research that has come to be practiced generally by forward-looking industries for the improvement of their products, and without which they would inevitably drop behind in the procession of progress.

The user necessarily has a double interest in such work, first as the recipient of its benefits, and second, as the one who must pay the bill. He must resign himself to the fact that, though the individual parts of a perfected machine may cost no more to produce than those of earlier models, a higher price must be paid for them to cover the indirect costs of their development. The costs of precision research are very high even for apparently simple factors; where there is sufficient quantity of production there may occur parallel improvements in manufacturing to return the research costs without price advances. However, for power-plant and similar special machinery, the manufacturing costs change but little and research can only be repaid by price increases, a single lot at times bearing a good part of the costs for its development. That the result usually justifies the charge is evidenced by the practice of many enlightened large users to encourage research by bonus, outright grant, or purchase of a model for test purposes, all with a view to removing development from the competitive business of machinery production, that would otherwise penalize the manufacturer who had done most to improve his product.

At first glance, it might seem that such intensive and detailed research would relieve the user of the necessity for initiative and careful selection in the purchase of his equipment. That it does not is because research is largely functional in character, while the user's interest is in both the functional performance and the mechanical details, as related to the period for which maximum performance will continue, the simplicity of its restoration, and freedom from petty annoyances. There are also to be considered such individual elements as transportation and evaluation of the penalties of shutdowns, compared to the costs of reducing them.

Thus, while research has assured the user of apparatus that will perform as specified and relieved him of attempting detailed specifications for the major dimensions and materials to be used, the element of competition requires that he be very specific in regard to general arrangements, replaceable wearing parts, corrosion- and wear-resistant materials, conveniences for assembly and dismantling, etc., in order that bids may be submitted on a uniform basis, as it is in those elements that the greatest variations may occur in functionally equal machines. Thus the practical specification may properly become little more than a description of the conditions to be met and the degree of refinement desired in a machine whose performance has been assured more effectively by research than anything the user might specify.

Research is a true partnership between supplier and user, of equal benefit to both, and to which every encouragement should be given.

J. D. Galloway.[15] From the context of this paper, it is evident that a close study is being made to increase the efficiency of the impulse turbine.

The use of straightening vanes to decrease the turbulence in the water as it emerges to form the jet is a new device, the value of which may be open to question. A correction in the form of the jet is obtained by the straightening vanes in some of the cases cited, but it would seem that the vanes would offer some resistance to the flow of the water and thereby act to reduce the efficiency of the unit. It would also seem likely that considerable wear would occur in such vanes, impairing their ability to form a perfect jet. However, as a means of correcting some old-time installations, where the water passages are incorrectly designed, the straightening vanes might prove to be a good device, even if occasional renewals were necessary. Experience over a term of years with the device would furnish the best evidence for or against its value. Curve four, Fig. 5, indicates that with properly designed water passages the vanes are unnecessary.

Some stress is laid by the author upon the improvement of the jet illustrated in Figs. 1 and 4. It might be questioned as to how much additional improvement in this direction is necessary, mainly for the reason that the needle is placed as close as possible to the wheel buckets and the jet is not of great length before it strikes the lip of the incoming bucket. That distance is at least only a fraction of the length of jet shown in the illustrations.

It would seem to the writer that the greatest advance has been made in the form of buckets, the results of which are indicated in Fig. 12. The recent type there shown has an efficiency of more than 1 per cent above that of the older type, which is a material gain. The efficiency curve is also much flatter at the upper portion, demonstrating that the higher efficiency holds over a wider range.

The author's remarks regarding the value of collaboration between the engineers of the power plant and the turbine designer, in order to produce the best design of the water passages, is well taken. To illustrate this point, some examples may be cited. One old-time power plant has several pipe lines leading into a manifold placed at right angles to the pipe lines. From the manifold several units are served, and none of the water passages to the nozzles takes off from the manifold opposite the pipe lines. As the units are of different capacity, one may wonder what is happening to the water in that manifold. However, it was recognized some time ago that a direct flow of the water from penstock to nozzle is necessary, and with increasing size of the power units, one penstock to each unit would be the proper design. A Y is required and also a gate valve back of each nozzle. The physical situation has an important bearing on the design of the water passages. Some plants, as at Tiger Creek, are so located that there is ample space in which to lay the penstocks in a desirable manner. On the other hand, at Bucks Plant, the situation was such that the power station was located close to a granite cliff,

[13] U. S. Letters Patent, No. 1,290,722; January 7, 1919; application filed December 27, 1913.

[14] Hydraulic Engineer, Placer Management, Ltd., San Francisco, Calif.

[15] Consulting Engineer, San Francisco, Calif.

where the horizontal dimensions were very limited. However, in all cases, collaboration between the plant engineer and the turbine designer is advisable in order to produce the best results.

High efficiency of turbine units is of value, even in a plant serving a single system. In such a plant, with several units, load changes resulting in part loads would be carried by one unit, the others being operated at or near full load. In a system of several plants, in different locations, governing is done at one plant, the others operating at full load, at least on the units in service.

The improvement that has taken place in the design and construction of impulse turbines has been well indicated by the author, who has summarized in a clear manner the recent progress and the manner in which it has been achieved.

J. P. JOLLYMAN.[16] Our experience with impulse turbines, having a wide range of heads and horsepowers, tends to confirm the author's conclusions.

The first wheels constructed for heads above 1000 ft were from 1500 to 3000 hp with wheel-to-jet diam ratios 45 to 33. Forces on the buckets were small and little trouble was experienced with bucket attachments or with bucket cracking. The need for larger units and the availability of higher-speed generators soon led to 7000-hp units with wheel-to-jet ratios as low as 10. Trouble with bucket attachments soon developed on account of the greatly increased stresses on the bucket bolts. Wheel-jet ratios have since been increased to 12 or more and the design of bucket attachments has progressed to the point where no trouble is had or expected with attachments. The hydraulic design of buckets is a problem in the relative motion of the buckets and the water. The problem cannot be solved by theoretical consideration alone, since it is impossible to predict the exact path of each particle of water after it enters the bucket. The experimental research described by the author has been of great value in assisting the hydraulic design.

The need for and the economy of large units has increased the size and loading of buckets to the point where fatigue cracking of buckets has been encountered. The hydraulic and mechanical requirements conflict at the entering lip. Hydraulically thin sections with sharp edges are desirable, whereas the necessary mechanical strength requires thicker sections and rounded edges. We are hopeful that the latest designs will largely overcome this difficulty.

The path of water in curved channels at high velocities is different from its path at low velocities, and this is recognized in Francis-turbine design. This difference may account for the change in characteristics from the model impulse turbine at low heads to the field unit at high heads.

The excellent efficiencies obtained at Tiger Creek confirm the advantages of the features of design mentioned in the paper. All of these features are included at Tiger Creek except the 90-deg needle nozzle which had not been developed at that time.

S. LOGAN KERR.[17] Two important factors affecting the performance of impulse turbines are brought forth by the author: First, the effect of turbulence in water passages leading to the nozzle and, second, the correlation between laboratory and field tests. While these two factors seem dimly related, they are, in the writer's opinion, one and the same.

Some 8 or 9 years ago, when the writer was closely associated with the inception of the research program which the author describes, the writer felt that the approach conditions in the conduits leading to the needle nozzle were left too much to the symmetrical eye of the draftsman and were given too little attention by the engineer. The transition from a single penstock to a divided conduit, thence to further divisions, all at high velocity, leading to the individual jets of a double-overhung wheel, could result only in turbulence in the free-discharging jets, causing them to reach the buckets in a manner far removed from the neat circle shown in the textbooks. One only needs to examine Fig. 1 of the paper to be convinced that the transfer of energy from the lower jet to the wheel buckets can hardly be efficient.

Coupled with this matter of turbulence is the exaggerated effect of small distortions in the needle-nozzle body when the flow is at high velocity. Each bump, crack, or minor roughness tends to distort the flow from the ideal pattern.

With reaction and propeller turbines, the relation between the model-test results and the field performance has been established so well that the Moody formula may be applied with reasonable accuracy for predicting performance of the full-sized units with assurance, so long as the conditions are held homologous with respect to flow and cavitation conditions. The agreement between true laboratory results, stepped up to full-scale units, and field performance is amazing and usually falls within the accuracy of test methods employed.

Unfortunately this same agreement is not true of impulse turbines and, as the author states: "The reason for this change in characteristics as yet has not been fully explained." Could not this be the basis for additional research? In the writer's opinion, more attention should be paid to the effect of absolute velocities on performance as compared with test velocities proportional to the square root of the head or determined from the geometric stepdown of the dimensions of the full-scale unit.

Consideration of the effect of high-velocity flow in the water passages immediately in advance of the orifice should be considered. These are the flow conditions that affect the jet directly, and approach conditions to an orifice should receive as much consideration as the approach conditions to a weir. Small disturbances at low velocity can change the flow pattern over a weir and cause marked changes in its accuracy. Is not the problem made much more acute when dealing with the discharge from needle nozzles with a usual velocity ten times or more that of the approach velocity to a weir?

Early in the research program described, the writer was able to correlate the field characteristics with laboratory tests by the introduction of artificial turbulence in the approach conduit; the point of uncertainty being only the required degree of turbulence.

Tests on a model unit under a range of heads from laboratory conditions up to field conditions should indicate the change in performance due to changes in absolute velocities. Such tests might pave the way for further improvement on large-scale units and supply a reliable basis for comparing laboratory and field test results.

E. F. MARYATT.[18] The author's comments regarding forces applied to the modern water-wheel bucket, in which he uses as an illustration the 25,000-hp wheel, each bucket of which is, in 10 years' time, subjected to 2,000,000,000 working cycles with an impulse of 50,000 lb applied to each cycle, very effectively bring to our attention, the magnitude of the problem. The question might logically arise: Why use such large-sized units? Why not buy more smaller units of the same total output and, by so doing, eliminate some of the tremendous fatigue problems. The answer is, of course perfectly obvious. The increasing size of our systems

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[17] United Engineers & Constructors Inc., Philadelphia, Pa. Mem. A.S.M.E.

[18] Assistant Engineer, Division of Hydroelectric and Transmission Engineering, Pacific Gas and Electric Company, San Francisco, Calif.

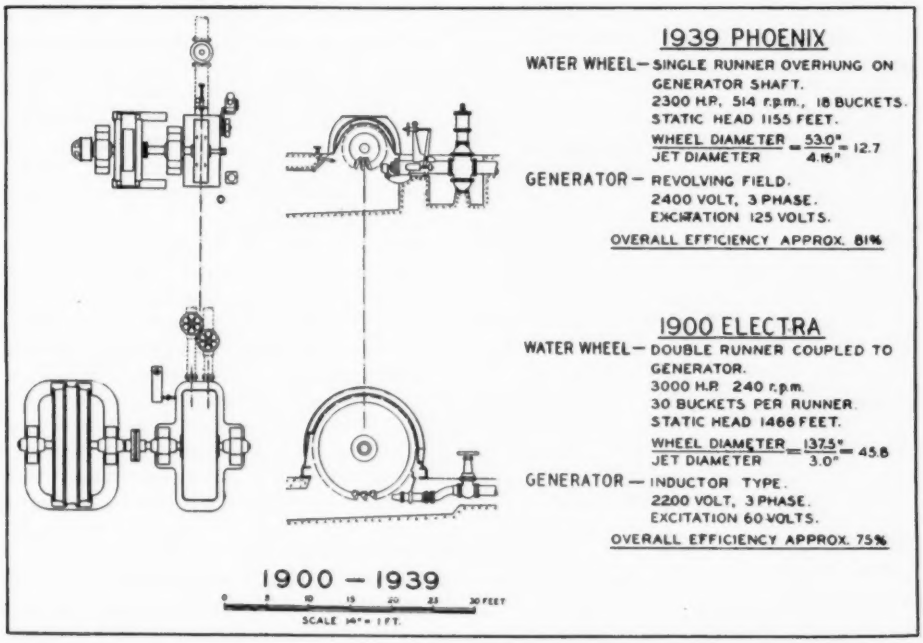


FIG. 18 PHOENIX 2000-KVA POWER UNIT AS COMPARED WITH OLD ELECTRA UNIT OF SAME OUTPUT RATING

with the accompanying increased length and voltage of our transmission lines requires these larger units.

If we build a powerhouse at the remote end of a 200-mile 220,000-v transmission line such as, for instance, the Pit lines of the P.G.& E. Co., each unit in the plant must be capable of supplying sufficient power to charge the line. In the case of the Pit lines, this amounts to approximately 50,000 kva. The Tiger Creek lines require approximately half of that amount, or 25,000 kva. Therefore, a double-overhung unit with two wheels, each of which is of the size of the one used in the author's illustration, would hardly be of sufficient capacity to charge the line if located at a site similar to Pit No. 1, for instance. Two units operating with generators in parallel are, of course, unstable for use in charging a 220,000-volt unloaded line.

The question might also be asked: Why not put a whole row of wheels on one shaft? The author may be forced to resort to some such scheme someday if the size of the required units continues to increase. In the very early stage of water-wheel development, this scheme was resorted to, but for one reason or another has grown out of general use. The most striking illustration of multiple wheels on a single shaft, which has come to the writer's attention, was at Snoqualmie Falls, Wash., where he was in 1910 on the construction of plant No. 2. Plant No. 1 had been built and placed in service in 1899. In plant No. 1 four units were installed, each consisting of a 1500-kw, 300-rpm generator driven by a so-called Doble water motor. The water motor consisted of six impulse wheels, side by side on a single shaft, each equipped with two nozzles and they were grouped with three wheels in one housing. In this unit there was a total of 12 jets, six wheels all on one side of the generator, and the rating was only 2500 hp. Those units violated many of the fundamentals of hydraulic design which the author has pointed out, and more. The head, being under 300 ft, was far more suitable for a Francis turbine. This was soon discovered, and 11 years after the original installation the impulse wheels had already been partly superseded by a horizontal single-discharge Francis turbine.

About 1 year after the Snoqualmie No. 1 plant was placed in operation, the Electra plant (now of the Pacific Gas and Electric Company), was built. In this plant the units were rated 3000 hp 240 rpm. Here the static head of 1466 ft was more suitable for impulse wheels. The Electra units consisted of a 2000-kva generator, driven by twin impulse wheels with one nozzle each. The two wheels were located side by side and in the same housing. The generators were the Stanley inductor-type, rated at 2000 kw.

At the Phoenix powerhouse, a new unit of the same output rating as the old Electra unit (2000 kva) is now being installed. Since the output ratings are identical and the hydraulic features are somewhat similar, the two installations make a very interesting comparison between the unit built in 1900 and the unit built in 1939.

Fig. 18 shows the two units to the same scale. The Phoenix unit has embodied within its design the latest known fundamentals mentioned in the paper, even to the inclusion of the 90-deg nozzle. From the drawing may be noted the straight-line approach of the needle in the Phoenix unit, while the Electra-unit needle approach includes a definite offset. However, the most striking contrast appears in the relative sizes of the two units. Attention should be called to the fact that the Phoenix unit has a direct-connected exciter on the end of the main shaft. Also space was provided for end shift of the generator stator. If it were not for these two items, the Phoenix unit would appear even smaller in the comparison.

We know that for the same unit, the speed should vary with the square root of the head. It is interesting to note that Electra, with 1466 ft static head, runs at 240 rpm, while the Phoenix unit, with 1155 ft static head, is now designed for 514 rpm. Very striking, also, is the comparison of the wheel-to-jet diam ratio, which for Electra is 45.8 and for Phoenix is 12.7. If the Electra unit had been designed with the same wheel-to-jet diam ratio as Phoenix, the Electra speed would have been increased to a theoretical speed of 830 rpm. The nearest synchronous speed is 900 rpm, which is almost 4 times the original speed.

J. D. SCOVILLE.[19] The author's discussion and illustration of the impulse-wheel jet indicate graphically the importance of proper design of the nozzle and the necessity for the introduction of straightening vanes to prevent spreading of the jet before it

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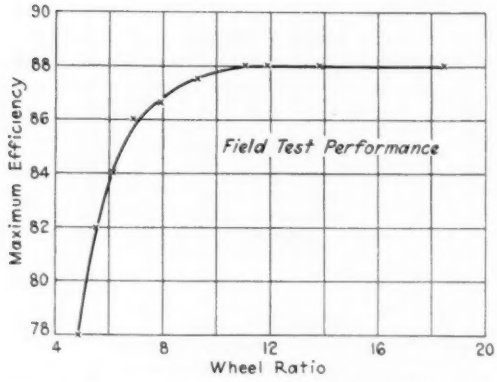


FIG. 19 CHARACTERISTIC CURVE FOR IMPULSE-WHEEL BUCKET

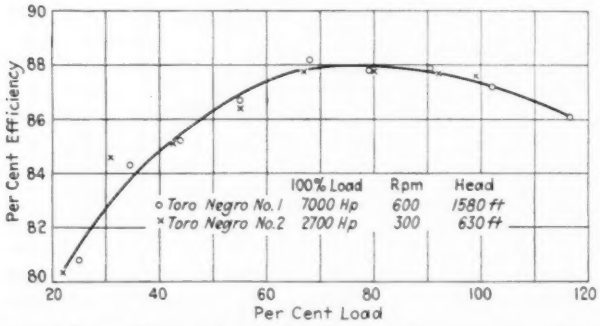


FIG. 20 FIELD TESTS OF SINGLE-JET IMPULSE WHEELS; PUERTO RICO RECONSTRUCTION ADMINISTRATION

reaches the runner. Proper design of elbows and approach passages is also required to get an efficient jet. The author illustrates various shapes of buckets and discusses briefly the importance of design on efficiency and cavitation. We think he does not dwell enough on one point which has a bearing upon efficiency, namely, the number of buckets. It is obvious that a runner with too few buckets will allow part of the water to pass without doing any work. It is also true that if too many are used, it becomes increasingly difficult to design the shape so that part of the water does not strike the back of adjacent buckets and so reduce the efficiency. Bolted construction limits the number of buckets on account of mechanical stresses. The use of solid-cast runners and very careful design of the shape permits the maximum number to be used and the maximum efficiency. This is especially important with wheel ratios below 8 on high-specific-speed runners. Fig. 19 shows maximum wheel efficiency plotted against wheel ratio. This represents tests of runners with more buckets than those of Fig. 13 in the paper.

Fig. 20 shows field tests of single-jet impulse wheels for the Toro Negro Plants in Puerto Rico. These are tests made on two different units of about the same specific speed in two plants of the Puerto Rico Reconstruction Administration. The runners of these units also have more buckets than shown in Fig. 13.

AUTHOR'S CLOSURE

The writer is indeed grateful for the many excellent ideas advanced in the discussions.

Mr. Bragg has pointed out the desirability of some form of adjustable plain nozzle which would eliminate the losses due to the needle bulb of the needle nozzle. Many tests have been conducted in the laboratory investigating the possibilities of such a nozzle but, on account of the many practical difficulties in handling the heavy loads, leakage, etc., no satisfactory design has as yet been evolved.

In Fig. 17, Mr. Bragg clearly shows the lack of symmetry of a power jet, resulting from the disturbance in the nozzle body. This disturbance becomes more acute as the relative radius of curvature is reduced, or as the elbow is made more abrupt, and is further accentuated as the velocity of approach is increased. The presence of such elbow losses has been appreciated for many years and it is now customary, for example, in the design of penstocks, to favor elbows of large radius rather than those having abrupt turns.

The problems of bucket and needle-nozzle cavitation and erosion have been studied very carefully both in models in the laboratory and in the large units in the field. Important progress is being made in correcting these difficulties both by changes in the design and in the use of tougher modern materials. The exceptionally long life of buckets found in the case of a few old units has been given a great deal of study. In all probability there was an unusually favorable combination of water, moderate loading, and homogeneous materials which accounted for the excellent performance. The efficiency of these units, however, has not been of the best and does not compare with current practice.

Mr. W. A. Doble is to be thanked for his excellent review of the early history of high-head impulse-turbine design. He has pointed out a number of interesting problems and the manner in which they were studied and solved at that time.

The practical problems of cooperation between manufacturers and users of hydraulic-power machinery has been very ably presented by Mr. Foulds. The hydraulic turbine is a custom-built or tailor-made machine designed for the particular hydraulic conditions of each installation. Consequently, model studies are of great value in verifying the results of the complicated calculations and analyses. Further cooperation covering the production and financing of model studies is to be encouraged and will do much to promote further improvements in the art of hydraulic-turbine design.

The review of past and current practice in the design of approach piping and penstock manifolds, presented by Mr. Galloway, shows the important progress made in recent years in a better understanding of the hydraulic problems involved. For example, the old practice of using a large manifold outside of the powerhouse, acting as a header from which a number of right-angle lines serve the individual units in the station, has been discontinued.

Mr. Jollyman has pointed out a difficulty confronting the designer in providing a water-wheel bucket having both high efficiency and low stresses or a long working life. Laboratory and field results demonstrate the advantages of a sharp splitter and entering lip for high efficiency. Sharp edges of this type may be nicked or damaged readily by debris in the water, thus introducing regions of local stress concentration which start further cracks and corrosion fatigue. When the entering lip of the bucket is made thicker and heavier in order to strengthen this part of the bucket against local stress concentration, the problems of entering the bucket into the power jet without turbulence and interference are greatly increased. Bucket design, therefore, must be a compromise between strength and efficiency. The trend of recent research has been to develop revised forms of bucket lips of high strength and acceptable efficiency.

Mr. Kerr suggests a more careful consideration of absolute velocities in studying the performance of model impulse turbines. Further information on the performance of a given unit when tested under a wide range in heads should throw more light on this problem. The reaction turbine and the centrifugal pump may be tested conveniently over a comparatively wide range in heads, since these two types of hydraulic machines are totally enclosed so that the flow is restricted into very definite channels and may be tested under heads varying from a few feet to 200

to 300 ft without much difficulty. However, the impulse turbine to be tested over its normal working range should be tested under heads of say from 200 to 2000 ft, which is beyond the facilities of most laboratories. These factors are fully appreciated and every opportunity will be used to explore further the influence of head.

Fig. 18, submitted by Mr. Maryatt, shows the trend of modern impulse-turbine design. A wheel ratio of about 12 is still the minimum which is most favorable for high efficiency. This is again confirmed, in Fig. 19, by Mr. Scoville. Smaller wheels with lower ratios may reduce the first cost of both the hydraulic turbine and the generator but usually are not the most economical when the entire water supply is to be converted into useful power.

In the early days of the industry, the large ratios resulted from the generator design, primarily, as it was fully appreciated even then that a ratio of 12 was advantageous.

Determining the most favorable number of buckets, consistent with high efficiency, good performance, and working life, involves problems of mechanical and hydraulic design. Laboratory testing, as carried out by the company with which the author is associated, incorporates the use of wheel centers and buckets, permitting complete flexibility in order to determine the most favorable number of buckets, lead, speed, jet position, and other such factors. The number of buckets used in the cases set forth in the paper is that producing the most favorable combination. Mr. Scoville points out that the tests with which he is familiar used a greater number of buckets. This point is fully appreciated as it has been found that the most favorable combination depends upon the bucket design, so it is to be expected that the appropriate number will change with the type.

The use of water-wheel buckets integrally cast with the wheel center is a practice of long standing and is well known to the art. Such buckets however are not satisfactory for laboratory tests as they do not permit of adjustments necessary to establish the most favorable assembly.

The efficiencies, noted in Fig. 20 of Mr. Scoville's discussion, are excellent and it is hoped that an opportunity will develop to obtain more complete details of the research which he has mentioned so that a further comparison may be made with the facts presented in the author's paper.

Joint Use of Diesel Engines and Utility Power

By EDGAR J. KATES,[1] NEW YORK, N. Y.

This paper shows that there are various ways in which a consumer of electric power may advantageously use both Diesel engines and public-utility service.

A brief review is first given of the different uses that steam and hydroelectric public utilities have themselves made of Diesel-engine power on their own systems.

The ways in which private power consumers have used Diesel engines and outside utility service jointly are discussed under the following classifications:

(*a*) Utility service as a stand-by for Diesel power; (*b*) Diesel power as stand-by for utility service; (*c*) Diesels to restrict the maximum demand on utility service; (*d*) utility service to check Diesel frequency and synchronous time; (*e*) utility service as auxiliary to Diesel power; (*f*) utility service to carry momentary load swings.

Data are given on the diversity factor of the use of utility breakdown service by a group of private power plants. The factor exceeds 10/1. The author proposes a new form of rate for breakdown and auxiliary service which he considers more equitable to all concerned than the usual forms of rate for such service.

The advantages to the power user of operating his Diesel plant electrically paralleled with the utility service are cited. The author discusses in detail the technical factors involved in parallel operation with which the power company is concerned. He concludes that if simple precautions are taken no difficulties should occur either on the utility system or in the Diesel plant. Actual experience confirms this, and such parallel operation is increasing.

Greater cooperation between private-plant owners and public-utility companies in facilitating the joint use of Diesel engines and utility service will benefit both parties.

THE ENGINEERING of steam power joined with public-utility service has been thoroughly expounded by several authors (1, 2, 3, 4, 5, 6, 7).[2] Also the use of Diesel engines in conjunction with steam engines in order to effect a balance between power loads and heating loads has been covered in a paper before this Society (9). The excuse for the present paper is that little has been said about the use of private Diesel power combined with public-utility service. While the paper has been written primarily from the viewpoint of the power user, an attempt has been made to consider the utility's aspect also.

Public-Utility Uses of Diesel Engines

Of course Diesel engines have long been used by public utilities themselves. Not only are there thousands of Diesel engines in the United States generating the entire output of public utilities (under corporate ownership as well as municipal), but Diesels have also been installed by the utilities to run in conjunction with their own steam or hydro power. Since these public-utility uses of Diesel engines combined with other sources of power are similar in some respects to the private applications to be discussed, they are here briefly reviewed:

(*a*) Diesel engines to carry peak loads in steam central stations. Here the primary object is to avoid the stand-by losses of banked boilers and idling turbines. This application has been mainly confined to Europe. There, several huge Diesel engines have been installed, such as the two 11,700-hp engines at Hennegsdorf near Berlin, Germany, the 15,000-hp engine at Hamburg, Germany, and the 22,500-hp engine at Copenhagen, Denmark (10, 11, 12, 13, 14, 15, 16).

(*b*) Diesels have been proposed for emergency "house power" in steam central stations. When a flood or hurricane completely shuts down a station and destroys its interconnections, the plant can be more quickly restored to service if a moderate amount of power is available at once to supply the station lighting, run the auxiliaries, provide initial excitation, operate the switchgear, and the like.

(*c*) Diesels for stand-by use on hydroelectric systems. The Diesels serve at times of low water and also for emergency reserve. They are particularly applicable in locations where the water flow is so irregular that the banking losses in a steam stand-by station would be excessive. The rapidity with which the Diesel plant will take over the load is an important feature. In factory tests on one of the three 3750-hp units installed in 1927 at Miraflores Locks, Panama Canal Zone, the unit took over its full load in 32 sec after being started cold (10, 11, 12, 13). This application of Diesels for hydroelectric stand-by is common; one manufacturer alone has installed ten such plants. A recent example is that of the Salt River Valley Water Users' Association at Phoenix, Arizona, where two 7000-hp Diesel stand-by units were installed last year to operate in conjunction with an eight-plant hydroelectric system.

(*d*) Diesels to carry the load of an isolated locality until the electrical demand becomes large enough to justify extension of a transmission line. During the period of utility expansion between 1915 and 1930, a number of such installations were made in the Southwest, where growing utility systems would acquire a small inefficient steam plant in outlying territory, and by serving the area at low rates by means of a Diesel plant would eventually build up enough load to warrant the investment in a high-tension transmission line from a large steam station. In fact, in some cases, such as at Paris, Tyler, and Brownwood, Texas, Diesel stations originally installed with the intention of subsequently removing them to other locations were found so to improve the operating conditions near the ends of the transmission lines that they were kept in use in their original locations for more than ten years (10, 15, 17).

(*e*) Diesels located at distant points on transmission lines to reduce peak loads in the main generating stations and on the transmission lines, reduce transmission losses, improve voltage regulation, increase power factor, and provide local service in case of transmission-line outage or main-station trouble. Diesel plants, designed to serve some or all of these purposes, long ago proved their value, and the practice, which became common in the Southwest some ten years ago, subsequently spread to other

[1] Consulting Engineer.

[2] Numbers in parentheses refer to the Bibliography.

Presented at the National Meeting of the Oil and Gas Power Division of The American Society of Mechanical Engineers, Ann Arbor, Mich., June 19–22, 1939.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

sections such as Florida, Iowa, and Minnesota. In the latter region, the Otter Tail Power Company is operating a system comprising 36,000 hp in steam, 7000 hp in hydro, and 6000 hp in Diesel units (15, 17, 18).

Private Use of Diesel Engines With Public-Utility Service

The foregoing indicates the various uses that steam and hydroelectric utility systems have themselves made of Diesel-engine power within their own stations or out on their lines. Private power consumers, also, have already discovered ways of profitably using both Diesel engines and outside utility service in the same plant, and new uses are continually developing. The following types of applications will be discussed:

(*a*) Utility service as a stand-by for Diesel power.
(*b*) Diesel power as stand-by for utility service.
(*c*) Diesels to restrict the maximum demand on utility service
(*d*) Utility service to check Diesel frequency and synchronous time.
(*e*) Utility service as auxiliary to Diesel power.
(*f*) Utility service to carry momentary load swings.

In some of these applications it will be evident that when the private-plant owner purchases public-utility service for joint use, he obtains a large benefit while the utility company merely sells current at standard rates. The question may therefore arise as to whether the utility company should not be entitled to charge higher rates for such service. The answer is explicit in the fundamental theory under which public utilities are constituted, i.e., that public utilities are "affected with a public interest," and in return for the exclusive privileges granted them, they are charged with the responsibility of furnishing the best possible service to *all* consumers at a price commensurate only with its actual cost. The fact that private-plant owners generate power for their own use which they might have bought from a public utility is no reason, under our basic law, why this group of consumers should be expected to pay the utility company a greater profit on the current sold them than on the current sold to any other group of consumers.

(*a*) *Utility Service as Stand-By for Diesel Power*

This application, which is the oldest of all, was the natural result of the occasional breakdowns of early Diesel engines during their development stage. Of course, most power users want power available continuously, but actually the need for unfailing service varies widely, from the gravel plant which can be shut down at any time with practically no economic loss, to the lighting and elevator services in places of public assembly such as department stores.

The practice of using utility service in this manner, which is generally known as "breakdown service," varies with the circumstances, but it is probably safe to say that the proportion of Diesel plants using it is steadily decreasing. One reason is that the records of modern Diesel plants show them to have high reliability. An analysis made by M. J. Reed of the A.S.M.E. Oil Engine Power Cost Reports for 1929 to 1935, inclusive, covering over four million engine-operating hours, showed that the enforced shutdowns aggregated only 0.8 per cent of the operating time. Another reason for the lessened use of utility stand-by service is the rather surprising fact that the price for this type of service has been greatly increased in some important areas. In these areas, reserve Diesel units are now generally found to be cheaper than the stand-by charges for utility service.

Different power companies have quite different policies as to the desirability of breakdown-service customers, possibly because there is a lack of precise data on the service characteristics of this group of consumers. Little information is available even on so important a factor as the diversity of the demands of a large group of users of breakdown service. Some interesting facts about this diversity were revealed in testimony presented before the New York Public Service Commission in 1936; this showed, for a one-year period, the date, hour, duration, and the electric load for each time utility breakdown service used by 118 private power plants of all kinds, steam and Diesel, industrial and commercial (19). The greatest coincident demand at any time (which happened to be in May) was only 9.7 per cent of the total breakdown service for which the group had contracted, and the actual coincident demand in December, the month of the utility-system peak, was only 2 per cent of the total power contracted for. In other words, the diversity factor, based on the total contracted demand, always exceeded 10/1.

If this had been a group of Diesel plants only, it is probable that the diversity factor would have been even greater, in view of the fact previously cited that the average Diesel plant is out of service because of enforced shutdowns less than one per cent of the time.

Utility rate schedules for so-called breakdown service are generally faulty in form, because they do not give separate consideration to the two distinct ways in which this service may be used by the private plant: First, stand-by service, used only in case of an enforced shutdown of the private plant, which may not occur for months or years, and second, auxiliary service used voluntarily.

One common form of rate is a minimum dollar charge, based on the amount of stand-by protection contracted for, which also covers the use of energy at the regular firm-service rates without additional cost up to the minimum charge. Such a form of rate is unfair to the utility company if the customer uses auxiliary service up to or beyond the minimum charge, as in that case the utility is providing him with stand-by protection free of charge.

The other common form of rate is a minimum demand charge, in which the standard firm-service demand charge per kilowatt is applied, not to the kilowatts actually used, but to the kilowatts contracted for. If any energy is used, either for breakdown or auxiliary service, it is charged for at an additional rate. This form of rate is unfair to the customer who wants stand-by protection only, but who rarely if ever uses any auxiliary service. Such a customer pays a price for stand-by protection which is greatly in excess of its cost to the utility because of the high diversity factor of breakdown service by itself.

In the author's opinion, a form of rate for breakdown and auxiliary service which would be equitable to both parties should recognize the distinction between these two kinds of service and should consist of, first, a charge for stand-by protection and, second, a charge for actual use. The first would be a monthly charge merely for the privilege of using current if desired (but not for the current itself). It would be a fixed sum based on the amount of stand-by service contracted for, and the charge per contracted kilowatt would be arrived at upon an actuarial basis founded upon the diversity of breakdown use as shown by past experience. Assuming a diversity factor of 10/1, as previously cited, and allowing for bookkeeping and other costs, the stand-by charge might properly be, say, $0.50 per kilowatt of stand-by protection contracted for. It should be noted that this is not a demand charge, because it is the charge paid only when there is no demand.

The second part of the suggested rate would be a charge for actual use in any month, whether the use were for breakdown service or auxiliary purposes. This part of the rate could be the usual demand and energy charges in effect for general consumers, and would be applied to the actual demand and energy consumption recorded by meter for the month. If no current were

used during a month, only the stand-by charge would be paid.

Under such a form of rate each customer would pay a fair price for the particular kind of service he used, without requiring any differentiating in the metering itself. In other words, standard metering equipment could be used.

The author believes that if rates for utility service to the private plant were of this form, the use of such service would increase considerably, with profit to all concerned.

(*b*) *Diesel Power as Stand-By for Utility Service*

This application, which is the reverse of the one just discussed, is a more recent development. Its origin and growth is a tribute to the reliability, compactness, and low cost of the modern Diesel engine, and its use has been accelerated by the 15 or more unfortunate but extensive outages of utility service that have occurred since January, 1936, in important urban areas like New York, Pittsburgh, and Buffalo. These outages have caused the managements of hospitals, schools, theaters, department stores, office buildings, apartment houses, hotels, telephone companies, and airports to become concerned with the problem of providing stand-by protection for their utility service. In fact, laws are already in force, e.g., the Fire and Panic Act of Pennsylvania (20), compelling the installation of emergency lighting systems in many classes of buildings.

Among the Diesel plants acting as stand-by for utility power may be mentioned the 200-hp unit installed in 1923 in the Bell Telephone Central Office at Harrisburg, Pennsylvania, several plants installed in 1937 and 1938 in various central offices of the New York Telephone Company, a 40-kw set installed in 1938 for lighting and radio use at the Syracuse, N. Y., airport, and several installations in processing plants for motion-picture film in southern California.

(*c*) *Diesels to Restrict the Maximum Demand on Utility Service*

The purpose of this application of Diesel engines is to reduce the cost of utility service by limiting the demand imposed upon the utility lines. In addition, the engines serve as stand-by against utility outages. The engines may be used in two ways: First, where the load rises at certain times of the day, the engines absorb the hump of the curve and improve the load factor of the utility service and, second, where the utility company has a low off-peak rate, the engines take all or most of the load during the utility's peak period.

Two illustrations of this application are:

1 The Dennison Manufacturing Company, Framingham, Mass., installed in 1931 a 350-kw Diesel unit to operate in conjunction with its steam plant and utility service (21). Prior to installing the Diesel, the steam-engine load was adjusted for maximum production of by-product power, and the balance of the load was carried by the utility. However, this imposed a highly variable load on the utility lines, with a correspondingly high demand charge. The Diesel engine improved the setup by smoothing out the utility load while still permitting the steam plant to generate maximum by-product power. At the same time the Diesel provided valuable stand-by service.

2 The Union Stock Yards, Chicago, installed in 1932 a 2250-kw Diesel unit to supply power along with a 6000-kw steam-turbine plant and also utility power (22, 23). The total-load peak was 12,000 kw and the total annual consumption was 54,000,000 kwhr. Prior to the installation of the Diesel, the limited capacity of the steam-boiler plant made it necessary to purchase about 42,000,000 kwhr with a maximum demand of 6000 kw, i.e., a load factor of 80 per cent. The installation of the Diesel greatly improved the over-all performance. The Diesel supplied 10,000,000 kwhr and 2000 kw, the steam plant generated 10,500,000 kwhr and 6000 kw, while the utility load was reduced to only 33,500,000 kwhr and 4000 kw, which improved the utility load factor to 95 per cent.

In both of these plants the Diesel engines as well as the steam plants are electrically interconnected with the utility systems, which is highly desirable to simplify the allocation of load among the several power sources.

(*d*) *Utility Service to Check Diesel Frequency and Synchronous Time*

Because of the extensive use of synchronous clocks by utility customers, most utility systems maintain an exceedingly uniform cyclic frequency together with periodic correction of accumulated deviations in time. By introducing a small service from such a utility system into a Diesel alternating-current power plant, the same accurate control of frequency and time may readily be transferred to the Diesel system at a trifling cost. The Diesel-plant frequency can easily be checked against the utility system by means of the regular switchboard synchroscope, or even with a pair of lamps. Clock time can be readily checked by comparing two synchronous clocks, one connected to the utility system, the other to the Diesel-plant bus. Numerous installations of this kind are giving good service.

(*e*) *Utility Service as Auxiliary to Diesel Power*

This falls into two general classes: First, where the utility service is used at a different time from the Diesel power; second where both power sources are used simultaneously (but not necessarily electrically paralleled).

In the first class will be found many examples in office and loft buildings where such an arrangement permits the Diesel plant to be shut down at night and over week ends, when the load is small. Where this plan is adaptable, the savings in Diesel fuel, attendance, and wear exceed the cost of utility service at the light load. If the load is very small (such as for a few hall lights), and the utility rate for breakdown and auxiliary service is of an unfavorable form, it may be advisable to install a separate wiring system and purchase the current at the utility's regular rate for firm service.

In the second class are plants where the Diesel-engine capacity is sufficient to handle the normal load but utility service is used (either on a separate circuit or in parallel) to help out during the peaks. The use of utility service to carry part of the plant peak load is economical only when the utility rate schedule makes it feasible to use the same service for several other purposes as well, such as supplying the valleys of the load curve when the Diesel plant is shut down, and furnishing stand-by protection. In such cases, less Diesel capacity is needed than otherwise, and more efficient operation is obtained because of the better running-engine capacity factor. The author has elsewhere described a successful installation of this kind (24).

However, on many rate schedules it is impossible to obtain breakdown protection together with auxiliary service economically, and since utility demand charges alone are generally larger than Diesel fixed charges, it is cheaper in such cases to use a Diesel engine to carry the peak load than to buy utility service for this purpose only.

(*f*) *Utility Service to Carry Momentary Load Swings*

In some plants the load curve is uneven due to heavy current inrushes of short duration, say from $^1/_2$ sec to 4 sec, due to motor starting, electric welding, and the like. Such inrushes sometimes create momentary loads that exceed the average load. If the power is supplied by an isolated Diesel plant, it necessitates investment in excess plant capacity and also operation at a small (and less efficient) load factor. Under such conditions, utility service can be used to great advantage if the utility company will permit the Diesel plant to operate in parallel with its own service,

i.e., electrically interconnected. By suitably setting the Diesel-engine governors, the engine power is adjusted to carry the base load, and the utility lines supply the remainder. The adjustment of load is surprisingly simple, because the utility frequency is quite constant regardless of load, while the engine governors have (or can easily be made to have) a drooping speed characteristic so that the speed drops with an increase in load. Consequently, once the engine governors are set for a given load, the engines will generate the same amount of power steadily, regardless of the total load that is being supplied.

As this method of operation puts the uneven part of the load on the utility service, the layout must of course be such as to keep the swings within the limits permitted by the utility's standard rules.

Paralleled utility service was used in this way to stabilize the load on the first Diesel engine installed in 1937 in the Simonds Saw and Steel Company's file plant located at Fitchburg, Mass.

The reason why it is economical for the consumer to carry momentary load swings on utility service is that utility demand charges are based on demand-meter readings showing the integrated consumption for intervals of generally 15 or 30 min, so that the demand charge is not that of the momentary swings themselves but the integrated average of the swings and the steady load. For a simple example, suppose the only load on the utility's meter is an elevator which starts every half minute and causes an inrush of 60 kw for 4 sec. The demand meter will not show 60 kw, but only $^4/_{30}$ of this, or 8 kw. This is not unfair to the utility, though at first glance it may appear so. The utility system serves many consumers and consequently benefits by the diversity in the time of their demands. This diversity is exceedingly large in the case of momentary swings, so that the merged load of a group of consumers having uneven loads is far smoother than their individual loads. Consequently the capacity required in the various components of the utility system, all the way back to the generating station, is but a fraction of the arithmetic sum of the individual consumers' momentary swings. It is therefore entirely equitable that demand charges for this class of service, the same as for others, should be assessed on 15- or 30-min average loads instead of on momentary swings.

Parallel Operation

In several types of joint operation of Diesel engines with utility service the full benefits can be enjoyed by the power user only if the Diesel plant is electrically interconnected with the utility system. Such parallel operation is, in fact, essential in one class of service already described, i.e., where the utility service carries the momentary swings of an uneven load curve.

Parallel operation is advantageous in most situations where both sources of power are used simultaneously, or where there should be no interruption in service when changing from one to the other. This is the case where the utility supplies auxiliary service to a Diesel plant, or where Diesel engines are used to restrict the utility maximum demand. Without parallel operation it is necessary to use double-throw switches, causing a momentary interruption of service as well as voltage and frequency changes whenever the switches are thrown. If the load must be distributed between the two sources, a complicated and expensive system of multiple switching is required, and even then the load must be shifted suddenly in large blocks. Parallel operation eliminates all of this. The switching arrangements become simple, and by merely manipulating the Diesel-engine governor, the load is gradually transferred with no service interruptions and with no change in voltage or frequency.

When a Diesel plant is to be electrically interconnected with a utility system, there are several factors with which the power company is rightfully concerned:

1 *Economic Effect of Feedback*

If the total load should fall below the output of the Diesel plant, it is physically possible for the Diesel plant to feed current back into the utility system. Such feedback would be economically unfair to the utility if no precautions were taken to prevent it from subtracting from the reading of the kilowatthour meter. The effect would of course be to give the consumer an excessive price for his dump power. In cases of power interchange between utility systems and large industrials generating by-product steam power, additional metering equipment is installed to measure the feed-back energy so that it may be credited at a proper value (3, 4, 5, 6, 7). However, in Diesel plants, such arrangements are unnecessary, because the cost of Diesel generation is greater than the value of dump power. All that is required to prevent economic loss to the utility is to fit a ratchet on the meter shaft to prevent reverse registration.

2 *Electrical Effects of Feedback*

Since feedback through a ratcheted meter would be a direct loss to the private-plant owner, it would never be done deliberately. But what would be the electrical effects of feedbacks that might occur accidentally? This might happen, for example, on account of an unnoticed large decrease in power consumption, or because of a large drop in frequency of the utility service. The effects would vary somewhat with the method of distribution used by the utility.

(*a*) The most common method of distribution is by the alternating-current radial system, whereby one or several consumers are supplied by a feeder emanating from a substation. The usual protective equipment is a simple overload circuit-breaker at the substation end of the feeder, and also of course the ordinary fuse or circuit-breaker protection at the entrance to each consumer's premises. (*b*) The alternating-current network which is the more general system for large urban distribution, consists of a network of interconnected street mains fed at numerous points by banks of transformers the primaries of which are supplied by high-tension feeders. The transformer secondaries are connected into the network through network switches which automatically open if the power flow is reversed. (*c*) The direct-current networks which exist in a few cities, such as New York and Chicago, are supplied by feeders from converter substations. These feeders are protected by circuit breakers at both ends, which will open upon a reverse of polarity.

A feedback from a private plant to an alternating-current radial feeder simply reduces the amount of energy fed into that feeder from the utility substation. Other consumers connected to the same feeder continue to receive their normal service, though part of it comes from the private plant. In the case of alternating- or direct-current networks, a feedback similarly supplies part of the requirements of other near-by consumers connected to the same street mains, thus reducing the load on the feeders supplying that portion of the network. If the feedback were of excessive magnitude, there is a theoretical possibility that it would actuate the reverse-power relay at the street end of the feeder and open the circuit breaker. However, this could not occur unless all of the other consumers supplied from the same feeder were already being adequately supplied from the private plant, and therefore there would be no interruption or diminution of their service. The relatively small capacity of the usual Diesel plant compared to that of the local utility system makes it extremely unlikely that a feedback large enough to open a network switch would ever occur, and in fact the author knows of no such instance. However, if it should be deemed necessary, feedbacks

exceeding any given amount could be automatically prevented by installing a reverse-power relay on the consumer's premises which would open the interconnection between the utility system and the Diesel plant as soon as the feedback reached a predetermined magnitude.

3 *Paralleling Derangements*

Because of accident or carelessness, derangements sometimes occur which cause a momentary but heavy current inrush over a paralleling connection. For example, a heavy inrush might occur when synchronizing, if the main switch were closed while the circuits were out of phase. Such an occurrence, however, does not seriously disturb the utility system or the service to other consumers, as an excessive inrush simply opens the overload circuit breaker on the paralleling connection and disconnects the private plant's utility service. In other words, the other consumers in the vicinity would not be affected to any greater extent than they would be, under nonparallel conditions, by a short circuit on some other premises.

The possible objections to parallel operation have been discussed here at some length because some utility companies have been unwilling to permit private plants to operate in parallel with their service. As a matter of fact, inquiries made during the preparation of this paper show that different power companies have quite different policies as to parallel operation, some excluding it altogether while others have definitely provided for it in their rate schedules. Extracts from several power-company letters on this subject are given in the Appendix, to show the wide diversity of policy.

The situation in New York City is interesting. Parallel operation is now prohibited by the power company except for an instantaneous throwover limited to 1 kw. (This exception was made in order to avoid stopping electric clocks in an apartment house when throwing over.) The power company testified, in the Public Service Commission hearings previously cited (19), that in its opinion, parallel operation was not feasible under a network system of distribution. On the other hand, the author has made actual tests of Diesel power plants operating in parallel with both the alternating- and direct-current networks in New York, and in no case did any difficulties occur either in the Diesel plants or on the utility lines.

There is little doubt that paralleling of private steam and Diesel plants with utility service is growing. In fact, some five years ago, H. Drake Harkins stated (7):

> The simplest case of cooperation between utility and industrial in the generation of electrical energy occurs when the industrial secures part of its demand from the utility and operates its own generating equipment in parallel with the utility but does not feed back. So many installations of this kind exist in the United States that it may be called common practice, although a few utilities have earned the ill-will of their customers by refusing to cooperate to this extent. This attitude is untenable, as can be proved by the entire success of the numerous existing installations. The utility in one large eastern city states that it has twelve customers operating in parallel, the largest taking 6500 kw from the power company and the smallest 100 kw.

Diesel engines run as readily in parallel with utility service as do steam engines or turbines, and it is to be hoped that those power companies that have heretofore forbidden parallel operation with Diesel engines will no longer stand in the way of engineering progress.

Conclusion

There are already many instances where Diesel engines are used, or can be used, jointly with public-utility service. This joint use is increasing in several types of application because of the marked benefits it confers upon the power user in the form of improved service and lower costs. The benefits are not one-sided, as the power companies also profit from the increased business at fair rates which flow from such joint use.

Parallel operation, i.e., electrical interconnection between the Diesel plant and the utility lines, is often advantageous for the plant owner and has no harmful effects on the utility system.

The all-too-frequent antagonistic feeling between private-plant owners and public-utility officials is greatly to be deprecated, as it often prevents both parties from gaining the full benefits of joint service which could be achieved by cooperation.

Engineers engaged on power studies should carefully investigate the possibilities of joint utility and Diesel service.

Appendix

Extracts from power companies' letters in response to inquiries as to circumstances under which they permit a customer to operate his own private electric plant in parallel with their service.

1 Consolidated Gas, Electric Light, and Power Company of Baltimore, Baltimore, Md.

> We do not have any rates, practices, or plans for customer generating plants to operate in parallel with our service with the exception of our interconnection agreements with other utilities.

2 Virginia Electric and Power Company, Richmond, Va.

> The Virginia Electric and Power Company has been studying this matter in a small way for the past several years, and at the present time has a few small interconnections. In each case the customer is operating back-pressure generating equipment in order to produce steam at low pressure or hot water for process. However, no specific rates or arrangements have been developed for this type of service.

3 Philadelphia Electric Company, Philadelphia, Pa.

> Where a customer requires low-pressure steam for his actual manufacturing processes which can be most economically supplied by exhaust or bled steam from a prime mover, we permit parallel operation.
>
> You understand, of course, that our proposition with respect to such service must include the protection necessary to prevent the possible troubles inherent in parallel operation from adversely affecting our operations and consequently the service to other customers.

4 Delaware Power and Light Company, Wilmington, Del.

> Of course, it is first presumed that the customer's generating equipment is capable of being placed in synchronism with our line without any adverse effects to other customers connected to the line.
>
> Before arranging for a parallel connection, we would first make a study of the customer's economics involved. Assuming that the economics point to partial generation by the customer and partial purchase, we would be agreeable to a parallel connection on the basis that the customer would own and maintain his substation and purchase not less than 100 kw; the actual capacity of the customer's substation not to exceed the capacity in kilowatts for which he had contracted. In other words, there would be no element of free stand-by for the generating capacity used by him. Of course, if a customer desires stand-by capacity for his own generating plant, this may be purchased under the appropriate rate schedule.
>
> We now have several industrial customers whose generating equipment is, or can be, operated in parallel with our lines.

5 Commonwealth Edison Company, Chicago, Ill.

> In cases of this kind we inspect the plant and if we find it is, in our opinion, being operated with proper supervision and if the class of business will not cause trouble if operated in conjunction with our service we permit them to so operate.

6 The Detroit Edison Company, Detroit, Mich.

> I am pleased to advise that this company does permit parallel operation of customer's generating equipment with its lines under certain conditions.
>
> If the customer has a reliable operating staff and will guarantee a monthly minimum load of 500 kilowatts, parallel operation will be allowed.
>
> We do make certain recommendations that the customer provide

equipment for his protection in case of possible failure of our supply. It is further understood that under no conditions will we be held liable for damage to equipment for loss of production due to this parallel connection.

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Discussion

CHARLES SNYDER.[3] We have operated several Diesel plants in isolated communities to carry loads while the load was building up to where a transmission line might be justified. We have also operated Diesels at the end of rather long transmission lines. In the ordinary case, the installation of a Diesel plant works out successfully for carrying a community until a line can be built, although it is quite costly because, usually, the load and load factor are poor. The cost of hanging engines on the end of transmission lines can hardly be justified, except where the engines are available, in which case, we have found that it helps the situation for a time, until service can be bolstered.

In Mr. Kates' discussion of utility service tied in with private industrial plants, he has taken the position that very largely the rate charged should perhaps be lower than most utility companies think it should be, because of the diversity involved. The writer cannot agree with Mr. Kates on the matter of diversity, because, on most utility systems, diversity is not felt by the utility until it gets back to the generating plant. On the local distribution system, and even in so far as networks are concerned, diversity has little or no effect.

As an example, the New York Public Service Commission, in a case with which, it is believed, Mr. Kates was identified, said in effect the same thing, that diversity was not felt until it practically got back to the generating station, and that the rate which should be charged for breakdown or auxiliary service should be different from the one which is charged for full-time customers.

The Wisconsin Public Service Commission, in the case of the municipal plant at Lodi, said the same thing, that the rate for part-time or temporary service should not be the same as the rate for full-time customers.

It seems to the writer that both of those conclusions have been based on the fact that the regular customers of the utility are paying for service on a different basis, naturally, from what the part-time customer would pay for service. In other words, they are buying service on the basis on which the utility company has made its rate, that is, full-time service. It is a well-recognized fact that the utility company cannot afford to supply stand-by service for the demand portion of its rate. Its rates are based on the use of energy. Regular rates contemplate the real use of service and are designed accordingly.

Such items as accurate frequency control and well-regulated voltage involve real expenditures on the part of the utility. The cost of these items is reflected throughout the entire rate structure. If a customer were to purchase a relatively few kilowatt-hours at regular rates merely to stabilize the operation of an individual plant, he would, in effect, be subsidizing his own plant at the expense of the regular customers of the utility. If this practice became at all common, it would, in my opinion, not be long before the utilities would be called upon by the various state commissions to answer the charge of unjust discrimination.

Now in the matter of the utility company's handling momentary swings of load, Mr. Kates pointed out that it was a matter into which diversity again entered. The same reasoning, at least in the writer's opinion, would apply in such a case, in that diversity is not effective until getting back to the power plant. For example, it would be practically ruinous to our distribution system, which serves a thinly populated area, including many scattered communities, to take such loads as mentioned in the paper.

The point the writer is trying to make is that all of these are particular cases. Each individual case requires sound engineering study from the point of view of the private plant, the Diesel plant, and the utility.

It might have been better if Mr. Kates had cited more really up-to-date cases than he has listed in his bibliography. It is also rather regrettable that the author did not go into more engineering detail, rather than the sales angle. The utility companies and the private plants have need of such information.

OLIVER F. ALLEN.[4] The writer does not remember any previous paper before this Society in which the technical aspects of the interconnection of the isolated unit with the utility system have been so clearly and broadly defined and analyzed. It can be concluded safely, partly from this paper and partly from general knowledge, that there are no technical difficulties inherent in the interconnection of isolated plants and the central-station service. There are, however, economic questions, which some Diesel enthusiasts have not always evaluated; at least some of which might prevent Diesel installations from being made. As has been pointed out, the cost of having central-station service

[3] Monongahela System, Fairmont, W. Va.

[4] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

available, as well as the cost to the private-plant owner of having his own service available all the time but used only part of the time, are features to be considered.

One private plant in the Middle West which was shut down more than three quarters of the operating week, had a load factor which was very low. Studies of this plant were made not very long ago, which indicated that the cost of operating the plant was not properly evaluated. The owner was under a misconception as to his actual operating costs.

On the other hand we all know of cases which are very successful economically, particularly where the isolated plant is of substantial size. There are also many other instances in which there is interchange of power between utilities and private plants, such as department stores, factories, and chemical or other process works, where the interchange of power goes on automatically and profitably for all concerned. In some plants there is no difference in rate. In others there is a rate differential. The meters are arranged so one set registers current coming from the private plant, while another set registers current coming from the central station. Rates are adjusted according to relative production costs.

Here again, the writer would emphasize the importance of an honest and complete study and of real cooperation between the utility, the owners, and the engineer who designs the isolated plant. The owner should have an engineering consultant to study the problem and not depend upon the overanxious salesman. The salesman is frequently a competent engineer, but obviously he is interested in making his sale, whereas a man in the author's position is able to analyze the problem and be more or less of an arbitrator. Such an independent engineer can then tell the owner whether he really should make the investment, because it is not simply a question of initial investment, labor charges, and fuel costs, but one of embarking in another business, requiring the diversion of funds from his own business. Before he enters the field of power generation, he should be sure that such a venture would be economically sound.

H. C. Forbes.[5] This paper seems to suggest that the rates of some utilities for breakdown, or stand-by service, to private plants are inequitable and, in substantiation of this view, the author refers to data submitted in a case before the New York Public Service Commission which purported to show that the diversity factor for breakdown service to a small group of customers was at least 10 to 1. From this diversity data, he infers that the cost of supplying such customers with breakdown service must be relatively low so that they ought to be entitled to special consideration in the matter of rates. Having some familiarity with the data to which he refers, the writer would like to point out that, inasmuch as the service to the group of customers in question, during the period covered by the data, was fundamentally auxiliary service rather than breakdown service, there is good reason to doubt whether it is at all indicative of the diversity which might be expected from a group of customers employing utility service as a breakdown supply in the usual sense of the term.

Be that as it may, there is no reason to conclude that, simply because under ordinary conditions the diversity among a group of breakdown customers is large, the cost of supplying them with breakdown service would necessarily be small. It must be realized that such a group of customers will not be located in a restricted area on the distribution system such as to make it possible to take advantage of their diversity to reduce the distribution-system investment. On the contrary, they may be expected to be widely scattered. It is generally necessary to install for each customer just the same distribution facilities for the customer's occasional use as would be required to deliver firm service of the same magnitude as that required for the breakdown supply. In general, therefore, it is not to be expected that the diversity will be a factor of any considerable importance in so far as distribution investment is concerned.

The author also ignores the fact that a common emergency may well operate substantially to eliminate the diversity among the breakdown customers. An interruption in the supply of Diesel fuel, due to strikes, or a general strike of private-plant operating labor, would undoubtedly have the effect of throwing the entire group of breakdown customers simultaneously upon the utility service at maximum capacity. Indeed, the recent coal strike showed conclusively enough, in the case of coal-burning private plants, that this is precisely the result to be anticipated, and the utilities must have adequate generating capacity to cope with such an emergency. Whatever the diversity may be under normal conditions among a group of breakdown customers, there is good reason to doubt, therefore, that the utilities are able in consequence to effect any substantial investment economies in supplying them with breakdown service.

From a casual reading of the paper, it might be thought that the auxiliary service, which is frequently associated with utility breakdown service to private plants, would constitute a desirable load for the utility. For obvious reasons, it often turns out to be quite the contrary. Usually the private-plant operator, in the interest of cutting down the amount of equipment required for his plant, will plan to use the utility service to carry his peak. As a general thing the private-plant peaks are likely to coincide pretty much with the utility-system peak so that the result is to impose an additional load of very poor load factor upon the utility system, causing its peak to occur at the worst possible time from the standpoint of the utility.

The author also labors to show that the parallel operation of private plants with utility systems is not undesirable. Under conditions which obtain on radial systems and, in the case of high-tension paralleling arrangements, there would seem to be no objection to such parallel operation. The operation of private plants in parallel through the low-tension networks of the utility systems, however, involves certain objectionable conditions which the author mentions in his paper without glossing them over very successfully. It is difficult, indeed, to see any reason why the utility companies should take upon themselves the duty of guarding against the possibility of harmful results which might arise through indiscriminate parallel operation, especially when their firm-service customers would be the ones to suffer in the event of a disturbance due to improper operation by the private plants.

It is to be noted finally that the breakdown rates of the utilities, of which Mr. Kates is so critical, have generally been subjected to an impartial review by the Public Service Commission having jurisdiction, before they were permitted to be put into effect, and it seems a little presumptuous to challenge them as inequitable without supplying more convincing supporting evidence than his paper contains.

Author's Closure

The author is heartily in agreement with Mr. Snyder's statement that the rates charged for utility service supplied to private-plant operators should be based on sound engineering study of this class of service. It is believed that in the majority of cases, such rates have been arrived at rather arbitrarily, and have been based more upon commercial and competitive considerations than upon the actual cost of the service.

Mr. Snyder is correct about the variations in diversity factor which may occur in different places. Certainly, on a long feeder line, serving only one or two customers in some isolated section,

[5] Consolidated Edison Company of New York, New York, N. Y.

there will be a much smaller diversity effect upon the distribution system and transmission lines than in the network system of a large city, where, within an area of a few blocks, a number of private plants are all served from the same network.

Again the author agrees with Mr. Snyder that the rate for breakdown and auxiliary service to private plants should be different from that for general consumers. With this in mind, a two-part rate for breakdown and auxiliary service was proposed. One part would be the same as the rate for general service, based on the actual demand and energy as shown by the meter. The other part of the rate would be an additional charge to the breakdown user consisting of a stand-by charge which he would pay in addition to the charge for the actual use of current. This rate for stand-by protection should be based upon actual cost, taking into account the diversity and other characteristics of that particular system.

Such a two-part rate was in fact proposed to the New York Public Service Commission in the case cited in the paper. It is gratifying to note that the Commissioner who heard the case concurred in principle, and in his printed opinion he proposed a charge for stand-by protection of two thirds the demand charge for actual use.

Referring to utility rates for swing loads, time service, low-load-factor service, etc., Mr. Snyder seems to advocate a higher charge to the private-plant operator than to the general consumer for identical service. This would be discriminatory indeed. The rate schedules for general consumers are supposed to impose fair charges, based upon actual cost, on those general consumers who happen to have swing loads, those who want time service only or those whose loads have a low load factor. If the prevailing rates to general consumers for service of such characteristics give the utility a proper return on cost, there is, in the author's opinion, no valid reason why the same rates should not apply to private-plant operators who want utility service of the same characteristics.

Mr. Forbes believes that the diversity factors of breakdown service, cited in the paper, applied to auxiliary service rather than to breakdown service. He is mistaken; the testimony upon which these figures were based are the answers received from 140 breakdown- and auxiliary-service customers to questionnaires[6] issued by the engineering staff of the New York Public Service Commission, calling for detailed data on each occasion where utility breakdown service was used by compulsion because of a plant breakdown. No auxiliary service was included.

Mr. Forbes believes the author is "presumptuous" in criticizing breakdown rates which, Mr. Forbes states, have generally been impartially reviewed by the Public Service Commissions having jurisdiction, before being put into effect. Yet in the same New York case to which Mr. Forbes refers, the Commission permitted the company's proposed rate to become operative because the Commission had no power to compel a further suspension, and it went on record[7] as follows: "The Commission has not found that the demand charge or the energy rate were just and reasonable rates. The Commission has left these matters open for later determination when it has sufficient evidence to determine what is a fair demand charge and what is a fair energy charge." Here is an instance of an unapproved rate that has been in effect for 3 years.

Mr. Forbes' discussion is itself evidence of that antagonistic feeling between public-utility officials and private-plant owners which was referred to in the paper. Surely the engineering problem of coordinating two different sources of power can better be solved by objective analysis and factual engineering studies.

[6] N. Y. State Public Service Commission, Albany, N. Y. Case 8622, Exhibits 35 and 37.

[7] Case 8622, Decision of October 29, 1936, p. 4.

An Improved System in the Application of Noncondensing or Extraction Turbines

By H. W. CROSS[1] AND E. S. WELLS, JR.,[2] CHICAGO, ILL.

The purpose of this paper is to demonstrate methods by which the electrical output of noncondensing or extraction turbines may be increased. The electrical output is definitely established by the energy drop between initial and exhaust conditions and by the quantity of steam flow. By increasing the total steam temperature at the throttle for a given pressure, the available energy is increased and consequently the electrical output. The application of a combined desuperheater and feedwater heater at the exhaust is shown to be advantageous in realizing gains from the use of higher-temperature steam. Examples demonstrating the possibilities inherent in such systems are illustrated and explained.

IT HAS BEEN the practice in the application of noncondensing turbines and extraction turbines to choose an initial steam condition, i.e., a total temperature at the selected inlet pressure, that will give the desired temperature at the exhaust or extraction opening of the turbine to suit the low-pressure system which is being served. As most applications are made in connection with old low-pressure-turbine equipment, the design of which limits the temperature at which they may be operated, or process systems where little or no superheat is desired, the total temperature at the throttle for the new turbine is thus definitely limited.

A considerable gain in the electrical output from the high-pressure, noncondensing portion of this cycle may be realized by increasing the total steam temperature at the throttle. The electrical output is definitely established by the energy drop between initial and exhaust conditions and by the quantity of steam flow. By increasing the total steam temperature at the throttle for a given steam pressure, the available energy between initial and exhaust conditions is increased and the electrical output is proportionately increased. However, the increase in initial temperature causes an increase in the temperature at the exhaust and results in an exhaust temperature higher than desired. This gain in electrical output may be obtained if a desuperheater is installed on the exhaust system to reduce the temperature of the exhaust steam to suit the low-pressure system.

In systems of this nature, it is common practice to install a feedwater heater served by the exhaust steam from the high-pressure turbine, in order to obtain a greater electrical output by utilizing more heat from the exhaust steam, thereby increasing the flow of steam through the high-pressure unit.

As an improved system for realizing this gain by the use of higher steam temperatures, the application of a combined desuperheater and feedwater heater at the exhaust is suggested. This would consist of two sections. One section would be a surface type of heat exchanger, designed on a counterflow basis and arranged for free flow of exhaust steam through it. The other section would be designed as a condensation-type feedwater heater.

The feedwater being returned to the boiler would be passed first through the heater section. If this section were of the surface type, it would be proportioned with sufficient surface to raise the feedwater temperature to within a reasonable terminal difference of the saturation temperature of the exhaust steam. This heater might be of the open type. However, the position in the system and the pumping arrangement would seem to dictate the choice of the closed-type heater. The feedwater would then flow through the tubes of the other section of the heater, where it would absorb the heat represented by the superheat in the exhaust steam. In reducing the temperature of the superheated exhaust steam, the temperature of the feedwater would be elevated above the saturation temperature of the heater. Steam of the desired temperature would be delivered to the low-pressure system, and the feedwater would be delivered to the boiler at a higher temperature than would be obtained from an ordinary saturation-type feedwater heater alone.

By transferring the high-level heat represented by the superheat in the exhaust steam to the feedwater supplied to the boiler at a temperature above the saturation temperature of the heater, the use of exhaust heat in the cycle is increased, with a consequent increase in heat flow through the turbine and a greater electrical output in conjunction with serving the necessary heat demand of the low-pressure system.

The design of the thoroughfare-desuperheater section would be determined by the temperature desired for the steam supply for the low-pressure system, which would establish the terminal difference between the feedwater entering this section and the steam leaving it. It would be desirable to design the desuperheating section to maintain a somewhat lower temperature than that required by the system, in order to provide a range for automatic control of the temperature.

Fortunately, where saturated steam is required for process work, approximate saturation is generally acceptable and a small amount of superheat does not appear to be objectionable. This is evidenced by the fact that where superheat is not controlled, the increase in temperature of the exhaust steam, occasioned by the lower efficiency of the prime mover under partial loads, is accepted. This margin in the system may be utilized to good advantage in decreasing the size and cost of the thoroughfare desuperheater by providing a higher terminal difference in the heater.

With certain choices of initial temperature and pressure, it is possible that, in absorbing the heat of the superheat in the exhaust, the feedwater to the boiler might attain too high a temperature to be conveniently handled. The temperature of the feedwater from the desuperheater can be regulated and held at the maximum allowable temperature by a by-pass around the feedwater-heater section, with a temperature control. This control would limit the temperature rise at the heater by by-passing a portion of the feedwater around it.

The thoroughfare desuperheater and the feedwater heater might be built as two separate sections working together as a

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

unit, or the two sections could be combined in a single shell, depending upon physical limitations or design characteristics. In either case, it would consist of a thoroughfare-desuperheating section and a feedwater-heating section. Between the heater section and the desuperheater section, provision should be made for detraining steam condensation from the thoroughfare section, to prevent its being carried into the low-pressure system. Under certain conditions of control or certain load conditions, it is possible that a portion of the surface of the desuperheater section might act as a condensation heater, resulting in the formation of some condensate.

An appreciation of the relative outputs of these systems may readily be gained from the accompanying illustrations. The steam pressures and load conditions were arbitrarily selected as being in the realm of normal usage. The turbine efficiencies used are based on normal commercial values which vary with the steam conditions, the size of unit, and the loading of the unit.

Fig. 1 shows a conventional application of a noncondensing turbine supplying 50,000 lb per hr of saturated steam at 150 lb gage pressure to process or to other equipment. It is assumed that this 50,000 lb per hr is returned as feedwater at a temperature of 210 F from other feedwater heaters. A high-pressure feedwater heater is indicated, proportioned to heat the feedwater to a temperature of 356 F when supplied with 150 lb gage steam from the exhaust of the high-pressure turbine. With a pressure of 650 lb gage at the throttle and a turbogenerator efficiency of 64.6 per cent (turbine efficiency alone of 68.5 per cent), it is necessary to limit the total temperature at the throttle to 596 F in order to deliver saturated steam at the exhaust. This requires a steam flow of 58,646 lb per hr and gives an electrical output of 1390 kw. The heat supplied by the boiler for this combined steam-and-electrical load is 55,887,000 Btu per hr.

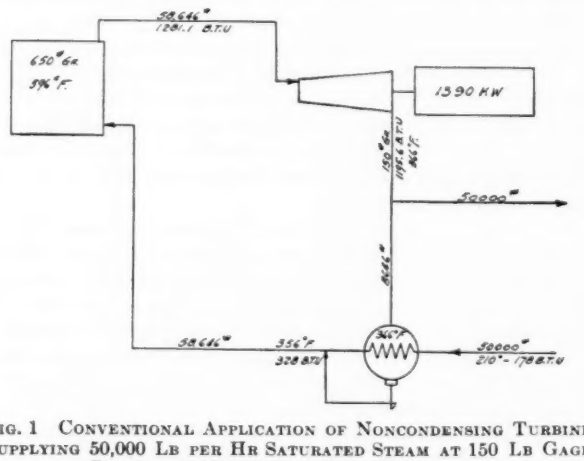


Fig. 1 Conventional Application of Noncondensing Turbine Supplying 50,000 Lb per Hr Saturated Steam at 150 Lb Gage Pressure to Process or Other Equipment

Fig. 2 shows an application to supply the same 50,000 lb of saturated steam per hr at 150 lb gage pressure, in which 650 lb gage at the throttle is used with a total temperature of 850 F. With a turbine efficiency alone of 70.5 per cent, the exhaust steam from the turbine at 150 lb gage would have a total temperature of 582 F. With a conventional type of desuperheater as shown, when supplied with feedwater at a temperature of 210 F, a flow of 44,730 lb per hr of exhaust steam would be sufficient to furnish 50,000 lb per hr of saturated steam to the low-pressure system. This requirement, together with the necessary 150 lb steam to serve the conventional high-pressure heater to heat the feedwater to a temperature of 356 F, requires a flow at the throttle of 51,527 lb per hr, which produces an electrical output of 1680 kw. This increase of 290 kw is an increase of 20.9 per cent in the power output incident to the furnishing of the same quantity of low-pressure steam, as in Fig. 1, and is obtained from this application of the increase in initial steam temperature from 596 to 850 F. The heat supplied by the boiler for this combined steam-and-electrical load is 56,930,000 Btu per hr.

Fig. 3 illustrates the application of the thoroughfare desuperheater and feedwater heater to supply the same 50,000 lb of saturated steam per hr at 150 lb gage pressure, with initial steam conditions as in Fig. 2, namely, 650 lb 850 F at the throttle. To allow for a 5-lb pressure drop through the desuperheater, the back pressure at the turbine would be increased to 155 lb gage. With a turbine efficiency alone of 72.8 per cent, the exhaust steam at 155 lb gage pressure will have a temperature of 581 F. The 50,000 lb of steam per hr, required by the process in passing through the thoroughfare-desuperheater section, is desuperheated by the feedwater and is supplied to the system at saturated temperature. Exhaust steam is absorbed in the feedwater-heater section in sufficient quantity to elevate the feed temperature from 210 to 356 F. The feedwater, in passing through the desuperheating section in contact with the thoroughfare flow of steam in counterflow direction, would be heated to a temperature of 465 F for return to the boiler, in absorbing the heat of superheat of the exhaust steam. The total flow required from the turbine is 58,646 lb per hr, which gives an electrical output of 1921 kw, a gain of 531 kw over Fig. 1, and a gain of 241 kw over Fig. 2, or 38.2 per cent and 14.3 per cent improvement, respectively. The

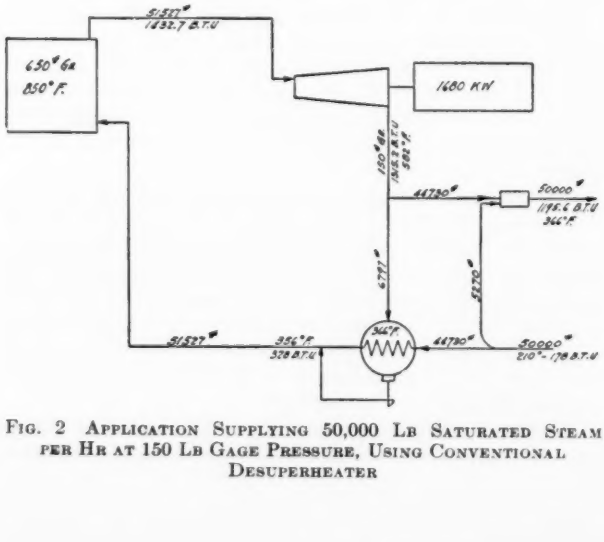


Fig. 2 Application Supplying 50,000 Lb Saturated Steam per Hr at 150 Lb Gage Pressure, Using Conventional Desuperheater

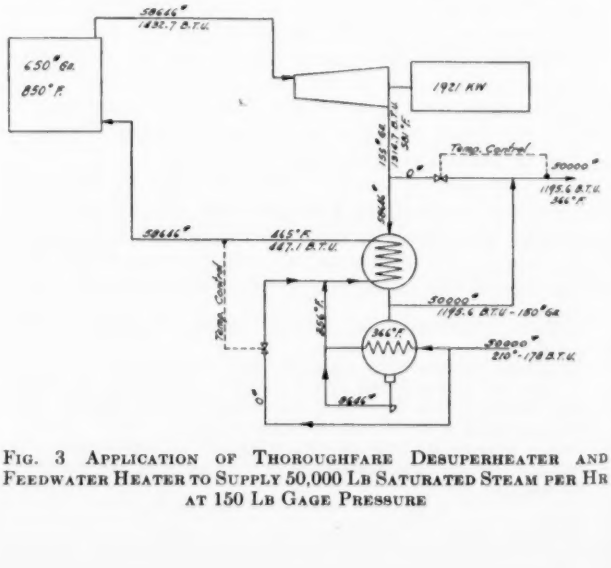


Fig. 3 Application of Thoroughfare Desuperheater and Feedwater Heater to Supply 50,000 Lb Saturated Steam per Hr at 150 Lb Gage Pressure

heat supplied by the boiler for this combined steam-and-electrical load is 57,800,000 Btu per hr.

Figs. 1, 2, and 3 are presented to demonstrate the increase in power available in connection with supplying low-pressure steam and therefore are measured in terms of electrical output. The increase in boiler capacity to accomplish this gain is small, since it is but necessary to supply the heat represented by the added kilowatt output at the rate of approximately 3600 Btu per kwhr. The relative amounts of heat supplied by the boiler, therefore, the relative boiler capacities required, compare as follows: Fig. 1, 100 per cent; Fig. 2, 101.8 per cent; Fig. 3, 103.4 per cent.

For installations where some superheat is desirable in the steam being furnished to the low-pressure system, the thoroughfare desuperheater might be designed with a greater terminal difference and equipped with a temperature control which would be arranged to control the proportion of exhaust steam routed through the desuperheater. Sufficient exhaust steam would be by-passed around the desuperheater so that, when mixed with the balance of the exhaust steam, the resultant mixture would have the desired temperature. The by-pass temperature control would be simple. It would consist of a damper or regulating type of valve which would be automatically positioned to proportion the flow through the desuperheater and around the desuperheater in a manner to give the necessary mixture to maintain the required temperature to the low-pressure system.

Similar gains and an improvement in thermal efficiency of the plant may be realized in the application of this scheme to condensing-type extraction turbines. The same proportional increase in output may be expected from the steam extracted for use in the process or the low-pressure system, which reduces the flow of steam to the condenser for a given output. In addition, the efficiency of the use of the condensed steam is improved, resulting in a gain from that portion of the flow commensurate with the gain on any condensing cycle by the increased inlet temperature.

Fig. 4 shows an application for an assumed requirement of 5000 kw electrical energy and a low-pressure demand for 50,000 lb of steam per hr at 50 lb gage, 25 deg superheat (323 F total temperature).

With 450 lb steam at the throttle, a total temperature of 625 F is required to give 25 deg superheat at the extraction opening at this load. With two stages of feedwater heating, the high-pressure heater served from the 50-lb extraction point to give a final feed temperature at the boiler of 288.7 F, and with 2 in. absolute back pressure at the condenser, a flow of 92,970 lb of steam per hr is required. The heat supplied by the boiler would be 98,455,000 Btu per hr.

Fig. 5 shows an application with load conditions similar to those shown in Fig. 4, except that a total temperature of 825 F is maintained at the throttle and a thoroughfare desuperheater is used to reduce the temperature of the 55-lb-gage extracted steam to the desired 25 deg superheat. In this case the steam as extracted from the turbine would have a total temperature of 484 F (181 deg superheat). In reducing the temperature of this extracted steam in the thoroughfare desuperheater to the desired 25-deg superheat the feedwater would be heated to 336 F. The total flow of steam required from the boiler would be 83,670 lb of steam per hr. The heat supplied by the boiler would be 93,710,000 Btu per hr.

As the steam-process load and the electrical load are held constant in the examples demonstrated in Figs. 4 and 5, the improvement in operation would be measured by decrease in fuel consumption. The saving of 4,745,000 Btu per hr in heat supplied by the boilers represents a reduction of 4.7 per cent in the fuel cost for supplying the steam-and-electrical load.

On noncondensing extraction turbines, the application of the thoroughfare desuperheater would be made at the extraction

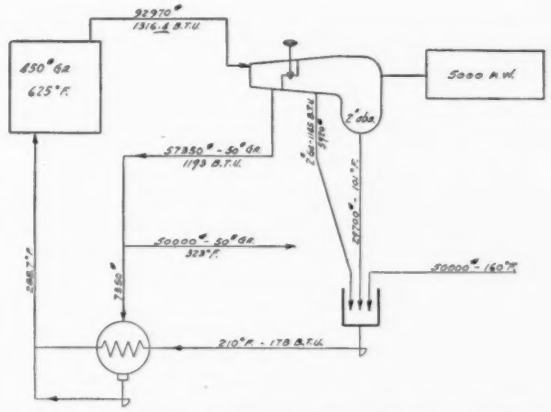


FIG. 4 APPLICATION OF CONDENSING-TYPE EXTRACTION TURBINE TO MEET REQUIREMENT OF 5000 KW ELECTRICAL OUTPUT AND LOW-PRESSURE DEMAND FOR 50,000 LB STEAM PER HR AT 50 LB GAGE PRESSURE, 25 DEG SUPERHEAT

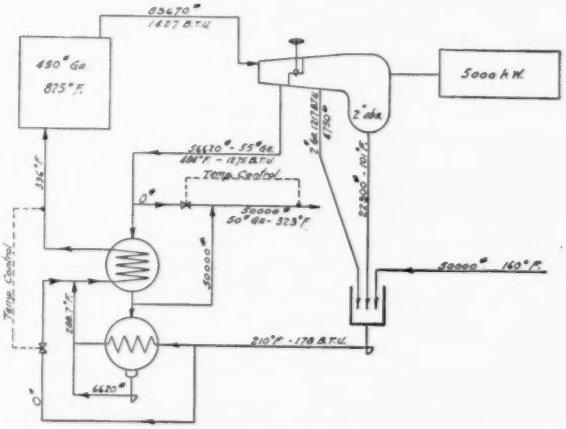


FIG. 5 SIMILAR LOAD CONDITIONS TO FIG. 4, EXCEPT THAT THOROUGHFARE DESUPERHEATER IS USED

opening or at the exhaust, depending upon which low-pressure system served to limit the inlet-steam temperature. The results from the application would be in all respects similar to the noncondensing-turbine application.

No attempt has been made to determine the economic factors in the examples given. In evaluating the applications, consideration must be given to the costs due to the modified boiler flows, the added superheat, the increased turbine capacity required, and the cost of the desuperheating equipment. As the resultant increased output from the noncondensing portion of the cycle is at a rate of approximately 3600 Btu per kwhr, the allowable cost for the suggested application could be the same, per kilowatt of increased capacity, as is found to be justified for the so-called "topping" installations. The surrounding conditions are so diverse in applications of this nature that each case must be considered on its own merits. It is recognized that a thoroughfare desuperheater designed to a close terminal difference would be large. As previously pointed out, however, the operating margin usually will permit the use of a design that will serve to decrease the size.

The thoroughfare-desuperheater and feedwater-heater combination introduces no complex apparatus and includes the customary feedwater heater. It may be equipped with all the necessary controls to hold constant temperatures of steam and feedwater from it when supplied with varying temperatures of both steam and water. The scheme shows substantial increases in

electrical power production in supplying a lower-pressure steam demand over conventional adaptations of these cycles, and merits consideration in situations where the added output is of advantage.

Discussion

W. F. RYAN[3] AND E. W. NORRIS.[4] The system described by the authors has been applied by the writers' company for several years and there is, therefore, a considerable background of actual experience supporting the mathematical demonstrations in this paper. While we have used the system in the majority of our central-station designs since 1933, the opportunity to use it in a back-pressure industrial plant did not arise until 1937. The advantages are much greater in the latter field, but its use has been restricted by a not unjustified reluctance on the part of industrial-power-plant operators to use very high steam temperatures.

The heaters required for this desuperheating cycle are usually small and inexpensive and no major difficulties have been experienced in operation or maintenance. The operating temperatures are frequently too high to permit the use of copper or copper-alloy tubes. Stainless steel is entirely serviceable but its low heat-transfer rate results in a bulky as well as expensive heater. For some high-temperature applications, chrome-vanadium steel, S.A.E. 6120, has been used. If properly heat-treated, tubes of this material are satisfactory under normal service conditions. Where it is possible to do so, the cycle should be designed to give temperature conditions which will permit the use of copper-alloy tubes in the desuperheating heater.

The authors have demonstrated the efficiency of this system for the back pressure and topping installations, but it also has marked advantages in the more limited application to the straight regenerative condensing central-station cycle. The latest installation of this type is in the South Meadow Station of the Hartford Electric Light Company, in connection with a 40,000-kw condensing-turbine generator, operating at 825 lb gage throttle pressure and 900 F total steam temperature.[5] At full load, steam is extracted from the eighth stage at 165 lb gage and 580 F (207 F superheat). This extracted steam, together with the high-pressure gland leakoff which is at a much higher temperature, is passed through a closed heater with a total heating surface of 353 sq ft and is condensed in an open feedwater heater and in the coils of the station evaporators. The feedwater is raised to saturation temperature (373 F) in the open heater and is pumped through the desuperheating heater en route to the boiler. The increase in temperature in the desuperheating heater is approximately 10 F and, due to decreasing efficiency of the high-pressure stages of the turbine at partial loads, this temperature increase is practically constant at all loads from 6000 kw to 45,000 kw, in spite of the fact that the throttle temperature drops at low boiler outputs. The use of the desuperheating cycle increases the total extraction by about 4000 lb per hr at full load, resulting in a yield of more than 150 additional kw generated as a "by-product" of the steam used for feedwater heating and make-up evaporation.

The desuperheating cycle becomes more effective as the portion of steam passing through the desuperheating heater increases in comparison to the total amount of steam in the cycle. For this reason it is particularly effective when applied to noncondensing high-back-pressure turbines or to extraction-type turbines where a large portion of the total steam is extracted. Obviously, the effectiveness of the cycle varies with the amount of superheat in the steam extracted or exhausted from the turbine. Where turbines operate at very high initial pressures and exhaust or extract at relatively low pressures, very little superheat remains, even with high initial temperatures, and the gain in power output and economy through the use of a desuperheating heater is relatively small. Conversely, in those installations, where steam is removed from the cycle at relatively high pressures and, especially, where it is impracticable to use initial pressures that are high enough to give a large amount of by-product power, considerable superheat may remain in the steam extracted, even with moderate initial temperatures. Under these conditions, the desuperheating cycle becomes very effective and enables a desirable increase to be made in the amount of by-product power generated under these unfavorable circumstances.

In many installations where noncondensing turbines are used to top existing power plants, the use of a condensing-type heater, drawing steam from the topping-turbine exhaust is considered questionable as, in effect, this heater operates on steam drawn from the low-pressure-boiler plant and transfers load from the high-pressure to the low-pressure boilers. The introduction of the desuperheating heater, however, is not subject to this criticism as the steam passing through it is exclusively from the high-pressure cycle and contributes materially to its efficiency.

The effects of the desuperheating cycle are not confined to the turbine-and-heater portion of the power plant, but also bring about some interesting changes in the design and operation of the boiler. As compared to the straight extraction plant with low-pressure heaters and moderate feed temperatures, the plant, equipped with a condensing heater connected to the extraction or crossover pressure with a desuperheating heater, operates at extremely high feedwater temperatures. While it requires the generation of a considerable amount of additional steam, due to the high feed temperature and the nature of the cycle, much of the heat in the steam is recirculated and the increased output of the boiler as measured in Btu per hr is small, as compared to the increase measured in lb per hr. For this reason the physical dimensions of the boiler remain practically constant, in spite of the fact that the high-pressure plant has materially increased power-generating capacity. A slightly increased furnace is required in order to burn the fuel needed for the increased by-product power generated. The convection water-heating surface, however, remains approximately the same, and only the superheater surface needs to be materially increased. The steam releasing space must be increased to correspond with the increased evaporation. This requires a larger drum, but usually has no material effect upon the space requirements of the boiler and relatively moderate effect on its cost.

Other changes required for the desuperheating type of plant are increased pipe sizes and increased capacity of boiler feed pumps. Fan and pulverizer sizes should remain practically the same as with the simpler cycle.

AUTHORS' CLOSURE

The discussion by Messrs. Ryan and Norris brings out several interesting features of the thoroughfare desuperheater scheme. It shows that similar arrangements have had some cases of application which had not come to the attention of the authors and apparently the cost of the equipment is not more than for ordinary conditions. The discussion is also in agreement with the claims and calculations of our paper.

Messrs. Ryan and Norris do not discuss the proposed features of automatic control. For ordinary power-station conditions, the temperature and pressure can be allowed to increase and decrease with load conditions, but for steam for process work they must be maintained fairly constant. This can be easily accomplished by appropriate control as mentioned in the paper.

[3,4] Stone & Webster Engineering Corporation, Boston, Mass. Members A.S.M.E.

[5] "South Meadow's New 40,000-Kw Extension," by William F. Ryan, *Power Plant Engineering*, February, 1939, pp. 96–102.

Performance Characteristics of a Mechanically Induced Draft, Counterflow, Packed Cooling Tower

By A. L. LONDON,[1] W. E. MASON,[2] AND L. M. K. BOELTER[3]

Predicted and test results of a packed cooling tower are presented in this paper. The geometrical form of the air-water interface may be described as a sheet conforming to the streamlined shape of the tower packing, in contrast with jets and drops encountered in other types of towers. Tower performance is expressed in terms of its effectiveness as an energy exchanger, as compared to the performance of an infinitely high tower. The effectiveness was found to vary from approximately 0.3 at low water rates and high air rates to 0.8 at high water rates and low air rates. Experimentally determined unit conductances for diffusion vary with the 0.48 power of the gas rate, for Reynolds numbers of from 7000 to 24,000, which is in substantial agreement with published thermal-convection data for similar shapes. The height of the transfer unit (HTU) was found to vary from 3.3 to 6.3 ft, depending predominantly upon the fraction of wetted packing area and the air rate. A simple method, analogous to that employed for counterflow heat-exchanger calculations, is presented for predicting tower performance analytically. This method is based upon the integration of the energy and material-balance equations together with the rate equations, subjected to the simplification that the unit over-all conductance for thermal convection may be approximated by the product of the air-water-vapor-mixture unit-heat capacity and the unit over-all conductance for mass transfer.

THERE is a dearth of basic heat-transfer and diffusion data for cooling towers. As a consequence, rational design is in many cases impossible. The objectives of this paper are:

1 To present experimentally determined heat- and mass-transfer data together with tower performance as an energy exchanger for a particular system.

2 To show that the basic data may readily be used to predict tower performance analytically by integration of the fundamental energy and material-balance equations, together with the material and thermal-convection transfer-rate equations.

Description of the Apparatus

The tests were conducted on a mechanically induced draft, counterflow, packed cooling tower (1).[4] Figs. 1, 2, and 3, indicate the general experimental arrangement. In Fig. 4, an idealized sketch of the tower reveals the type and distribution of the packing. The equipment and instruments were arranged so that over-all material and energy balances could be readily accomplished.

Water circulation during a run was maintained in a closed system; the water from the tower basin was pumped through a shell-and-tube-type heat exchanger back to the tower distributing main. A calibrated orifice in the line to the tower was used to meter the water rate. The rate of change of basin water level with respect to time, as indicated by point-gage readings, served to determine the tower evaporation rate.

Water distribution within the tower was provided by a 3-in-inside-diam redwood-pipe distributing main, running perpendicular to the packing length. Diametrically opposed holes on the horizontal plane, as indicated in Fig. 4, directed the water jets to the grooved top-flat of the distributing deck, which served further to spread the water over the tower cross section. Only the distributing deck was provided with grooved top-flats.

A calibrated nozzle located downstream from the fan discharge yielded the necessary air-rate information. The nozzle location is indicated in Fig. 1, and Fig. 3 reveals the nozzle details. Air-flow control was obtained for runs 1 to 6 and 28 to 60, Table 1, by varying the voltage to the compound-wound direct-current fan motor, while for the other runs an a-c motor drive with damper control was used. The power supplied to the fan motor was measured by means of a wattmeter.

The cooling-tower water was heated by a shell-and-tube-type exchanger with steam as the heating medium in the shell. The rate of heat exchange was controlled by throttling the steam to a constant indicated gage pressure on the high side of a small orifice in the inlet line.

Calibrated thermocouple and thermopile arrangements were used to measure temperatures. These were:

1 Water temperature into the tower; by means of 2 four-couple thermopiles with distributed junctions.

2 Water temperatures in the tower basin; by means of 2 four-couple thermopiles with distributed junctions.

3 Temperatures of the air into the tower, wet and dry bulb; by means of single junctions located at both entrances.

4 Temperatures of the air out of the tower, after passing through the fan, wet and dry bulb; by means of single junctions located at the air-nozzle outlet.

Nomenclature

a = packing area per unit packed-tower volume, sq ft per cu ft

c = unit heat capacity of liquid water, Btu per deg F per lb

$d(\ )$ = notation indicating a differential quantity

e = base of natural logarithms

f_o = over-all unit thermal conductance for convection, Btu per sq ft per deg F per hr

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[3] Professor of Mechanical Engineering, University of California, Berkeley, Calif. Mem. A.S.M.E.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] Numbers in parentheses refer to the Bibliography.

f_c' = unit film conductance for convection, air side of interface, Btu per sq ft per deg F per hr

f_{cL}' = unit film conductance for convection, water side of interface, Btu per sq ft per deg F per hr

f_D = over-all unit conductance for diffusion, lb per hr per sq ft per (lb/lb)

f_D' = unit film conductance for diffusion, lb per hr per sq ft per (lb/lb) air side of interface

$F(\ )$ = notation indicating a functional relation

g = acceleration due to gravity, ft per hr per hr

h_{fg} = latent-heat vaporization of water at 32 F, Btu per lb water

h = air-vapor-mixture enthalpy, Btu per lb dry air (32 F datum), $h = Hh_{fg} + s(t - 32)$

h_i = air-vapor-mixture enthalpy saturated at temperature T, Btu per lb dry air (32 F datum), $h_i = Hih_{fg} + s(t - 32)$

Δh = mean-effective-enthalpy potential difference for energy transfer between phases Btu per lb dry air

G = dry-air flow rate, lb dry air per hr

H = air humidity, lb of water vapor per lb of bone-dry air

H_i = interface humidity, assumed saturated at temperature T, lb water vapor per lb of bone-dry air

ΔH = mean-effective-humidity potential difference for diffusion, lb per lb

K = dimensionless coefficient

l = height of tower packing, ft

L = liquid water flow rate through the tower, lb per hr

n = exponent

R_{DL} = difference of over-all unit resistance and gas-film unit resistance for diffusion (hr ft² lb/lb) per lb

s = unit heat capacity of the air-water-vapor mixture at constant pressure, Btu per deg F per lb; $s = 0.24 + 0.45\ H$

S = average cross-section area of air flow, sq ft

t = air-main-body temperature, deg F

t_{WB} = air-main-body wet-bulb temperature, deg F

Δt = mean-effective-temperature potential difference for convection, deg F

T = water-main body temperature, deg F

u = wetted perimeter of packing cross section in the direction of the air flow, including trailing edge, ft

V = tower volume occupied by packing, cu ft

Δ = (delta) indicates an average potential difference, generally the logarithmic mean of the terminal potentials

μ = (mu) absolute viscosity of air mixture, lb per hr per sq ft

$\cong$ = denotes sensible equivalence

Dimensionless Variables

ϵ_h = (epsilon) tower effectiveness as an energy exchanger = $\dfrac{G(h_1 - h_2)}{G(h_{i1} - h_2)}$

ϵ_c = (epsilon) tower performance as a liquid cooler = $\dfrac{(T_1 - T_2)}{(T_1 - t_{WB2})}$

η = (eta) fraction of packing-surface area, wetted by the flowing water

Re = Reynolds' number for the packing shape, $\dfrac{Gu}{Sg\mu}$

$\dfrac{f_D aV}{G}$ = number of transfer units

Subscripts

1 denotes tower conditions at water entrance and air exit
2 denotes tower conditions at water exit and air entrance
c denotes cooler and thermal convection
D denotes diffusion
h denotes energy (enthalpy) exchange
i denotes interface conditions
L denotes liquid side

Summary of Experimental Results

The range of variables included in the test agenda were:

1 Water rates L, from 3600 to 12,600 lb per hr, which correspond to rates of 8.6 to 30 lb per hr per sq ft of packing surface.



Fig. 1

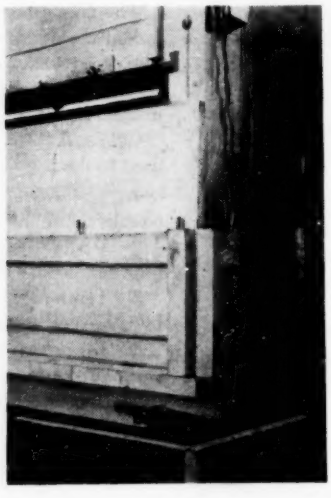


Fig. 2



Fig. 3

Fig. 1 View of Cooling Tower, Showing General Arrangement and Location of Air-Metering Nozzle

Fig. 2 View of Tower Basin, Showing Location of Well for Point-Gage Measurements of Basin Water Level, and Position of Psychrometer, Measuring State of Air Entering Tower

Fig. 3 Details of 8-In. Air-Metering Nozzle, Showing Pitot Tube Used for Calibration, and Arrangement of Psychrometer for Determining State of Air Leaving Tower

TABLE 1 EXPERIMENTAL DATA AND RESULTS

Run no.	1 Flow rates lb per hr Water L	2 Flow rates lb per hr Air G	3 Mass velocities, lb per hr per sq ft L/aV	3a Mass velocities, lb per hr per sq ft G/S	4 Temperatures in deg F Water In T_1	5 Temperatures in deg F Water Out T_2	6 Temperatures in deg F Air In t_2	7 Temperatures in deg F Air In tWB_2	8 Temperatures in deg F Air Out t_1	9 Temperatures in deg F Air Out tWB_1	10 Humidities milli-lb of water per lb of dry air H_1	11 Humidities milli-lb of water per lb of dry air H_2	12 Humidities milli-lb of water per lb of dry air H_{i1}	13 Humidities milli-lb of water per lb of dry air H_{i2}	14 Over-all material balance per cent	15 Sensible heat from motor Btu/hr	16 Over-all energy balance, per cent	17 Tower performance ϵ_e	18 Tower performance ϵ_c	19 Log mean of terminal potentials Deg F Δt	20 Log mean of terminal potentials Milli-lb per lb ΔH	21 Unit over-all conductances Btu per deg F per hr per sq ft f_e	22 Unit over-all conductances Lb/[hr ft² (lb/lb)] f_D	23 Unit over-all conductances (Btu per deg F per lb) fc/f_d	24 Height of transfer unit HTU ft
1	3600	2970	8.61	540	89.8	75.8	65.2	58.3	79.0	79.0	21.6	8.7	30.8	19.3	+ 2.3	180	+ 0.2	.58	.44	10.8	9.9	2.2	9.3	.24	4.4
2	3600	3460	8.61	629	88.4	74.1	65.7	58.4	77.0	76.9	20.0	8.7	29.4	18.3	+ 2.3	250	+ 1.8	.54	.49	10.0	9.5	2.3	9.9	.23	4.8
3	3600	4250	8.61	775	86.6	72.4	66.5	59.1	74.8	74.3	18.4	9.0	27.7	17.2	+ 5.2	380	+ 1.4	.49	.52	8.7	8.7	2.3	11.0	.21	5.3
4	3600	6590	8.61	1198	82.5	67.8	64.5	57.9	70.5	69.4	15.5	8.8	24.2	14.7	+ 1.8	940	− 4.3	.39	.60	6.9	7.2	3.1	14.7	.21	6.2
5	3600	8430	8.61	1533	85.1	67.8	66.2	59.4	69.8	69.0	15.2	9.3	26.4	14.7	+ 7.8	740	+ 5.4	.33	.67	6.2	7.9+	2.6	15.0	.17	7.7
6	3600	10120	8.61	1840	86.3	67.6	66.3	59.9	70.0	69.0	15.1	9.5	27.5	14.6	+ 7.9	1190	− 0.6	.28	.71	6.1	8.2	3.2	16.5	.19	8.4
7	4500	3220	10.76	585	93.9	80.1	68.2	64.3	84.4	83.3	24.7	12.1	35.4	22.3	− 0.7	1200	+14.1	.62	.47	11.4	10.4	2.5	9.3	.27	4.7
8	4500	4850	10.76	882	90.7	76.9	67.8	64.3	78.4	78.2	20.9	12.2	31.8	20.0	− 1.2	1140	+10.9	.49	.52	11.0	9.3	2.5	10.8	.23	6.2
9	4500	5260	10.76	956	86.2	74.3	67.4	64.0	76.7	76.7	19.7	12.3	27.6	18.3	+ 3.9	1050	+ 2.9	.49	.54	8.5	6.9	3.2	13.5	.24	5.4
10	4500	6010	10.76	1093	89.4	73.5	68.5	64.4	77.5	77.4	20.5	12.0	30.4	17.9	− 9.8	980	+ 8.3	.49	.64	8.2	7.7	3.7	15.9	.23	5.2
11	4490	6670	10.74	1213	90.6	74.9	72.2	66.1	78.6	77.8	20.6	12.4	31.6	18.7	− 1.1	950	+ 5.1	.43	.64	6.4	8.7	3.6	15.0	.24	6.1
12	4500	7400	10.76	1345	83.7	70.5	68.0	62.6	73.3	73.0	17.6	10.9	25.3	16.2	− 2.2	900	− 2.6	.42	.63	5.7	6.4	3.7	18.6	.20	5.5
13	5430	3240	12.99	589	90.2	80.3	70.6	65.1	84.4	83.7	25.1	12.1	31.3	22.6	+ 1.9	1200	− 0.7	.66	.39	8.4	8.2	2.9	12.3	.24	3.6
14	5420	4070	12.97	740	88.2	78.2	70.9	65.2	81.5	80.3	22.4	12.0	29.2	20.8	− 2.1	1190	+ 0.4	.59	.43	7.6	7.7+	3.0	13.0	.23	4.3
15	5430	5150	12.99	936	89.0	76.1	73.8	62.9	80.4	79.5	21.7	9.7	30.0	19.5	− 5.2	1090	− 3.2	.54	.49	5.0	9.0	3.5	16.4	.22	4.3
16	5410	6070	12.94	1104	84.3	71.7	74.4	59.5	76.6	74.0	17.5	7.5	25.7	16.9	+15.0	980	+ 3.2	.52	.51	...	8.8	...	16.5	...	5.0
17	5440	6480	13.01	1178	86.8	75.4	73.0	67.4	78.8	78.1	20.6	13.2	28.1	19.1	+ 3.3	960	+ 5.4	.50	.59	4.9	6.7	4.1	17.1	.24	5.2
18	5410	6500	12.94	1182	85.2	71.5	70.3	61.7	76.2	75.5	18.9	9.8	26.6	16.7	− 1.5	920	+ 4.7	.54	.58	4.0	7.3	5.1	19.4	.26	4.6
19	5430	7270	12.99	1322	84.9	72.9	69.6	64.3	76.3	76.3	19.6	11.7	26.2	17.6	− 1.0	910	− 8.9	.47	.58	5.7	6.2	4.7	20.2	.23	4.9
20	5440	7330	13.01	1333	84.4	73.0	67.6	65.1	75.2	75.2	19.0	12.8	25.9	17.7	− 2.9	1010	+ 2.3	.47	.59	7.4	5.8	4.2	18.7	.22	5.4
21	5440	7590	13.01	1380	79.9	66.1	61.3	56.4	70.1	70.1	15.9	8.5	22.1	13.8	−13.0	990	+ 0.5	.52	.59	7.2	5.7	5.2	23.6	.22	4.4
22	6250	4590	14.95	835	87.2	74.6	64.4	56.1	78.7	78.7	21.2	7.7	28.6	18.6	− 6.0	2260	− 0.5	.62	.40	10.3	9.0	3.3	16.5	.20	3.8
23	6300	5360	15.07	975	88.4	78.2	69.7	66.2	80.9	80.5	22.4	13.2	29.4	21.0	+ 7.3	1050	− 2.3	.55	.46	8.4	7.4	4.0	16.0	.25	4.6
24	6290	5440	15.05	989	87.2	73.8	67.0	59.2	77.8	77.8	20.8	9.0	28.6	18.1	− 7.5	2130	+ 4.8	.61	.48	8.7	8.4	3.5	18.3	.19	4.1
25	6310	6540	15.09	1189	88.4	73.5	68.3	60.5	77.6	77.6	20.4	9.5	29.4	17.9	−12.9	950	+ 6.2	.56	.53	7.9	8.7	4.3	19.7	.22	4.6
26	6270	7020	15.00	1276	87.3	73.2	66.1	61.5	76.0	75.7	19.2	10.7	28.4	17.7	+ 4.4	890	+ 9.6	.53	.55	9.3	8.1	4.2	17.6	.24	5.5
27	6260	7450	14.98	1355	83.9	69.1	61.1	55.2	72.5	72.5	17.4	7.8	25.4	15.3	−14.5	890	− 2.9	.51	.52	9.8	7.7	5.0	22.2	.23	4.6
28[a]	7200	2880	17.22	524	103.4	88.6	69.8	61.6	94.9	94.8	36.6	9.8	48.0	29.6	− 0.6	180	+ 7.8	.76	.35	13.1	15.2	3.3	12.2	.27	3.2
29	7200	3420	17.22	622	87.9	79.6	73.4	61.3	82.2	81.7	23.7	8.9	28.9	22.0	+ 1.0	230	− 1.0	.71	.31	6.1	8.6	2.9	14.1	.21	3.3
30	7240	4280	17.32	778	92.3	81.4	72.7	63.5	83.8	83.8	25.3	10.5	33.5	23.3	− 1.6	370	+ 1.3	.64	.38	8.7	10.3	3.2	14.7	.22	4.0
31	7200	5400	17.22	982	90.9	78.9	73.8	63.0	81.8	81.1	23.2	9.8	31.9	21.5	− 7.0	600	+ 0.2	.58	.43	7.1	10.1	3.4	17.1	.20	4.3
32	7240	6340	17.32	1153	89.3	76.4	73.9	61.3	79.9	78.3	20.6	8.7	30.3	19.7	− 1.7	990	+ 6.5	.56	.46	5.4	10.3	3.8	17.3	.22	5.0
33	7240	8420	17.32	1531	82.5	71.8	67.4	59.8	72.9	72.1	16.9	9.2	24.2	16.9	+ 3.4	740	− 1.6	.47	.47	6.8	7.5	3.8	20.6	.18	5.6
34	7240	10050	17.32	1827	82.0	70.9	68.6	60.4	72.1	70.8	16.1	9.4	23.8	16.3	+ 3.3	1200	+ 2.8	.43	.51	5.4	7.3	3.4	22.0	.15	6.3
35[a]	9180	2880	21.96	524	103.4	91.8	67.5	60.8	96.0	95.8	37.8	10.0	48.0	32.9	− 0.2	180	+ 2.0	.76	.27	14.4	15.7	3.4	12.2	.28	3.2
36	9220	3380	22.06	615	87.8	80.8	72.3	61.4	82.5	82.1	24.1	9.2	28.9	22.9	+ 5.0	230	+ 5.2	.78	.27	6.9	8.5	2.9	14.2	.20	3.3
37	9070	4210	21.70	765	93.2	83.4	70.3	61.9	84.9	84.9	26.2	9.9	34.6	25.0	+ 0.3	380	+ 2.2	.67	.31	10.7	11.4	3.4	14.4	.24	4.0
38	9250	5360	22.13	975	92.1	80.8	71.1	60.4	82.2	81.7	23.7	8.8	33.3	22.9	− 1.7	610	+ 6.5	.63	.36	10.0	11.7	3.4	16.3	.21	4.5
39	9220	6230	22.06	1133	81.6	74.1	66.3	59.8	74.1	73.7	18.0	9.4	23.5	18.3	+ 6.1	390	+ 1.7	.60	.34	7.8	7.1	3.6	18.0	.20	4.7
40	9250	6480	22.13	1178	88.4	78.3	70.7	59.9	80.0	79.4	21.8	8.6	29.4	21.0	− 3.2	960	−10.1	.55	.35	8.3	9.8	4.1	20.9	.20	4.2
41	9220	8350	22.06	1518	84.8	75.3	67.6	60.4	74.9	74.5	18.4	9.6	26.1	19.0	− 0.1	750	− 4.4	.49	.39	8.9	8.5	3.9	20.7	.19	5.5
42	9220	10050	22.06	1829	84.5	74.2	69.1	60.2	74.0	72.8	17.2	9.1	25.9	18.4	+ 4.8	1160	− 1.7	.44	.42	7.7	9.0	3.5	21.6	.16	6.4
43	10840	2950	25.93	536	90.3	84.5	72.0	60.9	85.3	85.0	26.3	9.1	31.3	25.9	+ 1.6	180	0.0	.76	.20	8.3	9.7	2.8	12.5	.22	3.2
44	10660	3020	25.50	549	94.8	86.9	65.8	59.1	88.3	88.3	29.6	9.2	36.4	28.0	+ 0.5	170	+ 3.3	.78	.22	12.6	11.8	3.2	12.5	.26	3.3
45	10840	3460	25.93	629	90.4	83.8	71.5	61.1	84.2	84.0	25.6	9.2	31.4	25.3	+ 1.6	220	+ 1.6	.74	.23	9.1	10.1	2.8	13.4	.21	3.5
46	10840	4180	25.93	760	88.8	81.6	69.1	59.9	81.8	81.5	23.6	8.8	29.8	23.5	+ 2.9	350	+ 0.3	.70	.25	9.7	9.8	3.2	15.1	.21	3.8
47	10800	5440	25.84	989	89.8	81.2	75.1	61.6	81.7	80.7	22.6	8.6	30.8	23.2	+ 3.0	640	+ 5.2	.63	.30	7.3	11.1	2.7	16.4	.16	4.5
48	11160	5790	26.70	1053	86.8	78.6	64.8	57.2	77.7	77.4	20.3	8.1	27.9	21.2	− 3.7	730	− 0.7	.60	.28	11.6	10.1	3.7	16.7	.22	4.7
49	10760	6080	25.74	1105	90.3	81.2	68.2	60.2	80.5	80.5	22.7	9.3	31.3	23.2	− 1.5	400	− 5.8	.56	.30	11.5	11.0	3.8	17.7	.21	4.7
50	10800	6440	25.84	1171	90.3	80.8	75.6	62.7	81.5	80.6	22.6	9.2	31.3	22.9	−11.6	970	+ 3.5	.59	.34	7.1	11.0	2.9	18.7	.16	4.7
51	11160	6480	26.70	1178	85.6	76.9	63.6	56.8	76.2	76.0	19.4	8.2	26.8	20.1	− 1.8	1000	+ 1.7	.60	.30	11.6	9.5	4.0	18.3	.22	4.9
52	10760	8350	25.74	1518	85.5	77.0	67.7	60.3	75.4	75.1	18.9	9.7	26.7	20.1	+ 4.6	750	− 4.6	.49	.34	9.9	9.0	3.7	20.4	.18	5.6
53	10840	10120	25.93	1840	82.1	74.1	67.1	59.4	72.6	71.8	16.7	9.0	23.9	18.3	− 1.1	1150	− 8.3	.44	.35	8.4	8.2	3.6	22.8	.16	6.1
54[a]	12490	2950	29.88	536	94.5	88.1	66.3	59.5	88.1	88.1	29.3	9.4	36.1	29.1	− 9.9	180	+ 3.1	.78	.18	12.7	12.1	3.0	11.6	.26	3.5
55	12530	3420	29.98	622	88.4	83.4	72.6	61.9	82.7	82.4	24.2	9.4	29.4	25.0	+ 2.4	230	+ 1.7	.73	.19	8.2	9.5	2.5	12.7	.20	3.7
56	12560	4320	30.05	785	90.0	83.7	67.2	59.8	82.2	82.2	23.9	9.3	31.0	25.2	− 5.9	370	− 3.5	.64	.21	11.8	10.9	3.2	13.8	.23	4.3
57	12600	5400	30.14	982	89.6	82.5	66.9	59.1	80.3	80.3	22.3	9.0	30.6	24.2	− 0.3	650	− 3.6	.58	.23	12.5	11.4	3.3	15.1	.22	4.9
58	12530	6440	29.98	1171	93.0	83.7	65.8	59.1	80.8	80.8	22.9	9.1	34.3	25.2	− 3.6	940	0.0	.54	.27	15.2	13.6	3.7	15.6	.24	5.7
59	12460	8310	29.81	1511	86.9	79.0	69.2	59.9	76.5	76.0	19.2	8.9	28.0	21.5	− 8.8	750	− 5.6	.48	.29	10.3	10.6	3.4	19.3	.18	5.9
60	12530	9970	29.98	1812	84.1	76.6	69.9	60.3	74.5	73.4	17.6	9.0	25.5	19.9	+ 1.6	1170	− 6.3	.45	.32	8.3	9.3	3.0	22.0	.14	6.2

[a] Slight fogging noticed at air exit.

2 Dry-air rates G, from 2880 to 10,200 lb per hr of dry air, giving mass velocities ranging from 520 to 1840 lb per hr per sq ft of average-flow cross section.

3 Entering-water temperatures T_1, from 80 to 103 F.

4 Air dry-bulb temperatures entering the tower t_2, from 61 to 76 F; and wet-bulb temperatures t_{WB_2}, of 56 to 66 F.

5 Air absolute humidities entering the tower H_2, from 0.008 to 0.013 lb water vapor per lb of bone-dry air.

The steady-state test data Table 1, are tabulated in columns 1 through 9 and column 15. The Bulkeley psychrometric chart was used to determine the air humidities, which are entered as columns 10 and 11. The Keenan steam tables, together with the perfect-gas law, were used to compute the saturated humidities

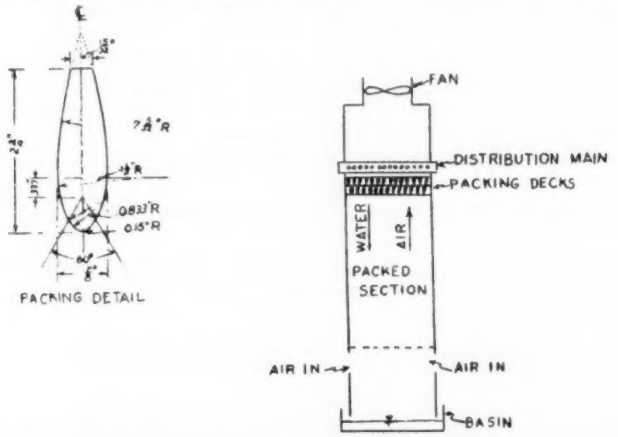


FIG. 4 SCHEMATIC DIAGRAM OF PACKED, COUNTERFLOW, INDUCED-DRAFT COOLING TOWER AND DETAILS OF PACKING USED

(Critical dimensions of the system are: Packing cross-section periphery, 0.525 ft; packing cross-section periphery, including dotted trailing edge, $u = 0.607$ ft; effective packing length, 2.67 ft; average packing area per deck, 19 sq ft, using 0.525-ft periphery; total packing area, $aV = 418$ sq ft, using the 0.525-ft periphery; number of decks, 22; height of packed-tower section, 5.75 ft; column cross-section area, 7.6 sq ft; mean cross-section area with respect to air flow, $S = 5.5$ sq ft; distance of free fall of drops, 0.302 ft per deck.)

corresponding to the water temperatures T_1 and T_2, columns 12 and 13 (2). No corrections were made for the variations in atmospheric pressure for, at the approximate sea-level conditions at the University of California in Berkeley, observations of the barometer indicated negligible deviations from the standard atmosphere. It is believed that the foregoing procedure results in humidities within 2 per cent of the values obtainable from more precise methods of calculation, which is within the accuracy of the wet- and dry-junction data.

The accuracy of the experimental results is revealed by the over-all material and energy balances, columns 14 and 16 in Table 1. The average magnitudes of both of these indicated differences are about 4 per cent.

Conductances for Thermal Convection and Mass Transfer

Unit over-all conductances for thermal convection and material transfer were computed, employing logarithmic-mean temperature and humidity potentials, corresponding to the terminal data. Interface conditions were assumed as saturated at the main-body-water temperature. This assumption yields over-all magnitudes of the conductances because there is some resistance to heat transfer in the water film and, as a consequence, the actual interface temperature is lower than the main-body-water temperature.

Use of mean potentials based on the logarithmic mean of the terminal values is exact if the variation of air-water-vapor-mixture (gas) temperature t is linear with T-t, and the humidity H varies linearly with H_s-H. These conditions do not prevail in the tower. However since the corresponding terminal differences are in the ratio of the order of 3 to 1 or less, it is believed that the logarithmic mean is a good approximation to the actual average potential.

The computed unit conductances are shown in Table 1, columns 21 and 22. Average magnitudes for each air rate, with varying water rates equal to or greater than 17 lb per hr per sq ft referred to the packing area, together with the number of transfer units $f_D aV/G$, are plotted in Figs. 5 and 6, as a function of Reynolds' number and the air-mass velocity G/S, expressed as lb of dry air per hr per sq ft referred to the average-flow cross section. Magnitudes for water rates less than 17 lb per hr per sq ft of packing area are not used for at these rates incomplete wetting of the packing led to much smaller apparent magnitudes of the conductances.

The equations defining the computed conductances are:

Material transfer

$$f_D = \frac{G\,(H_1 - H_2)}{aV\,\Delta H} \qquad [1]$$

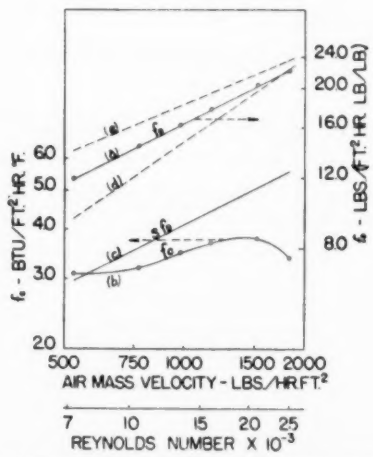


FIG. 5 VARIATION WITH AIR-MASS VELOCITY AND REYNOLDS' NUMBER OF UNIT CONDUCTANCES FOR MATERIAL AND HEAT TRANSFER f_D AND f_c

Curve (*a*) Experimentally determined f_D
Curve (*b*) Experimentally determined f_c
Curve (*c*) Experimentally determined sf_D
Curve (*d*) $f_D' = f_c'/s$ based on extrapolated thermal-convection data of Hughes (5)
Curve (*e*) $f_D' = f_c'/s$ based on extrapolated thermal convection data of Winding (6)

(Each point represents the average of 4 runs at approximately the same air-mass velocity and in all cases at a water rate equal to or greater than 17 lb per hr per sq ft.)

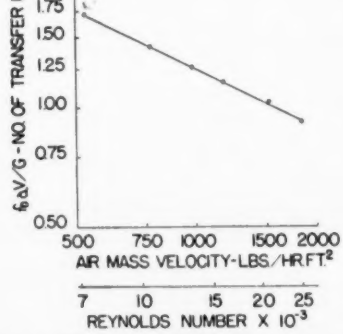


FIG. 6 VARIATION WITH AIR-MASS VELOCITY AND REYNOLDS' NUMBER OF THE NUMBER OF TRANSFER UNITS $f_D aV/G$

(Each point represents the average of 4 runs at approximately the same velocity and in all cases at a water rate equal to or greater than 17 lb per hr per sq ft.)

Convective heat transfer

$$f_c = \frac{sG(t_1 - t_2)}{aV\,\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

In computing f_c, the indicated gain in air sensible heat and the exit-air temperature were adjusted to account for the fan-motor input.

Effectiveness of the Tower

It was found that a plot of the dimensionless modulus

$$\epsilon_h = L_1 c\,(T_1 - T_2)/G\,(h_{i1} - h_2) \quad \ldots\ldots\ldots\ldots [3]$$

as a function of the air-mass velocity G/S, or as a function of the number of transfer units $f_D\ aV/G$, served to represent by a single curve the experimental data for water rates of 15 lb per hr per sq ft packing area and greater. An additional curve was required for each fixed water rate less than this value. The experimental effectivenesses ϵ_h, given in Table 1, column 17, are plotted in Fig. 7.

If, as normally is the case, the cooling range is such that $T_2 \gtrsim t_{WB_2}$, then, $L_1 c\,(T_1 - T_2) \cong G\,(h_1 - h_2)$ and the effectiveness as now redefined becomes

$$\epsilon_h = G\,(h_1 - h_2)/G\,(h_{i1} - h_2) \quad \ldots\ldots\ldots\ldots [4]$$

That is, ϵ_h represents the ratio of actual tower energy exchange between phases to the energy exchange which would result,

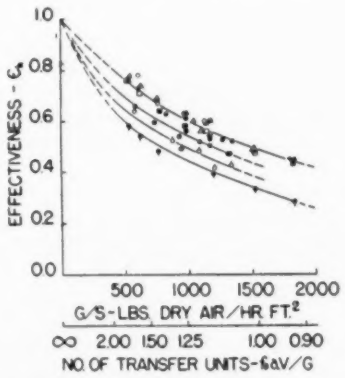


FIG. 7 EXPERIMENTAL RESULTS OF TOWER EFFECTIVENESS ϵ_h, AS A FUNCTION OF AIR-MASS VELOCITY G/S, LB PER HR PER SQ FT (The curves are the same as given by Equation [6] using the unit conductance f_D from curve [a] in Fig. 5.)

$$\epsilon_h = \frac{G(h_1 - h_2)}{G(h_{i1} - h_2)}$$

Key	
L/aV lb per hr per sq ft referred to packing area	
8.6 ♀	17.2 □
10.8 ♂	22.0 ○
12.9 ⊖	25.8 △
15.1 ●	30.0 +

provided the discharge air was saturated at the temperature of the entering water. This modulus compares the performance of the actual tower as an energy exchanger to that of a counterflow tower of infinite height and thus may be termed the effectiveness of the finite tower as an energy exchanger.

A consideration of the effectiveness concept, as will be demonstrated later, leads to the conclusion that ϵ_h may be represented empirically by an exponential equation of the form

$$\epsilon_h = 1 - Ke^{-\eta f_D aV/G} \quad \ldots\ldots\ldots\ldots [5]$$

where η represents the fraction of the packing area aV covered by the flowing water.

An analysis of the experimental data on this basis, using the magnitudes of f_D according to curve (*a*) Fig. 5, revealed a variation of η with water rate as given in Fig. 8. For water rates below 15 lb per sq ft per hr of packing area, complete coverage is not obtained and, for $(L/aV) = 8.6$ lb per sq ft per hr, the fraction covered is only 71 per cent.

The intercept of the straight-line extrapolation of the curve, representing the fraction of wetted area, shown in Fig. 8, is not 0, as might be expected, but rather about 0.40. This result is probably due to unequal water distribution to the two halves of the tower cross section from the distributing main, which was not under pressure, for the lower rates of flow. Such a condition may easily have been brought about by a slight rotation of the main on its axis, so as to make the horizontal line of radial distributing holes on one side come at a higher elevation than the diametrically opposing holes, serving the other half of the tower cross section. In this light the extrapolated line should not be straight, but curved so as to have a 0 intercept as indicated in Fig. 8.

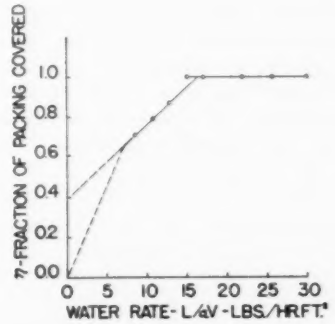


FIG. 8 FRACTION OF WETTED PACKING AREA η, AS A FUNCTION OF WATER RATE PER UNIT PACKING AREA, L/aV

The dimensionless constant K of Equation [5] was determined from the experimental results to equal 1.40 for the range of water rates covered. The following magnitudes of η substituted in the empirical equation

$$\epsilon_h = 1 - 1.40e^{-\eta f_D aV/G} \quad \ldots\ldots\ldots\ldots [6]$$

yield the tower effectiveness as an energy exchanger for the various water rates:

30 lb per hr per sq ft $\gtrsim L/aV \gtrsim$ 15 lb per hr per sq ft; $\eta = 1.00$
$L/aV = 13.0$ lb per hr per sq ft; $\eta = 0.87$
$L/aV = 10.8$ lb per hr per sq ft; $\eta = 0.79$
$L/aV = 8.6$ lb per hr per sq ft; $\eta = 0.71$

These relations, Equation [6] and the above values of η, are plotted in Fig. 7, using the abscissa scales of air-mass velocity G/S, and also the number of transfer units $f_D aV/G$.

The usual definition of cooling-tower performance is based on the ratio of the actual cooling range $(T_1 - T_2)$ to the cooling range which would obtain were the water discharged from the tower at the entering-air wet-bulb temperature $(T_1 - t_{WB_2})$. These magnitudes of $\epsilon_c = (T_1 - T_2)/(T_1 - t_{WB_2})$ are listed in Table 1, column 18. It may be seen that, when the effectiveness of the tower as an energy exchanger is low, the tower performance as a water cooler is high, and vice versa, that is, to a rough approximation

$$\epsilon_h + \frac{(T_1 - T_2)}{(T_1 - t_{WB_2})} = 1 \text{ (approximately)}$$

The tower effectiveness as an energy exchanger served to correlate the results more satisfactorily than the tower performance as a water cooler. As will be shown later, the use of ϵ_h is partially justified analytically. Further, if its effectiveness as an energy exchanger is known, the tower performance as a water cooler may be determined. Two restrictions apply to this calculation:

(*a*) The water-carrying capacity of the air stream in the infinite exchanger must not exceed the water rate, that is

$$L \leqslant G\,(H_{i1} - H_2) \quad \ldots\ldots [7]$$

which means that the water rate must be sufficiently large so that it will not suffer complete vaporization in the infinite tower.

(*b*) Cooling cannot occur below the dynamic equilibrium temperature at the air intake, which is established when the thermal convective heating of the water is just equivalent to the evaporative cooling by mass transfer. This equilibrium temperature is essentially the entering-air wet-bulb temperature, that is

$$T_2 \geqslant t_{WB2} \quad \ldots\ldots [8]$$

Any additional energy exchange which occurs in the actual tower above the amount required for the cooling range $(T_1 - t_{WB2})$ serves only to vaporize the water, reducing its enthalpy $(Lc\,t_{WB2})$ but further temperature reduction does not result.

Normal cooling-tower conditions are such that the operation is well within the limits defined by Equations [7] and [8].

Basic Equations

The differential equations for the material and energy balances and transfer rates will be stated for a perfectly insulated tower. For details of derivation the reader is referred to (2). The equations are written for a differential height of the packed section of the tower, corresponding to a differential volume dV and transfer area adV.

The energy balance expresses the fact that the gain in air enthalpy equals the loss in water enthalpy as the two streams pass counter to each other through the differential tower volume dV, that is

$$d\,[Lc\,(T - 32)] = G\,dh \quad \ldots\ldots [9]$$

The material balance yields

$$dL = G\,dH \quad \ldots\ldots [10]$$

The rate of material transfer from the air-water interface to the main gas stream is, taking V as increasing when proceeding downward in the tower

$$G\,dH = -f_D'\,(adV)(H_i' - H) \quad \ldots\ldots [11]$$

where f_D' is the unit film conductance for diffusion, defined on the basis of the humidity potential from the interface to the main gas stream $(H_i' - H)$.

The expression for the thermal-convection transfer from the interface to the gas stream is analogous to Equation [11] and may be written as

$$G\,s\,d\,t = -f_c'\,(adV)\,(t_i - t) \quad \ldots\ldots [12]$$

where f_c' is the unit thermal conductance for thermal convection.

Another equation is required to express the rate of heat transfer between the main body of the water at temperature T and the interface at temperature t_i. This relation is

$$d\,[Lc\,(T - 32)] = -f'_{cL}\,(adV)\,(T - t_i) \quad \ldots\ldots [13]$$

In addition to Equations [9] to [13], inclusive, a basic postulate usually made to relate the interface humidity and temperature is that H_i' is saturated at t_i . . . . . [14]

If over-all conductances f_c and f_D defined on the basis of the potentials $(T - t)$ and $(H_i - H)$, respectively, are used the necessity for Equations [13] and [14] vanishes and the rate Equations [11] and [12] are replaced by

$$G\,dH = -f_D\,(a\,dV)\,(H_i - H) \quad \ldots\ldots [15]$$

$$G\,s\,d\,t = -f_c\,(a\,dV)\,(T - t) \quad \ldots\ldots [16]$$

Unit Conductances

Unit conductances revealed in Fig. 5 are average over-all values. Since the packing shapes are staggered, at any tower deck approximately half the water rate will be in drop form. However, the drops for the most part were observed to possess relatively large diameters so that, even at water rates of 35 lb per hr per sq ft of packing surface, assuming a spherical drop diameter of 0.01 ft, the drop surface area would be about 6 per cent of the packing area. Also, since the results shown in Figs. 5 and 6 did not indicate any trend of f_D with water rate, if computed on the basis of a constant packing area of 418 sq ft, it was concluded that the effect of drop area was sufficiently small to be neglected.

The unit over-all diffusion resistance $1/f_D$ is the sum of the gas-film resistance $1/f_D'$, and an equivalent unit water-film resistance R_{DL}, introduced because f_D was based on H_i corresponding to the main-body-water temperature T, rather than on the interface temperature t_i.

The data on which Figs. 5 and 6 were based reveal no trend of f_D with water rate for a given air rate. This fact tends to indicate a constant value of R_{DL} with both air and water rates for, if the water-film resistance changes at all, it would be expected to change with water rate, and possibly also with the temperature conditions.

The presumption that f_D' varies as a power of G is reasonable and, since Fig. 5 also indicates that f_D varies as G^n, it follows that the magnitude of R_{DL} must be comparatively small if it is to remain constant as concluded.

There is some experimental evidence to the effect that mass-transfer coefficients vary as a power function of Reynolds' number in the same manner as do heat-transfer coefficients for similar flow systems (3, 4). Thermal convection data of Hughes (5) for a single streamlined shape, considerably more slender than the tower packing, indicated an exponent $n = 0.68$. The more recent data of Winding (6) for a bank of streamlined tubes yielded a value of $n = 0.42$. The slenderness ratio of the tube form used by Winding corresponded more closely with the tower packing than that of the shape used by Hughes. Thus the data of Winding appear to be more representative of the cooling-tower system under consideration in this paper, and the exponent 0.42 compares favorably with the slope of the unit conductance for mass transfer ln f_D versus ln G/S, curve Fig. 5, namely, 0.48.

The assumption of a constant liquid-unit resistance R_{DL}, equal to 10 per cent of the total unit resistance $1/f_D$ at a mean value of $G/S = 980$ lb per hr per sq ft, results in a unit thermal conductance f_D, varying exponentially with the 0.55 power of the Reynolds number. This magnitude appears high in comparison with the $n = 0.42$ from the data of Winding. The above assumption of the magnitude of R_{DL} also results in a logarithmic plot of f_D versus G, which possesses a slight curvature. This fact together with the larger average slope suggests that the water-film resistance is probably less than 10 per cent of the total.

The foregoing reasoning is not offered as conclusive proof of the comparatively small magnitude of the water-film resistance, but rather as an indication of its possible magnitude.

Considerable experimental evidence, obtained in part from psychrometric experiments (3), supports the Lewis equation

$$f_e' = s f_D' \quad [17]$$

for the air-water system only. The previously mentioned thermal film convection data of Hughes (5) and Winding (6) are compared with the experimental over-all diffusion data from the tower in Fig. 5, curves (*d*) and (*e*). Since the range of Reynolds' numbers for which the aforesaid data were given did not correspond to the tower values, a straight-line extrapolation of the logarithmic plot was used to determine the thermal unit film conductance f_e' over the desired range. The magnitudes so obtained were divided by the average air-mixture unit heat capacity $s = 0.25$ Btu per F per lb yielding the estimated unit film conductance for mass transfer f_D'. As may be seen from Fig. 5, curves (*d*) and (*e*) show agreement of about 10 per cent with the experimental f_D, curve (*a*). This comparison is good in view of the assumption made with respect to the magnitude of the perimeter used in Reynolds' number *Re*, and the fact that the data of Hughes refers to a single shape rather than to staggered banks.

Curves (*b*) and (*c*) of Fig. 5 reveal a comparison of the experimentally determined unit thermal conductances f_e, with the product of the mixture unit heat capacity s and the experimental unit mass-transfer conductance f_D. As may be seen the agreement is good at the lower air-mass velocities, but at $G/S =$ 1800 lb per hr per sq ft, sf_D is 40 per cent greater than f_e. An explanation of this difference may rest on the premise that small drops formed by splash are maintained or carried along by the air stream and suffer complete vaporization.

The unit over-all conductance f_e for thermal convection was computed subject to the assumption that the air-stream sensible-heat gain resulted from convection cooling of the main water body only. Thus the computed f_e may be considered an apparent value. As the complete vaporization of small drops requires the absorption of part of the sensible heat of the air stream, the net gain in sensible heat is less than the thermally convected transfer from the main water body, resulting in an apparent f_e smaller than the actual.

The small drop vaporization does not contribute to the cooling of the main water body, but merely changes the enthalpy distribution of the air stream so as to increase the latent-heat rate, $Gh_{fg}H$, at the expense of the sensible-heat rate, Gs $(t - 32)$. In effect, while the apparent unit conductance for heat transfer f_e is less than the true value, f_D as computed is greater than its true magnitude. However, as may be seen from Table 1, if small drop vaporization results, at the higher air rates, in a reduction of the air sensible-heat rate by about 4000 Btu per hr, the true f_D would be only about 5 per cent less than the computed values, whereas the true f_e would be 30 to 50 per cent greater than the apparent magnitudes, which is comparable to the product (sf_D).

The curves (*b*) and (*c*) of Fig. 5 indicate that the suggested effect of small drop vaporization increases with air rate, as shown by increasing difference between the f_e and sf_D curve ordinates. Further, at the low air-mass velocity of 530 lb per hr per sq ft, f_e is slightly greater than sf_D. At this rate, however, some fogging was apparent in the air stream issuing from the nozzle, and thus small drop condensation may serve completely to overcome the effect of small drop vaporization on the sensible-heat content of the air stream, and subsequently on the apparent unit conductance for thermal convection.

In normal cooling-tower operation thermal convection accounts for not more than 20 per cent of the energy transferred between phases. Thus an error of 20 per cent in the magnitude of f_e results in less than a 4 per cent error in the total energy exchange. If then, for the air-water system

$$f_e \cong s f_D \quad [18]$$

without any significant inaccuracy with respect to energy exchange, considerable simplification of the basic equations is possible. On this basis the rate Equations [15] and [16] combine with the energy-balance Equation [9] to yield

$$d\,[L\,c\,(T - 32)] = G dh = -f_D\,(a\,dV)\,(h_i - h) \quad [19]$$

This equation expresses the fact that the enthalpy difference $(h_i - h)$ of air saturated at the main-body-water temperature T and the unsaturated main air stream is the potential for energy exchange between phases; and f_D may be considered as the unit over-all conductance for this energy transfer. Equation [19] is not quite correct in that, when the potential is 0, thermodynamic equilibrium is not attained, for the liquid-water temperature is less than the dynamic-equilibrium wet-bulb temperature (2). As a result, the water temperature will tend to rise even though the net enthalpy transfer between phases is 0 at the 0 potential state. The temperature rise will continue until the wet-bulb temperature is reached. At this condition, a positive potential exists for which the enthalpy transfer from the liquid phase by evaporation exceeds the enthalpy transfer to the liquid phase, as a result of thermal convection, by the magnitude of the enthalpy of the liquid being evaporated.

For normal cooling-tower operation $c(T - 32)\,dL$ is about 3 to 4 per cent of $LcdT$ and, since c is constant, Equation [19], as a good approximation, may be written in the form

$$L_1\,c\,d\,T \cong G\,d\,h = -f_D\,(adV)\,(h_i - h) \quad [20]$$

This equation may be integrated if f_D = constant to yield

$$\frac{L_1 c}{G}\int_{T_1}^{T_2}\frac{dT}{(h_i - h)} \cong \int_{h_1}^{h_2}\frac{dh}{(h_i - h)} = \frac{-f_D\,aV}{G} \quad [21]$$

Equation [21] corresponds with the form of the equation defining the number of transfer units (2, 3, 7), a concept used in the design of absorption and extraction towers. The over-all values of the height of a transfer unit for the particular packing arrangement used in the tower are then

$$H\,T\,U = l/(f_D\,aV/G) \quad [22]$$

where l is the height of tower packing, 5.75 ft.

Table 1, column 24, lists the over-all values of the height of the tower transfer units, as computed from the experimental f_D. Fig. 6 summarizes the number of transfer units as based on average values of f_D at each air rate for those water rates sufficient to provide complete packing coverage.

It is seen, since f_D varies with the 0.48 power of the air-mass velocity, that the variation of HTU must be as the 0.52 power of G/S. Many of the data for gas-liquid systems, in which the gas-film resistance controls, correlate on the basis of the 0.8 power of gas-mass velocity or packing Reynolds number (3, 7, 8), rather than the 0.48 power as obtained for the packing used in the tower tested. This difference probably lies in the nature of the gas flow over a streamlined shape as compared to the flow over packings such as spiral tile, wood grid, broken quartz, and coke for which the 0.8 exponent was obtained. As is seen from Fig. 5, available thermal-convection data confirm the 0.48 exponent provided $f_D \cong f_e'/s$.

Effectiveness of Tower as an Energy Exchanger

If the water rate is infinite so that the water temperature T

remains constant, then the enthalpy at saturation h_i is constant and Equation [21] may be integrated, yielding

$$(h_i - h_1)/(h_i - h_2) = e^{-f_D aV/G}$$

or

$$\epsilon_h = \frac{G(h_1 - h_2)}{G(h_i - h_2)} = 1 - e^{-f_D aV/G} \quad [23]$$

The right-hand side of Equation [23] expresses the ratio of the gain in enthalpy rate $G(h_1 - h_2)$ of the air stream to the gain which would result in an infinite tower $G(h_i - h_2)$, for which the air would leave saturated at the entering-water condition. Since enthalpy changes represent the energy exchange between phases for the constant-pressure continuous-flow-tower system, this ratio is the tower effectiveness as an energy exchanger for the condition of constant water temperature T.

In the actual tower T is not constant but tends to approach the entering-air condition while descending over the packing. The degree of approach depends upon the potential for energy exchange and the water rate. Equation [23] suggests the form of an empirical expression which conforms to the experimental data of the actual tower. This form is given by Equation [5] and the empirically determined constants are as expressed by Equation [6]. For the experimental water rates, which resulted in complete packing coverage, Equation [6] is

$$\epsilon_h = 1 - 1.40\, e^{-f_D aV/G} \quad [24]$$

The coefficient 1.40 of this expression should not be constant but should be some function of the variation in water temperature throughout the tower, for in Equation [23], for constant water temperature, the magnitude is 1. The experimental data (see Fig. 7) are apparently of insufficient precision to reveal this variation in ϵ_h. However, the magnitudes of the effectiveness are not sensitive to the value of the multiplier K. For instance, for $\eta = 1$, the variation of ϵ_h about a mean of 0.50 is only approximately 7 per cent for $1.3 \leq K \leq 1.5$.

This consideration indicates that a single curve of effectiveness versus air-mass velocity (see Fig. 7 or Equation [24]) will serve as a performance characteristic for the tower within a limited operating range of water rates. The restrictions in the use of such a performance curve are expressed by Equations [7] and [8], and the additional fact that the water rate must be at least sufficient to give complete packing coverage.

With packing shapes which result in f_D, varying as the 0.8 power of the air-mass velocity, the variation of the number of transfer units would be to the —0.2 power of the air-mass velocity. As a consequence, a tower so packed would have an effectiveness as an energy exchanger which would not change greatly with air rate. For instance a threefold increase in air rate would result in but a 25 per cent decrease in effectiveness, ϵ_h for an initial value of $\epsilon_h = 0.5$.

Analytical Prediction of Tower Performance

The basic Equations [9] and [10], expressing energy and material balances, and the rate Equations [15] and [16], together with a knowledge of aV, f_D, f_c, c, and s will allow a prediction of tower performance for a given set of operating conditions. The solution may be accomplished in the form of a step integration, but such a procedure involves considerable labor. However, a proper algebraic combination of the equations mentioned, including the approximation $f_c \cong s f_D$, results in the expression

$$\int_{T_2}^{T_1} \frac{dT}{F(T)} = f_D \frac{aV}{G} \quad [25]$$

By plotting $\frac{1}{F(T)}$ as a function of T starting at one terminal value, say T_2, a trial-and-error solution for T_1 may be made which will give an area under the curve equal to the number of tower-transfer units at the given air rate $f_D\, aV/G$. This magnitude of T_1 establishes the tower-cooling range for the chosen operating conditions. The indicated integration can also be accomplished numerically.

Equation [25] was derived as follows: The energy-balance Equation [9] may be written, to a good approximation, as

$$L_1 c\, dT \cong G\, dh \quad [9a]$$

Integration from T_2 to T yields

$$h = h_2 + (T - T_2)\frac{(L_1 c)}{G} \quad [9b]$$

Then upon substitution into Equation [21], which combines the two rate Equations [15] and [16], the result may be expressed as

$$\int_{T_2}^{T_1} \frac{dT}{\frac{G}{L_1 c}(h_i - h_2) - (T - T_2)} = f_D \frac{aV}{G} \quad [26]$$

Since

$$h_i = s(T - 32) + h_{y_2} H_i$$

is a function of T only and h_2 is constant for a chosen entering-air condition, Equation [26] is of the form of Equation [25] where

$$F(T) = \frac{G}{L_1 c}(h_i - h_2) - (T - T_2) \quad [27]$$

The integration of Equation [26] in the manner described was performed for chosen magnitudes of air and water rates L_1 and G, entering-air state t_2 and H_2, and exit-water temperature T_2. The appropriate magnitudes of the number of transfer units $f_D\, aV/G$ were taken from Fig. 6, corresponding to the chosen value of air-mass velocity. No allowance was made for drop cooling for reasons previously mentioned. Table 2 and Fig. 9 summarize these computations made for water rates, L/aV of 17.2 and 34.4 lb per hr per sq ft referred to the packing area, for air-mass velocities, G/S, of 655, 1310, and 1865 lb per hr per sq ft referred to the flow cross section, and various initial air states and exit-water temperatures.

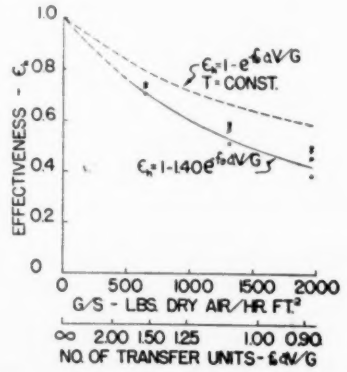


Fig. 9 Analytically Predicted Tower Performance Compared to Empirical Equation [24], Representing the Experimental Results

Key

L/aV	T_2 °F	t_2 °F	H_2 lb/lb
17.2	75	70	0.01○
34.4	75	70	0.01●
34.4	80	70	0.01+
34.4	85	70	0.01⌕

TABLE 2 COMPUTED TOWER PERFORMANCE BASED ON EQUATION [26]

(Unit conductances for diffusion taken from Fig. 5. Data for Fig. 9)

Assumed operating conditions					Computed performance		Comparison of f_D		
L/aV, lb per hr per sq ft	G/S, lb per hr per sq ft	T_2, deg F	t_2, deg F	H_2 lb per lb	$(T_1 - T_2)$, deg F	ϵ_h	Magnitude used lb/hr ft² lb/lb	Computed from log mean Δh lb/hr ft² lb/lb	Difference, per cent
17.2	655	75.0	70.0	0.01	5.61	0.705	13.48	13.4	−0.6
17.2	1310	75.0	70.0	0.01	11.6	0.51	18.9	18.7	−1.1
17.2	1960	75.0	70.0	0.01	18.0	0.385	22.8	21.7	−4.8
34.4	655	75.0	70.0	0.01	2.36	0.74	13.48	13.0	−3.6
34.4	1310	75.0	70.0	0.01	4.38	0.59	18.9	18.9	0.0
34.4	1960	75.0	70.0	0.01	6.18	0.50	22.8	22.8	0.0
34.4	655	80.0	70.0	0.01	3.54	0.745	13.48	13.4	−0.6
34.4	1310	80.0	70.0	0.01	6.62	0.58	18.9	18.8	−0.5
34.4	1960	80.0	70.0	0.01	9.57	0.485	22.8	22.9	+0.1
34.4	655	85.0	70.0	0.01	4.94	0.735	13.48	13.43	−0.4
34.4	1310	85.0	70.0	0.01	9.4	0.565	18.9	18.6	−1.6
34.4	1960	85.0	70.0	0.01	13.8	0.455	22.8	22.2	−2.6

$S = 5.5$ sq ft.
$aV = 418$ sq ft.

The comparison shown in Fig. 9 confirms the previous conclusion that the constant K of Equation [5] depends upon the variation of water temperature T, which in turn is a function of the water rate and driving potential. At the higher water rates, the water temperature changes less than for the lower water rates with the same terminal potential at the water-exit section. Thus, as a consequence, the effectiveness ϵ_h, is higher, approaching the value pertaining to a constant water temperature, Equation [23]. With an increase in enthalpy potential, as obtained by decreasing either or both t_2 and H_2, or increasing T_2, the water-temperature variation is greater (for a given water rate) and the effectiveness is thereby reduced, approaching the magnitude given by Equation [24].

A value of $K = 1.25$ in Equation [5] ($\eta = 1$) results in an empirical expression adequately representing the predicted performance for the higher water rate, while $K = 1.5$ is required for $L/aV = 17.2$ lb per hr per sq ft.

If the logarithmic mean of the enthalpy terminal potentials, Δh, serves to approximate the average potential with respect to energy exchange between phases, then a further simplification in predicting cooling-tower performance is possible, for Equation [21] may be integrated, yielding

$$\frac{h_1 - h_2}{\Delta h} = \frac{f_D a V}{G} \quad \ldots\ldots\ldots\ldots\ldots\ldots [28]$$

The prediction of the tower performance as an energy exchanger is then strictly analogous to the computation for a counterflow heat exchanger for which flow conditions and terminal states at either section 1 or section 2 are chosen. For instance, the following is a computation procedure which may be used:

(*a*) Choose h_2, T_2, G, and L_1.

(*b*) Determine f_D, and hence $f_D\, aV/G$ from the flow conditions.

(*c*) Estimate h_1.

(*d*) Compute h_{i1} corresponding to T_1 as determined from the energy-balance equation

$$L_1 c\,(T_1 - T_2) + (L_1 - L_2)\,c\,(T_2 - 32) = G(h_1 - h_2) \cong L_1 c(T_1 - T_2)$$

(*e*) Compute the logarithmic mean

$$\Delta h = [(h_{i1} - h_1) - (h_{i2} - h_2)]/\log_e \frac{(h_{i1} - h_1)}{(h_{i2} - h_2)}$$

(*f*) Substitute Δh in Equation [28] to verify the initial assumption of h_1. If this value does not result in an identity repeat the procedure with a new estimate of h_1.

(*g*) On establishing h_1 for the given conditions the energy exchange between phases is $G\,(h_1 - h_2)$.

The use of the logarithmic mean of the terminal potentials, while not exact, is justified as a good approximation. This conclusion may be verified by comparing the enthalpy conductances f_D, employed in the graphical integration Equation [26], with f_D computed from the logarithmic-mean terminal potentials, using the predicted conditions at section 1. The comparison, as outlined, was made for the conditions given in Table 2 and is indicated in the last three columns. As may be seen, the agreement for the most part is well within 3 per cent.

Conclusions

On the basis of the experimental results and the foregoing analysis, the following conclusions may be drawn:

1 For the cooling-tower air-water system in which the gas film presents the controlling resistance, over-all conductances for diffusion may be estimated from film conductances for thermal convection obtained from similar flow systems.

2 The correlation of tower effectiveness, as an energy exchanger versus air rate (or the number of transfer units), may be used as a performance curve for approximating either energy exchange or tower cooling. However, the application of a single curve is restricted to a limited operating range of water rates.

3 The basic transfer equations are simplified by the introduction of the approximation $f_e \cong sf_D$. Any error so introduced is not significant with respect to energy exchange in normal cooling-tower operation. This simplification results in an energy-transfer-rate equation, which uses as a potential the enthalpy difference between the main gas stream and saturated-air conditions at the main-water-body temperature. The equation so obtained may be readily integrated by graphical or numerical methods allowing a prediction of tower performance, providing the number of transfer units, $f_D \dfrac{aV}{G}$ is known.

4 The use of a logarithmic-mean enthalpy potential, based on terminal states, is a good approximation to the actual average with respect to energy exchange. This fact allows further simplification of performance calculations, the resultant procedure being analogous to the method now in general use for predicting counterflow heat-exchanger behavior.

Acknowledgments

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Quantitative Analysis of Single-Capacity Processes

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This paper is a continuation of a previous one[2] in which the author gave methods of quantitative analysis for five types of single-capacity processes with three types of regulators. The present paper analyzes and compares the various equations obtained in the previous paper and lays the basis for critical analysis of the entire subject.

Concrete examples are utilized as a basis for the graphical analysis to complement the mathematical analysis. A number of criteria are discussed critically to emphasize the importance of each, and examples are cited to indicate the light that each of the criteria is capable of shedding on the entire problem. Derivations and solutions of important factors are given in an Appendix.

IN THE author's previous paper[2] on the same subject, "single-capacity systems" were discussed together with the "hydraulic analogy" for continuous processes. To review, a single-capacity system consists of a single storage of energy, bounded by two resistances to flow of energy and characterized by a single potential level. It has the unique characteristic that any change in supply or demand immediately causes a measurable change in the potential level. Three basic types of regulators were considered and equations were derived to characterize the reaction of various combinations of these regulators and single-capacity processes to a "standard sudden disturbance." The equations describing these reactions were tabulated and in general it was shown that these reactions depend on (*a*) the mechanical constants of the regulators, and (*b*) the physical constants of the process such as its reaction time and the degree of self-regulation.

In this present paper the solution of the afore-mentioned equations will be considered and with the aid of concrete numerical examples certain general conclusions will be drawn. It is important to keep in mind the limiting assumptions made in the previous paper, so that the results herein described will not be generalized beyond their legitimate scope.

Scope of the Mathematical Solutions to Be Studied

The reaction equations in the previous paper[2] were expressed as differential equations of the first, second, and third order. This present paper deals only with the solution of equations of the first and second order, and excludes, therefore, certain combinations of floating-plus-proportional regulators even though combined with single-capacity processes.

The nomenclature given in the previous paper is repeated here in Table 1 for convenience, but a column is added to show the equivalent symbols used by Mason.[3] Mason's work is concerned mainly with multiple-capacity processes and if accurate substitution of equivalent terms is made, exact mathematical agreement between Mason's and the author's equations will result. Note that Mason does not bring in the concept of load z or of the initial flow condition Q. This is not the result of any conflict of theory, but results merely from his using purely quantitative units for flow equations instead of dimensionless ratios.

There are certain special cases of proportional-plus-floating regulators with single-capacity processes which can be expressed as second-order differential equations. The most important

TABLE 1 NOMENCLATURE

Dimensionless units:

	Spitzglass	Mason
γ	$= \phi/\delta$ = (per cent pilot-valve opening)/100	
δ	$= \phi_{max}$ = deviation corresponding to full pilot-valve opening	
θ	= characteristic of proportional regulator $= \phi/\mu$ = (spring force for full stroke)/h_2	
μ	$= \Delta d/d_{max}$ = (per cent deviation of valve area)/100	
ϕ	$= \Delta h/h_2$ = (per cent deviation)/100	$= \frac{T_a - T_{a0}}{T_{a0}}$
ϕ_p	= value attained by ϕ if process be permitted to reach equilibrium	$= \frac{T_p - T_{a0}}{T_{a0}}$
ψ	$= \Delta h/h_{1_0} = \phi(h_2/h_1)$ = (per cent deviation)/100 with respect to equilibrium value of h_1	
ψ_p	= equilibrium value of $\psi = \phi_p(h_2/h_1)$	
q	$= \Delta Q/Q$ = (per cent initial change in Q)/100	$= \frac{Q_a - Q_{a0}}{Q_{a0}}$
r	$= h_2/h_1$	
x	$= \Delta d/d$	= Same as q
z	= (per cent load)/100 $= Q/Q_{max} = d/d_{max}$	$= \frac{Q_{a0}}{Q_{max}}$

NOTE: μ', μ'' $= \frac{d\mu}{dt}, \frac{d^2\mu}{dt^2}$ ..... ϕ', ϕ'' $= \frac{d\phi}{dt}, \frac{d^2\phi}{dt^2}$

Time factors, sec:

	Spitzglass	Mason
T_a	= reaction time at full-load flow	$= z\,A_a\,R_a$
T_c	= reaction time at existing flow	$= A_a\,R_a$
T_s	= traversing time of regulator (corresponding to $\phi = \delta$)	
T_f	= reset time of floating component for full stroke of piston relative to dashpot (when $\phi = \theta$)	

Quantitative units:

	Definition	Hydraulic analogy	Mason Symbol	Mason Units, temperature system
A	= capacity between supply and discharge resistance	sq ft	A_a	Btu/deg F
Q	= flow rate at equilibrium valve position	cu ft/sec	Q_{a0}	Btu/min
Q_{max}	= Q at full valve opening	cu ft/sec		
R_a	= $1/dD$ = discharge resistance factor	sec/sq ft	R_a	min deg F/Btu
R_s	= $1/d$ = supply resistance factor	sec/sq ft		
t	= time after sudden disturbance	sec	t	min
h_1	= differential across supply valve	ft		
h_2	= differential across discharge valve	ft	T_{a0}	deg F
Δh	= deviation in h_2 at any time t (measured by regulator - measured variable)		$T_a - T_{a0}$	deg F
d	= regulator-valve opening area factor (supply)	sq ft/sec		Btu/sec deg F
dD	= discharge opening area factor	sq ft/sec		Btu/sec deg F

[1] Consulting Engineer, Hubbard Engineering Co. Mem. A.S.M.E. Mr. Spitzglass was graduated with an E.E. degree from Rensselaer Polytechnic Institute in 1922. Until 1937 he was vice-president in charge of engineering and research of Republic Flow Meters Company, Chicago, Ill.

[2] "Quantitative Analysis of Single-Capacity Processes," by A. F. Spitzglass, Trans. A.S.M.E., vol. 60, November, 1938, paper PRO-60-9, pp. 665–674.

Contributed by the Committee on Industrial Instruments and Regulators of the Process Industries Division and presented at the Annual Meeting, New York, N. Y., December 5–9, 1938, of The American Society of Mechanical Engineers.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] "Quantitative Analysis of Process Lags," by C. E. Mason, Trans. A.S.M.E., vol. 60, 1938, paper PRO-60-1, pp. 327–334.

special case is that based on the assumption that the regulator traversing time T_s is negligible as compared to the other time factors involved. A glance at the equations in Table 9 of the previous paper[2] shows that the coefficients of the third-order term always include as a factor, T_s; therefore, when this is a negligible factor, the third-order term drops out entirely. This leaves a second-order equation which could be analyzed with the direct methods of solution. This equation

$$\phi'' + \frac{1}{\theta T_a}\phi' + \frac{1}{\theta T_a T_F}\phi = 0 \quad \ldots\ldots[1]$$

is important, but not of sufficient significance to warrant separate consideration; therefore, the entire critical analysis of floating-plus-proportional regulated systems must be reserved until the subject can be treated in its entirety.

The Solution of Third- and Higher-Order Equations

Mason[3] has suggested an extension of the hydraulic analogy which the author feels offers the best solution for this type of third-order equation as well as equations of still higher order. Instead of considering these time factors as elements of the regulator equation, they are included as elements of the process and the regulator is assumed to perform its functions instantaneously. A single-capacity process analyzed this way becomes a multiple-capacity process, and it actually makes no difference whether these reaction times or "time lags" are introduced by the process or the regulator.

This method of analysis might have been used on the proportional regulator in connection with single-capacity processes. This would give a first-order equation for all such single-capacity processes and there would be no stability problem to consider. However, since the proportional regulator is frequently used on single-capacity processes of low natural period, the traversing time T_s too often plays a very significant role; therefore, the form in which the equation is considered by the author represents a very important class of process. Since this can be analyzed by direct solution it is included in this analysis. At a later time, when all modes of solutions have been publicized, it will be easier to decide which of the two methods is the more logical approach to the solution of regulation equations.

The following combined regulator and process equations, derived in the author's previous paper,[2] will be discussed in detail.

Processes Without Self-Regulation:

Floating

$$\phi'' + 0 + \frac{1}{\delta T_s T_a}\phi = 0 \quad \ldots\ldots[2]$$

Proportional

$$\phi'' + \frac{\theta}{\delta T_s}\phi' + \frac{1}{\delta T_s T_a}\phi = 0 \quad \ldots\ldots[3]$$

Processes With Self-Regulation:

Floating

$$\phi'' + \frac{ze}{T_a}\phi' + \frac{1}{\delta T_s T_a}\phi = 0 \quad \ldots\ldots[4]$$

Proportional

$$\phi'' + \left(\frac{ze}{T_a} + \frac{\theta}{\delta T_s}\right)\phi' + \frac{1}{\delta T_s T_a}\phi = 0 \quad \ldots\ldots[5]$$

These equations all take the general form

$$\phi'' + 2c\phi' + c_0\phi = 0 \quad \ldots\ldots[5a]$$

Note that the coefficient c_1 of the previous paper has been replaced by $2c$. This change was made for convenience since this coefficient always appears as $c_1/2$ in the solutions. It is obvious that the entire characteristics of the equation and oscillation depend on the values of c often referred to as the "damping coefficient," and c_0. The relative value of these two coefficients determines all the stability conditions and the absolute values determine the other factors such as amplitude and period.

The damping coefficient of each of the Equations [4] and [5] contains the term ze/T_a which is not present in Equations [2] and [3]. This term represents the effect on the process of the self-regulation of the system. The other term ($\theta/\delta T_s$ in Equation [5]) represents the damping due to the regulator itself. Equation [4] has no regulator damping term, showing that a floating regulator depends entirely on the process for its stability.

The self-regulation of a process is defined as the tendency of a process, after it has been disturbed, to balance input and output energy flows, as a result of the change in its potential level.

Two types of self-regulation are discussed by Stein and Wünsch,[4,5] namely, (*a*) back-pressure damping and (*b*) flow damping. The latter is caused by flow resistance occurring within the system and consequently presupposes a multiple-capacity system. The former only concerns itself with condition changes (pressure, in this case) across the supply and discharge resistances, and therefore, refers to single-capacity systems. Therefore, only back-pressure self-regulation will be considered at this time.

It will be shown that the relation of the two coefficients determines the type of vibration which will occur and that this may fall into two general classes, each of which gives rise to three specific types.

Thus, aperiodic regulation results:

(*a*)	If reaction time $T_a = 0$	$c = 0$	Floating or proportional
(*b*)	If traversing time $T_s = 0$	$c = 0$	Proportional regulator only
(*c*)	If very large damping exists	$c^2 \geqq c_0$	All types

Periodic regulation (vibration) results if $c^2 \geqq c_0$, and may be either of three types. From the Appendix the solution for ϕ, when $c^2 \geqq c_0$, is given by

$$\phi = Ae^{-ct}\sin mt \quad \ldots\ldots[6]$$

Therefore, if

(*d*)	$c > 0$,	e^{-ct} decreases with time	Damped vibration
(*e*)	$c = 0$,	$e^{-ct} = 1$	Sustained vibration
(*f*)	$c < 0$,	e^{+ct} increases with time	Resonant vibration

Since cases (*e*) and (*f*) do not represent practical solutions, we will not concern ourselves with them any further. If they occur at all, the values of c must be revised until they are greater than zero.

Cases (*a*) and (*b*), in which the second-order term drops out, revert to the first-order equation

$$c_1\phi' + c_0\phi = 0 \quad \ldots\ldots[7]$$

whose solution is very easily obtained as

$$\phi = e^{-(c_0/c_1)t} \quad \ldots\ldots[8]$$

It is necessary to consider in detail then only the two cases,

[4] "Regelung und Ausgleich in Dampfanlagen," by T. Stein, Julius Springer, Berlin, 1926. Also: "Selbstreglung ein neues Gesetz der Regeltechnik," by T. Stein, *Zeitschrift des Vereines deutscher Ingenieure*, vol. 72, Feb. 11 and 18, 1928, pp. 165 and 209.

[5] "Regler für Druck und Menge," by G. Wünsch, R. Oldenbourg, Munich, 1930.

where the second-order term remains and where $c > 0$. These two cases require individual treatment and differ only in the relative values of c and c_0.

Solution of Aperiodic Processes ($c^2 \geqq c_0$)

From the Appendix we get as the solution of these aperiodic processes

$$\phi = \phi'_0 \times \frac{1}{2\sqrt{(c^2 - c_0)}} \left(e^{\omega_1 t} - e^{\omega_2 t}\right) \quad [9]$$

or

$$\phi = Ae^{\omega_1 t} - Ae^{\omega_2 t} \quad [10]$$

where

$$\phi'_0 = qz/T_a \quad [11]$$

is the initial rate of change of the variable expressed by Equation [1], and

$$\omega_1 = -c + \sqrt{(c^2 - c_0)} \quad [12]$$

$$\omega_2 = -c - \sqrt{(c^2 - c_0)} \quad [13]$$

These roots are real, since the radical is positive and ω_1 is always larger than ω_2; therefore, the sign of ϕ never changes and it always remains at one side of its equilibrium value and reaches this value, $\phi = 0$, when $t = \infty$.

It is possible to calculate the maximum amplitude of the variable deviation, which, from the Appendix, is

$$\phi_{max} = \phi'_0 \frac{1}{2\sqrt{(c^2 - c_0)}} \left[e^{\frac{\omega_1 \log (\omega_2/\omega_1)}{\omega_1 - \omega_2}} - e^{\frac{\omega_2 \log (\omega_2/\omega_1)}{\omega_1 - \omega_2}} \right] \quad [14]$$

This solution is obviously rather unwieldy, but the analysis can be very easily converted to a graphical one which gives a complete solution.

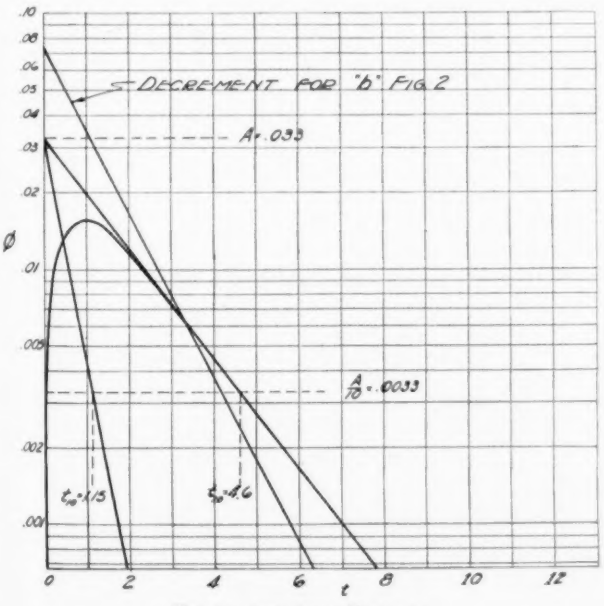


Fig. 1 Aperiodic Damping

Equation [10] consists of two distinct terms $Ae^{\omega_1 t}$ and $-Ae^{\omega_2 t}$. These exponential functions are very readily plotted on semilog paper. Fig. 1 shows a plot of example 1, a typical single-capacity process, which is solved as follows:

Example 1: Proportional Regulator—No Self-Regulation. Let $\theta = 0.25$, $\delta T_a = 0.10$, $T_a = 10$, $z = 0.5$, $c = \dfrac{\theta}{2\delta T_a} = 1.25$, and $c_0 = \dfrac{1}{\delta T_a T_a} = 1.0$. Then

$$\phi'_0 = qz/T_a = (1 \times 0.5)/10 = 0.05$$

$$A = \phi'_0 \frac{1}{2\sqrt{(c^2 - c_0)}} = \frac{0.05}{2\sqrt{(1.25^2 - 1.0)}} = 0.033$$

This represents the value of ϕ when $t = 0$, and is plotted as the initial ordinate.

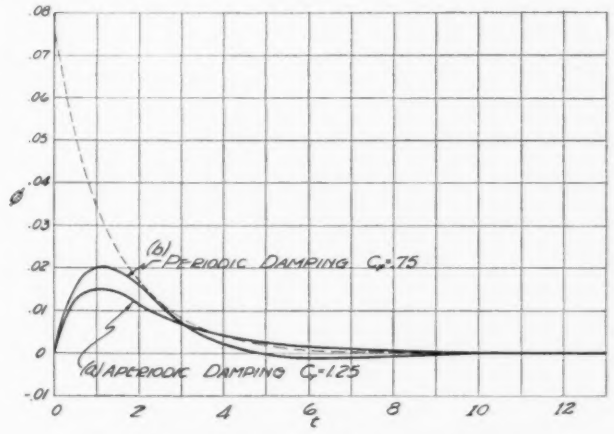


Fig. 2 Aperiodic and Strong Periodic Damping

The slope of the two curves is easily determined by solving for the time (t_{10}) required to reduce the variable to one tenth its initial value. Since the initial value of $e^{\omega t} = 1$, when $t = 0$, the time for $e^{\omega t} = 0.1$, or t_{10}, is obtained by solving directly for t in this last equation. That is, since $e^{\omega t} = 0.1$

$$t = \frac{1}{\omega} \log_e 0.1 = -\frac{2.3026}{\omega} \quad [15]$$

Now

$$\omega = -c \pm \sqrt{(c^2 - c_0)}$$

Therefore, in the example

$$\omega_1 = -1.25 + \sqrt{(1.25^2 - 1.0)} = 2.0$$

and

$$\omega_2 = -1.25 - \sqrt{(1.25^2 - 1.0)} = 0.5$$

Therefore

$$t_1 = \frac{-2.3026}{-2} = 1.15 \text{ sec}$$

and

$$t_2 = \frac{-2.3026}{-0.5} = 4.6 \text{ sec}$$

These values are plotted for values of $\phi = A/10$ or 0.033 and the total value of ϕ is obtained by subtracting the first curve from the second giving the resultant curve below from which any derived data may be taken directly.

Fig. 2 is a replot of this resultant curve on standard coordinates to bring out the more familiar contour of the aperiodic vibration.

Solution of Periodic Oscillating Processes

By far the most common and most significant group of proces-

ses are those that fall between the limits $c > 0$, and $c^2 \leqq c_0$, and whose equation of change is characteristic of a damped periodic oscillation.

The solution of this equation differs from that of the aperiodic process in that the roots are imaginary (see Appendix) and is represented by the typical equation for a damped oscillation, that is

$$\phi = Ae^{-ct} \sin mt \quad [6]$$

In Fig. 3 a typical process following this law is plotted. In the figure, the three factors fundamental to every damped vibration are clearly shown. These are

A, the initial or zero intercept of the decrement curve.

e^{-ct}, the "logarithmic decrement" or the curve representing the damping effect.

$\sin mt$, the factor that imparts the periodic character to the vibration, where $m = \sqrt{(c_0 - c^2)}$ radians per sec.

The constants c and m are determined directly by referring to the coefficient of ϕ' in the combined equation.

Note that the value of e^{-ct} reaches zero only at $t = \infty$. Therefore, in all regulator motions that damp out (periodic or aperiodic), the regulator continues to vibrate indefinitely, although with constantly decreasing amplitude. The fact that in practice the regulator actually stops in a finite time indicates the presence of a "dead zone" within which mechanical forces such as friction are sufficient to overcome the residual working forces in the power element of the regulating mechanism.

The Initial Coefficient A. This coefficient is obtained by analyzing the conditions that exist when $t = 0$, at which time

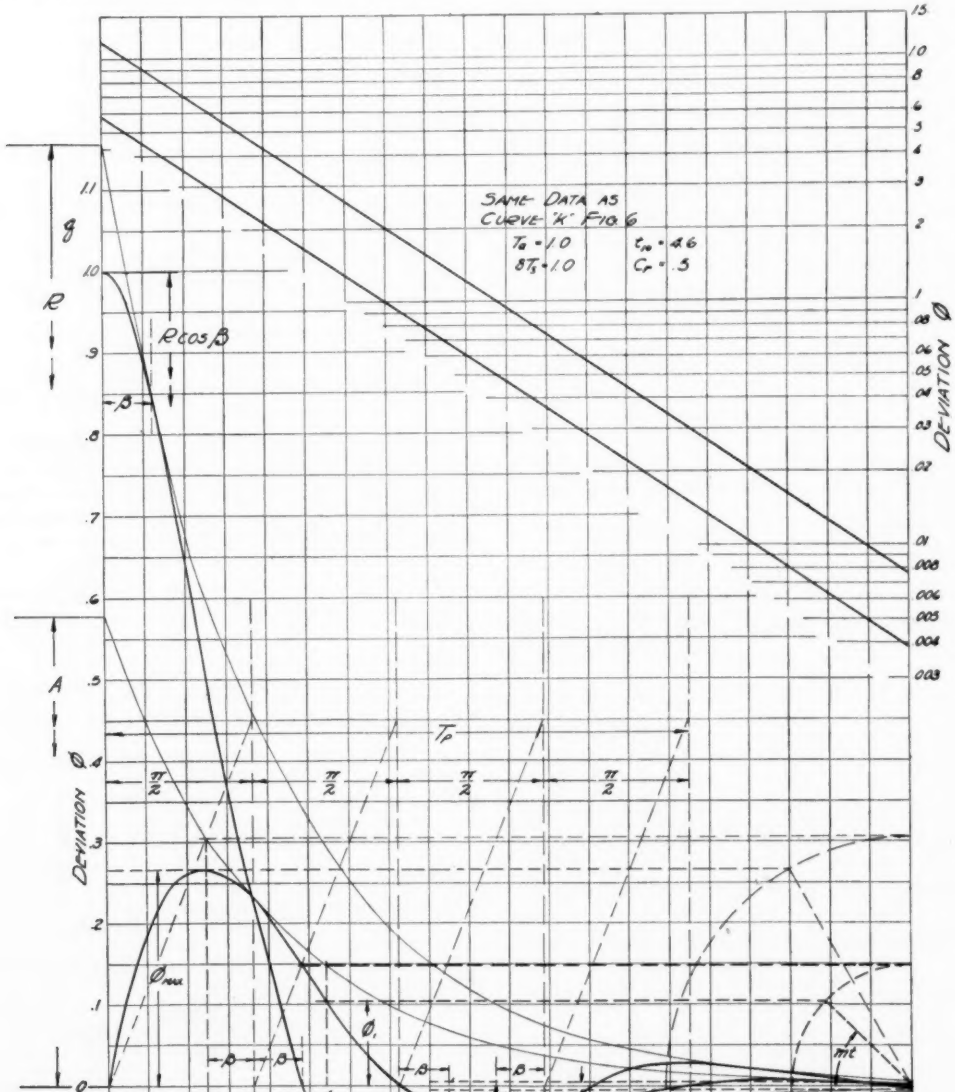


FIG. 3 GRAPHICAL ANALYSIS OF REGULATOR VIBRATION

$\phi = A$. The rate of change of ϕ or ϕ' is obtained by differentiating Equation [6] thus

$$\phi' = -Ace^{-ct} \sin mt + Ae^{-ct}\, m \cos mt \quad [16]$$

and at $t = 0$, $\sin mt = 0$, $e^{-ct} = 1$, $\cos mt = 1$, and $\phi' = \phi'_0$.

Therefore

$$\phi'_0 = Am \quad [17]$$

or

$$A = \phi'_0/m = zq_0/mT_a \quad [18]$$

The plotting is simplified greatly by first drawing in the decrement curve Ae^{-ct}. To do this an auxiliary curve is first drawn on semilog paper to the same time scale, as in Fig. 3. Ae^{-ct} is represented by a straight line in these coordinates, starting at A, for $t = 0$, and passing through $A/10$ at a time t_{10} sometimes referred to as the "time for tenthing," or "interval."[6]

Time for Tenthing. This time t_{10} is determined by calculating the time required to give 0.1 A. Thus

$$0.1\,A = Ae^{-ct_{10}} \quad [19]$$

or

$$0.1 = e^{-ct_{10}}$$

therefore

$$t_{10} = \frac{\log_e 0.1}{c} = \frac{2.3026}{c} \quad [20]$$

This defines the straight-line decrement on the semilog paper and the points can be projected along the time ordinates to the standard coordinates. The periodic curve is then generated graphically, the individual points being determined by multiplying the ordinate at any time t by its corresponding sine function.

The mathematical solutions for the various critical points of the curve will be given together with the discussion of its qualitative aspects.

Maximum Deviation. This deviation Q_{max} is sometimes the most significant quantity we have to determine, and is not quite as simple a function to calculate in the case of damped vibration as it would be if the variable followed an ordinary sine curve. Its value is obtained by differentiating Equation [16], which gives the following form, and equating ϕ' to 0. Thus, Equation [16] becomes

$$\phi' = Ae^{-ct} \cos(mt + \beta) = 0 \quad [21]$$

where

$$\beta = \tan^{-1}(c/m) \quad [22]$$

Therefore, mt must equal $(\pi/2 - \beta)$ in order that $\phi' = 0$, and β is the angle by which ϕ_{max} precedes the tangent point to the decrement curve, which occurs at $\pi/2$.

Solving Equation [6] for this value, $t = (\pi/2 - \beta)$ gives

$$\phi_{max} = Ae^{-\frac{c}{m}\left(\frac{\pi}{2}-\beta\right)} \cos\beta \quad [23]$$

where $\cos\beta = m/\sqrt{c_0}$.

The Period T_p. This is the time for 2π radians, or one complete cycle, that is, $mt = mT_p = 2\pi$. Therefore

$$T_p = \frac{2\pi}{m} = \frac{2\pi}{\sqrt{(c_0 - c^2)}} \quad [24]$$

[6] "Automatic Regulators, Their Theory and Application," by E. S. Smith, Trans. A.S.M.E., vol. 58, 1936, paper PRO-58-4, pp. 291-303.

The period is a constant quantity for a damped vibration, regardless of the disturbance or the time, although the vibration is not symmetrical. The zero intercepts occur successively at 0, π, 2π, etc., but the maximum amplitudes precede the points $\pi/2$, $(3/2)\pi$, etc., by the angle β. It actually takes the variable longer to get back to equilibrium than to reach its maximum departure, even though the cycle is perfectly isochronic.

This, incidentally, is true for all processes with continuous-type regulators, which represents another point of distinction between them and the discontinuous types, such as the two-position regulator. The period of such a discontinuous regulator varies with time as well as with the extent of the disturbance.

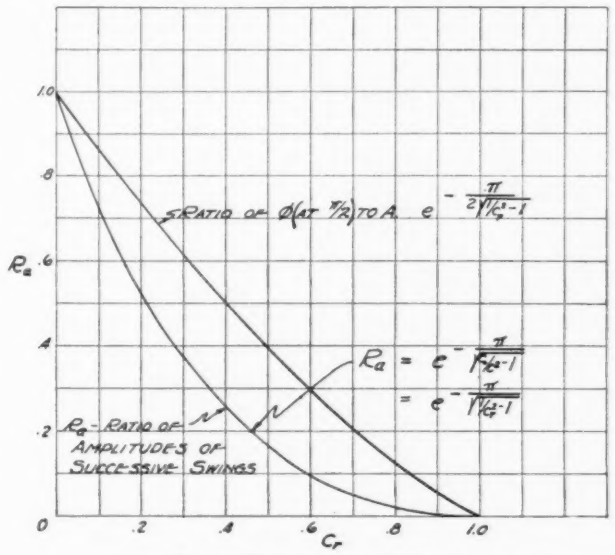


FIG. 4 RATIO OF AMPLITUDES

Ratio of Amplitudes. The ratio of any two successive amplitudes R_A is a very good criterion for stability, and may be represented by the two values ϕ_2/ϕ_{max}. This ratio is equal to the decrement for t = one half swing = $T_p/2 = \pi/m$, that is

$$R_A = \phi_2/\phi_{max} = e^{-c\frac{\pi}{\sqrt{(c_0 - c^2)}}} \quad [25]$$

$$R_A = e^{\frac{-\pi}{\sqrt{\frac{c_0}{c^2} - 1}}} \quad [26]$$

Damping Constant C_r. To simplify matters, the ratio of amplitudes R_A is plotted against a damping constant $C_r = c/\sqrt{c_0}$ as in Fig. 4. This constant C_r is a much simpler function to calculate than R_A and it has therefore been suggested by Wünsch[5] as the logical single criterion for the stability of a process (as a substitute for the more involved function R_A).

Fig. 4 presents the case where C_r is 0.5 and R_A is 0.16, which according to Wünsch[5] is a convenient criterion for good damping. If R_A is 0.16 then a disturbance is reduced to $(0.16)^2$ or 2.56 per cent in one full oscillation (two half swings), which can usually be considered as satisfactory damping. We shall investigate, however, to determine whether this is a sufficiently general criterion by which to judge the entire regulation process.

The values of C_r for typical regulator and processes are

Regulator	No self-regulation	Self-regulating process
Floating.....	0	$\frac{ze}{2}\sqrt{\frac{\delta T_a}{T_a}} = C_r$

TABLE 2 DATA FOR FIGS. 5 AND 6[a]

Fig. no.	Curve	Process z	Process e	Process T_a	Regulator θ	Regulator δT_s	c	c_0	T_P	t_{10}	A	C_r
5	a	0.5	2	10	0.1	0.1	0.500	1.000	7.26	4.6	0.0570	0.5000
5	b	0.5	2	10	0.1	0.1	0.050	1.000	6.30	46.0	0.0500	0.0500
5	c	0.5	2	10	0.1	1.0	0.050	0.100	20.10	46.0	0.1580	0.1580
5	d	0.5	2	10	0.1	1.0	0.100	0.110	19.90	23.0	0.1580	0.3016
5	e	0.5	2	10	0.1	10.0	0.005	0.010	62.80	46.0	0.5000	0.0500
5	f	0.5	2	10	0.1	10.0	0.050	0.011	72.60	46.0	0.5770	0.5000
5	g	0.5	2	10	0.6	10.0	0.030	0.010	66.00	76.8	0.5250	0.3000
6	j	0.5	2	1	0.1	1.0	0.050	1.000	6.30	46.0	0.5000	0.0500
6	k	0.5	2	1	0.1	1.0	0.500	1.000	7.26	4.6	0.5770	0.5000
6	c	0.5	2	10	0.1	1.0	0.050	0.100	20.10	46.0	0.1580	0.1580
6	n	0.5	2	100	0.1	1.0	0.050	0.010	68.20	46.0	0.0577	0.5000
6	o	0.5	2	100	0.1	1.0	0.005	0.010	72.60	46.0	0.0500	0.0500

[a] The nomenclature in Table 1 defines the terms used herein.

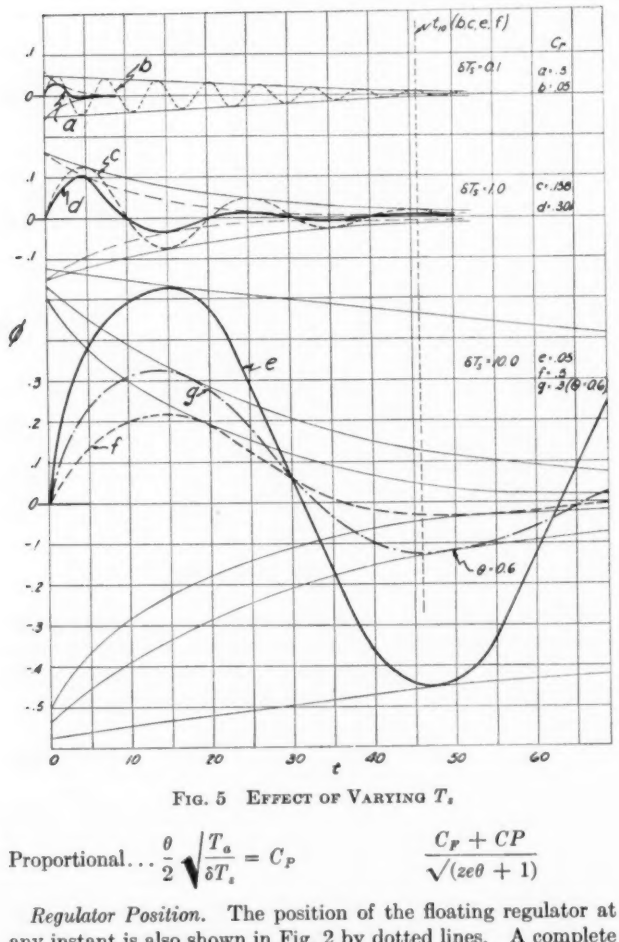


FIG. 5 EFFECT OF VARYING T_s

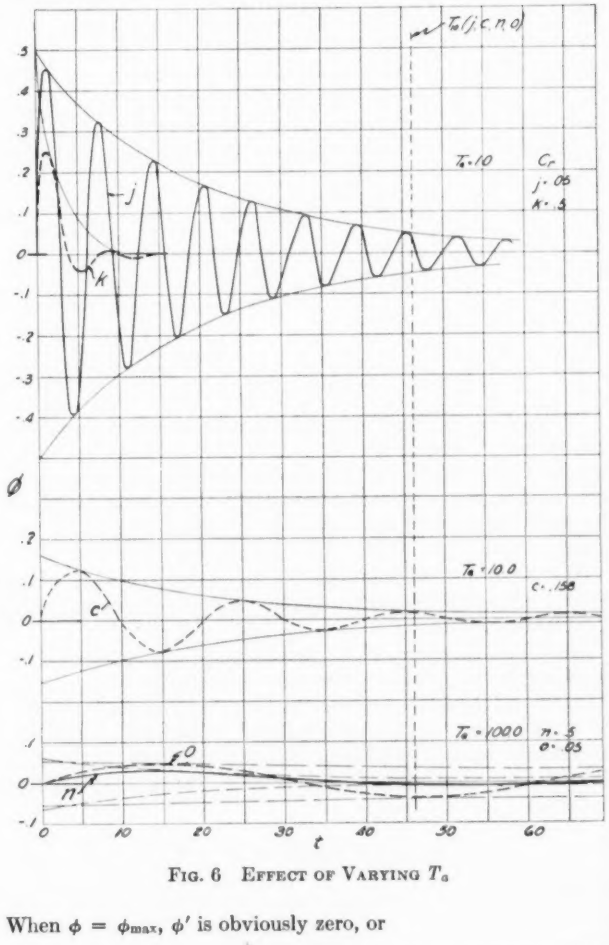


FIG. 6 EFFECT OF VARYING T_a

Proportional... $\frac{\theta}{2}\sqrt{\frac{T_a}{\delta T_s}} = C_P$ $\qquad$ $\frac{C_P + CP}{\sqrt{(ze\theta + 1)}}$

Regulator Position. The position of the floating regulator at any instant is also shown in Fig. 2 by dotted lines. A complete discussion of the regulator position is not possible here, but several interesting aspects are worthy of note.

The phase relation between the position, which is referred to in terms of the supply flow q, it produces, and the variable ϕ, is not 90 deg as might be expected, but 90 deg — β. The angle β is negligible only when the damping is negligible, as in a sustained oscillation.

Another point to notice is that the regulator does not go through its equilibrium position when the variable reverses, or ϕ_{max}, because the nature of the process has made the supply flow decrease, and the discharge increase, each by an amount depending upon the change in ϕ. Where $e = 2$, that is, when supply and discharge self-regulation effects are equal, ϕ_{max} corresponds to a position of the valve $q = \phi e = 2\phi$ from the equilibrium position. This valve position can be checked from the process equation

$$T_a \phi' = q - \phi e \qquad [27]$$

When $\phi = \phi_{max}$, ϕ' is obviously zero, or

$$q = \phi e \qquad [28]$$

If no self-regulation existed and a proportional regulator were shown, then the regulator would go through zero at ϕ_{max}.

EXAMPLES OF REGULATED PROCESSES

The more the question of regulation is investigated, the more it becomes evident that the process and regulator are part of one inseparable problem, and that their reactions must be studied together to lead to the most useful conclusions. To illustrate this, Fig. 5 has been drawn so that several regulator characteristics are compared with reference to a single process, and Fig. 6, so that a variety of processes are compared for a single regulator. Data for Figs. 5 and 6 are given in Table 2.

The curves show the effect of a sudden "supply" disturbance, caused by unbalancing the regulator an amount equal to q, and then letting the system come back to equilibrium. Therefore,

there is no change in load. It must be kept in mind that the change in the variable with load for the proportional regulator cannot be detected on these curves although its effect on the vibration is taken into account.

These curves have been verified experimentally by several observers and substantial agreement established, well within the required limits of accuracy.

Effect of Regulator Speed or Traversing Time T_s. The traversing time T_s which is the reciprocal of the regulator speed and δ, the pilot valve or speed characteristic, are found together in all the equations. Their combined effect will, therefore, be considered under the heading of effect of regulator speed.

Since δ is the value of ϕ at which the regulator will travel at full speed, discontinuity results whenever ϕ is larger than δ, and the speed does not increase any further. This happens only when very large disturbances occur but if it does, the reaction that takes place is neglected in the calculations while ϕ is greater than δ. As soon as the regulator has traveled sufficiently to bring ϕ within this range, then the equations are applicable. This imposes another set of limitations on the analysis. It is not a serious one, since, even under extreme conditions, the process gets back into this range in a relatively short time. If it does not, then the value of δ itself should be increased even though the product δT_s is satisfactory.

The term δT_s is the theoretical time per 1 per cent of stroke when $\phi = 1.0$. Of course, this time is never attained because ϕ never gets larger than δ, which is always a fraction, but it is an important quantity because as long as ϕ stays within δ we can imagine that the maximum regulator speed will finally go to this value, and treat it as the theoretical maximum speed.

Proportional Regulator Speed. Fig. 5 shows six cases for the same process having 10 sec reaction time, in which both extremes of stability and accuracy are obtained by varying the regulator characteristics. Thus, curve *a* shows a very stable regulation process with a proportional regulator that also gives a good deviation characteristic. An extremely fast regulator is required, however, one with a δT_s of 0.1 sec or a speed of 1 sec per stroke.

Curves *c* and *e* illustrate what occurs when the proportional regulator is slowed up 10 times and 100 times, respectively. The last two curves show a much lower stability, longer period and "time for tenthing," and much larger ϕ_{max}. These curves emphasize the importance of speed in proportional regulators and show why it cannot be neglected as a factor.

Curve *n* in Fig. 6 shows what the regulator of same T_s as curve *c*, or 10 sec, does on a process with longer reaction time. Obviously, the result is much improved, since the speed of the regulator, relative to that of the process has been increased. Therefore, it becomes obvious that the regulator speed must be considered in its relation to the other constants of the process.

Floating-Regulator Speed. If the process possesses self-regulation and a floating regulator is to be used, then the effect of the speed on damping is quite the opposite to the effect for proportional regulators. Thus, curve *f* in Fig. 5 shows that with the slowest of all the regulators in the figure, very strong damping results. As a matter of fact, speeding up the regulator has the effect shown in curve *b*, namely, the regulator is acting so fast that the self-regulation of the process has insufficient time to take effect strongly; consequently, a bad oscillating condition results.

Combined Effect of Proportional Regulation With Process Self-Regulation. When a proportional regulator is used on a system having strong self-regulation, as in curve *f*, the added damping due to the proportional characteristic is negligible. Likewise, if the proportional damping is very strong, then the added damping due to the self-regulation is negligible.

The third case, however, in which both regulator damping and self-regulation are rather weak, is shown in curves *c* and *d*. The factor $\theta/\delta T_s$ for the proportional-regulator damping coefficient equals 0.1 and ze/T_s, the self-regulation damping coefficient equals 0.1. Consequently, the two curves are identical and do not exhibit great stability, as shown by curve *c*. Curve *d* shows the effect of plotting the sum of these two effects, or a combined coefficient of 0.2; the result is considerably more stable than curve *c*.

This does not happen if we combine the damping of curve *b* with *a* because it is so negligible compared to *a* that it would not change the characteristic of the more strongly damped curve. A simple calculation bears this out.

It must be kept in mind that we cannot always use the combined equation since not all processes exhibit self-regulation due to discontinuity in the process, even though the constants of the process appear to be favorable. A process having pressure drops greater than critical is a case in point. Therefore, we may state the following as a practical and general résumé of the foregoing comments:

(*a*) If the damping due to self-regulation of the system be strong, the addition of a proportional characteristic will not improve the result but will merely introduce the error due to the band (droop or characteristic).

(*b*) If the damping due to self-regulation be weak, and a strong regulator damping be required, then the effect of the self-regulation of the system would be negligible and disregarding it in the calculation would introduce no serious error.

(*c*) If the self-regulation of the system be moderate (C_r = 0.15 to 0.3) and a regulator damping of about the same amount be required, then the total damping would be approximately the algebraic sum of the two.

For example, in curve *c* the C_r of the process exhibiting self-regulation equals 0.158, and the C_r of the regulator damping equals 0.158. For the combined process the value of C_r is 0.302, or approximately the sum of the two.

On the other hand, C_r for curve *e* equals 0.05 while for curve *f* it is 0.5, and the vector sum equals 0.52 which would give practically the same curve as curve *b*.

Effect of Reaction Time T_a. Fig. 6 presents a parallel analysis in which a single regulator characteristic is applied to three processes having widely different reaction times. As in the first figure, we see that the larger time constants of the process cause an appreciably larger period of vibration. Again the effect of the reaction time is considerably different depending upon whether a floating or proportional regulator is considered. Curve *j* shows how unstable a proportional regulator would be on a quickly responding process. Of course, this does not mean that a proportional regulator could not be made to work on this process, but its speed in order to be satisfactory would have to be increased about 100 times. For the speed shown, which is fairly high for normal regulator practice, the regulator hunts considerably.

On the other hand, with a large reaction time of 100 sec the proportional regulator gives a very small deviation and strong damping, although a very long period. Again this is largely due to the fact that the relative speed of the regulator is very high.

Where self-regulation is present, the floating regulator curve *k* damps out very quickly although the relative speed is still such that considerable deviation of the variable occurs. As a matter of fact, on a process having such a low reaction time, tremendous speeds of a different order entirely from those customarily used would be required if in addition to strong damping it was necessary to hold the maximum deviation of the variable within narrow limits.

On the system shown in curve *o*, the floating regulator shows practically no damping at all and ordinarily would continue this oscillation indefinitely. The oscillation is not very large, it is

true, but this is merely because the system has such large capacity that it would change very slowly even if no regulator at all were applied. Although the variable does not oscillate much, the regulator itself moves almost wide-open and closed. As a matter of fact, the results shown in curve *o* would not be the best solution even if they were not objectionable, since the simpler two-position regulator could be made to give results which are equally as good, if not better, provided the load variations were not too great.

The Effect of Proportional Characteristic θ. When the speed of a proportional regulator is insufficient and the process exhibits no self-regulation, then the values of θ must be increased as shown in curve *g*. Curve *g* shows a fair stability characteristic but a θ of 0.6 is required and this, of course, introduces considerable variation of the variable with change of load. This is not nearly so satisfactory a solution as speeding up the regulator as illustrated in curve *a*. Even curve *c*, which is relatively unstable, is better in some ways than curve *g*, due to the shorter period, since the actual deviation is less and of shorter duration.

Sometimes a pressure drop within the system can be utilized to introduce a proportional characteristic into the equation without necessitating its presence in the regulator mechanism. When this is possible a floating regulator may be used, but its effect is practically that of a proportional regulator. This type of damping is known as "flow" self-regulation, but the full discussion of it belongs under the heading of "Multiple-Capacity Processes."

Effect of Load. The load factor *z* as used here refers to the ratio between the equilibrium flow at the given time and the maximum flow to which the process may be subjected without introducing any discontinuity in the process reaction. It affects the various processes in several ways:

(*a*) With a proportional regulator the equilibrium value of the variable varies directly with the load by an amount determined by the characteristic θ. This is not brought out by the examples which are all taken at constant load.

(*b*) With a proportional regulator, the damping and the decrement are independent of the load.

(*c*) While the reaction time T_a is a constant, the "apparent" reaction time T_e actually decreases with load. It obviously takes the process longer to react for a given percentage of change at low loads than at high loads and this is confirmed by the equation for the damping constant

$$C_r = \frac{ze}{2}\sqrt{\frac{\delta T_a}{T_a}}$$

for self-regulating systems, which indicates that the damping is directly proportional to the load.

This is another limitation on the floating regulator. Unless the damping is so large at high loads that it is still sufficient at low loads, some proportional characteristic may be required merely to take care of the low-load condition.

Judging Quality of Results

Many attempts have been made to judge the quality of a regulation process by simple criteria such as stability or deviation. These will be discussed to determine whether any one of them is a true criteria of quality.

Stability. Stability refers to the ability of the regulator to damp out its oscillation quickly. This can be considered to mean either that the damping per cycle is strong, that is, C_r is large, or that the damping per unit time is strong, that is, t_{10} is short.

Now, curves *a* and *k* satisfy both these criteria, that is, C_r is large and t_{10} is small, and obviously they are examples of stable strongly damped vibration.

Curves *n* and *f* have large values of C_r but their period is so large that the t_{10} is also large; therefore, they would not satisfy the conditions of a fairly rapidly changing process, because the vibration though well damped might not damp out entirely between load changes. In fact, six of the curves in Figs. 5 and 6 have the same t_{10} value (46 sec) and yet there is no similarity between any of them in so far as their effect on the process is concerned.

It can hardly be said, therefore, that the t_{10} value forms a logical criterion for regulator operation. This does not mean that it is not a significant figure. It is a valuable index if only for the fact that it is very useful in preparing the rest of the graphical solution.

The damping constant C_r is a more informative criterion, although as has been pointed out, it is not safe to judge the process by this value alone. However, none of the processes with a large value of C_r is likely to cause much disturbance to the process, as this criterion is definitely associated with the tendency of the process to return to a normal position within one or two cycles.

When $C_r = 1.0$ or more aperiodic damping exists and while this is often very desirable there are many cases where it is preferable to have a slight oscillation. In Fig. 2, a second curve *b* has been drawn for the same set of conditions as the main curve *a*, except that $\theta = 0.15$ instead of 0.25. This gives a C_r of 0.75 instead of 1.25, or a very strongly damped periodic oscillation instead of one that is aperiodic. Curve *b*, the less damped, actually returns the variable to its equilibrium value much sooner, and though it overshoots it slightly, its average value after about 3 sec is much better than that of curve *a*. Furthermore, its final value is apt to be more accurate than in the case of the aperiodic curve, since it is more likely to stop in the middle of the "dead zone" while the aperiodic vibration usually stops at the edge of the dead zone.

Period. The period of a damped vibration, it has been pointed out, remains constant for a given process regardless of the magnitude of the disturbance. It depends almost entirely on the product of the regulator traversing time and the process-reaction time. With a large-capacity process, a large period is unavoidable unless the regulator traversing time is made extremely low. Very often, however, this is not undesirable since large capacities are usually present, at least partially, for the purpose of preventing sudden changes from occurring. It is important to note that if the period is large due to the slowness of the regulator, as in curves *e* and *f*, considerable deviation of the variable results. If it is the process-reaction time that is very large, relatively small deviation results even though the period is large, as shown by curves *o* and *n*.

Again, if C_r is very large, a very small period is helpful. However if C_r is small, a small period causes a fast vibration as in curves *b* and *j*.

Maximum Deviation ϕ_{max}. This quantity is a little more involved to determine mathematically but in a graphical analysis its value becomes self-evident. There is again no general rule which can indicate the extent of this deviation. Obviously, it is a direct function of the disturbance or change in load. Its relation to the other constants, however, is quite indefinite and is entirely different with a self-regulating process from that with a process with proportional regulator damping.

Graphical Methods. If instead of trying to determine individual criteria the entire results are plotted as in Fig. 3, the solution becomes evident at a glance. All the factors that enter in the picture are embodied in the resultant curve and it only remains to judge the process on the basis of the results required.

Requirements of the Process. The entire analysis so far has been made on the basis of a standard disturbance which consisted of moving the supply valve an amount necessary to change the flow by a value of *q*. It has been pointed out in the author's previous article[2] that this disturbance is typical of all types of

disturbances for single-capacity processes. This means that if the process will satisfactorily damp out a sudden disturbance in supply, it will take care of any other type of disturbance at least as well.

The actual disturbance against which we must compare our result is always, in some manner or other, the equivalent of a change in load. Any change in condition can be translated in terms of its equivalent in input flow or load.

The load factor then is plotted against time, and sections of the curve selected to represent both typical and extreme load disturbances. On these is superimposed the curve already obtained for equivalent standard disturbances, and the final decision, as to whether "satisfactory" regulation of the process exists, becomes a matter of judging the results and comparing them with the ultimate requirements.

Usually it is sufficient to perform these last-named operations mentally as a visual comparison of the load curve and the sudden-disturbance analysis should tell the whole story.

Résumé of the Complete Analysis of a Single-Capacity Process

1 Determine T_a, z, and e of the process (check to determine that the process is single-capacity and is not discontinuous or involving critical supply pressure drop).

2 Assume values of δT_s and θ to test the process.

3 Calculate the following coefficients:

	Floating	Proportional	Proportional with self-regulation
(*a*) c	$\frac{ze}{2T_a}$	$\frac{\theta}{2\delta T_s}$	$\frac{ze}{2T_a} + \frac{\theta}{2\delta T_s}$
(*b*) c_0	$\frac{1}{\delta T_s T_a}$	$\frac{1}{\delta T_s T_a}$	$\frac{ze\theta + 1}{\delta T_s T_a}$

(*c*) C_r or

$$\frac{c}{\sqrt{c_0}} \qquad \frac{ze}{2}\sqrt{\frac{\delta T_s}{T_a}} \qquad \frac{\theta}{2}\sqrt{\frac{T_a}{\delta T_s}} \qquad (ze\theta + 1)\left[\frac{ze}{2}\sqrt{\frac{\delta T_s}{T_a}} + \frac{\theta}{2}\sqrt{\frac{T_a}{\delta T_s}}\right]$$

4 From 3(*c*) determine which type of regulator to analyze first.

5 Calculate

(*a*) $m = \sqrt{(c_0 - c^2)}$
(*b*) $T_p = 2\pi/m$
(*c*) $T_{10} = 2.3026/c$
(*d*) $A = zq/mT_a$
(*e*) $\cos\beta = m/\sqrt{c_0}$

Of assistance but not necessary for calculations are

(*f*) $R_A = e - \dfrac{\pi}{\sqrt{\left(\dfrac{1}{C_r^2} - 1\right)}}$ (or see Fig. 4)

(*g*) $\phi_{max} = Ae - \dfrac{c}{m}\left(\dfrac{\pi}{2} - \beta\right)\cos\beta$

6 Plot e^{-ct} on semilog paper as straight line defined by

(*a*) $t = 0$, $\phi = A$
(*b*) $t = t_{1/10}$, $\phi = A/10$

7 Transfer to standard coordinates and multiply by periodic factor $\sin mt$.

Conclusions

There is no property or combination of properties of the process that justifies the assumption of the existence of an "index of controllability" or of satisfactory regulation. The true solution lies in the complete representation of the entire process reaction, and the recognition of the fact that once a regulator and process have been combined, they must be studied as a unit.

Certain generalizations are possible (with reference to single-capacity systems) but they must be restricted carefully to the limitations imposed on the problem. A few of the more important of these are:

(*a*) Certain processes exhibit a property of self-regulation, so as to impart an inherent stability to the process.

(*b*) When self-regulation is present in sufficient degree, a floating regulator will operate in a stable manner.

(*c*) When self-regulation is not present, then no stability exists unless it is supplied by the characteristics of the regulator itself, as in the case of a proportional regulator.

(*d*) Stability may be present at the expense of other desirable characteristics such as speed and accuracy; therefore, it is not alone a satisfactory criterion of operation.

(*e*) The dynamics of process regulation follows the well-known principles which govern all vibrating systems and then, by applying the principles of dynamic similarity, a general statement of the problem is possible through the use of dimensionless ratios and time constants.

(*f*) Knowing the constants of a particular process and those for the regulator, the characteristics of the regulated process can then be determined by applying the methods discussed in this paper. The results obtained will be sufficiently accurate for all practical purposes.

It is not always possible to determine the constants of the process with accuracy. This, however, need not impose too stringent a limitation to the broad application of the method herein outlined, since the results need only be approximations in order to be of value to the process designer. Very often it is sufficient merely to know the order of the quantities involved in order to make a satisfactory approximation.

Considerable literature dealing with the qualitative aspects of process regulation has been published, and the great need is for comprehensive quantitative data. It would be of invaluable aid to the future development of regulation technology, if those who are discussing the application of regulators to specific processes would describe these processes in terms of the time constants involved, and help form the basis for a quantitative approach to the whole subject.

The reasoning utilized here may be extended to multiple-capacity processes and to other types of regulators giving equations of the third and higher order. It is hoped that this extension will be published soon under the auspices of the Committee on Industrial Instruments and Regulators.

Appendix

General equation

$$\phi'' + 2c\phi' + c_0\phi = 0 \qquad \text{[A-1]}$$

Let

$$\phi = e^{\omega t} \qquad \text{[A-2]}$$

Substituting in Equation [A-1], dividing through by $e^{\omega t}$

$$\omega^2 + 2c\omega + c_0 = 0$$

Solving this quadratic equation gives the two roots

$$\omega_1 = -c + \sqrt{(c^2 - c_0)} \qquad \text{[A-3]}$$

and

$$\omega_2 = -c - \sqrt{(c^2 - c_0)} \qquad \text{[A-4]}$$

If $c^2 > c_0$ there is aperiodic damping.

The general equation for ϕ from Equation [A-1] is

$$\phi = Ae^{\omega_1 t} + Be^{\omega_2 t} \quad \text{[A-5]}$$

where A and B are constants of integration.

The values of A and B are determined from the initial conditions. Thus, when $t = 0$, $\phi = 0$, and $\phi' = \phi'_0$.

Then from Equation [A-5]

$$\phi = 0 = A + B \quad \text{or} \quad A = -B$$

and

$$\phi' = \omega_1 A e^{\omega_1 t} + \omega_2 B e^{\omega_2 t}$$

Let

$$t = 0, \ \phi'_0 = \omega_1 A + \omega_2 B \quad \text{[A-6]}$$

$$\phi'_0 = A\left(-c + \sqrt{c^2 - c_0}\right) - B + c + \sqrt{(c^2 - c_0)} = 2A\sqrt{(c^2 - c_0)} \quad \text{[A-7]}$$

Therefore

$$A = \frac{\phi'_0}{2\sqrt{(c^2 - c_0)}} \quad \text{[A-8]}$$

Likewise

$$B = -\frac{\phi'_0}{2\sqrt{(c^2 - c_0)}} \quad \text{[A-9]}$$

Substituting in Equation [A-7]

$$\phi = \frac{\phi'_0}{2\sqrt{(c^2 - c_0)}} (e^{\omega_1 t} - e^{\omega_2 t}) \quad \text{[A-10]}$$

If $c^2 < c_0$, then vibratory process exists.

Let

$$m = \sqrt{(c_0 - c^2)}$$

Then

$$\omega_1 = -c + im \quad \text{[A-11]}$$

And

$$\omega_2 = -c - im$$

where

$$i = \sqrt{-1} \quad \text{[A-12]}$$

Then the integrated value of ϕ in Equation (A-1) is

$$\phi = Ae^{-ct + imt} + Be^{-ct - imt} \quad \text{[A-13]}$$

where A and B are again constants of integration. Also

$$\phi = e^{-ct}(Ae^{imt} + Be^{-imt}) \quad \text{[A-14]}$$

Replacing the unit vectors e^{imt} and e^{-imt} by their sine and cosine functions

$$\phi = e^{-ct}[A(\cos mt + i \sin mt) + B(\cos mt - i \sin mt)]$$

$$\phi = e^{-ct}(K_1 \sin mt + K_2 \cos mt) \quad \text{[A-15]}$$

where $K_1 = i(A - B)$ and $K_2 = (A + B)$.

Let

$$K_1 = a \sin \theta \text{ and } K_2 = a \cos \theta$$

then

$$\phi = e^{-ct}(a \sin \theta \sin mt + a \cos \theta \cos mt)$$

or

$$\phi = ae^{-ct} \cos (mt - \theta) \quad \text{[A-16]}$$

where θ = angle at $t = 0$, or initial phase angle, measured from the peak of the first vibration.

Transferring the time axis $-[(\pi/2) - \theta]$ gives

$$\phi = Ae^{-ct} \sin mt \quad \text{[A-17]}$$

Discussion

H. L. MASON.[7] The author has well coordinated a number of theoretical quantitative treatments of the critical stability of a regulator-managed process. The equations can be widely used, but will become so only to the extent to which engineers and process designers can furnish numerical values for the process constants which enter into them. Hence, it seems pertinent to emphasize the basic concept—flow, which underlies all understanding and measurement of these constants, and point out that our present terminology may usefully be broadened.

Fluid passing through a typical industrial process, whether the "fluid" quantity be cubic feet of liquid, pounds of gas, or heat units of thermal energy, encounters two hindering actions —resistance and storage. The rate at which it gets through the resistance depends on the "head" across such resistance and is in many cases proportional to it. This head may be a difference of liquid levels, a difference of pressures, or a difference of temperatures between entrance and exit, and it may be expressed in feet of water, pounds per square inch, or degrees Fahrenheit. The constant of proportionality may be termed a "conductance" and expressed in units of fluid quantity passed per unit time per unit head; when multiplied by head it becomes flow.

The rate at which the "fluid" accumulates in a storage depends not on any property of the storing element, but solely on the inflow to and the outflow from it. We are more interested in the "level" of the stored "fluid," measured from an arbitrary datum and expressible as feet of water, pounds per square inch, or degrees Fahrenheit, because it is generally the variable we seek to control. Quantity stored is proportional to the "level," and the constant of proportionality may be termed "capacitance," and expressed in units of fluid quantity per unit level.

With the terms "fluid," "head," and "level" generalized in such fashion, the fundamental dynamic relation

Rate of change of level = inflow — outflow

will be useful for a variety of industrial processing systems. Each of its three terms represents an obvious physical reality, the numerical value of which is the product of obvious physical quantities, determinable both by computation and by direct experiment. The relation appears to be basic to a rational explanation of the so-called "time constants," and valid for any value of self-regulation, whether of supply or discharge, and (if limited to small deviations) for any process in any medium involving temperature, pressure, or flow. It may readily be extended to complex systems with a number of storages and resistances by applying it in turn to each storing element in combination with the resistive elements adjoining it. A plea is made, therefore, for the use of the foregoing concepts in arriving at numerical values for process constants.

C. P. BOSOMWORTH.[8] One of the author's primary assumptions states that "standard sudden disturbance" in a single-capacity system can be simulated by an instantaneous change in supply caused by the movement of the supply valve. It is doubtful whether such a disturbance can be found in the normal operation of process equipment. Generally, the disturbance occurs over

[7] Director of Research, Taylor Instrument Companies, Rochester, N. Y. Mem. A.S.M.E.

[8] Akron, Ohio.

a definite time interval and the change occurring in the characteristics of the system does so at a varying rate.

The author takes this into account in his further analysis of a single-capacity system when the time characteristics of the controller and process are made a part of the equation shown. It may be that the difference in the answers obtained to the problem using "standard sudden disturbance" assumption instead of the more or less gradual load change which occurs is relatively unimportant. Perhaps the author would give his opinion on the analysis if such a standard disturbance was assumed to occur in a specified time with a uniform change of demand.

A previous paper[2] by the author stated that Dr. Ivanoff used a standard disturbance consisting of ".....a continuous sinusoidal change in demand. This standard disturbance leads to a solution which is difficult to apply and generalize." It is probably true that such a type of change would be difficult to incorporate in a simple mathematical form; however, it appears to the writer that there should be something between the two assumptions of standard disturbance which would be a little more consistent with the usual simple single-capacity process.

WALLACE D. WOOD.[9] The author is to be commended for making available in English the work of Stein[4] and Wünsch[5] and for ably extending the work of control analysis through fundamental concept and mathematical development. The paper rightly emphasizes that the quality of control depends upon the combined influence of the characteristics of the process and of the controller.

The writer feels that if the author had chosen his "sudden standard disturbance" as one of load rather than supply that his equations would have been more useful. They would then have represented the actual changes that occur in practice. The quality of droop which is a characteristic of proportional control and not floating control would have been brought out in the mathematics. The proportional-control equations would then have indicated the factors which affect droop and the quantitative effect of the factors on the amount of droop. In extending the analysis to multiple-capacity systems, it will be preferable to assume a load disturbance. Therefore, it seems desirable to analyze the single-capacity system under similar assumptions.

It is the opinion of the writer that the author in his previous paper[2] did not choose the most representative type of proportional and proportional-plus-floating controllers. It is the experience of the writer's company that only in a very small percentage of its proportional-control installations is the traversing time of the valve slower than the speed of changes in the process, i.e., the rigid connection between the valve position and the pen position on the chart is maintained during motion of the pen. Should this be generally true, then the author's second- and third-degree equations for proportional and proportional-plus-floating control become first- and second-degree equations. This would alter the scope of the critical analysis as well as lead to somewhat different conclusions. For instance, a proportional controller regulating a single-capacity process could not produce periodic damping as the general equation of control would be of first rather than second degree.

The writer is not in agreement with the author's acceptance of the Wünsch damping constant C_r. In practice R_A, the ratio of the amplitudes, is simply observed and serves as a logical criterion of damping. Those who are making a mathematical analysis of control will find it only slightly harder to calculate R_A than C_r. Also the use of R_A is in keeping with log decrement as used generally in vibration phenomena.

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With reference to the purpose of control analysis, it may be argued by some that the easiest way to "synchronize" an instrument to a process is to install the instrument and adjust it by cut and try to the optimum control that it will give, and that generally it will be easier and more economical to install an instrument in this manner than to first determine the constants of the process and then apply a preadjusted instrument. But, one must not lose sight of a goal of this group which is to analyze and classify processes and instruments so that when a manufacturer desires an instrument to control a particular process and maintain a quality of product within given limits, he will know whether to choose a floating, a proportional, or more complicated type of controller.

With such an end in view, the author, at the close of his paper, emphasizes the need for comprehensive quantitative data of existing processes. The work of this group may well be directed to the determination of what data should be known about processes and how it should be obtained. A collection of such data representing a good cross section of existing processes would then be available for a complete analysis. The author has set up factors which describe the characteristics of a process. It would be useful to have other papers presented on this same subject with alternate methods of describing process characteristics. From these factors this group might select those most easily and generally measurable and use them as a basis for data leading to analysis. C. E. Mason[3] and the author have made a fine start in this direction.

E. S. SMITH.[10] While the author's two papers[2] involve a great amount of work and constitute a definite advance in the art, they may be criticized as being too short, particularly in their failure to set up the physical reasoning back of that useful tool, the Anlaufzeit. To the writer, this concept is a physically meaningless abstraction for the case where no self-regulation is involved, the "area" only being there needed. The author's closure could well include a brief introduction of the Anlaufzeit from a physical standpoint. For the occasional user, it would seem better to use Mason's more direct approach[2] instead of the ratio notation followed in these papers. The ratio treatment tends to weaken the use of dimensional analysis, i.e., use of the π theorem of Buckingham, in attacking the physical problems involved; but this is not of much consequence since regulation is more a matter of the mechanics of damped oscillating systems than of physics.

Some of the general conclusions of the paper under discussion appear to be limited to the particular sort of regulator illustrated, e.g., one should apply the analysis with caution to self-operating devices. The treatment of the dead zone due to mechanical friction is inadequate and should be expanded to bring out the advantage of the earlier response (to a change) due to a narrower dead zone.

AUTHOR'S CLOSURE

The discussions on this paper deal mainly with qualitative rather than quantitative concepts which the paper has attempted to emphasize. The author feels that this is a general tendency in the field of automatic control and is overemphasized in most of the discussions taking place.

The author's aim was to stimulate discussion of the results of all types of quantitative analyses, so as to promote a study of the quantitative constants involved and to begin to talk on the subject of regulation in terms of numerical values. Furthermore, we must get away from the realm of our personal experiences, since the tendency to view regulation problems against one's own

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limited experience often blinds us to the broader generalizations that are possible.

H. L. Mason offers the statement "the rate of change in level equals inflow minus outflow," and asks that it be accepted as a fundamental dynamic relation "for a variety of industrial process systems." He also suggests that the processes equation given by the author will come into wide use only to the extent that numerical values can be obtained for the "process constants" which enter into these equations.

The two remarks seem to be contradictory since the first is purely qualitative and places no significance on the dimensions or properties of the processes. Generalizations such as this in which the dimensions of the process are omitted entirely are more apt to confuse than clarify the problem. We all know that the rate of change in level is not the same for different processes, even if the inflow and outflow are exactly the same.

The author feels that less attention should be given to names and qualitative concepts, and more attention to the concise mathematical definition of each element. The names can well be selected by a Committee on Terminology, once we have decided upon the physical dimensions of each quantity. It matters little if we call a certain property the "reaction time," "Anlaufzeit," "capacity time constant," or any other term. After all, the important thing is to realize that we are talking about the ratio of q to ϕ', if we know exactly where q and ϕ' are numerically. The best test is the one so often quoted by C. E. Mason in his well-known "let's put some numbers in it, then we will all understand it."

Mr. Bosomworth's objection to the "standard sudden disturbance" on the ground that it does not often occur in practice is again an evidence of a qualitative point of view. What difference does it make whether it occurs in practice as long as it is a satisfactory mathematical test and one that can be checked experimentally? The reasons for this approval have been covered in previous papers by the author and others.

The objection raised by Mr. Wood to the use of the "supply" disturbance instead of the "demand" or "load" disturbance was seriously considered by the author before deciding on the method finally adopted. It is quite possible to bring in the effect of a sudden permanent change in load and derive an equation or plot a curve in which there is a permanent change or "droop" in the control point. This introduces further complication into the analysis, however, and adds nothing to our knowledge of the regulator action except what could be very simply determined by a simple and separate calculation using the "static" regulator equation, namely

$$\mu = k\phi$$

which tells us that the static value of the variable, or the control point, varies with the regulator position, or the "load," when a proportional regulator is used.

Frankly, this is a question on which opinion may very justifiably be divided and the author would gladly accept either method if it were to promote consistency between various writers in studying this subject.

There does not seem to be much validity in the statement that the regulators chosen in the analysis were not the most representative type. In the first place, these regulators were of no "type" whatsoever, the equations being perfectly general to *any* floating or proportional regulator, whatever the speed. This discussion in referring to proportional regulators of one type introduces a limitation which was never present in the paper itself.

The statement "It is the experience of our Company that only in a very small percentage of its proportional-control installations is the traversing time of the valve slower than the speed of changes of the process," indicates, first, a limited range of application and, second, a misconception of the effect of speed. In all turbogovernors and in many flow and pressure controls, high speed is of vital if not controlling importance. The writer knows of many problems where δT_e values of about 0.10 to 0.25 second were needed.

Furthermore, even if the speed is greater than the speed of change of the process, poor regulation may result when the two are about the same order of magnitude. This confusion results from generalizing from observations based on temperature-control problems where the process changes so very slowly due to the relatively large capacities involved, that the relative regulator speed is very high.

Obviously, certain single-capacity processes with proportional regulators of high relative speed reduce to a first-degree equation and will not vibrate. But this follows from the general equation very clearly and has been brought out in the paper, and it is not necessary to alter any conclusions or stretch any theory to be completely consistent.

E. S. Smith criticizes the failure to set up a physical reasoning back of the term "Anlaufzeit" and calls it a meaningless abstraction. If the writer could have supplied a photograph of this "time constant" of the process, it might be clearer, but time constants, while they cannot be seen, are very concrete, real quantities, and irrevocably related to and characteristic of all single-capacity processes. In the hydraulic analogy it is the time it would take to fill a tank of area A to a height h, at a rate of flow Q, or as stated in the simple Equation [4] of the first paper

$$T_e = \frac{Ah_2}{Q}$$

Mr. Smith notes that only certain regulators are treated and the treatment appears to be limited. The author purposely limited his treatment to "inertialess regulators," as clearly brought out in the first paper,[2] and this obviously eliminates any practical consideration of self-operating regulators.

As to the nomenclature, the author has no great preference, having used dimensionless and ratio nomenclature only because it tends to generalize the equations and simplify them. If the A.S.M.E. prefers other notations, the change-over is easily effected, and to this end a table has been included to relate the author's nomenclature with that of C. E. Mason, who uses quantitative units throughout.

Finally, it is hardly to the point to criticize the treatment of "dead zone," since this factor was assumed to be zero in this discussion and the treatment carefully omitted with malice a forethought.

In conclusion, the author wishes to point out that this work represents the merest introduction to the study of processes treated as vibrating systems, and the general quantitative approach to the whole problem of process analysis. He hopes that discussions of processes will gradually become more quantitative. Only in this way may the "art of process control" gradually develop into a "science of process operation."

Some Problems in the Design and Operation of Merchant Vessels

By ROBERT C. LEE,[1] NEW YORK, N. Y.

This paper presents certain technical aspects of ship design and operation as viewed from the standpoint of the executive administration of the merchant marine. The author evaluates the recent developments in hull and machinery design and construction in terms of economics. From the records of the Scantic Line and the American Republics Line, operating-cost figures are cited to demonstrate the relation of the various factors involved in ship operation. A review of shipbuilding costs and sales values of ships is included. Among other important matters discussed in the paper are safety provisions, crew training and personnel relations, and finally mention is made of the need for better and more efficient methods of cargo handling. The problems are proposed without solutions being available for many of them, in the hope that subsequent discussion may develop answers.

NO APOLOGIES need be offered for modern marine machinery. The industry is keeping well abreast of power developments. The average shore engineer would be favorably impressed with the power plant aboard any modern ship. Of course, in viewing a ship's installation one must keep in mind the limited space and the inevitable conflict which must occur in reconciling the different demands upon that space and the weight allowance for machinery in any given model. Certainly the fundamental axiom of ship construction is that each demand must be compromised in order to arrive at a buoyant, stable, and workable ship. Furthermore, it should be axiomatic that there must be employed afloat much larger factors of safety than are needed in shore design.

Just as in shore establishments, afloat there are steam plants, Diesel plants, combination steam and Diesel, and a constantly increasing use of electricity for the distribution of power to distant units, and for reduction gears.

For the main propulsive units, which account for 80 per cent or more of the power utilization, there are steam plants driving turbines through mechanical reduction gears; steam plants driving generators, which in turn drive the propeller through a motor or so-called electric drive; the straight Diesel drive; the Diesel drive through mechanical reduction gears; and the Diesel drive through electrical reduction gears, if they may be so termed. The race between the Diesel and the turbine is so close, from the standpoint both of reliability and economy, when the twenty-year life of a vessel is considered, that the choice between them becomes largely a matter of personal preference. The same condition is not quite true of the choice between mechanical or hydraulic reduction gears and electrical coupling. There is no question but that electric losses of such connections are somewhat greater than the friction losses of well-made reduction gears. On the other hand, the author is of the opinion that the flexibility and the elimination of vibration inherent in the electric gear more than compensate for the difference in efficiency in the case of many merchant ships and afford the electric connection much attraction for military purposes. It is assumed, of course, that the matter of reducing by gearing or other means the high rotative speed of the propelling units to the necessarily slow revolutions of the ponderous propellers of a modern ship, in order to secure reasonable efficiencies, is a familiar one.

In departments of the ship other than the engine room, there seems to be slight justification for any but electrical equipment. Electric winches, electric steering engines, electric galleys, because of the flexibility of electric-power conduits, as compared to the awkwardness and risks of conducting steam about a ship, have left the steam-driven winch and the oil-fired galley range in the limbo of the past.

Early Days of the Turbine in the Navy

As a young midshipman, the author vividly recalls a sunny morning at sea, leaning over the engine-room hatch of the old cruiser *Chicago*, while a friendly first-class man explained the rudiments of the steam turbine. At the time we were watching the slow turning of the weird-looking horizontal reciprocating engines. By the time of graduation from the Academy, two battleships, sister ships of the dreadnought class, had been placed in commission. One, the *Delaware*, was driven by reciprocating engines, and the other, the *North Dakota*, was propelled by turbine machinery. The author was ordered to the reciprocating ship, the *Delaware*. The *North Dakota* spent much of her first year of service at various Navy yards undergoing turbine-reblading operations. It almost seemed that the steam turbine could be discounted as a marine propulsion system. However, wise heads saw that the fault in the machinery of the *North Dakota* was structural and not one of principle. The *Delaware*, in spite of her performance and economies of operation, was one of the last reciprocating-engined ships built for the Navy.

Less than twenty years ago, the author knew of a case, where some very wise and able shipping men refused to consider any form of propulsion for ships of their fleet except reciprocating engines and Scotch boilers. Looking back now, it seems incredible that such a view could have been so tenaciously held by intelligent men and engineers.

Developments in Hull Construction

In hull construction great progress also has been made. The *Titanic* disaster, of course, brought about a complete revision of bottom construction and led to the adoption in the merchant marine of the double bottom long after that design had been applied to naval vessels. Flooding curves are now carefully studied and ships are compartmented with careful consideration of stability in a damaged condition. Practically no material is used in the construction of a ship these days that is inflammable. The author is by no means prepared to subscribe to the press

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

releases and other publicity which have recently proclaimed the advent of "nonsinkable, fireproof ships." That situation is still a long way off and probably always will be from the very nature of the enterprise. Nevertheless, great progress has been made in ship construction not only from a practical point of view, but in most cases from the artistic point of view as well.

When the average shipowner wants a ship built, he calls in his naval architect and advises him of the general characteristics and the limitation of cost. Then the freight department demands voluminous cubic and large deadweight carrying capacity; the passenger department demands all outside staterooms with bath; both departments demand top speed; the operating department demands machinery which will achieve the lowest possible fuel consumption, as well as innumerable auxiliaries, winches, booms, and other cargo-handling gear. The architect knows that there is no way in the world to provide such a ship and he begins to trade among the different departments in the owner's establishment, from which effort the final compromise emerges as a workable vessel. Most of the formulas used in designing a ship are inevitably arrived at by the trial-and-error method; therefore, the marine fraternity is a very conservative one, going forward only a little at a time. Changes are by slow evolution.

Less than three months ago, the new Cunarder *Mauretania* made her maiden voyage to New York. Though of comparable size to her famous predecessor, she is considerably slower. To be sure, there are the *Normandie* and the *Queen Mary*, superliners which dominate the Atlantic service, with both owners engaged in building or planning larger and faster ships. In the author's opinion this trend is likely to prove only a passing phase. The amount of power required to drive a vessel varies approximately as the cube of the speed. Excessively high speeds, therefore, become very costly. Also, there is no question but that a ship driven at high speed, in anything except good weather, causes considerable inconvenience to the passengers. The owner of one of these very high-speed ships told the author recently that, for at least one third of the running time, they were unable to use the full power of the vessel. It is the author's belief that the *America*, now under construction for the United States Lines, and other similar-type vessels of various shipping companies, are pointing the way for the future. Modern cabin liners of moderate size and moderate speed, compared with the superliners, are becoming increasingly popular. Transatlantic passage by air on a commercial basis is now an actuality, with every indication of increasing service. Probably overseas transportation of the future is likely to be conducted by moderate-sized, dependable, medium-speed vessels, leaving air transport to those passengers who would otherwise be the patrons of three-day ships, when and if they become an engineering possibility.

While the world tonnage of today, comprising some 30,000 craft of various sizes and refinements as to the means of propulsion, probably offers a fairly complete picture of the evolution of marine machinery from the simplest to the most complex type, the immediate interest and concern are in the advantages and limitations of the more modern machinery of the three principal types, the steam reciprocating engine, the turbine, and the internal-combustion engine. While the main competitors in the field of higher powers are the turbine and the Diesel, the reciprocating engine is still in the running and cannot be left entirely out of the picture.

Considerations in Selecting Propulsion Machinery

The modern shipowner, while constantly gambling with uncertainties characteristic of his business, cannot afford to gamble in the selection of his ships. He is conservative in his choice and will insist upon proved reliability and economy of his equipment. This is especially true of his marine power plants where failures are not only costly to repair but also can be very expensive in the matter of disrupted service and accompanying dissatisfaction of the shipping and traveling public. A wise shipowner will, undoubtedly, even sacrifice a certain amount of economy to obtain unquestionable dependability of his ships.

The combination of reciprocating engine and Scotch boiler, which so long appeared to be inseparable, is apparently at the end of its long trail of service. This is particularly true in the field of higher powers.

It is true also that the reciprocating-engine builders have made many valiant attempts to regain the lost ground but, if the supremacy of the turbine is being challenged, the effects of such a challenge are imperceptible. To overcome the thermodynamic disadvantages of the reciprocating engine the uniflow steam principle has been utilized, as well as the double-seat poppet valve, which is cam-operated and suitable for superheat. Preheated, low-pressure and exhaust steam turbines have also been interposed between the low-pressure cylinder and the condenser. To overcome the disadvantages of high weight-to-horsepower ratio, there have been refinements of design and materials, as well as a stepping up of engine speeds. Needless to say, all this, while reducing the operating cost of engines, at the same time has increased their purchase price.

This type of installation is unquestionably dependable and can stand a great deal of abuse. Furthermore it may be repaired readily in any part of the world. In these and other ways it embodies many features of desirability to the shipowner. Nevertheless, it is difficult to overlook the disadvantages of poor thermal efficiency, mechanical losses, and high weight per horsepower inherent in this type of machinery. The use of higher pressures and superheat introduces difficulties of cylinder lubricants in the feedwater. Consequently, in the present-day trends toward bigger and faster ships, the reciprocating engine is being left behind to do its work in the lower-power class, where the characteristics and economics of this type of marine power plant still offer certain advantages and serviceability.

With the advent of higher steam pressures and temperatures, steady progress has been made by geared-turbine machinery. The high dependability and economy of such a power plant renders it very desirable in marine work. Better and better economies are being obtained with installations of this type. Undoubtedly, the progress will continue for many years to come, as the metallurgist finds new and better materials which will insure unfailing reliability at higher pressures and temperatures. At the same time, developments must be forthcoming in the future which will cut down maintenance costs by eliminating the more common failures in the blading, packings, gears, shafts, and casings. Water-tube boilers have been developed to a high state of efficiency and dependability.

Speed-Reducing Systems for Propulsion Machinery

In order to take full advantage of the excellent efficiencies which the high-speed turbine offers, which must be combined with the low-speed propeller, various ingenious systems of gearing have been utilized. Of the three principal methods used, mechanical gearing is the most common, while strong claims have been put forth for hydraulic coupling. The electrical method, while having advantages of its own, apparently for the time being must be restricted to special cases.

Mechanical gearing may be either single- or double-reduction, the advantages to the shipowner being in the economies. Both types are reliable. While the double-reduction gear may be slightly less efficient than the single and involve more of an engineering risk, the higher turbine speeds permissible with double-reduction result in better over-all efficiency. This system of gearing makes possible the use of a turbine of smaller size, fewer

blades, greater rigidity, and one that is less likely to be susceptible to distortion, misalignment, and defective workmanship. As against this, must be considered the slightly greater reliability of the single-reduction gear. Also, because of the practical limits to the gear size, the application of single-reduction gears becomes more suited to cases where high revolutions and high powers per screw are desired.

The turboelectric drive has a high degree of maneuverability. It also reduces engine-room noise and helps to eliminate vibration, since torsional vibration is practically absent from this type of machinery. At reduced speeds, turbine efficiencies are poor and, consequently, where operation at low speeds and powers is required for any considerable length of time, it may be of real advantage to cut out one turbine while the others run at high efficiency. In the turboelectric drive, the astern element with its mechanical problems is also eliminated. Turboelectric drive is therefore of considerable value in naval vessels.

On the other hand, the first cost of the electric drive is apparently higher and the requirements of floor space greater. The fuel consumption is higher than that for the mechanical gear drive as a result of transmission losses, excitation, ventilation, lubrication, etc. Only special conditions of service would seem to justify the higher capital and running costs of this type of installation. Particularly it can be recommended for large passenger ships where elimination of vibration is of importance.

This type of marine installation, when compared with the geared turbine, has provoked many heated arguments among the experts. Whatever the outcome of these discussions, the shipowner must as usual decide the matter for himself. He cannot afford to overlook the basic value of any type of power plant, since it may help him to survive in his hazardous and highly competitive business.

Developments in Marine Diesel Engineering

It seems to be generally agreed that, if the Diesel had no other justification for coming into being, it has fully earned its place by providing healthy competition for the steam turbine, thus forcing the art of steam engineering into more advanced and efficient stages. In the course of its development, this type of power plant has progressed from the heavy, single-acting, four-stroke-cycle engine to the lighter, single- or double-acting, two-stroke-cycle type. During the course of its evolution many refinements in design have been made. As in the case of the steam turbine, inherent metallurgical problems will have to be solved to make possible further and more telling advances.

Diesel engineering has reached a high state of dependability. There are many Diesel installations which have rendered excellent service for many years. While in the United States the adoption of this type of power plant for marine uses has been comparatively insignificant and behind most of the European maritime nations, notably Norway, Denmark, Sweden, and Holland, where the motorship tonnage has reached roughly from 60 to 75 per cent of their respective totals, American shipowners cannot afford to overlook some of the real economic advantages which the Diesel affords.

Since its efficiency is equally high for all sizes and because it is adaptable for direct, mechanical, electrical, or hydraulic drive, the range of serviceability of this type of machinery is unusually wide. The excellent thermal efficiencies can be increased further by the utilization of exhaust heat for auxiliaries, as is being done regularly with varying degrees of success and economy. Proposals have been made to use multiple, high-speed, Diesel-electric units where high powers are required, since it is merely a question of time until the practicable limits for Diesel size will be established. Such installations undoubtedly will be made in the future. As the engines grow in size and power, the heat problems grow with them. Consequently, there are certain metallurgical and engineering limitations upon size, and new ways of utilizing the efficiency of the Diesel will have to be found if it is to continue into the field of higher powers.

At present, the first cost of a Diesel ship in this country is higher than that for steam. In certain cases, the weight and floor-space requirements of this type of power plant appear to be greater than would seem to be desirable. Because of the comparative inexperience of some of the Diesel-engine builders of the United States, there may occur an occasional failure of cylinder covers and crankshafts; there may also be some instances for reasonable complaints as to liner and piston-ring wear, fractured bedplates, or misalignment of shafts. However, the difference in maintenance cost between Diesel and steam drives is apparently not too great.

Fuel economy for the Diesel may vary somewhat according to the heat content of fuels used and the design of the engine. The Diesel will always have some advantage over steam in the matter of fuel consumption, regardless of how greatly steam machinery is improved, for the obstacles to further economies appear to be common to both types. Too often, however, the Diesel has been compared with some older steam installation and a false value derived.

During the present program of rehabilitating the American merchant marine, the problem of proper choice of marine machinery is being placed acutely before the operator. The question of steamship versus motorship must be decided. While the engineering profession can supply voluminous technical data on the two types and thus aid in arriving at a proper answer, the operator cannot overlook certain broader aspects of marine-machinery economies.

While the question of fuel itself appears to be neither a serious nor troublesome factor of operation, its importance in determining the relative advantages of the motorship and the steamship is undeniable. Experience indicates that the price differential between motorship- and steamship-fuel grades is important in the comparison of the two types, and that this differential never remains constant.

In Europe, where the fuel-price differential has been smaller than in America, the motorship with its favorable fuel-consumption rate has been economically more desirable. Therefore, the Diesel engine as a power source has been much more widely adopted on the Continent than in America. The fuel-price differential apparently is increasing in Europe; what it will be in the future is unpredictable. This differential will largely determine which of the two types of power will have the advantage and to what degree.

The strategic value of fuel oil cannot be denied; also its availability is apparently not limitless. Undoubtedly, political and economic factors of the future will have a profound influence upon the economics of ship operation. Enthusiasm for either steam or Diesel propulsion should be tempered by this consideration. It would be necessary to penetrate the future, to find the answer with regard to these two types of power plant.

Power as a Factor of Operating Costs

Among ship operating costs, the cost of power is not the greatest nor the most troublesome. During the design and construction stages of a vessel, the broad aspects of power selection, as they affect the economic life of the vessel, undoubtedly are of great importance, demanding considerable study. When the operating stage is reached, however, the cost of power is a stable factor requiring a lesser degree of vigilance on the part of the operator than other operating items. A typical example of the breakdown of the operating-expense dollar for an 8500-ton, deadweight, 12-knot cargo vessel in American Scantic Line service is:

Crew wages	15¢
Crew subsistence	3
Supplies and equipment	4
Repairs and maintenance	6
Insurance	7
Port charges and other port expenses	12
Cargo expenses	33
Agency expenses and brokerage	5
Fuel	15
	100¢

The cost of power, as made up of fuel, lubricants, engine supplies, engine-room crew, and repairs and maintenance of machinery, aggregates roughly 29 cents of each operating dollar expended and, consequently, is next in importance to the cargo-handling expenditures.

In the American Republics Line, where a de luxe passenger service to South America is conducted with modern turbo-electric vessels of 17,000 shp, developing sea speeds of 18 knots, a typical example of the breakdown of the disbursement dollar is as follows:

Crew wages	21¢
Passenger and crew subsistence	13
Supplies and equipment	6
Repairs and maintenance	6
Insurance	6
Port charges and other port expenses	10
Cargo expenses	17
Agency and brokerage expenses	9
Fuel	12
	100¢

The cost of power in this instance constitutes only 23 cents of each operating-disbursement dollar.

For tramp tonnage, handling bulk commodities, for tankers and similar tonnage, power costs, of course, may assume quite different proportions, going as high as 50 per cent of the entire costs. This largely accounts for the popularity of slow-moving ships, usually with reciprocating steam machinery.

Of the cost of power components, the fuel expense is the greatest but, while important in itself, only routine vigilance is required to control it. The same applies to the problem of repairs and maintenance.

Generally speaking, it may be said that the question of whether or not the cost of power is to become troublesome from the viewpoint of economics depends largely upon the wisdom and judgment of ship operators in the selection of ships to fit the requirements of various trade routes. The tools must fit the job if the job is to be well done. There is no such thing as a ship satisfying the requirements of all routes. Consequently, in a broader sense, good engineering, as applied to the field of marine transportation, may be considered as not being confined merely to the measure of thermal and mechanical efficiencies of marine power plants, but to the degree with which these efficiencies meet the economic dictates of the multitude of arteries through which the life blood of international commerce flows.

Ship Construction Costs

Needless to say, one of the principal factors entering into the design of any vessel, and one not to be found in textbooks of naval architecture, is that of construction cost. By all odds this is the most puzzling factor in the United States. Undoubtedly shipowners in all parts of the world think that their shipyards are charging too much. It is an established fact that wages and living conditions are on a higher scale in the United States than elsewhere. From this it may readily be deduced that American ships operating on the high seas in competition with unrestricted foreign tonnage must be subsidized. The amount of this subsidy can be fairly well established by a comparison of wages and similar costs, with those of competing foreign lines. It is the author's opinion that the differential is a good deal higher at present than it will be in the future, owing to the gradual leveling of wages and living conditions throughout the world. The subsidy is higher than it need be under any circumstances, if and when the American public actually begins to support its own flagships. It is hoped that the taxpayers, from whom these subsidies emanate, eventually will awaken to the advantage of supporting to the fullest extent their own merchant marine, which is indeed a part of their national defense.

When it comes to establishing the cost differential between an American ship and a similar ship built in a foreign yard, the matter becomes more confusing and there cannot help but be a feeling in the minds of many shipowners that all is not as it should be in our shipyards. Our shipyards build good ships. They are the equal of ships built anywhere in the world, and the superior of many. An explanation has been offered that, since most of the building has been Navy work, there have been very few ships commercially built in the United States and that part of the excessive cost is attributable to the application of Navy requirements to commercial construction. There is not much to this argument. When the United States Maritime Commission sent out its first invitations to bid on commercial tonnage, some startlingly high bids were received. The favorite explanation was that many unusual requirements were specified. The author is reliably informed that the Maritime Commission promptly met this situation by asking what reduction in price would be offered for the revision or elimination of any or all items or restrictions which imposed unnecessary expense. The resulting prices from the shipyards indicated that the effect of naval requirements on the bids had been negligible.

The National Council of American Shipbuilders, presented a very interesting document[2] to the Committee on Merchant Marine and Fisheries of the House of Representatives on February 29, 1928. In this document, it was stated:

"The principal reason for the greater cost of American vessels is the high wages paid American labor, which are nearly double those paid in Great Britain. . . . . There is some difference in the cost of material in the United States and Great Britain, but this difference is not as great as the difference in cost of labor. As shown by the *Iron Age*, steel plates, as of February 9, 1928, are only 10 per cent higher f.o.b. Pittsburgh than f.o.b. mill, Great Britain, while shapes and bars are 20 per cent higher."

In this valuable document,[3] the cost of building a 10,000-ton deadweight 14-knot cargo ship, equipped with high-pressure-steam machinery, is broken down. A figure of $171 per deadweight ton is deduced of which 50.2 per cent is material and 39.94 per cent is labor; the remaining 9.86 per cent being for customary miscellaneous items. At another point[4] in this document, a price of $125 per deadweight ton is given for a 10-knot freighter. At that time, namely, 1928, the building price of this class of tonnage in Great Britain varied between $42 and $45 per ton, which leaves in the author's mind, at least, an unexplained discrepancy of excessive cost in American yards of about $25 a ton. The National Council of American Shipbuilders further stated:

"Upon the assumption that both the foreign and American shipyards have a normal amount of work on hand, so that the efficiency of the employees of each class of yard is substantially the same, and upon the further assumption that each class of yards has the same percentage of overhead expense, the National Council of American Shipbuilders, after careful investigation, finds that it costs 59 per cent more to build a 10,000-deadweight-ton cargo steamer in the United States than in Great Britain. . . . Under present conditions the above assumptions do not exist, because there is no shipyard in the United States now engaged

[2] "Statement of Conditions of the Shipbuilding Industry in the United States," by H. Gerrish Smith, vice-president of the National Council of American Shipbuilders, New York, N. Y.

[3] Ibid, Exhibit VII, p. 38.

[4] Ibid, p. 15.

in new construction that is operating at over one third of its normal capacity. Therefore the present-day prices show an even greater differential than the percentages given above."

This might seem at firsthand, at least in some degree, to account for the unexplained $25 per ton. However, on May 10, 1938, one of the major shipyards in the United States signed a contract for a very fine ship, the first of a group of fifty or more being built by the United States Maritime Commission, at a fixed contract price of $2,080,292. From then on began a tremendous upswing in the volume of tonnage being built, not only by this yard, but by all the major shipyards in the United States. So great became the building program that the Maritime Commission has taken steps to encourage the expansion of shipyard facilities in order to proceed in an effective manner with its program. In view of this condition, it would seem that a point had been reached at which American shipyards might be considered to be operating under what should be their maximum efficiency. From personal inspection, it is a fact that this shipyard had every way filled on July 11, 1939, a little over a year later, when a bid was entered upon a similar ship but 1000 tons smaller and about 2 knots slower. The price submitted was almost identical with the earlier contract price cited, namely, $2,020,000.

The Merchant Marine Act of 1936, under which these ships are being built, definitely limits the top salaries that can be paid to anyone in the yard or, at least, that can be charged against the ship. Further, it requires that there shall be paid back to the government any profit in excess of 10 per cent of the total contract price. The motive is therefore not excess profits. If any remedy is to be found for the seemingly excessive high cost of tonnage, it must lie entirely in a more economical operation of our yards.

It is not proposed that there should be any change in the standard of wages or living conditions, either in United States shipyards or on American ships. The remedy cannot lie there. As a matter of fact, perhaps this idea of standard of living is a little out of perspective. Measured by automobiles and radios, it is high, but measured by many of the other things that go to make up the life of a family, there may not be so much to boast of concerning living conditions in America. As previously noted, a leveling-off process in living standards is undoubtedly occurring throughout the world. The International Labor Office at Geneva, probably about the only department of the League of Nations which still functions, has for its avowed purpose that objective.

Support by the American public can cure the situation afloat. In the case of the new American Republics Line, this support has been given by the traveling public. It is hoped, eventually, that similar support will be received from the shipping public. Correcting the high cost of shipbuilding, however, is an internal problem and one to which the general public may not contribute. The National Council of American Shipbuilders has said: "Give us some ships to build and then we will get the cost down." That has been done and the reverse has happened.

There seems to be but one answer to this high cost of American shipbuilding and that is the incompetency of the shipyards themselves. This incompetency does not apply to the quality of their work, but only to their ability to organize themselves so that economical work may be performed.

Admitting that a ship is probably the most complicated, comprehensive, and difficult creation of modern civilization; admitting that all the difficulties which enter into any kind of construction are encountered in the construction of a ship, to which must be added the difficulties of dropping the ship into the water; and understanding fully that the vessel must be so complete she can sever all ties with the land for a long and arduous voyage far from any help except her own; admitting all the problems thus presented, there still seems, to the average layman visiting the usual American shipyard, a lack of organization and a certain amount of inertia between the several departments which are wholly unreasonable.

The capital costs of a ship are inescapable. Whatever else happens, these costs go on and on. A ship earns money only when her wheel is turning over with pay load aboard, but she can lose money with alarming rapidity tied to a dock and charged only with interest and depreciation. Therefore, one of the vital elements of a successful maritime industry is the matter of capital cost.

A noteworthy technical efficiency has been developed in the United States. The genius of men like Henry Ford has revolutionized the world in the last forty years; and what a strangely different civilization it would be were it not for the automobile industry. Some thirty years ago, there was grave danger that the automobile industry would develop into just a small enterprise turning out a few high-priced cars a year—almost in the class of museum pieces. Today, despite all the high costs of American labor, the major factor in the export trade of the United States is the finished product of this industry, the American automobile. It is the best and the preferred car in any market in the world and presents an amazing value for the dollar paid. It is the author's hope that the technical efficiency which has produced the American automobile will be applied to produce an American merchant marine.

Range of Ship Prices in Great Britain

The fluctuation of price in building ships has been ably illustrated recently in a British publication.[5] The steamer for which the diagram was prepared, and which may be considered as typical, is a single-deck type, with bare specifications, having a bridge, poop, and forecastle, and carrying about 7500 tons deadweight. From this diagram, the building cost at the beginning of the World War, that is, the second half of 1914, was something over £7 per deadweight ton. It rose rapidly as the war progressed, so that by the end of 1915 it had doubled to about £15. The following year it went to £23 then to £26, and actually reached its peak at the end of 1920, when the price rose to £30. The slump of 1921 brought it to £13 by the end of that December, and to £9 by December 1, 1922. It stayed in that vicinity for the succeeding fourteen years, so that, in June, 1936, a price of £9 12s prevailed. It rose slowly through 1936 and 1937 and by June, 1938, had reached £14 8s. By June 30, 1939, it slipped back to £13 6s 8d.

On the same chart another curve is drawn showing the sale values of similar tonnage during the same period of time. While this follows in general the building cost, the sale cost being affected not only by the cost of building but also by the state of the market when built, it occasionally fluctuated considerably above or below the actual cost of replacement tonnage. For example, in 1932 and 1933, when there was almost no cargo in the world to be moved, this tonnage had a sale value of £32,000 although it cost to build in the same period £62,000 to £63,000. The diagram shows clearly that in periods of poor freight markets, the sale price of tonnage is less than the replacement cost; while in good periods it may greatly exceed replacement cost. For example, it is well known that a spurt of shipping activity occurred during the second half of 1937. For a few months then it was impossible to get tonnage. The building cost showed from £78,000 to £100,000, but in about the same period the sale price advanced from £60,000 to £125,000.

In the same article[5] the rather significant statement is made:

[5] "Fluctuations in Shipping Values," *Fairplay*, July 6, 1939, supplement, p. 23.

"We asked a well-known shipbuilder what he thought the future held, and he replied, 'So long as we have a continuous demand for armaments I cannot see much hope for any reduction in the price of ships over the next year or two. The cost of social services is always increasing and there is never any hope of getting alleviation from this part of the cost of a ship as time goes on.'"

Personnel Relations a Vital Factor

As previously noted, the marvelous creation which is a ship has reached a high state of development. Opportunities for advancement in the art of ship construction still remain, but probably far greater opportunities lie in that field of endeavor having to do with human relationship.

Incorporated in a modern vessel are fireproof materials, watertight bulkheads, smothering equipment, and even machines for steering the vessel automatically, but in the end disaster can come and a ship will fail to function without the skillful guidance of a trained crew. Suppose an absolutely fireproof ship were built. Before she could move it would be necessary to introduce into her highly inflammable fuel. Before she could earn revenue her holds would have to be filled with inflammable cargo, and her public rooms with passengers careless with cigarette butts, careless with cleaning fluids, careless with electrical fixtures, all of which are necessary. Therefore, no matter how fireproof the ship may be to begin with, by the time she is loaded and ready to embark, she has added all the hazards which in building were so carefully avoided.

Essentially, the crews which man merchant ships have been more or less allowed to develop themselves. In the United States this is particularly true where no effort at all is made to train any except a handful of officers in the two or three state training ships which are in commission, although the Maritime Commission has recently attempted to develop an extensive training program.

A practical application of the social sciences and more enlightened personnel relations are the evident needs of the immediate future. Among personnel there is a general lack of understanding of the problems of management which confront a ship operator. It is not possible to blow a whistle at noon on Saturday and go home and leave the ship until Monday morning. Because of some dispute with the management, the ship may not be abandoned in some foreign port, or in mid-ocean until the dispute is settled. In the teeth of a gale, there is no opportunity to discuss overtime demands. A ship goes for days and weeks, and seemingly nothing happens and then, in an instant, disaster threatens. There must be complete organization, military in its authority, inexorable, and inflexible. The Soviet, attempting to operate its vessels by committees, encountered one disaster after another, and finally gave up the idea. Much that is perfectly fair and proper and reasonable to be considered in the relationship between employer and employee, or boss and workman on shore, must be abandoned entirely by the sailor. A new social science must be devised.

There is also much yet to be done in the matter of systematic management. The unknown factors in operating ships will always be numerous. The reduction of known factors to a workable understandable basis should be prosecuted with all haste. Discussions of this kind assist. Probably more effective than anything else is the rapidly increasing availability of sound practical university courses in various phases of the marine-transportation industry. Young men completing these courses, if given proper subsequent training at sea, can eventually become of utmost value in the shore establishments. They are needed for analyzing the effect of all the factors that enter into the operation of a ship; for studying fuel consumption, the loading and discharging of a ship, her scheduling and turnaround, and the accumulation and disposition of cargoes.

Cargo-Handling Problems

Probably the problem that has had the least attention, or at least has undergone the fewest changes, is that of loading and unloading cargoes. At least 25 per cent of a ship's revenue goes into this operation, if she operates in the usual trades. In addition, while the vessel is being loaded or discharged, she is necessarily idle at a dock. The methods of loading, stowing, and discharging are in about the same state as they were generations ago. The winches are a little faster and may be electrically driven, making them easier to control, but the old burton is still dropping down into the hold where a few men with hooks, crowbars, or their bare hands manage to hook on a sling load for hoisting and discharging on the dock. Or a sling load comes down to them, is dropped in the middle of the hatch, and is lugged by the men to some corner for stowage. There is a better way to do most of this that will occur to someone some day. Some experiments have been tried, but these have been limited to peculiar situations. For example, in the *Seatrain*-type ships, an entire carload of freight is loaded into the ship, car and all. Some cargo has been strapped to platforms which can be lifted out of railroad cars by spade trucks, run over the dock to the ship's side, and easily slung on board, where frequently another truck picks it up in the hold.

In the "Good Neighbor Fleet" of the American Republics Line, escalators are used, for discharging and loading fruit and coffee, in conjunction with continuous roller conveyers. A cargo elevator is now being installed in the blind hold of each of the three ships. Some cargo is now being handled by the line in containers. Of course, the railroads, particularly abroad, are doing a very large container business. However these are only isolated cases, pointing the way to better methods which will be developed in the future. Now, only an infinitesimal part of the world's cargoes are handled by methods which in any way may be considered efficient.

Discussion

George H. Rock.[6] The author and the writer had similar assignments after graduating from the Naval Academy, as each was assigned at different periods to the same ship, the U.S.S. *Chicago*. When the writer graduated in 1889, she was the flagship of the "White Squadron." Probably the author did not have the same interesting experiences, however, when wanting to go ashore and asking permission from the executive officer, who was an exceptionally fine and able seagoing officer but gloried in being something of a martinet. On being asked for permission to leave the ship, he would look at the writer quizzically for a full minute and then ask: "Tell me how you would pump out A-19 in the event of damage in that compartment." If the correct answer was not forthcoming, there was no shore leave that day, nor the following day, unless the question propounded then was answered. It was always sure to be a different problem.

The author makes the inference in his paper that it is his belief the *Titanic* disaster resulted in double bottoms being incorporated in the designs of later ships. To correct the record, it is necessary to state that double bottoms were fitted in ships long before the *Titanic* was built. She had a double bottom extending throughout her length. In the wake of the machinery compartments, the double bottom extended up the sides to a height

[6] Rear Admiral (CC), U. S. N., Retired; President, Webb Institute of Naval Architecture, New York, N. Y.

of about 7 ft above the keel. The height of the inner bottom elsewhere was in general about 5 ft. The *Titanic* had been advertised as an unsinkable ship, but of course such advertisement would be taken with reasonable understanding of the design. Her damage, when she struck the iceberg at about 10 ft above the keel, extended over six compartments, i.e., the two forward boiler rooms and the four compartments forward of them. With these compartments full of water, no ship built at that time, nor at the present time, could remain afloat. Whether a double bottom, extending up the sides beyond the line of damage, would have confined the water to the outboard compartments, or whether, if the bulkhead deck had been watertight instead of weathertight, she would have remained afloat until the arrival of rescuing ships, are matters which we cannot know, but certainly they would have had some effect.

The writer is in sympathy with superliners and in that particular does not agree with the author. It is his belief that superliners are here to stay, and that the *America* only points the way for the ships of the future. If history is any guide, the *America* is far from the last word. The length and other dimensions of ships have increased at a slow but an unvarying rate from the beginning, and the changes looking to smaller ships have been for a short time only, when the increases in size again have been made. This trend was so well established in the minds of the Navy Department that some 12 or 13 years ago, when asked by the War Department for opinions as to the size of the new Panama Canal locks, the recommendations of the Navy Department were that they should be about 1300 ft long by 150 ft wide by 45 ft draft, in order to provide for what appeared to be the inevitable size of the ships about the end of the current century. Studying the records over the last 100 years discloses a steady and fairly uniform increase in length and breadth of 8 ft in beam and 80 ft in length for each period of 10 years. Probably sizes will not increase as rapidly in the future as in the past, due to considerations of cost, depth of harbors, etc. The Navy Department estimated an average increase of 5 ft and 50 ft, respectively, per decade during the remainder of the century. Incidentally, it should be noted that the *America* is appreciably larger than the *Washington* and *Manhattan*.

The author refers to the efforts of naval architects to produce nonsinkable and fireproof ships. We do not say that we can build nonsinkable ships, but we do say that we can build fireproof ships in so far as all materials entering into the construction are concerned. This, of course, as the author points out, does not make the cargo or the fuel oil fireproof, but ships are fitted with very efficient smothering devices for extinguishing fires in the cargo holds and, furthermore, the principal disasters at sea, due to fire, are not those in which the fires have started in cargo holds or fuel-oil compartments. They have been fires due to the use of combustible materials in the constuction of the ships, such as was the case in the *Morro Castle*.

The speaker refers to the slight justification for other than electric power for auxiliaries, and mentions the steering gear and galley ranges. It seems such a short time ago, about 25 years actually, when we in the Navy had to remove the electric-galley installations that had been made because they failed so quickly and completely in use. Fortunately, the electric-supply companies, by studying the particular requirements of galleys on board ship, succeeded in designing resistances, connections, etc., which made them practically foolproof. After a brief interval of time, the electric galleys were reinstated, until now, as the author informs us, they have almost completely displaced the oil-burning type of galley.

As to the steering gear, it seems also but a short time ago when we were spending so much time and energy in making the telemotor gear function, but, fortunately, before we had to give it up as an impossible job, the electric steering gear came into general use.

The speaker stresses the advantage of electric drive because of the elimination of vibration. That does not appear to have applied to the *Normandie*.

J. F. METTEN.[7] The author apparently has presented this paper as a résumé of the present conditions and trends in operation, design, and building of ships and some of the causes responsible therefor. Such papers naturally represent the considered opinion of the author, and their value largely depends upon the integrity of the basic facts which are used to support these opinions.

Under the heading "Developments in Hull Construction" reference is made to the *Titanic* disaster, which is stated to have brought about a complete revision of bottom construction, leading to the adoption of double bottoms in the merchant marine long after that design had been adopted by the Navy. The *Great Eastern*, completed in 1859, was provided with a double bottom carried up to the lower deck. Double bottoms were adopted in transatlantic liners built long before the *Titanic* was thought of. The *Paris* and *New York*, completed in 1890, had double bottoms with advanced transverse subdivision, no access openings of any kind being provided through transverse bulkheads below the main deck. The *Titanic* was provided with a double bottom. The disaster resulted from contact with ice above the inner-bottom margin, thereby opening up a series of compartments. All the transatlantic liners of that day were vulnerable in this respect as, when wing coal bunkers were provided in the wake of boiler compartments, it was recognized that power-operated watertight bunker doors were of doubtful value in such emergencies. The change from coal to oil fuel and elimination of open bunker doors was the development which permitted a more rational approach to the matter of effective subdivision.

Reference is made by the author to "the elimination of vibration inherent in the electric gear." He states: "The turbo-electric drive has a high degree of maneuverability, reduces engine-room noise, and helps to eliminate vibration, since torsional vibration is practically absent from this type of machinery."

This reference to the matter of vibration is somewhat misleading, since there is no inherent advantage in this respect possessed by the electric drive over the geared-turbine drive. Both, so far as the driving end is concerned, are completely balanced units provided that they are properly designed and constructed. Uneven torque at the propeller, arising from variable wake conditions in the propeller path, may cause torsional vibration in line shafting regardless of whether the shafting is driven by turbogeared or turboelectric propelling units. Moreover some of the most serious cases of vibration in the wake of passenger quarters arise from this variable propeller condition. The *Normandie*, a turboelectric-drive ship, is a good example. The very satisfactory correction of the vibration, experienced on the first trips of that vessel, involved the correction of unfavorable propeller conditions and interrelated structural factors.

The author's analysis of the cost of American-built ships and his claim that the differential between foreign- and American-built tonnage is largely due to the incompetence of American shipyards, seems to be based principally upon the fact that American automobiles are built as cheaply as foreign cars. He apparently feels that the quantity-production methods, special tool investments, conveyer assembly lines, etc., permitted in building 500,000 or 1,000,000 duplicate automobiles should be applied in a shipyard which is lucky indeed if it can get 4 dupli-

[7] President, New York Shipbuilding Corporation, Camden, N. J.

cate ships to build at one time. A proper and unbiased analysis will show that the differentials in cost between American and foreign flagship operation are not as great as the differentials in cost the American shipbuilder has to face in comparison with a foreign builder. Therefore, if the author's contention in respect to American shipyards is based upon sound reasoning, the writer would suggest that he might dispense with any form of government subsidy to the steamship lines he represents, by applying the same measure of efficiency to their operation.

B. E. MEURK.[8] A serious situation facing the shipowner, which has not been commented upon by the author, and which the writer believes should be given some thought, is the conditions of regulation and inspection of vessels imposed by the government on the shipowner. Shipbuilding and ship operation are of greater interest internationally than any other industry in existence. It has been found, therefore, of advantage to make rules and regulations in connection with the construction and safe operation of vessels somewhat uniform among nations. To create a mutual and common understanding of these problems, international conferences are held at which technical experts from the various maritime nations seek agreement upon construction features of ships and machinery in line with advances in the art. The most important conference of recent years was held in London in 1929. In the United States, however, there has been a tendency during subsequent years to exceed greatly the standards of safety agreed upon at that conference.

When accidents or catastrophies occur at sea, it is the practice of American newspapers to report the incidents in a sensational manner, which tends to play on the emotional side of the public. Senators and Congressmen feel it to be their duty to submit bills to Congress for the purpose of preventing the recurrence of such disasters. The laymen and average citizen never realizes that ships at sea constitute the safest means of transportation in existence and that these accidents are relatively small considering the magnitude of the business of ship operation. Of late, Congressional investigating committees have caused excessive regulation in construction of ships and control of steamship lines, which has resulted in increased shipbuilding and operating costs. Furthermore, it has interfered with the design and construction of a proper ship to meet strictly competitive trade conditions with foreign ships, particularly with reference to suitable cargo holds and the net registered tonnage which can be obtained for a certain cubic or cargo-lift capacity.

Due to the fact that the United States has imposed a far greater burden and responsibility on the shipowner than, for instance, the government of Great Britain has done, embarrassing conditions are likely to occur when chartering vessels of that country's flag. To illustrate this point, at the present time the author's company is having an important case tried in the U. S. Federal Court, in which the cargo underwriters are prosecuting an American steamship line as charterer of a British ship, for the loss of their cargo. After the casualty had occurred, the shipowner, the seaworthiness of the vessel, and the conduct of its crew were investigated by the British Board of Trade, and the vessel was declared seaworthy, while the shipowner was exonerated and exempted from responsiblity. However, as the cargo was owned by American interests and the party issuing the bills of lading was an American steamship company, the seaworthiness of this particular vessel and the owners' responsibility must now again be established in court under the American laws. It is easy to understand how embarrassment may occur if the laws and regulations of one nation establish different requirements from the laws governing steamship operation in another. Inevitably this will occur if we depart too greatly from international opinions in regard to seaworthiness and regulations of safety of a vessel.

E. H. RIGG.[9] Referring to the section, "Developments in Hull Construction," the author states that the *Titanic* disaster brought about a complete revision of bottom construction and led to the adoption in the merchant marine of the double bottom long after that design had been applied to naval vessels. This is a somewhat misleading statement. The earliest forms of merchant-ship double bottoms go back to about 1855; the modern double bottom is recognizable as far back as 1875, since which date they have been fitted in increasing numbers and improved forms. At the time of the *Titanic* disaster in 1912, they were well-nigh universal in deep-sea passenger ships, whatever may have been the case as to coastwise vessels. At that time, there doubtless were many single-bottom deep-sea survivals but not so for ships then or recently building. Warships may well have been ahead of merchant ships in this respect; it is doubtful if many double bottoms of date prior to 1860 could be found in warships. The widely known *Great Eastern*, 1858, was built to a design involving much that was ahead of its time, among other things, a full double bottom.

The author refers to flooding curves, inferentially, as a modern, post-*Titanic* development. They have been used and applied in their present form since 1890 and doubtless in earlier forms before that, especially in warships.

Stability in damaged condition has always been with us; in recent years, oil fuel has increased the dangers and complicated their determination, but practical methods of assessing the adverse effects of damage on a ship's stability have been devised and made available.

The author draws a picture of the difficulties of the naval architect in satisfying all the demands of the owner's different departments, in one comprehensive design; one reply is that the owner should know enough of his business to talk directly to his architect and handle his department heads himself. Leaving it to the hapless architect to "trade" one departmental chief against another does not help the owner much in getting a good ship; it might well increase the costs.

The statement is made that "most of the formulas used in designing a ship are inevitably arrived at by the trial-and-error method." That word "inevitably" is well placed. For a ship floating in smooth water, we need little or no "trial and error;" like the bridge engineer, we can design quite closely; then our troubles begin. Even harbor vessels bump against piers—how hard? A clumsy tugmaster can, in an instant, make the cleverest mathematics look sick. He does not necessarily have to be clumsy; he may have the bad luck of wind, tide, fog, and other vessels getting him in a tangle with but little, if any, fault of his own.

Just what margins over and above "accidentless" operating stresses is the owner willing to pay for? This still leaves our old friend "corrosion" out of the picture entirely. The big trial and error is in the inconsiderate action of the ocean. Any clever skeptic, who goes to sea and watches a first-class Atlantic storm, will come home considerably sobered as to his ability to calculate the stresses set up in the hull structure by the action of the supporting water.

Improved educational and training facilities have been giving us a steady stream of well-educated young men for many years past. In spite of this, we still hear from time to time, that our old friend the sea has caused "cracks" to develop in ships. The main point is that the clever young men are not quite clever enough yet and the "conservatives" still have their uses in ship structural design.

[8] Superintendent Engineer, Isthmian Steamship Company, New York, N. Y.

[9] Naval Architect, New York Shipbuilding Corporation, Camden, N. J.

Another former trial-and-error process of the first importance was that of hull form and the power necessary for a given speed. Thanks to William Froude and his disciples, that phase has yielded to treatment; all the competent naval architect needs nowadays is a reasonable time and cost allowance for model experimentation. Any questions not settled by model tests, to an accuracy of within 2 or 3 per cent, may well be left for experience to determine. With regard to such testing, we do not claim to know it all, but the progress of the last two generations has been great.

H. Gerrish Smith.[10] From the standpoint of a ship operator, this paper presents an informative analysis of the characteristics of present-day merchant ships, particularly as to their types of machinery; the evolution of ship design over a period of years; the fundamentals of design; and the problems of the designer in reconciling the many demands of the various branches of ship operation.

The writer can agree with many of the observations of the author until he reaches the section on "Ship Construction Costs;" even then, there is agreement with some of his statements in that section. However, the writer wholly disagrees with his conclusion that the high cost of ships is due to "the incompetency of the shipyards themselves." Such a conclusion is considered to be unfair and unreasonable.

The author asserts that "our shipyards build good ships" and that "they are the equal of ships built anywhere in the world and the superior of many." The writer would go further and say that the ships we are now building are of more exacting specifications and, therefore, are in many respects superior to those building anywhere abroad for similar service, and are far ahead of those built in this country 10 years or more ago.

Since the previous war, the speed of cargo vessels has been increased on the average from 11 knots to $15^1/_2$ knots. On the basis of earlier designs, such an increase in speed would require more than double the power and, therefore, double the fuel consumption. However, improvements in form, propulsion, and fuel per hp-hr have kept the fuel consumption per knot to practically the same for the $15^1/_2$-knot ship as for the earlier 11-knot ship.

Reduction in the weight of the ship by the use of welding and the use of more efficient machinery have resulted in a greater deadweight carrying capacity within the same over-all dimensions.

These improvements have made it possible for 3 ships of today's design to do the work of 4 ships of the older design—a very important point to be kept in mind in any discussion of this paper.

Other features of the design to meet owner and government requirements or to comply with new acts of Congress to make a more workable and a safer ship are as follows:

1 Improved accommodations for all members of the crew, including locker rooms, washrooms, and recreation spaces.

2 Extension of electricity to the point where virtually everything but the main engines are operated electrically. Expensive but efficient electrical equipment has replaced less expensive and less efficient steam-driven cargo winches, steering gear, anchor gear, capstans, and engine-room auxiliaries. Its use has made possible the development of desirable equipment unknown in the older ships, such as automatic alarm systems, remote controls, fans, and mechanical galley equipment.

3 The simple cowl ventilators for natural ventilation have virtually disappeared from the latest ships and in their place are complex mechanical ventilating systems. Cargo holds, once without ventilation, are now well aired by special air-conditioning equipment in order to minimize the spoilage of perishable cargoes.

4 Ratproofing for the preservation of health of passengers and crew and the protection of cargo.

5 Greatly extended use of fire-resistant materials; passenger ships are divided into zones to localize fire should it break out, requiring the insulation of many bulkheads and decks within these spaces which would not otherwise be needed; noncombustible material instead of wood in stateroom construction.

6 Greater subdivision of ships for safety.

7 Higher standards for lifesaving equipment; automatic watertight doors; greatly increased fresh-water-evaporating capacity.

8 Radio direction finders; electrical sounding devices; gyrocompasses and mechanical steering apparatus to take the place of the compass and the lead.

9 More outside rooms for passengers on passenger vessels.

The additional requirements must be warranted by a resultant increased efficiency and safety or a reduced operating cost, but certainly the increased cost due to their addition cannot be laid at the shipbuilder's door.

The cost of ships, however, does not stop with these added owner and government requirements for the following reasons:

Wages today are higher than at any time in the past, including the war period and 1929. Shorter working hours required by law have increased the time to construct a vessel because many highly skilled operations cannot be carried on by second and third shifts. A longer time of construction increases the overhead expense. Overtime allowances have been increased. Vacation with pay is now quite general. Increased taxes, both local and federal, and social-security taxes, where none existed a few years ago, have added to cost. Increased labor questions following passage of the Wagner Act, the Walsh-Healey Act, and the Wages and Hours Act, and the increased expense of workmen's compensation, by broadening the scope of state laws, have added to cost.

National-defense features in some cases require more power and speed than are necessary for peacetime operation, thereby adding to cost; some defense features are an integral part of the ship's structure and add to cost.

These various elements of increased labor cost are not peculiar to shipbuilding alone, but are experienced likewise by the suppliers of material from whom the shipyard must purchase such items as steel, boilers, turbines, navigating equipment, and innumerable other items. Practically all classes of material cost more now than in the past. There has also been an increased demand for special and higher-grade materials.

Delays occasioned by government inspection and approvals are vastly greater now than a few years ago. At the present time, plans of an ordinary cargo ship are subject to approval by the Maritime Commission, Navy Department, Public Health Service, and the Bureau of Marine Inspection and Navigation under the Department of Commerce. Plans are also subject to approval by a classification society, such as the American Bureau of Shipping, Lloyd's Register of Shipping, or both.

The increase in shipbuilding costs is not limited to the United States. They have gone up abroad as well as here but, as stated, the requirements and specifications for American vessels are much more exacting than for foreign vessels in a similar service.

Coming to the subject of shipbuilding versus the building of automobiles, the author appears to have reached his conclusion that shipyards are inefficient on the assumption "that the technical efficiency which has produced the American automobile" is applicable to the production of an American merchant marine. In his own language, he admits that a ship is a highly specialized structure, as he states: "The tools must fit the job if the job is to be well done. There is no such thing as a ship satisfying the requirements of all routes." He must know that to the

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contrary an automobile is a highly mechanized product. Wherein the latter is produced in great numbers, often as high as hundreds of thousands or even a million from the same designs, patterns, jigs, and fixtures, a ship is a custom-built product, of which only a few are ever fabricated from the same design. An entirely new set of plans and new patterns are required for each new type of ship. Furthermore, the use of costly jigs and fixtures for individual parts of a ship is not permissible except to a very limited degree, because of the high cost of production of such jigs and fixtures and the small number of ship parts that could be produced from them.

Limited study of the problems involved in the construction of a made-to-order structure such as a ship, as compared with a mass-production article, such as an automobile, will show clearly to anyone, who cares to investigate the subject, the essential differences in the methods of production employed for each and the effect of mass production on reduction of cost. A specially designed automobile, not standard, would cost many times as much as a standard automobile produced in quantities.

Mr. Ford apparently overlooked the difference between automobile construction and ship construction when he undertook to build 200 Eagle boats for the Navy Department during the previous war (about half of which were completed), on an attempted construction basis similar to that used in building automobiles. If the writer's information is correct, Mr. Ford lost money on that contract and it is noticeable that he discontinued shipbuilding.

There are a few specific statements, in the author's section on the cost of ships, requiring comment as, for example, the following:

"When the United States Maritime Commission sent out its first invitations to bid on commercial tonnage, some startlingly high bids were received. The favorite explanation was that many unusual requirements were specified. The author is reliably informed that the Maritime Commission promptly met this situation by asking what reduction in price would be offered for the revision or elimination of any or all items or restrictions which imposed unnecessary expense. The resulting prices from the shipyards indicated that the effect of naval requirements on the bids had been negligible."

The last sentence in this statement would seem to have no bearing on the subject of the elimination of items, but the records show a very substantial reduction due to eliminations, running as high as 13 per cent.

Reference is also made to the bid of one of the major shipyards on the C-1 cargo vessels. The gist of the statement is that ".... this shipyard had every way filled on July 11, 1939, a little over a year" from the time bids were submitted on the C-2 vessels until "a bid was entered upon a similar ship but 1000 tons smaller and about 2 knots slower. The price submitted was almost identical with the earlier contract price cited, namely, $2,020,000."

In the first place, a reduction of $60,000 from the bid of $2,080,292 on the C-2 vessel is hardly an "almost identical" bid and, in the second place, a careful review of the specifications will show that the C-1 vessel had additional requirements over the C-2 vessel which are estimated to cost not less than $150,000 per ship.

On the occasion of the opening of the bids for the C-1 vessels, one of our shipbuilders was accompanied by a British shipbuilder who remarked that he would like to take a contract for the construction of these vessels at half the bids submitted. He was asked, "To the same specifications?" and he replied, "Oh no." What the Britisher had in mind was a ship built to an entirely different specification.

While foreign ships as built are undoubtedly satisfactory to their owners they would not pass present American specification standards; the fuel economy would not be acceptable to the American owner; ratproofing and fresh-water arrangements would not be satisfactory to our Health Department; accommodations and many other features of the foreign vessels would not meet the standards of our Maritime Commission and of the Bureau of Marine Inspection and Navigation.

At another point in his paper, the author states: "The National Council of American Shipbuilders (in 1928) has said: 'Give us some ships to build and then we will get the cost down.'" He goes on to say: "That has been done and the reverse has happened." This statement is, of course, in error as the actual quotations, within the last 2 years, for ships for the Maritime Commission and private operators, show that, in every case where bids are called on 2 to 8 ships, the bid per ship for more than one has been less than for one ship of a design. The average reduction in bids for 4 ships of the C-2 type is about 13 per cent each less than the cost of 1 ship; the average reduction for 4 ships of the C-3 cargo type about 10 per cent less; the C-3 passenger-and-cargo type, about 11 per cent less; and the C-1 cargo type about 16 per cent less.

An analysis of the bids on the C-1, C-2, and C-3 cargo ships, the C-3 passenger-cargo ships, ships for the Seas Shipping Company, for the Export Line, and for the Mississippi Shipping Company, shows that, for 60 of these ships where bids were submitted for group construction as well as for a single ship, the actual saving on group-construction bids would be in excess of $18,000,000; in other words, a sum sufficient to build 9 additional C-1 ships.

The Maritime Commission has performed a commendable job, through a substantial degree of standardization of ship forms and characteristics, which has made possible the ordering of groups of ships of the same design, and has thereby effected considerable savings, as indicated for each ship of a group, under the cost of a single ship of a given design. By limiting the number of designs it is permitting the program to be carried out with a much smaller technical staff of shipyard employees than would otherwise be the case.

The author is also at a loss in his paper to reconcile certain figures presented by the National Council of American Shipbuilders before the Committee on Merchant Marine and Fisheries in February, 1928, in which statement the National Council presented certain figures as to differentials in shipbuilding costs in the United States and abroad. This statement of 1928 has little to do with the present-day cost of ships. However, to reconcile the figures of the National Council it should be noted that the statement referred to a price of $125 per ton for a 10-knot freighter, built to American design and specification. The author states that at that time "The building price of this class of tonnage in Great Britain varied between $42 and $45 a ton." Taking account of the National Council's figure for cost differential, he cannot reconcile an apparent difference unaccounted for of $25 per ton. The answer to this apparent discrepancy is simply due to the fact that the type of 10-knot freighter referred to by the National Council is a far different type of vessel from the simplest type of freighter on which quotations appear from time to time in the British press.

The shipbuilder offers no apologies on the subject of efficiency. There are numerous shipyards in the United States in different geographical locations, owned by different interests, run by different managements, and each is doing its best to devise ways and means of building ships at less than the cost to the other fellow. Competition alone is a sufficient incentive to each shipyard to do its best to produce at a minimum of cost.

Every conceivable method to increase shipyard efficiency has been studied and those judged to be sound have been adopted. Some yards have built up extensive organizations for producing

plans covering every detail of construction, for arranging schedules for the orderly progress of each part of the work, and with ample supervision to see that each workman knows exactly what he has to do and that he does it at the proper time to dovetail into other work under way. Other yards have tried and are trying the plan of keeping down organizations, issuing only the least possible instructions, and leaving it as much as possible to foremen and workmen to build the ships. Some yards have production departments of considerable extent and a production manager whose sole duty it is to provide for the most efficient possible production.

The shipyard practices of the different shipyards are well known each to the other and there is at least a general knowledge in American yards of the practices in foreign shipyards. It is incredible that those same shipyard managements which have produced efficient designs and "good ships" have not devoted equal energy to the development of efficient methods of production.

As a matter of fact, the going shipyards of the United States have kept their equipment up to date since the last war, many times when there has been promise of but little work ahead for them. They also have given close attention to the development of efficient methods of shipyard operation.

Anyone, of course, might obtain an opinion about the efficiency of shipbuilding by a careful investigation and study of the industry, and that is always invited. It is quite apparent, from the analysis that has been made in the foregoing of some of the statements in the paper in question, that the author has not studied conditions in the shipbuilding industry carefully or he would be more conversant with the facts. There appear to be no facts to justify his conclusion of "shipyard inefficiency." The writer is quite sure that any disinterested investigation of shipbuilding will show that the industry is well abreast of other American industries in its efficiency, and far in advance of many. In relatively few industries has the progressive development of both the product and manufacturing methods been so noted in the last 20 years. It might be concluded that the operation of ships is inefficient, but the writer's opinion would not make it so. The writer's background of experience in ship operation is not sufficient to justify reaching such a conclusion.

The shipbuilding industry is appreciative of the problems confronting the ship operator and has every reason to be sympathetic toward their solution. No shipbuilder profits in the long run by a curtailment of the volume of his business. Every shipbuilder therefore has every reason for reducing the costs of shipbuilding in order that his yard may remain active. But the shipbuilder has to provide what is ordered, not what is least expensive. His bid is controlled by the specifications. He is confronted in his operations by many matters largely beyond his influence. His own best interests are served by a policy of fairness to all elements concerned in the problem of upbuilding the American merchant marine. If there were anything inherently unsound in his methods, he would be the first to desire to correct it.

The shipbuilding industry is proud of its achievements in the design and construction of vessels, and in shipyard operation and believes it has been well at the forefront in the advancements that have been made by industry in the 20-year period since the previous war.

Author's Closure

In this closure I wish to express my appreciation of the considered discussions. Frankly, one of my purposes was to develop argumentation, particularly on the question of American costs of shipbuilding, and at least in this design a success must be admitted. In this closure it is not my purpose to prolong disputation, but merely to explain my point of view. Necessarily, in preparing a paper such as this, to be delivered in a limited time, one should eliminate all material possible. Even with this having been done the paper remains too long.

Doubtless the best refutation is a disclosure of error on the part of the opponent. I was careless in my reference to the *Titanic* and neglected to explain that I used the popular conception of the disaster rather than the technical one. However, Admiral Rock and others have carefully rectified this error, and there is nothing to be gained by a further discussion of it.

Mr. Metten has placed deductions in my paper which I do not think can fairly be found there. The paper was prepared for the consideration of a distinguished gathering of engineers. With men of this caliber it certainly would have been a waste of time to have elaborated my reference to the efficiency of an American automobile by an explanation that the building of a ship and the building of an automobile were not similar in so far as one could make use of quantity production methods. One of the speakers referred to the Eagle boats assembled by Mr. Ford during the war as a shocking example of what a mistake it would be to apply any automobile technique to the building of a ship. The fault with the Eagle boats was not with their assembly, which was remarkably well done. The fault was entirely one of design. I speak from the knowledge of having been the master of an Eagle boat on several extensive voyages.

In the discussion it was stated that a proper and unbiased analysis would show that the differentials in cost between American and foreign flagship operations were not as great as the differentials in cost the American shipbuilder has to face and that, therefore, if my deductions with regard to American shipyard cost are based upon sound reasoning, I as an operator might dispense with any form of government subsidy by applying the same method of efficiency. In my paper I stated that I believed that an operating subsidy for American ships would gradually disappear. On the other hand, I have yet to see this proper and unbiased analysis that in any way justifies the cost of building in American shipyards.

Mr. Smith has kindly agreed with much that I have said, and I in turn agree with practically all that he has said. Certainly American shipyards and operators are burdened with a lot of requirements that add to the cost of the ship. Many of these requirements are of questionable value. The problem of cost of ship construction as developed in my mind, however, accepted all of these many requirements and still found the American cost too high. How could I come to such a conclusion? For several years there was doubt in our minds as to whether or not it would be possible to go along with an American shipbuilding program. During those years we submitted several times to good builders in the United States and to good builders abroad identical specifications. These specifications included all the extraordinary requirements of American-built tonnage. The differences in the bids received at the same time and on the same specifications were startling. They reflected no credit on American shipbuilding costs.

From time to time this question of American costs has been discussed by me, informally to be sure, with the various shipbuilders. They have always assured me that there was complete justification for their costs. If such a justification exists, it seems to me that it ought to be very easy to put it down in a provable analysis. To prove this cost the shipbuilders should not have to rely on mistakes made in my paper with regard to the *Titanic*, nor the date of flooding curves, nor the date of the first double bottom, nor the failure of Mr. Ford to build ships, nor the cost of ratproofing an American ship not required in a foreign ship, nor to the fact that a ten-knot freighter is a far different type of vessel from a fifteen-and-one-half-knot vessel, nor to the self-evident fact that shipbuilding and automobile manufactur[illegible] are different processes. Mr. Smith states that a disinterested

investigation of shipbuilding will show that the industry is well abreast of other American industries in its efficiency, and far in advance of many. If this be true, it strikes me as curious that such a disinterested investigation has not been made, or if it has been made, it is not better known to those of us who ought to know.

Mr. Smith states that the shipbuilding industry is proud of its achievements and believes that it is well in the forefront of advancements that have been made by industry in the twenty-year period since the war. Complacency and self-glorification sometimes obscure the vision needed for self-discipline and correction. The Shipping Act of 1936 unfortunately places a premium upon inefficiency in the shipbuilding industry. The shipbuilder is entitled to ten per cent profit on his cost. Hence, the greater the cost the greater the profit. I made it quite evident in my paper that I was viewing the situation within the shipyards from a layman's point of view and without adequate experience in the yard. To my inexperienced eye there was a great deal of lost motion in every direction. It is possible, even probable, that I am entirely mistaken. In all fairness, however, are we not entitled to some tangible evidence of this? My purpose, frankly, was to arouse discussion in the belief that discussion leads to enlightenment, and enlightenment to improvement. In view of the volume of discussion, it seems, at least to that extent, the paper has been successful.

Problems of the Highway User in the United States

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The present-day highway problem is made up of many elements, namely, cost of construction, ability of highway users to pay for highways as they are built or during their life expectancy, the kind of taxes providing revenue to pay for highways, types of roads required, competition between transportation systems, and kindred matters. No single solution exists for all of these. The author analyzes comprehensively each of the elements of this nation-wide problem and suggests practical approaches to their solution. Of all the factors making up this problem that of cost is probably dominant. How the costs are to be met is the crux of the situation as it now stands, with the ever-expanding highway needs pressing for action. It is the author's view that a recent report of the Federal Works Agency, Public Roads Administration on the general question of "Toll Roads and Free Roads" offers the soundest long-range plan for national highway development yet advanced. A comprehensive analysis of this report has been included and appears as an Appendix to the paper.

IN A PAPER presented by the author at the World Engineering Congress in Tokyo on November 4, 1929, the application of motor transport to the movement of freight and passengers was dealt with. In the ten years that have intervened, much progress has been made in this field of transportation, but largely along the lines of design, material, and construction of the vehicles, as compared with new uses to which they have been put—application. For example, ten years ago concrete was being "mixed in transit," not so efficiently or economically perhaps, but being done.

In the process of making these rapid strides in the use of motor vehicles, many problems, some quite serious, have arisen to confront the users of the highway. It is interesting to note that, in spite of this, the desire for individual transportation—the automobile—is so strong and the necessity for the truck and bus transportation services so vital to all of us that the march is ever onward.

An engineer need not always approach every problem of public importance from those technical points of view in which, by his training, he is particularly informed. An engineer is a citizen as well as being a technician; consequently, when broad national problems confront the citizens of a nation, technicians of all types properly are expected to view such problems not alone from their technical aspects but from the economic, social, recreational, educational, and other characteristics which many of our problems now have come to possess.

Cost of highway construction; the ability of highway users to pay for the highways in the main as they are built, but surely during their life expectancy; the kind of taxes from which revenue is to be derived to pay for highways; the varieties of roads to be built; the economic welfare to be subserved by highways; and the maintenance of competition among and between transportation media; constitute a main portion, but surely not the whole, of the nation-wide problem which now confronts our highway users.

It is a recognizable fact that the highway users in the United States cannot single out one phase or aspect of our national highway problem and maintain with assurance that the solution of the particular segment of the problem is the solution for the whole. Different individuals naturally select different phases of the problem as being most important but, for the purpose of the present discussion, it may be stated that the cost factor in the future highway problem of the United States is, if not the most important, at least an extremely dominant one.

None of us could realize twenty to thirty years ago, at which time the modern highway era was first being planned economically, technically, and legislatively, that it would sweep forward with that speed of construction and use which is now evident. Strong words are necessary to describe the 1939 highway situation in the United States; perhaps stronger words are necessary in any attempt to describe the highway era to be further developed during the next two decades. Gigantic costs already have been met; gigantic costs are yet to be met. Spectacular mileages of all types of roads, serving all parts of the nation and all citizens therein, have already been laid down and yet we continue to look forward to many great and necessary improvements in our highway facilities.

Construction and Maintenance Costs

In considering the cost factor relative to highway building, maintenance, and amortization, it is natural to observe that it divides into two parts: first is the continuing and annual expense in building new highways; second, is the constant annual cost of maintenance.

Referring again to the initiation of our highway era not much more than two decades ago, our arithmetic of cost at that time was all centered on the building of new roads. That was natural in what may be called the pioneer era of building highways in the modern manner. We then estimated total costs in the nation as a whole in millions of dollars. We now actually face highway costs which have been elevated into the billion-dollar classification.

Furthermore, we are now realizing that maintenance of highways, the cost of which we are beginning to realize after two decades of use of the highways first constructed, is the increasing feature in our national highway financing. Two or three decades ago the uninitiated thought that highways were practically everlasting and that the initial cost was the only cost, but as the years move along maintenance costs of highways are probably destined to equal or surpass costs of original construction.

Right here a word of advice, of warning, needs to be sounded in regard to the highway costs of the future. We can run

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

riotously into highway costs which will prove to be an extreme burden on the present and future highway users of the country if we build new roads, of whatever type they may be, far beyond present or prospective future traffic needs. Particular point may be made in this connection in regard to the transcontinental super toll roads, which some sources are now advocating be flung across the country, east and west, north and south, quite regardless of original cost, of traffic needs, and of assured methods of payment.

An important service—it may be stated an epochal contribution—to current thinking on this question has been rendered in the recent report of the Federal Works Agency, Public Roads Administration (formerly the United States Department of Agriculture, Bureau of Public Roads) on the general question of "Toll Roads and Free Roads." Little comfort is found in this report by those who, with superenthusiasm, advocate super-transcontinental highways. Indeed the report of the Road Administration recognizes that engineers can build highways of the most elaborate kind and cost, if only some source of revenue will provide the money. But that is not the question. The real question is: Do we need these superhighways? Connected with this question is the correlative one relating to the ability of highway users to pay for such roads.

The high spots of the Road Administration's report are as follows:

Transcontinental superhighways cannot be supported by tolls.

There is no habit of transcontinental travel by highway.

Less than 6 per cent of all trips outside of cities are longer than 50 miles.

Only a small portion of present traffic could be attracted to a toll system.

Motor-vehicle traffic on main roads in 1960 will be virtually double that of today in miles traveled.

There should be a system of direct interregional highways with necessary connections through and around cities.

By-passes will not relieve congestion within and near cities.

Ninety per cent of the traffic on main highways near cities is bound to or from the cities themselves.

Express highways cutting directly into and through the centers of big cities are needed.

An independent Federal Land Authority should be created, empowered to acquire lands needed for public purposes. It should be implemented to acquire, by purchase or condemnation, necessary lands for adequate urban and rural rights of way at reasonable costs, which would be sold or leased to the state or municipality on an amortized basis. It should also be *empowered to acquire and sell excess lands* for the purpose of recoupment.

Such "excess taking" would require changes in federal and state laws.

In addition to the foregoing, this report is of such profound importance not only to all those connected with highway transportation but to every citizen of this country as well, that an appendix is being included with this paper which gives a comprehensive analysis of the Road Administration's findings and recommendations. The author urges that it be read carefully.

No criticism is being directed at multiple-lane highways where they are needed, but it logically may be concluded that our future building of new highways in the nation, and maintaining of our roads, will be a gradual evolutionary process, very much like the building of the present so-called primary highway system has been. It has taken us approximately two decades to get the primary roads built; to get them connected state to state; to get them flung across the country north and south, east and west.

It is in regard to highway costs that the Report of the Road Administration is a monument to that sound economic thinking which we must not surrender, unless we are willing almost wholly to disregard the ability-to-pay principle, as applied to the highway users of the nation.

Here it should be pointed out that one of our most serious problems in the expansion and improvement of our highways is that of securing rights of way. It is no exaggeration to say this is the crux of the problems facing us. Unless and until means are found to secure rights of way within a reasonable time and at reasonable costs, amortized over a period of years, it will not be possible for us to proceed very rapidly in this important undertaking.

Ability-to-Pay Principle of Taxation

Closely akin to the cost factor in highways is that personal and economic factor frequently applied to taxation technique in all its forms, namely, ability to pay by those who are expected to pay taxes. Statistics have been compiled—which need not here be repeated—which show that the average highway user in the United States enjoys only a modest annual income. More than half of all family automobiles are owned by families which have an annual income of $1500 or less. In addition to this, the highway user drives a car the present average value of which is only $197. On this car he is already paying an average annual total tax of $53, more than enough to confiscate the value of his car in 4 years' time. Less than 5 per cent of all cars are owned by families having an annual income of more than $5000. Less than one third are owned by families with an income in excess of $2000. From trends which, fortunately, seem at least partially stopped by the joint activities of many highway-user groups, it has been concluded that the highway users of America could pay any taxes which any administrator or legislator cared to enact and to impose. On the contrary, for example, to the motorcar owner with an income of less than $1500 a year, a toll of one cent per mile would very seriously increase his cost of operation and hence retard the usefulness to him of his motor vehicle.

In former years it was the usual plan, when more money was needed, to increase the gasoline tax, to raise the registration fee, to enact a ton-mile tax, or to penalize the highway user further with constantly increasing charges for driver's permits. These trends, not wholly eliminated but definitely retarded at least, have forced us to recognize that the average driver of the family car, or of the truck, is a person who is making his living from day to day by honest toil and is not blessed with fabulous wealth which can be taken from him by the imposition of various forms of motor imposts.

Not the least occurrence in recent years which has caused the highway user to rise up in rebellion against constantly increasing costs of highway use, is his realization that the money he pays for highway revenues has been used in too large quantities for almost every other imaginable purpose of government. The question of the diversion of highway funds, iniquitous as it is in a highway era, has, however, served the useful purpose of awakening the highway users of the nation to a militant position; first, that revenues from motor imposts must be used for highway purposes; and second, that there is a limit of taxation, already reached, taking the nation as a whole, which limit is the ability-to-pay principle. This is always a safe yardstick to apply to any form of taxation. When this principle is disregarded by lawmakers and administrators, the point of diminishing returns in tax revenues is reached, and public opinion characteristically solves the difficulty by taking into its own hands policy making in regard to taxation, as is now evident in regard to highway taxes and the dedication of highway revenues wholly to road purposes.

To the highway users of the United States this is rapidly be-

coming a serious menace to their interests and, therefore, it seems desirable to explain more fully what is involved. The use of the revenues from the taxation of the motorists for purposes other than highways customarily is termed "diversion." To understand the popular attitude toward diversion, attention must be devoted to the basic economic theory underlying the taxation of the motorists.

Benefit Taxation

Originally, fees for registration of motor vehicles were required as a method of regulation in the use of the highways. In recent years, however, the intent has changed somewhat. The current conviction seems to be that the payments of such fees, in general, should vary according to the weight and the carrying capacity of the vehicle.

Beginning twenty years ago the fee for registration was supplemented by the state tax on gasoline. It is contended that consumption of gasoline is a valid index of the economic benefits derived from use of the highways. If this contention is sound, such a levy taxes the motorist in direct proportion to his use of the roads. The economists commonly refer to it as taxation in accordance with benefit.

Both these taxes, therefore, originally were intended to compensate for use of the highways. Because of the very nature of the tax on gasoline, an exemption from the tax or a refund customarily is granted on all gasoline which is not actually consumed in the propulsion of motor vehicles on the highways. On the same grounds, the proceeds of the tax logically should be devoted entirely to the construction and the maintenance of roads.

Early Recognition

The principle that revenue from the taxation of the motorists should be used only for roads was recognized fully in several states some years ago. In 1920 the electorate in Minnesota approved an amendment to the state constitution establishing a system of trunk highways and creating a sinking fund for the retirement of obligations issued to finance highways. This amendment required all funds from the taxation of motor vehicles to be deposited in the sinking fund. When the gasoline tax was adopted in Minnesota in 1924, the Trunk Highway Fund and the State Road and Bridge Fund were established. All of the revenue from this new levy was required to be deposited in these two funds.

The tax on gasoline first was imposed in Missouri in 1928. At that time, the electorate approved an amendment to the state constitution requiring all of the revenues from the taxation of the motorists to be used exclusively for highways.

When Kansas first levied the tax on gasoline in 1928, a somewhat similar provision was inserted in the constitution of that state.

Some discussion of these constitutional amendments seems appropriate. In this country it is the custom to embody the fundamental principles of government into a single written document. The federal government, of course, possesses only those powers specifically granted it in the federal Constitution. All other powers, therefore, are reserved to the states.

Rigid limitations upon the taxing power of the elective representatives always have been considered legitimate subjects for inclusion in state constitutions. In fact, such limitations are quite common and quite general.

Tendency Toward Diversion

In some states for simplification in accounting, a single general fund has been established and, as the state revenues are collected they are placed in it. The appropriations for the financing of the state government then are made from this same fund. However, in some of these states special safeguards have been erected to guarantee that the taxes collected from the motorists are expended for roads. Statutory provisions require the appropriation from the general fund of a sum equivalent to the yield of the levies upon the motorists.

During the depression, there was a marked decline in the yield of the established sources of revenue for the financing of general governmental functions. The revenues from the taxation of the motorists, on the other hand, showed a relatively stable yield. As a result, there was a strong temptation to divert the funds from the taxation of the motorists to supplant the decline in the revenues from other sources. This tendency naturally was strongest during the depth of the depression, but it still persists. In 1937, the most recent year for which detailed information is available, such diversions amounted to $161,413,000 or 13.7 per cent of the total collections of $1,177,827,000.

Legal Safeguards

Recently there has been definite evidence of a reversion to the original concept that the funds should be used only for highways. As the electorate has become concerned over such diversion, its concern logically has become evident in a movement to amend the state constitution. This was the case in Colorado, for example. Extensive diversions of the highway funds in that state, during the early years of the depression, prompted the initiation of a constitutional amendment. The measure was approved at the election in November, 1934, prohibiting all diversion after July 1, 1935. Similar constitutional amendments were approved in California, New Hampshire, and Michigan at the election in November, 1938.

The Congress of the United States similarly has disapproved diversion. No doubt many of you have some knowledge of England's system of subventions and grant-in-aid made by the central government to the local units. Our system of federal aid to the states for highways is quite similar. It is the author's understanding that the grants in England almost invariably are rigidly circumscribed, and that certain conditions usually must be fulfilled by the local governments before the funds can be obtained. In this country, certain requirements likewise must be met. One of the earliest was that the funds be matched 50–50.

In 1934 the Congress imposed further restrictions upon the granting of federal aid for highways. The language of the Act makes it apply to "Those States that use at least the amounts now provided by law for such purposes in each State *from State motor-vehicle registration fees, licenses, gasoline taxes, and other special taxes on motor-vehicle owners and operators.*" The maximum penalty for failure of compliance was to be the loss of one third of the annual allotment of federal aid. Two states, Massachusetts and New Jersey, thus have been penalized.

The general public, therefore, fully is convinced of the logic of strict adherence to the basic tenet in taxation of the motorists in accordance with benefit. This conviction, in turn, is reflected in the acts of the electorate's duly designated representatives in the state legislatures and in the Congress. The current trend clearly is toward more rigid adherence to the principle of taxation solely for benefit.

Highway Tolls No Solution

But what kind of taxes are we to have in our necessity to maintain all our present and future highways, and to build new roads? This question can be answered negatively and positively. The negative answer, briefly stated, is that the nation is not inclined to turn history backward and to resurrect and reinstate highway tolls. Some say casually that highway users once paid tolls so why not have them again. But such a casual state-

ment—one can hardly dignify it with the use of the word argument—fails to realize that roads, upon which tolls once were assessed and collected, were built in the main by the use of private capital. Roads now are built by the use of public revenues. Highway users will not tolerate the imposition of tolls on highways for one simple reason. The people pay for the roads in motor imposts in the main. They will not pay tools for the use of an instrumentality for the construction and maintenance of which funds have been provided from public taxes. That is the whole answer to the highway-toll proposition. History is not going to repeat itself in regard to toll roads in the United States. Free roads, along with free schools, are two foundations of a representative democracy and our national forefathers, who started us going in the form of government we are now maintaining, made provisions for these two fundamentals of government.

It is in this connection that reference again may be made to the report of the Public Roads Administration, "Toll Roads and Free Roads." This report does not condemn a toll if it is charged for a specific highway purpose, as for instance, a bridge over a navigable stream which, when paid for by tolls, becomes free. The exceptional conditions rendering toll collections feasible include also situations in which there is either an absence of a competitive facility or so great an advantage in the toll facility as to justify, in the minds of users, payment of the toll required. But the nation-wide application of the toll principle to superhighways is condemned.

The report does not state, but one may reliably conclude, that, if tolls should be relied upon to pay for any or all highways, ultimately the tolls failing in this regard, recourse would necessarily need to be taken to increase motor imposts which are already as burdensome as the average highway user can afford to pay.

The positive answer to the form of taxes which highway users are to pay may be found in the statement that the motor imposts, which are now being paid, seem to be the most logical method of highway taxation ever devised, if only in the nation and locally highway users will be on guard to prevent these imposts getting out of all bounds, in so far as the ability of the highway user to pay taxes is concerned.

The gasoline tax is an equitable and rational method of requiring the highway user to pay in proportion to his use of highways. Obviously, all motor vehicles must be registered with some proper governmental authority. But the cost of such registration is a problem which public opinion, concentrated among the highway users, should decide rather than to rest such decision with administrators.

Farm-to-Market Highway

The questions of the taxes paid by highway users and the motor revenues derived therefrom are directly related to the kinds of roads which are to be built in this nation. Reference has already been made to the multiple-lane or the divided highway, where traffic density is known to require such a road. On the other hand, so to speak, of highway types, is the country road or, as it is now named in federal statutes, the farm-to-market highway. Between these two extremes, the country road on the one hand, the multiple-lane highway on the other hand, is comprised the future program of highway building and maintenance. To build one of these classifications of roads without the other would be an unbalanced program. To build them both in compliance with the information contained in the Report of the Public Roads Administration, as well as the highway surveys now rapidly nearing completion in the states, is the part of wisdom.

Again public opinion must not be disregarded in highway matters. Public opinion gives recognition to the farm-to-market highways, although it required two decades to accomplish this. Public opinion recognizes also that congested traffic nearer metropolitan areas is a modern problem which, for purposes of highway safety, as well as factors of economics and pleasure, must be supplied with modern-type multiple-lane roads.

Fifty thousand communities in the United States have no ingress or egress for persons or for products other than by highways. Twenty-five million automobiles, approximately, serve the families of America and the business of the nation every minute of every day. Approximately 4,250,000 trucks roll to markets, factories, and stores with products of all kinds. Here, for instance, are a few: The entire milk supply of 17 cities ranging in size from San Diego to Detroit. Nearly three fourths of all the milk used in New York and Philadelphia. More than 95 per cent of all used in Atlanta, Baltimore, Pittsburgh, and San Francisco. All the milk that is consumed in thousands of smaller cities and towns from coast to coast.

More than half of all fresh fruits and vegetables eaten in the thickly populated northeast section of the country. Ninety-five per cent of all that is hauled 300 miles or less to market. More than 5,000,000 tons of livestock, upward of 27,000,000 animals, from farm to market. Fifty-five thousand tons of butter to Boston, New York, Philadelphia, and Chicago alone. Nearly 4,000,000 cases of eggs—upward of a billion eggs—to the same four cities every year. Buses make their valuable contribution to the transportation services of the nation to an extent which was undreamed of ten years ago; there are 130,000 now in use.

Modern Transportation Methods

It is idle for anyone to state that highway transportation has not taken its place alongside the other methods of transport which the nation enjoys at the present time. Highways, railways, waterways, airways, and pipe lines are the quintuplets of transportation in the United States. In other words, the economic welfare of the nation does not revolve around one transport facility, highways or otherwise.

The railroads are in a competitive era. The people will be served in transport matters. They demand transportation at the lowest possible cost. They advocate also high qualities of service, which cannot be given them by any one method of transportation. Accent again may be given to the thought that public opinion is always a controlling influence in public affairs. The public believes in competition; the public is against monopoly in transportation or otherwise, because it is the public that pays the bill. Any plan, therefore, that would peg or freeze rates or carrying charges, thus killing competition, which in the case of transportation means efforts to render a better service at a lower cost, is not in the public interest and should be fought to the last ditch, and in this country, at least, it is the author's belief that it will be.

Entering the decade of the 1940's, we are at a period during which this question of competition in transportation will be adjusted, if not solved. Such adjustment or solution unquestionably will be such as to benefit the average citizen if he, as a highway user, stands firmly for his own rights. But if, on the other hand, competition in transportation, through regulation and legislation, is made impossible and, if the public should be taught to believe that competition no longer is the life of trade in transportation and otherwise, then, inevitably, the citizen will pay more in the future for transportation than he should pay, or than he is able to pay. He will get a poorer quality of service and will eventually discover that, after having paid to a large extent for every method of transportation we have, he is destined to get no competitive benefits from them.

Unfortunately regulation by government has come to be, in recent years, an answer to almost every public question. Some

years ago, the railroads began to discover that they had competitors in the transportation field and that their half century of monopoly was ending. Immediately, those who spoke for the railroads—and in a most natural way—began to clamor that all other methods of transportation should be regulated to the same degree as the rails have come to be in a half century of development under the Interstate Commerce Commission. In other words, the solution of the railroad problem, as voiced by railroad representatives with their slogan "Equality of Opportunity," is to regulate the competitors of the railroads into that position of incapacity wherein they would be unable to render a cheaper and more flexible transportation service by highway.

Less Regulation of Transportation Methods Required

The people are beginning to realize—or perhaps they definitely do realize—that the solution of the highway problem in the United States is not to overcome, to subdue, and to neutralize competitive factors in transportation by the process of imposing extreme regulations on transport media, but lies rather in the other direction; to relieve the five methods of transportation from much of their regulations so that the people may be served more adequately by the very initiative of free, self-controlled management of free business enterprise engaged in transportation. But in recent years we have seen such extremes of regulation sought to be imposed against highway users as: Ports of entry at state borders; the denial of reciprocal arrangements among states; the diversity of sizes and weights for commercial vehicles; the threat of increased taxation; and encouragement given to diversion of highway funds.

Recently we have seen forty-four state legislatures in session in as many states. Actually thousands of bills have been introduced having to do with highway transportation, many of which have sought to impose more regulations or more taxes on highway users, or both. Too many of these measures were negative in character; they sought to destroy; they were not written and introduced to aid one method of transportation, but primarily to harm a competitor. Few of these negative trends in state legislation were enacted into law, but it has required constant and almost superhuman activity on the part of highway-user organizations to keep the battle line of defense intact, and to prevent ridiculous and extreme regulations being imposed on highways.

Highway use requires only two general categories of regulation. One is to provide for highway safety; the other is to protect the highways themselves by establishing uniform standards in sizes and weights of motor vehicles. When these two regulatory requirements are in effect, any further advance in regulating highway traffic reduces its competitive influences and, to the same degree, denies to all of us those benefits from highway transportation which otherwise we would enjoy.

It is not a sensible argument in a great nation like the United States to state that competition in transportation, the easing up on regulation rather than magnifying it, will ultimately eliminate one or more methods of transport. Some are even so inaccurate as to state that competition in transportation means that the nation would ultimately have only one method of transportation left. Such a conclusion is fallacious in the extreme, even though it is true that more than a hundred years ago one method of transportation, the newly developing railroads, did make obsolete the then existing methods of transportation, the canals, and covered-wagon-type facility.

But now we have 130,000,000 people who are moving from place to place to extents which were undreamed of by our forefathers. We have commodities now, both raw and manufactured, of such tonnage, volume, and variety that one method of transportation could not possibly handle. We have uses for transportation now which formerly were seldom thought of, such as recreation, travel, pleasure, enlightenment, which no one method of transport is wholly qualified to give. As a result of this competitive era in transportation, all citizens will be benefited; highway users will be allowed more economically to enjoy and to profit from the highways of the nation; and the entire economic structure of the country will be strengthened.

All of this question of regulation and legislation, centering in recent months in state capitols, more recently has revolved around Congress on Capitol Hill at Washington, where the effort for ten years has been to put all methods of transportation under the Interstate Commerce Commission. In the earlier years of the present decade, the slogan was "coordination." Later this was changed in words but not in meaning to "equalization of transportation opportunities." Both of these expressions meant, reflected in proposed bills and statutes, that by the methods of regulation competition in transportation either would be reduced to a minimum or wholly eliminated, spelling higher transportation costs.

Consequently the complete program of coordination of transportation media in the Interstate Commerce Commission has not been accomplished except in part. Motor carriers were placed under the Commission in 1935 with some support from certain motor carriers. Now it may be stated with some accuracy that those who represented motor transportation in 1935 and in prior years and who sought federal regulation in the Interstate Commerce Commission alongside its rail regulation, with the experience of several years, are questioning the wisdom of their former position. This questioning attitude is strengthened by a significant statement in the last annual report of the Interstate Commerce Commission, wherein the record is made that, as a result of the Motor Carrier Act of 1935, *highway transportation is more comprehensively regulated than are the railroads.* This statement would appear publicly to give recognition to the startling fact that, in less than one half a decade of federal regulation of motor vehicles, a strait-jacket of regulation has been drawn as tightly, or more tightly, around motor transport than had been accomplished in regard to the railroads in half a century.

Efforts to Regulate All Transportation

In the last session of Congress, the regulation-type solution of the problems of all users of transportation in the United States continued to be the one most relied upon in the consideration of so-called railroad relief bills. The effort was first made to put air transport under the Interstate Commerce Commission—that failed. The effort was made to impose on highway users the commodities clause which for a quarter of a century has been in operation for the railroads and which, if imposed on other methods of transport, would, for instance, deny a baker, a farmer, a carpenter, or any other citizen having his own motor vehicle the right to haul his own products to market. The effort to put the commodities clause on highways, waterways, air lines, and pipe lines was indeed the nth degree of the ambition to cure all troubles by the process of regulation. That effort failed, as it deserved to fail. But the problem of regulation and legislation, both at Washington and in the forty-eight state capitols is a continuing one and largely revolves around the question whether or not competition in transportation is to be continued.

New communities, new enterprises, and new opportunities have everywhere followed the development of new and better means for moving goods and people. But progress in transportation—the reduction of distance in terms of time and cost—is in a larger sense only a symbol of expanded horizons in every field of activity, a symbol of the opportunities that have been created by the capitalization of technological progress, and indicative of the future contribution of science and industrial research.

As an expression of this concept of the economic and social functions of transportation and its impact on industry as a whole in the world of tomorrow, the author wishes to direct your attention to General Motors' New York World's Fair exhibit of the last season, which many of you visited and which was built around the theme, "Highways and Horizons"—broadening highways of transportation, with expanding horizons of industry.

The "Futurama" in effect is a panoramic tour of a cross section of the country as it might appear in 1960. From moving sound chairs the visitor sees, as from an airplane, a dramatic conception of the towns, cities, and farms of twenty years hence. New highways link the cities. Feeder roads tap fertile agricultural areas. Produce of the farm moves swiftly to market. Industrial products are carried more economically to consumers in every community.

The cities themselves are decentralized. With convenient suburbs taking the place of congested areas, with city traffic freed by wide, open thoroughfares leading into and through metropolitan areas, a new type of city community is made possible. Workers are enabled to live at a distance from their employment. Suburban and farm dwellers have new access to town and city conveniences. Everywhere are evidences of quickened economic and social life. Better homes, better schools, more plentiful parks and playgrounds, new industries, more healthful surroundings, have in this fantasy of 1960 expanded horizons of living for all. Truly a marvelous conception of the possibilities for economic and social progress.

A free interpretation of the forecast into the next two decades given us in the Public Roads Administration report, "Toll Roads and Free Roads" would be that highway transportation will move forward in the nation gradually; that highway costs, both for construction and maintenance, will be constantly kept in mind; that no great departure from practices now well established in regard to free highways will be tolerated; that, although it is possible for highway engineers to build any type highway, anywhere, economics and the ability of highway users to pay taxes will be observed in the construction and maintenance of all roads; that the question of rights of way for the future, more than in the past, will be of growing importance; that such rights of way, with the wider highways now needed in certain areas, might cost more than the ability of the highway user to pay will permit, unless proper steps are taken to expedite right-of-way acquisition; and that the highways, being here to stay, must be allowed to be of the greatest possible economic use to the people of the nation.

Appendix

Congress, under the Federal Highway Act of 1938, directed the Bureau of Public Roads to investigate the feasibility and cost of building a system of transcontinental superhighways to be supported by tolls—such a system to be composed of not to exceed three roads east and west and three north and south.

The report in its final form was transmitted to Congress by a special message from the President on April 27, 1939:

"I recommend the report for the consideration of the Congress as a basis for needed action to solve our highway problems."

With this closing sentence, the President placed his stamp of approval upon findings and recommendations of the Bureau.

The following language appears elsewhere in his message:

"It shows that there is need for superhighways, but makes it clear that this need exists only where there is congestion on the existing roads and mainly in metropolitan areas."

The report is properly divided into two main parts, each replete with tables, maps, and plates setting forth statistical information. The bureau has drawn extensively from the traffic counts and other basic data developed during the last two years by the state-wide highway-planning surveys now being conducted in forty-six states.

1—FEASIBILITY OF A SYSTEM OF TRANSCONTINENTAL TOLL ROADS

Important Conclusions

The report asserts that a system of transcontinental superhighways cannot be supported by tolls; that the factual evidence clearly indicates, "direct-toll highways cannot be relied upon as a sound solution of the problem of providing adequate facilities for the vital and necessary transportation of the United States or to solve any considerable part of this problem."

From a purely engineering standpoint, the report shows that such a system might be entirely feasible, physically, and the construction could be completed by 1945, with a capacity sufficient to handle the traffic that would be developed by then and for the period 1945 to 1960.

A toll superhighway must be so constructed as to offer a special attraction to traffic. It must have an adequate number of lanes to accommodate comfortably the peak load. It must offer advantages which loom larger than the tolls charged. The width or number of lanes must depend upon the maximum rather than upon the average use.

The report finds that 400 vehicles per hour, divided equally in both directions, is a maximum which may safely be carried over a two-lane road without inconvenience. It has been found that a peak load of 400 cars per hour for certain hours will approximate a total of 1500 vehicles per day, on the same road. It also finds that three-lane pavements are undesirable; they show no marked increase in the volume of traffic above that which can move comfortably over a two-lane road.

Traffic exceeding 1500 vehicles per day should be carried on divided four-lane highways.

The year 1960 has been selected as being far enough in the future to permit of careful long-time planning. Furthermore, it is estimated that by that year the motor-vehicle-traffic mileage will be double what it is today.

Estimated Cost

Pursuant to the directions of the statute and after a thorough analysis of the wealth of basic data concerning volume, character and range of traffic, the condition of existing highways, and the need for new facilities, the bureau selected approximate locations for six superhighways, the total length of which as projected would be 14,336 miles. The estimated cost of constructing them to desirable standards would be $2,899,800,000 or an average of $202,270 per mile.

The report answers any possible suggestion that proposed toll rates might be increased to provide the required revenue. It emphasizes the fact that the average motorist has an income of only $1500 per year. It also points out that even a superhighway toll charge of 1 cent per mile would actually double the cost of operating a small automobile. The more money people spend for tolls, the less they have left to spend for operation of their vehicles.

Toll rates which would probably produce the most favorable rate for the system as a whole would average 1 cent per mile for each passenger car and 3.5 cents per mile for each motorbus or truck, with the latter rate varying considerably with the weight and capacity of the vehicles. It is assumed that bus and truck traffic constitutes 20 per cent of the total traffic. An approximate average rate would be $1\frac{1}{2}$ cents per mile for each vehicle regardless of type.

Potential Toll-Paying Traffic

The total utilization of these six superhighways in their entirety

would not be expected to exceed 4,544,000,000 vehicle-miles of toll-paying traffic per year. The study indicates that 3,635,000,000 vehicle-miles would be by passenger automobiles and 909,000,000 by motor trucks and buses. This averages 12,450,000 vehicle-miles per day, or an average of 699 passenger vehicles and 175 motor trucks and buses on each mile of superhighway per day.

"The most optimistic estimated average daily toll-paying traffic for the period varies from a maximum of 5998 passenger automobiles and 1500 motor trucks and buses for the section from Jersey City, N. J., to New Haven, Conn., to a minimum of 96 passenger automobiles and 24 motor trucks and buses for the section from Spokane, Wash., to Fargo, N. D."

The cost of construction, maintenance, and operation of these six turnpikes between the years 1945 and 1960 would average $184,054,000 per year. On toll rates of 1 cent per mile per passenger car and 3.5 cents per mile per bus or truck, the revenue for that period would only be $72,140,000 per year. "It is therefore concluded that a direct-toll system on these six superhighways in their entirety would not be feasible as a means of recovering the entire cost of the facilities."

Only a small portion of present traffic could be attracted to a toll system. Long-distance travel constitutes only a small fraction of the total. The majority of trips outside of cities is for less than 10 miles, and less than 4 per cent of all such trips are longer than 50 miles. Only 300 passenger vehicles cross a given line daily in transcontinental travel. There is no habit of transcontinental travel by highway.

Vast portions of rural and interurban travel would never reach the nearest superhighway and could not be considered as even potential traffic for the system. Vehicles traveling shorter distances than the spacing between access points would not use the toll roads.

A major share of the longer-range traffic could not be counted upon when adequate free highways approximately parallel the toll roads. The bulk of traffic would continue to use the main public highways. The remainder of potential patrons who are able and willing to pay the toll "in practically all cases is too small to support the cost of the desired facility except at a rate exceeding the willingness or ability of even these."

Heavy-Traffic Sections

The portion of the proposed system estimated to be most nearly self-supporting is the 172 miles from a point near Philadelphia to a point near New Haven. With the increase in traffic expected by 1960, this portion alone would earn slightly more than its estimated cost for that year. Twenty-five other sections are analyzed upon which the anticipated revenues for 1960 would produce from 91.8 per cent down to 50.5 per cent of the estimated cost for that year. Thus it will be seen that only 4000 miles of the entire 14,000 miles would come within 50 per cent of paying its way.

The report suggests that, if Congress desires to make a test, a section of highway properly located between Washington and Boston would offer the best possibilities of attracting traffic in sufficient volume to approach financing through the toll method.

3—A MASTER PLAN FOR FREE-HIGHWAY DEVELOPMENT

Component Parts of the Plan

The bureau, in addition to making a study of a suggested toll superhighway system, supplemented its report with a proposed "Master Plan for Free-Highway Development" throughout the nation. This calls for appropriate action by federal, state, and local governments. As desirable joint contributions of the federal and state governments, the report lists the following undertakings:

1 The construction of a special, tentatively defined system of direct interregional highways, with all necessary connections through and around cities, designed to meet the requirements of the national defense in time of war and the needs of a growing peace-time traffic of longer range.

2 The modernization of the federal-aid highway system.

3 The elimination of hazards at railroad grade crossings.

4 An improvement of secondary and feeder roads, properly integrated with land-use programs.

5 The creation of a Federal Land Authority empowered to acquire, hold, sell, and lease lands needed for public purposes and to acquire and sell excess lands for the purpose of recoupment.

In addition to these enumerated undertakings, the bureau concludes that:

The federal government should recognize this national problem by advancing the funds essential to the purchase of lands necessary to this capital investment.

These lands should be leased to the states and their subdivisions for a period of years on a basis which will result in amortization of the cost.

Construction of a special system of direct interregional highways, as referred to in item 1, would tentatively include 26,700 miles of roadway. Existing main highways can be modernized to form a large part of that system, but some new highways will be needed to provide directness of travel. Although these roads represent less than 1 per cent of the total mileage of rural roads, the bureau estimates that they would serve, when improved as indicated, at least 12.5 per cent of the total rural-vehicle mileage.

Transcity Connections and Express Highways

One of the striking characteristics common to all highway-traffic maps is a sharp enlargement of the bands representing the volume of traffic on the important highways as they approach the larger cities. Obviously, these enlargements have a local cause. In fact, they are caused by a multiplicity of short movements into and out of the city; and it is not uncommon to find that the traffic on a main route approaching the city is swelled to several times its volume a few miles from the city limits.

Ninety per cent of the traffic on main highways near the entrances to large cities is bound to or from points in the cities themselves and cannot be by-passed around them. A large part of that traffic is destined to or bound from points in the very heart of the city or points conveniently reached by going through the center of the city. Express highways cutting directly into and through the centers of big cities are greatly needed, not only for through traffic, but also for daily in-and-out movement of local traffic between downtown sections and suburbs. This can best be handled on overhead or depressed highways, preferably the latter.

By-passes may not be regarded as means for the relief of congestion of this kind. Even if the entire through traffic on the main highway could be diverted over a by-pass route, the volume of traffic on connecting streets at the center of the city would not be reduced by any appreciable proportion. Only a small percentage of the existing traffic at the city's entrance can be diverted. The bulk of the traffic that is bound to or from the center of the city can best be served by transcity connecting routes and express highways. By-passes are recommended for the smaller towns, while a new type of belt-line distribution road around cities is proposed.

Using the city of Baltimore as an example, it was found that

the great problem is to provide relief for the in-and-out movement of local generation. For instance, there are over 9000 cars per day traveling between Washington and Baltimore but only 1068 of these cars travel between points beyond Washington and points beyond Baltimore.

The report deals with the subject of slow decay in those parts of the city that are adversely affected by existing traffic conditions. Greater space is needed for the unfettered circulation of traffic. Plans must be made to remove the impediments that embarrass the movement of twentieth-century traffic through eighteenth-century streets.

"Excess Taking"

The most spectacular addition to the accepted highway methods of the last three decades is the proposal to empower federal and state governments to embark upon a large program of purchase and condemnation of wide strips of land in both rural and urban areas.

The report points out that the early obsolescence of rural highways previously built is due in a large degree to restrictions imposed upon their design by inadequate rights of way. Major revision is also needed in the present archaic street plans in order to permit the free flow of modern traffic. Farsighted improvements of both are blocked everywhere by the inability of the state and local governments, unaided, to provide required rights of way. There is danger that "expedient measures forced by the irresistible pressure of traffic will result in heavy new investment destined for early obsolescence." Needed new facilities will involve a heavy present investment but it should prove to be a permanent one and should pay large dividends in the avoidance of future expenditures.

A Legislative Problem

Many state and local highway authorities operate under definite statutory restrictions as to the maximum width of right of way that may be acquired. These statutory widths often were adopted many years ago, and it is difficult for the authorities to acquire rights of way in excess thereof, regardless of what traffic conditions to be met may require. The United States Supreme Court has indicated that the use to which the land is to be put is subject to definite limitations in order to avoid infringement of rights guaranteed by the Constitution. Consequently, legislative changes must be made, both state and federal. The Ohio Supreme Court has said: "We cannot sanction an arbitrary and unreasonable taking of excess private property."

The report recommends that the statutes of each state be carefully examined and revised where necessary; that such revisions should eliminate maximum widths which may be acquired and, if any specific limit is prescribed, it should be the minimum not the maximum; that discretionary powers should be conferred upon highway authorities to determine the width desired with specific authority for such right of way to include lands in excess of those actually needed for the physical location of the facility so as to provide for safety, roadside development, recreational, sanitary, and other facilities, and for future expansion of the transportation facility to meet prospective future needs.

"When we consider that the highway widths provided for in existing statutes were adequate for all practical purposes less than twenty-five years ago, it is not difficult to understand why we now find the right-of-way problem one of the most difficult with which we have to deal."

Federal Financing

By supplying capital for investment in adequate rights of way, on a scale sufficient to protect the facilities and provide amply for their expected growth, the federal government can lend practical aid.

"Such rights of way, acquired with federal funds at the request of a state highway department and in accordance with state and federal laws, could remain the property of the federal government subject to lease by the state over a period of 50 years on terms that would in that period amortize the initial cost.

"In view of the predominant national importance of such a system, the federal government could reasonably contribute to its construction in a proportion materially larger than that in which it contributes under the Federal Aid Act."

The creation by Congress of a Federal Land Authority is recommended. It should have a corporate status with authority to issue obligations and power to acquire, hold, sell, and lease lands for stated purposes, such as, adequate width for future rights of way, safety factors, roadside development, recreational areas, sanitary provisions, and removal of unsightly obstructions.

Such an authority would also be an effective means for promoting reasonable coordination with other federal agencies that must purchase land for their own use, such as, public buildings, parks, and slum clearances. The proposed aid of the federal government could not be effectively employed in any state in the absence of constitutional authority.

Extent of Acquisitions

"Acquisitions by such agency should include all areas, the need for which might reasonably be anticipated for the physical construction of the improvement and for necessary roadside development, including probable future expansions." Authority should also be given to solve the remnant problem, i.e., small parcels that are separated by the road improvement from the main body of land to which they were originally attached and the value of which is impaired to the owner by the separation. It is far cheaper to buy such lands outright in many cases than to pay the actual or proved damages.

Detailed investigations should be made leading to the designation of a system of reasonably direct interregional highways with appropriate connections through and around cities and limited in total extent to not more than 1 per cent of the total mileage of the rural highways in the United States.

The report of the bureau includes in its appendixes a copy of the "Restriction of Ribbon Development Act, 1935" of Great Britain. This provides for the acquisition of any land within 220 yards from the middle of any road, which is declared necessary for purposes of construction or improvement or for preventing the erection of buildings detrimental to the view. This provides a strip 1320 feet in width.

Other Recommendations

The Master Plan, according to the report, is a 20-year program, and any delay at this time means increased congestion by 1960. Outside of city limits, there is need of modernizing the main highways to ease curvature, reduce grades, and extend sight distance. Near the cities, there must be a steadily increasing mileage of four-lane divided highways.

Such improvements are required on most of the federal-aid and state highway systems, especially those parts built before the comparatively recent increase in travel speed of motor vehicles. For the most part, the report shows these improvements involve only local changes in existing roads.

The suggested improvement of secondary and feeder roads would include a careful selection of mileage from among the 2,650,000 miles of roads outside of the state and federal-aid systems. This 78 per cent of the country's total road and street mileage at present serves only about 10 per cent of the total

vehicle mileage of traffic; but located on them are the homes and working places of about 75 per cent of the rural population. The choice of these secondary roads to be improved is important, therefore, in the promotion of beneficial land use. The states should select not to exceed 10 per cent of the rural mileage for such improvement. These thoughts are incorporated in the following recommendations:

By continuance of cooperation with the states in the improvement of the federal-aid highway system and the elimination of hazards at railroad grade crossings, with annual authorizations commensurate with those previously provided.

By continuance of the program of secondary- and feeder-road construction, with appropriations equal to, or larger than those authorized for the fiscal years 1940 and 1941, to be expended in such a way as to give effect to the principles enunciated in this report.

Comments

There can be no doubt that the proposal of excess taking is the crux of the Master Plan. It should be noted that the President's message strongly emphasized the principle as a means of preventing excessive purchase and condemnation payments, and of self-financing through the resale and rental of excess parcels along the right of way. In fact, the President would apparently carry out the idea much farther than the report seems to recommend. The Executive message reads in part:

"Under the exercise of the principle of 'excess taking' of land, the government, which puts up the cost of the highway, buys a wide strip on each side of the highway itself, uses it for the rental of concessions, and sells it off over a period of years to home builders and others who wish to live near a main artery of travel. Thus, the government gets the unearned increment and reimburses itself in large part for the building of the road"—rather than providing a profit "to a mere handful of lucky citizens and which is denied to the vast majority."

This theory of excess taking and recoupment was espoused by the President a year ago when he made his first pronouncement in discussing a toll superhighway system.

The report sets forth the fact that "attempted takings by condemnation in excess of that actually necessary for the physical location and construction of definitely contemplated improvements have been held unconstitutional by the courts." With that fact in mind, the question naturally arises as to whether the situation can be remedied through new legislation, either federal or state. It may require constitutional amendments in many jurisdictions, and that is something which cannot be hurried.

Possible Legislation

It is understood that bills are being drafted for the creation of an independent agency to be called the Federal Land Authority composed (probably) of the Secretary of Agriculture, Secretary of War, and the Administrator of the United States Housing Authority with the Chief of the Bureau of Public Roads as executive director. This authority would have corporate powers with a capital stock of $100,000,000 pledging the full credit of the United States. It would be empowered to issue bonds and securities not exceeding $500,000,000 outstanding at any one time.

Such bills will, no doubt, include the right of eminent domain and of condemnation for the purchase of lands to be used for highways and the power to purchase adjacent lands. Purchase or condemnation will probably require prior approval of the state highway departments and the Bureau of Public Roads. The authority will doubtless be authorized to sell lands adjacent to condemned or purchased rights of way and to enter into contracts with municipalities before condemnation or purchase of urban extensions; the municipality to repay the authority the purchase price, plus interest, over a period of forty years. It is understood that at least one such bill would clothe the authority with power to collect tolls.

The idea of "taking" lands in excess of those absolutely necessary for use in developing the facility will have to be worked out with the greatest care. The political implications must be thoroughly analyzed in order that they may be avoided. It will not be sufficient, in the minds of many, to authorize broad powers of condemnation simply for the purpose of transferring the profits of the unearned increment from the individual property owner to the government.

New Landmarks

In "excess takings," including excess condemnations, a new term has been added to the highway lexicon. It is a term that denotes a new idea in highway technique and one that deserves careful and intensive study.

The report indicates a definite alteration in engineering thought relative to by-passing cities. By-passes have been urged and recommended by engineers for two decades, to permit through travel to avoid downtown traffic. Now it is the local traffic, to and from the downtown area, itself, that must be relieved. The data developed by the state-wide highway-planning surveys are responsible to a large degree for this changed thinking.

Roads, once thought of as being rural, are coming to possess more and more urban characteristics. City streets, instead of being considered incidentally as urban extensions of main rural highways, are now advanced to a more dominant position in the road program.

Among other questions raised with reference to the report, and its recommendations, are the following:

Are there adequate grounds for the assumption that highway traffic will be doubled by 1960? There are those who predict a lessening of highway travel due to improved home conditions, such as air conditioning and television. Any appreciable increase in cost of travel would also act as a deterrent.

A question is raised as to the amount of existing highway mileage that will be suitable for incorporation into the twenty-year interregional-highway plan. This is particularly questioned with reference to radial highways, belt-lines, by-passes, overheads, and depressed highways.

The report contains no estimation of probable costs of the recommended Master Plan. There is, of course, the assumption that a major portion of land-acquisition costs to be borne by the federal government may be recouped through the sale of excess parcels, the government profiting by the unearned increment. The fact remains, however, that there will have to be vast sums appropriated by the federal government for the proposed Federal Land Authority and that the federal government's immediate share will probably reach 75 per cent of the initial total cost. There is apprehension on the part of some that this might result in increased federal gasoline taxes.

A Contribution of Great Value

Practically all observers believe that the Master Plan seeks to solve the future problems of highway construction with a forthright and constructive program. There is no gainsaying the fact that the problem is of vital interest to the entire nation. In an editorial comment,[2] it is noted as being "one of the most important guide documents of public policy in developing modern road transport since the federal-aid laws of 1921." Further commenting on the report, the editorial states: "Its great con-

[2] *Engineering News-Record*, May 11, 1939.

tribution is bringing measured fact to bear on the problem and relegating to well-deserved limbo the fantastic schemes for transcontinental superhighway grids that often find voice in Congress."

The report* has been prepared by the most expert minds now dealing with the problem. Common business judgment demands that it receive most careful and serious consideration by automobile owners, truck and bus operators, business representatives, and legislators of both state and federal governments.

There is an annual diversion of special motor taxes to other than highway purposes that should go far toward financing this plan almost in its entirety. If the program is to be adopted, the antidiversion campaign should become an integral part of the whole idea.

Discussion

THOMAS H. MACDONALD.[3] Engineers as a class have been too content to let other elements of our social structure carry the flag where public policies are concerned. It is highly desirable for engineers to assume a leading role in economic discussions involving the facts which are, after all, based upon data with which the engineer works continually and knows how to evaluate, perhaps to a greater degree than any other element of our social structure.

In this field of highways and highway transportation, engineers have a particularly fertile field for the exercise of leadership, based on sound facts. Fortunately, accurate information is becoming more widely available through the efforts of city and state planning bodies, as well as other organizations which are coordinating the information from various sources, as a basis for establishing sound policies in the future.

The problem of developing freeways, i.e., free-flowing highways throughout the cities and in the metropolitan areas, in my opinion, is the major problem in which to become most profitably interested in the future. The matter of securing rights of way, adequate for that development, will be a serious one financially. So far as may be foreseen, this problem can be met only by the use of all the credit facilities available, both nationally and by states and municipalities.

It is particularly serious when the number of cities which have reached their maximum debt limit is realized. Changes in their constitutions are necessary to provide authority to borrow when saddled with debts of that character. So we must turn to the state and federal governments if we are to supply the credits essential to securing and amortizing these rights of way over a long period of time. Viewed from that angle, it appears to be a tremendous undertaking, which it actually is. However, looking at it from the angle of what will happen otherwise, it is much more serious.

There is no doubt of the authenticity of this freeway conception of passing through metropolitan areas, as a solution of the problem of rehabilitating such areas. It is the only solution for the serious migration of values taking place in our metropolises.

The principal question of the moment is: What will be the future of the cities? If that question could be answered, many of the problems of planning and laying out road systems in metropolitan areas to provide for future demands could be solved. Will the city be a compact area with population highly concentrated, or will it be more loosely knit with the inhabitants spread out over greater territory and with metropolitan highways acting as connecting links between principal centers within the metropolitan area? The discussion on the present paper will unquestionably develop some thoughts on this phase of the problem.

There is one detail in connection with the use of present-day highways upon which the writer would comment, namely, the overemphasis placed on criticism of the truck and slow-moving vehicle in traffic. In our own driving, we are conscious of such vehicles and have the feeling that, if they were removed from the highways, we might enjoy pleasanter and more expeditious trips.

That is true to a limited extent. However, the question generally arises where highways have sufficient traffic that the slow-moving vehicle is an obstacle to the free flow of passenger-car traffic. For example, a two-lane highway when overloaded would be no better off if all the truck traffic were removed.

For 3 years, we have been conducting studies of the utilization of two-lane highways by automobiles and trucks. Instruments have been developed which automatically record the speed, position, and other characteristics of the behavior of vehicles or drivers in given areas on different types of roads. From the records obtained, we are establishing factual data on highway conditions as the first essential step toward rectifying such conditions as offer traffic hazards.

While the writer is more concerned about the economics of the situation than planning or specifying the type of highways to build, there is some question of the effect of buses and trucks on the life of highways. Deterioration of highways from this cause usually may be accounted for by poor foundations rather than poor surface construction. Structurally, the problem of providing durability in highways has largely been solved by soil stabilization. Formerly much of the destruction was caused by solid-tired trucks and heavy vehicles. With soil stabilization and the use of pneumatic, particularly balloon, tires combined with better construction and inspection, the structural problem of providing durability in a highway is no longer serious.

Turning to the matter of imposing tolls as a general tax upon those who use the highways, in general, this would seem to be an unsound proposal. Certain cases may be justified on the basis of the highway functioning as a time-saving facility. It appears that the American public will pay for time-saving as they will pay for no other commodity within their reach. The question of metropolitan highway development, which at present confronts us in an acute manner, must be settled independently for each city to meet the particular needs of the community. The city highway-traffic problem is urgent and will require the best thought that we can muster for its solution.

The matter of development of adequate roads in the rural districts will be solved largely by planning and by better engineering. Soil stabilization is one of the two answers to that problem. In time, the farmers can be served, if the roads are properly planned so as to limit the mileage to the least possible.

HAROLD M. LEWIS.[4] The author states that highway use requires two general types of regulation, one to provide safety along the highway, and another to protect highway surfaces by regulation of sizes and weights of vehicles. These types of regulation are both for vehicles. The writer would suggest supplementing these by an additional type of regulation which would apply to the right of way, rather than to the vehicles, i.e., restrictions to protect the carrying capacity of the highway, (*a*) by regulating the roadside uses along the highway, and

* NOTE: Copies of the full text of the report, together with all maps and plates, may be obtained.

[3] Commissioner of Public Roads, Federal Works Agency, Public Roads Administration, Washington, D. C. Mr. MacDonald acted as chairman of the meeting at which Mr. Horner's paper was presented.

[4] Chief Engineer and Secretary, Regional Plan Association, New York, N. Y.

(*b*) by regulating access from such uses to the actual roadway of the highway.

Cases are familiar in which highways, constructed as important major routes and hailed upon their opening as providing express movement between municipalities, have, within 10 or 15 years, become so clogged, due to the fact that business and other uses have been built up along their borders, that it has become necessary to consider replacing them entirely by parallel routes.

One of the best examples of this condition is Route 25 in New Jersey, between Newark and Trenton. The State Highway Commission now admits that it will be necessary to provide an entirely new route in order to carry traffic without interruptions and at speeds which prevailed along the present state highway at the time when it was opened.

The most complete solution to this problem is what has become known as the freeway, or limited access highway. So far, legislation authorizing highway departments to construct this new type of highway has been adopted in only four states. Those are the states of Rhode Island, New York, California, and Connecticut. Other states have considered this legislation, but so far as the writer knows, none has adopted it.

What is meant by a freeway, or limited access highway? It has been generally defined as a strip of public land, devoted to movement, over which the abutting owner has no right of light, air, or access. In other words, it is exactly the same as a parkway with the distinction that the parkway is a strip of park property, devoted essentially, in theory, to recreation. Although parkways as we know have also been devoted to movement, they are limited to passenger vehicles, while the freeway supposedly would be open to all types of traffic.

Such freeways would accomplish two definite financial savings over a long-time highway program:

1 By constructing roads in this way, it would not be necessary to pave local lanes either in the form of extra width for local traffic, or in the form of service roads paralleling an express highway, because the roadway would be dedicated to the use of comparatively long-distance or through traffic.

2 It would avoid the necessity of duplicating roads which may later become clogged by roadside uses, and thus constitute an important means of saving funds in construction costs of new highways.

The Bureau of Public Roads Report, parts of which appear in the Appendix of the paper, refers to the possibility of excess condemnation and resale along these highways. It would seem to be almost essential that any such program should be coupled with some such regulation of roadside uses as the writer has outlined. Otherwise, the process of condemnation and resale would encourage intensive development along such rights of way and, if access were not properly controlled, would lead to a rapid decrease in the efficiency of those highways.

All main highways in a state highway system need not be constructed in the form of freeways. The writer believes that a considerable mileage of existing main highways can be made permanently useful and efficient by adequate zoning along those routes, combined with the establishment of substantial setback lines for any building development which may take place along them. Thus, in rural areas the state system may largely utilize existing roadways.

A freeway is particularly needed in suburban areas on the edges of metropolitan districts, or as main access highways either to or through large urban areas. The writer visualizes a main-highway pattern in a metropolitan area as including a series of radial parkways and freeways outside the central business district. As these come closer in and perhaps approach some major waterway crossing, a parkway and a freeway might well be merged into a single route, that is, the parkway might join the freeway and the parkway traffic travel over the latter for some distance before arriving at a main waterway crossing.

Where parallel freeways and parkways will not be justified in the suburban areas because there is insufficient traffic, it might be possible to develop a type of route which would combine these two by functioning as a freeway during week days, and as a passenger-traffic parkway exclusively during week ends and holidays. While there is no record of this scheme having been tried, it is being seriously considered by a county in a section of the New York metropolitan district.

The importance should be stressed of making the most use of existing facilities. The writer believes the best means for attaining this end is to have not only a master plan for federal highways, but one for municipal, state, and county units as well, which may be dovetailed with the master plan for federal agencies.

A word of caution should be given against making estimates of too great growth of motor-vehicle traffic. Population in this country is becoming stable, as is also the curve of number of persons per motor vehicle. By this it is not meant that traffic will not increase. It will increase because, in the writer's opinion, the use of the individual car, as measured by mileage per year, will continue to increase as improved facilities are provided. We shall use our cars more, although the registration will tend to stabilize as the population also stabilizes.

The following is an example of the dangerous tendency continually to propose the addition of new facilities. Ten years ago, the New York Regional Plan was published, which presented a picture of main highway routes existing and proposed for the New York metropolitan district. Within the time, approximately 50 per cent of the mileage of major routes shown on that plan have been rebuilt or built in new rights of way, so that that system is fairly definitely fixed. We find that, as this tremendous rate of highway construction has taken place in the New York region, regional-plan proposals for additional new routes, which were considered radical and extreme by many persons ten years ago, are now considered as being relatively conservative. Many other proposals have been advanced which, in multiplicity of routes and extravagance of construction, would far exceed what was proposed by the regional plan.

CHESTER H. GRAY.[5] Highway transportation is a part of the general transition in transportation which is taking place rapidly in the United States. During such a transition period, new developments occur or older methods assume new forms. Mention might be made of a few of the new aspects of the transportation problem, not particularly as they would impress the farmer, but as they would appear to all citizens.

It appears that we are in a period of maintenance which in dollars and cents, reflected in taxes assessed and to be collected from the highway user, is quite likely in the future to amount to more annually than the money spent in construction. This fact is not generally realized.

For two decades we have been building highways in the modern manner; at first, under the slogan that they would be permanent. It was soon realized that there is no such thing as a permanent highway. Now we are entering into the period of intense maintenance cost of these older highways. The maintenance cost of existing highways will continue, but added to it will be the upkeep cost on highways yet to be built. Most of the latter will cost more per mile than the ones thus far constructed.

Therefore, a new aspect of highway financing and of highway taxation, namely, the measure of ability to meet maintenance costs for ourselves and future generations, must be considered

[5] Director, National Highway Users Conference, Washington, D. C.

when we evaluate and plan the highways of the future. Maintenance sometimes begins the year after construction and undoubtedly within 5 years after construction, no matter how well-built the highway may be. Once it commences, maintenance continues perpetually, until the road is worn out or must, for one reason or another, be displaced with newer construction in its entirety. Thus, two factors must be carefully considered, maintenance cost of all highways old and new, and amortization of new construction.

In the future, two types of highways will take precedence, i.e., the multiple-lane or express highway through country areas or metropolitan districts and the farm-to-market highway. Where the necessity exists for such highways because of density of traffic for persons and products, and ability to pay for such construction over a reasonable length of time is evident these roads must be built.

In the main, payment will be made by highway users, principally in special imposts, the most prolific revenue-producing source so far developed being the gasoline tax.

In the case of the farm-to-market highway, which entails lower construction costs, public opinion will demand that they be built in increasing numbers.

Accompanying increased construction of these two types of highways will be a continuation of road construction as it has been known in the last two decades.

Another consideration, which is assuming some importance, is that of tolls. For certain causes and for certain limited projects, tolls may be permissible as, for example, the payment in a reasonable number of years of a tunnel under a river or through a mountain. In the case of bridges over navigable streams, the toll principle had assumed the proportions of a political racket; so much so that farm organizations among others in recent years have opposed it to the point where such construction has practically ceased. The ability-to-pay principle within reasonable time limits should be the measure of necessity for practically all projects, be they tunnel, bridge, highway, or some other. If funds which users of the highways already provide were applied to highway purposes and not diverted to other uses, the problems of construction and maintenance would be greatly reduced. In other words, emphasis should be given to the warning that we must not go too far and too fast in approaching the toll principle which some people now are advocating as the cure-all for financing highways.

A word of caution is necessary, in connection with acquiring rights of way for new highways, that the cost of this land does not too greatly exceed the actual cost of construction. With the liberal attitude toward public funds that has existed in Washington in recent years, the tendency of holders of land, required for rights of way to widen old roads or build new ones, has been to demand handsome payment from the government. There are many instances where such costs have been 50 or 75 per cent of the total cost of a highway. Twenty years and more ago, landowners were generous in this matter and held it as a matter of pride to donate land for this purpose. Legislation is needed to correct this tendency, so that funds provided for highways will largely go toward construction and maintenance and not into purchase of rights of way. Such legislation should be formulated in Congress, so that in January, 1941, when the legislatures of practically all states will again be in session, bills on this matter may be formulated and enacted.

This is a period of highway planning during which roads should be built of the proper materials and type to meet the traffic and other conditions to be imposed. In the future roads should be built so that subsequently they will not require straightening, leveling, or grading; they will proceed around or through cities as the need exists, and as funds and public demand make possible. They will be built in the country and, above all, a scheme must be developed by which a considerable proportion of the cost may be paid as they are laid, and the balance amortized over a reasonable period. Mr. Horner has done well to bring these important road matters to public attention.

L. W. Horning.[6] The author states that the two big problems confronting highway users are the continuing expense of building new highways and the annual cost of maintenance. There is also a third problem, that of rebuilding the roads which have been destroyed.

The ability of highway users to pay for them in the main as they are built, but surely during their life expectancy, has been referred to. It is a fact that in many cases highways are torn up and destroyed long before the bonds which financed them have been retired. The writer recently discovered a series of bonds which will not be retired until 1947, while the road which they financed was rebuilt last year.

There are two classes of highway users, i.e., private-car owners, mostly having modest incomes, about 25,000,000 in number, including the farmer and the merchant with their small trucks, and another group of bus and truck operators who use the highway as a place of business for private gain. Certainly there is no community of interest between the private motorists and these others. The writer does not believe that the average motorist has occasioned the rebuilding of 22 per cent of our highway systems at a cost, reported by the American Association of State Highways in a hearing before Congress in 1938, of $1,607,609,000.

The current conviction seems to be that the payment of motor-vehicle fees, referring to gasoline taxes and license fees in general, should vary according to the weight and carrying capacity of the vehicle. With that principle, the writer largely agrees.

However, he disagrees with the statement that gasoline taxes, federal, at least, were originally intended to compensate for the use of the highways. In a recent discussion before Congress, Senator Barkley said, discussing the gasoline tax, "Just for the record, I think it ought to be stated that, when the tax on gasoline was enacted, it had no direct connection with building roads." It was not enacted specifically for the purpose of using it to build highways. This excise tax, therefore, represents a contribution for the support of general government.

We do heartily agree that where a tax is levied upon gasoline or upon motor vehicles for the express purpose of constructing, maintaining, or repairing highways, that every red cent of that money should be used for that purpose. But we also maintain that there is nothing so sacred about a gallon of gasoline or a motor truck that taxes should not be levied against them for the support of general government. All this talk about diversion may backfire one of these days and the ordinary homeowner and taxpayer may demand that his taxes be not diverted either. Where would our highway users be, if you please, if those millions of people who are not highway users should demand that we spend on highways only the income from motor-vehicle levies?

With Senator Barkley's conception of the purposes to which gasoline taxes may be put, the writer is in full agreement.

At another point the statement is made that the motor imposts now being levied seem to be the most logical form of highway taxation yet devised. The writer believes that, generally, the principle is correct in so far as the private-car-owner class is concerned, but certainly it is not a fair measure of highway use nor a fair and equable tax for the business user, unless, for ex-

[6] Regional Director, Competitive Transportation Research, American Association of Railroads, New York, N. Y.

ample, the gasoline tax provided for a high rate when it was to be used in a 10-ton or a 5-ton truck as the case might be. A ton-mile tax might be a fairer measure of highway use by such vehicles.

Speaking of farm-to-market roads, it is a fact that the railroads are keenly interested, since about 14 per cent of railway taxes are used for that purpose. Such roads act as feeders for the railroads. In a recent statement of policy, highway users announced that farm-to-market roads should be paid for by taxes on land, not by levies on the users. The author referred to thousands of communities located on such roads served only by motor transportation. It is these communities which are largely located on the secondary or farm-to-market roads and still the highway users would have this class of road maintained by "taxes on land" and not on the users.

The author states that highways, railways, waterways, airways, and pipe lines are the quintuplets of transportation in the United States. In other words, the economic welfare of the nation does not revolve around one transport facility, highways or otherwise. With that the writer heartily agrees. Every one of these has a definite place in the transportation scheme of things.

Then he refers to our cry for equality of opportunity. What is wrong with that? Isn't every American business, isn't every American, no matter what business he is in, entitled to equality of opportunity? Certainly, one railroad is entitled to equality of opportunity with its competitors and, certainly, the railroads are entitled to equality of opportunity with their competitors who operate on the public highways, upon the waterways, and elsewhere. Today, that is all the railroads contend for.

One vitally important method of regulating highway users, not mentioned by the author, is that of regulating rates and services. The writer believes that most of the common carriers, engaged in transporting freight upon the public highways, would welcome regulation of the functions which other highway carriers perform as contract carriers. In fact, many of them demand the enactment of legislation to that end. So it has not always been railroad-inspired legislation that would have imposed regulations on the carriers upon the highways. These carriers themselves, have demanded and are still demanding such legislation and here and there are highway carriers "for hire" who have suggested that the private carriers should be regulated too.

In 1933, representatives of the highway users and the railroads in conference agreed upon certain fundamental principles which were applicable to highway transportation, among which was the matter of regulation. The conference was unable to agree upon the uniformity of weights, sizes, and rates.

The railroads contended, and still do, that there can be no uniformity in the size of motor vehicles until there is uniformity in the highways, uniformity in the ability of the people to pay for the highways, and uniformity of traffic. As regards reciprocity, the highway users agreed that trucks from one state traveling into another should pay something for the use of the public highways. They said that reciprocal agreements should be entered into at "equitable rates." Now they contend, not for equitable rates, but for free reciprocity.

Much has been said about the regulation applied to common carriers as highway users. A recent statement of the Interstate Commerce Commission is quoted to the effect that highway transportation is more comprehensively regulated than are the railroads. It is not quoted exactly. The statement was: "These

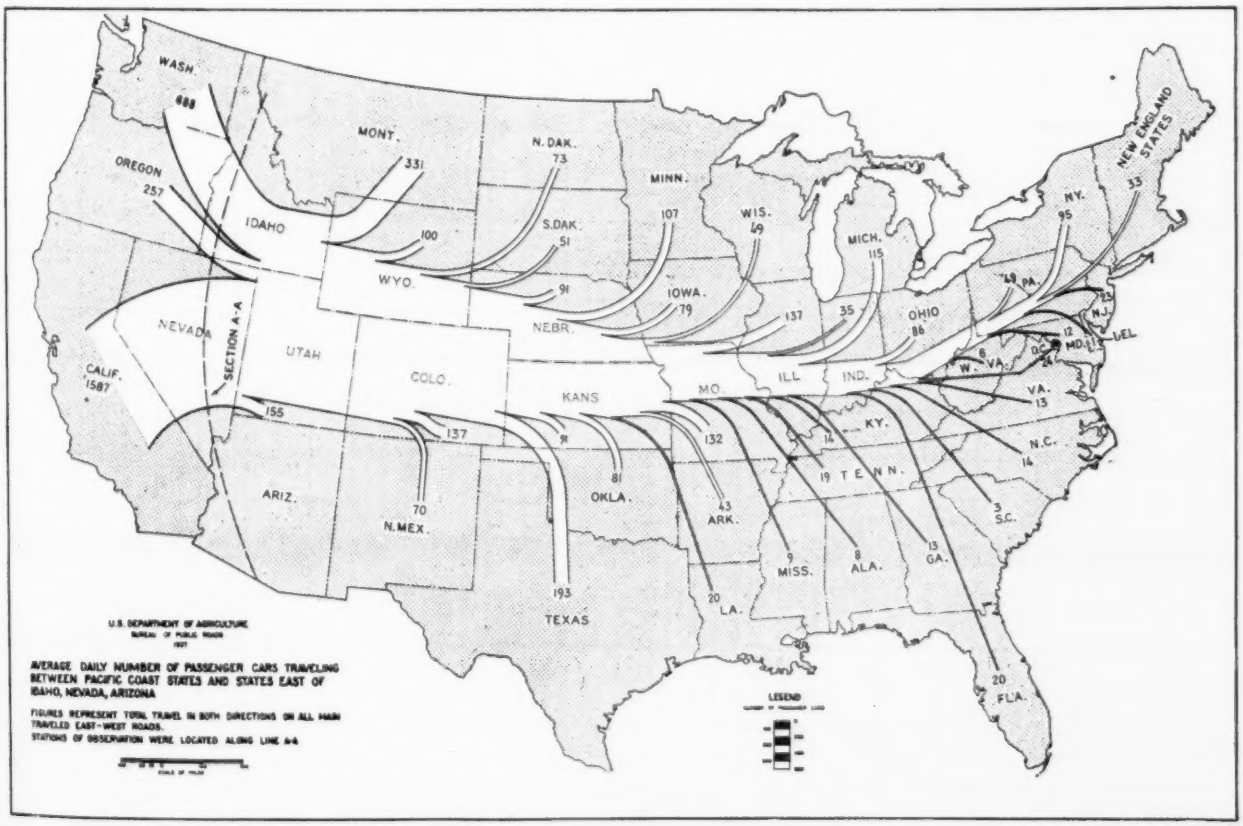


FIG. 1 DIAGRAM SHOWING THE STATES OF ORIGIN AND DESTINATION OF TRAFFIC ON ALL MAIN EAST-WEST ROUTES TRAVERSING THE STATES OF IDAHO, NEVADA, AND ARIZONA
(Traffic local to the named States omitted.)

two acts (the Motor Carrier Act of 1935 and the Civil Aeronautics Act of 1938) provide for the motor carriers and the air carriers, respectively, a system of regulation which is, if anything, more comprehensive than that which has been provided for the railroads." Possibly that is true. The writer believes such carriers should be, because they are operating on public highways, while the railroads operate on private rights of way.

H. S. Fairbank.[7] The report of the Public Roads Administration to which reference has been made in Mr. Horner's paper was in response to a definite question directed to the Administration by Congress. The report therefore took its character from that question. The question was whether or not it is feasible to build a system of three transcontinental roads running east and west, and three running north and south, from the Canadian border to the lower extremity of the United States, and whether or not it would be feasible to support such a system of six through or transcontinental roads by means of tolls. That was a definite question and the report first addressed itself to that question.

The conclusion at which it arrived, namely, that the building of transcontinental toll roads is not feasible, was the first answer that we attempted to give to that question and it was in the negative. In giving that answer, we did not mean to imply that there is no room, in the financing of highway facilities, for the use of a toll method. In fact, we specifically referred to the feasibility and possibility of using the toll method for financing certain portions of a highway, and especially for the financing of bridges and tunnels and other relatively short sections of highways, particularly in metropolitan areas where a toll facility is feasible because it offers a service greatly superior to that afforded by the competing free-highway facilities.

Having given the negative answer to the question of the feasibility of a toll system, the report proceeded to attempt to indicate what might be a positive answer in dealing with the general problem of providing adequate highway facilities for the United States. That answer was made along the following major lines:

[7] Chief, Division of Information, Public Roads Administration, Federal Works Agency, Washington, D. C.

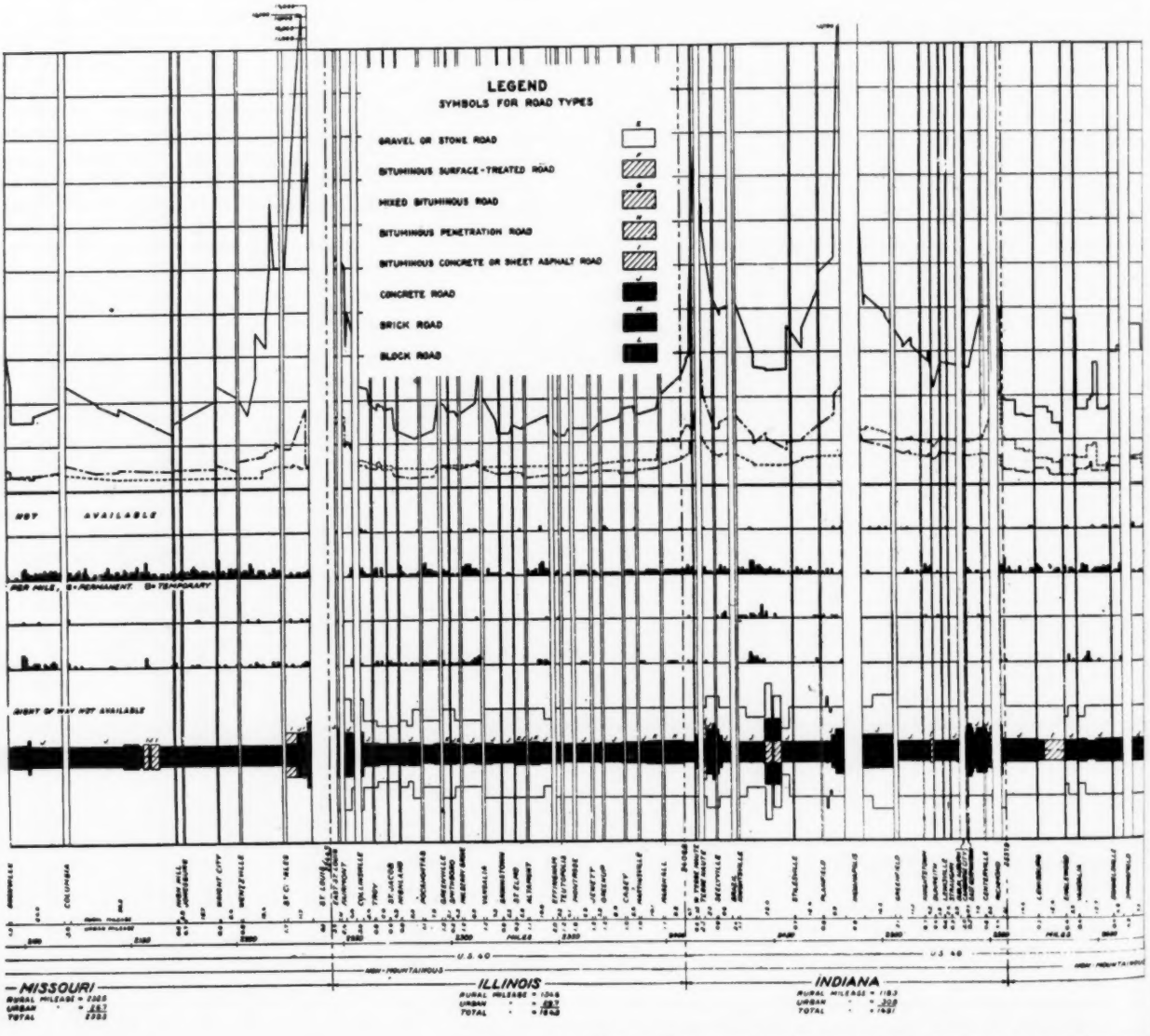


Fig. 2 Straight-Line Diagram of a Portion of the Principal Existing Conditions, as Revealed by the

First, the report pointed out that the most serious problem has developed on our highways in the vicinity of the metropolitan areas, and in the cities. It also pointed to the necessity for a modernization of the main highways of the customary type, which were built largely in the previous decade, in order to fit them for safe use by vehicles now operating at speeds very much higher than were foreseen when the roads were built. This the report treated as the first element of a proposed master plan.

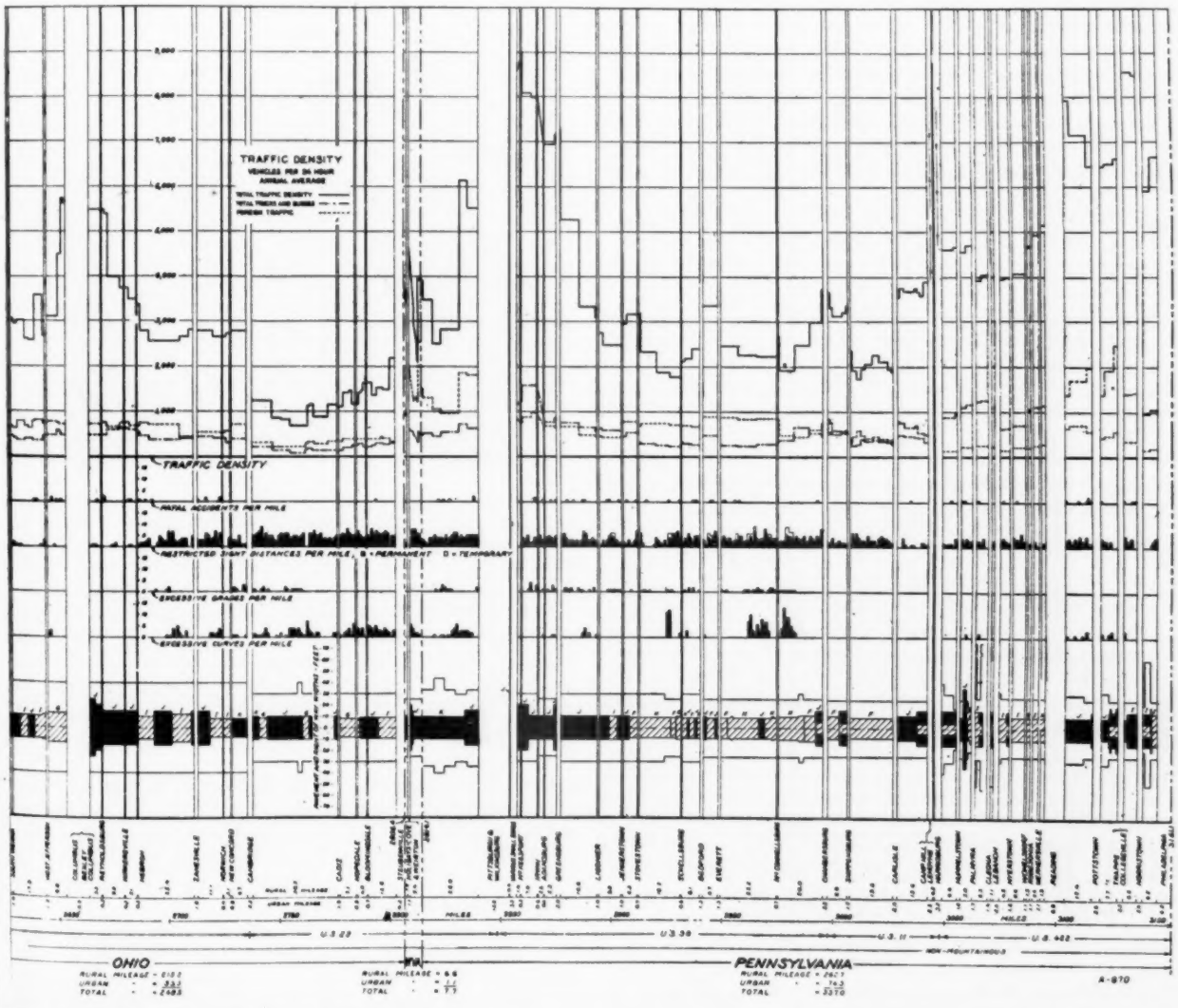
Next it was indicated that the principal location of such modernization of the main highways would be necessary along lines which might be extended into the formation of an interregional highway system of fairly limited extent, suggested as the second element of the plan. The report fixed the probable extent of that system as about 30,000 miles.

The third element referred to the secondary and "feeder" roads, local roads which are the feeders not only of the main highways, but also of the railroads. The three elements then were coordinated to form, in our estimation, a well-rounded plan of highway development which would meet all of the requirements of modern transportation, as they might develop. This plan was submitted as an alternative to the one-sided proposal which contemplated the construction of six main transcontinental highways. The building of such highways would only superficially touch the real problems that have developed through the growing use of our highways.[8]

Unquestionably, there has been built up in the popular mind an impression that the volume of transcontinental traffic to be served is large. To determine the actual dimensions of transcontinental traffic, a study was made of all main east-west highways, traversing the width of the country from the Canadian line to the Mexican border. Traffic counts were conducted in the states of Idaho, Nevada, and Arizona.

The result of questioning all but local traffic indicated that, on an average day, there passed along all of these highways, a total volume of about 2532 vehicles. Of these 2532 vehicles,

[8] At this point in the discussion Mr. Fairbank, by means of slides from the Public Roads Administration Report, illustrated major points developed by the author in his summary of this report. Copies of the report are available on request from the Administration.



Route Between Philadelphia and San Francisco Showing State-Wide Highway-Planning Surveys

1587 were bound to or from points in California; 257 were bound to or from points in Oregon; and 688 to or from points in Washington. The volume of traffic having its origin or destination on the East Coast amounted to but 300 per day. Thus transcontinental traffic is a rather thin trickle of our total traffic on the highways. (See Fig. 1 on page 87 of this paper.)

Conversely, the bulk of traffic on our highways is of a highly local range of movement. For example, based upon traffic observations, vehicles crossing into New York or out of New York, over the Hudson River by all the means of crossing from the Battery to Tarrytown, total on the average 82,000 daily. Beyond the metropolitan area of New York and New Jersey, nearly 5000 a day go to or come from points in New York State. Nearly 5000 a day go to points in Pennsylvania, or come from those points. Nearly 3000 come from or go to other points in New Jersey. Of the total of 82,000 crossing into or out of New York City, only 3100 remain as that portion of the large volume of traffic which either comes from or is destined to points more distant from New York City than the local counties and states. It is generally true that only about 6 per cent of total traffic throughout the country has a range greater than 50 miles per trip length. In sparsely populated areas, where distances are long and towns are far apart relatively light traffic flows on the main roads.

The areas of greatest traffic density are in the eastern and in the north central states, where distances are short and where there is a great concentration of short-distance movements.

Reference has been made to the fact that our present highways were built during a period when the speed of vehicles was less than it is now and, because of this fact, there is considerable

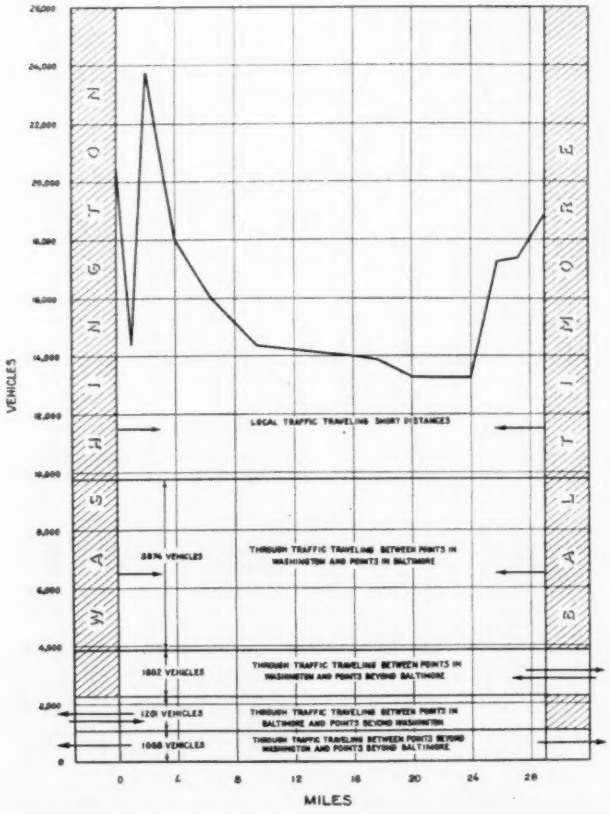


FIG. 3 DIAGRAM SHOWING THE ORIGINS AND DESTINATIONS OF TRAFFIC ON THE BALTIMORE-WASHINGTON ROAD, AND INDICATING THE SMALL PERCENTAGE THAT COULD BE BY-PASSED AROUND EITHER CITY

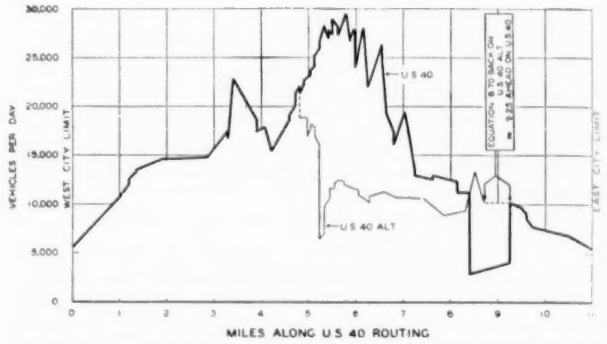


FIG. 4 TRAFFIC PROFILE ON THE ALTERNATE SECTIONS OF U. S. ROUTE 40 THROUGH COLUMBUS, OHIO, INDICATING THE LARGE EXTENT TO WHICH THE PEAK OF TRAFFIC ON THE ROUTE AT THE CENTER OF THE CITY IS DUE TO LOCAL CITY TRAFFIC

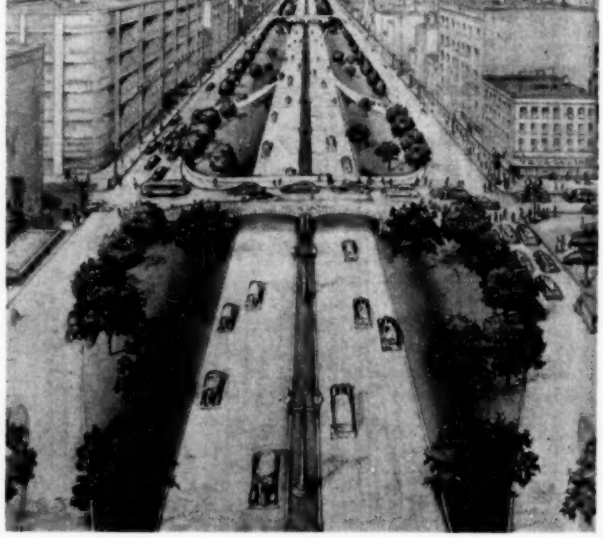


FIG. 5 SUGGESTED FORM OF A DEPRESSED EXPRESS ROUTE THROUGH A LARGE CITY

obsolescence of roads that were built during that earlier period. This is unquestionably a fair general statement of conditions.

One of the purposes of the so-called state-wide highway-planning surveys, which have been undertaken in most of the states, has been to determine just what that condition of obsolescence is, just what our problem of rebuilding may be, where that problem is located, and to what extent it is influenced by the density of traffic and the character of the provision that has been made previously (Fig. 2).

One of the facts developed by the surveys is that the greater percentage of traffic approaching a city is destined to, or emanates from, the city itself. Any proposal to by-pass that traffic would fail as a solution of congestion in the city, because the traffic cannot be by-passed, since most of it is generated by the city or induced by the city (Fig. 3).

A certain proportion of traffic might be by-passed around a city, but actually the greatest percentage of the movement has its origin or destination in the city. For example Columbus, Ohio, is traversed by two transcity extensions of U. S. Route 50. One of these is the main highway and the other an alternate route. The main highway carries about 5000 vehicles at the borders

of the city, while inward, traffic builds up to nearly 30,000 vehicles a day. If all the border traffic were to be by-passed, only a small percentage would be removed from the city. Therefore, by-passing as a remedy for traffic congestion within the city confines would be of but little account (Fig. 4). Actually the congestion is caused by the increasing volume toward the heart of the city being forced into streets diminishing in width. This is a condition generally experienced in older cities which have developed around narrow streets.

Where new and broader highways have been built, traffic will follow the line of least resistance, but when it must be diverted into the older and narrower streets of a business section, congestion occurs.

One solution to the problem of congestion within cities is the use of the depressed highway for bringing the major arteries into them (Fig. 5). There is no fixed preference for the depressed highway, but, in general, we believe that the depressed form of artery, rather than the elevated, is more consistent with the requirements of the modern motorcar. It would seem to make possible the extension or the wider distribution of settlement and does not necessitate the crowding or the concentration which might be indicated by the use of the elevated type of artery.

As we enter a city along such routes as those described, before long we penetrate a band or a zone within the city which represents the old residential sections. As the motor vehicle has made possible an outward movement of residential areas, we have left behind, in the band adjacent to the downtown sections, properties which were once good residences. These descend from one class of property owner to a lower class and finally become the slums of the city.

That condition exists in most cities and represents one of the tremendous tasks that must be dealt with in considering their future. As the necessity of providing through arteries is considered, the existence of decadent areas of old residential property seems to offer some hope that it might be possible to acquire rights of way for superior types of arteries, without involving too great expense, since the values of the old properties are declining steadily.

Beyond the cities, interregional roads have become a necessity. While such a system would comprehensively cover the country, it is not to be regarded as a transcontinental system. It becomes transcontinental only by the connection of local sections.

WILLIAM G. GROVE.[9] Mr. Horner's paper has emphasized many of the points brought out in the report "Toll Roads and Free Roads" prepared by the Federal Bureau of Public Roads, and the writer wishes to compliment Mr. MacDonald and the Bureau of Public Roads upon the thoroughness of that report. It is a masterpiece, and its careful perusal by anyone whose duties are connected with highway traffic is sincerely recommended.

For the last few months the writer's activities have been connected with the organization of the toll station on the Merritt Parkway and hence this discussion will reveal something of Connecticut's experience in the collection of highway tolls.

In June, 1938, the first section of the Merritt Parkway was opened to the traveling public. This parkway is a continuation of the Westchester County parkway in New York and, when completed, will extend from the New York state line to the Housatonic River, a distance of approximately 35 miles. The parkway is a dual type of highway; each traffic section is 27 ft wide and the dividing central island is 21 ft wide.

The Merritt Parkway will cost about $20,000,000, of which $15,000,000 is covered by a bond issue, now being refunded at the rate of $1,500,000 annually from the highway funds. The $5,000,000 remainder will also come from highway funds. The Merritt Parkway will be completed to the Housatonic River by July 1, 1940, and at the same time the bridge over the Housatonic River is scheduled for completion. This $1,000,000 structure is on the Wilbur Cross Parkway and is being financed through a P.W.A. grant covering 45 per cent of the amount; the remainder is to be taken from the regular highway funds.

The Wilbur Cross Parkway will eventually extend from the Housatonic River to the Massachusetts state line. The portion from Hartford south to Berlin is now under construction and will be paid for from the regular highway funds. Certain sections between Hartford and the Massachusetts state line have been built and other sections are now under construction.

The section from the east bank of the Housatonic River to Berlin, a distance of about 35 miles, could be completed in four years if funds were available. This stretch will cost approximately $20,000,000, and will require 20 years to complete on the basis of appropriating $1,000,000 a year from the regular highway funds for that purpose.

This, then, was the situation confronting the Highway Department at the first of the year, 1939. In order to complete the Wilbur Cross Parkway to Berlin by 1944, instead of 1960, three courses of procedure were considered: First to take $5,000,000 a year from the regular highway fund; second, to increase the state gasoline tax from 3 cents to 4 cents per gallon; and third, to float a bond issue in order to provide for the necessary amount of money and to amortize these bonds over a period of 20 years with money obtained by placing tolls on the Merritt Parkway and also on the Wilbur Cross Parkway as the latter parkway is completed.

Connecticut has a motor-vehicle registration of about 450,000 vehicles of all types. On the assumption that these vehicles average 10,000 miles a year and consume gasoline at the rate of 15 miles to the gallon, the approximate gasoline consumption at the present time is 300,000,000 gallons per year. The present state tax of 3 cents per gallon yields about $9,000,000; registration fees on 450,000 vehicles yield an additional $6,000,000, and these, with some miscellaneous revenues, provide an annual fund for the highway department of about $17,000,000 to $18,000,000 a year.

If the cost of the Wilbur Cross Parkway from the Housatonic River to Berlin were financed at the rate of $5,000,000 a year from the highway fund, the regular road program for both maintenance and new construction would be seriously crippled. Furthermore, a state tax of an additional 1 cent a gallon on gasoline consumption would provide only $3,000,000 a year.

The third plan that suggested itself was to raise the necessary funds by means of tolls. The 1939 session of the state legislature passed a bill, which was signed by the governor, authorizing tolls on the Merritt Parkway and, as commercial vehicles are not permitted on this parkway, a flat rate of 10 cents per car was specified in the bill. In order not to work unnecessary hardship on motorists who use the parkway regularly, a commutation rate of $10 was specified for any car licensed in Connecticut, which would entitle that car to unlimited passage through one toll station on the Parkway for a period of 1 year. The bill provides that one toll station shall be located in the Town of Greenwich and a second toll station located in the Town of Milford, and that there shall be such additional toll stations as the highway commissioner may deem necessary.

A study of the traffic over the Merritt Parkway near the New York state line indicated that approximately 5,000,000 cars would pass the Greenwich toll station annually, thus yielding $500,000. When the toll station at Milford could be placed in operation, after the completion of the new bridge over the

[9] Associate Highway Engineer, State Highway Department, Hartford, Conn.

Housatonic River in June, 1940, it was estimated that 3,000,000 cars would pass through that toll station annually, yielding an additional income of $300,000 a year.

The act further provides that 5 per cent of the highway funds shall be used for the Wilbur Cross Parkway and also authorizes the flotation of a bond issue, not to exceed $4,000,000. During the two-year period until the next session of the legislature in 1941, there should be available for the Wilbur Cross Parkway $850,000 a year from the regular highway funds of $1,700,000, $4,000,000 from the sale of bonds, and approximately $1,300,000 from tolls making a total of $7,000,000, or more than one third of the total amount required, which will be sufficient to purchase the right of way and to have under contract the construction of a considerable portion of the parkway from the Housatonic River to Berlin.

The 1941 session of the legislature will then be in a position to authorize the flotation of additional bonds, which, when added to $1,700,000 from the highway funds and $1,800,000 from tolls, will cover the total additional cost necessary to complete the Wilbur Cross Parkway to Berlin by 1944.

The act further provides that all monies collected from tolls shall be earmarked and placed in a separate fund known as the Wilbur Cross Parkway Fund, which fund shall be used, after deducting the cost of collecting the tolls, exclusively for paying the current bond interest and amortizing the bonds. After the Wilbur Cross Parkway has been completed to Berlin and paid for, and all the bonds have been amortized, the legislative act provides that no further tolls shall be collected.

The parkway toll bill was passed by the legislature on June 6, and signed by the governor on June 16. On June 21, at 8 a.m., the collection of tolls started at the Greenwich toll station and up to the present date (Oct. 31, 1939), approximately 2,350,000 vehicles have passed the station, which at 10 cents per vehicle amounts to $235,000 in 133 days, an average of almost $1800 per day.

During the summer the average traffic was 130,000 vehicles a week. After Labor Day, this weekly average dropped to 110,000 during September and October. The approximate monthly traffic was 590,000 for July, 570,000 for August, and 550,000 for September. The October traffic will be about 470,000 cars.

Sunday traffic averaged 30,000 cars for July, August, September, and October, with Saturday traffic averaging from 20,000 to 25,000 cars for the same four months. The greatest daily peak occurred on Labor Day when 39,855 vehicles passed the toll station. Based on present figures and allowing for decreased seasonal traffic during November and December, the three-million-vehicle mark should be reached about Jan. 1, 1940.

While the report prepared by the Bureau of Public Roads condemns the general use of toll roads throughout the United States, the report does state that in certain sections of the country toll roads would be self-liquidating and these sections are listed in the order in which the most profit is indicated. The number-one section on this list is from Jersey City, N. J., to New Haven, Conn. It is fortunate in that the beautiful Merritt Parkway is located on this number-one section. Traffic over the parkway is at the rate of 5,000,000 vehicles a year past the toll station when an alternate free route (the Post Road, U. S. No. 1) which parallels the parkway, is only 5 miles away. This is the answer to critics, that the public is willing to pay for superservice, which provides, first, safety, and second, a saving of time through the absence of red lights over a distance of 35 miles.

V. P. Ahearn.[10] Mr. Horner has surveyed the entire range of the question of sound public policy in the development of our highway systems. However, there are some points which, while not new, deserve special emphasis.

One of these is the tremendous threat to national unity which is presented in the multitude of state-highway regulatory laws, like the Port of Entry Law, the Milk and Food Inspection Law, and the like. Anyone who has read Dr. Buell's article[11] on this subject will have an uncomfortable realization that laws of this character, each prompted by the desires of special interests to monopolize local markets, will eventually reproduce in the United States some of the sore spots which are at the bottom of European controversies.

It seems to the writer of this discussion that there is real need for a dispassionate evaluation of our methods of land acquisition. Obviously, present public authority is admittedly inadequate even for the building of multiple-lane highways in some areas.

If regulation is inevitable, and if hopeless confusion is to be avoided, it would appear that uniformity of regulation is necessary. We should consider the question: "Is uniformity desirable?" If it is desirable, federal regulation would seem to be the only alternative. If that is true then it would be unwise to oppose such legislation solely on the ground that the powers of the federal government would be increased.

W. L. Waters.[12] The diversion of highway taxes has been much criticized. But we do not ask that cigarette taxes be used to provide smoking rooms or beer taxes to build saloons. So we cannot insist that motor-vehicle taxes be used exclusively for highways. The government taxes where it can, and gasoline like beer is ideal for the purpose.

As to highway construction, civil engineers are now working on soil mechanics and stabilization, and it is likely they will soon be able to develop a technique of building economically almost anywhere highway foundations equal to those of the Roman roads of 2000 years ago. So ten years hence highways may consist of a stabilized earth foundation covered with a thin flexible mat to protect it from wear, and the costly concrete type of highway will be obsolete.

It is also possible we are emphasizing too much the need for heavy expenditures on alignment and grades. Probably not one man in a hundred is physically and temperamentally fitted to drive at 50 mph in traffic; and long straight stretches of low-gradient highway are a temptation for the lunatic driver to speed at 70 or 80 mph.

As to highway routes in outlying suburban and rural districts, this problem is not difficult. It is merely a question of planning and spending money. But in the cities it seems hopeless. Some years ago I saw quite a disturbance in a village on the border of Burma and China which resulted from two caravans meeting in a street so narrow that the laden donkeys could not pass one another. So evidently traffic congestion is nothing new, and the history of large cities from time immemorial has probably been that transportation difficulties choked them so that they decayed, were abandoned, and replaced by new cities elsewhere which again went through the same cycle. And as our facilities which were sufficient in the horse-and-buggy days are now obsolete because of the demands of motor-vehicle transport, the same threat of decay hangs over our own large cities. Conditions can be somewhat bettered by substituting organized public-utility transport for much of the city traffic, by handling at night all trucking except household deliveries, by garaging automobiles in the suburbs, by severely restricting parking, by closing many street crossings, and by similar improvements. It is a question

[10] Executive Secretary, National Sand & Gravel Association, Washington, D. C.

[11] "Death by Tariff," by R. L. Buell, *Fortune*, August, 1938, pp. 32–35.

[12] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

for each city to decide whether it will submit to such restrictions or be gradually choked to death by traffic congestion.

The paper is most valuable, and it is to be hoped that it will result in a really common-sense national economic discussion of the highway problem. If it does, and if we can eliminate politics and introduce into our program some of the intensely commercial spirit which has been responsible for the phenomenal success of the motor-vehicle manufacturers, we shall, except in the cities, probably be able to develop most of the facilities required without greatly increasing taxation.

W. D. Ennis.[13] Not merely are roads expensive—major highways now cost as much per mile as railroads. In 1937, road expenditure absorbed about 3 per cent of our national income.

We argue about diversion, comparing total motor-vehicle revenues with highway expenditures therefrom. We began by postulating the application of all such revenues to major highway construction and maintenance. It was a new kind of postulate. Revenues from real estate, by contrast, were never wholly applied for the benefit of real estate. We are still collecting taxes from real estate to be spent on streets and roads. Diversion is not a one-way phenomenon.

Much confusion arises from the term "highway." Sometimes it means a major interurban thoroughfare. If we talk more broadly of "roads," public ways, including county and township roads and city streets, there is a lot of road cost which motor vehicles never pay.

The statement that a (uniform per-gallon) gasoline tax is an equitable and rational method of distributing highway cost in proportion to highway use is one that may be questioned. Such a tax might be fair if the vehicles all required the same character of road. But a large proportion of total road cost is due solely to a limited number of heavy vehicles. Lighter vehicles, including all private passenger cars, should pay no part of that excess cost. In New Jersey, in 1933, less than 5 per cent of all vehicles weighed over 7000 lb, yet it is for such vehicles that roads are planned. Fairer taxes on heavy trucks would open the way toward reduced taxes on vehicles of normal weight.

"Excess taking," the acquisition and sale by government of excess lands in connection with rights of way, is not really a new term or new idea, even in the highway lexicon. It has been discussed in road engineering for years, and experience is that it may require changes not merely in federal and state laws, but also in their constitutions. (Benefit assessments, as distinguished from local assessments of "cost," apparently not mentioned in the paper, are a related device to which there are similar obstructions.) It may be assumed that a moderate program of excess taking will be found most acceptable constitutionally. The Bureau of Public Roads program is moderate, if loose. It contemplates liberal provision for safety, roadside development, recreation, sanitation and other facilities, and future expansion (additional lanes).

The British strip width of 1320 ft (maximum) goes to the other extreme. Constitutional limitations, in our sense, do not exist in Great Britain. Even more extreme is the President's broad advocacy of seizing the unearned increment (equivalent to the benefit-assessment idea) and government speculation in land. If these policies are sound, their application should be broader than in the single field of highway construction.

[13] Humphreys Professor, Economics of Engineering, Stevens Institute of Technology, Hoboken, N. J. Fellow A.S.M.E.

Wear of Diesel-Engine Cylinders and Rings

By PAUL S. LANE,[1] BALTIMORE, MD.

This paper considers wear mainly from the viewpoint of cylinder and ring materials, rather than from the design or operating angle. The types of iron used, though important factors, have received only limited attention in published data. Existing wear rates in medium- and heavy-duty Diesel and gas engines in various fields of service are reviewed, followed by several examples of cylinder wear as affected by the hardness and structure of the iron alloy. By means of laboratory wear tests, correlated with service experience, the nature and characteristics of the materials used in cylinders and rings are surveyed and discussed in detail, covering the effects of section size, hardness, analysis, and structure. Initial engine wear, as influenced by bore finishes and wear-resistant or chemical surface treatments, is considered, together with the action of bimetallic or composite rings, in retarding scuffing and over-all wear rates.

THE WEAR of Diesel-engine cylinders, liners, and rings is a subject of mutual interest to engine designers, builders, and operators as well as to piston and ring manufacturers. Everyone even remotely connected with Diesel engines and their maintenance realizes that this subject deserves attention and study, particularly in view of the higher operating pressures, speeds, and temperatures which make lubrication problems more difficult. Even though improvements are being made in Diesel lubricating oils, the bearing and wearing characteristics of the rubbing parts will continue to have an influence on wear rates.

A review of published data on Diesel-engine cylinder wear gives considerable information as to the effects of lubricating and fuel oils, air and oil cleaners, design features, etc. However, only a few scattered references are made to cylinder and ring materials, and their influence upon over-all wear conditions.

In this paper it is proposed mainly to discuss and review present-day practice in regard to cylinder and ring materials, considering their wear performance not from the standpoint of any particular type of engine or service, but solely from the metal-structure viewpoint. An attempt is also made to illustrate the scuffing and scoring tendencies of various irons, and the possible effects of hardness, surface finish, and chemical treatments. Brief mention will be made of service results obtained through the use of piston rings of the Gold Seal or "insert" type, in reducing wear problems in various Diesel applications.

Rates of Wear

Before considering the possible influence of the foregoing factors, it seems advisable to offer the present interpretation of normal and abnormal cylinder wear in various engines.

Although it is agreed that the usual wear rate is greater than desired, yet this is difficult of measurement in service due to the gradual nature of the change. For example, it has been stated by Shidle[2] that, when a 5-ton truck finally wears out, it has actually lost in weight something in the neighborhood of 5 lb.

In the case of cylinder wear in medium- and slow-speed engines, the figure of 0.001 in. wear (diameter increase at top-ring travel) per 1000 hr is often considered normal. This means a wear per hour of 0.000001 in., which requires considerable amplification before it is possible of measurement with ordinary micrometers.

Table 1 shows wear figures submitted by Lee Schneitter,[3] which logically take into consideration a life expectancy of 6 years for liners, based on 8000 hr operation per year. Such figures must of course represent an average condition; the values appear to conform with general experience.

TABLE 1 LINER WEAR OF SLOW-SPEED ENGINES, BASED ON LIFE OF SIX YEARS

Cylinder diam, in.	Wear (maximum), before liner replacement, in.	Wear per year, 8000 hr running, in.	Normal wear, per 1000 hr in.
25	0.180	0.030	0.004
20	0.150	0.025	0.003
15	0.100	0.017	0.002
10	0.060	0.010	0.0013

Table 2 shows additional actual wear figures from a group of large stationary Diesel engines operating in pipe-line service in Oklahoma. These figures have been kindly contributed by J. B. Harshman.[4]

TABLE 2 LINER WEAR OF DIESEL ENGINES OPERATING IN PIPE-LINE SERVICE

	Four-cycle engines	Two-cycle engines
Number of engines	44	33
Number of cylinders	180	120
Size range, bore diameter, in.	16¼–22	15–20¾
Speed range, rpm	164–225	180–257
Usual load, per cent of rating	80–95	80–95
Average hours running per 0.001 in. wear	869	992
Average wear per 1000 hr, in.	0.0011	0.001

Two leading engine makes included in the two-cycle group of Table 2, have a service record of 250 to 500 operating hours per 0.001 in. wear (or 0.002 to 0.004 in. per 1000 hr). One engine, also included, gives 2000 to 3000 hr operation per 0.001 in. wear (or 0.0005 to 0.00033 in. wear per 1000 hr).

A recent service experience, involving a 925-hp, four-cycle Diesel engine, further illustrates the fact that very low wear rates are sometimes encountered. This engine, in ferryboat service, operates 24 hr per day every day in the year, except for two 2-week periods in dry dock for scraping and bottom painting. This engine is of the solid-injection type, 18 in. diameter × 24 in. stroke, running at 200 rpm.

New rings were installed in October, 1936, at which time the cylinders showed an average wear of 0.003 in. When recently examined in February, 1939, the maximum wear was found to be 0.009 in. or 0.006 in. increase in approximately 18,000 hr running; this is equivalent to a rate of 0.00033 in. per 1000 hr. Lubricating-oil consumption is reported to have been less than 5 gal per day or approximately 4400 hp-hr per gal.

In automotive engines, a cylinder wear of 0.001 in. to 0.003 in. diameter increase per 10,000 miles seems the normal experience with American engines. Piston-ring wear generally averages 0.003 in. to 0.005 in. per 10,000 miles.

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[1] Metallurgical Research Engineer, American Hammered Piston Ring Division, Koppers Company.

[2] "Five Pounds of Importance," by N. G. Shidle, *Automotive Industries*, vol. 66, 1937, p. 449.

Presented at the Twelfth National Meeting, Ann Arbor, Mich., June 19–22, 1939, of the Oil and Gas Power Division of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Ebasco Services, Inc., New York, N. Y.

[4] Master Mechanic, Stanolind Pipe Line Company, Tulsa, Okla.

Aircraft engines, even with very hard nitrided cylinders, frequently give a barrel wear of 0.0025 in. per 1000 hr. In this case, however, the time is roughly equivalent to 150,000 miles. Rings are of course always renewed at the overhaul period of 400 to 600 hr.

While all of the foregoing wear figures are offered as average values, undoubtedly there will be disagreement as to their being typical, since they may not be in accord with individual experience. It is this element of doubt which justifies employing some form of laboratory wear-test apparatus in an effort to eliminate the many variables experienced in actual service, and to rate the wearability in arbitrary values.

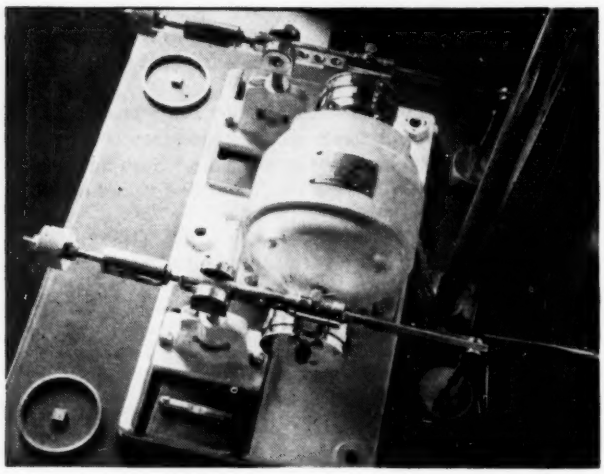


FIG. 1 TOP VIEW OF WEAR-TESTING UNIT USED FOR LUBRICATED TESTS UNDER HIGH PRESSURES AND LOADS

LABORATORY WEAR TESTING

During recent years, as a result of work done by various investigators relative to the effect of metal structure on wear resistance, it is now quite generally accepted that structure, rather than hardness, mainly determines wear resistance.

In spite of a wider knowledge of wear-resisting materials and alloys, the nature of wear itself yet remains an elusive quality, since it appears that each individual type of installation (having apparently its own particular set of conditions) reacts differently in relation to its wearing parts.

These conditions are further complicated by the fact that there is no universal or accepted method for testing or measuring wear resistance. In fact, it has been stated that a standardized wear-test machine is highly undesirable, as such an apparatus cannot possibly simulate conditions of all classes of service, on much the same basis that no standardized corrosion test has been found entirely reliable.

Due to this fact, many different types of machines have been used with varying degrees of success. All of these machines have failed, in one respect or another, to duplicate exact operating conditions. Nevertheless, it is believed that each of these attempts at measuring wear has added a bit more toward the total knowledge of wear characteristics in metals.

In a previous paper,[5] the author described certain wear results obtained with a machine of the brake-shoe type, wherein a flat specimen is held against a revolving drum, using a fixed load, no lubricant being supplied to the surfaces. A feature of this procedure is that, at the start of the test, the unit pressure is extremely high (momentarily at infinity), as the specimen makes only line contact with the drum. As wear takes place, unit pressures are lowered, depending upon the rate of wear of each specimen.

Fig. 1 shows the wear-testing apparatus, additional details of operating procedure may be found by reference to the paper.[5] Table 3 gives arbitrary wear values used to rate piston-ring-iron specimens.

TABLE 3 WEAR VALUES FOR RATING IRONS

Rating	Weight loss in 1 hr, mg
Excellent	18 and under
Good	20–24
Fair	26–30
Poor	over 30

Fig. 2 shows a schematic view of results on an extended wear test, comparing "time to seat-in" for a "good-" and for a "poor-" wearing iron. This illustrates the possible range encountered in the method of testing under consideration.

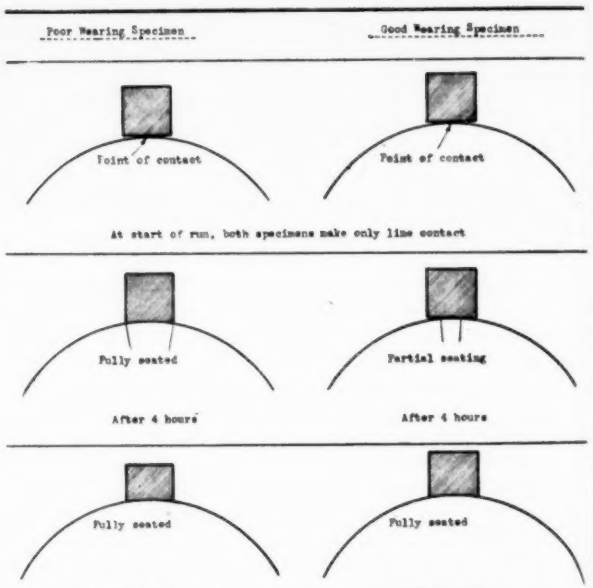


FIG. 2 DIFFERENCE OF SEATING-IN TIME FOR GOOD- AND POOR-WEARING IRONS

Gray cast iron is markedly affected by the speed of cooling. This in turn, is largely determined by the section thickness of the casting. This action is illustrated in Table 4, which shows the effect of section on the same ladle of iron poured in different size test bars, the combined carbon and hardness increasing as the section size decreases.

TABLE 4 EFFECT OF SECTION SIZE ON GRAY IRON CASTING

Cross section, in.	2	1½	¾	⅝	½
Graphitic carbon, per cent	2.95	2.90	2.80	2.65	2.40
Combined carbon, per cent	0.45	0.50	0.60	0.75	1.00
Brinell hardness	160	180	210	260	320

The change in hardness also means a change in other properties such as graphite size, strength, and wear, or abrasion resistance. This latter action is mentioned for the reason that it will frequently be referred to throughout the remainder of this paper.

DIESEL-ENGINE CYLINDER CASTINGS

One of the early investigations, in which the author partic-

[5] "Some Experiences With Wear Testing," by Paul S. Lane, Transactions, American Foundrymen's Association, vol. 45, 1937, pp. 157–199.

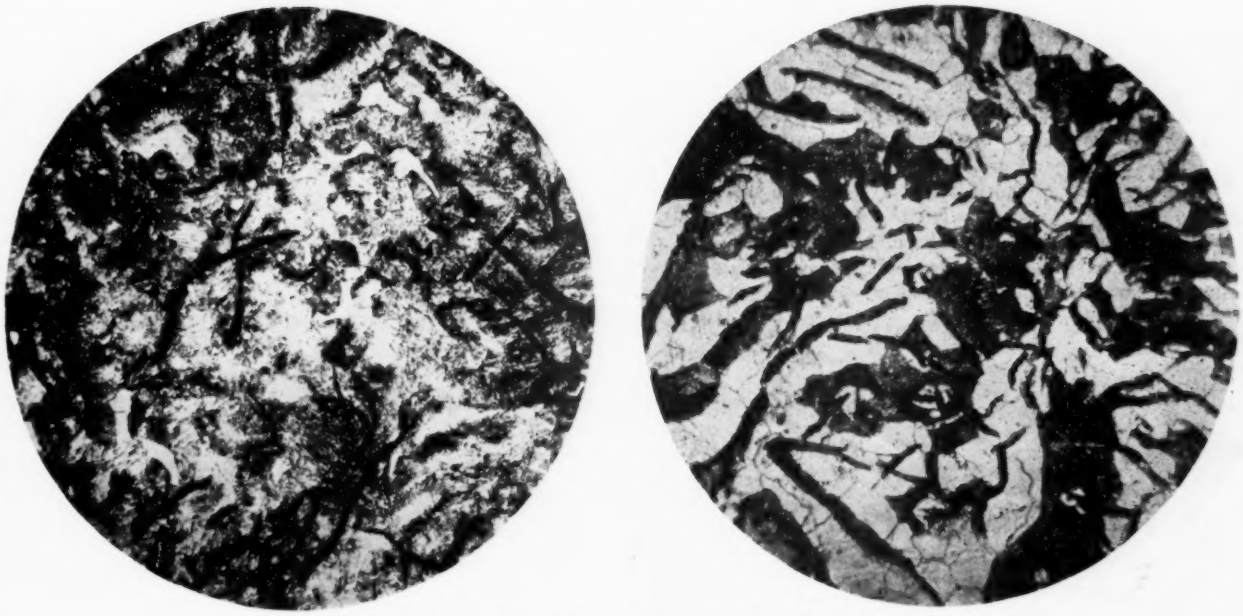


FIG. 3 DIESEL-ENGINE BORE STRUCTURES, ETCHED, ×200

pated, had to do with small Diesel-engine cylinder castings. Two sizes of engines were involved, one having a bore of about 3 1/2 in., the other a bore of 4 1/2 in. The section thickness of the bore proper was 3/16 to 3/8 in. for the smaller engine, and approximately 1/4 in. for the larger. The smaller engine showed normal wear in service, while the larger bore was giving high wear and scuffing troubles. Data on these castings are given in Table 5 and their microstructure in Fig. 3.

TABLE 5 WEAR DATA ON DIESEL CYLINDER CASTINGS

	Bore, 3 1/2 in.	Bore, 4 1/2 in.
Silicon, per cent	2.10	1.92
Sulphur, per cent	0.061	0.097
Phosphorus, per cent	0.19	0.14
Manganese, per cent	0.68	0.69
Total carbon, per cent	3.11	3.28
Graphitic carbon, per cent	2.55	2.86
Combined carbon, per cent	0.56	0.42
Nickel, per cent	0.57	...
Chromium, per cent	0.08	0.10
Molybdenum, per cent	0.06	0.31
Average Brinell hardness	185	158
Rockwell range	87–91B	80–85B
Wear value, mg	15.8	18.6

The wear value on the larger-bore engine is satisfactory from a weight-loss angle and may be viewed as not correlating with service results. On the other hand, it shows that the normal ferrite-graphite structure, if accompanied by large graphite flakes, resists wear (due to its toughness) in so far as weight-loss values are concerned. However, this very toughness accounts mainly for the scarring and scuffing tendencies of such structures. While this is not always clearly defined, it was noticed that marked roughening of the contact surfaces occurred with this specimen. A similar matrix, if accompanied by fine eutectic graphite, which more completely breaks up the continuity of the structure, would show a much higher wear rate.

As to the cause for such undesirable structures, this is due to incorrect analysis for the section thickness. Thick or heavy sections cool slowly, which may allow time for the structure to reach the so-called "stable" and undesirable ferrite-graphite composition (as shown at the right in Fig. 3), instead of one of the normal pearlitic forms. The same iron, only with lowered silicon content to adjust for cooling rate, would result in a structure similar to that in the 3 1/2-in. bore.

In connection with these results, it seems appropriate to mention that such conditions are apt to occur where castings of varying types and cooling rates are made under a broad chemical specification. When this is true, the foundryman should not be held entirely responsible. Rather, the fault lies with the management which may sometimes fail to appreciate the importance of mass effect and demand that all castings fall within a specified composition. This illustrates the need for close cooperation between the engineering and foundry departments, which is being urged more and more by the various technical societies.

VARIATION IN STRUCTURE AT PORT AREAS

Another example of the complications resulting from section thickness and design is the case of a cast-iron port section from a two-cycle Diesel engine having a bore of about 10 in. The ribs which separate or form the port openings have a relatively thin section, which is further exaggerated if the ports enter the cylinder tangentially rather than radially. The tangential design gives a thin tapered section immediately at the bore surface.

Under test and in service, these engines showed normal wear at the areas above and below the ports, where the metal thickness was approximately 3/4 in. Excessive wear was experienced along the port ribs, where the metal thickness varied from 1/16 to 3/8 in. Aside from any consideration of the metal structure, for many reasons it is true that more wear would naturally be expected around the port areas. The possibility of maintaining an oil film at this point is greatly lessened, and unit loading is higher. However, the wear on the ribs was so pronounced that metal-

TABLE 6 TESTS ON CAST-IRON PORT SECTION OF TWO-CYCLE DIESEL ENGINE

CHEMICAL ANALYSIS

Silicon, per cent	1.30	Manganese, per cent	0.90
Sulphur, per cent	0.14	Total carbon, per cent	3.20
Phosphorus, per cent	0.12	Chromium, per cent	0.12

WEAR TESTS

Specimen location	Mark	Section thickness, in.	Weight loss, mg	Hardness, Rockwell	Hardness, Brinell
Full section	A	1/8 to 3/4	15.2	89.7B	185
Port section	B	1/16 to 3/8	35.6	90.7	190
Port section	C	1/16 to 3/8	30.4	89.2	180
Rib-port section	D	3/8	23.5	89.5	183
Full section	E	1/8 to 3/4	15.3	88.0	175

structure and wear tests were studied. The results are shown in Table 6, together with the chemical analysis and hardness.

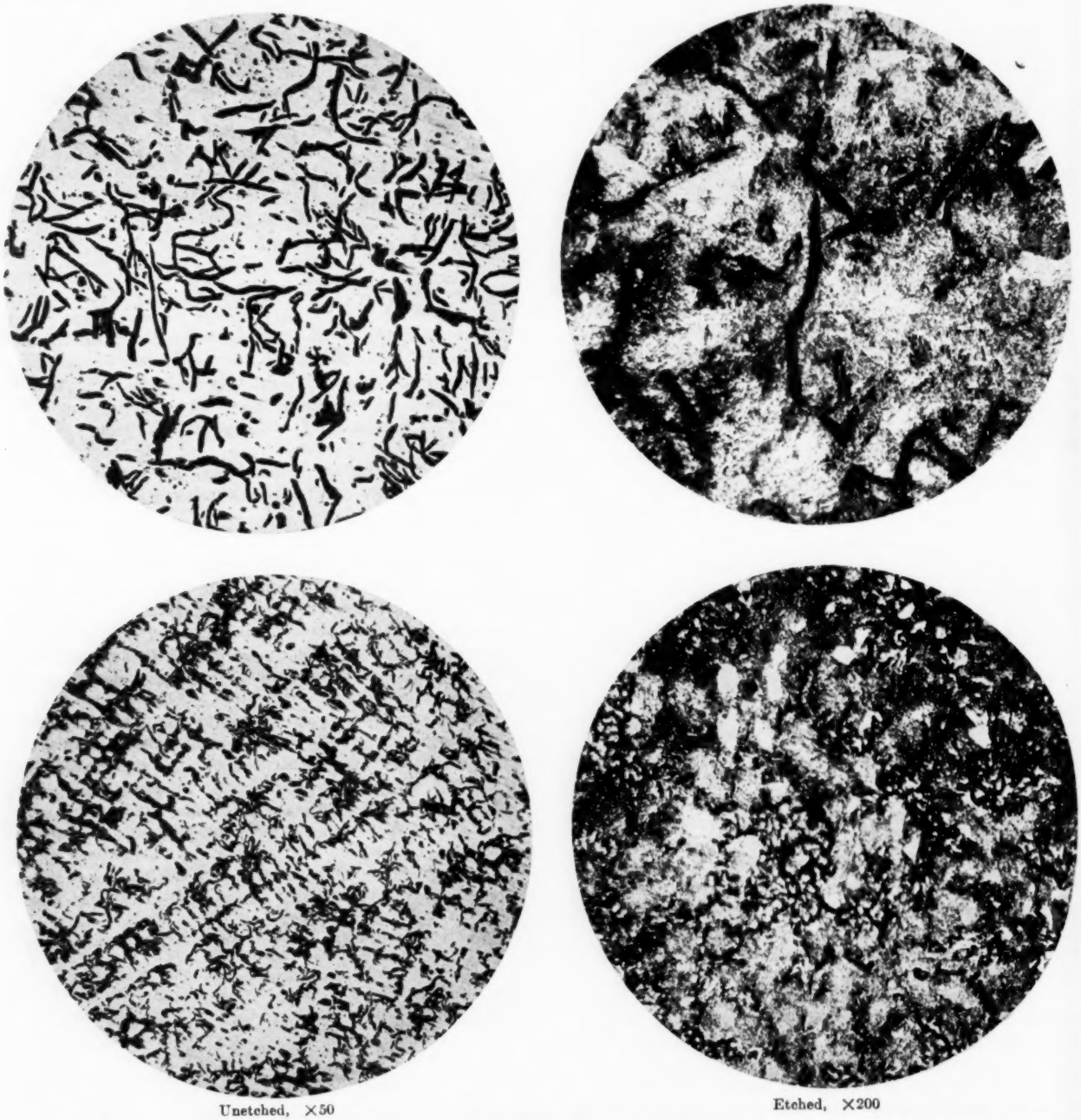
From the results shown in this table, it is seen that a 2 to 1 ratio in wear rate is indicated with only a small hardness variation. Fig. 4 illustrates the structure existing at the full sections and rib sections. The heavy section (top) has a normal eutectoid structure, rather coarse graphite (slowly cooled) and good indicated wear. The rib areas (bottom) have the fine dendritic graphite along with numerous ferrite spots; an abnormal structure with lowered abrasion resistance.

These tests have not as yet been brought to a conclusion, so it cannot be stated definitely just how important a part the metal structure will play in the solution of this problem. However, more recent wear tests, conducted on sections poured under different foundry conditions, are shown in Table 7; note the lowered wear rate in some of the rib areas.

TABLE 7 WEAR TESTS ON SECTIONS POURED UNDER DIFFERENT FOUNDRY CONDITIONS

Cast no.	Wear wt loss in mg: Full section, mg	Wear wt loss in mg: Rib section, mg	Rockwell B: Full section	Rockwell B: Rib section
A	14.7	26.8	90.7	89.0
B	16.4	26.1	90.2	89.2
C	11.2	18.0	98.2	97.0
D	10.4	16.6	96.5	96.5
E	16.0	25.3	88.5	89.0
F	12.0	22.4	95.2	94.7
G	15.5	20.8	95.5	94.0
H	14.0	29.5	87.7	89.0

Conditions such as the foregoing, that is, structural and wear



Unetched, ×50 Etched, ×200

FIG. 4 LARGE DIESEL-ENGINE CYLINDER STRUCTURE; (TOP) THICKNESS AT FULL SECTIONS, 3/4 IN.; (BOTTOM) THICKNESS AT RIB OR PORT AREAS, 1/4 IN.

TABLE 8 CHEMICAL ANALYSIS AND HARDNESS OF CYLINDER LINERS

Engine	Finished bore section, in.	Brinell	Silicon, per cent	Sulphur, per cent	Phosphorus, per cent	Manganese, per cent	Total carbon, per cent	Nickel, per cent	Chromium, per cent	Molybdenum, per cent	Copper, per cent
A	1/4	185	2.10	0.060	0.19	0.68	3.11	0.57	0.08	0.06	..
B	1/8	350+	(Case-hardened on surface—no analyses run)								
C	1/8	235	2.00	0.10	0.09	0.58	3.12	0.29	0.66	0.45	Nil
D	3/8	215	1.80	0.14	0.16	0.50	3.06	0.40	0.80	0.42	Nil
E	5/16	210	2.10	0.095	0.28	0.63	3.18	0.27	0.50	0.42	Nil
F	3/4	215	1.45	0.10	0.15	0.90	3.12	..	..	..	1.04
G	3/4	175	1.45	0.10	0.21	..	3.20	..	..	..	..
H	1	145	1.70	0.11	0.14	..	3.53	..	..	..	..
I	1 1/8	150	2.01	0.11	0.08	..	3.25	..	..	..	..
J	1 1/4	175	1.87	0.17	0.51	0.80	3.08	0.97	0.40	Nil	Nil
K	1 1/4	165	1.78	0.18	0.28	0.70	3.02	0.40	0.10	Nil	Nil
L	1 1/4	185	1.35	..	..	..	2.90	..	..	..	..

NOTE: (..) No determination made.

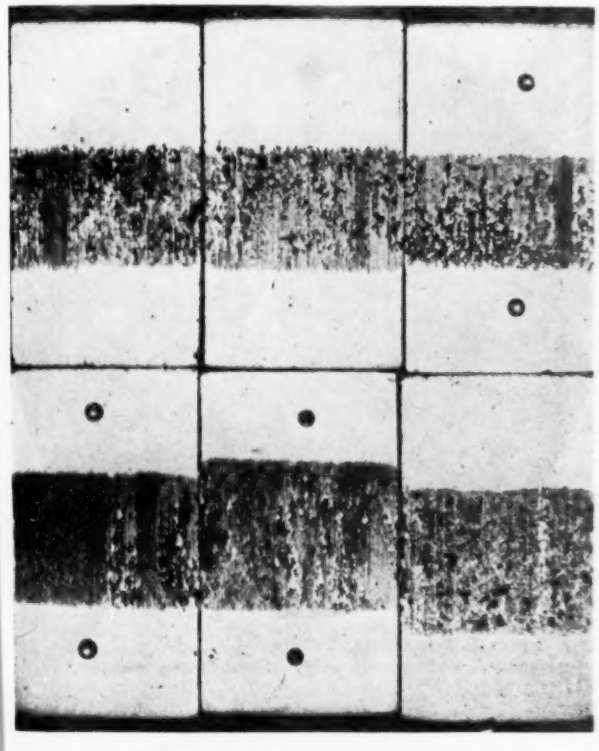


FIG. 5 APPEARANCE OF WORN SURFACE OF CYLINDER IRON; LOW WEIGHT LOSS BUT HIGH SUSCEPTIBILITY TO SCUFFING

variations within a single bore, are less prone to occur in liner or sleeve castings where the section thickness is more regular. However, it appears that present-day liners cover a rather wide range of hardness and chemical composition, as shown in Table 8 along with the section thickness.

NATURE OF LINER STRUCTURES

For the most part, liner irons for larger-bore engines are typical of the usual gray-iron casting used for various purposes. The matrix is mostly pearlitic. Generally, though not always, the liners are devoid of sizable amounts of free ferrite or cementite. Most American and foreign investigators are in agreement that a fully pearlitic structure gives relatively lower wear than other machinable types of structure. Another characteristic of such irons is that they usually have relatively coarse graphite, so they may be classified as of the "coarse-grained" variety.

Such structures, under friction conditions, generally show relatively low weight loss but with decided glazing or scuffing tendencies. This statement is based on a testing experience covering hundreds of various structures during the last few years. Fig. 5 shows the appearance of the worn surfaces of six cylinder-iron specimens. The light areas are dragged or "worked" metal similar to those encountered when metal-to-metal contact is present in rubbing surfaces. Early in wear-test work, the author observed this feature. Subsequent testing has developed a somewhat better understanding of this action, although films and surface effects are far from being entirely understood.

It is believed that the added ductility and toughness of coarsely pearlitic structures largely account for this worked surface. Irons of this type have certain bearing-like characteristics in that, under frictional forces, intimate contact with the mating surface is seldom obtained but rather, high and low points exist similar to the duplex structure in a babbitt. This means that the high points carry the load, while the low points hold the lubricant. It also means that areas of high local stress exist at the "tops" or "peaks" and, when conditions become sufficiently critical, scuffing or seizure occurs.

Pearlite, the dominating microconstituent in cylinder irons, is made up of alternate layers of ferrite (which is soft and ductile with Brinell around 100) and cementite, or iron carbide, having a Brinell of 600, with good strength, but of a brittle nature. Under conditions of boundary lubrication or dry rubbing, there is sufficient flow of the ferritic material to initiate scuffing or incipient galling. This action may tend to strengthen and support the surrounding crystals of cementite. A similar action is also believed to take place in the case of the hard phosphorus or steadite constituent when this is present in the microstructure. In order to explain this action, it appears that the pulling away of the particles of ferrite from the matrix also allows them to roll against and pack around the harder constituents, thus reinforcing the base of these compounds and actually strengthening them.

The ferrite movement between the laminations of cementite may also tend to lower unit pressures on the latter. It is believed that this surface is constantly changing upon itself. It is this quality which leads to the belief that the coarse pearlitic irons tend to "bruise easily and heal quickly," providing the point of critical loading is not exceeded. This theory appears to tie in satisfactorily with the low weight losses commonly experienced in the case of the relatively soft and weak irons. Another observation which should be mentioned is the greater abuse to the contact surface which obtains when testing irons of the cylinder type. This appears to be due, in part at least, to a tendency to load with wear products. In this instance, the nature of the structure is such that the tendency toward "imbeddability" of abrasive particles may be readily understood (Figs. 6 to 9).

Thus, on the basis of the foregoing reasoning, the viewpoint may be taken that cylinder or liner irons, at least those with which the author is familiar, generally have the following characteristics:

1 Coarse graphite and laminated pearlite matrix, generally near eutectoid composition.

2 Low dry-wear values based on the weight-loss method, but with proneness to scuffing.

3 Surface toughness or "flowability" indicated by character of friction surface. Wear products tend to adhere rather than slough away.

4 A "bruise readily and heal quickly" quality, probably a desirable feature if unit loading does not become too high.

5 Imbeddability, which may account for high abuse or lapping action to mating surfaces.

In actual service most Diesel cylinders show an excellent polished or glazed condition in those areas where lubrication is normal and resultant pressures lower. Near the tops of the cylinders, however, higher wear oftentimes accompanied by scuffing is sometimes encountered. Aside from the other causes to which this is frequently attributed, some of the characteristics mentioned may also bear on this phenomenon.

Nature of Piston-Ring Irons

Although piston rings, as well as liners, are of gray cast iron they are very different structurally from the cylinders just discussed. First the graphitic carbon is present in a finely divided form which breaks up the continuity of the metal. Second, though they are for the most part pearlitic, this constituent is of an extremely refined form approaching a granular rather than a laminated make-up. Figs. 10 and 11 illustrate the usual ring-iron structure. A comparison with the cylinder irons, shown in Figs. 6 to 9, brings out clearly the marked differences. Fig. 12 shows the usual appearance of ring-iron wear-test specimens.

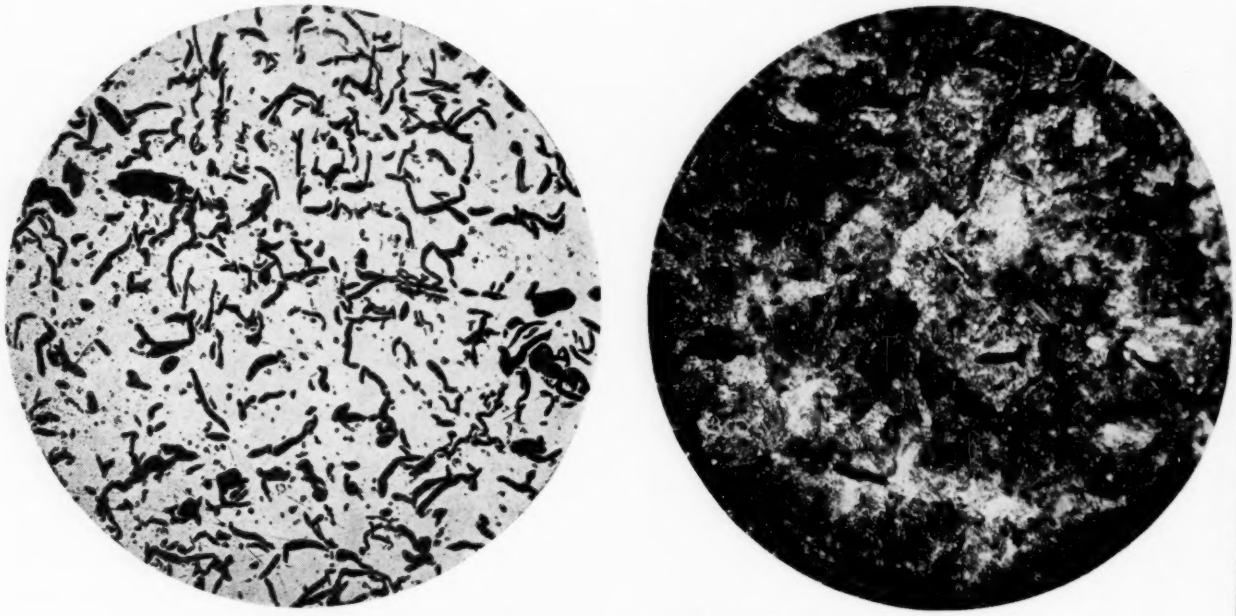


Fig. 6 Microstructure of Cylinder Liner of Engine F, 8-In. Bore; 220 Brinell
(Wear value, 15 mg; drum wear, 90 mg; total wear, 105 mg. Medium graphite, low phosphorus, pearlitic, slight scuffing tendency.)



Fig. 7 Microstructure of Cylinder Liner of Engine G, 10-In. Bore; 175 Brinell
(Wear value, 10 mg; drum wear, 115 mg; total wear, 125 mg. Very coarse "chunky" graphite low phosphorus, coarsely pearlitic, decided scuffing and welding properties.)

Irons of this group show relatively high wear as tested under the same conditions as cylinder compositions. This is due to the fact that they are "finer grained" and "less crystalline," both of which expressions are erroneous in metallurgical terminology but are used in the hope that they will convey the meaning. These ring irons, however, have two very distinctive and desirable qualities: First, they are exceptionally free from scuffing inclinations, and second, they are sacrificed to maintain the integrity of the cylinder.

The freedom from scuffing inclinations is due primarily to the form of graphite, which results in a ready sloughing away of wear products, permitting the material continually to present a smooth unblemished face to the mating surface. Particles of the surface, cut or abraded by wear forces, do not build or "ball up," but rather are prone to powder away in a fine dust. To illustrate: soft steel, due to its toughness and ductility, is infamous for its scuffing proclivities. Under dry rubbing, the wear products weld to the surface eventually resulting in galling and seizure; this being the converse of ring-iron characteristics.

Tests also have shown that, when wearing against a wide range

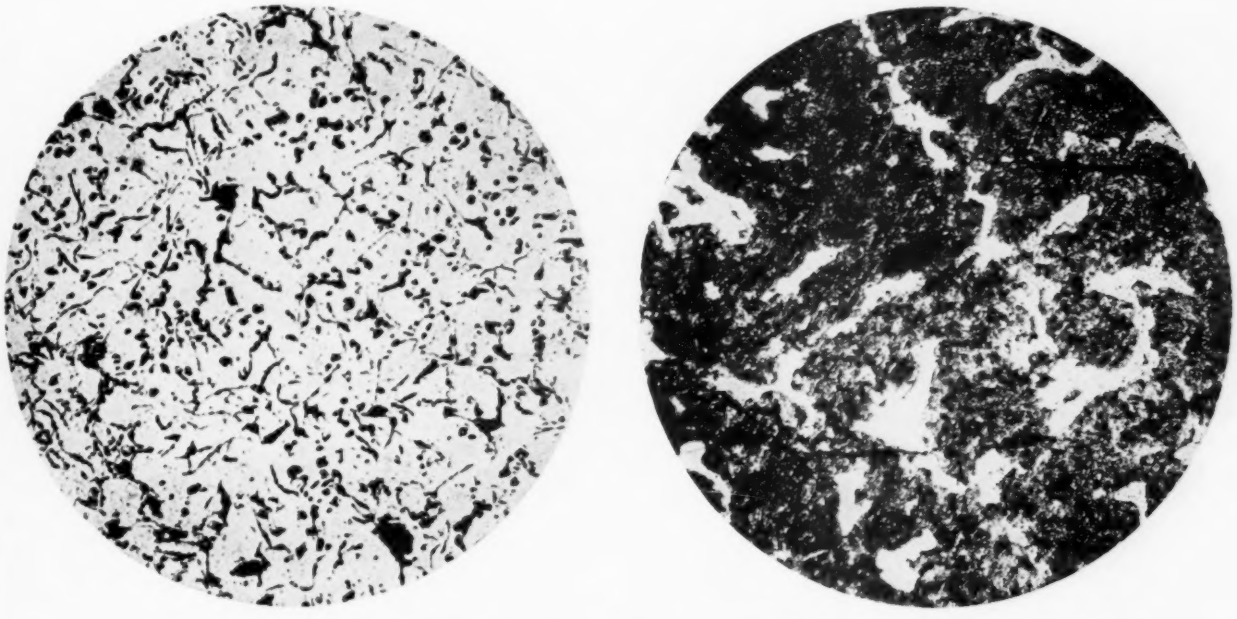
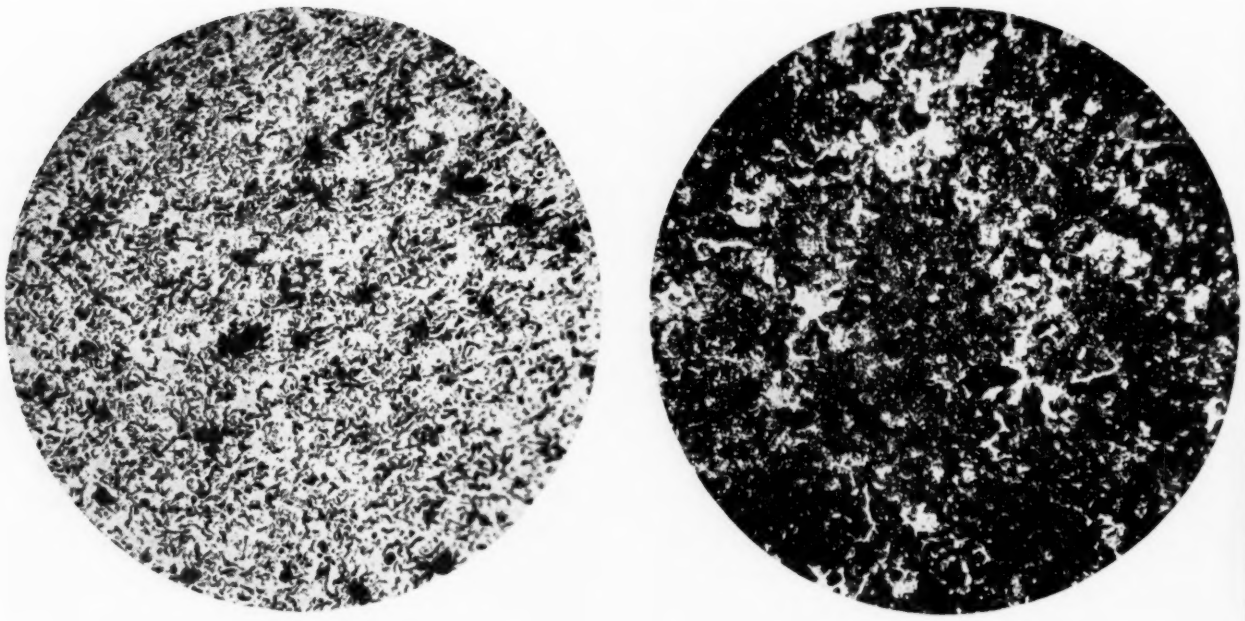


FIG. 8 MICROSTRUCTURE OF CYLINDER LINER OF ENGINE D, 5-IN. BORE; 215 BRINELL
(Wear value, 8 mg; drum wear, 116 mg; total wear, 124 mg. Fine graphite, alloyed iron, free carbides, structure has cutting and scoring proclivities.)

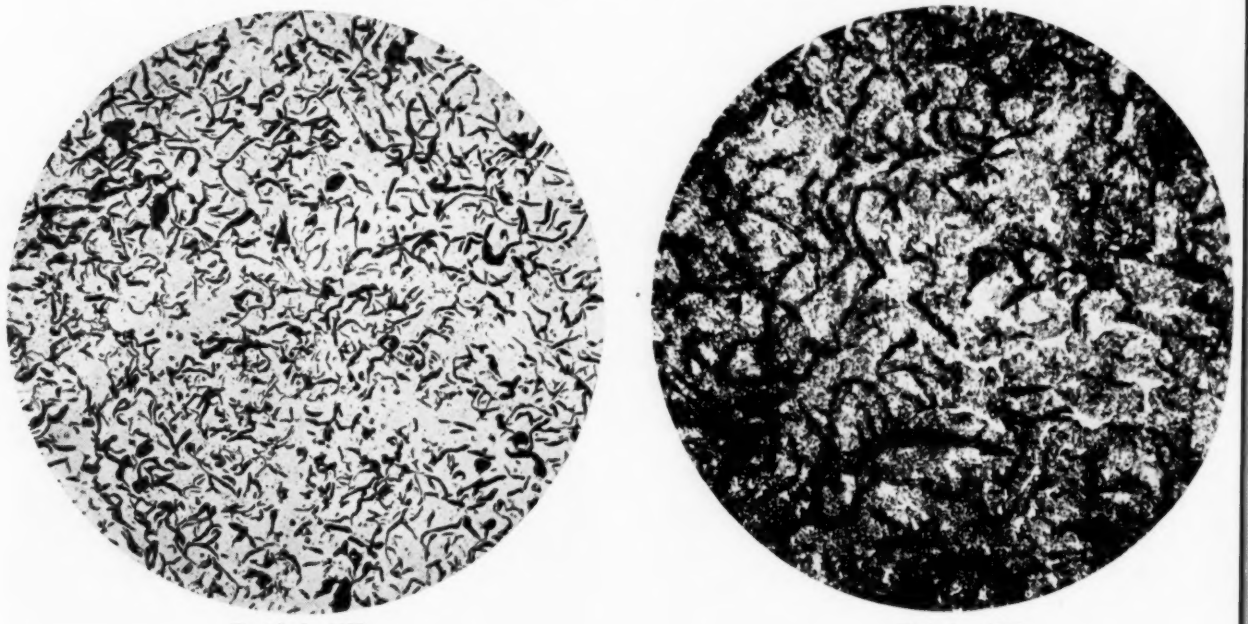


FIG. 9 MICROSTRUCTURE OF CYLINDER LINER OF ENGINE I, 14-IN. BORE; 150 BRINELL
(Wear value, 17.4 mg; drum wear, 62 mg; total wear, 79.4 mg. Medium to fine graphite, medium pearlite, good antiscuff properties.)



Unetched, ×50 Etched, ×200

FIG. 10 MICROSTRUCTURE 8-IN DIESEL-PISTON-RING IRON
(Wear value, 32 mg; drum wear 38 mg; total wear, 70 mg.)



Unetched, ×50 Etched, ×200

FIG. 11 MICROSTRUCTURE 14-IN. DIESEL-PISTON-RING IRON
(Wear value, 27 mg; drum wear, 47 mg; total wear, 74 mg.)

of mating materials and finishes, ring irons do not load or abrade. Instead, they are themselves worn, taking the major part of frictional abuse.

Piston-ring irons, on the basis of the foregoing, may be said to possess the following characteristics:

1 Very fine graphite and sorbitic-pearlite matrix of eutectoid composition.

2 High dry-wear values, based on weight loss only, but with good freedom from scuffing proclivities.

3 A granular surface tending to free itself of wear products, thus presenting a smooth unblemished contact face even under adverse loading.

4 Tendency toward glazing is less than in coarser-type irons, such as are often used for cylinders and liners.

5 Slight proneness to imbedding of abrasive matter so that lapping action is not present on the ring face.

In present-day practice, coarse-grained irons are utilized for cylinders and liners, and fine-grained irons for rings. In both

instances, this is likely a result of foundry practice rather than of choice. Fig. 13 shows the difference in grain or cell size of a Diesel-engine liner and a ring. It is apparent that structurally these irons are very different, this difference being one of cooling rate rather than of hardness and analysis. Fig. 14 further illustrates this condition showing the difference in graphite size between a ring and a cylinder.

The apparent theory is that coarse-grained irons are best for cylinders since they will glaze and polish, due to their natural surface toughness. This results in low wear if lubrication is always present. When oil supply is erratic for one reason or another, metal-to-metal contact occurs. When this happens, normally the piston ring will take the abuse and protect the cylinder. If the dry condition becomes too severe, however, scuffing starts. It is believed that this is usually initiated on the cylinder surface and is then transferred to the rings, with the result that, unless conditions change or relief in the form of oil supply takes place, serious galling or seizure ensues.

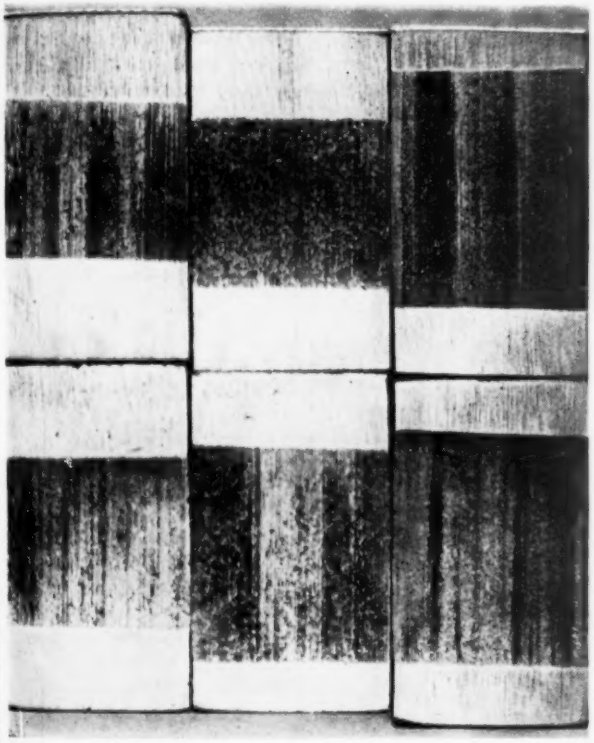


FIG. 12 APPEARANCE OF WORN AREAS OF PISTON-RING-IRON SPECIMENS

(Relatively high weight loss, but with ability to slough off wear products without surface mutilation.)

TABLE 9 DIESEL CYLINDERS—WEAR VALUES

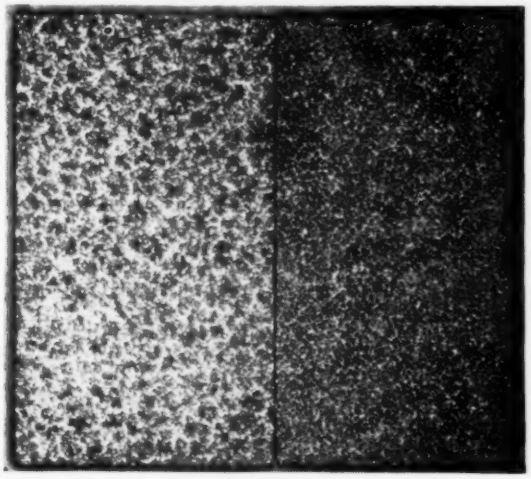
Engine	Cylinder bore (approx), in.	Cycle	Brinell	Wear, mg			Surface
				Cylinder	Ring	Total	
A	3½	4	185	15.8	59.0	74.8	Smooth
B	3½	4	350+	5.5	140.0	145.5	Scuffed
C	4¾	4	235	11.2	68.8	80.0	Scored
D	5	2	215	8.0	116.4	124.4	Scored
E	5½	4	210	13.2	75.6	88.8	Smooth
F	8	4	215	15.0	90.0	105.0	Slightly scuffed
G	10¾	4	175	10.1	115.0	125.1	Scuffed
H	10¾	4	145	11.2	143.0	154.2	Scuffed
I	14	4	150	17.4	62.0	79.4	Smooth
J	14	2	175	15.0	73.6	88.6	Smooth
K	14	2	165	17.5	60.5	77.5	Smooth
L	16	4	185	12.6	75.1	87.7	Scuffed

NOTE: Ring irons run against cylinder irons average 30 and 45 mg and give total wear of 75 mg without surface scarring.

Table 9 gives wear-test results of cylinder and liner irons operating against surfaces representing average piston-ring structures. Although weight-loss values are relatively low, it is seen that there is considerable abuse to the mating surface (ring wear) accompanied in most cases by scuffing or scoring of the cylinder specimen.

SUGGESTED STRUCTURAL CHARACTERISTICS FOR CYLINDERS

As mentioned early in the paper, this research work has been in the nature of a survey or review of existing practices. It would be presumptuous (if not outright dangerous) for the author to recommend a specific analysis, hardness, and structure for cylinders and liners. This will be obvious when the many factors involved are considered, e.g., castability, machinability, strength,



Etched, ×5

FIG. 13 DIFFERENCE IN GRAIN SIZES OF CYLINDER AND RING STRUCTURES

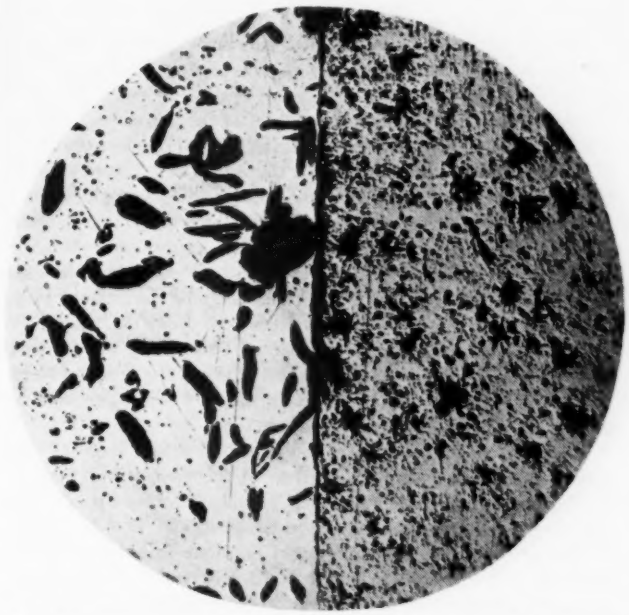


FIG. 14 COMPARISON OF GRAPHITE SIZES IN CYLINDER IRON (LEFT) AND RING IRON

and suitability for a given design, variations in finishing, loads, field of service, lubrication, etc. All of these must of necessity be fitted into the proper balance for any one engine casting.

However, the photomicrographs, Figs. 6 to 9, were selected as being typical (at least from the limited studies made) of the characteristics described. The author feels that, since this phase of the subject has received only limited attention, the trends and characteristics indicated may act as a starting point for further investigations in conjunction with service experience.

Ring and Cylinder Hardness

A matter which is frequently discussed is the relative hardness of rings and cylinders. Often the viewpoint is taken that similar

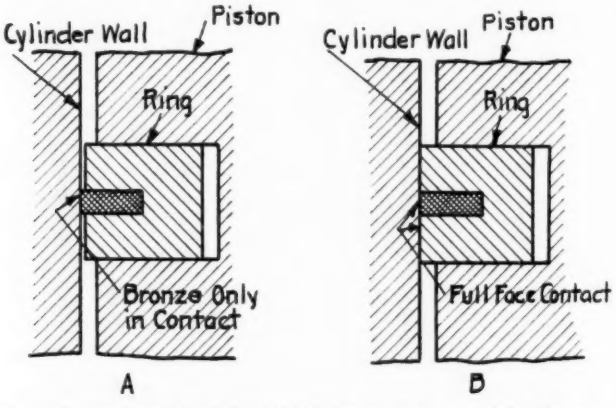


Fig. 15 Insert-Type 8-In. × 1/4-In. Piston Ring—28 Lb Tension (*A*, During run-in, unit pressure is high, 43 lb per sq in.; giving quick sealing and seating, and conditioning cylinder surface. Oil may accumulate between contact face, aiding in sealing action and furnishing lubrication. The bronze can carry high loads without scuffing. *B*, After complete seating, unit pressure drops to 14 lb per sq in. ring and cylinder surfaces are now mated. Filming action of bronze continues, and wear is retarded on both surfaces.)

hardness should in no case be used, this belief probably being derived from the early experiences of running like metals together in many instances with poor results, as compared with over-all wear when dissimilar metals are rubbed together. Then again the belief exists that ring hardness should be less than that of the cylinder or liner to protect the latter and throw wear on the rings. A third thought is that, ring surfaces being so small in relation to the cylinder area over which they travel, rings should be of greater hardness to get the proper wear balance. There is also a feeling that rings should be harder than the liner because of the imbeddability of cylinders, and their tendency to pick up foreign material, such as silica from dusty air, thus forming a lap which would wear the rings. This theory was held before the advent of air cleaners and oil filters in the automobile industry. It also holds under certain unusual conditions such as in oil-field service, hot oil pumps, etc.

Experience and the data here submitted would indicate that more consideration should be given to the compatibility of the mating materials, based on their structural make-up, allowing hardness to come as a secondary consideration. It is reasonable to conceive that an attempt might be made to reduce cylinder wear by utilizing softer rings, based solely on a hardness measurement. Depending upon the type of structure used, the exact reverse of what is desired might result.

Composite and Bimetallic Construction

Even though gray cast iron is a series of alloys with widely different wear properties, for one reason or another it is often difficult to make it satisfy all the requirements demanded by present-day usage. For example, modern high-speed engines, particularly automotive engines, are composed of a variety of metals substituted in the various wearing areas. For many years, engine bearings have been lined with bronze or babbitt, as these allow ready replacement, together with lowered wear rates. Valve-stem guides are now for the most part inserted with a material better suited for the particular heat-and-wear problem in the valve chamber. The same is true for valve seats and cylinder sleeves. Thus, in many cases, the original block casting acts mainly as a rigid frame, upon which are located various alloys, each to handle a specific type of abuse.

Another illustration of this trend is the modern aircraft-engine valve itself, which performs satisfactorily for several thousand hours under extremely adverse temperature conditions. Yet the steel from which this valve is made, though the best known for the purpose, fails to satisfy all the types of abuse encountered.

The wearing properties of the stem are not just right, so the surface is nitrided; the tip, subject to impact, is a special inserted alloy, while the valve face itself is stellite coated. Thus, even this relatively small but important engine part is of composite construction.

In the case of piston rings, the so-called insert or Gold Seal

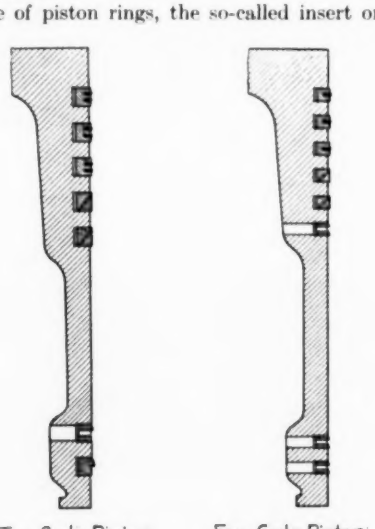


Fig. 16 Typical Insert-Ring Installation

type is quite widely known. This design consists of a cast-iron ring section, having one or two grooves machined in the cylinder-contact surface, into which is pressed a phosphor-bronze bearing strip. This band is permitted to project a few thousandths of an inch so that, during the seating-in period, only the bronze contacts the cylinder, but with relatively high unit pressure. This gives a rapid and complete sealing during the critical seating period and yet the initial scoring and scuffing, so frequently experienced during the run-in operation, is eliminated. This design is illustrated in Fig. 15, which shows the insert ring before and after complete seating. For larger rings with a wide contact face, the two bronze strips form an oil seal and also aid lubrication of the top rings. Furthermore, a bronze film is deposited on the cylinder surface, which promotes friction reduction and retards scuffing action. Such a design seems admirably suited for operation against surfaces which, as indicated in previous data, have scuffing tendencies. Fig. 16 shows typical insert rings for a 2- and a 4-cycle engine.

Fig. 17 shows in graph form the results of laboratory ring- and cylinder-wear tests using plain iron and "insert-ring" specimens. It is seen that besides the marked reduction in ring wear, cylinder wear is both considerably lower and more uniform.

Table 10 indicates results obtained on a $12^1/_2 \times 13^1/_4$-in., auxiliary, marine Diesel engine after the first 1450 hr operation. New liners, with a turned finish were installed, which gives a rather adverse condition from an initial-wear standpoint. The marked reduction in cylinder wear with insert rings is nevertheless significant.

Though ring-wear figures are not available from this test, it was reported that, at the time of inspection, the iron portion of the insert rings was just starting to make contact with the cylinder. Referring to special iron 20C, this was an alloy composition with

TABLE 10 RING AND CYLINDER SERVICE RESULTS ON AUXILIARY MARINE DIESEL ENGINE—1450 HR OPERATION

	Actual service results; cylinder wear, in.	—Laboratory wear tests— Ring wear, mg	Cylinder wear, mg
20A Insert rings	−0.00225	12.5	24.0
20B Regular iron rings	−0.0037	34.6	37.4
20C Special iron rings	−0.0045	11.2	77.3

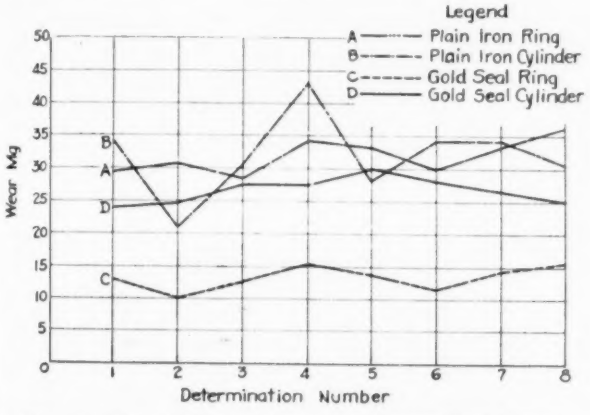


FIG. 17 WEAR TESTS OF PLAIN AND INSERT-TYPE RINGS

a coarse grain structure and Brinell hardness of 170. It is seen that under test and in service, cylinder wear is high, while indicated ring wear is very low. Prior to the above test installation, liner wear for approximately 22,000 hr running was reported as being 0.08 in. or 0.0037 in. per 1000 hr.

A recent service confirmation of the reduction in cylinder and ring wear through the use of insert rings is the experience with a two-cycle submarine engine which showed a maximum cylinder wear of 0.003 in. after approximately 10,000 hr running. Normally in such service a wear of three times this amount is considered satisfactory.

RUBBING-SURFACE FINISHES

During the last few years, remarkable improvements in finishing rubbing surfaces have been witnessed. This has been brought about through a better understanding of friction conditions, action of lubricants, and methods for evaluating or measuring surface roughness. The profilometer, developed by Dr. E. J. Abbott,[6] has already broadened the knowledge on this subject and provided a far better realization of the vital importance of the degree of surface irregularity. This instrument measures, in millionths of an inch, a running average of the height of the irregularities which constitute the roughness of the surface.

Present-day practice, particularly in the smaller-bore engines, tends more and more to honing of cylinder surfaces. There is already evidence that, through these improved surfaces, initial wear is greatly lessened. In larger-bore engines, however, a great number of cylinder finishes is still used, ranging from a so-called "fine-turned," to varying degrees of finish-grinding, with some few manufacturers using or considering honing as the final treatment.

Unfortunately, cylinder finishing is generally done so that the cutting or grinding is normal to ring and piston movement, resulting in a marked "file effect," with consequent high wear. The author recently examined several ground Diesel liners after approximately 6000 hr running, most of the grinding marks, except at the top-ring belt area, were still in evidence. Incidentally, excessive cylinder wear was a problem on this engine. It is likely that considerable improvement might be obtained through better cylinder finishing.

Aside from the rapid wear experienced with poorly finished bores, there is also the matter of the action of lubricating films on various surfaces. A honed finish characterized by a dull appearance seems to give good adherence or adsorption of oil films. Though much remains to be learned of this action, it is believed that surfaces glazed by grinding, chromium-plated surfaces and nitrided surfaces do not "wet" readily, whereas honed or matte finishes, and etched surfaces show good "blotter-like" or wetting action.

As an illustration of improved surfaces, Fig. 18 shows photomicrographs at 100 diameters of piston-ring surfaces. View (*a*) is the conventional side-face finish from a rotary grinder; this surface having a pleasing appearance and luster. However, the surface roughness is between 20 and 30 microinches (expressed 0.000030 in.). View (*b*) is a lapped side-face finish of 2 to 3 microinches. This finish is so smooth that some structural details are plainly visible. A similar type of finish is shown in (*c*), this being the lower contact edge of a tapered aircraft-engine ring which has been lapped to line contact prior to shipment to the engine builder. The cylinder-contact face of a worn piston ring after 4 yr service is shown in (*d*). The similarity between this photomicrograph and those of (*b*) and (*c*) is interesting. This ring had an excellent surface condition free from scoring or scuffing, the surface of the ring having been merely washed with naphtha and alcohol to remove grease and stains prior to photographing.

The improved face finishes on aircraft rings are reported to retard ring sticking and groove wear, reflect favorably on oil consumption, and also provide a ring of better flatness.[7] Piston grooves should, however, also come in for attention when using these improved finishes.

CHEMICAL SURFACE TREATMENTS

The foregoing discussion of metal surfaces would naturally be incomplete without some mention of the wider application of chemical treatment to friction surfaces. C. G. A. Rosen presented a paper[8] on this subject in 1937. Incidentally, his company pioneered in the application of chemically treated surfaces. Mr. Rosen mentions that "piston rings which normally required about 100 hr to obtain full ring-face conditioning, and 100 per cent bearing face, can be run into comparable surface contacts in 3 hr." The basic purpose of all treatments is the more rapid seating and better sealing during the run-in period. Numerous surface treatments are now available so there is a wide range of theory, color and composition at hand, from which the most suitable may be selected to fit a particular case.

Four general classifications might be mentioned: (*a*) metallic

[6] "The Profilometer," by E. J. Abbott, S. Bousky, and D. E. Williamson, *Mechanical Engineering*, vol. 60, March, 1938, pp. 205–216.

[7] "Piston-Ring Performance and Its Relation to Lubrication," by A. L. Beall, Society of Automotive Engineers (Baltimore Section), Feb. 2, 1939.

[8] "Lubricating Problems in Connection With High-Speed Diesel Engines," by C. G. A. Rosen, Trans. A.S.M.E., vol. 60, 1938, paper OGP-60-4, pp. 145–151.

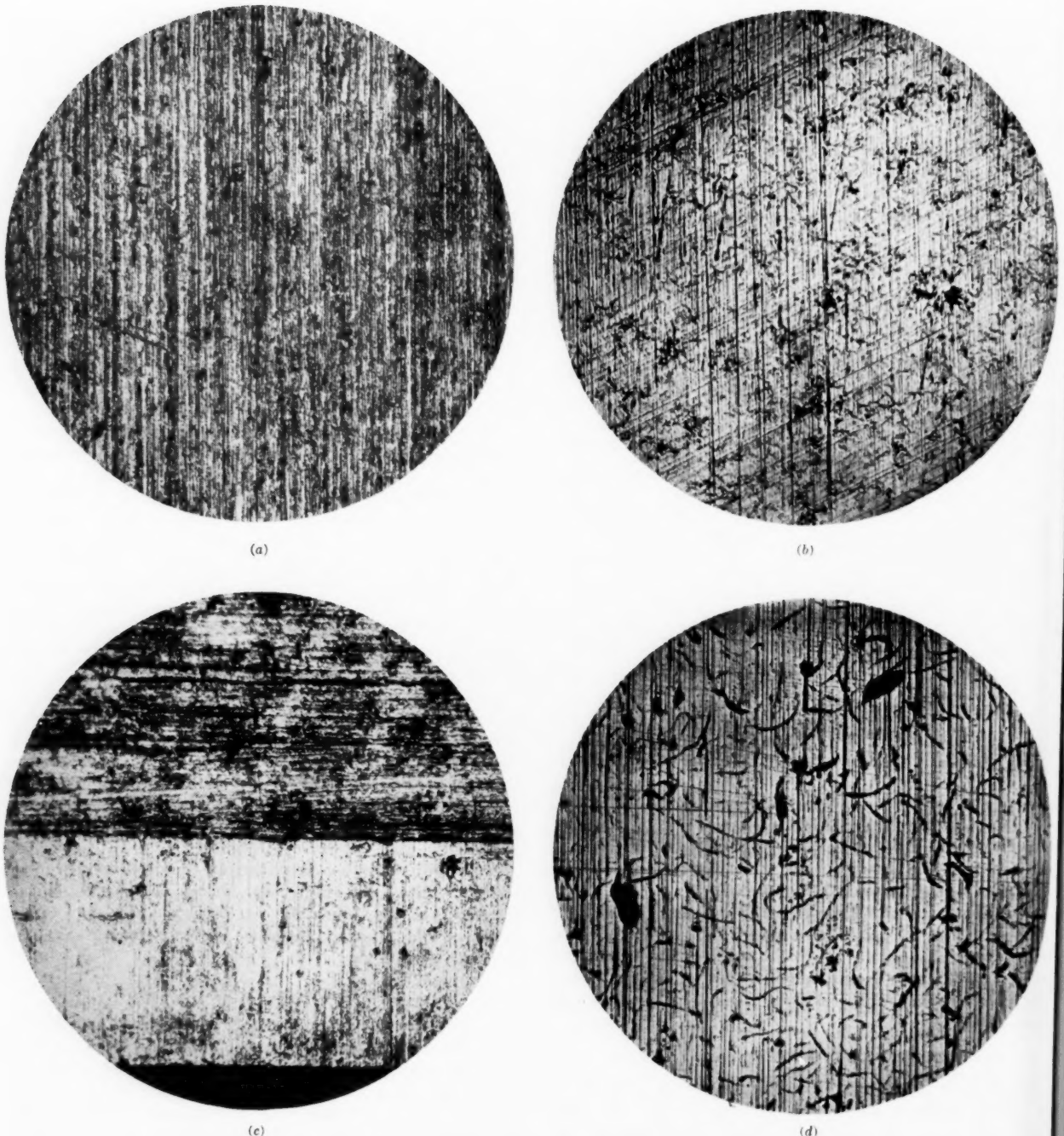


FIG. 18 VARIOUS TYPES OF FINISHES ON RINGS, ×100

[(a) Usual face finish, 30 microinches. (b) Hypro-lapped finish, 4–6 microinches. (c) Outside diameter of tapered ring. (d) Outside diameter of Diesel ring after 4 yr service. Surface cleaned and buffed lightly.]

plating, (*b*) oxide coating, (*c*) phosphate coating, (*d*) chemical sulphidizing. It should be mentioned that all of these are short-lived, being designed primarily for the run-in period. The treatment penetration or thickness is usually about 0.0005 in. or less. However, there is no doubt but that they can and often do enhance run-in operations. This of course can affect subsequent wear rates to an appreciable extent.

The metallic platings reduce friction and correct for slight errors in ring and cylinder fitting, this being common also to the other types. If low-melting-point metals are used, there is some question as to their stability, particularly in gas engines operating above the melting point of the coating. Their wetting action is satisfactory and it has been reported that a definite weight increase is obtained when rings so treated are immersed in hot oil. The basic theory for their use is the better friction properties as long as they might endure.

The oxide coatings have the disadvantage that they are generally obtained by high-temperature treatments (around 1000 F) so that distortion may be encountered, particularly since this treatment must be obtained after final finishing. Their wetting

action is good, and they have been widely used in automotive work on rings and valve tappets. In general, this treatment is deeper than the others; the underlying theory being to supply a fine abrasive oxide which acts as a lap and polishes the mating surfaces.

The phosphate coatings were originally developed for rustproofing and paint-adherence purposes. This is really an etching treatment and gives a matte finish with good oil filming. It is quite inexpensive and has received rather wide application since many plants had previously used it for the preparation of painted surfaces and merely applied it to various rubbing parts. A further attempt has been made to enhance its action through the application of colloidal graphite to the surfaces. Tests to date indicate that good success has accompanied this improvement. It has the advantage that temperatures under 200 F are required for treating most ferrous materials.

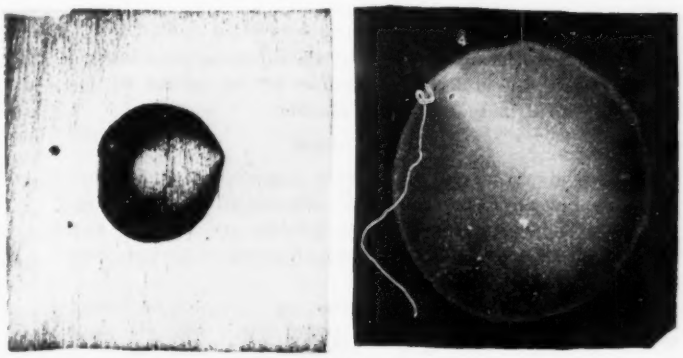
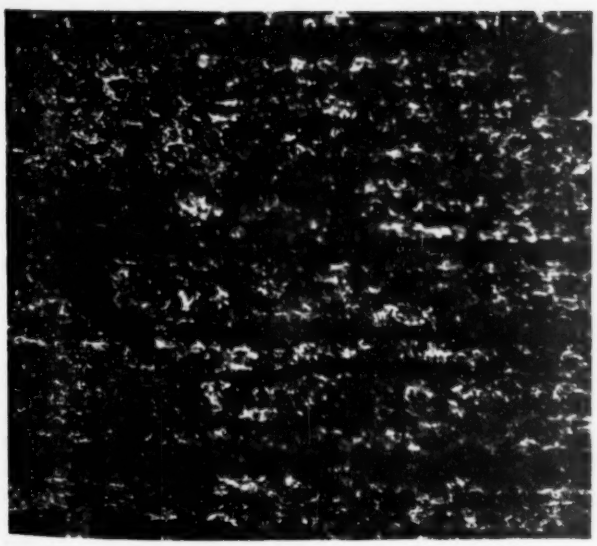


FIG. 20 DIFFERENCE IN OIL-WETTING CHARACTERISTICS OF PLAIN AND TREATED SURFACES



(a)



(b)

FIG. 19 APPEARANCE OF CYLINDER-CONTACT FACE OF RING BEFORE AND AFTER SULPHIDIZING

[(a) Automotive piston ring, untreated; turned finish, ×100. (b) Same ring after applying surface treatment, ×100.]

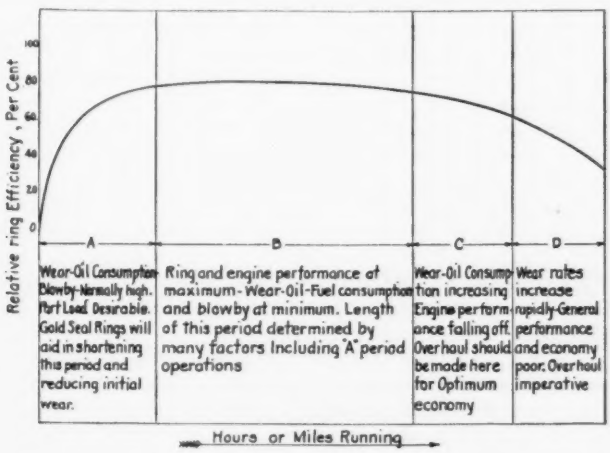


FIG. 21 RING EFFICIENCY CURVE

Chemical sulphidizing is a process developed by engineers of the Standard Oil Company of California. This treatment has found broader application in the Diesel field than any of the others, one leading manufacturer of high-speed, Diesel-tractor equipment having used it for rings and liners for more than a year.

Fig. 19 shows a smooth-turned surface before and after sulphidizing. Besides the removal of certain microconstituents, undesirable from a wear standpoint, the treatment also removes loose metal fragments which induce scratching and scoring. This is strikingly illustrated in Fig. 19.

Fig. 20 shows the difference in the "wetting-with-oil" action. These are two specimens of the same material, the one at the left being untreated. These specimens are $1/2$ in. square and magnified approximately $3\,1/2$ times. An equivalent drop of light engine oil was placed in the center of each specimen, the photograph being taken after 7 min had elapsed. It is apparent that the treated surface permits better spreading of an oil film. After 15 min, the treated-specimen face was completely covered with the oil film, whereas very little change occurred with the plain surface.

Though this treatment has been used mainly for rings, even further improvement results when the cylinder is also treated. The sulphidized surface has extreme pressure characteristics, so that scuffing of areas under high unit pressure is retarded. The adherence of oil films to this surface promotes good sealing against blowby between ring and cylinder and also of the side faces in the groove.

The wetting action and extreme pressure characteristics of the treated surface are largely results of the chemical action during

treatment, since similar properties are also imparted to compounded oil through chemical effects.

Though, as mentioned previously, the foregoing practices are relatively new, the underlying principles are logical and on this basis warrant further study and application.

Ring Efficiency

Fig. 21 shows a theoretical curve of ring efficiency. This refers to sealing, oil distribution and consumption, fuel consumption, and wear. For top efficiency, blowby and wear are at a minimum for a given set of conditions, other related factors being in proper balance.

As seen from the curve, relatively high wear exists during period A or run-in. To a certain extent this is desirable, since it is in this manner that final mating of the friction surfaces is obtained. If there were no wear or surface adjustment whatsoever, top efficiency would not be attained.

Anything done to facilitate this action by shortening the time required to reach the beginning of period B seems worth while. Better ring and cylinder materials and contours, improved finishes and surface treatments—all tend toward reduction of time required, and give better final mating. Unfortunately, even though it is recognized that A is a critical period, it is often given secondary consideration due to apparent need for such things as "immediate application of full load."

The length of period B is definitely influenced by what happened during the A interval. The longer the B period the better; at best it will be too short-lived. During B both ring and cylinder wear will be at a minimum, other factors as noted. This assumes, of course, reasonable care and attention by the operator.

As the C period is approached, indicated by a falling off in performance, and an increase in fuel and lubricating-oil consumption, wear rates again increase. Consideration should be given to overhaul and replacement rather than permitting conditions to reach the D period. This graph while possibly theoretical, seems nevertheless to represent a logical sequence to be followed.

Conclusion

No broad summary nor specific recommendations will be attempted as a result of the data reported herein. It is the author's opinion which seems to be substantiated by the work reported in this paper, that in some cases a possible reduction in wear rates of 2 to 1 may be expected through proper balance of ring and cylinder structures. There is further evidence that with existing conditions, a reduction in cylinder wear of roughly 2 to 1 may be obtained through the use of bimetallic and/or chemically treated rings. Thus, theoretically at least, there appear to be possibilities for an over-all decrease in wear rates in the order of 4 to 1; that is, wear seems possible of reduction to one fourth of existing averages. This seems worthy of consideration, even though the attack on such a problem may probably be somewhat different for each individual case. The author reiterates, that the primary purpose of this paper has been to review and consider the possible effects of metal structures on the over-all wear problem.

Appreciation is expressed to the various engine builders and operators, whose kindness and cooperation in supplying samples and data accounts in a large degree for whatever value may be found in this work, and to the author's associate, Richard Taber, for his care in performing tests and preparing photographic illustrations.

Discussion

Ralph Miller.[9] After studying Mr. Lane's paper, the writer is tempted to conclude that the problem of piston-ring and liner wear might be solved by giving surfaces a high degree of finish before assembling in the engine; for if this can be done, the structure or the kind of material lose their significance.

The oil film under the top piston ring is, of course, very thin and, if the rough projections on the surface are higher than the thickness of the oil film, naturally, scoring will result.

Now, attempts are apparently being made to select materials that can be run in or smoothed down without causing damage to the surfaces. To this end, wear- and corrosion-resistant qualities are being sacrificed.

H. A. Gehres.[10] Referring to the case of the 25-in. cylinder worn 0.004 in. as against the small one worn 0.0013 in., as given in Table 1, it would seem to the writer that the cases should be reversed because the small engine should be the higher-speed engine. Logically the wear should be in proportion to the number of trips the ring makes across the surface. Also it has been the writer's experience that the higher-speed engine runs with higher peak pressure, which would again indicate that it should show somewhat greater wear.

To corroborate and amplify Mr. Miller's statements on corrosion in liners, we have one installation which showed excessive cylinder wear, while other installations with exactly the same type engine have even shown subnormal liner wear. We have been forced to the conclusion that some corrosive substance in the air adjacent the steel plant where this engine is in operation caused the excessive wear, since the liner will wear as much as 0.014 in. while the ring will wear scarcely at all. Apparently deterioration is taking place on the surface of the cylinder which can be scraped off easily by the rings. This material cannot be very abrasive, otherwise it would cause the rings to wear.

About the time we had reached this conclusion, Mr. Van Der Horst of Holland discussed with us his method of chrome plating liners to reduce cylinder wear. He verified the opinion that 80 to 90 per cent of cylinder wear was due to corrosion and that, while the chrome-plated surface was extremely hard, the main benefit derived was not from the hardness but rather from the corrosion-resistance quality.

E. T. Vincent.[11] Cylinder-liner corrosion has been studied rather completely by the Institution of Automobile Engineers of Great Britain, which organization published test results three or four years ago. These results confirm the remarks made by Mr. Gehres. It is entirely feasible for corrosion to be concentrated along the path of the rings, because this surface is being continuously wiped clean by the rings, whereas other portions of the liner become coated with a protective layer of carbon. Therefore, when corrosion does not occur on parts other than the ring travel, it does not necessarily prove that corrosion does not exist.

In this paper a ring setup for four- and two-cycle engines was shown in which the latter were fitted with rings 50 per cent wider than the former. Is this change a definite recommendation, or is it accidental? The writer sees no reason, based on actual experience, why ring width should be changed for either type of engine.

E. A. Haskins.[12] Fortunately, we have experienced no serious difficulty from piston, cylinder, or liner wear. Corrosion has been no problem except in the case of a few experimental applications in which an effort was made to use the harder types of liners. There has been more difficulty with these than with our standard liner, which has a hardness of 200 to 215 Brinell. Possibly it is

[9] National Supply Company, Springfield, Ohio.

[10] Cooper-Bessemer Corporation, Mt. Vernon, Ohio.

[11] University of Michigan, Ann Arbor, Mich.

[12] Hercules Motors Corporation, Canton, Ohio.

our use of aluminum-alloy pistons with their better heat conductivity which keeps ring temperature down.

J. B. JACKSON.[13] The writer has observed cylinder wear which might be attributed to the thermal conductivity of the material, thereby creating higher surface temperatures on the liner and then contributing to the destruction of the oil film. Some of the liner wear which was reported in the case of the older engines may be attributed to a high thermal conductivity of the iron because it is not alloyed.

Another thing that influences ring wear is the surface temperature of the liner, as indicated by the appearance of liners and rings in supercharged engines. The writer has examined pistons removed from engines that were supercharged to 100 per cent above their original power. The pistons and rings appeared to be in far better condition than when they were removed, after operation at standard rating without supercharging. It is the writer's belief that excess air has a tendency to cool the liner surface and give better lubrication conditions.

The question arises as to how thin rings may be made before they become wavy and cause blow-by? The writer advocates thin rings, because of the weight of the ring riding in a relatively soft material. We are successfully using $^1/_8$-in. rings on pistons as large as 8 in. diam and $^3/_{16}$-in. rings on pistons nearly 13 in. diam. No definite evidence of increased cylinder wear is apparent with narrow rings.

A. W. MORTON.[14] Mr. Jackson stated that he would like to see narrower rings come into still greater use. There is a limit to which we can go in this direction. If the ring is made too narrow, it dishes as it is put into the cylinder. Obviously, extremely narrow rings will dish or buckle when compressed and, in the groove, contact occurs only at the edges of the rings. If the face of the ring is reduced, the wall thickness will also have to be reduced somewhat, in order to obtain intimate contact with the land of the piston when the ring is compressed and placed in the cylinder.

With the aid of machine-tool builders, we have been able to produce rings with a flatter surface than heretofore, which enables the ring to lie flat against the land of the piston, when made even narrower than formerly. The oil film between the land and the ring, however, is thicker than that between the ring and the cylinder. Therefore, the insulation value of the oil film is the limiting factor which controls the conduction of heat from piston land to the ring; the less the insulation, the greater the conductivity between the piston and the ring.

There is one point which has not been mentioned and that is cleanliness. When an engine is assembled, the condition of roughness of the edges of the rings, the cylinders, the pistons, and every other metallic part has a bearing upon the initial seating in and upon consequent scuffing or wear. Too much care cannot be taken in cleaning all parts of an engine assembly.

E. J. ABBOTT.[15] When speaking of unit pressures, one ordinarily thinks of the entire face or land of the ring as the working area. On second thought, such obviously is not the case. The surface profile shows many fine irregularities, and the peaks carry the load. Hence, unit stresses of primary interest are those occurring at the peaks of the surface serrations. The roughness of the surface has a tremendous effect upon these local unit stresses; therefore, roughness has an important relation to lubrication, wear, corrosion, and load-carrying capacity. These facts have been thoroughly demonstrated in the last few years.

[13] Aluminum Company of America, Cleveland, Ohio.

[14] American Hammered Piston Ring Division, Koppers Company, Baltimore, Md. Mem. A.S.M.E.

[15] Physicists Research Company, Ann Arbor, Mich.

The widths of the serrations on typical ring and cylinder surfaces are usually a few thousandths of an inch, and their total heights usually between 10 and 400 microin (3 to 100 microin rms; 1 microin = 0.000001 in.). Measurements show that surprisingly large differences exist in production, and that these differences are important.

Measurements of the actual dimensions of surface profiles show that the proportions of the surface are more favorable on smooth surfaces than on rougher surfaces. It is popularly conceived that smoother surfaces simply have smaller and more closely spaced peaks. Measurement shows that the spacing is not much closer on smoother surfaces, but the heights are often very much smaller. This makes the smoother surfaces duller as well as lower, and this is a decided advantage from the standpoint of load-carrying capacity.

Until recently, the measurement of surface roughness was either impossible, or a slow and costly laboratory job. However, the profilometer mentioned by Mr. Lane is a practical shop instrument which allows roughness measurements to be made rapidly and accurately on almost any specimen without damage to it. It is no longer necessary to guess the relative roughness of two or more surfaces. Experience thus far has thrown much light on the relation of surface roughness to the factors under discussion.

P. H. SCHWEITZER.[16] Mr. Morton mentioned the importance of cleanliness. It is a general procedure when we service engines to take a piston out and scrape it, including the lands. There is always a film of accumulated carbon. Some experts discourage cleaning or scraping the lands, claiming that the carbon formation is in consequence of an excess clearance put there by mistake or from figuring that operating conditions would be more severe than those the engine actually encountered. In the space available, hard carbon builds up. Allow it to remain and there is less blow-by. It is as if that much more metal were present.

AUTHOR'S CLOSURE

Mr. Miller asked whether or not wear might be reduced by using smoother surface finishes? That is not only true, but it is also the trend. The author believes the usual present-day practice in finishing Diesel-engine cylinders can be improved, particularly in the larger-bore engines. Unfortunately, due to the fact that the cylinder material range is very different, the same finish cannot always be obtained. Many do not realize this. Given two grades of cast iron, it is almost impossible to put the same finish on both of them.

There is much to be said about smoother finishes from the standpoint of distribution of pressure. We know that in the case of a threaded finish with a given loading, the unit pressure on the edge of the thread or at the tip can be enormous. When the thread is worn off, the pressure drops and we come to what is called a seat. One feature of such a seat is, of course, that the pressure is distributed. If we start with a smoother surface, we will immediately start with better load distribution, without the areas of high load concentration.

However, this matter must be approached rather cautiously. We know that it is possible to get the finish too smooth. It might be desirable to have surfaces smooth from standpoints of wear and corrosion but, having once attained such a high finish, it might be difficult to control the lubrication.

There cannot be much doubt that, as we improve the surface condition, we automatically improve corrosion resistance. In the case of worked material, we find, once it is glazed, that it reacts very differently to acid and to corrosion. If we could ob-

[16] Professor of Engineering Research, The Pennsylvania State College, State College, Pa. Mem. A.S.M.E.

tain such a surface originally, it would probably help to resist corrosion.

When we lap in a ring against the cylinder, we automatically smooth both surfaces. However, it is important to realize, at the same time, that we are changing the direction of the surface roughness. When a ring is lapped, the unit pressure is distributed, the surfaces are polished, and the directional properties of the surface are changed.

In reply to Mr. Gehres, the values, shown in Table 1, for the wear of the engines mentioned, were based on relatively slow-speed engines. Also, there is possibly some difference in material between the small-bore engines and the large-bore engines.

Regarding corrosion, Mr. Gehres' theory was very prevalent in England some two or three years ago, but now it is not quite so popular as heretofore. Unfortunately, our experience has been that the materials which have good corrosion-resisting qualities often fail to have good wearing qualities. It is believed that the oil film between the rubbing parts is of the boundary type so that the wearing qualities of the materials are important. The austenitic alloys are reported to have given good success in high-speed engines in England. They are quite corrosion-resistant, and yet in this country, we cannot seem to get the same results.

Another point that might be mentioned is that corrosion-resistant materials often lack what might be termed emergency-run qualities. Cast iron does have that. It incorporates graphite which represents roughly 10 per cent by volume, and when the dry periods come on, cast iron has enough soapiness or greasy quality to carry over the lean period. This feature is quite desirable.

In reply to Professor Vincent's query concerning the heavier rings on pistons of the two-cycle engines in Fig. 16; they are illustrated for two reasons: One is to show that, as the rings get larger, we insert two bands; and second, because in general the two-cycle engines do carry somewhat heavier rings to enable them to pass over the ports with less danger of port clipping than with narrow rings.

On the matter of heat conductivity, some thought and investigation might be given to improvement in ring side faces; that is, lapping does a great many things which many of us probably did not realize it would do.

In operating a ring with a ground side-face finish, very often high and low points of contact develop around the ring face, showing that it touched the groove only here and there, as compared with a lapped face ring, which makes a much more uniform contact with the piston groove. This means that heat conductivity may be aided through the use of rings which do fit into the grooves more closely. Of course, when rings of this type are used some consideration must be given to the improvement of the ring grooves themselves.

Another thing that the improved side face offers is a freer breathing ring, that is, a ring less prone to sticking, because, with greater smoothness of the surface, carbon deposits are not so likely to build up. It has been reported that in aircraft engines, a decided improvement in oil consumption has been realized mainly through the improvement in the ring-side faces. With flatter ring surfaces and a better fit, there is an improvement in the maintenance of an oil seal between the ring and the piston groove.

Where Does Interpretation Begin?—Analysis of the Oil-Engine Power Cost Report

By P. H. SCHWEITZER,[1] STATE COLLEGE, PA.

This paper is an analysis of fuel-oil and lubricating-oil economy based on the 1937 A.S.M.E. Oil-Engine Power Cost Report. Both analyses are divided into two sections, the factual and the interpretative parts. The nature of the factual analysis does not permit drawing any conclusion beyond the one that, in making more judicious use of graphical representations, the usefulness of the Cost Report can be greatly increased. Conclusions from the interpretative analysis are as follows:

1 Engine load has a great effect on both fuel- and lubricating-oil economy.

2 Engine type has a great effect on lubricating-oil economy and a small effect on fuel-oil economy.

3 Year of manufacture has a great effect on both fuel- and lubricating-oil economy.

4 In both cases the improvement is wholly due to advance in design and construction.

5 Engine size has practically no effect on either fuel- or lubricating-oil economy.

6 Engine speed has practically no effect on either fuel- or lubricating-oil economy.

7 Fuel gravity has practically no effect on fuel economy.

8 Other factors may have some small effect on fuel- and lubricating-oil economy but they have not been sufficiently investigated.

FOR THIS PAPER to be intelligible, familiarity with the Oil-Engine Power Cost Reports is a prerequisite. The graphs and comments are based on the latest report, covering the year 1937. No data given in that report have been arbitrarily omitted from the plots or tables, no matter how improbable they may appear to be.

The Oil-Engine Power Cost Report now enters its tenth year. More than ever it is a carefully assembled and competently edited report. The comprehensive data included are of great value to the cost analyst. At the same time, it must be admitted that the average reader will derive but a limited amount of information from the 52,000 figures and three charts it contains.

A most legitimate object of the Power Cost Report is to enable a prospective purchaser of a Diesel power plant to estimate his probable operating costs. This is admittedly impossible for the reader who has but a limited time to study the report. Since at least 95 per cent of the readers fall in this category the resultant loss is obvious.

It has been repeatedly discussed how the usefulness of the Power Cost Report might be increased. Generally the conclusion has been that it is only possible by an interpretation of the data. Since, however, no interpretation is completely free from bias, the A.S.M.E. Oil-Engine Power Cost Committee does not wish to lend its authority to any interpretation. However, the committee does not discourage other groups or individuals from an interpretation of the "tabulated data if they have the engineering and statistical background for such a study."

Nobody can disagree with this stand, unless to the word "interpretation" too broad a meaning is given. In a sense, every graphical presentation is an interpretation because, although the points shown may be factual, the abscissa is selected at the pleasure or judgment of the analyst. The data of the Cost Report are tabulated in no less than thirty numerical columns. Any of these can be plotted against any other. When the analyst presents a plot of one quantity against another, as in Fig. 1 of the Cost Report, he already uses judgment by assuming that the plant-running capacity factor has a bearing on lubricating-oil economy. Yet no one considers that as an interpretation in the practical sense. A most important question to decide is where factual presentation ends and interpretation begins.

The report states: "Certain of the data are plotted in chart form for illustrative purposes, but for that reason only. Such median and boundary lines as are shown are included to assist the eye in subdividing the plotted data but should not be assumed to indicate trends." That is a very conservative stand but the author believes the users of the Power Cost Report would be the gainers if interpretation which must be excluded were less broadly defined.

If listed data are plotted and a trend is apparent, it would seem to be entirely reasonable to draw a curve. If the data plotted are not listed directly but can be computed without any technical or economic assumptions, that is still factual presentation, rather than interpretation. If in plotting a chart an assumption is used or trend curves derived from other charts are utilized, but the assumption used or the trend established is obvious and not of a controversial nature, then the presentation should still be termed factual rather than interpretative. For example, most business charts are corrected for seasonal trend. If they were not, a wrong impression would be created in comparing the textile production in March with that of February. The correction factors are empirical. They have been derived from plotted data. Yet such a presentation is never understood as an individual interpretation of the statistics. Only if controversial assumptions are used in charting the data, then interpretation really begins. Examples of both types of presentation in reference to the Power Cost Report will be given in this paper. The work has been limited to fuel-oil and lubricating-oil economy. Other cost items should receive similar treatment.

Factual Analysis of Lubricating-Oil Economy

The result of lubricating-oil analysis is shown in Fig. 1. Rated kilowatthours (the horsepower rating of the engine × number of hours operated × 0.746 × 0.9) per gallon of lubricating oil are plotted against year of installation of the engine. The plot shows a shotgun pattern but trends become distinct if the chart is broken up according to engine types. This is done in Fig. 2. The straight lines are median lines showing the trend in lubricating-oil economy of new and older engines.

In Figs. 1 and 2, three factors are recognized, i.e., engine load, engine type, and year of manufacture. The index for engine load is the running-engine capacity factor which is defined as

[1] Professor of Engineering Research, The Pennsylvania State College. Mem. A.S.M.E.

Presented at the Twelfth National Meeting Ann Arbor, Mich., June 19–22, 1939, of the Oil and Gas Power Division of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

$$\frac{\text{Engine output in gross kwhr} \times 100}{\text{kw rating} \times \text{number of hours operated}}$$

Thus the running-engine capacity factor is the average load which the engine carried, while it was in operation, in per cent of its rating.

It is a common observation that the total lubricating-oil consumption per hour is the same at part load as at full load. Both the A.S.M.E. Cost Report and the British Diesel Engine Users Association recognize this by listing the lubricating-oil economy in terms of rated kilowatthours per gallon rather than actual kilowatthours per gallon. The former is the kilowatt rating of

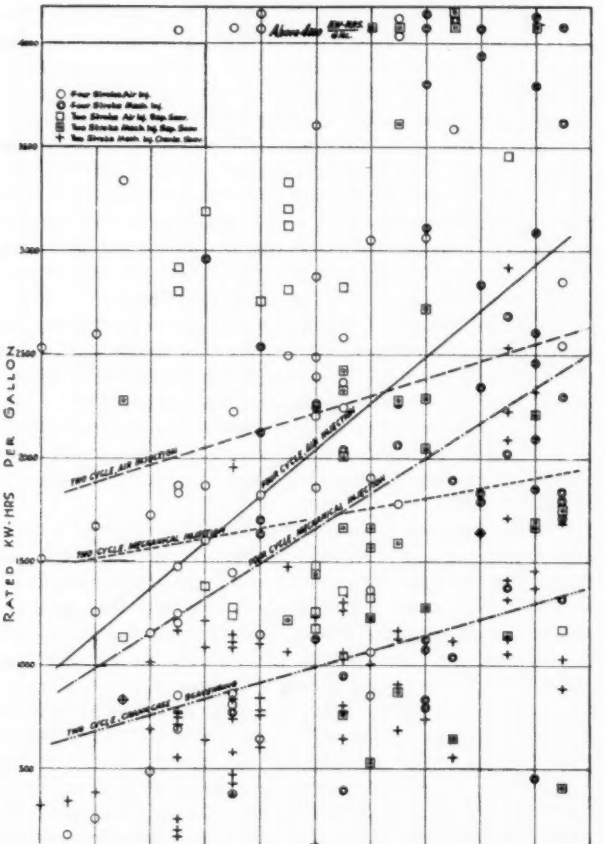


FIG. 1 LUBRICATING-OIL ECONOMY OF 226 DIESEL ENGINES

an engine generator set multiplied by the number of hours operated, or the kilowatthours output had the engine carried full load all the time it was running. The assumption which is implied by listing rated kilowatthours per gallon of lubricating oil is explicitly expressed in Fig. 1 of the Report (see Appendix) by the straight lines going through the zero point.

As to engine type, the Cost Report uses five classifications: (1) Four-stroke, air-injection; (2) four-stroke, mechanical-injection; (3) two-stroke, air-injection; (4) two-stroke, mechanical-injection, separate-scavenging; (5) two-stroke, mechanical-injection, crankcase-scavenging, which has been followed. In Fig. 2 the lubricating-oil performance of the five types of engines is separately plotted and medians drawn. By plotting rated kilowatthours per gallon, the load factor or running-engine capacity factor may be ignored and the abscissa becomes free for another relevant factor, e.g., the year of manufacture (or installation). The economy values plotted in this way in Fig. 2 show an unmistakable trend. With every type of engine, the lubricating-oil economy shows improvement in recent years. The lines indicate that the improvement is still in progress.

While the straight-line medians in Figs. 1 and 2 represent the probable lubricating-oil consumption of various-type engines, the lubricating-oil figures actually observed deviate considerably from the medians. The average deviations have been calculated as follows:

Four-stroke, air-injection, per cent	57
Two-stroke, air-injection, per cent	40
Four-stroke, mechanical-injection, per cent	75
Two-stroke, mechanical-injection, per cent	65
Two-stroke, mechanical-injection, crankcase-scavenging, per cent	33

The most important cause of these great deviations is, no doubt, the personal factor which has a considerable effect on lubricating-oil economy.

Figs. 1 and 2 constitute a limit in factual presentation, but even so they represent a step ahead from Fig. 1 of the Report. The differences are as follows:

1 Engines are plotted rather than power plants. If power plants are plotted, a small plant of one engine and a large plant with ten engines are both represented by one point, which gives undue weight to the engines used in small plants. True, almost half of the engines in the Report had to be ignored because no separate lubricating-oil figures were given. Even so, the fidelity of the plot is improved.

2 The engines are plotted according to their year of manufacture, which enables the reader more correctly to estimate the lubricating-oil performance of a contemplated engine. The idea gained from the Report in this respect is misleading. From the Report, it will be concluded that the probable lubricating-oil economy will be 1560 kwhr per gal at 100 per cent load. According to Fig. 1 of this paper, the probable lubricating-oil economy will be 2410 kwhr in case of a newly installed four-stroke, mechanical-injection engine and with other types more or less, accordingly.

3 In the Report gross kilowatthours are plotted; in Fig. 1 the rated kilowatthours. The two are equivalent if the straight-line relationship indicated in Fig. 1 of the Report is accepted. It is easy to obtain the gross kilowatthours from the rated kilowatthours by multiplying the figure by the load factor or running-engine capacity factor.

4 In Fig. 1 of this paper, instead of a median line for all the engines, separate median lines are shown for every engine type. Fig. 2 shows the separate plots for each engine type.

INTERPRETATIVE ANALYSIS OF LUBRICATING-OIL ECONOMY

The lubricating-oil consumption depends upon many factors, i.e., engine type, size, make, year of manufacture, workmanship, speed, load, hours of operation, condition of the engine, kind of lubricating oil used, its treatment, quality of attendance and management, and others. Any prediction based on a limited number of factors is liable to differ considerably from the figures actually observed. But without undesirable complications, only a small number of these factors can be considered. An important factor, namely, the human element, does not lend itself to plotting. Nevertheless, if the prediction is based upon a few carefully selected factors, the hitting average can be made much better than it is now. How to find the most pertinent factors? A reasonable guess may be made but, whenever possible, statistics should be used.

It is a safe guess that the engine uses no oil while it is standing still, therefore the lubricating-oil consumption per hour is inde-

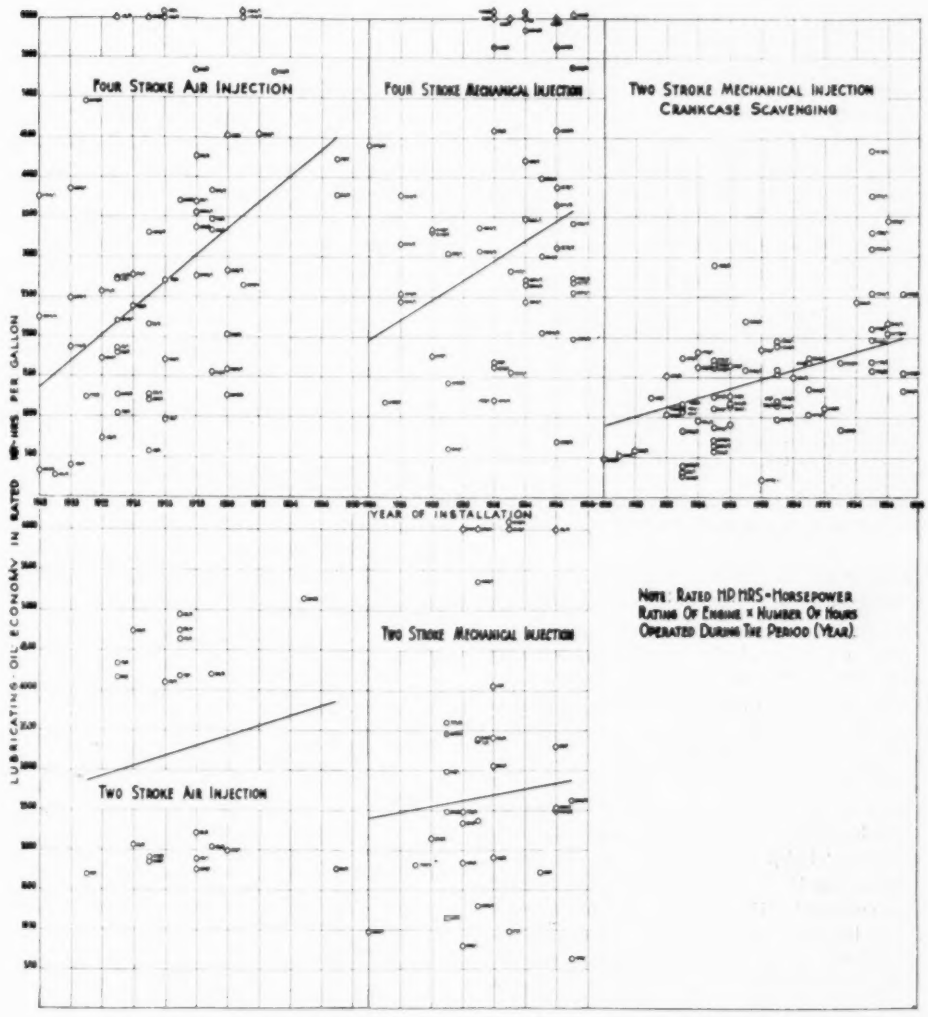


FIG. 2 LUBRICATING-OIL ECONOMY, DATA SEPARATED ACCORDING TO TYPES

pendent of the hours of operation. The evidence also indicates that the load has no effect on the lubricating-oil economy if the latter is expressed in rated kilowatthours per gallon. The engine type and year of installation were found to have a distinct effect as shown in Figs. 1 and 2. Now it is in order to investigate the other factors.

Effect of Engine Age. Fig. 2 shows the remarkable improvement in lubricating-oil economy in every type of engine. Taking a 15-yr period from 1922 to 1937, the increases are as follows:

Four-stroke, air-injection	from 1350 to 3050 rated kwhr	125 per cent
Four-stroke, mechanical-injection	from 1150 to 2400 rated kwhr	108 per cent
Two-stroke, air-injection	from 1970 to 2600 rated kwhr	32 per cent
Two-stroke, mechanical-injection, separate-scavenging	from 1560 to 1900 rated kwhr	22 per cent
Two-stroke, mechanical-injection, crankcase-scavenging	from 750 to 1340 rated kwhr	78 per cent

This improvement is significant. The question arises: Is the improvement due to the better mechanical condition of the new engines or to improvement in design and construction? The answer is that better design and construction are almost entirely responsible. M. J. Reed has investigated the lubricating-oil economy of 74 identical engines over a period of 6 years and obtained Fig. 3, which shows an insignificant variation in lubricating-oil economy as the engine gets older. Of course it must not

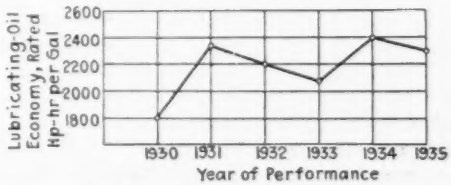


FIG. 3 LUBRICATION PERFORMANCE OF THE SAME 74 DIESEL ENGINES IN THE YEARS 1930–1935 (REED)

be understood that the lubricating-oil economy of an engine once installed remains constant. It usually varies in cycles. In the second year the lubricating-oil economy is, as a rule, better than in the first year because the engine has run in. Then for 4 or 5 years, the economy slightly deteriorates because of cylinder and ring wear. After 5 or 6 years, there is a general overhaul, involving replacement of piston rings, and sometimes, liners. After that the lubricating-oil economy is about equal to that of the new engine and the cycle repeats. Therefore the engine age as such can be ignored as a factor in lubricating-oil economy.

Effect of Engine Size. It is generally believed that large engines are more economical on lubricating oil than small engines. Against this, the Report discloses such glaring cases as a 320-hp engine No. 1167/2 with 45,702 rated hp-hr per gal,

a 300-hp engine No. 1128/3 with 7219, an 80-hp engine No. 1273/3 with 7008 rated hp-hr per gal, on one hand, and a 1200-hp engine No. 1083 with 1283, a 3750-hp engine No. 43/7 with 1875 and another 3750-hp engine No. 43/9 with 1900 rated hp-hr per gal. Of course some of these may be erroneously reported and some others overlubricated.

To obtain information on the effect of engine size, the engines of each type have been divided into three groups, 0 to 500-, 500 to 1000-, and over 1000-hp sizes and the average lubricating-oil economy in each group calculated. The result is shown in Table 1.

TABLE 1 EFFECT OF ENGINE SIZE ON LUBRICATING-OIL ECONOMY

Engine type	Average lubricating-oil economy, rated hp-hr per gal 0–500	500–1000	Over 1000
4-stroke, air-injection	2200 (30)	3027 (30)	4435 (10)
2-stroke, air-injection	(0)	3489 (5)	3110 (15)
4-stroke, mechanical-injection	4605 (27)	3830 (19)	3819 (4)
2-stroke, mechanical-injection	3775 (4)	2971 (15)	4140 (12)
2-stroke, mechanical-injection, crankcase-scavenging	1456 (82)	(0)	(0)

Note: The number of engines in each group is indicated in parentheses.

According to this table the lubricating-oil economy of four-stroke, air-injection engines improves with size; of two-stroke, air-injection and four-stroke, mechanical-injection engines decreases with size; and with two-stroke, mechanical-injection engines the trend is uncertain. These results, of course, should not be taken at their face value. First, because the number of engines on which the statistics are based is too small. Second, because certain factors, which also exert an influence on the lubricating-oil economy, have been ignored. For example, in the case of four-stroke, air-injection engines, the small engines are much older than the large ones. The average year of installation is 1918.3 for the 0 to 500-hp group, 1926.5 for the 500 to 1000-hp group, and 1929.7 for the horsepower group above 1000. This in itself explains sufficiently the 100 per cent increase between the smallest and largest engines without the engine size having any effect. With the other types, the trend is the other way or uncertain. The conclusion is that engine size has no pronounced effect on lubricating-oil economy.

The same conclusion has been reached from a graphical analysis. For each type, the rated horsepower-hours per gallon of lubricating oil, corrected for obsolescence, were plotted against the horsepower rating of the engine. No positive trend could be discerned. The respective graphs have not been reproduced for this paper. The result of the analysis is that engine size can be ignored when predicting lubricating-oil economy.

This does not conform with the general opinion that large engines are more economical in lubricating oil. The probable explanation for such a belief is that, if no differentiation is made between engine types, the large engines really show a superior economy. The reason is that crankcase compression engines, which are the poorest in lubricating-oil economy are built only in small sizes, while air-injection engines, which are best on lubricating oil, are built only in large sizes. This is the basis for the idea that large engines have better fuel economy. However, if the types are separated, it will be observed that it is the type and not the size which influences the economy.

Effect of Engine Speed. All of the engines listed in the Report are stationary engines connected to electric generators. Eighty-five per cent of them have a rated speed between 200 and 300 rpm. Only forty engines, listed with lubricating-oil figures, have lower than 200 rpm and twenty-five higher than 300 rpm. These figures are too scant to form the basis of a statistical analysis. Such individual cases, as No. 1220/3, a 27-hp, 1000-rpm engine giving 2922 rated hp-hr per gal of lubricating oil, lead to the belief that there is no correlation between engine speed and lubricating-oil economy in the field of generator-connected stationary engines.

Effect of Other Factors. The treatment of lubricating oil in the plant, such as settling, filtering, centrifuging, and chemical treatment, undoubtedly affects the lubricating-oil consumption. However, with a minor exception (3.86 per cent), all plants use some kind of purification (272 out of 441 centrifuge), and the difference in lubricating-oil consumption with various treatments is not pronounced.

The quality of the lubricating oil must also have some effect on the consumption but there is no ready way to measure lubricating-oil quality. The same thing is true to a greater degree of the quality of attendance and management. These are the factors which cause the wide spread in the plotted data.

Probable Lubricating-Oil Economy of Newly Purchased Engines

Fig. 4 sums up the results of the analysis of the lubricating-oil data in a simple, useful manner. It gives the probable lubricating-oil economy of newly purchased Diesel engines of various types. The figure is derived from Fig. 1 by taking the 1937 figures for 100 per cent engine-running capacity factor and connecting them with a straight line to the zero point. While

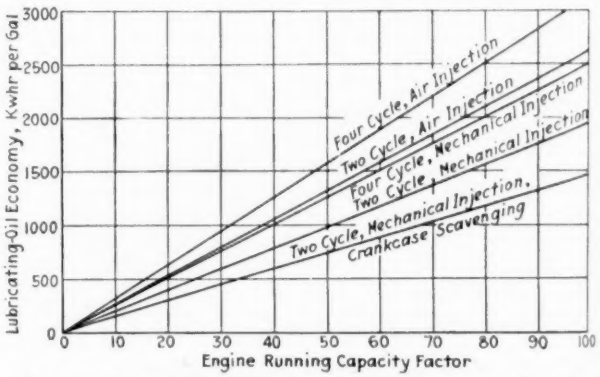


Fig. 4 Probable Lubricating-Oil Economy of Newly Purchased Diesel Engines

Fig. 4 shows probable consumption figures, it has been shown that considerable deviations from the listed values are possible. Consequently, in using Fig. 4 for predicting the lubricating-oil economy of a newly purchased engine, it should be realized that the probable deviation from the given values is about 50 per cent and the possible deviation more. This degree of uncertainty detracts from the value of the results. Nevertheless, it would be a mistake to consider the lubricating-oil statistics valueless, because without them knowledge of the matter is much less.

Factual Analysis of Fuel-Oil Economy

The result of fuel-oil analysis is shown in Fig. 5. The corrected kilowatthours are plotted against years of installation of the engine. In Fig. 6 the engine types are shown separately. The lines shown are ironed-out median lines and indicate the probable fuel consumption of engines.

Three factors have again been recognized, (1) engine load, (2) engine type, (3) year of manufacture.

Engine load is known to have an effect on fuel economy. It is unilluminating to compare the fuel economy of an engine running at half load with one running at full load. Fair comparison can be made either by separating the engines according to running capacity factor as was done in Fig. 3 of the Cost Report (see Appendix) or by correcting for running capacity factor (load) as

was done in Figs. 5 and 6. The latter course was chosen in order to permit the recognition of another relevant factor, the year of manufacture, in the chart.

Correction can be made for the load if, instead of plotting the actual kilowatthours produced by 1 gal of fuel, the kilowatthours are plotted that 1 gal would have produced if the engine had been running full load all the time. For such conversion, an empirical curve is required which shows how the fuel consumption in the average Diesel engine changes with the load. The typical curve the author has used is shown in Fig. 7.

There may be some inclination to accept the median line of the fuel-economy chart, Fig. 2 of the Cost Report, as a typical economy curve. This would be incorrect for the following reason: Older engines are more frequently operated at a low running capacity factor and modern engines at a high running capacity factor. Consequently, the fuel economy for points at the left end of the curve is low not only because of the low load but because of greater obsolescence. If correction is made for obsolescence, a curve is obtained, Fig. 8, which is almost the same as Fig. 7.

Figs. 5 and 6 were so constructed that the actual fuel economy of each point was multiplied by a fraction, the numerator of

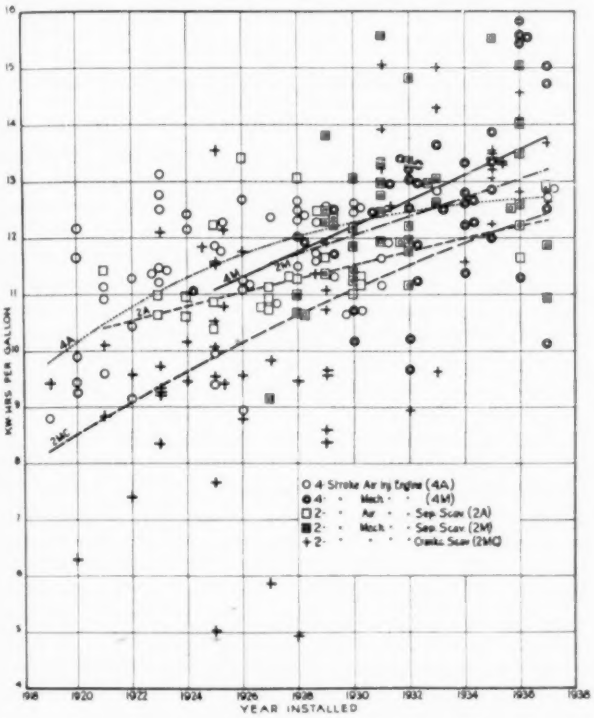


Fig. 5 Fuel Economy of 223 Diesel Engines, Corrected to 100 Per Cent Running-Engine Capacity Factor

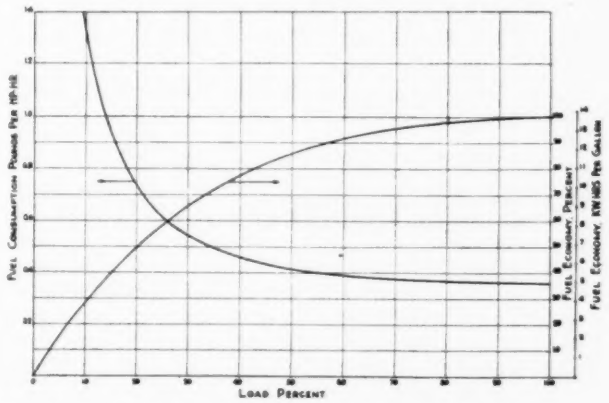


Fig. 7 Relative Fuel Economy of a Typical Modern Diesel Engine at Various Loads

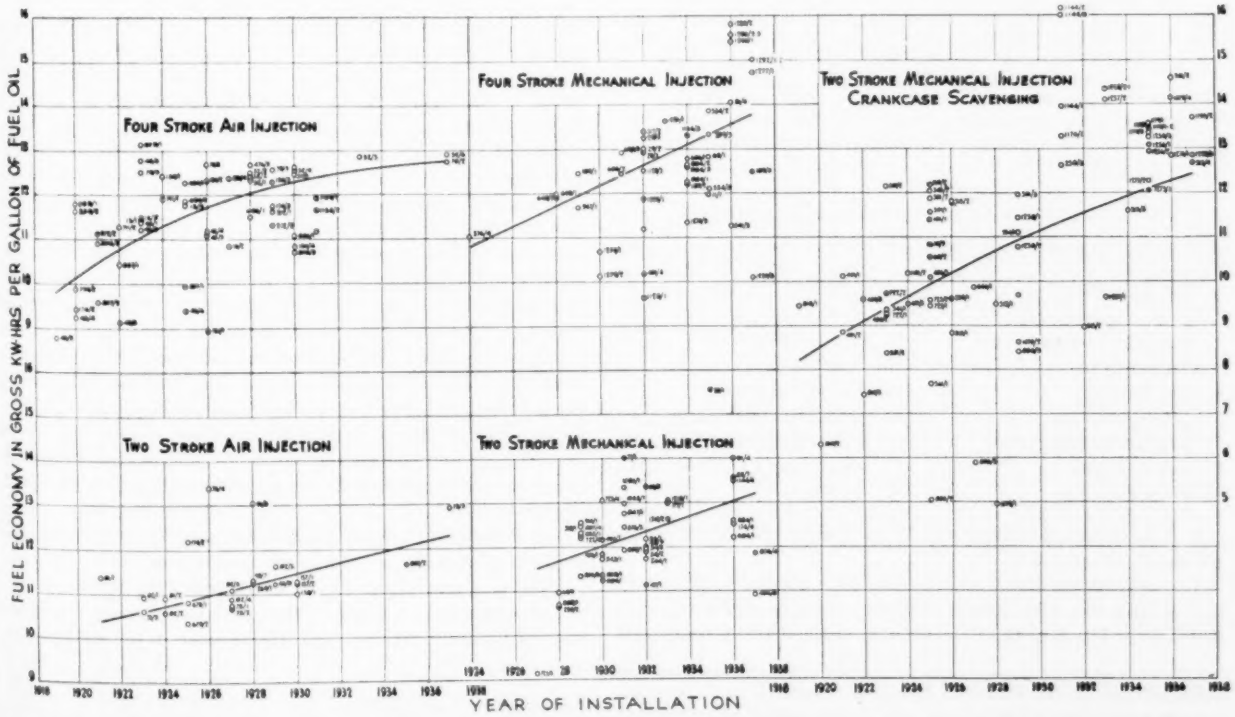


Fig. 6 Fuel Economy, Corrected to 100 Per Cent Engine-Running Capacity Factor; Data Separated According to Types

TABLE 2 EFFECT OF ENGINE SIZE ON FUEL ECONOMY

Engine type	No. of engines (1)	Average hp of all engines (2)	Average hp: Large engines (3)	Average hp: Small engines (4)	Average hp: Above median (5)	Average hp: Below median (6)	Ratio: Above–below / Large–small (7)
Four-stroke, air-injection	54	743.1	1055.9	430.3	774.3	701.0	0.074
Two-stroke, air-injection	23	1440.4	1957.2	940.9	1633.3	1372.3	0.257
Four-stroke, mechanical-injection	42	547.1	821.1	273.1	608.4	478.9	0.235
Two-stroke, mechanical-injection	41	1135.0	1607.2	675.1	1412.8	843.2	0.612
Two-stroke, crankcase-scavenging	68	228.4	298.8	158.8	229.2	227.2	0.014
						Average	0.238

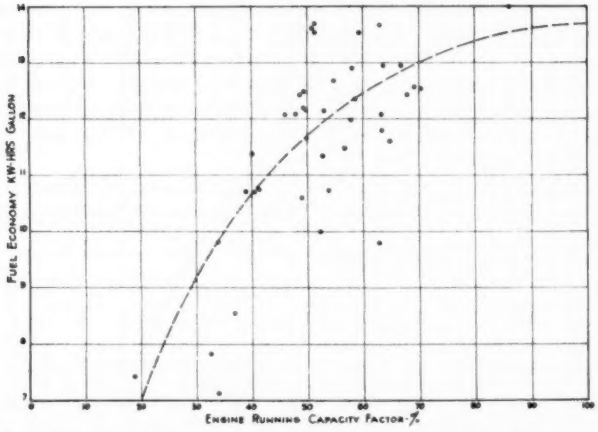


FIG. 8 FUEL ECONOMY OF FOUR-STROKE, MECHANICAL-INJECTION DIESEL ENGINES (CORRECTED FOR OBSOLESCENCE)

which is 1 and the denominator the height of, the typical curve, Fig. 7, at the corresponding running-engine capacity factor.[2] Giving consideration to the engine type, they permit a fair estimation of the fuel economy of an engine. The actually observed fuel economies listed differ from the curves by an average of 7 per cent, the actual deviations being as follows:

Four-stroke, air-injection, per cent	6.82
Two-stroke, air-injection, per cent	4.84
Four-stroke, mechanical-injection, per cent	7.76
Two-stroke, mechanical-injection, per cent	7.64
Two-stroke, mechanical-injection, crankcase-scavenging, per cent	14.1

The curves drawn represent the probable fuel economies. The spread between the probable curve and the individual points is relatively small.

INTERPRETATIVE ANALYSIS OF FUEL-OIL ECONOMY

The factors which might possibly influence fuel economy are:

1 Load conditions
2 Inlet air pressure (altitude) and temperature
3 Engine type
4 Size (rating)
5 Make and model
6 Year of manufacture
7 Age of engine
8 Workmanship
9 Condition of moving parts
10 Fuel gravity
11 Heat content
12 Viscosity
13 Cetane number
14 Quality of attendance
15 Instruments used (pyrometer, smoke meter, flowmeter, etc.)

It is a problem to determine which of these numerous factors are the most pertinent.

The importance of the load conditions is generally recognized by relating the fuel economy to the running-engine capacity factor as in Fig. 3 of the Cost Report. Figs. 5 and 6 have been corrected for running-engine capacity factor by means of a typical curve, Fig. 7.

The effect of inlet air pressure (altitude) and temperature on fuel economy is probably small in so far as stationary engines are concerned. The item was not suitable for statistical investigation.

The effect of engine type has been investigated and the result shows that it is not very pronounced. According to Fig. 5, of the recently manufactured engines the four-stroke-cycle, mechanical-injection engines have the best and the two-stroke-cycle, air-injection engines the poorest fuel economy but the total difference is less than 7 per cent. Of the earlier manufactured engines the two-stroke-cycle, crankcase-scavenged engines showed an appreciably poorer fuel economy but they have improved considerably in later years.

Effect of Engine Size. Whether the engine size has any appreciable influence upon the fuel economy can best be decided on a statistical basis. In Fig. 6 the fuel economies of various types of engines are plotted and median lines drawn. About one half of the engines are above and one half below the median lines. If the average horsepower of the better than median engines and the average horsepower of the poorer than median engines are calculated and they do not differ greatly, then it is justifiable to conclude that the effect of engine size is unimportant. However, if there is a great difference between the two averages, then the engine size probably has a considerable effect on the fuel economy. The result of the calculation is shown in Table 2.

In Table 2, columns (1) and (2) show the number of engines of the particular type listed in the Cost Report and their average ratings. Column (3) shows the average horsepower of the larger half of all engines of the type and column (4) that of the smaller half, assuming that the engines were divided in two parts strictly according to size. Columns (5) and (6) show the average horsepower of the engines above and below the median lines, Fig. 6. Column (7) is the ratio of columns (5–6)/(3–4).

It is clear that, if fuel economy would consistently increase with engine size, all large engines would be above the median and all small engines would be below it. Then the figure in column (7) would always be 1. On the other hand, if engine size were to have no effect whatever upon fuel economy, large and small engines would group about equally on both sides of the economy medians and the figure in the last column would approach 0. The average ratio is 0.238 which shows that the engine size has only a small influence on fuel economy. With two-stroke, mechanical-injection, separate-scavenging engines it is the greatest, probably because the design of this type of engine in small sizes has not yet been mastered. The effect of size on the economy of four-stroke, air-injection engines and two-stroke, crankcase-scavenged, mechanical-injection engines is practically negligible.

The make and model of the engines probably have some effect on the fuel economy but it was not investigated.

Effect of Engine Age. It was noted from Fig. 5 that the year of installation has a marked effect on fuel economy. Taking a 15-yr period from 1922 to 1937 the increases of the medians were as follows:

Four-stroke, air-injection ..... from 10.8 to 12.8 kwhr—18.5 per cent

[2] Five out of fifty-one separately scavenged two-stroke engines had independently driven blowers. The power required to drive these blowers was estimated as 10 per cent of the output and deducted from it in plotting fuel economy

Four-stroke, mechanical-injection ..... from 10.4 to 13.8 kwhr—32.6 per cent
Two-stroke, air-injection ..... from 11.2 to 12.3 kwhr—10.0 per cent
Two-stroke, mechanical-injection, separate-scavenging ..... from 10.7 to 13.4 kwhr—25.2 per cent
Two-stroke, mechanical-injection, crankcase-scavenging ..... from 9.1 to 12.4 kwhr—36.2 per cent

The mechanical-injection engines demonstrated a phenomenal improvement in their fuel economy and took the leadership from the air-injection engines, although the latter recorded some improvement also. It is obvious that estimating the probable fuel economy of a modern engine from statistical figures which included engines of various ages would be very misleading.

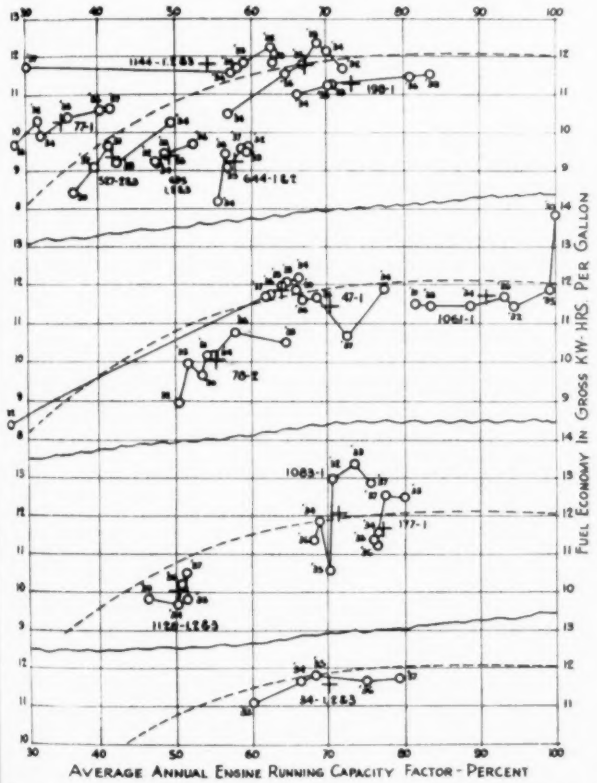


FIG. 9 ANNUAL AVERAGE FUEL ECONOMIES OF 25 DIESEL-GENERATOR SETS, OPERATED OVER 10,000 HR OR MORE EACH (REED)

The question arises, is the improvement due to the better mechanical condition of the new engines or to improvement in design and construction? The answer is that the improvement is almost entirely due to better design and construction. M. J. Reed has investigated the history of the twenty-eight engines which have reported fuel economies successively for 5 years or more. Of these, three operated for less than 10,000 hr total in the longest continuous period and were therefore eliminated. The data for the remaining twenty-five units are shown diagrammatically in Fig. 9.

Examination of Fig. 9 does not indicate that fuel economy deteriorates progressively. Eighteen out of the twenty-five units show better economy in the last year than the average reported. This should prove that the engine age as such has no pronounced effect on the fuel economy.

No data are available on the workmanship and the condition of the moving parts of the engines listed in the Cost Report.

Of the fuel properties probably several have an influence on fuel economy but the evidence is inconclusive.

Effect of Fuel Gravity. If the heat content of 1 lb of fuel is considered constant, which is approximately correct, the heat content of 1 gal of fuel increases with an increase in its specific gravity. An empirical formula expresses the relation of heat content to A.P.I. gravity. Higher heat content Btu per gallon =

$$\frac{1180[49(\text{deg A.P.I.} - 10) + 18650]}{\text{deg A.P.I.} + 131.5}$$

According to this formula the heat content of various gravity fuels is as follows:

Gravity, deg A.P.I.	20	24	28	32	36
Heat content, Btu per gal.	148400	145800	143200	140900	138700

Theoretically, therefore, the 20-deg A.P.I. fuel should give 7 per cent better fuel economy than 36-deg A.P.I. fuel. Since heavy fuel does not cost more than light fuel (if anything it is cheaper), it appears that there would be an incentive for using heavy fuels.

Of the 441 engines listed, fuel gravity was reported for 177. The gravities reported vary from 19 to 40 deg A.P.I. The average gravity of the fuels used in air-injection engines was 26.2 deg A.P.I., while that in mechanical-injection engines was 31.3 deg A.P.I. Otherwise, the type had practically no effect on the fuel used, the averages being 31.4, 31.2, and 31.3 for four-stroke, two-stroke, separate-scavenging and two-stroke, crankcase-scavenging engines, respectively, among the mechanical-injection engines.

Separating the engines according to their fuel economy, with the exception of the four-stroke, mechanical-injection engines, no appreciable difference is noted in fuel gravity, shown by Table 3.

With the four-stroke, mechanical-injection engines, the heavier fuels show better than average economy but the differences in gravity are moderate. In other types there is no appreciable correlation between gravity and fuel economy. As a whole, the data failed to substantiate the theory that heavy fuels have a better fuel economy. The probable reason is that, while the heat content of the heavy fuels is naturally greater, they burn less completely, which neutralizes the gain in heat content.

EFFECT OF FUEL VISCOSITY

Out of 441 engines listed only 125 reported fuel-oil viscosity. The viscosities reported vary from 33 to 112 sec Saybolt Universal at 100 F, except two engines with 1500 sec Saybolt Universal. The average viscosity used is 79.1 sec Saybolt Universal. Separating according to type of engines, the effect of viscosity on fuel economy is indicated in Table 4.

TABLE 3 EFFECT OF FUEL GRAVITY (A.P.I.) ON FUEL ECONOMY

Engine type	No. of engines (1)	Average gravity (2)	Average heavy (3)	Gravity light (4)	Average gravity—Above median (5)	Average gravity—Below median (6)	Ratio—Above-below / Heavy-light (7)
Four-stroke, air-injection	42	26.4	23.0	29.7	26.5	26.1	−0.06
Two-stroke, air-injection	12	26.0	19.7	32.4	26.9	25.5	0.11
Four-stroke, mechanical-injection	31	31.4	29.1	33.9	29.5	35.3	0.82
Two-stroke, mechanical-injection	38	31.2	28.1	34.3	30.3	32.1	0.29
Two-stroke, crankcase-scavenging	54	31.3	28.1	34.4	30.9	31.8	0.143

TABLE 4 EFFECT OF VISCOSITY ON FUEL ECONOMY

Engine type	No. of engines	Average vis sec SU at 100 F	Average viscosity Above median	Average viscosity Below median
Four-stroke, air-injection	32	132.2	85.3	200.7
Two-stroke, air-injection	11	61.7	73.3	44.4
Four-stroke, mechanical-injection	26	66	67	65
Two-stroke, mechanical-injection	22	56.1	68.3	41.1
Two-stroke, crankcase-scavenging	34	59.7	62.8	53.2

There is not much that can be learned from Table 4. Viscosity apparently has no deciding effect on fuel economy so long as it is not too high. In some four-stroke, air-injection engines (like No. 212/1 and No. 212/2) viscosity apparently was too high. The small number of viscosity data do not permit serious analysis.

Effect of Ignition Quality on Fuel Economy

The number of stations reporting fuel-ignition quality is still smaller, including only 42 engines, 23 of which reported cetane numbers, and 25 Diesel index numbers and 6 both. Changing Diesel index numbers to cetane numbers, the average ignition quality reported is 46 cetane numbers. Separately, according to engine types Table 5 gives the information.

TABLE 5 EFFECT OF IGNITION QUALITY (CETANE NUMBER) ON FUEL ECONOMY

Engine type	No. of engines reporting: Better than average fuel economy	Poorer than average fuel economy	Cetane (max)	Cetane (min)	Cetane no. (average)
4-stroke, air-injection	11	2	55	30.4	43.4
2-stroke, air-injection	..	..	..	..	...
4-stroke, mechanical-injection	7	1	45	30	36.75
2-stroke, mechanical-injection	1	5	65	36	49.9
2-stroke, crankcase-scavenging	12	3	61.5	40	51.62

The extremes reported are 30 and 65 cetane numbers. It is significant that out of the forty-two engines that reported fuel-ignition quality, thirty one were better than average and only eleven poorer than average in fuel economy. But this only proves that fuel economy is better with a wide-awake management. No significant differences appear between various engine types in Table 5.

Obviously, quality of attendance and management and the intelligent use of instruments, such as pyrometer, smoke meter, flowmeter, water thermometers, etc., for setting and correcting engine adjustments and ordering repairs have a considerable effect on the yearly fuel economy. It is unfortunate that the data available did not permit such a study.

Probable Fuel Economy of Newly Purchased Engines

Ignoring the factors, the effect of which are unknown or negligible, Fig. 10 shows the most probable fuel consumptions of newly purchased Diesel engines separated according to type and plotted against running-engine capacity factor. The fuel

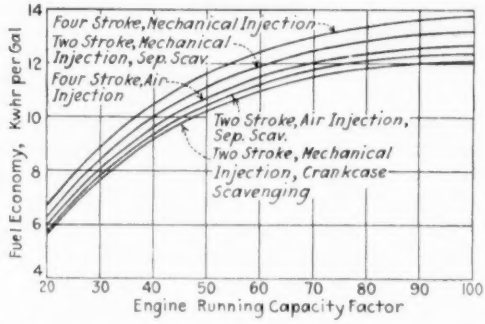


Fig. 10 Probable Fuel Economy of Newly Purchased Diesel Engines

economy actually obtained may differ from these values a certain amount. The variations will probably be less than 7 per cent one way or the other.

Although they mean little to the engine user or prospective purchaser, a tabulation of the average fuel-oil and lubricating-oil consumptions of the plants included in the A.S.M.E. Report follows:

Total number of plants reporting fuel-oil consumption	154
Their total fuel consumption, gal	31,223,511
Their total gross output, kwhr	394,794,626
Average fuel economy, kwhr per gal	12.6
Total number of plants reporting lubricating-oil consumption	146
Their total lubricating-oil consumption, gal	252,139
Their total gross output, kwhr	352,073,079
Average lubricating oil, kwhr per gal	1390

In view of the foregoing, these figures cannot serve as guides to engine users or prospective purchasers, but they may be of some interest to the oil vendors.

Acknowledgment is made to L. A. Ward and Hans Stern who assisted the author in carrying through the calculations in this paper.

APPENDIX

REPORT ON OIL-ENGINE POWER COST FOR 1937

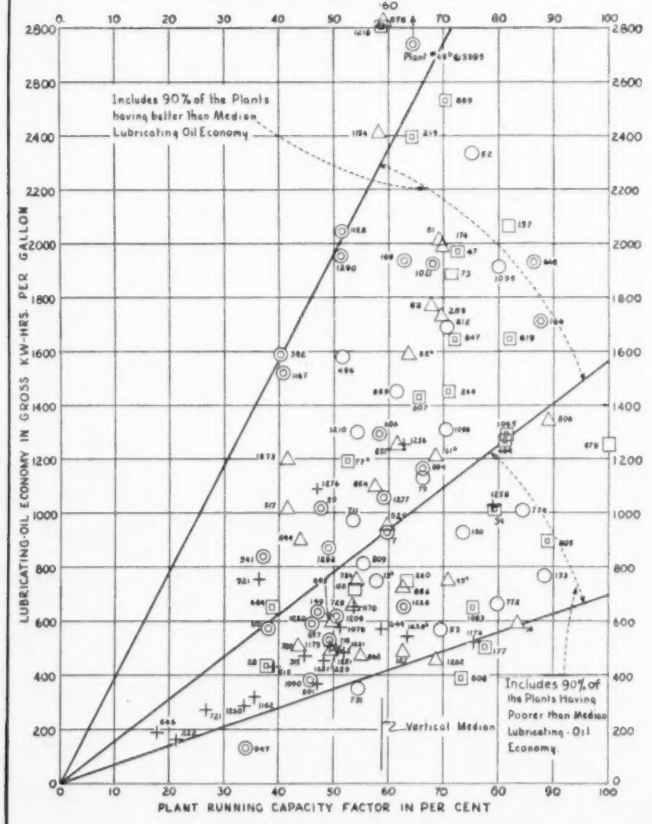


Fig. 1. Lubricating-Oil Economies of 111 Full-Diesel Plants, Not Including 29 Full-Diesel Plants, Plotting Data For Which Are Not Available.
(For Key to Plant Symbols See Fig. 3.)
(Notes. (a) Gross kwhr. includes power for scavenging blower; (b) gross kwhr. includes power for motor-generator excitation)

REPORT ON OIL-ENGINE POWER COST FOR 1937

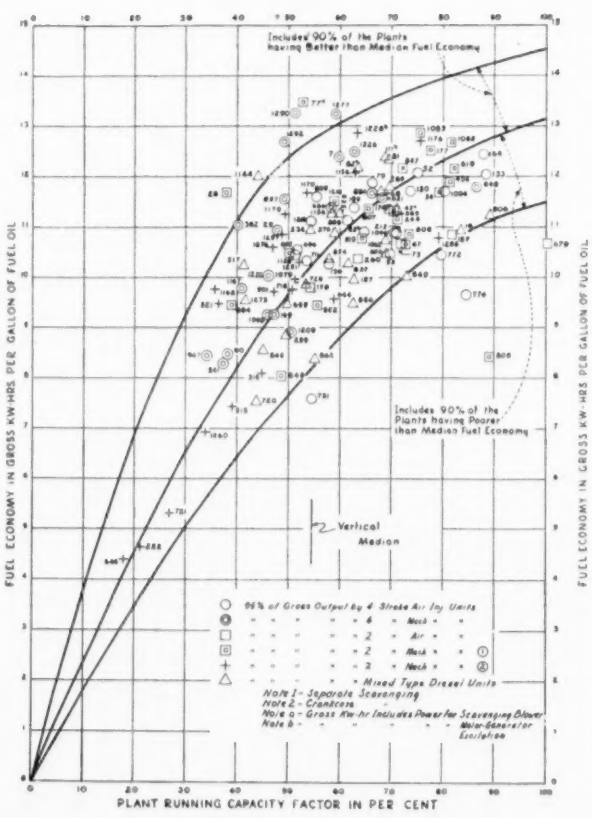


Fig. 2. Fuel Economies of 117 Full-Diesel Plants, Not Including 14 Full-Diesel Plants, Running-Capacity Factors for Which Are Not Available.
(Not Corrected for Heat Content of Fuel or for Altitude)

REPORT ON OIL-ENGINE POWER COST FOR 1937

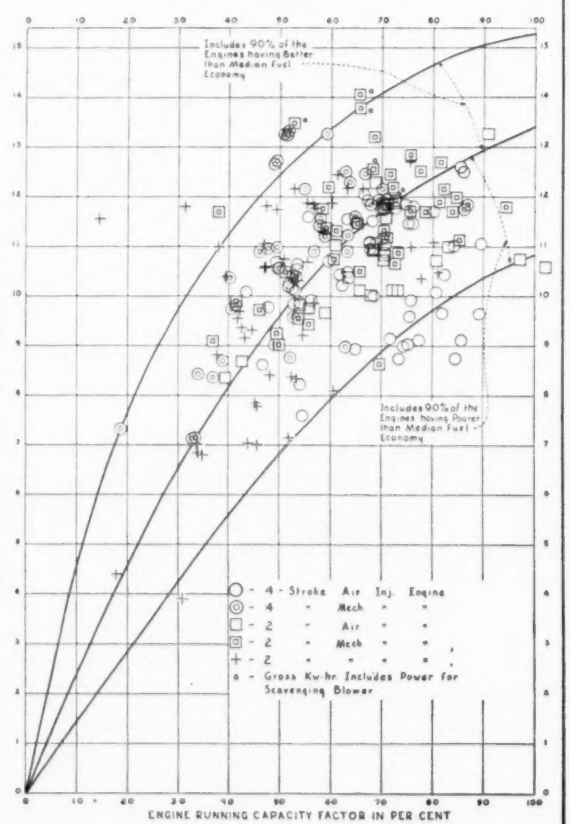


Fig. 3. Fuel Economies of 217 Full-Diesel Engines, Not Including 165 Full-Diesel Engines, Separate Plotting Data for Which Are Not Available.
(Not Corrected for Heat Content of Fuel or for Altitude)

Discussion

L. H. MORRISON.[3] The author points out that the engine built today has a better lubricating-oil consumption than older engines. Likewise, he makes a declaration that the modern engine has a better fuel consumption than an engine built 10 years ago. However, the writer believes he has fallen into error in not considering that the engines built 10 years ago have been running 10 years.

If he wished to plot these factors indicatively, he should have taken the lubricating-oil consumption this year of an engine that was placed in service last year, and then the lubricating-oil consumption of 9 years ago of an engine that was installed 10 years ago. This would constitute the lubricating-oil record for an engine 1 year after it was installed. If that were done, he would find that many of the discrepancies would disappear. Plotting on such a basis will show that the lubricating-oil consumption of a particular engine will gradually increase and then, due to a major overhaul, there is a short improvement in lubricating-oil consumption at the end of some definite period.

We find lubricating-oil consumption and fuel-oil consumption plotted by plants. The writer does not believe that to be indicative. Such plots should be made by engines because, in plotting by plants, a record of operation is shown but not a record of the engines.

It is the writer's belief that the report of the Diesel Users' Association of England is of great value because it contains statements of the operators and managers of plants on why certain figures appear in the cost report. It is highly illuminating to know that, in a certain plant this year, one of the engines had a major overhaul, another experienced piston trouble or what not. It would be illuminating to users if the A.S.M.E. Oil-Engine Power Cost Report contained similar information.

EDGAR J. KATES.[4] The nature of the discussion that has already arisen in connection with this paper justifies the practice of the Oil Engine Power Cost Committee in not itself interpreting the data which it collects and publishes. Probably everyone agrees that the data may be made more helpful to the general user if served in predigested form. However, most analyses and interpretations require the application of opinion or judgment and, since there are so many different ways in which the same data may be interpreted, it would seem better that the work of the Cost Committee itself be confined to gathering, tabulating, and publishing the data in purely statistical form (including simple graphs), and that the interpretations be left for individuals to do in the way Professor Schweitzer has done in the paper under discussion.

The author has made a fine effort to interpret fuel- and lubricating-oil efficiencies, and the suggestion is made that he or someone else should endeavor to apply the same interpretative methods to maintenance and attendance costs.

There are several minor points in connection with Professor Schweitzer's analysis that seem worthy of mention.

In order to show the effects of improvements in design and construction upon the lubricating-oil consumption, the author has assumed that the lubricating-oil economy of any one engine, as expressed by rated horsepower-hours per gallon, does not vary with the load of the engine. In other words, he has referred all of his data to a basis of 100 per cent load, and has assumed, in so doing, that the rated horsepower-hours per gallon at any load is a constant. The writer does not believe that to be a fact. At least, many records have been studied which show that lubricating-oil efficiency is poorer at heavy loads than it is at light loads; consequently, conclusions based on this assumption may be in error.

A similar question arises with respect to the fuel-oil economy. Again the author, endeavoring to correlate fuel economy with the period of design of the engine, eliminates the factor of load by the use of a curve to show the relative fuel economy of the "typical" modern Diesel engine. That curve is subject to wide variations for different engines. It will vary with the type of combustion system used in the engine, and by that is meant the entire shape of the curve will vary, not merely that the whole curve will be proportionately higher or lower. Further, the shape of the curve will vary with the engine manufacturer's rating policy. One manufacturer, who rates his engine conservatively, will get the best economy at full load, as is indicated by the curve in the paper. Another manufacturer, who rates less conservatively, will have his full-load point beyond the 100 per cent point in Fig. 7, and the fuel economy of his engines will be slightly worse at full load than around three-quarter load. Now obviously, if the shape of a curve varies for the different engines reported on, the proportioning which the author does to find the fuel economy at full load so that he may make comparison of the fuel economies at full load of engines built in different years will not be valid. It seems to the writer that the problem has been somewhat oversimplified by using such a curve.

T. M. ROBIE.[5] Referring to Figs. 2 and 6, the writer would like to call attention to the median lines for the crankcase-scavenging engines in respect to the number of plants involved for the later years. The author states that, while the straight-line medians, Figs. 1 and 2, represent the probable lubricating-oil consumption of various type engines, the lubricating-oil data actually observed deviate considerably from the medians.

Now, in constructing any median lines, the individual writer of the paper has to assume certain factors. You will note in the upper right-hand corner of Fig. 2, there are 4 engines below the median line and 12 above it. The engines that are above it are quite far above, so that the median line for the two years at the right is certainly very much too low and does not show a true picture of the present development of two-cycle crankcase-scavenging engines, as regards lubricating-oil consumption.

Similarly, for the fuel consumption of the two-stroke mechanical-injection crankcase-scavenging engine, there is only one engine below the median line in Fig. 6 for the last three years and about 12 above it.

The writer's company is probably the largest contributor to the engine installations on that curve, and we feel that our fuel consumptions are second to none in our present-day crankcase-scavenging engines, so it is felt that the median lines as drawn do not present a true picture.

That brings up another point in connection with a study of these reports or their interpretation. While it is true that they are important, still the manufacturer's guarantees of the engines which he is proposing for sale are also important and should be considered. There is the possibility that a more or less careless salesman may show some of these median lines to an unsuspecting purchaser and, without going into a detailed explanation, lead the purchaser to believe that for a given type he can expect results in accordance with the median lines, whereas reputable manufacturers are willing to demonstrate with tests the actual performance of the engines which they are proposing.

R. D. CAMPBELL.[6] The writer wishes to ask if the author obtained all the data which were used in this analysis directly from the Oil-Engine Power Report? My thought is that if all of this

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[5] Fairbanks-Morse & Company, Chicago, Ill.
[6] Shell Oil Company, St. Louis, Mo.

information were available in the reports, then anyone can plot any curve in which he is interested.

It is conceivable that an engine designer would be interested in types of combustion chambers. Once having identified the engine, he can proceed to plot the economy against either the type of combustion chamber, the piston speed, or any other variable he may see fit to select. It can also be conceived that a man in Mr. Kates's position as a consultant would be interested in knowing whether it is a stand-by plant or in irrigation service or a municipal power plant, and so he would study the different load curves against a different type of engine; in other words, the service of the engine against its type or any other factor, because from his standpoint he wants the over-all economies, whereas the engine designer may be interested in only the technical side of it.

So it seems to the writer that the Committee has chosen a rather wise course in incorporating a wealth of information in the report and drawing as few conclusions as possible. A very voluminous report might be made if the Committee wished to plot all the curves and draw all the conclusions in which any one of us might be interested.

LEE SCHNEITTER.[7] The author has given an excellent analysis and interpretation of certain conditions revealed by fuel and lubricating data in the Oil-Engine Power Cost Report. It is felt, however, that the author has missed the real purpose of data presented in reports during the last 9 years. As the writer sees it, the author devotes all of his paper to proving that engines built in 1937 are more economical in use of fuel and lubricating oil than engines built several years earlier. While the author's analyses are most interesting, the purpose of presenting oil-engine cost data by years for a large number of plants is far different from what the author indicates.

In the third paragraph of the paper he states: "A most legitimate object of the Power Cost Report is to enable a prospective purchaser of a Diesel plant to estimate his probable operating costs." The writer agrees that this is the real purpose of the report and that data on plants listed in the report over a long term of years can be used as a guide to forecast operating costs of a given plant over a reasonable period, perhaps for the economic life of the plant.

In referring to Fig. 7, the writer is sure that the author has committed a serious error in selecting a fuel-consumption curve for "the average Diesel engine." There is no standard curve that will suit all engine types. The shape and characteristics of such a curve will vary with engine make, speed, design, and type.

The author's conclusions are presented in two curves, Figs. 4 and 10, showing, respectively, probable lubricating-oil and fuel-oil economies for new engines. It is felt that a prospective purchaser of an engine would much prefer to obtain these data in the form of guarantees as customarily offered by all engine manufacturers. A series of calculated curves, as presented by the author, would not be as reliable as data obtained from manufacturers on specific engine types.

In presenting Fig. 10, it is suggested that the author specify heating value of fuel, if output in kilowatthours per gallon is to mean anything. Also, is the output in kilowatthours gross or net?

On the subject of lubricating-oil consumption, we must agree that there has been much improvement. Consumption has been reduced in the last 6 or 8 years. Betterments are due to a number of factors already mentioned in the paper. Operators are able to obtain better lubricants; in addition, the oils last longer, being of higher grade. Purification systems have been improved and engine designs have been modified to overcome some of the old problems of lubrication. Consequently, it is felt that the Oil-Engine Power Cost Committee can take cognizance of the fact that the records show some improvement in consumption of lubricating oil.

The author states that he found no difference in the economy of lubricating oil on large and small engines. The statement is made that size is not important, but type is. It seems to me that engine size is somewhat interrelated with type so that it is almost impossible to discriminate between them. Other factors must be considered when analyzing figures on lubricating-oil consumption and, in the case of the very small engine, speed is an important factor. In fact, an average of data presented by the author in Table 1 shows that engines under 500 hp gave about 3000 rated hp-hr per gal while those over 1000 hp in size produced almost 4000 rated hp-hr per gal. This would indicate that larger engines are more economical in lubricating-oil consumption.

V. L. MALEEV.[8] The writer does not quite agree with the author that he is doing an interpretation. He simply does a breakdown analysis of figures. In respect to Diesel engines, the most important subdivision is one that Professor Schweitzer pointed out, that is, dividing by the type of engines: Mechanical-injection two-stroke cycle, air-injection four-stroke cycle, crankcase-scavenging, and separate-pump scavenging. With these subdivisions, we get groups which can be compared rather than interpreted, simplified, or used for a different purpose.

Someone raised the question: "How valuable is a median if it is only within a plus or minus 50 per cent?" It may not be as valuable as we would like to have it. Unfortunately we have cases when this wide deviation is unavoidable. For example, at one time, we had two identical 6-cylinder 300-hp marine engines in operation, which were differing by 30 per cent in their lubricating-oil consumption. At first we were puzzled as to the explanation. Then we happened to notice that one was an installation on a fishing boat; the second was in a yacht and was equipped with counterweights. The yacht engine had to operate as smoothly as possible and, although it did not need counterweights, it operated more smoothly with them. The installation of counterweights increased the oil consumption by 30 per cent.

Also, as far as lubricating-oil consumption is concerned, there is a very great influence from the human element. In another plant, the writer installed 2000-hp gas-engine-driven compressors, using ten 200-hp engines. The installation was made with newly designed engines which had been developed and tested very thoroughly in the factory. We had given a guarantee for lubricating-oil consumption, and instructions on how to run the engines. A few months after the engines were installed, we had a complaint that the lubricating-oil consumption was almost twice as much as that guaranteed. These engines were furnished with equipment for regulating the number of drops of lubricant to each feed line. Our instructions stated that for some points of lubrication, there should be 2 drops, in some 3, and in some 5 drops might be permitted. But the operator believed in safety, so he gave it all the oil he could, and not an outlet was feeding less than 15 drops, some up to 25 drops per min. When this condition was adjusted, the lubrication was all right.

We do not have control over what engine operators are doing. Quite often they will do very foolish things.

The writer is in agreement with one of the discussers that, probably, the curve representing lubricating-oil consumption, Fig. 2, of the crankcase-scavenging engines may be a little less favorable than it should be. In trying to sketch it according to his ideas, the writer finds that the left-hand end should be slightly dropped and the right-hand end raised. Instead of being

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[8] Research Professor of Mechanical Engineering, Oklahoma A.&M. College, Stillwater, Okla. Mem. A.S.M.E.

about 15 deg from the horizontal, probably it would be about 30 deg; it would not change materially. The relative position would be the same.

In this connection it is suggested, in order to avoid arguments as to how such a curve should be drawn, to use the method of least squares, determining the probable errors of observation. If one takes a curve which should represent a median the best way, it should be checked by taking the vertical distances of all points, squaring them, and then adding the sums of those which are above and below. If the sum of the squares of distances above is equal to the sum of the squares below, there is no human element in the curve. If one draws such a line and finds, for instance, the sum of the squares of distances above not equal to that below, one can readily adjust it by checking and get a curve to which nobody can object.

Author's Closure

There is really only one challenge in Mr. Morrison's remarks; he suspects that it may be the engine's age and not its obsolescence which made the difference between the showing of the more recent engines and the older engines, and he suggests that should be taken into consideration.

Fortunately, this has been done. The answer is contained in Figs. 3 and 9 of the paper. These are the result of an investigation by Mr. Reed who determined how much of the improvement is due to better design and how much is due to a younger, more vigorous age; the result is rather conclusive.

In Fig. 3, the lubricating-oil performance of the same 74 Diesel engines has been compared by Mr. Reed in the years from 1930 to 1935. This demonstrates that engines get better with age instead of getting worse.

The same thing is true with fuel-oil economy. The age itself does not seem to make the engine bad, as shown by Fig. 9. It must be realized that every engine is overhauled at periodic intervals, every 5 or 6 years. Rings are replaced and frequently liners are either replaced or rebored; the engine is then again as good as new. As a matter of fact, the engines, in both lubricating-oil and fuel-oil economy, improve a little in the first year, then they get worse in the second, third, and fourth years, until they have a major overhaul; then the cycle repeats. It is improvement in design, no doubt, which makes recent engines better than older ones.

The author is in agreement with Mr. Kates regarding the lubricating-oil economy varying with the load, if the assumption that the rated kilowatthours per gallon of lubricating oil is practically independent from the load is incorrect. The explanation is that the author simply followed the Power Cost Committee, which has followed the British Diesel Engine Users' Association in this assumption. That association always lists the rated kilowatthours, and if not 100 per cent correct, at least it is approximately so. It is doubtful if any better assumption can be suggested.

The same thing holds true for the fuel consumption shown in Fig. 7. It should be realized that variations in the actual consumption per horsepower-hour do not affect the results, provided the per cent variations for various loads are the same. Here again the typical curve, which was felt to be necessary, checked so well with other authoritative sources that there was no hesitation in using it. The errors committed by the use of a single typical curve are probably negligible.

The attitude can be taken that a simplification by which the economy of various engines at various loads may be brought to a common denominator is neither permitted nor justified. That is a perfectly justifiable attitude but the opposite attitude may also be assumed that simplification is necessary in order to dispel the ignorance of those who know nothing about economy figures on Diesel installations. If we are to have simplifications, then the author would ask Mr. Kates, or others, what else can be proposed to take the place of the typical curve which has been used?

Mr. Robie complains about what he considers is an unfair presentation of the crankcase-scavenging engine in recent years. He did not object to the drawing of the lines in the period, say, before 1934 or 1935, but he thinks that the crankcase-scavenging engines have been making better showings than indicated on the curves in 1936 and 1937. Not only Mr. Robie but others will have objections regarding certain medians that have been shown.

The crankcase-scavenging engine does show remarkable improvement in the last 2 years. Perhaps he would have liked it better if the curve had bent up sharply for these years. That would have fitted the points better than the straight line does. The lines were drawn conservatively with no attempt to bend up any curves.

In answer to Mr. Campbell's inquiry, all of the types are indicated in the reports. However, in making up the Power Cost Report the prospective user was forgotten. The reports are valuable to the engine makers, to the oil salesman, to the utility salesman, and to the manufacturer. They are of some good to those who operate power plants. But it is felt they are of no practical value to the man who is interested in installing a Diesel power plant. That is the man the Power Cost Committee would do well to remember. The author approves of omitting interpretation from the reports, but a few more graphical presentations, judiciously plotted, would do a great deal of good to those most important readers of the Power Cost Report who want to use it in order to determine what they can expect of the engine they have in mind.

With reference to Mr. Schneitter's comments, these curves were drawn from the raw material available. The Btu content of the fuel used is not listed in the Power Cost Report. These are not assumed or proposed curves but are the results of actual statistical evidence. There is a certain amount of interpretation in the second part of the paper which is so termed. This may be out of place in the cost report.

The cost report includes individual engines, and only the data for individual engines have been used in the chart. Btu content has been ignored because there was no information available, but the next best thing is gravity. There is a rather close correlation between Btu and gravity in ordinary petroleum fuels. In general the heavier fuels are no better or worse but about the same as the lighter fuels in economy. My conclusion was, again interpretatively, that there was no effect of fuel gravity, generally speaking, on economy. It is true the heavier fuels have more heating value, but generally their combustion is less complete, which about makes up for the difference in Btu content.

The author is in agreement with Mr. Schneitter that, if size and type cannot be separated, then there is justification in the notion that the large engines are better in lubricating-oil economy than the small engines. But, if separated as was done in the paper, it becomes clear that within the types there is no correlation. If the crankcase-scavenging engine is studied alone or the two-cycle air-injection engine alone, then one cannot say that the large engine is better than the small engine. They are about the same, broadly speaking.

In connection with Professor Maleev's interpretation of Fig. 2, there is no doubt that others can and would draw the curves differently. Perhaps some of the material that was put under the heading of factual presentation should have gone into interpretation, though nothing was done but present the points as they were taken from the Power Cost Report, leaving out none no matter how improbable or erroneous some of them might be.

If figures such as Figs. 2 and 6 should be included in the Power Cost Report, the author would agree to leave out the lines altogether. These lines were drawn conservatively. While we have

not used the method of least squares, we have used something approaching it. If medians are used rather than averages, then the least-squares method is probably out of order.

Professor Maleev is perfectly correct that, when it comes to lubricating-oil consumption, the human element and some uncontrolled factors play a great part. We know that many operators overlubricate their engines, sometimes to be on the safe side and sometimes because of ignorance. If we know the probable lubricating-oil economy, plus or minus 50 per cent, that is worth more than nothing, and when it comes to fuel-oil economy, then the estimate narrows down to about plus or minus 7 per cent.

Stability of Crankcase Oils

By W. F. WEILAND,[1] LINCOLN, NEB.

In this paper the author attempts to answer two vital questions concerning lubricating oils, i.e., performance and useful life. Oxidation and corrosion tendencies constitute the major elements in the deterioration of crankcase oils. The essential requirements for conducting oxidation tests are outlined in the paper and the procedure and results of tests are indicated. Observations on the behavior of oils in service also offer a means for predicting oil performance. Acid-vapor tests to determine oil breakdown are discussed and the results of corrosion studies detailed.

IN CONNECTION with lubricating oils, there are two questions which engineers would like to have answered: (1) How can the performance of an oil be predicted? (2) When is an oil worn out?

The advent of the modern high-compression internal-combustion engine has brought to light many lubrication problems which formerly were not apparent. Foremost among these problems are the oxidation and corrosion characteristics of the crankcase oils. More or less rapid oxidation of the lubricating oils, subjected to temperatures as they exist in modern engines, results in the formation of gums and sludges which in themselves constitute an endless source of trouble. Furthermore, as oxidation progresses corrosion difficulties arise due to a gradual lowering of the temperature at which corrosive products of oxidation form.

An oxidation test to be successful must simulate engine conditions as nearly as possible. The duration of the test should be within reason to make it commercially practical as a routine test; it should be simple to perform; different observers should be able to check results within limits; and last, but not least, a close correlation should exist between test results and oil performance in an engine.

A suitable temperature at which to run an oxidation test must be selected largely by deduction. If one considers the jacket temperature, a temperature gradient through the cylinder walls, the usual polished and unburned surface of the piston-swept walls of an engine in good condition, and the rate of deterioration of the crankcase oil, then, with all of these factors taken into account, an oxidation temperature of 300 F is at least reasonable. The logic of this assumption will be further emphasized later.

A second condition to be applied in an oxidation test is aeration. The question sometimes arises whether oxygen from the combustion chamber contributes to disintegration of the oil, but this presumption may not be a major contributing factor. Abundant air is whipped into the oil as it is splashed around and churned in the crankcase. It is only necessary to expose this oil saturated with air to the heat of the piston and cylinder walls to accelerate oil breakdown.

Other conditions which may affect oxidation are pressure, foreign solids such as road-dust and carbon particles, dilution, products of incomplete combustion, and metallic catalysts.

A laboratory setup which at least partially fulfills the conditions enumerated has been described previously by the author.[2]

Measuring the Extent of Oxidation

The extent to which oxidation has progressed is based on the relative change in color density in the oxidized oil compared with the original oil. To measure this color change, fill one glass-bottom tube of the comparison apparatus to a depth of 1/4 in. with the oxidized oil. Pour from the original oil into the other measuring tube until the two oil columns show the same color density as observed by looking down through both columns simultaneously. The ratio of the height of the column of the original oil to the height of the column of oxidized oil is the "oxidation number" for that particular oxidation period.

The oils reported in this paper have been given code numbers which can be partially deciphered, namely, the hundreds digit indicates the S.A.E. viscosity, while the tens and units digits indicate the house brand. For instance, oil number 124 is S.A.E. 10 oil of brand 24, and 224 is S.A.E. 20 oil of the same brand.

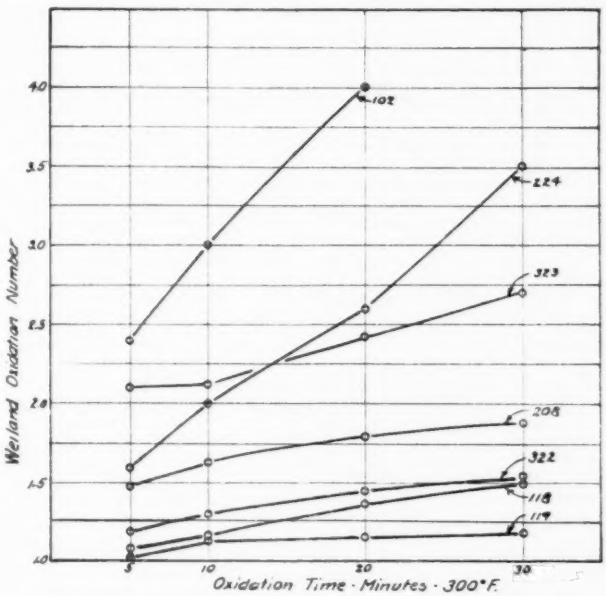


Fig. 1 Variations in Oxidation Characteristics of Oils Commercially Available

Fig. 1 illustrates the wide variation that can be expected in the oxidation characteristics of oils on the market. Oxidation numbers at 300 F are plotted against the oxidation period in minutes. A considerable percentage of the usual oils falls between Nos. 118 and 224. In so far as oxidation is concerned, oil No. 322 still has acceptable characteristics but oils above this curve must be used cautiously; oils above No. 224 had better not be used at all for internal-combustion-engine lubrication.

Since Fig. 1 includes a miscellaneous assortment of house brands, it shows that in general the weight of the oil is no index of its resistance to oxidation. It will be noted that both of the ex-

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

[2] "Measuring Sludging Characteristics of Lubricating Oils," by W. F. Weiland, *Oil and Gas Journal*, Jan. 20, 1938, pp. 83, 86, and 93.

treme curves are S.A.E. 10 oils. Even oil weights of one brand do not necessarily fall in logical order as is shown in Fig. 2. Here the S.A.E. 40 oil, except for short oxidation periods, is less resistant than No. 324. No. 524 is of doubtful quality because of the initially high oxidation rate. Even No. 624 falls above No. 322 of Fig. 1.

On the other hand, Fig. 3 represents the S.A.E. 10, 20, and 60 code No. 1 oils which are superior in quality. The boiling range of each weight is controlled within narrow limits in contrast to the code No. 24 oils represented in Fig. 2, where the several oil

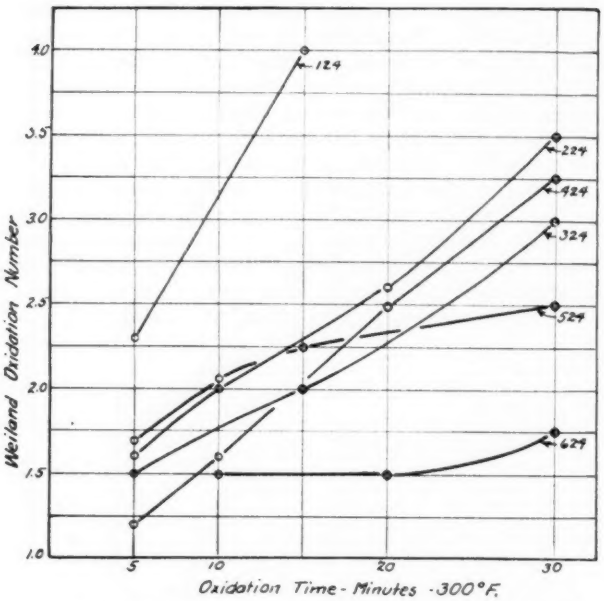


FIG. 2 OXIDATION CHARACTERISTICS, OIL BRAND 24

weights are obtained by blending. Oxidation numbers at 350 F instead of 300 F for code No. 1 oils were plotted principally to separate the curves more than would be the case at 300 F oxidation temperature.

EFFECT OF TEMPERATURE ON OXIDATION

This brings up the question of the effect of temperature on oxidation which is illustrated in Fig. 4. Oxidation numbers for 5 min, 15 min, and 30 min are given for several oils oxidized at temperatures ranging from 275 to 350 F. The oxidation numbers may be visualized as oxidation rates for the time specified. It is apparent that the oxidation numbers may be closely approximated by a more or less complex exponential function of the temperature of the general form

$$n = A(T + C)^x$$

where

n is the oxidation number
A is a factor which will include such variables as aeration, pressure, effect of catalysts, etc.
T is the temperature in degrees
C is a temperature factor

The exponent x in turn is a variable of the general form

$$x = \frac{n^k}{(T + C)}$$

This equation for the oxidation number as a function of the temperature will in general involve too much labor to be of any practical value. It is inserted only to give a conception of the increase in rate of oil breakdown with temperature, a fact which many engineers have probably sensed.

Investigators have observed changes in properties of some oils exposed to atmospheric conditions for long periods of time. Some tangible evidence of such changes is shown in Table 1. Air was bubbled vigorously through these oils at atmospheric pressure and room temperature, for periods indicated in column two. No attempt was made in these tests to obtain quantitative results. Column three shows the usual oxidation numbers for a 30-min period at 300 F. Comparison of aerated samples with the new oil shows only a faint to no color change, but an entirely different picture presents itself after a 30-min oxidation period at 300 F as indicated in the last column. Preliminary aeration has little or no effect on oils Nos. 309, 411, 412. The ratio of the oxidation numbers after aeration to the numbers before aeration for oils 214, 215, and 226 is approximately 1.3, while for oils 101, 105,

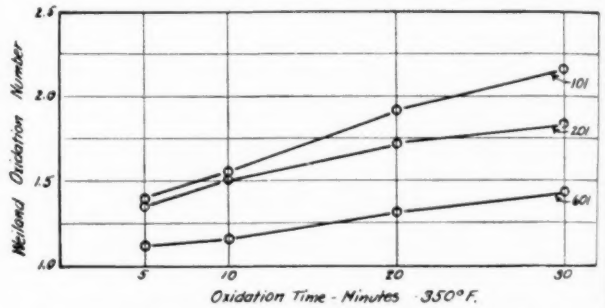


FIG. 3 OXIDATION CHARACTERISTICS OF S.A.E. 10, 20, AND 60 CODE NO. 1 OILS

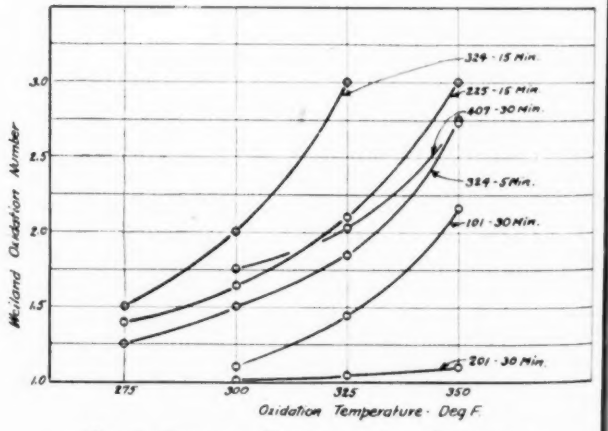


FIG. 4 EFFECT OF TEMPERATURE ON OXIDATION

and 408, the ratio is slightly over 1.4 and for oil 204 it is nearly 1.9. It is evident then that some oils will not stand up as well under the beating they get in an internal-combustion engine.

The upper curve of Fig. 5 is a service demonstration of this point. This oil had been used 2000 miles in an oil-bath air cleaner.

TABLE 1 EFFECT OF AERATION ON OXIDATION

Oil	Aeration period, hr	Oxidation numbers — 30 min 300 F before aeration	Aeration only	30 min 300 F after aeration
101	27½	1.12	1.00	1.64
204	26½	1.02	1.12	1.92
105	26½	1.04	1.00	1.48
408	26½	1.92	1.08	2.76
309	26½	1.28	1.08	1.32
411	26½	1.52	1.00	1.52
412	26½	1.20	1.00	1.20
214	33	1.28	1.00	1.56
215	21½	2.84	1.04	3.68
226	26½	1.04	1.04	1.34

TABLE 2 OXIDATION NUMBERS WITH COPPER STRIP

Oil	30 min 300 F	30 min 300 F with copper strip
101	1.12	13.70
203	1.56	13.48
206	1.28	4.34
206	1.80[a]	12.16[b]
208	1.88	5.00
112	1.32	4.36
120	1.32	1.96

[a] 6 hr 300 F
[b] 5½ hr 300 F with aluminum strip.

The 30-min 300-F oxidation number is 4.3 times the similar number on new oil.

Presence of metals in the oil sample during oxidation has a variable effect which is quite alarming with some oils. The oxidation number obtained in the usual way is positively no indication of the number obtained by oxidation with a copper strip as shown in Table 2. Oxidation of oil No. 203 for example has proceeded so far in 30 min ($n = 13.48$) that the oil has become supersaturated with sludge even though this is an oil with high gum-solvent characteristics. Oil 120 is far superior to the other examples in resisting what is very likely a catalytic action of copper. No information is available at present regarding the behavior of this oil in the presence of other metals.

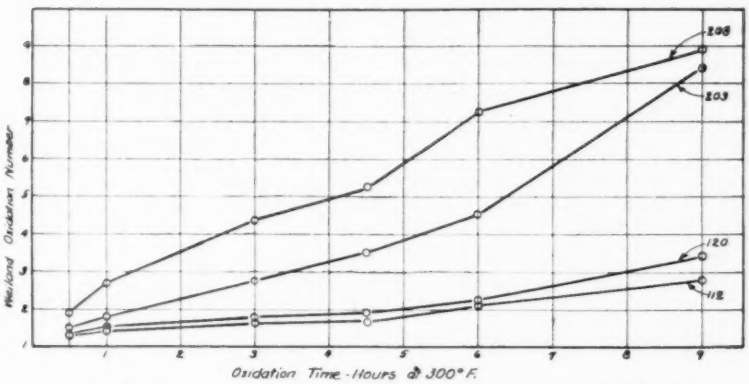


FIG. 6 RESULTS ON OILS OXIDIZED AT 300 F FOR PERIODS UP TO 9 HR

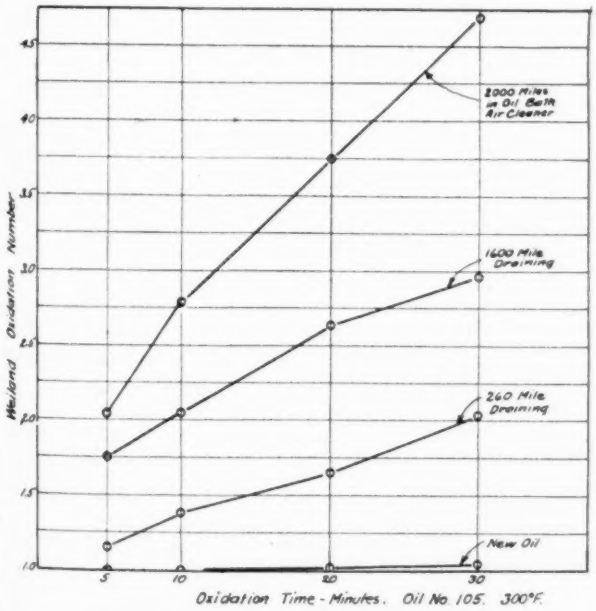


FIG. 5 SERVICE RESULTS WITH OIL NO. 105

Oil No. 206 behaves none too well in the presence of copper and it likewise performs badly in the presence of a strip of aluminum alloy cut from a new piston. This oil shows a high resistance to oxidation when the test is conducted in the usual manner.

It may seem rather strange to some that a mere change in color density of an oil is an indication of the extent to which oxidation has progressed. Some rather conclusive evidence has been presented[2] which shows the relation of the oxidation numbers to a sulphuric-acid test and also to Indiana values. To provide further and more tangible evidence, a number of oils were oxidized at 300 F for long periods of time. Results of some of these oils up to 9-hr oxidation time are included in Table 3 and Fig. 6. It is noted from the table that oxidation numbers for 5 min, 10 min, and 30 min are continuous with oxidation numbers for longer periods of time. Also that the most abrupt change occurs in the first 5 min, with very few exceptions. This last statement is made on the basis of tests on nearly 100 oils, representing over 60 trade brands.

The last column in Table 3 is highly significant in that it indicates the limit of solubility of the products of oxidation in the oil. It is probable that external influences may readily change the solubility characteristics. Nevertheless, when these gums can no longer be carried in solution, they will be deposited on vital engine parts and, by a process of accumulation, gradually build up to a dangerous point.

TABLE 3 LONG-TIME OXIDATION, 300 F

(Oxidation numbers—acid-vapor temperature, degrees F)

	Minutes			Hours					
Oil	5	10	30	1	3	4.5	6	9	Visible[a] sludge
203	1.28	1.28	1.56	1.88	2.80	3.52	4.52	8.46	10+
280°	270°	270°	260°	260°	220°	..	220°	..	
206	1.16	1.28	1.28	1.4	..	1.76	1.8	2.86	10+
290°	290°	270°	270°	280°	..	..	260°	..	
208	1.48	1.64	1.88	2.72	4.40	5.24	7.25	8.9	8
310°	290°	265°	265°	265°	250°	..	260°	230°	
112	1.12	1.16	1.32	1.48	1.60	1.68	2.16	2.80	10+
250°	250°	250°	240°	240°	..	..	240°	..	
120	1.16	1.17	1.32	1.52	1.80	1.92	2.28	3.44	10+
270°	260°	230°	265°	250°	240°	..	210°	..	

[a] Visible sludge, as indicated by muddy color or by black streaks in the oil, appears when the oxidation number reaches the approximate figure given in this column.

Concentration of gums or, more generally, the products of oxidation, need not reach the saturation point in the entire batch of oil in the crankcase before deposition begins. Local supersaturation may occur at any time in regions in the engine where these gums form most readily and especially where the oil is not generously and continuously replaced. The danger point of an oil may be reached even though the stick to all appearances is "clean."

PREDICTING OIL PERFORMANCE

Observations on the behavior of oils in engines, examination of the drainings, filter cartridges, gasoline used, and examination of the sludges have led to an attempt to predict the performance of an oil at least as far as oxidation is concerned. It must be clearly understood that oil performance is subject to many variables and that any predictions must likewise be accepted with an open mind. The estimate is based on the average slope of the oxidation number-time curve, stated as a percentage and is obtained as follows:

Assume an oil with oxidation numbers 1.20, 1.32, 1.40, and 1.40 for 5 min, 10 min, 20 min, and 30 min, respectively. Then

Change in the first 5 min = 0.20

Change per min $= \frac{0.20}{5}$ = 0.040

Change from 5 min to 10 min = 1.32 — 1.20 = 0.12

Change per min from 5 min to 10 min $= \frac{0.12}{5}$ = 0.024

Change from 10 min to 20 min = 1.40 — 1.32 = 0.08

Change per min from 10 min to 20 min $= \frac{0.08}{10}$ = 0.008

Change from 20 min to 30 min = 1.40 — 1.40 = 0.00

Change per min from 20 min to 30 min = 0.000

Average rate from 0 to 30 min = (0.040 + 0.024 + 0.008 + 0.00) ÷ 4 = 0.018, which stated as a percentage is 1.8 per cent.

This oxidation percentage divided into 100 gives 56, a number closely related to the maximum hours of useful service which the oil may be expected to give; and if a speed of 40 mph is assumed, the useful service is 2200 miles.

As a check on these calculations, Table 3 gives oil No. 206 as showing visible sludge at an oxidation number of slightly over 10. The oxidation per cent for the first 30 min for this oil is 1.87 per cent or the equivalent of 2100 miles. Table 4 shows oil No. 206 after 2000 miles of service. The oxidation number of these drainings after filtering through the usual laboratory filter paper is given in column 4 as 10.1 which checks well with Table 3.

TABLE 4 OXIDATION OF DRAININGS

Oil miles	New oil 30 min 300 F	Drainings 30 min 300 F to drainings	Drainings to new oil	Oxidation new oil per cent	Oxidation drainings per cent
206–2000	1.28	1.68	10.10	1.87	53.3
107–1600	1.46	2.16	9.93	2.60	68.3
208–1500	1.88	5.21	20.00	4.63	[a]
119–1100	1.19	2.67	4.26	0.85	34.6
119–1400	1.19	1.96	4.05	0.85	23.1
322–37 hr	1.56	2.50	5.65	2.20	43.7
327–39 hr	1.40	3.30	3.25	1.80	32.4
101–446	1.12	1.72—Ford V-8 moderate speed—long trip			
101–519	1.12	1.68—Ford V-8 city driving			
101–1150	1.12	1.41—Ford V-8 city driving—cold weather			
101–1000	1.12	2.42—Ford V-8 transport truck			

[a] This oil has outlived its usefulness. The calculated oxidation per cent for the drainings is well over 300.

A further check of the drainings is of value. The filtered drainings are oxidized 30 min at 300 F in the usual manner. The oxidation number as compared with the drainings is given in column 3 of Table 4 as 1.68. Then proceeding as in the calculation for oxidation percentage of the new oil:

$$\frac{(1.68 \times 10.1) - 1}{30} = 53.3 \text{ per cent}$$

Observations of drainings indicate that the oil has passed its service peak and is rapidly declining in usefulness as a lubricant when this "oxidation per cent" of the drainings passes the 50 per cent mark.

Oil No. 107 from Table 4 has outlived its usefulness. The casual observer would come to this conclusion from a mere inspection of these drainings.

Oil No. 208 in Table 4 has far outlived its usefulness. The oxidation number of the filtered drainings is 20.0 while Table 3 shows this oil reaches its useful limit at a sludging number of 8.

The 1100-mile draining of oil No. 119 is in worse condition than the 1400-mile draining of the same oil. This is readily explained in that the effective compression ratio of the engine from which the 1400-mile drainings were taken is considerably less because of poor valve timing than the engine using the same brand oil for 1100 miles. The better engine, as would be expected, is more severe on the lubricating oil. Both automobiles were operated under very similar conditions.

A still further example of the variable oil performance to be expected, even in the same or similar engines, is shown in the four examples of drainings of oil No. 101.

Fig. 5 is of interest as it illustrates the progressive increase in oxidation rate of oil in service. Both drainings were obtained from the same automobile operating in each instance under similar conditions.

All readings were made with a Weston electric meter on oil depths of $^1/_4$ in. and for this reason are not exactly comparable to other oxidation numbers reported in this paper. An electric-meter reading is a measure of the amount of light that passes through the oil, irrespective of color or color density. Furthermore, sludges and solid particles tend to disperse the light and may actually increase the meter reading in some instances and, at other times, give a lower reading because of the simultaneous blocking effect of colloidal particles. Many tests were made with the electric meter on various new oils and on drainings, but the method was finally abandoned in favor of visual inspection.

The per cent oxidation computed in the usual manner for the new oil, Fig. 5, is 0.15 per cent, for the 260-mile draining is 3.5 per cent, for 1600-mile draining is 7.55 per cent, and for the 2000-mile air-cleaner draining is 13.85 per cent. Total oxidation based on the new oil is 9.8 per cent for the 260-mile draining, 50.5 per cent for the 1600-mile draining, and 18.2 per cent for the air-cleaner draining. Sludge formation in the 1600-mile draining during the laboratory oxidation was noticeably rapid, which is further evidence of the desirability of an oil change when the oxidation per cent of the drainings reaches the 50 per cent point.

Acid Products From Oil Breakdown

It has been known for some time that oil breakdown is attended by the formation of more-or-less acid products. The exact nature of these products, general rules governing their formation, methods of their detection, and their effects are not so well known. In the practical analysis of this problem, their exact nature is of less importance than a knowledge of the conditions under which these products are formed and evidence of their effect. It must be clearly understood that the standard acid test for lubricating oils is of no value in the detection of these acid products, as they are given off by the oil principally as vapors, hence, the designation "acid vapor."

Acid-Vapor Test. The oxidation apparatus is slightly modified by placing a 1-in. diameter × 3-in. length glass tube in a vertical position in the suction line leading from the test tube and as near the test tube as practical. An acid indicator such as blue litmus is suspended in this tube.

Procedure. Heat the oil bath to 350 to 360 F and maintain the temperature within these limits. Place about 25 cc of oil in the test tube and adjust the air suction to 4 in. of water. When the first trace of color change appears on the litmus, note the temperature of the oil sample. This reading is recorded as the "acid-vapor temperature."

In the average run of oils, the acid-vapor temperature drops rapidly as oxidation proceeds. The lower row of figures under each oil in Table 3 gives some of these temperatures for the long-time laboratory oxidation tests. The acid-vapor temperature of the new oil is no definite prediction of what happens later. However, a strong acid-vapor indication in a new oil containing fatty additives is open to suspicion. Such is the case of the oil of Fig 5, which showed a relatively high resistance to oxidation by the usual test, but the 1600-mile draining broke down very rapidly.

Fig 7 shows graphically the trend in acid-vapor temperatures of three of the oils of Table 3. Oxidation for several points on these curves was forced to obtain a few of the acid-vapor temperatures not included in the table. The acid-vapor temperature appears to reach a near-minimum value and, when this occurs,

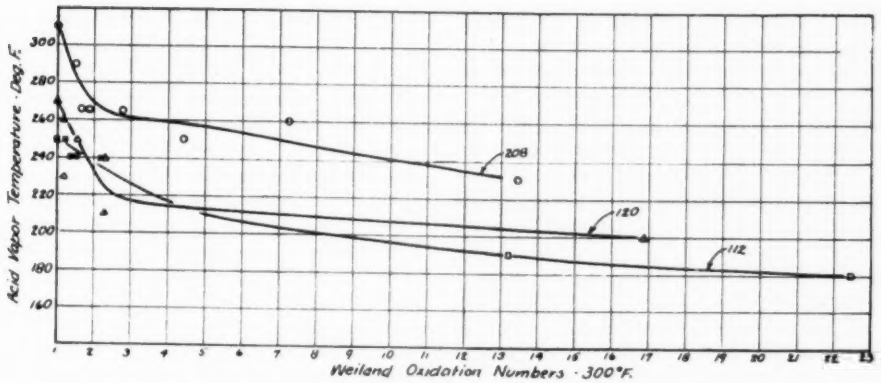


FIG. 7 GRAPHIC INDICATION OF TREND IN ACID-VAPOR TEMPERATURES

visible sludges begin to make their appearance in the oil, the oil being oxidized in the usual manner.

For low oxidation numbers the acid-vapor temperatures may be more or less erratic, especially for built-up oils containing a miscellaneous assortment of additives, as oil No. 120. Such oils are likely to approach the near-minimum, acid-vapor temperature much sooner than a straight-cut oil and, therefore, are again open to strong suspicion.

Acid-vapor temperatures as low as 180 F have been observed in oils oxidized in the laboratory, and as low as 140 F in crankcase drainings. There is no record as yet that any of these oils has shown the presence of acid by the standard acid test.

The minimum acid-vapor temperature and possibly the rate of drop in acid-vapor temperature are in themselves only an indication of what may be expected to happen in an engine specifically from the point of view of corrosion. To throw some light on this question, a number of new oils and drainings were heated at 300 F in the oxidation apparatus with air bubbling through in the usual manner for 3-hr periods. Carefully weighed strips of metal were placed in each sample of oil. Tests were made with cadmium-copper-silver bearing-metal strips, and repeated with fresh oil samples on polished copper strips as well as on pieces of aluminum alloy. At the close of the test period, the metal strips were removed and carefully washed with benzol and with carbon tetrachloride and allowed to dry. The metal strips were weighed again and the loss in weight in per cent calculated. It is well appreciated that there may be a slight increase in weight of the metal due to possible adsorption of hydrogen and nitrogen, or chemical combination or alloying with carbon, oxygen, sulphur, etc. A metallographic study or even a chemical analysis may be advantageous in the final study of this problem.

TABLE 5 CORROSION STUDIES, 300 F

Oil	Acid-vapor temperature, F	Loss in weight in 3 hr, per cent: Cadmium-copper-silver	Copper strip	Aluminum permite No. 3
101 new	290	0.0000	0.0208	...
101–1000 miles	230	0.0000	0.0208	...
102 new	285	0.0000	0.0386	0.0053
102–1000 miles	220	0.0390	0.0560	...
403 new	320	....	0.0110	0.0140
403–450 hr	240	0.0026	0.0322	0.0032
105–1600 miles	165	0.2950	0.0680	...
107–1000 miles	250	0.1240	0.0000	...
410 new	270	0.0013	0.1520	...
410–450 hr	200	0.0000	0.1180	0.0046
417–450 hr	210	0.0000	0.0234	0.0036
119–1100 miles	210	0.0440	0.0000	...

A few instances are reported in Table 5 which indicate no loss in weight. Attention is directed to the outstanding losses of the 410 new oil on copper strip, and the 105 and 107 drainings on the cadmium copper silver. The oxidation curves for oil No. 105 have been referred to in Fig. 5. Oil No. 107 is similar to oil No. 208 of Figs. 6, 7, and 8. With the notable exception of oil 410, the drainings generally show more rapid corrosion than the new oil, which is to be expected. Exceptions need explanation. Oil 410 is a Diesel oil with additives. When the additives have been disintegrated, the oil assumes a more normal status. However, in the meantime, as the oil wears down to a normal condition, the engine must suffer. Such was actually the case. The engine using this oil showed excessive wear and was ready for a major overhaul

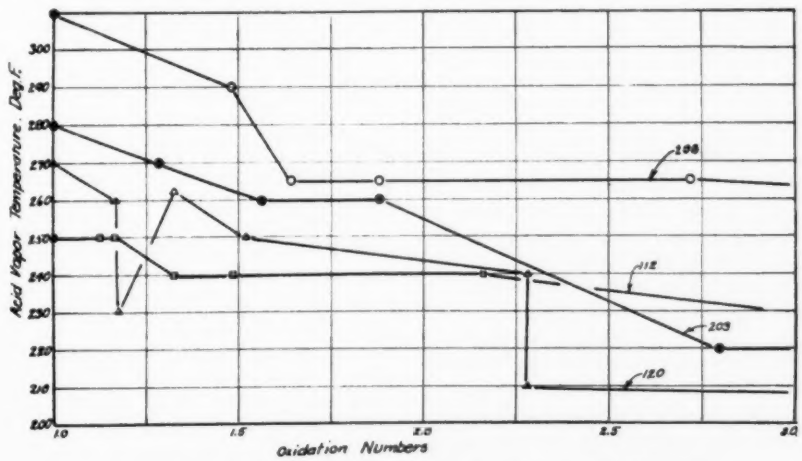


FIG. 8 ACID-VAPOR TEMPERATURES FOR LOW OXIDATION NUMBERS

in 450 hr. When oil No. 410 was replaced by oil No. 403, engine wear became negligible at least for the 450-hr period.

Loss in weight of the aluminum alloy was not anticipated. These losses are slight but they nevertheless exist, and lead to the question of possible corrosion with other metals used in the internal-combustion engine.

Drainings that show the lowest acid-vapor temperature are not necessarily the poorest oils, at least as far as the metals reported in these tests were concerned as in examples Nos. 107, 119, and 417.

It is quite probable that dilution has some bearing on the acid-vapor temperature and on the oxidation characteristics of the drainings. To the author's knowledge, no satisfactory method has been devised to remove dilution without the danger of changing the oil characteristics.

In conclusion, it is felt that the question of stability, or better, the lack of stability of crankcase oils has been only partially solved. Oxidation characteristics of the oils are the starting point from which to attack the problem. In this respect the performance of an oil can be predicted with only a fair degree of certainty. As the temperatures of engines go up, a temperature exponent may be introduced to modify the oxidation numbers. More information is needed concerning the catalytic action of metals on oil oxidation. In the final analysis, either the metals or the oils may be standardized; as a makeshift, oil A may be definitely recommended for engine x.

Oil breakdown is very definitely associated with a near-minimum, acid-vapor temperature, which in turn has its bad effects on at least some of the engine metals.

Formation of visible sludge is closely associated with oil breakdown and near-minimum, acid-vapor temperature. Evidence indicates that visible sludge is a warning to change the oil.

The acid-vapor problem throws an entirely new light on oil filters. Filters can be made to remove solids and some sludges; they can be chemical in action to neutralize acids that might exist in the oil. But these acid vapors which form more or less continuously at vital points in an engine never come near the oil filter.

As a final thought, perhaps the oil should be changed more frequently.

Discussion

GEORGE ROUND.[3] Every oil refiner is deeply interested in the subject of this paper and many dollars have been spent on it, but as yet without any conclusive answer being obtained. Whether the author has discovered the final answer with the particular oxidation test which he has developed is difficult to say, but we venture to hazard a guess that he himself would not be so bold as to say that it would be a substitute in every case for engine tests.

Most of the oxidation tests, which have been developed thus far, represent an attempt to predict, from a relatively short-time test, what is going to happen in an engine in a few hundred or a few thousand miles. The author's test takes less time than any other of which we know for making such a prediction. Any such accelerated test is always open to question even though in this instance the temperatures are not abnormal and below those of several oxidation-test procedures developed by others.

Before any test of this sort can be used with any degree of assurance whatsoever, it must be correlated with very extensive field and laboratory test work, particularly the former. In stating this, we wish to emphasize that it is the appearance and condition of the engine itself rather than that of the crankcase oil which counts. Hence, reliance on the analysis of crankcase-oil samples alone in judging the performance of oil is likely to be misleading. We must know what happens to the engine itself.

While it is possible to develop an oxidation test which will give reasonably consistent results on a single oil, the engine itself gives widely inconsistent results. For example, we could cite a case where a certain make of engine under certain operating conditions gave a great deal of trouble from lacquer deposits on certain parts, the engine and the filter otherwise being exceptionally clean. Laboratory oxidation tests indicated that the oil was exceptionally stable. When an attempt was made to duplicate the field condition in the laboratory, using the same make of engine and oil from the same containers, the engine ran perfectly and no sign of lacquer developed. Why this should be still remains a puzzle which apparently cannot be answered by any laboratory test, yet it presents for solution a very important commercial problem. If the author's oxidation test will give any clue toward answering the question of why the same oil gives satisfactory results in some engines and not in others, or in the same engine under one set of conditions and not in another, even under accelerated operating conditions, he has made a very valuable contribution to a lubrication problem.

There are some statements in the paper which we believe are not always consistent with the facts. For example, the author states, "the better engine, as would be expected, is more severe on the lubricating oil." This is not always the case. In fact, we find that in many instances, the oil deteriorates much less rapidly and the engine stays much cleaner when the rings are new and in good condition than they do when these parts have become worn. The answer appears to be in the fact that products of incomplete combustion and decomposition of the fuel get into the crankcase and contaminate the oil much more rapidly in worn engines than in those in good condition. Thus far it does not appear to be possible to differentiate between oxidation products from the fuel and from the lubricating oil.

The author speaks of oils containing fatty additives. We know of none being offered today for motorcar lubrication.

The author again refers to additives in Diesel oils as affecting oil performance in an adverse way. We wonder if he appreciates that it is impossible to lubricate satisfactorily certain makes of Diesel engines with oils not containing additives.

There is an implication in these references to additives which might lead one to feel that oils containing them are not all they should be. The facts of the matter are that with present and expected developments in both carburetor- and Diesel-type engines, it is going to be necessary to use additives of various sorts, because it is impossible to build into lubricating oils all of the characteristics that are required by these new engines.

T. H. ROGERS.[4] As indicated in his introduction, the author attempts to answer two questions:

1 How can the performance of an oil be predicted? A comparative test for color formation at 300 F is proposed as a measure of the performance but little or no data are given to show that this test correlates with performance of the oil.

2 When is an oil worn-out? The author at various points indicates limits of usability of oils based on the color-stability test, but provides no adequate basis for the limits selected.

It has long been appreciated by oil technologists that color formation is a qualitative indication of the oxidation behavior of lubricating oils, but few of them would agree that comparative color formation is an adequate quantitative measure of such behavior. In certain cases, for example, by the use of some inhibitors, this qualitative correlation is entirely upset. The oxidation of lubricating oil is of itself an extremely complicated

[3] Socony-Vacuum Oil Company, New York, N. Y.

[4] Standard Oil Company of Indiana, Whiting, Ind.

phenomenon, aside from the varied conditions involved in the service of such oils. We definitely question whether a brief color-stability test, under comparatively mild conditions, properly takes account of the situation. We note that the author states that his tests correlate with the Indiana oxidation test. However, the comparison made was with one single point on the asphaltene-time curve afforded by the Indiana test. In presenting the Indiana test and in our subsequent use of it, we have stressed that a single point on this curve is not a complete measure of the oxidation behavior of the oil, and we feel that various other aspects of the oxidation behavior are necessary to evaluate the oil completely.

To summarize briefly the status of the problem of the stability of crankcase oils, we would say that the following aspects are apparent in terms of the performance behavior of the oil: (*a*) Asphaltene formation, (*b*) viscosity increase, (*c*) corrosiveness toward certain metals, and (*d*) varnish formation.

These several aspects of oil oxidation or deterioration require separate consideration from the performance standpoint, and variations in the conditions of use will change the emphasis which is to be placed upon the different aspects. Furthermore, from the standpoint of oxidation tests, it is definitely known that these four aspects of oil oxidation do not correlate with each other; that is to say, it is not possible to measure asphaltene formation (or any other one feature) and then predict any one of the other three. No one of the aspects of oxidation can be used as a single criterion. Also, it has not yet been found possible to study the problem of corrosiveness by measuring the amount and strength of acids, but actual corrosive behavior toward metals is necessary.

The oxidation number, based on color development, used by the author, is not a real measure of any one of the previously mentioned aspects, but may be considered to be somewhat related to (*a*). On the basis of these considerations, it must be said that the calculations of the author as to per cent oxidation and the limits of usability of oil based on the color-stability test seem quite arbitrary. The observations of the author regarding the vapor temperature of acidity development are interesting. It would appear to be desirable to correlate these with actual corrosion tests.

H. M. Rugg.[5] For nearly two years, the Pennsylvania Grade Crude Oil Association has been conducting a very intensive and fundamental study of oxidation. The results of this study seem to disprove the statements that the extent to which oxidation has progressed may be predicted by the change of color density in the oxidized oil, and also that the grade of oil is no index of its resistance to oxidation.

In this research work, carefully prepared fractions of oil of varying molecular structure are studied to obtain variations in their susceptibility to the absorption of oxygen. To avoid the effect of other variables, pure oxygen has been used, thus, at the same time, accelerating the tests so that the results are equivalent to many times the expected use of an oil. Even though the Pennsylvania region has been manufacturing and studying petroleum products for over 75 years, much new knowledge is being developed in this research. Analysis of these oxidized fractions shows that the process of oxidation is a very complicated one. Data are obtained showing the distribution of oxygen into such products as carbon monoxide, carbon dioxide, water, volatile and fixed acids, and sludge, the sludge being further divided into lacquer-type, oil-soluble, and naphtha-insoluble fractions. The different fractions show varying tendencies in the absorption of oxygen; also, temperature vitally affects these tendencies.

These points are mentioned to show that, where such large differences occur in carefully controlled laboratory tests, any attempt to make quick analyses of oils used under the promiscuous operating conditions of an engine is simply futile.

Then there are the differences in the lubricating oils themselves which will undoubtedly make a color comparison of oxidation unreliable. This is probably why, in Fig. 1 of the paper, the oxidation rates of different oils seem to cross. With differences in crude source, where quite wide variation in oxidation stability occurs, differences in process of manufacture, conventionally refined, solvent-refined, long residuum oils, together with blends of these products, color comparisons will be of little value for oxidation data. We can find heavily oxidized oils light in color that are unfit for use and, in contrast, oils dark in color that are not oxidized to harmful degree.

A study of present-day engine designs, as well as future tendencies, and a study of the practices of a majority of automotive-vehicle owners and users indicate very plainly that a return to the former recommendations for frequent draining and occasional flushing will, in the end, be less expensive and give less trouble, notwithstanding the development of any special tests such as the author has described.

The automotive industry wants and receives from petroleum something that will burn freely and clean, producing all possible power. This is accomplished by uniting it chemically with oxygen. In other words a high-grade fuel is required. At the same time, it happens that this same medium, petroleum, has proved that a part of it makes a good lubricant. Therefore, it is required to take from this medium something else which will resist the reaction with oxygen, will not burn or oxidize, when such a reaction is really both natural and potent.

The petroleum industry is doing and has done a very good job in meeting these conditions but the users of these products must take into consideration the fact that petroleum, like all other materials prepared from matter, has certain physical limitations. The petroleum industry has no data, as yet, on what the "factor of safety" is in lubricants. For that reason, how do we know how much it will resist oxidation or what the thin film of oil on the cylinder wall will do? We do not like to see engine builders or designers go to the point of pressing that film until the thickness has to be measured in molecules rather than fractions of an inch, and the circulation drastically reduced to obtain "oil economy," then expect it to withstand the high temperatures and pressures of present-day designs.

A great deal of work has been done with additives, and many jobs seem to require them. However, all will agree that, where additives have been used to improve or correct one condition, almost invariably other difficulties have been encountered. We are not convinced that additives are absolutely necessary, but believe that, through our research studies, it will be possible to produce the proper qualities, namely, oxidation resistance, corrosion resistance, etc., in the lubricating oil by the method of processing it, and avoid the worries and troubles resulting from the use of chemical nonpetroleum additives.

E. T. Vincent.[6] The writer would like to indicate something of the engine-users' points of view, with the purpose of establishing some relation between the oxidation tests and actual performance. We had a problem some time ago involving an air-cooled aircraft engine in which considerable trouble from gum and carbon formations was experienced. One of the regular oxidation tests was tried. Unfortunately, the oil which performed best in the engine, giving almost no gum, was practically a tarry mass soon after the oxidation tests had started. It is therefore believed that engine tests should be conducted at the

[5] Pennsylvania Grade Crude Oil Association, Oil City, Pa.

[6] University of Michigan, Ann Arbor, Mich.

same time to establish some basis of comparison with the oxidation tests, because apparently they do not correlate.

The question of temperature is often raised in connection with oil engines. Everyone says the Diesel engine is hot, and that is why we get so much gum. The writer does not think that is the case, for several reasons. Jacket losses are much lower than with the gasoline engine, therefore, the temperature difference between the inside and the outside of the cylinder oil film must be lower. It is well known that heat flow depends upon temperature difference so, if the heat flow is less, temperature difference is less. A gasoline engine, comparable in size to a given Diesel engine, has higher piston temperatures. Therefore, the question is not wholly one of temperatures.

An experience of the writer's some years ago attests to this fact. While testing a completely new engine, using a high-grade lubricating oil, we were motoring it by means of the dynamometer, with no fuel in the engine. Therefore, the temperature differences were extremely low; just whatever the jacket-water temperature happened to be, due to the compression. We motored the engine for about 4 hr and then went to lunch. On returning, the engine could not be turned around. The gum formation which had occurred in 4 hr was so great that, when the engine stopped, it was actually stuck together. We used solvents to get the engine dismantled.

That same engine, with the same lubricating oil, was then put on fuel. With fuel oil and with higher temperatures, running on load, gum formation did not occur. Therefore, we can definitely rule that temperature is not the only cause of such formation. (The engine was of the sleeve-valve type and probably sensitive to blow-by, until the sleeves and rings were sealed in.)

In the writer's opinion oxygen content is the chief cause of gum formation. In the Diesel engine, when running on load, there is an excess of air which the gasoline engine does not have. The gasoline engine is usually running on a rich mixture with practically no oxygen at all in the blow-by gas. That consideration should be taken into account when dealing with Diesel engines.

L. A. Blanc.[7] In corroboration of Professor Vincent's opinion about oxygen in the blow-by, we have analyzed a number of blow-by gases on Diesel engines and have found that in all cases oxygen is never less than 20 per cent, regardless of load on the engine, and over a wide range of blow-by.

Author's Closure

There seems to be some misunderstanding in regard to color change in the oil. It will be noted that the paper did not say "color change" but rather considers and measures oxidation as a change in density of a given color, as compared with the original oil.

In regard to life of the oil, tests up to 12 and 24 hr have been run on oils of Fig. 6, since this paper was written. Oxidation of oil No. 208 continues approximately as a straight line until at 12 hr the oxidation number is 13.5 and, at 24 hr, it is 31.

Oil No. 120 seems to be quite good up to about 9 hr, when oxidation breaks sharply to give 12.8 at 12 hr and 90 at 24 hr. Visible sludge begins to form between 9 and 12 hr. This is a built-up oil which seems to be quite stable until the additives begin to break down. Thus, in predicting the useful life of an oil from short-time oxidation tests, the prediction holds fairly well, except for oils with additives, when longer oxidation periods must be used.

A study of oxidation rates and acid-vapor-formation temperatures in the presence of metallic catalysts should give some information relative to the effect of additives. It is true that some additives will perform well in certain engines but poorly in others. Reference is made to an oil not reported in the paper. This oil is built up especially for engines containing copper-base bearing metals. When this oil is oxidized in the presence of copper, very little change occurs, but other metals such as mild steel, cast iron, aluminum, and babbitt, all act as powerful catalysts to accelerate oxidation and reduce the acid-vapor temperature. Tests show that metallic catalysts, as a general rule, will lower the acid-vapor-formation temperature a good deal faster than the oxidation number increases. It is believed that tests in the presence of catalysts will in part answer some of the discrepancies which so often occur in engine tests, as well as in laboratory tests, and help to enumerate more of the variables that affect oil performance.

A question was raised by Mr. Round relative to performance of oil in the better engine. The term "better" refers to engine design, as both engines were relatively new and operating conditions were similar. The effective compression ratio of one of these engines was much below the nominal because of relatively poor valve timing.

It appears that Mr. Rugg contradicts his own statement that grade of an oil is an index of its resistance to oxidation. The statement is true for close-cut oils, such as in Fig. 3, but for the general run of oils on the market, viscosity cannot be considered an index of resistance to oxidation (refer to Figs. 1 and 2 of the paper).

The feasibility of predicting the useful life of an oil from these oxidation tests was questioned by several discussers. Their attention is called to the last column of Table 3, listing the oxidation number at which visible sludge appears, and to the subsequent section "Predicting Oil Performance." Furthermore, the appearance of visible sludge is very closely associated with the minimum acid-vapor-formation temperature which appears to be just as important as the oxidation number, as a gage to determine the useful life of an oil.

It is not necessary to assume blow-by as a specific contributor to oxidation. The air in the crankcase is beaten into the oil during operation and, upon subsequent heating, oil oxidation results. Table 1 shows that, in general, oil deteriorates faster when it is saturated with air.

In conclusion, a point was raised in regard to the effect of pressure on oil breakdown. A few tests have been made in an extreme-pressure machine. As an example, it was found that for oil No. 208, 6-min operation in the extreme-pressure machine was equivalent to 30-min oxidation, both as regards oxidation number and acid-vapor temperature.

[7] Caterpillar Tractor Company, Peoria, Ill.

Whip Stress in a Locomotive Main Rod at 100 Miles per Hour

By M. W. DAVIDSON,[1] VERMILLION, S. DAK.

Older railway motive-power units are being operated at, and new equipment is being designed for, higher speed than has prevailed in the recent past. The paper presents a precise method for the determination of inertia flexural, or whip, stress in locomotive main rods.

One hundred miles per hour is the speed that is chosen for illustrating the method, and for exhibiting the stresses which are developed; since at the present time that seems to be the general operating goal for fast rail traffic. Also, speeds of this order are frequently attained by the main rod and the mechanism in general when locomotives are slipping.

Accurate equations for the inertia forces and couples are obtained for use in determining the inertia reactions and loads. These equations are of universal applicability to all the parts and fractional parts of the mechanism.

The rod bending moments and accompanying stresses were determined through the use of the external wrist-pin reactions in combination with the distributed rod-inertia loading. The moments and stresses were found for six or seven sections along the body of the rod for each of twelve crank positions spaced at 15-deg intervals. Finally the total tensile unit stress is found for each section and crank position, by combining that which is due to whip, with that which results from rod axial-inertia forces and the force of the steam.

RAILWAY TRAINS which operate at sustained speeds approaching a hundred miles per hour are rapidly increasing in number. Because of this trend it has interested the writer to make something of a detailed study of the flexural or "whip" stress which develops in a conventional locomotive I-section main rod when the locomotive is running at 100 mph, and also to follow this with an examination of the total tensile stress which results when the whip stress is combined with that due to the inertia of the reciprocating parts, the fraction of the main rod itself, the steam force, and the component of the normal guide reaction which is due to the inertia of the rod.

In order to make this study accurate, it seemed desirable to employ accurate characteristic expressions, which apply to the slide-crank mechanism for the determination of its inertia forces and couples. These characteristic expressions may readily be obtained by the use of the differential calculus for the construction of formulas which give the linear acceleration of a point on the axis of the rod, and the angular acceleration of the rod. The procedure is first to express in terms of the crank angle the displacements of a point on the rod axis from its zero-displacement or dead-center position, both along the engine axis, and transverse to that axis. Then the second derivative of displacement with respect to time for each of the two expressions will give, respectively, expressions for the point's axial and transverse accelerations, and the product of these latter by the rod's mass will give the expressions for the corresponding components of its linear-inertia force. In like manner the angular acceleration of the rod may be obtained, if the same treatment be given the expression for the angular displacement. The product of this acceleration by the rod's moment of inertia about its mass-center axis yields the expression for the angular-inertia torque or couple about that axis. With these formulas the dynamic effect of the rod as a whole, or of any part of the rod may be completely determined.

The form of the expressions used herein departs from the conventional, in that the variables are expressed as lengths of the mechanism diagram, rather than as functions of the crank angle. These forms are thought to lend themselves conveniently to the determination of the desired quantities either by simple computation or by diagram scaling.

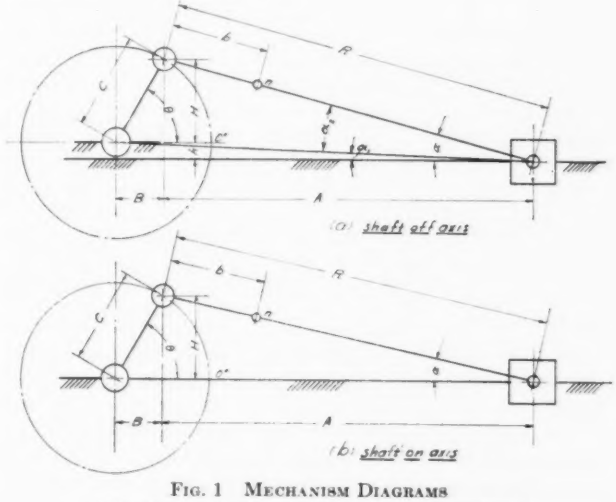


FIG. 1 MECHANISM DIAGRAMS
(The stroke-end value of α_1 is used in the equations.)

Fig. 1*a* represents the arrangement of the slide and crank with the crank center offset from the path of reciprocation, and Fig. 1*b* the usual arrangement with the crank center on that path. In addition to the quantities shown in Fig. 1, where all distances are in feet, the following notation is used:

- F_A = the horizontal or axial component of the linear-inertia force of any mass of which the mass center is at any point n in the axis of the connecting rod, lb
- F_v = vertical or transverse component of the same force, lb
- T = the angular-inertia couple about its mass center of any mass of which the mass center lies in the axis of the connecting rod, lb-ft
- V = linear velocity of crank-pin center, fps
- W = weight of the accelerated mass, lb
- g = acceleration due to gravity, ft per sec per sec
- π = the constant 3.1416
- N = number of revolutions per sec

[1] Professor of Applied Science, University of South Dakota. Mem. A.S.M.E.

Contributed by the Railroad Division and presented at the Annual Meeting, Philadelphia, Pa., Dec. 4–8, 1939, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

I = moment of inertia of the accelerated mass about the mass-center axis which axis is perpendicular to the plane of motion of the connecting rod, slug-ft²

The desired equations are[2]

$$F_h = \frac{WV^2}{gC^2}\left[B + \frac{b}{R}\left(\frac{R^2B^2}{A^3} - \frac{H^2}{A} - \frac{kH}{A}\right)\right] \quad \text{Fig. 1}a\ [1]$$

$$F_h = \frac{WV^2}{gC^2}\left[B + \frac{b}{R}\left(\frac{R^2B^2}{A^3} - \frac{H^2}{A}\right)\right] \quad \text{Fig. 1}b\ [2]$$

$$F_v = \frac{WV^2}{gC^2}\left[H\left(1 - \frac{b}{R}\right)\right] \quad \text{Figs. 1}a, 1b\ [3]$$

$$T = 4\pi^2N^2I\left[\frac{H}{A^3}(R^2 - C^2) - \frac{kB^2}{A^3}\right] \quad \text{Fig. 1}a\ [4]$$

$$T = 4\pi^2N^2I(R^2 - C^2)H/A^3 \quad \text{Fig. 1}b\ [5]$$

In applying these equations, distances to the right and left of the crank axis are respectively plus and minus, and those above and below that axis are likewise respectively plus and minus.

Examination of the equations shows that if b, the distance from the crank-pin center of any point n in the rod's axis be made zero, then n becomes the center of the crank pin, and F_h and F_v are simply the axial and transverse components of the centrifugal-inertia force of a mass centered at that point. If b be made equal to R, then n becomes the wrist-pin center, and F_h will give the axial-inertia force of any mass, such as the reciprocating weights, which may be centered there, and F_v will become zero as there is no transverse acceleration of the mass so located. If b be taken as the distance to the rod's mass center, then n will be its mass center, and F_h and F_v will give, respectively, the axial and transverse components of the linear-inertia force of the connecting rod as a mass unit, and T will give its angular-inertia torque about the mass-center axis which is perpendicular to its plane of motion. Also if b be taken for *any* point in the rod, the axial and transverse components of the linear-inertia force and the angular-inertia torque of a unit mass so centered may be determined accurately by the use of the equations.

The Problem

The particular connecting rod, reciprocating weights, general dimensions, and quantities used in the study are from a class F-3, Chicago, Milwaukee, St. Paul and Pacific locomotive, having cylinders 23 in. in diameter and 28 in. stroke; driving wheels 79 in. in diameter; reciprocating parts weighing 1126 lb; I-section connecting rod, 116½ in. between wrist-pin and crank-pin centers, and weighing 877 lb. The mass center of the rod with journal brasses in place was found at 38 in. and 78½ in., respectively, from the crank-pin and wrist-pin centers. The moment of inertia of the complete rod about its mass-center axis was experimentally determined by observing its time of oscillation as a compound pendulum when suspended upon a knife-edge support set parallel to the wrist-pin axis and which made contact with the outer wrist-pin brass along its extreme outer bearing element. By checking the results obtained with geometrically shaped masses with the results obtained with the integral calculus for the same masses, the author has found this method for determining mass moment of inertia very accurate when carefully performed. The value of the rod's moment of inertia first obtained for the axis of suspension and then transferred to the parallel gravity axis gave a value about the latter axis of 405 slug-ft².

[2] The Appendix shows the derivation of the equations for the case of Fig. 1*b*.

TABLE 1 PRINCIPAL DATA

θ, deg	H, ft	B, ft	A, ft	A^3	$\frac{H^2}{A}$	$\frac{R^2B^2}{A^3}$	$\frac{H}{A^3}$
360 – 0	0	1.167	9.71	916	0	0.1210	0
330 – 30	∓ 0.5833	1.010	9.70	912	0.0351	0.1053	0.0006390
315 – 45	∓ 0.8250	0.8250	9.675	904	0.0703	0.0709	0.0009125
300 – 60	∓ 1.010	0.5833	9.650	899	0.1058	0.0361	0.0011230
285 – 75	∓ 1.127	0.3025	9.633	894	0.1320	0.00963	0.0012600
270 – 90	∓ 1.167	0	9.630	893	0.1412	0	0.0013060
255–105	∓ 1.127	−0.3025	9.633	894	0.1320	0.00963	0.0012600

C = 1.167 ft, C^2 = 1.36, R = 9.71 ft, R^2 = 94.25, R^2C^2 = 92.89, V = 52 fps, V^2 = 2704, ω = 44.55 radians per sec, V^2/gC^2 = 61.65, N = 7.09 rps, N^2 = 50.3, W_{rod} = 877 lb, I_{rod} = 405 slug-ft², g = 32.16 ft/sec²

Rod section, 1 in. long	Area, sq in.	I-section, in.⁴	I_{mass}, slug-ft²	Weight, lb	b, ft	$\frac{b}{R}$	$\frac{1}{\text{Sec mod}}$
a	36	243	0.0149	10.2	10.156	1.047	....
b	36	243	0.0149	10.2	8.948	0.9225	....
c	10.54	54.24	0.00332	2.99	8.375	0.8630	0.05585
d	10.83	61.57	0.00377	3.07	5.542	0.5710	0.05225
e	11.07	68.45	0.00418	3.14	3.167	0.3263	0.04930
f	11.29	78.68	0.00482	3.20	1.167	0.1202	0.04486
p	15.00	195	0.01430	4.25	9.710	1	
k					7.208	0.7450	0.05500

The principal data for substitution in the equations for force and torque are given in Table 1. The friction effect upon the well-lubricated mechanism is relatively slight, and was considered to be zero. With this assumption the reaction of the crosshead guides at the wrist-pin end of the rod is normal to the path of reciprocation.

Using Equations [2], [3], and [5] for each 15 deg of crank angle, the data of Figs. 2, 3, 4, and 5 were computed. These give, respectively, the values for the rod's angular inertia in

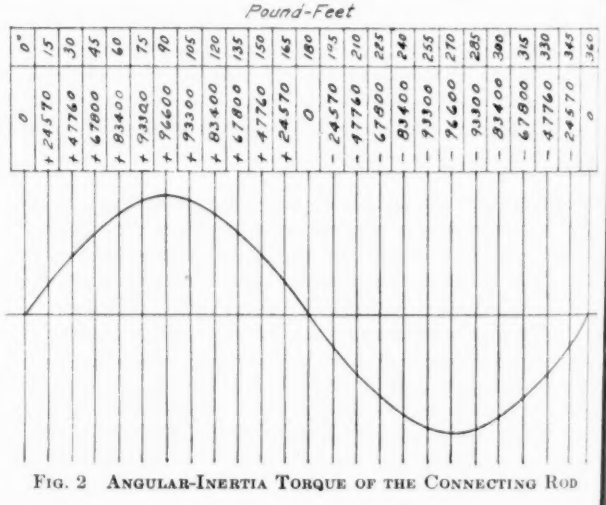


Fig. 2 Angular-Inertia Torque of the Connecting Rod

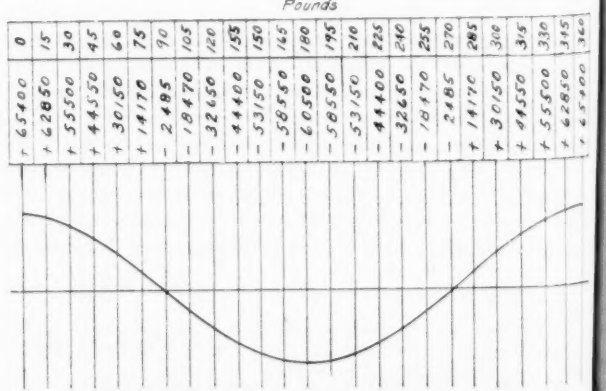


Fig. 3 The Axial Component of the Linear Inertia of the Connecting Rod

pound-feet, the component of its linear-inertia force parallel to the reciprocating path, the component of its linear-inertia force transverse to the reciprocating path, and the linear-inertia force of the reciprocating mass, all in pounds. Fig. 6 represents the net steam force for driving the piston, as obtained from a locomotive indicator card taken when the locomotive was running at high speed, though not at 100 mph.

Method for Determination of Whip Stress

The flexural or whip stresses which develop in the rod because of its inertia can be determined at any point in the rod and for any position in its motion cycle when the external bearing reactions and the distributed load due to its own inertia are known.

The d'Alembert principle was used as illustrated in Fig. 7 to determine the bearing reactions at twelve angular positions of the crank. The positions chosen were those which range through the arc which contains the position of maximum stress,

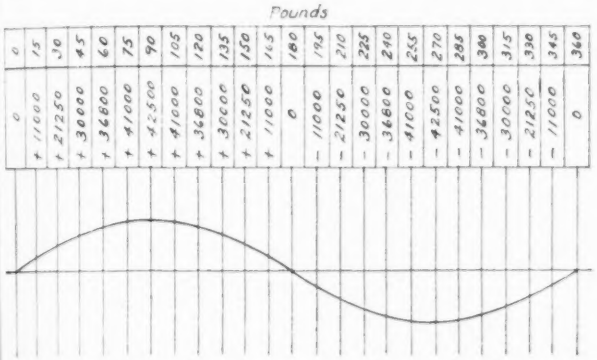


Fig. 4 The Cross Component of the Linear Inertia of the Connecting Rod

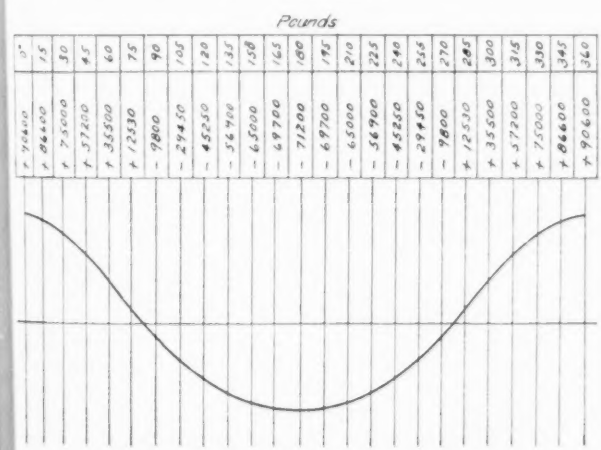


Fig. 5 Inertia of the Reciprocating Mass

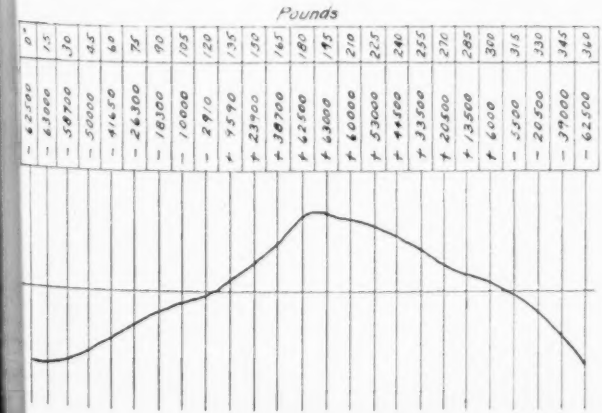


Fig 6 Effective Steam Force

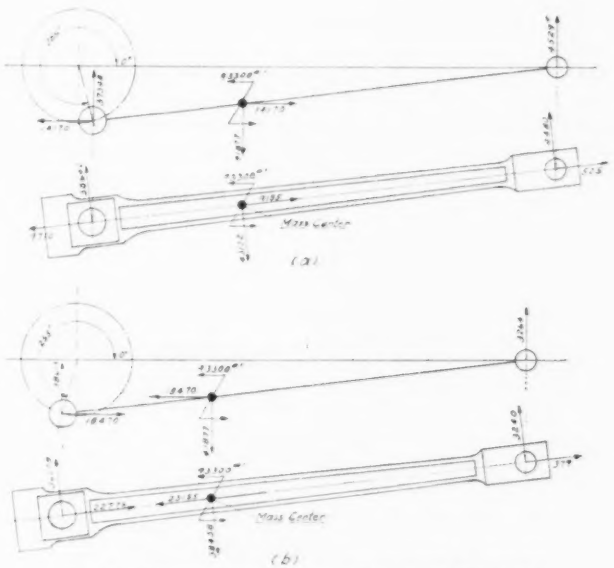


Fig. 7 Bearing Reactions, d'Alembert Principle
(Forces in pounds, couples in pound-feet.)

both when the rod is above and below the engine axis. These crank positions were spaced 15 deg apart and symmetrically with respect to that axis. This figure shows the application of the components of the rod's linear inertia at the mass center, the application of the angular-inertia torque for the mass-center axis, and the resulting reactions at the crank-pin and wrist-pin bearings when transformed into components which are parallel and perpendicular to the rod's axis.

In order to determine the manner of distribution of the axial- and transverse-inertia loads, beginning at the wrist-pin end of the rod the values of F_A, F_v, and T at seven points along the rod were determined for sections of the rod 1 in. in length. Using the first two of these values, the axial and transverse components of the 1-in. sections were found. The former for use in finding the rod's tensile effect upon itself, and the latter for determining the distribution of the transverse flexural load. The angular-inertia couples of the 1-in. sections have cumulatively very little flexural effect, but to make the study complete they were determined and used.

Fig. 8 shows the locations of the 1-in. sections for which the determinations were made, and gives the values as first obtained by the equations, and as transformed into components parallel and perpendicular to the rod's axis for the 285-deg position. As the weight of the rod has a slight effect upon the external reactions at the bearings, and also upon the distributed load, this was added to the inertia load, as it should be for rod positions below the engine axis. However, for rod positions above the axis the weight should be subtracted. And to that small extent the results which are correctly found for the lower positions are in error for those above.

Using the transverse unit loads, the six diagrams of transverse-inertia load were determined, Fig. 9; with the axial unit loads the four diagrams of axial-inertia load, Fig. 10, were constructed; and with the angular unit torques or couples the six angular-

load diagrams of Fig. 11 were determined. The angular positions called for on all of these diagrams refer to the angle θ in Fig. 1*b*.

As already stated, the sections chosen for the unit-load determinations are shown in the various figures, and it should be noted that section *e* represents the mass-center position of the rod. Four points were chosen along the I-section body of the rod, and three in the enlarged wrist-pin end. The load distribution over the body of the rod is accurately represented by the diagrams, and the section loads of the wrist-pin end are also accurate. Still the diagram at the end is not accurate, since the outline of this part of the load is formed by connecting with straight lines the points where the unit determinations are made. However, it seems to the author that the degree of accuracy employed here is ample when consideration is given

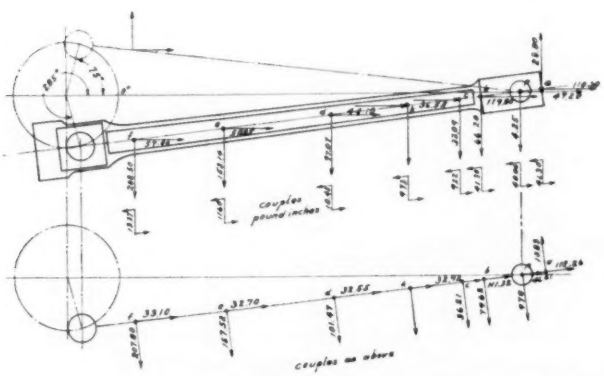


FIG. 8 INERTIA FORCES AND TORQUES FOR SECTIONS OF CONNECTING ROD 1 IN. LONG

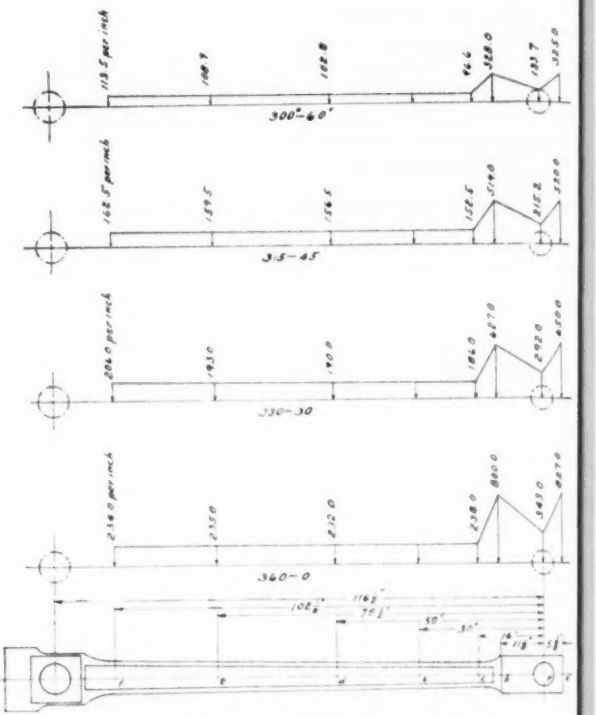
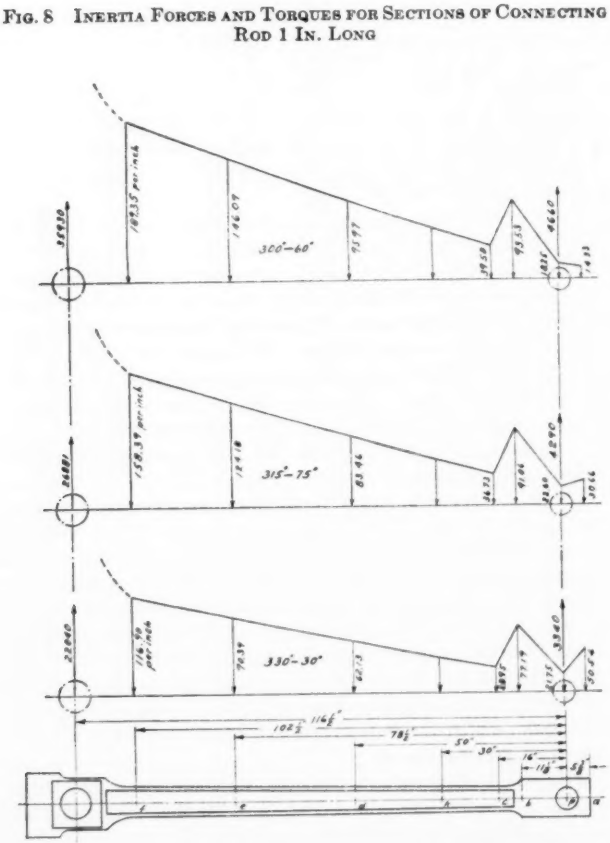


FIG. 10 DIAGRAMS OF AXIAL-INERTIA LOAD



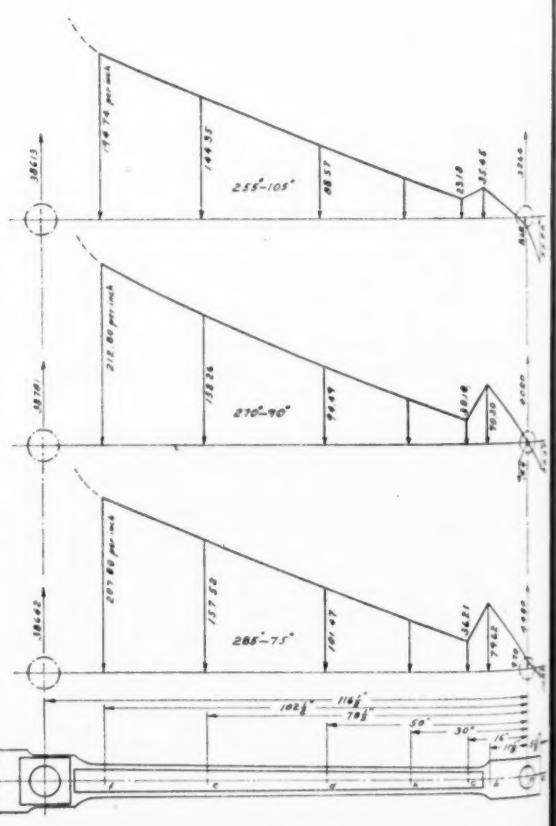


FIG. 9 DIAGRAMS OF TRANSVERSE-INERTIA LOAD

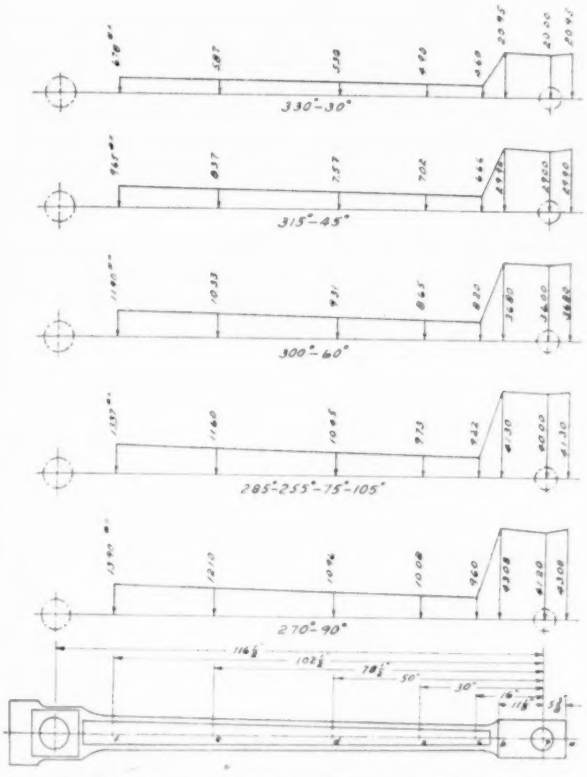


FIG. 11 DIAGRAMS OF INERTIA TORQUE OF 1-IN. SECTIONS

to the slight effect any further accuracy would have upon the results obtained. No determination of the inertia load of the crank-pin end of the rod was made, since this was not needed for finding the bending moments.

Referring to the various illustrations thus far given, and particularly to the diagrams of Fig. 9, it will be seen that the bending moment at any section of the rod will result from the combined effect of the wrist-pin reaction which is transverse to the rod, the action of the distributed transverse load, and the small cumulative effect of the torques of the individual 1-in. sections which are exerted about their individual mass-center axes.

In order to determine the magnitude of the transverse load from the rod end to the point of stress determination, the diagram area was found by planimeter. The center of pressure of the distributed load was found by carefully tracing the diagram upon drawing paper, cutting this out, and balancing over a knife-edge set perpendicular to the axis of the rod. When balance was obtained, the line of balance was accurately and quickly located by smartly striking the balanced paper load over the knife-edge with a weight.

The method of finding the bending moment is illustrated as follows: It will be seen in Fig. 9 for the 255- and 105-deg positions, that the transverse force at the wrist pin is 3264 lb upward. It is desired to find the bending moment at point *f* which is 102.5 in. from the wrist-pin center. The distributed load to this point is composed of a downward load which extends from point *f* almost to the wrist-pin center, and a small double-triangular upward load extending from the end of the downward load to the rod end at point *a*. The downward load amounts to 9380 lb, and its center of pressure is $33^1/_2$ in. to the right of section or point *f*. The small upward load amounts to 160 lb, and its center of pressure is 104.8 in. to the right of section *f*. From Fig. 11 the cumulative angular-inertia torques of the 1-in. sections total to point *f* 1783 lb-in., and these act to augment the moment resulting from the external reaction force of 3264 lb. Then the bending moment at section *f* will be represented by the following equation

$$M_f = 3264 \times 102.5 + 1783 + 160 \times 104.8 - 9380 \times 33.5 = 39{,}133 \text{ lb-in.}$$

After the bending moments at the various sections along the body of the rod for each of the six double-angular positions were found in this manner, the flexural or whip unit fiber stresses were obtained in the usual manner by multiplying each moment by the reciprocal of the corresponding section modulus. These values are plotted as ordinates for the six curves of Fig. 12. The maximum fiber stress and its position in the rod was then noted

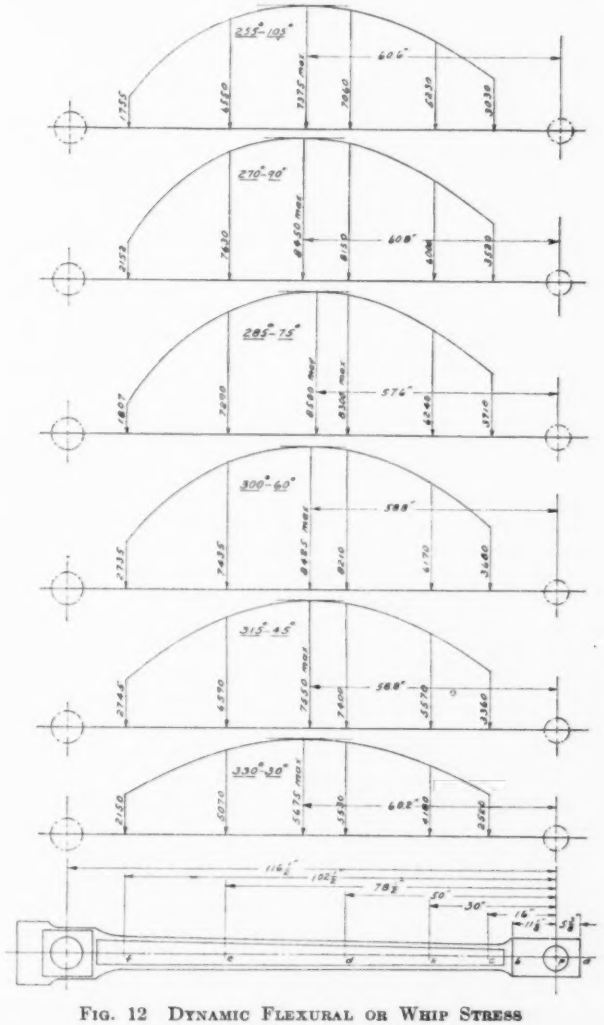


FIG. 12 DYNAMIC FLEXURAL OR WHIP STRESS
(Pounds per square inch.)

for each of the angular positions by carefully drawing tangents to the tops of the curves. These maximum values were then plotted as the ordinates of the curve shown in Fig. 13. The maximum ordinate of Fig. 13 should represent the maximum unit flexural fiber stress developed in the rod and with some accuracy give the precise crank angle of its occurrence.

It will be noted that the maximum flexural stress due to inertia for every angular position is to be found at about 60 in. from the wrist-pin center, that the maximum stress determined is 8500

psi at the 285- and 75-deg positions, and that this is very close to the maximum value as shown by the maximum curve of Fig. 13. Also with the exception of the slight difference due to the weight of the rod which has already been mentioned, it is seen that with the engine running forward the fiber stress on the under side of the rod varies from a maximum compressive flexural unit stress of 8500 psi at the 75-deg position to a maximum

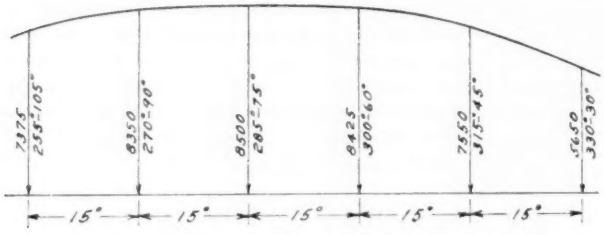


FIG. 13 CURVE OF MAXIMUM UNIT WHIP STRESS
(Pounds per square inch.)

tensile flexural unit stress of like value at the 285-deg position, thus giving a movement of 150 deg as the least space for the reversal.

THE COMBINED TENSILE STRESS

To determine the total tensile stress in the connecting rod, the tensile effect of the inertia of the fraction of the rod on the wrist-pin side of the stressed section, the tensile effect of the inertia of the reciprocating parts, the tensile effect of the steam force, and the tensile component of the normal guide reaction which is due to the inertia of the rod, all must be combined with that due to whip.

A study of these elements seemed to show that the combined stress reaches a maximum within the angular space between 300 and 360 deg. Therefore the rod-fraction axial-inertia diagrams of Fig. 10 for the angular positions 300, 315, 330, and 360 deg were constructed from the axial-inertia-force components of the 1-in. sections previously described. This element of the total force at any section is represented by the diagram area extending to that section. The tensile effect due to the inertia of the reciprocating mass was found by the use of the values of Fig. 5, the tensile effect of the steam force was obtained from Fig. 6, and the tensile effect of the rod-inertia reaction at the guides from the values as illustrated in Fig. 7. The tensile stress due to these forces was then combined with that due to whip for the four angles given, and the unit stresses resulting from the combination are shown by the curves of Fig. 14.

Examination of Fig. 14 shows that the combined stress reaches a maximum of about 13,650 psi at the crank positions of 45 and 315 deg, and at a distance approximately 61 in. from the wrist-pin center. However, for this value to hold for the 315-deg position, the engine would have to be in reverse, since for forward running the steam force would act to place the rod in compression. But though the position of the steam force with respect to the sides of the piston varies with the direction of rotation for any specified angular movement of the crank, thus causing either tension or compression in the connecting rod, the inertia forces do not thus vary. Irrespective of the direction of rotation, the horizontal-inertia effect of the connecting rod and reciprocating masses during the major part of the cylinder-side half of the revolution of the crank is to subject the connecting rod to tensile stress. For during the completion of most of the half of the stroke which ends at the forward dead center these masses are being decelerated, and throughout the first part of the succeeding stroke they are being accelerated. The force for this alternating acceleration and deceleration must be supplied at the crank pin, when not supplied by the fluid pressure.

In Fig. 15, the maximum values of the combined stress for the four angular positions are plotted as ordinates for a curve of maximum combined unit stress. And here again it appears that at about the 45- and 315-deg positions the combined stress is a maximum. Also it should be noted that the combined stress occurs not far from the point in the rod where the whip stress is maximum.

For the 360-deg position the combined stress was determined for the conditions "steam on" and "steam off with no compression" as shown for that position in Fig. 15. The values obtained

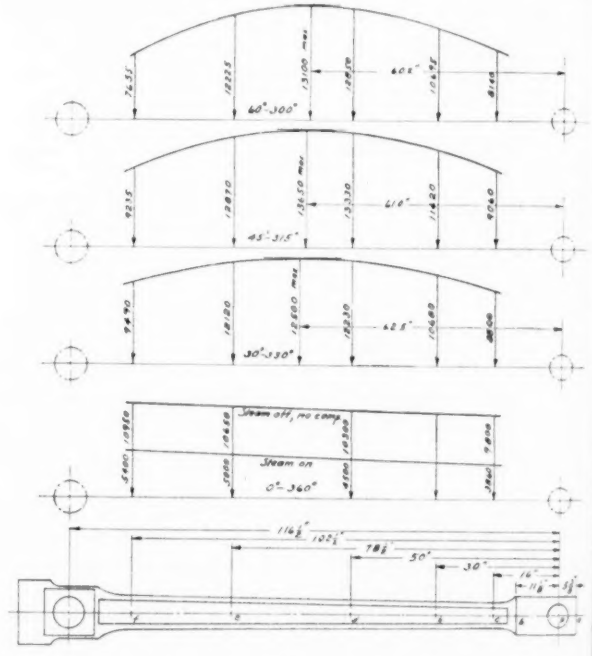


FIG. 14 DIAGRAMS OF COMBINED UNIT TENSILE STRESS

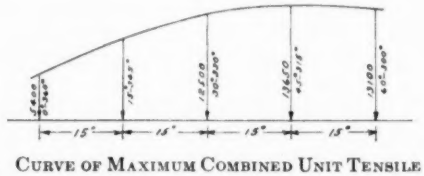


FIG. 15 CURVE OF MAXIMUM COMBINED UNIT TENSILE STRESS

are 5400 psi when operating with steam on, and 10,950 psi with steam off. The latter condition occurs when the locomotive is drifting with cylinder cocks open.

CONCLUSION

The results of the study seem to show that for 100 mph the maximum whip stress that can be expected in a conventional I-section locomotive main rod is of the order of 8500 psi, and that the maximum combined tensile stress will be of the order of 14,000 psi.

As the elastic limit of the usual carbon steel of which these rods are made is about 48,000 psi, the factor of safety with this as base is about 3.5. If $3\frac{1}{2}$ per cent nickel steel of the same carbon content were used for the rod material, the elastic limit would be about 60,000 psi and the factor of safety with this as base would be 4.38.

Appendix

Derivation of the Equations for Forces and Torques Given in the Paper

In addition to the notation given in Fig. 1*b* and the paper the following symbols are used:

Subscripts h and v refer to components in the plane of motion parallel and perpendicular, respectively, to the line of reciprocation

s = displacement of any point n of the connecting-rod axis from its dead-center position, ft

v = linear velocity of any point n, ft

ω = angular velocity of the crank, radians per sec

a = linear acceleration of any point n, ft/sec²

α = angular displacement of connecting rod from dead center, radians

ω_R = angular velocity of connecting rod, radians per sec

ϕ = angular acceleration of connecting rod, radians/sec²

The horizontal displacement of any point n of the connecting-rod axis from its dead-center position is

$$s_h = C + b - C\cos\theta - b\cos\alpha = C + b - (b/R)A - C\cos\theta$$

where $A = (R^2 - C^2\sin^2\theta)^{1/2}$

$$\therefore\ s_h = C + b - C\cos\theta - (b/R)(R^2 - C^2\sin^2\theta)^{1/2} \ldots [6]$$

The horizontal component of the linear velocity of n is

$$v_h = \frac{ds_h}{dt} = C\sin\theta\frac{d\theta}{dt} + \frac{b}{R}\frac{C^2\sin\theta\cos\theta}{(R^2 - C^2\sin^2\theta)^{1/2}}\frac{d\theta}{dt}$$

Since $d\theta/dt = \omega = V/C$ where V is the linear velocity of the crank-pin center

$$v_h = V\left\{\sin\theta + \frac{Cb}{R}\left[\frac{\sin\theta\cos\theta}{(R^2 - C^2\sin^2\theta)^{1/2}}\right]\right\} \ldots\ldots [7]$$

The horizontal component of linear acceleration of n is

$$a_h = \frac{dv_h}{dt} = V\left\{\cos\theta\frac{d\theta}{dt} + \frac{Cb}{R}\left[\frac{(R^2 - C^2\sin^2\theta)^{1/2}(\cos^2\theta - \sin^2\theta)(d\theta/dt)}{R^2 - C^2\sin^2\theta} + \frac{(\sin\theta\cos\theta)(R^2 - C^2\sin^2\theta)^{-1/2}(C^2\sin\theta\cos\theta)(d\theta/dt)}{R^2 - C^2\sin^2\theta}\right]\right\}$$

$$a_h = \frac{V^2}{C}\left\{\cos\theta + \frac{Cb}{R}\left[\frac{\cos^2\theta - \sin^2\theta}{(R^2 - C^2\sin^2\theta)^{1/2}} + \frac{C^2\sin^2\theta\cos^2\theta}{(R^2 - C^2\sin^2\theta)^{3/2}}\right]\right\}$$

Multiplying the expression in braces by C, dividing the term outside by C, and substituting from Fig. 1*b* gives

$$a_h = \frac{V^2}{C^2}\left[B + \frac{b}{R}\left(\frac{B^2 + H^2}{A} + \frac{H^2B^2}{A^3}\right)\right]$$

$$= \frac{V^2}{C^2}\left[B + \frac{b}{R}\left(\frac{B^2(A^2 + H^2) - A^2H^2}{A^3}\right)\right]$$

$$= \frac{V^2}{C^2}\left[B + \frac{b}{R}\left(\frac{R^2B^2}{A^3} - \frac{H^2}{A}\right)\right] \ldots\ldots [8]$$

The horizontal force necessary for the acceleration of a mass at n is

$$F_h = \frac{W}{g}a_h = \frac{WV^2}{gC^2}\left[B + \frac{b}{R}\left(\frac{R^2B^2}{A^3} - \frac{H^2}{A}\right)\right] \ldots\ldots [2]$$

The vertical displacement of any point n is

$$s_v = C\sin\theta - b\sin\alpha$$

$$\sin\alpha = (C/R)\sin\theta$$

$$s_v = C\sin\theta - (bC/R)\sin\theta \ldots\ldots [9]$$

The vertical component of linear velocity of n is

$$v_v = \frac{ds_v}{dt} = \left(1 - \frac{b}{R}\right)C\cos\theta\frac{d\theta}{dt} = \left(1 - \frac{b}{R}\right)V\cos\theta \ldots [10]$$

The vertical component of linear acceleration of n is

$$a_v = \frac{dv_v}{dt} = V\left(1 - \frac{b}{R}\right)\left(-\sin\theta\frac{d\theta}{dt}\right)$$

$$= -\frac{V^2}{C}\left(1 - \frac{b}{R}\right)\sin\theta = -\frac{V^2}{C^2}H\left(1 - \frac{b}{R}\right) \ldots [11]$$

The vertical component of the force necessary for the acceleration of a mass at n is

$$F_v = \frac{W}{g}a_v = -\frac{WV^2}{gC^2}\left[H\left(1 - \frac{b}{R}\right)\right]^* \ldots\ldots [3]$$

The angular velocity of the connecting rod is

$$\omega_R = \frac{d\alpha}{dt} = \frac{d\{\sin^{-1}[(C/R)\sin\theta]\}}{dt}$$

$$= \frac{C}{R}\left[\frac{\cos\theta}{\left\{1 - \left(\frac{C}{R}\sin\theta\right)^2\right\}^{1/2}}\frac{d\theta}{dt}\right]$$

$$\omega_R = \frac{\omega C}{R}\left[\frac{\cos\theta}{\left\{1 - \left(\frac{C}{R}\sin\theta\right)^2\right\}^{1/2}}\right] \ldots [12]$$

The angular acceleration of the connecting rod is

$$\phi = \frac{d\omega_R}{dt} = \frac{\omega C}{R}\left(\frac{\left\{\left[1 - \left(\frac{C}{R}\sin\theta\right)^2\right]^{1/2}(-\sin\theta)\right\}\frac{d\theta}{dt}}{1 - (C^2/R^2)\sin^2\theta} - \frac{+\cos\theta\left[1 - \left(\frac{C}{R}\sin\theta\right)^2\right]^{-1/2}\left(\frac{C^2}{R^2}\sin\theta\cos\theta\right)\frac{d\theta}{dt}}{1 - (C^2/R^2)\sin^2\theta}\right)$$

$$\phi = \frac{\omega^2 C}{R}\left[\frac{-\sin\theta}{\left(1 - \frac{C^2}{R^2}\sin^2\theta\right)^{1/2}} + \frac{C^2(\sin\theta\cos^2\theta)}{R^2\left(1 - \frac{C^2}{R^2}\sin^2\theta\right)^{3/2}}\right]$$

$$= \omega^2\left[\frac{-H}{(R^2 - H^2)^{1/2}} + \frac{B^2H}{(R^2 - H^2)^{3/2}}\right] = \omega^2\left[\frac{H}{A} - \frac{B^2H}{A^3}\right]$$

$$= -\omega^2(A^2H - B^2H)/A^3 = -\omega^2(H/A^3)(A^2 - B^2)$$

$$\phi = -\omega^2(H/A^3)(R^2 - C^2) \ldots\ldots [13]$$

The torque for the angular acceleration of a mass with a moment of inertia I about its axis of rotation is

$$T = I\phi = -I\omega^2(H/A^3)(R^2 - C^2)$$

$$T = -4\pi^2N^2I(H/A^3)(R^2 - C^2)^* \ldots\ldots [5]$$

* The minus sign indicates that the acceleration is in a direction opposite to that chosen for displacement.

Viscosity of Coal-Ash Slags[1]

By P. NICHOLLS,[2] and W. T. REID,[3] PITTSBURGH, PA.

This paper describes a viscometer for measuring the viscosity of coal-ash slag up to temperatures of 2900 F. The investigations disclose that slags in only a relatively small portion of the field of compositions of coal ash have the melting characteristics of glasses and that, for most compositions, there is a distinct liquidus point. From the limited data available for this progress report, plots for viscosity are given for 2800, 2600, 2400, and 2200 F, as well as showing the liquidus lines for each temperature. The relation between "flow temperature," as determined in earlier investigations for use with slag-tap furnaces, and viscosity and liquidus temperature is derived.

When complete trilinear diagrams are available, accurate predictions can be made on the effect of mixing the ash of two coals or of the addition of the main constituents. The authors emphasize that predictions of the relative fusion and clinkering characteristics of coals from the softening temperatures of their ashes may give erroneous comparisons, but that the initial and fluid temperatures can be some additional guide.

Introduction

THIS REPORT is a continuation of those previously presented to the Society on results of investigations on coal-ash slags and their properties. The last three reports were based on work done in cooperation with an A.S.M.E. Special Research Committee on Removal of Ash as Molten Slag From Powdered-Coal Furnaces and, although the committee has been disbanded, this may be considered as a final report for it. The previous reports covered three phases of the general investigations: (1) Studies[4] at one of the first plants using a wet-bottom furnace, the object being to determine the factors involved in this then new method; (2) studies[5] to determine the possibility of adding materials to the slag in a furnace, or to the coal itself, which would flux the slag and permit of its being tapped if it were too refractory to be molten at the temperature of the bed; (3) studies[5,6] to determine the relation between the temperature at which slags would tap with equal ease and their chemical composition under oxidizing and under reducing conditions.

These investigations necessitated some method of measuring the fluidity of the slags and, in particular, of determining the temperatures at which they could be tapped with a relatively similar degree of ease. The straightforward procedure would have been to adopt some method for determining absolute viscosities against temperature. However, at that time no method was developed to study slag associated with other processes, such as steelmaking or glassmaking, that could be used safely for coal ashes with assurance of its suitability, nor were existing methods appropriate for control of the atmosphere from oxidizing to reducing. Thus, it appeared that the development of some new or modified method would be necessary if absolute viscosities were to be determined. It was predictable that such a development would require a long time and the output of determinations would be slow. Therefore, it was not allowable to follow this procedure, because some general information on the properties of slags as related to slag-tap furnaces was required in the shortest possible time.

The solution adopted was to obtain relative measures of the fluidity of the slags by a method of "feel;" the slag being melted in a crucible and using a platinum rod. Three degrees of melting were standardized and indicated by the temperatures at which they occur:

1 The sticky temperature, which is the lowest at which slag on the end of the rod will stick to the slag in the crucible.

2 The penetration temperature, which is the lowest temperature at which the rod will penetrate the slag with a pressure of approximately 6 oz.

3 The flow temperature, which is that at which the rod can be moved through the molten slag and give indications of the same degree of fluidity.

Although such measurements would not be expected to be exact to a high degree, experience showed that different observers could duplicate each other; great exactness not being necessary, because the composition of the slag that occurs at any one station from day to day and the temperature conditions are not accurately predictable.

By this method, the entire field of compositions of coal ashes was covered and the data for relationship of flow temperature to composition were shown in previous studies[7] for melts in an atmosphere of air and[8] for melts under furnace conditions.

Although these results sufficed for immediate requirements, it was realized that the data had only limited application and that a knowledge of the absolute viscosities of coal-ash slags was required. For many years much has been imagined and written as to the meaning of standard cone-fusion temperatures, but the only way to settle the matter is by knowing the viscosities. The same applies to clinkering and to slagging of surfaces. Also, the flow temperature only indicates the temperature required for a certain ease of tapping and of itself does not show the change in fluidity of the slag above and below the flow temperature.

It was therefore decided to extend the investigation to the determination of viscosities of slags as related to their chemical compositions. The difficulties and time required were understood, but the experience and the knowledge of the behavior of the slags, acquired in the earlier work, would be of great help. Preliminary studies of methods were begun in October, 1937, but the development and construction of the furnace and viscometer were not completed until May, 1938.

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[1] Published by permission of the Director, Bureau of Mines, United States Department of the Interior.

[2] Supervising Fuel Engineer, Bureau of Mines, Central Experiment Station.

[3] Associate Fuel Engineer, Bureau of Mines, Central Experiment Station.

[4] "Study of Some Factors in Removal of Ash as Molten Slag From Powdered-Coal Furnaces," by Ralph A. Sherman, P. Nicholls, and Edmund Taylor, Trans. A.S.M.E., vol. 51, 1929, paper FSP-51-51, pp. 399–413.

[5] "Fluxing of Ashes and Slags as Related to the Slagging-Type Furnace," by P. Nicholls and W. T. Reid, Trans. A.S.M.E., vol. 54, paper RP-54-9, 1932, pp. 167–190.

[6] "Slags From Slag-Tap Furnaces and Their Properties," by P. Nicholls and W. T. Reid, Trans. A.S.M.E., vol. 56, 1934, paper RP-56-3, pp. 447–465.

Contributed by the Fuels Division and presented at the Joint Meeting, Columbus, O., October 5–7, 1939, of the Coal Division of The American Institute of Mining and Metallurgical Engineers and the Fuels Division of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[7] Reference (5), Fig. 7.

[8] Reference (6), Figs. 2 and 3.

The preparation for and completion of a test requires a minimum of 2 days, so relatively few have been made to date, as compared with those required to cover the five main constituents of coal ash. However, the tests have been spread over the field and throw enough light on relationship, upon which knowledge is required, to justify a progress report.

The main object of this paper is to suggest and discuss general conclusions. Many more tests will be required to obtain a complete set of numerical values of viscosity against chemical composition.

Choice of Method

The type of viscometer and design of furnace to be used were considered carefully and some experiments were made. The requirements were (1) that the maximum temperature at which the slag could be held should be not less than 2900 F; (2) that it should be possible to control the atmosphere to which the slag is exposed; (3) that the quantity of slag required for a test should not be excessive, because of the time required to prepare it and to melt it in the crucible; (4) crucibles of suitable size and cost. Coal-ash slags high in fluxes are very corrosive. They destroy and are contaminated by ceramic crucibles, which can be used with safety with other slags; also, when the iron is highly reduced or metallic iron is present, they are destructive to platinum. Consequently, the size of crucibles necessary and their cost were important factors.

An argument on methods and a discussion of the literature on viscosity measurement are not included; most work on viscosity at high temperatures has related to glasses and is well summarized by Morey.[9] The method adopted was that of the oscillating bob. Its principle is that a bob suspended in the slag by a torsion wire is made to oscillate and the damping of this oscillation by the slag is measured. Viscosity is a function of damping, of the dimensions of the bob and the containing vessel, of the inertia of the rotating system, and of the torsion of the suspension. This method was adopted because it could cover a large range of viscosities from 1 to 10^6 poises and would allow the furnace to be sealed at the bottom, which is necessary for the control of the atmosphere. It has the disadvantage that the friction factor, as dependent upon the shape of the bob, cannot be computed accurately enough and, therefore, must be determined by calibration with liquids of known viscosity.

Recent researchers who have used the oscillating-bob method are Dantuma[10] and Endell[11,12] and his co-workers. Dantuma did not require a temperature higher than 1000 F and he developed very exact methods, including a photographic record of the oscillations. Acknowledgment is made of the suggestions obtained from these publications.

A description of the apparatus as constructed, its calibration, and some troubles encountered are given as an Appendix. Accurate determinations of viscosity over the entire range are made possible by (*a*) the use of two suspensions, one above the other, so that the change from one to a combination of both can be made by a clamping arrangement; (*b*) by two additive weights, whereby the moment of inertia of the system can be controlled; and (*c*) by changing from damped harmonic motion to damped aperiodic motion for viscosities higher than 100 poises.

Results of Tests

Values for viscosity will be given in the absolute measure of cgs units, or poises. Some idea of the fluidities corresponding to a range of values in poises is shown by:

Water at 68 F	0.01 poise
S.A.E. 10 motor oil at 68 F	1.00 poise
Castor oil at 68 F	10.00 poises

There is no commonly used liquid having a viscosity of 100 poises at normal temperature.

Most of the data will be shown as plots of viscosity against temperature and, because the range of values of viscosities is so large, the scale of log viscosity will be used.

Most of the tests to date have been of melts in air, but enough have been under reducing conditions to show the effect of reduction of the iron oxides. The term "ferric percentage" was introduced in the previous reports as a numerical value to express the state of oxidation of the iron in the slags; it is the ratio of the ferric iron to the total iron expressed as a percentage, and is given by

$$\text{Ferric percentage} = \frac{Fe_2O_3}{Fe_2O_3 + 1.11\,FeO + 1.43\,Fe} \times 100$$

Melting of Typical Coal-Ash Slags

All these tests are concerned with the melting and properties of slags, as distinct from the first melting of the heterogeneous mixture of materials of which a coal ash is composed. Therefore, in all tests, the first procedure is to obtain a complete melting of the charge before any readings are taken. Thus, all readings begin at what is usually the highest temperature that will be used.

A short discussion at this point of the melting of slags will facilitate understanding the data for those who are not familiar with the subject. Slags in general are of two types: (1) Those that, as they are cooled slowly, maintain their glassy, uniform state and do not form any crystalline particles, and (2) those in which crystals of some definite composition begin to form at some temperature during cooling. The temperature at which crystals begin to form is called the liquidus temperature. This temperature has a definite value if cooling is slow enough, but it may require maintenance at each temperature for many hours to obtain a definite value, because the initial formation of crystals may be slow. However, as the temperature is lowered below the true liquidus value, the speed of crystallization increases. Similarly, it requires time for the crystals to redissolve in the slag, if the temperature is raised only slightly above that of the liquidus; solution is more rapid as the increase in temperature above that of the liquidus becomes greater. This action is simple when a melt has only two components but is complex when there are many, as in a coal-ash slag; one kind of crystal may begin to form at one temperature and, at lower temperatures, other types of crystals may appear. The term "solid phase" is more general in its meaning than is crystals.

If no solid phase exists, the melt will be a true fluid and will give true values for viscosity. If solid particles are present, they will increase the internal friction materially and the substance will not act as a true fluid. Its properties will tend to be more those of a plastic substance as the quantity of solid phase increases. It requires time for solid phase to form, and it may take longer for it to redissolve. For example, in a high-alumina slag, crystals of corundum were formed that did not redissolve. Thus, measures of "apparent viscosity," obtained at a temperature below the liquidus, will vary with time; this variation being greater if the crystals tend to interlock or to fuse together.

[9] "The Properties of Glass," by G. W. Morey, American Chemical Society Monograph, Reinhold Publishing Corp., New York, N. Y., 1938, chapter 5.

[10] "The Exact Determination of the Coefficient of Internal Friction of Metallic Salts," by R. S. Dantuma, *Zeit. anorganische allgemeine Chemie*, vol. 175, 1928, pp. 1–42.

[11] "The Dependence of Viscosity on the Temperature in the System Na_2O-SiO_2," by G. Heidtkamp and K. Endell, *Glastechnische Berichte*, vol. 14, 1936, pp. 89–103.

[12] "Concerning the Fluidity of Calcium Silicates, Calcium Ferrites, and Basic Siemens-Martin Slags up to 1625 C," by K. Endell, G. Heidtkamp, and L. Hax, *Archiv für das Eisenhuttenwesen*, vol. 10, Sept., 1936, pp. 1–6.

Fig. 1 shows the test data of a typical slag having a silica-to-flux ratio of 1.36, and a total flux content (iron oxides, lime, and magnesia) of 32.3 per cent. The slag was melted and the first measurement of viscosity was 2765 F, the value being 10 poises. The slag was then cooled in steps by setting the temperature controller to the lower temperature required; sufficient time, usually 15 min, being allowed to elapse to insure constancy before the viscosity determination is made. The points are plotted as the test proceeds and the round dots are those of this cooling cycle. When the temperature of 2225 F of point *A* was reached, it was found that, if the slag was maintained at that temperature, the viscosity increased with time. The actual procedure now used to determine point *A* more exactly is that, after cooling 100 deg, for example, and determining the viscosity, the slag is then heated 50 deg and another determination made. If the liquidus range had been reached, this last viscosity would lie above the plot of the values previously determined.

The slag was kept at the temperature of point *A*, gradually becoming more viscous. After 40 min, the bob was immovable. The temperature was then increased to 2280 F, but the bob still remained immovable. At 2310 F an apparent viscosity of 350 poises was obtained and, at 2325 F, the viscosity was nearly down

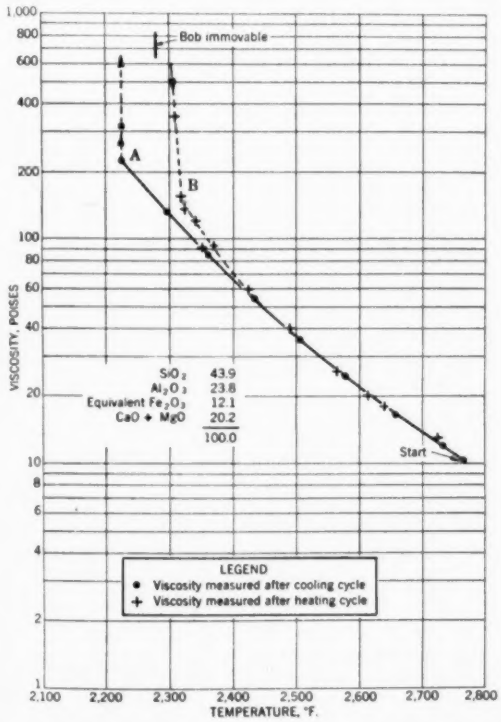


Fig. 1 Viscosity-Temperature Plot of a Typical Slag Having an Indefinite Liquidus Temperature

to that of the cooling curve. If enough time had been allowed, it would probably have given the value of the curve.

In this test the liquidus region was in evidence in a range *A* to *B*, or 100 deg; there is some question as to what temperature should be designated as the liquidus. That of point *A* is too high, because the viscosity changed from that of *A* to the point above it in 5 min; it is probable that the slag was supercooled and that the movement of the bob started crystallization. In many tests a retrial is made to determine a new temperature for point *A* by reheating the slag; the reheating must be thorough to insure that all the crystals are recombined in the slag, or they will act as "seeding" and raise the temperature of the liquidus.

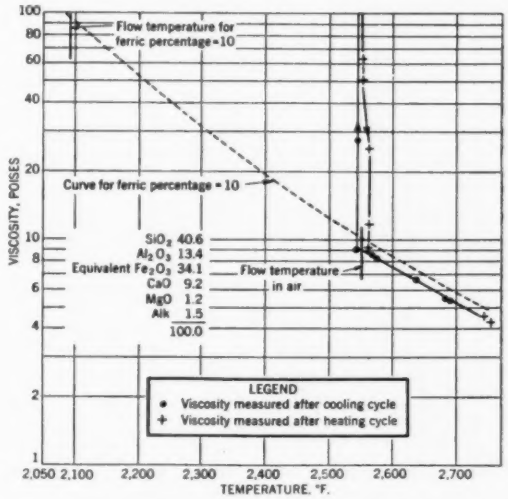


Fig. 2 Viscosity-Temperature Plot of a Typical Slag Having a Definite Liquidus Temperature

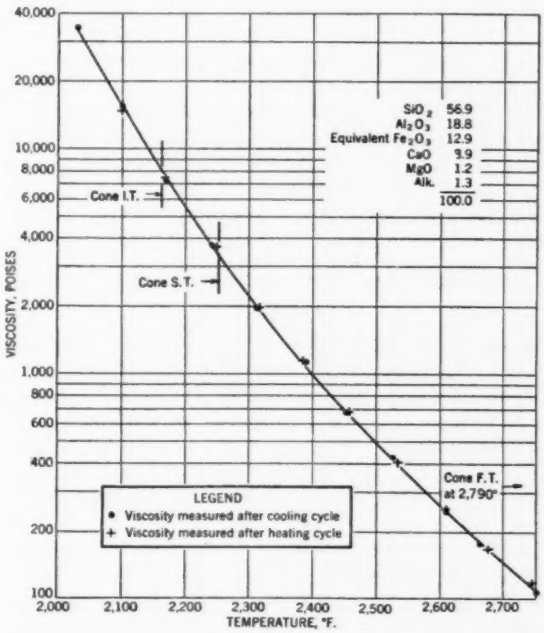


Fig. 3 Viscosity-Temperature Plot of a Typical Glassy Coal-Ash Slag

In this type of test there must be some question as to the true value of the liquidus temperature. For this slag it was recorded as 2300 F.

The solid lines of Fig. 2 show the test data of a melt in air of a slag having a silica-to-flux ratio of 0.91, 34.1 per cent equivalent ferric oxide and 10.4 per cent lime plus magnesia. This represents a slag from the Illinois field and is an example of a narrow liquidus range; that is, a sharp melting point. The liquidus temperature was 2550 F, and its viscosity just above the liquidus temperature was 9 poises, as compared with 100 poises of the slag of Fig. 1.

Fig. 3 is a melt in air of a glassy slag having a silica-to-flux ratio of 2.47; equivalent ferric oxide, 12.9 per cent; and lime plus magnesia, 10.1 per cent. Its viscosities are higher than the previous slags, being about 100 poises at 2760 F and 40,000 at 2020 F.

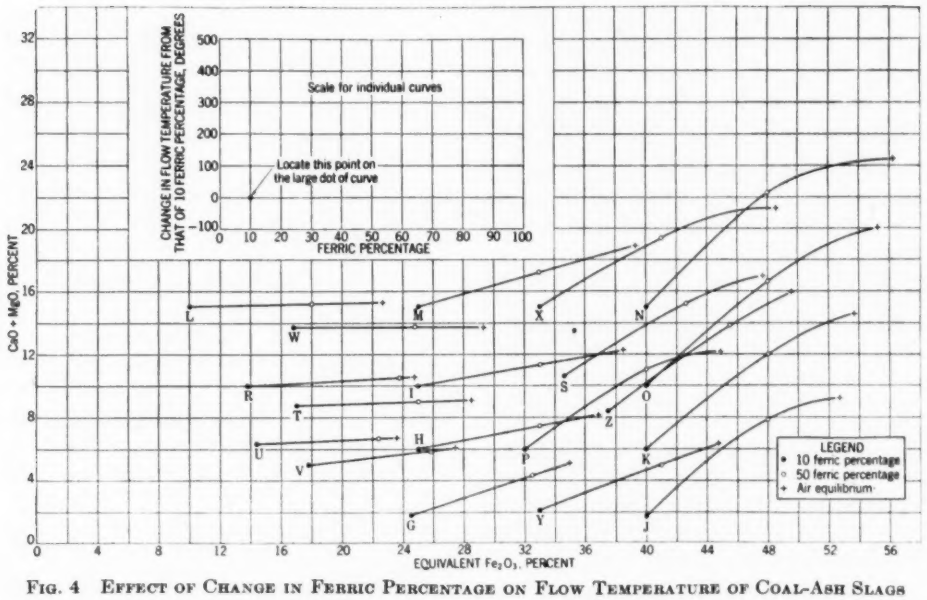


FIG. 4 EFFECT OF CHANGE IN FERRIC PERCENTAGE ON FLOW TEMPERATURE OF COAL-ASH SLAGS

The agreement of the points of the cooling and heating cycles shows the absence of solid phase. Having reached 40,000 poises at 2020 F, no solid phase would be expected to form at lower temperatures, but such a statement is limited to the conditions of this test with the relatively rapid cooling, because there might be crystallization if the cooling were slow enough. However, such a possibility has no importance because the cooling of slags and clinker in practice is always relatively rapid.

EFFECT OF FERRIC PERCENTAGE ON VISCOSITY AND LIQUIDUS TEMPERATURE

Although most of the tests were made in air, with a resulting high ferric percentage, enough were made in nitrogen, using previously reduced slags, to show the effect of a low ferric percentage (that is, high percentage of ferrous iron) on the viscosity and the liquidus temperature. The object of using the neutral atmosphere of nitrogen is to prevent a slag that already has been reduced to a low ferric percentage from being oxidized during the test.

The effect of ferric percentage on the flow temperature was determined in the earlier investigations. Further tests were made after the last report, and Fig. 4 includes all results; the relationships it shows are important.

The heavy dots give the compositions of the slags tested. The curve beginning at a heavy dot gives the change in flow temperature with increase of ferric percentage. The coordinates and scales for these curves are shown in the inset; a tracing can be made of this inset and it can be placed over a curve so that the heavy dots coincide, but this is not necessary.

The ferric percentage for melts in air are shown by crosses and, because no melts were made in oxygen, they are the highest ferric percentages obtained.

Fig. 4 shows that changes in the flow temperature are dependent upon the iron content and that changes in the lime content have little effect. Up to 12 per cent equivalent Fe_2O_3, the ferric percentage has little effect on the flow temperature. With higher ferric percentages, the flow temperature increases with increase in ferric percentage. The order of these increases for about 10 per cent lime are:

Equivalent Fe_2O_3, per cent	15	25	35	40
Increase of flow temperature from 10 ferric percentage to that of air, F	20	100	300	500

Fig. 5 shows the viscosity and liquidus temperatures in air and in nitrogen of a low-iron slag with 13.3 per cent equivalent Fe_2O_3; the ferric percentage in air was 32 and in nitrogen 18. The curve for the melt in nitrogen lies above the other, but parallel to it. From the test data, as shown on the plot, the liquidus temperature for the melt in air is 30 deg lower than that for the melt in nitrogen. The percentage difference in viscosity between the two curves is greater than in other tests.

Fig. 6 shows the data for a slag having a medium iron content of 19.8 per cent equivalent Fe_2O_3. The figure is rather crowded because it shows three tests. The test for the melt in air is shown by the short curve; the ferric percentage was 56. The liquidus range of *A* is not wide and the liquidus temperature is about 2530 F.

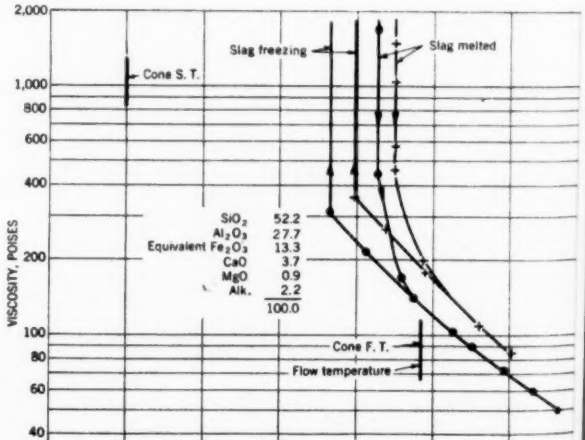


FIG. 5 EFFECT OF CHANGE IN FERRIC PERCENTAGE ON VISCOSITY AND LIQUIDUS TEMPERATURE OF A SLAG HAVING A LOW EQUIVALENT Fe_2O_3 CONTENT

There were two tests in nitrogen and the test points of both fall on the upper curve. With one the slag had a ferric percentage of 30; the test points are shown by circles. The liquidus range is short and the liquidus temperature is about 2410 F, a decrease of 120 deg from that of air.

In the second test in nitrogen the ferric percentage was 13. The test points are shown by crosses and range the full length of the curve. The liquidus range is not shown in the same manner as for the other tests; in this test the solid phase did not begin to separate until the viscosity was over 1000 poises, probably at 1600 poises, at a temperature of 2150 F. The time required for

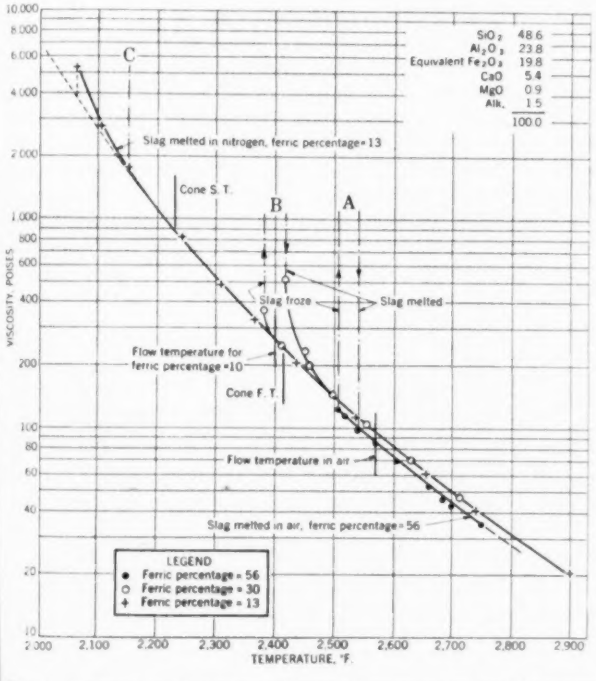


FIG. 6 EFFECT OF CHANGE IN FERRIC PERCENTAGE ON VISCOSITY AND LIQUIDUS TEMPERATURE OF A COAL-ASH SLAG HAVING MEDIUM EQUIVALENT Fe_2O_3 CONTENT

crystals to form will be greater as the slag is more viscous, and the weight of crystals formed in a given time will be less than if the slag is more fluid. Therefore, there was not the abrupt increase in viscosity that occurred in the other tests at *A* and *B*.

Fig. 6 also illustrates the effects of decrease in the ferric percentage. The viscosity is changed only a little, and the shape of the viscosity curve for the reduced slag is similar to that for the oxidized. As the ferric percentage was decreased from 56 in air, the liquidus region was moved to a lower temperature—to *B* for a ferric percentage of 30 and to *C* for a ferric percentage of 13.

The positions of flow temperature in air and for 10 ferric percentage are shown but will be discussed later.

From Figs. 5 and 6 and from other tests, the effects of lowering the ferric percentage from that of air are shown to be relatively simple and are summarized as follows:

1 The shape of the viscosity curve is not changed by a decrease in ferric percentage, but the entire curve is raised about 20 per cent (on the viscosity basis) above that for the melt in air.

2 The main effect of lowering the ferric percentage is that it decreases the liquidus temperature.

3 There is little change in the liquidus temperature with change in ferric percentage for slags having an equivalent Fe_2O_3 content of 12 per cent and lower. As the iron content increases, the change in the liquidus also increases, but the exact amounts have not yet been determined for the entire range of coal ashes.

At this time no tests in nitrogen have been made of high-iron slags because of difficulty in obtaining a crucible that will withstand their erosion, but the nature of the results can be predicted closely. This has been done in Fig. 2 for a slag with 34 per cent equivalent Fe_2O_3. The predicted form for a slag having a ferric percentage of 10 is shown by dotted lines. With this slag, the flow temperature in air is 2550 F, and for 10 ferric percentage it is 2090 F. The dotted curve has a viscosity of 90 poises at 2090 F; from Fig. 4 it can be deduced that the liquidus temperature of this slag would be about 2050 F.

GENERAL RELATIONSHIP OF VISCOSITY TO CHEMICAL COMPOSITION

The major components of coal-ash slags that may affect the viscosity are silica, alumina, iron oxide and its ferric percentage, lime, and magnesia. The effect of ferric percentage is now known. Previous reports have shown that lime and magnesia can be taken as one factor for the flow temperature when the percentage of magnesia is small, which it generally is.

Plots were made to determine whether there were other relationships by which the number of factors could be reduced further. They showed for slags entirely in the liquid phase (*a*) that if (equivalent Fe_2O_3 + CaO + MgO) were taken as one factor, the plots were little affected by the ratio of Fe_2O_3 to (CaO + MgO), but only by their sum; (*b*) that if the alumina were eliminated and the composition reduced to SiO_2 + equivalent Fe_2O_3 + (CaO + MgO) = 100, the plots were closely independent of the ratio of silica to alumina within the limits of 1.5 to 3.

Fig. 7 shows that these assumptions are justified as a first approximation. It is a plot for 2600 F of log viscosity against silica when the composition is expressed in the form just given, which omits the alumina from the plot. A few points are marked

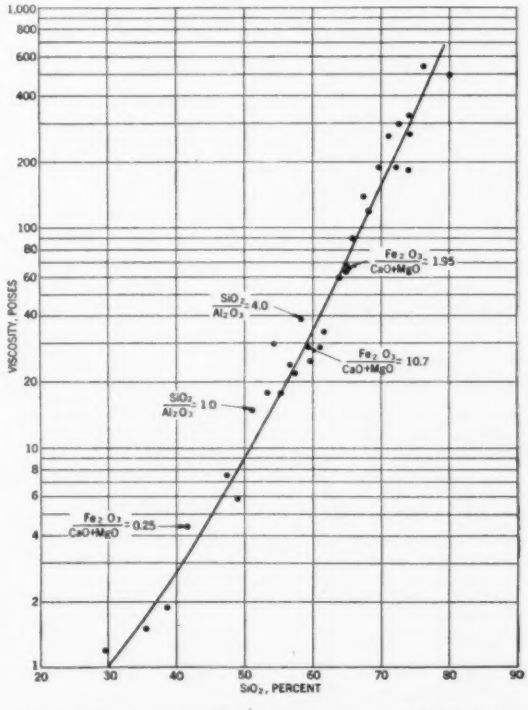


FIG. 7 VISCOSITY OF COAL-ASH SLAGS AT 2600 F IN AIR
(Plotted against SiO_2 content on the basis of omitting Al_2O_3 and making SiO_2 + equiv Fe_2O_3 + CaO + MgO = 100.)

to show that the plot includes points having a silica-alumina ratio of 1 and 4, and iron-lime ratios of 0.25 to 10.7, and yet the points are within a relatively narrow band. The plot shows that log viscosity tends to be proportional to the silica percentage when the alumina is eliminated from the method of plotting, although it is present in the slag.

The reasons for the departure of some of the points from the mean curve have not been determined; no investigation has been made as yet on the separate effects on viscosity of magnesia or the alkalies. However, the points conform nearly enough to a common line to justify making plots on the silica-iron-lime-plus-magnesia basis, because then the first order of relationship of viscosity to composition can be shown by a few diagrams, whereas many would be required if another variable were included.

Fig. 8 shows trilinear plots of viscosity against composition on the basis that SiO_2 + equivalent Fe_2O_3 + (CaO + MgO) = 100. There are four diagrams for temperatures of 2800, 2600, 2400, and 2200 F; the areas for 0 to 20 per cent silica are omitted. The diagram for 2800 F shows an area within the dotted lines that can be taken as the field of composition of normal coal-ash slags, although some may be found to fall outside this area.

Each plot shows the lines of equal viscosity for the range of compositions that are liquids without solid phase at the specified temperature.

Plottings showed that the liquidus temperature is not independent either of the silica-alumina or the iron-oxide-lime-plus-magnesia ratios; consequently, a separate set of trilinear diagrams is required for each of several narrow ranges of silica-alumina ratios. A large number of tests will be required to obtain sufficient data. From the limited number of tests already made, the liquidus values for a silica-alumina ratio of 2.0 to 2.3 were determined, and lines *A-B* follow the compositions of such slags, the liquidus temperature of which melted in air is that of the diagram, 2600 F, for example.

Lines *A-C* are for slags of 10 ferric percentage and are deduced from other data, but if the plots are to represent such reduced slags the isoviscosity lines should be extended to *A-C*.

No liquidus line is shown in the 2800-deg plot because the liquidus temperature of all slags tested was below 2800 F, although it is probable that there would be a separation of solid phase for compositions very high in silica or lime.

As the temperature decreases, the area inclosed by the liquidus lines decreases; that is, the slags that will be liquid at that temperature are reduced to a smaller range of compositions.

There is no claim that these diagrams have a high order of accuracy, especially for the position of the liquidus lines for the 10 ferric percentage. The effects of the minor chemical constituents are not known, nor have there been enough tests of the main constituents. On the other hand, it is safe to use the plots for general information to determine whether a slag of a stated composition will be liquid at one of the temperatures and what will be its approximate viscosity. The procedure is first to eliminate the alumina, thus:

Composition of slag per cent				
SiO_2	47	47 + 0.77	=	61.0
Al_2O_3	23	...		...
Equiv Fe_2O_3	25	25	=	31.5
CaO + MgO	5	5	=	7.5
		77		100.0

The point on the 2400-F diagram that corresponds to the com-

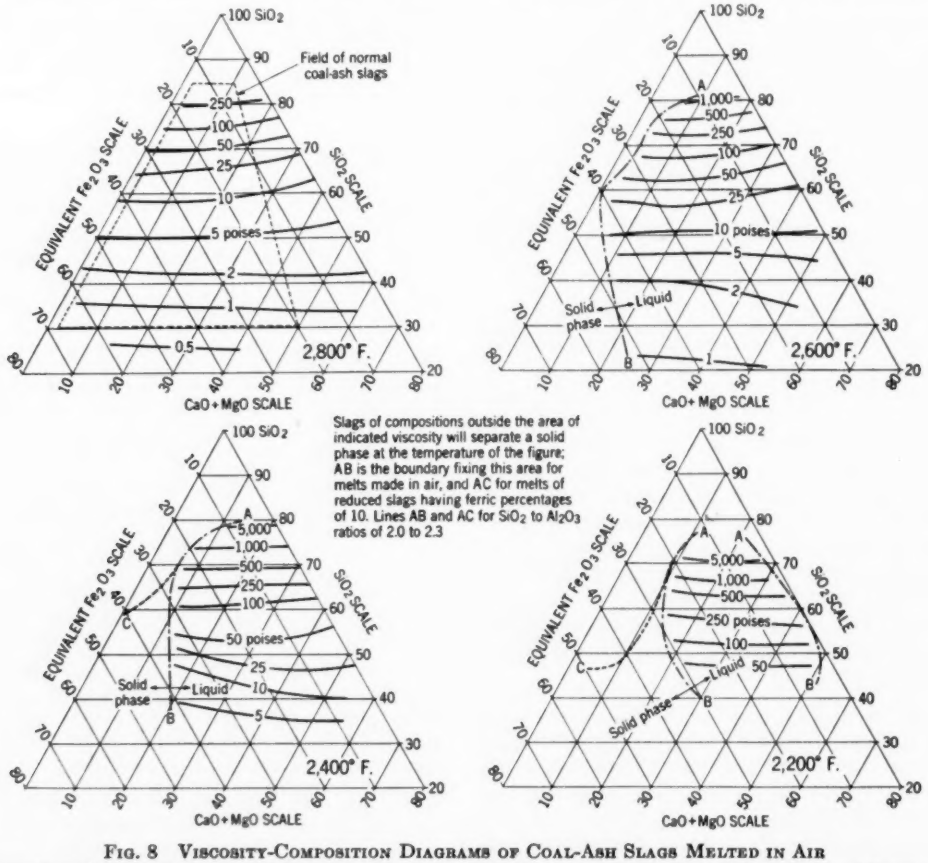


FIG. 8 VISCOSITY-COMPOSITION DIAGRAMS OF COAL-ASH SLAGS MELTED IN AIR

(Plots give the composition of the slags with the Al_2O_3 omitted and on the basis of SiO_2 + equiv Fe_2O_3 + CaO + MgO = 100 per cent; tests indicate that the viscosity of slags is approximately independent of the Al_2O_3 content for SiO_2-Al_2O_3 ratios of 1.5 to 3.0; alkalies not more than 2.5 per cent of original slag.)

position of the last column in the foregoing table is at the intersection of the 100-poise line with the line *A-B*. A slag will tap easily at 100 poises, but if this slag had a high ferric percentage, corresponding to the melt in air, it would give trouble in tapping because it is on the liquidus line. However, slags from furnaces have a low ferric percentage, and because the point is not near the line *A-C*, this slag would tap at 2400 F.

Mean-Viscosity Curve

The plot of Fig. 9 has been made by superimposing the individual curves of a number of tests. The principle used was that each curve must keep its correct position on the viscosity scale but can be moved parallel to the temperature scale, so that separate curves make the best matching. The arrowed lines distinguish the different curves by test number. The temperature scale represents temperature differences and not actual temperatures. Assuming that a mean curve were drawn, then tangents to that curve would be rate of change of log viscosity with temperature. The values for the slopes are the chief interest of this plot and the deductions from them are:

(*a*) The rate of change of viscosity with temperature of coal-ash slags depends mainly upon the viscosity and not upon the temperature.

(*b*) The rate of change of viscosity increases with increase in viscosity.

The object of including this figure is that it fixes a mental picture of relationships. Expressed numerically, the increase of the rate of change of viscosity with increase in viscosity is very large, as is shown by the following approximate values:

at	1 poise,	rate of change of viscosity per 100 F =	0.28 poise
at	10 poises,	rate of change of viscosity per 100 F =	4 poises
at	100 poises,	rate of change of viscosity per 100 F =	58 poises
at	1000 poises,	rate of change of viscosity per 100 F =	850 poises

Relation of Flow Temperature to Viscosity

Some of the slags used for viscosity determinations were identical with those used in the earlier investigation and, for those slags, the flow temperature is shown on the diagrams. Data on other slags were as follows:

Flow temperature, F	Viscosity at flow temperature, poises	Liquidus temperature, F
2550	9	2550
2350	16	2300
2580	20	2550
2685	78	2280
2730	80	2700
2390	80	2150
2570	85	2500
2560	90	2200
2500	120	2400
+2730	(100)	2760

The deductions on the relation of the flow temperature, determined by the feel method, to viscosity and the liquidus limitations are:

(*a*) Normally the flow temperature corresponds to a viscosity of about 80 poises (see values for air in Fig. 6).

(*b*) The flow temperature cannot be less than the liquidus temperature. Thus, if the viscosity at the liquidus is less than 80 poises, the viscosity at the flow temperature will also be below 80 poises. In Fig. 2, the viscosity at the flow temperature for melts in air was 9 poises. The observer could get very close values for flow temperature in a slag like this, because the melting range of slags high in iron is very short and the slag changes from plastic to liquid in the range of a few degrees.

(*c*) If the slag acted like a glass and had no liquidus point, or if the liquidus occurred at a viscosity much above 80 poises, the reaction of the observer was to decide on a value for the flow temperature when the slag had a viscosity greater than 80 poises.

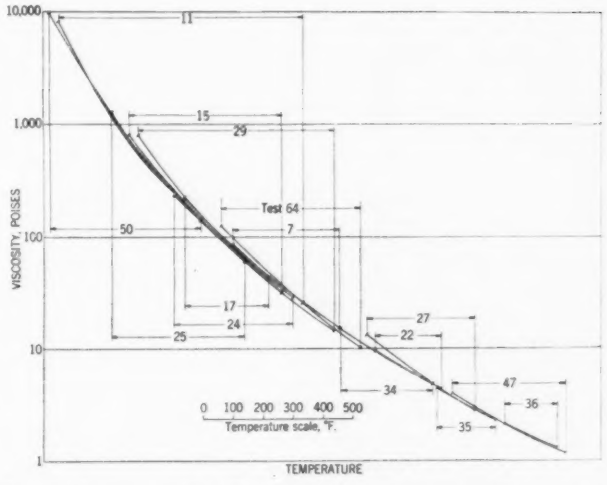


Fig. 9 Curves of Log-Viscosity Superimposed by Shifting of the Temperature Scale

The flow temperature for a ferric percentage of 10 in Fig. 6 is an illustration.

In the investigation on flow temperatures, the aim was to fix its values so that different slags would tap with the same relative ease. This aim was fulfilled, even though the viscosities at the flow temperature are not all alike. Types of slags covered by item (*b*) will flow faster because of their low viscosities; to offset this advantage, they will solidify with a small decrease in temperature and thus form "pipes" of slag, which clog the holes and decrease the flow. Slags covered by item (*c*) will flow more slowly because of higher viscosity but will be less hampered by rapid solidification and the clogging of the hole.

Ash-Cone Temperatures and Viscosity

In the A.S.T.M. cone test for the fusibility of coal ash, the measure of the fusibility of the finely pulverized ash is the temperature at which the cone deforms to specified shapes when the temperature is raised at a specified rate, these shapes being approximately spherical for the softening temperature and spreading to a flat layer for the fluid temperature. The factors that fix the shape the cone will take are:

1 The time-temperature history which, by specification, is that the rate of rise of temperature be a definite and constant value.

2 The states of fluidity of the ash, which are the properties that the cone temperatures are intended to measure.

3 Density of the slag, which will be approximately the same for all slags.

4 The surface tension of the slag. There are no published data on coal-ash slag. The surface tensions of molten glasses[13] range from 250 to 350 dynes per cm at 2200 F; they decrease slowly with increase in temperature. For comparison, at 68 F and against air, the surface tension of water is 72.7 and mercury 465 dynes per cm.

5 As applying more to the fluid temperature, the absorption of the slag by or its reaction with the base. These can be important factors.

The shaping of the cone at different temperatures is a complex function of these factors. The procedure of recording the height of the cone is more definite than is shaping, as is also the Bunte-Baum method[14] used for obtaining a graphical record of height

[13] Reference (9), chapter 7 (footnote 7).

[14] "The Significance of the Inorganic Constituents of Coal on Combustion," by K. Baum, *Glückauf*, vol. 64, 1928, pp. 1733–1741.

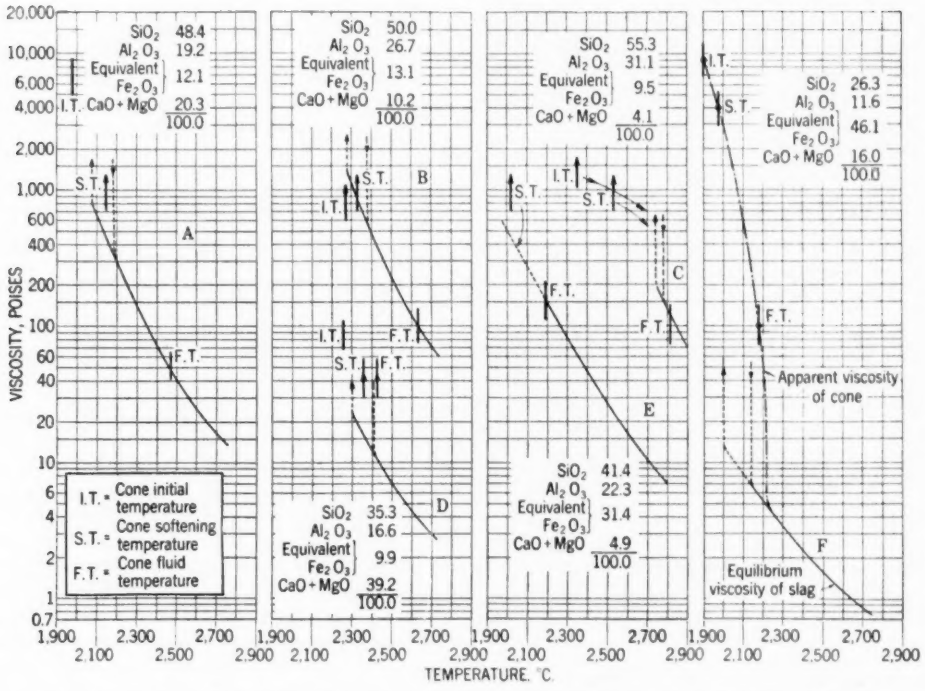


FIG. 10 COMPARISON OF CONE-FUSION TEMPERATURES WITH VISCOSITY-TEMPERATURE CURVES OF SIX COAL-ASH SLAGS

against temperature of a cylinder of ash of 1 in. diam and height.

There are two types of slags:

1 That in which the particles of the original cone are homogeneous and melt to form a true liquid without solid phase. For viscosities below that of the annealing range ($+10^{12}$ poises), there is no measurable time delay for the viscosity of a glass to change to its correct value for a new temperature. With such a true liquid, if held long enough at one temperature, the molten slag would take a final shape fixed only by the factors other than the viscosity; that is, independent of it. The time required to reach the equilibrium shape at a constant temperature will decrease as the viscosity decreases.

2 The type of ash in which some of the particles of the original cone melt to a true glass and others remain solid or fuse to a lesser degree; that is, the slag is not a true liquid. With such a slag, the solid present will affect the shape, and time has the additional importance that the solution of the solid increases with time.

It is not suggested that the shape of the cone against time could be predicted even if numerical values for the factors listed were known, but the principles of what the cone temperatures measure should be recognized when an attempt is made to interpret their meaning.

All standard cone tests are made under reducing conditions and the slag will have a low ferric percentage; therefore, the viscosity tests with which they are compared must also be for slags with low ferric percentage. Only a few such tests are available.

Fig. 3 is for a test in air but, as the slag melted as a true glass and showed no liquidus region down to 2000 F, it can be used for comparison, because the curve of a melt for a low ferric percentage would be parallel to it and about 20 per cent above, and also would have no liquidus. From this figure the cone-fusion temperatures and their viscosities at their intersections with the low ferric percentage curve are:

Initial temperature,	2160 F, 9000 poises
Softening temperature,	2250 F, 4000 poises
Fluid temperature,	2790 F, 100 poises

Because the slag had been premelted and was a glass, all the particles of the cone would be of the same composition; therefore, the first deformation, recorded as the initial temperature, would be due to an equal degree of melting of each particle of the whole cone, which is not so when an unmelted coal ash is tested.

The three viscosities listed for this glassy slag are a reference of comparison for other slags. The range of compositions will be very small for coal ashes, the slags of which are not true glass and yet, at the liquidus temperature, have a viscosity greater than 4000 poises, that is, for most slags the softening temperature is fixed by the liquidus temperature.

Fig. 10 gives the viscosity plots for six slags of widely different compositions and shows their cone-fusion temperatures. The plots illustrate the statement just made that the softening temperature is usually lower than the liquidus temperature, as judged from the heating cycle. The plots show that the softening temperatures are close to the liquidus temperature, as fixed by the line for the heating cycle, except for *F* and *C*, for which they are considerably below. The relationships shown by *C* are confirmed by another refractory slag (Fig. 5); these low-flux premelted slags have glassy characteristics but are not true glasses because an extension of curve *C* to higher viscosities would not pass near 4000 poises at the softening temperature. Slag *F* evidently has a wide liquidus range.

If the liquidus occurs at a viscosity of 500 or more poises, it would be expected that all the solid phase in the cone will have been absorbed by the time the viscosity is 100. Thus, the slag of the cone will act as a true liquid and the fluid temperature will be observed as at about 100 poises, unless reaction with the base affects the flattening of the slag. The viscosity at the fluid temperature of slag *B* was exactly 100 poises, and of *C* and *E* about that much. That of *A* is low, possibly because the slag did not wet the base as well; special tests and technique would be required to prove such explanations, because the base must be quenched at once or deformation will continue.

For slags for which the liquidus occurs at less than 100 poises, the fluid temperature will occur at a temperature a little higher than the liquidus temperature; slags *D* and *F* are examples. On

the plot of F, a dotted curve has been added to show what may be the apparent viscosity of the cone as its temperature increases.

The viscosity plot could be approximated if the liquidus temperature and the viscosity at one temperature were known. Table 1 lists values taken from the plots arranged in the order of the viscosities at 2600 F.

TABLE 1 COMPARISON OF VISCOSITIES WITH CONE-FUSION TEMPERATURES

Slag of Fig. 10	Viscosity at 2600 F, poises	Liquidus temperature F	Liquidus temperature Order	Cone-fusion temperatures Initial, F	Cone-fusion temperatures Softening F	Cone-fusion temperatures Softening Order	Cone-fusion temperatures Fluid, F
F	1.2	2120	3	1900	1970	1	2180
D	4.4	2400	6	2260	2360	6	2430
E	17.0	(2100)	2	1880	2015	2	2195
A	25.0	2185	4	2085	2150	3	2470
B	140.0	2380	5	2270	2335	5	2635
Fig. 3	260.0	Glass	1	2170	2250	4	2790
C	600.0[a]	2780	7	2350	2530	7	2815

[a] From extension of curve below liquidus temperature.

With the exception of slag D, the fluid temperature increases in the same order as the viscosities; slag D is an uncommon slag with very high lime. One point of the viscosity curve would be known if it were assumed that the viscosity at the cone-fluid temperature is 100 poises; but this does not hold for slags like D and E. It is possible that the plot of fluid temperatures against viscosities at 2600 F may give points within a band narrow enough so that the plot could be used for predicting viscosities at 2600 F, when the fluid temperature is known. However, indications from these data are that such predictions could be much in error.

The liquidus temperatures fall somewhat in the same order as the softening temperatures and have higher values. Low-flux slags would be an exception to this approximate rule. A glassy slag with no liquidus can be predicted from the cone temperature because it will have an interval of about 500 F between the softening and fluid temperatures.

As a summary, curves and rules could be derived from experimental data from which the liquidus temperature and one point on the viscosity-temperature curve could be predicted from the cone-fusion temperatures, but there will be the possibility that some predictions may be much in error.

No definite rules for the interpretation of the cone temperatures are given at this time, because data are too few and these cone fusions are for premelted ash; conversion factors are required to convert the values for unmelted ash into those for melted ash. The difference in the cone values is only large for the low-flux ashes; for example, the softening temperature of the unmelted ash of slag C was 2795 F, against 2530 F, for the melted slag.

In addition to such rules, a mean curve of viscosity such as that of Fig. 9 is required as a guide in drawing the viscosity curve through the one known point.

One must not expect that very close interpretations of the cone-fusion values will ever be possible. The criterion of change of shape and the several factors which produce those changes are too indefinite, but it should be possible to get better information than when only the softening temperature is used. For example, the softening temperatures of slags B and D of Fig. 10 differ by only 25 deg, and by that criterion their clinkering would be expected to be similar. However, although both are fluid at 2400 F, the viscosity of B is 470 poises and that of D is 12 poises; that is, D is 40 times more fluid than C. Under most conditions, such difference should certainly make D a worse clinkering coal than C.

The first step toward a better understanding of the cone temperatures was stated to be the ability to translate them into viscosity curves. That will not mean that the viscosity curves will be of greater practical help in prediction or explanation of clinkering or other ash actions, except as far as we can learn to use these tools with skill. They will be of direct help and application in relation to slags in a fluid state, such as slag-tap furnaces. They are needed, and should be helpful, in connection with clinkering, and more so if the analysis of the complicated actions in the fuel bed follows the life history of the coal from the time of firing.

Mixing of Ashes or Slags

The properties of a slag obtained by mixing the ashes of two coals can be determined from the trilinear diagrams of Fig. 8 by drawing a straight line between the two points representing their compositions and reading the values of viscosity, or of the crossing of the liquidus line, for percentage positions between the points. Thus, it is possible to predict the effects of mixing ashes or of fluxing by the addition of iron oxide or lime. Note, however, that these plots do not give any information on slags in the area of solid phase.

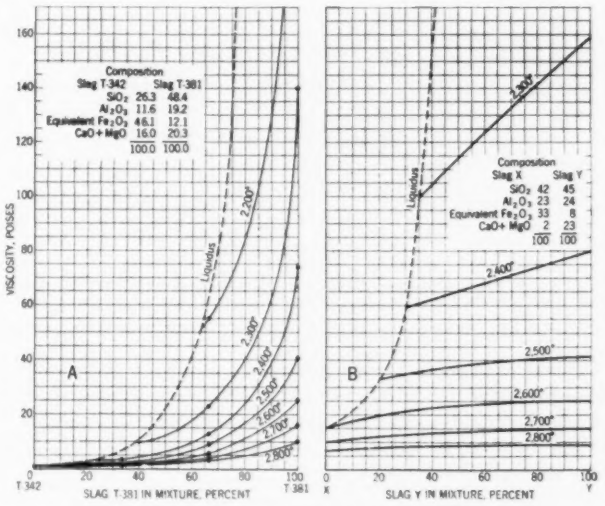


FIG. 11 EFFECT ON VISCOSITY AND LIQUIDUS TEMPERATURE OF MIXING TWO SLAGS

(*A*, having widely different viscosities; *B*, having similar viscosities.)

Such plots are valuable in that by inspection they show which of the three components, silica, iron oxide, or lime, should be increased to produce a desired change. Considering the 2600-F plot, for any composition in the liquid phase inclosed by the boundary *A-B*, the addition of lime will decrease the viscosity and move the composition farther from the liquidus line; both changes will make tapping easier.

To add iron only will decrease the viscosity by about the same amount as will the addition of lime, but now the tendency will be to approach the liquidus line, which is undesirable.

In general, the addition of silica will increase the viscosity, but for a low-silica, high-iron slag it may be beneficial in that it can move the slag toward an area of the field of composition where the liquidus temperature is lower.

Fig. 11 shows the results of mixing slags. Plot *A* is from experimental data. Slag T-342 had a viscosity of 1 at 2700 F and had its liquidus at 2600 F; slag T-381 had a viscosity of 16 at 2700 F and acted as a glass. The data for plot *B* were obtained from the trilinear diagrams of Fig. 8; the slags have somewhat similar viscosities and the same differences in liquidus temperature as those of plot *A*. These plots are self-explanatory.

Effect of Heating on Ferric Percentage

It is important to understand the effect of heat and temperature on the state of oxidation of the iron content of ashes and

slags. The method used in the chemical analysis evaluates the iron oxides as ferric (Fe_2O_3) or ferrous oxide (FeO). Metallic iron is not included, because it will not result from the heating of a slag in the absence of carbon or a reducing gas.

Certain tests were made to determine the factors influencing oxidation and reduction. The primary purpose of these tests was to obtain information required for the conduct of tests of this investigation and not to determine phase-equilibrium data.

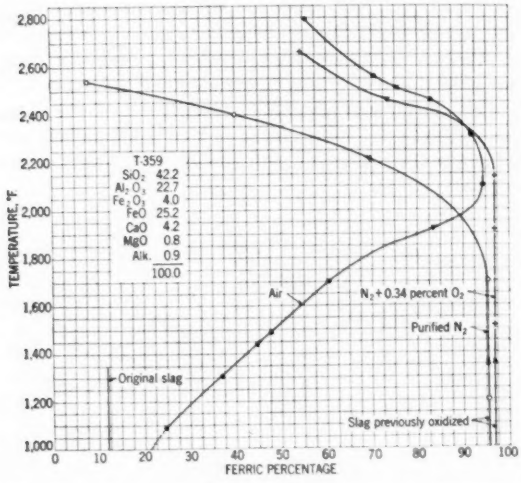


FIG. 12 OXIDATION AND THERMAL DISSOCIATION CURVES FOR A COAL-ASH SLAG
(Original slag ground to pass 60 mesh; 1-hr exposure at each point.)

The slags used had been premelted and were well reduced. They were ground to 60 mesh, and some were oxidized in the same manner as is a coal ash. The tests were made in a platinum-wound furnace, in which the atmosphere could be controlled; they were of three types:

1 Heating the reduced slag in air, stirring until prevented by eventual sintering of slag.

2 Heating the oxidized slag in an atmosphere of unpurified nitrogen containing 0.34 per cent O_2 and traces of water vapor.

3 Heating the oxidized slag in an atmosphere of nitrogen, purified by slow passage over hot copper gauze and drying with calcium chloride, and admitted to the furnace through a carbon tube.

Fig. 12 shows the tests on one slag. The method in all tests was to raise the temperature by steps and maintain it at each temperature for 1 hr. At the end of the hour a sample was taken, immediately quenched in water, and used for the determination of ferric percentage. Samples can only be taken before the pulverized material sinters and after it is at least partly liquid; the larger temperature intervals show when a sample could not be taken.

In the test in air, the slag began with a ferric percentage of 12. The percentage increased at a uniform rate until it reached a maximum of 94 per cent at 2100 F. Further increase in temperature caused dissociation, oxygen being driven off; at 2800 F the ferric percentage was reduced to 56.

In the test in unpurified nitrogen, the original ferric percentage was 97. There was no change in ferric percentage below 2130 F. The final ferric percentage differed little from that in air.

In the test in purified nitrogen, reduction began at 1700 F and at 2540 F the ferric percentage was 8. There is a possibility that this low value may be partly due to carbon particles from the tube being deposited on the slag.

Other slags gave the same type of result, except that the final ferric percentage of slags heated in air was smaller for slags with low iron-oxide content.

Time is required for oxidation, but reduction occurs quickly with the evolution of gas. Rate of oxidation depends upon area of surface, whereas reduction is independent of surface. Thus, when in practice, the ferric percentage of a bed of slag has been lowered by raising its temperature, the slag will retain that lower value after the temperature has been decreased, because oxidation is limited to action at the surface of the bed, where, over most of the area, the atmosphere will not be oxidizing.

GENERAL DISCUSSION

The authors believe that these tests and the deductions from them clear up some of the fog that has surrounded conceptions of the fusion characteristics of coal-ash slags, but the results have included no definite numerical values for the state of fluidity of slags cooled below the liquidus temperature.

The oscillating-bob viscometer is good in that it can cover a range of from 1 to even 10^8 poises, provided the slag remains a true liquid, but its action is extremely sensitive to the presence of solid phase.

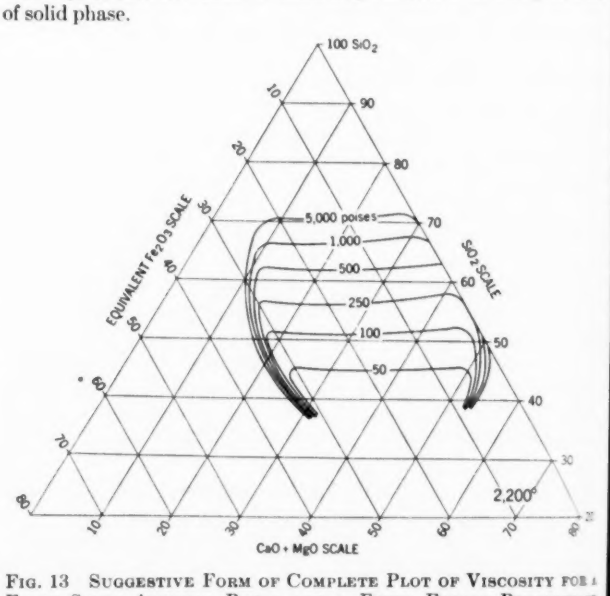


FIG. 13 SUGGESTIVE FORM OF COMPLETE PLOT OF VISCOSITY FOR A FIXED SILICA-ALUMINA RATIO AND A FIXED FERRIC PERCENTAGE

There is a great distinction between measurements of the state of fluidity at temperatures above and below the liquidus. Above the liquidus temperature, the slag is a true liquid and the same value for its viscosity will be obtained independent of the method used. Below the liquidus, the problem is entirely different. The numerical results obtained will depend upon the method and apparatus, the procedure and the time-temperature history of the thermal procedure. For example, the oscillating method would give higher values for equivalent viscosity than would the method of a bob rotating at a constant speed. In the procedure, the numerical value depends upon whether the slag is agitated to break up crystal formations. In these tests, measures of apparent viscosity at constant temperature have increased 100 per cent during a 15-min interval.

Thus, any method of measurement when solid phase is present in the slag will be arbitrary; for results to be duplicated there must be a rigid specification for procedure and for its relation to time.

The method of test to be used for temperatures below the liquidus has not been fixed because the correct choice is still in doubt. Three main possible principles are (1) rate or quantity of

flow under specified conditions, (2) force or work required to produce shear in the slag under specified conditions, and (3) deformation under specified conditions.

If the initial and softening temperatures are added to the viscosity-temperature plots, they show whether the initial melting is slow or fast and enhance the value of the plot.

Numerical values by any method could be expressed as apparent viscosity, under the definition that the apparatus and method are calibrated against a true glassy slag for which the viscosity-temperature plot has been determined.

Fig. 13 suggests the type of plot that would give complete data on the fusibility of coal-ash slags. The isoviscosity lines, approximately horizontal for the true liquid area are the same as those of the 2200 F plot in Fig. 8. The extensions of those lines begin from the liquidus line of Fig. 8 and are the apparent viscosity values. Their shapes would be as shown, but the distance between them is not based upon data, except that it is known they will be crowded closer together at the lower part than at the upper part of the plot. One plot would be restricted to one temperature, one ferric percentage, and a restricted range of silica-alumina ratios.

Acknowledgments

This investigation has been carried out under the authorization of A. C. Fieldner, chief engineer, Coal Division, United States Bureau of Mines. Paul Cohen, junior fuel engineer, assisted from the inception of this phase of the general investigation. The many chemical analyses required were made in the Chemical Section by F. H. Gibson under the direction of W. A. Selvig.

Acknowledgment is made to H. R. Lillie of the Corning Glass Works, Corning, N. Y., for supplying samples of glasses and slags with their viscosity curves; also to the Gulf Research Laboratory, Pittsburgh, Pa., for supplying samples of oils with their viscosity data.

Appendix

Description of Apparatus

Fig. 14 is a cross section through the furnace and viscometer; all the essential parts are labeled. Fig. 15 shows the complete apparatus; the switchboard in the background is for control of heating, and a multiple-tap transformer is below it; the apparatus beneath the table is for the purification and control of nitrogen for the atmosphere of the furnace.

Furnace

Experience in this investigation had shown that the operating temperature of a platinum-wound furnace should not be higher than 2750 F or its life would be short, as the temperature of the winding would be at least 2850 F. As the maximum operating temperature required was 2900 F or more, the upkeep of a platinum-wound furnace would be excessive.

The design adopted for the heater was a silicon-carbide tube 2 × 3 in. diam and 12 in. long. The end terminals are water-cooled copper plates, the spring pressure being on the upper plate. Turned lightweight refractory blocks surround the tube and are enclosed in a shell of high temperature sheet iron; the space between this and the furnace casing is filled with sil-o-cel powder. The casing is of welded construction, and the assembly is gas tight except for the opening at the top around the tubular element.

The regulation of the current is by means of a multiple-tap transformer of 3-v steps with variable resistors for adjustment between steps. The maximum load is about 3 kw.

The control and operation of the furnace has been satisfactory,

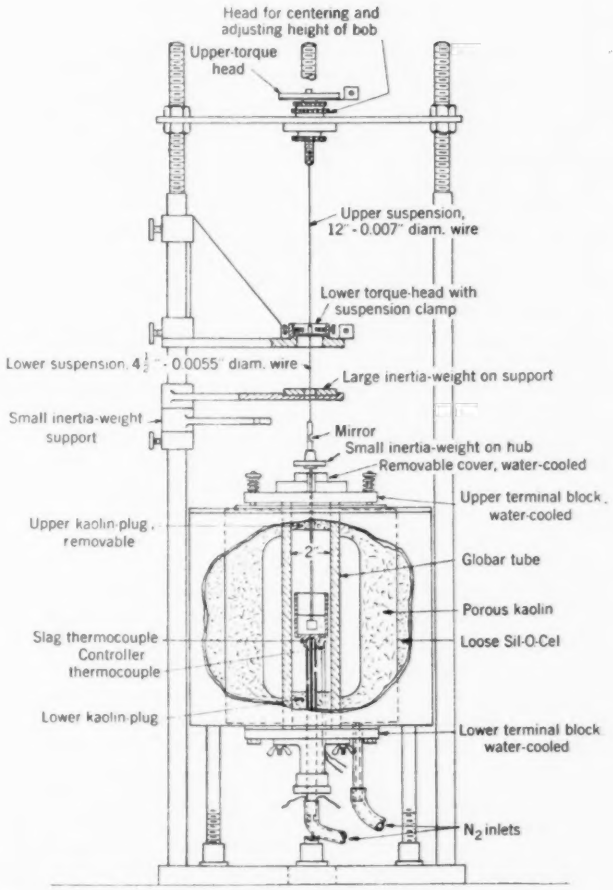


Fig. 14 Furnace and Viscometer for Determination of Viscosity of Coal-Ash Slags

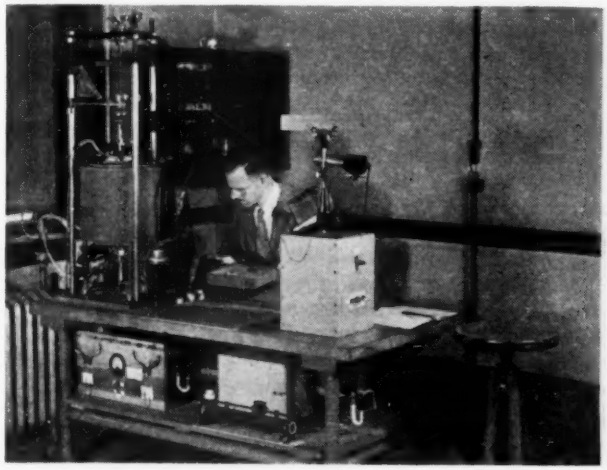


Fig. 15 General View of the Viscometer Setup

but the average life of the heating element has been only about 200 hr, or 15 to 20 heats, in not more than 5 of which a maximum temperature of 2930 F would occur. Failure has been caused by development of hot spots or cracks. Such local high temperature involves the additional risk that silicon vapor may be evolved and damage any platinum in the furnace.

Crucibles—Temperature Control

The crucibles have an inside diam of $1^1/_2$ in. and a depth of $2^1/_4$ in. Porcelain crucibles of special design are satisfactory for slags of total flux content of 15 per cent and a maximum temperature of 2900 F, or for a flux content of 25 per cent and a maximum temperature of 2800 F. Platinum crucibles are used for higher-flux slags or long exposure to high temperature.

A platinum 10 per cent rhodium thermocouple is used for temperature control, the hot junction being exposed to direct radiation of the furnace tube. Thus, with its small heat capacity, the thermocouple rapidly follows changing temperature of the tube and, through a potentiometer-activated temperature controller, maintains the slag at any fixed temperature within a maximum variation of 2 F. Thus it was possible to bring the slag to a given temperature and to hold it at that temperature indefinitely, without the furnace requiring constant supervision. At temperatures near the liquidus, this was of great value because momentary unobserved cooling of 10 F is sufficient, with some slags, to cause abrupt freezing of the melt.

Viscometer

The bob of the viscometer is $^1/_2$ in. in diam $\times$ $^1/_2$ in. long and has a $^3/_{32}$-in. spindle, both of platinum 20 per cent rhodium. The spindle extends outside the furnace and passes through a removable water-cooled cap which closes the furnace at the top and prevents heating of the upper part of the viscometer.

The viscometer has several ranges; two suspensions are connected in series such that both can be used as a unit, or the lower suspension only by clamping the junction of the suspensions at the lower torque head. The two inertia weights provided are of such moment of inertia that the smaller weight, in conjunction with the two suspensions, or both weights with the short suspension, form systems having a period of approximately 10 sec when oscillating in simple harmonic motion. This period of 10 sec was fixed as the shortest for which an observer could determine accurately the amplitude of an oscillation by the scale reading at the point where the oscillation reversed direction. For viscosities of less than 1 poise, both weights are removed and the period is about $2^1/_2$ sec.

When the damping effect of the viscous slag is so great that readings are not possible with harmonic motion, the inertia of the system is made its minimum and aperiodic motion is utilized, the time for the bob to rotate over a given angle being measured. The torque heads are equipped with magnets, so that a given twist of the suspension from the 0 position can be maintained until the bob has rotated to a preassigned position. A key switch is then operated, releasing the magnet so that the bob is free to rotate under a definite torque and, simultaneously, the electric timing clock is started. When the bob has rotated to the second preassigned position, the key switch is again operated, stopping the clock and reenergizing the magnet for the next timing cycle.

Viscometer Calibration

The original calibration was made with castor oil, the viscosity of which was checked in a modified Ostwald viscometer. Later, calibrations were made against a polymerized mineral oil having a much higher viscosity and also were measured in an Ostwald viscometer. These calibrations, made between 40 F and 180 F, could be compared directly and showed differences of about 5 per cent. Two calibrations were made at high temperatures with a synthetic iron-free slag and a glass, both of which had been measured previously in a Margules-type viscometer by H. R. Lillie, of the Corning Glass Works, who furnished the samples. Fig. 16 shows the comparison between the viscosity computed from the constants originally determined

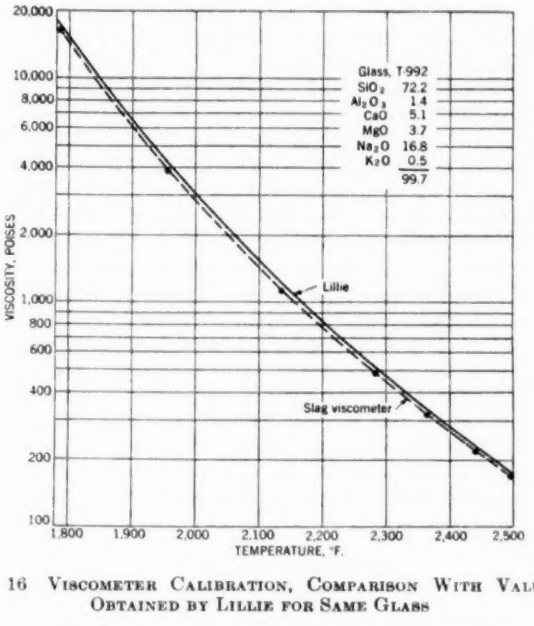


Fig. 16 Viscometer Calibration, Comparison With Values Obtained by Lillie for Same Glass

with castor oil and the viscosity for the same glass as reported by Lillie; the slag viscometer curve is approximately 5 per cent low. The constancy of deviation indicates that calibrations with castor oil are a sufficient check on the continued operation of the viscometer. The synthetic slag, which was of lower viscosity, has a similar agreement. The indicated accuracy of the instrument itself is approximately ± 5 per cent.

Two series of 50 consecutive determinations of the viscosity of two oils in this viscometer indicate that, for identical conditions of temperature and viscometer arrangement, with harmonic motion, the standard deviation from the true average will not exceed 1 per cent 99.9 per cent of the time, nor, for aperiodic motion, will the standard deviation exceed 3 per cent.

Discussion

R. M. Hardgrove.[15] The characteristics of slag are very important in the design and operation of large boiler furnaces fired by pulverized coal and stokers.

At the time the authors started this work, it was thought that the ability to tap the molten slag was the important feature. It still is necessary to tap the slag, but, as furnace designs have advanced, resulting in higher furnace liberations and the more general use of two-stage furnaces which have higher temperatures in the primary furnace, the difficulties in tapping slag have been reduced to a point where it is not of major importance.

The coating by slag of secondary-furnace walls, slag screens and superheater tubes is, however, a problem of major importance in both stoker-fired and pulverized-coal installations, and the determination of the characteristics of slag under these conditions is a subject worthy of intensive investigation.

The primary purpose of a boiler is to absorb heat. Slag coatings on the water-cooled walls reduce heat absorption and require operating labor to maintain the units in satisfactory working condition.

In boiler furnaces, there usually exist a reducing atmosphere in the primary furnace and an oxidizing atmosphere in the secondary furnace, in varying degrees. It is very essential that more

[15] Engineer in Charge, Design Department, The Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

research be carried on covering the full range from reduced to fully oxidized atmospheres. The writer would like to suggest that further work should approach the problem from that angle. The curves, such as Fig. 8, if completed to cover the possible degrees of oxidation, would be in a very useful form. Fig. 7 is a very interesting plot. In order to obtain smooth curves of constant temperature, it is not necessary to omit the Al_2O_3 as it can be added to the SiO_2 and the sum of SiO_2 and Al_2O_3 used as abscissa just as satisfactorily.

Viscosity and liquidus point may have some relation to the tendency to stick to furnace walls and tubes, but it is not clear just how it can be used. Perhaps the sticky characteristics at some temperature below the liquidus would be more analogous.

The authors have conducted a very thorough research and we surely hope they will find ways of continuing the program.

Authors' Closure

This opportunity is taken to add a figure which simplifies comparison of the properties of the slags indicated in Fig. 10 of the paper. Fig. 17 shows the viscosity curves and the cone-fusion temperatures of 8 coal-ash slags. The viscosity curves are for reduced slags of low ferric percentage, and the fusion temperatures are for the premelted slags and not for the original coal ash. Table 2 gives the chemical composition of the slags which are lettered in order of decreasing flux content; this designation is not the same as for the same slags in Fig. 10.

TABLE 2 COMPOSITION OF SLAGS OF FIG. 17

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
SiO_2	26.3	35.3	41.4	48.4	50.0	57.5	53.3	55.3
Al_2O_3	11.6	16.6	22.3	19.2	26.7	19.1	28.3	31.1
Equiv Fe_2O_3	46.1	9.9	31.4	12.1	13.1	13.2	13.7	9.5
CaO + MgO	16.0	38.2	4.9	20.3	10.2	10.2	4.7	4.1
	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Fe_2O_3 + CaO + MgO	62.1	48.1	36.3	32.4	23.3	23.4	18.4	13.6

Fig. 17 shows clearly the relationships between cone temperatures and viscosity for each slag, as well as those between the properties of slags having a wide range of composition.

Since the completion of the paper, viscosity and cone-fusion tests have been made on two glasses so that the results might be compared with those obtained for the glassy coal-ash slag. The values for the two glasses and for the glassy slag *F* of Fig. 17, are given in Table 3.

TABLE 3 VALUES FOR SLAG AND TWO TYPES OF GLASS

	Slag *F*		Pyrex		Soft glass	
	Deg F	Poises	Deg F	Poises	Deg F	Poises
Initial temperature	2160	9000	2020	60000	1480	100000
Softening temperature	2250	4000	2220	7000	1600	50000
Fluid temperature	2790	100	2900	600	2220	700

Table 3 shows that these three widely different types of glasses do not have the same viscosities at the initial, softening, and fluid temperatures, respectively. Thus the cone-fusion test is not a consistent measure for viscosity even for glassy slags. However, the relative positions of the I.T., S.T., and F.T. points of slags *F*, *G*, and *H* of Fig. 17, indicate that the corresponding viscosities of glassy slags of the coal-ash type may not differ so widely as do the glasses.

The authors take issue with some of the points raised by Mr. Hardgrove. Although tapping of slagging furnaces is not the problem that it was in the earlier days, yet he will agree that, even in recent installations, considerable trouble and expense could have been avoided if knowledge of the properties of the slags, such as that given by Fig. 17, had been available when the equipments were designed.

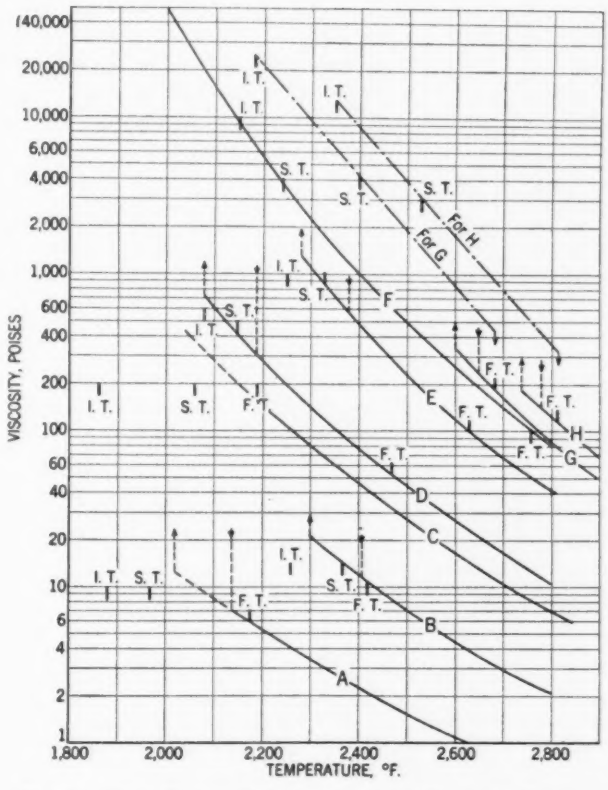


Fig. 17 Cone-Fusion Temperatures and Viscosity Curves for 8 Coal-Ash Slags

Again, data on viscosity and cone fusions, similar to those of Fig. 17, for any slag give primary characteristics for slagging properties much in advance of any previously available. As stated in the paper, more exact measures than those given by the cone, initial, and softening temperatures are required to express the properties of the slag below the liquidus temperature but, until such data are available, the cone-fusion temperatures give some indication of the mobility or plasticity of the slags.

Fig. 4 is a complete indication of the effect of ferric percentage over its entire range. These data were published in 1934,[6] and that paper should be consulted for the full application of Fig. 4. In addition, the authors have available similar data, as yet unpublished, on the effect of the ferric percentage on the "penetration" and the "sticky" temperature for the entire range of coal-ash slags.

Methods other than those omitting the Al_2O_3 can be used to reduce to three components for the plots of Fig. 8. The omission of the Al_2O_3 was found to be most satisfactory because the plot then shows the effect of silica content on viscosity, but silica plus alumina can be used for plots limited to a narrow range of ratio of silica to alumina.



Discussion

Cavitation Characteristics of Centrifugal Pumps Described by Similarity Considerations[1]

J. W. Macmeeken.[2] The authors prove in this paper the importance of the Thoma-Moody parameter $\sigma = \frac{H_{sv}}{H}$, for determining the safe lower limits of the absolute suction pressure above the vapor pressure of the fluid. Their discussion of this parameter as part of the general laws of similarity of machines of the same hydraulic type proves that no other criterion is necessary for determining the absolute pressure and suction-lift characteristic, provided consideration is limited to particular models and to series multiplications of these models.

However the whole field of centrifugal-pump applications would require an infinite number of models to cover the entire range of specific speeds. Often several different types of models for each specific speed are required to cover the variations of characteristic that are called for in centrifugal pumps in commercial use. Therefore it is necessary to have a knowledge of the variation of values of σ with specific speed and also to have a knowledge of the variation of σ in models of different hydraulic characteristics of the same specific speed.

The writer would like to emphasize that specific speed is only one of several type determinants in centrifugal pumps. This is true very particularly with reference to suction-lift or absolute-pressure parameters.

The curve shown in Fig. 8 of the paper represents a different value of σ for each specific speed throughout the range shown and the variation of σ value is continuous.

The importance of different values of σ for the same specific speed is illustrated in Figs. 1 and 2 herewith. Fig. 1 shows the characteristic curve of an 8-in. centrifugal pump, which delivers 2600 gpm against a total head of 180 ft at a speed of 1750 rpm. A breakdown occurs when the suction lift is 16.6 ft. Fig. 2 shows another impeller in the same pump, which delivers the same capacity and head but in which a breakdown does not occur until the suction lift is 26.1 ft. In Fig. 1, the value of σ is

$$\frac{32.7 - 16.6}{180} = \frac{16.1}{180} = 0.0895$$

but in Fig. 2 the value of σ is

$$\frac{32.7 - 26.1}{180} = \frac{6.6}{180} = 0.0368$$

These figures, taken from a popular size of commercial pump, show that a very great difference in the value of σ can exist in the same pump with two different impellers, both of which have the same specific speed.

Fig. 3 shows the characteristic of a centrifugal pump for the same capacity and head as before but with a speed of 3500 rpm instead of 1750 rpm. This shows a breakdown when the suction lift is 1.7 ft, which is equivalent to a σ value of $\frac{32.7 - 1.7}{180} = \frac{31}{180} = 0.172$. This shows that the value of σ is higher for higher specific speeds.

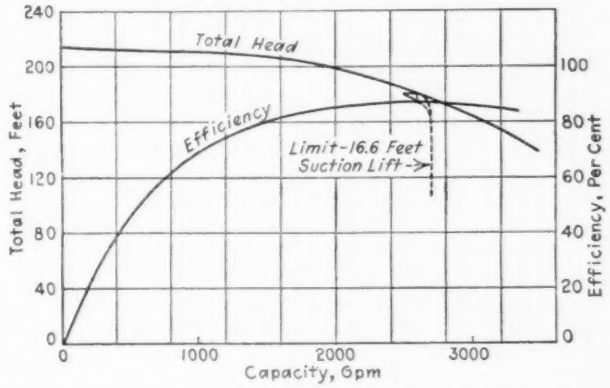


Fig. 1 Performance Curve, 8-In. Double-Suction Pump; 1750 Rpm Impeller—A

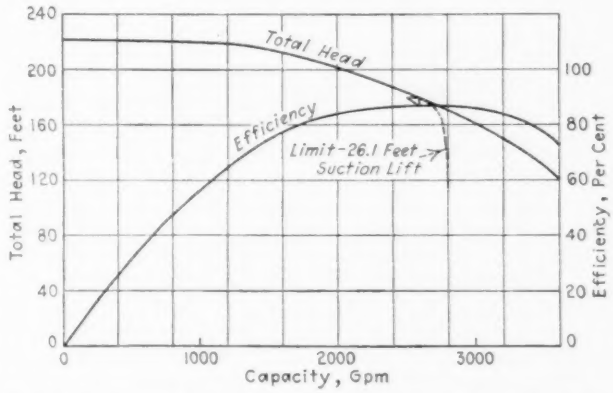


Fig. 2 Performance Curve, 8-In. Double-Suction Pump; 1750 Rpm Impeller—E

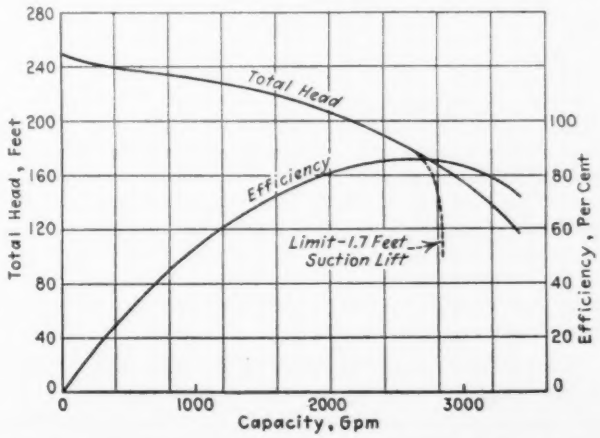


Fig. 3 Performance Curve, 8-In. Double-Suction Pump; 3500 Rpm

[1] Paper by G. F. Wislicenus, R. M. Watson, and I. J. Karassik, published in the January, 1939, issue of the A.S.M.E. Transactions, vol. 61, no. 1, pp. 17–24.

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It has been common practice among pump designers to calculate the absolute pressure required on the suction of centrifugal pumps by the use of dimensionless coefficients. The use of such coefficients and their limiting values permits the designer to lower the limits of suction pressure required with a fair degree of accuracy below average commercial practice, when suction lift is of great importance. The lowest possible suction pressures are required in condensate-removal pumps for surface condensers, for chemical-process pumps in conjunction with evaporators, and in well pumping, where submerged pumps are not required.

The parameter $S = \dfrac{N \times \sqrt{Q}}{H_{sv}^{3/4}}$ (Equation [6] of the paper), combined with the provision S = const and N_s = const (Equation [7]), appears on analysis to be inconsistent with observed tests.

Starting with Equation [6]

$$S = \frac{N \times \sqrt{Q}}{H_{sv}^{3/4}} = \text{const when } N_s = \text{const}$$

$$\therefore H_{sv}^{3/4} = N\sqrt{Q} \times \text{const, for a particular } N_s$$

$$\therefore H_{sv} = (N\sqrt{Q})^{4/3}$$

Now let

$$N = n\sqrt{H} \quad \text{and} \quad Q = q\sqrt{H}$$

n and q are sometimes called unit speed and unit capacity, respectively.

$$\begin{aligned} \therefore H_{sv} &= (N\sqrt{Q})^{4/3} \times \text{const} \\ &= (n\sqrt{H} \times \sqrt{q \times \sqrt{H}})^{4/3} \times \text{const} \\ &= (nH^{1/2} \times q^{1/2} \times H^{1/4})^{4/3} \times \text{const} \\ &= n^{4/3}q^{2/3} \times H \times \text{const} \qquad [1] \end{aligned}$$

$$\therefore \frac{H_{sv}}{H} = n^{4/3}q^{2/3} \times \text{const}$$

now let U_1 = peripheral velocity of D_s in ft per sec

and $K_{u1} = \dfrac{U_1}{\sqrt{2gH}}$

also let V_{su} = velocity of entrance at D_s in ft per sec

and $K_{su} = \dfrac{V_{su}}{\sqrt{2gH}}$

$$\begin{aligned} \text{Then } \frac{H_{sv}}{H} &= \left(\frac{K_{u1} \times 1840}{D_1}\right)^{4/3} \times \left(\frac{\pi}{4} D_1^2 \times K_{su}\right)^{2/3} \times \text{const} \\ &= K_{u1}^{4/3} \times C_1 \times K_{su}^{2/3} \times C_2 \times \text{const} \\ &= K_{u1}^{4/3} \times K_{su}^{2/3} \times C \qquad [2] \end{aligned}$$

which is the form of the equation in terms of dimensionless coefficients.

$$\begin{aligned} \therefore \text{Also } \frac{H_{su}}{H} &= \frac{U_1^{4/3}}{H^{1/2 \times 4/3}} \times \frac{V_{su}^{2/3}}{H^{1/2 \times 2/3}} \times C \\ &= \frac{U_1^{4/3} \times V_{su}^{2/3}}{H} \times C \qquad [3] \end{aligned}$$

Now two impellers of the same D_2 but with slightly different D_s may have the same N_s and the same capacity and head. The two impellers, the characteristics of which are shown in Figs. 1 and 2, are examples. Now consider the effect on the writer's Equation [3] of a slight change in D_s, as illustrated in Fig. 4. It is known from tests that a slight change in D_s results in a change in σ and S.

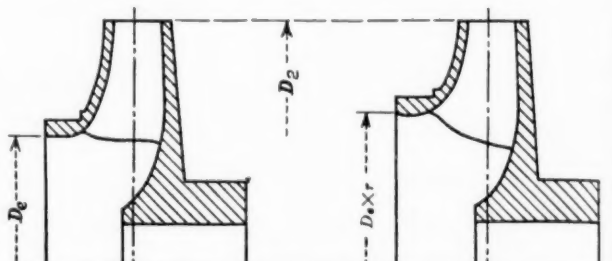


FIG. 4 EFFECT OF CHANGES IN INLET DIAMETERS OF IMPELLERS

Referring to Fig. 4, let D_2 be the diameter of both impellers and let the entrance diameter of impeller No. 1 be D_s, and let the entrance diameter of impeller No. 2 be $D_s \times r$, where r is a linear ratio, say, slightly greater than unity. By Equation [3]

$$H_{sv} = U_1^{4/3} \times V_{su}^{4/3} \times C \qquad [4]$$

where U_1 is the peripheral velocity of D_s in impeller No. 1 and V_{su} is the entrance velocity of flow at the eye of impeller No. 1. Then for impeller No. 2

$$\begin{aligned} H_{sv} &= (rU_1)^{4/3} \times \left(\frac{1}{r^2} V_{su}\right)^{2/3} \times C \qquad [5] \\ &= r^{4/3}U_1^{4/3} \times \frac{1}{r^{4/3}} V_{su}^{2/3} \times C \\ &= U_1^{4/3} \times V_{su}^{2/3} \times C \end{aligned}$$

the same as H_{sv} for impeller No. 1 (see Equation [4]).

That is, Equation [4], which is the equivalent of Equation [6] of the paper is such that, in impellers of the same capacity and specific speed, a slight change in D_s only, makes no change in H_{sv} or in the parameters σ and S.

If this were true and in agreement with actual tests, S would indeed be a valuable parameter and the calculation of σ would always be very simple, because it would be independent of the value of D_s. But unfortunately tests show that a change in D_s only, changes the value of σ. Hence the writer believes that the parameter $S = \dfrac{N\sqrt{Q}}{H^{3/4}}$ is proved to be of incorrect form by the method of reductio ad absurdum.

Instead of the Equation [6] of the paper and its derivatives such as the writer's Equation [2], it may be proved that

$$\frac{H_{sv}}{H} = K_{u1} \times K_{su} \times C_1 \qquad [6]$$

and then instead of [5], we would have

$$\begin{aligned} H_{sv} \text{ (for impeller No. 2)} &= rU_1 \times \frac{1}{r^2} V_{su} \times C_1 \\ &= \frac{U_1 V_{su}}{r} \times C_1 \end{aligned}$$

Now H_{sv} (for impeller No. 1) $= U_1 V_{su} \times C_1$

Therefore σ for impeller No. 2 is smaller than σ for impeller No. 1 in the ratio $\dfrac{1}{r}$: 1, or the impeller with the larger D_s has the lower value of σ. This is in agreement with a large number of

tests of many different impellers. The few impellers which are not in agreement have some feature of design which causes an interference such as too high entrance ratio, or too many vanes.

The writer has found that the variation of σ with N_s is best expressed by the following equation

$$\sigma = K \times C \times N_s^{4/3} \quad [7]$$

where K is a performance coefficient and is dependent on hydraulic efficiency, type of suction approach, length of suction pipe, degree of saturation of the fluid with air or gas, and on the inlet ratio of impeller vanes.

On the other hand C is a constant which is a measure of the relative size of inlet and has well-defined limits. For example, large C represents a relatively small D_s and low value of C represents a relatively large D_s, where D_s is the entrance diameter of the impeller. The value of C therefore determines, if it is within certain limits, whether the efficiency may be affected.

Values of K may be placed in three classifications: (1) When absolute suction pressure is sufficiently high that the pump efficiency is not affected; (2) a certain lower value of K means that the pump efficiency is slightly affected; (3) a still lower value of K is obtainable only with deaerated water, such as is obtainable in condensate-removal pumps, in evaporators, and in process pumps, where there is no quantity of gas dissolved in the fluid.

The derivation of the writer's Equation [7], with theoretical and practical values of K and C, is too lengthy to be given in detail in this discussion.

The development of Equation [7], however, differs from that of Equation [6] of the paper in that Equation [6] of the paper is a function of rotational head, whereas the writer's Equation [7] is derived from the rotational head, modified by the conditions of velocity of approach in the suction element of the pump. The velocity of approach is a function of capacity and not of rotation.

The writer would emphasize that the use of parameter S, given by the authors, is capable of serious error in the determination of limits for σ throughout the whole range of specific speed, because it merely represents a hydraulic condition and not the ability of any particular model to cope with the hydraulic condition.

In particular the authors' parameter S is, from its form, not influenced by change of impeller D_s which is contrary to test experience.

If the use of the authors' parameter S is restricted to machines of true similarity, it is totally unnecessary because then the Thoma-Moody parameter σ is all that is required.

The writer appreciates fully the very consistent mathematical discussion of the authors on this subject, which contributes to a better understanding of the subject; and he offers his criticism of the parameter S, merely because he finds it is not in agreement with test results and, because its use may result in serious confusion and possible errors in determining the best size and most economic speed of a centrifugal pump for the absolute suction pressure available.

L. F. Moody.[3] The occurrence and causes of cavitation in hydraulic machinery seem to have been first studied in the fields of marine propellers and hydraulic turbines. But little published material has appeared concerning cavitation in pumps. This cannot be explained by the absence of cavitation from centrifugal pumps, because no such happy state exists. While its occurrence in pumps is usually of somewhat less consequence than it is in great modern turbine units, its presence even in small pumps is far from welcome.

The authors have investigated the cavitation characteristics of centrifugal pumps as related to specific speed, as has previously been done empirically for turbines.[4] In so doing they have introduced a new cavitation coefficient, having the same dimensions as specific speed, and have deduced, by somewhat intuitive reasoning, an approximate functional relationship between the Thoma cavitation coefficient σ and specific speed. The relationship amounts to the proposition that, at least in low-specific-speed pumps, σ should vary as the four-thirds power of the specific speed, very nearly; or σ = (a constant) $N_s^{4/3}$. For this constant they have used their new cavitation coefficient S, putting $\sigma = (1/S^{4/3})N_s^{4/3}$.

They recognize that with material changes in design characteristics, as with widely different specific speeds, S may not be strictly constant, but they justly argue that S should change very slowly, and perhaps inappreciably, over wide ranges of N_s.

The writer has for several years been attempting to collect results of pump cavitation tests and to plot σ values vs. N_s. The data so far obtained seem to bear out in a general way the authors' thesis; a curve drawn through the centers of gravity of the mass of plotted points agrees well with the functional relationship they present. The writer also recalls arriving very nearly at the same conclusion in a theoretic analysis in which he showed that, for turbines and pumps of similar design characteristics, part of the cavitation factor, represented by velocity head at the runner throat, varies as $N_s^{4/3}$. Curves were plotted on this basis.[5] This is merely mentioned in partial confirmation of the authors' conclusion. The writer considers that the authors in their analysis have made a useful step in advance.

The practical difficulty in applying the results of such an analysis seems to be greater for pumps than for turbines. While limiting safe values of σ for Francis turbines have been found to vary rather consistently with specific speed, so that a fairly definite envelope could be drawn and usefully applied in practice, the spread of points in the field of propeller turbines is so great that such a curve can be used in that field only as a rough preliminary guide. It has been recognized that thorough cavitation tests for each individual design are the only dependable index to safeguard a new propeller-turbine installation.

In the pump field, however, the points, collected from various sources and from pumps by different builders, show so great a scattering throughout the entire field that it becomes a matter of judgment where to draw an envelope that will give reasonable safety in general practice.

The writer has come to believe that a general chart for pumps of any and every make cannot be a safe (and at the same time economical) guide. Only for a particular line of pumps of consistent design characteristics can a general curve or chart be practically applied as a safe and useful index. For example, a curve forming an envelope of the zone of reasonably safe practice for the general run of commercial pumps is widely different from one representing pumps of special design, such as the class of highly efficient pumps furnished by several American builders for the Metropolitan Water District of Southern California and tested with great accuracy at the Pasadena Laboratory. The σ for certain pumps plotted in the chart are as much as double those of other pumps of the same specific speed. The writer feels that the Utopia to be sought is the happy day when each individual pump will be tested not only for capacity and efficiency but for its cavitation coefficients as well.

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[4] "Inter-Relation of Operation and Design of Hydraulic Turbines," by F. H. Rogers and L. F. Moody, *Engineers and Engineering*, vol. 42, 1925, pp. 169–187.

[5] "The Present Trend of Turbine Development," by L. F. Moody, Trans. A.S.M.E., vol. 43, 1921, p. 1134.

It is far more calamitous to a customer to secure a pump which is found to operate in the cavitating region than to lose a per cent or two in efficiency. When it is considered that many a pump might have been installed at a slightly lower elevation, often at no extra cost, and thereby freed from serious operating difficulties such as noisy and unstable operation and, still worse, frequent shutdowns and impeller replacements, it should be realized that it is at least as important to know the safe value of σ as to know the efficiency. According to a chart, a pump may figure out as safely applicable to a given case, but the pump itself may be unaware of the fact and refuse to conform.

It is also felt that many tests which are thought to determine the cavitation factor do not really do so. For example, the practice sometimes employed of depending on the value of ultimate capacity or "breakdown point" for a given suction head is not a proper criterion, as shown in Fig. 2 of this paper. Its use probably results in many pumps operating in the cavitating range.

A. J. Stepanoff.[6] The fact that similarity of suction conditions as to cavitation for pumps of different specific speeds can be expressed by a parameter, similar in appearance to the expression for specific speed, is interesting but, the writer feels, its practical usefulness is very limited.

For the development of the connection between the cavitation parameters S or σ and specific speed, the authors make an assumption that the suction conditions, or the suction absolute pressure above the vapor pressure when cavitation starts, remain the same for the same capacity, if the impeller diameter is cut and the speed is constant. This means that the absolute pressure at which cavitation starts will depend on capacity only or, since the suction passages are fixed, it will be a function of the velocity through the impeller eye. Furthermore, when the pump speed is changed, the variation of the head, capacity, and power is the same as when the impeller diameter is cut at constant speed. Therefore, with equal reason it can be stated that in this case also the absolute pressure at cavitation conditions is a function of the capacity or radical velocity through the impeller eye only. This is confirmed very closely by actual tests on pumps of low specific speed, and only approximately on pumps of medium specific speed.

For correlating the experimental data on cavitation of centrifugal pumps of different specific speeds, accumulated in the course of several years, the writer has plotted the velocity heads, calculated for average radial velocities at the impeller inlet against the total suction lift, measured at the suction nozzle, and referred to the pump-shaft center line. These curves were published[7] and discussed in February, 1933. The writer has used these curves for several years since and has found them to be satisfactory for predetermination of pump cutoff capacity for given suction conditions for commercial pumps of low specific speed of about 1200 or less (single-suction pumps) when pumping water. It was found that, with pumps of low specific speed, the head-capacity curve dropped so suddenly under cavitation conditions that no decrease in efficiency or head was observed before the pump actually cut off. For slightly higher specific speeds, 1200 to 1500, a slight drop in head and efficiency was observed considerably before the maximum capacity was reached. For small commercial pumps this was disregarded as unimportant. This method of plotting the test data on cavitation was suggested by the observation that, for pumps of low specific speed, the maximum capacity, at constant suction conditions, does not change when the impeller diameter is cut. Also, it was found that the cutoff capacity changes very little, or not at all, when the speed of the pump is changed and the total suction lift kept constant.

This method of plotting cavitation-test data was used by Defeld in his book,[8] published in 1930. Later the writer prepared similar curves, based on the limited number of observations for boiling water and petroleum oils, which he has found to be useful in his work.

The writer does not share the authors' enthusiasm about the usefulness of the cavitation parameter S.

$$S = \frac{NQ^{1/2}}{H_{sv}^{3/4}} \qquad [8]$$

because it is possible to establish a much simpler relationship between the cavitation parameter σ and the specific speed N_s. Von Widdern[9] developed such a relationship over three years ago. He made essentially the same assumptions as the authors, that two impellers of different diameters and different total heads for the same capacity and at the same speed produce the same dynamic depression, or require the same suction head to suppress cavitation

$$\Delta h = \frac{c_1^2}{2g} + \lambda \frac{w_1^2}{2g} \qquad [9]$$

where $c_1^2/2g$ is the absolute velocity head at the impeller entrance and $\lambda w_1^2/2g$ is the drop of pressure below average on the back side of the vane, due to the power applied to water by the vane; w_1 is relative velocity at the impeller entrance; λ is an experimental coefficient similar to the lift coefficient of an airfoil. By definition

$$\sigma = \frac{\Delta h}{H} \qquad [10]$$

For two impellers of different outside diameters otherwise identical, operating under the same suction conditions, the following equation can be written

$$\frac{\sigma_2}{\sigma_1} = \frac{H_1}{H_2} \qquad [11]$$

In the expressions for the specific speed of the two impellers the speed N and capacity Q are the same. Therefore

$$\frac{N_{s1}}{N_{s2}} = \left(\frac{H_2}{H_1}\right)^{3/4} \qquad [12]$$

Combining Equations [11] and [12] we obtain

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{N_{s2}}{N_{s1}}\right)^{4/3} \qquad [13]$$

which is the desired relationship between the cavitation parameters for two impellers defined above in terms of specific speed. Von Widdern has also published experimental data on cavitation, based on a large number of observations, showing the validity of the relationship as given by the writer's Equation [13], if the difference in specific speed of the two pumps being compared is not too great.

The advantages of using parameter σ over the authors' parameter S is that σ has a definite physical meaning, well known to hydraulic engineers. Calculations when using σ are simpler.

[6] Assistant Chief Engineer, Byron Jackson Company, Los Angeles, Calif. Mem. A.S.M.E.

[7] "Sudden Break-Off of the Head Capacity Curve, Etc.," *Byron Jackson News Letter* no. 15, vol. 6, Feb. 15, 1933.

[8] "A Practical Treatise on Single and Multistage Centrifugal Pump," by Raymond Defeld, Chapman & Hall, Ltd., London, 1930.

[9] "On Cavitation in Centrifugal Pumps," by H. Cardinal von Widdern, *Escher-Wyss News*, January–March, 1936, p. 14.

Besides, there are considerable experimental data on cavitation expressed in terms of σ already accumulated by the pump and turbine designers.

The relationship given by Equation [13] is only implied by the authors' Equation [4]. By combining Equation [13] of this discussion and [4] of the authors', it can be shown that the parameter S is a constant for impellers of different specific speed, obtained by variation of the outside diameter only.

Both the authors and von Widdern have established the relationship between the cavitation parameter and specific speed of pumps referred to the operating point. However, when specific speed is used as a pump-type number, this is referred to the best efficiency point on the head-capacity curve. When the impeller diameter is cut, the normal capacity for the best efficiency point decreases in direct ratio of impeller diameter or square root of the normal head.

$$\frac{Q_{1n}}{Q_{2n}} = \left(\frac{H_{1n}}{H_{2n}}\right)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [14]$$

Following Bischoff and Thoma[10] the cavitation parameter σ for all points on the constant-speed head-capacity curve for convenience can be referred to the normal head H_n.

$$\sigma = \frac{\Delta h}{H_n}$$

and for the same dynamic depression Δh, the ratio of the cavitation parameters for two impellers at the same capacity and speed but referred to the normal head will become

$$\frac{\sigma_2}{\sigma_1} = \frac{H_{n1}}{H_{n2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [15]$$

The ratio of the specific speeds for the normal conditions will be

$$\frac{N_{s1}}{N_{s2}} = \left(\frac{Q_{n1}}{Q_{n2}}\right)^{1/2} \left(\frac{H_{n2}}{H_{n1}}\right)^{3/4} = \left(\frac{H_{n2}}{H_{n1}}\right)^{1/2} \quad \ldots\ldots\ldots [16]$$

and the ratio of the cavitation parameters in terms of the specific speed at the point of best efficiency will become

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{N_{s2}}{N_{s1}}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [17]$$

It is interesting to note that, if the relationship as given by the writer's Equation [17] is plotted on log scales, the curve will have a slope intermediate between that of the border lines on the Pasadena Laboratory tests reproduced in Fig. 7 of the paper and the curve based on the Hydraulic Institute Standards chart.

In their attempt to demonstrate the usefulness of the parameter S, the authors go so far as to suggest "factoring" of the impeller suction passages from one specific speed to another. This evidently is not acceptable to anybody familiar with pump design. There is already a well-established experimental relationship between all of the controlling impeller proportions including suction area, number of vanes, and entrance angles, for different specific speeds. This is so rigid in consistent designs that the ratio D_1/D_2 of the impeller eye and outside diameters was used as a type number in the past in the same manner as specific speed is universally adopted now. To destroy these proportions will immediately impair the best possible pump peak efficiency. Besides, it is well known that the points of minimum pressure under cavitation conditions are located in back of the impeller vane, some distance from the vane inlet tips. This low-pressure zone in pumps of medium and high specific speed may extend as far as half of the vane length. Thus, if the portion of the impeller vanes affecting cavitation is factored, it would be impossible to design the outer half of the impeller to match the impeller suction and obtain the desired head capacity with good efficiency. Apparently water pumps are, and will be, designed for the best possible efficiency, but the selection of pumps and the layout of pumping plants will be made with due regard to cavitation conditions.

It should be noted that the relation between the cavitation parameter σ and specific speed, established by von Widdern, applies equally to hydraulic turbines.

The authors' paper contains a number of authoritative statements which are either contrary to the facts, or are generalizations not justified in the light of the present state of knowledge of centrifugal pumps. For instance, the statement is made: "The combined influence of the changes in Reynolds' number and in the relative roughness is reasonably well known through test results with geometrically similar pumps of different sizes and for different speeds, and it is to be expected that the influence of these factors on the cavitation behavior of a pump will be of the same order of magnitude as their influence on capacity, head, or efficiency." Is it meant here that the effect of viscosity on the head, capacity, and efficiency is "reasonably well known" when pumping viscous liquids, such as petroleum oils? If not, why mention viscosity, Reynolds' number, "fluid," "machine," and not confine the discussion to water centrifugal pumps?

With water pumps and turbines, the question of determination of full-size pump efficiency from model tests, or efficiency of the same pump at different speeds, even when the suction conditions are properly adjusted, is still debatable. The difficulty lies in the fact that the change in efficiency with pump size and speed are of the same order as the accuracy of commercial testing instruments.

At another point, the general statement is made that "for most practical cases we need not include the temperature, vapor pressure, or nature of the fluid in cavitation formulas or charts, except in so far as these factors have an effect on the type of pump used." If we take petroleum oils, liquids next to water in importance in the pump business, the behavior of pumps under cavitation conditions is entirely different from when pumping water. Any formulas, charts, or test data obtained on water cannot be directly applied to oil-pumping problems.

On the other hand, the writer does not see why the established parameter σ or S, and formulas giving their connection with specific speed, cannot be applied with equal reason to hydraulic turbines, as the authors claim; or, why "the total turbine head and the net suction head are more intimately related, so that a separate consideration of suction and discharge flow conditions may not offer a useful approximation?"

The fact that the lines of constant parameter S in Fig. 7 happen to be in "excellent agreement" as to slope of the shaded area, the writer considers as a plain coincidence and not a "startling verification" of the practical usefulness of the cavitation parameter S, because the shaded area includes a limited number of points obtained with pumps of different specific speeds and of different makes. The actual slope of the lines drawn through two or more points corresponding to pumps of different specific speeds but of the same design and make may be different from the slope of the bordering line enclosing the whole shaded area.

W. M. White.[11] The cavitation characteristics of centrifugal pumps have been rather neglected in the development of the

[10] "Untersuchungen über das Verhalten Einer Kreiselpumpe bei Betrieb im *Kavitationsbereich*," by Armin Bischoff and D. Thoma, *Mitteilungen des Hydraulischen Instituts de Technischen Hochschule*, Munich, vol. 8, 1936, p. 56.

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industry. The first real step in their study was that undertaken by the Hydraulic Institute in order to establish recommendations for limiting specific speeds to eliminate troubles from vibration and pitting.

The original work on this study was done without the use of the Thoma-Moody parameter σ, the data being assembled by plotting specific speed against total head and drawing lines of constant suction conditions through the critical points. The use of the σ-values would have aided this work materially. The authors have added considerably to this by their new cavitation parameter S. It is believed that, for low-specific-speed pumps, the relations set forth should be reliable. However, it should be emphasized particularly that at higher specific speeds the use of this method is apt to be in error.

There is no assurance that the limiting value of S should be constant, regardless of the specific speed. The writer has plotted numerous pumps on the specific speed-S basis and can find no definite limiting value of S. On the other hand, when these same values are plotted on a specific speed-σ basis, the curves show a very definite trend from specific speeds of 500 up to 10,000 rpm, which covers the range of centrifugal, mixed-flow, and propeller-type pumps.

Fig. 10 of the paper is a good way to present the limiting capacities of various suction heads. There is danger, however, that it would be used for pumps in the higher specific-speed range and, therefore, would give misleading results.

M. SEDILLE.[12] The coefficient S introduced by the authors appears to mark considerable progress in the presentation of test results of cavitation in pumps. It is particularly important for the similitude calculations of pumps having similar inlet waterways and different outside dimensions, in other words, different values of N_s.

As the authors have shown, the advantage which this coefficient offers of being independent of what happens at exit from the impeller holds good only for impellers where this independence is clearly assured, i.e., for pumps having a low or moderate specific speed.

It is, therefore, not surprising that the authors of the paper have found by tests that S is practically independent of N_s in the region of corresponding specific speeds.

There exists another method of presentation which has the same properties as the coefficient S, as far as independence from the phenomena at impeller exit is concerned, but which starts out from an entirely different order of ideas. It is derived directly from the coefficients introduced by Rateau in the theory of turbomachines.

Referring to Fig. 5, it is well known that Rateau's characteristic coefficients have the form

$$\mu = gH/u_1^2 = \text{manometric coefficient} \quad [18]$$

$$\delta = Q/u_1 r_1^2 = \text{capacity coefficient} \quad [19]$$

H signifies the pump head and Q its capacity. There exist other coefficients which will, however, be ignored at this time.

Concerning the inlet waterways of the pump, the introduction of a supplementary variable, the suction head, requires a new dimensionless coefficient to follow this variation. It is natural to select as the new variable the analog of the manometric coefficient referred this time to the pump inlet

$$\mu_0 = gh_0/u_0^2 \quad [20]$$

where h_0 is the pressure necessary to make the water penetrate into the pump.

[12] Ingenieure AM et ECP. Ingénieur à la Société Rateau, Paris, France.

It is now possible to express the variations of μ_0, as for example, those of μ as functions of δ. In this study it is advisable to replace the variable δ by the analogous variable δ_0 referred to the inlet diameter of the impeller

$$\delta_0 = Q/u_0 r_0^2 \quad [21]$$

The pressure required to make the water penetrate into the pump may be defined as the difference between the dynamic pressure at the pump inlet and the minimum static pressure in the impeller, in other words

$$h_0 = \frac{V_0^2}{2g} + \left(\frac{w^2 - u^2}{2g}\right)_{\max} - \frac{w_0^2 - u_0^2}{2g} \quad [22]$$

This head is thus independent of atmospheric pressure and of the temperature of the water.

It is impossible to measure h_0 directly. It can be determined

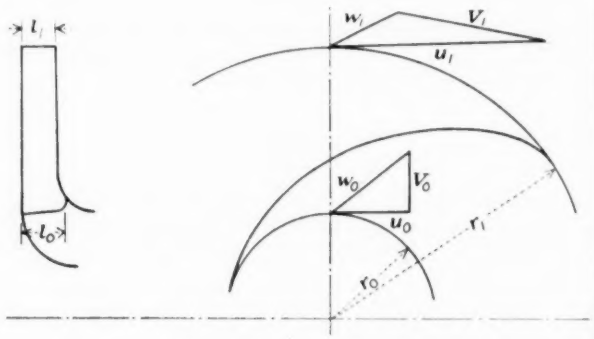


FIG. 5 NOTATION DIAGRAM



FIG. 6 CURVES OF μ VERSUS δ FOR DIFFERENT VALUES OF μ_0

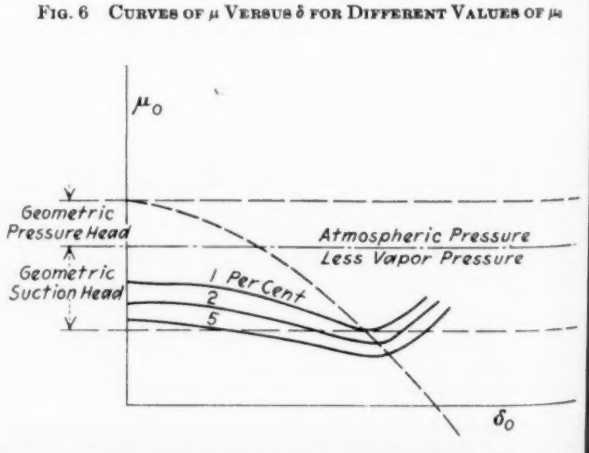


FIG. 7 CURVES OF μ_0 VERSUS δ_0 FOR DIFFERENT CAPACITY REDUCTIONS WITH CONSTANT INLET OPENING

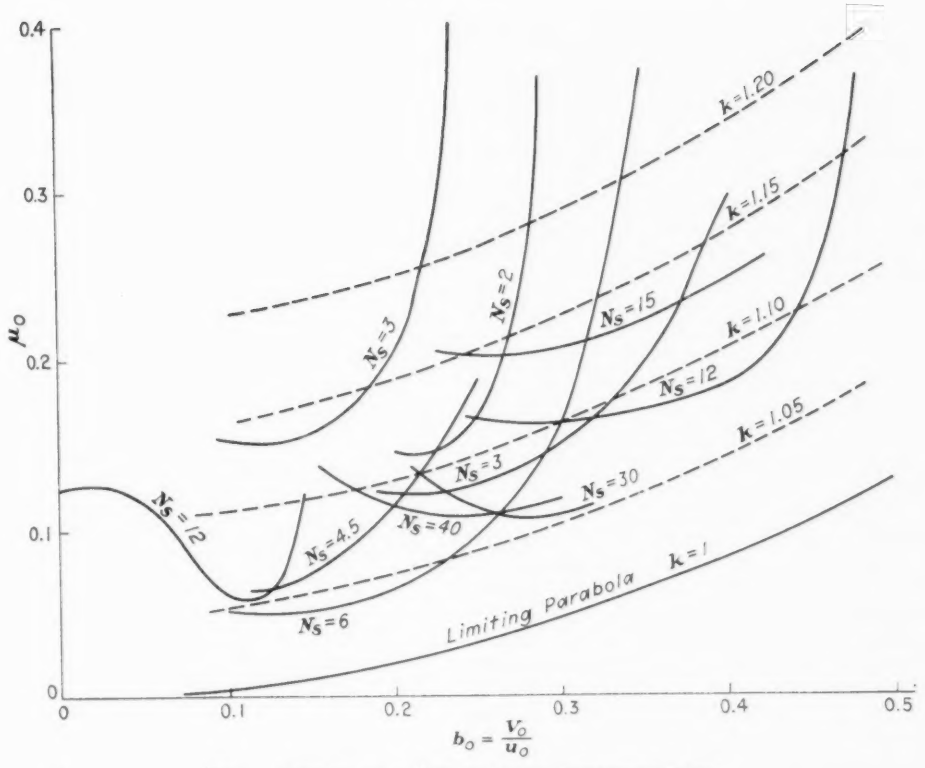


FIG. 8 CURVES OF μ_0 VERSUS b_0 FOR DIFFERENT PUMPS

only as a difference by measuring the suction head of the pump. There remains, however, the difficulty of establishing accurately the beginning of cavitation.

It is known that the curve (μ, δ) of a pump is strongly affected by cavitation as soon as the latter appears. If the suction lift is increased, the curves are noticeably deformed in the region of large capacities which is the most affected. This is due to the well-known drop in pump head and to the fact that the capacity rapidly approaches a limiting maximum value. Some pumps are also affected in the region of low capacities.

If a curve of capacity as a function of suction head is plotted for an opening of known properties, it is difficult to determine the point where cavitation starts to cause a reduction of capacity. For this reason the practice has been adopted of defining the suction head as the head for which the capacity is reduced by a certain percentage, 5 per cent for instance. There is no difficulty in establishing by tests the corresponding suction head and possible arguments can be easily avoided.

The performance of a given pump can then be completely defined by the family of curves (μ, δ) as a function of the parametric value μ_0, Fig. 6, and by the different curves of μ_0 as a function of δ_0 for the desired percentage reduction of capacity for constant opening. The latter curves can be easily derived from points of the former, Fig. 7.

The curves μ_0 make it possible to follow suction performance under different conditions. If the suction circuit has only a geometric static suction head (mine pumps) the characteristic of the circuit is horizontal, Fig. 7, and if the curve μ_0 of a pump has the form shown in the figure, an interruption of the flow may take place, either by increase, or decrease of the capacity. In the more general case the pump circuit has a suction head corresponding to the flow in suction pipe. The characteristic of the circuit will, in this case, be a parabola with its concavity turned downward.

The comparison of curves (μ_0, δ_0) of different pumps gives interesting data concerning the forms of the inlet openings and of the impeller near the inlet.

In order to eliminate entirely the influence of the impeller width at the inlet one can plot cavitation curves by referring μ_0 not to the variable δ_0, but to a variable $b_0 = V_0/u_0$ which is a direct function of the vane angle at the inlet.

The resulting curves, the general character of which is shown in Fig. 8 and which have been used for a long time by the Rateau Company, make it possible to obtain directly interesting data concerning the part played in the total head by the two partial heads

$$h_{01} = V^2/2g$$

$$h_{02} = \left(\frac{w^2 - u^2}{2g}\right) \max - \frac{w_0^2 - u_0^2}{2g} \quad \ldots\ldots\ldots [23]$$

which is, in fact $\mu_{01} = b^2/2$

$$\mu_{02} = \mu_0 - (b^2/2) \quad \ldots\ldots\ldots\ldots\ldots [24]$$

The simple form of the chart giving S as function of N, does not permit this separation. It is, further, necessary to know the relative dimensions of the impeller inlet of the tested pump.

In particular, for axial-flow pump

$$h_{02} = \frac{w^2}{2g} \max - \frac{w_0^2}{2g} = \frac{w_0^2}{2g}(k^2 - 1) \quad \ldots\ldots\ldots [25]$$

where k is a coefficient greater than one, which measures the ratio between the maximum velocity along the blades and the inlet velocity.

This separation makes it possible to obtain directly the value of the velocity coefficient k. The latter is influenced by the profile of the waterways, the wall thickness of the inlet opening, the radii of curvature near the edge of the inlet, and so on. It is possible to plot directly on the cavitation chart lines k = const.

As the coefficient b_0, as abscissa, depends only upon the shape of the inlet opening, the cavitation curves of pumps of different specific speeds are located in various regions of the chart. It is worth noting that in the same region of the chart are located the cavitation curves of entirely different pumps. At the same time, pumps with the same N_s can be treated in the same way though they may be situated in different regions of the chart.

To sum up it might be stated that the μ_0-b_0 chart permits judging directly the properties of the inlets of an impeller and comparing inlets which are otherwise quite different, having, for instance, different values of S and N_s.

The S-N_s chart possesses another inconvenience. The N_s of a pump varies with the location of its operating point. It happens sometimes, despite the precautions taken by the designer, that the location of highest efficiency does not coincide with the location of maximum suction. In such cases it is necessary, to indicate which are the values of capacity and of head used for the calculation of these coefficients, and it would be for instance, illogical to calculate S on the basis of values corresponding to location of maximum suction and N_s on the basis of values corresponding to maximum efficiency.

All this, however, does not detract from the usefulness of the coefficient S recommended by the authors and it is quite possible to use both methods which complement each other. The S-N_s chart will be especially useful for users of pumps and sales departments, while the μ_0-b_0 charts have a greater interest for engineers. On the other hand one might criticize the coefficients S and N_s as not being dimensionless and to changing their value according to location and the system of units used.

It is, however, easy to select instead of S and N_s analogous and proportional coefficients which will remain constant, provided that their different components are expressed in a coherent system of units.

In the writer's opinion, the value simplest to define is

$$N_s = \frac{N(t/\text{min})Q^{1/2}}{(gH)^{3/4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [26]$$

which can be expressed immediately by means of the Rateau coefficients as

$$N_s = \frac{30}{\pi}\,\delta^{1/2}\mu^{-3/4} \sim 10\,\delta^{1/2}\mu^{-3/4} \quad \ldots\ldots\ldots [27]$$

Similarly, a cavitation coefficient can be expressed by

$$s = \frac{N(t/\text{min})Q^{1/2}}{(gh_0)^{3/4}} \sim 10\,\delta_0^{1/2}\mu_0^{-3/4} \quad \ldots\ldots\ldots [28]$$

The coefficient N_s, as defined long ago is still in current usage, especially in the design of hydraulic turbines. It has not yet gained general recognition in centrifugal-pump engineering. The writer hopes that the authors will make efforts to introduce, at least in this field and, if possible, for general use, a dimensionless coefficient of specific speed of the type just described.

PAUL BERGERON.[13] Thanking the authors for mentioning in their paper the application previously made in France of the properties they have developed, the writer wishes to remark that since 1931 he has frequently and satisfactorily used the principle of adapting the impeller of a pump known with regard to cavitation to a pump of different specific speed. The principle is exact and safe if the specific speed of the pump which has to be designed is lower than the specific speed of the pump taken as a starting point, in other words, if the impeller vanes of the new pump have to be made longer than those of the model pump.

[13] Ingenieur École Centrale des Arts et Manufactures, Paris, France.

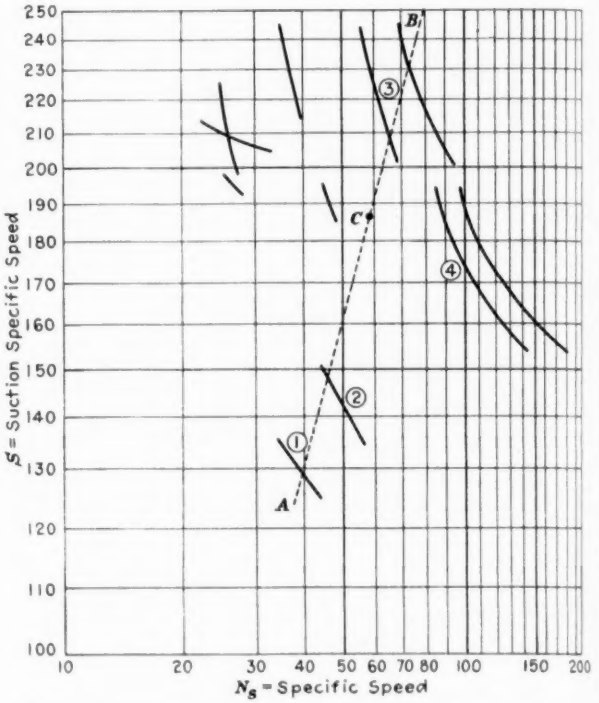


FIG. 9 CURVES OF S VERSUS n_s FOR A PUMP WITH ONE INLET
(Capacities in cubic meters per second, head in meters.)

In the contrary case, the application of the principle is difficult, especially if it has not been preceded by tests of successive cutting down of the impeller of the model pump to determine the zone where the discharge is affected by the suction.

Except for pumps of very low specific speed, the zone affected by the suction is very near the outlet diameter, which latter is determined by the suction possibilities of the pump. In fact, it should be noted that the same head-capacity characteristic may be obtained with two pumps of quite different outside diameter. If these two pumps of identical specific speed are determined, the one which has the greater outside diameter will have a higher suction capacity than the one with the smaller outside diameter, while in general, its efficiency will be less because the friction losses will be higher.

From this it follows that: (1) For a pump designer having sometimes to obtain high efficiency and sometimes to solve problems of high suction lift, pumps of the same specific speed may have quite different suction capacities. (2) Pumps with high specific speed will have a suction capacity (a capacity determined by the coefficient S) lower than that of pumps with low specific speed; in fact, the outside diameter of pumps with high specific speed is necessarily small, which limits the possibilities of developing the suction passages.

As a consequence of the great variety of problems to be solved, the pump designer usually establishes a considerable number of different pumps of which many may have the same specific speeds but different suction capacities. For classifying these different types of pumps, the coefficient S (suction specific speed) becomes important. A pump is, in fact, entirely determined by its specific speed and the coefficient S. If, then, one determines, at each point in the utilizable zone of the head-capacity characteristic of each pump, the value of the specific speed n_s and of S, and if one plots for each pump on a chart the values of S as a function of n_s one can show in a simple diagram all types of pumps available to the designer with all their possible applications.

Fig. 9 shows a partial abstract of a chart which has been used for nine years for the classification of Bergeron pumps. The practical value of such a chart is considerable. Normally, a pump to be designed is determined by the total head H, the capacity Q, and the suction head H_{sv}. From these values one obtains

$$S/n_s = (H/H_{sv})^{3/4} \quad [29]$$

which is represented on a chart by a straight line, for instance, in a specific case, by the straight line AB, Fig. 9. All pumps situated above this line may be taken into consideration: The pumps 1, 2, and 3 cut by this straight line are the extreme pump types which can solve the problem. Each corresponds to a speed obtained from the values of

$$S = nQ^{1/2}/H_{sv}^{3/4} \quad [30]$$

$$\text{or } n_s = nQ^{1/2}/H^{3/4} \quad [31]$$

Questions of speed (in the case of coupling with an electric motor), of efficiency, or manufacturing cost determine the choice between the several admissible types. Let it be assumed that pump 3 is the best suited. If the motor speed nearest to that given for this pump leads to values of S and n_s determining the point C, it will be possible to build a pump strictly similar to 3 which has the same specific speed, but a suction capacity higher than required. If pump 3 did not exist, it would be possible to start out from pump 4 by lengthening the impeller vanes in order to reduce the specific speed, as indicated in the example analyzed in detail by the authors.

In general, it is clear that such a chart makes it possible without further research to utilize all the properties of the pumps which the designer chooses.

In conclusion, without wishing to dwell any longer on this question, it might be said that the idea of combining capacity and suction head with the speed, while disregarding the pump head, is of great practical value because it permits the experienced designer to determine immediately at what maximum speed a pump of given capacity must run to produce a certain suction head.

H. H. Anderson.[14] This paper has been of particular interest and value to the writer. He has not used the parameter S in the form described by the authors but has for some time used the split form, as follows: Instead of

$$S = NQ^{1/2}/H_{sv}^{3/4} \quad [32]$$

plot N^2Q against H_{sv}, Fig. 10.

This gives a curve

$$H_{sv}^{3/2} = N^2Q \quad [33]$$

$$\text{or const} = N^2Q/H_{sv}^{3/2} = S^2 \quad [34]$$

It is convenient to show on the charts of N^2Q the permissible suction conditions of the Hydraulic Institute (American) so that particular test points can always be compared with quoting standards.

The plotting of Hydraulic Institute limits to a basis of N^2Q indicates readily the variation in permissible suction lift due to increase of total head for a given value of N^2Q. This variation is mentioned by the authors: "For the present the additional assumption is that changes in the outer waterways . . . do not affect flow conditions in suction passages. . ."

It will perhaps be useful to see the nature of these variations

[14] Chief Hydraulic Designer, The Harland Engineering Company, Limited, Alloa, Scotland.

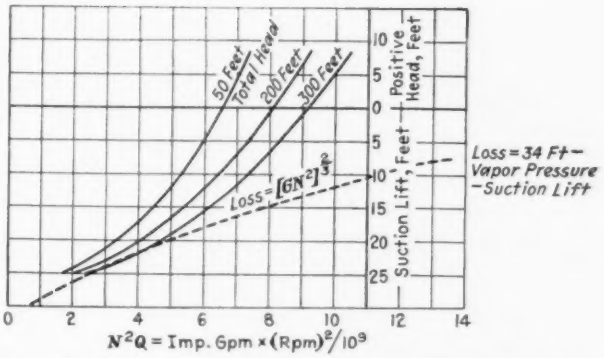


Fig. 10 Maximum Suction Conditions for Single-Entry Impellers
(For double-entry impellers, double gpm.)

from the suction chart based on N^2Q. The writer hopes therefore that his use of $N^2Q = H_{sv}^{3/2}$ will be of interest to the authors in emphasizing the practical value of their paper.

Authors' Closure

Before answering the various discussions the authors wish to call the reader's attention to the fact that the numbers and titles of Figs. 8 and 9 of the paper must be interchanged in order to agree with the illustrations.

The authors wish to thank all those who have given of their time and knowledge in presenting discussions of this paper. There can be no doubt that these contributions from such an outstanding group of engineers form a most valuable addition to the literature on the cavitation problem in general.

Because many of the points brought out in the various discussions have been answered in one or more of the other discussions, the authors will restrict the closure to those important points which remain answered only partially or not at all, and to those points which it will be helpful to reemphasize. In particular the relative merits of the σ- and S-representations and their practical usefulness have been well presented in the discussions by Bergeron, Sedille, and Anderson, and require no amplification.

Mr. Macmeeken in the first part of his discussion analyzed the effect of changes in the impeller inlet diameter on the cavitation performance of the pump. Unfortunately his careful derivations are based on a fundamental misunderstanding of the text. As stated in connection with Equation [8] of the paper the relation S = const must necessarily be true only if the inlet flow conditions are similar. However, if for the same speed of rotation and capacity the inlet diameter is increased, the flow conditions cannot remain similar because the peripheral velocity at the eye of the impeller increases while the flow velocity through the eye decreases. Hence, a discussion of the effect of a change in the inlet diameter based on the equation S = const contains from the beginning a contradiction and must necessarily lead to a contraditcory result.

Neither σ nor S have but one value for a given specific speed. As brought out in the discussion by Bergeron their variations under otherwise constant conditions classify the respective pumps with regard to their cavitation performance in the same way as pumps are classified by their specific speeds with respect to speed, head, and capacity. The classification according to the values of S, however, is approximately independent of the specific speed, while a classification with respect to σ has a meaning only if quoted together with the specific speed. By calculating the S-values for the cavitation points at the conditions given on the curves presented by Mr. Macmeeken, it is found that the second

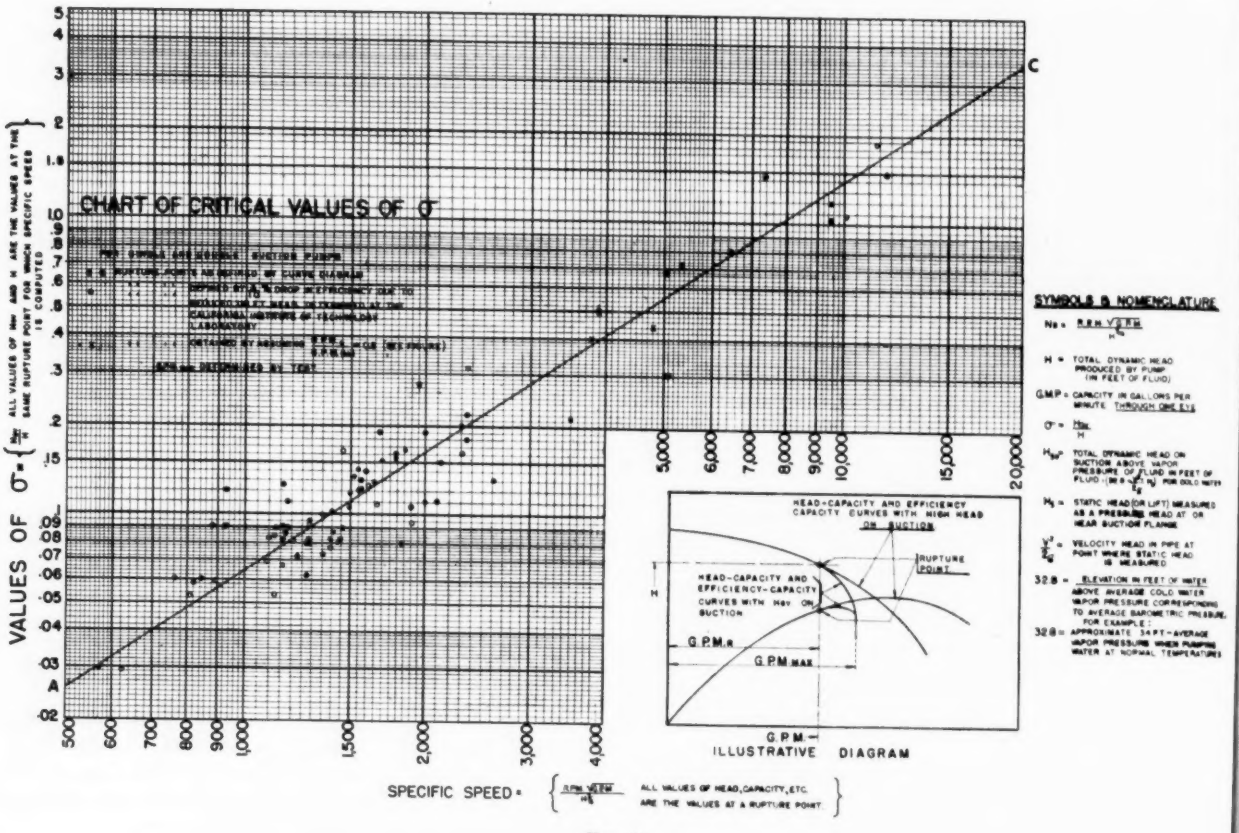

CHART OF CRITICAL VALUES OF σ
VALUES OF σ
SYMBOLS & NOMENCLATURE
HEAD-CAPACITY AND EFFICIENCY CAPACITY CURVES WITH HIGH HEAD ON SUCTION
HEAD-CAPACITY AND EFFICIENCY-CAPACITY CURVES WITH Hsv ON SUCTION
RUPTURE POINT
G.P.M.R
G.P.M. MAX
G.P.M.
ILLUSTRATIVE DIAGRAM
SPECIFIC SPEED =

FIG. 11

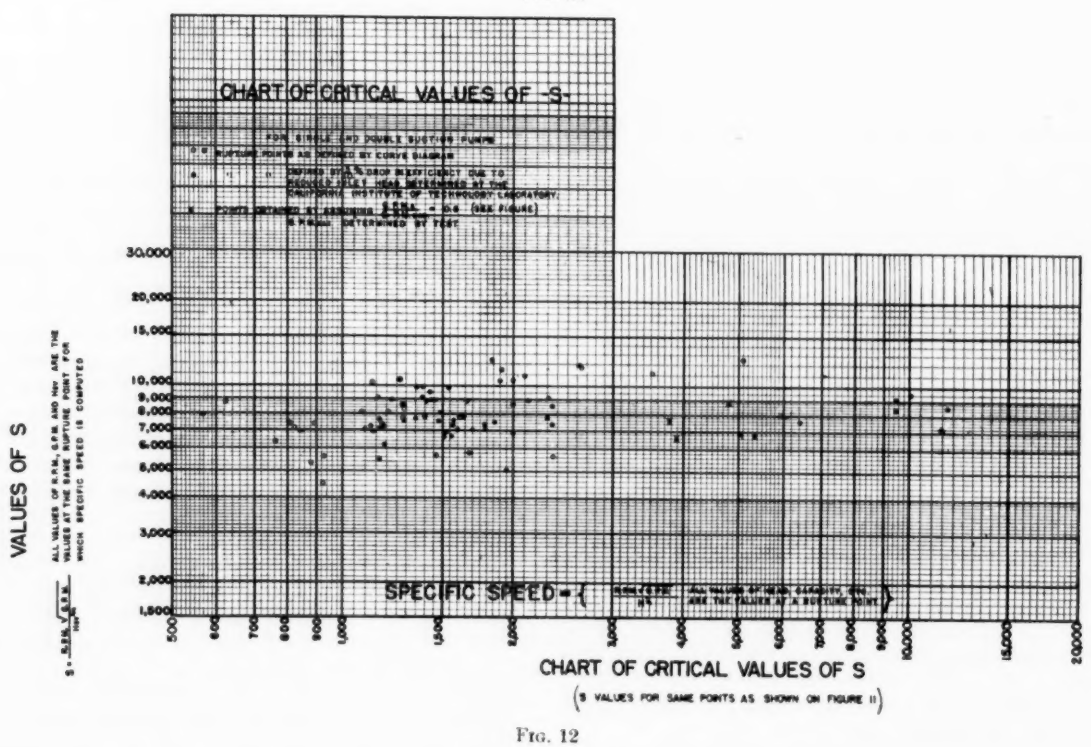

CHART OF CRITICAL VALUES OF -S-
VALUES OF S
SPECIFIC SPEED =
CHART OF CRITICAL VALUES OF S
(S VALUES FOR SAME POINTS AS SHOWN ON FIGURE 11)

FIG. 12

example refers to a pump specially designed for high-suction qualities while the other two pumps represent cases of average-suction performance. The comparison on the basis of the respective S-values has the advantage that it shows whether the design of the inlet can or cannot be similar and in which general respects the designs must differ. Had these tests however been made at different instead of the same speeds or on pumps geometrically similar to the ones tested (but of differing physical dimensions), the comparison would not have been so directly obvious but would have been made evident by the simultaneous comparison of the values of σ and N_s, or by the comparison of the S-values alone. It is indeed in the very nature of the parameter S that any changes in the form of inlet design and flow conditions are reflected by corresponding changes in S, while geometrically similar or perfectly equivalent inlet flow conditions are characterized by the same value of S.

The authors have not found that the relation between the net inlet head H_s and the inlet diameter can be expressed generally by Mr. Macmeeken's Equation [6], which equation leads to the conclusion, that, other factors being constant, the inlet head at breakdown is inversely proportional to the first power of the eye diameter. On the other hand it is noted that Mr. Macmeeken independently arrived at Equation [7] which is equivalent to the fundamental relation derived in the paper. It may be of interest to compare Equation [7] of the discussion with Equation [14] of the paper because the latter in connection with Equation [11a] may be written in the form

$$\sigma = \frac{1}{S_N^{4/3}} \times \frac{1}{S_Q^{2/3}} \times N_S^{4/3} \quad \ldots\ldots\ldots\ldots [35]$$

which has the same structure as his Equation [7].

Mr. Macmeeken's caution regarding the use of the parameter S seems to be based largely on the previously discussed misunderstanding regarding the effect of changes in the impeller diameter. Beyond that the authors should like to refer in this respect to Fig. 12 of their closure and to the discussions by Bergeron, Moody, and White.

Regarding the discussion by Professor Moody the authors would like to point out that much of his collection of empirical cavitation-limit points has been included in the accompanying Figs. 11 and 12. It is interesting to note that again the four-thirds-power relation between σ and N_s has been derived previously by somewhat different methods, apparently indicating that this relation is a fundamental one.

The latter fact is once more borne out in the discussion by Stepanoff quoting a publication by von Widdern. Equation [9] of this discussion has been investigated in the past at the Pasadena Laboratory and has also been used by Spannhake in Germany. The subsequent method of deriving the four-thirds-power relation between σ and N_s is interesting but not fundamentally different from that given by the authors.

Referring the σ-values of a pump only to the pump head at best efficiency, H_n, was the procedure originally used at the Pasadena Laboratory, but was dropped for practical reasons. The derived relation of Equation [17] of the discussion is indeed interesting but can be misinterpreted too easily if the underlying definitions are momentarily overlooked.

The authors do not agree with Mr. Stepanoff regarding the fact that all controlling impeller proportions are so well established as to prohibit some changes in the inlet dimensions without corresponding changes in the remaining dimensions. Such changes if carried out by an experienced pump designer have been found to be possible without adverse effects on the efficiency. Naturally, however, if such changes are carried too far they will ultimately reduce the efficiency of the pump, but such sacrifices are made rather frequently in order to obtain exceptional suction performance or a minimum over-all cost.

The authors naturally made sure that the agreement between the shaded area and the lines S = const in Fig. 7 of the paper was not just a coincidence. This was borne out by individual points within the area obtained with pumps furnished by the Worthington Pump and Machinery Corporation. The agreement shown in Figs. 7 and 12 is considerably better than the authors originally expected, and in view of the great simplicity of the underlying principle may well be considered as a startling one.

Mr. Stepanoff's objection to the inclusion of cavitation-performance data on water and *all* oils in one consideration is well founded, inasmuch as the derivation in the paper is based on fluids with well-defined vapor pressures. In other words, the statements made apply principally to one-component liquids, a condition met by but few of the petroleum derivatives. However the discrepancy, with the proper determination of the "head above vapor pressure," is slight and usually well on the side of safety. For highly viscous liquids the failure to hold similar flow conditions between the highly viscous liquid and the nonviscous liquid in geometrically similar inlet passages immediately precludes the possibility of bringing them into one consideration.

Concerning the application of the factor S to turbines, the authors wish to point out that the basic principles apply equally to pumps and turbines, but they are not certain at present whether or not results will be equally valuable.

The discussion by Dr. White must be considered as a particularly valuable one because of the large amount of test data and general experience backing his statements. Figs. 11 and 12 are intended to answer the question as to the relative appearance of a plot of σ- and S-points, respectively, using N_s as a common abscissa. Because of the rigid mathematical relation between σ, N_s, and S expressed by Equation [14] of the paper, both plots (σ-N_s and S-N_s) have to be equally consistent. Regarding the collection of points given, the authors wish to emphasize that this collection intentionally covers a wide variety of designs with greatly differing dates and places of origin. (Only condensate pumps and process pumps for extremely low inlet pressures have not been included because the cavitation limits for such pumps are defined in a different way than for pumps of standard applications and therefore would show misleading extreme values on the diagram.) If only pumps of a consistent series of designs were included the resulting scatter of points doubtlessly would be considerably reduced. On the given diagrams this scatter is further increased by the fact that the points are limits of "safe" operation, which are more difficult to determine than the ultimate-breakdown limits.

It is appropriate to call for caution in the use of any diagram such as shown in Fig. 10 of the paper. In view of the collection of points shown in Fig. 12 such a diagram however, may be used for a quick estimate of cavitation limits over a wide if not the whole range of specific speeds. This fact is illustrated by line A-C in Fig. 11, representing a constant value of S = 7800, on which value the whole diagram of Fig. 10 of the text was based. As pointed out by Professor Moody no diagram or general equation can be expected to give exact cavitation limits for given designs. For such purposes the experimental method doubtlessly will continue to form the only reliable basis for considerable time to come.

The coefficients used in the discussion by Dr. Sedille are physically equivalent to the coefficients S_n and to the ratio S_s/S_n introduced in the paper. The latter ratio was used by the authors when these methods were originally presented before the Hydraulic Institute. It was not included in this paper because the authors wished to maintain a complete analogy between their coefficients and the coefficients already in general use by centrifu-

gal-pump engineers with respect to pump head and capacity. For the same reason the authors did not use the strictly dimensionless form of the new coefficients but they fully agree with Dr. Sedille that such a dimensionless form of all pump coefficients would be highly desirable to use, particularly in international communications.

The form of representation described by Dr. Sedille refers in particular to questions of pump design by including design data of the impeller. On the other hand the authors as well as Dr. Bergeron have focused their attention in this paper primarily on the question of performance characteristics rather than design characteristics. The purpose of this restriction was to make the present paper more concise and easier to understand than would have been possible had a discussion of design characteristics been included. The authors naturally have already extended their investigations into the latter field and find Sedille's contribution in this direction most helpful and appropriate. The authors have been informed that promising work along the lines mentioned in this discussion has already been in progress for considerable time at the Pasadena Laboratory.

Regarding the details of Dr. Sedille's discussion the authors merely wish to point out that the use of Equation [22] seems to require further theoretical investigation with respect to the relative flow conditions in radial-flow impellers. They feel that the rotational character of the relative flow may require some modifications of this equation.

As pointed out in the paper, Dr. Bergeron already has used the parameter S for a considerable number of years. His discussion therefore, presents a mature and elegant interpretation of this parameter. Bergeron points out that the performance of any pump is completely defined by two parameters, N_s and S, so that the performance of all pumps can be represented in a two-dimensional field, i.e., on an S-N_s diagram. By the sample given he indicates that the performance of every pump is described by a curve on this diagram, and he shows how a collection of such curves, corresponding to the points on Fig. 12, can be used for the selection of a pump for given operating conditions. The two parameters N_s and S may not be completely independent of each other, depending on the behavior of S as a function of N_s. Bergeron is of the opinion that, for high and medium specific speeds, S tends to increase only if N_s is reduced. This statement is probably true if, as he assumes, the pump of higher specific speed is developed from the pump of lower specific speed by a reduction in the length of the vanes. However, the specific speed may be increased also in different ways such that the average vane pressure decreases, in which case the change of S as a function of N_s may not follow the trend given in the discussion. The authors therefore feel that this relation between S and N_s depends entirely on the method of design by which the changes in N_s are produced. In addition to a certain freedom of choice on the part of the designer this method will depend largely on the type of pump, i.e., the range of specific speeds in question so that the trend of changes of S as a function of N_s may not be the same for the whole range of specific speeds. Finally it has to be pointed out that Bergeron's conclusions apparently are derived on the basis of cavitation limits directly related to the ultimate cavitation breakdown (ultimate capacity) of the pump. His conclusions might have been different if the considered cavitation limits were based on the conditions under which the influence of cavitation begins to be noticeable. Generally speaking it is therefore necessary to state always the type of cavitation limit considered in order to define completely the physical meaning of subsequent conclusions.

The discussion of Mr. Anderson is very helpful and once more confirms the fundamental four-thirds power relation derived in the paper and so frequently appearing in the discussions.

A Supersensitive Governor for Hydraulic Turbines

By W. M. WHITE,[1] MILWAUKEE, WIS.

A supersensitive governor is defined in this paper as one that will move the gates of a hydraulic turbine in the corrective direction by reason of a change of speed of 0.02 per cent and will then move the gates back by reason of a change of the speed back to the former speed. The author outlines the elements essential to a supersensitive governor in order to accomplish this fine degree of regulation. The application of these principles to governor design is then outlined.

THE essential elements in a supersensitive governor, as defined, to accomplish the fine degree of regulation of the gates of a hydraulic turbine, as well as for governing the turbine under other conditions, are:

1 A frictionless, sensitive, powerful flyball.

2 Means for causing rotation of the flyball in exact ratio to the rotation of the turbine shaft or, as an alternative, in exact ratio to the cycles of the system.

3 Means for translating the minute axial motion of the flyballs, due to a change of speed of 0.02 per cent, to proportionate movement of the turbine gates without causing an opposed force to the axial movement of the flyball sufficient materially to affect the ratio of speed and axial motion of the flyball when free.

4 Means for causing a predetermined droop in speed in proportion to the gate opening of the turbine.

5 Momentary means for increasing greatly the ratio of speed change to gate motion on sudden changes of load, thus making use of the connected flywheel effect for stabilizing the speed during the slowing up of the rate of movement of turbine gates to prevent disastrous pressure changes, caused by the inertia of the water columns to and from the turbine.

The frictionless flyball of the supersensitive governor is a combination of springs and weights so related that movement of the actuating stem of the flyball is caused by the flexure of the springs without any moving joints. The centrifugal force of two revolving flyballs is resisted by coiled springs between the two, and this combination is driven and supported by multiples of flat steel springs between the fixed driving head and the flyball weights. Axial motion to the floating collar is imparted by other multiple flat springs fixed between the flyballs and the axially moving collar which is supported only by, and revolves in, a sleeve bearing. The axially moving collar surrounds but does not touch the supporting shaft between the fixed driving head and a lower bearing. The axial motion of the floating collar is imparted to a pilot-valve stem through a slot and hole in the governor shaft, being held in position at its upper end only by the connection to the axially moving collar and at its lower end by the bearing of this revolving pilot valve in its ported sleeve. By this combination is secured a pilot valve rotating in its sleeve and the actuating collar revolving in its sleeve bearing, thus having no friction to axial motion imparted by flexure of multiple springs rigidly fixed to the flyball.

Therefore, the force produced by the weights of the flyball under minute speed changes are translated to the rotating pilot valve with only the resistance of the molecular friction of the springs and the floating rotating spring support.

The sensitivity of the flyball is secured by driving it at the high speed of 900 rpm. Because of this high speed and terrific power of the flyball weights at this speed, the slightest change of speed provides sufficient power to give immediate movement to the pilot valve. A powerful flyball is secured by making the flyball and particularly the weights of such size that a minute change of speed supplies considerable energy to move the pilot valve.

The curve shown in Fig. 1 gives the ratio of a change of speed

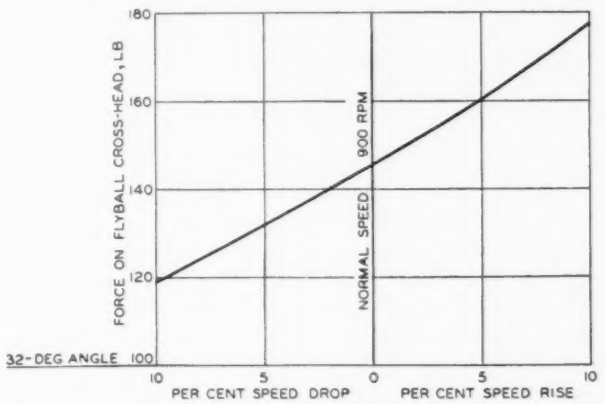


FIG. 1 ENERGY CURVE FOR 900-RPM SUPERSENSITIVE FLYBALL

and force at the pilot valve. From this curve, it may be seen that the range of the flyball is through a speed of 10 per cent above normal down to 10 per cent below normal and that the power of the flyball is 35 lb on the stem, when the stem is held in a position of normal speed and the flyball speeded up to 10 per cent above normal speed. When, however, a change of speed of 0.02 per cent takes place, it produces a force of 0.05 lb on the pilot valve. Because of the requirement of action within a very small speed change, the force from the flyballs to move the pilot valves is greatly reduced and means for amplification of this force is shown hereafter. The transverse coil spring provides means for adjusting the flyballs to the desired range of speed change of the flyball for maximum travel of the pilot valve. A more detailed description will now be given so that the action of the flyball may be followed by reference to Fig. 2.

DESCRIPTION OF FLYBALL-GOVERNOR MECHANISM

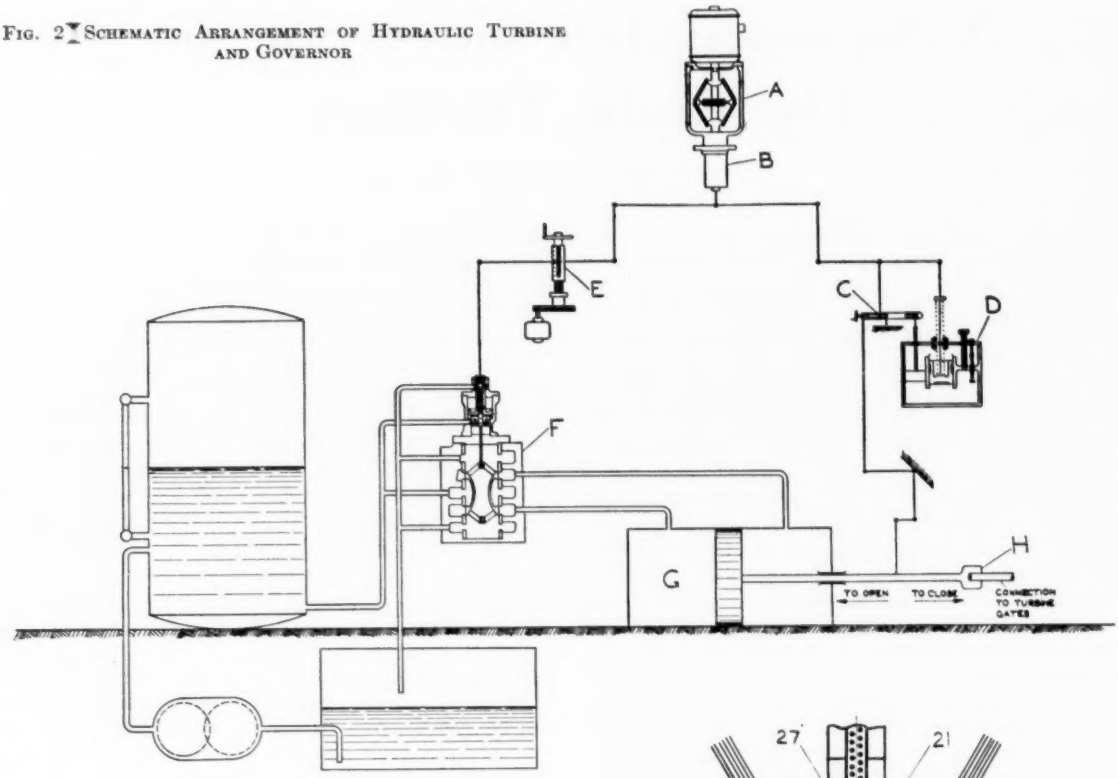
In Fig. 3, the upper fixed support of the flyball (4) is rigidly doweled to the driving motor shaft and held in position by the bearings of the motor. Between the movable flyball support (21) and the fixed support (4) are supported the flyball weights (13) by flexible springs (14) as indicated. Support (21) revolves in the sleeve bearing (25), also shown in enlarged detail in Fig. 4. The flat springs (14) act only as connecting members between the flyball weights, the fixed support, and the movable support.

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Contributed by the Hydraulic Division and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

FIG. 2 SCHEMATIC ARRANGEMENT OF HYDRAULIC TURBINE AND GOVERNOR



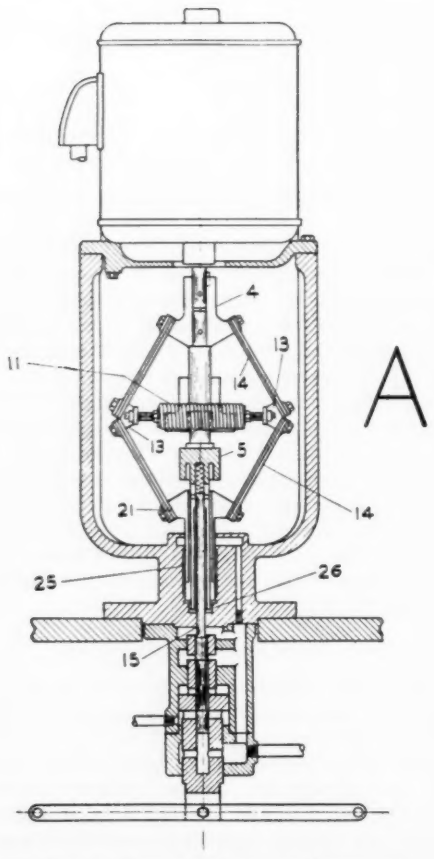


FIG. 3 DETAIL OF FLYBALL GOVERNOR

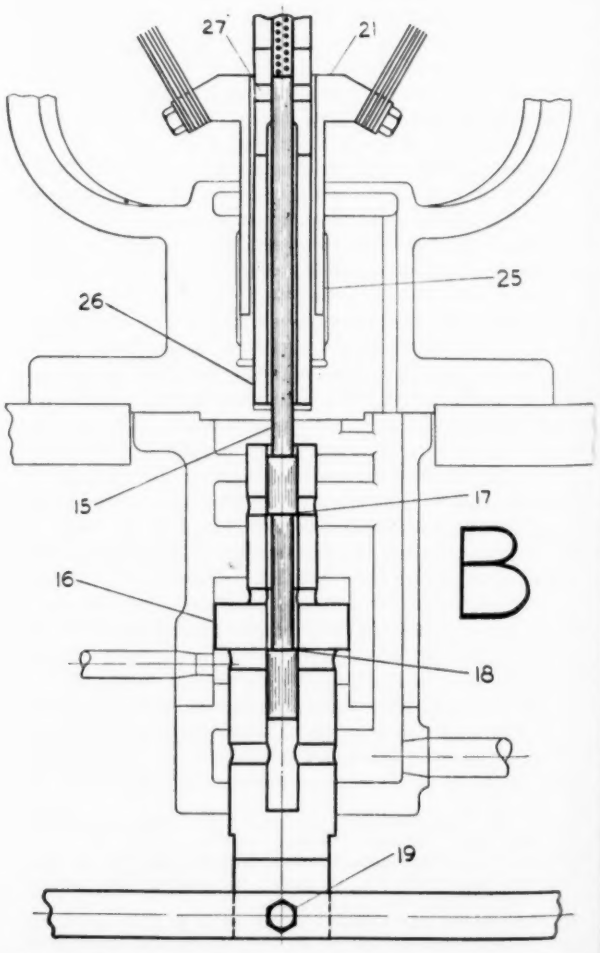


FIG. 4 ENLARGED DETAIL OF FLYBALL-GOVERNOR BEARING

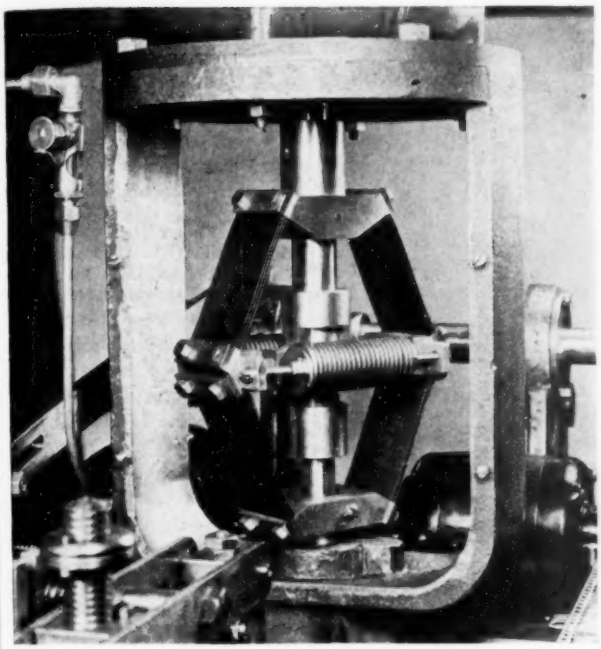


FIG. 5 ASSEMBLY OF NEW SUPERSENSITIVE FLYBALL FOR HYDRAULIC-TURBINE GOVERNOR

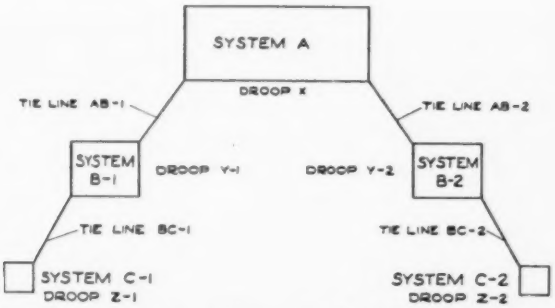


FIG. 7 SCHEMATIC ARRANGEMENT OF FIVE HYDROELECTRIC SYSTEMS INTERCONNECTED BY FOUR TIE LINES

Transverse springs (11) serve to hold the flyballs in fixed position for a given speed. The moving parts are held in the axis of rotation by the shaft (5), one end of which is doweled to the motor shaft, the other end being supported in a lower bearing located centrally of, but not touching, the axially moving support (21). The shaft (5) is bored to carry within but not to touch the pilot-valve stem. It is also slotted, through which a cross pin fixes the axially moving collar (21) to the pilot valve.

By means of the double sleeve bearing (25), shown in Fig. 4, supporting the shaft (5) and support (21) apart from each other, static friction is thus avoided which is one of the essential features of this flyball. It is this double bearing feature which makes the flyball frictionless in respect to the actual motion of the pilot

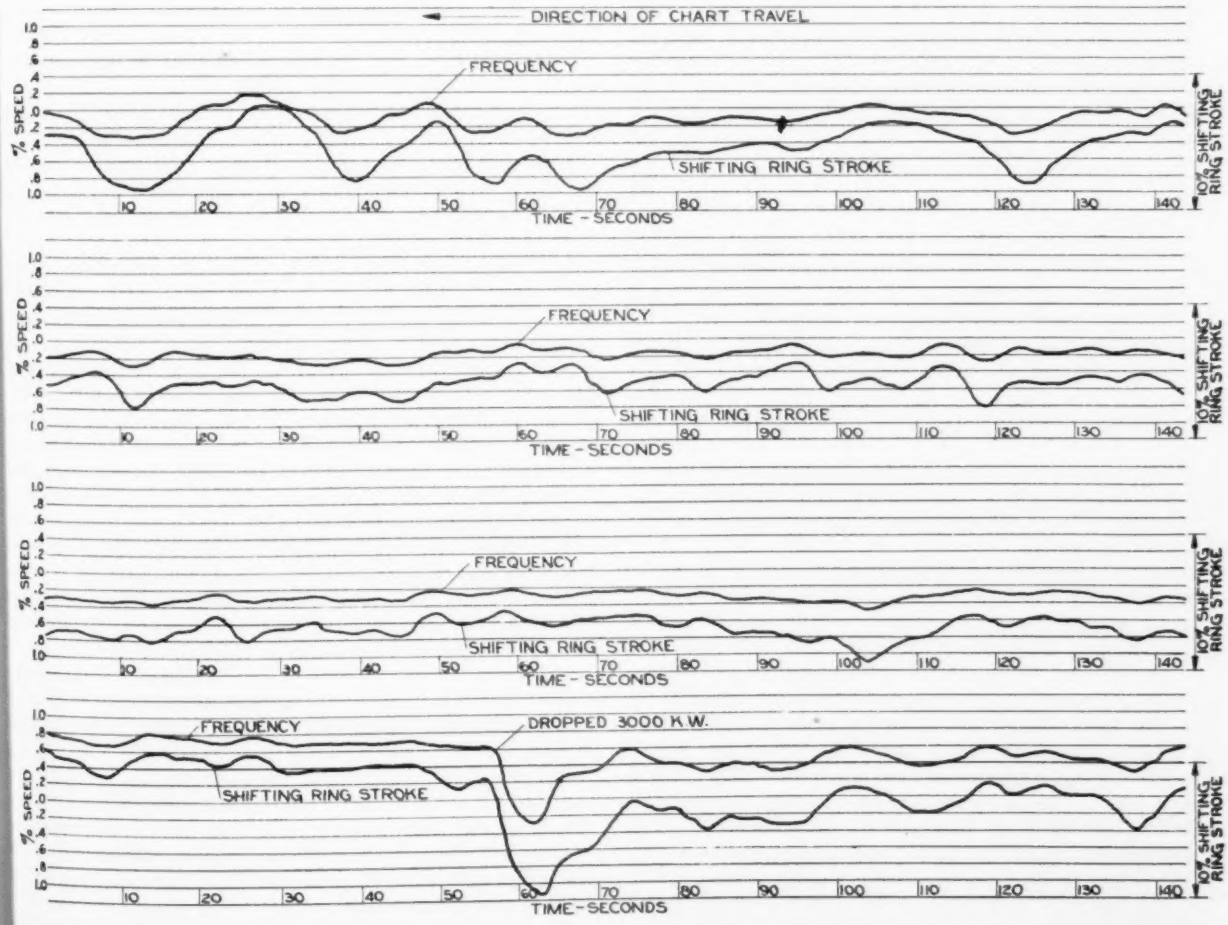


FIG. 6 SIMULTANEOUS RECORDINGS OF FREQUENCY AND SHIFTING STROKE OF A SUPERSENSITIVE-GOVERNOR INSTALLATION



FIG. 8 FRONT VIEW OF CONTROL BOARD USED ON GOVERNOR FOR PICKWICK LANDING DAM, TENNESSEE VALLEY AUTHORITY, APRIL, 1938

valve and enables such fine degree of regulation to be secured.

A properly designed and proportioned belt-driven flyball has heretofore provided a degree of regulation within 0.5 per cent but, as the demands for closer regulation have increased, a more exact ratio of speed of flyball to rotating turbine shaft has been necessitated, or a more exact ratio of the speed of flyball to the cycles of the system.

A separate generator for driving the motor-driven flyball is the best known means at present of maintaining a fixed ratio between the flyballs and turbine shaft. Transformers connected to the system for driving these flyballs is the best known method of providing a fixed ratio between the speed of the flyballs and the cycles of the system. A fine degree of regulation has been attained by both of these methods. The author believes that the latter method will ultimately prove to be preferable for maintaining the exact ratio between cycles of the system and rotation of the flyball essential to the most perfect regulation. As the load changes and, particularly, as the power factor of the generator changes, as it does during the load change and during the power-factor change, a difference occurs between the normally fixed ratio of rotation of turbine shaft and cycles of the system because the pole pieces take up different positions under the foregoing conditions with respect to the rotating field. It is true that this difference is minute but, since we are straining for the most minute regulation possible, it may be that we will have to have that definite relation between rotation of flyball and cycles of the system which is afforded by using transformers for driving the motor-driven flyballs.

Means for translating the minute axial motion of the flyball collar, caused by a change of speed of 0.02 per cent with little resistance to its movement, comprises a pilot valve $^1/_2$ in. in diam, controlling admission of oil pressure to two sides of a $2^1/_4$-in-diam piston. The diameter of the piston is made small so that only a minute volume is displaced by a movement of the pilot valve corresponding to a change of speed of 0.02 per cent. The reaction around the sharp edge of the pilot valve and its corresponding port which causes axial force on the pilot valve is, of course, in proportion to the velocity through the port, the amount of opening of the port, and the diameter of the valve. By making the diameter small, the volume and consequently the velocity is kept so low as to avoid unfavorable reactions.

By this hydraulic multiplication, the axial force is multiplied 1000 times for use in moving the main pilot valve of the governor. The piston follows the pilot valve so closely that measurements made show a difference between the position of the two in normal operation not greater than 0.001 in. The pilot valve (15), Fig. 3, coacts with a combined sleeve and piston (16) having ports (17) and (18). In the old type of governor, the flyball spindle was directly connected to the floating lever at point (19). This hydraulic multiplying device gives the required force at the point (19) to operate the floating lever of the governor with the minute speed change.

Fig. 2 is a diagram of a governor in which the supersensitive flyball is illustrated at A, the multiplying device at B, and the means for causing speed droop at C. The essential elements of the remaining parts of the governor are the same as heretofore used, except that the restoring mechanism is made unusually rigid to provide for an exactness of motion which will permit division of load between the units when the speed droop is of the order of 0.5 per cent.

Fig. 6 gives the plottings of simultaneous recordings of frequency and shifting stroke of a supersensitive-governor installation. From these curves it is readily seen that the gate motion follows a change of frequency of 0.02 per cent.

The supersensitive governor, by reason of its fine degree of regulation, provides for a division of load between such units with much less speed droop than has heretofore been secured in practical operations. It is believed that the supersensitive governor will afford an additional means of operating systems in parallel with one another over tie lines of less kilowatt capacity than either of the interconnected systems.

Reference is made to Fig. 7, in which the five blocks represent five hydroelectric systems which are interconnected by four tie lines as indicated. The supersensitive governor provides means for moving the gates of all of the turbines in all of the stations simultaneously with a speed change of 0.02 per cent. The speed droop of the governors in each of the systems may be made different from that of the others to cause such relative change of gate in each unit in the system as will result in a minimum change of power in the connecting line.

Because of the sensitivity of the governor, the speed droops may be set as low as 0.1 per cent in some groups with fair division of load between units and as high as 6 per cent in another group to provide a large gate movement for a given change of speed in one group with a resulting smaller change of gate opening in another unit of the system.

The possibilities of the supersensitive governor for the regulation of systems of tie lines have not been fully explored.

Power Swings in Hydroelectric Power Plants[1]

By W. J. RHEINGANS,[2] MILWAUKEE, WIS.

Power swings chargeable to hydraulic turbines have been found in hydroelectric plants built as far back as 1912. However, it has only been during the last ten years that much thought has been given to such swings, their characteristics, source, and elimination. The purpose of this paper is to supply technical data on the subject and to start such discussion as may lead to a better understanding of the problem. The paper shows that power swings are due to draft-tube surges. It deals with the characteristics of such surges, their relation to power swings, the elimination of surges and power swings in the field, if necessary, and a consideration of the factors of the design of a hydroelectric unit which will prevent small draft-tube surges from producing excessive power swings. However, the paper does not present a theoretical analysis of the true origin or source of draft-tube surges, which is a matter for further observation and study and for future discussion.

THE power swings referred to in this paper are the fairly regular swings readily apparent in hydroelectric power plants on the switchboard indicating instruments, such as the ammeters indicating generator-armature current and the indicating wattmeters. A distinction is made between power swings and load swings in that the former is defined as being a pulsation in power output of a generating unit, while a load swing is the fluctuation on a transmission and distribution system due to change in load demand.

Power swings may be identified by their regularity and frequency, which is roughly one third of the speed of the unit. Thus they are too fast and too regular to be accounted for by any of the usual normal changes in demand for load. The amplitude of the power swings may be anywhere between 5 per cent to practically 100 per cent of the output of the unit. Such swings are found only in connection with Francis- and propeller-type turbines, which eliminates impulse turbines from consideration.

Although power fluctuations can be produced by governor hunting, those which are to be discussed occur with the turbine guide vanes held in a fixed position, either mechanically or by the governor.

Power swings in hydroelectric power plants are not a new occurrence of recent years, since they are found in plants built as far back as 1912. In those days they attracted but slight attention because they were causing no trouble and were not considered harmful to the operation of a power system. Today, with the development of long transmission lines, power swings, while actually having caused trouble in only a few cases, are sometimes suspected of being the source of increased transmission-line instability.

Sensitive Switchboard Indicating Meters Developed

One factor responsible for centering attention upon power swings recently has been the development of sensitive electrical switchboard indicating meters which have very little damping effect. The meters in the older plants were quite sluggish, thus minimizing any existing fluctuations. The newer meters are built for greater accuracy and greater sensitivity; thereby, the damping effect has been reduced. They will overtravel during a quick power change, the overtravel being particularly pronounced when there are comparatively rapid and regular fluctuations in power output. Units having any tendency toward swings cause the meters to indicate greater power fluctuations than actually exist. Tests at one plant showed that the indicating meters were overtraveling 20 to 30 per cent during power fluctuations.

This is not meant as a criticism of the electrical meters, since they function correctly under ordinary circumstances and do not overtravel for slow changes. It does mean that in many cases the overtravel of sensitive switchboard indicating meters calls attention to power swings which, under other circumstances, would be disregarded.

However, sensitive undamped meters do not account for the several cases during the last ten years where the magnitude of the power swing was so great as to make it impossible to keep the units in parallel with other units on the line. It is therefore only natural that engineers are giving more and more thought to locating the source of power swings and to their elimination or prevention.

A search of the literature on the subject shows practically no mention of power swings as thus defined. During some of the discussions at National Electric Light Association meetings about ten years ago, power swings were mentioned as a probable contributory source of electrical instability, but no effort was made to analyze them in detail. Therefore, it was thought a paper at this time on the characteristics, source, elimination, and prevention of power swings, based on ten years' experience in their observation, study, and analysis, would be a valuable contribution to the art.

Power Swings Closely Related to Draft-Tube Surges

There is now no longer any question but that power swings, such as are chargeable to the hydraulic turbine, are closely related to and dependent upon surges in the draft tube. This is definitely established from the characteristics of the swings and of the draft-tube surges.

Draft-tube surges do not occur in all turbines. Where they are absent, there are no power swings that are chargeable to the turbine.

Power swings are accompanied by surges in the draft-tube vacuum or pressure of exactly the same frequency and by an audible swish in the draft tube of similar frequency, Figs. 1 and 8.

Power swings always occur over a short range of gate openings. They build up quickly to a maximum amplitude and disappear at higher or lower gate openings. Fig. 2 shows a typical curve of amplitude plotted against per cent gate opening. The surges in the draft-tube vacuum or pressure follow this same tendency of building up to a maximum at the same gate opening at which the

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[1] The field data used in this paper were obtained from a number of confidential sources. For the purposes of this paper, therefore, it has been deemed both undesirable and unnecessary to reveal the identity of the plants used as illustrations.

[2] Test Engineer, Hydraulic Department, Allis-Chalmers Manufacturing Company. Mem. A.S.M.E.

Contributed jointly by the Hydraulic Division and Power Division and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

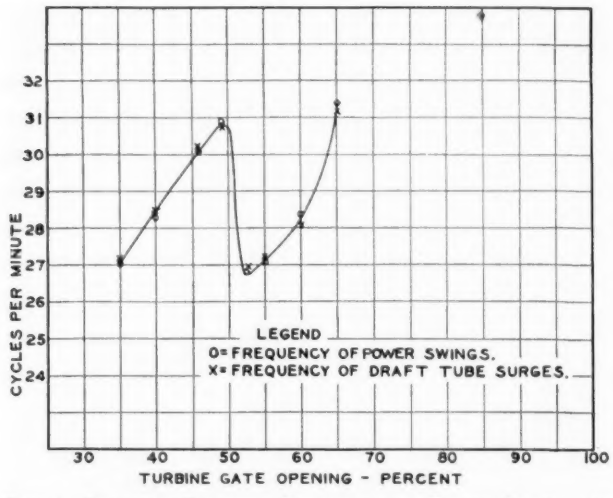


FIG. 1 RELATION BETWEEN FREQUENCY OF POWER SWINGS AND DRAFT-TUBE SURGES FOR VARIOUS GATES ON A TURBINE
(At gates below 35 per cent and above 65 per cent, power swings and draft-tube surges disappeared, therefore, no frequencies could be noted.)

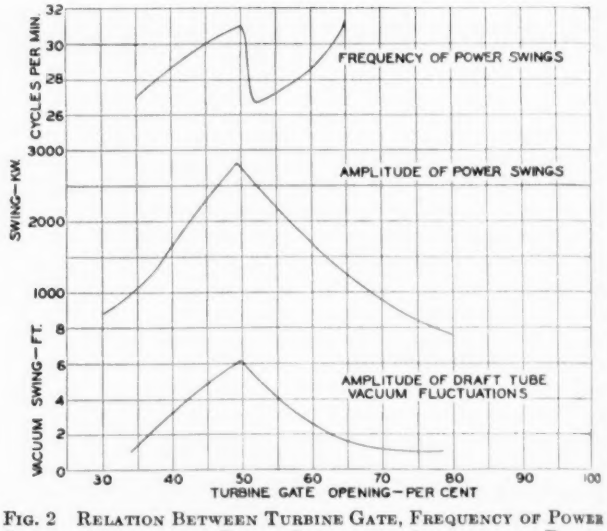


FIG. 2 RELATION BETWEEN TURBINE GATE, FREQUENCY OF POWER SWINGS, AMPLITUDE OF POWER SWINGS, AND AMPLITUDE OF DRAFT-TUBE SURGES
(Below 35 per cent and above 65 per cent gate, power swings, and draft-tube surges were so small and irregular that no frequencies could be determined.)

maximum power swing occurs. The same is true of the audible swish in the draft tube.

The range of frequency of the power swings eliminates the possibility of their being a direct function of the turbine speed in combination with the number of guide vanes or runner vanes.

The deciding factor, however, is that the power swings can be eliminated or greatly reduced by admission of air to the draft tube, or by making physical changes in the draft tube, thereby changing the flow conditions. This would not be possible, however, if the power swings were not dependent upon the draft-tube surges.

The manner in which the draft-tube surges produce the power swings is as follows: The fluctuations in draft-tube vacuum or pressure cause a fluctuation in the effective head on the turbine and thereby cause a fluctuation in the discharge. The fluctuations in discharge then produce a further fluctuation in penstock pressure due to water hammer. This is proved by the fact that draft-tube surges are always accompanied by penstock pressure fluctuations of similar frequency. In short penstocks the amplitude of the penstock fluctuation is small. In long penstocks the amplitude is always an appreciable amount and, under certain conditions of length of penstock as related to frequency of fluctuation, may assume serious proportions, Fig. 3. Since the elimination or change of draft-tube surges always eliminates or changes the penstock pressure fluctuations, it is certain that the draft-tube surges produce the penstock pressure fluctuations and not vice versa.

Observation has shown that the penstock pressure fluctuations are usually in phase with the draft-tube surges. As the vacuum in the draft tube increases, the penstock pressure increases and vice versa. Thus the penstock pressure fluctuations may combine with the draft-tube surges to create a change in head on the turbine, which produces a variation in torque on the runner. Since the turbine is connected to a generator, the electrical connection of which to a transmission system is in the nature of a spring, it is apparent that a regular variation in torque on the turbine runner can produce a corresponding oscillation in the generator output.

MAGNITUDE OF POWER SWINGS

The magnitude of the power swings also depends upon the relation between the frequency of the draft-tube surges and the natural frequency of oscillation of the generator connected to the power-transmission system. By causing a fluctuation in the torque on the turbine runner, the draft-tube surges will impress corresponding variations on the generator output. The generator connected to the transmission system has a natural frequency of its own which should not be confused with the alternating-current frequency of the system. Since the electrical connection between the turbine and the transmission system is in the nature of a spring as noted, the unit, when tied to the transmission system, will follow the laws of vibration. These laws state that a periodic disturbing force will produce forced vibrations in a vibrating body and that the amplitude of the forced vibrations will depend upon the magnitude of the disturbing force and upon a magnification factor. As the name implies, the magnification factor is a multiplier, applied to the amplitude of vibration, which amplitude corresponds to the disturbing force acting upon a vibrating body. The size of the magnification factor depends upon the amount of damping in the vibrating body and upon the relation between the frequency of the disturbing force and the natural frequency of the vibrating body.

When the periodic disturbing force has a low frequency compared with the natural frequency of vibration of the vibrating body, the magnification factor approaches unity. When the disturbing force has a very high frequency compared to the natural frequency of the vibrating body, the magnification factor approaches zero. However, when the frequency of the disturbing force approaches the natural frequency of the vibrating body, the magnification factor becomes very large. This relation is illustrated in Fig. 4, where the magnification factor is represented as a function of the ratio between the disturbing-force frequency and the natural frequency of the vibrating body.

The amount of damping in the vibrating body also affects the value of the magnification factor, especially when the disturbing force approaches resonance with the natural vibrations of the vibrating body. This is illustrated in Fig. 4, where the magnification factor approaches infinity at resonance, when there is zero damping.

In the case of a hydroelectric unit, the draft-tube surge is the periodic disturbing force and the rotating element of the turbine and generator connected to the transmission system is the vibrating body. This presents a problem similar to that of a pulsation

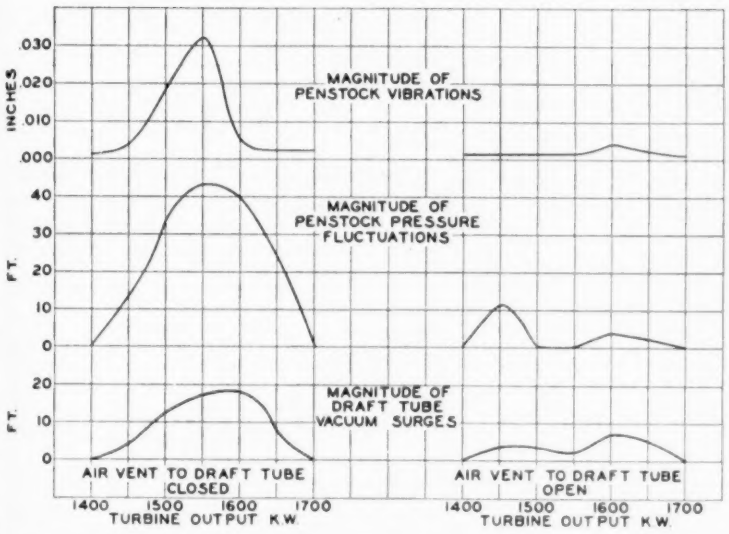


FIG. 3 RELATION BETWEEN MAGNITUDE OF DRAFT-TUBE VACUUM SURGES, PENSTOCK PRESSURE SURGES, AND PENSTOCK VIBRATIONS AT VARIOUS LOADS WITH AIR VENT TO DRAFT TUBE CLOSED AND THEN OPENED

(Turbine, 1700 kw, 260 ft head, 900 rpm. No power swings present; frequency of vibrations, surges, and fluctuations 320 per min.)

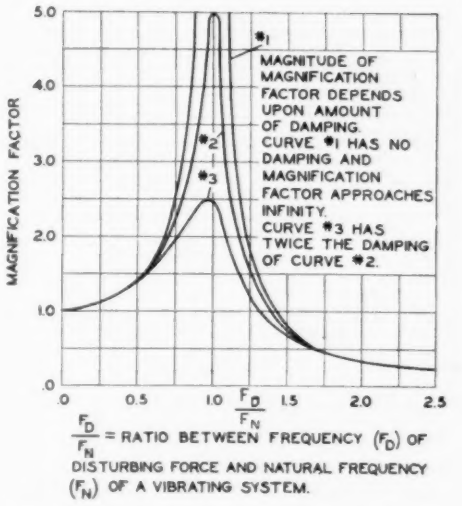


FIG. 4 VARIATION IN MAGNIFICATION FACTOR FOR DIFFERENT VALUES OF RATIO BETWEEN FREQUENCY OF DISTURBING FORCE AND NATURAL FREQUENCY OF A VIBRATING SYSTEM

in power where an internal-combustion engine (reciprocating drive) is used to drive a generator.

At this point, it should be emphasized that, in making this comparison, the vibrations of the rotating element refer to an oscillation in the speed of the rotating parts and do not refer to a mechanical vibration of the unit.

In the hydroelectric unit, when the frequency of the draft-tube surge approaches the natural frequency of oscillation of the generator connected to the transmission system, the magnification factor will be large and a small fluctuation in torque on the runner can produce a very large forced swing in generator output. It is a condition of this sort, combined with low damping in the generator, which produces power swings approaching the full rated output of the turbine.

Damping in a generator may be increased by use of amortisseur windings which would reduce the magnification factor and thus prevent serious power swings. However, it should be noted that damping is effective only when the disturbing force is near resonance with the natural vibrations of the vibrating body. When the frequency of the disturbing force is more than 30 per cent higher or more than 30 per cent lower than the natural frequency of the vibrating body, damping has very little effect on the magnification factor. Therefore, so long as resonance conditions are avoided, it is not practical to increase the damping of the generator for the sole purpose of reducing possible power swings.

The author knows of only one case in which the combined draft-tube surge and penstock-pressure fluctuation was of sufficient magnitude to produce a fluctuation in torque on the runner corresponding to a power fluctuation greater than 10 per cent of the rated turbine output. In this one exceptional case, the fluctuation in torque on the runner was 30 per cent of the rated output, but the frequency of the torque fluctuation was so high compared to the natural frequency of the generator that there were no power swings, Fig. 3. In those cases, where the power swings exceeded 10 per cent of the rated turbine output, the fluctuations in torque were being multiplied due to conditions approaching resonance.

PREVENTING RESONANCE TO ELIMINATE POWER SWINGS

Thus, if it is possible in the design of a hydroelectric unit to prevent resonance conditions, the possibility of obtaining large power swings will be eliminated. In order for the designer to determine whether a condition of resonance will be approached in a proposed hydroelectric unit, he must know the frequency of the draft-tube surges, if present in the proposed turbine, and the natural frequency of the generator as connected to the transmission system. The frequency of the draft-tube surges can be determined approximately from an empirical formula based upon the following observations:

In one case the speed of the turbine was varied while carrying a water-rheostat load and the frequency of the draft-tube surge

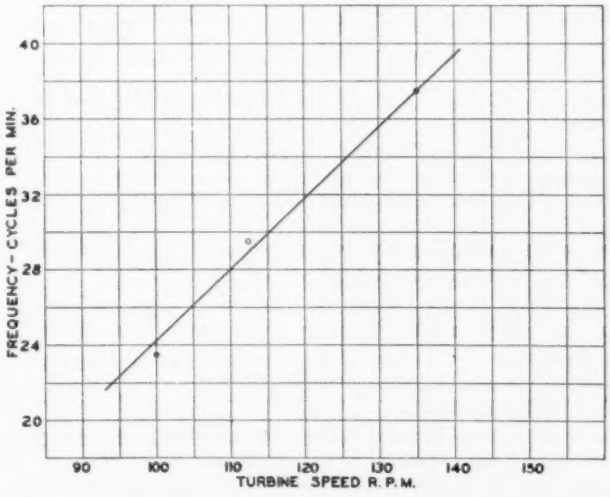


FIG. 5 RELATION BETWEEN SPEED OF A TURBINE AND FREQUENCY OF DRAFT-TUBE SURGES

(Data taken from one turbine by varying its speed while on water-rheostat load.)

was determined for various speeds. The results of this test are shown by the curve Fig. 5, indicating that the frequency of the draft-tube surge varies directly with the speed of the turbine.

Based on extensive data obtained from various sources, it was

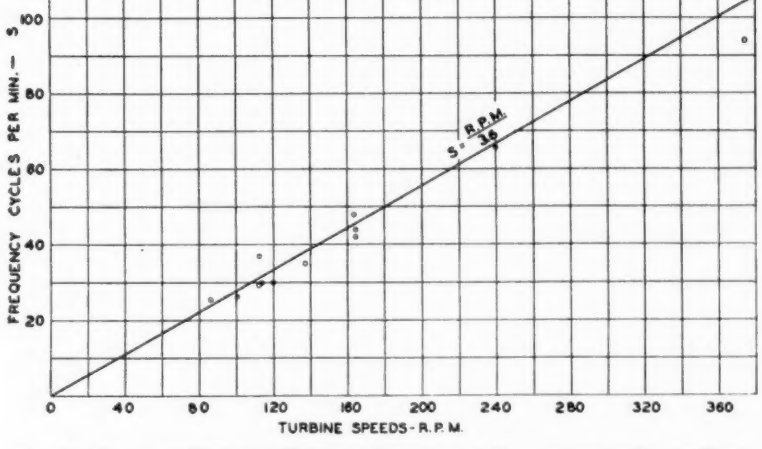


FIG. 6 RELATION BETWEEN TURBINE SPEEDS AND FREQUENCY OF DRAFT-TUBE SURGES AND POWER SWINGS
(Data taken from field observations on turbines in various power plants.)

also found that the frequencies of draft-tube surges and power swings have a rough relationship to the speed of the unit for various types and sizes of turbines, Fig. 6. This relationship can be expressed empirically as

$$S = \frac{\text{rpm}}{3.6} \quad \ldots\ldots [1]$$

where S = approximate frequency of draft-tube surge in cycles per min
rpm = speed of turbine in revolutions per min

The natural frequency of the generator connected to a transmission system depends upon the generator characteristics and transmission-line characteristics. This formula is

$$F = \frac{35200}{\text{rpm}} \sqrt{\frac{Pr'' \times f}{WR^2}} \quad \ldots\ldots [2]$$

where
F = natural frequency in cycles per min
rpm = revolutions of unit per min
f = alternating-current frequency of system in cycles per sec
WR^2 = weight in lb of entire rotating mass of unit times the square of its radius of gyration in ft

$$Pr'' = \frac{Pr\ Pr'}{Pr + Pr'}$$

where
Pr = synchronizing power of generator in kw per electrical radian
Pr' = kw per electrical radian displacement between generator voltage and transmission-line voltage. This displacement is a function of transmission-line characteristics such as line reactance, line resistance, and load carried on line.

It is not within the scope of this paper to explain the various combinations of generators and transmission lines in detail, since the calculations can be made by anyone familiar with the theory and design of synchronous machines. However, Equation [2] may be simplified by assuming the generator to be connected to an infinite system, i.e., a system having constant voltage and constant alternating-current frequency. Equation [2] will then be

$$F = \frac{35200}{\text{rpm}} \sqrt{\frac{Pr \times f}{WR^2}} \quad \ldots\ldots [3]$$

In the design of a hydroelectric unit, a regulation constant is used to give an indication of the degree of stability and the regulating qualities of the unit under consideration. The formula for the regulating constant is

$$C = \frac{WR^2 \times (\text{rpm})^2}{\text{hp}} \quad \ldots\ldots [4]$$

where C = regulating constant
hp = rated output of turbine in horsepower

From Equation [4]

$$WR^2 = \frac{C \times \text{hp}}{(\text{rpm})^2} \quad \ldots\ldots [5]$$

Substituting this value of WR^2 in Equation [3]

$$F = \frac{35200}{\text{rpm}} \sqrt{\frac{Pr \times f \times (\text{rpm})^2}{C \times \text{hp}}}$$

$$F = 35200 \sqrt{\frac{Pr \times f}{C \times \text{hp}}} \quad \ldots\ldots [6]$$

Although Pr, the synchronizing power of the generator, depends upon the load on the generator, the generator voltage, and power factor, as well as upon generator-design characteristics, for rough approximation, it can be assumed that

Pr = 2.3 kw rating of the generator

But the kw rating of the generator corresponds approximately to the output of the turbine.

Therefore

$$Pr = 1.7 \text{ hp (approximately)} \quad \ldots\ldots [7]$$

and Equation [6] becomes

$$F = 35200 \sqrt{\frac{1.7 \text{ hp} \times f}{C \times \text{hp}}}$$

$$F = 46000 \sqrt{\frac{f}{C}} \quad \ldots\ldots [8]$$

This shows that regardless of the size, type, power-output rating, speed of the unit, or head on the turbine, the natural frequency of its generator connected to an infinite transmission system is a function of the system frequency and the regulating constant C. The natural frequency of generators connected to a finite system will also be a function of the system frequency and regulating constant C, but due to factor Pr'', the natural frequency in these cases will be lower than if the generator were connected to an infinite system.

ELIMINATING CONDITIONS OF RESONANCE

Thus, when the designer finds that he will have the unfortunate combination of draft-tube surge frequency and natural frequency of the generator connected to a particular distribution system which gives resonance conditions, he can change the speed of unit without changing the regulating constant, he can change the regulating constant without changing the speed of the unit, or be

can change both, in order to prevent such resonance conditions.

If the designer changes the speed of the unit but finds it desirable to retain the same regulating constant, a change in WR^2 of the unit will have to be made. With the regulating constant unchanged, the natural frequency of the generator will remain the same but the draft-tube surge frequency will change, due to the change in speed of the unit.

If it is found possible to change the regulating constant as well as the speed of the unit, a change in speed would accomplish both, without requiring any change in the WR^2 of the unit. This would be desirable because a change in speed in this case would have opposite effects upon the draft-tube surge frequency and upon the natural frequency of the generator. For example, a decrease in speed of the unit would decrease the draft-tube surge frequency as indicated by Equation [1], and would also decrease the regulating constant C, as indicated by Equation [4], provided the WR^2 remains unchanged.

However, a decrease in the regulating constant would increase the natural frequency of the generator as indicated by Equation [8] and, thereby, the net effect of a change in speed would be away from resonance conditions.

If it is undesirable to change the speed of the unit, the regulating constant would have to be changed by changing the WR^2 of the unit. This would change the natural frequency of the generator but not the frequency of the draft-tube surges.

The regulating constant C used for the various hydroelectric installations has no fixed relation to the turbine size, output, speed, or head on the turbine. It does depend upon the type of turbine, the transmission system, and upon the regulation desired. However, as a general rule, the regulation constant will be between 4,000,000 and 16,000,000. Exceptions to the upper limit are the very large low-speed units, where the generator rotor has a large WR^2 for structural reasons.

Confining this discussion to the smaller units and to a 60-cycle system frequency, the very general statement, based on Equation [8] may be made that natural frequencies of hydroelectric generators fall within the comparatively small range where the highest natural frequency to be expected is about twice the lowest. The natural frequencies of the very large units, with a large regulating constant, and of units on 50- and 25-cycle alternating-current systems would be somewhat below this range.

Accordingly, in low-speed units, i.e., units with low rpm, the draft-tube surge frequency is likely to be low enough to be below the natural frequency of the generator and, while resonance conditions may not be present, the per cent power swing will at least be equal to the per cent torque fluctuation on the runner.

Higher-speed units are likely to be in a critical range where resonance conditions may be set up.

Above this range of speeds, the draft-tube surge frequency will be greater than the natural frequency of the generator, giving a magnification factor approaching zero, Fig. 4, and making high-speed units remarkably free of power swings. This latter condition has been frequently observed in the field.

It should be emphasized that the foregoing are only generalizations and that many individual exceptions can be found. It is for this reason that no attempt has been made to set the actual speeds at which critical conditions are likely to obtain, nor to fix the limits of the natural frequencies of the generators, especially since the latter depend upon transmission-line characteristics as well as upon the generator characteristics.

Variation of Draft-Tube Surge Frequency With Change in Turbine-Gate Opening

A characteristic of the draft-tube surge frequency is its variation with a change in turbine-gate opening, Fig. 1. A change in gate opening will bring the draft-tube surge frequency either closer or farther away from the natural frequency of the generator. This explains in part why the amplitude of power swings sometimes varies so greatly for a small change in gate opening.

The frequency of the draft-tube surges remains practically fixed for a given gate opening and for small changes in head on the turbine. However, the natural frequency of the generator may vary depending upon the generator load, power factor, transmission-line characteristics, number of units in parallel, etc., as previously mentioned.

Thus one combination will bring the natural frequency of oscillation of the generator closer to the frequency of the draft-tube surges and will produce greater power swings than another combination.

The amount of power swing which can be safely allowed in a hydroelectric unit depends largely upon local conditions. In so far as the generator alone is concerned, the A.S.A. rules for rotating electrical machinery allow a current pulsation of 66 per cent of the rated current in motors driving reciprocating machinery. No rule is given for generators but, in the design of generators to be driven by reciprocating engines, it is usually considered that, if the power swings are kept below 50 per cent of the rated output, there will be no serious heating or dangerous increases in generator stresses.

Effect of Power Swings on Transmission System

As to the effect of power swings on a transmission system, it has been demonstrated in actual practice that, if the swings are not too large, they have no serious effect on the system. There are units in operation today which have power swings of 10 per cent of the rated turbine output or 16 per cent of the turbine output at the gate where the swings occur. Although these units are connected to a 200-mile transmission line, exhaustive tests have shown that the swings had so little effect upon the operation or stability of the system it was not thought desirable to go to the slight expense of making the changes which tests had demonstrated would practically eliminate the swings.

Power swings are sometimes transmitted over transmission lines to all parts of the system. In one case, power swings on a unit were transmitted over about 600 miles of transmission line, causing power fluctuations in the output of units at the other end of the line. Even this condition did not affect the stability of the system.

Power swings if sufficiently large can produce voltage swings. This is objectionable because usually there are numerous connections on a system demanding constant voltage. Since the fluctuations in voltage as produced by the power swings are quite rapid, a heavy burden is placed upon the voltage regulator and, although the regulator is constantly trying to correct the voltage fluctuations, it can never quite keep up with them.

Power swings constantly tend to change the transmission-system frequency. However, on extensive systems with a large total WR^2, the power swings are relatively so small that the change in system frequency is infinitesimal. Therefore, it is very seldom that power swings have any effect upon the system frequency. So long as power swings do not affect the system frequency materially, they will not affect the operation of the turbine governor. However, there have been a few instances in which a single unit with power swings was connected to an isolated distribution system and produced a large fluctuation in the system frequency which affected the operation of the governor. As an illustration of this condition, the following case may be cited:

A 250-kw 60-cycle unit was carrying the entire power load in a small city. The turbine had a power swing at 75 per cent gate of about 25 kw. So long as the system was tied in with a larger system, these small power swings could not change the frequency noticeably and therefore did no harm. When the system was cut

loose from all interconnections and operated as an independent system the frequency would vary as much as three cycles in synchronism with the power swings. This change in frequency was reflected in the governor operation. The governor would vary the turbine gates, trying to correct for the frequency fluctuations. However, the system frequency fluctuated 58 times per min and the governor could never catch up with these comparatively rapid fluctuations. The governor varied the turbine gates about 10 per cent at the rate of 10 times per min. This movement of the turbine gates had a tendency to accentuate the power swings and the system-frequency swings.

The relation between the power swings and system-frequency swings in this case was such that, when the power increased, the system frequency increased and, when the power decreased, the system frequency decreased. This immediately indicated that the source of the trouble was in the output of the generating unit and not in a fluctuation of load on the system.

Fins Added to Draft Tube to Reduce Power Swings

After some experimentation, fins were added to the draft tube, Fig. 7, which reduced the power swings and system frequency fluctuations 75 per cent.

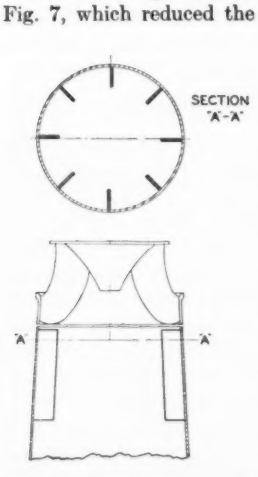


Fig. 7 Fins Installed in Draft Tube to Reduce Draft-Tube Surges and Power Swings

When a unit with draft-tube surges is connected to a water-rheostat load, the generator output remains fairly constant. The fluctuation in torque on the runner manifests itself in a change in the speed of the unit. The change in speed is possible under these conditions because the only WR^2 in the system is that of the generator and turbine.

As explained and illustrated previously, draft-tube surges react upon the penstock pressure in such a manner as to cause fluctuations in the penstock pressure. The increase in penstock pressure is not sufficient to cause excess stresses in the penstock itself. In some cases, however, the continued and regular fluctuation in penstock pressure causes a penstock vibration which may become serious. The following case is cited as an illustration of this condition:

Two 1700-kw, 900-rpm units, operating under a head of 260 ft were both connected to a single penstock 765 ft long. At outputs between 1500 and 1600 kw on either or both units, there was a violent draft-tube vacuum surge of 18 ft, a penstock-pressure surge of 43 ft, and a penstock vibration of $^1/_{32}$ in., all at a frequency of 320 per min. In this particular case, there were no power swings because the frequency of the draft-tube surge was considerably higher than the natural frequency of the generator connected to the transmission system, which gave a magnification factor of practically zero, Fig. 4. This is an excellent example of a high-speed turbine being free of power swings despite large draft-tube surges. It was found possible to eliminate the draft-tube surges, the penstock pressure fluctuations, and the penstock vibrations by admission of air to the draft tube, Fig. 3.

Mechanical vibration and rough operation of the turbine sometimes accompany draft-tube surges. Whether the draft-tube surges cause the rough running and vibration has not been definitely established. There is reason to believe that there is some relation between them because the roughness of operation usually occurs at the same turbine-gate opening at which the draft-tube surges are a maximum and because it has been found possible to reduce vibration and roughness of operation by admission of air to the draft tube, which also reduces the draft-tube surges.

Field Methods for Reducing Draft-Tube Surges and Power Swings

After a unit has been designed, built, installed, and placed in operation and then develops draft-tube surges and power swings, it becomes necessary to use economical methods in the field to eliminate or reduce them. The first criterion of such a method is its effect upon power output and efficiency. One of the most reliable methods is the admission of air to the draft tube, preferably to the center, either as free air or compressed air, Fig. 3. This method invariably reduces the draft-tube surges and thus the power swings, but sometimes it affects the power or efficiency of the turbine, especially on low-head installations.

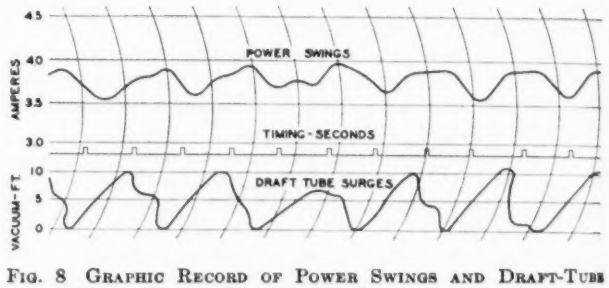


Fig. 8 Graphic Record of Power Swings and Draft-Tube Surges Before Making Any Changes in Turbine

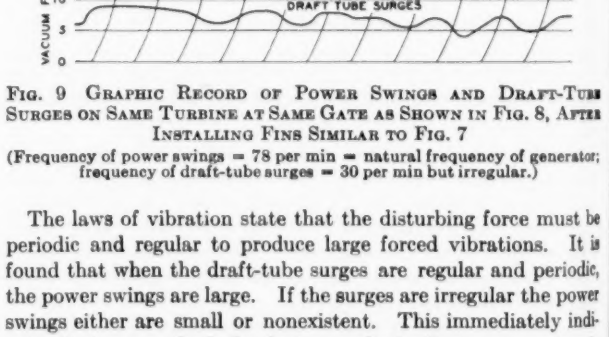


Fig. 9 Graphic Record of Power Swings and Draft-Tube Surges on Same Turbine at Same Gate as Shown in Fig. 8, After Installing Fins Similar to Fig. 7

(Frequency of power swings = 78 per min = natural frequency of generator; frequency of draft-tube surges = 30 per min but irregular.)

The laws of vibration state that the disturbing force must be periodic and regular to produce large forced vibrations. It is found that when the draft-tube surges are regular and periodic, the power swings are large. If the surges are irregular the power swings either are small or nonexistent. This immediately indicates that one method of reducing or eliminating power swings is to break up the regularity of the draft-tube surges.

Admission of air, besides reducing the draft-tube surges, always tends to break up their regularity. It thus serves a dual function in reducing power swings. Another advantage of the admission of air is that it can be controlled by the gate movement and admitted only at such gates as required.

The disadvantages are that the admission of free air is affected by large variations in the tailrace levels. In some cases, where the level of the tailrace is above the turbine runner, it becomes necessary to force compressed air into the draft tube. It may not always be convenient to supply compressed air for this purpose.

Certain changes in draft tubes have also reduced the surges and power swings considerably in many cases. Among these, the most successful method has been the installation of fairly short fins on the walls of the draft tube directly below the runner, Fig. 7. The fins break up the swirl in the discharge, change the flow condi-

tions, and cause the draft-tube surges to become irregular, in addition to reducing their amplitude, thereby reducing the magnitude of the load swings, Figs. 8 and 9.

A low cone in the draft tube, if it is of the concentric type similar to W. M. White's hydraucone, has also been successful in many cases.

In conclusion, the following facts may be stated as having been definitely established:

1 That the power swings chargeable to the hydraulic turbine are always accompanied by draft-tube surges and penstock pressure fluctuations, but that draft-tube surges and penstock pressure surges are not always accompanied by power swings.

2 That the power swings are produced by fluctuations in torque on the runner, which in turn are caused by the draft-tube surges and penstock pressure fluctuations.

3 That the magnitude of the power swings depends upon the magnitude of the change in torque on the runner and upon the relation between the frequency of the draft-tube surges and the natural frequency of the generator connected to the distribution system.

4 That by avoiding resonance of draft-tube surges and natural vibrations in the generator during design of a proposed hydroelectric unit, serious power swings can be avoided.

5 That power swings as much as 10 per cent of the rated output of the generating unit can be tolerated without harm to the operation of the unit or stability of the transmission system.

6 That the power swings can be greatly reduced or eliminated in the field by admission of air to the draft tube or by making physical changes in the draft tube.

Discussion

A. F. Darland.[3] The writer believes that the author was present at the Cushman No. 2 hydroelectric plant of the City of Tacoma at the time difficulties were encountered because of power swings. On that occasion the difficulty was entirely overcome by venting the turbine cover plates through small air valves, about $1^1/_2$ or 2 in. in diam. Not only were the power swings eliminated but a small increase in output of the generators was realized due to the smoother flow in the draft tubes, resulting from the opening of the air vent. The power swing occurred at about 0.6 or 0.7 gate opening and the air valve was arranged to open automatically at these values of gate openings.

McNeely DuBose. Hydroelectric operators have seemed to accept that a water-wheel unit should have a "rough spot" or a "jumping point" somewhere in its range of operation. At that particular point there might be an uncomfortable vibration, an alarming heaving of the penstock, or an occasional step-bearing failure, but the point could be quickly passed over. Operators learned so well to keep their machines away from it that a year or so after installation, this condition was practically forgotten by the engineers.

But with passing years, these unstable points have become less easily overlooked, for larger generators with longer transmission lines seem to be more susceptible to recurrent hydraulic surges than were the smaller units with shorter lines of 30 years ago.

Nor is the problem of power swings necessarily restricted to some particular gate openings which can be avoided. There have been cases where swings originated at comparatively small gate openings and continued in greater magnitude up to full gate, if the machines could be kept on load up to that point.

Mr. Rheingans points out that little or nothing on this subject has appeared in technical literature. That might be amended by saying "public" technical literature, for reams of paper have been used in correspondence on this subject. Probably this reticence is natural. A hydroelectric unit consisting of both water wheel and generator may be on occasions a prolific source of "buck passing," which is heartily indulged in by engineers in the field, but in general has had so little firm ground to stand on that no one has wanted to post it up for studied criticism. As a result, the operators have sometimes been caught between the upper and nether millstones. In the end their difficulties were straightened out, but after months or even years of inconvenience or worse.

So the writer is heartily in favor of anything which tends to call this matter to attention and brings out for the general benefit experience which so far has been locked away in filing cabinets.

An alternating-current generator is a machine which will oscillate. Its period is infinitely long at no load and decreases as the load builds up. Larger machines have a longer period than small machines. The period of even the largest machine at full load is seldom slow enough to synchronize with impulses originating in a water wheel, although inductive load and the reactance of transmission lines may slow down the rate of oscillation of a generator to where it will fall in step with the hydraulic oscillations.

Speaking of the rate of swings, a comparison of calculated and observed rates may be of interest. For instance, take the case of a group of 65,000-hp units, operating at full gate, connected through 28 miles of heavy transmission line to a group of 45,000-hp units, operating at about 0.8 gate. About 100,000 hp is passing through the line. The calculated period of the larger units at full noninductive load by the formula given by the author is 85 cycles per min. The actual observed period of the individual generators varies between 77 and 84 per min. The oscillation period of the 45,000-hp units at full load is calculated to be 100 cycles per min. The observed period of the individual units varies between 92 and 116 per min. The effect of the transmission line is obviously small. At lighter loads, the frequency is observed to drop some 20 per cent.

This example of hydroelectric-generator swings should not be confused with system swings. Take the case of two systems each with some 750,000 hp of generating capacity connected through a weak transmission line 215 miles long. With 30,000 hp passing through the tie, the period of the swing is 24 cycles per sec and the amount of the swing is some 15,000 hp. But while this is going on, the individual generators of the system are swinging at their own higher periods.

Hydraulic-turbine oscillations are always at a comparatively slow period. The writer has never observed recurrent surges in a low- to medium-head turbine at a rate much lower than 30 or greater than 60 per min, and doubts if it is possible for them to exceed the latter rate by much. However, it should not be forgotten that impulses may be transmitted materially to the generator when its period is a multiple of that of the water wheel.

The hydraulic designers are the ones who must deal first with the problem of power swings. They should receive the full collaboration of the operators in so far as their intentions go regarding generator characteristics and system connections. However, requirements of system stability and the geographic locations of power sites often permit very little leeway for helpful changes in generator design, other than the addition of damper windings. But that is a case of attacking the symptom instead of the disease.

J. P. Growdon[4] and D. J. Bleifuss.[5] It has long been known

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[5] Assistant Hydraulic Engineer, Aluminum Company of America, Pittsburgh, Pa.

that occasionally objectionable power swings are observed in hydroelectric units, but to the writers' knowledge, the author is the first to discuss them publicly. Heretofore, such difficulties have been treated as dark secrets between the purchaser and the manufacturer, otherwise the purchaser has suffered in silence.

The author states in conclusion that six facts are definitely established. Proceeding to a discussion of these points in the same order, the writers offer the following:

1 It is probably true that power swings chargeable to a hydraulic turbine are always accompanied by draft-tube surges, but perhaps not necessarily by penstock pressure fluctuations. It is likewise true that draft-tube surges do not always result in power swings. This may be because the surges themselves are irregular, or because the natural period of the generator does not coincide with that of the surges, i.e., there is no resonance. However, the absence of power swings in the presence of draft-tube surges is not evidence that the unit has been correctly designed; on the contrary, the presence of the surges is evidence of incorrect design. It is certain that no design could result in absolutely smooth flow; it is equally certain that bad design will result in very rough flow.

2 The writers agree that swings are caused by variation in torque on the runner, caused by draft-tube disturbances. These disturbances, of course, are due to unstable flow conditions, which in turn are the result of excessive swirl in the water or improper distribution of flow. The ideal condition would be that in which all water flowed vertically as it left the runner, and the draft tube was filled with water flowing at a uniform velocity. For any given runner, at a given speed, this condition is approximated at only one combination of gate opening and head, but divergences from this condition are minimized by correct design.

3 The writers agree that the magnitude of the swings is dependent upon the relation between frequency and magnitude of disturbances on one hand and the natural frequency of the unit on the other, if there are no contributing conditions in the connected system.

4 The writers agree that by avoiding resonance between frequency of surges and vibrations of the generator, serious swings may be avoided. At this point, however, they wish to emphasize that this is not the complete story. Although the swings may be thus avoided, the tendency may be there, and the efficiency of the unit consequently less than should be expected. It would seem that a far better method of prevention would be to design the turbine so that surges do not occur.

5 The writers think that while power surges of 10 per cent of the rated capacity of a unit may be tolerable, they are still most undesirable.

6 The writers agree that power swings can be reduced or eliminated by admitting air to the draft tube. They realize that normal operation of a Francis turbine involves admission of certain amounts of air at certain gate openings. Admission of air in greater amounts than these, and for the purpose of reducing or eliminating swings which have developed, must involve some loss of power. As to draft-tube alterations, the writers have no experience, but wish to indulge in a little speculation. It appears that the difficulties of placing fins in a draft tube already built might be enormous. Consider a unit discharging 4000 cfs, with maximum velocities of 40 ft per sec, and the problem of putting in fins which will stay is a real problem. Further, swirling in a draft tube indicates that the water is leaving the runner with some inclination from the vertical; this inclination varies with the gate opening, and it would appear that when it is a maximum the fins must cause considerable disturbance and consequent loss of power.

The writers wish to point out that either admission of air, or alteration of the draft tube is a palliative or partial remedy, and to remind the profession of the old adage concerning the desirability of "an ounce of prevention."

In explanation of the writers' position, they wish to cite a specific example, where power swings on large units were sometimes of such magnitude as to preclude operation at full load.

Each unit consisted of a Francis-type turbine and an a-c generator. Turbines were rated at 55,500 hp at 135-ft head, the speed was 120 rpm, and the WR^2 about 66,920,000. Using the author's formula for a generator connected to an infinite system, the natural frequency is found to be about 85.5 cycles per min. The draft tube was of the simple elbow type, divided for some distance in from the mouth by a vertical pier.

When the first unit was put on the line, it was allowed to carry 8000 kw for several hours. Then the load was increased to 15,000 kw and it carried this load overnight, although swings of about 2000 kw were noted. The next day load was increased by increments of 5000 kw to the maximum of about 39,000 kw. At a load of 30,000 kw, power swings of an amplitude of 6000 to 10,000 kw were observed, increasing slightly above this load. These swings had a frequency of about 80 cycles per min. They were not consistent. They would at times arise and die without apparent cause, when there was a constant load on the system. The electrical system was investigated for possible resonance or disturbing conditions; nothing was found which could in any way contribute to the swings. Swinging occurred over the entire gate-opening range.

The draft-tube discharge was very turbulent; at the higher loads most of the water appeared to be leaving through the right one half of the right one half of the draft tube. At lower gates, there was actually re-entrant flow in the left half of the tube. Variations of several feet in tail-water elevation demonstrated that it had no effect on the power swings; they were observed at high tail water as well as low.

The portion of the draft tube just below the runner was lined with steel. On the upstream side of this liner there is a mandoor. A pitot tube was inserted horizontally through a stuffing box in this door, and conditions of flow were studied at an elevation 7.5 ft below the center line of the runner or, rather, the center line of the scroll case. The pitot tube was so constructed that it could be rotated, and thus the angle of flow as well as velocity could be determined. A mercury manometer was used in connection with the pitot tube. Observations were taken in as far as 3 ft from the draft-tube liner; since the draft tube has here a diameter of about 14 ft, conditions of flow were determined in the outer 67 per cent of the draft-tube area. Results of a Gibson test on the unit were available, so that the discharge for any given gate opening and head was known. Thus, knowing the vertical component of velocity over 67 per cent of the draft-tube area, it was possible to estimate very closely the vertical component of velocity in the remaining 33 per cent.

For each set of measurements, the turbine gates were blocked at a certain opening, and conditions were such that head and tail water did not vary. A constant discharge was thus assured. Readings were started with the pitot-tube orifice pointing straight up; the tube was then rotated, stopping every 30 deg for an observation. The difference between the legs of the manometer, in inches of mercury, was recorded at each observation. These readings were plotted as follows:

A circle was drawn, the circumference representing a zero reading of the manometer. Pressures were plotted radially outward from this circle, vacua radially inward, at the directions observed. A smooth curve connecting the plotted points resulted in a characteristically pear-shaped figure, the major axis of which represented the direction of flow.

The water whirled with the runner at all gates; the inclination of flow from the vertical varied, for instance, from 51 deg at 0.3 gate to 18 deg at full gate, 15 in. inside the liner. Vertical components of velocity varied as follows:

1 At the liner, 19 ft at 0.3 gate to 9 ft at full gate.
2 Two feet in from the liner, 7 ft at 0.3 gate to 21 ft at full gate.
3 At the center, zero at 0.3 gate to 0.5 gate, a maximum of 45 ft at full gate.

Thus two things were apparent (1) there was excessive swirl, (2) there was a very uneven distribution of velocity. The first indicated that water was leaving the runner at an incorrect angle; the second, that it was leaving the runner in the wrong places. From the figures stated, it will be noted that, at full gate, the vertical component of velocity varied from 9 ft at the liner to 45 ft at the center. Most of the discharge was concentrated at the middle of the tube.

It was speedily found that, by admitting air to the space underneath the runner through the hollow turbine and generator shafts, the power swings could be greatly reduced or entirely eliminated; this fact, with other observations, led to the conclusion that the trouble was caused by disturbances in the draft tube.

Unfortunately, the admission of air resulted in a power loss of from 1500 to 2000 kw as determined by an integrating watt-hour meter, and was accompanied by a distinctly objectionable whistling noise.

The vacuum gage on the draft tube was unsteady, but never indicated a variation regular enough to account for the power swings, and penstock pressure never varied more than 1 lb on the gage.

The turbine manufacturer studied the problem thoroughly, experimenting both on the units in the field and on models, and finally decided on an alteration of the runner. Each runner was cast steel, 155 in. in diam, with 15 buckets. At the lower edge of each bucket a roughly triangular piece was removed, about $24^1/_2$ in. long, and varying in width from $9^1/_2$ in. at the band to 0. This piece was drilled and chipped out at its junction with the band and burned elsewhere. The remodeled runner operates satisfactorily with no air coming through the hollow shaft, and there is a slight increase in power as compared with the original runner operating with no air. An automatic air valve in the cover plate admits air up to about 0.85 gate opening. Maximum amplitude of swing with the remodeled runner is 2000 kw.

The writers wish to state that the turbine manufacturers deserve great credit for their determined attack on what was, at the time, a new problem, and for their arrival at a fundamentally correct solution.

R. G. MacDonald.[6] The writer concurs essentially with the conclusions of the author. The bases of the writer's opinions are specific instances in which approximately the same phenomena and the same effects of corrective measures have been observed.

Unfortunately, at this time, the identity of the plants from which data were collected cannot be revealed, due to its confidential nature. However, without reference to the plants or the identity of the equipment manufacturers involved, some of the conditions which were observed before and after corrective measures were applied will be outlined briefly.

In one instance a unit of moderate rating, when connected to a very complicated system with extremely long interconnected transmission lines, and a very large system WR^2, at a "critical" turbine gate opening, were subjected to power swings of about 25 per cent of the load being carried at the time, and reflected swings of about 10 per cent of the load carried were observed at very large units which were at the time operating in parallel with the smaller unit. Upon separating from the last-mentioned system, swings on both systems almost completely disappeared. Bearing in mind the fact that the gate opening, magnitude, and frequency of the draft-tube swings remained unchanged, it is obvious that alterations of the system characteristics were alone responsible for the power swings.

After the separation there still remained at least 600 miles of transmission systems connected in parallel, but the WR^2 was a small fraction of that existing during the first test when all units and systems were in parallel. This combination resembled a three-tine fork, each tine being a transmission line of 250 miles or over in length, the handle representing the loading of each system. At the time of separation, the central tine was removed leaving a Y-shaped transmission-and-distribution system. No undue power swings were caused by the addition, at the junction of the two transmission lines, of another system approximately 100 miles long. The combined WR^2 in this case remained at a low value.

It was learned that changing the magnitude and/or the frequency of the draft-tube swings also altered the values of the load swings, and in most cases maneuvering completely removed objectionable power swings.

The writer is familiar with one plant where all draft tubes have nearly the same swings, with respect to value and frequency, where different units are operated over isolated transmission systems having dissimilar characteristics. In some instances power swings are in evidence and in others none exists. Therefore, it follows that units operating on a given system may have power swings on another, due to changes in system characteristics.

Considering the most troublesome unit in this plant, at approximately one-half load on the turbine, power swings equal to about $^5/_8$ of the rated capacity were present.

Oscillograms were taken to record the frequencies of the draft tube and power swings and the natural frequency of the system. It was found they had approximately the same period. The natural period of the system was determined by taking an oscillogram of the armature current and a timing wave during a paralleling operation with the incoming machine slightly out of phase with the transmission system. Excellent pictures can be obtained by the proper timing and operation of an oscillograph. Films can be superimposed making frequency relations obvious by inspection.

A companion unit of the same dimensions and capacity which had draft-tube fins did not produce power swings in excess of 10 per cent of the rated capacity. The fins used were similar to those described by the author. During the test, the turbine gates were set at the most critical point. Free air was admitted through the fins. Thus the installation of fins in the draft tube reduced swings to about $^1/_{10}$ of their former value.

It is interesting to note the reduced value of power swings obtained when two similar units, one with draft-tube fins and the other without, were operated in parallel. This appears to prove that reduced power swings result from both a change of system characteristics and the addition of a stabilizing unit, the draft-tube swings of which have been brought under control.

At this point, the writer wishes to supplement the author's sixth conclusion by the following statement: The corrective effect of air in the draft tube is proportional to the ability of the equipment to cause the air to reach the points where it is most needed.

Installation of hydraucones, fins, air, and other devices at least partially correct power-swing troubles arising from draft-tube swings. This indicates that factors in design can be considered

[6] Boulder City, Nev.

rather than changing the speed as would be indicated by Fig. 5 of the paper.

It appears that the only manner in which this problem can be solved, as a matter of prevention rather than correction, wherein much psychology must of necessity be involved, lies in newer and better methods of model testing. It appears to the writer that actual pressures and of course velocities, together with scale models of penstocks and draft tubes, must be employed in testing model runners. This sort of testing would be in addition to the orthodox "flume" test as now practiced. Much about cavitation could be learned at the same time.

L. F. Moody.[7] The author has made a useful contribution in calling attention to a problem which has received little attention in the literature, and in investigating and recording factual data and certain conclusions relating to the influence of some of the chief factors involved. The problem is a complex one, and as the author points out, further analysis is needed to establish a basis of theory. It is hoped that he and others will continue this investigation so that we have a rational framework behind the empirical conclusions.

Some preliminary hypothesis seems necessary if we are to try to account for the phenomena of power swings. It seems to the writer that we cannot explain the action as due primarily to such factors as elastic pressure waves in the water column, which have a constant frequency depending only on the length of the conduit and the wave velocity, both of which are constant, or torsional vibration of the turbine shaft and runner, which also has a constant frequency. Nor can we explain it as a slower surge of the water column as a whole, treated as an inelastic medium. Liquid in a U-tube, having free surfaces, can be made to oscillate harmonically; but the turbine water column terminates in fixed head- and tail-water surfaces. The writer has from curiosity set up the equation for accelerated flow in the system, imagining a momentary disturbance of equilibrium; but the solution is similar to that for a suddenly opened valve at the end of a pipe line. The flow approaches the steady value asymptotically along a smooth curve of logarithmic form, and is nonoscillatory.

These considerations lead the writer to suggest the presence of an elastic medium, such as an air or vapor cavity, somewhere in the system. The point where this may be expected is around the axis of the draft tube just below the runner. Under part gate operation, the reduced discharge through the constant runner areas of course results in reduced relative exit velocity and consequent forward absolute whirl of the outflow. The draft-tube vortex involves flow separation near the axis and invites the formation of a hollow core in which entrained air can accumulate; and probably in many cases where power swings and draft-tube surges are marked, a vapor-filled cavity is formed. After the foregoing suggestion was offered the writer had the opportunity of witnessing a motion picture which it is hoped can be shown again. This was contributed by I. A. Winter of the U. S. Bureau of Reclamation and showed the flow within a transparent turbine model. At part gate operation this illustrated clearly the presence of a vortex core, accompanied by instability of flow.

The surging of the draft-tube water column, resulting from the presence of air or vapor, may be of more consequence in its direct effects than in the power swings it causes in the generator output. If the turbine setting does not provide much margin over the critical cavitation coefficient of the turbine, a unit which would operate perfectly under steady conditions may cavitate when a draft-tube surge sets in, the cavitation then accentuating and maintaining the surge.

[7] Professor of Hydraulic Engineering, Princeton University, Princeton, N. J. Mem. A.S.M.E.

The method of combating surges, vibration, and power swings by air admission has of course been often used. Fortunately gate-controlled air admission is provided in most modern turbines of higher specific speeds, although special means may have to be added to get the air into the vortex core. The writer might mention one case where an air pipe was inserted through the draft-tube wall, extending near enough to the axis to reach the vortex core and effectively keeping the surges within bounds. Guiding fins on the draft-tube wall are an undesirable remedy, from considerations of turbine efficiency.

Since power swings and surges occur in turbines under fixed-gate operation, it is likely that similar phenomena may occasionally occur in pumps, particularly in propeller or mixed-flow pumps and, in general, whenever whirl may be induced in the suction space.

J. F. Roberts.[8] The six conclusions at the end of the paper form a complete summary, but it is believed the fifth is open to discussion. This fifth conclusion reads as follows: "That power swings as much as 10 per cent of the rated output of the generating unit can be tolerated without harm to the operation of the unit or stability of the transmission system." It is possible that the author is endeavoring to educate the users to expect this much power swing. However, while the writer would not state definitely that 10 per cent swing might do serious damage, he feels that it certainly would be objectionable and cause uneasiness to the operator. Considering that, with a 50,000-hp unit, this would mean a power swing in the order of 5000 kw, it would certainly occasion comment. Possibly, with a 3000- or 4000-hp unit, a power swing of 300 or 400 kw might not be considered objectionable; but in general, it is felt that 10 per cent is two or three times what could be passed without serious objection.

Regarding the fourth conclusion, that resonance between the draft-tube surges and the natural vibration of the generator may be avoided during design, this would involve a large amount of money if it were done by increasing the WR^2 of the generator. For instance, doubling the WR^2 of the generator would reduce the ratio between the frequency of the disturbing forces and the natural frequency of the electrical system from 1 to 0.75, or only 25 per cent; but such an increase in the WR^2 of the generator would probably amount to a considerable percentage of the original cost of the generator. If the result can be achieved by the use of fins installed in the draft tube or by the admission of air under the runner, these solutions are far more economical than the increase of WR^2 of the generator.

The author does not state whether or not the addition of the fins affects the performance of the draft tube and the over-all efficiency of the unit. The writer believes such a statement would be enlightening to the members of the Society.

R. E. B. Sharp.[9] The author has called attention to an important factor in the design of hydroelectric units, and has submitted valuable data for the study of the phenomenon of power swings. The writer is as yet able to offer only one instance of a power swing and its relation to draft-tube surges. In this instance: Rpm = 187.5; observed frequency of draft-tube swings at about 0.82 gate = 80 to 86 per min = S, whereas $\frac{\text{rpm}}{3.6} = 52$,

$$F = 46{,}000\sqrt{\frac{f}{c}} = 46{,}000\sqrt{\frac{25}{13{,}700{,}000}} = 62.2;$$ power swing

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[9] Chief Engineer, I. P. Morris Division, Baldwin-Southwark Corporation, Philadelphia, Pa. Mem. A.S.M.E.

5000 kw (full load about 46,000 kw); penstock pressure swings 115 to 125 lb per sq in.

The wide discrepancy between the author's value for S and that actually observed may be due to the large gate opening at which the draft-tube disturbances occurred in this instance. On the other hand, another unit in the same station of different design encountered draft-tube disturbances just as severe but at a much higher frequency, about the same as the normal rpm of the unit, namely 187.5, but with no power swings. The draft-tube design as well as the runners were different for the two units.

For the instance cited, at least, severe power swings resulted where S actually was more than 50 per cent greater than the value indicated by the author's empirical formula $S = \frac{\text{rpm}}{3.6}$. A more accurate method for determining this frequency, therefore, appears desirable. If the undesirable relation between F = natural generator frequency and that of draft-tube swings is to be avoided, where possible, in the design state it is naturally important to predict closely the dangerous zone to be avoided. While air admission valves are today a standard turbine accessory, and they are known to reduce or even eliminate draft-tube surges, it is, of course, wise where practicable to avoid the zone of resonance.

From a broad standpoint, it appears important to the writer that turbine manufacturers exchange complete experience data relative to draft-tube surges, both in cases where power swings have and have not occurred, in order to be in the most favorable position to reduce or eliminate, if possible, the tendency for such surges to occur. Due to the many factors which enter into this problem, this procedure should result in its quickest solution to the benefit of all concerned.

G. R. Strandberg.[10] The data collected to date on the subject of power swings have been rather limited since in most cases the load swing has been so small in comparison to the size of the system to which the units have been connected that system operation was not affected. The writer has observed load swings on units of several plants built from 1912 to recent years and has data showing a maximum load swing of about 20 per cent on units of one plant.

The author states that in cases of power swing there is always a pressure swing in the penstock accompanied by a vacuum swing in the draft tube and that maximum pressure in the first case occurs with maximum vacuum in the latter. If the surges in the draft tube were in phase over the entire draft tube, it would seem to the writer impossible for an increase in vacuum in the draft tube to cause a pressure rise in the penstock. In one installation, gages at quarter points in the draft tube showed that the surge was not in phase at all points. Diametrically opposite points were out of phase 180 deg and points 90 deg from each other were out of phase 90 deg. The phase rotation was clockwise on units rotating clockwise looking down on the unit. It is possible that the pressure swing in the penstock is caused by the rotation of the vacuum surge around the draft tube passing the nose point of the scroll case. It would be interesting to know whether pressure swing in the flume or load swing has occurred in any plant having closed flume setting with no scroll-case nose point connecting the wall to the speed ring.

In one plant, where two units were connected to a single transmission line, the indicated maximum combined load swing was 3 times the indicated maximum load swing of a single unit and the same indication was obtained at both the generating and the receiving ends.

Vertical steel-plate fins installed in the draft tube below the runner in an effort to correct load swings materially reduced them, and the reduction was increased as the width of fin increased. Tests indicated little or no change in reduction whether the fins were located immediately below the runner or several feet lower in the draft tube. With too wide a fin, however, the draft tube became noisier and the indications were that the width of fin would have to be limited for structural reasons. The fact that such fins materially reduce the load swing indicates that the rotating component in the unit discharge has a material relation to the load swings.

I. A. Winter.[11] The writer has had occasion to observe the phenomenon described by the author and is greatly impressed by the apparent agreement with the conclusions expressed and results obtained for installations familiar to the writer. The paper quite properly is limited to a discussion of power swings associated with resonant pressure changes in the draft tube, although the title would indicate a much broader field of application.

Power swings from other sources in an interconnected hydroelectric system are of a nature equally as involved and difficult of solution as the power swings described in the paper. Notable among these objectionable phenomena of regulation and control are those of incipient instability of hydraulic penstock systems with surge tanks, where the length of penstock is greater than 4 to 5 times the head on the unit, and the inherent inability of present-day governors to effect proper distribution of load between hydrounits operating in parallel as required for efficient operation, and to carry correct time throughout a 24-hr period as is now universally expected by electric-clock users.

A series of tests was conducted at the Guernsey power plant of the North Platte Project in Wyoming for the purpose of determining if the so-called incipient instability[12] of relatively long penstocks with surge tanks and under low heads and the draft-tube-surge phenomenon are tangible and inviolate or if other unknown factors could be responsible for the troubles now attributed to these phenomena.

The Guernsey power plant consists of two 3400-hp vertical-shaft, single-runner, Francis-type, spiral-casing turbines operating at 240 rpm under an average effective head of 80 ft. Each turbine is equipped with a conical-plate steel draft tube discharging into an open rectangular flume. Power water is supplied through a 12-ft-diam single penstock 850 ft in length, terminating with bifurcated branches, 8 ft 6 in. in diam and at a distance approximately 70 ft from the point of attachment to the turbine-scroll cases. In the 12-ft-diam penstock immediately above the bifurcated branch, there is located a simple surge tank, 22 ft in diam, and 85 ft in height with a restricted orifice in the bottom 7 ft 6 in. in diam. Units A and B in this plant were installed at different times and are of different manufacture.

Analytical investigation of the hydraulic characteristic of the penstock system made before the tests were started indicated that the period of the incipient surge should be approximately 55 sec. Using the author's Equation [1], it was estimated that the frequency S of the draft-tube surge if present would be approximately 67 cycles per min. Observations made during the tests established the period of the secondary surge in the penstock and surge-tank system as being 54 sec with an amplitude of 0.25 ft

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[11] Senior Engineer, Bureau of Reclamation, United States Department of the Interior, Denver, Colo.

[12] "Test for Incipient Stability," Hydroelectric Handbook by W. P. Creager and J. D. Justin, John Wiley and Sons, Inc., New York, N. Y., 1927, p. 533.

when discharging a total flow of 462 cfs, which is in substantial agreement with the calculated values.

Unit *A*, when operating at 30 per cent gate opening, developed a surge in the draft tube of 54 cycles per min with pressure variations from 5 to 10 ft of water. The draft-tube pressure gage on unit *B* was quite steady. The total load on the distribution system in which the units were generating amounted to only 5000 kw and no evidence of power swings coordinated with the draft-tube surge in unit *A* could be observed. It is, therefore, quite obvious that resonant pressure surges in the draft tube cannot alone be responsible for objectionable power swings in the distribution system. It was also demonstrated that the pressure surges are not a characteristic of the type of draft tube, since both units *A* and *B* have identical draft tubes of the straight conical type. A further conclusion which may be drawn from these tests is that the formation of resonant surges in the draft tube can be prevented by the proper design of turbine runner.

Field investigations of the draft-tube surges made at Guernsey had been preceded by experiments made on a 6-in. model turbine in the laboratory of the Denver office of the Bureau of Reclamation, using scroll cases and draft tubes made of pyralin, which permitted visual observations of the phenomena as developed both in the scroll case and draft tube. Motion-picture records made of these tests show clearly the formation of a void at the runner tip, which gradually tapers in size to a point and disappears as it reaches the horizontal discharge portion of the draft tube. The conical-shaped void has a spiral motion at its boundary identical with that of the ordinary surface vortex with the exception that the spiral has a second motion of rotation which is about the center of the draft tube and has a frequency of rotation at its tail described by the author as being approximately $^1/_3$ of the speed of the turbine. The use of pyralin models of the type described opens the field of investigation of homologous turbines for individual designs which will establish the characteristics of the runners with respect to the formation of the void at the tip of the runner. Should this procedure be followed, it is conceivable that runners could be designed which would be entirely free from the troubles now being experienced.

From the limited amount of data available, a magnification factor of 1.5 or less using curve No. 2 in Fig. 4 of the paper may be tentatively selected as indicating that power swings, resulting from resonant surges in the draft tube, will not cause operating difficulties. A study of the magnification factors for a number of units installed or contracted for is shown in Table 1, using notations identical with those of the author. Significant facts revealed by these figures are as follows:

Unit No. 1 with a magnification factor of 2.1 developed power swings of a magnitude such that the unit could not be operated in parallel with a system carrying a load in excess of 1,000,000 kw. This unit was corrected by the admission of air to the center of the runner, thereby permitting a more uniform volume to be maintained in the void forming immediately below the tip of the runner. This correction demonstrated that the formation of the void alone is not detrimental but it is the making and breaking of the void that creates the objectionable surges.

This solution of the difficulty has been fully described by the author but the writer does not agree that the admission of air breaks up the surge in the manner of the draft-tube fins shown in Fig. 7.

Of second interest in Table 1 is unit No. 2 with a magnification factor of 1.6. This unit developed trouble identical to unit No. 1, but of lesser magnitude, and was placed in successful operation in the same manner as described for unit No. 1.

Unit No. 3 in the table has a magnification factor of 5 and power surges developed so critically that the unit could not be operated throughout approximately 60 per cent of the power range. The use of compressed air forced in at the center of the runner failed to correct the troubles as described for units Nos. 1 and 2. Units Nos. 3 and 4 are units of identical design and capacity. When units Nos. 3 and 4 are operated in parallel, the magnification factor is reduced from 5 to 1.6 and it is possible to operate both units satisfactorily throughout their entire power range with a moderate amount of compressed air (350 cfm) admitted at the center of the runner. Unit No. 3 has fins installed as illustrated in Fig. 7, which are effective in breaking up the swirls and thereby reduce the draft-tube surges which permits operating the unit throughout its entire power range. Comparison of these two units, when operating singly and in parallel, is an outstanding example of the conclusion drawn by the author that the magnitude of the power swings depends upon the relation between the frequency of the draft-tube surge and the natural frequency of the generator.

Further examination of the units listed in Table 1 tends to demonstrate the conclusion that a magnification factor of 1.5 and below represents a reasonably safe installation factor. For example, unit No. 5 is the Guernsey unit previously described with a magnification factor of 1.2. No trouble of any nature could be traced to this unit in the development of unstable power swings in the generating system. Unit No. 6 has been given a preliminary test with the results as indicated in the table and it appears that correction by the admission of air to the center of the runner will be necessary to eliminate power swings effectively due to surges originating in the draft tube.

Units Nos. 7, 8, and 9 are units which have not been installed but are under construction or contract and will be placed in operation in the near future. It appears that units Nos. 7 and 9 may be expected to give no trouble from power swings originating in the draft tube. However, it is proposed to investigate unit No. 8 with models using pyralin draft tubes and scroll cases for the purpose of determining whether the runners as designed are favorable or unfavorable from the standpoint of the development of a spiral void below the runner.

For the convenience of those interested in investigating the effect of the electrical characteristics of a particular generator or transmission line, the following formulas are applicable:

The value of *Pr*, Equation [7] of the paper, for a particular generator may be calculated as follows:

$$Pr = \frac{dP}{d\delta} \text{ times kva rating of machine} \quad [9]$$

where

$$\frac{dP}{d\delta} = \frac{eI}{X_d}\cos\delta + \frac{e^2(X_d - X_q)}{X_d X_q}\cos 2\delta \quad [10]$$

e = per unit terminal voltage
I = per unit field current
δ = torque angle in electrical radians

TABLE 1 MAGNIFICATION FACTORS FOR VERTICAL-SHAFT SINGLE-RUNNER HYDROELECTRIC UNITS

Unit no.	Regulating constant C, $C \times 10^{-6}$	Natural frequency of generator F, cycles per min computed	Frequency of draft-tube surges S, cycles per min Computed	Observed	Ratio $\frac{S}{F}$	Magnification factor, Fig. 4, curve 2
1	12.0	64	71	85	1.33	2.1
2	30.6	79[a]	50	45	0.58	1.6
3	21.2	49	42	49	1.00	5.0
3 + 4	21.2	75	42	49	0.56	1.6
5	7.6	141[a]	67	54	0.38	1.2
6	9.5	105[a]	63	50	0.60	1.6
7	6.9	145[a]	50	..	0.34	1.2
8	13.4	72	38	..	0.53	1.5
9	12.3	77	26	..	0.34	1.2

[a] Transmission line not included.

X_d = per unit direct axis synchronous reactance
X_q = per unit quadrature axis synchronous reactance

and

$$P = \frac{eI}{X_d} \sin \delta + \frac{e^2 (X_d - X_q)}{2X_d X_q} \sin 2\delta \quad [11]$$

= per unit power output

The value of Pr' for a particular transmission line may be calculated as follows:

$$Pr' = \frac{dP'}{d\delta'} \text{ times kva base used in computing reactance} \quad [12]$$

where

$$\frac{dP'}{d\delta'} = \frac{E_s E_r}{X_{sr}} \cos \delta'$$

E_s = per unit voltage at generator terminals
E_r = per unit voltage at receiving end substation low-voltage bus
δ' = angle between E_s and E_r
X_{sr} = per unit reactance of system between E_s and E_r

and where δ' is determined by

$$P' = \frac{E_s E_r}{X_{sr}} \sin \delta' \quad [13]$$

= per unit power transfer on kva base used in computing reactance

In summary it may be stated that the author offers an empirical method of calculating the natural frequency of draft-tube surges, reviews the method of calculating the natural frequency of the electrical system, and concludes that the factor having the greatest influence on the magnitude of the power swings is the relation between the frequency of the draft-tube surge and the natural frequency of oscillation of the generator and its interconnected power-distribution system. When based on the meager experience with units as outlined in Table 1, the following additional conclusions may be drawn:

(*a*) A magnification factor of 1.5 or less indicates that power swings will not cause any operating difficulty.

(*b*) The effect of the transmission line is to lower the natural frequency of the electrical system.

(*c*) For the case of one generator connected to a long transmission line, the effect of the transmission line must be included.

(*d*) For the case of two or more generators operating in parallel, the maximum frequency approaches the maximum frequency of one generator connected to an infinite bus.

(*e*) A runner designed to avoid excessive whirl at the center of the draft tube throughout the critical range will not produce power swings of the character described in the paper.

Author's Closure

Mr. Darland mentions the venting of the cover plates at the Cushman No. 2 hydroelectric plant. This is one of the few cases where venting reduced the power swings without causing a material reduction in turbine efficiency. The power swings in this case were small, being less than 5 per cent of the rated output and, thus, a large reduction in the swings was obtained with a small amount of air admitted to the draft tube.

The close agreement Mr. DuBose obtained between calculated and observed values of the natural periods of oscillation of various generators is very gratifying and establishes the fact that the natural period of a generator may readily be calculated during the design stage.

Mr. DuBose questions whether it is possible for draft-tube surges to exceed 60 cycles per min. Fig. 3 of the paper mentions draft-tube and penstock pressure surges at the rate of 320 per min in a 1700-kw, 260-ft head, 900-rpm turbine. The frequencies of both the draft-tube and penstock surges were carefully observed at this plant and there is no question regarding their occurrence at this high rate.

Mr. Growdon and Mr. Bleifuss state that the difficulty of placing fins in draft tubes already built might be great. However, experience has shown that properly designed fins are quite easily installed either in low-head plants of large physical dimensions or in high-head plants where high velocities are encountered. Fins thus properly designed and installed will remain in place as long as the draft tube does.

Experience with models and field tests on draft-tube fins has also shown that, when properly designed and located in the draft tube, they will not reduce the maximum output of the turbine and will reduce the maximum efficiency only slightly if at all. This is explained by the fact that, at the best efficiency gate, the flow in the draft tube is very nearly axial, the fins, therefore, presenting but slight obstruction to the flow at this point. However, part-load efficiency may be reduced as much as 1 per cent, depending upon the amount of whirl in the draft tube.

The detailed description of Mr. Growdon and Mr. Bleifuss of their investigations of power swings in a 55,000-hp, 135-ft-head turbine is a valuable contribution and certainly indicates the true source of the draft-tube surges and disturbances.

Mr. MacDonald's description of the variations in load swings with a change in the characteristics and setup of the transmission and distribution system opens up a field for further and more thorough investigation.

Mr. Roberts is correct in stating that a 10 per cent power swing might be objectionable and might cause uneasiness to the operator. However, this is not true in all plants. There is a case on record where a number of duplicate units developed power swings of 10 per cent of the rated output of the turbine. It was demonstrated on one of the units that the power swings could be eliminated by the installation of fins in the draft tube with only a slight loss in efficiency. However, the owners of the plant preferred to operate the remaining units with the power swings present rather than to install the fins. These units have been in operation for nearly 10 years. There are numerous other cases where units have been in operation for 10 years or more with power swings of 10 per cent or greater.

The author agrees with Mr. Roberts that changing the WR^2 of the generator would be an expensive method of avoiding resonance conditions. A change in speed of the unit might be less costly and might be likely to give more definite results.

Mr. Sharp's apparently limited experience with power swings seems to indicate that, where the draft-tube surges occur at the larger gate openings, the frequency is somewhat greater than given in the author's empirical formula. One of the theories for the origin of draft-tube surges, which the author is investigating at the present time, seems to indicate that the frequency will increase with an increase in gate opening.

Mr. Winter's description of the draft-tube surges at the Guernsey power plant, which did not produce any power swings, and his table showing the magnification factors and field results of various installations, are an excellent illustration of the theory advanced by the author that resonance conditions must be present before the draft-tube surges can produce serious power swings. We should, therefore, not lose sight of the fact that the electrical system is a contributing factor in the production of power swings and that part of the responsibility should be so delegated, especially when exact resonance conditions are encountered.

It seems fairly well established at the present time that draft-tube surges are caused by the whirl of the water in the draft

tube, which whirl is produced by the angular flow of the water leaving the runner vanes. The consensus of opinion of those discussing this paper seems to be that draft-tube surges can and should be eliminated by proper runner design. However, a hydraulic-turbine runner can be designed to give axial discharge for only one gate opening and for only one speed. For any other gate or for any other speed there will be a whirling discharge. Since turbines usually have to operate at various gate openings and sometimes have to operate under a wide range of head at fixed speed (which corresponds to a varying unit speed), draft-tube whirl will be present under some of the conditions of operation. Also, in some plants the tailrace elevations may vary considerably with a consequent variation in draft-tube conditions.

Therefore, it may become exceedingly difficult to design a runner which will operate satisfactorily under a wide range of conditions with the high efficiency and high specific speeds demanded at the present time. It may then be necessary to make use of such devices as fins in the draft tube to assure smooth operation under the various conditions.

At the present time, dividing walls and splitters are used near the exit of the draft tubes. Fins and baffles are used in the suction pipes of pumps to straighten out the flow and increase the efficiency of the pump. Baffles are used to guide the flow of air and water around elbows with a minimum of loss and disturbance. Fins are used in impulse-wheel nozzle pipes to prevent whirling water from distorting or breaking up the jet. Similarly, fins in hydraulic-turbine draft tubes, properly designed and installed to counteract the bad effects of whirling water, may become a standard part of turbine installations as a definite insurance against rough operation and excessive power swings, regardless of improvements in runner design.

Generator Characteristics as Affecting System Operation

By SHERWIN H. WRIGHT,[1] EAST PITTSBURGH, PA.

In an interconnected power system the frequency regulation and tie-line power swings are influenced by the electrical characteristics of all electric equipment connected to the system, including generators, lines, and miscellaneous motors, as well as by the mechanical characteristics of all rotating apparatus. In this paper the interrelated action of these factors is visualized by means of the mechanical analogy developed in connection with power-system stability, and this is augmented by a discussion of angular displacement with load and kinetic-energy values for typical machines, generator reactance, and short-circuit ratio.

Introduction

THE OBJECT of this paper is to discuss some of the more important characteristics of synchronous generators which affect frequency regulation and tie-line power swings.

In observing the system operation of interconnected electric-power systems, with speed regulation and tie-line loading particularly in mind, one is struck by the great number of variables, interrelated in a complex manner, the composite action of which determines the actual frequency regulation and tie-line loading characteristics observed. For example, if at a given instant a frequency corrective action is initiated, the results obtained depend not only upon the electrical characteristics of generators, transformers, lines, and load, but also upon the mechanical characteristics of all rotating equipment connected to the generators and motors throughout the system. Likewise, the power swings on interconnecting transmission lines are in fact electromechanical oscillations which involve the same factors mentioned.

In visualizing and analyzing these matters, the electrical engineer is prone to, and justified in, reverting to concepts and methods developed in connection with power-system stability. As used here, "power-system stability" is concerned with the successful parallel operation of a-c machines as affected by the electric power transmitted; considerable attention was focused on it, and a practical analysis made, when long-distance power transmission came to be explored.

Mechanical Analogy

Since system stability has many elements in common with the problems of frequency regulation and tie-line power swings, it will be useful to refer to the mechanical analogy of power-system stability, developed by S. B. Griscom (1).[2] Fig. 1 shows a model of this mechanical analogy, and lists the electrical quantities involved in power transmission and their mechanical analogies. Some of these models are in use by utilities and universities.

This mechanical model is not limited to illustrating electric-power transmission in high-voltage transmission lines. It simulates electric-power transmission between any two points of known voltage connected by an impedance which can be taken as pure reactance (represented by the spring). In Fig. 1 there are two arms fixed to separate hubs which are mounted concentrically but free to turn independently. The cord passing through the pulley has one end fixed to each hub so that a weight on the pulley tends to separate the two arms, being opposed by the coil spring connected near the arm ends. The orthodox description of this analogy is in terms of voltage and current vectors, as suggested by the analogies listed in Fig. 1. However, it may be more convenient to look upon the two arms not as vector voltages but instead as radial chalk marks on the shaft ends of two synchronous machines, these chalk marks being viewed in the light of a stroboscopic lamp.

Suppose for example, we are considering the action of a single generator with respect to a large system of capacity relatively so great it may be considered as an "infinite system." Imagine the system simulated by a single huge equivalent synchronous machine which when running maintains exactly 60-cycle speed and constant terminal voltage. Imagine this machine to be beside the single generator. Suppose that, with both the generator and system machine at a standstill, we find they are 2-pole machines, and so place a radial chalk mark on the shaft end of each in line with the center of the North field pole. When running, the system chalk mark is represented by the arm in Fig. 1 labeled motor voltage, its terminal voltage by the radial distance to the point of spring attachment and, for an infinite system, this arm would be anchored. The generator chalk mark is represented by the arm labeled generator voltage; the inertia of this arm and hub simulate that of the actual generator. With the generator "floating" at no load, the chalk marks viewed in stroboscopic

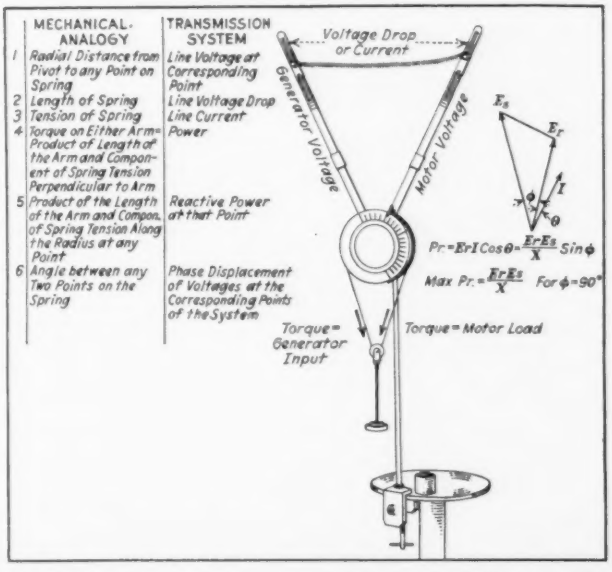


FIG. 1 MECHANICAL ANALOGY TO THE PROBLEM OF TRANSMISSION STABILITY

[1] Central Station Engineer, Westinghouse Electric & Manufacturing Company.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Hydraulic and Power Divisions and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

light would be in line, no angle between them and, in Fig. 1, the two arms would be coradial, the spring being absent. As the generator prime-mover gate is opened, the generator mark will shift ahead in the direction of rotation as seen in the light of the stroboscopic lamp. In the model, Fig. 1, this loading is simulated by placing a weight on the pulley, stretching the spring. If in the model the weight is added suddenly, there will be overswing, the spring stretching on its first swing beyond its final equilibrium position.

Should this generator, while floating at no load, be retarded as by the brakes on a hydroelectric unit, its mark would shift behind the system mark and the generator would draw power from the system, and this action is again simulated by the model.

The model in Fig. 1 is so constructed that the loading of the generator and the motor is simulated by means of a single weight connected by a pulley and cord in such a way as to produce rotational forces on generator and motor arms. The model might as well have been made with two separate but equal weights (each weighing one half that used in Fig. 1), one for generator load and one for motor load. However, unless the two weights are exactly equal a net torque would exist and a net rotation of the two arms would take place. This net rotation would simulate what actually takes place in a system when load is increased or decreased and system frequency changes slightly before the governors operate.

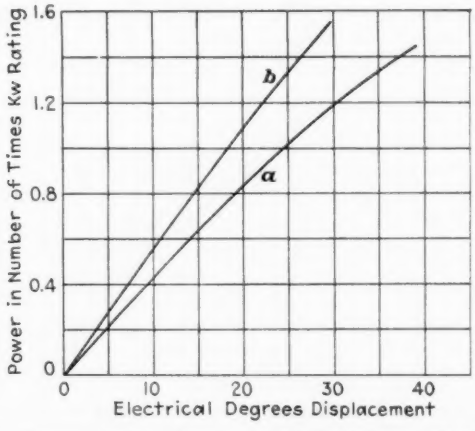


FIG. 2 POWER-ANGLE CURVE OF WATER-WHEEL GENERATOR; STEADY STATE, RATED VOLTAGE

(*a* Field-current constant at no-load value. *b* Field-current constant at value required for rated load at 0.9 power factor; this is 1.8 times value in *a*.)

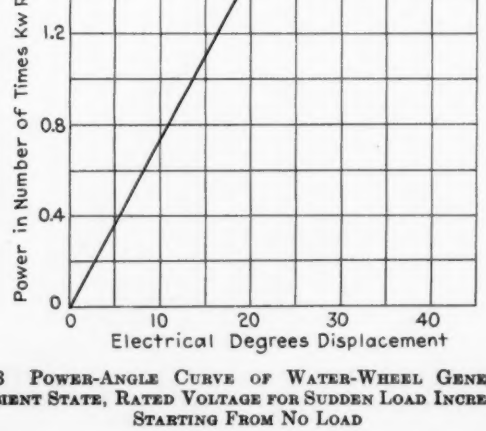


FIG. 3 POWER-ANGLE CURVE OF WATER-WHEEL GENERATOR; TRANSIENT STATE, RATED VOLTAGE FOR SUDDEN LOAD INCREMENTS, STARTING FROM NO LOAD

Here again the mechanical analogy allows visualization of both the electrical and mechanical factors, for both steady-state and oscillatory conditions.

ANGULAR DISPLACEMENT VERSUS LOAD IN TYPICAL GENERATING UNITS

The angular displacement in actual hydroelectric and turbogenerator units is illustrated in Figs. 2, 3, 4, and 5. A generator is electrically "stiffer" under suddenly changing load conditions than it is under gradually or slowly changing load conditions. This is due to the tendency of the field winding to maintain its flux linkages and therefore internal-voltage constant at the value existing before the sudden load change occurs. This is exemplified in Figs. 3 and 5 which show that to develop any given value of electric-power output, the angular displacement need not be as large for transient as for steady-state conditions. By sudden change is meant one, for example, involving an oscillation having a period of less than 1 sec or, to be more precise, less than the transient time constant of the machine.

The curves for transient conditions merely show the angular displacement that must exist in order for the generator to develop corresponding values of electrical kw output. They do not show the maximum angle through which a machine will swing under conditions of sudden load changes because, in this case, the mechanical inertia causes overswing.

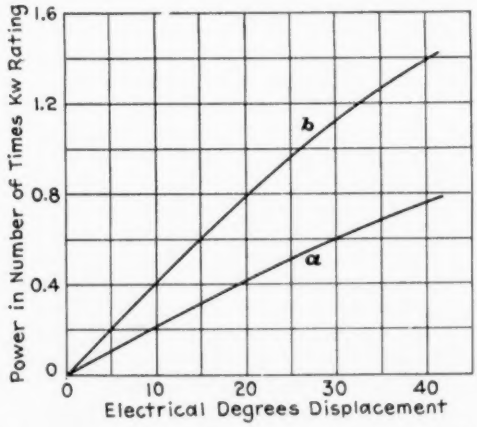


FIG. 4 POWER-ANGLE CURVE OF TURBOGENERATOR; STEADY STATE, RATED VOLTAGE

(*a* Field-current constant at no-load value. *b* Field-current constant at value required for rated load at 0.8 power factor; this is twice the value in *a*.)

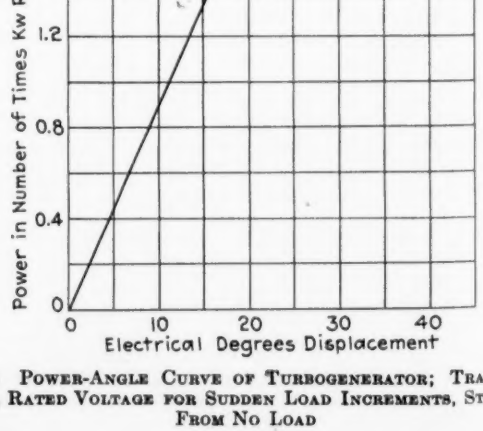


FIG. 5 POWER-ANGLE CURVE OF TURBOGENERATOR; TRANSIENT STATE, RATED VOLTAGE FOR SUDDEN LOAD INCREMENTS, STARTING FROM NO LOAD

Quantitative Analysis

In addition to visualizing these phenomena, it is pertinent to review some of the mathematical relations. As given in Fig. 1, the power P transmitted between two points having voltages E_s and E_r with the reactance X and phase angle ϕ between them, is $P = \frac{E_s E_r}{X} \sin \phi$. (For a 3-phase system, using line-to-line voltages in kilovolts and reactance per phase in ohms gives total power in megawatts (thousands of kilowatts).) This, of course, neglects damping torque which is generally small, being proportional to the slip frequency.

This equation then defines conditions completely, providing they are steady-state conditions so that inertial effects are absent. Under oscillating or transient conditions, however, an attempt at a formal mathematical solution leads to an elliptic integral. In multimachine transient-stability studies, this is circumvented by employing a step-by-step solution, usually with the aid of the a-c-network calculator where the power system is set up in miniature. It is interesting, however, to investigate an approximate solution for the two-machine case when oscillations are small enough in amplitude so that the cosine of the angular change may be taken as unity and the sine as equal to the angular change itself. In this case, the period of the natural-frequency oscillation is

$$T = \sqrt{\frac{4\pi W}{f_0} \times \frac{X}{E_s E_r} \times \frac{1}{\cos \phi_0}} \text{ sec}$$

where

W = stored energy at normal speed, in kwsec divided by 1000
f_0 = normal system frequency in cycles per sec
E_s and E_r = fixed voltages of the two synchronous machines or systems in kv, line to line
ϕ_0 = initial phase angle between them
X = reactance between them in ohms per phase

When one machine or station is swinging with respect to a large or relatively "infinite" system, W is the stored energy of that one machine. When the two machines or systems involved have comparable values W_1 and W_2 of stored energy, W in the formula is replaced by $W_1W_2/(W_1 + W_2)$. A single unit or station swinging with respect to a large system may have a period of less than 1 sec, while two large systems may swing with a period of several seconds.

For this two-machine case, the amplitude of the oscillation is independent of the system inertia, if variations in prime-mover input and shaft loads are neglected. The period however depends upon the stored energy. The period may be increased by increasing reactance but, as seen from the previous power equation, this would handicap the tie by reducing the maximum power which could be transmitted. In the two-machine case the overswing can be determined readily, as in stability studies, by the "equal-area method" (3). Analysis of amplitude and period for any particular system becomes quite complicated when governor operation is involved. A practical solution probably necessitates carrying out a step-by-step process, with the aid of an a-c-network calculator.

Machine Reactance

The machine reactance involved in these system oscillations or changes depends upon the period and duration of the oscillation. A quantity used commonly in synchronous-machine short-circuit and stability calculations is "transient reactance." Its name suggests use under any transient condition. However, transient reactance accurately characterizes a machine only at the instant when a sudden change in armature current takes place. In practice it is usually applied only where the time interval being investigated is roughly less than the machine transient time constant. When generator loading is being changed at a slow rate such as 20 per cent load in 5 sec, transient reactance is not applicable. The machine reactance under such conditions changes with load because of saturation. If a voltage regulator changes field current, thus affecting internal voltage, this must also be taken into account. Furthermore, when load is changing slowly on a salient-pole machine (such as a water-wheel generator, in contrast to a cylindrical-rotor turbine generator) the two-reaction theory involving a direct axis reactance and a quadrature axis reactance is applicable.

Stored Kinetic Energy

The stored energy in a system may be estimated from a knowledge of the rating and type of all connected machines, including load as well as generating equipment. In the case of the generating equipment, this is a straightforward calculation when the stored-energy value for each individual unit is available. When the WR^2 in lb-ft^2 is known, the stored kinetic energy is given by

$$W = 2.31(10^{-7})(\text{rpm})^2(\text{lb-ft}^2) = \text{kwsec}$$

When individual values are not available, estimates may be made from the data given in the following section. The energy stored in motor loads may form a large percentage of the total system stored energy and, in such cases, should be included. In estimating these the connected-motor capacity should be used rather than the net power supplied. The section following gives representative values of stored energy for the three types of rotating-machine load, i.e., synchronous motors, induction motors and rotary converters.

Having determined the total stored energy in a system, its change for a given frequency change can of course readily be computed since it varies as the square of the frequency. When frequency changes, another factor to be considered is the variation of shaft torque exerted by the motor-driven loads themselves Few data are available on this point, partly because any system power measurements will be greatly affected by voltage variations, lighting load, for example, varying as the 1.5 to 2 power of the voltage. Some measurements indicated the total system load varying approximately as the square of the frequency. The rate at which frequency changes from one value to another determines the relative importance of this load characteristic with respect to the total system stored energy, the latter being the dominating factor when the frequency changes at a rapid rate. When attempting to evaluate this particular characteristic from system measurements at different frequencies, it is essential that any change in voltage be taken into account lest it give false conclusions.

Stored-energy values of generators are conveniently expressed in kwsec per kva of rating so that, knowing this value for any class of machine, the stored kinetic energy at rated speed of a particular unit is readily evaluated as the product of its kva rating and the kwsec per kva, the result being energy expressed in kwsec. Conversion to other units such as hp-hr or ft-lb can readily be made. For example, multiply kwsec by 737.6 to obtain ft-lb, or multiply kwsec by 0.369 to obtain ft-tons. Comprehensive stored-energy data are given in (2) and repeated here.

Fig. 7 gives the stored energy of hydroelectric units, total of the generator and the water-wheel runner, the water wheel being about 15 per cent of the total. Fig. 6 gives stored energy for large turbogenerator units, condensing as well as noncondensing. It is to be noted that while the 3600-rpm noncondensing turbogenerator units have values which are of the same order as for water-wheel units, the 1800-rpm and 3600-rpm condensing

units have kinetic-energy values much greater, twice as much or more. Thus while a 10,000-kva slow-speed water-wheel unit possesses about 1.75 kwsec per kva, a 10,000-kva 1800-rpm condensing turbogenerator unit has about 9.4 kwsec per kva. The energy in these 10,000-kva units at rated speed would be 17,500 kwsec (or 6450 ft-tons) for the water-wheel unit, and 94,000 kwsec (or 34,700 ft-tons) for the turbogenerator unit.

The effect of increasing water-wheel-generator inertia upon

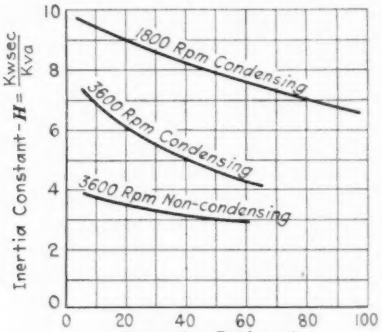


FIG. 6 INERTIA CONSTANTS OF LARGE TURBOGENERATORS; TURBINE INCLUDED

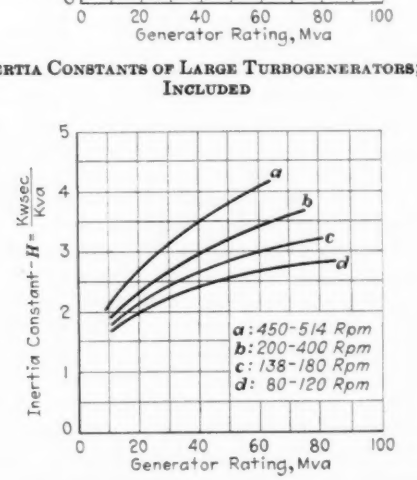


FIG. 7 INERTIA CONSTANTS OF LARGE VERTICAL-TYPE WATER-WHEEL GENERATORS
(Curves include allowance of 15 per cent for water wheels.)

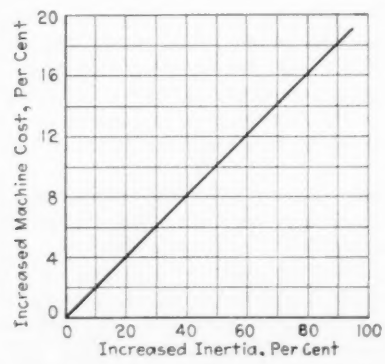


FIG. 8 APPROXIMATE COST OF INCREASING INERTIA OF LARGE VERTICAL-TYPE WATER-WHEEL GENERATORS

the cost is illustrated approximately in Fig. 8. Such increases in inertia are desired not only for hydraulic reasons but also occasionally for stability reasons.

For rotating machines other than hydroelectric and turbogenerator units, the following table affords average values from which stored energy may be estimated.

	Kwsec per kva
Condensers	
Large	1.25
Small	1.00
Synchronous motors	2.00
Induction motors	0.50
Rotary converters	2.00

SHORT-CIRCUIT RATIO

Mechanical energy is put into the shaft of an a-c generator and electric energy taken from the stator terminals. Between the rotor and stator is an air gap and, because of this, it is often said that all energy input to the shaft must "cross" the air gap. What allows this energy to cross the air gap is a magnetic field or flux which exists in it, called the air-gap flux. Anything that affects this air-gap flux also affects the power- or energy-trans-

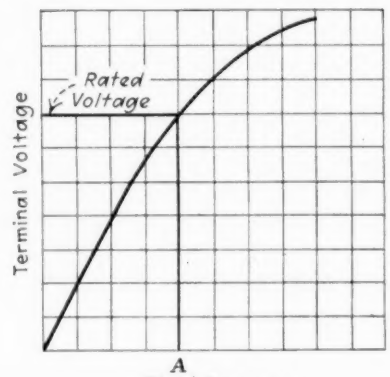


FIG. 9 NO-LOAD SATURATION CURVE

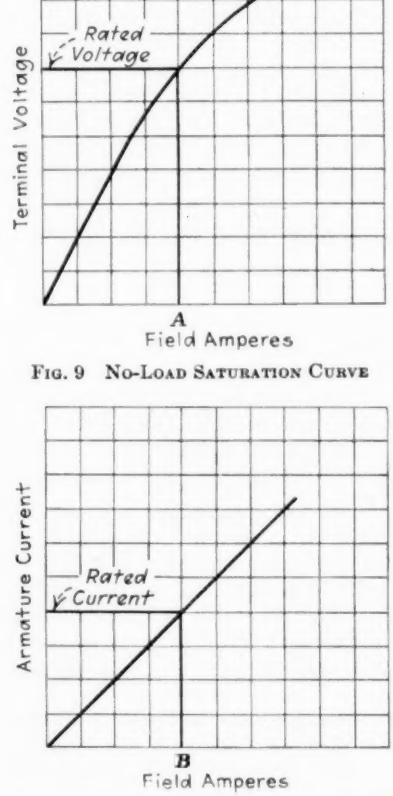


FIG. 10 SHORT-CIRCUIT SATURATION CURVE

mitting ability of the air gap and, therefore, the ability of the machine to put out electric energy.

Air-gap flux is usually thought of as being determined primarily by current in the field winding and this is illustrated by any no-load saturation curve, Fig. 9, which shows how, at no load (armature winding open), the armature voltage varies with field current. However, current flowing in the armature winding has a decided effect on the air-gap flux and a particular "type" of armature current has a strong demagnetizing or weakening effect on the air-gap flux. This "type" of armature current is commonly known as zero-power-factor lagging current or as demagnetizing current and is the kind that flows when the armature is shorted at its terminals or when it is connected to an inductive load. Under actual operating conditions in a system, a generator usually supplies current which has a demagnetizing component. Thus it requires more field current to produce any specified amount of

air-gap flux when armature demagnetizing current is flowing than when there is no armature demagnetizing current. Under actual operating conditions, the air-gap flux is the resultant of that which the rotor-field current tries to produce, minus the bucking effect of any armature demagnetizing current that is present.

In order to gain an idea of how much weakening effect a given amount of armature demagnetizing current has on a given machine the following test may be made:

Short-circuit the three armature terminals and then apply sufficient field current to cause rated armature current to flow. It requires a certain amount of air-gap flux to cause this armature current to circulate, even though the terminal voltage is zero. This test thus measures the amount of field current required to produce a certain amount of air-gap flux when being "bucked" by a specific amount of armature demagnetizing current. The greater this required field current, the weaker is the field in relation to the armature. A "strong" machine is thus one in which this required field current is low, since that indicates air-gap flux is determined principally by the field current. Suppose this required field current is 500 amperes. Since this is an abstract quantity with little meaning in itself, in order to compare machines, it must be possible to express this in per cent or per unit with respect to some reference level. The field current required for rated voltage at no load is a handy and definite reference; suppose it is 550 amperes. Just how can the no-load field current be made useful as a reference? The natural thing to do is to take a ratio and, if it is to be indicative of the relative strength of field with respect to armature, this ratio must be the no-load value of 550 amperes divided by the short-circuit value of 500 amperes field current, giving 1.10.

This ratio of (no-load-rated-voltage field current) to (field current required to circulate rated armature current on 3-phase sustained short circuit) is called the short-circuit ratio and the American Institute of Electrical Engineers definition is in these terms. Thus, from Figs. 9 and 10, short-circuit ratio $= \frac{A}{B}$. Because of the fact that, when the machine is short-circuited, the armature current is almost exactly proportional to field current Fig. 10, it turns out that short-circuit ratio is equal to the number of times rated armature current which flows on sustained 3-phase short circuit, when the field current is set at the value required for rated voltage at no load.

For a generator which is operated with field current manually controlled, short-circuit ratio is an indication of the steady-state stability characteristic of the machine, provided that the field current is normally set and left at a definite field value such as the no-load-rated-voltage value. However, if this same machine has its field current automatically controlled, as by a generator voltage regulator (as is almost always the case with water-wheel generators), then short-circuit ratio is not a direct indication of the steady-state stability characteristic because, as armature demagnetizing current increases and thus tends to reduce the air-gap flux, the voltage regulator will boost the field current. The automatic control of kw output and speed by a governor has its counterpart in the generator voltage regulator which automatically controls field current, using terminal voltage as an index.

Conclusions

The analysis of frequency regulation and tie-line loading in extensive power systems is a complicated matter which can be visualized to a fairly complete degree by means of the mechanical analogy developed in connection with power-system stability.

In studying frequency regulation in a system, the conditions are such that synchronous machines ordinarily cannot adequately be characterized by transient reactance as they are in transient stability studies. The action of voltage regulators and the effect of saturation on the synchronous reactance must be considered.

The a-c-network calculator which allows quantitative analysis of a power system in miniature, and which is used in steady-state as well as transient stability studies, affords a means of analyzing frequency regulation in an interconnected system.

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Steam-Turbine Governors

By REED J. CAUGHEY,[1] SCHENECTADY, N. Y.

This paper has been prepared as a contribution to a symposium on the subject of steam-turbine governors. The importance of the subject at present arises more from the increasing complexity of interconnection requirements than from any definite deficiency in existing apparatus. Attempts have recently been made to draft specifications for governing equipment that might be applicable both to steam and hydraulic prime movers. The purpose of this paper is to state briefly the essential characteristics, functions, and behavior of governing apparatus both for condensing and superposed turbines. Definitions are suggested for terms and phrases frequently used without definite significance. In conclusion the essential points of a brief specification are outlined for the purpose of discussion.

THE governing requirements for steam-turbine generators, in regard to stability and safety of operation, can generally be determined by the manufacturer and each turbine is supplied with a governing mechanism by means of which the manufacturer endeavors to meet these requirements. These requirements vary considerably with the type and size of turbines as will be explained later.

On the other hand, the characteristics of turbine-governing mechanisms to suit them best to the particular requirements of a purchaser's system and load usually cannot be so readily determined by a manufacturer. In addition the effect of turbine-governing characteristics on power-system stability is a controversial matter and warrants more investigation. This discussion has been planned with the idea of aiding the purchaser in determining and stating the governing characteristics desired for a particular turbine generator to suit it in the best way to his system, and to permit the manufacturer to interpret these requirements and make his design such as will meet them.

The satisfactory operation of the governing mechanism of a steam turbine depends upon the sensitivity and response of this mechanism.

The relation of these two characteristics of a governing mechanism to the regulation, rapidity of load change, and protection against overspeeding is useful in determining the design limits of a governing mechanism.

To develop the effect of these relations, this paper will discuss: (1) General principles of governing; (2) governors and their limitations as speed indicators; (3) governing mechanisms and their limitations due to regulation and rapidity of load changes; (4) protection of the steam turbine against overspeeding. In addition (5) a short description of the governing problems encountered in the design of superposed steam turbines is included with (6) a tentative specification for steam-turbine-governing mechanisms.

General Principles of Governing

The governor of a steam turbine is shown diagrammatically in Fig. 1. It may be a simple governor controlling the steam valves as shown in *A*, or it may operate the steam valves through the controlling mechanism as shown in *B*. It may also operate the steam valves through a primary relay and control mechanism as shown in *C*. In all of these arrangements, the governor of a steam turbine is designed to control the power output with a nominal variation in turbine speed and to prevent dangerous overspeeding of the turbine. The method of controlling the output will be discussed first.

As the load upon the turbine changes, there will be a change in turbine speed, providing the steam input, and hence the power output, of the turbine buckets remains constant. It is the function of the governor to measure this speed change and to change the steam input to suit the new load. To change the steam input, the governor is designed so that a change in its speed of rotation is converted into a corresponding displacement of the governor weights or equivalent. By means of any of the arrangements shown in Fig. 1, this displacement in turn actuates the steam valves controlling the steam input to the turbine. The amount of this displacement required to move the steam valves from wide-open to shut (total valve stroke) or vice versa is called the "effective stroke" of the governor. The regulation of the governor is the speed change required to cause the governor to move through its effective stroke.

Under steady-state conditions, Fig. 2 shows a definite relationship between governor speed and effective governor stroke. Fig. 3 shows a similar definite relationship between total valve stroke and the effective stroke of the governor. In Fig. 4 these same data are replotted showing the relationship between governor speed and total valve stroke. This last figure also shows that, for each speed of the turbine, there is a fixed valve position and, therefore, a fixed steam input.

Governors and Their Limitations as Speed Indicators

In Fig. 5 governor speed is plotted against the effective stroke of the governor. Line *AB* is the same as line *AB* in Fig. 2. A governor which has no internal friction and does no external work will have the characteristics shown by this line. In this paper this form of governor will be called a perfect governor. With a perfect governor, speed change and response are simultaneous.

An actual governor has internal friction so that a speed change is necessary to build up a force to overcome this friction. This change in speed without governor response is shown by the lines drawn through *C*,*C* on each side of the line *AB*. Lines are drawn on both sides of line *AB*, since the internal friction of the governor is resisting the governor response as its speed either increases or decreases. This change in speed *C* without governor response is a measure of the sensitivity of the governor.

Besides overcoming internal friction, any actual governor as shown in Fig. 1 has to perform external work in order to move the steam valves. This external work has the same effect as the internal friction and further reduces the sensitivity of the governor. This effect is shown by the lines drawn through *D*,*D* on either side of the line *AB*.

In order to reduce the external work done to a minimum, a power amplifier is required whose power is controlled by a valve moved by the governor. This valve is called the primary pilot valve and the complete power-amplifying device is called a relay mechanism. The combination of this relay mechanism and the

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

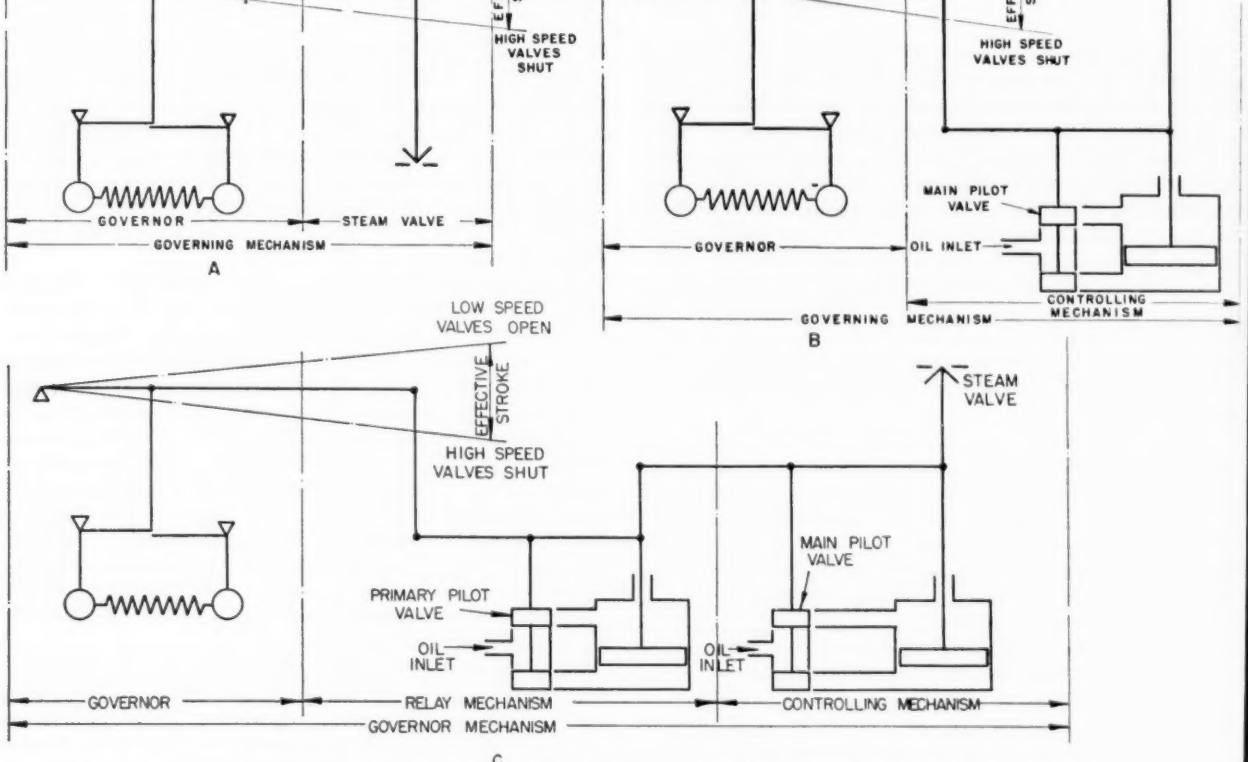


FIG. 1 DIAGRAMMATIC REPRESENTATION OF GOVERNING SYSTEMS

(*A*, Simple governor controlling steam valves; *B*, governing mechanism equipped with hydraulic power amplifier; *C*, governing mechanism equipped with double hydraulic power amplifier. The system of nomenclature used throughout this paper is shown in this diagram.)

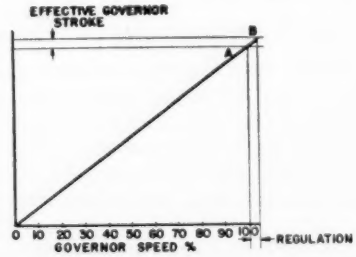


FIG. 2 RELATION BETWEEN GOVERNOR SPEED AND EFFECTIVE STROKE

(This diagram shows how a speed indicator functions as a governor. In this and succeeding diagrams straight-line relationships have been used for simplicity. In actual governors and governing mechanisms these relationships deviate more or less from the straight line. However, such deviations are not important to a correct understanding of governor behavior.)

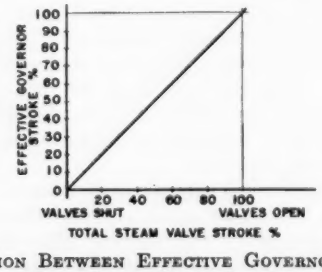


FIG. 3 RELATION BETWEEN EFFECTIVE GOVERNOR STROKE AND TOTAL STEAM-VALVE STROKE

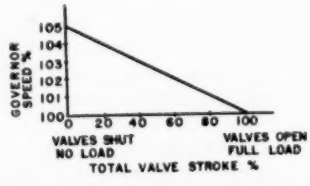


FIG. 4 RELATION BETWEEN GOVERNOR SPEED AND TOTAL VALVE STROKE

(The diagram is derived from the relations in Figs. 2 and 3, and shows that for each speed there is a definite valve position and steam input.)

governor is called a relay governor, and it is shown in Fig. 1(*C*). The effect of using the relay mechanism is to reduce the amount of external work that the governor has to do. In Fig. 5 the effect is to move the lines passing through *D*,*D* closer to the lines passing through *C*,*C*. This effect is true when the rate of change in speed of rotation of the governor is small. As the rate of change in speed of rotation of the governor increases, there will be a further delay in governor response as compared to the response of a perfect governor. This delay is called lag and is shown by the lines drawn through *E*,*E* at either side of the line *AB*. The greater the rate of change in speed of rotation of the governor, the greater the lag.

GOVERNING MECHANISMS AND THEIR LIMITATIONS DUE TO REGULATION AND RAPIDITY OF LOAD CHANGES

In the preceding section, the action of the governor is analyzed

The action of the governor when operating the steam valves will be discussed next. In order to show the effect of a change in steam input upon a turbine, the relationships between turbine speed and effective governor stroke, valve stroke, and time are shown in Fig. 6. If the turbine loses full load, the steam valves must move through their total valve stroke. In order to move the steam valves through their complete stroke, the governor must make its complete stroke as shown at *A*. The speed change required for this movement is the regulation as shown by the vertical ordinate for charts *A*, *B*, and *C*. In these charts the regulation is shown as 5 per cent. As the governor makes its effective stroke, the steam valves make their total valve stroke. The speed change for this total valve stroke is shown at *B*.

If the steam input to the turbine remains constant, the time required for the 5 per cent speed change will be as shown in *C* and will be fixed by the rate of acceleration of the turbine rotor.

In order to obtain a total valve stroke, corresponding to effective governor stroke, it is necessary to have the time required for the valve to make its total stroke no greater than the time required for the speed to reach the value represented by the regulation. This is shown at *D*.

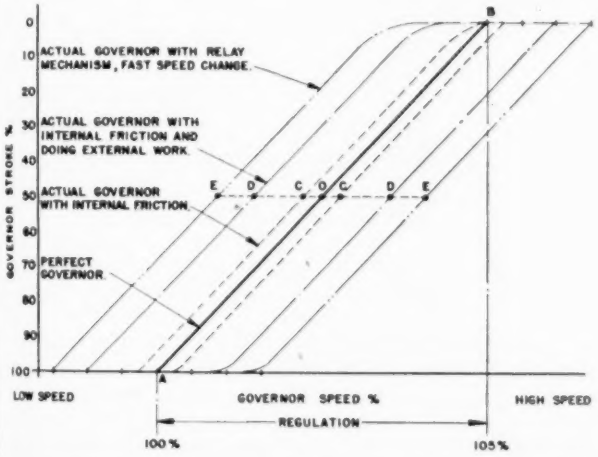


FIG. 5 COMPARISON OF ACTUAL GOVERNOR PERFORMANCE WITH PERFECT GOVERNOR BEHAVIOR AS REPRESENTED BY STROKE-SPEED LINE *AB*; ALSO SHOWN IN FIG. 2

(The "lag" is $= EO$ or less. The lines through *C,C* and *D,D* show elements contributing to lag. The distance $= CO$ is a measure of the sensitivity, being smaller the greater the sensitivity.)

In the ideal case, the time available for total movement of the steam valve under a full-load change will be the time required for the speed to change an amount equal to the regulation for which the governor has been set.

There are numerous practical considerations which make it difficult to approach this ideal case. In the case where a controlling mechanism is used Fig. 1(*B*), there will be a time delay, as time is required to initiate any motion which uses a fluid as the power medium. The magnitude of the time delay of a controlling mechanism is a function of the quickness and amount of movement of the main pilot valve. The power required to move this main pilot valve is supplied by the governor and, as shown in Fig. 5, this is obtained at the expense of sensitivity of the governor.

It will be seen, therefore, that if the load changes are slow with correspondingly slow speed changes, an extremely accurate governing mechanism may be designed. This is due to the fact that these slow speed changes require slow motion of the steam valves which may in turn be obtained by a small main pilot valve with a small stroke and with a correspondingly small amount of external work done by the governor. When, however, the load changes are fast, with correspondingly fast speed changes, then the design of an extremely accurate governing mechanism becomes difficult. To obtain a fast movement of the steam valve, a large main pilot valve with a large stroke is necessary, and this in turn requires a large amount of external work by the governor and a loss in its accuracy.

For this reason, relay governors have been developed which impose a relay mechanism between the governor and the controlling mechanism as shown in Fig. 1(*C*). This relay has an additional time delay which must be taken into account when determining the response of the governing mechanism.

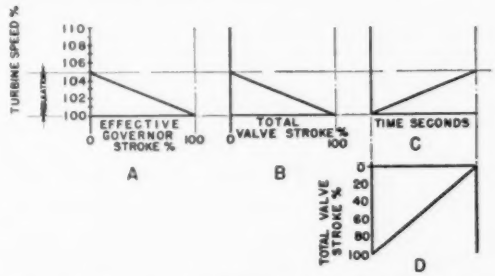


FIG. 6 IDEAL RELATIONS BETWEEN TURBINE SPEED, EFFECTIVE GOVERNOR STROKE, VALVE STROKE, AND TIME TO ACCELERATE THROUGH FULL REGULATION

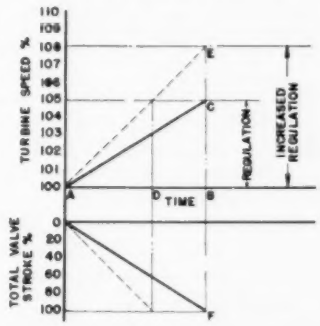


FIG. 7 RELATION BETWEEN VALVE-OPERATING TIME, REGULATION, AND ACCELERATION WITH AN IDEAL GOVERNING MECHANISM

(This diagram shows that the slower the valves or the greater the acceleration, the greater must be the regulation in order to have perfect governing.)

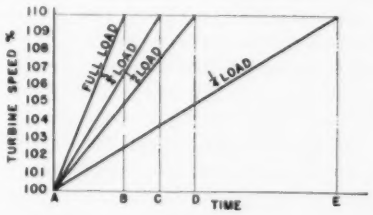


FIG. 8 RELATION BETWEEN AMOUNT OF INSTANTANEOUS LOAD CHANGE AND TIME REQUIRED TO ACCELERATE TURBINE ROTOR TO 110 PER CENT

(The smaller the instantaneous load change, the slower may the valves operate in order to follow the speed.)

It has already been noted that the steam valves must make a complete stroke from wide-open to shut in the time required for the steam turbine to change its speed an amount equal to the regulation. This is shown in Fig. 7, where the turbine speed *BC* has changed 5 per cent in the time *AB*. Since with a perfect governing mechanism the steam valves will be shut when the governor has made its effective stroke, the time for 100 per cent valve stroke will also be represented by time *AB*. If, however,

the speed increases 5 per cent in the time AD, then either the valve must be able to make its complete stroke in this same time, or the regulation must be increased as shown by the line BE so that the speed rise, represented by the regulation, will require as much time as the valve closing time AB.

A perfect governing mechanism locates the steam valve at the new load point at the same time that the steam-turbine speed reaches the speed corresponding to this new load.

As is shown in Fig. 8, a given turbine rotor will accelerate to a given speed in the time AB for a full load change. It will accelerate to the same speed in the time AC for a $^3/_4$ load change and in the time AD and AE for $^1/_2$ and $^1/_4$ load changes. This indicates that, for a given turbine rotor, the less the instantaneous load change, the slower the valves need to move in order to follow the speed.

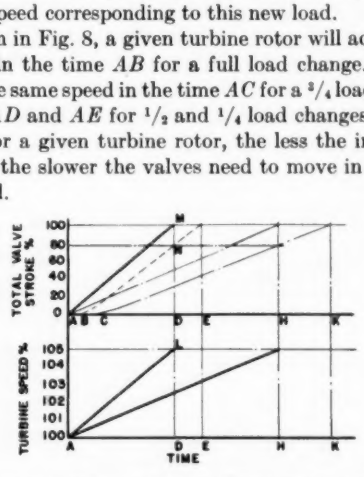


FIG. 9 RELATION BETWEEN VALVE-OPERATING TIME, ROTOR-ACCELERATION TIME, AND LAG OF GOVERNING MECHANISM
(This shows that the slower the valves move, the slower must be the turbine acceleration in order to obtain the same lag.)

In general, steam turbines fall into two classes as far as governing is concerned. These classes are, first, a single turbine in an isolated factory or power plant operating on a variable load and, second, a number of turbines in large central stations connected together on a common distribution system.

Steam turbines in actual service are not called upon to make full load changes continuously, except under abnormal power-system conditions. In view of the foregoing limitations, it is necessary for the designer to know the magnitude and rate of all load changes. This will depend upon the character of the load and the number and size of the turbines connected together. Knowing the magnitude and rate of all load changes, it is possible to design a governing mechanism which has any specified lag. This lag will not be exceeded as long as the magnitude and rate of the load changes are not more than has been specified. Besides the magnitude and rate of load change, the entrained steam between the steam valves and the turbine exhaust is another determining factor not only of the type of governing mechanism but also of the possible lag which can be obtained.

If the time required for 100 per cent total valve stroke is AD in Fig. 9 and the turbine rotor changes speed 5 per cent in this time, then there will be perfect following of the turbine speed by the valve, providing the regulation is 5 per cent. If, however, the valves have a time delay of AB and a speed of movement represented by the time BE, then there will be a lag of the valves of 20 per cent of the stroke as shown by the line MN. When the valves have a time delay represented by AC and a time for 100 per cent stroke of CK, then to obtain the same lag the time required for the rotor to accelerate must be AH which necessitates a smaller or slower load change.

The ability of the governing mechanism of a steam turbine to position the steam valves for a given load change without sustained movement of the steam valves is called stability. The stability of a governing mechanism depends upon the magnitude of the lag shown at MN, Fig. 9.

As this lag of a governing mechanism is a function of the speed of response of this mechanism, it follows that the regulation of the governor, as defined previously, materially affects the characteristics of the governor.

Stability of the governing mechanism, therefore, should be defined in terms of regulation and speed of response of the governing mechanism. Stability, of course, cannot be obtained without a certain sensitivity. In general, the slower the response of the governing mechanism, the greater must be the regulation of the governor if the sensitivity remains constant, in order that the governing mechanism may have a minimum lag.

In Fig. 10, if the steam valves can make their complete stroke in the time AB, then if the rotor acceleration is fixed by the line AC, a 3 per cent regulation will permit the steam valves to reach the position corresponding to the speed.

If the steam valves require the time AD to make their complete stroke then, when the speed reaches 103 per cent, the valves will have completed only 60 per cent of their stroke (or a lag of 40 per cent) and it will be necessary to increase the regulation to 5 per cent in order to permit the valves to be in the proper position corresponding to the speed.

As speed of response over a certain value is difficult to obtain, governing mechanisms must be designed with adequate regulation.

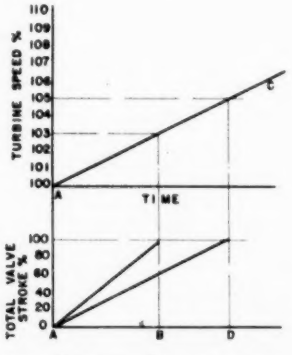


FIG. 10 RELATION BETWEEN VALVE-OPERATING TIME AND REGULATION
(This shows that with a fixed rate of acceleration the slower the valve-operating time the greater must be the regulation in order to have perfect governing.)

This means that, from the definition of regulation, a considerable speed change of the turbine is necessary to change the load from full load to no load, or from no load to full load. This characteristic of governing mechanisms has necessitated speed-correcting devices in governing mechanisms which permit correcting the speed of the steam turbine after a speed change which has automatically followed the load change. This speed-correcting device on steam turbines has taken various forms, either hand-operated or automatic. The latter fall into two general classes, i.e., speed-resetting devices or automatic frequency control, and self-restoring devices or droop correction.

The variation from normal speed which the steam turbine maintains, after the speed change due to regulation has been restored, is termed droop of the governing mechansim and is given as the per cent speed variation from normal speed.

PROTECTION AGAINST OVERSPEEDING

As has been stated previously, the governor of a steam turbine has another function besides the accurate proportioning of the steam input to power output. This function is to prevent a rise in speed sufficient to trip the emergency governor on loss of full load. If the loss of load is instantaneous, the rate of speed change will depend on the WR^2 of the steam-turbine and generator rotors. In this case, lag of the governor may be compensated for

by an increase in the speed of the controlling mechanism. With an accurate governor, the speed of the controlling mechanism may be slower as in either case the steam valves must be at no-load position when the speed has increased the specified amount.

If it is desired to have the valves closed when the speed reaches 110 per cent, as shown in Fig. 11 and, if the time required for this speed change is represented by AB, it will be possible to accomplish this by moving the valve through 100 per cent stroke in the time AB or delaying the starting of valve movement by the time AC and then moving the valve through 100 per cent stroke in the faster time CB.

The ability of the governing mechanism to prevent excessive speed rise on loss of full load depends upon the magnitude of the instantaneous load changes for which the governing mechanism was originally designed. In general, it may be said that unless the governing mechanism is designed to follow accurately

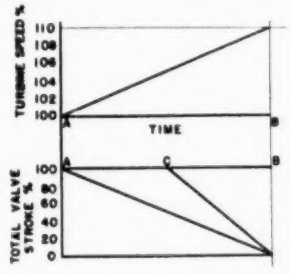


FIG. 11 RELATION BETWEEN TURBINE ROTOR ACCELERATION AND VALVE-CLOSING TIME

(This shows that the more lag there is, the faster must the valve move in order to prevent the turbine speed exceeding a set limit.)

an instantaneous full-load change, the amount of overspeed on loss of full load will be the limit in design of the governing mechanism.

With the governing mechanisms designed to prevent excessive overspeed, it will be found that automatically a smaller WR^2 high-speed steam turbine will be obliged to have a more responsive governing mechanism than a low-speed steam turbine having larger WR^2.

SUPERPOSED- AND EXTRACTION-TURBINE GOVERNING

Superposed and extraction steam turbines require governors and governing mechanisms, the functions and characteristics of which are similar to those used with condensing turbines. For controlling the steam input a pressure governor is used. This pressure governor responds to changes in pressure. For superposed turbines, it is not generally necessary to use the speed governor for controlling the steam input and, in this case, the speed governor is used as a pre-emergency governor only. The rapidity of the load change and the WR^2 of the turbine rotors are not used in the control-mechanism calculations for this class of turbine. In place of these effects, the rapidity of the change of steam flow and the volume of the back-pressure steam-distributing system are used. For superposed turbines, the governing mechanism is designed to prevent excessive overspeeds of the turbine. The back-pressure governor and the controlling mechanism are designed with the necessary sensitivity and response to produce stable operation.

Stability of operation on this type of turbine is more difficult to obtain than stability of a condensing turbine controlled by a governing mechanism. This is due to the fact that in general the volume of the exhaust piping between the superposed turbine and its associated low-pressure turbines is relatively small. The result of this small volume is the same as having a steam turbine with a very small WR^2 per kilowatt and means that the rate of change of pressure is much faster than the rate of change of speed on this same type of turbine.

The design of the pressure-governing mechanisms, therefore, following the same specifications as outlined for governing mechanisms, must be sensitive and responsive enough to provide stable governing. For the same reasons, when the speed of response has been made as fast as is practical, then the regulation of the pressure governor must be increased to a value which will produce stable operation. This increase in regulation, as in governing mechanisms, has made necessary pressure-resetting devices on pressure governors. These devices may be either hand-operated or automatic. The latter may be either pressure-resetting or pressure-restoring.

The design and operation of large superposed turbines have presented some problems which have not been encountered so far on either the smaller turbines of this type or on condensing turbines. These problems have been to determine the constants to be used in designing in order to fulfill the requirements previously outlined. These large superposed turbines have smaller rotors than condensing turbines having the same kilowatt rating. This means that the WR^2 per kilowatt is less and that the acceleration of the steam-turbine rotor and generator is much faster than has been encountered on other turbines. The size of these turbines requires, of course, large steam valves so that the steam-pressure drop may be held to a minimum and the efficiency improved. These large steam valves require large forces to operate them. They must also open and shut quickly so that the governing mechanism will function as outlined previously. The speed of operation of the steam valves on this type of turbine must be exceedingly fast in order to be able to follow the very fast speed change of these steam turbines.

It is not uncommon to use 100 hp to 200 hp for the movement of these valves. This power must be controlled by both the governor and the back-pressure governor with the proper sensitivity and response. This requires powerful and sensitive governors, which have a minimum of time lag and which can control this power accurately.

Extraction turbines differ from the superposed turbines since, in general, both governing and pressure governing are necessary in order to obtain the satisfactory control of power output and extracted steam.

All of the problems outlined previously, covering both governing mechanisms and pressure-governing mechanisms, apply to this type of turbine. It should also be noted that this type of turbine comes under the class of a turbine generally connected alone to a system, which condition may produce larger load changes on the steam turbine. The governing mechanisms for this type of turbine, therefore, must be more sensitive and more responsive.

This type of turbine, for this reason, is generally equipped with relay governors in order to secure the degree of sensitivity required. The relay governor produces more external work with less speed change, thus increasing the sensitivity and response and permitting very close speed control.

This type of governor is designed with a large regulation and is generally equipped with automatic speed control so that the speed is held within close limits.

GOVERNING-MECHANISM SPECIFICATIONS

At present no definite specifications covering requisites for governing mechanisms have been laid down by various steam-turbine users. It has, therefore, been necessary to design the governing mechanisms so that they will function in the most efficient manner, based upon the manufacturers' best information as to operating requirements.

We, as manufacturers, have in the past asked groups of steam-

turbine users to discuss with us their requirements for governing mechanisms. However, until recently for the most part we have had to form our own ideas as to these requirements.

Within the last few months, however, a group of utility operators under the leadership of the Hydraulic Power Committee of the Edison Electric Institute has submitted a tentative governing specification for prime movers. Using this specification as a basis of approach, we have attempted to formulate a governing-mechanism specification for steam turbines used in central stations. The minor changes made are the result of considering the transient requirements and the complete governing mechanism rather than the governor alone.

To the standard governing mechanism used by central stations at present, there has been added in the proposed specification regulation and droop control. A discussion of the value of these features in central-station operation might be in order following this paper.

These specifications have been written to cover all of the points brought out in this paper. The several headings apply to the various requirements for governing mechanisms as outlined. Each heading has first a definition and then a specification.

Sensitivity. The ability of the steam valves to respond to a certain sustained speed change. The sensitivity of a governing mechanism should be such that response of the steam valves will occur with not more than 0.04 per cent speed change of the turbine. The sensitivity of the governor should be such that response of the governor will occur with not more than 0.02 per cent in speed change.

Droop. The steady-state change in speed for a full-load change. Droop shall be adjustable from zero to the regulation of the governor and this adjustment will be indicated.

Regulation. The speed change necessary to move the steam valves from zero load to full load with no droop correction. The regulation shall be adjustable from 2 per cent to 6 per cent and will be indicated.

Stability. The ability of the governing mechanism of a steam turbine to position the steam valves for a given load change without sustained oscillation of the steam valves.

Overspeeding. The turbine speed shall not exceed 110 per cent speed on the loss of full rated load.

Response. Perfect governing-mechanism performance is the exact following of the speed-load position by the steam valves. In practice the steam valves always lag behind the speed-load position. For a 6 per cent governor regulation, this lag should be not more than 20 per cent of the total steam-valve stroke when the instantaneous load change is not more than 50 per cent of full rated load.

Speed Adjustment. Possible from 98 per cent at no load to 102 per cent speed at full load.

Load Limit. Adjustable and its position shall be indicated.

It is hoped that this paper, together with the foregoing tentative specifications of a governing mechanism will enable steam-turbine operators and designers to formulate more definite specifications for governing mechanisms which will permit the better functioning of steam turbines in all central-station service.

Discussion

J. C. Hobbs.[2] Governor engineers are to be commended for their intricate analytical work in devising a mechanism which functions in a manner responsive to some motivating force. When every part functions perfectly according to the fundamental theory without lost motion, without wear, without dirt, without falling apart, without breaking, and without lubrication or without any one of hundreds of other troubles, reliable satisfactory operation is secured; but, unfortunately, nothing is perfect. Beginning even with the governor-engineers' design and following with the manufacturers' workmanship and material, the erectors' technique, and the operators' treatment, there always seems to be some minor cause for what sometimes becomes a major accident. Today, manufacturers' organizations appear to be striving more for the complex and, as a result, illustrations could be cited in great number where complexity has caused complications while the manufacturer remains complacent.

No criticism should ever be made without at least voicing a constructive thought or, better yet, a solution. Therefore, without attempting to provide a detailed design for any of the myriad governor problems, it is suggested that the first and fundamental requirement of any governor, where safety and reliability are paramount, is that the governor design and construction shall be as simple as it is possible to construct and still obtain a practical performance of the desired functioning. Each and every governor design should be critically analyzed with a view to determining whether this group or that group of parts might not be replaced with a single part and so on until the minimum combination of moving elements remains. This suggestion is made in the interests of both the operators and the manufacturers, since whatever results in good operation also results in lowered costs for manufacturers, due to reduction in service expense and, in addition, life and property are safeguarded.

R. W. McLaughlin.[3] This paper presents a well illustrated résumé of the problems connected with the design of turbine-governor mechanisms. The brief statement of the case surrounding extraction-type turbines and of the fact that they require the most sensitive governing mechanisms available is well founded. However, due to the high sensitivity and responsiveness of these governors, it is not always necessary to have the governors set for broad regulation, which requires an automatic frequency control to prevent excessive speed variations.

It has been found in numerous instances, where good frequency control was desirable but not of sufficient value to warrant purchase of automatic frequency-controlling equipment, that the extreme sensitivity and rapid response of the governors made it entirely satisfactory to set them for 2 to 3 per cent regulation. That is, with no manual correction of speed whatever, should the electrical load and extracted steam flow either or both together, gradually vary from zero to the maximum for which the machine was designed, their resultant variation in speed would be less than 1 cycle above or below normal. Thus, with the normal variations of load and extracted steam flow which, though often sudden, are only a fraction of the maximum capacity, manual correction of frequency is required only at protracted intervals. Perhaps one of the most difficult applications of this type which has been worked out successfully is the case of an extraction turbine which operates alone in an industrial plant. The governing mechanism on this machine is taxed quite severely, since it serves an electrical load, the bulk of which consists of spot-welding equipment and plant-lighting equipment, neither of which provides the system with any rotating inertia. The surge in power which results from several spot welders striking simultaneously necessitates continual rapid movement of the governor, since the system inertia is so small that the turbine speed is affected by these power surges, even though they are of short duration.

Probably these results could not have been accomplished without the use of a fully compensated valve gear, by means of which either the governor or extraction pressure regulator can

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[3] Turbine Engineering Department, Elliott Company, Jeannette, Pa.

correct variations in the medium it controls without disturbing the other. A further requirement is the use of a double oil relay governor somewhat different in arrangement but similar in principle to that illustrated in Fig. 1 (*C*) of the paper. Furthermore, the accomplishment of such results can scarcely be expected without the designing engineer being fully equipped with advance information furnished by the purchaser with respect to the anticipated details of operating requirements before the design is started, which, as the author suggests, is all too often not the case, as future operating requirements are often unpredictable.

R. B. Smith.[4] This paper presents a summary of the salient features of the governor problem from the static viewpoint. Undoubtedly, the author would be one of the first to agree that a satisfactory solution to the problem necessitates a study dynamically in which the time lags as a result of inertia are included. In many respects there is a similarity between the governor and the vibration problem. Since the question is essentially dynamic, it seems rather unfortunate that more analytical and experimental analyses are not common in our technical literature. A start was made some years ago by Tolle (1)[5] and Stodola (2). This was extended in both a practical and analytical way by Freudenreich (3), and more recently by the efforts of Stein (4) and Luoma (5). In this country, the attention of the Process Industries Division of the A.S.M.E. in its subcommittee on Industrial Instruments and Regulators has been devoted to fundamental study of this problem along analytical lines, and with promising results.

While it has been pointed out in this symposium that a satisfactory governor is in the end one which meets the requirements of the customer is it not also true that a satisfactory governor can be more properly described as one which is dynamically and transiently stable? Dynamic instability, usually termed hunting, meets the author's definition of stability. Its determination necessitates a consideration of the time lags outlined in the author's Fig. 5 and a consideration of these lags in respect to the prime-mover system as a whole. Transient instability arises when, under sudden changes, the governor does not function properly or fails to keep the machine from overspeeding or tripping.

Either of these two types of instability may arise from two sources—a forced instability or an inherent instability. Forced instability can be thought of as arising from external causes such as excessive backlash or from friction; it is indicated in part by the lag *OC* in Fig. 5. An inherent instability arises when the various time lags which exist within the system, and which are attributable largely to inertia, bear an improper relation to the dynamics of the rest of the prime-mover system. It is particularly in the investigation of these dynamic lags that analytical in combination with experimental analysis has proved helpful. As a general statement, it can probably be said that a few years ago instability of a forced nature was the most common governing difficulty. With the advent of the high-back-pressure machine, having a high rate of maximum initial acceleration, and particularly when it operates in conjunction with lower-pressure turbines, the difficulties arising from an inherent instability of the system have assumed a much more important aspect.

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3 "Untersuchung der stabilität von Regelvorichtungen," by J. von Freudenreich, *Stodola Festschrift*, Orell Füssli, Zurich, 1929, pp. 172–179.

4 "Self Regulation," by Th. Stein, *Zeit. V.D.I.*, vol. 72, Feb. 11, 1928, pp. 165–171.

5 "Superposed Turbine Regulation Problem," by A. F. Schwendner and A. A. Luoma, Trans. A.S.M.E., vol. 58, 1936, paper FSP-58-8, pp. 615–620.

W. Trinks.[6] The illustrations of Fig. 1 (*A*, *B*, and *C*) are apparently quite diagrammatic, because the springs between the centrifugal weights would have to be so wound that they have less than no length when free from tension; or else they must be so wound that they exert considerable tension when coiled tightly. Both feats are difficult.

The reason for this may be readily seen: In an isochronous governor, centrifugal force is proportional to distance of center of weights from the axis. The spring must be free from tension when the centers of gravity of the weights are in the axis. Since the spring is shown shorter than the distance between the weight centers, it must have a negative length, when free.

It may be possible to coil the spring so as to exert tension when coiled tightly. If so, the writer would appreciate directions on how to do it.

With regard to nomenclature, the following may be mentioned: Although the Power Test Code Committee has settled on certain names, attention should be given to the fact that within the Society there is another committee which deals with regulation and control and which is busily engaged in standardizing names for instruments such as are used to control pressure, temperature, rate of flow, and the like. It would be unfortunate if two committees within the same society selected different names for the same quantities. They should cooperate.

S. H. Hemenway.[7] The opinion has been expressed that all governors on a system should respond to a speed change of 0.01 per cent to avoid tie-line load swings; also, that the purchaser should specify governing characteristics to take care of the magnitude and rate of load changes on the system. This will stand further study, of which the following are indications:

Many types of governors have obtained stability by having a temporary speed droop *OC*, Fig. 12 of this discussion, greater than the settled speed droop *OA*. This was accomplished on the flyball governor, Fig. 13, by a dashpot on the operating lever. The effect of having the governor respond to smaller speed changes can be seen by disconnecting the dashpot, or by allowing the dashpot some lost motion *Y*. This dashpot arrangement for securing stability has been used on pressure regulators as well as speed governors. In a few cases where it was permissible to try this lost motion, a rapid hunting resulted. Variable droop can be obtained by adjustable automatic speed-resetting means.

A governor in which the speed-sensitive element is of the no-travel type is shown in Fig. 14. In this case, with movement of the governor, the compensating piston *E* increases or decreases the pressure over the dashpot piston *G*. Properly proportioned, this arrangement is nearly equivalent to the dashpot of Fig. 13. It is as stable and reliable as the governor having appreciable travel of the speed-sensitive element, and there would appear to be no reason for prejudice against it on this score. A similar dashpot *H*, Fig. 15, is shown for a no-travel pressure transformer-type of governor.

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[4] Development Engineer, Elliott Company, Jeannette, Pa. Mem. A.S.M.E.

[5] Numbers in parentheses refer to the Bibliography at the end of this discussion.

[6] Professor of Mechanical Engineering, Carnegie Institute of Technology, Pittsburgh, Pa. Mem. A.S.M.E.

[7] Moore Steam Turbine Corporation, Wellsville, N. Y. Mem. A.S.M.E.

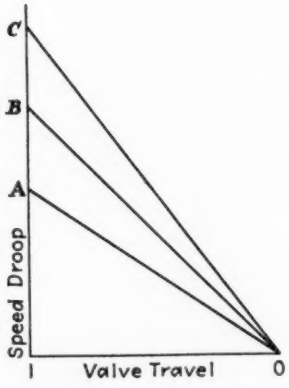


FIG. 12 SPEED-DROOP CURVES OF FLYBALL GOVERNOR

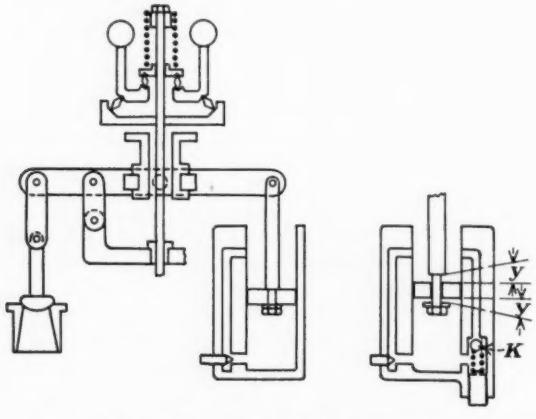


FIG. 13 TYPE OF FLYBALL GOVERNOR

With any of these governors, the temporary speed droop in the closing direction can be reduced from *OC* to *OB*, Fig. 12, by a spring-loaded relief valve *K*, Figs. 13, 14, and 15, to limit the dashpot action in the closing direction, thus limiting the speed jump with loss of full load.

The governors described in the papers[8] show the trend to more intricate and involved mechanisms to satisfy the increasing requirements. If purchasers specify in detail the governor characteristics, there will be a tendency to "play safe" and result in governors of greater intricacy and difficulty of maintenance. Before proceeding in this direction, it would be well to get together those experienced in these problems to outline the sensitivity and speed of response that is desirable considering the following:

1 Tie-line load change
2 Load changes due to "faults"
3 Resulting change in boiler output

[8] This committee on discussion applies jointly to the present paper and the paper, "Constant System Speed and the Steam-Turbine Governor," by A. F. Schwendner, which appears on page 199 of this issue of the Transactions. With the discussion of Mr. Schwendner's paper there is also a joint discussion by Francis Hodgkinson of the present paper.

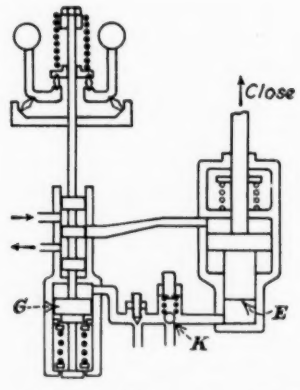


FIG. 14 GOVERNOR IN WHICH SPEED-SENSITIVE ELEMENT IS OF THE NO-TRAVEL TYPE

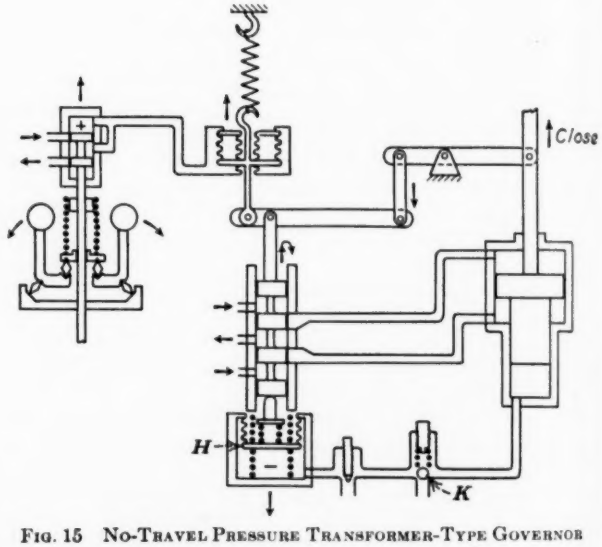


FIG. 15 NO-TRAVEL PRESSURE TRANSFORMER-TYPE GOVERNOR

4 Maintenance of voltage and electrical stability.

Theoretical studies of these factors can also be made with profit, before radical changes are made in governing apparatus. The writer believes there will be agreement that there are many conditions under which slow response has a stabilizing effect.

There has always been a question of the accuracy of measurement of speed, load change, pressure changes, and governor movements. If specifications can be drawn up to provide the sensitivity at different frequencies of equipment for measuring these values, and such equipment can be produced to these specifications by instrument makers and used for all such testing, it will clear up many discrepancies.

[Mr. Hemenway submitted with his discussion suggested terms for governor and control apparatus which have been turned over to the A.S.M.E. Power Test Code committee on Speed, Pressure, and Temperature Responsive Governors. This Committee is shortly to revise and enlarge the present Code.—EDITOR.]

Constant System Speed and the Steam-Turbine Governor

By A. F. SCHWENDNER,[1] SOUTH PHILADELPHIA, PA.

The demand for constant system speed is a current problem of power-generating systems. Electrical stability, governor, governing-valve characteristics, and other factors play their part in the problem. In this paper only the effect the governors and governing valves have in maintaining or disturbing the system speed is discussed. New governing systems are described to show the extent to which the turbine manufacturers go in trying to meet the demand for better system speed. The author points out that, when the governor characteristics of all units in a power-generating system are known, the combination of units used for certain load blocks can be determined not only for the best generating economy but also for better system speed. This information can be used to specify required governor characteristics for new units to further improve the system speed. Attention is called to the need for uniform governor performance and testing specifications.

THE last twenty years have brought many changes in the power-generating field. From small isolated power plants, large interconnected power-generating systems have developed. System frequency has improved from $^1/_2$ to 1 cycle total change, decreasing to 0.2 cycle maximum allowable change, and time errors have decreased to within 1 sec. Most of these improvements have been obtained without material changes to the then existing governors. The control-room operators have more work to do. Automatic frequency-control equipment has been installed, actuating one or several of the synchronizing motors of the governors. The increased use of hydraulic power has brought new problems, but the large systems have taken care of these without changing the governor (specifications); in most cases by increasing the turbine-governor speed variation, that is, the change in speed between no load and full load on a unit.

With the increased use of faster and more sensitive frequency recorders, short period frequency swings are shown which did not appear on the less sensitive instruments previously used. Load changes are observed between units in a power plant which do not correspond with system load changes or total plant output. There are several reasons for this performance. Neglecting the effect which electrical stability, low short-circuit ratio of generators, and harmonic pulsations of load may have in the frequency and load swings, only the part the governors and governing valves have in the problem will be considered.

Governor performance in the past was judged by its synchronizing ability and the amount of frequency change when carrying reasonably steady load with the unit on the line by itself. On larger systems where more than one unit was required to carry minimum system load, a governor was judged good if it synchronized well and showed very little change on the steam-flow chart. From the manufacturer's point of view, a governor is good if it satisfies the customer's requirements. Specifications very seldom called for more than a governor with 4 per cent speed regulation (a change in speed of 4 per cent from no load to rated load). If a governor could synchronize reasonably well with 4 per cent speed regulation and was sluggish enough not to show too much steam-flow fluctuation on the chart, it was considered satisfactory. The frequency recorders did not show momentary frequency swings, and slower changes of 0.2 cycle plus or minus were then accepted as satisfactory.

Sensitive Governors Required

The introduction of the superposed turbines and the larger-capacity 3600-rpm units brought a new type of steam-turbine generating unit into the power systems. The small rotor inertia and consequent high rate of acceleration inherent in this type of unit require fast and sensitive governors so that the units may be synchronized well. A sensitive governor which follows the smallest changes in frequency does not have a smooth steam-flow chart. If a unit with a sensitive governor is put on the line with a unit which has a sluggish governor, the steam-flow changes shown on a recording meter will increase, and will increase further with the additions of other units with sluggish governors.

A sluggish governor or governing system requires a certain amount of speed change before the governing valves begin to move. The centrifugal force of the governor weights and the governor spring force balance at any speed point. If there is a larger amount of friction between the governor parts, the speed will have to change until the difference between the centrifugal and spring force is sufficient to overcome the friction. The same is true with an oil governor. The oil pressure produced by the governor impeller acting on a relay or bellows is balanced by a spring force in the same manner as in a flyball governor. Between the governor and the governing valves there may be one or several servomotors and their linkage. Lack of response in relays and backlash in the linkage alone, or coupled with a governor having friction, may require considerable speed change before the governing valves begin to move. The speed change required to move the governor is not necessarily constant through its total stroke. The change in sluggishness and the difference in load change for the same frequency change, but at another governor or load point, require a thorough study of the governing characteristics of each unit, before a study of system control characteristic is attempted.

A 50,000-kw unit with a governor adjusted for 4 per cent speed regulation will usually show approximately 2000-kw load change with 0.1 cycle change in frequency, assuming straight-line load and governor travel characteristics. This is not always the case, however. Some 50,000-kw units will show 4000-kw load changes with 0.1 cycle change in frequency at some load points while, at other load points, the load change may be less than 1000 kw. Very few governing valves have straight-line flow and lift characteristics. Fig. 1 shows a governing-valve characteristic curve with four governing valves. The governor-servomotor travel, or governor-sleeve travel with flyball governors, governor-impeller oil pressure with oil governors, or rotor speed are shown on the vertical axis, while the steam flow or, with some correction, kilowatt load is plotted on the horizontal axis.

[1] Westinghouse Electric & Manufacturing Company.

Contributed by the Power Division and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Line *A* is the theoretical speed variation of the governor while the valve curves 1, 2, 3, and 4 show the amount of deviation per valve. A change of 0.1 cycle in frequency shows a 2000-kw change on the *A* line, while the same frequency change shows 3000-kw load change on the No. 1 valve line between 0.3 and 0.4 and 750 kw between 0.6 and 0.7 cycle. This load change in terms of speed variation would mean that when the unit load changes 3000 kw with 0.1 cycle speed change it has less than 3 per cent speed regulation, and with 750-kw load change 11 per cent.

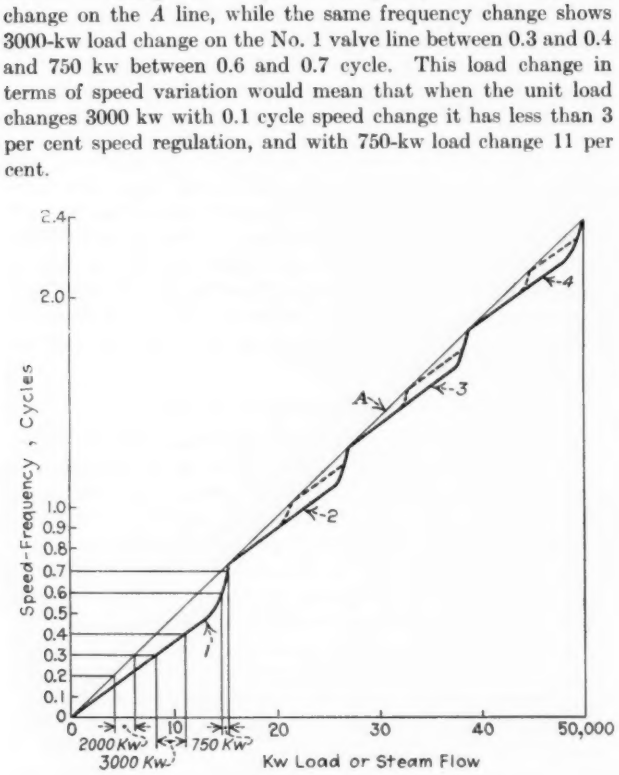


FIG. 1 GOVERNING-VALVE CHARACTERISTIC CURVE

The tail end of the governing-valve curve is determined by the minimum pressure drop allowed across the valve to obtain maximum full valve efficiency. The effect of this curve on the governing of the unit can be reduced by increasing the number of governing valves as shown by the dash lines, by the use of cams or by setting each governing-valve opening closer to the preceding valve. There is very little difference in the rate of load change on the newer units, but some of the older units may show considerable load change between different points of the governor travel.

EFFECT OF SLUGGISH AND SENSITIVE GOVERNORS

Fig. 2 shows the difference in load change between units with 4 per cent and 6 per cent speed-regulation governors, also the effect of a sluggish and a sensitive governor on a system which has only two units on the line with 50,000-kw capacity each. To be able to show the effect of a sluggish governor better, it is assumed that the unit having a 4 per cent speed variation has a sluggish governor which will not respond to a frequency change of less than 0.3 cycle, and that the unit having 6 per cent speed variation has a very sensitive governor. Following the line showing 0.2 cycle change on the 4 per cent governor, it is seen that the turbine load should increase 4000 kw while the unit with the 6 per cent governor should pick up 2800 kw. With only the two units on the line, each carrying a certain load with 60-cycle frequency, a system load increase of 6800 kw is assumed which normally will drop the system frequency 0.2 cycle. With the 4 per cent governor not responding to a change less than 0.3 cycle, the system frequency must drop 0.3 because the 6 per cent governor picks up only 4000 kw out of the total of 6800 kw with that frequency change. However, as soon as the 4 per cent governor moves at that frequency change, it picks up 6000 kw or assumes a valve position equal to it. The system load increases only 6800 kw while the combined increase in valve openings can carry 10,000 kw. The excess steam flow will increase the system speed and so reduce the load on the 6 per cent governor to 800 kw with a system frequency drop of 0.057 cycle from its original value.

The steam-flow chart on the unit with 4 per cent speed variation will show only one change in flow equal to a load increase of 6000 kw. The flow chart on the unit with 6 per cent speed variation will show a flow increase equal to 4000 kw, then a decrease of 3200 kw. The frequency recorder will indicate a decrease in frequency of 0.3 cycle, then an increase of 0.243 cycle. When the frequency and steam-flow changes on each unit are compared with the system load change, the first impression may be that the unit with the 4 per cent governor is the better of the two as it picked up more of the load change with only one change on the flow chart, and very little final change in frequency. However, as pointed out previously, without the 6 per cent governor the frequency changes would have been considerably larger.

The condition just described is multiplied many times on a large interconnected system. The units on the line will have some sensitive and some sluggish governors. There will be units carrying load at full-open-valve points while others run at intermediate points. A small frequency change may move a few

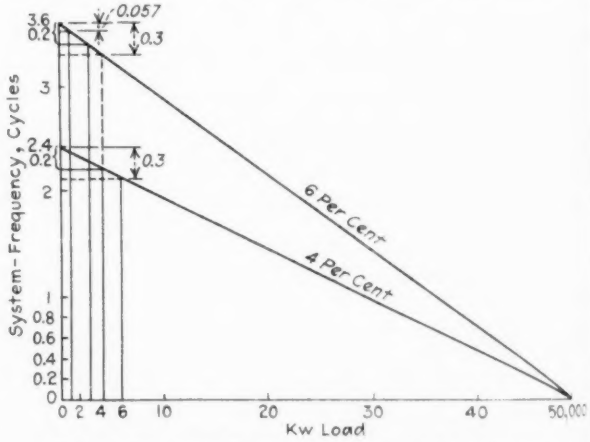


FIG. 2 GOVERNOR SPEED VARIATION AND LOAD CURVE

sluggish governors while others will start as soon as the frequency changes a trifle more. The sensitive governors in the meantime will try to correct for the movements of the other governors.

EXHAUST PRESSURE REGULATORS

The superposed turbines operating with exhaust pressure regulators will add to the present disturbances. Suppose the high-pressure unit has 50,000-kw capacity and exhausts into a header feeding the two 50,000-kw units described. Assume that the contract calls for a 4 per cent governor in the high-pressure unit. It will be sensitive in order to be able to synchronize properly. Repeating the previous performance, the system load is increased by 10,800 kw or the total change of load with 0.2 cycle frequency drop. Dropping the frequency 0.3 just before the sluggish 4 per cent low-pressure-unit governor begins to move, there will be 6000 kw on the high-pressure and 4000 kw on the 6 per cent low-pressure units. A 6000-kw increase on the high-pressure unit will increase the low-pressure header pressure because only one unit is increasing its steam demand and only the amount required for the 4000-kw increase in load. The exhaust

pressure regulator will start to decrease load on the high-pressure unit. The time lag depends upon the low-pressure header volume. Just then the low-pressure 4 per cent governor will begin to move and will pick up 6000 kw, which in turn drops the low-pressure header pressure and so starts to increase load on the high-pressure unit. The governing valves on all three units will now be open to carry an increase in load equal to about 16,000 kw and the frequency will rise, unloading the high-pressure and 6 per cent low-pressure units. Due to the time lag in the low-pressure header, the frequency may change three to five times before equilibrium is established against the two changes described.

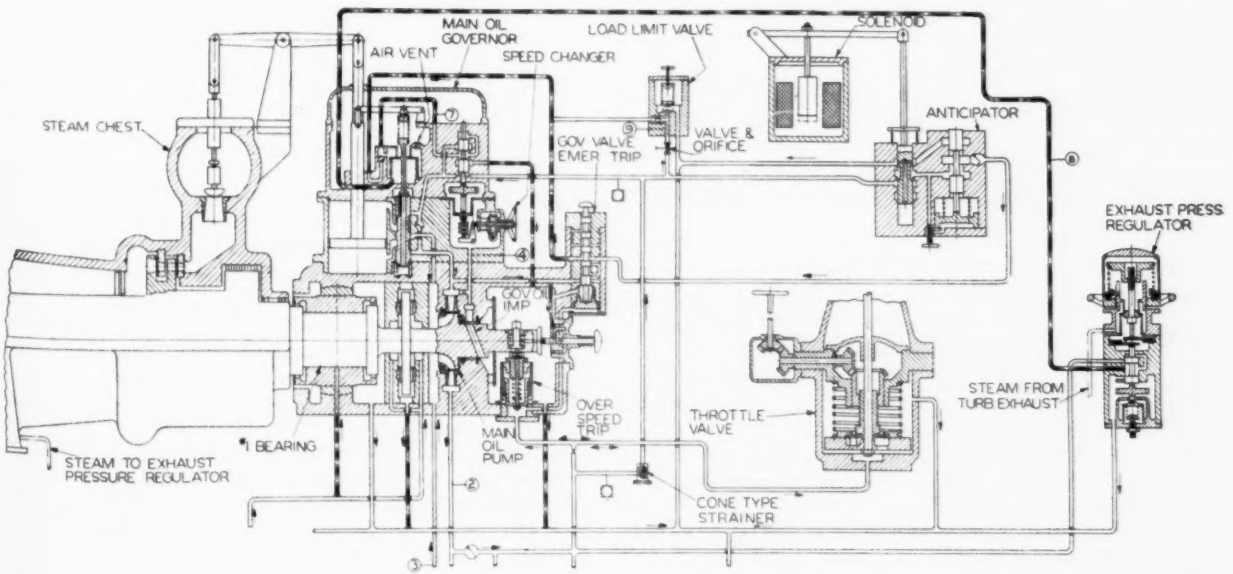
The high-pressure unit, when exhausting into a low-pressure system with sensitive governors and having sufficient capacity to take all the steam from the high-pressure unit, can be balanced so that with load changes the exhaust steam equals the steam demand of the low-pressure units with very little correction by the exhaust pressure regulator. But at times the high-pressure unit may have only one or two low-pressure units taking exhaust steam. The high-pressure unit following the frequency changes will change the exhaust-steam flow beyond the requirement of the low-pressure units. Hence, the exhaust pressure regulator must correct the steam flow after each frequency change and, because of the header lag, will cause additional frequency changes.

There are other factors which can disturb the system speed, but their effect is relatively small and the cycle long. The practice of using the newer turbines as base-load units does not help the system speed. To be able to obtain the best efficiency of a base-load unit, the load set on the governor should not change with small frequency changes of the system. The governor speed variation is normally increased above the other units in the system. That by itself does not help if the base-load unit has a sensitive governor while the majority of the other units in the system have sluggish governors. The new units have very little rotor inertia and so the governors cannot be made very sluggish before they become unstable.

Eliminating Effect of Sluggish Governors

Most power-generating systems have a high percentage of old units in the power plants. At the time of installation their performance was satisfactory. Some of the governors on the old units are sufficiently sensitive not to cause frequency disturbances. A good many of the others can be made reasonably sensitive. There may be a few that will require new control equipment. The capacity of the sluggish units in relation to the system capacity and the number of sensitive governors on the system will determine largely the need for new control equipment. To eliminate the effect of the sluggish governors on the system speed, the simplest method is to take the units off the governors by a load-limiting device in such a manner that the governor will cut in, in case of a rise in system speed above the speed set by the limiting valve, but will not increase load when the system speed drops. In itself this would decrease the available generating capacity in case of system disturbances. To overcome this objection the load-limit device can be made to release its limiting effect if the system speed drops below a set speed. With this equipment, the load on the unit will not be affected by the normal speed change of the system, but will still be available for emergency operation.

Control equipment has been developed for the growing demand for better system speed variation. To reduce the effective changes in speed variation due to governing-valve characteristics, the multivalve steam chest with diffuser-type valve seats was developed. The flow-lift curve of a diffuser-type valve seat is a straight line up to 85 per cent of its full steam flow. The length of the curved section is determined by the allowable pressure drop through the valve. The curved section is relatively small even



LIST OF PRESSURE GAUGES

1-BEARING OIL
2-OPERATING OIL DISCHARGE
3-OPERATING OIL SUCTION
4-GOVERNOR OIL
5-AUX OIL DISCHARGE
6-HYD GLAND SEAL OIL
7-GOVERNOR REGULATING OIL
8-EXHAUST REGULATING OIL
9-LOAD LIMIT VALVE REG OIL
10-HYDROGEN GLAND SEAL ALARM
11-BEARING OIL PRESS. REGULATING

SYMBOLS

EXTERNAL OIL PIPING
INTERNAL OIL PIPING
CHECK VALVE
GLOBE VALVE
STRAINER
ORIFICE
GAUGE MOUNTED ON GAUGE BOARD
GAUGE MOUNTED ON PIPING
RELIEF VALVE

Fig. 3 Load-Limiting Valve

with very low steam-pressure drops with this type of valve and seat. A cam in the servomotor follow-up linkage can correct the slight deviation from the straight line, but the error is so small that the use of a cam has not been found necessary.

The speed-responsive element in the governor is designed to give a maximum sensitivity and fast response. Fig. 3 shows an oil-governor control system. The rotation of the turbine shaft establishes a pressure in a chamber surrounding the shaft. This pressure is maintained by the centrifugal force of the oil in the inclined hole drilled in the shaft connecting the pressure chamber to drain. Oil is supplied through an orifice into the pressure chamber from the main oil pump, also located on the turbine shaft.

The small amount of oil flowing through the orifice into the pressure chamber maintains a pressure which varies as the square of the turbine speed. To increase the sensitivity and response of the servomotor to respond to very small speed changes, the relatively small pressure changes produced by the centrifugal force of the oil in the inclined hole, otherwise called the governor impeller, are magnified in the governor transformer. By the use of a rotating relay or valve and the use of bellows, the small governor-impeller pressure changes are increased in the ratio of 5 to 1 without friction or measurable time lag. The transformed or secondary pressure controls the servomotor-operating-piston position through a pressure-responsive device in the follow-up linkage. An increase in the control pressure applied in the pressure-responsive device will move the operating piston in the closing direction. Through a system of check valves, other pressure impulses controlled by regulators, responding to functions other than speed, may be applied in the pressure-responsive device of the servomotor. In Fig. 3 the load-limit valve, load-dump anticipator, and governing-valve emergency trip all act on the top chamber of the pressure-responsive device. The exhaust pressure regulator, as shown, is connected to the lower chamber of the pressure-responsive device.

The pressure-responsive device responds to the highest of all pressures applied or connected to the top chamber. The higher pressure will close the check valve in the other regulators maintaining a lower pressure. Any of the regulators can decrease the load regulated by another regulator by increasing the regulating pressure above the pressure of the controlling regulator. The regulator with the highest pressure will control the load and no regulator connected in the same regulating pressure system can increase the load.

Load-Limit or Steam-Flow-Limit Valve

The load-limit or steam-flow-limit valve shown in Fig. 3 is a spring-loaded cup-shaped valve which maintains an oil pressure corresponding to the setting of the compression spring.

By setting it to a valve opening equal to a single boiler capacity, the load on the unit cannot be raised above that with the speed changer or the exhaust pressure regulator. When the unit is required as base-load generating equipment, the load limit can be set to any desired load demand while the secondary pressure, regulated by the speed changer, can be set just below the pressure of the load limit so that normal frequency changes will not affect the load on the unit. When a superposed unit is controlled by the exhaust pressure regulator and it is desired to cut out the response of the governor to frequency changes, the load limit may be set in the same manner as above. In many installations the operation of the superposed unit, when controlled by the exhaust pressure regulator, is more satisfactory with the governor not responding to speed changes. To eliminate the need of the load limit for this condition, the exhaust-pressure-regulator control pressure can be applied on the top chamber of the pressure-responsive device in some cases. In this manner, while the exhaust pressure regulator controls the steam flow, all the other control features are cut out automatically.

The speed regulation of the governor may be changed by changing the fulcrum point of the servomotor follow-up link. This arrangement is used if the speed regulation is set to a permanent figure. In the future the need may arise to have governors which can change the speed regulation while the unit is carrying load, to satisfy the constant system-speed demand of the system with different combinations of units on the line. The governor can be furnished with a different transformer housing which allows the transformed or secondary pressure to enter the chamber located below the transformer relay or valve through an orifice and air bell. The regulating or admission valve is so constructed that the secondary pressure may be partly or fully applied below the transformer relay obtaining partial or full-speed compensation. The change is gradual and can be adjusted while the unit is in operation.

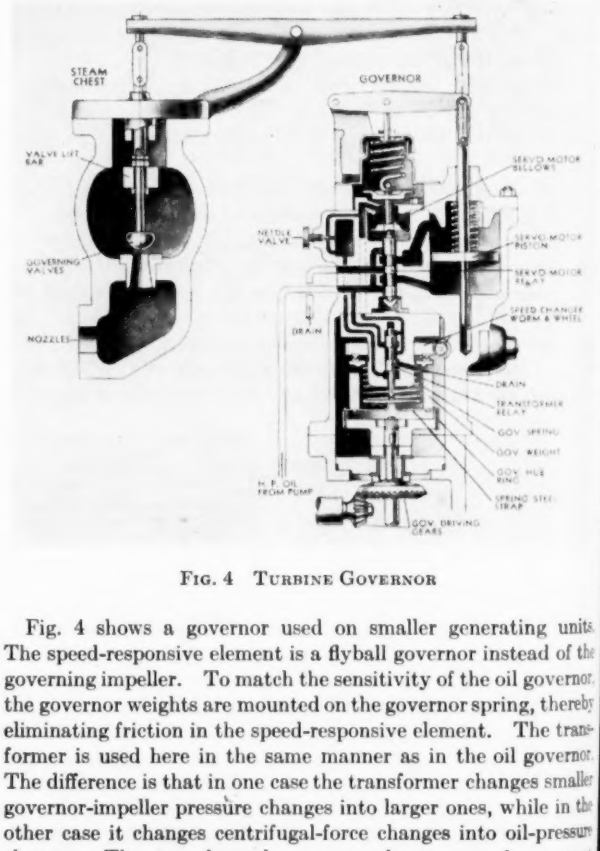


Fig. 4 Turbine Governor

Fig. 4 shows a governor used on smaller generating units. The speed-responsive element is a flyball governor instead of the governing impeller. To match the sensitivity of the oil governor, the governor weights are mounted on the governor spring, thereby eliminating friction in the speed-responsive element. The transformer is used here in the same manner as in the oil governor. The difference is that in one case the transformer changes smaller governor-impeller pressure changes into larger ones, while in the other case it changes centrifugal-force changes into oil-pressure changes. The transformed pressure change per 1 per cent speed change is the same in both governors and all the governor characteristics described apply also to the smaller governor.

Industrial power plants tied in with power-generating systems should be considered as part of the system. There are many industrial power plants with up-to-date generating equipment maintained in excellent operating condition. There are others, however, and some with quite large generating capacity where the equipment, particularly the governors, are not kept in as good operating condition. The reaction on the system speed caused by the faulty operation of these governors is determined by their capacity against that of the system. The governor performance characteristics of industrial plants, particularly the ones with large generating capacity, should be checked along with the generating units of the power system.

Systematic Study of Governor Performance Needed

It does not help the power systems when the manufacturers supply generating units with governors capable of the widest changes in control characteristics and no attempt is made to take advantage of it. A systematic study should be made of the existing governor performances in each power plant of an interconnected power system and existing governor performance corrected to suit system needs as much as possible with the existing equipment. When ordering new units, analyze the required governor performance with all practical combinations of units, power plants, and loads just as carefully as the best combination of steam conditions is determined. The required governor performance should then be specified, including possible changes in characteristics while the unit is in operation or shut down.

A committee should define standards in governor performance and set up methods of checking and testing governor performance.

The range of the variable characteristics should be determined, also which one should be susceptible of change with the unit in operation.

A definite governor-performance specification will help the turbine builders and a knowledge of the limitations of each governor will help the system operators.

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Discussion

I. C. Martin.[2] The author states that governing characteristics of all units on the system should be carefully studied and load conditions considered before specifying required governor characteristics for a new unit. It is agreed that this study should be made for the purpose of determining most desirable operating methods. However, it seems that the governor for any new unit should be built with maximum sensitivity, controllable speed of operation, speed-droop adjustability between zero and at least the greatest value for any of the existing units, and provision for any auxiliary controls such as the author mentions. Governors for existing units should be brought up to new unit performance rather than providing for poorer governor performance in new units. Only in this way can maximum system flexibility be obtained. Although unit characteristics and load conditions determine optimum operating methods, full advantage cannot be obtained unless governors on the various units can be adjusted as desired. Governors with the same range of adjustment, whether the units are steam, hydro, or Diesel, present the only means of obtaining maximum system flexibility for both current operating methods and possibility of future changes. Therefore, it would seem to be that the point of wide range of governor adjustment is most important whether selecting governors for old or new units.

E. E. Parker.[3] The author's explanation of the action of turbine generators with sensitive governors, when paralleled with units having less sensitive governors, should assist in eliminating misunderstandings on this matter. Because of the slower response of the hydraulic-turbine governing mechanisms, such operation has often been encountered in power systems where steam and hydroelectric generating equipment must work together. Failure on the part of a few turbine users to appreciate this has given rise to complaints of governing-mechanism instability where, as the author points out, the governing mechanism is actually doing a better job than are the mechanisms which they consider satisfactory.

Field experience over a number of years with turbine governing mechanisms has established that the slope of the kilowatt-load (or steam-flow) versus governor stroke curve is an important factor in determining whether or not the governing of a turbine will be stable. A number of analytical studies of turbine governing, including governors of both the flyball and oil-pressure type, have shown that the slope of this curve is very nearly inversely proportional to the minimum regulation at which a turbine may be operated with stability. For example, in Fig. 1 of the paper, if the actual governing curve has a slope (speed change per kw input) at any point equal to one half the average, then the minimum regulation for stability must be twice what would be required with the same turbine and governing mechanism if the slope were average at all points.

Thus, it can be seen that, for a given turbine governing mechanism and regulation, the stability of operation can be improved by reducing the variation of the slope of this curve from the average; or for a given degree of stability the regulation can be decreased as the slope of this curve at any point approaches the average. In addition to stability considerations, governing curves with minimum variations of slope are very desirable for good control (either automatic or manual) of the output of a turbine.

Fig. 5 of this discussion shows the governing curve for a tur-

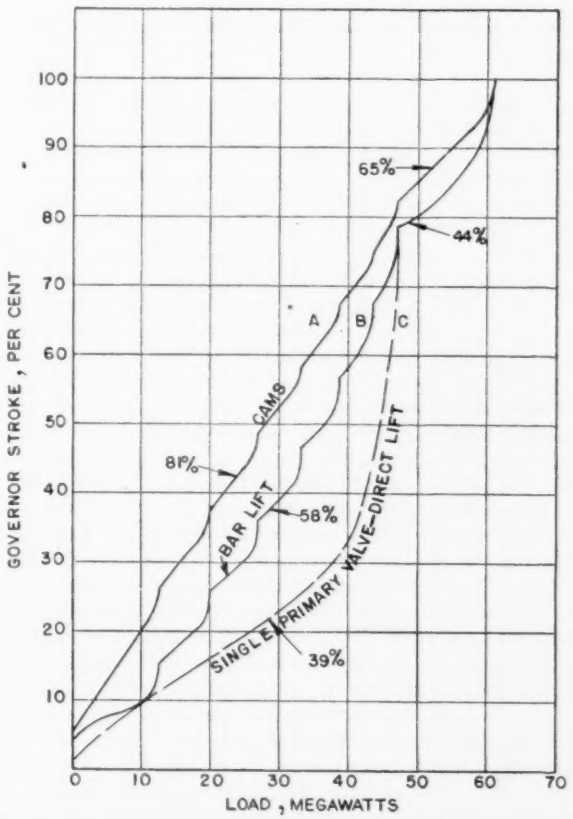


Fig. 5 Comparison of Governing Curves Obtained With Cam- and Direct-Lift Mechanisms

(Figures represent slope of curve at points indicated in per cent of average slope.)

[2] Woodward Governor Company, Rockford, Ill.

[3] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

bine of recent manufacture. Curve *A* shows the governing curve when each control valve is operated by an individual cam. Curve *B* shows this same curve for the unit if the same valves are lifted directly by the hydraulic piston or servomotor. For equal stability, the turbine may be operated with 30 per cent narrower regulation if cam operation of the valves is employed, because of the more uniform governing curve.

This particular turbine has eight primary valves and one overload valve under control of the governor. Each primary valve supplies an individual section of the first-stage nozzle. The valves have the so-called "diffuser-type" valve seats. This type of turbine valve seat was first developed to pass a given steam flow through a given valve-seating area with a minimum pressure drop.

The difference in governing curves obtained from cam- and direct-lift mechanisms becomes greater as the number of valves is decreased. For example, if the eight primary control valves used on the turbine considered in Fig. 5 are replaced by one valve, a governing curve as shown by curve *C* results for a direct-lift mechanism. On the other hand, if the single valve were operated by a cam, the minimum slope of the governing curve would be no worse than with multivalve cam operation.

By the use of pilot poppet-type valves, turbines with direct-lift (often called "bar-lift") valve mechanisms are built with governing curves equally as uniform as those obtained with cam-lift mechanism. Equally good governing curves can also be obtained with direct-lift mechanisms by overlapping the governing valves more. However, since increasing the overlap has a detrimental effect on the efficiency of a turbine it is desired to keep the overlap to a minimum.

The use of cam-operated governing valves also permits more latitude in the location and operation of the steam admission valves.

R. B. Smith.[4] The author has developed in some detail the case against the "sluggish" governor. It would seem, however, that the author's criticism is leveled more against the governor with a forced instability, that is, one having external friction and backlash in its mechanism which makes it unresponsive to a primary change until such a change is sufficient to overcome these external influences, rather than one that is sluggish. Actually, it is possible that a governor may be sluggish in the sense that the dynamic time lags inherent in the various parts of the mechanism are so great that it does not respond rapidly and, yet, at the same time, it possesses sensitivity to even small changes in the primary variable. If such were the case, it would undoubtedly be true that, in the author's Fig. 2, both turbines would show a change in the flow chart reading and in exactly the proportion they theoretically should.

The author's cure for difficulty of the nature he describes is a more refined hydraulic system with every effort expended toward the elimination of external effects. While this change is undoubtedly beneficial, it is not necessarily true that the design will result in a stable governor. Undoubtedly as the author is very much aware, the question of stability is a dynamic one, involving a consideration of the inertia effects in the whole turbine-generator system. In the end, it usually resolves itself into a determination of the various time lags in the mechanism, the prime mover, and other parts and a proper interpretation of these lags for the specific design under consideration.

The single-seated venturi-type valve, the characteristics of which are described in Fig. 1, has certain desirable features. It is an adaptation from the original proposal of Ferranti made some 30 years ago. While its use results in a straight-line travel characteristic up to a point higher than for an ordinary plug-type valve, it is difficult to understand how it can influence markedly the hump characteristic which occurs when a valve starts to open. Valve humps can be influenced by the valve lap but, in the main, they are a function of the pressure ratio at which the turbine operates. On condensing turbines, the humps are small but, on noncondensing machines, they are frequently of such an extent that they deserve special consideration in performance guarantees. Somewhat the same situation exists in regard to throttle-valve and governing-valve drops. From the efficiency standpoint, it is much more important to keep them small on noncondensing than on condensing turbines.

When governor difficulties arise, it would appear to be the author's belief that he attributes the source of these generally to the governor operating mechanism. Is this practically always the case or does the author know of difficulties which have their source in the primary element? In other words, are the characteristics of the impeller always as excellent for governing as they theoretically appear to be, particularly when a rapid response is called for?

H. Steen-Johnsen.[5] The author has called attention to the scramble which takes place between sensitive and sluggish governors when the load changes.

The effects of two distinct characteristics are considered, namely:

1 Rate of response or sensitivity (smallest speed change that will cause the governor and valve gear to move).

2 The speed regulation—change in speed from no load to full load.

The governor is an instrument which responds to changes in speed, that is, changes in frequency. From the analysis presented, it appears that for close frequency control the governors used must be sensitive, and have a low speed regulation.

This conclusion appears quite satisfactory when we think of frequency changes as caused by load changes. But we have seen from other papers presented on the subject of frequency control that there are several possible causes of disturbance.

In one of these it was shown how, in a large system with several generating stations interconnected and supplying a diversified load, oscillations will take place between the various points of supply and consumption points of supply and consumption of power, assuming steady load. These oscillations in speed will cause the governors to move the same as a change in load and, if the time lag in the governor action is suitable, it might amplify the swing, rather than reduce it. In other words, while a governor may operate perfectly under one set of interconnections, it may be very bad under another. It might prove interesting to hear the author's comment on possible corrections of this type of difficulty. If the amplitude of the swing is small, a sluggish governor might be advantageous or a heavily damped governor.

Another interesting feature of several power sources tied together is that they often tend to break up a system oscillation by acting as an anchor at a point of maximum swing. Such a condition was observed recently in an industrial plant tied in at the end of a long transmission line. The governor was 7 per cent and not particularly fast, but yet, the chief engineer said that, when the turbine was not on the line, the system was swinging so badly that he could not use his electric razor. Looking at the governor it did not appear to do any particular job. The load did not swing more than 200 kw.

[4] Development Engineer, Elliott Company, Jeannette, Pa. Mem. A.S.M.E.

[5] Turbine Division, Westinghouse Electric & Manufacturing Company, Philadelphia, Pa. Mem. A.S.M.E.

In the old types of flyball governor, the position of the flyballs varies with the governor opening. In the no-load position, the weights are out and, as the load is added, the unit slows down, the weights moving in, opening the governing valve and admitting more steam at a lower speed and lower centrifugal force. The rate of centrifugal-force change varies with the radius, and as a result it requires more speed change to move from no load to $^1/_4$ load than it takes from $^3/_4$ to full load.

This variation is eliminated in the constant-position flyball governor. In this type of governor, the flyballs move only a few thousandths to change the relay position and, therefore, the radius remains the same.

[Mr. Steen-Johnsen submitted with his discussion a nomenclature for steam-turbine-governor performance and characteristics. This has been turned over to the A.S.M.E. Power Test Code Committee on Speed Pressure and Temperature Responsive Governors. This Committee is shortly to revise and enlarge the present Code.—EDITOR.]

G. B. WARREN.[6] It seems to the writer that the author has given the most important criterion as to a governor's excellence, namely, it is a good governor when it satisfies the requirements of the user. In the writer's company, we have taken pains at various times to make sure that the governors as they were built would meet the requirements of the users. Recently, however, newer conditions, largely in connection with system interconnections, have apparently come into existence which are beginning to require governors of greater refinement. There is satisfaction in knowing that these requirements are being scrutinized carefully by a committee of turbine users under the auspices of the Edison Electric Institute, and that a tentative specification will probably result.

In this connection, the writer would suggest to those concerned in formulating this specification that most careful consideration be given to the extent of the refinements which are so stipulated, such as extreme sensitivity, flexibility, etc., in order to be certain that all of the qualities indicated as necessary are really needed or indeed desirable.

It is easy enough for a member of such a committee to say that the sensitivity requirements, or other requirements, should be of a certain character, and yet it may be difficult or disadvantageous from other standpoints for the manufacturers to meet the requirements so specified. The industry might thus be burdened with an unnecessary cost level for regulating equipment.

FRANCIS HODGKINSON.[7] It seems that these two papers[8] and the present interest in the subject have been brought about by the modern practice of superposing high-pressure-turbine elements over existing prime movers. These types of turbines, because of their range of steam expansion have small physical dimensions relative to their capacity or steam flow. Their WR^2 is relatively low; the writer is informed that, in several cases, the rate of acceleration of the rotor system with full steam flow and without load approximates 700 rps.

The author of this paper has pointed out that satisfactory governing devices, which are capable of conforming to any reasonable specification, except perhaps that of rapidity of valve motion, have been available for years. It is true that at least some manufacturers have applied governors to steam turbines which were without sensible friction, of extreme sensitivity, and which had close regulating characteristics with stability, commencing with the Parsons steam relay governing system of almost 50 years ago.

It has been pointed out that, in the case of multivalve turbines, a more nearly straight-line characteristic (speed versus output or steam flow) may be obtained if each valve is operated by an individual cam. In contrast to this, a case has been cited of a turbine, controlled by a single throttling valve, which gave anything but a straight-line characteristic and which could not be stable at all points of load except with extreme speed variation between zero and full load. Years ago, valves were arranged with ports in such manner that equal increments of flow were obtained with equal increments of valve travel and straight-line regulating characteristics obtained with single-valve turbines.

The authors of these papers and some discussers have emphasized the need of a standardized basis for a specification for governors, which would also set up appropriate nomenclature. This is desirable, since considerable confusion seems to exist on both matters.

The writer wishes to point out that among the A.S.M.E. Power Test Codes is one on "Speed-Responsive Governors." This is a test code with rules for tests and, as such, necessarily sets up nomenclature and bases for specification, the latter without numerical values. It would seem, therefore, that the Power Test Codes would be an appropriate channel for carrying out the suggestions which have been made.

As chairman of the main committee of the Power Test Codes, the writer has no hesitation in stating that the various committees will always seek the collaboration of other sections of the Society.

This particular code was issued 12 years ago and is now obsolete. It was written at a time when we did not have much conception of the purposes of codes. It requires revision, and a complete reorganization and enlargement of the individual committee to deal with the matter would seem to be advisable. Its scope should be increased to include pressure- and temperature-responsive devices limited, however, to performance and exclusive of details of construction of mechanisms.

NOTE: A joint discussion by S. H. Hemenway of this paper and the paper, "Steam-Turbine Governors," by Reed J. Caughey, appears on page 191 of this issue of the Transactions.

AUTHOR'S CLOSURE

The author is grateful to Mr. Martin for putting the important factors in system and governor-characteristic study in a short condensed form. We all agree that maximum sensitivity, straight-line load and speed characteristics, and all the other controllable features are required to obtain maximum system flexibility. The question arises what is the numerical value of maximum sensitivity and how straight the load and speed line should be for satisfactory operation. Mr. Warren and Mr. Hemenway stress the fact that to obtain the foregoing maximum performance characteristics, considerable and probable unjustified increase in the cost level may be necessary. It would help everyone concerned if the various committees of turbine users and turbine builders would jointly determine and define the required maximum sensitivity; and also the maximum allowable deviation from the straight load and speed line, with a method for testing, which would still be satisfactory to the turbine users and not put unnecessary burdens on the manufacturers.

Mr. Parker shows a considerable difference between the bar-lift and cam-operated valves. The author has had no difficulty in obtaining initial valve-lift load angles between 75 to 80 per

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[6] Designing Engineer, Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

[7] Consulting Mechanical Engineer, New York, N. Y. Mem. A.S.M.E.

[8] This discussion applies jointly to the present paper and the paper, "Steam-Turbine Governors," by Reed J. Caughey, which appears on page 191 of this issue of the Transactions.

cent without the use of the pilot poppet-type valves. Flexibility in the location and operation of steam admission valves can be obtained without cumbersome governor or power linkage by using individual valve servomotors. The servomotors are operating the valves directly and are connected to each other and the governor through the control pressure oil line. On one unit seven governing valves which were located at different points on or around the turbine cylinder were operated in this manner successfully.

Mr. Smith's discussion clearly shows the need for nomenclature defining forced instability, sluggishness, external friction, and so on. The author wants to point out that a sluggish governor as described in his paper is not necessarily an unstable one. He purposely exaggerated the sluggishness of the sluggish governor so that the effect could be shown better on the curve. In actual operation there are few governors which do not respond to a frequency change of 0.1 cycle. A governor with a so-called dynamic time lag will respond in the same manner as the sluggish governor described in the paper if the load changes are sudden. Only with slow load changes or after conditions are settled will the flow chart readings be proportional with the theoretical flows. The question of governor stability concerns the design of the control equipment only. The paper described the performance of governors in actual operation, and the turbine users will not accept an unstable governor. The author is thankful to Mr. Smith for bringing into the discussion the hump characteristic of the valves at opening points. It should be considered when maximum allowable deviations are considered from the straight load speed line. There were difficulties with the governor impeller in the beginning as with any other type of new control equipment. It has been corrected, but there was no trouble following rapid speed changes.

Mr. Steen-Johnsen discusses a few interesting system disturbances and their relation to governor performance. Nothing short of a complete check of governor performance on all the units in the system with tie-line load changes and the resultant frequency changes will give sufficient information to be able to correct the disturbance.

Mr. Hodgkinson is correct in stating that governors and governing systems have been built in the past which are able to satisfy up-to-date demands on sensitivity and regulation. Straight-line regulating characteristics can be obtained with ported single valves if high pressure drops are permissible. With low pressure drops a cam is required to bring the single ported valve up to straight-line performance.

The A.S.M.E. Power Test Codes on Speed-Responsive Governors seem to be the proper channel to set up nomenclature, method of tests, and performance specifications. It will have to work in conjunction with other committees so that its specifications are in line with the performance requirements.

Mr. Hemenway calls attention to a governor characteristic which is seldom considered. There is a considerable difference between the instantaneous or temporary speed regulation and the settled speed regulation on some governors. Mr. Allen of the Pennsylvania Water and Power Co. has some quite interesting tests on governor characteristics taken with his recording instrument.

The manner whereby a high temporary speed regulation can be added on existing governors which otherwise would be unstable with low speed regulation is quite clever. The desirability of having such characteristics on some units which may be tied to others without the same characteristic will have to be investigated. Mr. Hemenway and Mr. Warren stress the need of further investigation to establish minimum values which will be satisfactory from the system's point of view. Instruments for testing governor performance have been available and new ones are being developed so that testing governor performance will not be an impossible undertaking.

The Field of System Governing

By ALBION DAVIS,[1] SAINT LOUIS, MO.

The field of system governing lies between the problems of pulsating loads and electrical stability on the one hand and frequency or time control on the other. It is concerned with the short period swings or hunting action for which poor governors are usually responsible. With proper instruments, poor governor units may be detected readily and corrections made. Sensitive governors may be substituted for them. The paper discusses some of the elements of governing from the standpoint of the system operator and suggests certain essential requirements which new governors should meet.

A YEAR ago, the author appeared before the Prime Movers Committee of the Edison Electric Institute with a paper covering some of the system aspects of governing. A portion of the material presented at that time will be used again in this paper. The shortcomings of some governors will be contrasted against the good points of others in an effort to throw more light upon what governors really do on our systems and how much benefit may be derived from investigations and improvements in the field of system governing.

This paper is not concerned directly with the problems of electrical stability, the problems of relaying, or those dealing with harmonic pulsations of power output, resulting from hydraulic conditions in penstocks or draft tubes, or from the amount of "rubber" in the ties between the different rotating masses of the system. These transient effects usually occur within periods of less than 1 sec. They come within the field of system relaying—a field already comprehensively explored by electrical men. Nor is this paper concerned directly with the operating problems of maintaining accurate time and keeping average frequency within reasonable bounds for commercial purposes. This "frequency control" or "time control" is obtained by means of automatic or manual adjustment of the settings of the governor. Such adjustments are concerned with changes of speed which take place generally over periods of more than $^1/_2$ min.

The field of system governing lies in between the problems of transient pulsations on the one hand and frequency or time control on the other. Governors are primarily concerned with adjustments within periods from $^1/_2$ sec up to 1 or 2 min. They are concerned with, and in most cases responsible for, the short period swings of speed on isolated systems and swings of tie-line loadings on interconnected systems, both of which affect system economy and the quality of service to customers.

Over short periods, the system-load curve is what the governors and regulation by the operators make it. Inadequate governors produce swings which amount to a continuous but somewhat irregular hunting action. These swings, superimposed on the real changes of average speed due to load changes, make it impossible for either the manual or the automatic operator to detect the small changes of load trend quickly and to provide for them with correspondingly small changes of prime-mover output.

The wider the fluctuations caused by the governors, the more difficult it becomes for the operator to minimize the number of load adjustments made on the different units of the system. More frequent and heavier hit-and-miss prime-mover adjustments will result and plant outputs will fluctuate over a wider range; all of which spells reduced economy of generation.

Over and above the matter of rendering good service, steady and close regulation of frequency will pay actual dividends. In the case of steam units, load swings of any appreciable magnitude will be accompanied by adjustments of boiler controls to meet the new condition and, in the passage of the boilers from one stable condition to another, a loss of economy results. Suppose each such boiler adjustment costs $1 and suppose, because of a ragged speed curve, two or three extra adjustments are required each hour, then in a year the ragged speed curve has cost the system some $20,000. Losses may be of equal magnitude in a hydroplant due to excessive regulation.

On an interconnected system, where governing irregularities show up as load swings on the tie lines, the capacity value of these tie lines may be greatly reduced because of the load fluctuations. With a smoothing out of these load swings by improved governor action, the average tie-line loadings may approach closer to the stability limit of the line, thereby increasing its capacity value.

By eliminating the more violent swings of system speed or of tie-line loadings, the electrical stability of the system will be improved. Ties between generating units and the load will be strained less and may safely carry higher loads than is the case where governors on one section of a system are bucking those of another section over short intervals of time.

The capacity of governors is measured by their "sensitivity," the term being used in the sense of ability to move the control mechanism of the prime mover promptly the correct amount in the proper direction. It does not mean "flighty" or merely rapid, but action of the sort which results in small adjustments made without delay and without overtravel.

The nature of the short 10- to 20-sec swings of system speed, from which so many problems of regulation stem, is often not realized until system speed is recorded by a sensitive high-speed frequency recorder. The ordinary station frequency chart does not record instantaneous values of system speed. The chart may show the high and low points which have occurred and indicate the band within which frequency fluctuates, but the operator is provided with only a vague indication of the rapidity of the fluctuation. The operator may think of the short period swings as "inherent pulsations of load" but they are really fluctuations caused by the hunting action of the governors.

System Tests

Fig. 1 shows frequency charts taken on the Union Electric system. The system supplies a load which ranges from 200,000 kw to well over 400,000 kw. Regulation is divided among units of three plants, the main 300,000-kw steam plant in St. Louis and two 125,000-kw hydroplants connected with St. Louis by transmission lines about 150 miles long. At the left of Fig. 1 are two sections of chart taken from the Leeds and Northrup station recorder upon which the operator must depend in controlling speed. The interval between the vertical time-scale lines is 30 min.

Over the 10-min periods represented by the blocks shown on the charts at the left, there is but little to differentiate the fluctuations of one period from those of the other. However, when

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

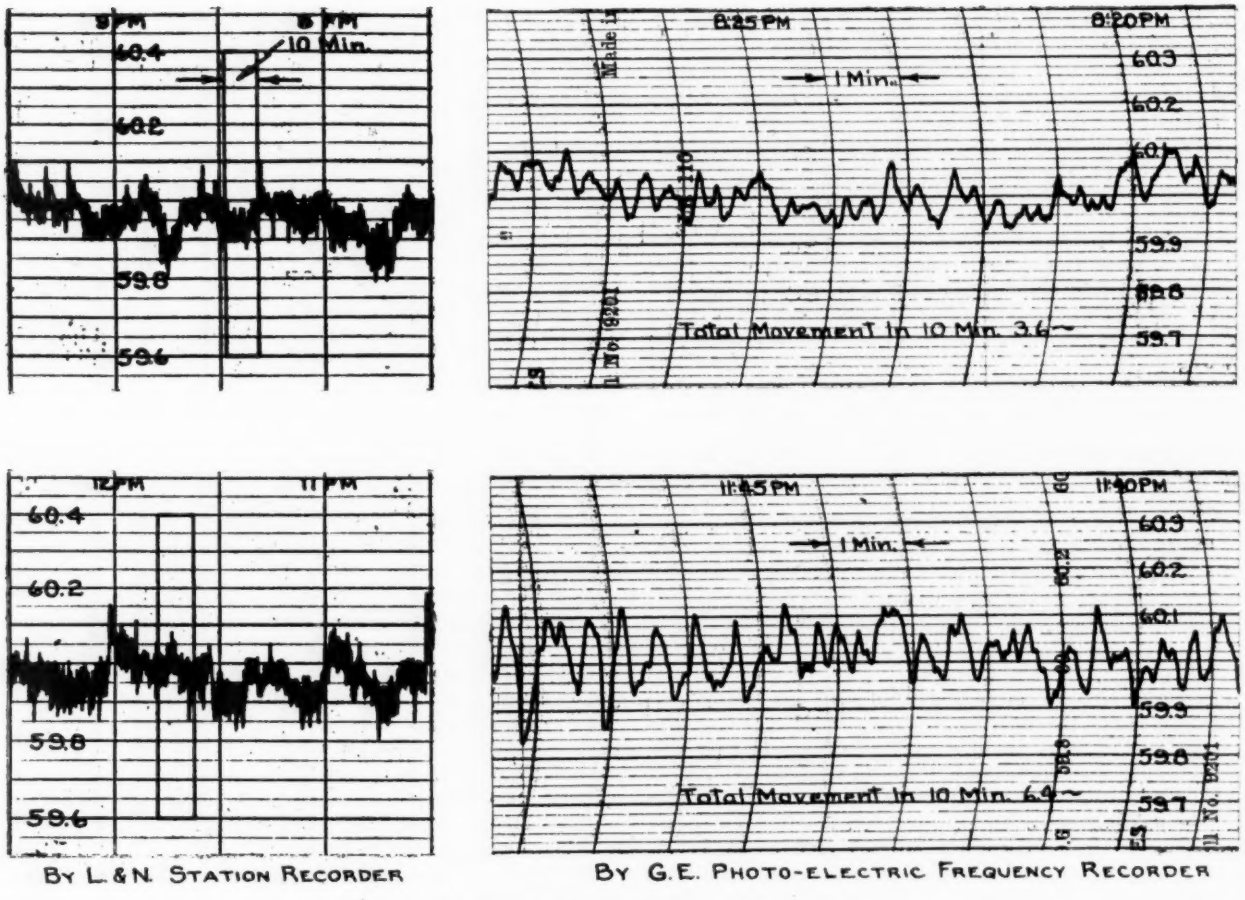


Fig. 1 Governing of Union Electric System Frequency

the high-speed charts at the right for the same 10-min periods are studied, a marked difference in the character of the regulation is noticeable. These charts were taken with a high-speed G.E. photoelectric recorder operating at a chart speed of $^1/_2$ in. per min. The total movement of system speed for the upper chart was only 3.6 cycles in 10 min, whereas that of the lower chart was 6.4 cycles in the same time, or nearly twice as great. Whereas the upper chart might be considered fairly good regulation for the system, the lower chart shows poor regulation. In the latter case, the speed is recorded as swinging up and down over a range of about 0.15 to 0.20 cycle.

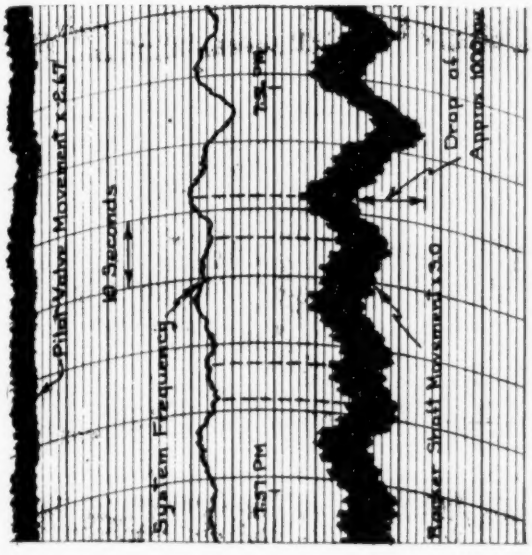
The difficulty of avoiding bad hit-and-miss adjustments on generating units with the system swinging as shown on the lower chart of Fig. 1 can be appreciated better when it is realized that an adjustment of only 0.02 cycle in speed is required to correct a 1-sec time error in 1 hr. On these swings of speed, the governors of the system pick up and drop from 6000 to 8000 kw of load; although, to correct a drift of average speed of 0.10 cycle in 10 sec requires only 1000 kw.

Total movement of speed, or the summation of all its fluctuations in a given period of time, has been used as a measure of good or bad speed regulation. All system governors, which are regulating, follow these swings. Total movement of system speed is not only a measure of the average rate of change of speed, but a factor which measures relative wear of the moving parts of all the control mechanisms on the system and, to a certain extent, the cost of their maintenance.

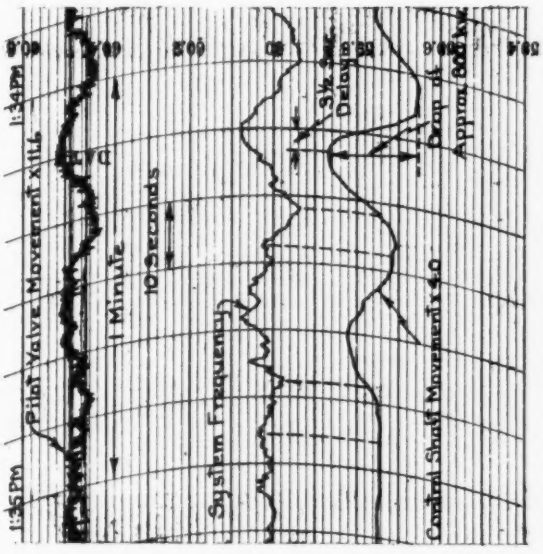
Fig. 2 shows the effect of individual steam governors on system frequency. The charts at the left were taken with the photo-electric meter running at a speed of $^1/_2$ in. per min. Conditions as to the number of units regulating, etc., were the same for both the upper and the lower charts, except for the interchange of steam units Nos. 1 and 2. The upper chart, showing regulation with steam units Nos. 2 and 6, shows roughly half the fluctuation of the lower curve, taken with steam units Nos. 1 and 6 regulating. The total movement of system speed with No. 2 on was 4.8 cycles in 10 min, whereas, with No. 1 on and No. 2 off the total movement of the system speed was 7 cycles in 10 min.

At the right of Fig. 2 are shown short sections of charts run at a speed of 3 in. per min to analyze the performance of the governors on steam units Nos. 1 and 2. These show plainly the reasons for the wide difference in system regulation. For this analysis two mechanical-motion pens were added to record the accuracy of governor response to speed changes. System frequency is recorded in the center of the chart, pilot-valve movement at the top of the chart, and movement of the valve-control mechanism at the bottom of the chart. All three pens travel on the same arcs. For correct governing, each of the mechanical motions should move in the same direction as the change of speed. It is difficult to detect any appreciable inaccuracy in the operation of the No. 2 governor.

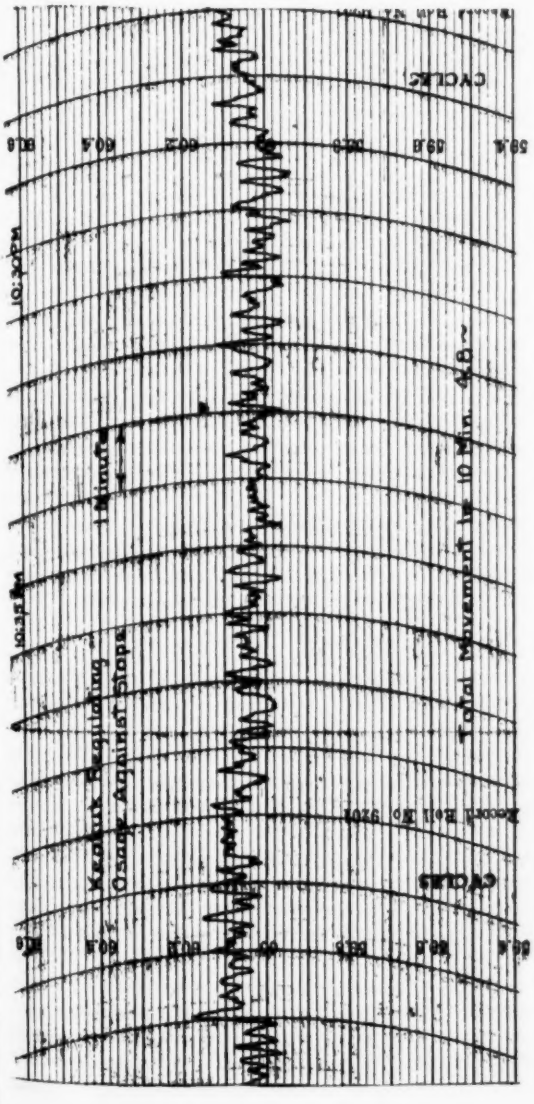
The lower chart records poor performance on the part of the No. 1 governor. It is to be noted that on the larger speed change at the right there was a lag of $3^1/_2$ sec between the peak of the frequency curve and the peak of the resultant gate movement. Two cases are shown where there was an increase of speed of more than 0.05 cycle in 6 to 8 sec and yet the governor, during that period, was opening the gates and picking up load; just the



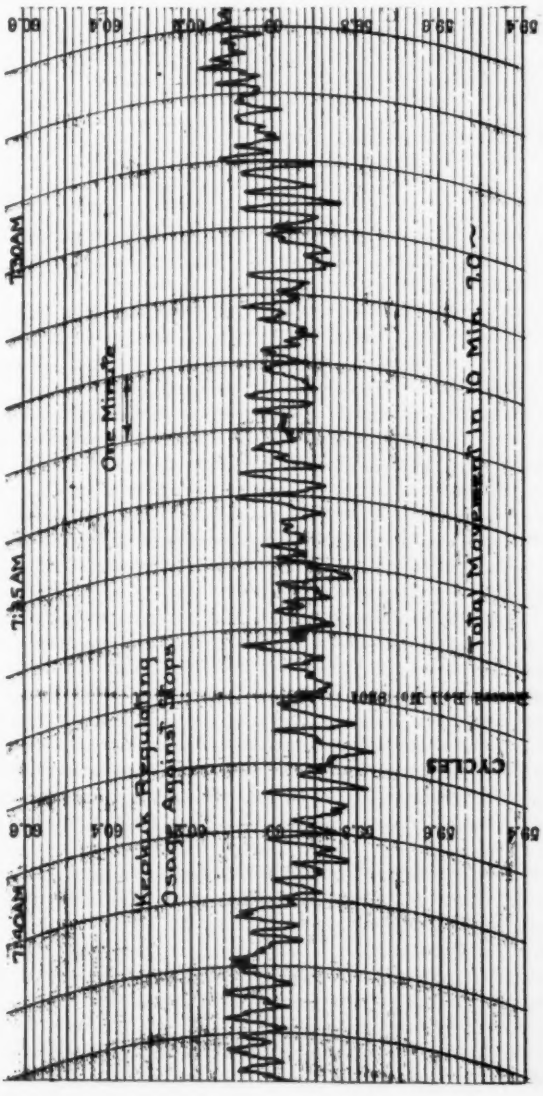
UNIT 2 GOVERNOR PERFORMANCE



UNIT 1 GOVERNOR PERFORMANCE



STEAM UNITS 2 AND 6 REGULATING



STEAM UNITS 1 AND 6 REGULATING

FIG. 2 EFFECT OF INDIVIDUAL STEAM GOVERNORS ON SYSTEM FREQUENCY; HYDROGOVERNORS UNCHANGED

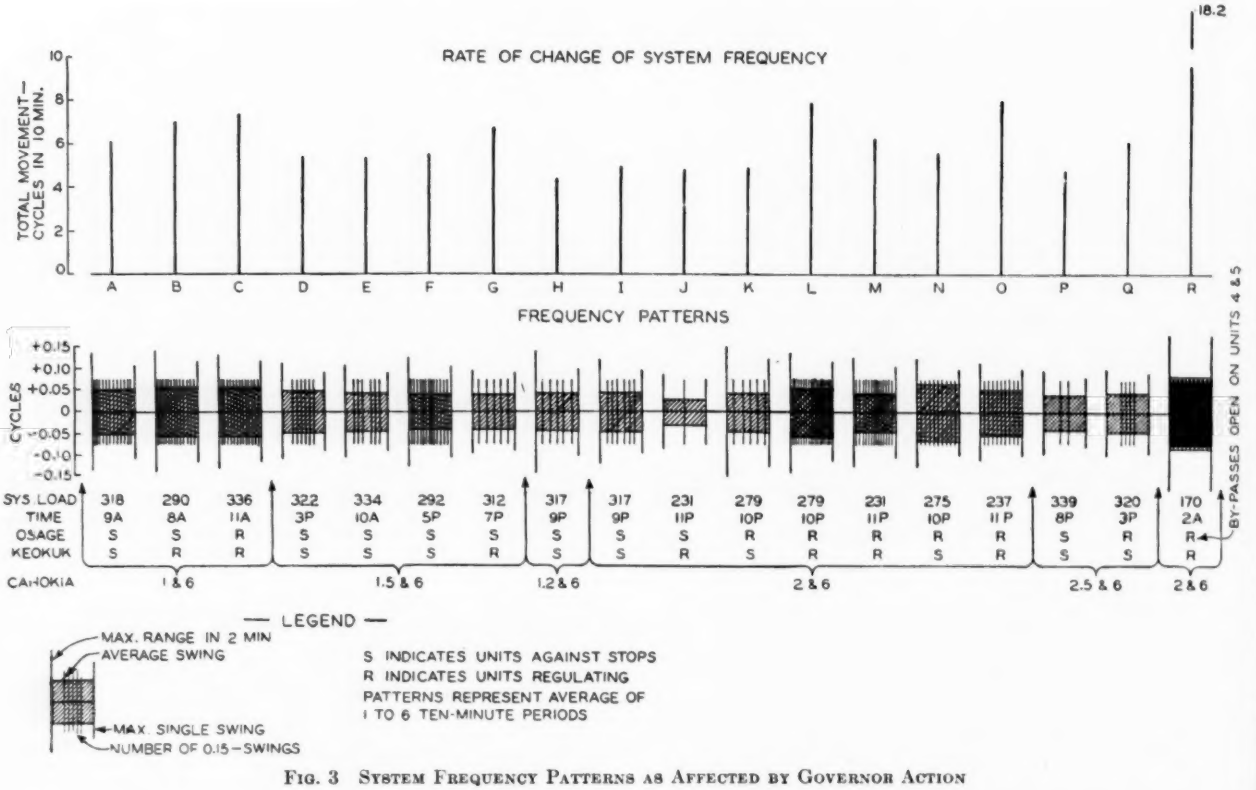


FIG. 3 SYSTEM FREQUENCY PATTERNS AS AFFECTED BY GOVERNOR ACTION

reverse of what it should have been doing. The difficulty with this governor at the time the chart was taken was apparently caused by excessive lap on the pilot valve.

The conclusion that most of the 10- to 20-sec swings of system speed are produced by governor action rather than by changes of load is based on a series of tests which are summarized graphically in Fig. 3. The typical frequency patterns resulting from some 18 different governing combinations are shown in condensed form. For each pattern, the maximum range of speed in any 2 min is shown at the left; the maximum single swing at the right; and in between, the average swing and the average number of 0.15-cycle swings in 2 min. Each pattern represents an average of from one to six 10-min periods. Above each frequency pattern the total movement of system speed in 10 min is shown.

Condensing the charts into these patterns makes it possible visually to differentiate the results of the different governing combinations. Direct comparison indicates that pattern *J* with steam units Nos. 2 and 6, operating with one hydroplant regulating and the other against the stops, is one of the best governing combinations. It is also significant that all of the patterns showing the lowest total movement of system speed occurred when the good steam unit No. 2 was regulating. In the case of pattern *R* at the right of Fig. 3, two hydrounits were operating with their dashpot by-passes open, resulting in a serious hunting of system speed.

Some of the hydrogovernors improved system regulation when working with good steam governors but, when working with poor steam governors or with other poor hydrogovernors, they exaggerated the fluctuations. Close analysis of these tests suggested that, if all governors of the system could be blocked during a period of fairly constant load, practically all of the short period fluctuations of speed would disappear.

To test this idea, advantage was taken of a period when a small section of the load was isolated from the system. The hydrounit, carrying the isolated load, was put on hand control for a short period in order to observe, on the high-speed frequency chart, the changes of speed which occur solely because of the load changes. The results of this test are shown in Fig. 4. At the left is a 10-min stretch of chart, during which the hydrounit was on hand control and there was no change of turbine-gate opening.

Ungoverned, the speed swung over a range of $\frac{1}{2}$ cycle, yet the total movement of speed in 10 min was only 6 cycles. The chart at the right of Fig. 4 shows the same load with the governor in operation, holding constant average speed but swinging the speed up and down over a range of 0.10 to 0.15 cycle, 6 to 8 times per min. Disregarding the section of the chart where the dashpot by-pass was opened, the total movement of the governed system speed in 10 min was 14.5 cycles, more than twice the rate of change which occurred when the load was ungoverned.

TESTS OF INDIVIDUAL GOVERNORS

Fig. 5 has been included to show the range of performance which may be observed on a system when tests of individual governors are first made. It is likely that some good governors and some poor governors will be found. The chart at the left of Fig. 5 is a record of performance of a fairly good hydrogovernor controlling a 25,000-kw vertical unit at a 95-ft head. System frequency is recorded in the center of the chart, gate movement at the left, and pilot-valve movement at the right of the chart. This governor, which is operating on a speed-droop setting of 3 per cent, registers fairly good performance. Gate movements are following speed changes with time delays on the reversals of only about 2 sec.

The chart in the center of Fig. 5 shows poor performance of a governor on a small low-head hydrounit. Gate positions wander badly because the governor is not sensitive to speed changes of less than 0.08 cycle. Time delays on the reversals range from 5 to 8 sec. This poor performance contrasts sharply with that

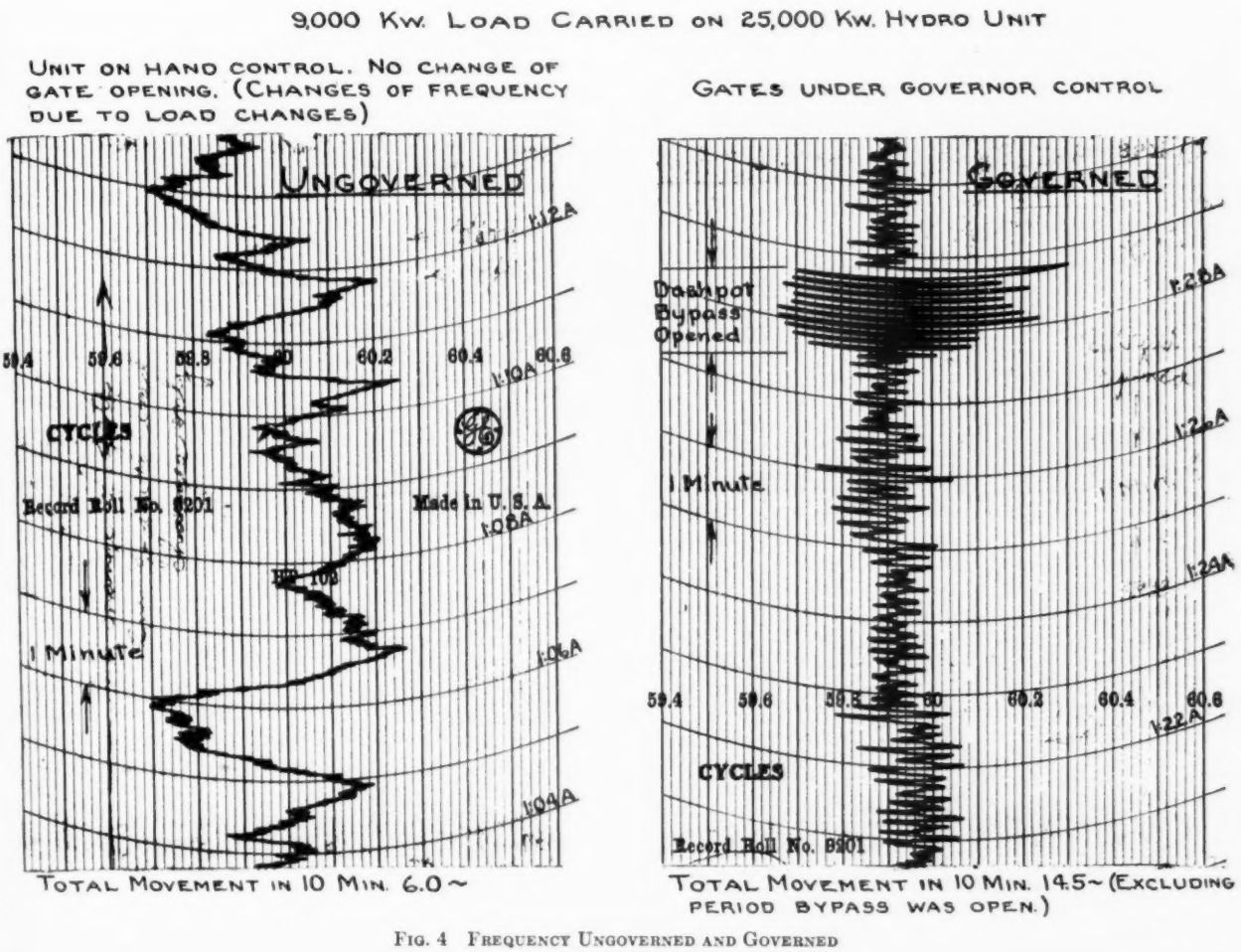


FIG. 4 FREQUENCY UNGOVERNED AND GOVERNED

for the Diesel governor shown at the right of Fig. 5. Movement of the fuel-control mechanism matches system speed with time delays of only about 1 sec.

The contrast of perfection and inadequacy in steam-turbine governors has already been mentioned in connection with Fig. 2. One steam governor was sensitive to less than 0.01 cycle speed change and the greatest time lag between speed changes and valve movement was about $^1/_2$ sec. The other governor was sensitive to only 0.05 cycle and time delays ran up to 3 sec or more. The point to be emphasized here is that there are good, bad, and indifferent steam governors. Likewise there are good, bad, and indifferent hydrogovernors and Diesel governors. The poor governors need to be weeded out and the good ones cultivated.

SOME OF THE REQUIREMENTS OF SYSTEM GOVERNING

At this point the field of governing should again be clearly differentiated from the field of the frequency controller and from the field of stability problems. Automatic frequency controllers are merely automatic operators. Their field is in keeping average frequency at the proper level. Inherently, they are not designed to do the job assigned to governors. Governors have the primary job of making load adjustments quickly and with stability, i.e., with a minimum of hunting. Their field is in making adjustments within periods from perhaps $^1/_2$ sec up to 30 sec. Generally speaking, speed swings having a period of 1 sec or less should not be touched by the governors. They come within the field of electrical stability problems and pulsating loads. Just as the governor should not lap more than a short distance into the stability field, so should the controllers not lap more than a short distance into the governor field. Frequency controllers should not attempt to correct the hunting cycle of governor action nor should governors attempt to correct the hunting cycle of load pulsations.

In controlling the flow of power from their prime movers into any system, governors may be called upon to perform five more or less separate functions as follows:

1 On loss of loads up to the capacity of the unit, the governor must close the throttle promptly enough to prevent any dangerous overspeed. This requirement determines the size, capacity, and timing of the governor.

2 The governor must be able to hold the idling speed of the unit steady enough for synchronizing. This determines the amount of damping action introduced to prevent hunting.

3 The unit must work satisfactorily in parallel with other units regulating normal speed. For this condition a high degree of sensitivity is required with very prompt, though limited, response on the smallest speed changes.

4 On the wider and slower changes of speed that are due to changes of load, the governor should bring on its full quota of load. For such speed changes the governor should bring on more load, proportionately, than it does on the short period governing swings.

5 The governor may be required to bring its unit on as reserve from the no-load idling position, up to partial or full load in a few seconds on speed changes as small as 1 cycle or less.

To attain all these objectives, many compromises in the ad-

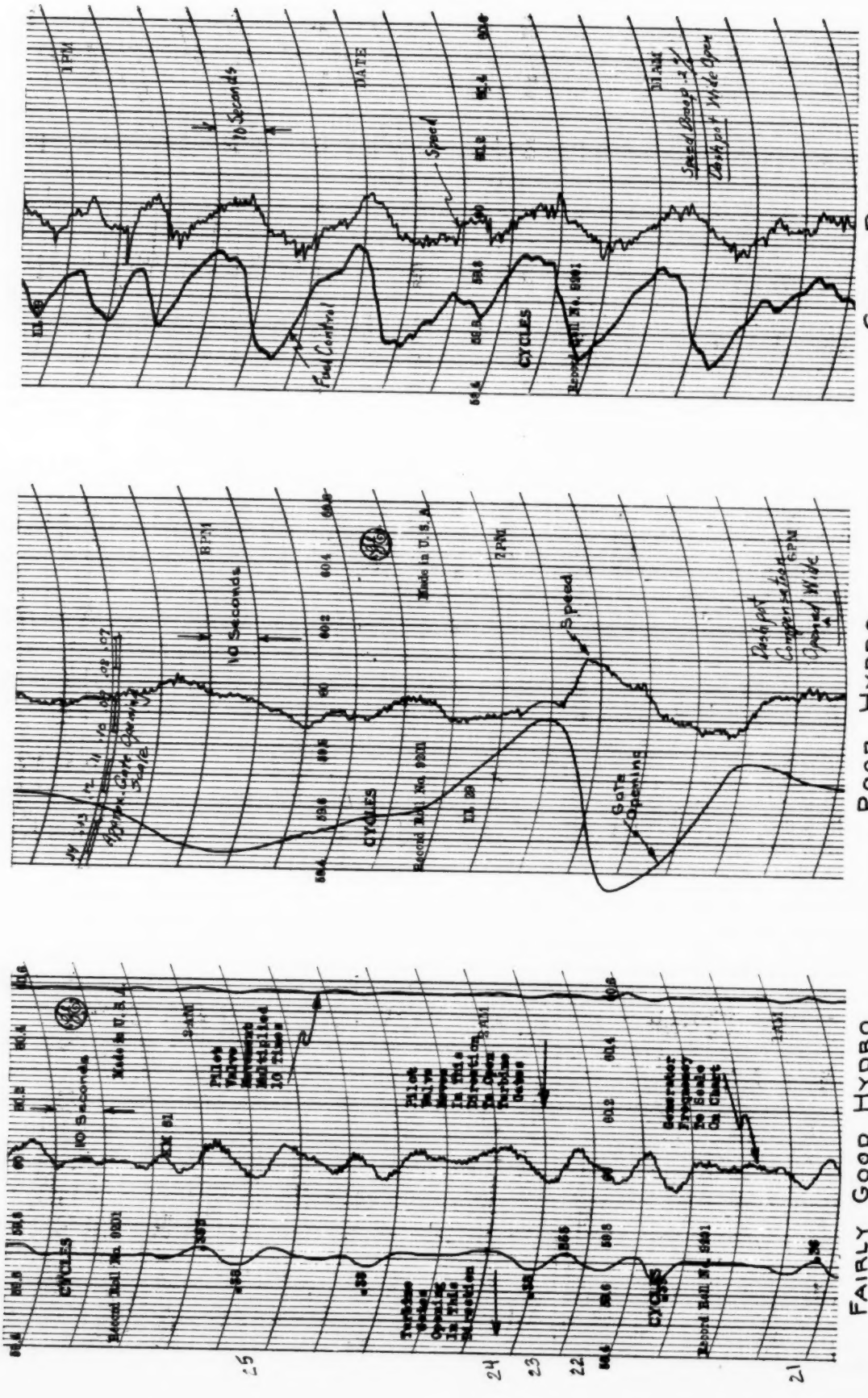


FIG. 5 HIGH-SPEED CHART RECORDS OF INDIVIDUAL GOVERNOR PERFORMANCE

justment and settings of the governors must be made. Often the attainment of one objective precludes the attainment of another. But fundamental to the attainment of all these objectives is built-in sensitivity of the governor—its intelligence, so to speak. Yet the responses resulting from this sensitivity must be properly controlled and adapted to the responses which other governors on the system are making.

Most prime movers utilize oil under pressure for the power to move the control gates or valves. Power is applied through a "regulating valve" controlling the flow of oil into an "operating cylinder" or servomotor. The governing element is actually applied to the regulating valve. This seems to be true of all governors regardless of the type of speed-responsive element used and regardless of the mechanism employed beyond the speed-responsive element.

The governing mechanism consists of three parts, the governor head or speed-responsive element, the regulating valve, and the operating cylinder. As used in this paper, the term "governor performance" applies to the response of the operating-cylinder piston, not to the load response. Although affecting system regulation to a limited extent, the relation between piston movement and power output is here considered a matter of turbine design and, as such, beyond the scope of this paper.

In order to present in more tangible form some of the requirements which might lead to better governing on our systems, the author proposes the following six items as some of the essentials to include in the specifications for performance of new governors on important system prime movers. These items refer to the performance of the governor, not to the design of the governor. Their purpose is to secure a governor sufficiently sensitive that it can perform with stability the five functions which have been listed, as well as to secure a governor which the operating department may readily adjust to the day-by-day requirements of the load.

1 A sensitivity of 0.01 per cent. On a speed change as small as 0.006 cycle (on a 60-cycle system), the governor will pass oil into the operating cylinder in the direction to produce a corrective movement of the control valves or gates.

This is a measure of the precision of the governor head up to the ports of the valve controlling oil flow to the operating cylinder. It is a measure of its ability to detect small changes of speed and translate them into positive movements of the relay valve.

2 Assuming that over the normal operating range the kilowatt output of the unit varies approximately in direct proportion to the movement of the operating-cylinder piston, then the travel of the piston per unit of speed change should be made the same over the full range from zero to full opening, regardless of the initial load setting of the turbine.

This is to obtain a reasonably simple, direct relation between speed change and average load change for the system operator's convenience in assigning normal speed regulation and reserves for maintaining speed in emergencies. On many of the present steam-turbine governors, piston movement per unit of speed change varies over a considerable range. At low loads the movement may be 4 or 5 times what it is at high loads.

3 Travel of the operating-cylinder piston per unit of instantaneous speed change or the "speed droop" of the governor should be made accurately adjustable by hand without the aid of tools while the unit is running. There should be an accurate dial indication of this adjustment. This speed droop of the governor should be made adjustable from zero to 10 per cent of the full travel of the operating-cylinder piston, "speed droop" being defined as the change in speed in per cent of normal required for the governor to move the operating-cylinder piston from the closed to the full-open position.

This is to provide the means of setting the governor at any time for the performance best suited to the system load conditions at the time, and permit convenient testing of the governor for system performance.

4 The governor should be capable of stable operation, either alone or in parallel with other units, when set on a speed droop of 0.5 per cent.

The amount of speed droop required for stable operation is a rough measure of the composite over-all effect of lost motion, friction, valve lap, any inadequacies of port design, slow operating-cylinder response, inertia, and time lag. These factors must be small to meet a specification of stable operation on low speed droop.

5 The governor should be provided with a load-limiting device which is adjustable by hand, without the aid of tools while the unit is running. There should be a dial indication of load and load limit on one scale calibrated in tenths and hundredths of the full travel of the operating-cylinder piston.

This is to permit testing system effects with the governor on or off regulation, yet with the governor left free to drop load on an overspeed. It is also an aid to economic system operation. Many conditions arise when it is decidedly advantageous to operate units on block load.

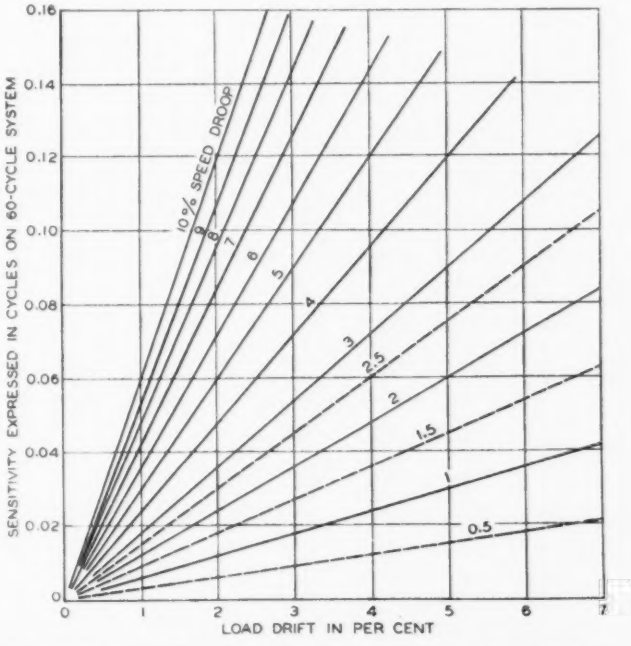


FIG. 6 RELATION BETWEEN GOVERNOR SENSITIVITY, SPEED DROOP, AND LOAD DRIFT

6 The governor should be equipped so that the unit may be synchronized from the switchboard room at any system speed from 5 per cent above to 20 per cent below normal speed, or from 63 to 48 cycles in the case of a 60-cycle system.

This is to make it possible for the switchboard operator to get available generating capacity on the line quickly when he needs it the most. Without this provision, in the event of an unforeseen system split and frequency below the range of the governor, he must either wait until the unit can be synchronized by hand from the governor floor or he must drop customers' load to bring the speed back to the point where he has control of the unit from the switchboard.

With a few governors meeting the above specifications distributed over the system, the load dispatcher has the means at

hand for smoothing out plant load swings as well as tie-line load swings. A few good governors combined with the intelligent scheduling of speed-droop settings by the system load dispatcher would iron out a lot of wrinkles in the load curve and would eliminate much of the seesawing which now goes on between the prime movers on the one end and the load on the other.

Fig. 6 is of interest in connection with the problem of limiting the wanderings of poor governors. It shows the relationship between load drift, sensitivity, and speed droop. For a governor of a given sensitivity, the amount of load drift may be reduced by increasing the speed-droop setting of the governor. The amount of load drift will decrease directly with increase of speed droop. Generally speaking, governors of high sensitivity will be given a greater burden of regulation by lower speed-droop settings, and governors of low sensitivity will be curtailed by higher speed-droop settings.

Work of the Operating Cylinder

The work which the operating cylinders have to do has an important bearing on the speed and accuracy with which the various

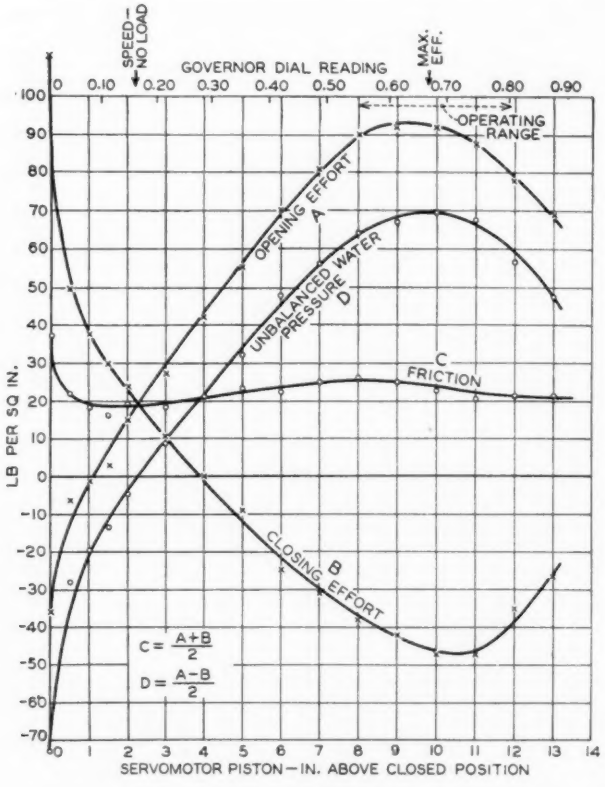


Fig. 7 Unit No. 4 Governing Effort
(25,000-kw hydrounit at 95-ft head; net piston area, 848 sq in.)

settings called for by the governor head are accomplished. The positioning of the regulating valve may be very accurate, and yet, if the porting of the regulating valve, the proportioning of the operating cylinder, and oil pressures are not suited to the effort required to move the gates, large factors of time delay may occur and considerable hunting result.

Fig. 7 gives the result of governor-effort tests made recently on a 25,000-kw, 95-ft-head hydrounit. Tests on this unit may serve to demonstrate elements of governing which are present in any type of system governor. Tests were made in the manner demonstrated by E. D. Searing in an article published in 1908.

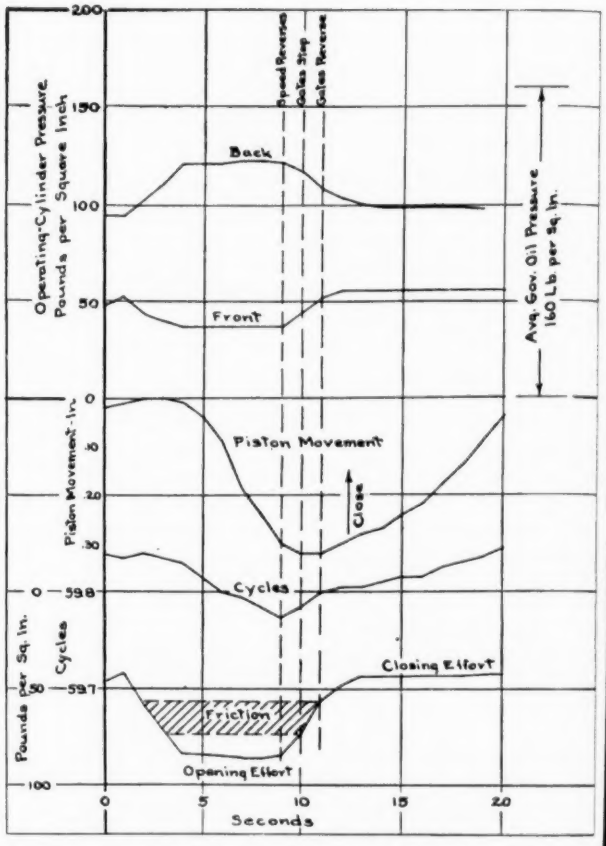


Fig. 8 Operating-Cylinder Pressures and Piston Movement During Normal Governing Cycle, Unit No. 4 Governor

Pressure indicator cards were taken on both ends of the operating cylinders throughout the full range from closed to full-open gate. Unbalanced water pressure and friction were figured from the differences between opening and closing effort. While the effort to overcome friction is practically a constant quantity, the unbalanced water pressure on the gates varies over a wide range. Variations of cylinder pressures, of course, modify the pressure drops available through the ports of the regulating valve and in turn affect the velocity of flow of oil to the operating cylinder.

Fig. 8 shows the variation of operating-cylinder pressures and of piston movements when the governor is responding to a normal system-speed swing. The sequence of events at the time of a speed reversal is of interest. One second after the speed reversed, the differential pressure on the operating-cylinder piston dropped from 85 to 73 lb and the piston movement stopped. In the next second, the differential pressure had fallen to 56 lb and the piston movement started in the reverse, or closing direction. In another 2 sec or less the differential pressure had fallen to 44 lb as the gates continued to close. While the sequence of events in this particular governor is somewhat slow, Fig. 8 demonstrates rather well the changes which occur, and the appreciable time required for oil to pass the ports of the regulating valve and move the gates, in a case where the change of speed is slight.

Fig. 9 shows what happens in the operating cylinder of the same 25,000-kw hydrounit when wide and rapid movements of the gates are called for. For this particular test, the gate opening was swung quickly by hand control from 0.72 gate up to 0.73 gate, then down to 0.55, back to 0.73, and down again to 0.72 gate, all within a period of 6 sec. The maximum rate of gate movement on this test corresponds to a full gate movement in 8 sec. For

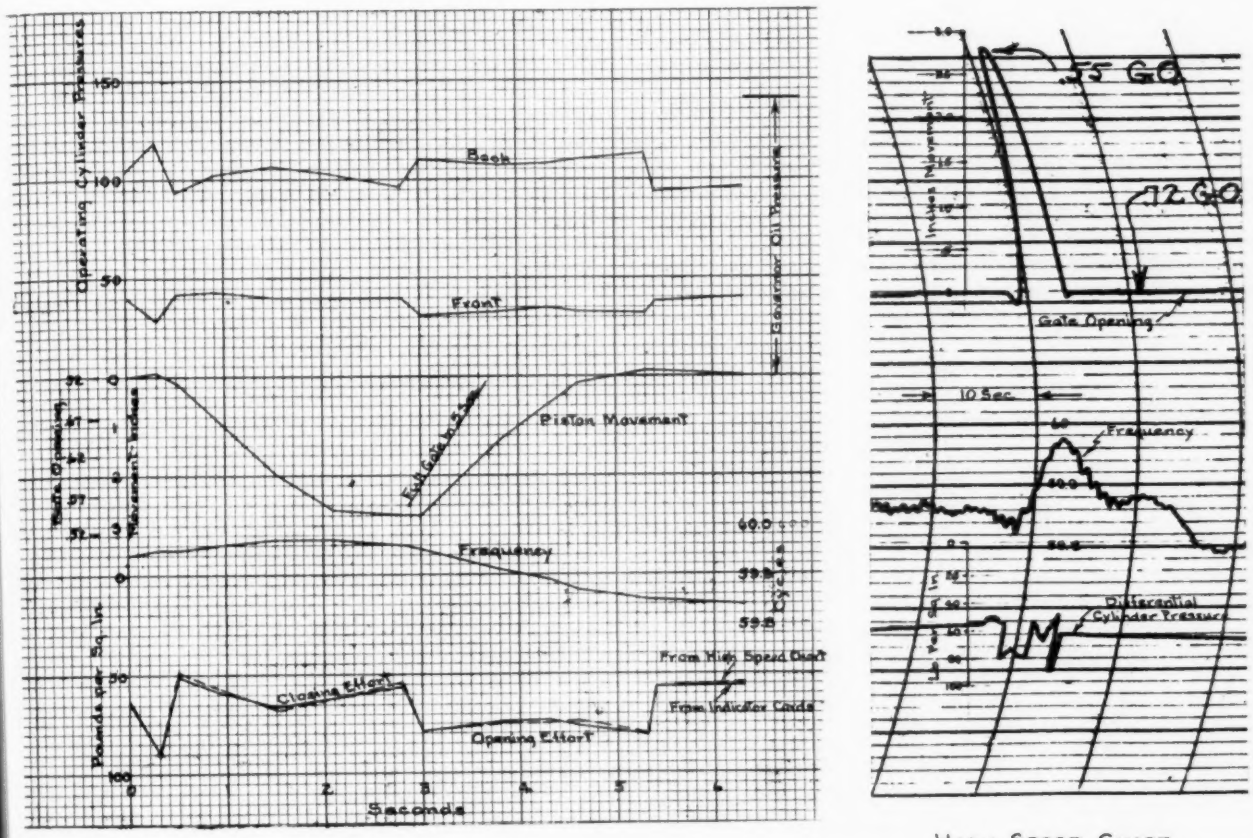


FIG. 9 OPERATING-CYLINDER PRESSURES AND PISTON MOVEMENT DURING RAPID GATE MOVEMENT, UNIT NO. 4 GOVERNOR

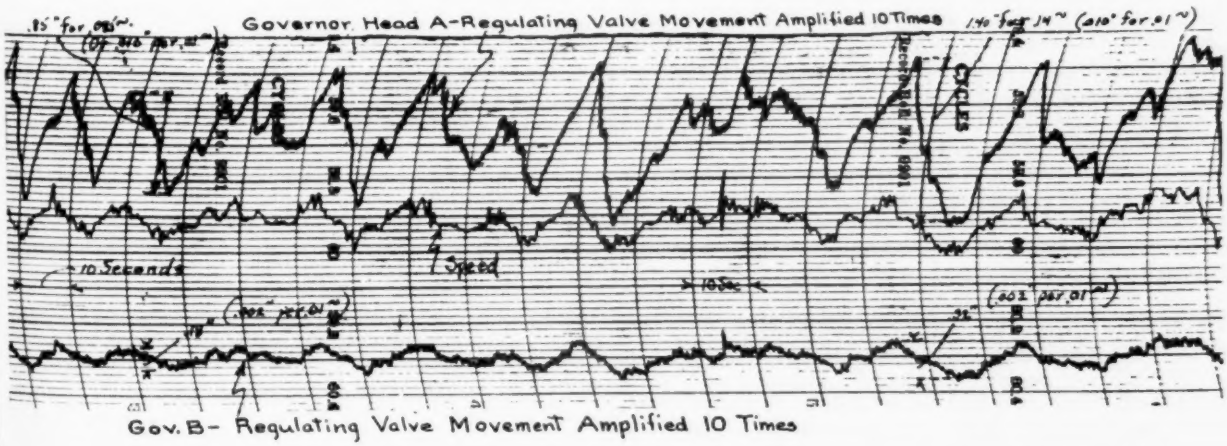


FIG. 10 COMPARISON OF MOVEMENT OF REGULATING VALVES, SHOWING PRECISION POSSIBLE IN SENSITIVE GOVERNOR HEAD

this test, steam-engine indicators were connected to the front and back of one of the operating cylinders, and indicator cards taken for the complete test cycle. The indicators were arranged so that the difference in pressure between the front and back of the cylinder could be recorded on the high-speed frequency chart, simultaneously with system speed and a record of gate movement. Differential pressures are recorded on the frequency chart at the right of Fig. 9.

For rapid movements such as this, there is a momentary inertia effect. On a quick opening the differential pressure swings up for an instant above that required for a normal slow opening movement. After this momentary increase, there is a marked decrease of pressure as the rate of gate movement increases, due to the decrease of penstock pressure and draft-tube vacuum. The opposite effect occurs when the gates start closing. Except for momentary gate inertia effects at the instant of starting a gate movement, there is a marked decrease in the differential pressure required to move the gates at the rapid rates.

These tests are cited to show that the work to be done by the operating cylinder varies over quite a range, which means that pressures across the ports of the regulating valve vary for different gate positions, thereby changing the rate of flow of oil into

the operating cylinder, and its rate of response. The rate of movement of the cylinder piston will vary not only with the port opening but also with the effort required to move the gates.

Control of Regulating Valve by the Governor Head

To make a large regulating valve move in exact synchronism with changes of speed calls not only for mechanical ingenuity but a high degree of manufacturing precision. The amount of movement which can be produced in the regulating valve without measurable error and without time delay measures the skill of the governor manufacturer and grades his product. It is, in effect, another measure of the sensitivity of the governor head.

To give some idea of the precision possible in the movement of the regulating valve, the high-speed chart in Fig. 10 is included. The center graph is speed and the graphs at the top and bottom are regulating-valve movements of two governors amplified 10 times. The governor at the top is producing accurate movements of the regulating valve of as much as 0.01 in. for each 0.01-cycle speed change. The regulating valve is reflecting speed changes accurately and without measurable time delay. In fact, there might be a question as to which was measuring the system speed more accurately, the sensitive frequency meter or the regulating valve. It should be noted that, in this particular instance, the movements were produced mechanically in a 6-in. relay valve. To the author, this is a truly remarkable accomplishment for any governor head. But what is done with these accurate movements of the relay valve in converting them into motion of the operating cylinder is a third phase of the governor story.

Factors Affecting Design of the Ports of the Regulating Valve

Suiting the design of the ports of the regulating valve to system requirements involves so many interrelated factors that it is difficult to say just what features are best. Yet improper features of design can spoil many good features. There are square ports, round ports, thin rectangular ports, vertical ports. Some regulating valves are of large diameter with small axial movement, some of small diameter with large axial movement. There is always the question of how much lap to allow. Should the valve admit oil to the operating cylinder on the very small speed changes and, if so, how much? All of these factors influence the responses of the operating cylinder. But to what extent?

To arrive at some sort of an answer to a few of these questions, a study was made of a governor for the 25,000-kw unit on which the tests were made. The study, which was carried on entirely by computation, was confined to operation of the governor from the regulating valve to gate movements, on the assumption that the regulating valve could be made to follow system speed exactly and with negligible time lag. Various ratios of movement of the regulating valve, ranging from 0.002 in. up to 0.016 in. per 0.01-cycle speed change were used. By first analyzing one factor at a time, methods were finally devised by means of which almost any governor action could be computed and analyzed.

Fig. 11 shows the results of some of these computations for a governor on the 25,000-kw hydrounit. The effect of speed droop was first studied with the governor following the normal system-frequency cycle shown at the top of Fig. 11. This frequency cycle has an amplitude of 0.14 cycle and a period of 20 sec. Using square ports and no compensation, governor action was found to lag a little less than 1 sec behind the speed when on 5 per cent speed droop and 2 sec behind the speed when on 2 per cent speed droop.

Response of the servomotor piston on 2 per cent speed droop is shown in the center group of curves in Fig. 11. Comparison with the curve marked "ideal governor response" will show the time lag. Note that the governor-response curve crests at the point where it crosses the ideal curve. Inherently, the lag of gate movement decreases as speed droop is increased. Doubling the speed droop results in half the time lag. Time lag governs overtravel and the amount of hunting.

Next, the effect of various port arrangements was studied. As shown by the curves marked "square," "round," and "A" ports, it is possible to change the amount of load pickup by changing the port design. It is also possible, by varying the size and

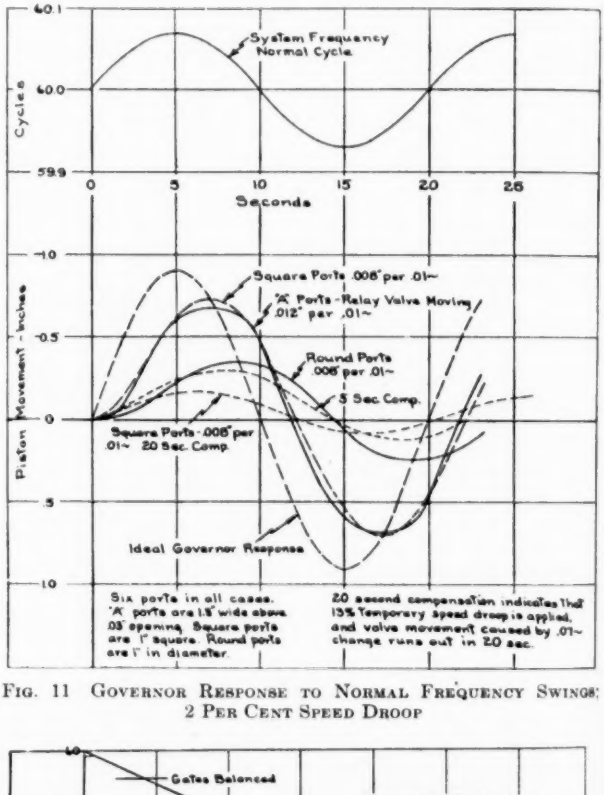


Fig. 11 Governor Response to Normal Frequency Swings; 2 Per Cent Speed Droop

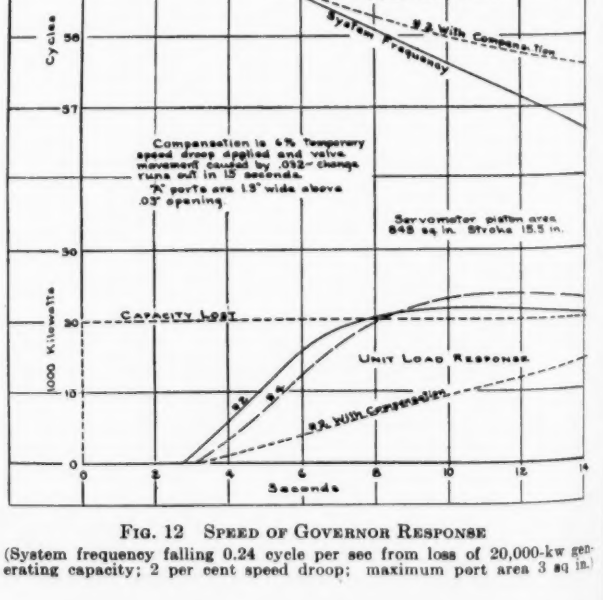


Fig. 12 Speed of Governor Response

(System frequency falling 0.24 cycle per sec from loss of 20,000-kw generating capacity; 2 per cent speed droop; maximum port area 3 sq in.)

shape of the valve ports, to control the time lag of gate movement behind speed. Square ports show much less time lag than do round ports of the same area. By using wider ports, the time lag is cut down further. Doubling the width of the ports results in half the time lag. Similarly doubling the movement of the regulating valve for a given speed change will give half the time lag. Doubling the oil pressure also will approximately halve the time lag. All this is true, if the restoring mechanism functions without lost motion. However, if the initial piston movement is made too rapid, lost motion in the restoring mechanism may leave the governor uncontrolled and "jumpy" or unstable on the small speed swings, particularly on the transient swings.

The A ports investigated are 1/3 wider than square ports above 0.03 in. opening and only about 1/4 the width of square ports below that opening. They were found to yield about the same time lag as square ports for the normal 10- to 20-sec speed swings which occur on our system. At the same time, they afford the advantage of slower movements on the very small speed changes, giving stability in the field of transient pulsations without sacrificing sensitivity to the slow swings of speed due to load changes.

Finally, the effect of adding compensation was studied. Compensation has the temporary effect of a large speed droop that is slowly removed as dashpot action runs out. With compensation, time lag can be reduced to a yet smaller amount and, at the same time, the amount of load picked up on normal speed swings can also be reduced to a figure more nearly comparable with that needed to correct the speed change, thus increasing the stability of the governor, provided there is no lost motion in the restoring mechanism and provided the timing of the dashpot action is suited to the speed swings which the governor is trying to correct. The adjustment of the dashpot is important because, while the best timing will reduce the time lag, short timing may increase the time lag seriously. Witness the two curves comparing 5- and 20-sec timing of compensation, shown in Fig. 11. Five-second timing showed a lag of 3 sec, while the 20-sec timing showed a lag of only 1.5 sec.

Fig. 12 shows the action of the governor on the system in an emergency with frequency falling 0.24 cycle per sec from the loss of 20,000 kw of generating capacity. The No. 2 governor with compensation will pick up load very slowly as shown, and the speed will drop to around 57 cycles before it is checked. Without compensation, the frequency is checked at 58.82 cycles within 6 sec after the gates begin to open. It was very apparent that to function properly as reserve capacity, compensation should be removed by some means when the speed falls below about 59.6 cycles. The line marked 4 shows the computed performance of another regulating valve with wide ports and a small movement of the regulating valve per unit of speed change.

Square ports obviously give quicker action than round ports, particularly at the smaller port openings. Apparently the advantage prompting the use of round ports is in making the governor slower and more stable on the small speed changes. But this slowness introduces more time lag and overtravel on the type of governing swings which occur on our system. Then too, round ports are a distinct loss in speed of operation when the governor is handling the gates for system reserve.

The studies made so far suggest the use of ports which have essentially the characteristics of round ports up to plus or minus 0.02 cycle speed change, the range within which transient pulsations occur. Beyond this point, it would appear that the ports should be rectangular and as wide as practicable, to reduce time lag and give quicker response in bringing on load as reserve. It also appears highly desirable to utilize the maximum movement of the regulating valve that can be obtained without measurable error or time delay, and with it, the smallest diameter of valve that will yield the required port area with the given valve movement.

It would seem that the best answer in a governor is a combination of high sensitivity translated into movement of the regulating valve, careful porting of the regulating valve to suit the location on the system, compensation to help reduce time lag, and the lowest speed-droop setting that can be used without hunting.

To this discussion, one important comment should be added. In speaking of system requirements and matters of design, we have tacitly assumed the governors to be in good mechanical condition. This is not always the case. It should be remembered that poor performance of a governor may be due entirely to lack of intelligent maintenance of the delicate parts of its mechanism, rather than to defects of design. The best governor this year may be the poorest one 5 years hence, if parts have become worn and lost motion has developed. Keeping governors up to a system standard of mechanical perfection is a very essential part of the problem of good governing.

It is realized that many of the foregoing statements are somewhat far afield. Yet such seemingly remote considerations have deep roots in system economy. The information is presented as suggestive of what may be accomplished by analysis and study. From what has been observed, there is plenty of room for investigation and improvement in the field of system governing.

Acknowledgments

In closing, the author desires to acknowledge his indebtedness to R. A. Garrett and to G. J. Vencill for much of the background material of this paper. Mr. Garrett made and analyzed most of the tests on individual governors. Mr. Vencill collaborated in the making of system tests, in developing the method of governor analysis by computation, and assisted in the preparation of this paper.

Discussion

P. W. Keppler.[2] The A.S.M.E. Committee on Industrial Instruments and Regulators (here abbreviated C.I.I.R.) has for several years sponsored study of automatic regulation. It has also accomplished much in standardizing terminology. It is hoped that a way may be found for this very active committee to give assistance to the excellent efforts of the author. This discussion is intended to help bring about such coordination of effort. First, however, it is necessary to clarify the terminology used in this discussion.

"Droop" is a well-understood term and is equivalent to "throttling range" of the C.I.I.R. terminology.

The conventional central-station steam governor without automatic frequency control is a "proportional" regulator (C.I.I.R. terminology). Its characteristics are conveniently made clear by imagining 2 jumps in frequency as shown in Fig. 13. These result in changes of governor position proportional to the changes in frequency curve No. 1, Fig. 13.

Next in importance is the "proportional speed floating" principle, sometimes called "automatic reset" (C.I.I.R. terminology). Automatic frequency controllers are based on this principle. It is illustrated by curve No. 2, Fig. 13. Rate of change of governor position is proportional to the total deviation of frequency, as shown.

The combination of these two most important principles gives the proportional plus proportional speed floating regulator, illustrated by curve No. 3, Fig. 13. This type of regulation, found in every steam central station if automatic frequency control is used by the system, has proved most successful in almost

[2] Assistant Engineer of Power Plants, Consolidated Edison Company of New York, New York, N. Y. Jun. A.S.M.E.

every field of automatic regulation, and basically all control problems are closely related.

The proportional speed floating regulator by itself is hardly suitable for frequency regulation, and need not be further discussed.

Before discussing the characteristics of the other two types, it

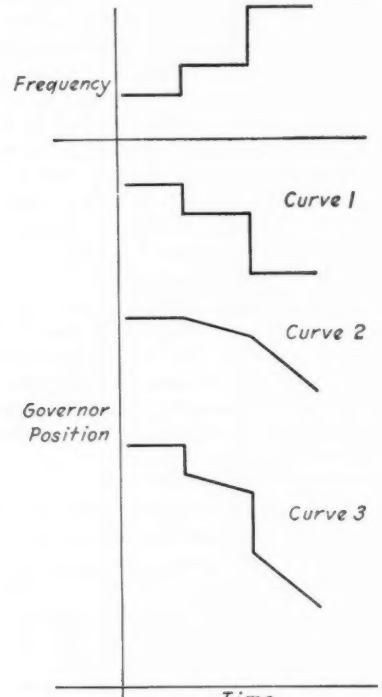


FIG. 13 MOTION OF THREE MOST IMPORTANT TYPES OF REGULATORS

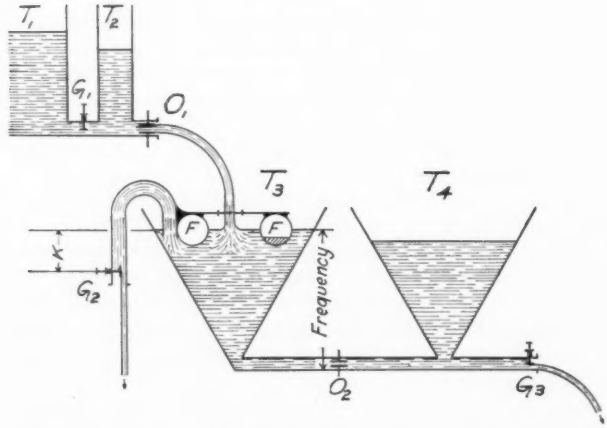


FIG. 14 HYDRAULIC ANALOGY OF ENERGY FLOWS TAKING PLACE DURING MODERATE LOAD CHANGE IN AN ELECTRICAL SYSTEM
Width of each tank is uniform

appears worth while to clarify the control problem by hydraulic analogy similar to that sponsored with great success by the C.I.I.R. In Fig. 14, tank T_1 represents energy stored in the boiler room and held at nearly constant level (constant steam pressure). This energy flows through the governor valves G_1. Straight-line or viscous flow is generally assumed where velocity is directly proportional to head, because it is sufficiently accurate for moderate changes and greatly simplifies visualizing and analysis. Energy cannot flow directly to tank T_3 which represents kinetic energy of the system and into which flows steam energy and from which flows electrical energy. First, the pressure and heat-energy content of the turbine must be raised before the increase in power can fully flow out of the coupling. Slowness or inaccuracy of the governor in obeying its own characteristic has the same effect. These lags are represented by tank T_2, Fig. 14. Time is lacking to determine if one tank is enough for representing this lag. Theoretically, every turbine stage should have a tank. Fortunately, where many small lag capacities occur in series, they may generally be combined into a few, and in our case even one tank is likely to be quite accurate. Tank T_2 cascades through the fixed orifice O_1 into tank T_3, the head of which is related to frequency. Cascading is here used because this frequency head does not appreciably affect energy flow out of T_2. In other words, with a blocked governor, the total useful work done by the steam is not likely to change with frequency.

T_3 represents the kinetic energy of all synchronous moving parts of the system, driving or driven, including machinery. Its head is frequency. Energy flows out at gate G_2 operated by the

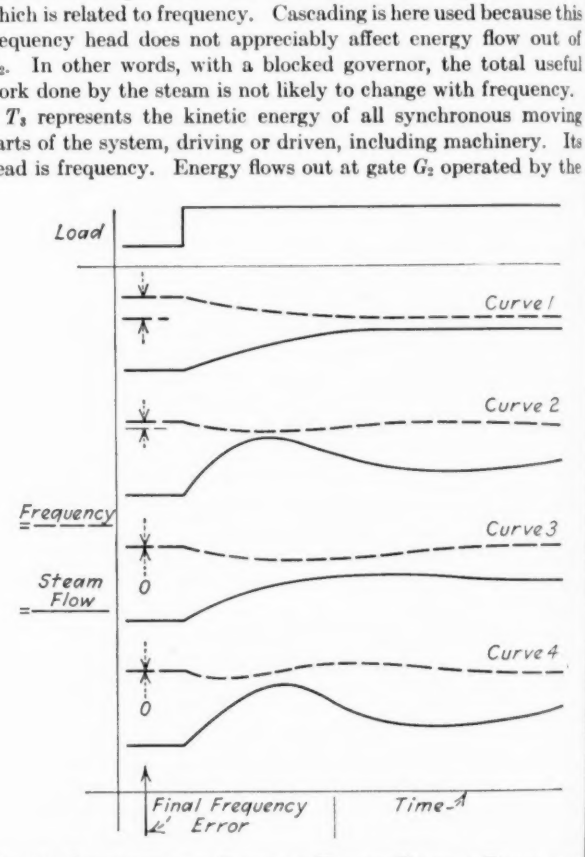


FIG. 15 CHARACTERISTIC CURVES OF VARIOUS TYPES OF REGULATION FOR AN ASSUMED JUMP IN LOAD

consumer, representing all output except a-c motors. This output is normally independent of frequency due to close voltage regulation. The float device F is therefore shown giving constant head K. Energy also flows through orifice O_2 into tank T_4 and finally leaves through gate G_3, again operated by the consumer. Tank T_4 represents the kinetic energy of induction motors and their driven machinery. Orifice O_2 is determined by slip characteristics of these motors. Both T_3 and T_4 vary in size but this may be assumed equal to the average value for any given moderate change of load. Their width is uniform so that, due to sloping sides, kinetic energy varies with the square of frequency. Fig. 14 represents the energy flows taking place during a moderate load change in an electrical system. Tank T_2 is an interference and should be kept small. The governor should respond quickly and accurately according to its proportional characteristic and the turbine should change its content of heat-and-pressure energy as little as possible with load. If both T_2 and T_4 could be

eliminated, then regulation would be ideally simple. But, while T_2 may be reduced, T_4 and O_2 can hardly be changed appreciably, and must be taken into account. It is entirely likely that mathematical analysis based on this analog is feasible and would result in optimum settings of governors for any given T_2, T_3, T_4, O_1, and O_2. At any rate, this analogy serves to clarify the problem. Of course the problem of governing, assuming this analog to be sufficiently accurate, is that of maintaining the level in tank T_3 as constant as feasible by adjusting gate G_1 smoothly, while G_2 and G_3 are being varied by the consumer.

The nature of the characteristics of various types of regulation for a "process" such as this is shown in Fig. 15, for an assumed jump in load. Curves No. 1 represent a "proportional" governor with sufficient droop and curves No. 2 with insufficient droop. Curves No. 2 are of course undesirable for the author's purposes of smooth regulation, due to excessive overtravel.

Curves Nos. 3 and 4, Fig. 15, illustrate the proportional plus proportional speed floating characteristics; curves No. 3 with optimum proportional characteristic and optimum floating speed; curves No. 4 without these. Curves No. 4 are again undesirable.

The "compensated" governor mentioned by the author is closely related to the foregoing one except that frequency is not brought back to its exact value but some residual droop is left in it. One advantage of this type is that any number of independent floating "heads" may be operated in parallel, although there is danger of some hunting. With true proportional speed floating, only one independent "head" device appears feasible, because with two heads in parallel, one would tend to go to full load, the other to zero load. Compensated governors should also be more stable for the same initial droop and floating speed settings, but they require manual or automatic elimination of the residual droop.

For steam central stations, the adoption of this type appears an unnecessary complication at least for the present. The "proportional" governor can be made very stable and smooth; its phase displacement with frequency ("time lag") may readily be made negligible, as witnessed by the author. If something better than a good proportional governor is desired, then careful application of automatic frequency regulation, giving zero final droop and eliminating manual interference, would appear a better step forward. Especially if load dispatchers were to change vital governor settings such as droop of "compensated" governors according to the load, as the author recommends, then regulation like curve No. 4, Fig. 15 would be likely to result.

The proportional plus proportional speed floating principle has proved satisfactory. Good regulator design plus careful setting is likely to produce accurate and smooth regulation.

Many steam governors are crude due to poor design or poor maintenance and this should undoubtedly be improved. But care should be used in writing specifications so that only essential values are asked for.

First, there should be a straight-line relationship between droop and output over the operating range as the author recommends. Otherwise, neither the governor nor the automatic frequency control can be given the optimum setting. To facilitate synchronizing, the droop at extremely low outputs may have to be made greater than that used in the operating range.

Second, there should be a sensitive "head," a minimum of lost motion, amply large oil pumps and pilot valves, rectangular ports without lap, in short the governor should come as close to its ideal (proportional) characteristic as possible. To make governors independent of harmonic-frequency pulsations is likely to affect their accuracy of motion and it would seem better to let them oscillate.

With improved governors, the optimum initial droop would become less, and optimum reset speed of frequency controllers would increase, resulting in improved accuracy and smoothness of regulation. Even with existing governors, however, these can be made quite good.

The writer wishes to commend the author, particularly for his excellent test work, and again express the hope that, by joint effort with the Committee on Industrial Instruments and Regulators, a substantial improvement in regulation may be brought about.

Regulation of System Load and Frequency

By HERBERT ESTRADA[1] and H. A. DRYAR,[2] PHILADELPHIA, PA.

This paper discusses the practices of the Philadelphia Electric Company system in the regulation of system load and frequency, including the scheduling and actual operation of generating capacity with regard to reliability and economy. A summary of the principles of economical load allocations on both steam and hydrogenerating stations is presented. The effect of changes in system load or generation on frequency, the self-regulation characteristic of the system, and the combined system governor action are discussed from the standpoint of the individual and the interconnected group of systems. The practices of the interconnection, of which the Philadelphia Electric Company is a member, in manually regulating the frequency and the incident problems thereto are considered. A discussion of governor instantaneous and average speed regulation, sensitivity, and response is given, with particular reference to their effect on frequency regulation.

THE problem of system regulation in its many phases has seemed to vary with the ever-changing operating conditions; it is probable in each instance that we have been satisfied with a partial solution of the existing problem. We may find the reason for such oversight, however, in the many other operating problems that were more persistently demanding a remedy.

Isolated Generating-Station Operation. When the system was supplied by a single generating station, the changes in load on the plateau of the curve constituted a large percentage of the total demand and, as a result, variations in the instantaneous frequency were such that industrial customers, particularly textile and paper mills, were subject to damage to their product. The average frequency was such that the production schedule of industrial plants, in quantity and quality, frequently could not be maintained. Frequency-measuring instruments of that day, however, were of such a broad scale that close average regulation was manually difficult.

Parallel Operation of Generating Stations. With the parallel operation of a number of generating stations, dependence on system inertia and governor response for system regulation seemed to provide a satisfactory solution. With this method, manual control was assigned to a single regulating station, the other generating stations maintaining their assigned output as an average loading, permitting the load to vary as the turbine governors responded to changes in speed. Aiding in the improvement of the system regulation was the diversity in the load swings which made the system load fringe a smaller percentage of the total load although larger in absolute magnitude.

Interconnected Operation of Systems. When interconnections were established and the problem of stability made its appearance, the settings of turbine governors were further damped and, in many instances, automatic frequency control was installed which tended to perform the normal function of the governors. If the tolerance of the controller was small, load variations on the regulating station were excessive and the station economy suffered. Electrical time service, however, was considerably improved and this was instrumental in the improvement of this type of service on systems having manual control.

Systems that favored that type of manual control, wherein dependence was placed upon turbine-governor response and the inertia of rotating equipment, to limit variations in frequency and to avoid burdensome changes in load on the regulating station, regarded the frequency indication as a guide in load regulation.

Limited Knowledge of Problems of System Regulation. In all these phases of system regulation, it would appear, as has been charged, there was a lack of sustained interest in this general subject, an interest that would apply itself to a complete solution of the problem rather than confining itself to a study of an existing phase and seeking a remedy for that.

Operating men favoring automatic frequency control as a remedy, regarded a narrow frequency band as essential as closely regulated voltage. Operating men who held to manual control of frequency objected to abnormal changes in load on a regulating station. The generating-station operating force was concerned primarily with economical performance. What ills there were in system regulation were regarded as inherent and system dispatchers "worried along" with them, striving to hold the frequency within reason and changes in station output to a minimum. The advent of interconnections may be said to have further postponed efforts looking to a solution, as continued interest in the problem was manifested usually by but one company of the group.

Scope of Study of System Regulation. In a study of system regulation, it is necessary to consider each of the following factors:

1 The scheduling of generating capacity
2 The actual operation of generating capacity
3 Load control
4 Frequency control
5 Governor characteristics.

Scheduling Generating Capacity

The two principal functions in the operation of an electric-power system, that of providing reliable service and attaining the highest efficiency of performance, demand that adequate generating capacity be scheduled for operation, based on the estimate of the system load, on the sum of the estimated loads of each area of the system, and the available transmission capacity. Fig. 1 is a schematic diagram of the main generating and transmission system of the Philadelphia Electric Company, indicating the distribution of the system load and the generating capacity which may be scheduled for operation.

Load Estimating. Service reliability and efficient operating performance demand accuracy in the forecast of the system load, in order that generating capacity, adequate to supply the system load and provide the necessary operating reserve, may be economically scheduled.

In the daily estimate of the system load, it is found that there is a base load for each day and for each hour of each day, the components of which are all the factors, with the exception of those affected by weather conditions, that may be considered as influencing customer demand and contributing to the diversity of such demand. When the components of the customer demand

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[2] Chief Load Dispatcher, Philadelphia Electric Company.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

FIG. 2 SCHEMATIC DIAGRAM OF LOAD DISTRIBUTION OF PHILADELPHIA ELECTRIC COMPANY'S POWER SYSTEM

that are the result of existing weather conditions are added to the base, the sum will be the system load. In making an accurate estimate of the system load, consideration should be given to the weather with the proper weight for degree of cloudiness, wind velocity, and temperature based upon the best available forecast.

Reliability. In the scheduling of generating capacity to supply the system load, the requirement of reliability of service is the first consideration. Reliability is fostered by maintaining the availability of the necessary generating capacity to each load center of the system. The degree of availability is deemed adequate if coverage is provided for the possible loss of the generating unit of largest capacity within a load area or the possible loss of the transmission capacity rendering available to the load area the largest amount of generating capacity.

Economy. In the scheduling of generating capacity to supply the system load and provide the necessary spinning reserve, economy as a factor in the operation of the system is secondary in importance only to that of reliability of service.

Economy in the operation of the generating system is effected when the cost per kilowatthour of the energy delivered to the bus of each load center is the lowest that may be obtained. Thus it is necessary not only to calculate the generating cost per kilowatthour, but to determine also the losses in transmission.

Spinning Reserve. The spinning-reserve requirement of an interconnected system is usually specified as equivalent to the capacity of its largest generating unit. This requirement, however, is an arbitrary measure and in some instances the reserve coverage provided is equivalent to the capacity of two or more generators.

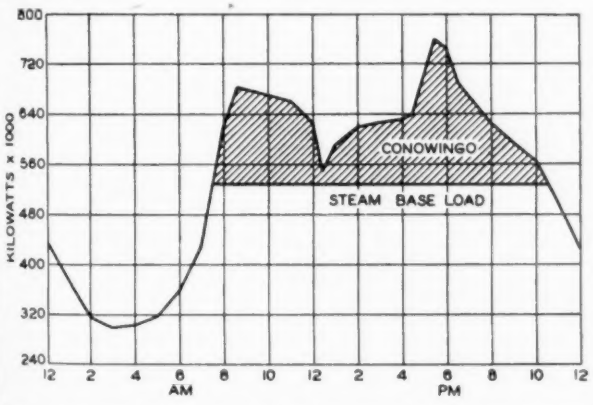


Fig. 3 Chart Showing System Daily Load With Portion Allocated to Conowingo

When systems are interconnected and the spinning reserve is pooled to provide coverage for the loss of the largest generating unit of the group, it will soon be evident that errors in the load estimates are much more frequently the cause of deficits in spinning-reserve capacity than the loss of generating capacity. Thus it has become necessary to provide spinning reserve as coverage for the loss of the largest generating unit or the possible load-estimating error, whichever is the greater.

Assuming that coverage on an interconnected group of three systems is to be provided for the largest unit or a 7 per cent load-estimating error, whichever is greater, it is seen that the reserve requirement will be based on coverage for the load-estimating error, if the capacity of the largest unit is less than 7 per cent of the load. The reserve requirement may be divided among the systems in accordance with some previously agreed upon formula.

When a system is uninterconnected, the hazard to reliability of a load-estimating error is obscured in the coverage for the largest unit of the system or in the number of units of smaller rating which provided an appreciable amount of reserve capacity above their efficient rating.

Actual Operation of Generating Capacity

Typical System Setup. To illustrate the actual operation of generating capacity on the system of the Philadelphia Electric Company, let us assume that the system load for which generating capacity has been made available is 760,000 kw, and the distribution of this load among the load centers as shown in Fig. 2 is:

Station	Load, kw
Chester	105000
Wilmington	37000
Paschall	19000
Schuylkill	163000
	324000
Delaware	87000
Richmond	145000
Westmoreland	87000
Plymouth Meeting } Barbadoes Island }	80000
Emilie	10000
Miscellaneous, including losses	27000
Total	760000

The capacity requirements to supply this system load and provide the necessary sectional and system spinning reserve would be:

Station	Capacity requirements, kw
Conowingo	245000
Richmond	255000
Delaware	60000
Schuylkill	135000
Chester	90000
Deepwater	54000
Total	839000

Coordination of Steam and Hydro. In determining how the load should be most economically allocated to the generating stations, it is necessary to know the performance characteristics of each plant in the system. However, the loading of Conowingo, a run-of-river hydro plant, supplying a large part of the system load, is determined first. Conowingo utilizes the full flow of the Susquehanna River for about 70 per cent of the year. During the remaining 30 per cent of the year, the river flows are in excess of what can be utilized by the present installed capacity of 252,000 kw. The plant generates approximately one third of the total energy output of the Philadelphia Electric Company system in a normal hydro year. It is highly important to allocate the load correctly to this station as it has much effect on the economy of the connected steam system.

Steam Base Load. Minimum steam-station costs are closely approximated by allocating to Conowingo a block of the system daily load, Fig. 3, of such proportions that the area of the block corresponds to the available energy to be generated, and the ordinate of the block corresponds to the economical hydro-plant capacity.

Referring to Fig. 3, the higher the position of shaded area, which represents the Conowingo energy output, the less will be the area of this block. The lower load limit of the shaded area, indicated as steam base load, increases as the energy output from Conowingo decreases. The steam base load may be defined as that system load above which Conowingo becomes the marginal station in system loading, taking all the variations in system load within the limits of its economical capacity. When the system load exceeds the steam base load plus the economical capacity of Conowingo, the excess must be carried as steam peak. When the system load is less than the steam base load, it is entirely carried by steam and Conowingo is shut down. By using the steam base load as the basis for loading Conowingo, advantage is taken of the Conowingo Pond in absorbing the variations which

occur in system daily outputs due to weather conditions and other factors affecting system load.

Determination of Steam Base Load. Corresponding to each value of Conowingo daily kilowatthour generation there is a definite steam base load for a particular day. This relation is utilized in determining the steam base load for each day by using the Conowingo loading chart for seven machines available at Conowingo, shown in Fig. 4. These curves determine the steam base load for a given daily Conowingo energy output and daily

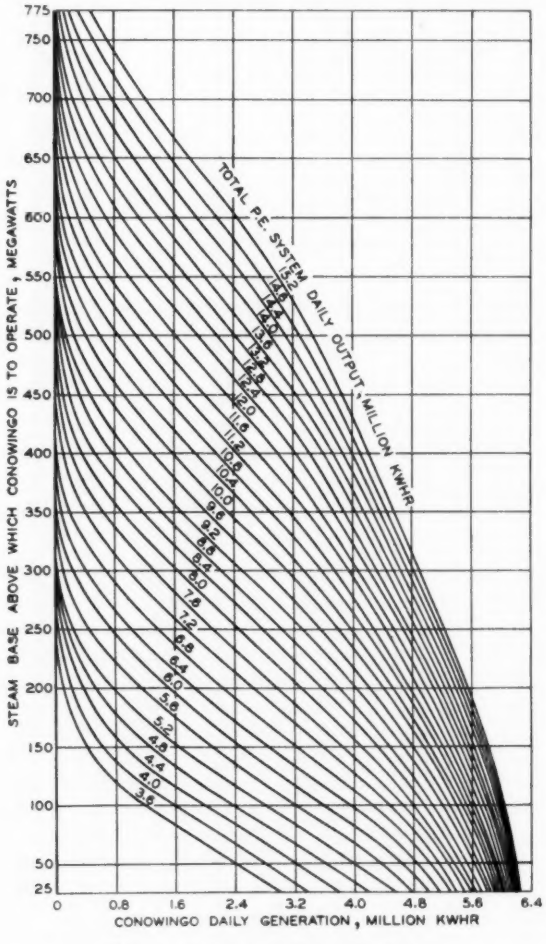


FIG. 4 CONOWINGO LOADING CHART

(Available peak prepared for = 259,000 kw plus steam base. If system peak is greater than this available peak, the steam peak prepared for must be increased to make up the difference.)

system generation. Similar charts are used if less than seven machines are available at Conowingo, one chart for each Conowingo capacity value. The curves are worked out from daily occurrence curves which show the generation in the load curve below each value of system load. The steam-base curves apply to the different seasons equally well in spite of the seasonal variations in the shapes of the daily load curve.

Variation in Hydro-Station Increment Rate. The foregoing relations have been based on substantially constant hydro-station increment rates. By hydro-station increment rate is meant the increase in station draft, or in rate of flow of water through the turbines, corresponding to a slight increase in hydro load divided by the increase in load. It can be conveniently expressed in terms of cfs per kw. Fig. 5 shows the variation in increment rate for Conowingo against station loading expressed in per cent of average hydro-station increment rate. The hydro increment rate remains practically constant up to the most efficient load for all hydro units on the line. Beyond this load, however, the hydro increment rate usually increases quite rapidly.

Economical Loading of Hydro. The principles of most economical hydro load allocations can be summarized generally as the utilization of the river flow so as to displace the maximum steam-station cost per cfs of river flow. The value of a small increment of river flow cfs is equal to the value of the corresponding small increment of hydro load Δ kw, in displacing steam generation.

The value of the river flow per cfs per hr $= \dfrac{I}{W}$

where $W = \dfrac{\Delta \text{cfs}}{\Delta \text{ kw}}$ = hydro-station increment rate

I = incremental cost of displaced steam generation

In order to load the hydro station in the overload region beyond most economical capacity, the value of $\dfrac{I}{W}$ must be equal to or higher than during the steam-base period of the day, as the total flow for the day is fixed and whatever extra water is used during

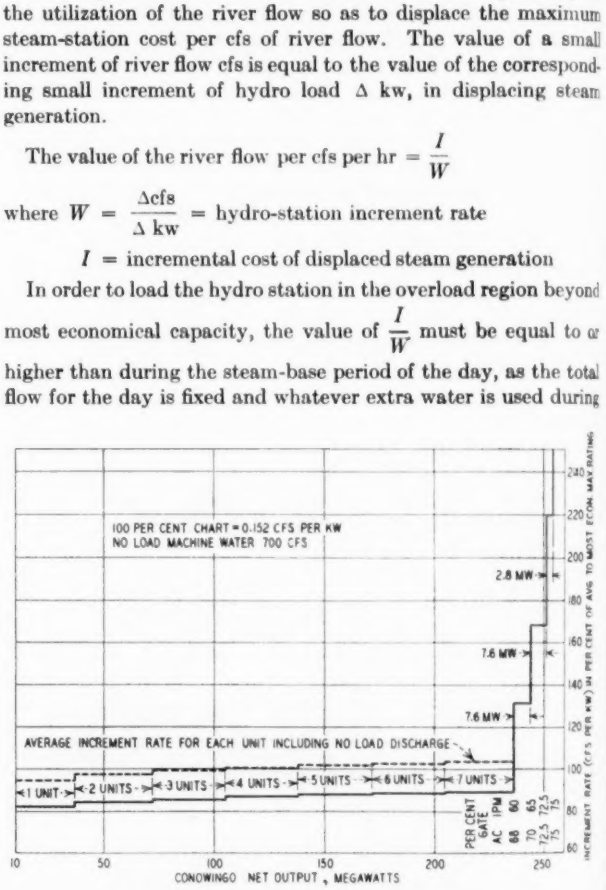


FIG. 5 CONOWINGO INCREMENT RATE

(To justify operation of hydro overload capacity the increment cost of steam stations, of energy displaced by hydro overload capacity, must be equal to or greater than steam-base increment cost multiplied by percentages on this chart.)

one period must be stored in the pond during some other period. Therefore, the hydro station is loaded incrementally beyond its most efficient rating in conjunction with the system to carry the peak portion of the load. In any case, however, the overload capacity of the hydro station, beyond most economical capacity, should be utilized as reserve on the system in order to reduce steam stand-by costs.

The incremental value of the hydro station in this region to be compared to the incremental cost of the steam is $I_B \dfrac{W}{W_B}$

where I_B = steam-base increment cost
W = hydro-station increment rate
W_B = average hydro-station increment rate

If W_B is taken as the average hydro increment rate up to the economical capacity, the curves in Fig. 5 then give the values of $\dfrac{W}{W_B}$ in per cent for the overload capacity.

Steam-Station Performance Curves. The steam-station performance curves are based on accurate performance tests of all

pertinent equipment, the tests having been conducted with the apparatus in the best possible operating condition. The heat-balance calculations, which are made by combining the tests of the individual equipment into station performance curves, are the same sort of performance calculations that are made in the design of a station. Daily performance curves are calculated from typical daily load studies by summating the individual hourly performances for the day, computed from the hourly plant performance curves. In working out the station performance curves, the most economical operating and loading schedules for individual equipment in the plant are developed.

Charts for Typical Station. Fig. 6 shows one of the plant-performance charts for a generating station having 2 turbogenerators and 12 boilers, giving the relation of hourly coal consumption to net kilowatt output for 32 F and 80 F condenser circulating water in injection temperatures, and for varying number of boilers under fire. The total coal consumption, based on coal having a standard heating value, is obtained from the sum of parts *A*, *B*, and *C* given by the curves. Part *A* gives the banking or no-load coal per hr for the number of boilers under fire. Part *B* gives the coal required per hr as a function of the net kilowatt output with all the boilers in the station under fire. Part *C* gives an adjustment for a less number of boilers under fire, the adjustment resulting from the decrease in boiler efficiency with increase in rating.

Increment Coal Rate. The heat-balance calculations for the station-performance curves are also used in obtaining the increment-coal-rate curves for the station. The increment-coal-rate curves greatly simplify the task of allocating properly the system load to the individual generating stations, and are also useful in determining increment costs of so-called economy power purchased from or sold to interconnecting companies.

The increment coal rate may be defined as the pounds of coal required to produce an increment of 1 kw in the net station output. It corresponds to the slope of the tangent at a given load on the hourly-coal-consumption curves shown in Fig. 6. The increment coal rate varies with station loading, condenser injection circulating-water temperature, and also with the number of boilers which are under fire.

In Fig. 7 are shown the increment-coal-rate curves for the generating station, the performance curves for which are shown in Fig. 6. The increment-coal-rate values expressed in per cent of average station coal rate are shown as a function of station load for 32 F and 80 F condenser injection circulating-water temperature. Adjustments are given below the main curves for the number of boilers which are under fire in the station. This adjustment is added to the value given by the main curves to obtain the total bogey increment rate. The main curves are for the maximum number of boilers under fire.

Increment Rate in the Overload Region of the Turbines. Sufficient turbogenerators are carried on the system at all times to maintain the load in the event of the failure of the unit of largest capacity. All machines, in general, are provided with overload capacity, where the increment coal rate is from 12 to 20 per cent greater than in the normal load region. This overload capacity should, therefore, be used as far as possible in providing reserve for the outage of the largest unit of capacity on the system. To make the increment-rate curves Fig. 7 more readable, the overload increment rate for the first main unit on the line has been omitted.

In other words, the curve is worked out on the basis that the next machine is put on the line just before the overload valves open on the machine already in service. The addition of the second unit on the line, Fig. 7, increases the coal consumption from 3100 lb per hr to 4400 lb per hr, depending upon the injection temperature. This increased coal consumption must be balanced against the increase in increment rate in the overload region to determine how much overload capacity should run before an extra unit is placed on the line in advance of reserve requirements.

Operating Factor. Inasmuch as the increment-coal-rate curves are derived from the same heat-balance calculations as the coal

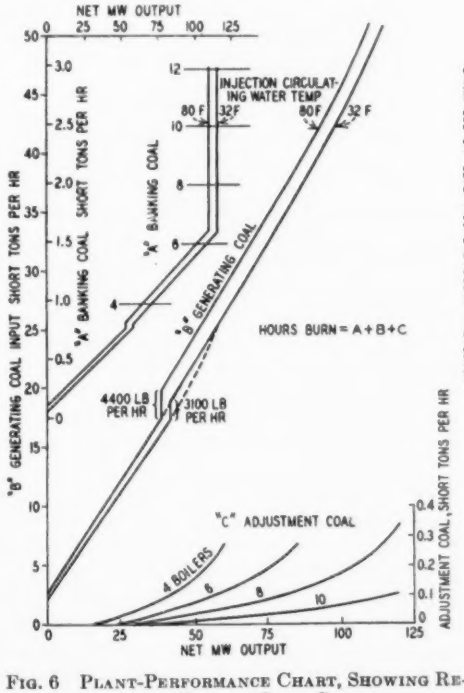


FIG. 6 PLANT-PERFORMANCE CHART, SHOWING RELATION OF TOTAL HOURLY COAL CONSUMPTION TO NET MW OUTPUT

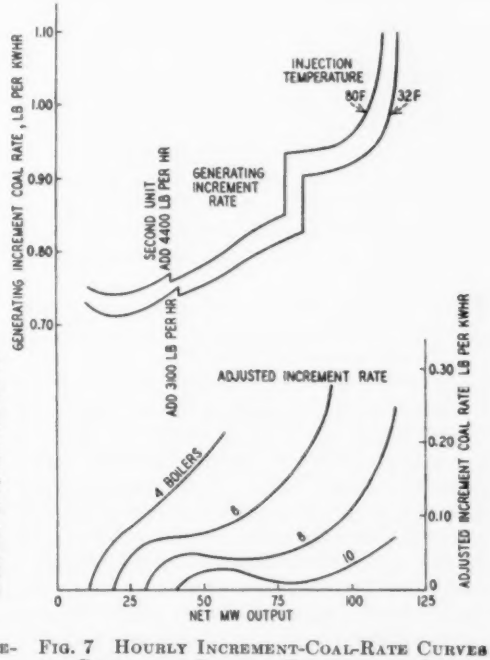


FIG. 7 HOURLY INCREMENT-COAL-RATE CURVES FOR GENERATING STATION; PERFORMANCE CURVES GIVEN IN FIG. 6

bogeys which are used to check the performance of the generating stations, the margin by which the actual coal consumption exceeds the bogey coal consumption should be used in correcting the bogey increment coal rate to actual conditions. This margin is approximately 2 per cent giving a factor of approximately 1.02 by which to multiply the bogey increment-coal-rate values in order to correct the actual plant conditions. The actual factor is known at all times from the check of operation in the plant.

Allocation of Load. The criterion for correct load allocation is that the load shall be increased on the steam-generating station having the lowest increment cost per kwhr. An important consideration with reference to load allocation is the number of main units to have on the line. At the lower loads, the number of units is determined by the amount of reserve for which it is necessary to stand by. In this case, the station increment-coal-rate value given by the curves in Fig. 7, corrected by the operating factor, is multiplied by the price of fuel giving the incremental cost of fuel per kwhr. To this is added the incremental maintenance cost to determine the total steam-station incremental cost which is used directly in determining the correct allocation of load. Assuming a river flow corresponding to a daily Conowingo output of 2,000,000 kwhr per day, and a daily system output of 13,000,000 kwhr, the steam base load corresponding to this is 529,000 kw. The most economical allocation of this steam base load to the individual steam-generating stations, Fig. 2, is as follows:

Station	Load, kw
Schuylkill	115000
Deepwater	54000
Richmond	250000
Chester	66000
Delaware	44000
Total	529000

With this steam base and the system load of 760,000 kw, Conowingo is loaded to 231,000 kw. As the system load changes, the load on Conowingo is varied to hold the frequency.

Regulation of Load

The system load is a composite function of the variable demands of the customers of a power company, the connected load or the maximum potential demand of which may be four or five times the actual diversified energy consumption, termed the system load. The continual change in the customer demand produces a fringe in the system load which is attended by a continual variation in the system speed. On an interconnected group of systems having a combined approximate load of 2,400,000 kw, the frequency variation because of this fringe may approximate 0.05 cycle. Accentuating this fringe and responsible for a greater frequency error are the swings of such loads as that of railroad or steel-mill demands.

These changes in the system load, however, should not cause the abnormal variations in frequency which, when corrective measures are applied, are sometimes reflected in extreme and continued changes in the output of the generating stations.

Frequency Variation a Guide to Load Change. A deviation from standard frequency under normal operating conditions generally may be considered as an indication of a change in system load. The major load changes, such as that of the morning pickup or the noontime drop, however, are anticipated from the normal shape of the load curve. Thus, in a system which regulates its load manually, the attention of the station operator is held to the frequency recorder in order that a sustained load change may be observed and the necessary change in station output made. Where the manual regulation is a responsibility of the load dispatcher, and the regulation is not assigned to a station, the allocation of the increased or the decreased demand will be made in accordance with the economic loading schedule. In this operation as in all regulating measures, attention of the station operator should be directed to the frequency, in order that errors in speed may be avoided.

Regulation of Tie-Line Loading. In the manual regulation of the load of an interconnected group of systems, it is usual to delegate to one company the responsibility of frequency control and to one or more of the other companies the duty of regulating the tie-line loading.

Regardless of the part in the general program of regulation that a company may be assigned, it should be recognized that control of the frequency is inherent in each regulating function. Slight changes in generation in an effort to return a tie-line loading to schedule, for instance, may improve the frequency or, if proper operating procedure is violated, may cause an abnormal frequency error.

Effect of Load Change on Frequency. The effect of a system load change on the frequency, when turbine-governor response tends to limit the variation, may approximate the following: Two per cent load change produces $^1/_8$ per cent frequency change or a system regulation of approximately 8 per cent.

These deviations in the frequency, with respect to system load changes, are those which remain after the turbine governors have responded to the initial frequency change and have partially corrected the variation in speed. These frequency deviations are, in themselves, responsible for an actual change in system demand. This change, when added to the change in generation effected by the governors, is equal to the initial load change at standard frequency. The correction of the remaining frequency deviation is effected by changing generation manually on the marginal station. Theoretically this manual change of generation should be comparable in amount to the change in load as the turbine governors of other stations act to restore their output to their assigned loading.

Frequently, however, there is little if any counteraction by governors and the manual change of generation on the marginal station to correct the remaining frequency deviation will effect a load change on all systems of the interconnected group in proportion to their respective system loads as the frequency is restored to normal.

Effect of Large Generating-Capacity Loss. If there were no turbine-governor response to a system load change, the variation in frequency for a given load change would be much greater in percentage deviation from standard. This deviation from standard frequency with respect to a system load change would be comparable to that which occurs when there is a deficiency of generating capacity with respect to the load at normal frequency.

The change in the system load as a result of a drop in frequency because of the loss of a large amount of generating capacity will approximate 2.5 per cent of the system load to 1 per cent change in frequency. In one instance, the system load at normal frequency was 720,000 kw and there was a loss of 230,000 kw generating capacity, the system load automatically reduced to equal the generating capacity remaining available. The generating capacity remaining available to the load was 580,000 kw and the reduction in load was 140,000 kw or approximately 20 per cent. The decrease in frequency was not quite 5 cycles, or approximately 8 per cent.

Effect of Frequency Reduction on System Load. The same relation between system load change and frequency variation, which exists in the event of the loss of considerable generating capacity, also applies when there is a reduction in system frequency. Reducing the frequency 0.6 cycle or 1 per cent on an interconnection having a load of 2,400,000 kw at normal frequency will reduce the combined load approximately 2.5 per cent or 60,000 kw.

Effect of Change in Generation on Frequency. This relation between load change and frequency variation is presented as forming a close approximation of the effect of the change of one factor on the other when there is no governor response to correct an error in the system speed. This same relation is also effective in the correction of frequency errors or the returning of tie-line loading to schedule. Where there are relatively small changes of

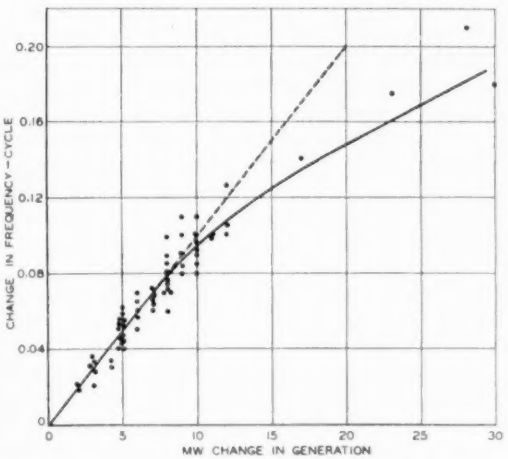


FIG. 8 EFFECT ON FREQUENCY OF CHANGE IN GENERATION
(Governor inactive; combined load of interconnected system = 2,400,000 kw.)

generation, there may be little if any counteracting effect of the turbine governors of other stations or systems. Thus it will be found on even large interconnections of approximately 2,400,000-kw load that a change of 10,000 kw in generation will frequently cause a change in frequency of 0.1 cycle.

Fig. 8 is a diagram of readings taken when the system generation was changed to note the effect on the frequency. These were approximately 40 per cent of the total number of readings taken. The other 60 per cent, which were not included as counteraction of governors or load changes on the system, were evidenced in the readings.

The company changing its generation 10,000 kw to correct tie-line loading may cause a frequency change of 0.1 cycle, and a company to correct an error of 0.1 cycle in frequency may have to change its generation only 10,000 kw.

Regulation of Simple Interconnection. Where only two systems are interconnected, proper operating procedure requires that manual readjustment of tie-line loading should be made as follows:

Generation should be increased or decreased first to correct any existing error in the frequency. Then, both systems must act together to complete the correction of tie-line loading, one system decreasing its generation as the other increases its output. In like manner, manual correction of abnormal frequency errors requires mutual action if "seesawing" of station loadings and of the frequency is to be avoided.

Interconnected-Group Systems. In the interconnected group of which the Philadelphia Electric Company is one, there are approximately fourteen operating companies. In the operation of the Pennsylvania-New Jersey Interconnection, of which the Pennsylvania Power and Light Company, Philadelphia Electric Company, and Public Service Electric and Gas Company of New Jersey are the member companies, instructions to change the generation are given by the interconnection load dispatcher who has the responsibility of maintaining the frequency. The allocation of the combined loading of the three companies forming this interconnection on the basis of economical performance improves the regulation of the frequency as the loading of the tie lines which are of ample rating may vary in amount and direction and requires no regulation. The Pennsylvania Water and Power Company and the Metropolitan Edison Company have the responsibility of regulating the tie-line loading.

Regulation of Interconnected Group of Systems. In the manual regulation of tie-line loading in an interconnected group of systems, proper operating procedure requires readjustment to be made in the following manner:

Generation first should be increased or decreased to correct any existing error in the frequency. The system to initiate this correction will be indicated by the relation of the tie-line loading to the schedule and the possible effect of the change on the frequency. Then, each system may have to change its generation, one or more decreasing its output as the others increase their output.

It can be said that correction of tie-line loading is not a duty

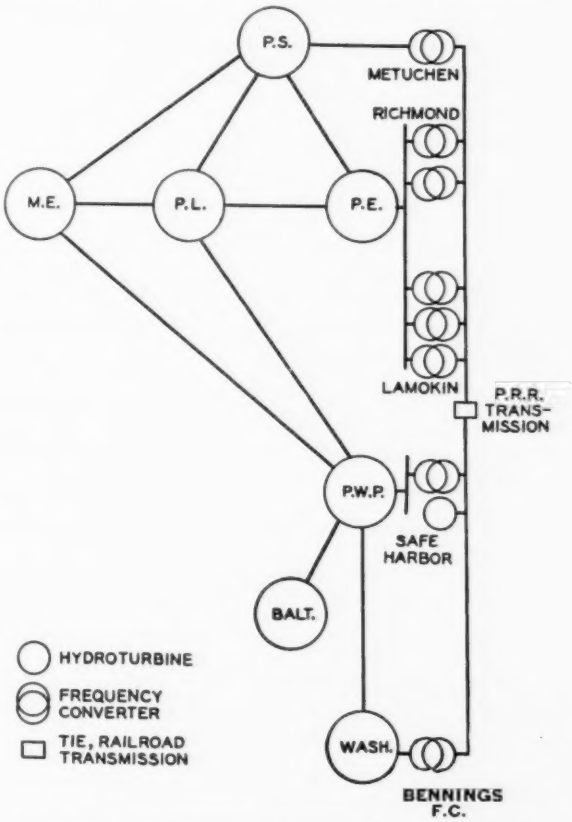


FIG. 9 REGULATION OF GROUP OF SYSTEMS, 60- AND 25-CYCLE LOOP; SIMPLIFIED DIAGRAM
(P.S., Public Service Electric and Gas Company; M.E., Metropolitan Edison Company, New Jersey Power and Light Company; P.L., Pennsylvania Power and Light Company; P.E., Philadelphia Electric Company; P.W.P., Pennsylvania Water & Power Company; Balt., Consolidated Gas & Electric Company; Washington, Potomac Edison Company.)

that may be performed by one system. Observance of the tie-line loading may be the responsibility of one system, but its regulation requires the cooperation of two or more systems.

In the same way, correction of abnormal frequency errors is a duty that may not usually be performed by one system. Mutual action by all systems of the interconnected group may be required to avoid "seesawing" of station loadings and of the frequency.

Attention to Frequency. In any adjustment of tie-line loading, it is desirable that the frequency be observed in order that an

existing frequency error is not aggravated, or a frequency error in the opposite direction is not induced, as subsequent correction of the frequency will again result in the tie-line-loading schedule being adversely affected and again requiring further adjustments.

In order that abnormal frequency errors may be avoided, it may be desirable that the application of corrective measures begin when the frequency departs more than 0.05 cycle from standard. This practice not only may reduce the number of abnormal errors, but will minimize the loading changes on stations and tend to prevent seesawing of station and tie-line loading.

Attention to Tie-Line Loading. Where a meter on the tie line is not available to the frequency-regulating system to indicate where a load change has occurred, the load dispatcher of this system must of necessity depend upon the dispatchers of those companies whose responsibility it is to observe and correct for tie-line loading. If there is a failure of companies to observe tie-line loadings, the burden is placed on the frequency-regulating system to ask for a check of the loading of these tie lines, and to await a report of these loadings before complete corrective measures may be taken to restore normal frequency. In the meantime, some changes in station output have been made and frequently these moves will be followed by changes in the opposite direction when correction by the tardy system has been made.

Interconnection Forming 25- and 60-Cycle Loop. Referring to Fig. 9, the load dispatcher of the Pennsylvania-New Jersey Interconnection, who supervises the allocation of the combined load of this operating group to each system (P.E., P.L., P.S.),[3] also has the responsibility of frequency control. The control of tie-line loading thus devolves on M.E. and P.W.P.,[4] with the greater part of this duty on the latter as P.W.P. must control the loading of the 60-cycle ties (P.W.P.-M.E. and P.W.P.-P.L.) and also the 25-cycle tie (P.W.P.-P.E.) formed by the frequency-converter supply to, and the transmission facilities of, the Pennsylvania Railroad.

Deviation from the scheduled loading of these ties frequently results from the frame shifting of the frequency converter at Safe Harbor or at Bennings. This operation may be performed to:

1 Regulate loading of 25-cycle tie
2 Increase the supply of power to Washington
3 Reduce overload on P.W.P.-Washington 60-cycle tie
4 Utilize output of hydro unit on 60-cycle system of P.W.P.

The movement of a converter frame, if necessitating the correction of tie-line loading by Washington and Baltimore, one counteracting the other in the effect on the frequency, should not cause an abnormal deviation in the system speed if the frequency is properly observed in the corrective measures.

[3] Philadelphia Electric Company; Pennsylvania Power & Light Company; Public Service Electric & Gas Company.

[4] Metropolitan Edison Company, New Jersey Power & Light Company, and Pennsylvania Water & Power Company.

Division of Railroad Load Swings. When there is an increase of the railroad load, regardless of the area in which it occurs, the supply of the increased demand will closely approximate the relative frequency-converter capacity in operation in each area, as the relative impedance of this type of equipment determines to a great degree direction of flow. In like manner, a reduction of railroad load will divide in approximate ratio to the frequency-converter capacity operating.

Regulation of Frequency

A frequency curve appears to have several components: A rapid, periodic variation of approximately 0.005 cycle of an approximate period of 2 sec; variations of approximately 0.03 cycle having a semblance of a periodicity of 30 sec; variations of 0.1 cycle or more from standard frequency that are irregular in occurrence. Fig. 10 is a copy of a high-speed Leeds and Northrup chart of the frequency of the Philadelphia Electric Company system for a 7.5-min period. The curve for this period is typical of the normal frequency curve.

Frequency Ripple. The rapid, periodic variation of approximately 0.005 cycle appears to be a rhythmic pulsation which may be ascribed to synchronizing power between generators and the load or between systems or parts of a system. The rapidity of the variation probably will not permit of regulation and the smallness of the variation does not warrant efforts to correct. This variation of 0.005 cycle follows the curve of the frequency as it responds to the other influences which cause changes in speed.

System Characteristic. Variations of approximately 0.05 cycle also are continuous. If the other causes of speed change were absent, these variations would probably be reflected in a curve which would be a succession of waves of an amplitude not exceeding 0.05 cycle that it would be inadvisable to regulate, because the load change on each generating station, as a result of the variation, is negligible, and any corrective measures might tend only to halve the periodicity of the variation without any reduction of the amplitude. This variation of 0.05 cycle also follows the trend of the frequency curve. As the deviation from standard frequency becomes greater, governors should respond to limit the changes in speed. If the governor regulation is 4 per cent and the frequency variation is 0.05 cycle or $^1/_{80}$ of the speed drop for 100 per cent load change, it may be possible that there will be no governor response.

This variation of 0.05 cycle in the system frequency may be ascribed to the change in the fringe of the load curve, which marks the great diversification in the use of appliances, lighting, motors, etc., throughout the day.

It is difficult to believe that these continual variations of approximately 0.05 cycle are a result of improper governor action as any erratic impulse from this source might be expected to cause an unevenness in the regulation that would render an average frequency of 60 cycles impossible.

That such average regulation may be considered as inherent

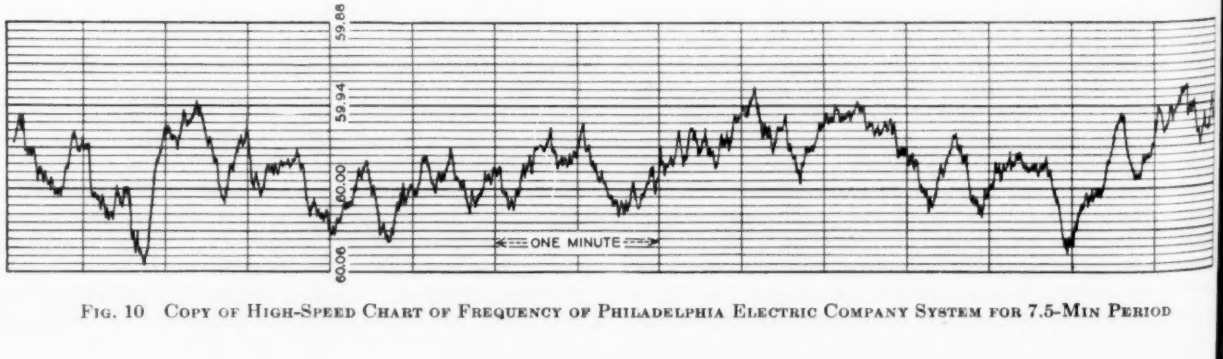


Fig. 10 Copy of High-Speed Chart of Frequency of Philadelphia Electric Company System for 7.5-Min Period

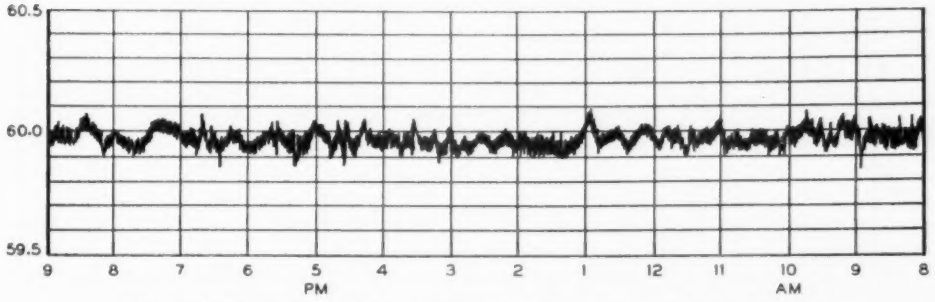


FIG. 11 FREQUENCY CHART OF UNINTERCONNECTED SYSTEM, MARCH 10, 1927

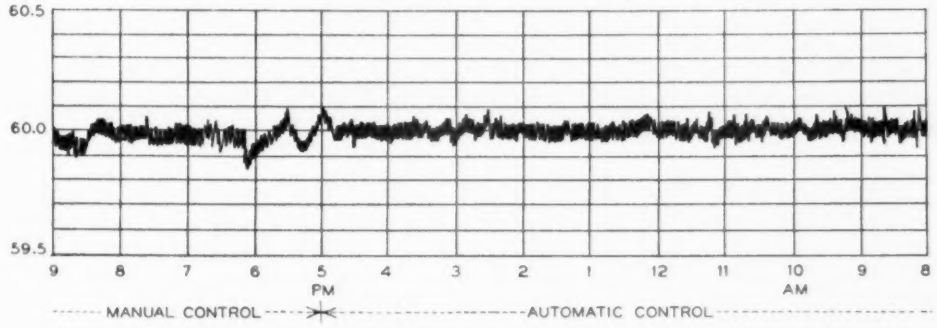


FIG. 12 FREQUENCY CHART OF TWO SYSTEMS INTERCONNECTED, APRIL 3, 1928

in a large system group is substantiated frequently in the operation of the combined system for intervals of several hours, during which periods there were no changes in generation to correct frequency deviations. This self-regulating property was evidenced particularly some years ago in an interconnection of systems having a combined load of approximately 4,000,000 kw. Frequency control on this group of systems had been automatic until 7:15 a.m. one day, when the regulating company transmitted a request to be relayed by telephone through six companies, to another company, which practiced manual regulation, to assume control. The message was not delivered and for more than 4 hr the frequency curve was similar to that of the previous hours when regulation was automatic. The lack of supervision of the regulation was discovered when the first regulating company asked the latter, "When did you install automatic frequency control?"

Similarity of Systems' Characteristic. Evidence of this component of the frequency curve is seen in the frequency chart of March 10, 1927, Fig. 11, when the Philadelphia Electric Company was operating as an uninterconnected system with a load of approximately 400,000 kw. This same variation is seen in the frequency chart of April 3, 1928, Fig. 12, when the system of the Philadelphia Electric Company was interconnected with that of the Pennsylvania Power & Light Company and in the frequency chart of October 14, 1929, Fig. 13, with Public Service Electric & Gas Company of New Jersey as one of the interconnected group. The similarity of the curves testifies to some one force that is inherent in the operation of a system or a group of systems, small or large, as productive of the variation. Such an impulse may have its source in the changing fringe of the load.

The chart of October 14, 1929, Fig. 13, indicates also the load of the regulating station, Conowingo. At that time the frequency control was always assigned to a generating station. As experience indicated that large interconnections with ties of large capacity were, to a great degree, self-regulating, the load dispatcher assumed direct supervision of the frequency and allocated sustained changes in load in accordance with the economic loading schedule. This practice may be observed in the chart of November 6, 1931, Fig. 14, which indicates the generated load of the marginal station which was directed by the load dispatcher to change its output when the change in frequency indicated a sustained load change.

Variations Demanding Correction. Also of interest in all these charts is the third component of the frequency curve which is irregular in occurrence. This component may be divided into two factors. One is the variation caused by changes in such load as the demand of railroads or steel mills. The other is the variation which is due to one of the following causes:

1 Loss of generating capacity
2 Units loaded regardless of frequency
3 Failure to anticipate system load changes
4 "Regulating" tie-line loading regardless of frequency.

Frequency variations due to such load changes as that of railroad or steel-mill demand may influence the setting of governors in order that governor action may maintain the frequency within reasonable limits for an appreciable change in load.

Variations in frequency which find their cause in departure from proper operating procedure may be said to be responsible for most of our present problems in regulation. It is probable that some improvement in governor performance would aid in limiting these variations. The improvement which would bring about closer attention of all operators to the frequency, however, would be fully as adequate. Maintaining satisfactory frequency then would be a matter of observing its trend and directing the necessary change in the output of the marginal station to correct deviation.

Nonobservance of Proper Operating Procedure. Of the failures to observe proper operating procedure, "regulating" tie-line loading regardless of frequency, appears to be the cause of approximately 75 per cent of abnormal frequency variations. The abnormal variations from this cause frequently occur on the hour or half hour or at the time readings are taken. Evidently, equipment loadings are then changed and, at times, regardless of the

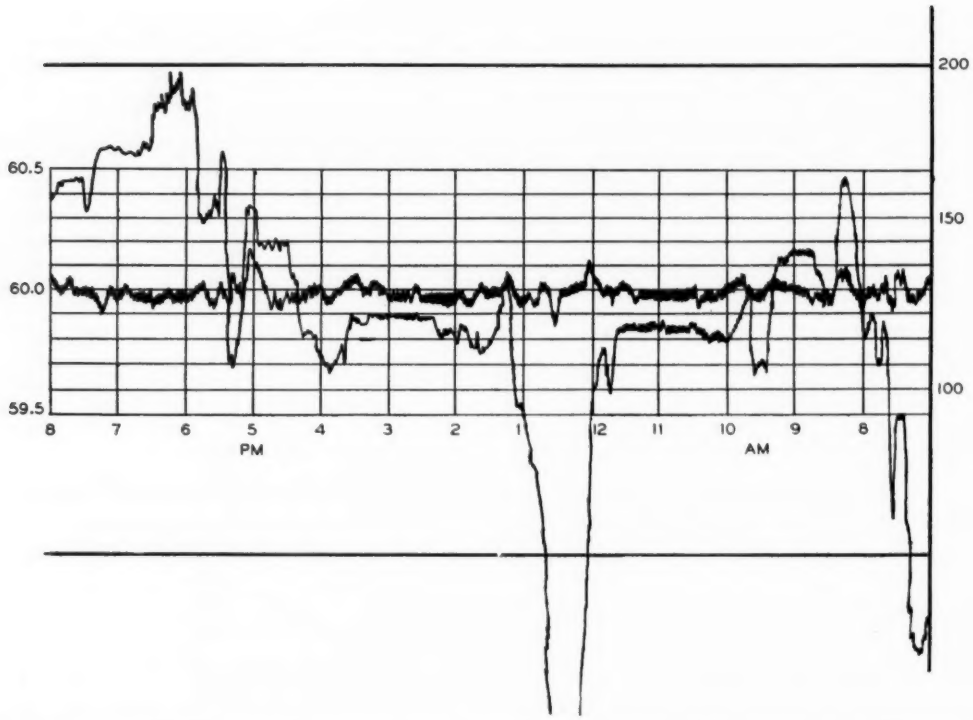


FIG. 13 FREQUENCY CHART WITH LOAD OF REGULATING STATION OF THREE SYSTEMS INTERCONNECTED, OCTOBER 14, 1929

frequency. It is commonly assumed that efforts in the correction of tie-line loading in its effect on the frequency will be counteracted by the action of the governors of units of other stations or systems, thus tending to maintain normal frequency. Often this is not the case, Fig. 8, and if the generation is changed without regard to the frequency, an abnormal frequency error may occur.

Speed Characteristic of Systems. In the regulation of the frequency and of tie-line loading, it is essential that this speed characteristic of the group of paralleled systems be known in order that the effect of changes in load and generation on the system frequency may be anticipated and the effect of variations of the frequency on the system load may be recognized.

On a group of systems, the combined load of which is 2,400,000 kw, this change of generation may be approximately 10,000 kw to effect a frequency change of 0.1 cycle. The reverse of this is also true. A change of generation of 10,000 kw, in an effort to correct tie-line loading, may cause a frequency change of 0.1 cycle.

Regulation of Tie-Line Loading Also Controls Frequency. Because of the appreciable effect on the frequency of a small change in generation, it is evident, if tie-line loading is to be corrected, that the creation of a frequency error must be avoided. Where this possibility is present, the system, which should counteract any adverse effect on the frequency, should be informed of the proposed action.

Frequently it happens that measures applied to correct frequency or tie-line loading have to be counteracted immediately afterward by measures of the same magnitude in the opposite direction. This type of operation constitutes the really objectionable problem in the regulation of the load and frequency. It evidences a failure to observe the trend of the load or of the frequency. It indicates that the remedy for the wide variations in frequency may be found in many instances in a minimum of applied corrective measures.

GOVERNOR CHARACTERISTICS

Governor Action. The action of the speed governors on the prime movers is a primary factor in maintaining the system fre-

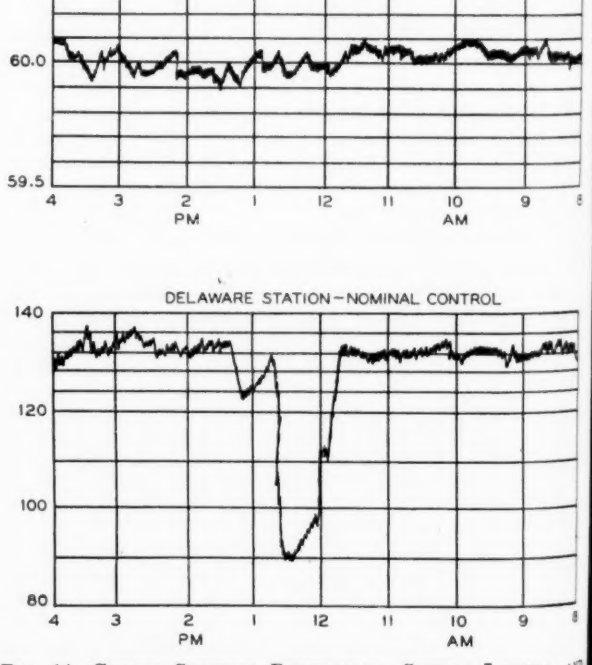


FIG. 14 CHARTS SHOWING REGULATION, SYSTEM INERTIA, AND GOVERNOR RESPONSE, NOVEMBER 6, 1931

quency within narrow limits. The governors should be sensitive to small speed changes and should hold the speed within narrow limits in accordance with their speed-regulation characteristic. The governors should do this without sacrificing stability, as it is part of the governor function to permit proper parallel operation of the prime movers.

Governor Sensitivity. The sensitivity of the governors is important for small changes in system frequency. If the governors do not respond to small changes in frequency, due to small load changes, the variation in frequency for small changes in load is then determined by the self-regulation of the system load. Fig. 8 shows the changes in frequency, resulting from changes in load less than 10,000 kw on a system with a combined load of about 2,400,000 kw. The average variation determined from the straight line in the illustration shows that the frequency varies 0.1 cycle for a load change of 10,000 kw, which on a 2,400,000-kw system is equivalent to 0.4 per cent load change for a change in frequency of $^1/_8$ per cent. This same approximate variation was obtained on several occasions when transmission-line failures resulted in a deficiency of generating capacity.

Theoretically it would appear that this is too large a variation in load for a given change in speed, but it is possible that a small percentage of the governors are acting to decrease the amount of speed change for a given change in load; it may also be that there is some slight variation in voltage which tends to contribute to the load change. By increasing the sensitivity of the governors, smoother operation of the governor function would be promoted and easier control of the frequency after load changes would result.

Governor Speed Regulation. For larger changes in speed, the variation in speed for a given change in load is determined by the speed-regulation characteristics of the governors. The speed-regulation characteristic may be defined as the change in speed in per cent of normal speed, which is required by the governor in order to vary the load from no load to full load. The governors on the system vary in speed regulation from two to eight per cent.

While the speed regulation over the range from no load to full load would indicate the frequency variation for a very large change in load on the system, the instantaneous speed regulation is a better indication of the effect on frequency of the load changes which are usually obtained on large systems.

Most governors have a wide range of instantaneous speed regulation for small changes in speed. This variation in instantaneous speed regulation is partly due to the difference in the amounts of steam admitted by a given movement of the control valve, depending upon the position of the valve. Other reasons for this variation are the changes in the efficiency of the turbine over the load range and on some governors the speed motor introduces a spring force which changes the speed regulation.

The position of the control valve has a large influence on instantaneous regulation. Thus, most control valves, which are just opening, will admit a large increment of steam for a given movement of the valve, while a valve which is nearly fully open will show a small increase in steam flow for a given movement of the valve. This is one of the reasons why as units approach full load, the instantaneous speed regulation is usually very high and these units participate very little in maintaining close frequency. The valve characteristics are sometimes changed by reshaping the ports, or by installing cams to control the movement of the valve.

Fig. 15 shows the instantaneous speed regulation plotted in relation to load for a turbine equipped with three overload valves. The instantaneous speed regulation is determined for each load point from the formula

$$\text{Instantaneous speed regulation in per cent} = \frac{C}{S \times \frac{\Delta \text{ kw}}{\Delta S}} \times 100$$

where C = turbine capacity
S = normal speed
$\frac{\Delta \text{ kw}}{\Delta S}$ = rate of change of load with speed

It is seen that the instantaneous speed regulation varies considerably over the load range and at full load is quite high. This turbine is equipped with cams on the primary admission valve, the secondary and tertiary admission valves, while the quaternary

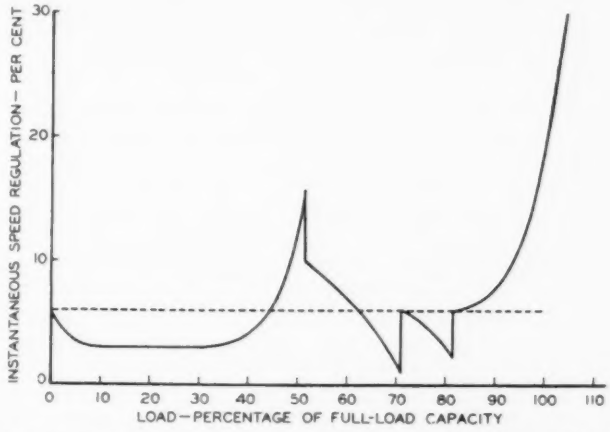


FIG. 15 GOVERNOR SPEED REGULATION; TURBINE EQUIPPED WITH FOUR ADMISSION VALVES

admission valve does not have a cam. On the valves equipped with cams, the instantaneous speed regulation is more uniform. Before the secondary valve has started to open at about 50 per cent load the instantaneous regulation is quite high and, if the machine is loaded at this point, there is less variation in load with a change in frequency. This may be desirable from an economy standpoint, as there is a sharp increase in the incremental heat rate of a turbine when the next admission valve is just opening. Any tendency to open the overload valve on load swings may result in loss of economy.

Combined-System Governor Response. The combined governor speed characteristic of the system results in a variation of 40,000 kw for a change in frequency of 0.1 cycle, which is about 2 per cent load variation for a change in frequency of $^1/_8$ per cent. This would correspond to a governor speed regulation of about 8 per cent. This regulation is obtained when the hydro units are on peak operation and their governors are assisting in the frequency regulation, although some of the governors on the steam-base-load units are more sluggish. When the flow on the Susquehanna River is at flood stage and the hydro units are operating wide-open with the governors against the stops, the speed variation on the system for a given load change is much greater, inasmuch as the governors are controlling speed on a smaller percentage of the generating capacity. This is especially apparent when the system load is a minimum and the hydro is maximum.

A governor with considerable inertia may become unstable and cause hunting if its speed regulation is made too small so that it tries to hold the speed within narrow limits, without sufficient dashpot action. It may also cause a swinging of the load at certain load points where the instantaneous regulation is too small. This hunting condition is usually self-evident in a steam-generating station where it occurs and may cause considerable trouble in the boiler plant. The operating forces in the steam-

generating station usually seek to correct such hunting in order to eliminate their trouble. Such hunting may be corrected by broader governor regulation and, from the system standpoint, it is better to eliminate such hunting even at the expense of broader regulation.

Effect of Governor Action on Tie-Line Loads. Where two large groups of systems are interconnected over a tie line, with a load change in one system, the governors on the other system will share in the load change in accordance with the variation in frequency and consequently cause a variation in the tie-line load. If the tie line is of relatively small capacity, the variation in tie-line load when superimposed on the normal tie-line load may cause the tie line to trip out due to overload or instability.

In some cases, the original variation in tie-line load caused by the governors may initiate a seesawing condition in attempting to correct manually the tie-line load. This additional variation may cause the tie lines to trip.

Effect of Governor Regulation on Economy. The regulating action of the governors on the base-load units would appear to result in an uneconomical allocation of system load if the frequency should change. For instance, if the frequency should increase, the most efficient base-load units in order to share in the frequency regulation would proportionately decrease in load, while theoretically for maximum economy the entire load drop should be taken by the marginal station. However, if all the units are sharing in the regulation, the change in frequency would be very small and the corresponding change in load on any station due to the governor operation would be very slight. Also, the more efficient units would in general be loaded nearer their maximum capacity and the control valves would be close to the fully open position where the governor regulation is much broader and the smaller change in load results from a given change in frequency. The frequency would then be restored to normal by repositioning the governors on the marginal station whose load should be changed in accordance with economic loading requirements. As the frequency is restored to normal, all the generating units with the exception of the marginal unit should return to their original loads before the load change occurs.

If an attempt is made to hold the frequency too close by regulating the frequency on a marginal steam station, all the load swings would be taken by the marginal station and would impose a heavy burden on the boiler plant of the station, with consequent loss in economy on the entire load carried by the station. If the marginal station is a hydro plant, the loss in economy would not be as great as on a steam plant.

Governor Load-Limiting Devices. Some governors on steam units are equipped with load-limiting devices which are used when the boiler capacity is less than the spinning-turbine capacity. Regardless of a decrease in frequency, the load-limiting device limits the loading on the turbine to any predetermined value corresponding to the setting of the load-limiting device. Should the outage of a boiler unit reduce the boiler capacity below the turbine capacity, the load-limiting device will prevent the possibility of water carry-over from the boilers to the turbine on a drop in frequency which would normally cause the governor to bring the turbine to full load.

Conclusions

1 The sensitivity of the governors should be increased as much as possible, in order to give smoother operation of the governors and provide a definite relationship between change in frequency and change in system load for small changes in load or generation.

2 The instantaneous speed regulation of the governors over the entire load range should be more uniform, in order to improve governor stability where the instantaneous regulation is lower and to limit the speed variation for a given change in load where the regulation is high. The instantaneous regulation, however, should be higher in percentage where sharp increases occur in the incremental heat rate of the turbine and at minimum loads on a plant where boiler-plant conditions may become unstable with extreme load changes.

3 The average speed regulation of the governors should be reduced as much as possible without, however, sacrificing stability of parallel operation. This measure should be applied generally throughout the systems of an interconnected group in order to limit the burden due to load variation on any particular station.

4 Where tie-line loads must be regulated, the regulation of frequency or tie-line loading is a mutual operation, requiring the cooperation of all system dispatchers. The regulation of the frequency must be performed with regard to the tie-line loading and the regulation of the tie-line loading must be performed with regard to the frequency.

5 Where interconnection tie lines require no regulation, frequency variations will indicate a change in system load and the interconnection dispatcher can allocate the load change on the basis of economical loading.

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Discussion

Albion Davis.[5] Under the heading "Regulation of Frequency," the authors state: "A frequency curve appears to have several components: A rapid, periodic variation of approximately 0.005 cycle of an approximate period of 2 sec; variations of approximately 0.05 cycle having a semblance of a periodicity of 30 sec; variations of 0.1 cycle or more from standard frequency that are irregular in occurrence."

The variations of 0.05 cycle are referred to as a "system characteristic." It is stated that these variations are continuous and follow the trend of the frequency curve. The periodic variation of 0.05 cycle is ascribed "to the change in the fringe of the load curve, which marks the great diversification in the use of appliances, lighting, motors, etc., throughout the day." Such regulation is considered by the authors as inherent in the operation of a system or a group of systems, a self-regulating characteristic.

The writer believes that tests of the response of governors to these 0.05-cycle speed swings will disclose the fact that most of the governors are following these speed changes, picking up and dropping load on the swings, but not exactly in synchronism with the speed. It is believed that many governors will show a con-

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siderable time lag on the reversals of speed and, because of the time lag, will register considerable overtravel. Tests will probably show that the composite effect of overtravel of all governors is the underlying cause of the regular system hunting action, rather than the characteristics of the load.

It is for this reason the writer has urged that more systems get the necessary instruments to measure governor performance and find out, not only what individual governors are doing, but also what different governing combinations are doing to system speed.

R. W. McLaughlin.[6] An interesting point brought out in this paper is that in a typical large power system a $2^1/_2$ per cent load change will cause a 1 per cent frequency change if the governors do not function at all to counteract the change in speed. In other words, the system regulation with no governor action is equivalent to 40 per cent for 100 per cent load change. Of course, the term "regulation" as used here means the average regulation existing for a relatively small load change and, as used further in this discussion in order to make it possible to refer to regulation as being constant for any given machine, the fact that the regulation at various loads is considerably affected by valve design and valve-opening point is disregarded. Furthermore, in order greatly to simplify the discussion and assume a simple straight-line relationship between speed changes and load changes, we will not attempt to include the effect of lag caused by lack of sensitivity, which is in itself a variable for different loads on any given machine.

One of the most prevalent causes of complete lack of governor response to speed change of a few per cent is the increasing use of load-limiting devices. Under normal operating conditions, load limiters are set to allow the machines to carry a small margin of load above that which they are scheduled to carry, and governors are allowed to function normally. In case of a heavy rapid load increase on a system, it is possible that for varying time intervals the load limiters on several machines in a system may be holding the admission valves at a fixed position, preventing the governors from helping to control the frequency of the system. Thus, when a load-limiting device takes control away from a speed governor, the apparent regulation of the turbine immediately jumps to 40 per cent, based upon the load that the machine is carrying and, if the load limiter is set at 80 per cent of the capacity of the machine, the regulation referred to full capacity of the machine immediately becomes 50 per cent, as compared to a probable regulation of between 4 and 8 per cent when operating on the speed governor. Therefore, when a load limiter comes into effect, the apparent regulation of the machine immediately becomes so broad that it will have but slight influence on the frequency of the system, although a change in frequency will cause a slight change in load on the machine operating at its limit. The foregoing comments indicate that, although a load-limiting device may be of distinct advantage with respect to the operation of the boilers, it has an adverse effect upon the instantaneous regulation or frequency control of the system as a whole.

In order to calculate the average regulation for a power system with respect to gradual changes in load, assuming that the capacity and regulation of each machine serving the system are known, the following simple equation is useful

$$\text{Average regulation} = \frac{A + B + C + \ldots.}{\frac{A}{R_a} + \frac{B}{R_b} + \frac{C}{R_c} + \ldots.}$$

in which A, B, C, etc., are the rated capacities of the machines in operation, while R_a is the regulation for the turbine of capacity A, R_b is the regulation for the turbine of capacity B, etc. It may be noted that the denominator is equal to the total load change equivalent to 1 per cent speed change, while the numerator is the total rated capacity of the machines in service, and not the total load being generated. While the actual regulation at any given load varies somewhat from the average, the variation in regulation for different valve openings on modern multivalve machines is not of sufficient magnitude to be of particular importance.

Although some progress has been made in an attempt to minimize the importance of good speed regulation by the use of automatic frequency-controlling equipment, the continued importance of regulation is manifest by recent discussions between representatives of power companies and turbine manufacturers regarding the advisability of building governors with an arrangement whereby the regulation can be adjusted between certain specified limits while the machine is carrying load. While many of these devices have been built, the value of such a device depends upon whether the station operators are able to use it to its best advantage. It appears that the principal advantage of having variable regulation control is that the operators can vary the regulation of any machine in operation until they find the regulation which apparently gives the best average result. Thereafter, it is doubtful whether the bother of making frequent adjustments to this device will be justified by any real improvement in operation.

F. Oppenheimer.[7] At recent technical meetings, there has been frequent reference to designing and operating governors with speed droops at or near zero. This is, no doubt, due to a recognition of the fact that existing governors are on the whole operating at such high speed droops that poor frequency regulation results, which in turn adds to the difficulty of load and tie-line control.

The writer has pointed out the practicability of reducing immediately the general level of speed droop on a system step by step, unit by unit, a method which minimizes the risk of instability and mechanical difficulties. Power systems would thus take advantage of available, if limited, characteristics of existing governors without waiting for the possible remodeling which might be feasible, or waiting until older machines have been retired and replaced by modern governors, such as have been described. From certain tests made (for another purpose) on the interconnected system described by the authors, the writer has figured that for the whole interconnected system the speed droop is about 11.5 per cent (excluding hydro plants). This figure was obtained as follows. The total capacity of the interconnected system was 2,400,000 kw; Conowingo decreased its output 200,000 kw which caused the system frequency to decrease 0.53 cycle. The Holtwood governors picked up 13,000 kw on 61,000 kw capacity, Safe Harbor 30,000 kw on 96,000 kw capacity. Taking these three plants out of the computation gives

$$\frac{2{,}043{,}000}{157{,}000} \times \frac{0.53}{60.00} \times 100 = 11.48 \text{ per cent}$$

This is radically different from the speed droop attained by the governor described by Dr. White or the system described by Mr. Holden which dealt with droops of 0.02 per cent and about 0.5 per cent, respectively. It is obvious that the desire of steam engineers for large speed droop has been attained and with advantages in the way of steam economy and ease of operation of the steam plant. However, it is hardly necessary to point out that if the speed droop of all the units on a system is reduced proportionally, the load swings will divide as before but there will

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[7] Pennsylvania Water & Power Company, Baltimore, Md.

be an improvement in system frequency. Also the frequency change of 0.1 cycle for each change in generation of 10,000 kw described in this paper will become proportionally less, and probably so small that an operator could make changes of 10,000 kw in generation with almost inappreciable effect on the frequency. This means he could correct tie-line loading without having to be concerned with its effect on frequency (except for large changes). This would be a great step forward in the operation of interconnected systems.

W. E. PHILLIPS.[8] Increasing the governor sensitivity, as stated in the paper, and decreasing the governor droop as brought out in the discussion, will undoubtedly improve the system frequency. This is a much desired objective. However, when all stations in an interconnected group are responding directly to frequency, it will cause greater swings in the interconnecting tie lines.

If we assume that the load comes on in the area of only one of the interconnecting companies, all stations will respond since there will be a drop in frequency. When the company in whose area the load comes on adjusts its generation to meet this new demand, it will be necessary for the other interconnecting companies to change their generation back to the value which existed before the change.

One generally accepted method of regulating an interconnection is to assign the control of the frequency to a centrally located company and to let the other interconnecting companies hold the tie lines toward the frequency-controlling station. If it were possible to make instantaneous changes in generation so that the tie line could be held absolutely constant, then the problem of the frequency-controlling station would be relatively simple, since generation changes would be required by it only to take care of load changes in its area. In actual practice, generation changes cannot be made instantaneously, consequently the tie lines swing and the load changes in the other areas are reflected in the frequency, the frequency-controlling or master station responding to them. This puts an added burden on the frequency-controlling station.

If, when the frequency deviates, all the interconnecting tie lines are made to deviate a small amount in a direction to help the frequency, then the burden of the master station is much less and the frequency is improved, since all stations are helping to control it. As the frequency returns to normal, all tie lines return to their normal scheduled load. The amount which any tie line deviates is definitely fixed for any given frequency deviation.

It may appear that the contribution made to frequency is essentially the same as that made when the governor sensitivity and droop are adjusted as indicated previously. However, the essential difference is that the amount contributed is not dependent upon where the units are operating on the speed-droop curve, Fig. 15 of the paper, or upon possible variations in governor sensitivity. The amount of contribution is definitely under control of the dispatcher. Thus the contributions of the interconnecting companies can be readily coordinated.

AUTHORS' CLOSURE

It is possible, in some cases, that the overtravel of governors may be responsible for a system hunting action that causes speed changes. However, a large number of tests that were made on the interconnected group of systems, of which the Philadelphia Electric Company is one, has indicated that governors frequently fail to respond to speed changes of as much as one-tenth cycle. It is apparent where the governor response is absent, that the speed changes are a result of changes in load.

[8] Engineering Department, Leeds & Northrup Company, Philadelphia, Pa.

Where these load changes are greater, and on the Philadelphia Electric system load swings of 50,000 kw are relatively frequent, governor response is more consistent. If there were overtravel of the governors in such cases the frequency would be extremely erratic, which is not the case.

Furthermore, the self-regulating characteristic of the systems, evidenced frequently for periods of several hours at a time, is difficult to reconcile with erratic governor action.

The following mathematical calculation indicates that the variations in frequency of approximately 0.05 cycle are due to changes in load demand. Let Δl = small instantaneous decrease in load demand; f_o = normal system frequency; f = system frequency at any instant; and L_o = system load. Then, the stored kinetic energy in system = Wf^2, where W is a constant.

Fig. 8 of the paper shows that for small sustained changes in load the frequency varies $^1/_6$ per cent for a load change of approximately 0.4 per cent. This indicates that the change in load demand in per cent is a little over twice the per cent change in frequency. The change in load demand due to frequency change $= 2L_o(f - f_o)/f_o$ approximately.

The small instantaneous decrease in load demand Δl results in an excess power input which is available for accelerating the system until the frequency changes sufficiently to absorb the excess power input. At any instant, Δl equals instantaneous power input for accelerating the system plus the increase in load demand due to increase in frequency. That is

$$\Delta l = \frac{d}{dt}(Wf^2) + 2L_o\frac{(f - f_o)}{f_o} \quad [1]$$

$$= 2Wf\frac{df}{dt} + \frac{2L_o(f - f_o)}{f_o} \quad [2]$$

For small changes in frequency f_o may be substituted for f in $2Wf(df/dt)$, or approximately

$$\Delta l = 2Wf_o\frac{df}{dt} + \frac{2L_o(f - f_o)}{f_o} \quad [3]$$

and

$$\frac{1}{2}\frac{\Delta l}{L_o}f_o = \left(\frac{Wf_o^2}{L_o}\frac{df}{dt}\right) + f - f_o \quad [4]$$

The solution for this differential equation is

$$(f - f_o) = \frac{1}{2}\frac{\Delta l}{L_o}f_o(1 - e^{-t/T}) \quad [5]$$

where $T = Wf_o^2/L_o$ kwsec per kw; T is the time constant in this equation and is equal to the stored kinetic energy in the system at normal frequency (Wf_o^2) per kilowatt of load. The approximate value of T for our system is 7 kwsec per kw, or 7 sec. In accordance with Equation [5], the change in frequency reaches about 63 per cent of the final sustained change in 7 sec. Portions of the curve in Fig. 10 of the paper have been reproduced on a large scale in Fig. 16. Section A refers to the last few seconds of the first-minute period, section B to the portion of the curve at 1½ min, and section C to 3½ min.

The exponential curve, Equation [5], was fitted to each portion of the system-frequency curve by measuring the frequency change at an interval of 7 sec after the beginning of the change. This increment is approximately 63 per cent of the total frequency change due to changed load demand. For example, considering section A, the values in Table 1 were obtained.

In this case the agreement was good during the first 8 sec of the change showing that the change was due to the inherent characteristics of the system rather than governor overtravel. The other two sections reproduced also show good agreement.

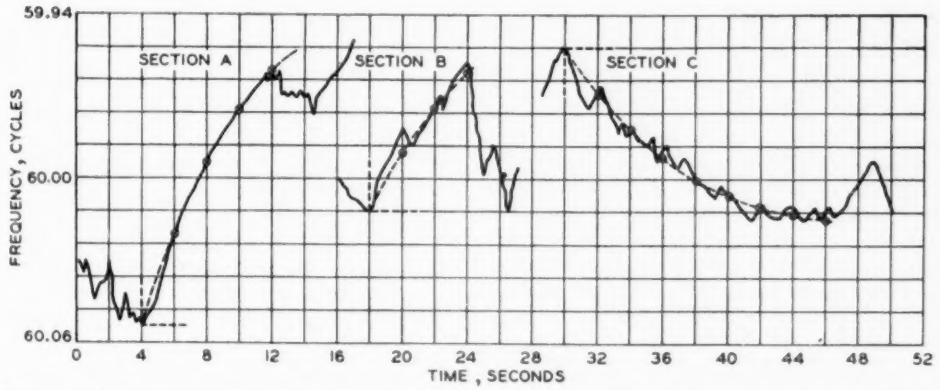


FIG. 16 HIGH-SPEED FREQUENCY CHART OBTAINED ON THE PHILADELPHIA ELECTRIC COMPANY'S SYSTEM

R. W. McLaughlin refers to the statements regarding the speed characteristic of the interconnected group of systems, of which the Philadelphia Electric Company is one. This characteristic is also evidenced in those instances of system trouble where there is a considerable deficit of generating capacity with respect to the normal system load.

Regarding the assertion that the paper has disregarded the fact that the regulation at various loads is affected considerably by valve design and valve-opening points, such effect was considered at some length in the body of the paper, and in the conclusions. In addition, Fig. 15 shows the variations in the instantaneous speed regulation throughout the load range of the unit.

TABLE 1 VALUES OBTAINED BY FITTING THE EXPONENTIAL CURVE TO SECTION *A* OF THE SYSTEM-FREQUENCY CURVE SHOWN IN FIG. 16

t, sec	$1 - e^{-t/T}$
2	0.243
4	0.436
6	0.577
8	0.680
10	0.760
12	0.821
14	0.865
16	0.899

$T = 7$ kwsec per kw

At 4 sec frequency, cycles..... = 60.054
At 11 sec frequency, cycles..... = 59.968
Frequency change, cycles..... = 0.086

Sustained change $= \frac{0.086}{0.63} = 0.136$

Points on the exponential curve:
At 2 sec, frequency change $= 0.136 \times 0.243 = 0.033$
At 4 sec frequency change $= 0.136 \times 0.436 = 0.059$

With regard to Mr. McLaughlin's comments on load-limiting devices, these devices in our system have been necessary to protect the boilers and also the turbines being supplied from these boilers in the event of a large change in frequency where the governor would normally cause the turbine to be loaded to its maximum capacity, which would be in excess of the available boiler capacity. However, where these devices are used as a stop on the maximum load which the turbine can carry, if the turbine is normally loaded at some point below this maximum load the governor should function in a normal manner for small changes in frequency, and it is only as a result of large decreases in frequency that the device becomes effective in limiting the load of the turbine.

In Fig. 8 it is shown that for small changes in frequency up to approximately one tenth of a cycle the per cent load change is two and one-half times the resultant per cent frequency change. Fig. 8 also shows that for load changes in excess of 10 mw, the frequency changes at a much slower rate, showing that the governors were responding when the apparent threshold of sensitivity of one-tenth cycle was exceeded. The fact that the governors respond for speed changes in excess of one-tenth cycle would indicate that load-limiting devices were not affecting the regulation.

Referring to W. E. Phillips' discussion, it may be said that whether governor sensitivity is increased proportionately throughout the interconnected group or remains as at present, the tie-line loading changes as a result of governor response should remain approximately the same.

Load changes occur in all load areas. Assuming a load change in one area, however, as this company makes the necessary change in generation, increased governor sensitivity will tend to more consistent automatic correction of the loading of the units of other companies.

Where the principles of interconnection operation as exemplified in the operation of such groups as that of the Pennsylvania-New Jersey interconnection are followed, tie-line loading, where the tie capacity is adequate, presents no problem.

Experiences in System Speed Regulation

By OTTO HOLDEN,[1] TORONTO, ONTARIO, CANADA

This paper describes operating conditions on four widely varying power systems. In one case, the means, adopted to reduce the wide variation in frequency of a small system serving two mines only, is discussed. The problems encountered in larger systems serving mines, paper mills, grain elevators, and municipal requirements are dealt with, concluding with the largest of the four, a system developing 1,400,000 hp at peak load, fed from nine generating stations. One plant of the system is assigned to regulate speed, the others conforming under instructions from a load supervisor. Automatic frequency-regulating devices are applied to some units at one plant, manual control being necessary at all others.

ILLUSTRATIONS of normal operating experience with regard to speed regulation in several of the systems of the Hydro-Electric Power Commission of Ontario, in which the results obtained vary greatly, are submitted in this paper. It is not intended to include details of the various phenomena or to offer any solution for the problems raised, but they are submitted in the hope that the description of the conditions prevailing may prove of interest. The paper considers only speed regulation as indicated by system frequency as this is affected by speed-controlling devices, inertia of rotating elements, and magnitude, diversity, and variation of loads. It does not deal with stability of the system, that is to say, its behavior under transient abnormal electrical conditions such, for example, as would be caused by short circuits.

The systems on which the regulation conditions are described are of various sizes and characteristics, ranging from one of small size, with one generating station supplying two customers, to one having a load of approximately 1,400,000 hp, supplied by nine or more generating stations with upward of 1500 miles of 110 to 220-kv transmission lines.

In each case, the type of load carried by the system, the relative magnitude of the sudden load changes, the generating and governing equipment, and various other factors, all have their effect on departure of system speed from its normal value. Comments on some of these factors will be found in the text. Sections of load and frequency charts reproduced supplement these comments.

The St. Joseph District

The St. Joseph District, the smallest of the systems considered, consists of a generating station with two units installed. The head is normally 16 ft, and one unit consists of a four-runner horizontal open-flume turbine, with a capacity of 1380 hp, connected to a generator rated at 2000 kva at 164 rpm and 6600 v. The other unit is a single-runner vertical propeller-type turbine of 1750 hp capacity, directly connected to a 1500-kva generator operating at 128.5 rpm and 2300 v.

A composite regulating constant for the plant, derived by adding together the product of WR^2 and square of the speed for each unit and dividing by the sum of their horsepower ratings, is 8,860,000. These units feed through banks of 6600/22,000- and 2300/22,000-v transformers to a 28-mile transmission line. The power supplies two gold-mining properties, after being stepped down at the local stations to 550 v.

Each of the mines has relatively large air-compressor and hoisting capacity. It is the operation of this particular equipment which gives rise to the large changes in load, with consequent wide variations in power characteristics. As both of these customers have been constantly increasing their production, the plant is at all times loaded to, or near, its maximum generating capacity.

When a relatively large increase in load occurs, due perhaps to the synchronizing of compressor operation and the starting of a hoist, the increased demand often exceeds the capacity of the plant. The speed of the unit is then reduced below normal, with consequent lowering of frequency, and a considerable period may ensue before normal speed is restored. Smaller changes in demand of, say, 25 per cent of the capacity of the plant are constantly occurring. The resultant frequency varies over a considerable range. The load and frequency charts shown in Fig. 1 indicate the conditions which obtain.

It is recognized that such regulation would not be suitable for other service but, owing to the fact that all the supply is used for mining operations, the conditions existing are more desired by the power customers than improvement at the cost of reduction in demand, with consequent lessening of production, since the operation of a high-speed automatic voltage regulator maintains voltage level within satisfactory limits.

Characteristics of motors used in mining practice and methods of reducing the load swings have been investigated. Discussion of these factors with manufacturers and operators has led to the readjustment of the time delays on the contactors controlling the motors, with some improvement in speed variation. On new motors being installed in the system, greater attention is being paid to the proper setting of the time delays in some instances and, in one case, a motor generator set with flywheel is being installed in conjunction with d-c motors on the mine hoists.

Load Conditions in the Abitibi District

In contrast to the foregoing are the conditions obtaining in the Abitibi District, a larger system, which also supplies, in the main, mining load. In this instance the generating station contains five units, the turbines being rated at 66,000 hp each, operating at 150 rpm under 237 ft head, and the regulating constant is 9,000,000. The turbines are supplied through 18-ft penstocks 262 ft long. The present 20-min peak load carried by the plant is approximately 135,000 kw, most of which, with the exception of some 30,000 kw used for steam generation, is mining load.

Transmission is at a normal potential of 110 kv. One large load of approximately 35,000 kw is located 250 miles from the generating station. Many loads of 2000 to 10,000 kw are connected. While the main load is derived from the operation of mines, a number of other important industries are also served. The ample generating capacity available, as well as the diversity in demand resulting from the large number of customers, particularly the large number of mines, results in power characteristics of the first order, which satisfactorily meet all requirements of service.

One of the turbine governors is adjusted to have zero speed droop, and the others a droop of 0.6 per cent. Stable operation

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Contributed by the Hydraulic Division and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

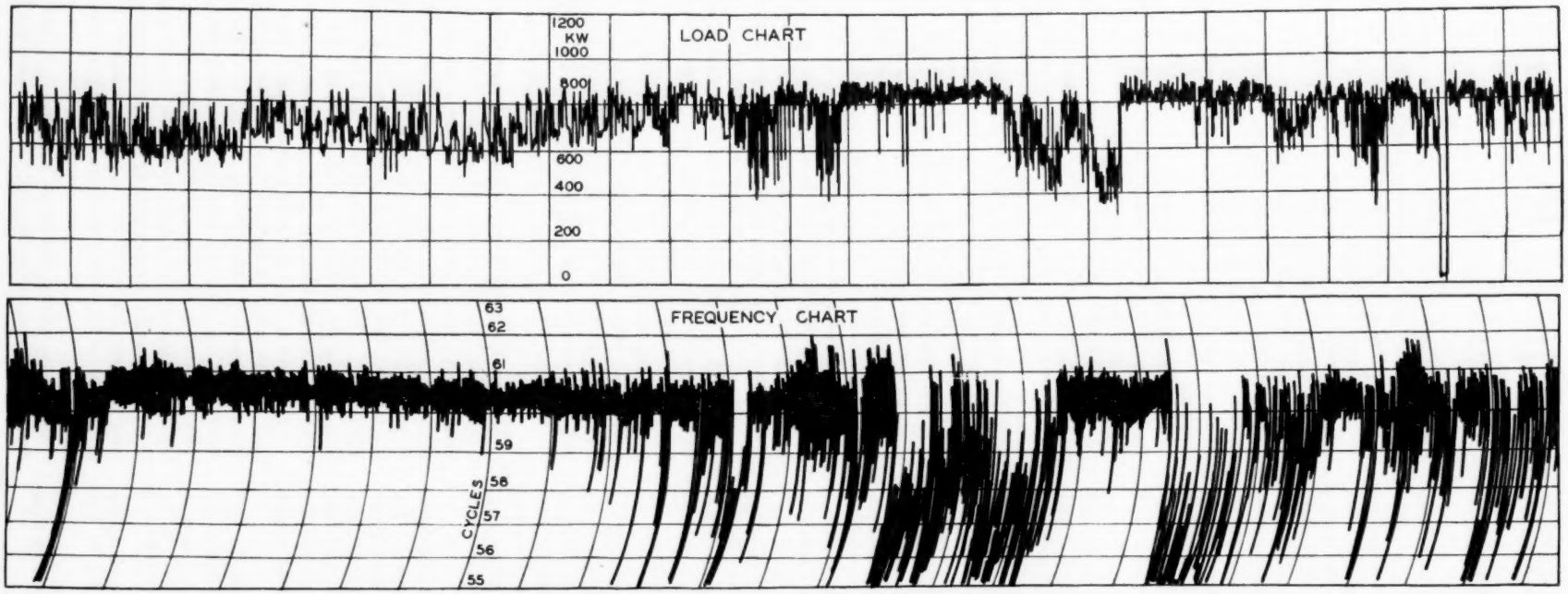


FIG. 1 TYPICAL LOAD AND FREQUENCY CHARTS; ST. JOSEPH DISTRICT

(The system load was approximately 2000 kw, all of which is for mining operations of two customers. Of this total a steady load of 1200 kw was being carried on unit No. 2, for which no graphic chart is shown. The average load on unit No. 1 was approximately 800 kw, variations from this being shown on the chart. Exceptionally violent fluctuations of frequency occur.)

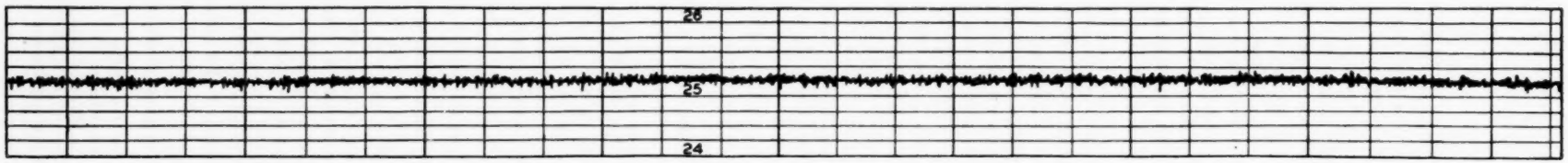


FIG. 2 TYPICAL FREQUENCY CHART; ABITIBI DISTRICT

(Power is supplied in this system to a large number of mines and other industries. The bulk of the load is mining load. The load for the selected hours was 90,000 to 100,000 kw.)

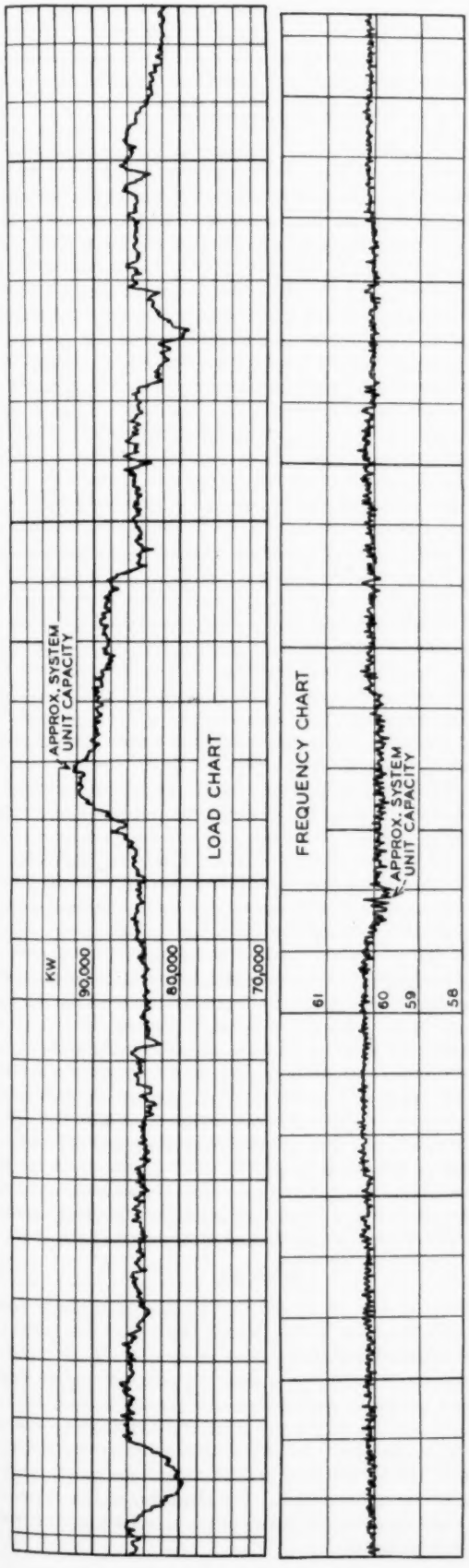


FIG. 3 TYPICAL LOAD AND FREQUENCY CHARTS; NIPIGON SYSTEM

(System load here is 80,000 to 90,000 kw, service being given to pulp and paper mills, grain elevators, municipalities, and mines.)

and satisfactory distribution of load among units are obtained with this arrangement. An examination of the frequency chart Fig. 2 indicates very little drift from normal frequency, although the momentary variations are quite similar in percentage to those in the following cases described.

CLOSE SPEED REGULATION NECESSARY ON NIPIGON SYSTEM

The Nipigon System provides an example where close speed regulation is necessary, but where difficulties in solution are introduced by the nature of certain of the loads.

Power supply for this system is secured from two plants on the Nipigon River. The first of these, at Cameron Falls, contains six units, each with a rating of 12,500 hp at 120 rpm under a head of 72 ft. The other plant, about 1 mile downstream, known as the Alexander Power Development, contains three units with a rating of 18,000 hp at 100 rpm, under a head of 60 ft. Composite regulating constant derived for nine units is 8,500,000.

The system peak has frequently exceeded the nominal rating of the power plants. The bulk of the power is transmitted at 110 kv to the cities of Fort William and Port Arthur, approximately 80 miles distant. Here, in addition to municipal and small industrial loads, there are several newsprint mills requiring, for satisfactory operation, speed regulation of a high order. At this point also are located the terminal grain elevators through which much of the grain grown in western Canada passes. In addition, at times a considerable block of power is used for steam generation, and the sudden fluctuation in this demand imposes additional duty on the regulating equipment. Transmission lines also reach out 90 miles northeasterly from the power plants to important mining districts east of Lake Nipigon and near Long Lake, where twelve or more mines use from 250 to 2500 hp each. Here, the effect of rapid changes in demand, which characterize mining loads, are offset somewhat by the number of separate customers and the relation that the size of the individual load bears to the total system load. In many instances the system load approaches within a few per cent the maximum capacity of the generating units. Under these conditions system speed is maintained within very narrow limits. Typical load and frequency charts are shown in Fig. 3.

NIAGARA SYSTEM REQUIREMENTS

Turning now to the largest system operated by the commission, the requirements to be met are of quite a different order; results are commensurate with these requirements. The Commission's Niagara System supplies power to the most populous part of Ontario, extending from Niagara Falls and Toronto on the east, to Windsor, Sarnia, and Goderich on the west, including 180 municipalities, 87 rural power districts, and a number of large industries supplied directly by the commission. The system extends beyond the easterly limits named, however, as a portion of its power supply is drawn from a power plant at Chats Falls on the Ottawa River, 200 miles east of Toronto, and from five others in the Province of Quebec, one of which is over 120 miles beyond Chats Falls.

Three power plants at Niagara Falls, having a combined capacity of 830,000 hp in 36 units, varying from 11,700 to 58,000 hp, supply the major part of the power for the system. The peak load in 1938 was approximately 1,400,000 hp, portions of which, however, are segregated from each other. The following comments apply to one portion of the system tied together electrically and having a peak load of about 800,000 hp. No value of the regulating constant is given for this condition, as power in large amounts enters the system from several power plants in the Province of Quebec, with varying numbers of units on this supply.

In such a system there are, of course, many industrial plants where very close speed regulation is essential and, because of

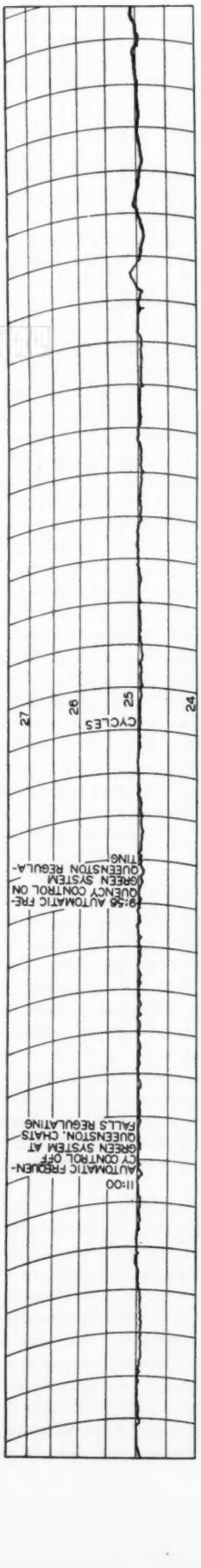


FIG. 4 TYPICAL FREQUENCY CHART; NIAGARA SYSTEM

(The load for which this frequency chart applies amounts to 500,000 to 600,000 kw, the power being distributed over a large area to many municipalities and industries. For the period from ten to eleven o'clock, automatic frequency regulation was in use.)

the size of the connected load and the diversity in demand, the requirements in this regard are readily met.

The system is of such size that variations in individual loads are of scarcely any moment. Even the sudden loss of the generating capacity of one of the large units, through some fault in the unit itself, would scarcely be noticeable throughout the system. The enormous amount of kinetic energy in the remaining units would prevent any material change in speed for the few seconds while other units were increasing their outputs to compensate for the capacity lost in the unit which had dropped out of service.

The problem in this case is one of sensitive and accurate governing to maintain at all times the normal frequency of the system. It has been the practice to assign speed regulation to one plant, other plants in the system increasing or decreasing their output in response to orders from the load supervisor located in a central office. Adjustment for minor variations in system load are automatically carried out by the governing equipment in the plant which at that time is responsible for speed regulation. The operators in this plant are required to give constant attention to the maintenance of correct frequency.

To improve the speed regulation further, devices to provide automatic frequency regulation have been applied to certain of the governors in the Queenston plant on the Niagara River. This equipment was built by the commission and is still in a state of development. A definite improvement has resulted from its use in the reduction of minor departures from the correct frequency, which otherwise would be unnoticed by the operator until the error had accumulated sufficiently to give an indication by comparison with whatever device is used to show constant frequency. It is significant that more nearly perfect uniformity of speed was experienced in operation of the coelostat controlling the telescope at the David Dunlop Observatory, north of Toronto, whenever automatic frequency control at Queenston replaced the manual control at other plants.

LOAD DISTRIBUTION

Procedure in distributing load among the plants supplying power to the system may be of interest at this point. As previously stated, power enters the system at the eastern boundary of the Province from five power plants in the Province of Quebec and from the Chats Falls plant on the Ottawa River. It is transmitted thence more than 200 miles at 220 kv to the Leaside Transformer Station at Toronto. Connection is made here to the 110-kv system extending outward from generating stations at Niagara Falls. There is no distribution of energy from the system between the eastern power sources and the Leaside Transformer Station. Apportionment of load among plants is determined by the load supervisor at Niagara Falls, who assigns to each plant the load that is to be carried at standard frequency, the amount carried by each plant being varied from time to time to suit the system-load curve, and various other circumstances.

In the portion of the system under consideration, two or three units at the Queenston plant may be in use, these usually being equipped with automatic frequency control. Operators at the eastern plants pick up the load assigned to their plant on instructions from the load supervisor, the load governors being set for the desired load at standard frequency. As normal variations in load in accordance with the daily load curve take place, all variations in output will take place on the regulated units at the Queenston plant connected in the system. As their maximum or minimum capacity is approached, as the case may be, suitable blocks of power are picked up or dropped by the eastern plants on instructions from the load supervisor.

Temporary disturbance in the system may cause the frequency to depart from normal. In such case, the load carried on each of the eastern plants may depart from that assigned by the load supervisor, but the operators at these plants are instructed to make no correction for changes in load caused by such temporary frequency departures. Restoration of system frequency will of course be made after the disturbance, but this is done by adjustments at the one plant then responsible for speed control. Usually, restoration of frequency redistributes the loads on the eastern plants to the amounts originally assigned by the load supervisor without action on the part of the local operators.

If disturbances in the system are of considerable moment, automatic frequency-regulating devices are disconnected automatically, and restoration of normal frequency accomplished manually.

At times, speed regulation is assigned to the Chats Falls plant on the Ottawa River, in which case manual control of frequency is necessary. The loads carried by other plants, including the Queenston plant, are in accordance with the assignments of the load supervisor. Fig. 4 shows that automatic frequency control is in use part of the time and manual control part of the time.

SUMMARY

The chart for the St. Joseph District illustrates a case where variations in frequency go far beyond the permissible limits for what is ordinarily known as satisfactory service. It accomplishes what is desired, however—it enables the mines served to obtain as great an output as possible from the available equipment.

The frequency charts for the other three systems provide an interesting comparison. In all cases satisfactory regulation is maintained, but certain peculiarities indigenous to the system are apparent in each instance. The character of the variations from constant speed are indicative of the peculiarities of the various controlling factors.

Two Years' Experience With High-Temperature High-Pressure Stations

The present review of operating experience with four typical high-temperature, high-pressure power-generating installations, among those originally presented in the "Power Panel Discussion" of December, 1937, deals principally with problems which have developed and means adopted for their correction. Great improvement in design has been accomplished and availability of both turbogenerator units and steam-generating units has increased materially. While in detail operating experience has varied largely, because of the variety of equipment involved, the composite picture presented of advance in the field of power generation, will serve admirably as a guide to designers of future equipment and to operators whose difficulties may have paralleled those contained in this joint discussion.

High-Temperature High-Pressure Superimposed Installations at Waterside Station[1]

By J. C. FALKNER,[2] NEW YORK, N. Y.

THERE have been very few changes in operating procedures at the Waterside Station since the installation of the high-pressure units two years ago. Such changes as have occurred have dealt mainly with the method of make-up, the method of watching water levels, chemical feed, sampling of boiler water, and the taking of steam samples.

Method of Operation Without Low-Pressure Boiler

It has not been possible, due to load conditions, to operate without some low-pressure boilers in service. A test of short duration with the high-pressure and low-pressure turbine units, running without low-pressure boilers, was made to check the operation of the back pressure regulator on No. 4 turbine and to familiarize the operating personnel with the difficulties to be expected when excessive swings occur. The tests proved very satisfactory. Loads were varied on both high- and low-pressure units without difficulty. An operating procedure has been worked out so that, as soon as conditions permit all low-pressure boilers will be shut down.

The distribution of load between the high- and low-pressure units will be made by the switchboard operators. In case of loss of low-pressure load, the back pressure regulator will automatically reduce the load on the high-pressure units but, in case load is lost on a high-pressure unit resulting in a reduction of low-pressure steam, the operator will reduce low-pressure load to a point where the low-pressure steam is normal.

High-Pressure Turbogenerator Units

Blade Troubles, Fouling Sources, and Cleaning Methods. Turbine generator No. 4, a 50,000-kw Westinghouse unit, was removed from service shortly after it was placed in operation September 10, 1937, in order to permit that company to remove the two rows of impulse blades. There were a few cracks in the shroud on the first row of impulse blades but in neither row were there any blade failures. These blades were removed on the advice of the company which had had trouble with other machines of similar design. With these blades out this machine operated without difficulty, except for a few shutdowns for inspection of the governor and bearings, until February, 1939.

On February 10, the turbine was taken out of service to examine bearings. Upon examination it was found that the bearing in the head end of the high-pressure turbine and the bearing near the flexible coupling between the main and feed-heating turbines showed cracking which necessitated their being rebabbitted at the Westinghouse Company's shop. When these bearings were replaced, the original shim positions were slightly changed.

In March, 1939, the unit was opened due to a rub being heard while the machine was being taken off the line. It was found that the coupling on the main-turbine side of the stub shaft between the main and feed-heating turbines had broken. The collar of the thrust bearing for the main unit is located on this stub shaft. The breaking of the coupling made the thrust bearing ineffective for the main unit and the spindle moved so that the high-pressure dummies rubbed and seized. One broken blade was found in the tenth stage and three pieces of shroud bands were found missing in the same row. The fact that the rotating parts of the high-pressure seal were integral with the spindle necessitated a complete new spindle for this unit. There is a difference of opinion as to the cause of the breaking of the coupling and as to whether the coupling broke before the rub appeared in the high-pressure dummies. The author is of the opinion that the coupling broke first, as there had been a slight change in alignment when the rebabbitted bearings were installed.

The new spindle with new blades, new dummies, and a new design of stub shaft coupling was installed in June, 1939, and the machine has run very satisfactorily since this change with the one exception of the governor which still has a hunting characteristic. This fact has necessitated the use of the load-limiting governor which has worked satisfactorily to date.

Until January, 1939, there was practically no evidence of blade fouling on either of the two high-pressure turbines.

Beginning in January, 1939, there was a capacity loss on the General Electric No. 5 turbine from 42,000 to 36,000 kw, taking place gradually over a 3-month period. The turbine was then taken off the line and allowed to cool for 12 hr, after which it was washed out with low-pressure steam from one of the boilers, without any water being admitted for desuperheating. This method of cleaning did not prove effective and on March 20 the machine was again taken out after provisions had been made to admit water with the steam. The washing lasted for a period of from 6 to 8 hr with a steam pressure of 15 to 20 lb per sq in., which was sufficient to turn the turbine. The steam temperature was 215 to 220 F. This method of cleaning proved satisfactory and the load on the turbine was increased to 42,000 kw. Ten days later the turbine again showed signs of fouling and it was again necessary to wash the turbine on June 10. After the third washing, the load was again brought back to 42,000 kw from 38,000 to which it had dropped.

[1] An Addendum to this paper appears on page 258 of this issue.

[2] Manager Production, Consolidated Edison Company of New York, Inc. Mem. A.S.M.E.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

No. 4 Westinghouse unit showed no sign of fouling when it was opened in March. It was first thought that the fouling of No. 5 General Electric unit was caused by the eliminator screens in the 5A and 5B boilers being plugged, thus allowing the solids to go out with the steam. Checks showed that the carry-over was less than 0.3 ppm which was as low as could be measured. A different method of blowing down the boilers has practically eliminated the fouling which proved that it is necessary to wash out lower waterwall headers and mud drums, while the boiler is off the line at intervals of not over two weeks.

Vibration Problems. On No. 4 unit there was a double frequency vibration on the generator end which was caused by the two-pole field of the generator. This trouble was eliminated when the machine went back into service as the field has been provided with slots to compensate for the lack of pull on the sides 90 deg from the field coils. No vibration has been experienced on either of the high-pressure turbines.

Protection Against Overheating With Loss of Load. There is a by-pass connection around the throttle valve of No. 4 unit which will pass a quantity of steam sufficient to keep the blades partially cool in case the throttle valve is closed and the generator is on the line operating as a motor. A steam gage in the high-tension control room gives indication to the switchboard operator of loss of steam at the boilers and, in case a throttle valve trips and the load starts down on the unit, the operator will trip out the generator in either case when the load has dropped to 2000 kw thus protecting the turbine rotor against overheating.

Consumption of Hydrogen. The Westinghouse unit uses an average of 10[3] cu ft of hydrogen per day, and the General Electric unit uses an average of 30 cu ft. There has been no trouble due to the use of hydrogen in either machine. The hydrogen was not used in the Westinghouse generator until July, 1939. A load of 45,000 kw at 0.9 power factor could be carried on this generator using air as a cooling medium.

Methods of Starting and Stopping High-Pressure Units. Both high-pressure units are started up integrally with the boilers; that is, as soon as there is a pressure of 200 lb in one of the boilers, the turbine is gradually brought up to 200 rpm, where it is held for approximately 1 hr. The exhaust from the turbine is used in the station auxiliary steam line at 5 lb back pressure. The pressure is then brought up and, when it reaches 600 lb, the exhaust steam is cut into the 200-lb header. From this point, 1 hr is allowed to bring the machine up to full speed.

In stopping the unit, no special precautions are taken other than those used on any low-pressure unit, with the exception, however, that the machine is kept on the turning gear for a period of time varying from 24 to 48 hr when stopping the Westinghouse machine and 24 hr for the General Electric machine.

Rise in Exhaust Temperature During Shutdown. There is a rise in the exhaust-casing temperature immediately after shutdown up to 670 F. This temperature has caused no trouble to date.

High-Pressure Boilers

Both sets of boilers for Nos. 4 and 5A units have operated successfully at 1300 lb and 450,000 lb of steam output, but to date it has not been possible to operate at the rated load of 500,000 lb per hr, without dissociation of water due to hot tubes in the top boiler-header sections or excessive carry-over.

Changes in baffle and washer designs in the drums have been made and it is expected that in a short time the guaranteed output will be obtained.

Trouble with refractory at the burner has been experienced and new carborundum blocks and refractory are now being tried out.

No trouble with slag-screen tubes has been experienced to date. The chrome ore over the slag-screen tubes has to be replaced every time the boiler comes off for cleaning.

Steam-Temperature Control. The temperature of the steam is 900 F and is controlled with a flue-gas by-pass around the superheater. This control has worked satisfactorily and it has been possible to maintain the desired temperature over all ranges of load. The outlet temperature of the exhaust steam leaving the turbine governs the steam temperature entering the turbine as the low-pressure units now installed in Waterside Station are not built to use steam at a temperature much over 525 F.

Influence of Ash Composition and Fluid Temperature on Cleaning Problems and Capacity. The ash composition and the fluid temperature have a great influence on the cleaning problems of the high-pressure boilers. Coal that has an initial ash-deformation temperature of not less than 2200 F has been determined upon after 2 yr of operation. At present, the cleaning of boilers is done by hand lancing and by taking the boiler off the line for a few hours, about twice a week, thus allowing the slag formation on the tubes and side walls to cool and fall off.

An experiment on the problem of cleaning the side walls of the high-pressure boilers by the use of a hand lance of special design, which has been used successfully at the East River Station for many years, is now being conducted. Also a nozzle to use water on the steam-generating tubes is being made up for trial.

Three men per watch of 8 hr each are continuously occupied in keeping slag off the boiler and superheater tubes. Soot blowers are used on the economizer tubes with good results.

Experience With Lighting Devices. Illuminating gas from the street mains is used in bringing the boilers up to pressure and also to ignite the pulverized fuel. A period of 5 to 6 hr is taken to bring the boiler up from cold. The gas is ignited by an electrical "glow bar" which is located directly in the path of the gas. The glow bars have burned out quite frequently, but as there are four burners per boiler, no trouble whatsoever has been experienced with the ignition of the coal. The induced-draft fans are always in operation before the gas is turned on the boiler.

Performance of Mills, Power Consumption, Life of Parts, and Sensitivity to Various Coals. The performance of the mills has been uniformly good. They give a fineness of 80 per cent through 200 mesh, 96 per cent through 100 mesh, and 99 per cent through 60 mesh. The mills run very quietly, and very little trouble has been experienced outside of the replacement of mill rollers after their normal expected life. The average life of the rollers is 85,000 tons; scrapers, 18,000 tons; and bull rings 85,000 tons. Only two grades of coal have been used on these mills, namely, Coleman "Colver" and Weaver "Revloc." As these two coals are the only coals used, the data on the sensitivity are rather limited.

Flame Stability at Varying Loads. The flame stability has been satisfactory at all loads above one third of the rated capacity of the boiler. A pulsation in the furnace has been experienced, the cause of which the boiler manufacturer is attempting to ascertain. A new primary fan inlet is being tried out as tests show that the initial pulsation starts at that point.

Feedwater Treatment, Boiler Concentration, and Internal Cleanliness. The Hall treatment is used in these boilers and consists of the following chemicals:

An average of 86 lb of disodium crystallized phosphate is used per 24-hr day for two boilers.

An average of 21 lb of sodium hydroxide is used per 24-hr day for two boilers.

The phosphate is introduced directly in the boiler drum by means of high-pressure pumps, while the sodium hydroxide is added by gravity to the boiler feed between the deaerator and the boiler-feed-pump suction.

Samples of boiler water are taken three times per day and ana-

[3] This system has been in operation for only a very short time.

lyzed for sodium hydroxide, sodium chloride, and sodium phosphate.

The boiler concentration is held at 450 to 500 ppm, but it is thought that by the use of boiler-drum baffles, which have now been installed, the boiler concentration can be carried as high as 800 ppm without excessive carry-over. Small amounts of scale were found in the boiler tubes, while the drum was free from scale, and very little deposit was found on the side-wall tubes.

Experience With Loss of Feedwater. There have been several cases where the lack of feedwater has necessitated the shutting down of the high-pressure boilers. On two occasions, the side waterwall of one boiler ruptured, and as the feed lines for both boilers for one unit come off the same header, the pressure dropped, thereby necessitating the immediate removal of the other boiler from the line. Another shutdown of the boiler due to the loss of feedwater occurred when the automatic valve, controlling the feedwater level in the drum stuck, and the water tender, while endeavoring to put the valve on hand operation, jammed the valve in such a manner that the valve closed, and the boiler had to be taken off the line. No shutdowns due to loss of feedwater to the boiler feed pumps have occurred.

Control, Availability, and Maintenance of Boiler Feed Pumps. Four of the six boiler feed pumps now in use at Waterside are motor-driven; the remaining two being driven by steam turbines. The two steam-driven pumps are so designed that they can be brought up to speed almost instantaneously. The motor-driven pumps are driven by constant-speed motors. The boiler feed pressure drop is taken across the boiler feed valve, there being no excess pressure valves used.

The availability of the Worthington boiler feed pumps has been 97.6 per cent. There has been practically no maintenance on these pumps other than the normal packing of the glands.

One of the difficulties experienced with steam-driven pumps has been the leakoff valve from the outlet header to the suction header, which failed on several occasions. The trouble was caused by an excessive drop across the valve and this was finally overcome by putting two valves in series with a pipe made with labyrinth orifices in it, in order to help in building up a resistance to the water flowing through the valves. The pump manufacturers asked that the steam-driven boiler feed pump always be operated with the by-pass valve partly cracked so as to permit some water to pass through the pump at all times.

Air Heaters

Means of Preventing Condensation During Starting on Light-Load Operation. There has been no means of preventing condensation in the air preheaters during the starting up of boilers or while operating at light loads, and this has caused considerable fouling of the air preheaters with the consequent loss of boiler capacity until the air preheaters were cleaned or washed. Due to load conditions one of the four boilers had to be taken off every night.

An experiment was tried using a movable air jet, which blew down from the air-inlet side of the air heater. The nozzle was so adjusted that it would move in and out so that all parts of the surface were blown every 2 or 3 hr. Compressed air at 125 lb pressure was used for this blowing. Steam-jet soot blowers originally installed with the air preheaters operated partially satisfactorily. A by-pass around the air preheaters has been installed on Nos. 4A and 4B boilers and, if it proves successful, a similar by-pass will be installed on Nos. 5A and 5B boilers.

Methods of Cleaning. The most successful method of cleaning air preheaters is by washing them with boiler feedwater which usually restores the draft drop through the air preheaters to its original figure.

Effect of Fouling on Capacity and Economy. Several times during the life of the boilers, it has been necessary to reduce the capacity of the boilers due to the lack of secondary air, because of the dirtiness of the air preheaters. When this condition occurs it is necessary to take the boiler off the line at the earliest opportunity and wash the air preheaters.

Deaerators

Effectiveness of Deaeration, Steam-, and Water-Flow Control. It has been found that, when the steam pressure in the deaerator is kept above 2½ lb, the deaerator will give almost 100 per cent deaeration. If, however, the steam pressure is allowed to vary for any reason, there will be a certain amount of oxygen fed into the boiler.

The steam to the deaerator is controlled by a regulating valve at the feed-heating tubine. The operation of this valve has been investigated and it was found that there is an intermittent flow of steam from the feed-heating turbine to the deaerator with this type of control. A new control valve will be installed at the deaerator to insure sufficient steam at all times for complete deaeration.

The water flow to the deaerator is controlled by a float valve which is actuated by the water level in the deaerator. Some trouble has been experienced in maintaining the water level in No. 4 deaerator, while no trouble has been experienced on No. 5. A new control valve, the same as on No. 5, will be installed at an early date.

Condenser-Hot Well Deaeration. The feedwater is deaerated in the condensers of the low-pressure turbines before being sent to the deaerators of the high-pressure units, but there is equipment through which some air may come in contact with the feedwater; such as leakage through the packing of the hot well pump, etc., which necessitates the use of a deaerator.

Deaeration of Make-Up Water. The deaeration of make-up water takes place in the condensers of the low-pressure units, and the make-up water comes over mixed with the condensate from the unit to which it was admitted.

Since the installation of the high-pressure units in Waterside, it has not been possible, due to load conditions, to cut out the low-pressure boilers. The supply of all make-up water needed in the station is fed to the low-pressure boilers, so that the high-pressure boilers have 100 per cent condensate.

Effect of Filling Idle Boilers on Deaerator Performance. On

TABLE 1 PERFORMANCE OF HIGH-PRESSURE INSTALLATION AT WATERSIDE STATION

High-pressure turbine generators	Unit No. 4	Unit No. 5
Date in commission	Sept. 10, 1937	June 29, 1938
Kilowatthours generated	325,909,000	258,260,000
Maximum kw load on generator	47000	52000
Service hours	11377.4	8250.6
Total available hours	12379.1	8980.3
Idle hours[a]	1001.7	729.7
Outage hours	4165.4	556.5
Period hours	16544.5	9536.8
Availability factor (ratio of available hours to period hours), per cent	74.8	94.2
Use factor (ratio of average load for service period to rated capacity), per cent	54.0	59.1
Capacity factor (ratio of average load for period to rated capacity), per cent	37.2	51.1

High-pressure boilers	No. 4A boiler	No. 4B boiler	No. 5A boiler	No. 5B boiler
Date in commission	Sept. 10, 1937	Sept. 10, 1937	June 29, 1938	July 25, 1938
Maximum steaming rate for 1 hr, lb	503300	501100	524800	509800
Service hours	9914.7	10546.8	6788.8	7292.6
Idle hours[a]	905.7	902.9	760.7	400.6
Total available hours	10820.4	11449.7	7549.5	7693.5
Outage hours	5724.1	5094.8	1987.3	1218.0
Period hours	16544.5	16544.5	9536.8	8911.5
Availability factor (ratio of available hours to period hours)	65.4	69.2	79.2	86.3

[a] Hours available but not in operation due to conflicting outage of dependent equipment.

several occasions an idle high-pressure boiler was filled too fast and thereby drained the deaerator to such a point that make-up had to be put into the deaerator from the city water connection, thus adding to the work imposed upon the deaerator, with a resulting decrease in deaerator efficiency. The boiler operators are cautioned against filling the boilers too rapidly, and also against a too rapid heating of boiler tube and drum surfaces.

High-Pressure Heaters

All of our heaters are of the vertical type with a floating head.

Tube Materials, Cleanliness, and Leakage. The only trouble experienced with cleanliness of the tube has been that of oil coming from the exhaust steam, necessitating the washing of the tubes.

Some trouble has been experienced due to leakage of the heads of the heaters. Due to the location of these heads in the boiler room, the work of removing leaking heads is very difficult. There have been 43 tube failures in the No. 5 heaters and no tube failure in the No. 4 heaters. As two heaters are operated in parallel, it is possible to take one heater out of service to make the necessary repairs without excessive drop in feedwater temperature.

Table 1 gives the performance record of the two 53,000-kw topping turbines and the four high-pressure boilers at Waterside Station, covering the period from the date of installation to July 31, 1939.

Fisk Station Topping-Unit Operation[1]

By A. E. GRUNERT,[2] CHICAGO, ILL.

THE FISK STATION high-pressure topping plant has at this time completed nearly two years of operation. The first two months of its operation have been previously outlined.[3] That discussion was largely a disclosure of operating difficulties which invariably reveal themselves with initial operation of new developments of this nature and of defects in design which caused these difficulties.

The Fisk Station topping unit was the first capacity addition to the Chicago system following several depression years during which no new generating equipment was installed. It naturally followed that the basic trend at that time toward higher steam pressures and temperatures influenced design. The installation is not particularly different in design when compared with those contemporary to it but, being an early topping installation, it is representative of development. The topping installation was first put into service on October 8, 1937, by a plant personnel having no previous experience with this type of equipment.

The period, following that covered in the previous discussion[3] and extending to the present time, is one in which early operating and design difficulties were largely overcome. In the following discussion an endeavor is made to log the various trials and tribulations experienced in the course of this operating period. It is felt that efforts made to correct conditions which caused difficulties, have been successful to a degree that future operation may now be viewed with confidence. It is consequently believed that subsequent availability and performance records will compare favorably with those of any installation of this type and of this kind.

[1] Covering the period October, 1937, to August, 1939.

[2] Superintendent of generating stations, Commonwealth Edison Company. Mem. A.S.M.E., Melville Medalist, 1931.

[3] "Discussion of Preliminary Operations of the Fisk Station Topping Unit of Commonwealth Edison Company," Power Panel Discussion on New High-Temperature High-Pressure Stations, Trans. A.S.M.E., vol. 60, FSP-60-13, July, 1938, p. 413.

Details of Installation

In brief, the installation consists of a 30,000-kw 1200-lb per sq in. 900-F turbine generator, supplied with steam by two 375,000-lb per hr boilers. All auxiliaries are electrically driven except boiler feed pumps. Turbines for these three pumps take steam from the high-pressure header and exhaust to the low-pressure header in parallel with the topping turbine. The capacity of this topping installation was so chosen that the exhaust would supply the steam demand of two 25-cycle horizontal units. This combination forms in effect a flexible and efficient 90,000-kw cross-compound unit. More detail of the equipment in the installation is contained in Table 1. Figs. 1 and 2 are, respectively, cross-sectional views of the steam generators and turbine that are used in the topping installation, and Fig. 3 is a flow diagram of the topping plant.

Fig. 4 shows graphically the record of availability of each of the two high-pressure boilers and of the turbine generator, which together comprise the topping plant. The cause of each case of outage is briefly stated and those cases which may be considered as forced outages, due to conditions of the particular boiler or the turbine generator, are specially indicated. The monthly production economy of the combined plant under the actual conditions prevailing in each month is also shown. A more detailed analysis of the thermal performance appears in Table 2. It will be noted that the normal indicated production economy of the combined plant will be approximately 12,800 Btu per net kwhr. Some reduction in this figure is anticipated when certain contemplated alterations in the low-pressure system are completed. The plant was originally designed for an actual performance of 12,500 Btu per kwhr and it is expected that this performance will eventually be attained. It should be noted in this connection that the low-pressure equipment in this plant is of a rather old type and design.

Turbine-Generator Operation

The turbine-generator unit itself had an availability factor of 85.7 per cent for the three months' operation in 1937, 90.7 per cent for the year 1938, and 100 per cent for the first six months of 1939. The outage chart extends one month beyond this period to August, 1939, and indicates the fact that at that time the topping plant was out of service for scheduled inspection and overhauling. During the 22-month period covered by the chart there were nine outage periods with but three attributable to the turbine itself. The two longest periods from July 24 to August 23, 1938, and July 6, 1939, through the remainder of the month, were for general inspection and overhauling. During both of these periods repairs to spindle blading were necessary as will be explained subsequently. The generator, since installation, has had a perfect availability record.

Referring to the outage chart for the turbine, Fig. 4, the outage between October 8 and October 13, 1937, was necessary for adjustment of a throttle-valve bushing and for minor adjustment and tune up of the governing gear. Further checks were made of the thrust bearing and shaft seals, and the dummy clearance was adjusted on October 23. The next outage which started on October 27, 1937, is not attributed to the turbine as it was caused by boiler carry-over deposits which caused the second inlet valve to stick. Heavy deposits of boiler carry-over were also found in the valves of the header system and these deposits were removed by flushing the header with water. The three subsequent outages of January 7, January 25, and May 20, 1938, are likewise not attributable to the turbine, as is indicated by the causes which are shown on the outage chart. The July 2, 1938, outage was necessary, however, when the first and second admission valves were found to be sticking. Upon removal and inspection of the valves the head of a $^3/_4$-in. cap screw was found lodged between the seat

TABLE 1 GENERAL DATA, FISK STATION HIGH-PRESSURE PLANT

STEAM GENERATING UNITS:	
Manufacturer	Babcock & Wilcox Co.
Number	Two
Max.Continuous Output (each)- lb/hr	412,500
Normal Max. Rating " "	375,000
Pressure- Superheater Outlet- lb.ga.	1275
Temperature- " " F	910
Feed Water Temperature F	325
BOILER:	
Heating Surface - sq.ft.	7106
Tubes- thickness	4 in.O.D. x 7/16 in.
Design Pressure- lb.ga.	1400
SUPERHEATER:	
Type	Convection in parallel flow circuit with the economizer.
Heating Surface- sq.ft.	19,800
Tubes- Thickness	2 in. #4 gauge & 2¼ x .26 in.
Design Pressure- lb.ga.	1400
ECONOMIZER:	
Type	Continuous tube in parallel flow circuit with the superheater.
Heating Surface- sq.ft.	12,000
Tubes- Thickness	2 in. #4 gauge & 2¼ x .26 in.
Design Pressure- lb.ga.	1490
AIR HEATER:	
Type	Tubular
Heating Surface - sq.ft.	56,200
Tubes- Thickness	2½ in. #11 gauge
FURNACE:	
Type	Two stage, water wall, slag-tap with water cooled floor
Heating Surface- sq.ft.	5070
Tubes- thickness	3¼ in. x 11/32 in.
Heat Liberation at 375,000 lb/hr. - Btu/cu.ft/hr.	31,600
PULVERIZED FUEL EQUIPMENT	
Manufacturer	Foster-Wheeler Corp.
Pulverizers- Type	D-S, Hardinge, Conical, Ball
Number per boiler	Two
Capacity- tons per hour	15
Exhausters- Type	Paddle Wheel
Burners- Type	Multiple intertube, Vertical firing.
Number per boiler	4
FORCED DRAFT FANS:	
Manufacturer	B. F. Sturtevant Co.
Type	Turbovane with vane control
Number	2
Capacity (each) cfm at 140 F	128,000
Static Pressure- In.H_2O	9.7
INDUCED DRAFT FANS:	
Manufacturer	B. F. Sturtevant Co.
Type	Double inlet, turbovane
Number	2
Capacity- cfm at 360 F	170,000
cfm at 400 F	222,000
Static Pressures	10.9 in. - 16.0 in.
Speeds- rpm	720 - 900
Motor Sizes- (two motors per fan)- hp	500 - 900

PERFORMANCE DATA OF STEAM GENERATING UNIT:		
Steam- lb/hr	250,000	375,000
Steam press.Suphtr.Outlet-#/sq.in.	1275	1275
Steam temp. " " -F	825	910
Water Temp. Econ. In -F	325	325
Water Temp. " Out -F	451	502
Gas Temp. Air Heater Out -F	318	350
Air Temp. " " " -F	472	486
Per cent CO_2 Furnace Out	15.3	15.3
Per cent CO_2 Air Heater Out	14.8	14.8

NOTE: Figures given are design or guarantee values.

PRECIPITATORS:	
Manufacturer	Research Corporation
Type	"Cottrell" electrostatic
Normal Gas Rating- cfm at 360 F	180,000
Guaranteed Recovery- Per cent	90
Maximum Gas Rating- cfm at 400 F	212,000
Pressure drop at Max.Rating- In.H_2O	0.25
TURBINE-GENERATOR:	
Turbine manufacturer	Allis-Chalmers Mfg. Co.
Capacity- kw	30,000
Power Factor- per cent	95
Kv-a	31,577
Frequency	25-cycles, 3-phase
Speed- rpm	1500
Steam Press.at Throttle-#/sq.in.ga.	1200
Steam Temp. at Throttle-F	900
Steam Press.at Exhaust- #/sq.in.ga.	220
Number of Stages	2-impulse, 34-reaction.

PERFORMANCE DATA OF TURBINE-GENERATOR:			
Load- kw	7,500	15,000	30,000
Exhaust Temp.- F	621	561	525
Exhaust Heat- Btu/lb	1329	1297	1278
Water Rate- lb/kwh	34.0	25.25	21.9
Throttle flow-M-lb/hr.	255	379.75	657

NOTE: Figures given are guarantee values.

BOILER FEED PUMPS:	
Pumps	Worthington Pump & Mfg. Corp.
Number	3
Type	Solid Barrel, Vertical Split
Capacity- gpm	1000
Discharge Press.- lb/sq.in.	1600
Turbines	Westinghouse Electric & Mfg. Co.
Number	3
Type	Impulse
Rating- hp	1135
Throttle Steam Press.- lb/sq.in.ga.	1200
Throttle Steam Temp. - F	900
Exhaust Press.- lb/sq.in.ga.	220
DE-SUPERHEATING AND PRESSURE REDUCING EQUIPMENT:	
Manufacturer	Republic Flow Meters Co.
De-superheater type	"Smoot," Venturi, Storage
Size	16 in. x 24 in.
Capacity- lb/hr.	700,000
Quick Opening Valve	"Smoot" Hydraulic - 10 in.
Press. Reducing Valve	"Smoot" Hydraulic - 10 in.
EVAPORATOR EQUIPMENT:	
Number of evaporators	2
Type	Horizontal, Submerged
Heating Surface- sq.ft.	298
Max.Capacity of two shells- lb/hr	27,000
Type of Evaporator Condenser	Horizontal, Floating Head.
Heating Surface- sq.ft.	1152

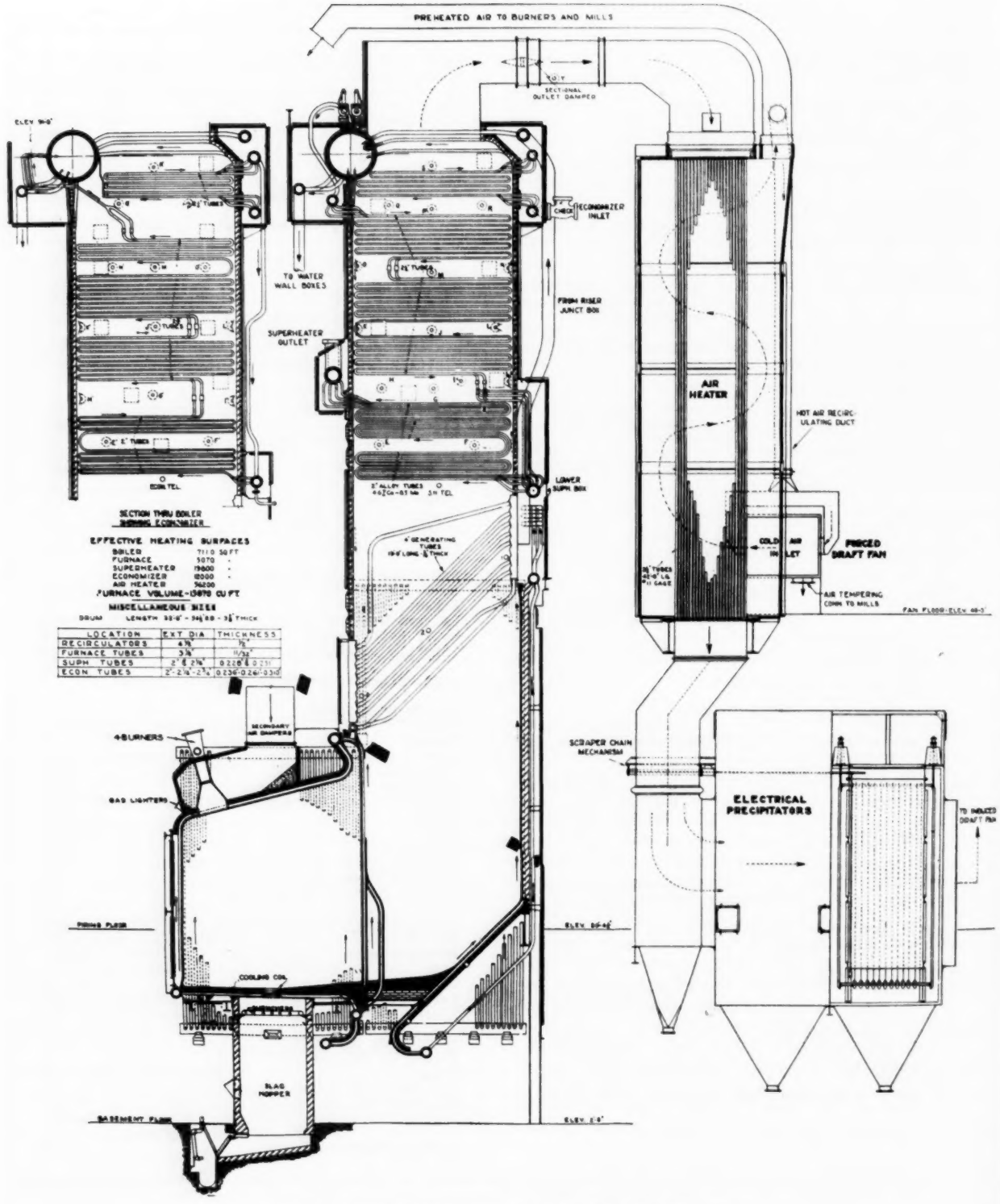


FIG. 1 GENERAL ARRANGEMENT OF BOILERS AT FISK STATION

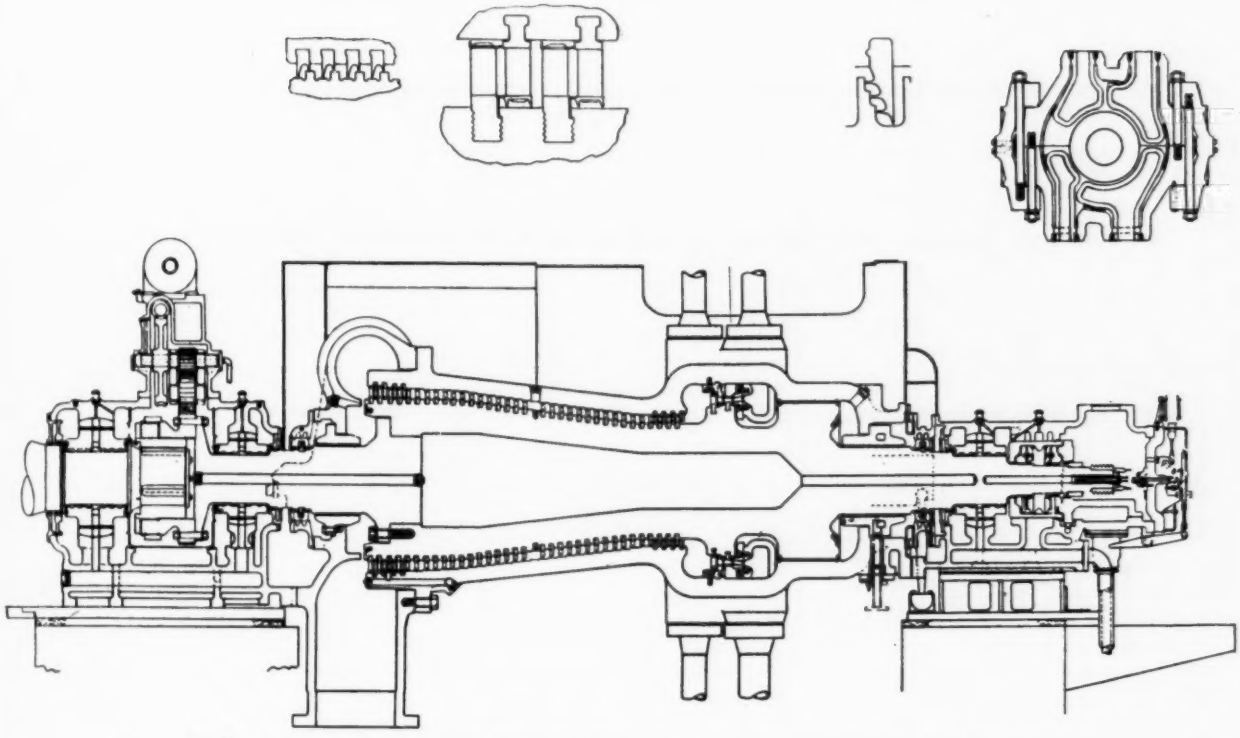


FIG. 2 CROSS SECTION OF HIGH-PRESSURE TURBINE USED IN TOPPING UNIT OF FISK STATION

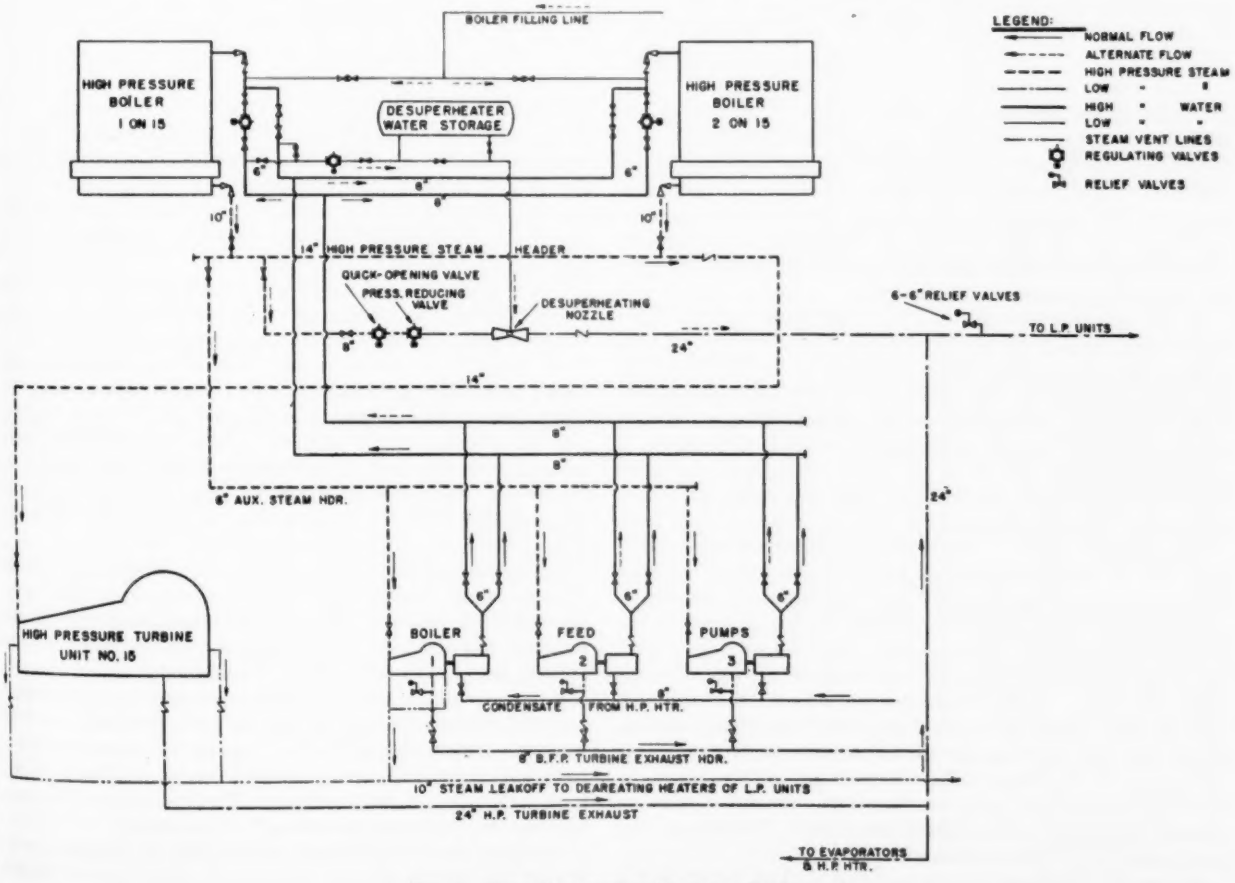


FIG. 3 FLOW DIAGRAM OF TOPPING PLANT

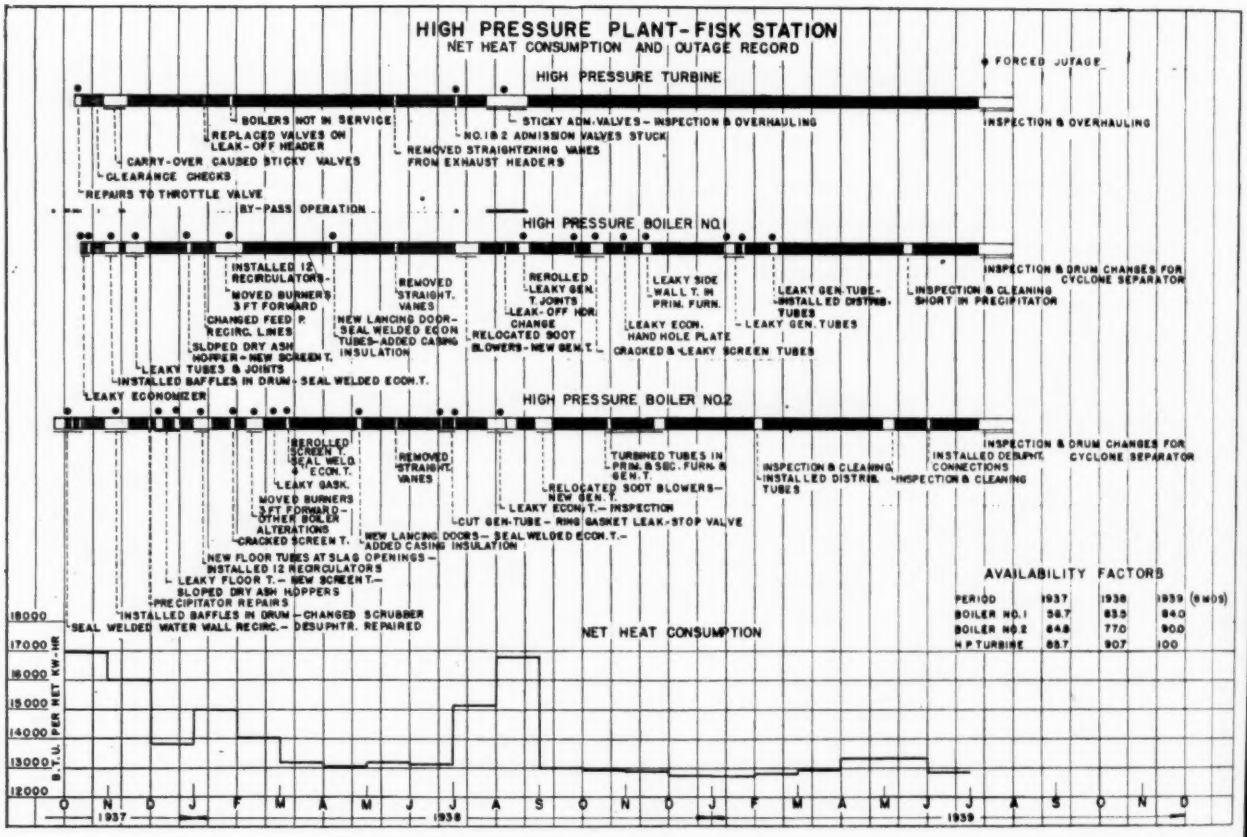


FIG. 4 NET HEAT-CONSUMPTION AND OUTAGE RECORD

and disk of the second inlet valve. Upon inspection of the throttle valve it was found that this cap screw head had broken from one of the screws which held the throttle-valve seat to the throttle-valve body. The sticking of the first inlet valve was caused by valve-stem packing which had lost its plastic qualities and had caused excessive friction. A combination die-formed resilient metal-ring and soft asbestos-ring packing was substituted but this packing lasted only three weeks.

As the first general inspection of the turbine was scheduled to occur soon, the date of the inspection was advanced to July 24, 1938, because of the failure of this packing. It was found that the metal-foil packing rings had disintegrated under high temperature and particles had adhered to the valve stems. To ease the duty of the packing the leakoff was changed from 225 lb per sq in. to approximately 4 lb per sq in. Soft packing in die-formed rings was then installed and the number of rings was decreased to that required by the new pressure. This change proved satisfactory.

The inspection made at this time showed that stellite seat facings were in perfect condition. Three broken $3^1/_4$-in. cylinder studs, which were judged to have failed due to defects in material or machining, were replaced in the bolting that is used for the turbine cylinder joint. The admission-valve operating mechanism needed correction where clearances did not correspond to design dimensions. Lapped admission-valve-cover joints made up without gaskets held tight under high pressures and temperatures. Crane ring-type joints on all admission inlet bends were found to be in good condition. The trip-gear mechanism, which on several occasions had operated sluggishly, was fitted with an Alemite lubricating system during this outage. This change has materially improved its operation.

During the inspection, it was found that the lock washer which had been released from under the head of the broken cap screw, mentioned previously, had gone through the valves and nozzles and had lodged in one of the steam passages of the second moving row of impulse blading. The inlet edges of the first moving row of impulse blading were damaged by the passage of the lock washer but the relatively slight damage done was corrected by local grinding.

The blade elements of the turbine consist of a 45-in-mean-diam 2-row impulse wheel, followed by 34 rows of reaction blading. Nozzle blocks for the impulse blading are built up of a series of blade elements in which steam passages are completely machined before the elements are welded together as a single nozzle-block unit. The first row of moving blades is $2^1/_2$ in. wide as shown in Fig. 5. Proportions of these blades illustrate the massive construction used to resist the heavy intermittent forces encountered. All blades are made of a standard alloy known as "Cyclops No. 17-A," a 19 per cent nickel, 8 per cent chromium alloy.

The integral shrouds of the first moving row of impulse blades were originally welded together in groups of five. During inspection it was found that many of the welded joints had cracked, although the welding between several pairs of blades still remained sound. The strength of the original welding, however, had been somewhat reduced during manufacture in securing a smooth cylindrical surface by turning down the blade tips after welding. The blades of the first impulse row were rewelded in pairs during overhauling and a much heavier welding bead was used than in the original assembly. The full section of the welding bead was left in place for strength in this repair.

A slight looseness was noted in the blades of the first moving row on the impulse wheel. The manufacturer did not feel that

TABLE 2 AVERAGE MONTHLY OPERATING DATA, COMBINED HIGH-PRESSURE PLANT

	1938												Average 1938	1939					
	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.		Jan.	Feb.	March	April	May	June
COAL, TONS PER MONTH																			
Boiler No.1																			
1. N Mill, Tons	3171.2	6216.4	6135.5	6264.0	6470.2	6861.4	2862.0	4714.6	6518.2	4644.8	5158.2	7413.6	5702.3	4658.0	5395.6	7598.4	6977.8	5809.4	6775.2
2. S Mill "	2949.0	5623.2	6169.4	6261.8	6648.0	7346.2	2929.2	4751.6	6732.2	4873.2	5308.8	7575.4	5764.0	4657.8	5391.6	7501.4	7046.4	5725.2	7161.0
3. N Mill Hours	300	564	738	602	653	661	311	511	651	446	501	707	524	444	520	726	683	549	671
4. S Mill "	303	531	739	601	665	701	316	516	666	456	505	726	560	447	518	718	683	548	687
5. N Mill Tons/hr.	10.6	11.0	11.0	10.4	9.9	10.4	9.2	9.2	10.0	10.4	10.3	10.5	10.3	10.5	10.4	10.5	10.2	10.4	10.3
6. S Mill " "	9.7	10.6	11.1	10.4	10.0	10.5	9.3	9.2	10.1	10.7	10.5	10.5	10.3	10.4	10.5	10.4	10.4	10.5	10.4
7. Mill System Kwhr/ton	26.0	24.7	24.5	28.7	28.5	27.4	30.2	30.5	28.6	27.2	27.3	27.2	27.4	26.7	25.5	26.1	26.5	26.3	26.1
8. N & S Mills - Fineness																			
% Thru																			
48 Mesh	-	-	99.6	99.0	99.6	99.9	99.8	99.8	99.9	99.8	99.9	99.8	99.7	99.8	99.6	99.8	-	99.4	99.7
100 "	-	-	95.0	92.5	95.6	96.8	96.5	96.8	97.4	96.9	94.9	97.2	96.0	97.4	94.8	96.5	-	94.3	96.6
150 "	-	-	87.4	84.3	89.0	90.7	89.8	91.4	92.3	91.5	86.4	91.6	89.4	91.9	86.6	89.3	-	85.1	91.6
200 "	-	-	75.8	73.4	78.8	80.7	80.3	82.9	85.6	84.4	75.0	84.0	80.5	83.8	75.2	77.5	-	80.2	84.1
Boiler No.2																			
9. N Mill, Tons	3651.0	3180.1	7122.2	6300.0	6242.8	6246.0	5078.6	2761.6	4947.8	6542.8	5318.6	7383.4	5386.1	6686.4	5447.6	7018.0	6327.6	5475.8	6357.6
10. S Mill "	3030.2	3094.0	7285.0	6274.8	6691.8	6406.0	5076.8	2563.6	4980.2	6885.6	5435.0	7898.8	5483.5	6678.2	5536.6	7286.6	6992.8	5482.0	6624.6
11. N Mill Hours	382	295	692	612	613	611	526	305	489	612	505	721	529	650	542	726	672	532	661
12. S Mill "	321	292	693	614	678	610	527	292	483	637	503	738	532	674	565	739	688	535	683
13. N Mill Tons/hr	11.1	10.7	10.3	10.1	10.2	10.2	9.7	9.1	10.1	10.7	10.5	10.2	10.3	10.1	10.1	9.7	9.4	10.3	9.6
14. S Mill " "	9.5	10.6	10.5	10.2	10.3	10.5	9.6	8.8	10.3	10.8	10.8	10.8	10.3	10.4	10.2	9.9	9.6	10.3	9.7
15. Mill System Kwhr/ton	26.0	27.6	26.4	28.1	27.1	26.9	28.4	31.1	27.6	26.4	26.4	26.6	27.2	26.6	26.7	27.4	28.3	26.6	27.6
16. N & S Mills - Fineness																			
% Thru																			
48 Mesh	-	-	99.8	99.8	99.6	99.6	-	99.9	99.8	99.8	-	99.8	99.8	99.6	99.7	99.7	99.7	99.7	99.9
100 "	-	-	97.4	97.0	94.8	95.2	-	97.6	96.5	97.4	-	97.2	96.7	96.4	95.8	96.7	97.2	96.6	98.2
150 "	-	-	91.3	89.2	86.7	89.6	-	93.5	89.9	92.8	-	93.0	91.4	90.1	89.1	87.8	92.6	91.6	96.0
200 "	-	-	82.1	83.2	76.8	81.0	-	87.5	80.3	84.3	-	84.7	83.5	80.3	77.5	79.1	86.8	81.7	94.2
ANALYSES																			
17. B.t.u./lb., Dry	11978	11818	12073	12563	12532	12073	11847	12033	12094	11929	12189	12065	12232	12272	12321	12189	12229	12116	12335
18. B.t.u./lb., as received	10574	10301	10742	11068	10828	10695	11158	11166	10939	10574	10966	10738	10776	10774	10867	10733	10637	10539	10973
19. Moisture, %	11.66	12.86	12.31	11.68	12.83	12.76	10.59	10.91	11.77	11.71	13.01	12.51	12.38	12.71	11.80	11.91	12.98	13.18	12.42
20. Ash	13.04	14.15	11.79	10.53	12.15	12.78	11.71	10.97	11.82	13.49	12.26	11.25	12.08	11.95	12.12	12.53	12.25	11.84	11.03
Boiler No. 1 Refuse																			
21. Furnace Slag - % Ash	-	-	99.75	99.78	99.60	-	99.69	99.72	-	-	-	-	99.71	99.72	99.73	99.73	99.71	99.66	-
22. Fly Ash "	-	-	99.77	99.88	99.82	-	99.84	99.77	-	-	-	-	99.78	99.56	99.79	99.69	99.48	99.62	-
Boiler No. 2 Refuse																			
23. Furnace Slag - % Ash	-	-	99.79	99.71	99.89	-	-	-	99.75	99.75	-	-	99.66	-	99.89	99.70	-	-	99.02
24. Fly Ash "	-	-	99.77	99.68	99.87	-	-	-	99.80	99.82	-	-	99.71	-	99.68	99.70	-	-	99.37
BOILER ROOM																			
25. Feedwater Million lb/month (To economizer)	198.4	263.9	478.9	400.3	413.5	417.3	266.3	296.0	382.6	352.0	340.1	500.0	358.3	378.6	358.6	477.0	429.9	348.1	442.3
26. Avg. Output - Boiler No. 1 M-lb/hr.	303	304	317	332	307	304	305	303	318	318	312	318	317	304	301	329	317	317	313
27. Avg. Output - " " No. 2 " "	310	309	324	326	316	301	319	290	328	318	320	320	317	324	331	327	307	317	323
28. Boilers Hours Steam/month	627	854	1435	1220	1330	1379	899	624	1187	1115	1029	1475	1116	1122	1056	1467	1387	1058	1385
29. Boiler " Cold/Month	861	480	52	218	98	61	639	664	255	373	411	33	348	344	288	21	51	390	52
30. Boiler " Steaming/Month, %	42.2	64.3	96.5	84.8	93.4	95.8	61.0	52.3	82.3	74.9	71.5	99.1	76.4	75.6	78.6	98.6	96.5	72.8	96.2
31. Boiler Hours Cold/Month, %	57.8	35.7	3.5	15.2	6.6	4.2	43.0	47.7	17.7	25.1	28.5	.9	23.6	24.6	21.4	1.4	3.5	26.2	3.8
32. Steam Press - Superheater Outlet, Lb. Ga	1198	1188	1213	1224	1227	1223	1232	1217	1217	1225	1228	1229	1220	1228	1228	1228	1242	1235	1232
33. Steam Temp/- " " " F	880	891	888	896	890	886	888	883	897	890	883	887	888	880	885	886	891	894	890
34. Feedwater Temp. F	307	324	323	323	320	320	317	320	318	319	318	317	319	317	320	316	317	318	316
35. Actual Evaporation, Lb.Steam/Lb. Coal	7.47	7.28	7.68	7.96	7.84	7.75	8.33	8.29	8.28	7.69	7.99	8.25	7.90	8.11	8.11	8.09	7.94	7.86	8.21
36. Added Heat, B.t.u./lb.	1150	1139	1138	1142	1143	1136	1142	1143	1139	1146	1140	1145	1142.5	1139	1139	1146	1146	1147	1147
37. Overall Boiler Room Efficiency, %	81.3	80.0	81.2	82.0	84.3	83.9	85.4	84.9	86.6	86.2	86.0	86.4	84.1	85.8	85.0	86.2	85.4	85.6	85.8
Gas Temperatures, F																			
38. Air Heater Inlet	693	681	704	700	681	688	698	682	678	678	678	671	682	673	671	673	674	653	695
39. Air " Outlet	325	339	407	403	373	391	381	378	376	374	370	369	378	370	369	365	373	365	395
40. Drop	394	322	297	297	308	297	317	304	309	305	304	302	304	303	302	304	301	293	300
Air Temperatures, F																			
41. Air Heater Inlet	81	97	100	88	91	95	98	102	103	110	109	95	97	96	91	102	111	114	118
42. Air " Outlet	517	532	562	524	500	522	570	580	580	585	550	556	555	547	545	566	572	561	565
43. Rise Thru Air Heater	436	435	462	436	409	427	472	478	477	475	441	455	458	451	454	464	461	447	447
TURBINE ROOM																			
44. Avg. Load kw	34700	42700	72200	63000	64900	63500	35600	29200	56800	52200	51700	73900	52500	55800	58900	69400	63900	50400	67700
Temperatures, F																			
45. Throttle Temp. - H.P. Unit	877	891	888	893	885	880	880	883	888	888	880	880	885	876	884	883	873	878	884
46. Exhaust " " "	565	580	588	580	576	576	575	578	588	560	553	575	576	566	580	586	574	576	580
47. Throttle " - L.P. Units	542	562	535	564	543	539	562	564	564	554	548	553	552	553	557	562	554	552	556
48. Condensate	66.2	61.4	54.6	55.5	53.4	53.6	55.5	58.1	55.5	59.0	64.2	57.6	71.8	71.5	57.2	62.8	64.5	65.6	64.9
49. Circulating Water Outlet	62	65	69	65	63	70	77	74	73	66	55	65	69.0	64	65	70	68	67	74.6
50. Circulating " Inlet	33	33	45	44	39	42	70	68	64	59	47	36	51.0	36	36	41	43	39	43.5
51. Circulating " Rise	9	12	9	9	8	8	7	6	7	7	8	9	8.0	8	9	9	9	8	9.1
Pressures																			
52. Throttle Press.- H.P. Unit, Lb. Ga	1194	1181	1203	1196	1198	1204	1211	1200	1170	1200	1200	1200	1197	1200	1212	1211	1213	1219	1207
53. Exhaust " " " "	204	203	214	209	219	217	213	216	219	219	218	220	215	216	217	219	221	219	217
54. Throttle " - L.P. Units " "	206	207	200	207	207	213	218	218	218	220	217	212	211	212	212	210	212	215	212
55. Exhaust " " " NO. ABS.	.661	.847	1.086	1.158	1.115	1.198	1.312	1.229	1.269	.872	.818	.758	1.036	.734	.776	1.058	1.017	1.115	1.145
ELECTRICAL																			
56. Output-High Press. Turbine, M-kwhr/Month	5263	8252	18468	15129	15682	15701	6217	3130	13580	12015	11466	18639	11969	12879	12368	16863	14378	10767	16165
57. Output - Low Press. Turbines " "	14462	20388	35232	30166	29483	30092	19990	18649	28108	26844	25833	36361	26262	28835	27013	34716	31441	26677	32988
58. Total Output - High Press. Plant, Gross/mo.	19703	28640	53720	45291	45165	45793	25767	21779	41648	38859	37299	54980	38226	41734	39381	51577	45819	37444	49153
59. Output- H.P. Turbine, % of Total	26.6	28.9	34.4	33.4	34.7	34.3	24.1	14.4	32.6	30.9	30.7	33.9	31.3	31.0	31.4	32.7	31.4	28.8	32.9
60. Total Output, Net M-kwhr/month	17858	24357	49841	42123	42315	43619	19639	18995	36143	34720	32803	51803	35526	38763	36886	48246	43115	34953	46257
61. Auxiliaries, " "	1851	2525	3879	3172	2828	2878	2148	2139	2752	2715	2569	3177	2700	2771	2995	2930	2704	2491	2926
62. Auxiliaries, %	9.4	8.1	7.2	7.0	6.3	6.3	8.4	9.8	6.6	7.0	6.9	5.8	7.1	6.7	6.5	5.7	5.9	6.6	5.9
63. Plant Water Rate, Lb./kwhr.	9.71	9.12	8.78	8.79	9.14	9.10	10.31	11.86	9.17	9.08	9.09	9.10	9.24	8.97	8.96	9.22	9.34	9.27	9.06
PLANT HEAT CONSUMPTION																			
Lb. of Coal/Net-kwhr	1.434	1.372	1.232	1.187	1.245	1.252	1.350	1.506	1.190	1.270	1.222	1.168	1.258	1.185	1.180	1.209	1.252	1.269	1.178
Brute, B.t.u./ gross kwhr.	13763	12988	11904	12200	12398	12362	13773	15168	12158	12087	12055	12030	12152	11912	12015	12234	12532	12868	12114
Net, B.t.u./ Net kwhr	15169	14133	13259	13171	13252	13150	14056	16816	13017	12907	12948	12771	13506	12767	12825	12975	13318	13369	12882
THERMAL EFFICIENCY, %	22.5	24.1	25.7	25.9	25.6	25.9	22.7	20.3	26.2	26.5	26.4	26.7	25.3	26.7	26.6	26.3	25.6	25.6	26.5

this looseness was harmful as the massive dimensions of the blades are such that tightness is assured by centrifugal force when the unit is up to speed.

The shrouding of the second moving impulse row was found cracked in a number of places and a short piece of shrouding had broken off completely. One blade of this row was broken off where the shroud section failed and the tenon of the adjacent blade was injured. These two blades were replaced.

The shroud sections originally installed on the blades of the second moving row of the impulse wheel were of sufficient length to include six blades. These shroud sections were punched to receive blade tenons, the shrouding being applied by riveting of the tenons. It was concluded that cracking of the shrouding was caused by high stresses set up during riveting, and that harmful stresses would be avoided by carefully fitting new shroud sections so that they could be assembled in place without initial stress, the new shroud sections being held by welding around the projecting ends of the tenons. This shrouding was designed to group three blades instead of six as in the original design, because of the temperature range through which this blading must operate. As a result of the failure of the original shrouding on the second moving row of impulse blading, it was decided that a replacement row of blades with integral shrouding should be designed and held in

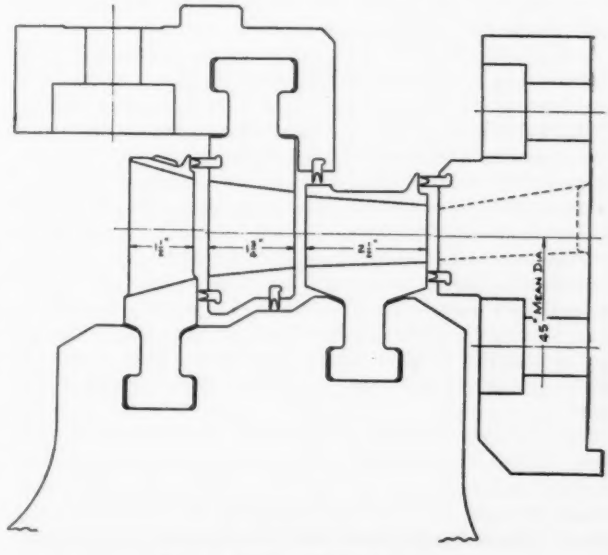


FIG. 5 IMPULSE-BLADE ASSEMBLY

readiness for installation should further difficulty be experienced in this section of the turbine.

The reaction blading was found to be in good condition on this inspection. A slight looseness was noted in some of the assembled segments toward the center of the spindle, but this looseness was not considered harmful as the blade segments become tight when up to speed. There were a few shrinkage cracks found in the silver soldering of the reaction-blade shrouding toward the low-pressure end. These were resoldered.

Following this overhauling period in August, 1938, there was no further outage for repairs of the turbine-generator unit during 1938 or the first six months of 1939. During March, April, and May of 1939, however, the steam rate of the topping turbine showed gradual increase. This was particularly noticeable in the first part of May when the steam rate of the turbine had increased to a corrected rate of 26.9 lb per kwhr at 23,000 kw load. This was reflected in the division of load between the topping turbine and the low-pressure turbines to which steam was supplied. During the same period, a reduction of capacity had been experienced in the auxiliary turbines driving the boiler feed pumps. Fouling of nozzles and blading of these small turbines was suspected and successful washing restored their capacity.

The foregoing experiences led to the washing of the topping unit. The washing was to be accomplished with saturated steam supplied at a rate slightly above no-load full-speed requirements with the unit electrically tied to the system. Saturated steam at full pressure was to be obtained by reducing the steam temperature as far as possible with boiler regulation and then removing the remaining superheat by injecting desuperheating water into the steam header at a point as far removed as practical from the turbine. A suitable nozzle was installed in one of the boiler risers for this purpose and thermoelements were inserted in existing wells at the turbine inlet and exhaust for accurate steam-temperature determination during the washing procedure. A steam-sampling outfit was set up which permitted checks to be made of conductivity of a continuous sample of condensed steam from the turbine exhaust.

With but one boiler in operation at reduced steam flow, steam-temperature reduction by means of boiler control was started at 11:00 p.m., June 3. Adhering closely to a temperature-reduction rate of 50 F per hr, a temperature of 650 F was reached at 3:00 a.m., June 4. This steam temperature represented the lower limit obtainable by means of boiler control. At 3:15 a.m. water was gradually injected into the header and by 5:00 a.m. a steam temperature of 590 F was reached.

At this point the conductivity of the exhaust-steam sample rose suddenly from 5.6 micromhos to 32,000 micromhos. As the washing progressed the steam-sample conductivity decreased to a value of 1000 micromhos after $3^1/_2$ hr of washing. During this period several samples of condensed exhaust steam were taken for chemical analysis. Analysis showed the blade fouling to consist of approximately 40 per cent soluble sodium silicate, 23 per cent sodium hydroxide, and 23 per cent sodium chloride. Other soluble inorganic salts made up the remainder of the deposit.

After the foregoing washing period, the temperature of the steam was gradually restored to normal and load was applied to the unit. At a load of 27,000 kw, the corrected steam rate of the topping turbine was found to be 24 lb per kwhr. The conditions existing during the washing operation are shown graphically in Fig. 6.

After a month of continuous operation subsequent to washing of the turbine, the topping plant was taken out of service for scheduled inspection on July 6, 1939. When the turbine was opened 26 blades in the second impulse-spindle row were found broken. The only evidence of this failure, noticeable while the unit was in operation, was an increased steam rate which had not returned fully to normal after completion of the washing procedure. Blade failure, however, was not definitely expected as the unit ran in good balance before the inspection.

As mentioned when discussing the July, 1938, turbine inspection, two blades and the shrouding of this second moving impulse row were replaced. The new blading, designed with heavier sections and integral shrouding made up for replacement following the 1938 inspection, was installed during the August, 1939, overhauling. The integral shrouding of these new blades was welded together so that groups of three blades are formed.

The shrouding on the spindle reaction blading was originally of pure nickel. Failure of this shrouding, due to cracking, required its replacement with new shrouding made of stabilized 18 per cent chrome, 8 per cent nickel material. This shrouding was arc-welded in place with 25 per cent chrome, 12 per cent nickel weld rods.

Four $3^1/_2$-in-diam cylinder bolts were found broken on this inspection. As a result of this and of the bolt failures found in the previous inspection, all of the larger studs for the turbine joint were replaced. The new studs will be made of Triplex No. 3

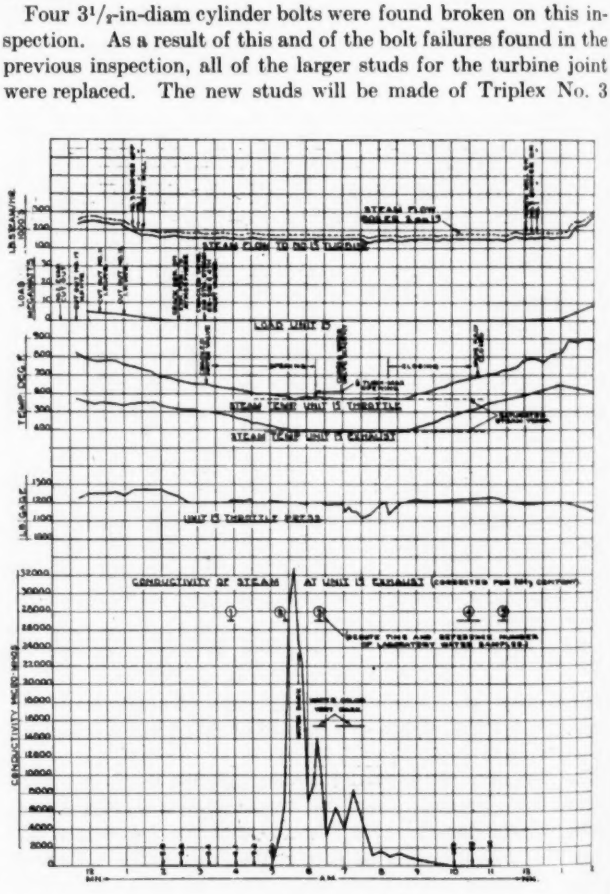


FIG. 6 DESUPERHEATING OPERATION FOR HIGH-PRESSURE TURBINE WASHING

bolt steel, a material which is now believed to be more suitable than the material originally used for this high-temperature-bolting service.

Although blading difficulties have been found in each of the two inspections of the turbine so far made, it is felt that the improvements accomplished will be found adequate, and that improved performance will result. Based on observed conditions and performance of the first impulse-spindle row, the changes made should limit future blading maintenance to minor repairs. It is expected that availability records with subsequent turbine operation will be further improved by shorter overhauling periods which possibly may become less frequent.

Steam Generators

The availability of the two steam-generating units of this topping installation has not equaled that of the turbine generator. This is apparent when the turbine and boiler outage charts in Fig. 4 are compared. The steam generators had an average availability factor of 61.2 per cent for their operation in 1937, 80.2 per cent for the year 1938, and 87.6 per cent for the six months of 1939. Steam supply, however, has been available from one or both steam-generating units during all except two short periods in the first four months of operation of the installation.

As the steam generators are similar in design, the major operating difficulties encountered with one were either indicated or experienced in the operation of the other. As a result, the improvements made in the boiler installations were made on each of the units in about the same sequence.

Steam was supplied to the high-pressure-header system on September 18, 1937. After preliminary adjustments were made to the feed pumps and other auxiliaries, operation was continued, the steam produced being passed through the pressure-reducing and desuperheating equipment to the low-pressure header, Fig. 3. In the early operation of the boilers, joint leaks developed where recirculators entered headers of the side-wall screens and also where economizer tubes entered the steam drums. Rerolling of these joints failed to prevent the recurrence of leaks during further service, and it was necessary to seal-weld the joints for tightness. Inability to hold rolled economizer joints tight was attributed to strains due to variable economizer temperatures. Recurring leakage of rolled recirculator joints was attributed to excessive joint strains set up by unequal expansion of recirculators and screen wall tubes. Seal welding, where necessary, has prevented joint leakage but strains at the joints persist. These strains have to the present time caused no apparent difficulty other than the leakage before joints were seal-welded.

During the initial operation of the installation, carry-over deposits were found to be accumulating in header valves and control valves of the turbine unit in quantities which were sufficient to interfere with valve operation. Tests for steam purity, made when units were steaming at the rate of 230,000 lb per hr, indicated that excessive carry-over was present when total solid concentrations in boiler water were maintained at 425 ppm. Lowering of the water level resulted in carry-over of less than 1 ppm at this rating but, when ratings were increased to 310,000 lb per hr, carry-over again became excessive. As 375,000 lb of steam per hr is required from each boiler for designed operation of the topping installation, improvements for better steam purity were necessary. With a changed drum baffle and steam-scrubber arrangement,[4] satisfactory steam purity has been attained at normal boiler ratings when total solids in the boiler water were maintained at 400 ppm. Operation was continued with this steam-scrubber arrangement in 1938 and the first six months of 1939.

As ratings were increased, difficulties with furnace slag accumulations on the sloping portion of the secondary-furnace wall were experienced. The original design provided for operation of the secondary furnace as a dry-bottom furnace, the slag falling through the opening across the bottom of this furnace, in a dry state, and being removed by a hydrojet system installed beneath the opening. This arrangement proved impractical as slag built up on the sloping secondary-furnace rear wall in thicknesses of 4 to 12 in. and at times avalanched down in quantities and sizes so large that removal by the hydrojet installation was impossible. During boiler outages in December, 1937, the secondary-furnace bottoms were changed, as shown in Fig. 1. With this arrangement all furnace slag is readily removed through the slag-drip holes of the primary furnace in liquid form.

[4] Reference (3), Fig. 4, p. 416.

Through this period difficulties were also experienced with failures of vertical slag-screen tubes and ruptures of tubes around the slag-drip openings in the primary furnace floor. The several changes necessary to correct these conditions required outages shown on the chart during January and February of 1938. The changes made consisted first, of the addition of 12 recirculating tubes between the upper slag-screen header and the lower distribution header to the screen; second, of offsetting alternate groups of 2 slag-screen tubes into the secondary furnace as shown in Fig. 1, instead of into the primary furnace as originally designed; and third, of moving the burners 2 ft 10 in. forward in the primary furnace. These changes had the effect of improving circulation and reducing the duty imposed on the tubes which previously had ruptured. A better balance in circulation between the screen-tube and floor-tube circuits also resulted. Further changes that were made to improve conditions in the floor-tube sections consisted of increasing the radius of tube bend in tubes around the slag-drip holes and the installation of water-cooling coils at these slag openings to retain a deeper protecting layer of molten slag over the floor tubes. These changes resulted in improved performance of these sections of the boiler installation.

During the outage of the steam-generating units shown in April, 1938, lancing doors of improved design with added insulation, improved hinges, and latching devices were applied in place of the original doors. It was necessary also to apply additional insulation about the boiler setting to reduce the temperature at lancing galleries and in other localities about the boiler where operators must work.

Repeated leaks from economizer-tube joints at the headers necessitated the completion of seal welding on all joints in the economizer sections at this time. During previous outages some seal welding of economizer tube joints had been accomplished.

On May 20, 1938, an outage of the entire topping installation was necessary to remove pieces of straightening vanes which had come loose from their position in the topping-turbine exhaust line. These straightening vanes had originally been installed to direct steam flow for more accurate metering through a topping-turbine exhaust orifice. As a result of the failure of this set of vanes, all straightening vanes in the header installation were removed to prevent a recurrence of failure, and possible resulting injury to turbine blading.

During the outage of the topping turbine for inspection and overhauling, the boilers were alternately taken out of service for inspection, cleaning, and repairs. At this time 9 economizer-tube sections at the boiler drum, which had been seriously eroded by spray from a joint leak, were replaced in the economizer of No. 2 boiler.

Difficulties with blistering and rupturing of tubes in the vertical slag screen, and in the primary-furnace floor and upper generating-tube sections, had caused questioning of the adequacy of boiler-water circulation and of the water treatment used in the installation. Apparently some of the tube failures were aggravated by more or less loose deposits which had accumulated at the points of tube rupture. These deposits had settled out of suspension in such quantities as to be evidence of sluggish circulation. Water lines in the interiors of the top-row tubes in the generating section were also evidence of inadequate circulation. In an effort to improve this latter condition, the top row of generating tubes was replaced with a changed design of tubes during outages of No. 1 boiler early in July, 1938, and of No. 2 boiler from August 27, to September 10, 1938. The original top tubes were parallel to the remainder of the tubes in the bank, while the new tubes were bent as shown in Fig. 1.

Soot-blower position changes were made during these boiler outages for more effective cleaning of the heating surface. The

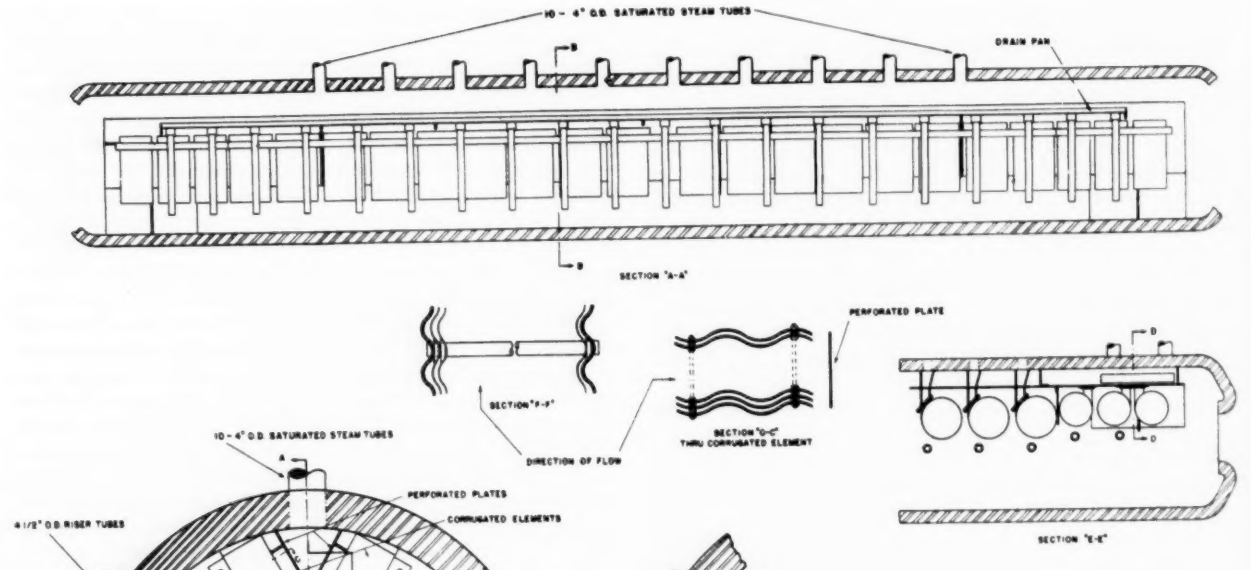


FIG. 7 IMPROVED STEAM-SCRUBBER ARRANGEMENT

soot-blower supply-header piping was also changed to secure higher pressure at the blower heads, resulting in higher jet velocities for more effective cleaning.

Operation of the boilers during the period from October, 1938, to February, 1939, was interrupted with several outages taken to clean tubes and make changes to improve circulation in the generating section. Restrictor plates in the uptake headers of this bank of tubes, altered design of top-row generating tubes mentioned previously, discontinuation of Akon treatment, and installation of tube cores in the upper ends of the top two rows of generating tubes were tried in efforts to prevent tube failures. These alterations failed to produce the desired result.

During the outage of No. 2 boiler between January 29 and February 6, and of No. 1 boiler between February 10 and February 17, 1939, distributor tubes made up of 1-in. tubing with a cone attached at the bottom were inserted in each of the uptake headers of the generating-tube section. The cone is arranged to divert the intense circulation of the bottom-row tubes through the 1-in. tube, while the 1-in. tube terminates near the top of the header and flow through it tends to induce circulation in the remaining tubes in the header. Sufficient equalization of circulation has been attained with this arrangement to prevent further tube failures in the generating-tube section.

With the changes mentioned, both boilers have operated for three-month periods and were taken out of service during May, 1939, only for inspection. No condition was found in these inspections which would have interfered with a longer service run on either boiler.

Due to difficulties experienced in other high-pressure-boiler installations with cracking of drum metal at joint seats, across which excessive variations in temperature occurred during operation, a changed design of economizer-tube joint at the drum and method of introducing feedwater were recommended by the manufacturer. The changed position of economizer-tube entry to the steam drum resulted from a decision to change the drum baffling and steam-scrubber arrangement from that shown in the previous paper,[4] to the cyclone-separator arrangement developed by the manufacturer. These drum changes were accomplished during the August, 1939, outage of both boilers, while the topping turbine was inspected and overhauled. The improved economizer-tube joint at the boiler drum and drum-cyclone arrangement is shown in Fig. 7. The installation of cyclone separators and changed scrubber arrangement is not expected materially to improve steam purity over that secured with the previous arrangement, but was adopted primarily to assure a maximum density in water supply for boiler circulation.

During the August, 1939, outage, the boilers were thoroughly cleaned externally, and the tubes in the generating sections and screen and wall circuits were turbined. The turbining was not considered necessary to remove deposits but was performed to make certain that the boilers would be in optimum condition for an extended service run.

In general it appears that the tube failures experienced during these correction and development periods have been largely due, directly or indirectly, to sluggish circulation. While there are many factors in operation and design which may contribute to this condition, it would appear that the matter of solid-water downcomer supply is particularly important as is also the matter of ample return-circulating-tube area. It would also appear that where relatively high heat transfers occur, parallel circulation with unequal flow resistances should be avoided as far as practical design will permit. The several changes in these boiler units have largely been made with the intent to improve one or the other of these conditions. The records of improved availability indicate that efforts along these lines have been sufficiently successful to regard regular boiler-operating periods of six or more months' duration as a reasonable expectancy. Turbine availability in this topping installation has exceeded this performance in previous operation. With these improvements a very desirable plant availability performance is anticipated.

Logan Operating Experience

By PHILIP SPORN,[1] NEW YORK, N. Y.

BEFORE beginning a review of the approximately two years of operating experience at the Logan Station of the Appalachian Electric Power Company, a brief description of the major equipment installed in 1937 will be of help as a background. The superposed extension consists of a 40,000-kw, 3600-rpm, water- and hydrogen-cooled generator supplied with steam at 1250 lb per sq in., 925 F, from a single boiler of 1,000,000 lb per hr capacity and exhausting at 200 lb per sq in. 510 F to five low-pressure turbines totaling 50,400 kw. The details of the equipment comprising the plant and the underlying design considerations have been published in various technical journals.[2] However, mention should be made that, at the time of its purchase in 1935, the 40,000-kw turbogenerator was the largest-capacity, 3600-rpm topping unit that could be proposed and the first hydrogen-cooled generator placed on order. It is also, so far as is known, the only water-cooled generator stator. The boiler is still the largest built to date.

Initial operation of the high-pressure boiler was started in October, 1937, and trial operation of the turbine, November 2, 1937. Operating experience during this initial period up to December 2, 1937, was reported as part of the Power Panel Discussion at the Annual Meeting of the Society, December 8, 1937.[3] The rather limited experience of one month of operation available at the time of that discussion justifies further comment on operating experience as the second year of operation nears a close.

Since the starting date until August, 1939, comprising 15,638 hr of elapsed time, the high-pressure section of the plant was available 8956 hr or 57.3 per cent of the time, while the service hours were 8750 or 56 per cent of the time. With a gross generation of 255,527,000 kwhr, the capacity factor for this period has been 40.8 per cent. The availability of the high-pressure turbine alone was 86.4 per cent and the service hours 56 per cent of the elapsed time. The single high-pressure boiler, which is used for the low-pressure section of the plant through the pressure-reducing and desuperheating system when not supplying the topping turbine, had service hours equal to availability amounting to 71.8 per cent of the period. Details of the outages on the high-pressure turbine and boiler will follow later. Since July, 1938, when the last major turbine difficulty was remedied, the record is more pleasant to report. Fig. 1 shows a graph of the maximum hour output of the high-pressure turbine since that time. The availability of the high-pressure section of the plant increased to 81.5 per cent, the service hours to 80.2 per cent, and the capacity factor to 61.8 per cent. For the turbine only the availability at 97.2 per cent and the service hours at 80.2 per cent have been quite satisfactory. The high-pressure-boiler performance is encouraging with availability and service hours increasing to 84.7 per cent in this recent thirteen-month period.

During the thirteen-month period, the station heat rate of 14,400 Btu per kwhr compares favorably with the design expectations, remembering of course that a serious shortage of circulating water in the summer months, necessitating spray nozzles and recirculation, is an uncontrollable influence on over-all plant economy. Under such conditions the condenser pressure attains as high as 3.4 in. Hg, and the additional large auxiliary power consumption to operate the spray-pond pumps amounts to 8 per cent of full-load generation.

Although the high-pressure turbine is rated 40,000 kw and the five low-pressure turbines aggregate 50,400 kw, making a station rated at 90,400 kw, much higher loads have been consistently possible because of hydrogen cooling of the topping unit and nozzle changes on the low-pressure units. Under favorable winter conditions, with ample circulating water, a gross station load of 103,000 kw has been attained, involving 48,500 kw on the 40,000-kw topping unit supplied with 1,000,000 lb per hr of steam from the single high-pressure boiler. This high load represents a reserve capacity available of 12,600 kw or 14 per cent over the name-plate rating of the units.

Operating procedure, as originally set up for this station at the time of initial operation late in 1937, has proved to be entirely satisfactory and no major modifications have as yet been found necessary. Basically, the system of starting and stopping is one in which the low-pressure boilers, the low-pressure turbines, the high-pressure boiler, and the topping turbine are treated as separate divisions, each of which is operated separately but in sequence. Whenever the high-pressure boiler is out of service at least two of the low-pressure boilers and two of the five low-pressure turbines are operated. The desired load on these units is maintained by backing down on the low-pressure boilers while steam flow is increased through the reducing valve as the high-pressure boiler is started and loaded. The high-pressure boiler could not be started without at least one of the low-pressure boilers in service because induced-draft fans are turbine-driven. Then the topping turbine is warmed through a manually operated atmospheric exhaust. When ready, the topping turbine under control of the speed governor is synchronized, then put under control of the back-pressure governor, and loaded while flow through the reducing station decreases. The reverse order is followed when taking the high-pressure equipment off the line. Under running conditions, the reducing valves are closed, the load is varied on the low-pressure turbines which, in turn, control the steam flow and load on the topping unit through the medium of the back-pressure regulator on the topping turbine.

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[1] Vice-President and Chief Engineer, American Gas and Electric Service Corporation. Mem. A.S.M.E.

[2] "Logan's Double Shell Turbine," by G. B. Warren, *Power*, vol. 81, June, 1937, pp. 302–305.
"The Logan Steam Station," by Philip Sporn, *Southern Power Journal*, vol. 56, June, 1938, pp. 26–34.
"Logan Steam Plant—a Landmark," by Philip Sporn, *Electrical World*, vol. 106, April 11, 1936, pp. 1017–1019, 1084–1086.

[3] "Logan Operating Experience," by Philip Sporn, Trans. A.S.M.E., vol. 60, FSP-60-13, July, 1938, pp. 421–422.

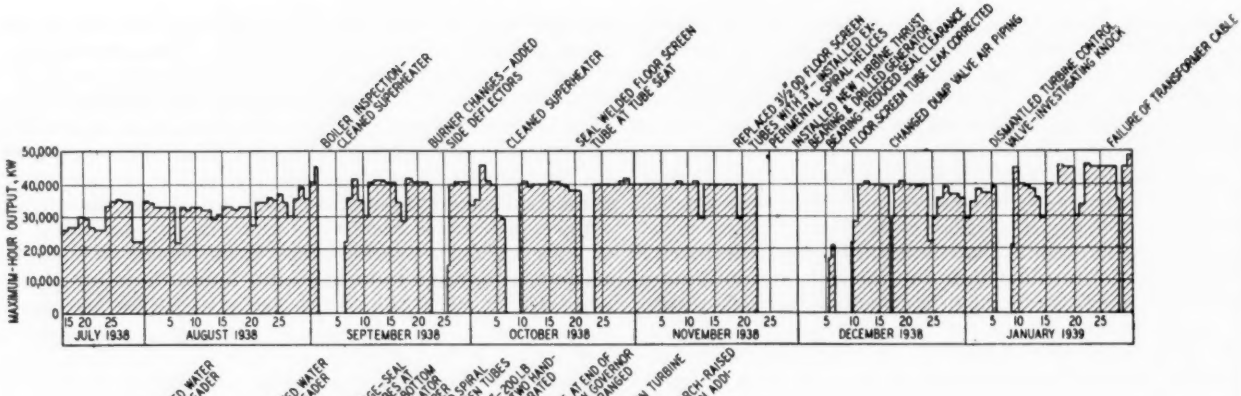


FIG. 1 MAXIMUM HOUR OUTPUT FROM JULY, 1938, TO AUGUST, 1939, LOGAN HIGH-PRESSURE TURBOGENERATOR

There have been several interesting experiences which indicate the flexibility of the station. On one occasion a test trip-out of 15,000 kw and an accidental trip-out of 36,000 kw of low-pressure unit capacity created no disturbance beyond control of the boiler operator and without popping of safety valves. There was an occasion when one of the induced-draft fans was lost and another, when a forced-draft fan was lost, both times with full load on the high-pressure boiler, but recovery to stable operation was accomplished without difficulty. Any system disturbance affects the load less on the topping unit than on the low-pressure units.

A most interesting incident occurred once when the entire station load was lost. The pressure in the 200-lb steam header, supplying the low-pressure units, dropped to as low as 60 lb, but this was sufficient to keep in operation the turbine-driven induced-draft fans on the high-pressure boiler supplied from this header without resorting to firing the low-pressure boilers.

HIGH-PRESSURE TURBOGENERATOR

The major outages on the high-pressure turbine have resulted from failure on two occasions of the first-stage shroud band. With the second shroud-band failure, the turbine manufacturer gave proper consideration to the extreme combinations of stress to which the first-stage blading is subjected, involving intermittent stress from partial peripheral admission and stress from centrifugal force coupled with high-temperature effect. Since the second repair was completed and the turbine placed on the line on July 15, 1938, it is gratifying to be able to report that no further trouble has resulted to date from the first-stage shroud band. It is to be noted that the turbine admission-valve system at Logan, shown in Fig. 2, continues to be, as it was initially, a partial admission system, the sequence being 1, 2, 3, 4, 5, 6. The only other serious difficulty occurred when the thrust bearing let go, but since its repair there has been no recurrence.

Overheating of blades in the topping turbine, caused from motoring of its generator with insufficient steam flow, when low-

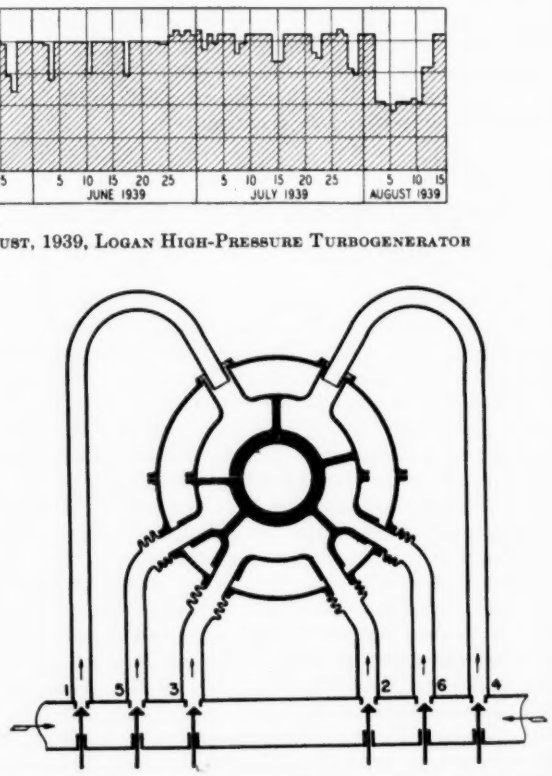


FIG. 2 DIAGRAM OF ADMISSION-VALVE SYSTEM, LOGAN HIGH-PRESSURE TURBINE

pressure units have tripped out, is prevented at Logan by a somewhat special reverse-power relay which the author helped to develop.

The scheme consists of a sensitive reverse-power relay calibrated to operate on power component only and to close its contacts to trip the generator off the line in case the power flow is from the bus bars into the generator with a magnitude of $\frac{1}{2}$ per cent of the kilowatt rating of the generator after a time of 1 min. In addition to this reverse-power relay, a timing relay is provided to give the desired time of 1 min. A single current transformer is employed with a ratio of approximately full-load rating of the generator to give 5 amp secondary current. Furthermore, a lockout, single-phase, reverse-power relay is provided to make the above sensitive reverse-power relay inoperative, provided the generator

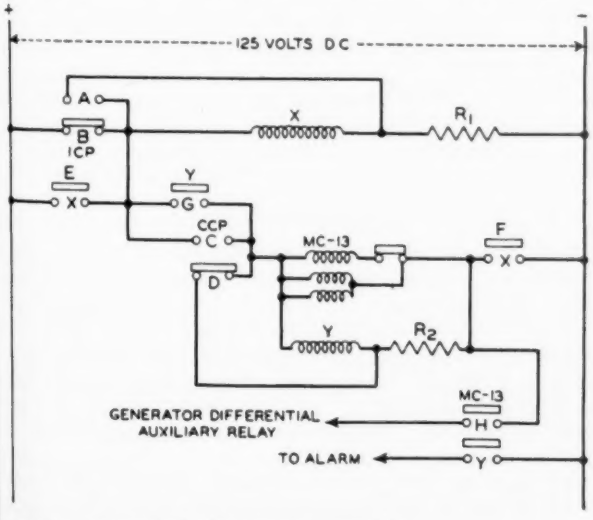
is supplying power to the bus bars of an amount equal to or greater than 40 per cent of its kilowatt rating. The installation of this lockout relay is provided to insure that the generator will not trip off the line unnecessarily due to improper operation of the sensitive reverse-power relay on surges or otherwise. In addition, an alarm circuit is provided to operate at the instant the generator starts motoring, so as to inform the operator that the generator is motoring and permit him to correct conditions within the period of 1 min before tripping occurs, if possible.

The detailed operation of the scheme may readily be understood from Fig. 3, which is an elementary control diagram of this system.

Under normal conditions, with the generator supplying power to the bus bars in excess of 40 per cent of its rating, contacts *A* of lockout reverse-power relay *ICP* are closed, short-circuiting coil of auxiliary relay *X*, causing contacts *E* and *F* of relay *X* to open, thereby removing negative supply to contacts of timing relay *MC*-13, preventing the operation of generator differential auxiliary tripping relay, regardless either of the operation of the sensitive reverse-power relay *CCP* or of the timing relay *MC*-13.

In case the generator is supplying power of less than 40 per cent of its kilowatt rating to the bus bars, contacts *B* of the lockout reverse-power relay *ICP* close, energizing auxiliary relay *X*, causing its contacts *E* and *F* to close. Due to the power flowing into the bus bars, contacts *C* of the sensitive reverse-power relay *CCP* are open, preventing energization of the timing relay *MC*-13. Also, contacts *D* of the sensitive reverse power relay *CCP* are closed, short-circuiting coil of auxiliary relay *Y*, causing contacts *G* and *H* to open, preventing the operation of the alarm circuit.

Under motoring conditions, power flow being from the bus bars into the generator, contacts *B* of lockout reverse-power relay



ICP = lockout single-phase reverse-power relay
CCP = sensitive motoring three-element single-phase reverse-power relay
MC-13 = d-c timing relay
X = type HGA d-c auxiliary relay
Y = type HGA d-c auxiliary relay
R_1 = resistor for *X* auxiliary relay
R_2 = resistor for *Y* auxiliary relay

Fig. 3 Elementary Control Diagram of Special Reverse-Power Relay, Logan High-Pressure Turbogenerator

ICP close, energizing auxiliary relay *X*, causing its contacts *E* and *F* to close. In addition, contacts *C* of the sensitive reverse-power relay close, energizing coil of auxiliary-relay *Y*, causing its contacts *G* and *H* to close. Then timing relay *MC*-13 starts to operate and, at the same instant, alarm circuit operates. After timing relay *MC*-13 has operated for 1 min, its contacts close, energizing generator differential auxiliary relay, tripping generator off the line.

During the last year there have been two occasions when the low-pressure units have tripped out from failure of the group excitation for these units. On both occasions the high-pressure unit was prevented from motoring by the successful functioning of the special reverse-power relay.

When the topping turbine is being shut down, the rise in exhaust steam temperature is negligible before the exhaust is opened to the atmosphere, but in bringing the turbine on the line after

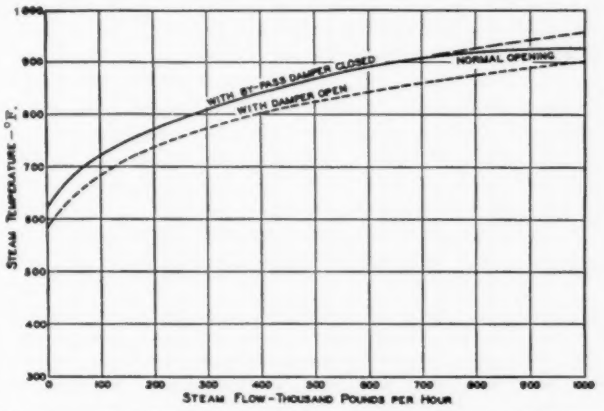


Fig. 4 Steam-Temperature Range With By-Pass Damper Control, Logan High-Pressure Boiler

a short shutdown period the stored heat raises the exhaust steam approximately 175 F until loaded and under control of the governor.

There have been no vibration problems with the topping turbine and the use of vibration and eccentricity recorders have proved helpful during starting periods. There is also no noise problem whatever.

The hydrogen-cooled generator has functioned most gratifyingly even under capacity load with the adverse condition of high cooling-water temperature. Although this generator is designed for a gas pressure of 15 lb per sq in. and has had test runs at this pressure, during which the shaft sealing and gas consumption were satisfactory and lower temperature of windings realized, day-by-day operation has been confined to a pressure of 0.5 lb per sq in. Even with cooling water at 90 F, a higher gas pressure was not needed to effect proper cooling of the windings. Hydrogen purity is easily maintained at 99 to 100 per cent, and gas consumption, averaging less than 40 cu ft per day, has been costing less than $300 per year.

Regarding the pad cooling arrangement for direct water cooling of the stator, outside of the original difficulty with the removal of air from the system, satisfactory performance has been obtained. There have been no signs of water leakage either in the pads or in the piping of the system. It has not been necessary at any time to remove and replace the condensate used as make-up for this pad-liquid system. Two heat exchanges are provided for the cooling of the pad liquid both of which are used during the warmer months but only one of which is needed during most of the year.

High-Pressure Boiler

The 1,000,000-lb per hr, dry-bottom, pulverized-fuel boiler has, of course, been subject to some outage—more outage than the turbine. Ash-screen tube failures resulting from impaired circulation and excessive heat input required replacement. To

remedy this difficulty, smaller-diameter tubes having spiral circulating strips and an increased slope were substituted and burner angles adjusted with satisfactory results.

West Virginia bituminous coal from four different seams near Logan with ash-fusion temperatures ranging from 2350 to 2700 F has been burned without difficulty. Slagging has not been serious and cleaning of boiler tubes by soot blowers and lancing has been easily accomplished. Load swings of 10,000 to 15,000 lb per hr are readily handled and, on occasion, it has been possible to drop load from 900,000 to 540,000 lb per hr momentarily. Steam temperature has been controllable within the prescribed range shown in Fig. 4 with the remotely operated by-pass dampers. With tangential firing, a minimum boiler load of 300,000 lb per hr can be attained without loss of flame stability. However, it is interesting to note that, with a fuel combination of gas and coal, a load of 175,000 lb per hr has been carried successfully for a 2-hour period. Four auxiliary natural-gas burners are used for starting with the aid of a kerosene torch and together are capable of carrying a boiler load of 50,000 lb per hr. The four Hardinge mills each of 15 tons per hr capacity have been rendering satisfactory service with a power input of 16 kw each at full capacity. To date these four pulverizers have handled a total of over 350,000 tons of coal. Experience indicates that exhauster blades made of boiler plate require replacement after a mill has handled 35,000 tons, and the wear on the ball charge amounts to 0.086 lb per ton of coal. The two Ljungstrom air heaters when cleaned once each shift with steam soot blowers have maintained their capacity and have exhibited no tendency to fouling. However, the exit flue-gas temperature, leaving the air heaters, remains about 30 or 40 F higher than the manufacturer's performance guarantee.

Feedwater make-up is supplied from an evaporator whose feed is coagulated river water in which temporary hardness usually predominates, forming a scale in the evaporator not readily susceptible to cracking. To offset this, approximately 0.2 ppm of tannin is added to the evaporator feed for 1 ppm of hardness. Total dissolved solids in the vapor leaving the evaporator is confined to 2 ppm or less. A pressure-type deaerator located midway in the feedwater-heating circuit, using exhaust steam from the turbine-driven induced-draft fans, has consistently maintained the dissolved oxygen of the feedwater at 0.005 cc per liter or less. Scavenging of final oxygen is obtained by continuous feeding of ferrous sulphate and caustic soda at the deaerator outlet in proportion to the dissolved oxygen in the feedwater. At no time does the boiler water show the presence of ferrous iron. Orthophosphates and caustic are fed in shots into the feedwater at the discharge side of the regulator for maintaining a pH value of 9.6 and a phosphate concentration of 10 to 25 ppm as PO_4. Boiler-water total solids range between 100 and 250 ppm.

The saturated steam from the boiler has a specific conductivity averaging from 0.5 to 0.7 micromhos. Even so, some fouling of turbine blades, as evidenced by increasing first-stage pressure, has been experienced after long periods of continued operation, suspected to be attributable to iron from the oxygen scavenger not detectable by conductivity determinations because of being too slightly ionized. Only recently the oxygen-scavenger feed was temporarily discontinued to determine the effect on first-stage pressure drop. Fortunately the deposit on the turbine blades seems to be removed or greatly reduced following a turbine shutdown. The only reason assignable to this is that the condensation on the blading during the subsequent turbine warming-up period is responsible for washing off the deposit.

Boiler Feed Pumps

Three identical 3600-rpm feed pumps are installed, one of which is motor-driven and two turbine-driven with the exhaust steam absorbed in the feedwater-heating cycle. Ordinarily, the turbine-driven pumps are used with the motor-driven as stand-by. These pumps have horizontally split cases and handle water at 350 to 390 F. Their performance has been satisfactory in every way except for failure of ring gaskets involved with the removable head at the discharge end of the pump where the balancing disks are located. Apparently, it is a question of slightly warped castings and selection of suitable gasket material. But this is a minor matter.

Reducing Valves and Desuperheaters

The two steam reducing valves for supplying the low-pressure section of the plant from the high-pressure boiler have been practically free of vibration and noise since the initial changes reported two years go.[3] When the station is operating normally with exhaust from the topping unit supplying the low-pressure units, motor-operated shutoff valves ahead of the reducing valves are kept closed. This is believed to be favorable to the life of the reducing valves. The automatic desuperheating equipment, though somewhat complex, has functioned very satisfactorily, and has maintained the steam temperature within narrow limits.

Heaters

In the feedwater-heating cycle, there are no heaters on the discharge side of the boiler feed pumps. Following the deaerator on the suction side of the feed pumps are two vertical surface heaters of the floating-head type, both handling water at 450 lb per sq in. and using steam at 100 and 200 lb per sq in., respectively. Admiralty tubes in the 100-lb heater and arsenical-copper tubes in the 200-lb heater have both functioned satisfactorily thus far without leakage or tube dirtiness. After some experience with various head gaskets, the toroidal type was definitely unsatisfactory, but copper and corrugated monel have proved to be much superior. Drain subcoolers built into heaters are not used at Logan. However, a separate drain cooler, used in conjunction with the 100-lb heater, has performed up to expectations with control of the drains on the outlet side so that no flashing can occur in drain piping between the drain cooler and the heater being drained.

Electrical Features

It will be recalled that the entire output of the Logan topping unit, as well as substantially all that of the five low-pressure machines, is taken care of by a single, 4-winding, 100,000-kva transformer bank with windings operating at 4000, 11,000, 44,000, and 132,000 v. Further, the larger station auxiliaries are fed through a newly developed magnetic air-type circuit-beaker switchboard. Both of these large and/or new developments have worked out most satisfactorily. No trouble has been experienced with the load-distribution or regulating characteristics of the multiwinding transformer. The air breakers showed their rating capability in an elaborate series of acceptance tests and have since functioned with a complete absence of any difficulty.

In a discussion of this kind there is only time for covering the high spots but there are, of course, many interesting detail problems which have been solved or remain to be solved. As a general conclusion it may be said that the selection of 1250 lb pressure, 925 F temperature for a topping extension to this station and the equipment designed for use therewith have proved a design, operating, and financial success. If we had the job to do over again, the experience gained of course would enable improvement in many details, but basically the design would be the same.

Superposed Installation at Omaha Station of Nebraska Power Co.

By LOUIS ELLIOTT,[1] NEW YORK, N. Y.

INFORMATION was presented two years ago before the Society concerning the high-pressure high-temperature superposed extension at the Omaha Station of the Nebraska Power Company.[2] At that time equipment operation aggregating 5700 hr was discussed. The present statement covers the period from initial operation in February, 1937, to May, 1939, and includes operating data and experience in addition to or differing from that detailed in the previous paper.

In brief, the extension includes one 10,000-kw 0.8-power-factor turbogenerator, designed for 1200-lb, 900-F operation, and one 265,000-lb per hr bent-tube boiler with economizer and regenerative air heater, fired by a 570-sq ft traveling-grate stoker. The high-pressure turbine is superposed on a 15,000-kw 200-lb condensing unit, with a nonautomatic reducing and desuperheating station discharging to a 325-lb header. Division of load between high-pressure and topped units is determined by maintenance of constant back pressure on the former. The station is operated normally with at least one 325-lb turbine and one or more boilers on the line, in addition to the superposed combination.

Operating time of boiler and turbine to May 1, 1939, has aggregated about 15,000 hr, equivalent to 78 per cent of elasped time since initial service. Availability (practically equal to "demand availability") of the high-pressure-turbine unit for the entire period has been 82 per cent, and for the last 7300 hr, since latest overhaul, 94 per cent. The unit was opened up in February, 1938; the next overhaul is scheduled for February, 1940. High-pressure boiler availability has been 87 per cent for the entire period, and 94 per cent since the latest general overhaul. Generation by the high-pressure unit has been at 78 per cent unit output factor (based on 10,000-kw rating) for the entire service period. Operation of the topping combination has been practically continuous since the early part of this year, and reliability is considered as substantially on a par with that of the 325-lb section of the plant.

Load on the combined unit is determined by consideration of over-all station costs, of service reliability, and of certain operating and maintenance difficulties experienced. As examples, loading is influenced by economy characteristics of the 325-lb units operated, by the formation of deposits on the blading of the 1200-lb and 200-lb turbines and on the tubes of the cross-over heater, and by slagging of boiler heating surfaces.

During 7300 hr ending April 30, 1939, generator had averaged 7050 kw, and heat rate of combined high-low-pressure unit averaged 13,700 Btu per kwhr net output with 84 per cent boiler efficiency and 60 F average deficiency in throttle temperature.

Design heat rate at 7500 kw (gross) load with 84 per cent efficiency and 900 F steam temperature is 13,050 Btu per kwhr net output, or 13,600 Btu with correction applied for actual steam temperature prevailing. It is therefore to be expected that when steam temperature is brought up to that originally intended, performance of unit will be substantially equal to design value.

High-Pressure Turbine

Reduction of turbine capacity from blade deposits was experienced prior to the February, 1938, overhaul, at which time blading was cleaned by blasting with fly ash because silica content and temperature conditions prevented removal by washing. Subsequently, boiler-water concentrations were reduced, and leakage of the low-pressure unit condenser is now checked regularly; stage pressures and load tests now indicate no appreciable scaling of blading, so that further ash-blasting may not be necessary.

The turbine unit originally showed some vibration, but with adjustments, made from time to time, conditions are now much improved and are quite tolerable.

As at first installed, the turbine water rate was 2 to 3 per cent high at lighter loads, caused in part by excessive high-pressure packing clearances, but largely by turbulence of steam in the space between the first-stage-wheel exit and smaller diameter nozzles of the second stage. A dam or baffle was installed in the first-stage shell space, extending from the inside of the shell to the rotor and attached to the second-stage diaphragm and to the inside of the upper half of the casing. This improvement eliminated much of the loss and permitted the turbine to meet its expected performance.

No automatic devices have been installed, or other provision made, to protect against overheating of the turbine with loss of load. The smaller turbines are less vulnerable in this respect than the larger.

Starting-up procedure has been modified as compared with the method reported two years ago. The 200-lb turbine is now started first, taking steam through the 325-200-lb reducing valve. The exhaust valve of the high-pressure turbine is also opened to admit 200-lb steam to the turbine casing. The high-pressure boiler is brought up to 600 or 800 lb pressure, passing 75,000 to 100,000 lb of steam per hr through the 1200-325-lb reducing valve. The high-pressure turbine is then started, exhausting to the 200-lb turbine. As the load is picked up, the 1200-325-lb reducing valve is closed, and the boiler pressure is raised to 1200 lb. This method obviates loss of steam to the atmosphere, inherent in the original starting method. Starts and stops have been relatively infrequent—only as occasioned by maintenance requirements.

High-Pressure Boiler

There is little occasion to make rapid changes in load on the combined unit, and the boiler output is maintained steady, except as the daily load curve calls for adjustment at certain hours.

Deficiency in total steam temperature persists, correction having been delayed. On account of this temperature condition, there has been no need for control of steam temperature, for which a manually operated by-pass gas damper is provided.

Slagging of boiler-unit heating surfaces must be fought continually, the Kansas coal burned having relatively high sulphur content and low ash-softening temperature; samples show initial deformation at 1850 F and a fluid condition at 2100 F or below. Because of these ash characteristics, difficulties of cleaning boiler and air-heater surfaces increase rapidly as maximum boiler capacity is approached. The boiler is now normally operated during the day period at an output of approximately 225,000 lb per hr, as slagging difficulties increase above 230,000- to 240,000-lb per hr output. It requires 14 man-hours per day for cleaning of the heat-transfer surfaces.

Relatively few large stoker-fired boilers have recently been installed in utility plants, and the question may arise as to comparison with pulverized-coal firing for conditions such as encountered at Omaha. At time of selecting firing method, no experience was available with burning of Omaha's basic coals in the pulverized form, under a boiler operating at high steam pressure and temperature. On the other hand, experience in this plant over a period of years with relatively large traveling-grate-fired boilers had been quite satisfactory, combustion

[1] Consulting Mechanical Engineer, Ebasco Services Inc. Mem. A.S.M.E.

[2] "Superposed Extension to Omaha Station of Nebraska Power Company," by Louis Elliott, Trans. A.S.M.E., vol. 60, FSP-60-13, July, 1938, pp. 418–419.

losses and stoker maintenance being low. Investment for a stoker-fired unit was estimated as lower than if powdered-coal equipment had been used, particularly considering that the dust-collecting problem for the latter would have been more difficult.

The slagging troubles that have been experienced with this steam generator, and especially with the superheater, have been more severe than on the older equipment, probably because of higher steam, gas, and metal temperatures required. However, while opinion might be held by some that these slagging troubles would have been less if pulverized coal had been used, there is as yet no experience to assure such a difference, particularly considering the much larger weight of ash carried up through boiler and superheater.

No circulation troubles have been experienced in boiler or furnace walls or arches, and the traveling-grate stoker has developed no operating or maintenance difficulties, notwithstanding its great size. Boiler water is conditioned to maintain an average pH value of 10.7, while keeping total solids at 250 ppm and total alkalinity at 75 to 100 ppm. A ratio of 0.5 between sulphates and carbonates is normal.

Air Heaters—Feed Pumps

Corrosion and plugging of the air heater at the top of the inlet side of the rotating element results from the sulphur and moisture content of coal. This necessitated installation of a by-pass around the air heater to permit cleaning. The upper 4-in. portion of the regenerative surface has been modified and is now made up of elements with corrugations designed to give a freer gas travel, and bound together in groups so as to permit ready removal of a corroded or fouled surface and substitution of new or cleaned sections.

The shaft of one of the 3600-rpm steam-driven feed pumps failed completely. The majority of opinion is that this resulted from imperfections in shaft material.

High-Pressure High-Temperature Superimposed Installations at Waterside Station

By J. C. FALKNER, NEW YORK, N. Y.

ADDENDUM[1]

FOR the last 2 years, high-temperature high-pressure equipment has operated at the Waterside Station. One Westinghouse 53,000-kw unit went into service September 10, 1937, and one General Electric 53,000-kw unit went into service June 29, 1938. Both units operate at 1200 lb pressure 900 F temperature, 3600 rpm and have hydrogen-cooled generators. Both units have a feed-heating turbine attached to the main-turbine shaft. Four Combustion Engineering Company boilers of 500,000 lb of steam per hr capacity are used; two boilers per turbine. There is no cross connection between the steam lines at the boilers or turbines; however, the boiler-feed-pump discharge headers are cross-connected.

At the present time, 52 old 650-hp 200-lb boilers are operating. These boilers are necessary for load on the station and also to supply the demand steam to the New York Steam Corporation. The heat rate on the station for both high- and low-pressure boilers taken from the weekly station-performance records for the months of October and November, 1939, shows an average Btu rate of 13,676.

Operating Record of Turbogenerators

There has been considerable trouble with the high-pressure equipment and many changes were made to the original installations before finally arriving at a satisfactory operating condition. With reference to the turbines, two rows of impulse blades of the Westinghouse unit were removed after a few weeks of operation at the request of the Westinghouse Company because of trouble on other machines. This unit operated as a straight reaction turbine from the time the impulse blades were removed until it was rebuilt in July, 1939, with practically no operating trouble. This turbine had to be completely rebuilt after the breaking of the coupling attached to the shaft on which is located the main thrust bearing. This break allowed the turbine rotor to change its position so that the high-pressure dummies rubbed and seized,

[1] Presented in connection with the contribution appearing on page 241 of this issue.

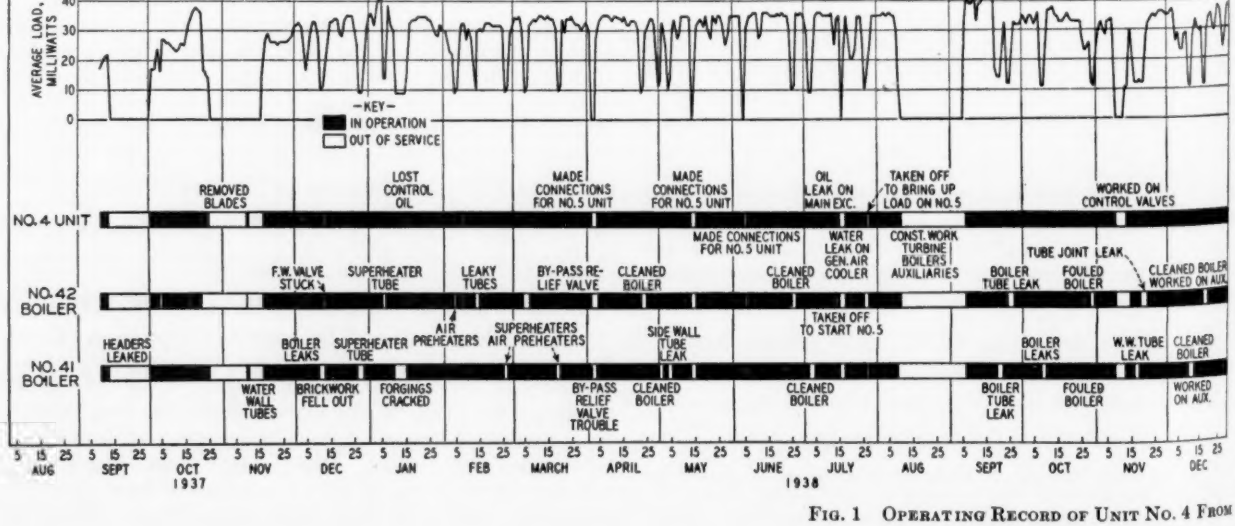


FIG. 1 OPERATING RECORD OF UNIT NO. 4 FROM

causing an outage from March 11 to July 4, 1939. The General Electric unit has not been opened since it was installed.

The Westinghouse unit has generated 420,000,000 kwhr and has an availability factor of 78.5 per cent. Since this machine was put back into service for regular operation on July 4, 1939, the availability factor has been 99.1 per cent.

The General Electric unit has generated 355,000,000 kwhr and has an availability factor of 95.7 per cent. Since July 1, 1939, the availability factor has been 99.2 per cent.

Both units have steam admission at full periphery. The Westinghouse unit has carried a maximum load of 49,000 kw; the General Electric unit 51,000 kw. At present, these units operate at 48,000 kw due to a high back pressure at the turbine exhaust. This is a temporary condition caused by pipe-line changes necessary for the addition of two high-pressure units, one to be installed in 1940 and the other in 1941.

The hydrogen-cooling equipment on both generators has given no trouble to date.

Figs. 1 and 2 show the operation of units Nos. 4 and 5 from their starting dates to December, 1939.

Performance of High-Pressure Boilers

Two of the four Combustion Engineering Company boilers went into service September 10, 1937; one on June 29, 1938; and one on July 25, 1938. The availability factors for these boilers have been as follows:

No. 4A boiler	79 per cent
No. 4B boiler	81 per cent
No. 5A boiler	82 per cent
No. 5B boiler	88 per cent

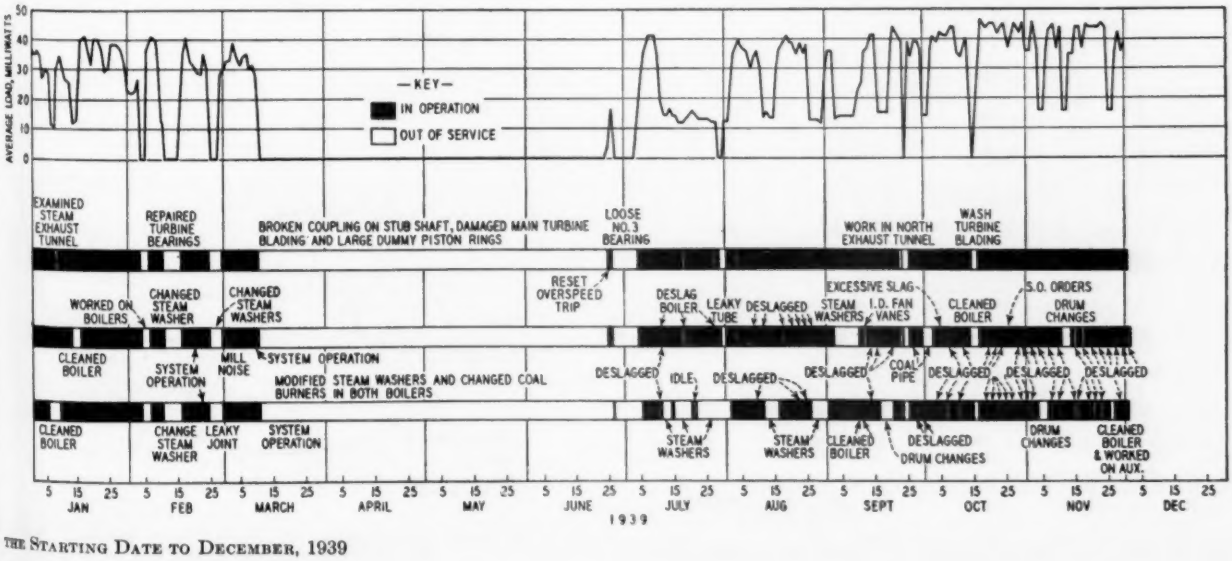
There has been circulation trouble on these boilers in the sectional headers, side-wall and superheater tubes. During the first few months of operation there were two side-wall-tube failures; this trouble was cured by welding sectionalizing plates in the intermediate waterwall headers. No trouble has been experienced with these tubes since the sectionalizing plates were installed. One superheater tube in No. 4A boiler ruptured; it was found that the trouble was caused by improper alignment of the superheater. The trouble was eliminated by reducing the space between the superheater tubes and the boiler side walls by the use of Fahrite plates.

Circulation trouble in the sectional headers developed as soon as the boilers were brought to full rating. These sectional headers are made up of 4-high and 3-high sections placed alternately across the boiler. Trouble developed on both the 4-high and 3-high header tubes, the failures being due to the overheating. Because of load conditions, it was impossible to make changes at that time so that the boilers were operated at 1150 lb pressure with a maximum rating of 450,000 lb of steam per hr, at which rating no circulation trouble was experienced.

Later, there were many experiments made by installing orifices or restrictors in the sectional headers and tubes; and changes were made to the baffling in the drum. These changes consisted of removing the screen-type washers, Fig. 3, which were originally installed in the boilers for the Westinghouse unit, and installing bubble-type washers in their place. Another change consisted of a scoop kicker baffle over the 3- and 4-high sectional header downcomer tubes in the drum. Another change was a partition baffle.

Several of these arrangements are illustrated in Figs. 4, 5, and 6. The present arrangement in the drum, Fig. 7, which was made about 2 months ago, consists of separate straight kicker baffles in front of the downtake tubes to the sectional headers. These kicker plates make use of the velocity head as well as the static head of the water in the drum, at the same time the steam is released upward. Also, three additional screens were placed in the drum ahead of the six original drying screens in order to help in the elimination of carry-over. Tests indicate that a total solid concentration of 800 ppm can be carried without excessive carry-over. The boilers are now operated at full load, full pressure, and full temperature, with a total solid concentration averaging 450 ppm.

Extensive tests were made on these boilers after each change was made in the drum, and thermocouple readings were taken on each of the top tubes of the 3- and 4-high sectional headers. These thermocouple readings showed many variations. At times there were no hot tubes but an excessive carry-over and vice versa. Temperatures as high as 850 F were shown, the saturation temperature being 580 F. Hydrogen readings checked the high tube temperatures, indicating that dissociation was taking place. Also the carry-over was checked by the steam-temperature-recording chart and by a sensitive cell placed at the turbine exhaust; density readings were taken by measuring the head of water from the boiler drum to the mud drum. Unless further circulating trouble develops no further changes will be



The Starting Date to December, 1939

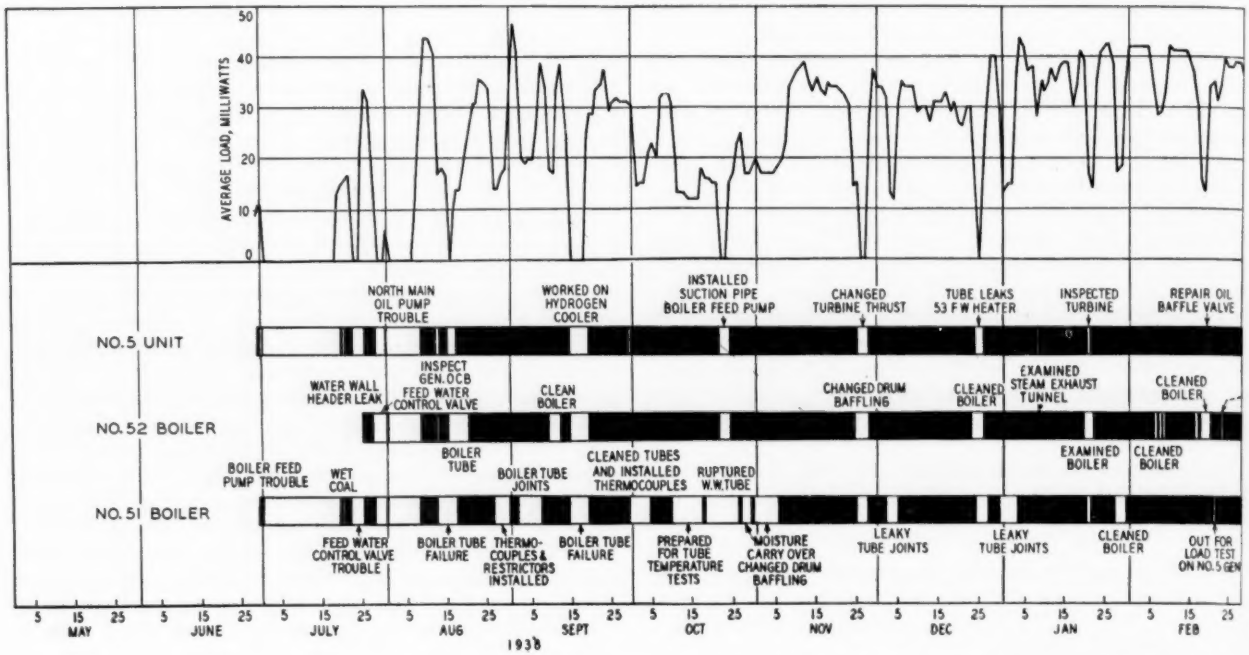


FIG. 2 OPERATING RECORD OF UNIT NO. 5 FROM

made in these boilers, although there are several schemes which have been advanced.

Considerable rerolling of tubes has been necessary since these boilers were put into operation, but the number is gradually diminishing so that at the present time it is seldom necessary to reroll a tube.

A continuous blowdown system is used on these boilers so that any desired concentration can be held within close limits.

Trouble has been experienced with burner tips. This trouble has been partially eliminated by decreasing the drop through the mill classifier from 2 to 3 in. which increases the primary air to the burners. This reduced the fineness of the coal from 80 to 76 per cent through a 200-mesh screen. It is felt that this reduction of fineness will not cause an increase in slag troubles and that the increased primary air will reduce burner maintenance. This change also reduces milling costs.

No trouble has been experienced with the use of gas for starting the boilers except in the case of the electric glo-bars used in igniting the gas. They have been a continuous source of trouble

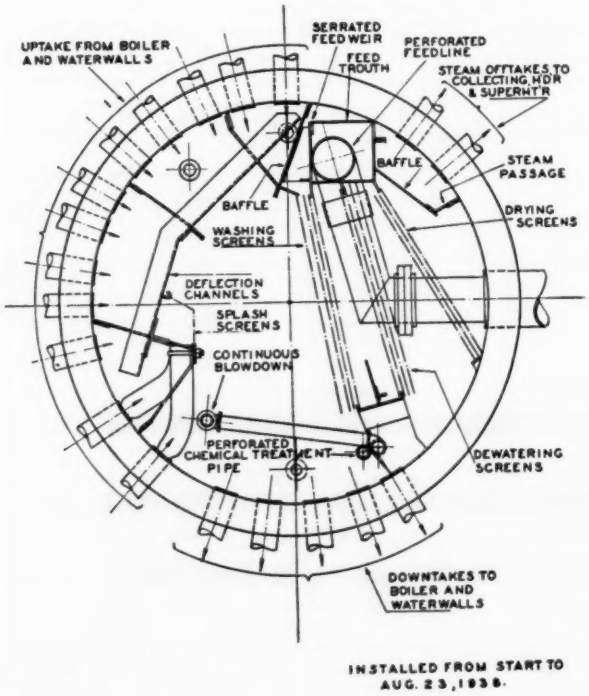


FIG. 3 ORIGINAL ARRANGEMENT OF DRUM STEAM WASHER

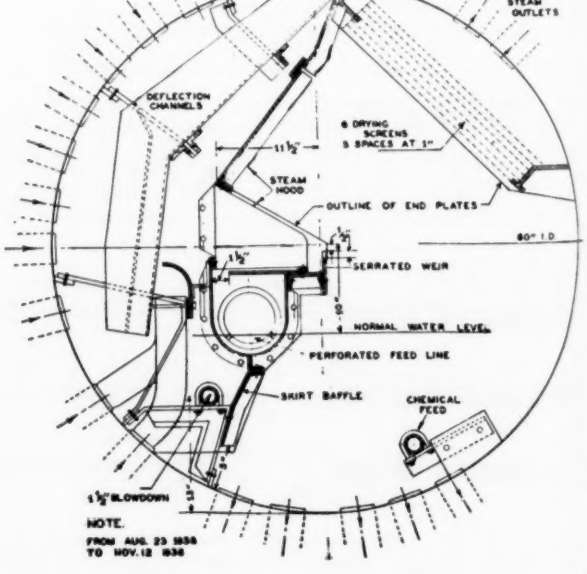
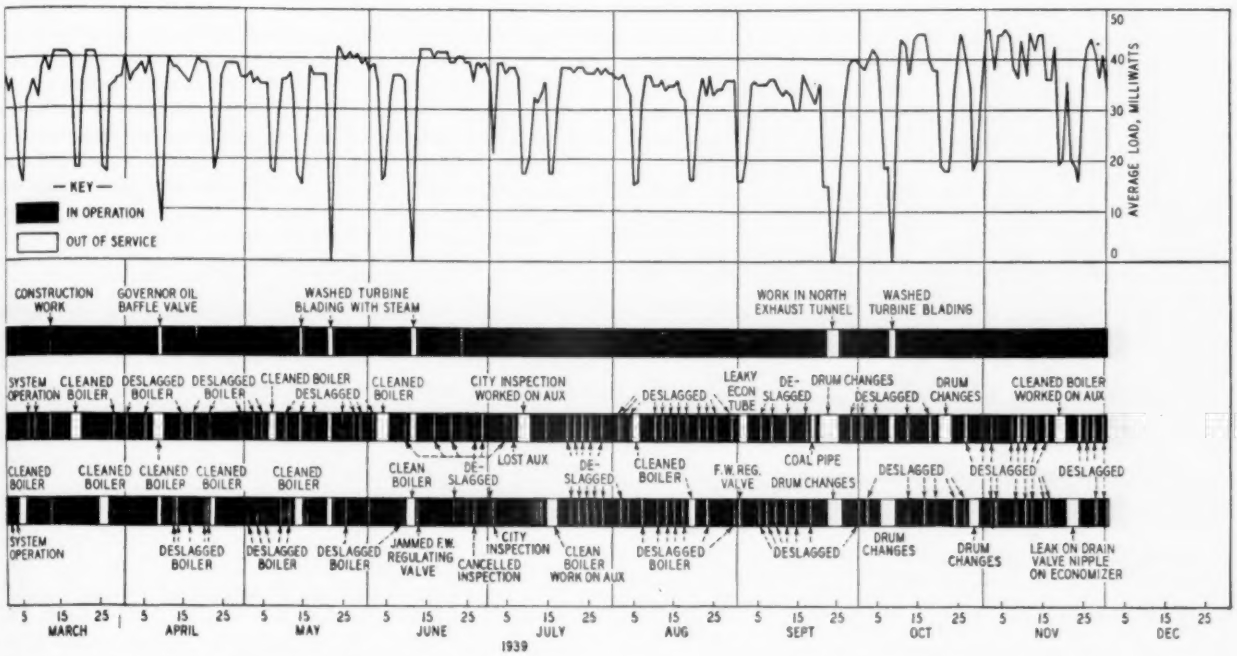


FIG. 4 TYPICAL ARRANGEMENT OF DRUM STEAM WASHER IN AN EARLY INTERMEDIATE STAGE



THE STARTING DATE TO DECEMBER, 1939

and an engineering study is being made to remedy this condition.

The continuous slag-tap arrangement has given very good service. There have been a few instances, however, where due to a change of coal, a great quantity of slag suddenly poured over the slag spout which necessitated taking the boiler off the line on two occasions, and only quick action on the part of the operator prevented another shutdown.

The cleaning of the boilers, superheaters, and furnace walls has developed into quite a problem and it is found necessary under present operating conditions to take one of the four boilers off the line for a 2-day period of cleaning every 4 weeks. In addition, there are three men per watch of 8 hr each who are continually occupied in keeping slag off the boiler and superheater tubes. An engineering study is being made for the installation of retractable soot blowers to aid in the elimination of slag formations.

In addition to the Bailey meters, gas-sampling lines for each boiler were run to a central point on the operating floor whereby the operator can take snap samples of CO_2 in the boilers. It has been necessary on a few occasions to reduce the load on

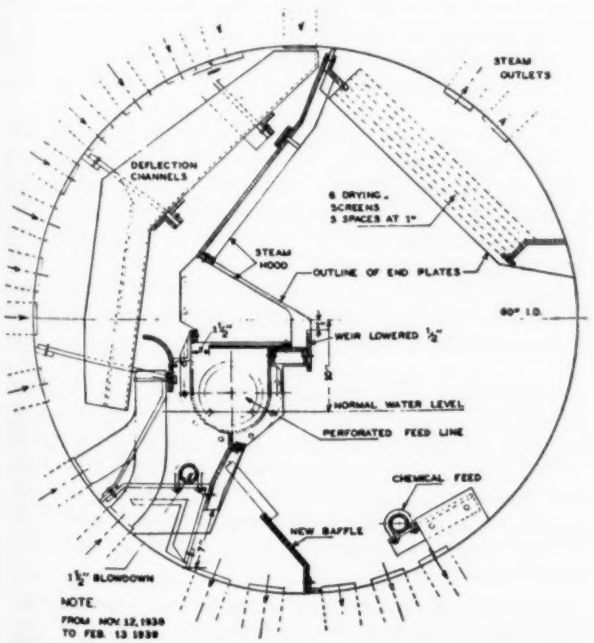


FIG. 5 TYPICAL ARRANGEMENT OF DRUM STEAM WASHER IN ANOTHER INTERMEDIATE STAGE

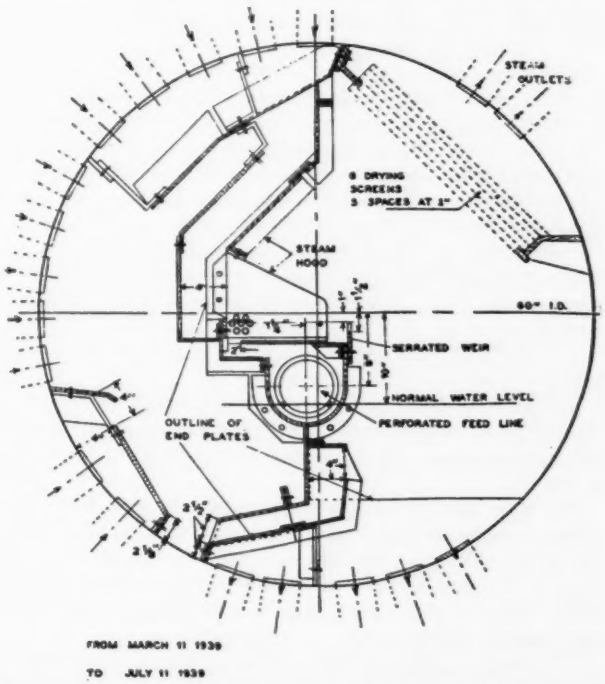


FIG. 6 TYPICAL ARRANGEMENT OF DRUM STEAM WASHER IN A LATER INTERMEDIATE STAGE

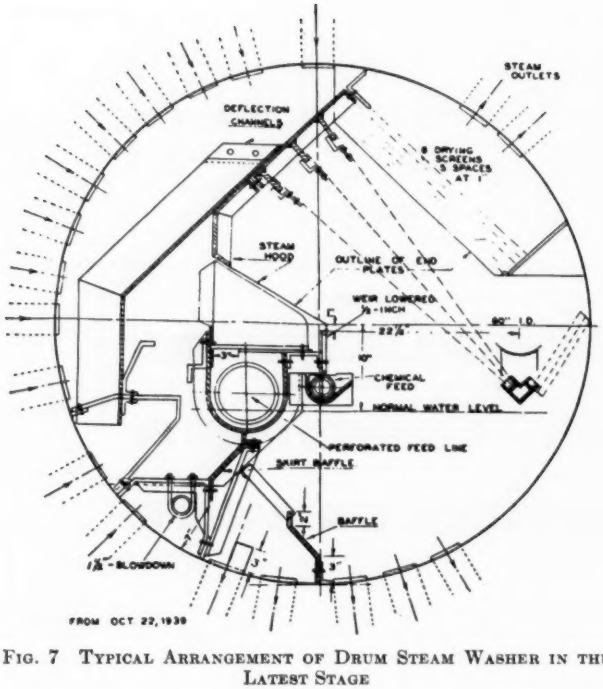


FIG. 7 TYPICAL ARRANGEMENT OF DRUM STEAM WASHER IN THE LATEST STAGE

boilers from time to time when it was found that the CO_2 was increasing to a point where the temperature would further aggravate the furnace conditions. At the boiler outlet, CO_2 is held at 14½ per cent.

The carry-over from the boilers is relatively small and cannot be read accurately on any meter but, over a period of months, there is a gradual fouling of the turbines which necessitates washing. This washing is done by admitting 200-lb saturated steam to the turbine and operating it for a period of several hours, care being taken not to cool the turbine down at a rate of more than 100 F per hr. These washings have resulted in bringing the machine back to normal.

Preliminary tests have been made on the quick pickup of these boilers and additional tests will be made to see what effect a sudden swing on this station will do to the boilers, particularly the water level. Preliminary tests indicate that no trouble will be experienced. The tests were made by running the General Electric turbine on one boiler and reducing the steam output of the boiler from 500,000 to 400,000 lb of steam per hr and then increasing the load on the turbine by rapidly operating the turbine control governor to the load corresponding to 500,000 lb of steam on the boiler. This test was repeated at 400,000 and 350,000 lb of steam per hr. An increase of not over 1 to 1½ in. water level in the boiler was experienced which is not considered dangerous. Additional tests will be made.

The pulverized-fuel mills have operated satisfactorily and no trouble has developed other than the usual maintenance, such as the replacement of mill rollers and rings after their normal expected life.

The other auxiliaries, such as air preheaters, fans, boiler feed pumps, precipitators, etc., have given some trouble, but nothing out of the ordinary. The slag conveyer from the pit under the boiler to the slag hopper has had a high maintenance, and another method of slag removal will be installed on the boilers for No. 3 and No. 4 high-pressure turbogenerators.

A Zeocarb system for water softening, having a capacity of 600,000 lb per hr, was put in service 2 months ago. This system is used for make-up to the high-pressure boilers when steam from the exhaust of the high-pressure turbines is sold to the New York Steam Corporation. This water has amounted to as much as 25 per cent of the total water to the high-pressure boilers.

On the whole, as indicated by the availability factors on the turbines, boilers, etc., even with the trouble as indicated, the high-pressure equipment at Waterside Station has operated in a satisfactory manner and it is felt that, since the boilers can now be operated at full capacity, an even better operating record will be made.

Discussion

E. G. BAILEY.[1] With respect to boiler operation, the papers and discussion have repeatedly developed three general points as being troublesome in the case of some of the units covered in the operating experiences of the last two or three years. The design of these units dates back to 1936, when a diversity of topping units was purchased, many for limited space conditions and other requirements beyond the experience of either engineers or manufacturers.

The three points most frequently mentioned are boiler circulation, carry-over in the steam, and slag. While the principles of circulation have been well known for a long time, there were many limitations in certain specific cases, such as a strong urge for a cheap unit and cramped space conditions, which resulted in unsatisfactory installations. These, however, have been rectified in situ, and some units, installed simultaneously with those, have been free from any circulation difficulties. We feel that the designs which are now being installed will definitely be free from circulation difficulties.

There are times when impurities in the water and the formation of scale are the cause of failures. It is believed that users and manufacturers will be able to distinguish between the different causes, and each will be able to meet satisfactorily the requirements and prevent outages due to tube failure in waterwalls and boilers.

The second point, impurities in the steam, is also well along toward a much improved solution, through the use of improved baffling and steam cyclones in the steam-and-water drums. These have been installed in a sufficient number of places, under enough conditions, with different water analyses, to assure us that the steam can be made quite clean and, furthermore, that the steam can be separated from the water, as well as the water from the steam.

The third point, slag, is one which cannot be so quickly summarized, because there is a diversity of coal and ash, with as widely varied characteristics. However, a great deal has been accomplished in the last 2 years toward surveying this condition more accurately and critically. It is believed that the improvement in furnace design is along the direction in which the slag problem will be kept within limits for satisfactory operation, at a minimum of cost for initial design, slag-cleaning equipment, steam consumption, and man-hours.

C. C. BALTZLY.[2] The high-pressure installation at our Schuylkill Station consists of two Combustion Engineering Company, 600,000-lb per hr tangentially fired boilers, supplying a Westinghouse 50,000-kw 3600-rpm high-pressure turbine at 1250 lb gage pressure and 900 F temperature, which exhausts into a low-pressure header at 230 lb per sq in. gage pressure and 550 F temperature, supplying low-pressure units having a combined capacity of 85,000 kw.

[1] Vice-President, The Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

[2] General Superintendent of Station Operation, Philadelphia Electric Company, Philadelphia, Pa. Mem. A.S.M.E.

Boilers. Initial operation of the two high-pressure boilers began in September, 1938, and during the first year operated approximately 80 per cent of the total time. Considerable trouble was experienced with the bunker-C fuel-oil burners installed for starting purposes, but this was finally corrected by installing a burner which has a fan-shaped spray employing either air or steam for atomization. Firing rings with directional air vanes are used with this burner. The boilers are fired with pulverized fuel, but the oil burners were used for a period of 1 month with an operating economy within $1^1/_2$ per cent of that obtained with pulverized fuel. The maximum rating obtained with 8 of these burners per boiler was 420,000 lb of steam per hr. No detrimental effects were noticed throughout the boiler system.

Leaks in the economizer-tube bends have been encountered, caused by faulty fabrication. These leaks were repaired by gas welding.

Leaks in the water-column connections to the drums and flanged joints on the superheater-outlet headers were found to be due to improper bolting materials. These were replaced with chrome-molybdenum-tungsten A.S.T.M. 193 Symbol B-13, which have proved satisfactory.

One floor tube failed due to erosion of iron in the slag and impingement of flame on the tube. The burner angle was changed, and no more trouble has been encountered.

On both boilers the fins on the floor tubes around the slag spout burned off, allowing the slag to come in contact with the tubes. A water-cooling coil was installed around the slag spout, and the slag level was increased from 2 in. to 4 in. over the entire furnace floor.

After about 2 months' operation, the beams supporting the economizer in both boilers were found cracked and broken. These beams were made of Fahrite material and were replaced with beams of the same material, but were heat-treated for less brittleness. No more failures have occurred.

Difficulty was experienced initially in maintaining the steam temperatures below the maximum operating limit of 925 F, at ratings over 500,000 lb of steam per hr. The following steps were taken: (1) Removed kicker baffles between the superheater and first bank of generating tubes; (2) lowered the front baffle 24 in.; (3) removed every other tile in the top row of the baffle below the superheater by-pass damper. However, it is still necessary to water lance the furnace walls and air lance the front bank of generating tubes daily to keep the steam temperature within the controlling range of the by-pass damper.

Trouble was experienced with the carry-over when injecting a chemical solution at a rapid rate into the rear drum. This trouble was overcome by injecting a more dilute solution over a more extended time. Expected performance on the screen-type washers was not to exceed 2 ppm carry-over at 600,000 lb per hr with 500 ppm drum concentration. Boiler concentration is now being held below 300 ppm for total dissolved solids and the carry-over is running around 1 ppm. In an additional effort to improve the carry-over condition, 37 of the steam-circulating tubes between the two upper drums were removed.

The Raymond bowl mills, each of 15 tons per hr capacity, based on 72 grindability coal, have given satisfactory service.

The fan blades, however, have to be changed after about 15,000 tons of coal. Different kinds of material were used, but it was found that the soft-steel blades have proved most effective.

Considerable trouble has been experienced with the cast-iron burner boxes, which have the Fahrite material tips. Two boxes have recently been installed made of steel plate having the plate extending through the Fahrite casting so that if the joint between the Fahrite and the box becomes loose there would still be no leakage through the joint. These were installed only a short time ago, so we are unable to say how successful they will be.

No. 3 Turbine. The high-pressure turbine was operated initially on September 14, 1938, without the impulse blading and the generator was cooled with air. The maximum load with these conditions was 42,000 kw.

Shortly after the initial operation, a severe rubbing was noted during the starting-up period. The turbine was dismantled and the spindle returned to the manufacturer's shop. The radial seals were found to be worn about 0.05 in., and the low-pressure shaft seals were worn and pitted. New radial and shaft seals were installed at that time. Considerable trouble has been experienced in the operation of the various governing devices. The machine is now operating satisfactorily on the back-pressure control and load-limiting device. The speed-governing mechanism is not yet functioning satisfactorily.

On two occasions, outages were caused by thrust-bearing failures, necessitating the installation of new thrust blocks. Changes in the design of the oil circulation were made to correct this condition.

In February, 1939, the unit was dismantled for installation of the impulse blading with full peripheral steam admission. The axial and radial sealing strips were found badly worn, 3 first-stage nozzles were found broken, seals were damaged, and several valve seats were found loose. The unit was returned to service on April 27, 1939. Hydrogen cooling was installed at that time and the machine was loaded to 50,000 kw.

The internal inspection at that time disclosed a heavy chemical deposit on the last 6 stages. Water-spray nozzles have been installed to permit washing of the blading with the turbine in service. Tests had indicated that the deposit could be removed by hot water or saturated steam.

In order to determine any indication of further fouling of the blading by chemical deposit, a daily check has been maintained on the drop in pressure from the impulse chamber to the turbine exhaust for any given load, and is compared with a prepared curve of standard performance. A gradual increase in this pressure drop indicated that washing of the turbine was warranted, and this was done on October 15, 1939.

Washing was carried on with the machine in service with about 1000 kw load. The machine was out of service about 10 hr for other necessary work, and the casing reached a temperature of about 700 F. Before the throttle was opened, the steam temperature was reduced by desuperheating to about 680 F. The machine was brought up to speed and synchronized in the normal manner with 1250 lb pressure steam at this reduced steam temperature. Desuperheating water was gradually increased until the steam ahead of the turbine throttle reached a condition of 10 deg superheat. Under this throttle steam condition, the exhaust steam was found to contain about 1 per cent moisture, and a conductivity cell in the exhaust indicated that the deposit was being removed. These steam conditions were then maintained for a 5-hr period, at which time it was indicated that the washing had been completed. The steam temperature was gradually brought up over a 3-hr period to normal throttle temperature. The water washing indicated a decided improvement in performance, and preliminary tests indicate that the machine is about 2 per cent better than the guaranteed performance at 50,000 kw load.

Pressure-Reducing Valves. Considerable trouble has been experienced with the pressure-reducing valves due to excessive vibration. Each valve was designed for 600,000 lb per hr flow, and is now limited to 400,000 lb per hr, and new valves of larger design will be installed.

Tank Pumps. Tank pumps are used to pump the plant water from an atmospheric deaerator to the boiler-feed-pump suction at about 450 lb per sq in. The submergence of these pumps is 30 ft. Three such pumps are installed. Three failures of

tank-pump shafts have been experienced, and these were replaced with shafts of heavier construction.

Drip Pumps. Three shaft failures were experienced on the 2 drip pumps. The failures were due, apparently, to too light a design and also to a highly fluctuating supply of water from the flash tank. The water supply to the pumps was steadied by resetting the drainer on the flash tank to operate within closer limits. The new shafts were of a heavier design.

Boiler Feed Pumps. Three turbine-driven boiler feed pumps are installed. Considerable trouble was experienced with shaft packing. Cooling water fed to the lantern gland apparently corroded the gland and attacked the shaft sleeve. Elimination of water feed to the lantern gland and substitution of a flat metal ring in place of the lantern gland have removed that source of trouble.

Leaking joints on the balance drum and stuffing-box bushing have occurred from time to time. The original copper (asbestos-insert) gaskets have been replaced by solid-metal gaskets.

On two occasions, a check valve on the pump discharge failed to close when the pump was taken out of service, resulting in reversing the pump and slight damage to the thrust bearings.

Automatically operated recirculating valves were provided to limit the minimum flow to 100,000 lb per hr, and these have given satisfactory service.

Deaerators. The atmospheric deaerating heaters have given very good service, and no trouble has been experienced in maintaining feedwater with zero oxygen content.

Evaporators. Double-effect evaporators are used which utilize steam bled from the high-pressure boiler-feed-pump turbines. Either effect can be run separately as a single-effect evaporator. A great deal of trouble has been experienced with the performance of these evaporators. Cracking-down was not sufficient and it was necessary to clean the coils by hand. Since that time, the shell water has been continuously treated with soda ash and city water has been used in place of river water, with a continuous blowdown of about 20 per cent.

Tests are being run by the manufacturer to determine whether or not these evaporators can be kept clean by the cracking-down process, without chemical and evaporating river water. However, we are of the opinion that this cannot be done, and plans are being made to install a zeolite softener ahead of the evaporator.

In conclusion, it is felt desirable to point out the fact that the difficulties encountered with this high-pressure high-temperature installation have been no greater than would normally be expected in placing another plant in operation where the steam conditions were of a more moderate nature.

C. B. CAMPBELL.[3] In this Power Panel discussion frequent reference has been made to the matter of carry-over from the boiler, with the resulting scale deposits in the high-pressure turbines. Deterioration of turbine capacity and efficiency have been mentioned. These are the obvious effects of restricting steam passages through the turbine blading. Of even greater importance, however, is the fact that depositing scale through part or all of the turbine blading changes the pressure distribution and, therefore, the net axial thrust on the rotor. This was the fundamental cause of the two thrust-bearing failures on the Schuylkill Station turbine mentioned by Mr. Baltzly.

It is evident from inspection of several installations that carry-over and blade deposits can be controlled so that washing is seldom, if ever, required. This entails constant vigilance, as once scale is deposited it will not be removed again by normal operation. In isolated cases, deposits have been cleared by shutting down long enough for the turbine to cool. The washing process, one scheme for which was described by Mr. Baltzly, has proved effective and may be safely performed when carefully planned and executed.

First-stage blade failures have occurred in high-pressure superposed turbines of large capacity, some of which were mentioned in the papers and in discussions. Experimental development is now confirming a logical mathematical analysis of the problem. As the next step in this development and its proof, the writer's company, with the valuable cooperation of the Philadelphia Electric Company, is installing a single-stage turbine at the Schuylkill Station, where superposed-turbine operation will be duplicated at full scale. The method of attacking the problem has been such as to investigate it fundamentally and for applications in the future at steam pressures beyond those with which this symposium was concerned.

J. M. DRABELLE.[4] The value of papers dealing with actual operating experiences with high-pressure, high-temperature stations cannot be overestimated. Theoretical considerations involving so-called studies of performance and economics are one thing but the acid test is "how does it work?" For the Society to have the benefit of actual operating experiences is invaluable.

At the Cedar Rapids Station of the Iowa Electric Light and Power Company, a high-pressure high-temperature topping unit was installed. Originally the boiler was fired with a stoker which proved to be unsatisfactory in performance due principally to difficulties with the high-ash, low-fusion-ash western coal, and also a length factor which resulted in a badly disturbed fuel bed as compared to the fuel-bed performance on smaller stokers. The boiler was changed over to pulverized coal and since that change has been operating with entire satisfaction. The back-pressure control on the turbine has performed satisfactorily.

The principal problem in connection with the operation of this unit is on account of the large amount of steam fed to the central heating system which results in high make-up water. Data on this matter will be developed during the winter season and will be made available to the Society at the next Power Panel discussion.

F. P. FAIRCHILD.[5] The superposed unit at the Essex Generating Station of the Public Service Electric and Gas Company has been in operation approximately 17 months. This installation consists of a 50,000-kw General Electric turbine generator, designed for 1250 lb per sq in., 950 F at the throttle, and two Babcock & Wilcox Company boilers, each having a maximum capacity of 605,000 lb per hr, equipped with Foster-Wheeler Corporation Hardinge mills. The turbine exhausts at a constant pressure of about 225 lb per sq in. into the station header. Addition of this unit has increased the capacity of the station from 193,000 kw to the present 243,000 kw.

The turbine is a 6-valve double-casing machine directly connected to a hydrogen-cooled generator operating at 3600 rpm.

The boilers are of the radiant type, with the generating and superheater tubes arranged essentially vertically, designed for direct pulverized-coal or fuel-oil firing.

During the first 2 months of operation the boilers were fired with fuel oil. Then for a period of approximately 7 months pulverized coal was fired regularly. This has been followed by fuel-oil firing, extending from April 1, 1939, to the present time. The type of fuel used depends upon the current fuel prices.

Considering the total elapsed time since the date of initial op-

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[4] Consulting Engineer, Iowa Electric Light and Power Company, Cedar Rapids, Ia. Mem. A.S.M.E.

[5] Chief Engineer, Electric Engineering Department, Public Service Electric and Gas Company, Newark, N. J. Mem. A.S.M.E.

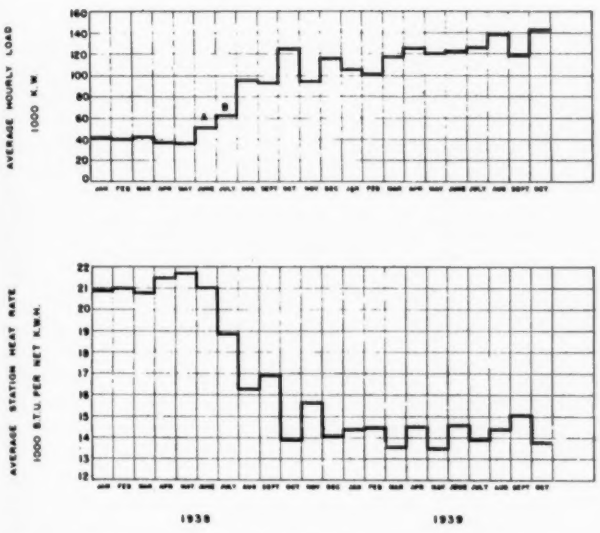


FIG. 1 RELATIVE MONTHLY LOADING OF ESSEX GENERATING STATION AND MONTHLY STATION HEAT RATES
(*A* = High-pressure boilers put into service with reducing value; *B* = high-pressure turbine put into service.)

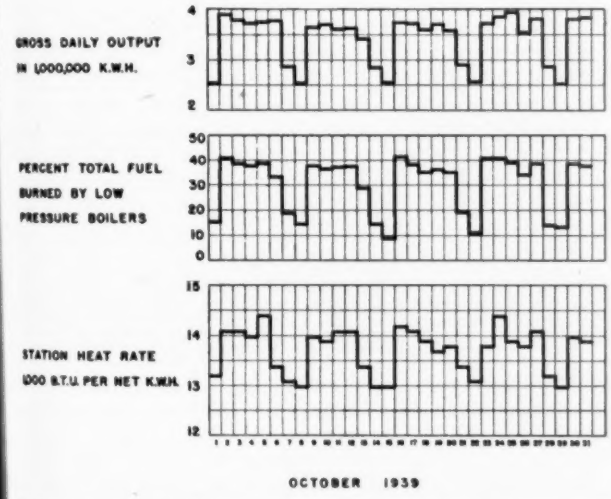


FIG. 2 TYPICAL MONTH'S OPERATION OF ESSEX GENERATING STATION WITH SUPERPOSED UNIT IN SERVICE

eration, the turbine has been in service 92 per cent of the time; the first boiler has been in service 83 per cent of the time; and the second boiler 88 per cent of the time.

Disregarding the first 6 months of operation, which may be considered as a preliminary adjustment period, and considering only operation from January 1, 1939, up to the present time, the turbine has been in operation 97.5 per cent of the time; the first boiler in operation 90 per cent of the time; and the second boiler in operation 91.1 per cent of the time.

Fig. 1 on this page indicates the relative monthly loading of the Essex Generating Station and monthly station heat rates from Jan. 1, 1938, to Oct. 31, 1939. This includes a period before the superposed unit went into service as well as the period since the superposed unit has been operating. It will be noted that the new unit was put into service during July, 1938. The average station heat rate decreased from a monthly average of approximately 21,000 Btu per net kwhr to values of between 13,000 and 14,000 Btu per net kwhr in the most favorable months. As a result of this improvement, the station loading was increased from approximately 40,000 kw to 120,000 kw. The station heat rate depends upon the portion of the total load carried by the low-pressure boilers.

Fig. 2 on this page shows a typical month's operation of the station with the superposed unit in service. The upper curve shows the average daily load throughout the month, the dips in the curve being the week-end periods. The middle curve indicates the portion of the total fuel burned in the low-pressure boilers of the station. The lower curve indicates the average daily station heat rate for the same period.

The unit has operated at the design pressure since the start. However, the steam temperature has been limited to 925 F to build up a background of experience at this temperature before going higher. With superheat control this lower-temperature operation is simply a matter of damper operation. Thus far there have been no indications of trouble from $1^1/_2$ years of operation at 925 F. It is anticipated that the temperature will be raised to 950 F if, at the next inspection of the unit, no evidence of trouble from the present operating temperature exists.

The cover of the turbine has not been lifted for about 13 months, but a complete inspection is scheduled for the spring of 1940.

The boilers, being of a new design, received considerable attention during the preliminary and early operating periods. Extensive testing equipment for measuring circulation, density of steam-and-water mixtures in various parts of the unit, water levels, and temperatures was installed. It is believed that this careful check, carried on principally by the boiler manufacturer, was in a great measure responsible for certain troubles not developing to a point where forced and serious shutdowns might have occurred.

Early operation of the boilers developed several things which were not right. Water-level detectors in the drums showed that levels indicated by the gage glasses were not true levels. Due to the low density and extremely turbulent condition of the mixture in the drums, only a limited rating, about two thirds the steam-water expected maximum, was possible without carry-over. The low-density mixture in the drums also resulted in considerable steam being carried into the downcomer pipes and thus reduced the circulation of solid water. Although there were no tube failures, except a few leaks in shop welds of economizer tubes, the ratings on the boilers were limited to a safe point. Installation of cyclone separators in the two drums of each boiler completely corrected the circulation and carry-over difficulties. Since installation of this equipment, it has been possible to operate the boilers at full load with solids in the steam under 1 ppm at boiler-water concentrations as high as 1400 ppm. Operation with higher concentrations may be possible, but there has been no occasion to carry over 600 ppm regularly.

During early operation, measurements indicated that the temperatures of the top sides of some of the furnace-floor tubes were considerably in excess of the saturated-steam temperature. This was due to steam pockets in the upper part of the tubes caused by insufficient circulation in the relatively flat tubes. Installation of diaphragms in the floor headers to improve water distribution, installation of cores in the floor tubes to increase water velocity, and baffles in the downcomer inlets to reduce the steam-water ratio in the downcomer pipes were inadequate to improve the floor-tube temperature condition appreciably. Solution of this trouble has been the installation of a water-cooled dam at the slag-drip ledge, in order to carry a slightly higher slag level on the furnace floor, with a resulting reduction of the heat input to the floor tubes.

Slagging of the heating surfaces of the unit has not been a serious problem. However, several alterations to the original

soot-blowing equipment have been made to improve the cleaning facilities. The first few rows of close-spaced superheater tubes showed a tendency to bridge across with a spongy form of ash with coal firing. No soot blowers were originally provided for this section of the superheater and, during the early period of operation, steam lancing was necessary to keep these tube spaces open. Later installation of telescopic and retractable blowers at this point eliminated the trouble. Steam at 225 lb pressure is used on most of the soot blowers, but on the superheater by-pass blowers it was found necessary to change to 450-lb pressure steam to provide effective cleaning. Retractable blowers on the furnace walls have proved valuable in maintaining equilibrium conditions in the furnaces.

Recent operation with fuel-oil firing has shown a tendency for the gradual depositing of a hard slag in the first section of the superheater. This slag is made up principally of vanadium oxide and sodium sulphate, and knits very tightly to the tubes. The soot blowers seem ineffective in removing it. It has been necessary to take the boilers from service at about 2-month intervals to hand clean this slag from the superheater tubes.

Thus far no harmful effects of the 925 F operating steam temperature have been found in any of the materials or equipment in the superheated-steam system. As a matter of inspection and to obtain some idea of the effects of $1^1/_2$ years of operation on the superheater-tube material, a section of one of the front legs of a superheater element was recently cut out. Internal inspection of the tube showed no evidence of corrosion, and photomicrographs showed no harmful effects to the metal structure during this period.

As a whole the entire unit, both turbine and boilers, has performed favorably. In the two new plant extensions which this company now has under construction, that is, at the Burlington Generating Station and the Marion Generating Station, equipment of similar design to that at Essex will be installed.

J. C. Hobbs.[6] "Topping" experience over 10 years has convinced the writer that most troubles are chargeable to poor engineering and can be avoided. An analysis of the various reports submitted today on other installations emphasizes a fundamental often overlooked, namely, that the best way to eliminate trouble with a particular feature is to eliminate the feature. It is sometimes possible to eliminate the entire piece of equipment. The writer's experience involving three sets of topping units confirms the great value of this policy. The elimination of those features which have been reported as causing trouble in other installations has made each successive higher-pressure topping unit more reliable than the former, in spite of the greater potential hazards. Briefly, the aim should be extreme simplicity in design, maximum availability for inspection, with a minimum of parts and time required for dissembly.

C. H. Spiehler.[7] The following are the essential particulars of the installation at the Millers Ford Station of the Dayton Power and Light Company, Dayton, O., upon which the writer will discuss the operation experience for the period from September 15, 1937, to September 15, 1939.

The superposed plant incorporates 2 Babcock & Wilcox steam-generating units having an output of 375,000 lb of steam per hr at 1250 lb per sq in. pressure and 900 F total temperature. One 30,000-kw General Electric turbogenerator set is installed operating on 1200 lb per sq in. 900 F steam, exhausting at 230 lb per sq in., and 500 F.

[6] Vice-President, in Charge of Manufacturing, Diamond Alkali Company, Painesville, Ohio. Mem. A.S.M.E.

[7] Results Engineer, Columbia Engineering and Management Corporation, Cincinnati, Ohio. Mem. A.S.M.E.

Originally the low-pressure plant consisted of three 1373-hp boilers delivering steam at 230 lb per sq in. pressure and 550 F (these boilers have been removed); and six 1965-hp boilers having the same steam conditions. Turbine generators include two 12,500-kw, three 20,000-kw, and one 25,000-kw machines, all operating on steam at 230 lb per sq in. pressure and 350 F.

An outline of the difficulties experienced during the first few months' operation of the Millers Ford high-pressure topping plant was given at the Power Panel discussion at the December, 1937, meeting. Those difficulties have been satisfactorily corrected. The reliability of this installation is now considered to be at least as good as that of the low-pressure plant. The turbine was operated for 16 months with only routine maintenance; the internal inspection at the end of that period found the machine in perfect condition. Operating periods of the steam-generating units of 4 or 5 months' duration have been obtained, and the operating department can see no reason why these periods should not be extended to 6 months.

The economy expected has been realized. This plant prior to the installation of the topping unit had a heat rate of 20,000 Btu per kwhr net generated send-out. With the topping unit supplemented by generation from the low-pressure boilers, the heat rate for the first 10 months of 1939 was 15,075 Btu, the best month being 14,000 Btu. The over-all steam-generating-unit efficiency for the 10-month period was 87.5 per cent.

A second high-pressure topping installation is now being made at this station. The new equipment will duplicate the present installation. Changes will be few and of minor nature as follows: The steam-generating units will have more cooling surface in the uptake waterwalls which will be accomplished by installing 2 $^{11}/_{16}$-in-outside-diam tubes between the $3^1/_4$-in-outside-diam tubes which are on 6-in. centers. The only Bailey block to be used in the furnace will be installed on the slope of the downtake waterwalls and below burners. Cyclone separators in the drums will be used in place of the present sprays and baffling.

The forced- and induced-draft fans will have approximately 15 per cent greater capacity. The Cottrell precipitators will have 34 ducts instead of 26 ducts as in the present installation.

Turbine-Generator Operation

The topping turbine was operated until June 1, 1938, with its maximum load limited to 25,000 kw. The present limit of 30,000 kw became effective on that date. During the 2-year period the topping unit has been available 92.37 per cent of the time. It has operated 87.28 per cent of the time and has a capacity factor of 45.09 per cent. Practically all of the inoperative available hours are charged against taking the turbine off to put a high-pressure boiler into service.

When the station load is such that the load on the topping unit is less than 25,000 kw, there normally are no low-pressure boilers in service. The exhaust is regulated manually by the switchboard operator during these periods. As the load on the topping unit approaches 25,000 kw, low-pressure boilers are brought onto the line. Additional station load is picked up by these boilers until they have enough margin to regulate the pressure. Then the topping unit is loaded to its maximum which is normally 29,000 kw. The reverse procedure is followed when the station load is decreasing.

There have been three major turbine outages, as follows:

1 February, 1938; the only indication of trouble was a substantial increase in pressure on the first stage. The turbine was continued in operation under this condition for several days before it was decided to make an internal inspection. Upon inspection it was found that a small section of the first-stage blade shrouding had failed, Fig. 3. The section which had broken away was about $^1/_2$ in. wide and 6 in. long. A 5-blade section



Fig. 4

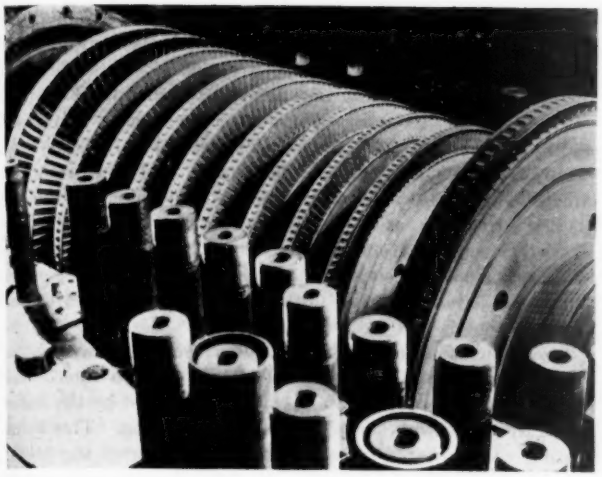


FIG. 3 DAMAGE TO ORIGINAL SHROUDING

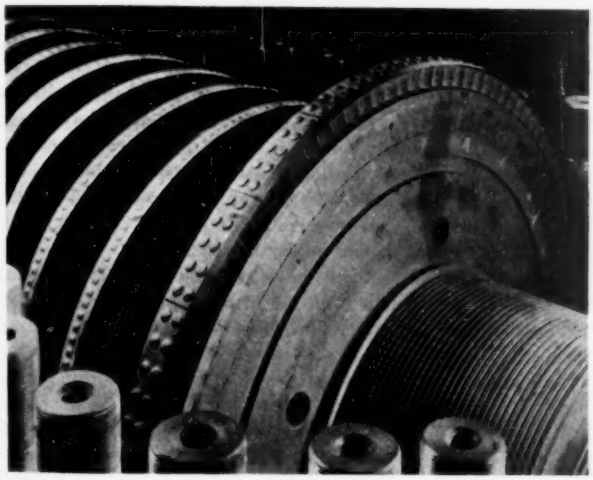


FIG. 4 NEW FIRST-STAGE BLADING AND SHROUDING; SECOND AND THIRD STAGES REBLADED

was attached to the shroud or cover plate by the usual rectangular tenons. New first-stage blading, Fig. 4, was installed with two round tenons and the new shrouding was considerably heavier. The blades on the second- and third-stage wheels had also been damaged and were replaced. Heavy deposits of boiler carry-over were found on the blades in the lower stages. Changes on the economizers and in the drums were completed during this turbine outage, which reduced the solids in the steam to less than 0.5 ppm.

2 This outage occurred in May, 1938, at the request of the manufacturer to inspect the new shrouding which had been installed. Everything was found to be in a satisfactory condition. The blades at this time were free of deposit. No changes were made.

3 An outage occurred in September, 1939, to install a new rotor Fig. 5, having blades with integral shroud caps and shrouding attached to the caps with single rectangular tenons. A change was also made in the high-pressure packing for the purpose of reducing the thrust. The thrust bearing was revamped, as a precautionary measure, to provide greater capacity in both directions.

No trouble had been experienced with the old thrust bearing, although changes were made at the request of the manufacturer.

The turbine was found to be in a satisfactory condition in every particular. The deposit on the blades after 16 months of operation since the last inspection was negligible.

This machine has always operated very smoothly. Vibration has never been a problem.

All other outages have been of a minor nature.

The rotor which was removed, although in perfect condition, is to be changed to duplicate the new rotor and will be used in the turbine now on order.

The only protection provided against overheating with loss of load is the temperature control of the dampers above the economizer passes. This control is designed to maintain 900 F inlet temperature or a maximum of 575 F in the steam leaving the turbine.

The rise in exhaust temperature, when the turbine is taken off the line, does not exceed 100 F.

The back-pressure governor is not used for normal operation.

Hydrogen consumption during this 2-year period has averaged 36.9 cu ft per day. For the month of June, 1939, the consumption went up to 52.8 cu ft per day. With the elimination of leaks, the consumption of hydrogen has been brought down to 25.6 cu ft per day and this rate has been maintained since June.

A reverse-power relay prevents the generator from motoring. The only trouble experienced with the electrical parts of the unit was with the brush holders on the pilot exciter.

OPERATION OF HIGH-PRESSURE BOILERS

From Table 1, it is apparent that boilers No. 7-1 and No. 7-2 would have had an availability of 85.6 per cent and 86.3 per cent, respectively, except for other causes not attributable to the boilers.

Alterations and repairs include changing the economizers from

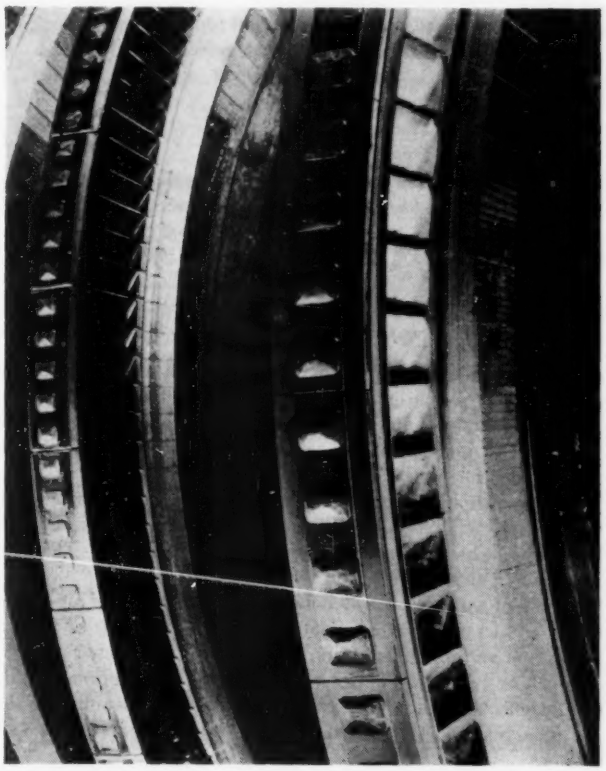


FIG. 5 NEW ROTOR INSTALLED SEPTEMBER 1, 1939, SHOWING NEW-TYPE BLADING AND SHROUDING

TABLE 1 OPERATING RECORD OF HIGH-PRESSURE BOILERS

	Boiler No. 7-1	Boiler No. 7-2
Initial start	May 3, 1937	July 27, 1937
Availability from start to Sept. 15, 1939	Per cent 76.1	Per cent 80.7
Alterations and repairs	14.4	13.7
Tube corrosion	1.7	1.9
Precipitator changes	1.7	1.7
Gate valves	1.7	1.2
Hydraulic couplings	1.9	....
Reducing valve	1.4	....
Unclassified	1.1	0.8

parallel flow to a combination of parallel-counterflow, changes of the baffles, sprays, etc., inside the drum, seal welding of economizer tubes in the drum and headers, changes in the construction of the furnace corners to obtain a better seal, cap leaks, internal cleaning, etc.

During the first year of operation, heavy sludge and analcite scale formed in the generating tubes and waterwalls. Frequent turbining was necessary. The principal cause of this difficulty was condenser leakage, which could only be corrected by retubing 3 condensers. In the meantime it was decided to use the Akon treatment. The Akon treatment was started July 27, 1938, and was fed at the rate of 2 ppm with a pH of 11 in the boiler water. The next internal inspection disclosed that several of the steam-generating tubes were deeply corroded and had to be replaced. The next inspection about 2 months later showed a similar condition. The Akon was then reduced to 1 ppm and the pH of the boiler water raised to 11.5. Under these conditions the rate of corrosion was materially accelerated. Two forced boiler outages were then caused by tube corrosion. The tubes which blew through were in each instance No. 5 tube in No. 7-1 boiler. The second outage occurred 33 days after the tube had been replaced. The Akon treatment was discontinued March 27, 1939; since that time no further corrosion has been noticed.

At present, the feedwater has a pH of 6.9 and a concentration of 1 ppm of solids. A blowdown of 0.1 per cent maintains a boiler-water concentration of about 150 ppm solids.

A typical analysis of boiler water at present is:

Analysis	Boiler water solids, ppm
SiO_2	10
R_2O_3	2
NaOH	31
Na_2HPO_4	30
Na_2SO_4	35
NaCl	9
Na_2CO_3	3
Organic	16
Total Solids	136
pH	10.6

A United States Bureau of Mines embrittlement tester has not indicated caustic embrittlement to be a problem.

Disodium phosphate is the only chemical added to the feedwater. The addition of a chemical to scavenge oxygen has not, as yet, been necessary.

The operation of soot blower, retractable slag blowers (added to original installation) and hand lancing require one man per 8-hr shift, which operations are generally effective. The coal used has an ash with a fusion temperature of 2850 F.

Traces of corrosion are evident on the upper rows of air-heater tubes on the gas side, air-inlet end. Its extent varies from a fraction of an inch in length on the lower of the affected rows, to as much as 2 ft on the upper row. It is not at all serious and no special effort is made to prevent it. Effective cleaning of the gas passages is accomplished by 6 valve-in-head revolving soot blowers.

The 6 Babcock & Wilcox type-B pulverizer mills have performed satisfactorily. Maintenance has been limited to replacing feeder plates, belts, three pinion shafts and adjustment of ball-spring tension. No balls have been replaced although one mill has pulverized more than 80,000 tons. The original carborundum feeder plates have been replaced with a sectional plate having carborundum imbedded in the metal. This plate wears better and sections can be replaced without a major outage on the mill. Air lines, installed to blow debris from the ledge below the stationary ring, caused trouble and were removed. An inclined plate which guided coal from the spout to the belt on the Richardson scales was moved to a more vertical position to prevent wet coal from arching over and failing to move down to the belt. Mill fineness is generally greater than 80 per cent through a 200-mesh sieve. Mill power varies from 18 to 18.5 kwhr per ton.

Two major failures of the hydraulic couplings have caused forced boiler outages. The first failure was on a forced-draft fan, in which the coupling was demolished, caused by the failure of the thrust bearing, located within the coupling. This thrust bearing was relocated on the outboard bearing of the motor. Trouble with this bearing on an induced-draft-fan coupling caused another outage. This time, however, the damage was limited to the internal thrust bearing.

The gas-burner lighters have been satisfactory in every respect. No trouble has been experienced with the stability of the flame at any load.

Arcing and burning off of electrodes on the electrostatic precipitators was corrected by changes in the design of the electrode shrouds and rapping mechanism. Low efficiency of dust removal was improved by installing baffle plates on inlet- and outlet-gas passages, addition of perforated plates to rod curtains and elimination of gases by-passing the cells by placing vertical baffle plates in each hopper under the cells.

Boiler-feed-pump outages have resulted from trouble with shaft-packing sleeves; erosion at breakdown bushing; leakage at horizontal casing joint, slow leaks from stud hole in casing flange. All outages on the feed pumps were scheduled. The excess-pressure regulators and the 1200-lb 900-F turbine drives have performed satisfactorily.

Carry-over was encountered in the initial operation of the double-effect Foster-Wheeler evaporators. Make-up water, erratic in quality, and in the amount of suspended matter from an overloaded softener, made the internal treatment and blowdown regulation difficult in the evaporators. Lime deposits fouled the steam scrubbers and gill separators.

The installation of a new 5000-gal per hr Cochrane hot-process lime-soda softener, replacing the old smaller softener, now produces a uniform and suspended-matter-free make-up water. A hood placed over the scrubber box eliminates flooding the gill steam-separator chamber. Continuous surface blowdown through a manifold extending across the evaporator shell and weekly cracking with cold water keeps the evaporators clean and free of scale. Internal treatment of Hagan phosphate and tannin is fed intermittently to the suction of the evaporator feed pumps. Carbon dioxide in the second-effect coil severely corroded the copper float of the discharge trap. A stainless-steel float appears to be satisfactory. About 24,000 lb per hr is the usual rate at which the evaporators are operated. With 10 per cent blowdown, the evaporator steam contains less than 1.5 ppm of solids.

The condensate and make-up pass through Worthington deaerating heaters. No trouble has been experienced with the float valves which control the flow of condensate to the heaters. The deaeration is effective. A pressure-reducing valve automatically makes up for any deficiency in the 5-lb exhaust-steam supply to the deaerators.

The high-pressure feedwater heater is a horizontal, lockhead, 4-pass type with $^3/_4$-in-outside-diam, No. 10 B.W.G. arsenical copper tubes. The heater is located on the discharge side of the

high-pressure boiler feed pumps and receives its steam from the boiler-feed-pump turbines exhaust. Most of the maintenance has been caused by gasket leaks and fouling of the water side of the tubes. Slight leaks developed and some of the tubes were rerolled. The pipe flanged joints at the inlet- and outlet-water connections have developed leaks from time to time. Slight grooves have been found in both the male and female flanges at one time or another. These were repaired and the joints seem to hold satisfactorily.

A deposit on the water side of the tubes, about $^1/_{16}$ to $^1/_8$ in. thick, largely composed of calcium ortho phosphate, decreased the heat-transfer efficiency of the cross-over heater. Since the evaporator performance was improved, condenser leakage corrected and boiler-water recirculation eliminated, no appreciable deposits of calcium ortho phosphate have been noted.

The high-pressure reducing valve first installed was unsatisfactory. Excessive vibration caused linkage failures and the fracture of the valve stem. This 6-in. valve was replaced with an 8-in. valve of much heavier construction, the capacity rating remaining the same, namely, 390,000 lb per hr. The new valve has performed in a satisfactory manner but is still noisy. On occasions when the valve is used, it is operated manually by remote control from the boiler control board.

The Bailey 3-element feedwater control maintains a stable water level ever since the changes were completed on the economizer. Before the changes were made the boiler had bad swinging water levels.

The Leeds & Northrup superheat control has performed satisfactorily. The Leeds & Northrup combustion control also performs satisfactorily since the closed manometers using oil were installed in place of the inverted bells. Formerly a deposit formed on the bells, causing the control to become sluggish. With large changes in the turbine load, the change in the pressure drop from the drum to the turbine inlet must be compensated for manually on the master control.

G. B. Warren.[8] To the writer, one of the most significant things developed by the discussion of this subject is that the reliability of the high-pressure high-temperature steam turbine is approaching that of the older low-pressure turbine, at least after the initial difficulties with the high-pressure turbines have been overcome. Availability factors of 96 to 98 per cent have been frequently mentioned.

Surveys of the Edison Electric Institute figures on turbine outage made at intervals have indicated that the outage record of the older high-pressure superposed turbines in service from 7 to 10 years now is better on the average than that of the low-pressure turbines of comparable age, and bear out this recent experience.

Another situation which may be of interest in this connection is that, based upon comparisons which we have made, the present-day materials in these high-pressure high-temperature turbines are being worked more conservatively than were the then available materials in use on lower-pressure and lower-temperature turbines 10 and 15 years ago. This is largely due to our increased knowledge of creep and other properties of materials at high temperature and our definite endeavor to keep within the limitations so imposed. This is quite in contrast to the situation which prevailed in the earlier period, when we had by no means as much information on the behavior of the materials we were then using at the temperatures which were coming into general application.

On the other hand, a word of caution might be in order on the tendency of materials to rupture under low strains when subjected for long periods of time to temperature and stress, a fact which the industry has only recently commenced to appreciate.

A few years ago we thought that the phenomenon of creep gave us an additional safety factor in that it tended to equalize stress, but now we are beginning to learn that, apparently, the excessive localized strains so resulting might be such as to start a rupture which would be hazardous so far as our present knowledge and observation go. Present practice with respect to temperatures and stresses is such we believe, as to preclude any serious consequences from this situation. Extensive studies are under way in the writer's company and in other places to accumulate additional knowledge relative to this situation. In the meantime, all designs are being scrutinized with this factor as well as the creep situation in mind.

The first-stage-bucket situation and, from the standpoint of the writer's company, the first-stage-bucket-cover situation primarily, have been matters of considerable concern. We feel, however, supported now by considerable operating experience on several turbines, that this situation is well in hand.

Louis Elliott.[9] Concerning experience at Deepwater Station of the Houston Lighting & Power Company, at Houston, Texas, during 1932 a topping extension was put into operation in this plant, which previously contained 135,000 kw in 325-lb turbines; this was among the earlier examples of high-pressure superposition plants. As the installation has been described both in technical papers and press, the present discussion is brief.

Equipment includes a 12,000/15,000-kw 1260-lb 710-F turbine, with steam-to-steam reheat in condensing and thoroughfare heaters, raising high-pressure turbine exhaust to temperature of steam in low-pressure header. Steam-generating plant comprises two 375,000-lb per hr 1350-lb 850-F gas-fired boiler units with economizers and air heaters. In 1930, when engineering on Deepwater extension was under way, American experience with utilization of temperatures above 750 F was quite limited, and manufacturers were not offering as standard equipment turbines adapted to a temperature much higher than this.

One or more low-pressure boilers are always in operation, supplying 325-lb steam "in parallel" with exhaust from topping turbine. An automatic by-pass is provided around high-pressure turbine, and this has functioned successfully upon the few occasions when the machine has tripped off.

Both boilers and turbines have shown good availability, and have been run under high average loads; contributing to this possibility has been large capacity upon which high-pressure unit is superposed. Boilers have shown outage hours, largely occasioned by necessary work on these units, totaling less than 6 per cent over recent 4.5-year period; there has been only about 4 per cent boiler outage during the last year. Turbine outage has averaged 1.4 per cent in same 4.5-year period, and less than 0.3 per cent during best year. For entire time since installation was placed in service, outage of high-pressure boilers has averaged about 12 per cent, and of topping turbine approximately 6 per cent of elapsed time, largely due to work on unit in first year.

Total generation by high-pressure unit since 1934 has aggregated 500,000 megawatthours, equivalent to an average generation of 12,800 kw (when operating); this means a unit output factor of 106 per cent based on 0.8 pf rating, or 85 per cent based on 1.0 pf rating. For the entire operating time since early in 1932, the average generation has been 11,900 kw, or nearly the 0.8 pf rating.

Lower availability of boilers and turbine during first year of operation resulted from the usual difficulties incident to starting

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up new equipment, and from a variety of troubles, caused particularly by the high pressure and by the limited experience of manufacturers in designing equipment for high-temperature high-pressure service. Apparatus troubles were experienced with heat exchangers, boiler feed pumps, and feed-line gate valves. High-pressure piping joints, which were of the flange type and of special carefully worked out design, were unusually successful; however, considerable trouble was experienced with high-pressure joints in equipment until proper operating procedures were developed. The record since 1934 shows that the early difficulties have been corrected, and details concerning them would hardly be worth giving at this time.

In view of blading deposits in low-pressure turbines when higher boiler concentrations were permitted, total solids in boiler water are now kept at about 400 ppm, with 100 ppm total alkalinity and 10.8 pH value. Silica is maintained at or below 15 ppm, and it has been found practicable to maintain sulphate carbonate ratio of 3:1.

Mr. Sporn's Closure

The discussions of the papers on "Two Years' Experience With High-Temperature High-Pressure Stations" confirm my informal introductory remarks at the Annual Meeting of the A.S.M.E. in Philadelphia. They show that this group of stations or units which had initially been thought of as ultradaring has become almost commonplace because of their gradual and natural integration with their systems until they are just other stations or units on the line. This is most gratifying. But let us not forget that they now can serve as another starting point for further advances in the art of power generation, advances that must and will proceed inevitably, regardless of the reluctance of any company or group of companies to keep up with progress.

Characteristics of Cloth Filters on Coal Dust–Air Mixtures

By A. R. MUMFORD,[1] A. A. MARKSON,[2] and T. RAVESE,[3] NEW YORK, N. Y.

This paper deals with the installation, operation, and maintenance of cloth filters for handling vented air from pulverized-coal mills. The experiences of the authors in solving problems in connection with the cloth-bag filter system, as used at the Kips Bay Station of the New York Steam Corporation, are the basis on which the subject is developed. Complete performance tests on a 40-ton-mill installation are cited.

TWO INSTALLATIONS of cloth filters have been successfully handling the vented air from two large pulverized-coal mills at the Kips Bay Station of the New York Steam Corporation for nearly three years.

In the application of such equipment to power plants, the magnitude and variation of the draft loss, when related to the frequency of the shaking cycle and to the capacity, assume greater importance than in smaller applications for air cleaning and, therefore, have been very carefully investigated.

Similarly the required maintenance is of greater importance because of the interdependence of the mill and other plant equipment and therefore this item has also been examined.

The presence of a fire hazard in the handling of combustible air mixtures adds to the complexity of the design and operation so the safeguards taken in the design and installation at this plant will be indicated.

Description of Plant

The Kips Bay steam plant was erected in 1926. The station was laid out for the storage system and this design was continued in the 1927 and 1930 extensions.

The millhouse contains seven pulverizing mills with a total capacity of 150 tons per hr. Raw coal entering the millhouse contains about 3½ per cent moisture and of this 2 to 3 per cent is removed in the process of preparation. The coal is dried in external-grid driers, using exhaust steam for four mills while in three mills the coal is dried in the mill by air heated by high-pressure steam in air heaters. Air circulated through each mill system conveys the pulverized coal to the primary cyclone where separation of the coal and air takes place. The conveying air is vented to the atmosphere through a secondary cyclone and dust washer. The air-flow circuit for the 25-ton ball-type mill is given in Fig. 1.

One to two per cent of the coal milled may escape with the air vented from the primary cyclone and 0.25 to 0.5 per cent may escape from the secondary cyclone. For further reduction of the discharge concentration, air washers have been generally used. The use of washers brought secondary problems which required continual attention. These are as follows:

1 Disposal of sludge water and the associated problem of maintaining drain lines free from clogging.

2 Corrosion of air washer and fouling of sprays.

3 Warm-air discharge, high in humidity, creates fog obstructing vision of roof operations.

4 Some spray water is discharged as a very fine mist, deteriorating building walls and creating a bad condition on the millhouse roof.

5 Loss of pulverized fuel as a result of the wet washing.

It is evident that, if the vent coal is to be reclaimed dry, a dry type of filter must be resorted to and further that such dry-filtering equipment must be nearly 100 per cent efficient under all load conditions or a potential nuisance will exist. One of the few types of filters on the market which fulfills the requirements is the cloth or bag filter system.

It was decided in 1936 to purchase two cloth-type filter collectors, each with four filtration compartments for the 25- and 40-ton per hr ball-type mills, eliminating from the vented-air circuit both the secondary cyclone and air washer.

The two filter units were installed under a single housing on the millhouse roof over the raw-coal bunker and adjacent to the south coal tower. The secondary cyclone and air washers on the ball-type mills were left intact after connecting the filter dust collectors so that, in the event of failure of the dry filters, which has not occurred to date, the vent system could be changed over to wet washing in a matter of hours. The dry-filter system air-flow circuit with the change-over provision for wet washing is shown in Fig. 2.

The dust filter compartments are in the same housing to sim-

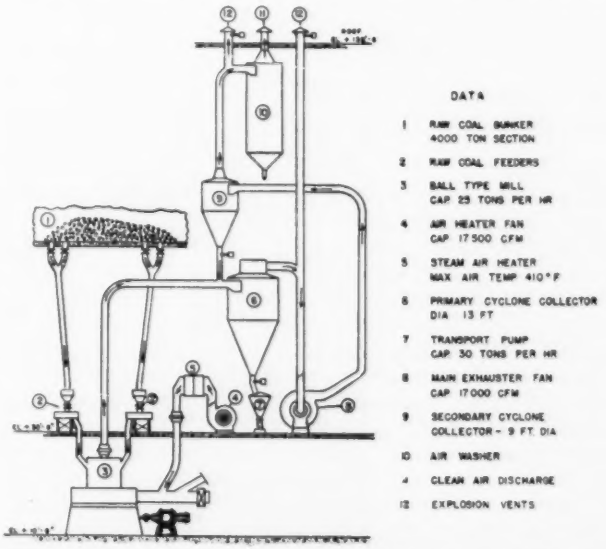


Fig. 1 Coal and Air Flow Circuit, 25-Ton Mill With Wet Washing of Vent

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

plify construction, although their operation is entirely separate. The path of flow of the coal-dust and air mixture through the collector is shown in Figs. 3 and 4. Flow of the mixture is up through the bottom of the collector, through a pipe located in the center of the filter then to the four compartments, two of which are located on one side of the inlet and two on the opposite side. At the inlet to each pair of filtering compartments is a gate-type damper which can be used to close off the flow to either compartment but never to both at the same time. Take-up springs are provided on the driving-shaft mechanism to prevent shearing of the damper shaft and also to insure a pressure fit of the damper against a $^1/_2$-in-thick soft-rubber gasket which lines the door frame.

The dusty air enters the filtering compartment and makes a turn of 180 deg to the outside of the filtering surface. The filtering surface consists of a number of bags each shaped as a rectangular envelope open on one end. The bag or envelope is slipped over a galvanized-wire screen, the opposite surfaces of which are held in correct spacing of $1^3/_{16}$ in. by metal struts. The 25-ton mill has 552 such bags and the 40-ton mill 840 bags, six rows high. The path of air flow is through the cloth and out the open end of the envelope. The coal dust builds up a dust layer on the cloth surfaces, which is shaken off periodically, and the cleaned air passes through to an opening provided in the housing. The open end of each bag screen is sealed to the collector casing, dividing it into a dust side and clean side.

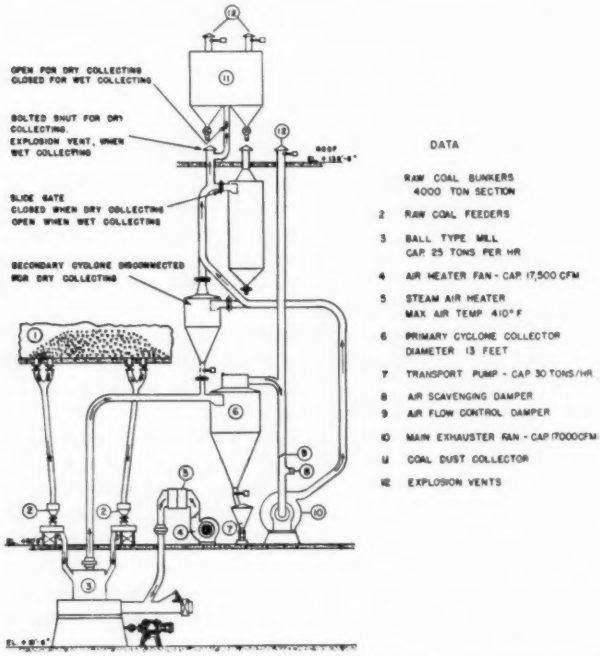


FIG. 2 COAL AND AIR-FLOW CIRCUIT, 25-TON MILL WITH CLOTH FILTERS

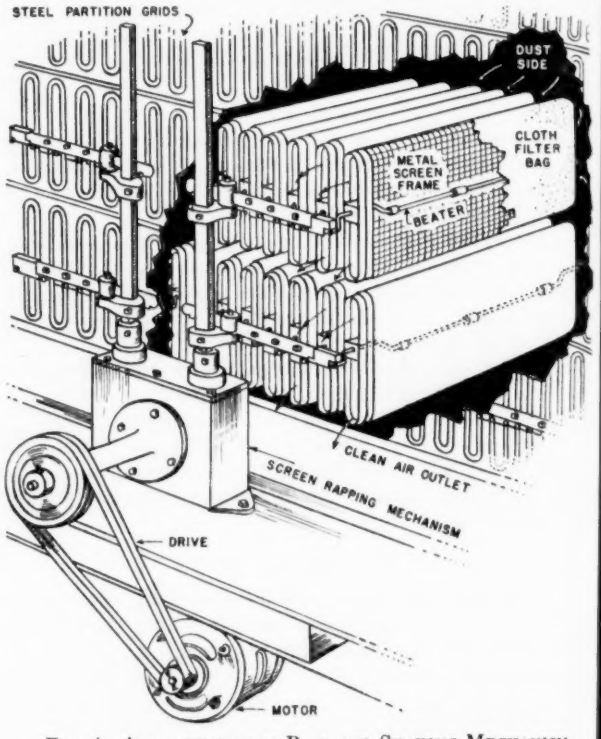


FIG. 4 ARRANGEMENT OF BAGS AND SHAKING MECHANISM

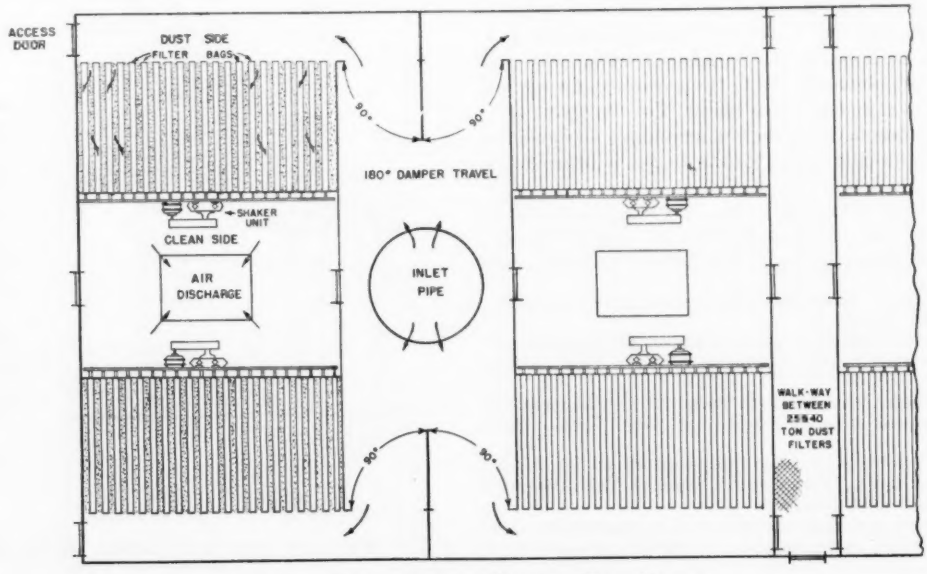


FIG. 3 AIR-FLOW DIAGRAM THROUGH DUST FILTER

The shaker consists essentially of a motor operating a rocker mechanism which transmits oscillating motion to beater rods located along the centers of the bag frames. Coal hoppers are located directly below the cloth surfaces to receive the loosened dust from the bags. A rotary valve, which is air-sealed by means of a rubber flap, rotates at slow speed and discharges the reclaimed coal to a return coal pipe which conveys it to the hopper above the transport pump for delivery with the main body of coal to the boiler fuel bins. Each compartment has its own shaker motor and shaking mechanism. The length of the shaking period and proper sequence is governed by the cam switches on the time-control panel.

Laboratory Tests

The successful installation of the collector depended on several factors, some of which had to be determined in the laboratory because the information was unavailable. The known factors were:

1 Vented-air volume.
2 Vented-air temperature and relative humidity.
3 Vented-coal quantity and screen sizing.
4 Maximum pressure loss allowable in operation.

The factors which had to be investigated because little or no information was available were:

1 Pressure-drop characteristic of the filter cloth.
   (*a*) The effect of flameproofing on the pressure drop.
   (*b*) The effect of coal-dust loading on the pressure drop.
2 Safety considerations.
   (*a*) Precautions necessary to insure safe operation of the cloth filters.

The known quantities as previously established are given in Table 1.

TABLE 1 KNOWN FACTORS OF COLLECTOR OPERATION

	25-Ton mill	40-Ton mill
Vented-air volume, cfm, max	15000	27500
Vented-air temperature, F	130	130
Vented-air, relative humidity, per cent	60	60
Vented coal, lb per hr, max	800	1600
Vented-coal sizing	100 per cent through 325-mesh Tyler screen	
Available pressure for cloth filters, in. water, max	6	6

Tests were made on a number of filter cloths submitted by manufacturers. The cloths were tested for air-flow resistance in the clean condition and also after loading with coal dust. The fabrics tested were made from cleaned cotton having a sateen weave with a count 96 × 64 threads per in. Two weights of fabric tested are reported; one with a weight of 1.05 yd per lb and the other 1.30 yd per lb; both fabrics 54 in. wide. The cloth samples were clamped tight without stretching to one end of a cylindrical chamber approximately 10 in. diam and 3 ft long. Metered clean air was blown in on the other end. Cloth resistance was measured with a U gage connected to the chamber approximately 2 in. from the cloth. The effect of several flameproofing solutions on the pressure drop was also established. The designations of cloth tests reported are given in Table 2.

TABLE 2 DESIGNATIONS OF CLOTHS TESTED

Curve symbol designation	Cloth thread count	Cloth width, in.	Cloth weight, yd per lb	Flameproofing
A	96 × 64	54	1.05	Untreated
A_1	96 × 64	54	1.05	Mixture of ammonium-phosphate and boric-acid solution
B	96 × 64	54	1.30	Untreated
B_1	96 × 64	54	1.30	Sodium-borophosphate solution
B_2	96 × 64	54	1.30	Sodium-ammonium-borophosphate solution

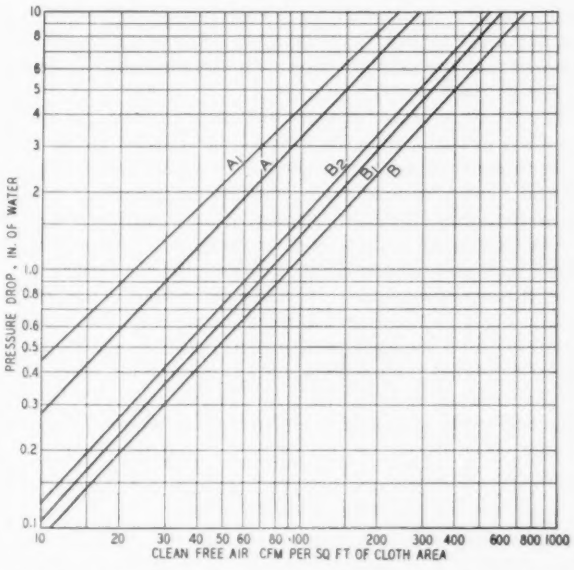


FIG. 5 PRESSURE DROP THROUGH CLOTH WITH CLEAN AIR

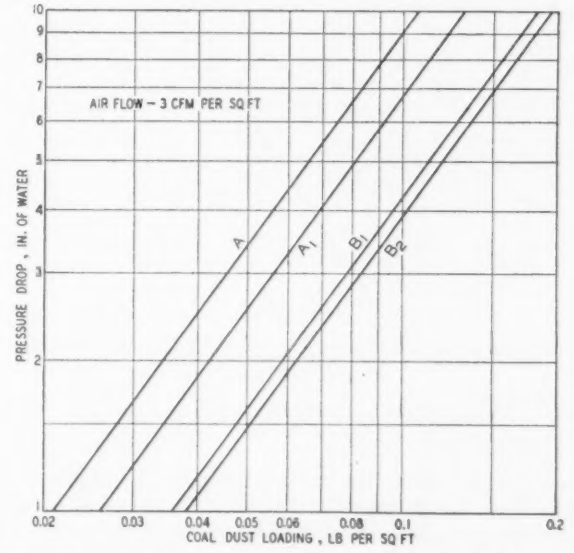


FIG. 6 PRESSURE DROP THROUGH CLOTH LOADED WITH COAL DUST

The results for clean free air are given in the curve in Fig. 5 and indicate that the flow-resistance curves are parallel, with the exception of curve A_1 which may be accounted for by the fact that cloth A_1 may have had an initial stretching.

The difference in the weight of the cloths A and B is reflected in their relative resistance to air flow. Cloth B (the lighter cloth) will pass approximately 2.7 times as much air as cloth A for the same pressure drop. Flameproofing increases the pressure drop of cloth A by about 24 per cent and that of cloth B by about 18 to 25 per cent. The effect of flameproofing on the flow resistance will depend largely on the composition, strength, and also the method of application to the cloth.

Continuing the foregoing test; some fine coal (screened through a 200-mesh Tyler screen) was blown against the cloth and pressure-drop data collected. These data were taken for a number of coal loading points so that it was possible to study the rate at which the pressure drop increased. Powdered coal was intro-

duced into the cylindrical chamber mentioned with an air injector, making it possible to secure a uniform coating of coal on the cloth, the amount of which was determined by weighing the cloth after each run.

The results of these tests are indicated in Fig. 6, which shows the effect of coal loading on the pressure drop for an air flow of 3 cfm free air per sq ft. The curves are plotted for cloths A, A_1, B_1, and B_2 and have the same letter designation given in Fig. 5. If, as an example, for cloth A a point at 0.05 lb per sq ft coal-dust loading is taken the pressure drop is 3.4 in. Comparing with Fig. 5, a pressure loss of 3.4 in. will pass 102 cfm free air per sq ft or 34 times as much air. It will be noted that, in line with the tests of Fig. 5, cloths B_1 and B_2 will allow more coal accumulation on the cloth for the same pressure-drop and flow conditions than cloths A and A_1. Thus for a 3-in. pressure drop, cloth A_1 will allow a coal loading of 0.056 lb per sq ft, whereas cloths B_1 and B_2 will allow 0.078 and 0.083 lb per sq ft, respectively. This, however, does not mean that the normal operating pressure of the cloth filters will be less with cloth B_1 or B_2 than with cloth A_1, but rather that cloths B_1 and B_2 will retain more coal on their surfaces than cloth A_1 and that the residual loading, after the cleaning period of cloths B_1 and B_2, is more than with cloth A_1. The normal operating pressure is approximately the same with either cloth as borne out by actual operating experience with both cloths.

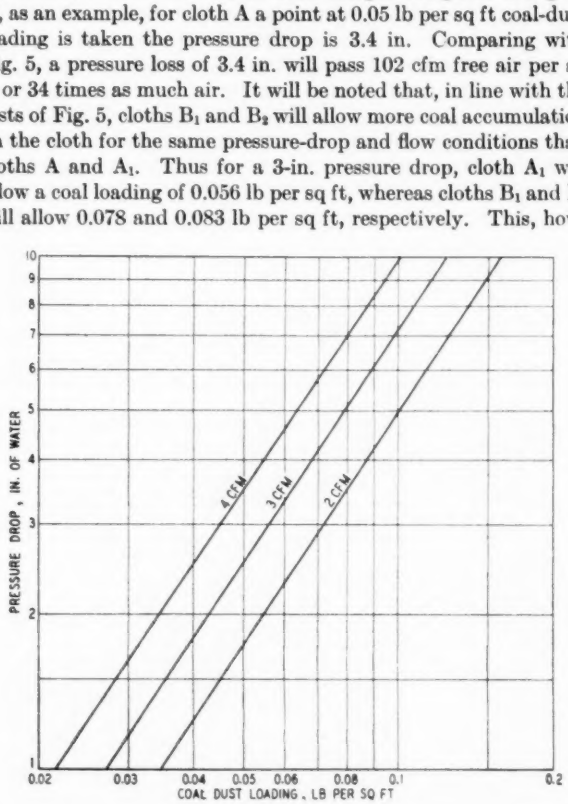


FIG. 7 PRESSURE DROP THROUGH CLOTH A_1 VERSUS COAL-DUST LOADING

Observations made during these tests indicate that coal dust passes through the cloth when it is clean until the pores become filled with dust particles to a point when no more coal dust passes through. The dust retained on the cloth provides a mat filtering medium. This effect is more noticeable with cloths B_1 and B_2 than with cloth A_1. No tests were made with cloth B (untreated cloth) because it had been decided to use flameproofed cloth in the installation.

Cloth A_1 was initially selected for the installation because of its superior mechanical properties. It was believed that the heavier cloth would withstand the effects of temperature and beater rods better than the lighter cloths B_1 or B_2. Flow curves were constructed for cloth A_1, as in Fig. 7.

The design for cloth-filter surface for both mills is contained in Table 3.

TABLE 3 CLOTH-FILTER SURFACE AS DESIGNED

	25-Ton mill	40-Ton mill
Coal milled, lb per hr	50000	100000
Coal to cloth filters, lb per hr	800	1600
Vented air, cfm	15000	27500
Free air cfm per sq ft cloth area (100 per cent area assumed)	2.1	2.6
Free air cfm per sq ft cloth area (75 per cent area)	2.8	3.5
Total cloth area, sq ft	7040	10710
Reduced area, 75 per cent of total, sq ft	5280	8030
Coal-dust loading, lb per M sq ft to give 4 in. normal operating pressure at reduced area	71	62
Total coal on cloth filters, lb	500	665
Shaking interval, min	37	25

From the figures given in Table 3, it is evident that the cleaning-period cycle should be less than 37 min for the 25-ton mill and less than 25 min for the 40-ton mill. It was recognized that the calculated figures are only approximations because it had been assumed that the filter cloths were thoroughly clean at the beginning of each cycle. The limited value of the tests, because of the number of uncontrolled variables, such as temperature, humidity, dust and sizing, did not justify more accurate determination of the cleaning cycle. To simplify the control for these operations, it was decided to operate both mills with the same initial cleaning cycle, a compromise between the two calculated cleaning cycles.

Each filtering unit has four compartments and is so constructed that one compartment at a time is removed from service for cleaning. It is, of course, important that the cleaning operation occur only while the compartment is dampered off. The setting adopted is given in Table 4.

TABLE 4 CYCLE SETTING ADOPTED

	25-Ton mill	40-Ton mill
Number of complete cycles per hr	2	2
Time required for 1 cycle, min	30	30
Total time, unit operating with 100 per cent filtering capacity, min	10	10
Total time per cycle when unit is operating with 75 per cent filtering capacity, min	20	20
Actual shaking time per compartment per cycle, min	3	3
Actual shaking time per compartment per hr, min	6	6
Actual shaking time all compartments per cycle, min	12	12
Actual shaking time all compartments per hr, min	24	24

NOTE: Time control for the cycle was obtained from a synchronous motor through reduction gears driving a shaft onto which were fastened bakelite cams opening and closing contact switches in proper sequence for damper and shaker motor operation. Provision was also made for manual operation of the control limit switches.

The validity of such testing with respect to the resistance of different cloths has a good degree of plausibility, since it is possible to test the cloths under practically identical Reynolds-number conditions as exist in the full-scale apparatus. These tests indicate that the effective filter resistance is principally determined by the filter mat which is formed by the cloth and the so-called residual dust loading. To a considerable degree, independence of filter resistance to the textile specification of the cloth exists, a fact of interest because cloths may be selected principally for their mechanical qualities.

If it should be found that such tests furnish an approximate basis for the design, they would assume considerable importance. The authors' results indicate that this is only roughly true. Since such correlation was not of primary interest at the time of carrying out the work, necessity for further and more extensive data is indicated, especially on different materials.

The laboratory tests indicate two apparent flow-resistance rules. First, at constant loading of the cloth the draft loss varies directly as the flow. This is reasonable enough as the flow through the filter itself must be viscous. Second, at constant air flow, the draft loss varies as the 1.5 power of the loading, a statement by no means so obvious and which must be confined to the limits of the experiments, because it involves several variables

connected with dust sizings, atmospheric conditions, and others not under controlled variation in the experiment.

Field Tests

The installation of the 25-ton-mill filter was completed in the spring of 1937 and placed in service immediately thereafter. For the first several days, operation of the collector was as expected. Initially some coal dust came through the cloth onto the clean side until a dust mat was formed on the cloth after which the air coming through the cloth was perfectly clean. Subsequent operation of the collector showed that the air resistance kept building up and no amount of shaking would bring it back to normal while the mill was in service, although the shaking was effective with the mill shut down. After numerous tests, it was decided to change the timing cycle. New cams were ordered, cut, and installed, which permitted more flexibility in the time-control variation by resetting the cams on the drive shaft and changing the motor reduction gears.

During the shaking cycle, the sequence of the cleaning operation is as follows:

1 Close off compartment to filtering by damper.
2 Time prior to shaking called "rest period."
3 Shaking period.
4 Time after shaking called "settling period."

Changes were made in the number of cleaning cycles per hour and the rest, shaking, and settling periods.

Some of the results of the tests have been plotted in curve form

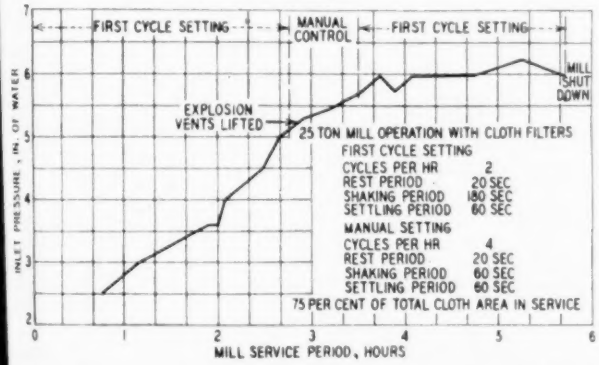


Fig. 8 Variation of Back Pressure With Time; First-Cycle Setting

and indicate the rate of pressure build-up. Fig. 8 is a typical curve of the initial cycle for 75 per cent filtering area. During part of this cycle 100 per cent filtering area was used. However, so that the test results of the original cycle may be compared with subsequent data on cycles which employ 75 per cent filtering area all of the time, the results were plotted for 75 per cent filtering area. In a period of 2 hr 5 min pressure built up to 4 in., the normal design pressure, and increased to $5^1/_4$ in. in an additional 47 min, resulting in a lifting of the explosion vents. After 2 hr 45 min of service the time control was changed to manual with 4 cycles per hr and 60 sec shake, but pressure continued to build up until the vents were continually open. The mill was shut down after 5 hr 45 min of service because of excessive back pressure.

Figs. 9, 10, and 11 are plots of results of the tests in which the cleaning cycles per hour, rest, shaking, and settling periods were changed as indicated on the curve sheets. All tests indicate the filter back pressure rises to 4 in. from 2 to $3^1/_4$ hr and, with the exception of one test (Fig. 10) in which 4 in. is reached in about $1^1/_2$ hr, continued to increase until the vents lifted at $5^1/_4$ in. and the mill was shut down. In each run the milled coal was 21 tons per hr, air flow approximated 14,000 cfm, dry-bulb temperature 125 F, and relative humidity 40 per cent. No effect of the cycle frequency on the filter back pressure is apparent except that the results with the 3-cycle frequency are worse than with

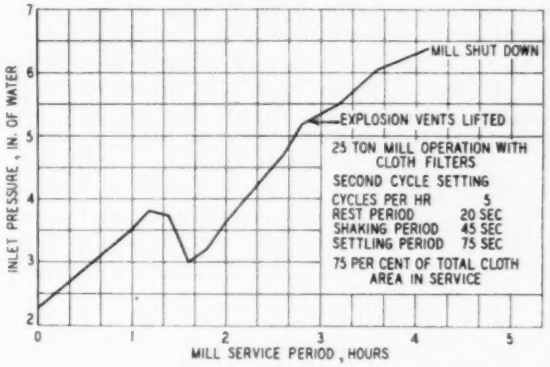


Fig. 9 Variation of Back Pressure With Time; Second-Cycle Setting

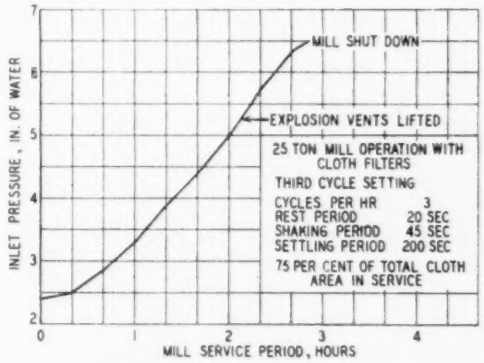


Fig. 10 Variation of Back Pressure With Time; Third-Cycle Setting

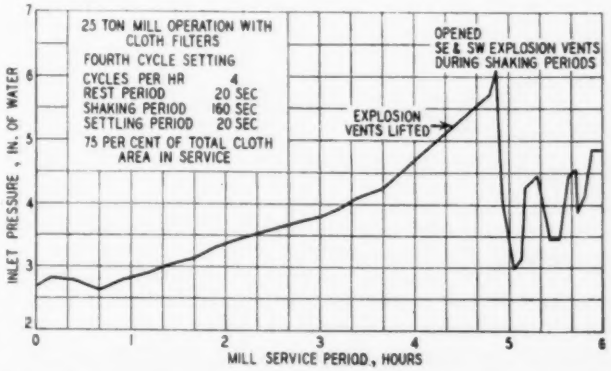


Fig. 11 Variation of Back Pressure With Time; Fourth-Cycle Setting

the 4 or 5 cycle. The duration of the shaking period had no effect on the pressure build-up.

Continued checking of the shaker mechanism and damper-operating mechanism indicated them to be in good working condition. When the damper closed off a compartment to air flow, preparatory to shaking, there was a gradual drop in pressure by virtue of the fact that flow occurs until the pressures on both sides of the filter cloth are in balance. If the time allotted for pressure balance is not sufficient or is offset by air leakage around

the damper seal, then shaking of the cloth no matter what the duration will be ineffective. A checkup of the compartment pressure, when dampered off, indicated a positive pressure of approximately 0.02 to 0.04 in. of water.

During the test run plotted in Fig. 11, two of the vent doors were lifted accidentally while the compartments were dampered off, thus allowing quick equalization of clean- and dirty-side pressures and also providing a sufficiently large area for escape of any inleakage without affecting the pressure balance. The effects were readily apparent. The filter resistance dropped from 6.10 to 4.10 in., after one of the compartment filters had been cleaned, and dropped to 3 in. after the cleaning of the second compartment filters. The pressure increased to 4.4 in. as the first then the second compartment on which the vent doors had been lifted were removed from service. The pressure dropped from 4.4 to 3.45 in. when both filtering compartments were returned to service.

Design of Residual-Pressure Release System

When the pressure filter system, as previously indicated, went into operation the resistance built to very high values. Increase

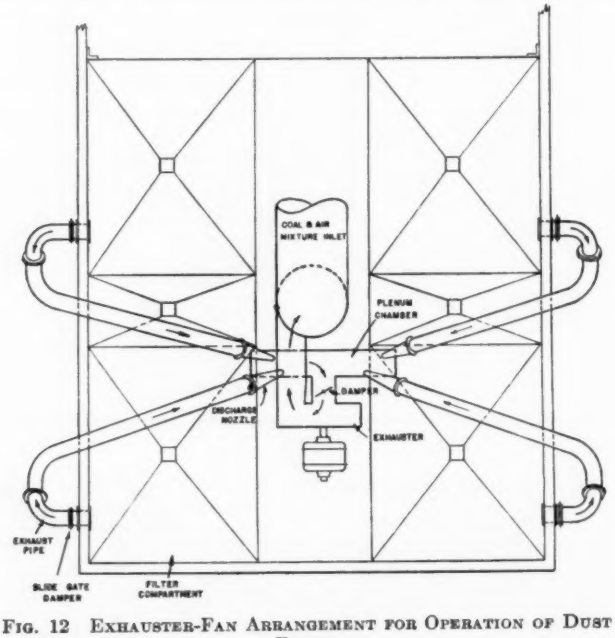


Fig. 12 Exhauster-Fan Arrangement for Operation of Dust Filter

of the number of shaking periods and of the time of shaking was ineffective in eliminating pressure build-up. One theory advanced was that the residual air pressure in the isolated compartment held the coal against the filter in spite of efforts to shake it free. Tests were run to determine the amount of evacuation necessary to allow the free discharge of the dust from the cloth. It was found by measurement of air flow that 275 cfm were required for the filters of 410 cu ft free volume, and 400 cfm for the filters of 590 cu ft free volume. A margin for damper leakage was also allowed.

After consideration this evacuation was carried out in the manner shown in Fig. 12. A single fan was connected to all compartments and continuously operated. Its capacity requirements were based upon the following criteria:

The operating suction at the fan inlet is selected so that variations in flow from operating and nonoperating compartments owing to differences in pressure will be swamped out to a desired degree. This necessitates the introduction of accurate resistances in the individual lines, which may be tapered nozzles, properly designed and discharging in the inlet plenum. The over-all system resistances including the nozzles should be calculated because high line velocities should be used. The total flow into the fan under operating conditions is readily figured and leads to the selection of a fan of proper P-V characteristics. Low velocities and flat spots must be avoided as shown in Fig. 12.

The power requirements of this fan are a charge against operation amounting to 7 hp for the 7040-sq ft filter and 12 hp for the 10,710-sq ft filter.

The necessity for some such installation would exist whether the filter were under pressure or suction. However, for a suction filter, a simple mechanical atmospheric break on the clean side might be provided for each compartment.

The reasoning indicating the necessity for evacuation is expressed mathematically in the following expression which offers a means for evaluating the time which must elapse before the particles may be shaken off in the face of a slight residual pressure.

If Q_t, $\overline{V}$, P_t are, respectively, the weight of air in the filter, the volume of the filter, and the absolute filter pressure at any time t

$$Q_t = k_1 \overline{V} P_t$$

and the time derivative is

$$\frac{dQ_t}{dt} = k_1 \overline{V} \frac{dP_t}{dt}$$

the rate of outflow from the compartment is

$$-\frac{dQ_t}{dt} = k_2 \frac{P_t - P_a}{X} = k_3 (P_t - P_a)$$

in which $\frac{P_t - P_a}{X}$ is the pressure gradient across the filter cloth, P_a being atmospheric pressure.

$$k_3 (P_t - P_a) = -k_1 \overline{V} \frac{dP_t}{dt}$$

from which $-T = K \log (P_t - P_a) + C$

$$\text{or } T_{t_1}^{t_2} = K \log \left(\frac{P_1 - P_a}{P_2 - P_a}\right) = K_4 \log \left(\frac{V_1}{V_2}\right)$$

where V_1 and V_2 are the velocities through the filter mat.

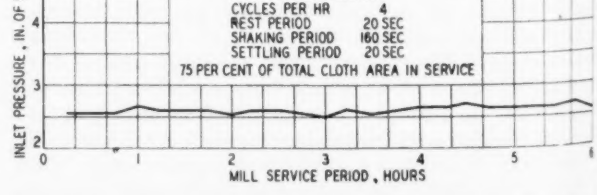


Fig. 13 Variation of Back Pressure With Time; Residual-Pressure Release System

If the time for decrease of the pressure to $^1/_2$ value is observed, the time for the volume rate discharge to change from V_1 to V_2 is given simply enough by the foregoing ratio, assuming that the inlet damper is tight. Theoretically, at least, if the terminal velocity of the particles were known, the time the filter was to be out of service before the pressure decreased to the point at which the particles would fall off the cloth would be determinate.

A consideration of the terminal velocity of fine particles, as discussed by Croft,[4] leads to the general conclusion that the air velocities for the particles encountered here must be less than about 3.5 in. per sec and may have to be as low as 0.001 in. per sec before shaking can be effective. This consideration indicates the necessity for residual-pressure release.

The effect of the residual-pressure release mechanism on the back pressure is clearly shown in Fig. 13 which should be compared with Figs. 8, 9, 10, and 11.

Inspection and Maintenance

Of particular interest in the operation of the dry dust filters is the number of man-hours involved in inspection and maintenance. Inspections while the equipment is in service cover the following:

1 Lubrication of bearing surfaces of coal-conveyer valves, evacuating fans, and motors.
2 Checking housing for leaks from dust side of collector.
3 Observation of clean-air discharge for evidence of torn or defective bags.
4 Checking reclaimed-coal lines for free movement of coal.
5 Checking operation of main exhauster damper control.
6 Checking pressure balance of cloth filter with draft gage.

Items 1 to 4 are covered by the regular millhouse operating crew and are made a routine matter along with other duties at similar locations about every 2 hr. The inspection is primarily a precautionary measure. Items 5 and 6 are looked after by test men in the technical department of the company. A monthly inspection of the damper control is all that is generally necessary and requires about 3 or 4 hr per month. Item 6 is included for the purpose of checking the auxiliary-fan operation and is made about every 3 or 4 months in about $\frac{1}{2}$ hr.

During the shutdown operation it is advisable to check the following and do general internal cleaning:

1 Shaker mechanism.
2 Inspect damper gaskets.
3 Inspect auxiliary-fan piping.

It is good practice to make a general internal inspection of the collector weekly.

Maintenance data on the dust filters are not as complete as they should be at this time because included in the maintenance figures are design changes. These changes in design will undoubtedly lower maintenance charges and decrease outage periods in the future. Figures subsequently cited are for the total operating hours given for each filter.

The largest single item of maintenance in the dust filters is the filter bag. Bag life is approximately 2 yr and the bags, with flameproofing applied, cost approximately $1 each. To remove and replace bags requires about 180 man-hr for the 25-ton mill and 275 man-hr for the 40-ton mill, assuming all bags are replaced. Defective bags are located from the clean side of the collector and are replaced from the dust side of the collector. Since there are 552 bags in the 25-ton-mill installation and 840 bags in the 40-ton-mill installation, this replacement is therefore an appreciable item and worthy of investigation.

At the present time, tests are under way to confirm a proposed decrease in shaking frequency and time. One other change has been made which should improve the life of the bags. Formerly the flow of clean air was downward but now the flow is upward through the top, in order to prevent clean air with high vapor content passing from one collector to the other when one of the collectors is out of service.

The next largest item of maintenance is general cleaning and gasket repair. Approximately 400 man-hr have been spent in this work but it is reasonably certain that this item will virtually disappear, because 90 per cent of the time devoted to cleaning would not have been necessary if the collector were dust-tight initially. For example, the access doors and coal-hopper casing leaked dust continually while the collectors were in service and required constant cleaning and patching, which otherwise would not have been necessary. Dust-tight access doors were made to replace the initial doors and the hopper casing was welded to make it dust-tight. Gasket maintenance is a relatively small item and has approximated 50 man-hr, which time is spent about once a year renewing gaskets on explosion vents, access doors, and damper frames.

Reclaimed coal drive systems have had approximately 330 man-hr maintenance, a considerable portion of which can be considered as due to the dusty atmosphere, resulting from leakage which in turn caused bearing maintenance and maintenance of the chain drive. Rotary valves which feed the coal into the screw conveyer and return coal pipe have had no maintenance. Some trouble had been experienced in the early days with flooding of the conveyer screw, causing plugging of the return coal pipe at the point where it connects to the screw, but this has been rectified by increasing the angle of inclination of the outlet coal pipe. At the present time the coal drive system runs continuously, but it is planned to try out intermittent operation of the drive system which, if successful, will decrease maintenance.

The shaker mechanism has needed 205 man-hr of maintenance which has been due largely to wear of the roll which fits in the cam groove of the rocker mechanism. The roll was worn egg-shaped because it did not rotate with the cam but slid in the cam groove. Some of the rolls were worn down to the pin in a period of 6 to 8 months but, when a grease-seal ball bearing was substituted as a cam follower in place of the hardened steel roll, very slight uniform wear resulted in a period of 18 months. There has been some loosening up and falling out of setscrews fastened to the rocker shafts which have been replaced with a self-locking type of setscrew.

Approximately 75 man-hr have been charged to resetting limit switches. This is not excessive, since it is good practice to check the limit switches on the gate-damper motors about once every 6 months. The purpose of the check is to insure a good seal of the damper against the door frame at all times so that the pressure balance of the cloth will be proper for effective shaking.

The 25-ton mill has been in service 6197 hr and the 40-ton mill 5759 hr since the installation of the dry filters. During this time it is estimated that 4400 tons of coal have been reclaimed by the dust filters. The operation of the filters has been normal and no major incidents in their operation have occurred.

Safety Measures

It is recognized that coal milling is a process which must be carefully handled to prevent fires during the operation. Protective equipment and safe procedures had been employed in the millhouse in line with best practice and this policy was extended to the dry filters. Numerous tests on filter cloth, both treated and untreated with flameproofing, indicated the desirability of treating the cloth, although it was recognized that such treatment would increase the flow resistance of the cloth slightly and perhaps affect its life. The effect of the flameproofing was found to be that of retarding the burning of the cloth and localizing it, although the time required for ignition of the treated and untreated cloth seemed the same.

Long horizontal runs of pipe are objectionable as are low pipe

[4] "The Calculation of the Dispersion of Flue Dust and Cinders From Chimneys," by H. O. Croft, Trans. A.S.M.E., vol. 57, 1935, paper FSP-57-1, pp. 5–10.

velocities. Accordingly in the layout of the pipe lines leading to the collector, the velocities are maintained at above 3000 fpm and the pipe runs are all practically vertical with the exception of the tie-in connection to the collector on the 40-ton mill, which is approximately 25 ft long. Exposed pipe above the roof is well insulated and provided with waterproof covering. In addition the horizontal run is provided with an inspection and access door. Dry-coal return lines are vertical with minor exceptions, where 45-deg bends were necessary. These were provided with cleanout plugs.

It has been mentioned that the dust filters were installed on the millhouse roof. The collector, therefore, had to be well insulated to prevent condensation. A 4-in. layer of rock wool backed up with $^1/_4$-in. transite sheet was provided, the collector itself being lined with No. 14 gage sheet metal. The temperature drop through the collector is therefore small, in the order of 10 F, and the exit-air temperature is above the dewpoint.

Each compartment is provided with an explosion vent in addition to those already existing on the mill proper. Each filtering compartment has an access door.

Published data on the subject of inflammability of coal dust–air mixtures are meager with reference to the problem at hand. Two general statements may be made:

1 The dust must be present in a cloud of inflammable density and composition.

2 There must be a source of ignition, such as freely burning coal or an electric spark.

These two basic conditions may be discussed in terms of the factors which determine them. With regard to any distinction between inflammability and explosiveness, investigators of the Bureau of Mines[5] find, as a practical matter, that it is impossible to distinguish between them and that it is inadvisable to attempt such distinction.

When the coal–air mixture at Kips Bay Station leaves the primary cyclone, the velocities and composition are as given in Table 5.

TABLE 5 VELOCITIES AND COMPOSITION OF COAL–AIR MIXTURE, LEAVING PRIMARY CYCLONE

	25-Ton mill	40-Ton mill
Grains of dust per cu ft of air at 130 F	6.2	6.8
Velocity in pipe, ft per min	3300	3400
Dust fineness	(100 per cent through 325-mesh Tyler screen)	

The question of mill drying by flue gases was studied at length and abandoned as economically impractical in this instance.

The regulation of air flow from the mill to the filters is accomplished by a high-quality automatic control which throttles the mill fan discharge to maintain a constant suction at the mill cyclone outlet. This function, the maintenance of constant mill air flow, has performed admirably with concurrent advantages in mill operation. Variations in filter pressure are not felt in the mill system.

On the receiving hoppers of the filter, mechanical rappers were installed and have worked very well. The rotary valves for discharging the hoppers are mechanically driven and isolate the hoppers at all times from the receiving screw conveyers.

The receiving screw conveyers were the scene of several fires in the early operation due to spontaneous combustion of packed coal in the barrels. This condition was eliminated by minor changes. Such precautions are also directed against fire due to spontaneous combustion.

[5] "Coal-Dust Explosibility Factors Indicated by Experimental Mine Investigations, 1911–1929," by G. S. Rice and H. P. Greenwald, U. S. Bureau of Mines, Washington, D. C., Technical Paper No. 464, 1929.

It is also necessary to insure against the hazards of static electric discharges. This insurance is largely inherent in the fact that this filter is a mass of metal mesh supporting the cloth, giving almost continuous grounding with a minimum free volume for the occurrence of intercloud discharges.

There is some evidence that high humidities reduce the inflammable character of the mixture, although publications of the Bureau of Mines declaim the futility of relying on this. For instance, Rice and Greenwald[6] unequivocally reject the influence of humidity in practical work. However, high humidities may reduce static-electricity hazard considerably.

Appendix

Three rather complete performance tests on the 40-ton-mill cloth-filter installation are given in Table 6. Of particular interest is the power consumption of the cloth filters and the fineness of the reclaimed coal. Power consumption of the cloth-filter installation is less than 0.2 kw per ton of milled coal, which is approximately 1.5 per cent of the total. Fineness of the reclaimed coal indicates that practically 100 per cent passes through a 325-mesh Tyler screen and also that the density of the reclaimed coal is approximately one half that of the milled coal. It is of interest to note that the efficiency of the primary cyclones is better than 99 per cent.

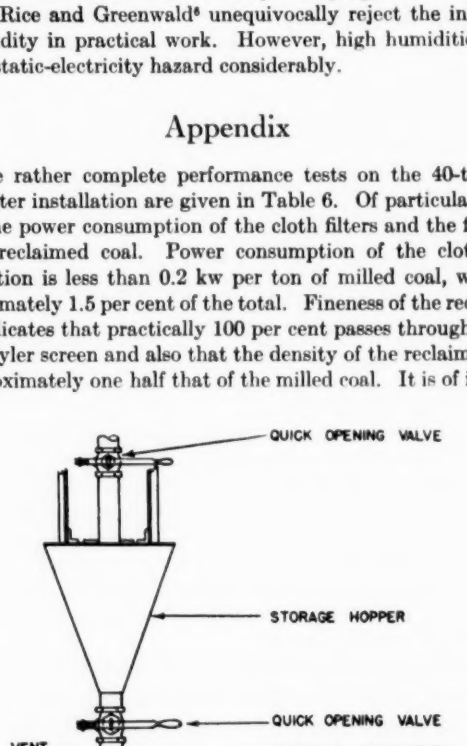


FIG. 14 RECLAIMED-COAL WEIGHING DEVICE

The tests were made with 4 cleaning cycles per hr on the cloth filters. The duration of each shaking period was approximately 20 sec per compartment for each cleaning cycle.

The reclaimed-coal weighing device used in these tests is indicated in Fig. 14. The particular advantage of the installation

[6] See reference (5), p. 8.

TABLE 6 PERFORMANCE TESTS OF 40-TON MILL WITH CLOTH FILTERS

Date	June 17, 1938	June 20, 1938	June 21, 1938
Test no.	2	3	4
Duration, hr	4.53	4.02	5.0
Total coal milled, tons	192.2	150.3	188.1
Coal milled per hr, weighed, tons	42.35	37.35	37.62
Power Consumption:			
Mill including heater fan, kwhr	377.0	371.8	381.8
Feeders and magnetic separators, kwhr	1.8	1.8	2.0
Main exhauster fan, kwhr	112.8	111.0	112.0
Transport pump, kwhr	9.8	10.8	9.6
Cloth-filter motors, inc. conveyer, kwhr	7.0	7.0	7.0
Total, kwhr	508.4	502.4	512.4
Mill including heater fan, kw per ton	8.90	9.95	10.13
Feeders and magnetic separators, kw per ton	0.04	0.05	0.05
Main exhauster fan, kw per ton	2.66	2.98	2.98
Transport pump, kw per ton	0.23	0.29	0.26
Cloth-filter motors, including conveyer, kw per ton	0.17	0.19	0.19
Total, kw per ton	12.00	13.46	13.61
Air-Heater Data:			
Fan-inlet temp, dry bulb, F	80	87	88
Relative humidity, per cent	50	38	44
Discharge pressure, in. water	8.9	9.0	9.0
Fan speed, rpm	1765	1760	1767
Heater-inlet pressure, in. water	3.7	3.6	3.6
Heater-outlet pressure, in. water	2.7	2.8	2.7
Heater pressure drop, in. water	1.0	0.8	0.9
Heater outlet temp, F	236	234	227
Hot air to mill, cfm	20700	20700	21100
Hot air to mill, lb per hr	69300	69200	71600
Steam to heater, lb per hr	2530	2380	2270
Btu available by steam in heater, M Btu per hr	2430	2381	2200
Heat-up by fan, M Btu per hr	47	47	49
Total Btu input, M Btu per hr	2477	2428	2249
Btu absorbed by air, M Btu per hr	2590	2440	2390
Radiation and unaccounted for, M Btu per hr	−113	−12	−151
Radiation and unaccounted for expressed as per cent	−4.7	−0.5	−6.7
Mill Data:			
Exhaust temperature, F	115	118	116
Pressure air entering mill, in. water	1.7	1.8	1.5
Pressure before classifier, in. water	−6.4	−6.6	−6.6
Pressure exhaust, in. water	−9.4	−9.6	−9.7
Air leaving mill, lb per hr	70800	72300	70800
Air inleakage, lb per hr	1500	3100	−800
Air inleakage expressed as, per cent	2.2	4.5	−1.1
Pressure drop across classifier, in. water	3.0	3.0	3.1
Pressure drop across mill, in. water	11.1	11.4	11.2
Primary Cyclone Data:			
Inlet pressure N, in. water	−10.5	−10.7	−10.8
Outlet pressure N, in. water	−11.5	−11.7	−11.9
Inlet pressure S, in. water	−10.5	−10.7	−10.8
Outlet pressure S, in. water	−12.0	−12.1	−12.3
Pressure drop N, in. water	1.0	1.0	1.1
Pressure drop S, in. water	1.5	1.4	1.5
Main Exhauster Fan Data:			
Disch. temp, dry bulb, F	130	131	129
Relative humidity, per cent	36	38	36
Inlet press. before control damper, in. water	−12.1	−12.1	−12.3
Inlet press. after control damper, in. water	−27.2	−27.2	−27.2
Discharge pressure,[a] in. water	2.7	2.6	2.7
Discharge pressure,[b] in. water	3.6	3.6	3.6
Fan speed, rpm	1180	1180	1185
Air discharged, lb per hr	70800	72300	70800
Air discharged, cfm	18600	19000	18600
Cloth-Filter Data:			
Inlet temperature, F	130	131	129
Inlet-pipe velocity, fpm	3800	3880	3800
Inlet pressure,[a] in. water	2.3	2.3	2.3
Inlet pressure,[b] in. water	3.1	3.0	3.1
Coal collected, weighed, lb per hr	440	458	447
Outlet temperature, F	124	123	123
Dust concentration, gr per cu ft air	2.8	2.8	2.8
Collected coal, per cent of milled coal	0.5	0.6	0.6
Coal-separation eff of primary cyclones, per cent	99.5	99.4	99.4

[a] Neutral period (all filter compartments filtering).
[b] Cleaning period (one filter compartment closed for shaking).

Mill Heat Balance:			
Input			
With hot air, M Btu per hr	2590	2440	2390
Mill motor power, M Btu per hr	1120	936	961
Total, M Btu per hr	3710	3376	3351
Output			
Sensible heat in coal, M Btu per hr	656	648	606
Heat-up and evap moisture, M Btu per hr	1861	2013	2007
Heat-up inleakage air, M Btu per hr	13	20	27
Heat in air leaving mill, M Btu per hr	596	457	333
Total accounted for, M Btu per hr	3126	3138	2973
Rad. and unaccounted for, M Btu per hr	584	238	378
Expressed in per cent:			
Sensible heat in coal	17.7	19.2	18.1
Heat-up and evap moisture	50.2	59.6	59.9
Heat-up inleakage air	0.4	0.6	0.8
Heat in air leaving mill	16.0	13.5	9.9
Total accounted for	84.3	92.9	88.7
Rad. and unaccounted for	15.7	7.1	11.3
Total	100.0	100.0	100.0
Moisture Removal:			
Calculated			
Total in air from mill, lb per hr	2480	2620	2570
Less moisture in inlet air, lb per hr	779	846	870
Moisture removed, lb per hr	1701	1774	1700
Total moisture removed by analysis, lb per hr	1610	1340	1260
Moisture removed, calculated, per cent	2.0	2.3	2.3
Moisture removed, by analysis, per cent	1.9	1.8	1.7

Raw- and Pulverized-Coal Data

Screenings:			
Raw coal (mill inlet)			
Over 1/16-in. opening, per cent	13.1	34.6	32.8
Over 1/18-in. opening, per cent	26.5	23.5	25.0
Over 1/32-in. opening, per cent	22.8	16.5	16.9
Over 1/60-in. opening, per cent	10.0	6.2	6.5
Over 1/84-in. opening, per cent	5.5	3.7	3.8
Dust through 1/84-in. opening, per cent	22.1	15.5	15.0
Over 40, per cent	10.6	7.9	7.0
Over 60 per cent	3.9	2.8	2.7
Over 80, per cent	1.6	1.0	1.0
Over 100, per cent	1.0	0.7	0.7
Over 200, per cent	1.9	1.3	1.5
Through 200, per cent	3.1	1.8	2.1
Powdered Coal (milled):			
Over 40-mesh Tyler screen, per cent	1.9	1.7	1.8
Over 60-mesh Tyler screen, per cent	4.9	5.1	5.0
Over 80-mesh Tyler screen, per cent	5.0	5.1	4.7
Over 100-mesh Tyler screen, per cent	5.8	6.1	5.9
Over 200-mesh Tyler screen, per cent	20.5	20.6	20.3
Through 200-mesh Tyler screen, per cent	61.9	61.4	62.3
Reclaimed Coal (collected by filter):			
Over 200-mesh Tyler screen, per cent	0.0	0.0	0.0
Over 325-mesh Tyler screen, per cent	0.1	0.0	0.0
Through 325-mesh Tyler screen, per cent	99.9	100.0	100.0
Moisture Analysis:			
Raw coal, per cent	3.0	3.0	2.9
Powdered coal, per cent	1.1	1.2	1.2
Removed by analysis, per cent	1.9	1.8	1.7
Coal returned by cloth filters, per cent	0.8	1.3	1.3

Raw Coal—Proximate Analysis:

	Wet basis	Dry	Wet basis	Dry	Wet basis	Dry
Moisture, per cent	3.0		3.0		2.9	
Volatile, per cent	29.5	30.4	33.1	34.1	31.5	32.4
Ash, per cent	6.5	6.7	6.7	6.9	6.6	6.8
Fixed carbon, per cent	61.0	62.9	57.2	59.0	59.0	60.8
Heat content, Btu per lb	14060	14490	13735	14160	13885	14330

Powdered-Coal Densities, A.S.T.M. Method:		
Milled coal, lb per cu ft	33.6	
Coal returned by cloth filters, lb per cu ft	17.8	

shown is that it permits continuous movement of the coal dust in the pipe line, because it incorporates a storage hopper in addition to the weighing hopper and also that it is dust-tight which, in this particular application, is rather important. It is, of course, important if the dust-conveying pipe is under pressure other than atmospheric that the weighing hopper be vented during the weighing period. For this purpose, a vent valve is provided in the weighing-hopper cover plate. This particular type of installation has also been used successfully in weighing flue dust from pulverized-fuel installations. The hoppers are fabricated from 3/32-in. plate and all seams are welded.

Discussion

J. E. FULWEILER.[7] It would be interesting to know if any serious objections to the installation discussed in this paper were offered by the engineers of the insurance company covering the risk; also whether the demonstrated safe operation of this completely dry equipment was satisfactory proof that sprays are not necessary when the other precautions described are taken.

In view of the tendency to insist upon sprays in connection with explosive industrial dusts, this example may prove of considerable value in the art, because some of the dusts involved are of value if kept dry and utterly ruined if wet.

The wetting of the dusts also seems to increase rather than minimize the disposal problem as well as to entail considerable trouble, due to the freezing of the water in cold weather. An installation of this size and importance, operating dry, is therefore of considerable interest to the writer.

RALPH A. SHERMAN.[8] The writer would like to suggest that supplementary information covering the proximate analysis and calorific value of the dust, which is collected in the bag filters, would aid in a comparison of the remainder of the coal collected in the cyclones.

R. F. THRONE.[9] There are upward of 100 installations of cloth filters in service, handling vents from pulverized-coal equipment. The general experience has been quite satisfactory. Some provisions have been necessary to drain off the static electrical charge, and some installations have required special attention to the slope of the inclined surfaces. Only a few of the installations use flue gas as an inert gas, the circulating gas in most cases being air.

Cloth-type filters have the highest reclaim capacity, especially in their ability to remove the microscopic sizes under 10μ. These sizes represent as high as 80 per cent of the total dust and constitute a major visual nuisance.

Precaution must be exercised to maintain the gas humidity below 100 per cent, as the resistance through the cloth greatly increases as it absorbs free moisture, as well as permits the coal dust to cake onto the filter surface, thus increasing the difficulty of restoring the filtering medium. The tubular free-hanging type of cloth filter, can be adequately cleaned in service merely by closing the outlet-gas damper without in turn closing the inlet-gas damper. The agitating mechanism is sufficient to shake the coal dust from the filter cloth without allowing a time period for equalization of gas pressure on either side. The most effective procedure is to have a reverse flow of the gas through the filter cloth during the shaking operation, but such reversal, either with the cleaned gas from other sections or with room air, cools the gas in the compartment being cleaned, to and below its dew point, causing condensation. Cloth filters have proved equally successful under negative pressures as high as 26 in., as well as under positive pressure. The major disadvantage of negative-pressure operation is the detection of air-inleakage areas.

Fire hazard is increased in the employment of cloth filters. This becomes an important factor, especially with the "younger" coals, for which the affinity for oxygen is high. The combustion of filtering cloth with the resulting flame adds a menacing factor to what would otherwise be an easily controllable condition. Flue gas as the circulating medium has not proved satisfactory in avoiding ignition of coal dust and the resulting loss of filter bags on the "western" coals. Even though a noninflammable filtering cloth material should be developed, considerable question remains whether with these "younger" coals, the large area available is not a definite hazard.

J. C. WITT.[10] The writer would like to know whether any installations of the type described have been involved in fires? He has had some experience with one all-steel dust-collector installation in which a fire occurred. Much of the equipment was rendered worthless by the heat, although nothing was burned. The question therefore arises: What benefit would come from fireproofing the cloth, in this instance, if a fire should start?

AUTHORS' CLOSURE

The authors believe that the successful operation of the cloth-filter installation described is in a large measure made possible by the safety features incorporated in the design and also because of the systematic inspection program carried out by the operating personnel. The insurance carrier, in the case of this installation, interposed no requirements with those on pulverized-coal installations in general.

The powdered-coal proximate analyses omitted in the performance tests and requested by Mr. Sherman have been obtained. Average laboratory results of the powdered-coal analyses are tabulated as follows:

POWDERED-COAL PROXIMATE ANALYSES

	Milled coal	Reclaimed coal (collected by cloth filter)
Moisture, per cent	1.4	0.8
Volatile matter, dry, per cent	34.4	33.8
Ash, dry, per cent	7.7	7.4
Fixed carbon, dry, per cent	57.9	58.8
Sulphur, dry, per cent	1.45	1.10
Calorific value, dry basis, Btu per lb	14110	14150

No appreciable difference in laboratory analyses of the two powdered-coal samples is evident. Although the primary cyclone efficiency is better than 99 per cent, it is not sufficient to detect differences in proximate analysis between milled coal and the vented coal, if such a difference exists. For this installation at least, the conclusion must be that the proximate analyses of the powdered coals are the same.

Mr. Throne points out that it is necessary to insure against the hazards of static electric discharges. This insurance is largely inherent in the collector described. The importance of maintaining the temperature of the coal dust conveying air in the collector above the dew point cannot be overemphasized. Careful selection of heat-insulating materials has eliminated difficulties of this kind.

Pressure balance of the cloth may be secured by closing either the inlet- or outlet-gas damper. It is not necessary to close both. In this installation, the inlet dampers were used because it simplified construction and outlet dampers were not provided because they were unnecessary. It has been found by experience that shaking in a screen-type collector is ineffective if the cloth pressure is unbalanced as little as 0.02 in. of water. Reverse gas flow is unnecessary and is of doubtful value.

Mr. Throne states that cloth pressure balance is unnecessary with the tubular free-hanging-type collector. It is quite unnecessary with all types if draft loss is no object. It would be of interest to know the magnitude of the pressure unbalance for the installation mentioned. It is certain that, if a pressure unbalance exists at the time of shaking, it is also true that more effective shaking is possible if the pressure unbalance is eliminated. Cloth maintenance will be reduced as a result. Needlessly high

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[10] Technical Service Manager, Marquette Cement Manufacturing Company, Chicago, Ill. Mem. A.S.M.E.

pressure drops should be avoided on large installations because fan-power requirements will be high and cloth maintenance will be a very costly item. Although the collector described operated at a pressure above atmospheric, the system will operate equally satisfactorily at a pressure below atmospheric. The pressure-release system in the latter case will be rather simple.

The authors agree with Mr. Witt that fireproofing the cloth does not protect the dust-filter equipment after a fire has started in the collector. As pointed out in the paper, the primary purpose of the cloth flameproofing is that of localizing a fire after it has started. At this stage, the chief concern of the operator would be to prevent spreading to other parts of the mill system and to adjacent equipment. From this point of view as well as the moral value, the slight additional cost of the flameproofing is justified. The authors do not know of any coal-dust cloth-filter installations of the type of design described in which fires have occurred. There have been fires in the type of collector described with bakelite-dust installations and other similar dusts. None of the fires were directly attributed to the collector but they were carried into it from other parts of the system.

Mean Temperature Difference in Design

By R. A. BOWMAN,[1] A. C. MUELLER,[2] AND W. M. NAGLE[3]

In heat-transfer apparatus the rate of heat flow from the hot to the cold fluid is proportional to the temperature difference between the two. For design purposes, it is necessary to be able to determine the mean difference in temperature from the inlet and exit temperatures. Numerous investigators have contributed analyses of the temperature difference for exchangers with neither countercurrent nor cocurrent flow. This paper coordinates the results of previous studies on the same basis to give as complete a picture as possible of all the various arrangements of surface and flow. Shell-and-tube exchangers with any number of passes on shell side and tube side are covered as are the crossflow exchangers with different pass arrangements and with mixed and unmixed flow. The special cases of trombone coolers, pot coolers, and batch processes, not previously published, are also treated in detail.

Nomenclature

THE following nomenclature is used in the paper:

A = area of heating surface
C = specific heat of shell-side (or hot) fluid
c = specific heat of tube-side (or cold) fluid
e = base of natural logarithms
F = correction factor, dimensionless
M = weight of fluid batch
N = number of shell-side passes
$P = (t_2 - t_1)/(T_1 - t_1)$, dimensionless
$p = (T_1 - T_2)/(T_1 - t_1)$, dimensionless
Q = quantity of heat transferred
$q = (t_2 - t_1)/(T_1 - t_1)$, dimensionless
$R = wc/WC$, or $(T_1 - T_2)/(t_2 - t_1)$, dimensionless
$r = \Delta t_m/(T_1 - t_1)$, dimensionless
r_0 = value of r for countercurrent flow
T_1 = inlet temperature shell-side (or hot) fluid
T_2 = outlet temperature shell-side (or hot) fluid
t_1 = inlet temperature tube-side (or cold) fluid
t_2 = outlet temperature tube-side (or cold) fluid
Δt_m = mean temperature difference
U = over-all heat-transfer coefficient
W = weight rate of flow of shell-side (or hot) fluid
w = weight rate of flow of tube-side (or cold) fluid
Θ = time

Introduction

Heat-exchanger design is based primarily on the equation

$$Q/A\Theta = U\Delta t_m \quad \ldots\ldots [1]$$

In this basic equation, the amount of heat transferred per unit of time and surface is shown to equal the product of the over-all heat-transfer coefficient U, and the mean temperature difference, Δt_m. The value of U may be estimated from correlated over-all coefficients determined in similar exchangers under similar conditions, or it may be obtained by combination of individual coefficients. The mean temperature difference Δt_m is calculated from terminal temperatures, the method of calculation varying with the type of exchanger and method of operation.

For simple heat exchangers in which there is steady-state operation and countercurrent flow of the hot and cold fluids, integration of the differential equation, relating the temperatures of the two fluids, leads to the well-known log-mean temperature difference

$$\Delta t_m \text{ (countercurrent flow)} = \frac{(T_1 - t_2) - (T_2 - t_1)}{2.3 \log_{10} \dfrac{T_1 - t_2}{T_2 - t_1}} \quad \ldots [2]$$

The integration involves the following assumptions:

1 The over-all heat-transfer coefficient U is constant throughout the heat exchanger.

2 The rate of flow of each fluid is constant.

3 The specific heat of each fluid is constant.

4 There is no condensation of vapor or boiling of liquid in part of the exchanger. Condensation of pure saturated vapor or boiling of pure saturated liquid throughout the entire length of the heat exchanger, resulting in $T_1 = T_2$ or $t_1 = t_2$, does not affect the integration if the first assumption remains true.

5 Heat losses are negligible.

For heat exchangers in which there is steady-state operation and cocurrent flow, the corresponding integrated average temperature difference is

$$\Delta t_m \text{ (cocurrent flow)} = \frac{(T_1 - t_1) - (T_2 - t_2)}{2.3 \log_{10} \dfrac{T_1 - t_1}{T_2 - t_2}} \quad \ldots\ldots [3]$$

For any set of terminal temperatures, the average temperature difference for cocurrent flow is always less than that for countercurrent flow, unless the temperature of one fluid stream is constant throughout the exchanger.

In the majority of industrial installations true countercurrent heat exchangers are not as economical as multipass and crossflow units. In multipass exchangers the flow is cocurrent in part and countercurrent in part and, as a result, the mean temperature difference lies somewhere between Δt_m for cocurrent flow and Δt_m for countercurrent flow. The mean temperature difference for the various types of crossflow exchangers is also less than that for countercurrent flow and greater than that for cocurrent flow. The relationships between terminal temperatures and mean temperature difference limit the performance of each type of exchanger. For a given set of temperature and rate conditions, the mean temperature difference for some types of multipass or crossflow exchangers may be zero, and the exchanger therefore inoperable, while a countercurrent or another type of multipass exchanger may operate satisfactorily. Formulas or curves are available for calculating the mean temperature difference from terminal temperatures in various types of heat exchangers. These are assembled in this

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

paper and, where possible, are expressed in the form of a correction factor, F, by which the log mean Δt for countercurrent flow is multiplied to give the true mean temperature difference.

$$\Delta t_m = (F)\ (\Delta t_m \text{ for countercurrent flow})$$

$$= (F)\ \frac{(T_1 - t_2) - (T_2 - t_1)}{2.3 \log_{10} \dfrac{T_1 - t_2}{T_2 - t_1}} \quad \ldots\ldots\ldots\ldots\ldots [4]$$

This method of presentation is believed to show most clearly the degree to which the mean temperature difference of any exchanger is inferior to the log-mean Δt for countercurrent flow.

Multipass Heat Exchangers

The widely used shell-and-tube heat exchangers are often provided with baffled heads to route the fluid inside the tubes back and forth from one end of the exchanger to the other. In some cases the shell-side fluid is also caused to travel the length of the heat exchanger more than once by means of longitudinal baffles. Differential equations for a number of such arrangements have been derived and integrated. In the integration the following assumptions are made:

1 The over-all heat-transfer coefficient U is constant throughout the heat exchanger.
2 The rate of flow of each fluid is constant.
3 The specific heat of each fluid is constant.
4 There is no condensation of vapor or boiling of liquid in part of the exchanger.
5 Heat losses are negligible.
6 There is equal heat-transfer surface in each pass.
7 The temperature of the shell-side fluid in any shell-side pass is uniform over any cross section.

The first five assumptions are those employed in the derivation of the ordinary log-mean temperature-difference formula. The sixth is in accord with usual heat-exchanger design practice and the seventh is essentially true where many transverse baffles are incorporated in the exchanger.

Multipass exchangers with an even number of tube-side passes per shell-side pass were studied by Nagle (1),[4] Underwood (2), Bowman (3), and Yendall(15). Their results are summarized as follows:

One Pass Shell Side; Two Passes Tube Side. The correction factor F for multipass heat exchangers, having one shell-side and two tube-side passes, is plotted in Fig. 1 against P and R. The latter are dimensionless ratios, defined as follows:

$$P = (t_2 - t_1)/(T_1 - t_1)$$

$$R = (T_1 - T_2)/(t_2 - t_1)$$

The curves of Fig. 1 are based on the integrated equations obtained by Underwood (2) and by Yendall, rather than on the graphical integrations of Nagle (1), from which they differ by as much as 2 per cent. The integrated equation for Δt_m is

$$\Delta t_m \text{ (one-two exchanger)} = \frac{\sqrt{(T_1 - T_2)^2 + (t_2 - t_1)^2}}{\log_e \left\{ \dfrac{T_1 + T_2 - t_1 - t_2 + \sqrt{(T_1 - T_2)^2 + (t_2 - t_1)^2}}{T_1 + T_2 - t_1 - t_2 - \sqrt{(T_1 - T_2)^2 + (t_2 - t_1)^2}} \right\}} \quad \ldots\ldots [5]$$

This equation may be transposed into the following form, in which the F factor for one-two exchangers is expressed in terms of P and R.

$$F_{1,2} = \frac{\sqrt{R^2 + 1}}{R - 1} \log_{10} \frac{1 - P}{1 - PR} \Bigg/ \log_{10} \frac{(2/P) - 1 - R + \sqrt{R^2 + 1}}{(2/P) - 1 - R - \sqrt{R^2 + 1}} \quad \ldots\ldots [6]$$

In Equation [6], and in subsequent Equations [7] and [8], the expression

$$\frac{1}{R - 1} \log_{10} \frac{1 - P}{1 - PR}$$

becomes indeterminate when $R = 1$, but the usual treatment for such indeterminates reduces this expression to

$$\frac{P}{2.3\ (1 - P)}$$

The correction factor is exactly the same whether the shell-side fluid enters at the fixed or the floating head.

One Pass Shell Side; Four Passes Tube Side. The correction factor for one-four heat exchangers is slightly less than that for one-two exchangers, but the difference is so small that separate curves are unnecessary. Underwood's integration is expressed in hyperbolic functions, and Yendall's in logarithms; but the numerical results appear to be the same. Based on Yendall's work, the correction factor may be expressed as

$$F_{1,4} = \frac{\sqrt{4R^2 + 1}}{2(R - 1)} \log \frac{1 - P}{1 - PR} \Bigg/ \log \frac{1 + V\left(\sqrt{4R^2 + 1} - 2R\right)}{1 - V\left(\sqrt{4R^2 + 1} + 2R\right)} \quad \ldots [7]$$

[4] Numbers in parentheses refer to the Bibliography at the end of the paper.

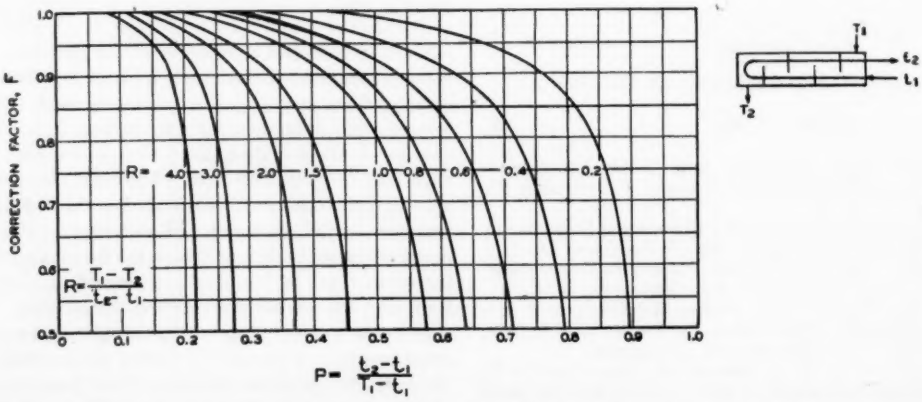


Fig. 1 Correction-Factor Plot for Exchanger With One Shell Pass and Two, Four, or Any Multiple of Tube Passes

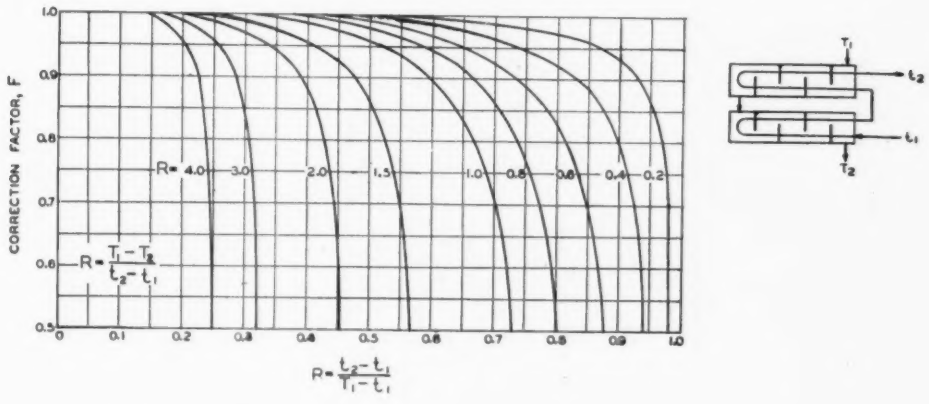


FIG. 2 CORRECTION-FACTOR PLOT FOR EXCHANGER WITH TWO SHELL PASSES, AND FOUR, EIGHT, OR ANY MULTIPLE OF TUBE PASSES

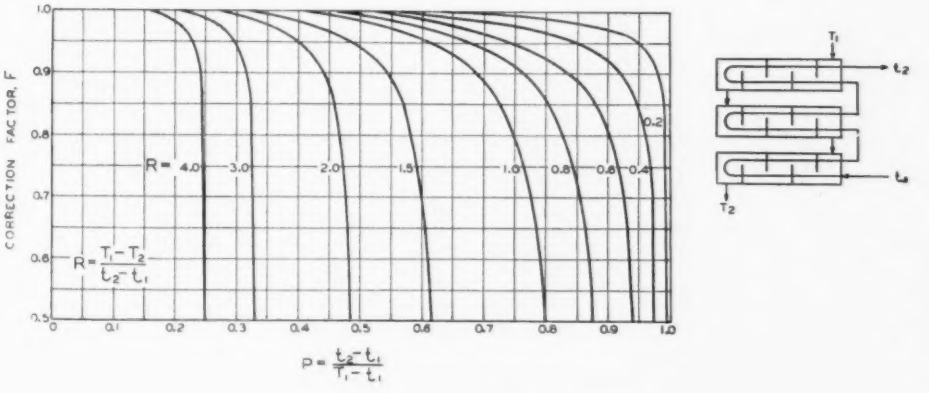


FIG. 3 CORRECTION-FACTOR PLOT FOR EXCHANGER WITH THREE SHELL PASSES, AND SIX, TWELVE, OR ANY MULTIPLE OF TUBE PASSES

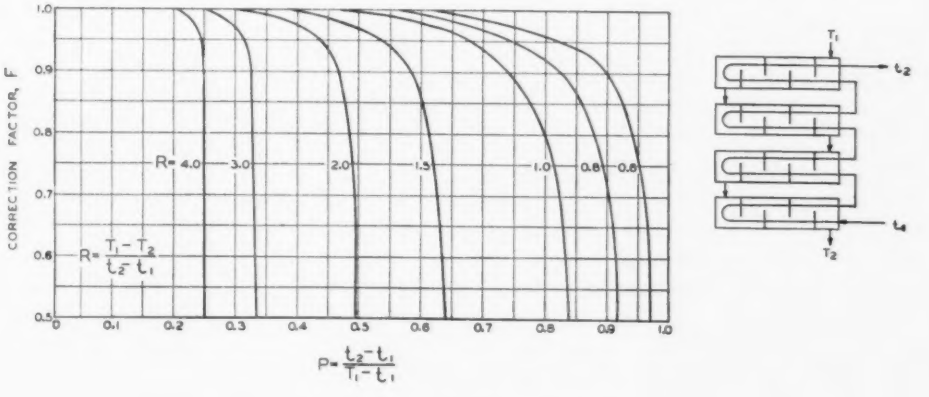


FIG. 4 CORRECTION-FACTOR PLOT FOR EXCHANGER WITH FOUR SHELL PASSES, AND EIGHT, SIXTEEN, OR ANY MULTIPLE OF TUBE PASSES

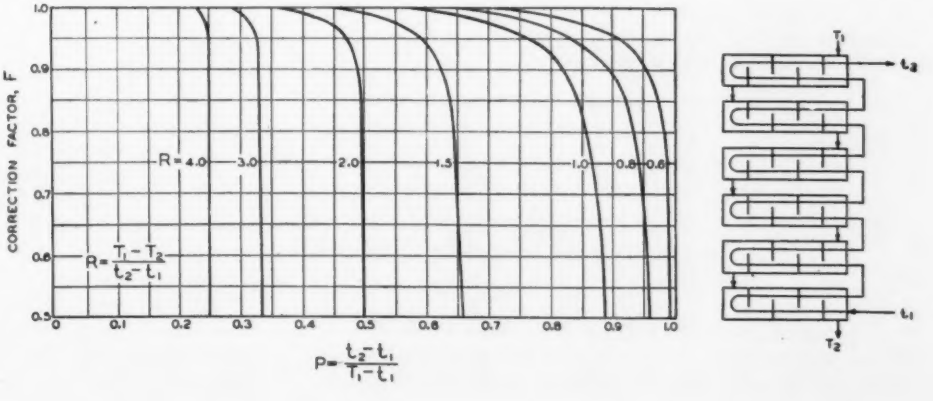


FIG. 5 CORRECTION-FACTOR PLOT FOR EXCHANGER WITH SIX SHELL PASSES, AND TWELVE, TWENTY-FOUR, OR ANY MULTIPLE OF TUBE PASSES

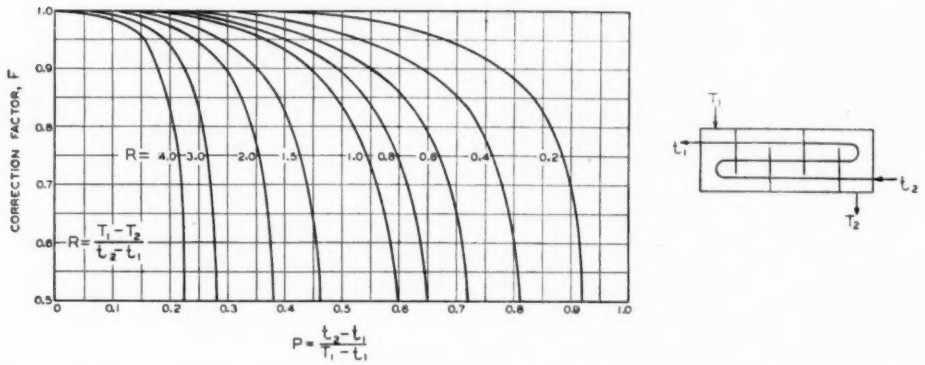


FIG. 6 CORRECTION-FACTOR PLOT FOR EXCHANGER WITH ONE SHELL PASS, AND THREE, SIX, OR ANY MULTIPLE OF TUBE PASSES; MAJORITY OF TUBE PASSES IN COUNTERCURRENT FLOW

where

$$V = \frac{t_2 - t_1}{4T_1 - (t_1 + 2t_i + t_2)}$$

and t_i is the intermediate temperature of the tube-side fluid where it leaves the second and enters the third tube-side pass. The value of t_i may be obtained by trial-and-error solution of

$$\left(\frac{t_i - t_1}{t_2 - t_i}\right)^{\sqrt{4R^2+1}} = \frac{1 + V\left(\sqrt{4R^2+1} - 2R\right)}{1 - V\left(\sqrt{4R^2+1} + 2R\right)}$$

One Pass Shell Side; Six Passes Tube Side. Nagle (1) found that the correction factors for one-six exchangers agreed so closely with those of one-two exchangers operating with the same terminal temperatures that the curves of Fig. 1 may be used for this case.

One Pass Shell Side; Infinite Passes Tube Side. It has been pointed out by Bowman (3) that the correction factor F for a multipass heat exchanger, having one shell-side pass and a very large number of tube-side passes, approaches as a limit the case of crossflow with both fluids completely mixed. Even in the limit, however, the value of F is generally only 1 to 2 per cent less than that of the one-two exchangers.

Two Passes Shell Side; Four Passes Tube Side. The foregoing discussion indicates that the correction factors for heat exchangers with one shell-side pass and two, four, six, and infinite tube-side passes are essentially the same. In many jobs that reach the designer, however, these types of exchangers are inefficient or entirely inoperative, and an arrangement giving higher values of F is required. This may be obtained by the use of two exchangers in series or of a single exchanger having two shell-side passes and four tube-side passes. The correction factor F for two-four exchangers piped in the usual manner to approach countercurrent flow is given graphically in Fig. 2 and algebraically by the following rearrangement of Underwood's equation for two-four heat exchangers

$$F_{2,4} = \frac{\sqrt{R^2+1}}{2(R-1)} \log_{10} \frac{1-P}{1-PR} \Bigg/ \log_{10} \frac{(2/P) - 1 - R + \frac{2}{P}\sqrt{(1-P)(1-PR)} + \sqrt{R^2+1}}{(2/P) - 1 - R + \frac{2}{P}\sqrt{(1-P)(1-PR)} - \sqrt{R^2+1}} \quad \ldots [8]$$

The derivations leading to Fig. 2 and Equation [8] involve the additional assumptions that there is no leakage of fluid or heat across the transverse baffle separating the two shell-side passes.

Three and More Shell-Side Passes. Bowman (3) developed a general method for calculating the F factors of three-six, four-eight, six-twelve, etc., heat exchangers from the correction factors of one-two exchangers. At any given values of F and R, the value of P for an exchanger having N shell-side and $2N$ tube-side passes is related to P for a one-two exchanger by the equation

$$P_{N,2N} = \frac{1 - \left(\frac{1 - P_{1,2}R}{1 - P_{1,2}}\right)^N}{R - \left(\frac{1 - P_{1,2}R}{1 - P_{1,2}}\right)^N} \quad \ldots [9]$$

For the special case of $R = 1$, Equation [9] becomes an indeterminate which reduces to

$$P_{N,2N} = \frac{P_{1,2}N}{P_{1,2}N - P_{1,2} + 1}$$

The results are given in Figs. 3, 4, and 5, which may also be used with little error for exchangers that are multiples of the one-four or the one-six, rather than the one-two exchanger.

From an inspection of Figs. 1 through 5, it may be seen that, for any value of P and R, the correction factor approaches unity as the number of shell-side passes is increased. This is to be expected, since a multipass exchanger with several shell-side passes approaches the ideal countercurrent heat exchanger more closely than does one with one shell-side and two or more tube-side passes.

In all the cases mentioned, the ratio of tube-side to shell-side passes is an even number, such as 2, 4, or 6. However, Fischer (4) has pointed out that there is some improvement in the mean temperature difference if the ratio of tube-side to shell-side passes is an odd number and the exchanger is so connected that the tube-side fluid is flowing counter to the shell-side fluid in over half the passes.

One Pass Shell Side; Three Passes Tube Side. The correction factors F, for one-three heat exchangers and for two-six, three-nine, and four-twelve exchangers have been derived by Fischer (4) and presented in the form of tables and charts. In each case, the value of F is greater than is found in exchangers having the same number of shell-side passes but an even numbered ratio of tube to shell passes. The improvement in mean temperature difference resulting from the use of three tube-side passes per shell-side pass instead of two or four is, however, by no means as great as that resulting from an increase of one in the number of shell-side passes. Correction factors for the one-three exchanger are given in Fig. 6.

Effect of Variation in Heat-Transfer Coefficient. The preced-

ing derivations are based on the assumption that the over-all heat-transfer coefficient U is constant throughout the heat exchanger. For the case of countercurrent flow in which U varies linearly with temperature, Colburn (13) has derived the general heat-transfer equation

$$\frac{Q}{A\Theta} = \frac{U_2 (T_1 - t_2) - U_1 (T_2 - t_1)}{2.3 \log_{10} \dfrac{U_2 (T_1 - t_2)}{U_1 (T_2 - t_1)}}$$

where U_1 is the value of U at the T_1 end of the exchanger and U_2 at the T_2 end. This equation has been combined by Sieder and Tate (*Industrial and Engineering Chemistry*, vol. 28, 1936, p. 1434) with the correction factor F for use with multipass heat exchangers

$$\frac{Q}{A\Theta} = (F) \frac{U_2 (T_1 - t_2) - U_1 (T_2 - t_1)}{2.3 \log_{10} \dfrac{U_2 (T_1 - t_2)}{U_1 (T_2 - t_1)}}$$

Until this equation has been tested more extensively its use should be limited to multipass exchangers having only one shell-side pass.

Examples Applying to Multipass Exchangers

The examples illustrating the variation of the mean temperature difference with type of heat exchanger are shown in Table 1.

TABLE 1 EXAMPLES ILLUSTRATING VARIATION OF MEAN TEMPERATURE DIFFERENCE WITH TYPE OF HEAT EXCHANGER

Example No. 1:
Where T_1 = 300 F, T_2 = 200 F, t_1 = 100 F, t_2 = 200 F
(R = 1, P = 0.5)

Exchanger	F	Δt_m, deg F
Countercurrent flow	1.000	100.0
Six-twelve multipass	1.000	100.0
Two-pass countercurrent crossflow, one fluid mixed	0.970	97.0
Horizontal film-type, helical connection	0.960	96.0
Two-four multipass	0.958	95.8
Horizontal film-type, return-bend connection	0.915	91.5
Single-pass crossflow, both fluids unmixed	0.910	91.0
One-three multipass	0.840	84.0
Single-pass crossflow, one fluid mixed	0.835	83.5
One-two multipass	0.804	80.4
One-four multipass	0.798	79.8
Single-pass crossflow, both fluids mixed	0.790	79.0
Two-pass cocurrent crossflow, one fluid mixed	0.405	40.5
Cocurrent flow	0	0

Example No. 2:
Where T_1 = 300 F, T_2 = 200 F, t_1 = 160 F, t_2 = 260 F
(R = 1, P = 0.714)

Exchanger	F	Δt_m, deg F
Countercurrent flow	1.000	40.0
Six-twelve multipass	0.970	38.8
Horizontal film-type, helical connection	0.795	31.8
Two-pass countercurrent crossflow, one fluid mixed	0.745	29.8
Single-pass crossflow, both fluids unmixed	0.682	27.3
Two-four multipass	0.645	25.8
Horizontal film-type, return-bend connection	0.395	15.8
One-three multipass	Impossible	
Single-pass crossflow, one fluid mixed	Impossible	
One-two multipass	Impossible	
Single-pass crossflow, both fluids mixed	Impossible	
One-four multipass	Impossible	
Two-pass cocurrent crossflow, one fluid mixed	Impossible	
Cocurrent flow	Impossible	

In the first example, all arrangements except that of cocurrent flow can be used. In the second example only countercurrent-flow heat exchangers, or multipass exchangers, which have two or more shell-side passes, can possibly give the desired results.

In the design of multipass heat exchangers, the general rules of Fischer (4) applying to mean temperature difference should be kept in mind. These are quoted as follows:

1 When the number of tube passes per shell pass is even, the mean temperature difference is independent of the direction of flow of the shell fluid in each shell pass.

2 When the number of tube passes per shell pass is odd, the mean temperature difference depends on the direction of flow of the shell fluid in each shell pass and is greatest when the counterflow tube passes exceed the parallel-flow tube passes in each shell pass.

3 In multi-shell-pass exchangers the over-all mean temperature difference depends on whether the flow of the fluids between the shell passes is counter or parallel flow.

4 The equations available for solving the true mean temperature difference applying to multipass heat exchangers can be made general by letting the T values represent the hot-fluid and t values represent the cold-fluid terminal temperature.

Use of Correction Factors in Design. From the standpoint of mean temperature difference alone, it is desirable to use the same number of shell passes as of tube passes since by so doing true countercurrent flow can be obtained. However, structural and servicing considerations limit the number of shell passes, two passes in general being the maximum number for one shell. On the other hand the minimum number of tube passes is set by the quantity of fluid, the allowable pressure drop, the available space, and the costs of construction. Any reduction in the number of tube passes below this figure will result in a lowered heat-transfer rate which may overbalance any gain in mean temperature difference.

The usual procedure in designing a heat exchanger is, then, to choose the number of tube passes on the basis of the foregoing considerations and then to select the number of shell passes which will give a satisfactory mean temperature difference. If the use of two passes gives too low a correction factor, the usual practice is to divide the required surface among two or more shells in series.

Probably the most noticeable feature of the correction curves is the fact that they all have a very steep gradient for the lower values of the correction factor. In other words, a relatively small change in temperature conditions will cause a large change of F and consequently of the mean temperature difference. It also follows that a small deviation from the assumed conditions on which the curves are based will cause an appreciable error in mean temperature differences derived from this part of the curves. For instance, an exchanger might be expected to work with a mean temperature difference of 50 per cent that of a countercurrent exchanger on the basis of the correction-factor curves, whereas, it may actually be an impossible condition, due to the fact that there is leakage of fluid or heat through the longitudinal shell baffle contrary to the assumption of zero leakage.

Consequently, the use of correction factors below 0.8 seems to be questionable from a design standpoint. In those cases where such low correction factors are indicated, one or more additional shell passes should be used. Also test data, obtained under conditions which give low correction factors, should not be accepted too readily for general application.

The use of exchangers with three tube passes per shell pass gives some improvement in mean temperature difference and there are applications where this is of value. Unfortunately, the greatest improvement comes at the lower values of correction factor where the use of the calculated data for any type of exchanger is none too safe. Furthermore, mechanical and thermal difficulties may prevent the use of this arrangement in the majority of cases.

When only the inlet temperatures, rates of flow, heat-transfer area, and coefficient are known, the method of Ten Broeck (5) is especially useful. The exit temperatures are easily obtained from his type of plot, eliminating the necessity of trial-and-error solution.

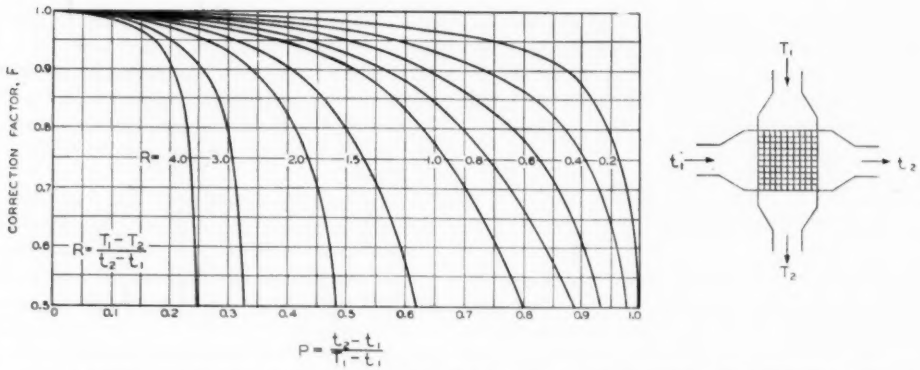


FIG. 7 CORRECTION-FACTOR PLOT FOR SINGLE-PASS CROSSFLOW EXCHANGER, BOTH FLUIDS UNMIXED

CROSSFLOW EXCHANGERS

Crossflow exchangers have the fluids flowing at right angles, therefore, neither the cocurrent nor countercurrent equations for mean temperature differences can be applied. The temperature of the fluid may vary in a direction normal to its flow as well as with the direction of flow. At the outlet, all infinitesimal sections of the stream may be at different temperatures as each was subjected to a different range of temperatures of the other fluid. The final outlet temperature is obtained by complete mixing of all infinitesimal sections. Mixing may take place within the exchanger and, if completely mixed, the fluid will have a temperature gradient only in the direction of flow. Therefore, the mean temperature difference is dependent upon whether either or both fluids are unmixed or mixed. In any practical crossflow exchanger, the case of unmixed fluids is more nearly approached than that of completely mixed fluids. Shell-and-tube exchangers with many transverse baffles are not crossflow exchangers because, even though the flow is normal to the tubes, the temperature change of the shell fluid is negligible for any baffle pass, the temperature gradient being parallel to the tubes. Conversely, as the number of shell-side passes in crossflow exchange is increased, the more closely will the case of cocurrent or countercurrent flow be approached.

Mathematical analysis of the mean temperature difference in a single-pass crossflow exchanger was initiated by Nusselt (6, 7). Smith (8) finished the analysis of the remaining cases of single-pass crossflow and extended it to two cases of two-pass crossflow exchangers. While others have presented equations or methods to determine the mean temperature difference, only Nusselt (7) and Smith (8) have carried out their calculations and presented tables or graphs from which the mean temperature difference may easily be computed from the known terminal temperatures.

There are several methods of presenting the results of the analyses of crossflow; each has certain advantages. Smith and Nusselt use a method in which the temperature-difference relationships depend upon three parameters which are defined as

$$p = \frac{T_1 - T_2}{T_1 - t_1}$$

$$q = \frac{t_2 - t_1}{T_1 - t_1}$$

$$r = \frac{\Delta t_m}{T_1 - t_1}$$

Substituting these parameters in the log-mean temperature-difference equation for countercurrent flow yields

$$r_0 \text{ (countercurrent flow)} = \frac{p - q}{\log_e \dfrac{1 - q}{1 - p}}$$

The method adopted for this paper is that of comparison to the log-mean temperature difference for countercurrent flow, which represents the maximum obtainable difference. This involves the use of F, P, and R which can be expressed in terms of the three parameters by the following relationships

$$F = \frac{r(\text{crossflow})}{r_0 \text{ (countercurrent flow)}}$$

$$R = \frac{p}{q} = \frac{T_1 - T_2}{t_2 - t_1}$$

$$P = q = \frac{t_2 - t_1}{T_1 - t_1}$$

By means of these relationships Smith's data (8) have been transposed into the curves Figs. 7 to 11. In the present form, the efficiency of the type of exchanger being considered is given by F. It is also easily apparent whether the exchanger will operate under the given conditions and what penalty is paid in the form of a poor correction factor F. For performance calculations on a known exchanger, the method of Ten Broeck (5) is especially useful.

The single-pass crossflow exchanger is the only case which has been completely investigated for all types of flow. Only several of the more important types of flow have been presented for the double-pass exchanger and the double-pass trombone exchanger. Mean temperature differences for more than two passes have not been determined except for the trombone exchangers. As the number of passes increases, the log-mean temperature difference for cocurrent or countercurrent flow will be approached, but no rule can now be given for the number of passes required before serious errors are introduced by use of these limits.

The integrations for mean temperature difference were made under the following assumptions:

1 The over-all heat-transfer coefficient U is constant throughout the heat exchanger.

2 The rate of flow of each fluid is constant.

3 The specific heat of each fluid is constant throughout the heat exchanger.

4 There is no condensation of vapor or boiling of liquid in part of the exchanger.

5 Heat losses are negligible.

6 There is equal heat-transfer surface in each pass.

7 The fluid is either unmixed or completely mixed normal to the flow, depending upon the type of flow assumed.

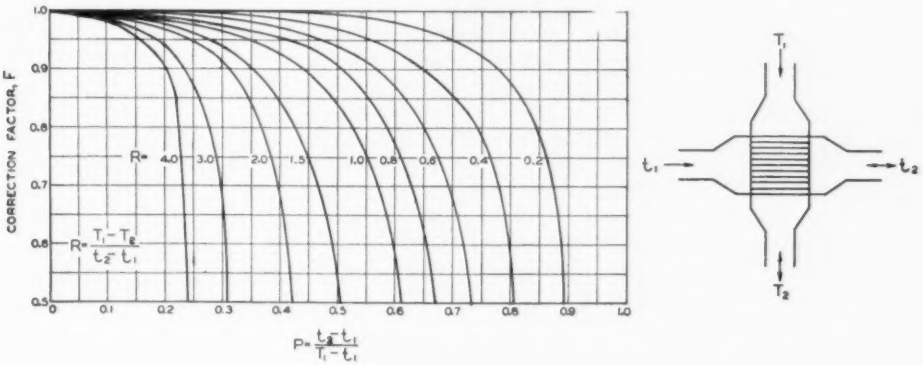


FIG. 8 CORRECTION-FACTOR PLOT FOR SINGLE-PASS CROSSFLOW EXCHANGER, ONE FLUID MIXED, OTHER UNMIXED

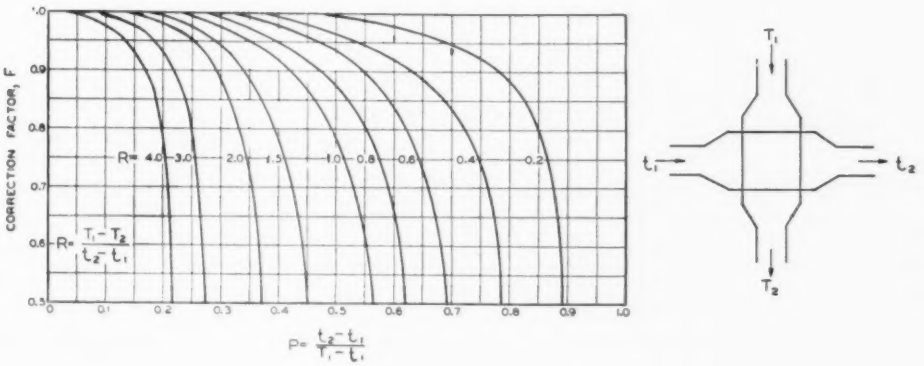


FIG. 9 CORRECTION-FACTOR PLOT FOR SINGLE-PASS CROSSFLOW EXCHANGER, BOTH FLUIDS MIXED

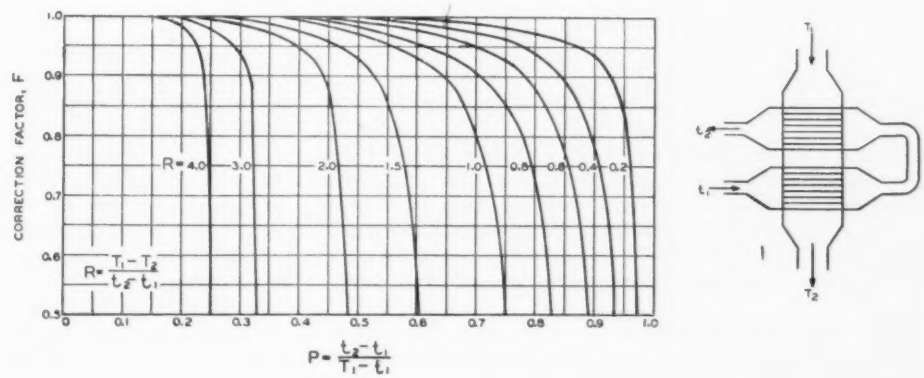


FIG. 10 CORRECTION-FACTOR PLOT FOR TWO-PASS CROSSFLOW EXCHANGER, SHELL FLUID MIXED, TUBE FLUID UNMIXED
(Shell fluid flowing across second and first passes in series.)

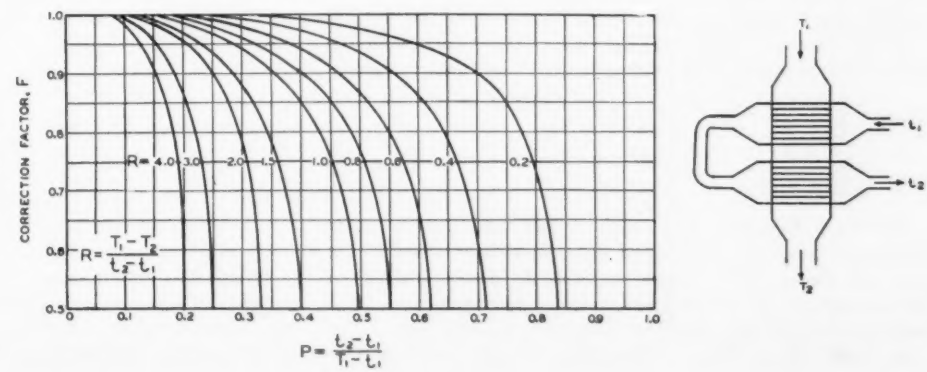


FIG. 11 CORRECTION-FACTOR PLOT FOR TWO-PASS CROSSFLOW EXCHANGER, SHELL FLUID MIXED, TUBE FLUID UNMIXED
(Shell fluid flowing across first and second pass in series.)

Single Pass; Both Fluids Unmixed. Each fluid crosses in a series of independent infinitesimal streams between which there is no mixing or heat transfer until the outlet is reached. Since each stream is subjected to a different range of temperatures of the other fluid, the streams will have different outlet temperatures, the final outlet temperature being the result of mixing of all streams. This assumption represents only the limit for any actual heat exchanger. However, it is nearly approached when one fluid flows through many tubes and the second fluid, crossing outside the tubes, is divided into many streams by baffles. In plate exchangers, mixing is prevented by baffles on each side.

This type of crossflow has been frequently investigated. Nusselt's first solution (6) contained slowly converging series and in a later paper (7) a more rapidly converging series was obtained. Roszak, Véron, and Tripier (9), Binnie and Poole (10), and Liebaut (11) have all presented solutions of varying complexity. Fig. 7 was obtained from values calculated by Nusselt and based on the equation

$$r = \sum_{u=0}^{u=\infty} \sum_{v=0}^{v=\infty} \left\{ (-1)^{u+v} \frac{(u+v)!}{u!(u+1)!v!(v+1)!} \left(\frac{p}{r}\right)^u \left(\frac{q}{r}\right)^v \right\} \quad \ldots\ldots [10]$$

Single Pass; One Fluid Mixed, Other Unmixed. In this case, one fluid is so completely mixed normal to its flow that there is no temperature gradient. The second fluid is assumed to consist of independent streams between which there is no mixing. This is approached by the first fluid flowing across tubes with no baffles to prevent mixing and the second fluid flowing through tubes. This type of flow is obtained in a single-pass trombone exchanger. Here, as in subsequent cases where one fluid is mixed, p refers to the mixed fluid, q to the unmixed. Fig. 8 is based on the graphs of Smith (8) which were calculated from the equation

$$r = \frac{q}{\log_e \dfrac{1}{1 - \dfrac{q}{p} \log_e \dfrac{1}{1-p}}} \quad \ldots\ldots [11]$$

Comparison of the graphs shows the correction factors F are lower than in the preceding case. If an exchanger is to be designed with predetermined terminal temperatures, but the choice of fluid which is to be mixed is open, a larger correction factor F is obtained if the fluid with the greater temperature change is chosen for the mixed fluid. However, the increase in most cases is small.

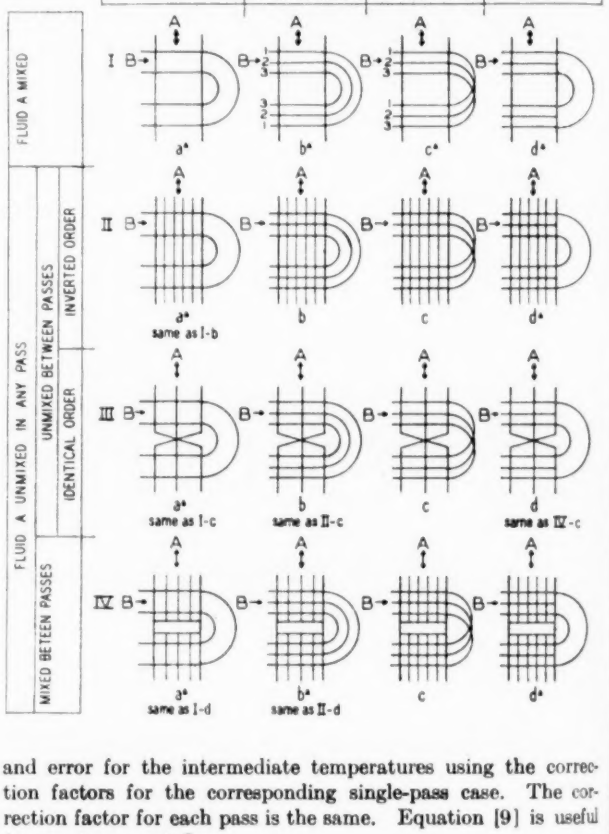
Single Pass; Both Fluids Mixed. The remaining limiting case for single-pass crossflow is where each fluid is completely mixed so there is no temperature gradient normal to its flow. In Fig. 9 the correction factor F, is lower than in either of the two other cases. Here it is possible to get into a region where reversed heat flow may take place, that is, near the outlet the original cold fluid may be hotter than the other fluid. Fig. 9 is based on the graphs of Smith (8) which were calculated from the equation

$$r \left\{ \frac{\frac{p}{r}}{1 - e^{-\frac{p}{r}}} + \frac{\frac{q}{r}}{1 - e^{-\frac{q}{r}}} - 1 \right\} = 1 \quad \ldots\ldots [12]$$

Two-Pass Crossflow Exchangers. Two identical crossflow exchangers may be connected in series in at least ten different ways, as shown schematically in Table 2. Each of these arrangements can be operated either co- or countercurrent. The cases which have been partly or completely solved follow:

Ia Both Fluids Mixed. This case can be solved by trial and error for the intermediate temperatures using the correction factors for the corresponding single-pass case. The correction factor for each pass is the same. Equation [9] is useful for countercurrent flow.

TABLE 2—SCHEMATIC ARRANGEMENTS OF VARIOUS CASES OF TWO-PASS CROSS-FLOW
(CASES MARKED WITH TRIANGLE ARE EITHER PARTIALLY OR WHOLLY SOLVED)



Ib Fluid A Mixed, Fluid B Unmixed, Passes Connected in Inverted Order. This is exemplified by the trombone cooler with return-bend connections.

Ic Fluid A Mixed, Fluid B Unmixed, Passes Connected in Identical Order. This is exemplified by the trombone cooler with helical connections.

Id Fluid A Mixed, Fluid B Unmixed, Except Between Passes. Here it is assumed that the unmixed fluid makes two passes through the tubes but is mixed to a uniform temperature between the passes. The second fluid is completely mixed at all times and flows across the tubes of the second and first tube passes in series. Fig. 10 was obtained from the graphs by Smith (8) which were determined from the equation for the countercurrent case

$$r = \frac{q}{2 \log_e \dfrac{1}{1 - \dfrac{q}{p} \log_e \dfrac{\sqrt{\dfrac{1-q}{1-p}} - \dfrac{q}{p}}{1 - \dfrac{q}{p}}}} \quad \ldots\ldots [13]$$

When the present case is changed to cocurrent flow it is possible to obtain reversed heat flow due to the temperatures of the fluids crossing. The correction factors F in Fig. 11 apply to the log-mean temperature difference for countercurrent flow

as this is the basis for all other graphs. Fig. 11 is based on the graphs of Smith (8) which were calculated from the equation

$$r = \frac{q}{2 \log_e \dfrac{1}{1 - \dfrac{q}{p} \log_e \dfrac{p + q}{q + p\sqrt{1 - (p + q)}}}} \quad \ldots\ldots [14]$$

IId Both Fluids Unmixed, One Mixed Between Passes. This case is treated by Schumann (14) for co- and countercurrent flow. The method of analysis developed depends upon calculating each pass separately. At the present time, no simple relationship has been developed to determine the mean temperature difference for both passes from the terminal temperatures. One pass is the same case developed by Nusselt, the other, however, is complicated by an initial temperature distribution entering the pass.

IVd Fluids A and B Unmixed But Mixed Between Passes. This is treated in the same manner as *Ia*.

Horizontal Film-Type Exchangers

Horizontal film-type exchangers (also called trombone or trickle coolers) are simply banks of horizontal pipes, one above the other, over which a film of fluid is distributed. These exchangers are frequently used because of their simple and cheap construction and the ease of cleaning the outside of the pipe.

The equations for any number of passes of horizontal film-type exchangers have been derived. The cases solved are for countercurrent flow and for the return-bend and helical systems of connecting the passes. The usual assumptions given at the beginning of this section are made. In addition, it is assumed that the temperature of the fluid, inside the pipe, T, is uniform throughout its cross section at any point. It is also assumed that the liquor flowing over the banks of pipes is not mixed laterally. The final temperature of the liquor, t_2, is the result of complete mixing of each independent stream, which was at a different temperature when leaving the bottom pipe.

Return Bends. Here the fluid in the tube in one pass flows in an opposite direction to the fluid in the pass immediately above or below. There is only one bank of pipes formed with this system of connections. Fig. 12 is based on curves obtained for two passes from the equation

$$\frac{1}{1 - p} = e^{Kp/q}\left[\cosh \frac{Kp}{q} + (1 - K) \sinh \frac{Kp}{q}\right] \ldots [15]$$

where $K = 1 - e^{-q/2r}$.

Helical Connection. In this case the pipe may be considered as an elongated coil forming two banks of pipes. The direction of flow through all pipes in a given bank is the same. Fig. 13 is based on curves obtained for two passes from the equation

$$\frac{1}{1 - p} = e^{Kp/q}\left(e^{Kp/q} - K^2 \frac{p}{q}\right) \ldots\ldots [16]$$

where $K = 1 - e^{-q/2r}$.

Mean Temperature Difference for Tanks in Series, Heated or Cooled by Coils

There are occasional processes where a fluid is heated or cooled in a series of tanks by coils. One fluid overflows from one tank to another, each tank being at a different temperature. Fluid in the coils may be in series countercurrent to the tank fluid or each coil in a tank may be fed separately. The problem in design or analysis of data arises when only the number of tanks and the terminal temperatures are known. A method has been derived for such cases. For this case, assumptions are made as follows:

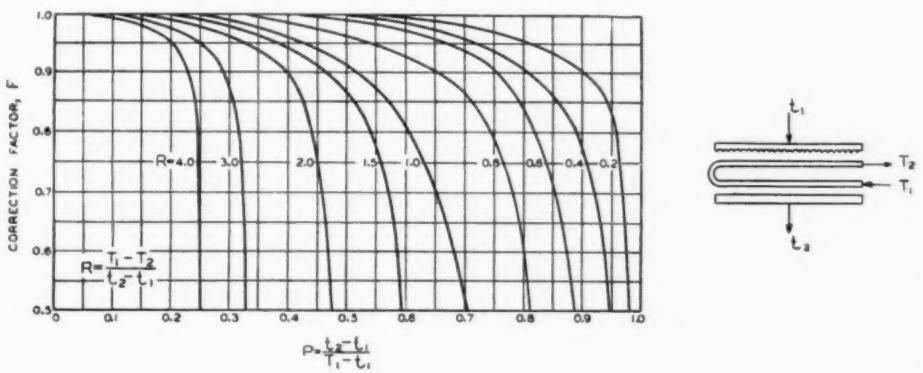


Fig. 12 Correction-Factor Plot for Two-Pass Horizontal Film-Type Exchanger, Return-Bend Connections

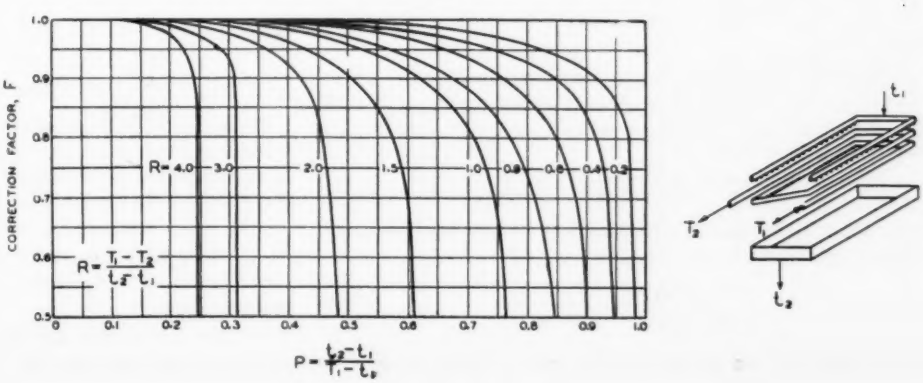


Fig. 13 Correction-Factor Plot for Two-Pass Horizontal Film-Type Exchanger, Helical Connections

1 Heat-transfer area and coefficient in each tank are the same.
2 Each tank is at a constant and uniform temperature.
3 Heat transferred is sensible heat.
4 Specific heats are constant.
5 Temperature and flow of fluids are constant.

Tanks in Series; Coils in Series Counterflow

Let $T_1, T_2, \ldots, T_n$ = temperature of feed to first, second, ... n tank
= temperature of coil liquid leaving the first, second, ... n tank
= temperature of liquid entering n tank

By a heat balance

$$\frac{wc}{WC} = R = \frac{T_n - T_{n+1}}{t_n - t_{n+1}} = \frac{T_1 - T_{n+1}}{t_1 - t_{n+1}} \quad \ldots\ldots [17]$$

for any tank, say tank No. 2

$$wc\,(t_2 - t_3) = UA\,\frac{(T_2 - t_3) - (T_3 - t_2)}{\log_e \dfrac{T_2 - t_3}{T_3 - t_2}} \quad \ldots\ldots [18]$$

and with UA constant for all tanks

$$\frac{UA}{wc} = \log_e \frac{T_{n+1} - t_{n+1}}{T_{n+1} - t_n} \quad \ldots\ldots [19]$$

From Equations [17] and [19] it may be shown that

$$R - \frac{R-1}{\phi} = \frac{T_1 - t_1}{T_2 - t_2} = \frac{T_i - t_i}{T_{i+1} - t_{i+1}}$$

where $$\phi = \frac{T_{n+1} - t_{n+1}}{T_{n+1} - t_n}$$

Therefore, the ratio of the terminal temperature differences for n tanks is

$$\frac{T_1 - t_1}{T_{n+1} - t_{n+1}} = \left(R - \frac{R-1}{\phi}\right)^n = Z^n \quad \ldots\ldots [20]$$

By adding Equation [18] for every tank and noting the relationship given in Equation [19]

$$wc(t_1 - t_{n+1}) = UA\,\frac{t_1 - t_{n+1}}{\log_e \phi}$$

but $wc(t_1 - t_{n+1}) = UAn\Delta t_m$ by definition, therefore

$$\Delta t_m = \frac{t_1 - t_{n+1}}{n \log_e \phi} \quad \ldots\ldots [21]$$

which may be expressed in terms of the terminal temperatures.

If R, ϕ, and the inlet temperatures are known, Equations [20] and [17] may be used in calculating the intermediate temperatures.

From Equation [20]

$$T_1 - t_1 = Z^n T_{n+1} - Z^n t_{n+1} \quad \ldots\ldots [22]$$

and from Equation [17]

$$T_1 - T_{n+1} = Rt_1 - Rt_{n+1} \quad \ldots\ldots [23]$$

solve for t_1 in Equation [22], substitute in Equation [23], and simplify to obtain

$$\frac{t_1 - t_{n+1}}{T_1 - t_{n+1}} = \frac{Z^n - 1}{RZ^n - 1} \quad \ldots\ldots [24]$$

These relations can be constructed into a simple graph. In Fig. 14, the line OG is drawn through point T_1, t_1 with a slope of R. The line OH is drawn to divide any horizontal segment GI between OG and the diagonal OI in such proportion that $GI/HI = \phi$. By similar triangles $GI/HI = GE/HJ$, which latter expression is ϕ by definition. The temperature in any tank is found by proceeding stepwise from T_1 and t_1 for n steps.

Tanks in Series; Coils in Parallel. In this case, let t_1 = inlet temperature of all coils and $t_2, t_3, \ldots t_{n+1}$ = outlet temperature for first, ... n tanks.

For any tank, say tank No. 1

$$wc(t_2 - t_1) = UA\,\frac{(T_2 - t_1) - (T_2 - t_2)}{\log_e \dfrac{T_2 - t_1}{T_2 - t_2}} \quad \ldots\ldots [25]$$

If UA is the same for each tank

$$\frac{UA}{wc} = \log_e \frac{T_{n+1} - t_1}{T_{n+1} - t_{n+1}} \quad \ldots\ldots [26]$$

From Equations [25] and [26], it is readily found that

$$R\left(1 - \frac{1}{\psi}\right) + 1 = Y = \frac{T_1 - t_1}{T_2 - t_1} = \frac{T_2 - t_1}{T_3 - t_1}, \text{ etc.} \quad \ldots [27]$$

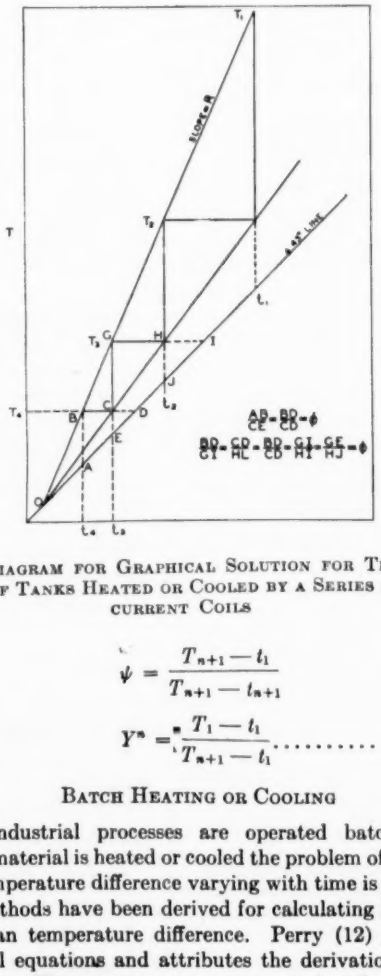


FIG. 14 DIAGRAM FOR GRAPHICAL SOLUTION FOR TEMPERATURES IN SERIES OF TANKS HEATED OR COOLED BY A SERIES OF COUNTERCURRENT COILS

where $$\psi = \frac{T_{n+1} - t_1}{T_{n+1} - t_{n+1}}$$

whence $$Y^n = \frac{T_1 - t_1}{T_{n+1} - t_1} \quad \ldots\ldots [28]$$

BATCH HEATING OR COOLING

Many industrial processes are operated batchwise; and where the material is heated or cooled the problem of determining a mean temperature difference varying with time is encountered. Several methods have been derived for calculating heat transfer or the mean temperature difference. Perry (12) presents one of the final equations and attributes the derivation to Boelter and Cherry. The equations were integrated with the following assumptions:

1 The rate of flow and inlet temperature of the heating or cooling medium are constant

2 The specific heat of each fluid is constant.

3 Only sensible heat is involved, there being no change of phase or heats of reaction.

4 The batch fluid is well agitated and of uniform temperature T.

5 Heat losses are negligible.

6 The over-all heat-transfer coefficient U is constant.

The general design equation is

$$Q = UA\Delta t_m\Theta = MC(T_1 - T_2) \quad [29]$$

and expressed differentially as

$$\frac{dT}{(T-t)_m} = \frac{-UAd\Theta}{MC} \quad [30]$$

where $(T - t)_m$ is the log mean of $(T - t_a)$ and $(T - t_b)$, t_a and t_b being the inlet and outlet temperature of the flowing fluid at time Θ. A heat balance on the batch and cooling media at time θ is

$$wc(t_b - t_a)d\Theta = -MCdT \quad [31]$$

Solve Equation [31] for t_b and substitute in the log-mean temperature difference $(T - t)_m$ in Equation [30]. This gives

$$\log_e \frac{T - t_a}{T - t_a + \dfrac{MCdT}{wcd\Theta}} = \log_e \frac{\Delta t_a}{\Delta t_b} = \frac{AU}{wc} \quad [32]$$

which may be rewritten and integrated over time period Θ, to give

$$Q = wc\Theta(T - t_a)_m \left(1 - e^{-\frac{AU}{wc}}\right) \quad [33]$$

or

$$-UA = 2.3\, wc \log_{10}\left(1 - \frac{Q}{wc\Theta\,(T - t_a)_m}\right)$$

where $(T - t_a)_m$ is the log-mean difference of the terminal batch temperatures and the inlet temperature t_a of the medium.

Either Equation [32] or [33] may be used depending upon the information available. Equation [32] is useful when the temperature rise of the heating or cooling medium is known at any time Θ and batch temperature T. With this equation experimental data may easily be investigated to determine the variation of the over-all coefficient U. If the over-all coefficient varies as a straight-line function of temperature T, then Colburn (13) has shown that the value of $U\Delta t_m$ should be the log mean of $U_1\Delta t_2$ and $U_2\Delta t_1$. Equation [33] is useful in design because it is not necessary to know the exit temperature of the flowing fluid. The foregoing equations were derived on the assumption of complete mixing of the batch at any time; this, however, is seldom realized in actual operation. In the other extreme of no mixing the problem is that of unsteady-state conduction. Methods of determining mean temperature for various degrees of mixing have not been developed.

Acknowledgment

The authors are grateful for permission to include the previously unpublished results of A. P. Colburn for the batch heating process, of T. B. Drew for horizontal film-type exchangers and tanks in series, and of E. F. Yendall on the one-two and one-four heat exchangers.

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15 Private communication.

Discussion

E. A. Schumann, Jr.[5] The writer is well aware of the enormous labor expended in the preparation of the various correction-factor plots given in this paper and feels that the authors are to be thanked for their contribution to heat-transfer-design data.

A comment seems appropriate on the significance of the dimensionless correction factor F in studies of the relative merits of counterflow and unmixed crossflow heat exchangers. It is readily seen that the correction factor represents a ratio of mean temperature differences.

$$F = \frac{\Delta t_m \text{ (crossflow)}}{\Delta t_m' \text{ (counterflow)}}$$

and further, that

$$F = \frac{U'}{U} \times \frac{A'\text{(counterflow)}}{A\text{(crossflow)}}$$

Since F is less than unity

$$UA > U'A'$$

and for identical rates of heat absorption in both types of exchanger, it is apparent that the extent of crossflow heating surface need not be greater than that in the counterflow exchanger, as is sometimes supposed.

On the contrary, designs which can be adapted to high mass-flow rates of the shell-side fluid (tubular-type unmixed crossflow exchanger) commensurate with economical pressure drop may be made to operate with higher shell-side fluid conductances than could be obtained in a counterflow installation.

Finally, any correction factor based on the authors' argu-

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ment P, and fluid ratio R, is only as reliable as the assumption made regarding the strictness with which the expected or actual flow directions adhere to the ideal crossflow.

Authors' Closure

The intent of this paper is to present a simplified method for determining the true mean temperature difference for various types of flows encountered in heat exchangers from the log-mean temperature difference by applying a correction factor. The correction factor F should not be interpreted as indicating the relative merits of countercurrent exchangers with other flow-type exchangers as this would hold only under the special conditions of identical flow rates, temperatures, and heat-transfer coefficients. While it may be possible by using an exchanger other than a countercurrent type to increase the over-all heat-transfer coefficient more than the lowering of the true mean temperature difference, there are, however, some types of exchangers for which certain temperature conditions cannot be met irrespective of the value of the heat-transfer coefficient; this is shown in example 2 of Table 1. No method of calculating the true mean temperature difference for the crossflow case has been developed for cases where the flow is not either completely mixed or unmixed; this was specified in the assumptions on which the equations are based.

Automatic Control in the Presence of Process Lags

By C. E. MASON,[1] FOXBORO, MASS., and G. A. PHILBRICK,[2] FOXBORO, MASS.

In this paper, the methods of "Quantitative Analysis of Process Lags" developed in a previous paper[3] are extended to include automatic control. Carrying on the idea of representation by liquid-level systems, a specific system is postulated as a model for a typical industrial process which may or may not be thermal. Continuing the quantitative approach, the mathematical machinery is demonstrated by which the operation of control on processes may be investigated. A selected succession of controllers, characterized by the laws of control which govern them, are considered in connection with their performance on the model system. Although no attempt has been made to formulate rules governing the proper application of the various types of controllers considered or governing the adjustment of any of these individual types, the authors consider that analyses of this sort may well serve as a guide to practical theorems in a technology of automatic control.

THE quantitative behavior of certain resistance-capacity systems was discussed in a former paper[3] presented before this Society. These systems exhibited characteristics similar to those of industrial processes in which control-resisting lags are found, and were thus proposed as convenient models for such processes. To bear out the implied analogy, the models themselves, considered as liquid-level systems, were developed side by side with equivalent thermal systems. In this way it was found possible to give definite analytic form to the all-important dynamic properties of equipment to which the application of automatic control is contemplated, and to justify a professed effort to fill the gap in a literature otherwise primarily concerned with the properties of the controls themselves.

Having already dealt with the inherent attributes of a few such model systems, the authors will show in the present paper, how the treatment may be extended to cover their behavior under automatic control. The procedure will be to choose a specific model process to which to apply a succession of types of controls (assumed to be pneumatically operated), the model being one which is representative of a typically difficult prototype system, involving a combination of capacity and transfer lags. For concreteness, this prototype system may be considered as a thermal process, although it could equally well involve hydraulic, pneumatic, or electrical phenomena, either singly or in combination.

Although the alleged purpose of an automatic-control installation is that of holding constant a certain process variable, this is in reality only accomplished by a series of recoveries from threatening unbalances, the continuity of which constitutes the record of control. In practice, the disturbances are normally gradual to some degree. It is this fact which makes possible the apparent elimination of the effects of upsetting influences.

Since it is obviously impractical to draw conclusions as to the over-all (transient) characteristics of the combined system through the application of any series of influences, it is desirable to select an isolated, suddenly applied, disturbing influence. The results of such a disturbance must involve the complete transient characteristics of the process and controller in combination.

The model system selected as representative is shown in Fig. 1. The capacity and resistance elements of which it is constructed are the same sort as previously described.[3] T_a, T_b, and T_c are

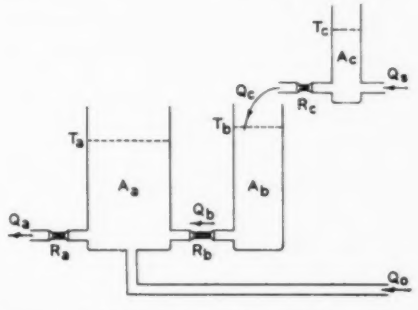


FIG. 1 HYDRAULIC MODEL OF A TYPICAL CONTINUOUS PROCESS

levels (potentials) in the three tanks, the equivalent areas[4] (capacities) of which are A_a, A_b, and A_c. These areas are interconnected through the restrictions (resistances) R_a, R_b, and R_c. The flows Q_a, Q_b, Q_c, Q_s, and Q_o are disposed as shown.

The level T_a in A_a is intended to be the controlled variable, while the supply flow Q_s is to be given the role of manipulated variable by means of which the control of T_a is accomplished. The auxiliary-supply flow Q_o provides a means for altering the load condition, for on the value of this flow depends the value of Q_s necessary to produce any given eventual (or potential) value of the controlled variable T_a. The output flow Q_a is directly dependent at any time on the value of T_a and in equilibrium is equal to the sum of Q_s and Q_o.

TABLE 1 SUMMARY OF PHYSICAL UNITS FOR HYDRAULIC MODEL AND THERMAL PROCESS

	Dimensional symbol	Units: Hydraulic counterpart	Units: Thermal counterpart
Quantity	[W]	Pound (weight)	Btu (heat)
Potential (T)	[P]	Inch (level or head)	Deg F (temp)
Time (t)	[T]	Minute	Minute
Flow (Q)	[WT^{-1}]	Lb/min	Btu/min
Capacity (A)	[WP^{-1}]	Lb/in.	Btu/deg
Resistance (R)	[$W^{-1}PT$]	In./(lb/min)	Deg/(Btu/min)

The physical units for the various constant and variable factors or quantities will be taken as those employed in the former paper,[3] although of course any consistent system of units could

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[2] Research Engineer, The Foxboro Company.

[3] "Quantitative Analysis of Process Lags," by C. E. Mason, Trans. A.S.M.E., vol. 60, May, 1938, pp. 327–334.

Contributed by the Committee on Industrial Instruments and Regulations of the Process Industries Division, and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] A factor proportional to the area, and numerically dependent upon the units used.

be chosen. Table 1 summarizes these units both for the hydraulic model and for the corresponding thermal process.

Since the model shown in Fig. 1 is similar in form to that given as case VI (Fig. 11) in the former paper,[3] differing therefrom only in the inclusion of the cascaded capacity A_c, it will be appropriate to assign the same numerical values to the constants of the system, in so far as this is possible (Table 2). The units to be assumed for these system constants are as given in Table 1, whether for the hydraulic model or for the thermal prototype.

TABLE 2 NUMERICAL VALUES OF SYSTEM CONSTANTS

$A_a = 32.00$	$R_a = 0.1250$
$A_b = 16.00$	$R_b = 0.2500$
$A_c = 4.00$	$R_c = 0.0625$

As pointed out, the original process, of which the hydraulic system of Fig. 1 is to be considered a model, might have any one of a variety of forms. Its nature has been taken arbitrarily as thermal. It might, for example, be thought of as a process identical with the two-capacity thermal system shown in Fig. 4 of the previous paper,[3] but with the modification that the electric heating agency is indirectly manipulated by means of an operation involving simple capacity lag, such as exists with a large air motor setting a current regulator.[5] In any event, for present purposes it may be assumed that the model has been faithfully constructed, and thus the hydraulic system as such may be dealt with directly. Incidentally, the possibility should not be overlooked that the original, or prototype, system might be of the same nature as the model, as for instance a large-scale industrial liquid-level process having transfer lag.

Before proceeding to the application of automatic controls, let us briefly discuss the properties of the specific hydraulic system (or analog) from the point of view of the former paper.[3] Since this is a three-capacity system, the differential equation describing the dynamic relation between the significant variables (T_a, Q_s, and Q_o) will be of the third order. The derivation of this equation will not be given in its entirety since it follows straightforwardly from the methods of the previous paper. The fundamental hydraulic equations are

$$\left.\begin{array}{ll} T_a' = (Q_b + Q_o - Q_a)/A_a & T_a = R_a Q_a \\ T_b' = (Q_c - Q_b)/A_b & T_b - T_a = R_b Q_b \\ T_c' = (Q_s - Q_c)/A_c & T_c = R_c Q_c \end{array}\right\} \quad \ldots . [1]$$

Eliminating all variables except the significant ones

$$(A_3)T_a''' + (A_2)T_a'' + (A_1)T_a' + T_a = R_a Q_s + R_a Q_o + R_a(B_1)Q_o' + R_a(B_2)Q_o''$$

$$\left.\begin{array}{ll} \text{where } (A_1) = A_a R_a + A_b R_a + A_b R_b + A_c R_c \\ (A_2) = A_a R_a A_b R_b + A_c R_c (A_a R_a + A_b R_a + A_b R_b) \\ (A_3) = A_a R_a A_b R_b A_c R_c \\ (B_1) = A_b R_b + A_c R_c \\ (B_2) = A_b R_b A_c R_c \end{array}\right\} \quad \ldots [2]$$

which is the complete process equation for the system of Fig. 1. In the present investigation, the auxiliary flow Q_o will be kept always at steady values except when suddenly changed from one steady value to another in simulation of an instantaneous upset; thus for the time intervals considered

$$Q_o' = Q_o'' \equiv 0$$

and the process equation becomes

$$(A_3)T_a''' + (A_2)T_a'' + (A_1)T_a' + T_a = R_a(Q_s + Q_o) \quad \ldots . [3]$$

[5] In this manner, the so-called "valving" lags may be included as parts of the process rather than as parts of the controls.

Upon insertion of the numerical values given in Table 2

$$4\,T_a''' + 18.5\,T_a'' + 10.25\,T_a' + T_a = 0.125\,(Q_s + Q_o) \quad \ldots . [4]$$

in which, by way of recapitulation,

T_a = controlled variable
Q_s = manipulated variable
Q_o = auxiliary supply flow

The supply flow Q_s is to be operated on by the controlling apparatus in its effort to maintain T_a at a constant value. The auxiliary flow Q_o will suddenly be changed from one value to another under otherwise steady conditions to provide a disturbance to the controlled system at an arbitrarily initial instant ($t = 0$).

As in the former paper, let the desired or normal value $(T_a)_{norm}$ of T_a be

$$(T_a)_{norm} = 80 \text{ (in. or deg)}$$

In the thermal counterpart of Fig. 4,[3] the auxiliary flow Q_o was the flow of heat energy—above 0 F—contained in the mechanical flow of 8 lb per min of water. Assuming again that initially the inlet-water temperature is 60 F, then the disturbance can consist of a sudden decrease in this temperature from 60 F to 40 F. Thus, for the thermal system, the flow Q_o had been initially $8 \times 60 = 480$ Btu/min, while the altered value of Q_o is $8 \times 40 = 320$ Btu/min. Hence for either system

$(t < 0)$ $Q_o = 480$ (lb per min or Btu per min)
$(t > 0)$ $Q_o = 320$ (lb per min or Btu per min)

Allow the range in which the variable Q_s may be changed by the controls to be given by

$$(Q_s)_{min} \leqq Q_s \leqq (Q_s)_{max}$$

where $(Q_s)_{min} = 0$ (lb per min or Btu per min)
$(Q_s)_{max} = 640$ (lb per min or Btu per min)

If the controlled variable T_a is to be at its desired value $(T_a)_{norm}$ in the balanced or undisturbed preinitial state, then, since the value of the auxiliary flow Q_o is already determined for that epoch, it will be seen from the steady-state version (all derivatives equal zero) of Equation [3] (or Equation [4]) that the initial balanced value of Q_s must be

$$(Q_s)_{init} = [(T_a)_{norm}/R_a] - (Q_o)_{init} = (80/0.125) - 480 = 160$$

Our system is thus prepared for the application of the controlling apparatus which will now be considered.

Application of Control Apparatus

The complete or interconnected control circuit, comprising both controlled process and controller, is symbolically shown in Fig. 2, in which the process is characterized by the group of its inherent system constants, and where the other various components of the controlled system are shown in their functional relationship to one another.

Fig. 3 gives a somewhat more realistic picture of the combined system. In Fig. 1 the elements of the process under control are shown in greater detail.

The manipulated supply flow Q_s is assumed to be under successful control by means of a flow controller, in which instrument the balanced value of flow, or the flow control point, may be set by the operating means (the pneumatic operating pressure) of the level controller, which is the direct analog of a temperature controller in the thermal counterpart. This instrument contains the measuring means and the control mechanism, while the flow controller acts as a positive controlling means. These assumptions will be followed for all types of control mechanism considered.

Suppose for simplicity that, as the operating air pressure P, Fig. 3, of the control mechanism varies from zero to its maximum value, namely, the supply pressure P_s, the resulting supply flow Q_s maintained by the flow controller varies proportionately from zero to its maximum value $(Q_s)_{max}$. Thus

$$\frac{P}{P_s} \equiv \frac{Q_s}{(Q_s)_{max}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

With the operating means of the control mechanism thus directly linked to the manipulated flow, the equation of the control mechanism may be expressed as a functional relationship between the controlled variable T_a and the manipulated variable Q_s, or as

$$Q_s = F_c(T_a) = \text{"controller function" of } T_a \ldots\ldots [6]$$

This corresponds to the form of the process equation which may be written

$$T_a = F_p(Q_s) = \text{"process function" of } Q_s \ldots\ldots [7]$$

Equations [6] and [7] are of course symbolic since the functions are in general differential expressions and involve time. They

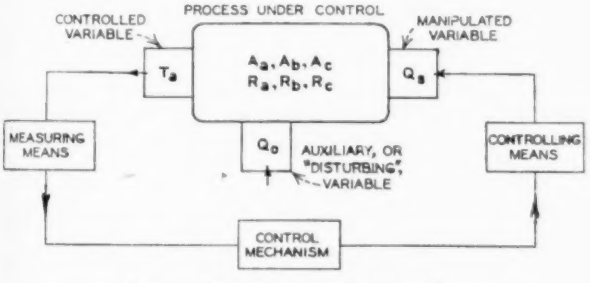


FIG. 2 DIAGRAMMATIC CONTROL CIRCUIT

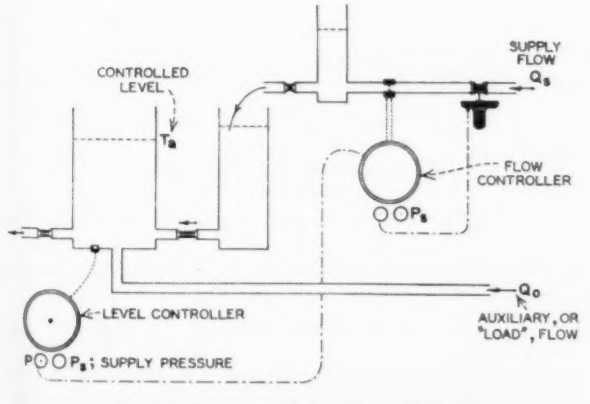


FIG. 3 COMPLETE CONTROLLED SYSTEM

serve, however, to illustrate the reversal of cause and effect in passing from process to controller or from controller to process. In tabular form:

	Cause	Effect
Process	Manipulated variable	Controlled variable
Controller	Controlled variable	Manipulated variable

In the "philosophy" of control, this concept may be considered basic.

The exact nature of the functional relation Equation [6] will depend upon the particular type of controller under consideration. The functional relation Equation [7] for the process is provided in the present case by Equation [3] (and/or [4]). For purposes of comparison, not only among the general controller types but between different conditions of adjustment for any one type of controller, the same initial conditions and also the same manner and magnitude of upset will be assumed throughout. Before taking up the controllers in order, it will be informative to determine the effect on the controlled variable T_a of the disturbance to be employed when this disturbance acts on the system in equilibrium and with the controls assumed inoperative. The resulting deviation of T_a will be a standard by which to ascertain the actual amount of correction brought about by the action of the controllers when operative on the process. Thus, it will be assumed that the foregoing initial conditions are in effect on the process and that, at $t = 0$, the chosen disturbance is inflicted. Since no controller is acting, the supply flow Q_s will remain constant at its initial value, while the auxiliary flow Q_o will have its new balanced value. Thus from the initial conditions and from the process Equation [4]

$$4\,T_a''' + 18.5\,T_a'' + 10.25\,T_a' + T_a = 0.125\,(160 + 320)$$

or

$$(T_a - 60)''' + 4.625\,(T_a - 60)'' + 2.5625\,(T_a - 60)' + 0.25\,(T_a - 60) = 0 \ldots\ldots [8]$$

The solution of Equation [8] to find the variation of T_a as a function of time is a procedure similar to those described in the previous paper.[5] It involves the roots of the auxiliary algebraic equation.

$$k^3 + 4.625\,k^2 + 2.5625\,k + 0.25 = 0 \ldots\ldots [9]$$

and the initial values of the first and second derivatives of T_a (rate of change and acceleration immediately following the disturbance).

The roots of Equation [9] are all real and negative and may be found by any desired method, although in general for speed and precision Graeffe's method[6] is recommended for any linear algebraic equation of higher degree than the second. As a matter of fact Equation [9] is directly factorable in this case (due to the cascaded nature of the analog process) and may be written

$$(k^2 + 0.625\,k + 0.0625)\,(k + 4) = 0 \ldots\ldots [9a]$$

The roots are thus

$$k_a = -0.125 \qquad k_b = -0.5 \qquad k_c = -4 \ldots\ldots [10]$$

The initial values of the first two derivatives of T_a may be determined, through the use of Equations [1], from the change in Q_o constituting the disturbance. Since the initial value of the third derivative of T_a will also be necessary in later examples and may conveniently be found in conjunction with the other two derivatives, all three have been found for the assumed upset and are listed as follows:

$$\left.\begin{array}{lll} (T_a')_o = -5 & \text{(in. per min or deg per min)} \\ (T_a'')_o = +1.875 & \text{(in. per min}^2\text{ or deg per min}^2\text{)} \\ (T_a''')_o = -0.859375 & \text{(in. per min}^3\text{ or deg per min}^3\text{)} \end{array}\right\} \ldots [11]$$

Reference to Appendix 1[7] shows the solution in this case to be of the form

$$T_a - T_p = T_a - 60 = C_a e^{k_a t} + C_b e^{k_b t} + C_c e^{k_c t} \ldots\ldots [12]$$

[5] "Mathematics of Modern Engineering," by R. E. Doherty and E. G. Keller, John Wiley & Sons, Inc., New York, vol. 1, 1936, pp. 98–130.

[7] The Appendixes referred to throughout the paper form a mathematical supplement which does not appear in this publication. A limited number of copies of this supplement are available, however, and may be obtained from the authors on request.

where the k's are the roots given and the C's are integration constants involving both the roots and the initial values of the derivatives in a manner shown in Appendix 1.[7] Inserting the numerical values there results

$$T_a = 60 + 13.33\,e^{-0.125t} + 6.67\,e^{-0.5t} + 0.00\,e^{-4t}$$

The integration constant C_c is reduced automatically to zero for the case chosen, since the system is essentially two-capacity with respect to changes in Q_o alone, Q_c Fig. 1 remaining constant along with Q_s for the period considered. Thus for the effect of the disturbance on T_a in the absence of control

$$T_a = 60 + 13.33\,e^{-0.125t} + 6.67\,e^{-0.5t} \quad \ldots\ldots [13]$$

$$\left.\begin{array}{ll} \text{Thus when } t = 0 & T_a = (T_a)_o = 80 \text{ (in. or deg)} \\ \text{and when } t \to \infty & T_a \to (T_a)_\infty = 60 \text{ (in. or deg)} \end{array}\right\} \quad \ldots\ldots [14]$$

The behavior of T_a following $t = 0$ is shown in Fig. 4. An even-

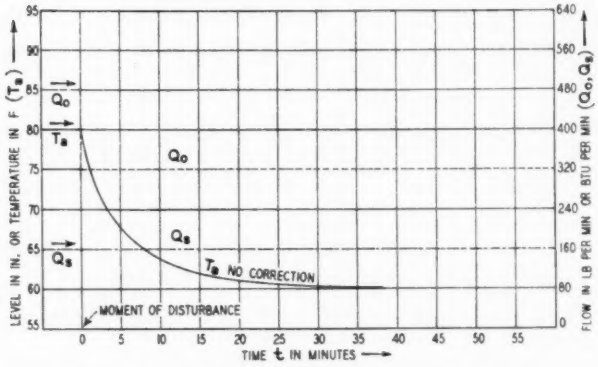


FIG. 4 EFFECT OF DISTURBANCE WITHOUT CONTROL

tual departure of 20 units from the desired value is evident. Plotting may be carried out directly from Equation [13] by means of exponential tables, or through the rather more convenient indirect graphical method used in the previous paper (Appendix 4[7]).

In either manually or automatically controlling this process (Fig. 1), the flow Q_s is changed in an attempt to counteract the effect of the disturbance on T_a, and eventually to restore this variable to its original value. The equation for T_a' (see Equations [1]) and the curve of Fig. 4 show that a change in Q_o has an immediate effect on T_a. On the other hand a change in Q_s, however great, can be shown to have no immediate effect on the value of T_a, due to the intervening parts of the process between the point of application of the manipulated flow Q_s and the location of the controlled variable T_a. Thus no matter what operations are performed on Q_s, a certain inevitable departure of T_a will be experienced, the magnitude of which will depend, not only upon the character (including transfer lags) of the intervening parts of the process, but also on the amount of the available corrective change in Q_s. This leads naturally to the subject of automatic control. First, that type of control will be considered which can produce the greatest initial change in the manipulated variable namely

"TWO-POSITION" CONTROL

Other terms commonly applied to this category are: "on-and-off," "open-and-shut," "fixed-position" control, and the various "....stats." The French "*tout ou rien*" seems particularly picturesque. Certainly there is no need to linger over this characteristic. The law of control may be summarized briefly, in connection with the system under consideration, as

$$\left.\begin{array}{ll} Q_s = (Q_s)_{max} = 640 & \text{for } T_a < (T_a)_{norm} = 80 \\ Q_s = (Q_s)_{min} = 0 & \text{for } T_a > (T_a)_{norm} = 80 \end{array}\right\} \quad \ldots [15]$$

Since in operation, the value of Q_s is constant except for discontinuous changes involving (theoretically) zero time intervals, the resulting behavior of T_a may be discovered by a series of solutions of the process equation itself using methods similar to those in the foregoing part of this paper and in the previous paper.[1] This procedure involves dividing the time following the disturbance into intervals between successive crossings of the desired value by the controlled variable. Fortunately the linearity of the equations permits superposing additively the individual results of the members of any combination of causes in determining their combined effect. Thus, in the beginning, it is merely necessary to add, to the disturbance deviation given by Equation [13] or Fig. 4, the deviation caused by suddenly increasing Q_s alone from its preinitial balanced value (160) to its maximum value (640). This deviation is similar to curve (*a*) of Fig. 12 in the former paper and is readily obtained, requiring merely a different set of boundary conditions to be imposed on Equation [4], namely,

$$\begin{array}{ll} Q_s = 640 & (T_a')_o = 0 \\ Q_o = 480 & (T_a'')_o = 0 \\ & T_p = 140 \end{array}$$

Reference to Equation [1] will show why the first two derivatives are initially unaffected by a sudden change in Q_s. The numerical

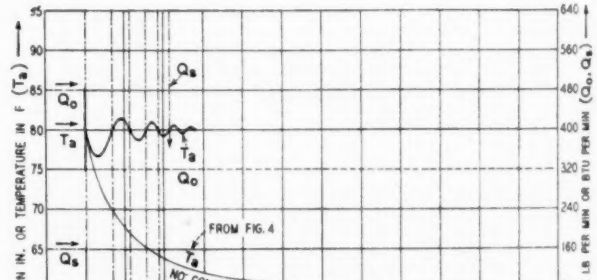


FIG. 5 RECOVERY TRANSIENT WITH IDEAL TWO-POSITION CONTROL

solution, which is found in a similar manner to that for the curve of Fig. 4, is

$$T_a = 140 - 82.59\,e^{-0.125t} + 22.87\,e^{-0.5t} - 0.28\,e^{-4t} \quad \ldots [16]$$

When this deviation is added to that occasioned by the disturbance alone, the first loop of the control transient is obtained, in which T_a passes through its first (inevitable) minimum and returns sharply toward a new potential value. This stage continues only until the desired value is attained, at which time another deviation must be added to the sum of the other two. This is the deviation caused by an independent sudden variation in Q_s from its maximum to its minimum value. This algebraic addition of a new deviation, at each crossing of the desired value (or control-point setting), to the sum of all previous deviations, may be continued as far as is practical or desired. Thus, at approximately 15 min in this case, we are substantially adding together 10 distinct curves. The accuracy of the result becomes increasingly questionable due to the difficulty involved in exact determination of the time intervals. The resulting control transient (or "recovery" transient) is shown in Fig. 5. The

intervals become increasingly shorter as time progresses while the amplitude tends toward zero. Limiting conditions of this sort are only true in an ideal sense, of course, since even slight imperfections, such as dead zones, friction, discontinuity, etc., will make them invalid. It may be noted that, even though the flow Q_s theoretically never becomes steady, its fluctuations are becoming increasingly fast and its average value in this ideal case must approach that required to give T_a the desired value. Thus

$$(Q_s)_{avg} \rightarrow 320 \text{ (lb per min or Btu per min)}$$

To avoid the irrational assumption that, preceding the disturbance, the flow hypothetically had been in a state of constant motion throughout its entire range, it will be assumed that the process was in equilibrium with the controls inoperative prior to $t = 0$, at which time the controller is made operative simultaneously with the imposition of the upset.

As already noted, the foregoing results apply only to an assumedly ideal two-position control mechanism. This condition may be approached only with a highly sensitive and responsive measuring means. It will be interesting to consider, as an example of what may come under this heading in general, the case of a two-position controller in which a certain degree of dead space is involved, for example in the detection of the controlled variable. Here the control mechanism might not sense the actual value of T_a itself but instead that of another variable in imperfect correspondence therewith. The imperfection meant here is one

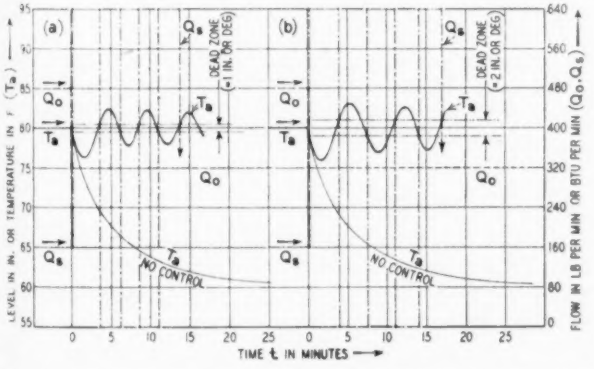


FIG. 6 RECOVERY TRANSIENTS WITH TWO-POSITION CONTROLLER HAVING "DEAD SPACE"

which allows the variable corresponding to T_a to follow T_a at a constant separation when the latter variable moves persistently in one direction and to remain unaffected when T_a reverses its direction until T_a has proceeded in the opposite direction by an amount equal to twice the constant separation. This behavior is similar to the action of two imperfectly meshed gears when the rotation of the driver is reversible. Let us determine the effect on the recovery transient.

The recovery transient for the two-position controller with dead space may be found in much the same way as for the ideal instrument, except that the correcting changes undergone by Q_s occur, not when the desired value of T_a is crossed, but when T_a has passed beyond that value by an amount equal to $^1/_2$ the dead zone. This merely requires, in plotting, that the individual effects of the changes involved are added together at different initial instants. Fig. 6 shows the results for 1 and for 2 units (in. or deg) of dead zone, assuming the same initial conditions, etc., as before. The oscillations of T_a can never die out in this case but will eventually settle down to a steady state of oscillation with an amplitude which is always greater than the dead zone and which may even be several times the width of this zone when complicated process lags are involved. The manipulated variable Q_s will continue to fluctuate regularly between its maximum and minimum values at definite intervals, which may be of objectionable duration.

TWO-POSITION-WITH-RATE CONTROL

This descriptive, though not particularly euphonious, name has been tentatively assigned to a class of control which is widely represented in relatively simple applications. Closely allied to the simple two-position class, it differs therefrom in that it gives to the action of the controlling means a definite constant rate when the controlled variable is on one side of the control-point setting, and gives (usually) the same rate in the opposite

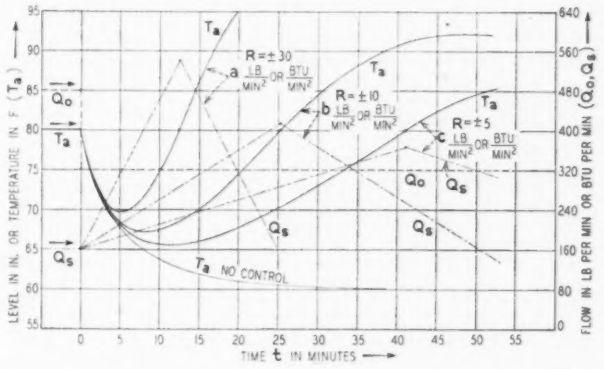


FIG. 7 RECOVERY TRANSIENTS UNDER "TWO-POSITION-WITH-RATE" CONTROL

sense when the controlled variable is on the other side of the control-point setting. This "law of control" may be expressed for the case under consideration, as

$$Q_s' = +R \quad \text{when} \quad T_a < T_{cp}$$
$$Q_s' = -R \quad \text{when} \quad T_a > T_{cp}$$

The quantity R is the constant rate at which the manipulated variable (the flow Q_s) changes. It should be added that this expression of the law requires that Q_s remain between the limits zero and $(Q_s)_{max}$. This type, it is evident, approaches two-position control as the rate R becomes progressively larger.

This characteristic is included primarily for the sake of completeness; its behavior in the face of process lags is markedly inferior. Thus we shall not give the mathematical procedure by which determination of its behavior on the chosen system may be found, but merely note that this involves solution—by stages—of the nonhomogeneous differential equation

$$(A_3)T_a''' + (A_2)T_a'' + (A_1)T_a' + T_a - R_a(Q_0 \pm Rt) = 0$$

Results of such determinations are shown in Fig. 7, in which the following rates are assumed:

$$\text{Case } (a) \quad R = 30 \left(\frac{\text{lb}}{\text{min}^2} \text{ or } \frac{\text{Btu}}{\text{min}^2}\right)$$

$$\text{Case } (b) \quad R = 10 \left(\frac{\text{lb}}{\text{min}^2} \text{ or } \frac{\text{Btu}}{\text{min}^2}\right)$$

$$\text{Case } (c) \quad R = 5 \left(\frac{\text{lb}}{\text{min}^2} \text{ or } \frac{\text{Btu}}{\text{min}^2}\right)$$

The relatively fast rate used in case (*a*) is seen to produce an excessive overshoot on return, while a considerably slower rate, as in case (*c*), delays recovery from the upset and permits a greater

deviation. It is evident that a compromise, as in case (*b*), is ineffective in avoiding these two evils.

The inadequacy of this type of control in the present instance is due to the inclusion of transfer lags in the process. This method of control has been used successfully in certain commercial applications, however, but its success has resulted from an abundance of capacity in that part of the process in which the controlled variable is measured. When such conditions prevail, two-position control would be even more successful in the degree of control obtainable, but the two-position-with-rate method allows much smaller fluctuations in the manipulated flow.

Illusion of an Immediate, "Exact" Correction

It may be interesting to note the effect, on the behavior of T_a following an upset, of an enforced immediate change in Q_s to a

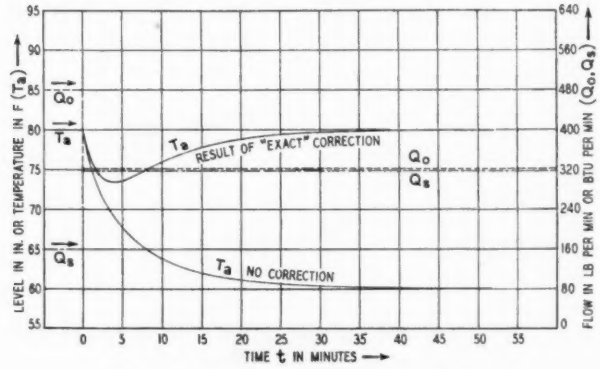


Fig. 8 Effect of an "Exact" Correction

new steady value of such magnitude as to produce eventual return by T_a to its desired value. Such a change in Q_s would necessitate a precise knowledge of the disturbance suffered, and might be thought to be an ideal correction. It is ideal, however, only in the steady-state sense. By the foregoing superposition method employed we may plot the resulting deviation by adding to that of Fig. 4 the result of suddenly changing Q_s from 160 to 320 units (an "exact" correction). Fig. 8 shows the consequent behavior of the controlled variable as a relatively slow aperiodic return to the desired value, involving a considerable maximum deviation. This type of correction would be satisfactory, in the transient sense, only if negligible lags existed between the manipulated and controlled variables. Furthermore, it does not lend itself to automatic execution since the measuring means can respond only to the result of a disturbance and cannot measure the cause which provoked it.

Proportional Control

This type is also called "throttling" control, due to the definitely allocated band of values (termed the "throttling band") in which the mechanism acts. The operating means of the control mechanism, here the value of the pneumatic operating pressure, is made to correspond linearly with the values of the controlled variable within the throttling band in such a way that the operating pressure has its maximum value when the controlled variable is at the lower limit of the throttling band and has its minimum value when the controlled variable is at the upper limit of that band. Whenever the controlled variable is below (or above) the throttling band, the operating pressure stands at its maximum (or minimum) value. When the controlled variable is precisely in the center of the throttling band, at a value arbitrarily called the "control-point setting," the operating pressure stands halfway between its extreme values. Since the manipulated variable Q_s is made coextensive with the operating pressure of the control mechanism, the foregoing remarks on the behavior of the operating pressure apply directly to that variable as well. Thus, assuming that the minimum value of the manipulated variable Q_s is zero, the law of control may be expressed as

$$\frac{Q_s - \frac{(Q_s)_{max}}{2}}{(Q_s)_{max}} = -\frac{(T_a - T_{cp})}{b}$$

where T_{cp} = control-point setting, or value of T_a at center of throttling band

b = absolute extent of throttling band in same units as T_a

For completeness the following boundary conditions should also apply

$$Q_s = (Q_s)_{max} \quad \text{for } T_a < T_{cp} - (b/2)$$
$$Q_s = 0 \quad \text{for } T_a > T_{cp} + (b/2)$$

It will be seen that the equations given merely embody the description of the mechanism's operation. Written in a more usable form

$$Q_s = \frac{(Q_s)_{max}}{2} - \frac{(Q_s)_{max}}{b}(T_a - T_{cp}) \quad \ldots\ldots [17]$$

or, with the assumed numerical datum as to $(Q_s)_{max}$

$$Q_s = 640\,[1/2 - (1/b)(T_a - T_{cp})] \quad \ldots\ldots [18]$$

The control-point setting T_{cp} and the throttling band b are of course "adjustables" of the mechanism. It may be said that the characteristic of this controller is a static one, in that the value of the manipulated variable at any time depends only upon the coincident value of the controlled variable and is independent of all previous or subsequent behavior of the latter variable.

The first step in the investigation of the performance of this type of control in connection with the specific process will be to combine the controller Equation [17] with the process Equation [3], obtaining

$$(A_3)T_a''' + (A_2)T_a'' + (A_1)T_a' + T_a = R_a Q_o + R_a\left[\frac{(Q_s)_{max}}{2} - \frac{(Q_s)_{max}}{b}(T_a - T_{cp})\right]$$

or

$$(A_3)T_a''' + (A_2)T_a'' + (A_1)T_a' + (1 + s)(T_a - T_p) = 0$$

where

$$s = \frac{R_a(Q_s)_{max}}{b} \quad \ldots\ldots [19]$$

$$T_p = \frac{R_a Q_o + s\,[T_{cp} + (b/2)]}{1 + s} \quad \ldots\ldots [20]$$

Now since s and T_{cp} will be constant during the time interval considered, the combined equation may be written as

$$(A_3)(T_a - T_p)''' + (A_2)(T_a - T_p)'' + (A_1)(T_a - T_p)' + (1 + s)(T_a - T_p) = 0 \quad \ldots [21]$$

The solution of Equation [21] under the usual initial conditions will give the behavior of T_a following the disturbance under proportional control. The meaning of T_p is shown (by Equation

[21]) to be the value at which T_a will eventually balance after the oscillations following the disturbance have faded. The controlled variable will not, except for one particular value of Q_o, balance out at the control-point setting. This general phenomenon is often referred to as "loss of control point." With this type of control the values of $(T_a)_{norm}$, T_p, and T_{cp} are distinct and different except for special cases.

Since the initial value of Q_o is known, Equation [20] will yield the value of T_{cp} necessary to give an initially balanced value of T_a, namely $(T_p)_o$, equal to the desired value $(T_a)_{norm}$. Thus

$$T_{cp} = \frac{(1+s)(T_a)_{norm} - R_a Q_o}{s} - \frac{b}{2} \quad \ldots\ldots [22]$$

Two separate cases will be considered in which the throttling bands are

$$(a) \ldots b = 10 \text{ (in. or deg)}$$
$$(b) \ldots b = 40 \text{ (in. or deg)}$$

Corresponding to these we have, from Equation [19]

$$(a) \ldots s = 8 \text{ (dimensionless sensitivity constant)}$$
$$(b) \ldots s = 2 \text{ (dimensionless sensitivity constant)}$$

Assigning to $(T_a)_{norm}$ the usual value of 80, Equation [22] gives, for the two cases, respectively

$$(a) \ldots T_{cp} = 77.5 \text{ (in. or deg)}$$
$$(b) \ldots T_{cp} = 70.0 \text{ (in. or deg)}$$

Thus, from Equation [20], the final balanced values are

$$\left.\begin{aligned}(a) \ldots T_p &= 77.78 \text{ (in. or deg)} \\ (b) \ldots T_p &= 73.33 \text{ (in. or deg)}\end{aligned}\right\} \quad \ldots\ldots [23]$$

The combined differential Equation [21] for controller and process becomes, respectively, in each of the two cases

$$4(T_a - T_p)''' + 18.5\,(T_a - T_p)'' + 10.25\,(T_a - T_p)' + 9\,(T_a - T_p) = 0 \ldots [24]$$

and $$4(T_a - T_p)''' + 18.5\,(T_a - T_p)'' + 10.25\,(T_a - T_p)' + 3\,(T_a - T_p) = 0 \ldots [25]$$

where the values of T_p are given by Equation [23].

The solutions of Equations [24] and [25] involve cyclically varying functions of time. The process of solution differs from

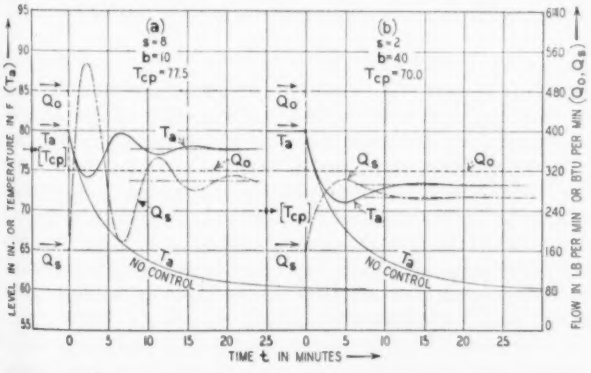


FIG. 9 RECOVERY TRANSIENTS WITH PROPORTIONAL CONTROL

those cases encountered heretofore in that the roots of the auxiliary algebraic equation are not all real. Appendix 2[7] shows a formal method of integration which may be used here, and which gives, for the two examples

$$T_a = 77.78 + 0.0472\,e^{-4.127t} + 6.549\,e^{-0.2440t}\cos(0.6959t + 1.231) \ldots [26]$$

and $$T_a = 73.33 + 0.0120\,e^{-4.036t} + 11.602\,e^{-0.2944t}\cos(0.3149t + 0.9599) \ldots [27]$$

These equations describe the behavior of T_a following the instant of upset and are plotted (Appendix 4[7]) in Fig. 9. It is essentially the exponential-cosine (or damped-sinusoid) term of the solution which governs the nature of the oscillations which the control transient comprises. The frequency (or period) and the damping factor are implicitly contained in this term.

For any given process, the roots of the auxiliary algebraic equation, and hence also the nature of the resulting damped sinusoid, are dependent exclusively upon the dimensionless constant s, which is the only variable of the controller entering the coefficients of Equation [21]. The factor s, which has been called the sensitivity constant of the combined system, depends directly upon both the output resistance R_a of the process and the maximum value $(Q_s)_{max}$ of the manipulated supply flow, and is inversely proportional to the extent of the throttling band b, Equation [19]. As the numerical value of the constant s is increased toward a value which would produce discontinuity for the particular upset, the period of cycling becomes shorter, the eventual departure from the desired value $[T_p - (T_a)_{norm}]$ becomes less, while the variation of the manipulated variable Q_s increases. The throttling band b is ordinarily an adjustable of this type of control mechanism, and is used to influence the sensitivity constant s. In practice, an optimum value for s (or for $1/b$) depends upon the permissible variation of Q_s for normal disturbances, and upon the mechanical perfection of the attendant apparatus.

Determination of the fluctuations suffered by the manipulated variable Q_s is fairly direct in this case since this may be accomplished from the already discovered behavior of T_a (Equations [26] and [27]) through the use of the controller Equation [17] or [18]. These fluctuations are also plotted in Fig. 9.

The effect of inertness or dead space in this type of control mechanism would be worthy of very intensive study, due to its inclusion in varying degrees in most commercial instruments. In the case of wide throttling bands, or when the magnitude of dead space is small compared to the throttling band, the effect is principally a proportionate increase in the amplitude and period of oscillation. When the dead space is commensurate with the throttling band, however, the effects may become far-reaching. This condition is especially threatening for small throttling bands; it will be seen that the detrimental effect of dead space as shown in the case of open-and-shut control becomes more influential as the throttling band approaches zero.

Once a substantial balance is attained with proportional control having dead space, the insensitivity of the controlling means to changes of the controlled variable within the dead space may result in complete inactivity of the controls during intervals of minor disturbances. Following larger disturbances, however, intervals of violent cycling can occur which may persist even after the cessation of the upsets. Many inexpensive regulators of this nature are in commercial use which apparently function ideally under small or slow disturbances, but which, when changes occur, calling for significant action by the control mechanism, seem to "lose their heads" and require manual assistance to regain their requisite inactivity.

"FLOATING" CONTROL

The name "integrating control" is sometimes given to this classification for reasons which will be evident. Except for the fact that some of the less complex regulating devices have approximately this characteristic, it is not in common use today. It is important, however, as a component of the more complex types, and has some unique features.

The functioning of this class of mechanism may be described by the statement that the action of the operating means of the control mechanism is a linear function of the integral of the deviation of the controlled variable from a desired value. Taking this value as the control-point setting T_{cp}, we may write, in terms of the operating pressure and using a proportionality factor c

$$\frac{P}{P_s} = -\frac{1}{c}\int (T_s - T_{cp})dt$$

or, with a definite integral

$$\frac{P}{P_s} - \left(\frac{P}{P_s}\right)_0 = -\frac{1}{c}\int_0^t (T_s - T_{cp})dt \quad \ldots\ldots [28]$$

A more universally pleasing form is obtained merely by differentiating either of these equations with respect to time; thus

$$\frac{P'}{P_s} = -\frac{1}{c}(T_s - T_{cp}) \quad \ldots\ldots [29]$$

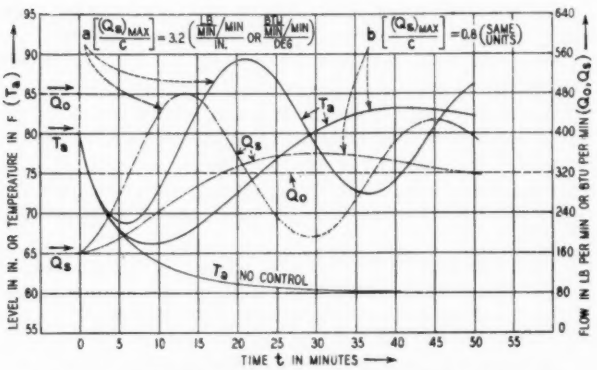


FIG. 10 RECOVERY TRANSIENTS UNDER FLOATING CONTROL

By the continued assumption that the manipulated variable is made coextensive with the operating pressure, we have from identity [5] and Equation [29]

$$Q_s' = -\frac{(Q_s)_{max}}{c}(T_s - T_{cp}) \quad \ldots\ldots [30]$$

Thus the floating characteristic may be described as one in which the rate of change of the manipulated variable is made proportional to the deviation of the controlled variable from a desired value. The constant factor $(Q_s)_{max}/c$ may be thought of as the rate per unit deviation, or as

$$\left[\frac{\text{lb}}{\text{min}} \Big/ \frac{\text{min}}{\text{in.}}\right] \quad \text{or} \quad \left[\frac{\text{Btu}}{\text{min}} \Big/ \frac{\text{min}}{\text{deg}}\right] \quad \ldots\ldots [30a]$$

The negative sign in Equations [28] to [30] assures the occurrence of the corrective action in the appropriate direction, and allows the proportionality factor to be an inherently positive constant.

The combination of the controller Equation [30] with the time derivative of the process Equation [3] gives the following fourth order differential equation (Q_c being constant disappears in differentiating):

$$(A_3)T_s'''' + (A_2)T_s''' + (A_1)T_s'' + T_s' + q(T_s - T_{cp}) = 0$$

where

$$q = \frac{R_a(Q_s)_{max}}{c} \quad \ldots\ldots [31]$$

or, since T_{cp} is constant

$$(A_3)(T_s - T_{cp})'''' + (A_2)(T_s - T_{cp})''' + (A_1)(T_s - T_{cp})'' + (T_s - T_{cp})' + q(T_s - T_{cp}) = 0 \quad \ldots\ldots [32]$$

A solution of Equation [32] is possible by methods similar to those used previously and specifically indicated in Appendix 3.[7] Two cases will be considered, in which different values are assigned to c. Before giving the results we may note a fact of great importance from Equation [32]. When the effects of all disturbances are balanced out, all derivatives become zero, and the equation shows that under such conditions $T_s = T_{cp}$. Hence this control characteristic eliminates any permanent or residual effects of changes in load; thus there is no "loss of control point." Although shown here in a specific case, this recognized fact is applicable in general. The shortcomings of this controller type lie in its inability to cope with the immediate effects of a disturbance.

Shown in Fig. 10 are the recovery transients, following the same disturbance as used throughout, for the following adjustments

(a) .... $(Q_s)_{max}/c = 3.2$ (units given by Equation [30a])

(b) .... $(Q_s)_{max}/c = 0.8$ (units given by Equation [30a])

It is evident, from Fig. 10 that, for a larger value than in (a) the extent of the first return loop, or first maximum, would become excessive. Instability, or oscillation of increasing amplitude, lies not far in that direction. For smaller values than in (b), on the other hand, the recovery would involve both an excessive deviation and an excessively slow return. At a certain smaller value of this adjustment, the recovery would become critically aperiodic; that is, the return curve would approach the desired value at the most rapid rate possible without crossing.

Only in the relative absence of transfer lags between the manipulated and controlled variables could the value of $(Q_s)_{max}/c$ be made sufficiently large to give satisfactory sensitivity to the controlling means.

The behavior of the manipulated variable Q_s, as shown in Fig. 10, may be obtained by solution of the original equations for Q_s instead of T_s, or more conveniently (although less accurately) by graphical integration from the recovery curves of T_s (see Equation [28]). The figure plainly shows the lagging phase relationship borne by Q_s to T_s, and that the rate of change of the manipulated variable is zero whenever the deviation is zero.

PROPORTIONAL-PLUS-FLOATING CONTROL

This is a type of control having a characteristic which combines properties of the proportional type and the floating (or integrating) type. Other names are, "throttling-plus-floating," "throttling-plus-reset," "proportional reset," etc. The use of this characteristic is fairly well known today. The success which practical controllers of this type have met with, and which they shall continue to enjoy, may be attributed to the fact that they inherit the particular advantages of their component devices, while the separate or distinct disadvantages of each are fortunately canceled in the combination.

To bear out this conception of its origin, we may show how the equation describing the law of control of this mechanism is synthesized from those of the proportional and of the floating controllers, namely, from Equations [17] and [30]. Differentiating Equation [17]

$$Q_s' = -\frac{(Q_s)_{max}}{b}(T_s - T_{cp})' \quad \ldots\ldots [33]$$

Equation [33] states that the rate of change of the manipulated variable is proportional to the rate of deviation of the controlled

TABLE 3 NUMERICAL VALUES OF ROOTS AND INTEGRATION CONSTANTS

Case	r	s	(b)	$+k_a$	$-k_b$	$-k_c$	$-k_d$	$+C_a$	$+C_b$	$-C_c$	$+C_d$
(a)	0.1335	13/12	73.85	0.07791	0.2210	0.1639	4.019	0.5822	54.95	45.91	0.00647
(b)	0.1335	2	40	0.2661	0.2255	0.1390	4.035	1.158	18.09	7.268	0.01165
(c)	0.1335	4	20	0.4448	0.2014	0.1356	4.069	1.317	10.93	2.763	0.02212
(d)	0.1335	8	10	0.6700	0.1788	0.1344	4.133	1.422	8.089	1.239	0.04013
(e)	0.4452	4	20	0.4831	0.04836	0.4648	4.063	−1.253	− 8.362	−2.594	0.02056
(f)	0.4452	1/8	160	0.1559	0.6362	0.4896	4.008	−1.225	−17.15	−5.805	0.00285

variable; this is merely a paraphrasation of the characteristic of the proportional controller. Now if Equations [30] and [33] were in effect additively, we should have, for the new value of Q_s'

$$Q_s' = -\frac{(Q_s)_{max}}{b}(T_a - T_{cp})' - \frac{(Q_s)_{max}}{c}(T_a - T_{cp}) \quad \ldots [34]$$

Equation [34] is the law of control for the proportional-plus-floating mechanism.[8] It shows the rate of change of the manipulated variable depending additively upon the deviation of the controlled variable from the control-point setting (desired value) and on the rate of change of that deviation. This equation may also be written

$$Q_s' = -\frac{(Q_s)_{max}}{b}[(T_a - T_{cp})' + r(T_a - T_{cp})] \quad \ldots [35]$$

in which

$$r = b/c \quad \ldots [36]$$

The constant r may be called the "reset constant." From Equation [35] it is apparent that this term governs the ratio between the influence of the deviation and that of the rate of deviation, or the ratio of the proportional to the integrating effect. The integrating (or "floating") influence brings about eventual equilibrium of the controlled variable at the desired value, and this action has been called "reset." The dimensions of r are (1/time), and its units are "inverse minutes." The constant b is the familiar throttling band of the proportional controller and has, when referred to immediate changes, the same meaning in the present mechanism. Both b and r (and of course T_{cp}) will be considered as "adjustables" of the mechanism.

To obtain the differential equation of the interconnected system, we combine Equation [3] for the process with Equation [35] for the controller, in a similar manner to that followed in obtaining Equation [32] for the integrating case, and obtain thus

$$(A_1)(T_a - T_{cp})'''' + (A_2)(T_a - T_{cp})''' + (A_1)(T_a - T_{cp})'' + (1+s)(T_a - T_{cp})' + (rs)(T_a - T_{cp}) = 0 \quad \ldots [37]$$

where s is given by Equation [19] and depends inversely on the throttling band, while the role of r has been just described. This equation describes the behavior of the controlled variable T_a under any circumstances (within limits of continuity) and may be solved under the usual specific conditions to show recovery transients obtained with this type of control. Several solutions of this sort will be carried out for various values of the adjustable constants r and s (or of $1/b$). It may be shown that there are rather definite values for r and s to give critically aperiodic return. Strictly definite values giving this critical state would be conveniently obtainable if the combined Equation [37] had been of the third order. This equation would have been of the third order but for the inclusion of the valving lag described earlier and represented in the hydraulic analog, Fig. 1, by the cascade capacity. Hence, by neglecting this lag, values may be computed for the constants r and s which are approximately critical for the complete system. Adjustments of r and s (or of $1/b$), below those values which would give critically aperiodic return, produce a control characteristic which has been called "averaging control." The effect obtained is to reduce the fluctuation of the manipulated flow at the expense of variations in the controlled variable. It has found wide practical application in level-control problems. Except in such applications, however, the adjustments giving aperiodic behavior following a disturbance are not the most favorable.

In the periodic, or usual, cases Appendix 3[7] shows the solution of Equation [37] to be

$$T_a = T_{cp} + C_d e^{k_d t} + C_c e^{k_c t} + C_b e^{k_b t}\cos(k_a t + C_a) \quad \ldots [38]$$

where the C's and the k's depend upon the coefficients of Equation [37] and hence are affected by the reset constant r and by the sensitivity constant s (or the reciprocal of the throttling band). Table 3 shows the numerical values of the roots and integration constants, for substitution in Equation [38], which result when certain chosen values are assigned to the constants r and s. Case (a) involves the values of r and s which give the substantially critical case, in which neither the magnitude of the throttling band nor the reset constant are such as to cause cyclic behavior. The recovery transient for case (a) is plotted as curve (a) in Fig. 11. Except for a greater deviation, curve (a) exhibits a similar recovery to that resulting from an exact instantaneous correction as described previously and plotted in Fig. 8. This similarity is due to the fact that the flow attains its final value essentially without overshoot in the comparatively brief interval of 11 min. Curve (a) of Fig. 12 shows the recovery transient

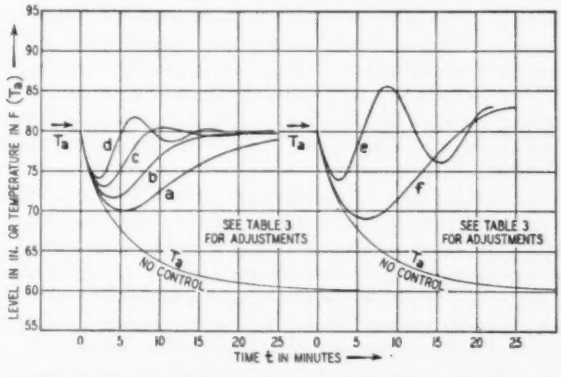


FIG. 11 RECOVERY TRANSIENTS UNDER PROPORTIONAL-PLUS-FLOATING CONTROL; CONTROLLED VARIABLE

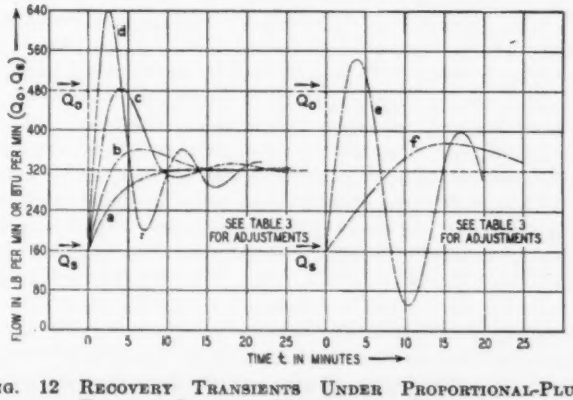


FIG. 12 RECOVERY TRANSIENTS UNDER PROPORTIONAL-PLUS-FLOATING CONTROL; MANIPULATED VARIABLE

[8] This equation could also have been developed from the mechanics of a particular control device faithfully embodying proportional-plus-floating characteristics.

for the manipulated flow in this case. The flow transients for this type of control may be found from the transients of the controlled variable by methods indicated previously.

The remaining cases selected for illustration were not chosen at random, for the results of such a procedure might well be confusing. The adjustments, giving the recovery curves shown, form an orderly sequence which, it is felt, will serve to demonstrate their separate influence on the control behavior.

Assuming for the moment that the adjustments giving the critical case (curves (*a*), Figs. 11 and 12) are in effect, let us consider, in a qualitative sense, the result of allowing b to remain at its critical value and of changing the adjustment r. Without resorting directly to actual solution, we may observe that when under such conditions, r is made smaller (slower reset rate), a longer time will be required for the manipulated flow to attain a substantially balanced state close to its final, or potential, value (320 lb/min or Btu/min). It is apparent, from the discussion previously given of the critical values of b and r, that a periodic recovery transient will result if the constant b is decreased, or the constant r increased, from its critical value. Considering separately an increase in the reset constant r, it is evident that such an increase will bring about an earlier crossing, by the manipulated flow Q_s, of its final or potential value. These relationships are best discernible in the light of the individual characteristics of the component devices, namely, of the floating and the proportional mechanisms, as exhibited in Figs. 10 and 9, respectively. The control influence of the floating part of the characteristic tends to make the rate of change of the manipulated flow proportional to the deviation of the controlled variable from its desired value; this influence tends to prevent reversal of that rate until the controlled variable has crossed its desired (and potential) value (Fig. 10). On the other hand, the control influence of the proportional part of the characteristic tends to reverse the rate of change of the manipulated flow in coincidence with the reversal of the controlled variable (Fig. 9). The desirability of this latter effect suggests that it might be informative to show the effect, on the recovery transients, of progressive reduction in the magnitude of the throttling band b (or of progressive increments in the sensitivity constant s) with the reset constant r remaining at its critical value.

The results of these adjustments are plotted in Fig. 11. Given in Fig. 12 are the curves showing the behavior of the manipulated variable in the corresponding cases. Starting with the practically aperiodic case (*a*), the reset constant r is left at its critical value and the sensitivity constant s is increased in geometrical steps above its original value. This initial series of adjustments may of course be considered to be obtained by successive reductions in the throttling band b. As s is increased, or as the throttling band is decreased, the cyclic period and the damping per cycle both become smaller, for the oscillations of Q_s as well as of T_a. The initial deviation of T_a is reduced by the successive adjustments, but the corresponding fluctuations of Q_s are seen to increase in magnitude. In each of the first four cases the promptness of return to the region of the desired value is creditable.

With this type of control, discontinuity should be avoided; thus, the choice of an optimum value, for throttling band (*b*) for any given process, will also be influenced by the available range of the manipulated variable Q_s as well as by the desired stability of that variable and the mechanical perfection of the apparatus.

Smaller values of r, or reset rates slower than the critical, would merely delay the return in each case. The effect of larger values of r (faster reset rates) is indicated by case (*e*), in which the magnitude of the throttling band is left the same (20 units) as in case (*c*), but in which the reset constant is increased approximately threefold. Although this adjustment slightly decreases the period of oscillation, as well as the magnitude of the initial deviation, the amplitude and damping of the subsequent oscillation is seriously impaired. Herein lies the only real danger arising from haphazard manipulation of this adjustment in the commercial use of this type of control mechanism.

From field experience derived from the application of control mechanisms which involve a proportional control effect (or which have a throttling band, in the usual sense), a general rule has evolved to the effect that cycling can be reduced or eliminated by the expedient of an increase in the magnitude of the throttling band. In the case of the proportional-plus-reset control mechanism, this rule is only of practical value provided that the reset rate (or the reset constant r) is reasonably near the critical value demanded by the lag characteristics of the process under control. Case (*f*) shows the futility of increasing the extent of the throttling band in an attempt to attain satisfactory control when the reset rate is excessively fast for the process under consideration.

The authors' experiences in the mathematical investigation of other types of processes, as well as actual field experience, indicate that there exists—for every process—a critical reset rate (or reset constant r) which is independent of the disturbances to which the process may be subjected and of the throttling band which may be employed in the mechanism.

In spite of the necessarily limited generality of the mathematical approach in the present paper, in that a single specific process has been dealt with, the authors suggest that it may at least serve to introduce a method by which the development of rational theorems may be made possible on an analytic basis. It is evident that the formulation of rules sufficiently practical for industrial use must be the result of sincere cooperative effort on the part of the technical groups both of the users and of the manufacturers of industrial control equipment.

For the present, experience alone can supply the necessary criteria in the application and adjustment of the more complex control mechanisms. Even this brief introductory treatment, however, would seem to establish certain of the properties characteristic of the reset adjustment. It is evident that, although an optimum reset rate may exist for any process, the determination of this optimum is not a simple routine deduction; furthermore, the usefulness of the reset feature in eliminating "loss of control point" is seen to be limited by the critical nature of its adjustment at values below the optimum.

This paper was prepared, as a continuation of the previous paper,[3] to give, together with that paper, a more complete exposition of a suggested method for a quantitative approach to the development of a science of automatic control. The authors wish to submit, for consideration by the A.S.M.E. Committee on Industrial Instruments and Regulators, the suggestions embodied in both papers, and also to offer their sincere cooperation in the accomplishment of the purposes of the committee. In this connection, they will equally welcome comment or critical discussion from any individuals interested in these purposes.

Acknowledgment

It is impossible to acknowledge in detail the assistance of the many engineers whose discussions of practical experiences have contributed to these developments. We wish to mention particularly, however, those of R. A. Rockwell of the Foxboro Company, who has throughout the entire development made valuable suggestions, conducted field experiments, and accumulated reliable data pertaining to the practicability of the mathematical analysis. Credit for the preparation of this paper for presentation is due in large measure to the encouragement given by John J. Grebe of the Dow Chemical Company. We are also indebted to A. F. Spitzglass and M. J. Zuerow for helpful discussions and for cooperation in pointing out basic similarities between the present approach and those of earlier, foreign writers.

Discussion

E. S. SMITH[9] AND C. O. FAIRCHILD.[10] This paper presents a valuable and straightforward treatment of a combined controller and plant, i.e., a controlled system for several familiar types of controllers and a relatively uncommon process or plant. While level control is used as a complete analog of temperature control, this does not appear to be adequate since the latter involves the important factor of attenuation due to "distributed capacity and resistance."

Since a process is a method, not an apparatus, the word "plant" should be used instead of "process" when the apparatus for carrying out the process is meant. The use of a volume unit instead of a weight unit as the basic "quantity" unit would seem preferable as appreciably simplifying the dimensional treatment.

The so-called philosophy of control cited in connection with Equation [7] does not appear to be entirely consistent with the essential unity of the plant and its controller implied in footnote,[5] the noted reversal being simply a matter of opposite sign or 180-deg phase difference.

Under two-position control, the conclusions that the intervals and amplitudes progressively decrease with time do not appear to be of value for the common practical case where metering lag exists, in which case the intervals and amplitudes become constant.

The two-position-with-rate control is more commonly known as constant-speed floating control. In Fig. 7 and its context, it is unfortunate that R should be used in another sense than resistance as it was elsewhere used in the paper. The last paragraph of the "Two-Position-With-Rate" section appears to be incorrect or inadequate, since a dead zone or meter friction is necessary to eliminate hunting, and an increase of capacity requires a slowing of the rate R of valve movement to avoid reaching the limits of valve travel.

Under the heading "Proportional Control," the s of Equation [19] might well be explained further as being the ratio of the head T_a for $Q_a = (Q_a)_{max}$ to the head of the throttling band b. Following Eq. [21], the "loss of the control point" is more commonly known as "drift" or "load error." In the absence of further explanation of T_p, T_{cp}, and $(T_a)_{norm}$ the final values of Equation [23] appear to be in error. The generality of the conclusion, "As the numerical value of the constant s is increased toward a value which would produce discontinuity for the particular upset, the period of cycling becomes shorter . . . .," must be questioned, since this does not seem to be so in temperature control, including where attenuation is involved. If the flow scale of Figs. 9 and 10 were inverted as in a paper[11] by one of the writers, a clearer picture of lags for proportional and floating control would be obtained. Thus, it is immediately apparent that the final control element moves in phase with the measured value of the head T_a in Fig. 9 and lags 90 deg behind such head in Fig. 10. The last sentence of this section apparently means that the throttling control simply breaks down into two-position control under unfavorable conditions.

Under the heading "Floating Control" (of a particular kind), a fuller treatment of q in Equation [31] in terms of c would give the reader a needed mental picture of this before he reaches Equation [36]; at least the dimensions of q and c should be stated. The use of the inverse of c, i.e., $1/c$ seems to the writers to be more nearly consistent with the speed of reset as used throughout the latter part of the paper. The reference to transfer lags in the penultimate paragraph of this section might well include a reference to the portion of Fig. 1 which is meant. The last paragraph might likewise state that the lag of a floating valve is 90 deg behind that of a valve with proportional control.

In connection with Equations [34] to [36], it might be worth while to bring out that $1/c$ is simply the speed of reset dimensionlessly expressed in terms of the maximum flow per unit of head departure $T_a - T_{cp}$, and that an increase of r is accompanied by a resultant tendency for the stability to decrease. Exception may be taken on the grounds of overgenerality to the statement, "Except in such applications (averaging level control), however, the adjustments giving aperiodic behavior following a disturbance are not the most favorable," a case cited in an unpublished paper by R. P. Lowe and one of the present writers presented before the Society in 1938 constituting an important exception in which the controller had to follow a storm flow smoothly.

The last paragraph of the first column on page 304 appears to imply that the throttling band b and the reset speed $1/c$ are adjusted together to maintain r, or b/c, constant. In the second paragraph beyond this, a question arises as to whether $1/c$ or r is meant. In the following paragraph, it would seem to be the users' turn next if the progress of the present paper is to be consolidated without the possible loss of anything more than a bit of higher mathematics.

While the addition of a cascaded reservoir A_a enabled the authors to show that their approach was not limited to simple cases, yet it is likely that a somewhat simpler plant would provide a clearer relative picture of the merits of the several sorts of controllers or regulators. This suggestion is borne out by the authors' simplification of the treatment of Figs. 11 and 12.

A somewhat different approach may be more helpful in certain cases. For example, the analysis of Ivanoff for temperature control of plants involving thermal lags and attenuation also led to some fairly general conclusions as to reset speed which have considerable practical justification. For another example, Ivanoff also determined a controlling apparatus to fit a desired law of control for a particular plant which presented unusual difficulties.

Time is of the essence in detecting a change and initiating a movement of a final control valve properly to offset the cause of the change. With a proportional controller, which operates without serious metering lag or dead zone, nonhunting throttling control exists with a single-capacity plant, since any change of head immediately acts to position the valve properly. In level control, it may be noted that the addition of a rate-of-departure component permits the controller to act as though it were governed by the net discharge from the reservoir and hence to set the control valve immediately to the proper position to maintain the set level, after a change in the rate of either inflow or outflow, without waiting for the level itself to depart objectionably. The addition of the rate component implies a high sensitivity which is generally obtainable only by the servo-operation of the portion providing such component, which portion may be actuated by either the meter or the control valve as convenient. This "departure-plus-rate-of-departure" controller thus functions in an entirely different manner from that of a throttling control and tends to be more stable; although theoretically, with a single capacity, the valve of a throttling controller will be stable even though its throttling band be indefinitely reduced. However, it is better in practice from the standpoint of stability to have a reasonably wide "initial throttling band" followed by a gradual reset to the set point.

The addition of one cascaded capacity also requires the addition of a rate component to let the control valve be properly positioned soon after a variable supply has changed, considering that the outflow be controlled.

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[10] Director of Research, C. J. Tagliabue Mfg. Co., Brooklyn, N. Y.

[11] "Automatic Regulators, Their Theory and Application," by E. S. Smith, Trans. A.S.M.E., vol. 58, 1936, PRO-58-4, pp. 291–303.

From the standpoint of promptness of response, the similar addition of a second cascaded capacity seems similarly to require the addition of an acceleration component, and so on. A general rule seems to be that an increase in attenuation in the number of cascaded capacities or in the order of the differential equation for the plant requires the corresponding addition of a higher derivative to the controller to make prompt control with accurate compensation possible. The use of a floating control valve instead of a throttling control valve likewise raises the order and requires the addition of a higher derivative. An increase in the power of time of the law of departure likewise in effect raises the order and necessitates the addition of a higher derivative if control is not to be lost while within the range of the controller. From these particular considerations, it appears that a controller may be straightforwardly selected to give the desired mode of control of a plant instead of trying empirically a number of different sorts of controllers in an effort to find one which has an acceptable performance with the particular plant.

The reset requirements of flow-rate controllers are of a different order from those of temperature controllers. If the authors have space in their closure, they might include a statement as to whether their more general remarks upon the effects of the reset speed on stability apply equally to flow controllers. The effects of cavitation with gassy liquids and of water hammer in long lines may well modify radically some of the general conclusions with flow-rate controllers where a premium may exist on making the flow changes as gradual as possible. This requirement also exists in other cases such as that in which a thermal gradient must be maintained without exceeding a critical temperature.

Since there is no true distinction in a control system between the plant and its controller and the paper includes a comparative treatment of controllers, it seems desirable to call attention briefly to generally related patent literature: Patent No. 2,113,-164 of A. J. Williams, Jr., contains an interesting analysis of a system which involves both the first and the second derivatives; and the C. E. Mason patent No. 1,897,135 (Re. 20,092) is for a device using the first derivative and having a coupling of a differential bellows to its nozzle and flapper. The latter also includes an interesting analysis. It is requested that the author's closure state whether this Mason patent for the "Stabilog" controller should be regarded substantially "proportional-plus-(first derivative) floating control" or as having a significant higher derivative effect due to close coupling, which effect is not included in the mathematical analysis of such patent.

W. D. Wood.[13] In the section of the paper covering "Proportional Control," the authors have used the term "Control band," and have developed from it a dimensionless sensitivity constant s. This proportional sensitivity constant is familiar to the readers of the published works of Ivanoff, who also derived the term "potential temperature." The sensitivity constant s may be simply defined as the potential temperature change caused by unit temperature change as measured by the control instrument. This constant may be aptly termed "proportional overall sensitivity," since it includes the sensitivities of all elements in the control circuit. Ivanoff has pointed out the relation between this sensitivity constant and "loss-of-control point" in proportional control. It is worth repeating since it is not readily apparent from the present paper. If a load disturbance is described in terms of the temperature change which would result if the controller were inoperative, then the actual equilibrium temperature change due to a disturbance during control is $\frac{1}{1+s}$ times the disturbance.

[13] Engineering Research Department, Taylor Instrument Companies, Rochester, N. Y. Jun. A.S.M.E.

One wonders why the authors introduced the auxiliary flow Q_a to effect load changes. In actual installations, load disturbances generally arise through a change in the outlet resistance. From Equation [19], it is seen that the sensitivity constant depends upon the outlet resistance. Thus the sensitivity constant normally changes with load changes. The need for characteristic flow valves, such as percentage or logarithmic valves, arises from this consideration.

In the section dealing with "Floating Control," the authors have introduced a constant c which corresponds to control band b for proportional control. In floating control, the valve travel depends upon the area of temperature deviation and time. The constant c has a real physical significance, namely, that on a time-temperature-deviation graph (which may be a temperature-controller chart) it represents the area necessary to cause complete valve travel.

Corresponding to the proportional sensitivity constant s the authors have derived a floating constant q, which may be termed "floating over-all sensitivity" and defined as the rate of potential temperature change for unit actual temperature change.

To the writer it seems desirable to describe an instrument's action in terms of the ratio of output change to input change. The narrower the control band, the greater should be the proportional effect since more valve action takes place for a given controller input change. Therefore, the proportional effect is proportional to $1/b$ and, correspondingly, the floating effect is proportional to $1/c$. On this basis the authors reset constant r, defined as the ratio b/c, should be called the ratio of the integrating to the proportional effect.

In connection with Fig. 11, it is noted that the reset constant was held unchanged and the control band decreased. It is of interest to note that, for several commercial pneumatic instruments, the mechanism is so designed that a change in the proportional adjustment affects the floating action, whereas a change in the floating adjustment does not affect the proportional action.

The authors' analysis has clarified the control effects of independent adjustments of the proportional and floating components of control. They point out that there is a critical reset rate for every process. Does this mean that there is one best floating adjustment, or one best ratio of floating-to-proportional adjustment, and how is this adjustment arrived at?

The ultimate value of this paper will depend upon its practical application. The authors have made a rational analysis of the control problem from which it is hoped valuable information will be obtained regarding guides to practical rules for industrial use. One might wish that the authors had devoted some space to the constants which arise from consideration of an actual process.

Authors' Closure

Messrs. Smith and Fairchild suggest that the model system shown in Fig. 1 of the paper is a relatively uncommon one. In the interest of clarity, it should be stated that no attempt was made by the authors to select an actual process for treatment. Perhaps the system selected is uncommon in that sense. However, it is both common and relevant in another and more important sense, since it involves and represents types of lag which must frequently be dealt with in the successful industrial application of control. In addition, the model system chosen has the valuable property that it submits gracefully to a relatively simple mathematical analysis, and requires a low-order linear differential equation for its complete dynamic description.

The authors have been particularly careful to avoid dealing with processes, either hydraulic or thermal, which might involve questions as to the practicability of the assumptions necessary to construct their differential equations. Under these conditions, the accuracy of the results may be relied upon to the same de-

gree as the mathematical manipulation, the latter being carried on according to accepted procedures. Until there is a thorough understanding of the behavior of nondistributed or lumped systems as such, the authors see no valid object in precarious generalizations. The possibilities inherent in the use of lumped parameters have been marvelously demonstrated over and over again in the electrical field, where highly complex networks have been represented to an uncanny degree of precision by such practical methods. In these cases, certainly, a complete knowledge of the simpler component systems was already available from previous work extending over many years. The infant science of automatic control can hardly disregard this established method of development.

The differential equation of the thermal process, of which Fig. 11 of an earlier paper[13] was taken to be an analog, was developed in that paper. Reference to that paper will show that the very reasonable assumptions which were made in the development of the thermal system will permit it to be considered as devoid of distributed capacities and resistances. Thus it is not encumbered by these complications; its thermal-lag characteristics are reasonably embodied in the structure of a second-order linear differential equation in terms of heat flow, temperature, and time.

Whether the system under control is called a "plant" or a "process" is a question which loses importance when models are under discussion. A great deal of incorrect but descriptive nomenclature has grown out of common usage in the field. This same usage has given a meaning to the word "plant" somewhat broader than would seem appropriate. The simplification obtainable in the hydraulic model by using a volume unit for quantity rather than a weight unit was foregone because of the attendant loss of vividness in the correspondences between the units of the hydraulic system and those of the various other systems which might be represented (for example, refer to Table 1 of the paper).

With regard to the "philosophy of control," the causal relationships between the manipulated and controlled variables, as pointed out in the paper, are held to be more fundamental than the simple polarity or phase criterion mentioned by Messrs. Smith and Fairchild.

Unless the term "metering lags" is interpreted as including discontinuities such as dead zones, we cannot agree that the addition of metering lag leads necessarily to a continual cycling under two-position control.

"Two-position-with-rate" control was given in the paper as a tentative name. We agree that this method of control is commonly known as "constant-speed-floating," but it was desired to emphasize its similarity to two-position control rather than to floating control. The last paragraph in the section "Two-Position-With-Rate Control" refers to a condition in which there is a predominance of capacity in that part of the process in which the controlled variable is measured. Under such a condition, the rate of corrective action could be materially increased if the variations involved in such action might be disregarded. In practice, however, dead zones are frequently employed to reduce valve activity. As pointed out by Messrs. Smith and Fairchild, this reduction in the valve variation is only accomplished by using extremely slow rates. On the other hand, if the rate of valve movement is increased indefinitely, this type approaches two-position control and the valve occupies its limiting positions practically all of the time. This was not borne out in Fig. 7 of the paper since, in none of the transients shown did the manipulated variable attain a limiting value, although it was shown in that illustration how extremely slow the rates of corrective action must be made to attain small flow variations.

It is certainly worth while to stress the physical meaning of the dimensionless sensitivity constant s, defined in Equation [19] of the paper. Even more generally (as pointed out in Mr. Wood's discussion), s may be identified as the ratio of an immediate change in "potential" level enacted by the controls to the original change in level which initiated such action. Unfortunately, there must be some misunderstanding about Equation [23] and/or the definitions of T_p, T_{cp}, and $(T_a)_{norm}$. In the remarks made in the paper, concerning the effects of an increase in the constant s, the term "period of cycling" was used to denote the cyclic period itself (or 1/frequency) which is invariably decreased by an increase in s. The orientation of the coordinate scales used for plotting control transients is naturally a matter of choice, but the straightforward method employed by the authors bears out graphically the 180-deg phase difference mentioned earlier in the discussion.

The constants q and c were given subordinate roles in the exposition; they were introduced in a natural manner in the development and were used to lend a logical symmetry to the equations. A joint consideration of the symbols used in Equations [19], [31], and [36] will show this symmetry.

In connection with Equations [34] to [36], the term $1/c$ cannot in any sense be considered dimensionless. The reset constant r, however, has the formal dimensions of inverse time. The fact that the value of r may be so chosen that an ultimate point stability is obtained without appreciable loss of oscillatory stability is of prime importance. Rather than being overgeneral, the statement in the paper concerning aperiodic behavior was perhaps not made general enough; further elaboration would have deviated from the main purposes of the paper.

The adjustments, demonstrated under the heading "Proportional-Plus-Floating Control," are summarized in Table 3 of the paper. Changes in the numerical value of b (or of s) have no effect on the numerical value of r, and vice versa. Their joint influence on control performance is implicit in the mathematics. The authors hope that the impression cannot be gained from the paper that the subject has had complete mathematical treatment. Such treatment has scarcely begun. Mathematics itself is only a means to an end. The progress which may be made through cooperation between user and manufacturer has been indicated by a wealth of practical evidence. Although practical experience will be invaluable in continuing the mathematical development toward some stage of practicability, it will be essential to heed the advice of Francis Bacon, given some 300 years ago: "... At the entrance of every inquiry our first duty is to eradicate any idol by which the judgment may be warped."

It is perfectly true that the original two-capacity system could have been employed. On the other hand, a more complex system might be considered necessary if a prototype, having considerable distributed resistance and capacity, were to be represented. The model chosen provided plausibly complete representation of a typical prototype involving some difficulty and, at the same time, was simple enough not to require an unwieldy analytical exposition.

The reference by Messrs. Smith and Fairchild to the work of Ivanoff is interesting and appropriate. That author's approach has been from the frequency standpoint. The two methods of treatment are equally valid and are certainly not incompatible, each having its distinct advantages. They may also serve as a check on one another in practice. The value of any method of analysis must depend upon its ability to yield laws useful in practical applications. In any case, the authors contend that there is a very real advantage in the concreteness of the representation obtainable with the liquid-level method of analogy; the more complex the system to be represented, the greater the desirability of this concreteness.

To obviate the possibility of misunderstanding, it should be

[13] "Quantitative Analysis of Process Lags," by C. E. Mason, Trans. A.S.M.E., vol. 60, 1938, p. 331.

reiterated that the application of a series of controller types to the model process was made in order to demonstrate an analytic procedure found by the authors to be effective, rather than to document a search for a satisfactory controller for the particular model, as implied in the Smith-Fairchild discussion. In practice, any criterion of excellence for control is affected by economic considerations relating to the environing equipment of the controlled system concerned.

As to flow control, the general conclusions of the paper encounter no exceptions in that field. For the most part, however, automatic-flow-control problems are considered to be on a lower plane of difficulty than that dealt with in the paper, an important exception arising in those liquid-flow cases where the mass of moving fluid imparts a momentum seriously restricting the action of the controls.

Reference to the patent literature by Messrs. Smith and Fairchild is acknowledged without comment. The analysis given in the paper was directed to the mathematics of automatic control, and was not intended to refer to the particular instrument designs of any of the various manufacturers. Thus, for example, the proportional-plus-floating controller was merely postulated as a mathematical type in the development.

Mr. Wood's elaboration of the fundamental meaning of the sensitivity constant was acknowledged in a previous statement. This meaning was not directly brought out in the paper since the corresponding portion of the analysis was couched principally in terms of the empirically measurable "throttling band." In an earlier paper[13] by one of the authors, the term "potential temperature" was acknowledged to have been originated by Ivanoff. The expression for equilibrium "loss of control point" is directly evident from Equation [20] of the paper, as

$$\Delta(T_p) = \frac{1}{1 + s}\,\Delta(R_a Q_o)$$

where $\Delta(R_a Q_o)$ represents the load alteration whether originating in the load resistance R_a or in the load supply flow Q_o.

The mathematical method shown by the authors applies equally well to all types of disturbances. Although, for some systems, load changes are more precisely represented by changes in R_a than in Q_o, it might be questioned whether such systems are in the majority. The practical benefits arising from the use of valves having logarithmic characteristics extends far beyond the mere compensation for variation of outlet resistance with load.

Mr. Wood points out the practical significance of the constant c, thereby providing an answer to one of the questions in the Smith-Fairchild discussion.

An answer to Mr. Wood's question on the best floating adjustment may perhaps be provided by the result (indicated by the mathematical analysis of the paper) that the optimum value of the reset constant r, once it is found, is very nearly independent of the value of the throttling band which it might be advisable to employ. A controller equation faithfully describing any particular commercial instrument must be written directly from the detailed mechanics of that instrument and, thus, such an equation may not necessarily take the exact form of any of the mathematically developed types shown in the paper.

The Influence of Crystal Size on the Wear Properties of a High-Lead Bearing Metal

By JOHN R. CONNELLY,[1] BETHLEHEM, PA.

This paper,[2] describes work done to correlate metallurgical aspects of a bearing metal with mechanical testing under conditions existing in a bearing. Previous work by Karelitz and Kenyon (1),[3] and by the author (2) and (3) have concerned effects of different composition of bearing metal and different loading and size of specimen. The present investigation was made to determine the effect on removal of the bearing-metal surface and the pressure at which the oil film breaks down as a result of variations in the crystalline structure of the bearing metal. In this work both the chemical composition of the bearing metal and the size of the specimen were constant. The crystalline structure was varied and the resulting change in properties determined.

The Problem

THE analysis of bearing conditions is usually associated with the hydrodynamic theory of lubrication. This theory has served very well to explain phenomena under conditions where it is applicable, namely, where the trueness of the surfaces, the speed, the load and the fluid are such that the forces which the fluid can develop are sufficient to float the journal and prevent wear. In a number of cases, the hydrodynamic theory is not applicable, at least, as it is understood at present. The fact that most bearings show some wear after service indicates that a fluid film may not be present at all times. Thus, many bearings, which normally are running under conditions quite similar to those assumed in the hydrodynamic theory, may occasionally depart from those conditions. Such departure may be due to loss in viscosity of the lubricant either by reason of high temperature or deterioration of the lubricant.

In addition, very slow speeds or very high loads may cause such departure. Under such extreme conditions, in the presence of a lubricant, the phenomena of dry rubbing surfaces may not appear. Such extreme conditions without dry friction are often referred to as "boundary lubrication" or similar descriptive terms. Some very slow disappearance of the mating surfaces accompanies boundary lubrication. This is called wear. In brief it may be said that the conditions of boundary lubrication are slow removal of one or both mating surfaces in the presence of a lubricant but absence of a lubricant film. It was the purpose of this investigation to determine the effect on removal of the bearing-metal surface or wear and the pressure at which the oil film breaks down as a result of variation in the crystalline structure of the bearing metal.

[1] Associate Professor of Industrial Engineering, Lehigh University. Mem. A.S.M.E.

[2] Abstract from a professional thesis submitted to the University of Illinois; published by permission of the Dean of the Graduate School.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Special Research Committee on Lubrication and presented at the Annual Meeting, Philadelphia, Pa., December 4-8, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Method of Investigation

A specimen of bearing material has one side machined to a plane surface. A constant holding force, hereafter referred to as the load, presses the machined surface tangent to a rotating steel cylinder, submerged in a bath of lubricant. For these tests the force was obtained by gravity. The specimen, which can move only in a direction normal to the machined surface, wears a cylindrical groove and the contact changes from line to a progressively larger area. This wearing away continues until equilibrium conditions are established between the forces causing wear and the forces resisting wear. The load divided by the projected area at any instant is referred to as the pressure and the pressure existing at equilibrium is called the final or ultimate bearing pressure or equilibrium pressure. Rate of wear is designated as volume of bearing material removed per length of travel of a point on the surface of the cylinder.

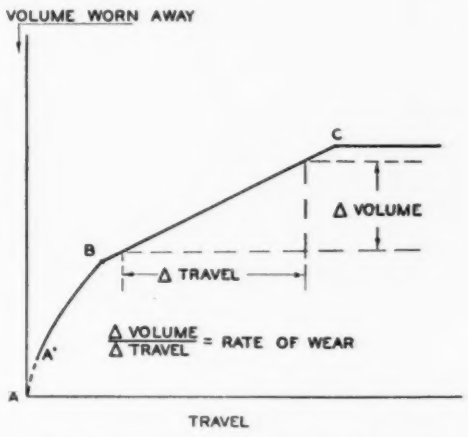


Fig. 1 Determination of Rate of Wear

This wearing away of the bearing material seems to occur in more than one phase, as illustrated in Fig. 1. From A to A' results are in doubt because of the difficulty of measuring the rapid movement. Within a few minutes after the test starts, the rate of wear becomes more stable and proceeds along the path $A'B$. To distinguish $A'B$ from the following phase BC, $A'B$ is referred to as unlubricated wear, although the phrase may not be completely descriptive. Data previously analyzed show rate of wear in phase $A'B$ proportional to the corresponding contact pressure. The phase BC of the rate of wear has a linear relation with travel for all data that have come to the author's attention. This constant rate of wear, for pressures just above the ultimate bearing pressure, fixes the rate of wear in phase BC as independent of the bearing pressure. The phase BC is referred to as lubricated wear.

PREPARING TEST MATERIALS

Bearing-Metal Specimens

The bearing metal selected for this investigation is known as

Cosmos metal and has an approximate composition of lead 76 per cent, antimony 15 per cent, and tin 9 per cent. This composition is in common use in industry. It is often used as a substitute for true babbitt metal because the high lead composition is less expensive. A further reason, for selecting this particular composition for this investigation, is that considerable variation in crystal size can be obtained by varying rates of cooling of the metal just prior to and including solidification.

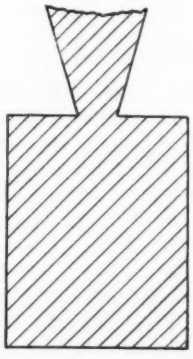


FIG. 2 TEST BAR AND RISER

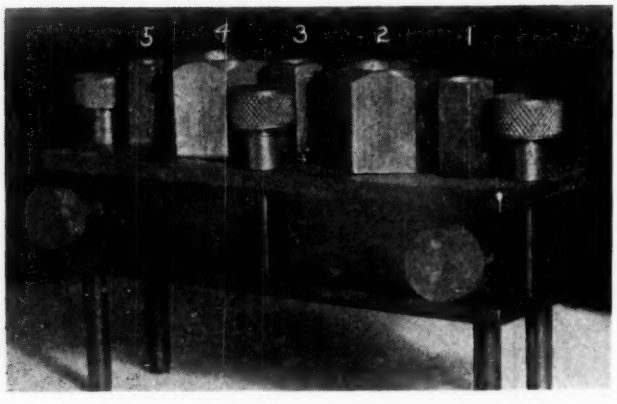


FIG. 3 MOLD FOR TEST BARS AND RISERS

A permanent mold of pieces of steel clamped together was constructed which cast a bar 1¼ in. by 1 in. by 6 in. Fig. 2 illustrates the cross section through a riser. When first made the mold had three risers which are Nos. 1, 3, and 5 in Fig. 3. Specimen bars contained cavities so risers Nos. 2 and 4 were added. Risers Nos. 1, 3, and 5 had a total volume of about 8 per cent of the bar. Risers Nos. 1, 2, 3, 4, and 5 have a column of about 31 per cent of the bar. With these additional risers no cavities appeared in the specimen bars. The risers were constructed as

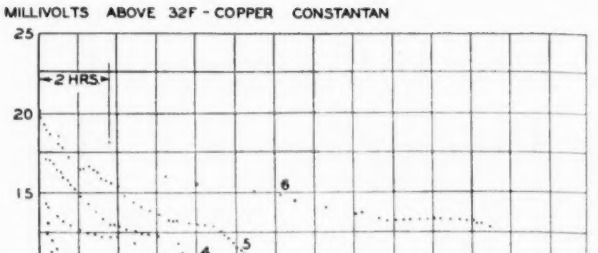


FIG. 5 COOLING OF TEST BARS

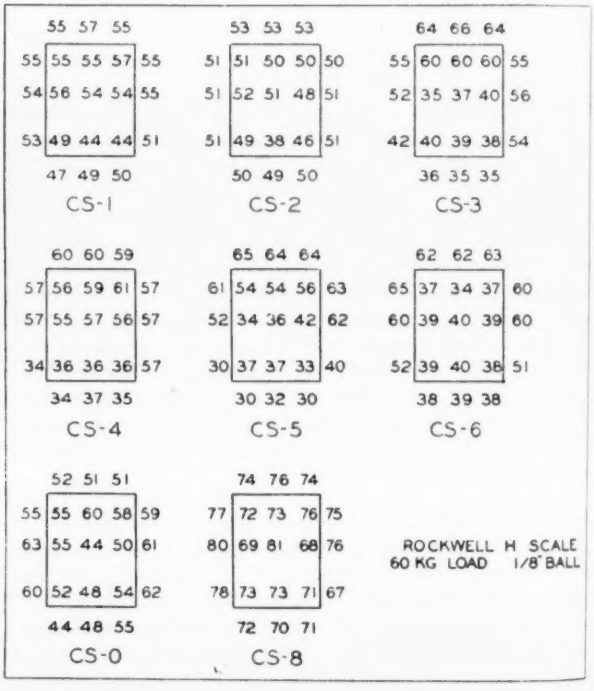


FIG. 6 HARDNESS READINGS; SECTION AND SURFACE OF TEST BARS

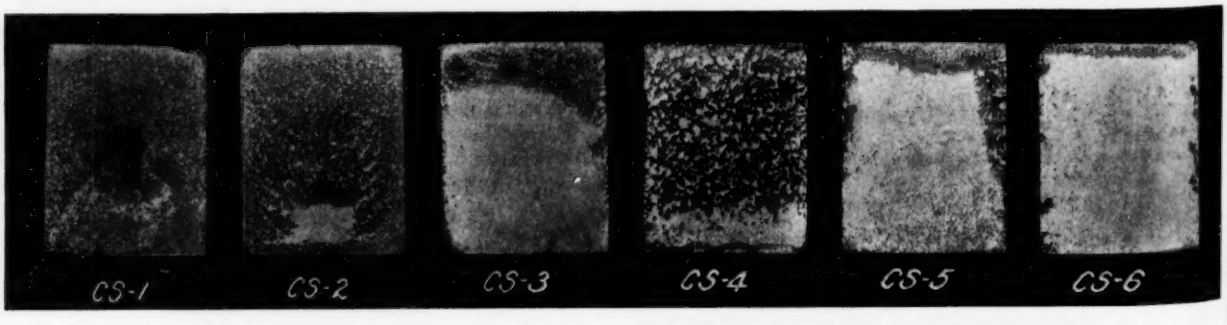


FIG. 4 MACROSTRUCTURE OF TEST BARS FOR INVESTIGATION WITH THEIR DESIGNATIONS

TABLE 1 CRYSTAL DISTRIBUTION IN SPECIMENS

Specimen	Area devoid of crystals
CS-1	4 per cent (estimated)
CS-2	8.4 per cent (by planimeter)
CS-3	71.3 per cent (by planimeter)
CS-4	Unobtainable (by planimeter)
CS-5	78.5 per cent (by planimeter)
CS-6	93.3 per cent (by planimeter)

inverted cones to assist in removing them without damage to the specimen bar. In Fig. 3, riser No. 3 is shown partially unscrewed; the unscrewing breaks the riser at the narrow section.

Six bars, hereafter designated as CS-1, 2, 3, 4, 5, and 6, were poured, Figs. 4 and 5. From these six bars were obtained (*a*) a broken section, (*b*) a polished and etched section to show microstructure, (*c*) a polished and overetched section to show macrostructure, (*d*) a section for hardness survey Fig. 6, and (*e*) a wear-test specimen. Of the above items (*b*), (*c*), and (*e*) were expected to yield the principal data. Items (*a*) and (*d*) are frequently given in metallurgical studies and for completeness were included. Also the value of that property indicated by hardness tests in this field is controversial and further information may be

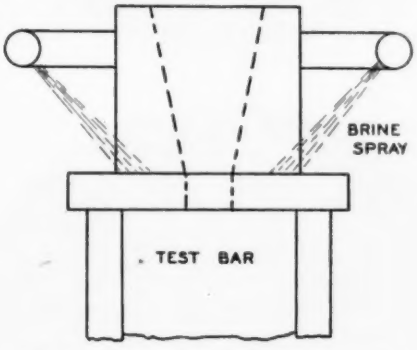


FIG. 7 METHOD OF CHILLING SPECIMEN

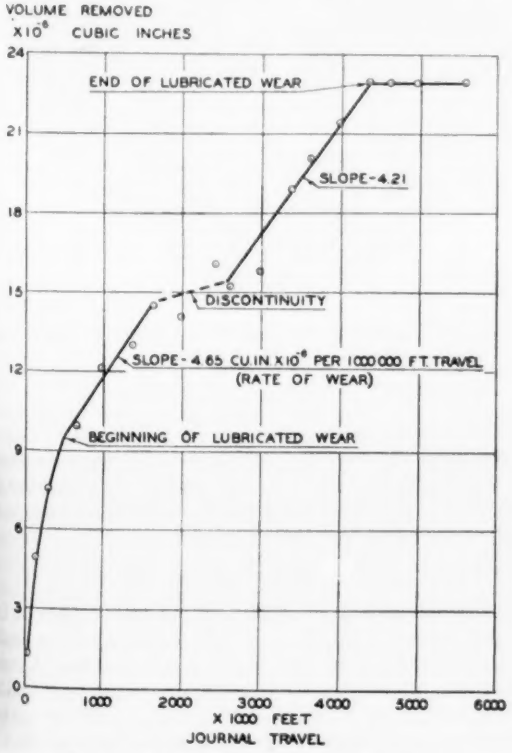


FIG. 8 DETERMINATION OF RATE OF WEAR

of value. The wear specimens were machined to shape by a new milling cutter used exclusively for this work. The $2^1/_2$-in. cutter had a face of 4 in. and 16 teeth. The test surface for each specimen was finished with the cutter rotating at 500 rpm and a linear feed of $^5/_8$ in. per min. Each wear specimen was machined to a length of approximately 0.4 in. Previous investigations indicate this length to be sufficiently beyond the range where end leakage for this type of test is a factor. Tests using the varying-wear method as described were run on these specimens.

Examination of the macrostructure of specimens CS-1, 2, 3, 4, 5, and 6, Fig. 4, shows that except for CS-4 there is a core practically devoid of crystals. The extent of crystalline structure is indicated in Table 1. This core increases in size as the rate of cooling decreases, as indicated in Table 1. The tin-antimony crystals solidify before the lead so that, when the solidification of lead is delayed, the crystals float toward the top of the bar. According to metallurgical explanation this excess of tin and antimony at the top of the bar will cause some of the antimony to crystallize as pure antimony. Antimony being a relatively hard substance, this will give a greater hardness at the surface, as

TABLE 2 RESULTS OF RATE-OF-WEAR INVESTIGATION

Bar and test	Equilibrium press., lb per sq in.	Rate of wear, in.[3] × 10^{-9}	Avg dimension, edge of crystal, in.	Press. at start of straight-line portion of wear, lb per sq in.	Length of specimen, in.
CS-1-2t	397	4.20(5) 4.65(4)	0.0025	535	0.414
CS-1-3t	600	9.70(3)	0.0025	...	0.414
CS-1-4t	278	22.40(7)	0.0025	...	0.414
	AM 425	AM 11.80			
	GM 405	GM 9.07			
CS-2-1t	835	0.65(4)	0.0015	905	0.401
CS-2-2t	662	1.10(2)	0.0015	680	0.401
CS-2-3t	610	16.80(4)	0.0015	710	0.401[a]
CS-2-4b	639	1.92(5) 2.04(4)	0.0015	...	0.401
CS-2-3b	681	2.34(3) 1.70(5)			
	AM 704	AM 1.63			
	GM 700	GM 1.49			
CS-3-1t	540	2.98(4)	0.0040	610	0.393
CS-3-2t	440	5.20(4)	0.0040	460	0.393
CS-3-3t	510	6.80(6)	0.0040	585	0.393
CS-3-4t	386	10.00(4) 8.60(7)	0.0040	535	0.393
	AM 469	AM 6.95			
	GM 465	GM 6.48			
CS-4-1t	340	13.50(4) 12.10(4)	0.0030 0.0030	495	0.413
CS-4-2t	304	9.20(3) 9.60(3)		335	0.413
CS-4-3t	292	13.75(4) 17.80(7)	0.0030	370	0.413
	AM 312	AM 13.5			
	GM 311	GM 13.2			
CS-5-1t	433	2.320(5)	0.0075	510	0.413
CS-5-2t	568	0.425(4)	0.0075	600	0.413
CS-5-3t	540	1.45(6)	0.0075	780	0.413
CS-5-1b	676	1.95(5)	0.0075		
CS-5-3b	578	1.50(3) 1.50(4)			
	AM 559	AM 1.57			
	GM 553	GM 1.41			
CS-6-1t	453	6.90(3)	0.0100		0.414
CS-6-2t	523	3.50(5)	0.0100	650	0.414
CS-6-3t	387	3.50(6)	0.0100	470	0.414
	AM 454	AM 4.22			
	GM 451	GM 4.05			
CS-8-1t	604	4.00(6)	0.0060	730	0.397[b]
CS-8-2t	630	2.76(7)	0.0060		0.397[b]
	AM 617	AM 3.33			
	GM 617	GM 3.28			
CS-0-1t	638	1.64(5)	0.0010	725	0.402
CS-0-2t	603	1.13(6)	0.0010	880	0.402
CS-0-4t	980	2.48(2)	0.0010		0.402
CS-0-5t	840	2.76(4)	0.0010		0.402
	AM 765	AM 1.82			
	GM 750	GM 1.71			

NOTE: Numbers in parentheses in this table denote number of plotted points which slope value represents.

[a] Neglected on basis of photomicrograph.
[b] Different composition.
t indicates top. AM signifies arithmetic mean.
b indicates bottom. GM signifies geometric mean.

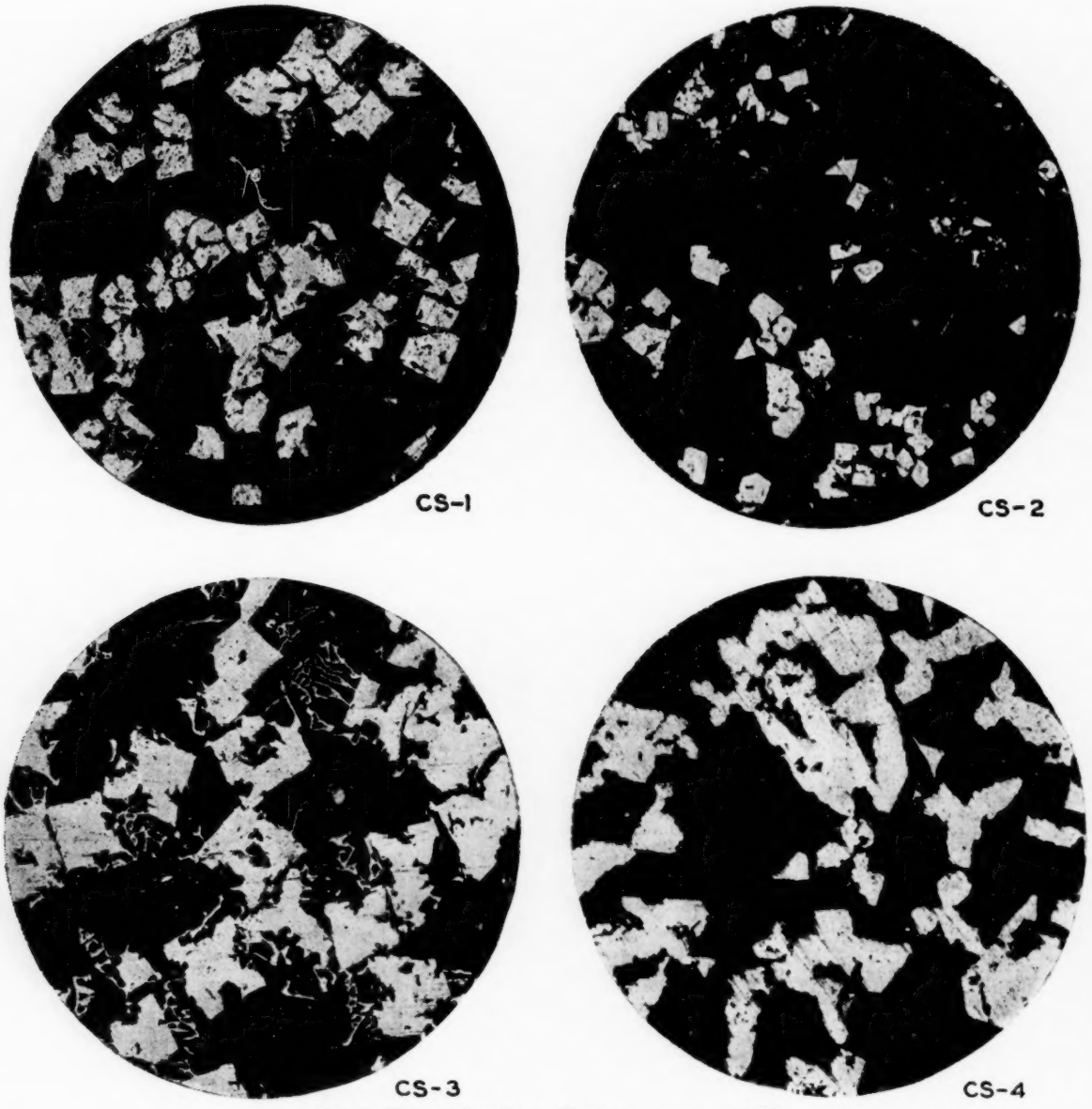


FIG. 9 TOP OF TEST BARS, MICROSTRUCTURE, ×100

shown by the hardness survey, Fig. 6. Such hardness might affect the test results materially and obliterate the effect of crystal size. To investigate this possibility bar CS-7 was poured having reduced percentages of tin and antimony and was allowed to cool overnight in the furnace. The intention was that tin and antimony in floating to the top would give about a normal concentration and composition in that region. Bar CS-7 was so soft that it was damaged in attempting to obtain a broken section. For this reason it was discarded. A new bar CS-8 was prepared in the same manner as CS-7 and packed in dry ice for several hours. A broken section was obtained without difficulty.

It is questionable whether such a broken section is indicative of structure. It was decided to cast additional bars and attempt to secure crystals smaller than previously obtained. CS-0 was poured of the same composition as CS-1, 2, 3, 4, 5, and 6 and chilled by spraying cold brine at 15 F onto the top of the mold, as shown in Fig. 7. The bars CS-8 and 0 were given the same series of tests as were CS-1, 2, 3, 4, 5, and 6. Etching of specimens was by ferric-chloride solution or a 3 per cent solution of nitric acid in ethyl alcohol, Fig. 9.

TEST JOURNAL

The test journal was a piece of $^1/_4$-in-diam drill rod. No treatment was given the rod but it had been in service some months before starting these tests using the same bearing-metal-and-oil combination. This use had polished the drill rod but the change, if any, in diameter was less than 0.0001 in.

LUBRICANT

The lubricant used was D.T.E. light turbine oil secured from the Socony-Vacuum Oil Company. As received it has a Saybolt viscosity of about 169 sec at 100 F. After some use the value rises slightly. This oil had been in use for several years and had a Saybolt viscosity of 173 sec. During the tests the oil was kept at an average temperature of 77 F and fluctuated between 75 F and 80 F.

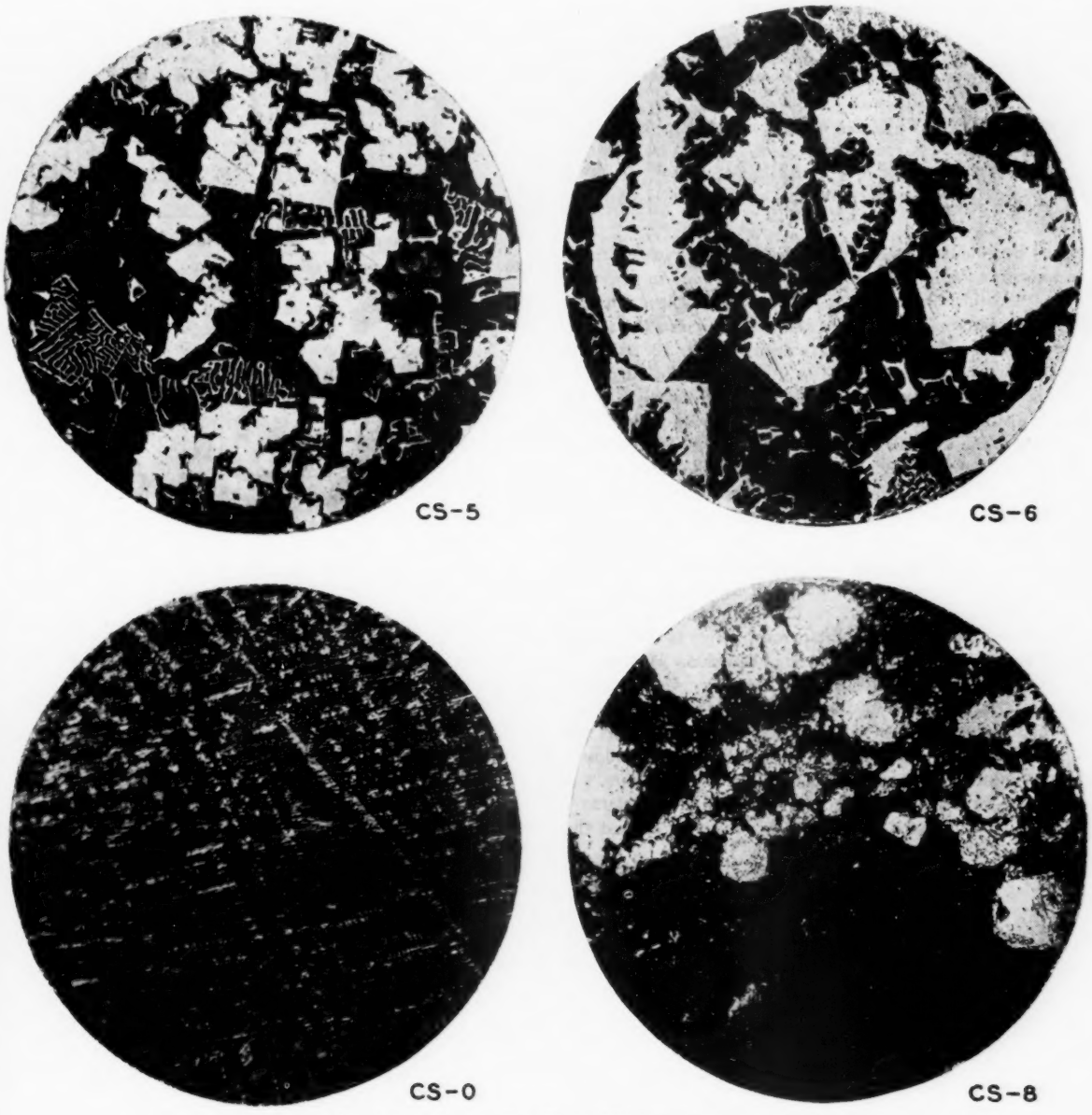


FIG. 9 (*continued*) TOP OF TEST BARS, MICROSTRUCTURE, ×100

SUMMARY OF TEST DATA

As previously described, the rate of wear is obtained by measuring the slope of the volume curve immediately preceding the attainment of equilibrium conditions. The unit of slope is arbitrarily taken as cubic inches of volume removed per million feet of travel of a point on the surface of the journal. Fig. 8 illustrates the method of procedure. Since this bearing metal is heterogeneous, regions of variable properties occur. Discontinuities in the volume removal data evidence that the rotating journal passed through such regions. These discontinuities do not occur in every case and seem to have no regularity when present. The test results are given in Table 2. In each case, the number in parentheses, following tabular values of rate of wear, denotes the number of plotted points which the slope value represents. The larger the number of points the more reliable the rate of wear.

The dimensions of the crystals given in the table were obtained with the aid of professional metallurgists. These values represent the inspection of more areas than would be feasible to present in a report.

A further item included in Table 2, showing results, is the pressure at which the change-over from unlubricated to lubricated wear takes place.

DISCUSSION OF DATA AND RESULTS

GRAPHICAL REPRESENTATION

Figs. 10, 11, 12, and 14 show results taken directly from the tabular values. Figs. 10 and 11 show equilibrium pressure and rate of wear versus the linear dimension of the tin-antimony cubical crystals. Fig. 12 presents rate of wear and equilibrium pressure against each other. Fig. 14 shows the relation between the pressures obtaining at the beginning and end of lubricated wear.

Fig. 13 shows the mass centers of results in Fig. 12. The x and y coordinates of these mass centers, referred to as $\bar{x}$ and $\bar{y}$ are

TABLE 3 COMPARISON OF RESULTS

Test bar	Crystal size: Edge, in.	Crystal size: Area, in.^{2} × 10^{-3}	Equilibrium, lb per sq in.: Mass moment	Equilibrium, lb per sq in.: Arith. mean	Equilibrium, lb per sq in.: Geom. mean	Rate of wear, in.^{3} × 10^{-6} per 1,000,000 ft: Mass moment	Rate of wear, in.^{3} × 10^{-6} per 1,000,000 ft: Arith. mean	Rate of wear, in.^{3} × 10^{-6} per 1,000,000 ft: Geom. mean
CS-0	0.0010	0.00100	667	765	750	1.66	1.82	1.71
CS-1	0.0025	0.00625	367	425	405	11.70	11.80	9.07
CS-2	0.0015	0.00225	682	704	700	1.78	1.63	1.49
CS-3	0.0040	0.01600	428	469	465	7.71	6.95	6.48
CS-4	0.0030	0.00900	310	312	311	13.70	13.50	13.20
CS-5	0.0075	0.05625	559	559	553	1.58	1.57	1.41
CS-6	0.0100	0.10000	446	454	451	3.92	4.22	4.05
CS-8	0.0060	0.03600	618	617	617	3.31	3.33	3.28

determined by calculations analogous to those used in determining the location of the centroid of a group of rivets in a riveted joint. In place of the area of a given rivet in the analogous method, the number of slope points minus 2 is used as the multiplier for the coordinates of points shown in Fig. 12. By the number of slope points is meant the number of plotted points which the slope value represents, as described previously. Since the slope is a straight line, it is determined by two points. Subtracting 2 from the number of points which the slope represents gives the number of agreeing values in excess of those required to determine the slope line. The quantity of excess agreeing values, obtained in this way, is a conservative measure of the reliability (4) of the slope line.

Table 3 is a summation of the results showing relation between crystal size and averages of rate of wear and equilibrium pressure. The arithmetic mean, the geometric mean, and the mass centers from Fig. 13 are included. Arithmetic mean refers to the method of dividing the sum of data by the number of values. Geometric mean or logarithmic mean is obtained by taking the antilogarithm of the arithmetic mean of the logarithms of the data values. This method is employed in computations where averages are desired for data in which the nature is such that there is a lower limit to the values but no upper limit. The geometric mean is considered to give a more representative average of such data.

Figs. 15 and 16 show the averages of rate of wear against the

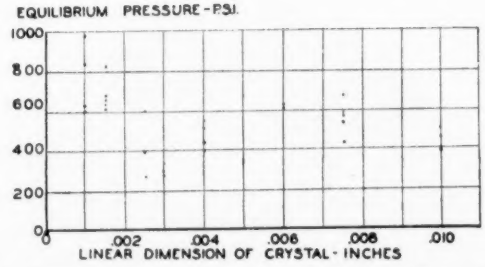


FIG. 10 EQUILIBRIUM PRESSURE VERSUS CRYSTAL SIZE; SCATTER DIAGRAM

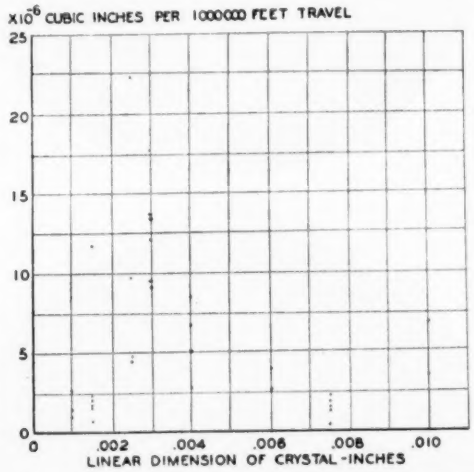


FIG. 11 RATE OF WEAR VERSUS CRYSTAL SIZE; SCATTER DIAGRAM

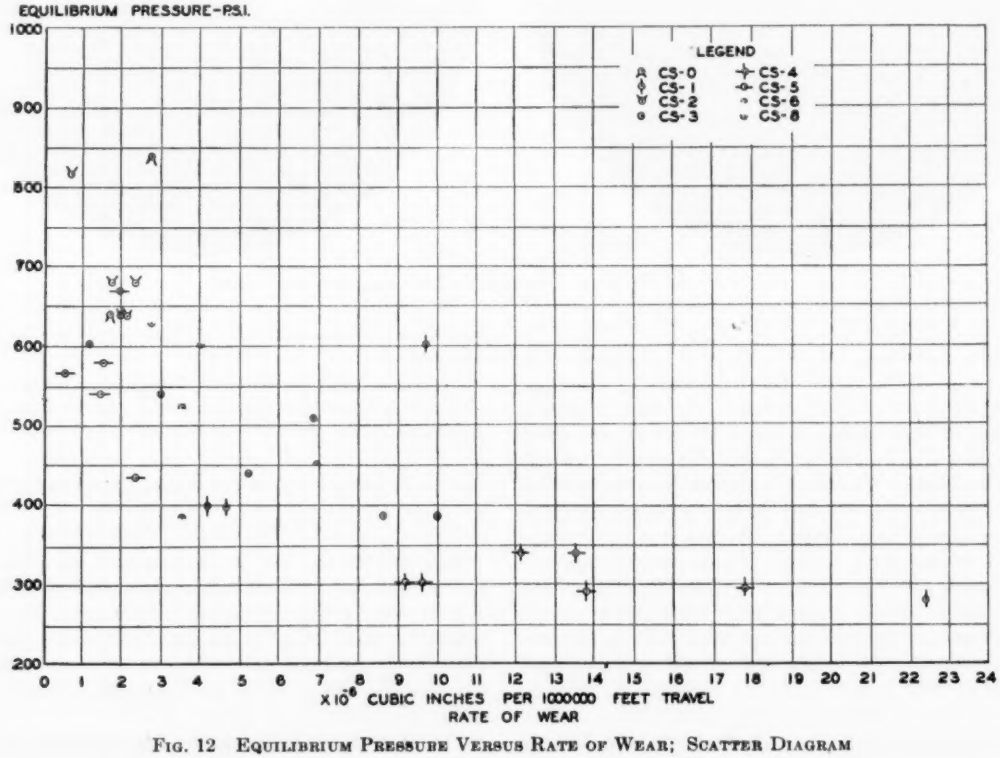


FIG. 12 EQUILIBRIUM PRESSURE VERSUS RATE OF WEAR; SCATTER DIAGRAM

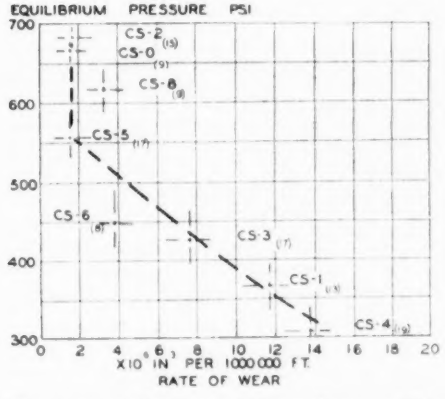


FIG. 13 EQUILIBRIUM PRESSURE VERSUS RATE OF WEAR; MASS CENTERS

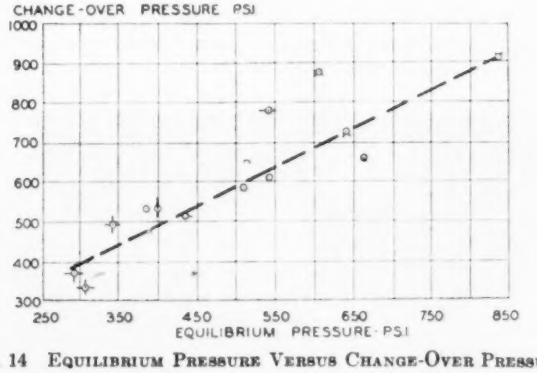


FIG. 14 EQUILIBRIUM PRESSURE VERSUS CHANGE-OVER PRESSURE

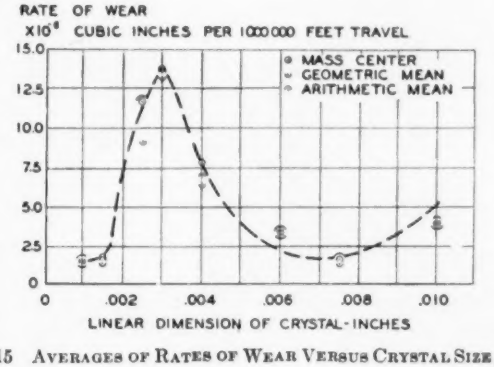


FIG. 15 AVERAGES OF RATES OF WEAR VERSUS CRYSTAL SIZE; EDGE

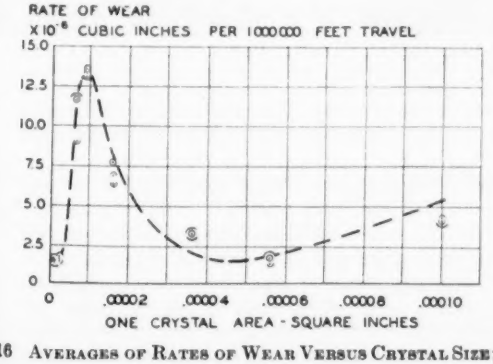


FIG. 16 AVERAGES OF RATES OF WEAR VERSUS CRYSTAL SIZE; AREA

linear dimension of the crystals and against the exposed area of one crystal that is the average crystal area. In passing a curve to represent the data, the mass centers have been favored, since the method of their determination takes account of the presumed relation between equilibrium pressure and rate of wear. The departures of samples CS-6 and CS-8 from the general trends indicated in Fig. 14 are also considered in passing the curve.

Figs. 17 and 18 represent the averages of equilibrium pressure against linear dimension of the crystals and against the average

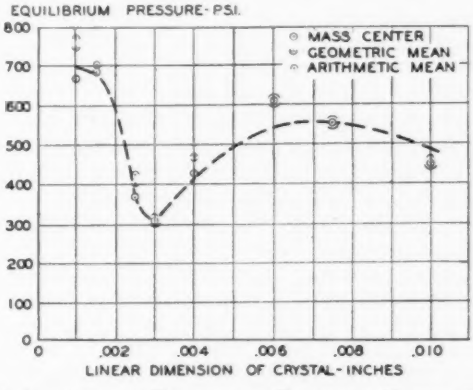


FIG. 17 AVERAGES OF EQUILIBRIUM PRESSURES VERSUS CRYSTAL SIZE; EDGE

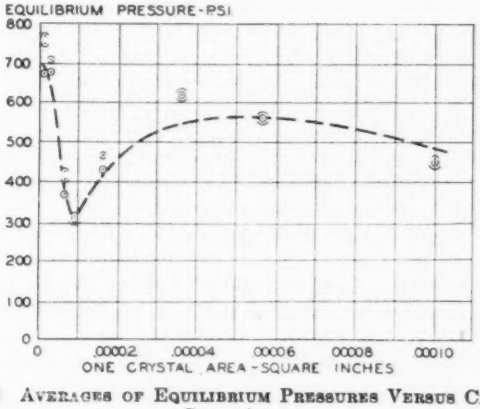


FIG. 18 AVERAGES OF EQUILIBRIUM PRESSURES VERSUS CRYSTAL SIZE; AREA

exposed crystal area. The passing of a curve to represent the data was done in the same manner as that described for Figs. 15 and 16.

COMMENTS

Whatever property of the bearing metal makes for a low rate of wear makes for a high equilibrium pressure and vice versa. On the basis of the mass centers, Fig. 13, it would seem that, below a certain crystal size, rate of wear changes little and the equilibrium pressure increases rapidly.

Fig. 12 shows that each pair of values of rate of wear and its corresponding equilibrium pressure are relatively consistent. The results are believed to give an accurate picture of the material tested, the spread of values for a given test bar being due to the heterogeneous bearing metal.

It is evident that, regardless of the averaging method employed, the general shape of the curves in Figs. 15, 16, 17, and 18 will vary little.

COMPARISON WITH OTHER INVESTIGATIONS

In a paper (1) published in 1937, Karelitz and Kenyon reported

some work by this method of testing on several compositions of bearing metal. In subsequent discussion (1) of this paper some values of rate of wear were worked out which are somewhat comparable to the data presented in the present paper. Karelitz and Kenyon report that according to their photomicrographs the specimens were overchilled. This evidence would place their data in the range of CS-0, although the various hardness values do not agree. For the lead-base specimen of Karelitz and Kenyon, the rate of wear in the same units as used in this paper is 0.92×10^{-6} cu in. per 1,000,000 ft of travel. The equilibrium pressure is approximately 700 lb per sq in. These check very well with the data here reported.

Conclusions

The results are evidence that, for the bearing metal tested, (*a*) crystalline structure is a factor and crystal size an important variable in thin-film lubrication, (*b*) rate of wear and equilibrium pressure are related, and (*c*) pressures at beginning and end of lubricated wear are related. The method of investigation used is a way of determining quantitative values of rate of wear, equilibrium pressure, and the pressure at the borderline condition between unlubricated wear and lubricated wear. Other things being equal a very small crystal size gives optimum values of rate of wear and equilibrium pressure.

In corroboration of the results obtained, it may be pointed out that the motorcar industry at present produces bearing inserts by a technique which gives very small crystals having a linear dimension of the order of 0.001 in. Since this is done deliberately we may infer that gross testing of over-all bearings has indicated the lower rates of wear obtained with small crystals.

Bearings that are poured in place, such as large pillow-block bearings, have crystal sizes in the range where the rate of wear and equilibrium pressure are decidedly not optimum. In the future, such large bearings could be better designed to make use of some type of insert that could be cooled rapidly upon pouring.

Of possibly greater importance than the specific results on the effect of crystal size are certain implications in connection with fundamental analysis of a journal-lubricant-bearing combination. The present concepts of thin-film lubrication take no account either of the material or crystalline structure of the mating surfaces.

In the light of this investigation, it would be appropriate to revise the phrase "hydrodynamic theory of lubrication," to be stated as the "theory of hydrodynamic lubrication." This would indicate correctly that the theory refers to a certain part of the field of lubrication and not to the entire field.

A theoretical explanation of the mechanism of wear should wait on further research in the field of thin-film lubrication. The explanation should be developed independent of the data in the range of thick-film lubrication.

Acknowledgments

The author wishes to express thanks to A. D. Shankland, engineer of tests, and M. W. Dalrymple, metallurgical supervisor, of the Bethlehem Steel Company for advice and help and for photomicrography, photomacrographs, and hardness survey of crystal specimens CS-1, 2, 3, 4, 5, and 6; Professor Allison Butts of Lehigh University for advice in preparation of a crystal specimen of reduced tin-antimony content; J. H. Vail, James Ward Packard Fellow in Mechanical Engineering 1936–1938, for a major part of the data from the varying-wear machine; R. L. Scott, James Ward Packard Fellow in Mechanical Engineering 1938–1940 for the remainder of the data on the varying-wear machine; Professor F. V. Larkin, head of the department of mechanical engineering at Lehigh University, for securing funds to carry on the investigation.

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Discussion

B. F. Hunter.[4] Recently our laboratories had occasion to examine a high-lead bearing that had failed in a steam turbine for the purpose of determining the cause of failure of the bearing metal. The bearing was made in two parts each half being composed of a steel shell into which the babbitt metal was cast. In the main portion of the bearing the lining was about $^1/_4$ in. thick, at the edges about $^1/_2$ in. thick.

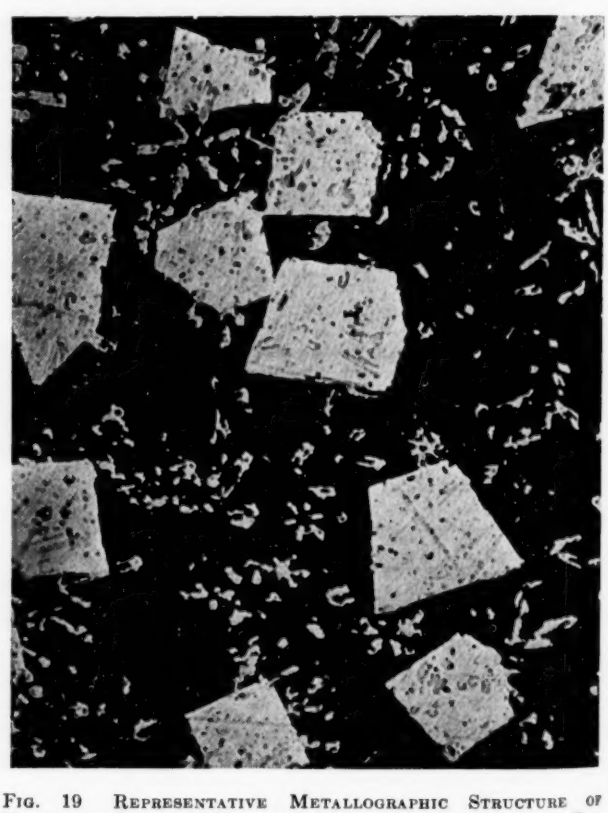


Fig. 19 Representative Metallographic Structure of Defective Bearing Metal, Showing Large Cubes of Tin-Antimony, and Short Thick Needles of Copper-Tin in Combination, ×100 (Etched With Ferric Chloride)

The upper half of the bearing showed only a few slight scores but the lower half was badly scored, in addition to being cracked and pitted. Numerous small particles had fallen out and others were loose and could easily be picked out of the lining.

Chemical analysis was not made since the manufacturer of the bearing material supplied the following specifications:

[4] Chief Lubrication Engineer, Gulf Oil Corporation, Pittsburgh, Pa.

Tin, per cent	84 to 86
Lead, per cent	0.035 max
Antimony, per cent	9.5 to 10.5
Copper, per cent	4.5 to 5.5

Sections from both halves of the bearing were given metallographic examinations. A representative structure is shown in Fig. 19 of this discussion. It contained extremely large cubes of Sn-Sb, and rather short thick needles of Cu-Sn in the alpha-solid solution of Sn and Cu and pseudoeutectic. At the magnification used, this latter constituent could not be resolved, but there was a considerable portion present.

The conditions attending the pouring and cooling of this bearing were not obtained and, since no analysis was made, it was assumed that the composition was correct.

That the pouring temperature was correct is revealed by the relatively short thick needles, some of which are arranged in a starlike shape. Also, the pouring temperature, except in so far as it affects the rate of cooling, is insignificant in its action toward the formation of such large cubes of Sn-Sb.

The size of these cubes, the area of each of which, as shown in Fig. 19, is about 10 times normal and their uneven distribution throughout the matrix are the only conditions present which can conceivably be the cause of the failure. The cause of these very large cubes can be attributed to the bearing alloy being too slowly cooled. The reason for the failure of the lower half was probably due to the much higher loads it had to carry; the absence of failure in the upper half was because of the comparatively light loads to which it was subjected.

The unsatisfactory service secured from this bearing appears to have been due to the presence of extremely large, poorly distributed crystals of tin-antimony compound in the bearing material, resulting from too slow a cooling rate after pouring.

This is one of many similar bearings we have examined and our findings would seem to substantiate very definitely the author's data.

F. C. Linn.[5] The author has presented an enlightening paper upon the crystalline structure and its effect upon wear and load-carrying capacity of the bearing when the operation is within the thin-film region.

Under "Conclusions" he states that "(*a*) crystalline structure is a factor and crystal size an important variable in thin-film lubrication." In reviewing Figs. 15 to 18, inclusive, it would appear that the linear dimension of the crystal has a very critical range, at which the rate of wear is large and the equilibrium pressure low. Outside of this range it is noted that the rate of wear is relatively lov and the equilibrium pressure high. This indicates that the size of the crystal should be held within reasonable limits during the casting cycle.

It is not quite understood how the author arrived at the conclusion (*c*) in which he states that "pressures at beginning and end of the lubricated wear are related." A further discussion of this point would be appreciated.

The writer requests that information be supplied on the following:

1 What effect has hardness upon wear?

2 What effect has hardness on rate of wear and equilibrium pressure?

3 Referring to Fig. 6, on what surfaces were the wear tests made?

R. G. Simard.[6] The scope of this investigation on the relation of bearing-metal wear to structural characteristics is especially commendable and, in this writer's opinion, constitutes a fruitful approach to the problem of wear phenomena. The directness of the method employed appears to have definite worth in fundamental investigations of this nature. Although there is no pretense that the work is in the category of simulated testing, the time is not considered inopportune again to remind ourselves that wear resistance is a consequence of service conditions rather than an inherent property of matter. Application to practice of the results of this or similar investigations should be attended only with extreme caution.

An investigation of the wear quality of lubricating oils in relation to representative bearing metals along lines suggested by the work of Karelitz and Kenyon at Columbia has been under way for the last 3 years in the laboratories of the research division of the Atlantic Refining Company. A battery of 6 identical machines, carefully constructed to eliminate running inaccuracies, was employed for the purpose. The standard metal specimens were prepared under rigid specification of composition and casting procedure. Results soon indicated, however, that reliance could not be attached to micrometer indication of rate of wear, since the depth of the wear groove was of such magnitude that its measurement at the end of a 5:1 lever was seriously disturbed by temperature fluctuations and slight inaccuracies of the shaft. Measurements have accordingly been confined to comparative total wear after an operating period sufficient to insure attainment of equilibrium.

The following data have been singled from our files as illustrative of that which predicated the change in procedure and these comments:

Metal	S.A.E. No. 62 bronze (hard cast)
Oil	S.A.E. No. 30 acetone-benzol dewaxed nitrobenzene refined motor oil (without) film-strength additive) at 130 ± 5 F
Shaft	⅛-in. ground tap and reamer stock
Rpm	1760
Load	60 lb (nominal)

Unit	Time (days)	Measured groove width, in.	Indicated groove depth, in.	Calculated groove depth, in.
1	5	0.0139	0.0003	3.75×10^{-3}
2	7	0.0173	0.0002	5.63×10^{-3}
3	9	0.0149	0.0005	4.37×10^{-3}
4	11	0.0166	0.0004	5.63×10^{-3}
5	13	0.0154	0.0005	4.37×10^{-3}
6	13	0.0143	0.0002	3.75×10^{-3}

Author's Closure

In discussing the photomicrograph of the failed bearing, Mr. Hunter has opened up a field about which comparatively little is known. Since preparing the original paper, the author has investigated several additional methods of etching and is at present of the opinion that no one method gives a complete picture of the metallographic structure for the high-lead metal described in the paper. The electrolytic etch described by Weaver[7] has been quite instructive. Whether the same applies to the high tin metal cannot be stated, but it is entirely possible that some further critical conditions were not revealed by the etching agent employed.

Conclusion (*c*), "pressures at beginning and end of the lubricated wear are related," was made after study of the data shown in Fig. 14 of the paper, where the ordinate is the pressure at the beginning of lubricated wear (see Fig. 8) and the abscissa is the pressure at the end of lubricated wear. Mr. Linn raises the question of the effect of hardness on wear. Referring to Fig. 6, it is seen that CS-0, the specimen having the lowest rate of wear

[5] Turbine Engineering Department, General Electric Company, West Lynn, Mass. Mem. A.S.M.E.

[6] Research and Development Department, The Atlantic Refining Company, Philadelphia, Pa.

[7] "Type Metal Alloys," by F. E. Weaver, *Journal of Institute of Metals*, vol. 56, 1935, pp. 209–233.

and highest equilibrium pressure, has a low but not the lowest hardness value, while CS-8 (which can best be understood by adding 100 to all other hardness numbers), which has the lowest hardness number, is intermediate as to rate of wear and equilibrium pressure. Further examples could be given but no relation seems evident by considering Rockwell numbers. However, present methods of taking hardness may be inadequate to indicate the hardness of certain critical components of the bearing metal.

The experience of Mr. Simard concerning the measurement of groove depth does not seem directly applicable to the present paper. Karelitz and Kenyon reproduced the method reported by the author in his first paper (3) on the subject. However, at that time it was pointed out that the Ames dial had been discarded because of its many failings in favor of an electrically indicated micrometer screw. The use of a substantially different bearing metal and a resulting groove width of quite a different order of size would not seem conducive to comparable results. The author has investigated the effect of temperature fluctuations and found that, while they are of measurable magnitude, the fluctuations resulting from the controlled oil temperatures are insufficient to vitiate the results. Further, on the basis of work done, since the original paper was prepared, the author is extremely cautious about accepting the phrase "specimens were prepared under rigid specifications of composition and casting procedure." This is no reflection on those who prepared the specimens, but is simply a warning that, present knowledge of nonferrous compounds of three components and higher being in such a state, we cannot be absolutely sure that a set of specimens is exactly as intended.

It seems that metallographic work will have to be done in parallel with work on wear of bearing materials and, as this is done, we must, as Mr. Simard suggested, be most careful of interpreting results. If others would care to work in the high-lead bearing metals, the author would gladly share his troubles.

Calculation of the Elastic Curve of a Helical Compression Spring

By H. C. KEYSOR,[1] CHICAGO, ILL.

Equations for the elastic curve of a helical compression spring are developed, without making the usual assumption of axial loading. The load is found to be eccentric, in general, being axial only under certain conditions. Elastic curves are given for both axial and eccentric loading. These curves are sinusoidal, showing that deflection along the bar is not linear, as is customarily assumed. A reversed deflection is found to occur in the closed end, back of the tip contact point, which increases the deflection of the spring as a whole, and shows that it is not correct to consider the entire closed end as inactive. An axial load is theoretically possible under certain conditions, but practically axial loading is difficult to obtain and cannot be counted on with any certainty. The effect of load eccentricity on stress is considered, and an approximate formula for estimating this effect is given. Reference is made to laboratory tests by another investigator[2] which are in good agreement with the theory here presented.

AN ANALYSIS of helical-spring deflection has been made by the author which yields three results:

1 The load is, in general, eccentric with consequent increase in stress.

2 The correct deduction for inactive end turns is shown to be approximately 1.2, based on the usual practice of taking the total turns as solid height divided by bar diameter.

3 The deflection at any point along the bar is obtained, thus permitting the elastic curve to be drawn.

The first two items have been discussed in a previous paper[3] by the author. The third item forms the subject of the present paper.

Fig. 1 shows the lower portion of a helical compression spring with ends closed and squared. Assume that the spring is compressed between parallel plane surfaces, as this is the usual condition of loading. Let P be the load. Any point on the bar, as point a', must therefore sustain the direct load P, a torque T, and a vertical bending moment M, the line of action of P being at some point Q. It is apparent that Q must be on a radial line through a, the longitudinal center point of the bar, for otherwise the forces and reactions on the top and bottom halves of the spring would not be symmetrical and the necessary condition of static equilibrium would not obtain. We do not restrict the position of point Q on the radial line through a; that is, we consider the load to be eccentric by some variable amount e, leaving the magnitude of e to be determined by the conditions of the problem.

The load P can be replaced by two component loads, P_1 at a and P_2 diametrically opposite a. If $P_1 + P_2 = P$ and the center of pressure of P_1 and P_2 is at point Q the torque, moment, and direct load at the general point a' will not be affected. The reactions which balance P are P_3 at the tip contact point b, and $(P - P_3)$, distributed in some manner in the vicinity of point O where the bar makes contact with the loading plane. There is also a torque reaction at point O. The angular distance v between b and O is roughly 180 deg for most springs; the determination of v will be explained later.

The location of the contact point b is subject to variation due to the manner of forming the tapered end, and due also to the load, for it is evident that, with increasing load, b will move away from the tip so that the angular distance between contact points will diminish. Consideration must be given to this fact in interpreting the following analytical results in their relation to actual springs.

From the foregoing discussion, it is evident that we require the deflection equations for a helix fixed at the origin and sustaining

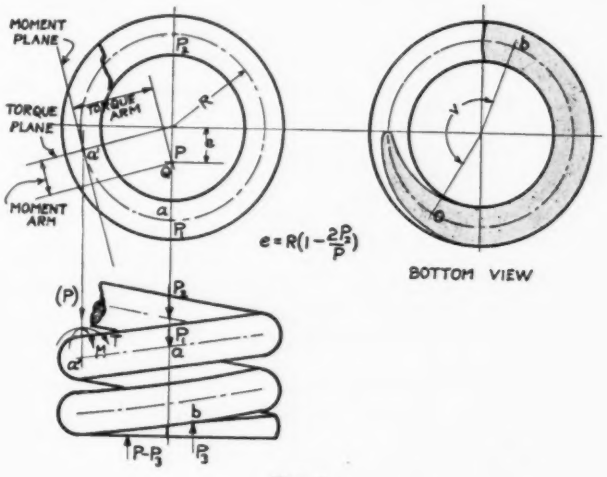


FIG. 1

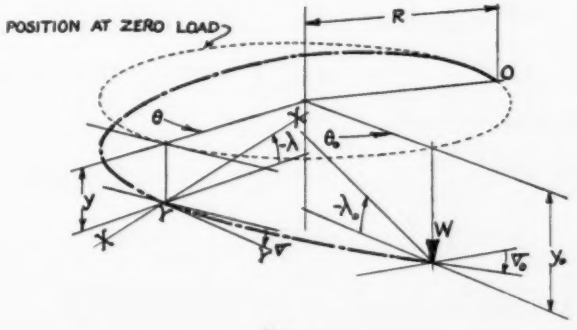


FIG. 2

[1] Mechanical Engineer, American Steel Foundries.

[2] "Helix Warping in Helical Compression Springs," by D. H. Pletta and F. J. Maher. Published in this issue, page 327.

[3] "Analysis of Deflection and Stress in Helical Compression Spring," by H. C. Keysor, presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, December, 1934; not published.

Contributed by the Special Research Committee on Mechanical Springs and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

a vertical load at any point along its length. The effects of bar curvature, direct shear, and angularity of the bar with the horizontal, are neglected, as these factors do not appreciably effect the results in the case of close-coiled springs.

Let O be the origin in Fig. 2 and W the load applied at the angle θ_0. At any point θ the deflection is y and the slopes in the circumferential plane and the radial plane are σ and λ, respectively. These three quantities, expressed as functions of θ_0 and θ, will completely define the position of the bar under load. They are obtained by the strain-energy method. It is shown in texts on elasticity that

$$y = \int \frac{Mm}{EI}\,dl + \int \frac{Tt}{GH}\,dl$$

where

M = bending moment, T = torque } due to applied load
m = bending moment, t = torque } due to unit load applied at the point whose deflection is y
EI = flexural rigidity
GH = torsional rigidity

Referring to Fig. 3, let θ be the coordinate of the point whose deflection y is required, and ψ be the coordinate of any point

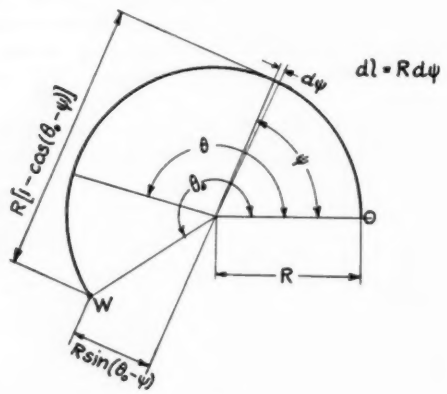


FIG. 3

between the origin O and θ. The load W is applied at θ_0, perpendicular to the plane of the figure.

At the point whose coordinate is ψ we have

$$M = WR\sin(\theta_0 - \psi)$$
$$T = WR[1 - \cos(\theta_0 - \psi)]$$
$$m = R\sin(\theta - \psi)$$
$$t = R[1 - \cos(\theta - \psi)]$$

$$y = WR^3\left\{\frac{1}{GH}\int_0^{\theta}[1 - \cos(\theta_0 - \psi)][1 - \cos(\theta - \psi)]\,d\psi + \frac{1}{EI}\int_0^{\theta}\sin(\theta_0 - \psi)\sin(\theta - \psi)\,d\psi\right\}$$

Reducing, we obtain

$$y = WR^3\left\{\frac{1}{GH}\left[\theta + \frac{1}{2}\theta\cos(\theta_0 - \theta) + \sin(\theta_0 - \theta) - \sin\theta_0 - \sin\theta + \frac{1}{2}\cos\theta_0\sin\theta\right] + \frac{1}{EI}\left[\frac{1}{2}\theta\cos(\theta_0 - \theta) - \frac{1}{2}\cos\theta_0\sin\theta\right]\right\} \quad [1]$$

The circumferential slope σ is found by differentiating Equation [1] with respect to $Rd\theta$ since

$$\sigma = \frac{dy}{dl} = \frac{1}{R}\frac{dy}{d\theta}$$

$$\sigma = WR^2\left\{\frac{1}{GH}\left[1 + \frac{1}{2}\theta\sin(\theta_0 - \theta) - \frac{1}{2}\sin\theta_0\sin\theta - \cos\theta\right] + \frac{1}{EI}\left[\frac{1}{2}\theta\sin(\theta_0 - \theta) + \frac{1}{2}\sin\theta_0\sin\theta\right]\right\} \quad [2]$$

The radial slope λ is shown in Fig. 2, and it is apparent that the originally horizontal radial axis X—X will slope upward toward the coil axis, while the circumferential Y—Y axis slopes downward. Therefore, if we take σ in Equation [2] as positive, λ will be negative. Let $d\phi$ be the angle of twist in the element dl at ψ, and di be the change in slope due to vertical bending, in the length dl at ψ. Projecting $d\phi$ and di upon the plane of θ, which makes the angle $\theta - \psi$ with the plane of ψ, we obtain

$$d\lambda = d\phi\cos(\theta - \psi) - di\sin(\theta - \psi)$$

It can be seen that the torque T which causes $d\phi$, tends to give a positive value of λ at θ, but the moment M would make $d\lambda$ negative. Since

$$d\phi = \frac{Tdl}{GH} \quad \text{and} \quad di = \frac{Mdl}{EI}$$

then

$$\lambda = WR^2\left\{\frac{1}{GH}\int_0^{\theta}\cos(\theta - \psi)[1 - \cos(\theta_0 - \psi)]\,d\psi - \frac{1}{EI}\int_0^{\theta}\sin(\theta - \psi)\sin(\theta_0 - \psi)\,d\psi\right\}$$

which reduces to

$$\lambda = -WR^2\left\{\frac{1}{GH}\left[\frac{1}{2}\theta\cos(\theta_0 - \theta) - \sin\theta + \frac{1}{2}\cos\theta_0\sin\theta\right] + \frac{1}{EI}\left[\frac{1}{2}\theta\cos(\theta_0 - \theta) - \frac{1}{2}\cos\theta_0\sin\theta\right]\right\} \quad [3]$$

The deflections and slopes at the load point θ_0 are obtained by making $\theta = \theta_0$

$$y_0 = WR^3\left\{\frac{1}{GH}\left[\frac{3}{2}\theta_0 - 2\sin\theta_0 + \frac{1}{4}\sin 2\theta_0\right] + \frac{1}{EI}\left[\frac{1}{2}\theta_0 - \frac{1}{4}\sin 2\theta_0\right]\right\} \quad [1a]$$

$$\sigma_0 = WR^2\left\{\frac{1}{GH}\left[\frac{3}{4} - \cos\theta_0 + \frac{1}{4}\cos 2\theta_0\right] + \frac{1}{EI}\left[\frac{1}{4} - \frac{1}{4}\cos 2\theta_0\right]\right\} \quad [2a]$$

$$\lambda_0 = -WR^2\left\{\frac{1}{GH}\left[\frac{1}{2}\theta_0 - \sin\theta_0 + \frac{1}{4}\sin 2\theta_0\right] + \frac{1}{EI}\left[\frac{1}{2}\theta_0 - \frac{1}{4}\sin 2\theta_0\right]\right\} \quad [3a]$$

These equations can be checked by observing that they must satisfy the condition

$$y_0 = y + \bar{y} + R\sigma\sin(\theta_0 - \theta) + R\lambda[1 - \cos(\theta_0 - \theta)] \quad [4]$$

where $\bar{y}$ is the deflection at θ_0 referred to θ as an origin, and is obtained from Equation [1a] by replacing θ_0 with $(\theta_0 - \theta)$. Reduction of the right-hand member of Equation [4] shows that it becomes identical with Equation [1a], thus proving that Equations [1] to [3] are correct.

The deflection and slopes of a point beyond the load point are required as shown in Fig. 4.

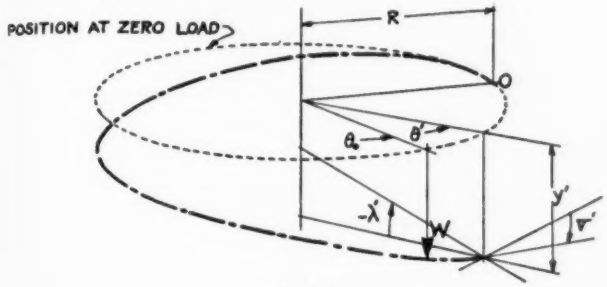


FIG. 4

Letting θ' be the coordinate of the desired point we obtain y' from Equation [4] by making $\bar{y} = 0$, $y_0 = y'$, $\theta_0 = \theta'$, and $\theta = \theta_0$:

$$y' = y_0 + R\sigma_0 \sin(\theta' - \theta_0) + R\lambda_0[1 - \cos(\theta' - \theta_0)]$$

Projecting the orthogonal slopes σ_0 and λ_0 on the plane of θ', that is, through the angle $(\theta' - \theta_0)$, we obtain

$$\sigma' = \sigma_0 \cos(\theta' - \theta_0) - \lambda_0 \sin(\theta' - \theta_0)$$

$$\lambda' = \lambda_0 \cos(\theta' - \theta_0) - \sigma_0 \sin(\theta' - \theta_0)$$

Substitute y_0, σ_0, and λ_0 from Equations [1a], [2a], and [3a]; reducing we get

$$y' = WR^3 \left\{ \frac{1}{GH} \left[\theta_0 + \frac{1}{2}\theta_0 \cos(\theta' - \theta_0) + \sin(\theta' - \theta_0) - \sin\theta' - \sin\theta_0 + \frac{1}{2}\cos\theta' \sin\theta_0 \right] + \frac{1}{EI} \left[\frac{1}{2}\theta_0 \cos(\theta' - \theta_0) - \frac{1}{2}\cos\theta' \sin\theta_0 \right] \right\} \quad \ldots\ldots [5]$$

$$\sigma' = WR^2 \left\{ \frac{1}{GH} \left[\frac{1}{2}\theta_0 \sin(\theta' - \theta_0) + \frac{3}{4}\cos(\theta' - \theta_0) - \cos(2\theta_0 - \theta') + \frac{1}{4}\cos(3\theta_0 - \theta') \right] + \frac{1}{EI} \left[\frac{1}{2}\theta_0 \sin(\theta' - \theta_0) + \frac{1}{4}\cos(\theta' - \theta_0) - \frac{1}{4}\cos(3\theta_0 - \theta') \right] \right\} \quad \ldots\ldots [6]$$

$$\lambda' = -WR^2 \left\{ \frac{1}{GH} \left[\frac{1}{2}\theta_0 \cos(\theta' - \theta_0) + \sin(\theta' - \theta_0) - \sin\theta' + \frac{1}{2}\cos\theta' \sin\theta_0 \right] + \frac{1}{EI} \left[\frac{1}{2}\theta_0 \cos(\theta' - \theta_0) - \frac{1}{2}\cos\theta' \sin\theta_0 \right] \right\} \quad \ldots\ldots [7]$$

The method of applying the equations to the calculation of the elastic curve will now be given. Fig. 5 shows the lower half of the spring, a being the longitudinal center of the bar. The angle $u - v$ is known, since it is one half the angle between tip contact points. Point O, which is taken as the origin, is the same as in Fig. 1, and its position must be left to be determined, that is, angle v is an unknown. The two load components, P_1 and P_2, and the reaction P_3, are all unknown, so that, with v, we have four quantities to determine. The four necessary conditions are:

1 $P_1 + P_2 = P$ where P is the given load on the spring.

2 The combined deflection at point b is zero, since the tip prevents downward movement of the adjacent coil.

3 The combined radial slope at point a must be zero to suit the imposed condition of parallel loading planes.

4 The combined vertical bending moment at point O must be zero, for the bar is not fastened to the loading plane and is, therefore, free to move upward, which it actually does. Suppose that at some point O' between O and b the bar were forced back to its no-load position by a force F. The reaction to F would be upward at b resulting in a vertical moment at O', which point would be the origin. Therefore, a moment at the origin would necessitate a downward force at that point to hold the bar against the plane, obviously a physical impossibility. It will be shown later that the torque at the origin is not zero, and that this condition is consistent with a zero bending moment.

The combined circumferential slope at a is not required, for whatever value it may have, the condition of parallel loading planes will be satisfied due to the symmetry of the top and bottom halves of the spring.

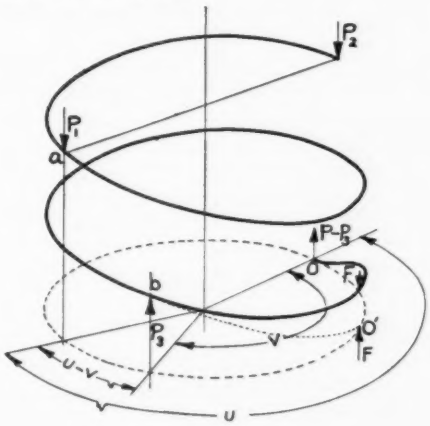


FIG. 5

To express the conditions (2) and (3), and to find the deflection at a, the component deflections are written from Equations [1] to [7]. Taking P_1 at a, we have $\theta_0 = u$ and $\theta = v$. From Equation [1a]

$$(y_a)_1 = P_1R^3 \left\{ \frac{1}{GH} \left[\frac{3}{2}u - 2\sin u + \frac{1}{4}\sin 2u \right] + \frac{1}{EI} \left[\frac{1}{2}u - \frac{1}{4}\sin 2u \right] \right\}$$

From Equation [1]

$$(y_b)_1 = P_1R^3 \left\{ \frac{1}{GH} \left[v + \frac{1}{2}v\cos(u - v) + \sin(u - v) - \sin u - \sin v + \frac{1}{2}\sin v \cos u \right] + \frac{1}{EI} \left[\frac{1}{2}v\cos(u - v) - \frac{1}{2}\sin v \cos u \right] \right\}$$

From Equation [3a]

$$(\lambda_a)_1 = -P_1R^2 \left\{ \frac{1}{GH} \left[\frac{1}{2}u - \sin u + \frac{1}{4}\sin 2u \right] + \frac{1}{EI} \left[\frac{1}{2}u - \frac{1}{4}\sin 2u \right] \right\}$$

Taking P_2 diametrically opposite a, we have $\theta_0 = u + \pi$ and $\theta = u$ for point a. From Equation [1]

$$(y_a)_2 = P_2R^3\left\{\frac{1}{GH}\left[\frac{1}{2}u - \frac{1}{4}\sin 2u\right] + \frac{1}{EI}\left[-\frac{1}{2}u + \frac{1}{4}\sin 2u\right]\right\}$$

From Equation [3]

$$(\lambda_a)_2 = -P_2R^2\left\{\frac{1}{GH}\left[-\frac{1}{2}u - \sin u - \frac{1}{4}\sin 2u\right] + \frac{1}{EI}\left[-\frac{1}{2}u + \frac{1}{4}\sin 2u\right]\right\}$$

For point b, $\theta = v$ and we have from Equation [1]

$$(y_b)_2 = P_2R^3\left\{\frac{1}{GH}\left[v - \frac{1}{2}v\cos(u-v) - \sin(u-v) + \sin u - \sin v - \frac{1}{2}\sin v\cos u\right] + \frac{1}{EI}\left[-\frac{1}{2}v\cos(u-v) + \frac{1}{2}\sin v\cos u\right]\right\}$$

Taking P_3 at point b and using the negative sign because P_3 acts upward, we have $\theta_0 = v$ and $\theta' = u$. From Equation [5]

$$(y_a)_3 = -P_3R^3\left\{\frac{1}{GH}\left[v + \frac{1}{2}v\cos(u-v) + \sin(u-v) - \sin u - \sin v + \frac{1}{2}\sin v\cos u\right] + \frac{1}{EI}\left[\frac{1}{2}v\cos(u-v) - \frac{1}{2}\sin v\cos u\right]\right\}$$

From Equation [7]

$$(\lambda_a)_3 = P_3R^2\left\{\frac{1}{GH}\left[\frac{1}{2}v\cos(u-v) + \sin(u-v) - \sin u + \frac{1}{2}\sin v\cos u\right] + \frac{1}{EI}\left[\frac{1}{2}v\cos(u-v) - \frac{1}{2}\sin v\cos u\right]\right\}$$

From Equation [1*a*]

$$(y_b)_3 = -P_3R^3\left\{\frac{1}{GH}\left[\frac{3}{2}v - 2\sin v + \frac{1}{4}\sin 2v\right] + \frac{1}{EI}\left[\frac{1}{2}v - \frac{1}{4}\sin 2v\right]\right\}$$

Combining the above results we have:

$$(y_b)_1 + (y_b)_2 + (y_b)_3 = 0 \ldots \text{Condition (2)}$$
$$(\lambda_a)_1 + (\lambda_a)_2 + (\lambda_a)_3 = 0 \ldots \text{Condition (3)}$$
$$y_a = (y_a)_1 + (y_a)_2 + (y_a)_3$$

This last equation is not required for the elastic-curve calculation, but is used in calculating the deflection of the spring as a whole and, hence, the deduction for inactive end turns. The condition equations can be simplified by the usual trigonometric transformations, and for round steel bars, a further simplification is made by putting $\frac{1}{EI} = 0.770\,\frac{1}{GH}$.

Simultaneous solution of the equations for conditions (1), (2), and (3) gives the results

$$\left.\begin{aligned} P_1 &= P\,\frac{BF + EC}{F(A+B) - 2C^2} \\ P_2 &= P - P_1 \\ P_3 &= P\,\frac{E(A+B) + 2BC}{F(A+B) - 2C^2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [8]$$

$$y_a = \frac{PR^3}{GH}\left[K\frac{P_1}{P} + L\frac{P_2}{P} - D\frac{P_3}{P}\right]$$
$$A = 0.885u - \sin u + 0.0575\sin 2u$$
$$B = 0.885u + \sin u + 0.0575\sin 2u$$
$$C = 0.885v\cos(u-v) + \sin(u-v) - \sin u + 0.115\sin v\cos u$$
$$D = C + v - \sin v$$
$$E = -C + v - \sin v$$
$$F = 1.885v - 2\sin v + 0.0575\sin 2v$$
$$K = 1.885u - 2\sin u + 0.0575\sin 2u$$
$$L = 0.115u - 0.0575\sin 2u$$

The angle v is now determined from condition (4) which is

$$\left[BF + EC - \frac{1}{2}F(A+B) + C^2\right]\sin u - \left[\frac{1}{2}E(A+B) + BC\right]\sin v = 0 \ldots\ldots\ldots [9]$$

This equation can be solved by trial. If we assign a numerical value to v, the terms containing only trigonometric functions of u can be grouped into one member, and the other member of the equation becomes merely a constant times u. To illustrate, let $v = 188$ deg, and the condition (4) equation becomes

$$\sin^2 u - 0.5494\sin u[2.9039\cos(u - 8°) + \sin(u - 8°) + \sin u + 0.0160\cos u] + 0.004226\sin 2u = -0.06502\,u$$

The left-hand member contains only sine and cosine functions of u and its graph will therefore be sinusoidal with waves of constant amplitude. The right-hand member is merely an inclined straight line, and since it is negative, we need consider only the negative portions of the sinusoidal curve. This curve repeats at intervals of π, therefore only the first negative wave is required, and we obtain the roots by drawing a series of inclined parallel lines separated by a vertical distance of $0.06502\,\pi$ as



FIG. 6

shown in Fig. 6. The first two roots, points (1) and (2), have no significance since they are less than v. The other seven roots are as shown.

Values of u and v obtained in this manner are plotted in Fig. 7. It is seen that v fluctuates about 180 deg as a mean and that, except for short springs, it differs but little from this value.

We can now work out any desired numerical case from Equations [8] and the graph of Fig. 7. It is found, in general, that $P_1 \neq P_2$ or the resultant load P is eccentric. The loading is axial, $P_1 = P_2$ only when

$$u - v = \text{integer} \times \pi + 92 \text{ deg}$$

and the eccentricity is maximum when

$$u - v = \text{integer} \times \pi \text{ (approximately)}$$

The maximum eccentricity diminishes with increase of u, as shown in Fig. 7. The abscissas are shown in terms of u and also the number of turns between tip contact points.

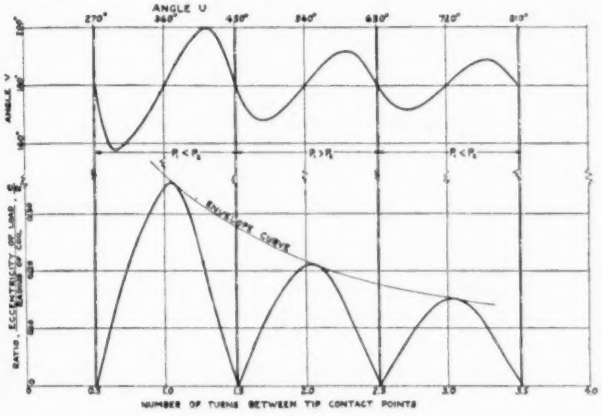


FIG. 7

The reactions between the coil end and the loading plane must now be considered, to show that the necessary conditions of static equilibrium are satisfied. The tapered tip which contacts the loading plane is shown in Fig. 8.

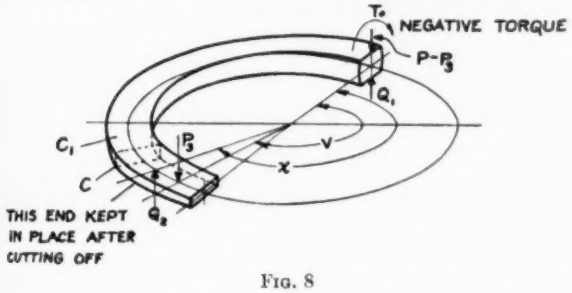


FIG. 8

A numerical calculation of P_1, P_2, and P_3 for various values of u shows that there is always a negative torque T_0 at the origin, as shown in Fig. 8, which would cause the tapered end to rise from the loading plane if it were not held down by the reaction P_3. The reactions between the tapered end and the loading plane are Q_1 and Q_2. Static equilibrium requires that the sum of torques, and the sum of bending moments at O must each be zero, and sum of reactions equal to the load on the spring:

$$P_3R(1 - \cos v) - Q_2R(1 - \cos x) - T_0 = 0$$
$$P_3R \sin v - Q_2R \sin x = 0$$
$$Q_1 + Q_2 = P$$

The terms P_1, P_2, and P_3 are calculated as previously explained, and T_0 from the equation

$$T_0 = R[-P_3(1 - \cos v) + P - (P_1 - P_2) \cos u]$$

Q_1, Q_2, and x can then be found from the foregoing three equations. The results of a sample calculation are as follows:

u = 387 deg 7 min	
v = 193 deg	x = 193 deg 55 min
P_1 = 0.32407 P	Q_1 = 0.33362 P
P_2 = 0.67593 P	Q_2 = 0.66638 P
P_3 = 0.71284 P	
T_0 = —0.09423 PR	

To test the truth of the foregoing reasoning, a test was made, cutting off the tapered ends at C (in Fig. 8) and using a spring with ends ground accurately parallel (to within 0.0005 in. in 6½ in.), and employing ground-steel plates for loading planes. Upon application of the load it was found that the end C_1 moved upward from the loading plane, thus proving that there is a negative torque at O if the ends are not cut.

The elastic curve can now be calculated for any desired length of spring. Taking first a case of approximately maximum eccentricity (u = 1080 deg and v = 180 deg) we obtain from Equation [8]

$$P_1 = 0.4520\ P$$
$$P_2 = 0.5480\ P$$
$$P_3 = 0.5755\ P$$

Letting θ be the coordinate of any point, we write the component deflections from Equation [1] using

$$\theta_0 = \begin{cases} 1080 \text{ deg for } P_1 \\ 1260 \text{ deg for } P_2 \\ 180 \text{ deg for } P_3 \end{cases} \quad \text{and } 0 < \theta < 180 \text{ deg}$$

Adding these deflections, in terms of P, we obtain

$$y = \frac{PR^3}{GH} \times 0.4245\ [\theta(1 + \cos\theta) - 2\sin\theta] \ldots\ldots [10]$$

which gives the elastic curve for points between the origin O and the tip contact point b.

For values of $\theta > v$ we require another equation because P_3 now comes between θ and the origin. Using Equation [5], and replacing θ' with θ, and θ_0 with v (180 deg) we obtain the component deflection for P_3. The component deflections for P_1 and P_2 are the same as used in the preceding paragraph. Combining

$$y = \frac{PR^3}{GH}\ [\theta(1 - 0.08489\cos\theta) - 1.8081 + 1.6175\sin(\theta + 81°37')] \ldots [11]$$

which applies between the tip contact b and the bar center point a.

The consistency of these results can be seen by noting that Equation [10] gives $y = 0$ for $\theta = 0$ or $\theta = 180$ deg, and Equation [11] gives $y = 0$ for $\theta = 180$ deg. Also the derivatives of Equations [10] and [11], taken with respect to θ, are equal for θ = 180 deg, thus satisfying the necessary condition of continuity.

The graph of Equations [10] and [11] is the required elastic curve and is shown in Fig. 9, deflections being plotted in terms of the deflection of the entire spring. It is seen that the deflection is reversed between the origin and tip contact point, which fact makes the deflection of the whole spring more than it would be if the bar were held fixed at the tip contact. The elastic curve has sinusoidal waves, the amplitude of which diminishes as the center of the spring is approached.

Taking the case of axial loading, we have

$$P_1 = P_2 = 0.5\ P \qquad u = \text{integer} \times \pi + 92 \text{ deg}$$
$$P_3 = 0.5305\ P \qquad v = 180 \text{ deg}$$

The method of calculation is the same as for the previous case, and we find

$$y = \frac{PR^3}{GH} \times 0.4695\ [\theta(1 + \cos\theta) - 2\sin\theta] \ldots\ldots [12]$$

$$0 < \theta < v$$

$$y = \frac{PR^3}{GH}\left[\theta - 1\frac{2}{3} + 1.4763\sin(\theta + 87°38')\right] \ldots [13]$$

$$v < \theta < u$$

Equations [10] and [12] are identical except for a small difference in the numerical coefficient, showing that the deflection back of the tip-contact point is practically unaffected by load eccentricity. The graph is shown in Fig. 10; it has sinusoidal waves of constant amplitude as can be seen from the form of Equation 13, whereas, with eccentric loading the amplitude diminishes at the bar center point. This is the distinguishing feature between axial and eccentric loading as far as the elastic curve is concerned.

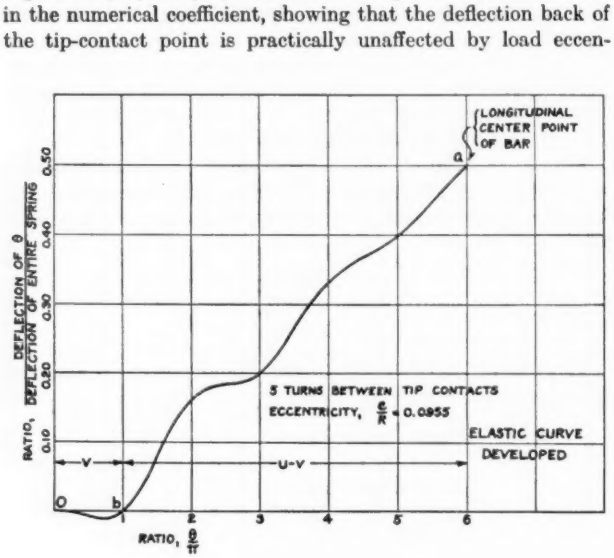


FIG. 9

An experimental determination of the elastic curve, covered in a current paper[2] by D. H. Pletta and F. J. Maher of the Virginia Polytechnic Institute, furnishes proof of the essential correctness of the analytical results.

Load eccentricity increases stress because the maximum torque is increased, the direct load being unchanged. If we let Z be the eccentricity stress factor, the relation between Z and the eccentricity e is given by

$$e = 1.123\,R(Z - 1) \quad \text{or} \quad Z = 1 + \frac{e}{1.123\,R}$$

Axial loading is theoretically possible, but actually it cannot be realized. This point is discussed in more detail in Professor Pletta's paper.[2] His experiments showed close agreement between the test and theoretical elastic curves in cases of maximum load eccentricity. For the theoretical number of turns corresponding to axial loading, the agreement between test and theory was not as close, indicating that axial loading was not realized in the tests in spite of all the precautions taken to obtain it. In actual springs the chances of getting axial loading would be even less, and it would therefore appear best for conservative design to calculate the eccentricity stress factor Z from the envelope of the e curves shown in Fig. 7. It can be shown that on this basis the envelope formula for Z would be

$$Z = 1 + \frac{0.5043}{N} + \frac{0.1213}{N^2} + \frac{2.0584}{N^3}$$

N = solid ht ÷ bar diameter

In order to use the foregoing analytical results in calculation, it is necessary to relate the angle $u - v$ to the number of turns in the spring. The actual average location of the tip-contact point b, as shown in Fig. 1, is somewhat advanced from the extreme tip, say about $^1/_{20}$ turn or 18 deg. Since the bar-tip thickness is $^1/_4$ of the bar diameter, the number of closed turns at one end is

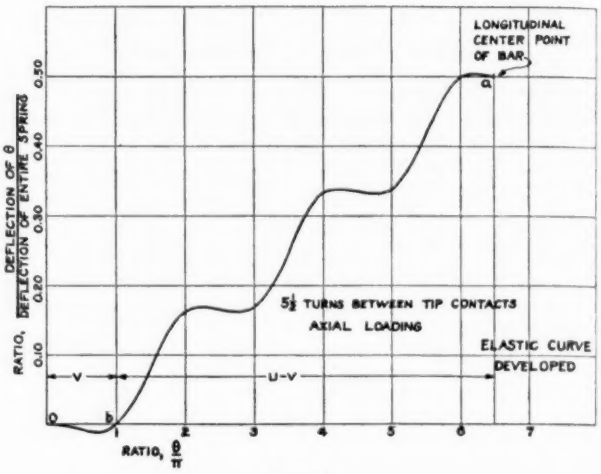


FIG. 10

$^1/_2 + ^1/_4 + ^1/_{20} = 0.8$, or 1.6 for the whole spring. Therefore

$$2(u - v) = 2\pi(N - 1.6)$$

$$\frac{u - v}{\pi} = N - 1.6 = \text{number of turns between tip-contact points.}$$

Using this equation in connection with any desired value of N, we can find the eccentricity from Fig. 7, or the envelope stress factor Z from the equation in the preceding paragraph.

The deflection for the whole spring is

$$y = \frac{PD^3\pi(N - 1.2)}{4GH}$$

$$y = \frac{8PD^3(N - 1.2)}{Gd^4}$$

for round bars, where $N - 1.2$ is the number of effective turns. The deduction of 1.2 for inactive end turns is obtained from the foregoing analysis by calculation of y_a.

Discussion

H. C. PERKINS.[4] Elastic curves of helical compression springs, as computed by the author, check so well with the experimental results[2] reported by Pletta and Maher that the essential validity of his theory seems definitely established. He is to be congratulated upon the success of his analysis and the accuracy of his calculations. Nevertheless, his formulas are inconvenient, especially in case his Fig. 7 is inapplicable.

Since the author's theory seems to be of sufficient interest and value to warrant an attempt to simplify the analysis and reduce its results to more convenient form, the following discussion of his paper has three objects:

1 To check the author's theory
2 To simplify his analysis
3 To derive formulas which could be used in design.

Fig. 11 of this discussion represents the lower half of a compression spring stressed and deflected by a force P with eccentricity e and supported by reactions P_3 and P_4 at the lower end coil as shown. Since the bending moment does not, in general, vanish at the point $V = \pi$, but is small and changing rapidly, the author chose to take the origin O at the cross section of zero bending moment and to regard V as undetermined.

[4] Assistant Professor of Mechanical Engineering, Cornell University, Ithaca, N. Y. Mem. A.S.M.E.

It therefore follows that the stress resultant at O (including any reaction between O and b) is a vertical force P_4 with action line intersecting a coil diameter through O, at an unknown distance C from the axis of the coil. If f represents the number of free turns of wire between end contacts b, then half the free length of the wire is equal to $Rf\pi$ as suggested in Fig. 11 of this discussion.

Three independent equations of equilibrium may be written. Thus, summing forces

$$P_3 + P_4 - P = 0 \quad [14]$$

Summing moments about a coil diameter through a

$$P_3 R \sin f\pi + P_4 C \sin U = 0 \quad [15]$$

Summing moments about a coil diameter through b

$$Pe \sin f\pi - P_4 C \sin V = 0 \quad [16]$$

But a moment sum about the coil diameter perpendicular to the one through a is also useful in case $\sin f\pi$ and $\sin V$ vanish as they do when f = integer. Thus

$$Pe + P_3 R \cos f\pi + P_4 C \cos U = 0 \quad [17]$$

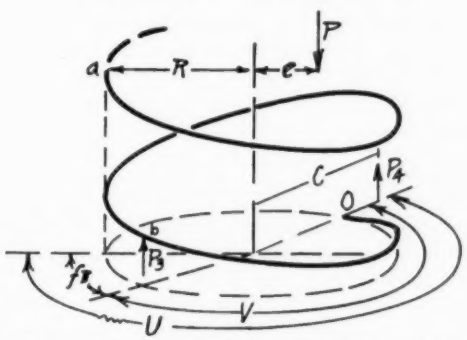


FIG. 11 LOWER HALF OF COMPRESSION SPRING STRESSED AND DEFLECTED BY FORCE P AND SUPPORTED BY REACTIONS P_3 AND P_4 AT THE LOWER END COIL

Before proceeding with the general analysis, consider two special cases.

In case 1, when $2f$ is an odd integer, the end contacts b are at opposite ends of a coil diameter. In this case, by symmetry of the upper and lower halves of the spring $e = 0$, since P must be equidistant from the two end contacts. Then by Equation [16] of this discussion, since $f\pi = 0$ and $P_4 C$ does not vanish, it follows that $\sin V = 0$, so $V = \pi$ and U = (integer) (π) + 90 deg. This is 2 deg smaller than the author's value. The error is not one of calculation but seems to follow from the assumption that the radial slope λ vanishes at the origin. This assumption is, in general, inconsistent with the fact that, as shown by the author, there are torques on most cross sections of the "dead end coil" of an actual spring, but the tests[3] of Pletta and Maher indicate that the consequent errors are probably smaller than those introduced in the manufacture of the spring.

In case 2, when f is an integer, $\sin f\pi = 0$, and since $P_4 C$ does not vanish, Equation [16] of this discussion requires that $\sin V = 0$, so $V = \pi$.

Given the value of V as in cases 1 and 2, the corresponding value of C/R is found from the condition that the deflection y must vanish at b. The deflection y at any point of the elastic curve may be formulated as follows.

The bending and twisting moments on any cross section located by angle θ from 0 or angle ϕ from b are

$$M = P_4 C \sin \theta + P_3 R \sin \phi \quad [18]$$

$$T = P_4 (C \cos \theta - R) - P_3 R (1 - \cos \phi) \quad [19]$$

and corresponding values for unit downward force on the elastic curve at angle α from O or β from b are

$$m = -R \sin (\alpha - \theta) = -R \sin (\beta - \phi) \quad [20]$$

$$t = -R [1 - \cos (\alpha - \theta)] = -R [1 - \cos (\beta - \phi)] \quad [21]$$

Substituting these values in the strain-energy formula

$$y = \int \frac{M m dL}{EI} + \int \frac{T t dL}{GH} \quad [22]$$

and integrating $dL = Rd\theta = Rd\phi$ from $\theta = 0$ to $\theta = \alpha$ or from $\phi = 0$ to $\phi = \beta$, the result is

$$y = P_4 R^3 \frac{c}{2R}\left[\frac{1}{EI} + \frac{1}{GH}\right](\alpha \cos \alpha - \sin \alpha) + \frac{P_4 R^3}{GH}(\alpha - \sin \alpha) + \frac{P_3 R^3}{2EI}(\beta \cos \beta - \sin \beta) + \frac{P_3 R^3}{2GH}(2\beta - \beta \cos \beta - 3 \sin \beta) \quad [23]$$

Note that by taking $\beta = \theta$, $P_3 = W$, $P_4 = 0$, and $\theta_0 = 0$, or taking $\alpha = \theta$, $P_4 = W$, $P_3 = 0$, $C = R$, and $\theta_0 = 0$, Equation [23] and the author's Equation [1] are reduced to the same form.

Since P_4 is the only reaction if the angle θ is less than V, the zero deflection at b is formulated by letting $P_3 = 0$ and $\alpha = V$ in Equation [23] of this discussion. It follows thus that

$$\frac{C}{R} = \frac{2EI}{EI + GH} \cdot \frac{V - \sin V}{\sin V - V \cos V} \quad [24]$$

or if $\frac{GH}{EI} = 0.77$

$$\frac{C}{R} = 1.13 \frac{V - \sin V}{\sin V - V \cos V} \quad [25]$$

In cases 1 and 2, $V = \pi$ and $C/R = 1.13$. In general, however, before C/R can be evaluated, V must be computed from the condition that, by symmetry, the radial slope λ must vanish at the mid-section. Thus, bending and twisting moments on any cross section are given by Equations [18] and [19] of this discussion and corresponding values for unit torque at mid-section a are

$$m = -\sin (U - \theta) = -\sin (f\pi - \phi) \quad [26]$$

$$t = \cos (U - \theta) = \cos (f\pi - \phi) \quad [27]$$

Substituting these values in the strain-energy equation

$$\lambda_a = 0 = \int \frac{M m dL}{EI} + \int \frac{T t dL}{GH} \quad [28]$$

integrating $dL = Rd\theta = Rd\phi$ from $\theta = 0$ to $\theta = U$ or from $\phi = 0$ to $\phi = f\pi$, solving for $\frac{-RP_3}{CP_4}$ and equating it to the corresponding ratio from Equation [15] of this discussion

$$\frac{-RP_3}{CP_4} = \frac{U \cos U - A \sin U}{f\pi \cos f\pi - \sin f\pi} = \frac{\sin U}{\sin f\pi} \quad [29]$$

in which $A = \dfrac{EI\left(2\frac{R}{C} - 1\right) + GH}{EI + GH}$

To check, let $\frac{GH}{EI} = 0.77$, and take $f = 1/2$ which comes under case 1. Then $\frac{C}{R} = 1.13$, $A = 0.87$, $\sin f\pi = 1$, $\cos f\pi = 0$, $\sin U = -1$, $\cos U = 0$. These values do not satisfy the equation. To estimate the error, regard $f\pi$ as undetermined. Then

$$\cot f\pi = \frac{0.13}{f\pi}$$

Solving by successive approximations, $f\pi = 94$ deg 30 min. The author found that $e = 0$ when $U - V = (\text{integer})\ (\pi) + 92$ deg. In general, by taking $A = 1$ in Equation [29] of this discussion, the error is eliminated in case 1 and perhaps in other cases as well. If this change is effected, the equation can be written in the form

$$\cot U = \frac{f\pi \cot f\pi}{U} \quad \ldots\ldots\ldots\ldots\ldots\ldots [30]$$

which is easily solved by successive approximations.

For example: let $f = 3.25$ so $\cot f\pi = 1$. Substitute $U_0 = 585 + 180$ deg on the right and solve for U_1 on the left. Thus

$$\cot U_1 = \frac{585\ (1)}{765} = 0.764$$

so $U_1 = 772.6$. Substitute U_1 on the right and find $U_2 = 772.9$ deg on the left. This completes the solution, since further refinement is useless for slide-rule computation. Now $V = 772.9 - 585 = 187.9$ deg, which seems to check with the corresponding ordinate in the author's Fig. 7.

Using this value of V in Equation [25] of this discussion, $C/R = 1.238$

Equations [14] and [15] of this discussion may now be solved to find $P_4 = 0.583P$ and $P_3 = 0.417P$ and Equation [16] solved to find $e/R = 0.101$ which is about 10 per cent less than the corresponding ordinate in the author's Fig. 7. Finally, the over-all deflection of the spring is obtained by doubling the deflection at mid-section, the latter being found from Equation [23] by taking $\alpha = U$ and $\beta = f\pi$.

A formula approximating the author's "envelope curve" may be found by taking $f =$ an integer and solving Equations [24], [14], [17], and [29] to find

$$\frac{e}{R} = \frac{-2}{f\left(3 + \frac{GH}{EI}\right) + 2} \quad \ldots\ldots\ldots\ldots [31]$$

Disregarding sign and taking $\frac{GH}{EI} = 0.77$, the results check the author's values of e/R for $f = 1$, 2, and 3 free turns, respectively. Increasing the numerator 5 per cent and, changing sign, the "envelope formula" is

$$\frac{e}{R} = \frac{2.1}{f\left(3 + \frac{GH}{EI}\right) + 2} \quad \ldots\ldots\ldots\ldots [32]$$

Since e, in Fig. 11 of this discussion, is alternately positive and negative as f is alternately odd and even, and passes through zero when $2f$ is an integer, the curve e versus $f\pi$ is periodic. It may be approximated by the formula

$$\frac{e}{R} = \frac{-2 \cos f\pi}{F\left(3 + \frac{GH}{EI}\right) + 2} \quad \ldots\ldots\ldots\ldots [33]$$

in which F is the integer nearest to the given value of f.

In the foregoing example, in which $f = 3.25$ and $\frac{GH}{EI} = 0.77$, $F = 3$ and the approximate eccentricity ratio is

$$\frac{e}{R} = \frac{1.414}{3 \times 3.77 + 2} = 1.06$$

This discussion seems to support the conclusion that, although simplifying assumptions have introduced some errors, the errors are probably small. The author's calculations also seem to be highly accurate despite their complication. However, as shown, the analysis need not be complicated and, judging by the illustrative example near the close of this discussion, the formulas therein derived could easily be used in the design of helical compression springs.

Author's Closure

Professor Perkins' discussion is of much interest as showing a different method of approaching the problem. His treatment is admittedly shorter than the author's, and possibly simpler, but it has been compressed to such an extent that it is more difficult to follow. For instance, Equation [23] is given for the elastic curve, but no equations expressing P_3 and P_4 directly in terms of P are given. Apparently it would be necessary to use Equation [15] which involves U, finding U from Equation [30]. The author admits frankly that the assumption of zero λ at the origin is not exactly true. However, it is very nearly true, physically, as is the case with many assumptions commonly accepted in elasticity calculations. It is not clear why the reaction P_4 has been taken as acting outside the coil.

The paper is concerned primarily with the determination of the elastic curve, as the subject indicates. No attempt was made to give working formulas for practical spring calculation. For such purposes, neither the author's paper nor Professor Perkins' discussion is suitable. A further simplification is necessary and the results must be put in tabular form so that the use of formulas is eliminated. Such tables have been prepared by the author and used for several years in routine spring calculation. They have proved to be rapid and accurate, containing the proper allowance for inactive end turns and load-eccentricity effect, as expressed in the envelope curve.

Helix Warping in Helical Compression Springs

By D. H. PLETTA[1] and F. J. MAHER,[2] BLACKSBURG, VA.

When helical springs are compressed between parallel planes, the load is usually eccentric with the longitudinal axis and the helix, in its deflected position, is no longer truly helical but assumes a sinusoidal or warped shape. The magnitude of the eccentricity and of the warping is a function of the number of turns in the spring.

The research reported on here illustrates this warping for a heavy helical spring, whose length was varied for the series of tests, and it also compares the test results with the theoretical behavior as outlined in the accompanying paper of H. C. Keysor. In general, the actual and theoretical behavior were in excellent agreement, thus apparently substantiating the theory. An analysis of the data also emphasizes the need of always assuming some eccentricity of load in practical design, to allow for the lack of perfect concentricity of the individual turns.

THE TESTS reported on in this paper were undertaken to determine the correlation between the actual behavior of loaded helical compression springs and the theoretical behavior as outlined in the companion paper[3] by H. C. Keysor. The theory indicates that helical springs, when compressed between parallel planes, are usually subjected to an eccentric rather than a concentric load, and that this eccentricity causes the helix to warp under load. The term warping, as used here, indicates that the deflected helix assumes a sinusoidal shape and does not remain truly helical in its deflected position as the ordinary spring formula leads one to believe. Fig. 1 illustrates this phenomenon. Curve *A* shows the unloaded helix "unwound" and projected on a vertical plane. Curve *B* indicates the deflected position of this helix after application of the load, the solid curve showing actual behavior, and the dotted line showing the commonly assumed position. The shaded area between curves *A* and *B* represents deflection and is replotted in curve *C* for only half of the helix.

The actual and theoretical behavior of compression springs may then be checked by measuring the actual eccentricity of load or by measuring the warping of the helix, and comparing these observations with the theoretical values. The tests as carried out included only observation of helix warping, mainly because apparatus capable of determining warping already existed and because it would have been somewhat difficult to measure eccentricity accurately on the heavy stiff spring used.

A single spring was used in this project. Hence, all variables such as differences in heat-treatment, moduli of elasticity, coil-and-bar diameter, etc., common to any series of separate specimens were eliminated or at least minimized. The bar as coiled consisted originally of about seven turns from each end of which equal lengths were cut as the spring was shortened for the series of tests. The spring had an outer diameter of 6.42 in. and a bar diameter of 1.336 in. A plain carbon steel (0.9 to 1.05 per cent C) was used, coiled hot on a slightly tapered mandrel, and subjected to an oil quench from 1525 F and a subsequent drawing to 850 F.

The spring lengths, as tested, were placed between special helical end blocks mounted to the base and movable head of a testing machine, Fig. 2. Very thin plaster of Paris cappings, as cast in place under load, insured continuous contact for the first 270 deg of the dead turn. The balance of the apparatus to measure the warping consisted essentially of a collar, concentric with and free to rotate about the lower end block, to which a vertical post was attached. An Ames dial reading to 0.001 in. was fastened to a sleeve which was free to slide up and down the post. A tapered pin, which fits through the sleeve and post simultaneously, could be inserted in any one of a series of holes in the post spaced 1 in. apart. The cantilever arm attached to the lower end of the Ames-dial plunger rested on the top side of the wire of the spring. With this arrangement, it was possible to determine the elevation of any point on the coil to 0.001 in. above a datum for any given load. A subtraction of two successive elevations at any given point gave the net deflection for that increment of load.

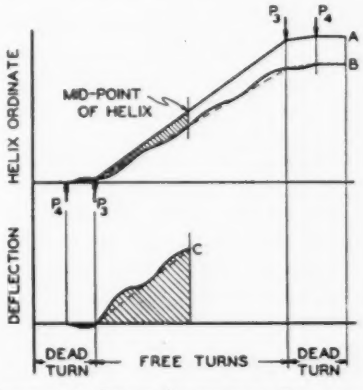


FIG. 1 SINUSOIDAL SHAPE ASSUMED BY DEFLECTED HELIX

During a test run, readings were taken at points spaced 30 deg apart along the helix for loads of 500 and 8000 lb. The cantilever arm was used to eliminate the error that could possibly result if points on the outside of the wire had been used. Any twist in the wire, when the coil was loaded, would have caused these points to deflect more or less than the helical axis, whereas any measurement made on a horizontal tangent to the top of the wire would always remain half a wire diameter above the axis. At each load the deflection of the whole spring was also observed by noting the movement of the head of the testing machine.

At the 8000-lb load, the upper and lower split wedge blocks were adjusted so that the deflection of the end of the dead

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[2] Instructor of Applied Mechanics, Virginia Polytechnic Institute.

[3] "Calculation of the Elastic Curve of a Helical Compression Spring," by H. C. Keysor. Published in this issue, page 319.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

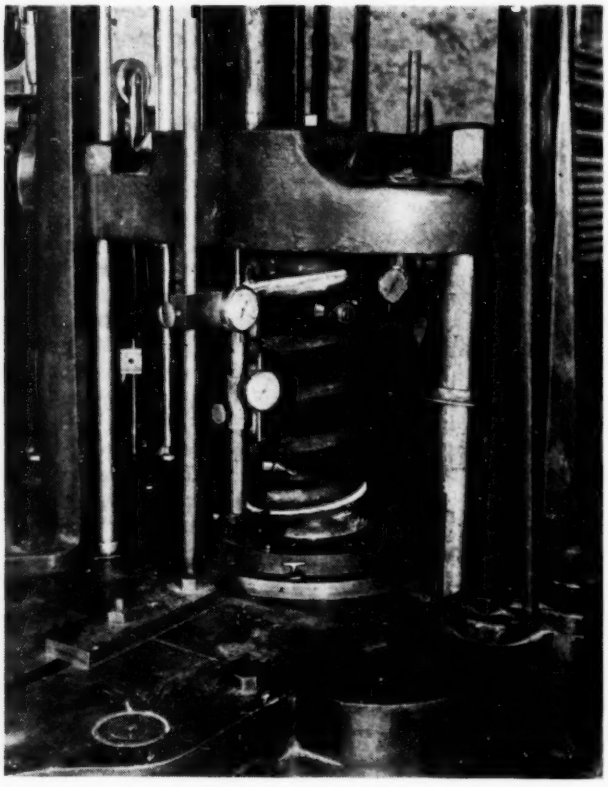


FIG. 2 METHOD OF MOUNTING SPRING LENGTHS IN TESTING MACHINE

turn (point 13) was reduced to zero relative to its seat. The wedges can be seen in Fig. 2 and are also illustrated in detail in Fig. 3. They resemble a bolt with the circular head of the bolt and washer tapered. Under the 8000-lb load the two ends of the dead turn closed several hundredths of an inch allowing point 13 to deflect. This movement was reduced to zero by tightening the nut, thus separating the ends of this turn. Various other nonadjustable supports were tried but all of them allowed some deflection due to the localized contact deformation. It was thought best to have this deflection zero in keeping with the theory, but it should be added that the small movement allowed by these other supports was not critical, for the deflections of points along the helix were only slightly affected.

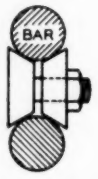


FIG. 3 SPLIT WEDGE BLOCK

Table 1 indicates the scope and some of the results of the tests. Seven tests were made, the bar length varying from 6 to 3.511 total turns. Three of these lengths were such that the eccentricity of the load was theoretically zero; whereas the others, with the exception of case 2, were such that the eccentricity was the maximum attainable. The theoretical spring deflection was calculated, using a shearing modulus of elasticity of 11,600,000 lb per sq in.

Fig. 4 shows the helix warping up to mid-length of the spring for the seven tests outlined in Table 1. The other half of the spring should, of course, be identical and tests indicated that it was, but there seemed little need of plotting the curves for the entire spring. The load increment was 7500 lb. The solid line represents theory. No attempt was made to draw a curve through the data points. In four of the tests the correlation between theoretical and actual behavior was almost perfect, whereas in the other three cases it was only fair. There are

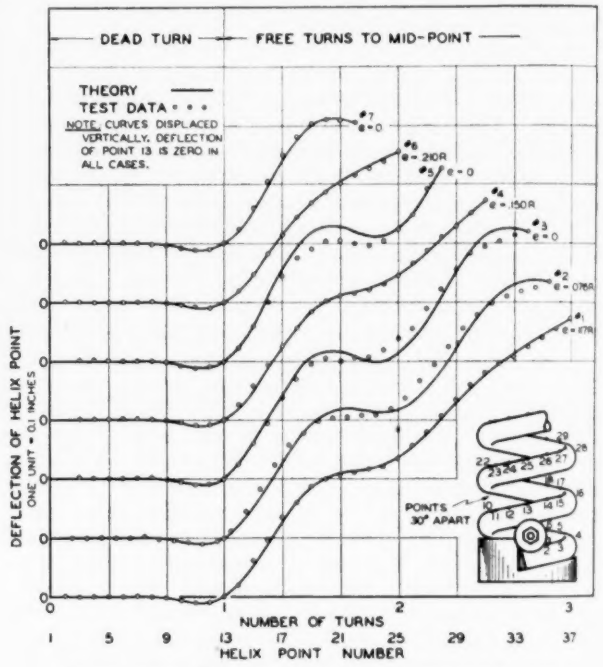


FIG. 4 HELIX WARPING UP TO MID-LENGTH OF SPRING

several reasons why the agreement in these last three cases should not be perfect. The measurement of the deflection on the top of the bar rather than along the neutral axis; the slight rocking of the bar on the split wedge blocks, tending to shorten the spring; the imperfect seating on the helical end seats; and the lack of perfect concentricity for the individual turns of the spring, all tended to modify perfect behavior. The last condition is the most important. It is practically impossible to coil heavy bar so that all turns are identical in diameter, pitch, and concentricity. If only one turn is displaced laterally, an axial load on the spring would be eccentric on this coil. Theoretically the warping decreases near the center of the spring as the ec-

TABLE 1 SCOPE AND RESULTS OF COMPRESSION TESTS ON HELICAL SPRINGS

Test no.	Number of turns: Free[a]	Number of turns: Active (theoretical)	Number of turns: Excess	Deflection in in.: Actual	Deflection in in.: Theoretical	Variation from theoretical, per cent	Theoretical eccentricity
1	4	4.415	0.415	0.947	0.938	+0.96	0.1171R[b]
2	3.75	4.194	0.444	0.888	0.892	—0.45	0.0763R
3	3.511	3.979	0.468	0.841	0.846	—0.59	0
4	3	3.399	0.399	0.732	0.723	+1.39	0.1503R
5	2.511	2.979	0.468	0.625	0.634	—1.42	0
6	2	2.371	0.371	0.507	0.504	+0.59	0.2096R
7	1.511	1.979	0.468	0.416	0.421	—1.19	0

[a] All test springs had two dead coils in addition.
[b] R = mean coil radius.

centricity of load increases. Hence the sinusoidal effect would tend to be damped out especially at the center of the spring. The measured eccentricity of individual turns on the spring amounted to several hundredths of an inch, which corresponded approximately to $e = 0.02R$ rather than zero. Naturally the magnitude of this error was relatively larger in cases 3 and 5 when the eccentricity should have been zero than in cases 1, 4, and 6 when e was appreciable. Yet in case 7, with $e = 0$ theoretically, the correlation is very good. The reason for this is that this spring had only 1.511 free turns and was therefore too short to be affected by any lateral displacement of a full turn, the short length practically requiring concentricity with itself.

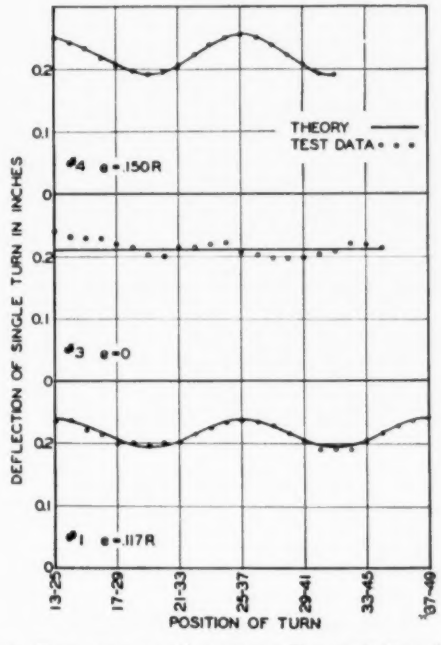


FIG. 5 DEFLECTION OF INDIVIDUAL TURNS 30 DEG APART

The actual eccentricity of individual turns in cases 3 and 5 was actually determined, but its existence would also be suspected when studying Fig. 5. Here the deflection of individual full turns 30 deg apart is plotted for cases 1, 3, and 4. The sinusoidal nature of the test data for case 3 is possible only when the load is eccentric. When considering this variation, it should be remembered that the total load on the spring, considered as a concentrated force, acts perpendicular to the diameter through the mid-point of the spring, and that it is generally displaced from the longitudinal axis of the spring.

Some objection to these tests and the theory might be raised because of the difference in the end conditions between these tests and those in commercial springs with their tapered ends ground square. Fig. 6 illustrates helix warping in a commercial spring tested and reported on previously.[4] Here again the sinusoidal effect is quite evident.

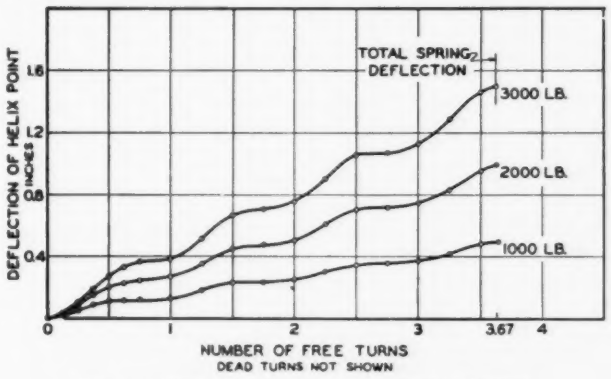


FIG. 6 HELIX WARPING OF A COMMERCIAL SPRING

It is the belief of the authors that the tests reported in this paper substantiate Mr. Keysor's theory on spring behavior and also emphasize the absolute necessity of always assuming some eccentricity for the load in commercial design to be on the safe side. In other words, an eccentricity based on the envelope of the theoretical cyclic variation in eccentricity,[5] rather than the theoretical eccentricity itself, should be used to allow for the lack of concentricity arising in the practical manufacture of springs. In the commercial design it should, of course, be remembered that eccentric loads produce greater shearing stress than the commonly assumed axial loads.

[4] "The Effect of Overstrain on Closely Coiled Helical Springs and the Variation in the Active Coils With Load," by D. H. Pletta, S. C. Smith, and N. W. Harrison, Bulletin 24, Engineering Experiment Station, Virginia Polytechnic Institute.

[5] Reference 3, Fig. 7.

Boundary Film Investigations

By SYDNEY J. NEEDS,[1] PHILADELPHIA, PA.

This paper describes attempts made in Dr. Kingsbury's laboratory to detect the influence of boundary surfaces on the viscosity of thin films of lubricant between two optically plane parallel disks. Investigations included both mineral and vegetable oils. Details of two testing machines are given and typical results with each machine are described.

Introduction

THE MANNER in which a liquid in contact with a solid surface is influenced by that surface has been the subject of considerable study in this country and abroad. It appears obvious that the molecules of the liquid actually in contact with and adhering to the surface, or to adsorbed layers on the surface, must necessarily behave differently from adjacent molecules of the liquid in bulk. But the nature of this surface phenomenon and the distance from the surface to which its influence extends are not well understood and experimental studies have given rise to widely divergent opinions on these particular points. Kingsbury (1)[2] pointed out some 37 years ago that the nature of this surface property was probably "an intensified viscosity in that part of the fluid within the region of attraction of the surface molecules of the metal." Hardy and Nottage (2) offer proof "that the attraction field of the solids modify the state of the lubricant throughout a layer many hundreds or thousands of molecules in thickness;" and they place the limit of boundary lubrication (3, 4) at a distance of 0.007 to 0.010 mm (0.0003 to 0.0004 in.), that is, the film must be of that order of thickness if any of the liquid is to be beyond the range of surface influence. Wilson and Barnard (5) found that capillaries as large as 0.3 mm (0.01 in.) diam would gradually clog up under the flow of oils and would close completely if a small percentage of oleic or stearic acid were added to the oil. This clogging was attributed to the gradual building up of adsorbed films on the walls of the capillaries.

In a carefully conducted series of experiments with capillaries of approximately the same diameter as those used by Wilson and Barnard, Bulkley (6) was able to show that the clogging was due to the fact that the oil had not been thoroughly filtered. From further experiments with highly filtered oils in fine glass and platinum capillaries having diameters of the order of 0.01 mm to 0.03 mm, Bulkley concluded "that the thickness of any fixed layer adjacent to the walls is not in excess of two or three hundredths of a micron, or approximately a millionth of an inch." Confirming results have been published by Bastow and Bowden (7), who conclude from measurements of the flow of liquids between optically polished flat plates that no sign of induced rigidity is to be detected in liquids at a distance of 1000 Å [$4(10)^{-6}$ in.] from the surface. There are many other reports in the literature which form, as pointed out by Bowden (8), "a large body of conflicting evidence and opinion as to the range of surface forces."

The present paper reports some of the results of a 4-year study of thin films of vegetable and mineral oils conducted in Dr. Kingsbury's laboratory under his direction. Aside from the problems of filtration, selection of the proper metal for the test surfaces and the preparation of the surfaces, the experimental work divided itself into two distinct parts. Part I of this paper describes attempts to discover indications of increasing viscosity in thin films of oil formed by two plane disks approaching each other under loads of the order of 0.20 lb per sq in. with no tangential motion. Part II describes experiments with plane surfaces approaching under loads up to 800 lb per sq in. with continuous relative rotation of the surfaces. It also describes a method of measuring the minimum film thickness reached under these conditions. Experiments reported in part II are extremely simple and may be readily reproduced. They add clarity to the results of part I and appear to show that steel surfaces have a profound influence on the behavior of lubrication films up to thicknesses greater than 60 millionths of an inch (0.0015 mm); the thickness depending somewhat upon the lubricant. These films have great mechanical strength and are capable of supporting loads at least as great as 800 lb per sq in. Under such loads, the film thickness quickly reaches a constant minimum value which remains unchanged indefinitely even with continuous relative rotation of the surfaces.

The influence of the surfaces on the film manifests itself in what might be termed "directional rigidity," a property, which to the author's knowledge, has not been previously observed. As mentioned, the condition of constant minimum film thickness is indicated by constant torque at a given rpm, provided the rpm is above some minimum value. As long as the film is under continuous shear at a rate above this minimum, it behaves as a purely viscous liquid in the circumferential direction and the torque is proportional to the rpm. In the radial direction, however, the liquid exhibits remarkable plasticity or rigidity since, under heavy loads, it will not flow radially from between the surfaces. Building up of an apparently similar rigidity in the circumferential direction may be observed by stopping rotation and running at infrequent intervals only long enough for a torque observation to be made. The torque will increase rapidly at first and then more slowly, finally reaching an upper limit within 1 to 3 days. This upper limit will be several times greater than the constant torque when the film is free from circumferential rigidity. During the rigid stage, the film will resist considerable circumferential stress without appreciable deformation. If continuous rotation is again resumed, the circumferential rigidity will be gradually removed; the torque will decrease rapidly at first and then more slowly until finally the same constant value is reached when rigidity has entirely disappeared, thus indicating no change in film thickness. In any particular test this cycle may be repeated as often as desired.

Oils used in the earlier tests were rotated for periods up to 50 hr in a clinical centrifuge. A small ultracentrifuge, operating at extremely high speeds, was next constructed and used. In the later tests the samples of oil were removed by pipette from near the centers of containers that had remained undisturbed for several months. It may therefore be assumed that the oils were used in the same condition as commerically produced. Centrifuging of the oils made no apparent difference in test results.

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[1] Service Manager, Kingsbury Machine Works, Inc. Mem. A.S.M.E.

[2] Numbers in parentheses refer to the Bibliography at end of paper.

Contributed by the Special Research Committee on Lubrication and presented at the Annual Meeting, Philadelphia, Pa., December 4-8, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Theoretical

Formulas for two plane, circular, parallel plates, approaching each other under the action of an external force and with the space between the plates filled with a viscous liquid, may be readily developed from Reynolds' (9) basic equation

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\mu\left\{(U_0-U_1)\frac{\partial h}{\partial x}+2V_1\right\}\quad[1]$$

Geometry of the surfaces is shown in Fig. 1. The notation used is as follows:

- a = radius of plane circular plates
- f = shear stress in liquid at surfaces of plates due to relative rotation
- f_r = shear stress in liquid at surfaces of plates due to radial pressure flow
- h = film thickness or uniform distance between parallel plates
- N = revolutions per unit time of rotating plate
- p = pressure at any point s in film above atmospheric or uniform ambient pressure
- r = radial distance from centers of surfaces to point s. Rectangular coordinates of s are x, z
- T = torque due to relative rotation of plates
- t = time required for any given decrease in film thickness
- U = $2\pi rN$ = linear velocity at s due to relative rotation of surfaces
- $U_0 - U_1$ = relative tangential velocity of surfaces
- V_1 = relative normal velocity of surfaces
- W = external load acting on surfaces
- μ = viscosity of the liquid between the surfaces; assumed constant and uniform.

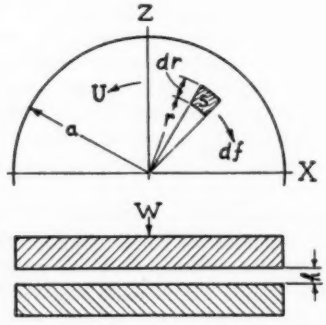


Fig. 1 Mathematical Notation; Plane Circular Plates

Since there is no relative tangential velocity of the surfaces in the case under consideration, $U_0 - U_1 = 0$; and, since the plates are parallel, there is no variation of film thickness; hence, $\partial h/\partial x = 0$. By definition $V_1 = dh/dt$; hence Equation [1] reduces to:

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial z^2}=\frac{12\mu}{h^3}\frac{dh}{dt}$$

a solution of which is

$$p=\frac{3\mu}{h^3}\frac{dh}{dt}(x^2+z^2-a^2)\quad[2]$$

The pressures are symmetrical with respect to the centers of the plates; hence, the total load W will be four times the integrated pressure over each quarter. Therefore

$$W=\frac{12\mu}{h^3}\frac{dh}{dt}\int_0^a\int_0^{\sqrt{a^2-x^2}}(x^2+z^2-a^2)\,dx\,dz\quad[3]$$

Successive integration gives

$$W=-\frac{3\pi\mu a^4}{2h^3}\frac{dh}{dt}\quad[4]$$

and

$$t=\frac{3\pi\mu a^4}{4W}\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right)\quad[5]$$

the time required for the surfaces to fall from h_1 to h_2. If h_2 is small compared with h_1, the initial film thickness may be neglected and Equation [5] becomes

$$t=\frac{3\pi\mu a^4}{4Wh^2}\quad[6]$$

from which

$$h=\frac{a^2}{2}\sqrt{\frac{3\pi\mu}{W}}\sqrt{\frac{1}{t}}\quad[7]$$

Equations [5] and [6] were first developed by Stefan (10) in his studies of the approach of plane, circular disks separated by a viscous liquid. Reynolds' Equation [1], published 12 years later is of great general utility since, in addition to the present case, it is also the basis of modern theoretical treatment of plane and cylindrical bearings.[3]

From (9) the shear stress at the surfaces of the plates, due to outward radial flow of the liquid, is

$$f_r=\frac{h}{2}\frac{dp}{dr}\quad[8]$$

and this stress will be the same on any radius passing through the common axis of the plates. Since $x^2 + z^2 = r^2$, we have from Equation [2]

$$\frac{dp}{dr}=\frac{6\mu}{h^3}\frac{dh}{dt}r$$

and from Equation [4]

$$\frac{dh}{dt}=-\frac{2h^3W}{3\pi\mu a^4}$$

Substituting in [8]

$$f_r=-\frac{2hWr}{\pi a^4}\quad[9]$$

the negative sign indicating that the shear stress opposes flow from between the surfaces. Since the shear stress is maximum when $r = a$

$$f_{r(\max)}=-\frac{2hW}{\pi a^2}\quad[10]$$

The film thickness may also be calculated by rotating one of the plates relative to the other at a known rate and measuring the required torque.

Referring to Fig. 1, the frictional drag opposing rotation at the differential area $2\pi r dr$ is df, and the moment is rdf. If N is expressed in revolutions per minute, the linear velocity U is $2\pi rN/60$. Then for a simple Newtonian liquid

$$df=\frac{2\pi\mu U r dr}{h}$$

[3] Reynolds' original paper contains many misprints, which unfortunately also appear in his "Collected Papers." His expression, similar to Equation [5], for the approach of parallel elliptical plates should have the coefficient 3/2 instead of 3.

and

$$r df = \frac{\pi^2 \mu N}{15h} r^3 dr$$

The total torque

$$T = \frac{\pi^2 \mu N}{15h} \int_0^a r^3 dr$$

from which

$$h = \frac{\pi^2 \mu N a^4}{60T} \quad \text{.....[11]}$$

At any time during a test Equation [11] may be used as a check on Equation [7].

EXPERIMENTAL

I—The Disk Viscometer

The disk viscometer, shown in Fig. 2, was constructed to measure the rate of approach of two plane disks under conditions that would enable a comparison to be made with theory. It consists essentially of two disks A with plane surfaces and the balanced beam B, with a micrometer screw C to measure the movement of a pin fastened in the end of the beam. The upper plane is $^7/_8$ in. diam and is firmly attached to the upper bar D of the rugged steel frame by means of a fitting electrically insulated from the frame. The lower plane is $^3/_4$ in. diam and has a steel ball placed so that the lowest point on its surface is located as near as possible to the center of gravity of the disk. The ball rests on a plane support attached to the beam, thus locating the disk and permitting its plane surface to adjust itself to the upper plane. A thin-steel cup E is provided to prevent loss of liquid flowing from between the approaching surfaces. The beam is supported by the knife-edge F. Load is applied by the $^1/_{10}$-lb sliding weight G and may be varied by changing the position of the weight. When the weight is against the knife-edge fulcrum H, the center of gravity of the beam and all its parts is brought as close to the knife-edge as possible by adjustments of the counterweights J. The cam K is used to raise the loaded end of the beam, thus keeping the surfaces separated until the start of an experiment.

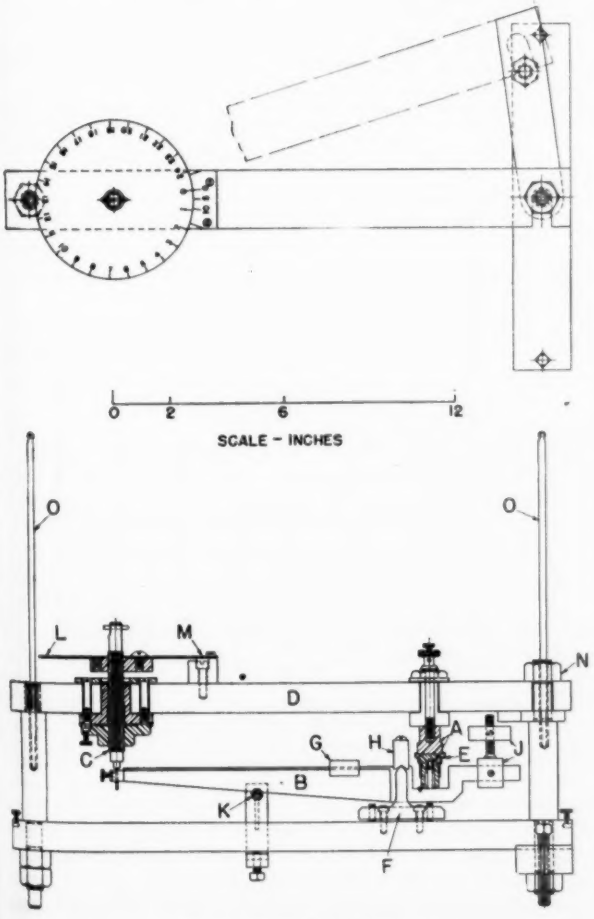


Fig. 2 Disk Viscometer

The micrometer-screw nut is fastened to the upper bar by four screws and electrically insulated with sheet mica. A lower nut is provided to eliminate lost motion in the screw. The micrometer screw is $^1/_2$ in. diam and carefully cut with 40 threads per in. Smooth operation is obtained by lapping the screw and nuts together. After assembly, the spindle in the end of the micrometer screw was lapped optically flat. A disk L, $5^1/_2$ in. diam and graduated into 25 equal numbered divisions, is attached to the upper end of the micrometer screw. Each division is subdivided into tenths and a vernier M is provided for reading to $^1/_{10}$ of each subdivision or $^1/_{100,000}$-in. vertical movement of the screw. The distance from the micrometer screw to the knife-edge is 10 times greater than the distance from the knife-edge to the center line of the plane disks. Each vernier division therefore represents a change in film thickness of one millionth of an inch. Contact between micrometer and beam is indicated by a milliammeter in a circuit closed when the micrometer spindle touches the pin in the beam. A similar circuit is provided to indicate contact between the plane surfaces. By loosening the frame nut N, the upper bar may be swung to one side permitting insertion of the lower disk or removal of the beam.

The viscometer is operated in a wooden box (not shown) covered with aluminum foil and heavy paper for heat insulation. Curtained inspection windows are fitted in the box and the necessary controls are provided for external operation of the instrument. The box is electrically heated by coils operated through a relay and a differential-expansion thermostat capable of holding the temperature constant to within 0.1 F. Viscometer temperatures are measured by thermometers O located in mercury wells in the vertical supports at each end of the instrument and also by a thermometer suspended in the atmosphere near the plane surfaces.

The plane disks were first made of normalized low-carbon steel and later a second pair was made of hardened tool steel. The surfaces were optically polished and flat to better than one millionth of an inch, including the edges. Turned edges were avoided by cementing the otherwise finished disk with beeswax in a ring about $2^1/_4$ in. diam made from the same bar of steel as the disk, and finishing the two as a single piece. The turned edge will then be on the ring and, if the diametral clearance between the disk and ring is of the order of 0.001 in., the plane surface of the disk will be continuous to its edge. After cementing the disks in the rings, the surfaces were ground flat, lapped on lead, and finally polished with optical rouge on pitch. Supports were then placed under the unpolished sides of the disks and the rings gently heated until melting of the beeswax permitted them to drop free, leaving the edges of the disks untouched.

Most of the difficulties encountered in polishing the surfaces to the required flatness proved to be simply questions of technique; eventually the point was reached where the scratching of a pair of surfaces, instead of actually threatening the success of the investigation, occasioned merely a relatively short interruption in the experimental program. However, researches for a



and

$$\text{rate of approach} = \frac{\pi^2 n_0 N}{15h} r^4 dr$$

The total torque

$$T = \frac{\pi^2 n_0 N}{15h} \int_0^{r_0} r^4 dr$$

from which

$$h = \frac{\pi^2 n_0 N a^4}{60T} \qquad [11]$$

At any time during a test Equation [11] may be used as a check on Equation [7].

EXPERIMENTAL

I—The Disk Viscometer

The disk viscometer, shown in Fig. 2, was constructed to measure the rate of approach of two plane disks under conditions that would enable a comparison to be made with theory. It consists essentially of two disks *A* with plane surfaces and the balanced beam *B*, with a micrometer screw *C* to measure the movement of a pin fastened in the end of the beam. The upper plane is $^3/_8$ in. diam and is firmly attached to the upper bar *D* of the rugged steel frame by means of a fitting electrically insulated from the frame. The lower plane is $^3/_4$ in. diam and has a steel ball placed so that the lowest point on its surface is located as near as possible to the center of gravity of the disk. The ball rests on a plane support attached to the beam, thus locating the disk and permitting its plane surface to adjust itself to the upper plane. A thin-steel cup *E* is provided to prevent loss of liquid flowing from between the approaching surfaces. The beam is supported by the knife-edge *F*. Load is applied by the $^1/_{10}$-lb sliding weight *G* and may be varied by changing the position of the weight. When the weight is against the knife-edge fulcrum *H*, the center of gravity of the beam and all its parts is brought as close to the knife-edge as possible by adjustments of the counterweights *J*. The cam *K* is used to raise the loaded end of the beam, thus keeping the surfaces separated until the start of an experiment.

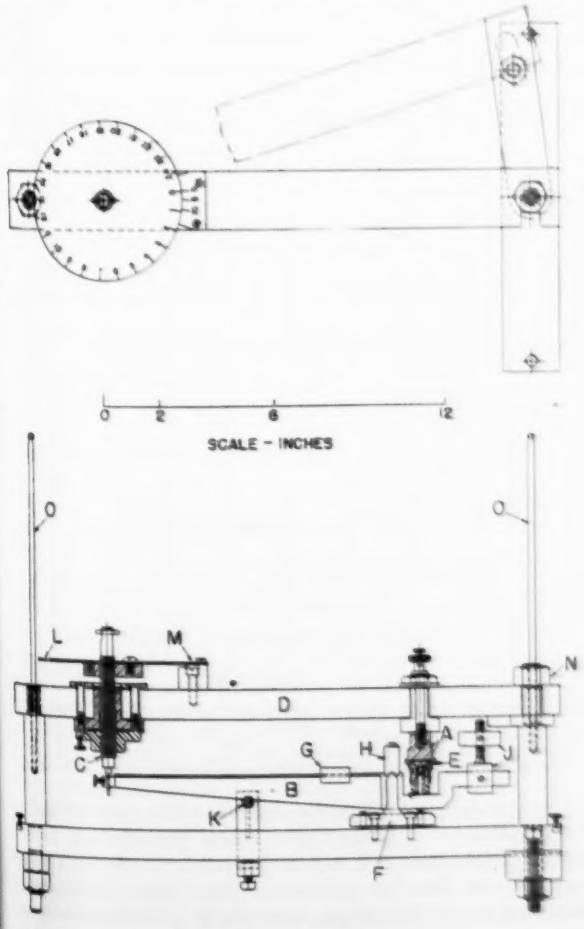


Fig. 2 Disk Viscometer

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satisfactory metal of which to make the disks were not so successful. There appears to be no metal which, when polished and examined even under low powers of the microscope, will present a surface of uniform appearance even remotely approaching that of well-polished glass. The metal surfaces invariably contained pits or nonmetallic inclusions, iron carbides or traces of manganese in the form of gray spots with well-defined but irregular edges. At times traces of corrosion were also visible somewhat resembling nonmetallic inclusions. No one steel of good enough grade to be considered for optical flats is likely to contain all the above flaws, but, by the very nature of the constituents of steel and its method of production, nonhomogeneous structures are to be expected on any plane. Experience finally seemed to indicate that the best surfaces were produced by high-speed tool steel or a high-chrome tool steel. The chrome steel is corrosion-resistant and will hold its shape over long periods of time but it warps badly during the hardening process and is somewhat difficult to polish. High-speed tool steel gave the most uniform surface of all metals investigated, and it was therefore used in the later experiments.

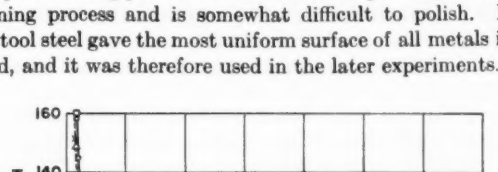


FIG. 3 APPROACH OF PLANE CIRCULAR PLATES; TESTS WITH CASTOR OIL

(Plates, 3/4 in. diam; oil temperature, 92.2 F; $\mu = 50.8(10)^{-6}$ lb sec per sq in.; load, 0.199 lb. The four tests were made under identical conditions.)

After polishing, the disks were prepared by washing with pads of absorbent cotton moistened with carbon tetrachloride. When all visible traces of contamination had been removed the disks were gently rubbed with dry absorbent cotton and finally touched with a fine camel's hair brush to remove any tiny pieces of lint left by the cotton or other contamination deposited from the air. The disks were then placed in the viscometer and the upper bar locked in position. Two carefully cleaned platinum wires approximately 0.001 in. diam were placed parallel across the lower plane approximately equidistant from its center. The beam was gently lowered until the surfaces were separated only by the wires. The box was then closed, outside operating attachments were connected, and the heat turned on.

When the temperature had reached the desired constant value, a micrometer reading was taken with the wires in place. If the diameter of the wires is known, it is then a simple matter to compute the micrometer zero, which is the micrometer reading when the disks are in actual contact. This seems to be the only practicable method of determining the zero reading. If the surfaces were brought together without the wires, a false reading would probably result due to possible surface contamination or to adsorbed air or moisture films on the surfaces. Attempts to obtain optical contact of the surfaces by wringing them together would doubtless result in scratches and would also introduce the problem of separation after the zero reading had been made. It is probable that the small-diameter wires cut through any adsorbed layers on the surfaces and came into actual contact with the metal. Assuming this to happen, it then became necessary to measure the diameters of the wires in order to establish



FIG. 4 APPROACH OF PLANE CIRCULAR PLATES; TESTS WITH OLIVE OIL

(Plates, 3/4 in. diam; oil temperature, 92.2 F; $\mu = 7.02(10)^{-6}$ lb sec per sq in.; load, 0.199 lb. The four tests were made under identical conditions.)

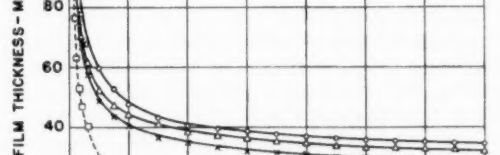


FIG. 5 APPROACH OF PLANE CIRCULAR PLATES; TEST WITH DTE LIGHT MINERAL OIL

(Plates, 3/4 in. diam; oil temperature, 92.3 F; $\mu = 5.23(10)^{-6}$ lb sec per sq in.; load 0.199 lb. The four tests were made under identical conditions.)

the micrometer zero. This was done by a filar micrometer used with a microscope. Many measurements were made at various magnifications with ordinary and monochromatic illumination and the wire diameter determined from these measurements. Assuming perfect microscopy, however, the limitations of the microscope would leave the final result open to an uncertainty of the

order of $^1/_2$ wave length of light and this is considerably greater than the film thicknesses it had been hoped to investigate.

After noting the zero reading, the micrometer screw was raised so that the beam could be lifted by the cam. The wires were then removed and oil introduced between the separated surfaces from a pipette. This was done with but slight temperature disturbance through a hole in the top of the box. When the temperature had again reached the constant value, the time was noted, the cam moved from beneath the beam, and the surfaces began to approach.

Observed rates of approach of the planes are compared with theory in Fig. 3 for the case of castor oil, and for olive oil and DTE light in Figs. 4 and 5, respectively. Calculations of rates of fall are based on the known bulk viscosities of the three oils and are shown by the broken lines. In general, the observed rates of approach are a fair approximation to theory but excepting the experiments with castor oil the film thicknesses at the end of 1 to 2 hr are found to be considerably greater than the calculated values. Satisfactory duplication of tests was found impossible, even though the lubricants were carefully centrifuged before using. After 8 hr, little if any reduction was observed in film thickness and, at the end of 24 hr, the film thickness was invariably found to have increased slightly above the reading at 8 hr. It is interesting to note that after a few hours the measured film thicknesses were of the same order regardless of the differences in bulk viscosities of the oils. Apparently the films were not behaving strictly according to theory. The probable causes of these discrepancies will appear from the results described under part II.

It was also observed that, when the films reached a thickness of approximately $(10)^{-4}$ in., they began to show indications of carrying current under a voltage of 0.025 volts. At thicknesses of about 60 $(10)^{-6}$ in., the resistance of the film had dropped to the point where it carried small currents as readily as the copper wire of the circuit, that is, the ammeter showed the same current flowing with and without the film in the circuit.

With loads of about 0.2 lb per sq in., the final film thicknesses were of the order of 30 to 40 millionths of an inch, subject to errors of approximately 10 millionths of an inch. Because of this high percentage of uncertainty, these figures are not entirely satisfactory. Nevertheless the above experiments indicate that there is some limiting film thickness reached by plane surfaces approaching under load. The limits of boundary lubrication placed by Hardy are considerably greater than those observed in the foregoing experiments.

II—Continuous Rotation

It is obvious from the results cited that loads much greater than a fraction of a lb per sq in. are required to bring the surfaces to within a few millionths of an inch of actual contact. The apparatus shown in Fig. 6 was therefore constructed and mounted in the same drill press used with the tapered-plug viscometer and the journal-bearing testing machine (11, 12). The lower disk A is located by the fitting B, and held against rotation by a set screw. This assembly is located by the ball bearing C and supported by the torsion rod D. Angular twist of D, measured by a graduated curved scale E, gives an accurate measure of the torque, due to rotation of the upper disk A'. The torsion rod is supported against buckling by a split bushing F set in a ball bearing to avoid binding at the support. The torque-indicating arm is counterbalanced by the weight G and vibrations of the arm are damped by a pan of oil H. At extremely low speeds, the movement of the indicating arm may be very small; hence a microscope has been found convenient for detecting minute deflections. The upper disk is held central and rotated by the sleeve J, the bore of which has been lapped to a close sliding fit on the spindle K. Load is applied by compression of the spring L which is held

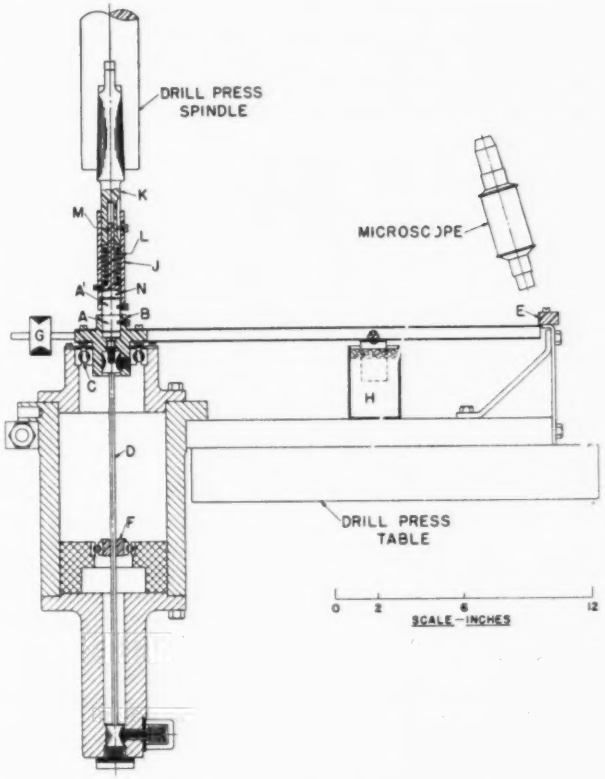


Fig. 6 Torsional Disk Viscometer

central by the plunger M. The lower surface of the plunger is hardened and ground flat to form a contact surface for the hardened conical point of the disk support N, thus providing for adjustment of the upper disk. Both plane disks are supported on annular rings, turned on the backs of the supports. Assuming a parabolic pressure distribution in the oil film, the diameter of the ring is such as to provide support at a point directly under the center of gravity of each radial half section of the parabola. The spindle K is fitted into the drill-press spindle in the usual manner and electrically insulated by paper.

The drill press is driven by a $7^1/_2$-hp, 1200-rpm synchronous motor through a chain-and-change-gear transmission. Friction clutches are provided for starting, stopping, and reversing. An independent drive for low speeds is furnished by a small synchronous motor through a speed reducer. Speeds as low as 0.0016 rpm are thus obtained. The loading springs were calibrated in position and the spindle graduated at the top of the sleeve. A spring is provided to cover the range up to 100 lb per sq in.; a second spring is used for loads from 100 to 400 lb per sq in.; and the heaviest spring possible with the present apparatus gives loads up to 800 lb per sq in.

The hardened plane disks, $^7/_8$-in. diam, were prepared as previously described, except that the final polish was made on lead instead of pitch. After cleaning, they were covered with a heavy film of oil, placed in the machine, and brought together until the oil formed a single film approximately $^1/_{16}$ in. thick. At zero time, rotation was begun and load applied. Almost immediately most of the oil was pressed from between the surfaces, forming a thick ring around the edge of the film as shown in Fig. 7; thus protecting the film from external contamination. The meniscus of this ring introduced a slight negative pressure at the edge of the film.

Assuming that the plane disks remain coaxial, it may be assumed that the rate of approach is independent of relative rota-

tion. A test was therefore made with olive oil at a load of 100 lb per sq in. with the upper surface continuously rotating at 11.06 rpm. Results of the test are shown in Fig. 8. The broken line represents torque calculated on the basis that the planes are approaching at the rate indicated by Equation [7], and the circled

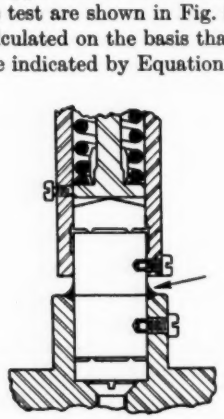


FIG. 7 OIL MENISCUS AT EDGES OF PLANE DISKS; TORSIONAL DISK VISCOMETER

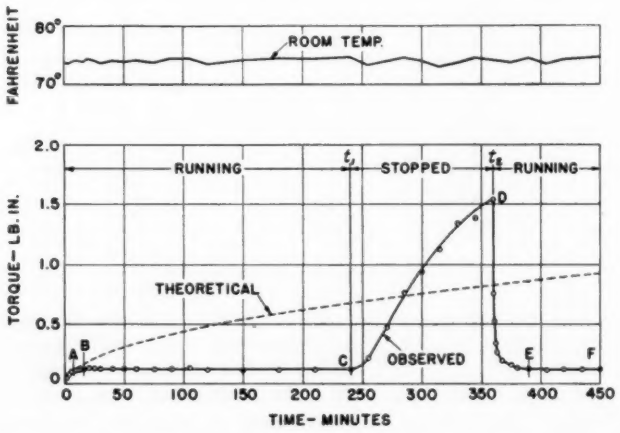


FIG. 8 TEST WITH OLIVE OIL; TORSIONAL DISK VISCOMETER
(Plane disks, 3/8 in. diam; 11.06 rpm; load, 100 lb per sq in.; average temperature, 77 F; $\mu = 10.1(10)^{-6}$ lb sec per sq in.)

points are actual torque observations. During the first 8 min, over the range *OA*, the observed torque agrees with theory. This indicates that, while the film is relatively thick, the rate of approach is the same as calculated. It is interesting to note, however, that after 16 min the torque becomes constant over the range *BC*, showing that the film thickness has reached a minimum value and approach of the surfaces has ceased. If rotation is stopped at time t_1 and resumed at stated intervals only long enough to take a torque observation, it will be found that the torque is progressively increasing. After rotation has been stopped for some time, it will also be found that a slight angular movement of the upper test surface will move the torque indicator from zero and it will not return, thus showing rigidity in the film. That the rigidity increases is shown by the continuous increase in torque with time over the range *CD*. If at time t_2, continuous rotation is resumed, the torque will drop rapidly at first and then more slowly until the previous constant value is reached at *E*. Rotation may be continued indefinitely over the range *EF* with no variation in torque, unless there is a change in film viscosity, due to variation of room temperature.

Fig. 9 shows the results of a continuous run of 52 hr with olive oil at 11.06 rpm and 40 lb per sq in. load, followed by periods of building up and removal of rigidity. Observations of torque versus speed at any time during the constant-torque period fall on a straight line passing through the origin, provided the speed range is low enough to avoid frictional heating of the film. Fig. 10 shows such a curve taken at 50 hr in the test shown by Fig. 9. When rigidity has not been entirely removed by rotation, the torque-speed curve may be a straight line or a curve which, at zero speed, gives a value of torque proportional to the rigidity. It is obvious that torque observations with rigidity present will not be consistent since shear reduces rigidity, thus reducing the torque. There is, however, some minimum speed below which rigidity will not be entirely removed. No attempts were made to discover this limiting speed, but it was observed with olive oil at 40 lb per sq in. load that rigidity would build up very slowly indeed at 0.9 rpm.

After these tests the surfaces appeared to be exactly the same as at the start. There were no traces of scratching or metallic contact even at high magnifications. The first indication that the film will carry small electrical currents occurs at calculated film thicknesses of about $(10)^{-4}$ in. Before constant-torque conditions are reached, the circuit with the film in series carries the same current as when the film is short circuited by a copper wire. Rigidity does not appear to influence the resistance of the film.

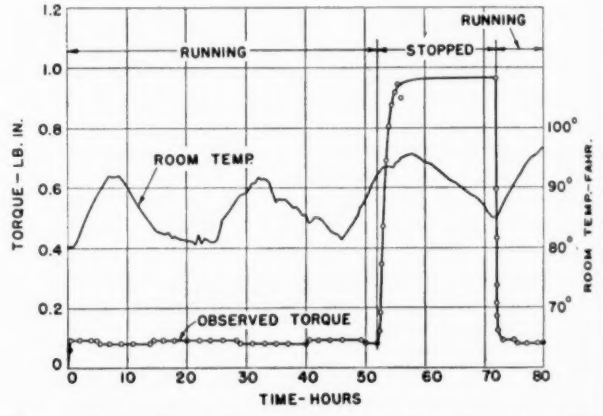


FIG. 9 EIGHTY-HOUR TEST WITH OLIVE OIL; TORSIONAL DISK VISCOMETER
(Plane disks, 3/8 in. diam; 11.06 rmp; load, 40 lb per sq in.; average temperature, 87.9 F; $\mu = 7.80(10)^{-6}$ lb sec per sq in.)

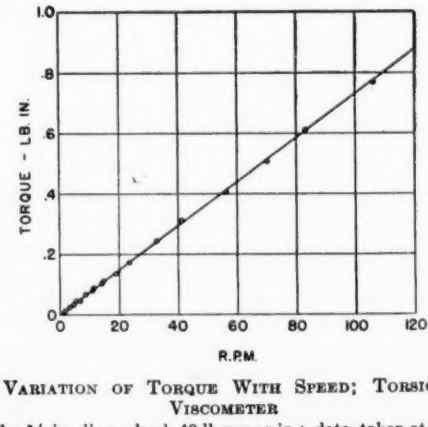


FIG. 10 VARIATION OF TORQUE WITH SPEED; TORSIONAL DISK VISCOMETER
(Plane disks, 3/8 in. diam; load, 40 lb per sq in.; data taken at 50 hr in test shown in Fig. 9.)

Previous reference has been made to the excess oil from the very thick starting film, flowing out under the applied load and forming a seal around the edge of the film. The meniscus of this seal is in such direction as to cause a slight tension at the edge of the film. This is of little importance except to point out that

there is no pressure restricting flow; in fact, the results are the same if all excess oil is removed.

To measure the final film thickness a $^3/_{32}$-in-diam pin with a spherical babbitt tip was fitted into a hole in the lower disk, as shown in Fig. 11. The descending upper plane forces the pin into the hole and the height of its tip above the lower plane is measured after the test by counting the interference fringes (13) between the surface and a small optical flat in intimate contact with an edge of the plane surface and the pin. Film thicknesses thus measured after tests with several oils at loads up to 800 lb per sq in. are shown in Fig. 12. For example, with Atlantic superheat cylinder oil and a load of 300 lb per sq in., the torque reached

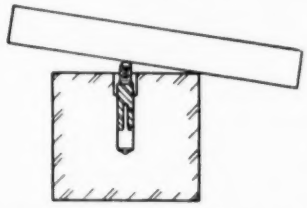


FIG. 11 PIN METHOD OF MEASURING MINIMUM FILM THICKNESS

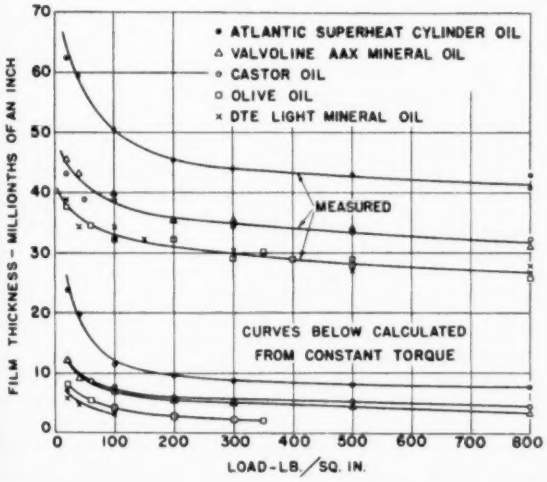


FIG. 12 MEASURED AND CALCULATED MINIMUM FILM THICKNESS WITH VARIOUS OILS; TORSIONAL-DISK VISCOMETER

(Plane disks, $^7/_8$ in. diam; 11.06 rpm. Atlantic superheat cylinder oil; average temperature, 72.4 F; $\mu = 670(10)^{-6}$ lb sec per sq in. Valvoline AAX mineral oil: average temperature, 71.2 F; $\mu = 113(10)^{-6}$ lb sec per sq in. Castor oil: average temperature, 72.2 F; $\mu = 115(10)^{-6}$ lb sec per sq in. Olive oil: average temperature, 73.6 F; $\mu = 11.04(10)^{-6}$ lb sec per sq in. DTE light mineral oil: average temperature, 72.4 F; $\mu = 9.42(10)^{-6}$ lb sec per sq in.)

a steady condition one hour after the load was applied and remained constant during the next hour. The test was then discontinued, the surfaces separated, washed with carbon tetrachloride and finally with alcohol. The optical flat showed the central pin to be $44(10)^{-6}$ in. high and this is the final constant film thickness under the test conditions. From the known bulk viscosity of the oil and the observed torque, the calculated film thickness from Equation [11] is found to be $8.8(10)^{-6}$ in. Variation of speed shows the torque to be directly proportional to the speed and there is no rigidity in the film in the direction of motion as long as rotation is continued. The lubricant is therefore behaving as a purely viscous fluid in so far as the direction of rotation is concerned. From the observed torque and the measured film thickness, the effective viscosity in the direction of motion has been increased 44/8.8 or five times the ordinary bulk viscosity. In the radial direction, however, the effective viscosity has been apparently increased to infinity, since no oil will flow from the film $44(10)^{-6}$ in. in thickness supporting a load of 300 lb per sq in. This may be explained by the fact that rigidity will build up unless the rate of shear is above some critical value. The speed of rotation is sufficient to prevent the formation of rigidity in the direction of motion except perhaps to a small distance from the center. The rate of shear in the radial direction, however, is due to radial oil flow only and this decreases as the surfaces approach. Apparently when the rate of shear in the radial direction falls below the critical value, rigidity rapidly builds up, thus stopping further radial flow from the film. The surfaces are therefore unable to approach and a minimum film thickness is established.

Preparations for the foregoing tests were made under ordinary room conditions; hence, we may not exclude the possibility of contamination from the atmosphere. It is also probable that the commercial lubricants used were not chemically clean. A mass of particles of undefined nature would hold the surfaces apart, but the experimental evidence is against this possibility. If the particles were hard enough to hold the surfaces a definite distance apart the intensity of pressure on the larger particles undoubtedly would be great enough to cause scratching. If the particles were soft enough to change shape under load, they would also be expected to break down under continuous motion and permit closer approach of the surfaces. This would be evidenced by increasing torque. The presence of particles would also tend to increase friction to values well above those observed. Probably the most convincing argument against surface separation by foreign particles is the fact that, immediately after rotation has ceased and before rigidity has become appreciable, if the torque indicator is moved from its zero position, it will immediately spring back. This would be impossible if the surfaces were separated by particles under sufficient pressure to support the load. It seems beyond question that the surfaces are not held apart by particles in the film and any particles present must be smaller than the distance between the surfaces.

The fact that time is required to build up rigidity in the film and time is also required to remove it by motion suggests molecular rearrangements in the film. In such cases, it is reasonable to expect that both building-up and tearing-down actions would be rapid at first and then proceed more slowly as constant conditions are approached. Torque observations indicate that some such phenomenon actually occurs. It would also be expected that the building up and removal of rigidity would be more rapid in the less viscous fluids where molecular motion is more free. This is also observed to be the case. But the less viscous the oil the thinner the film under a given load, and presumably surface influence would become more pronounced as the film thickness is reduced.

Referring again to Fig. 12, the calculated values of film thickness were based on the known bulk viscosity of the oil and the constant torque along the range *BC* or *EF* of the curve in Fig. 8. At loads up to 350 lb per sq in., constant-torque conditions with olive oil were reached in which the friction coefficients were of the order of 0.007. At loads above 350 lb per sq in. with olive oil, and above 100 lb per sq in. with DTE light, no constant-torque conditions could be found. For the first few minutes of the test, the torque followed the theoretical law and then gradually increased to comparatively high values where it remained unsteady, fluctuating between values giving friction coefficients from 0.15 to 0.30. Measured film thicknesses under these conditions, however, fell on a smooth curve consistent with previous measurements at lower loads and constant-torque conditions. It is interesting to note that lard oil, which has about the same viscosity as olive oil, carried a load of 800 lb per sq in. under constant-torque conditions. Oleic acid and a mineral oil of somewhat higher viscosity than DTE light also carried the 800-lb per sq in.

load under constant-torque conditions. This is also true of white Russian mineral oil, the latter presumably nonpolar. Addition of 2 per cent of an ester of phosphoric acid to the above mineral oil made no difference whatever in torque or final film thickness.

Upon separation of the surfaces after a test under constant-torque conditions, the residual films appeared to be evenly spread over the entire area and of variable color depending somewhat upon the angle of view. Immediately after separation, castor- and olive-oil films showed the effects of surface tension by drawing up into tiny circular droplets. These arranged themselves in a geometrical pattern not unlike honeycomb but lacking its regularity of shape. About 10 min were required for the complete formation of the pattern. The final appearance was that of a piece of spider web dropped over the surface. Russian

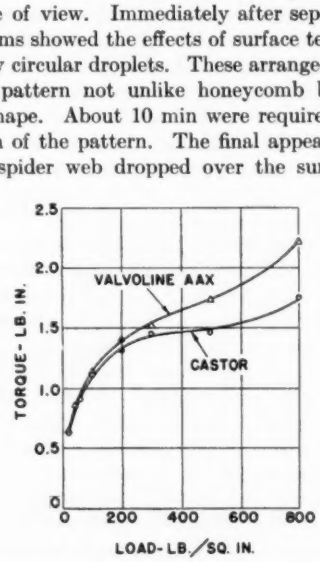


FIG. 13 VARIATION WITH LOAD OF CONSTANT TORQUE AT A GIVEN LOAD; TORSIONAL DISK VISCOMETER
(Plane disks, 7/8 in. diam; 11.06 rpm; $\mu = 115(10)^{-6}$ lb sec per sq in.)

mineral oil and lard oil displayed similar tendencies. The pattern formed by oleic-acid film was much finer than the others and much more rapidly arranged. The commercial mineral lubricating oils showed no similar tendencies; the residual films remaining fixed in the presence of air and thoroughly wetting the surfaces.

In tests with olive oil at the heavier loads, where no constant-torque conditions could be found, the forces set up by rotation were strong enough to tear the films away from the surfaces and break them up into tiny white fragments. These fragments were so small that in some cases the separated surfaces had the appearance of being streaked with paraffin wax. At high magnification, the tiny white fragments were seen to be spots of oil exhibiting characteristic color fringes at their edges, but instead of being spherical segments like the droplets formed by surface tension, they were of irregular shapes with three to eight sides. Generally some round droplets were also present. Even though the film had broken up, the surfaces invariably showed no signs of scratching or wear. Apparently metallic contact had not occurred even though the friction coefficients were as high as would be expected with unlubricated surfaces. Similar tests with DTE light mineral oil generally resulted in faintly scratched surfaces or circular bands visible because of a higher polish than the remainder of the surfaces. Apparently the mineral oil was not as clean as the olive oil and the particles present were hard enough to scratch in the former case and soft enough to give a polishing effect in the latter. The films showed no tendency to break away from the metal as in the case with the olive oil.

Since the ordinary bulk viscosities of castor oil and Valvoline AAX are closely the same at the test temperature, and the measured film thicknesses are about the same, it would be expected that the same torque would be found at the same loads and speeds with the two oils. From Fig. 13 this is seen to be true at low loads but, as the load is increased, the mineral oil gives greater friction than castor oil. At 800 lb per sq in. the friction with the mineral oil is approximately 27 per cent above that with castor oil. Increased viscosity due to film pressure (12) would account for about 8 per cent greater friction with the mineral oil. Corrections have been made for the slight differences in room temperature at which the various tests were made.

DISCUSSION AND CONCLUSION

Results of part I indicate that under very light loads a minimum film thickness is reached which is about 30 to 40 millionths of an inch. The order of these measurements is substantiated by the direct measurement at heavier loads described in part II. It now appears that the apparent increase in film thickness observed overnight in part I might possibly be due to the building up of film rigidity. From the results of part I, it appears that, at film thicknesses of about $(10)^{-4}$ in., the electrical resistivity of the film began to decrease. At thicknesses of about $60(10)^{-6}$ in. the film carried small currents as readily as the copper wire in the circuit. This also agrees with similar observations in part II.

The comparatively simple technique of the experiments described in part II and the ease with which results may be reproduced, offer an attractive method for further study of friction in thin films. The method is free from the complications and uncertainties introduced by abrasion and wear. An improved method of measuring film thickness is desirable.

It has been shown that, when two optically smooth plane steel surfaces are brought together in the presence of oil in such manner that the formation of a parallel film is possible, the distance between the surfaces will reach some definite minimum value depending upon the viscosity of the oil and the external force acting upon the surfaces. At room temperature, with surfaces 7/8 in. diam, this minimum film thickness varies from about $30(10)^{-6}$ in. with olive oil to about $60(10)^{-6}$ in. with a heavy cylinder oil. The minimum film thickness is not greatly changed by loads up to 800 lb per sq in. of surface area.

The nature of this separating film is not known but experimental evidence indicates that the proximity of the metal surfaces has a profound influence upon the forces normally acting between the molecules of the liquid. This gives the film a high degree of mechanical stability, as evidenced by its ability to support considerable load without radial flow. The film also assumes the curious property of directional rigidity which seems to depend upon the direction, rate, and duration of shear.

At the present time there is no direct experimental evidence that rigidity is due to oriented molecules. But the rate at which rigidity builds up and the rate at which it is removed by shear points to the possibility of molecular rearrangement. If molecular orientation is due to surface influence it would be expected that the plane of least influence would be midway between the surfaces. The molecules at the center would be the first dislodged by shear thus causing a rapid reduction in friction. Resistance to dislodgment would increase with each molecular layer removed since the orientating forces would become stronger as the surfaces are approached. This would be evidenced by a decrease in the rate of torque reduction. Eventually a level would be reached where the orientating forces holding the molecules in place would be sufficiently strong to balance the shearing forces and dislodgment of molecules would cease; hence, the observed condition of constant torque at calculated film thicknesses well below the actual distances between the surfaces.

From the foregoing may be visualized a film composed of a fixed layer or layers of molecules firmly attached to each surface, and a central layer in which the molecules are kept free of surface influence by continuous shear. The central layer is there-

fore in the same molecular state as the liquid in bulk as far as the circumferential direction is concerned. Direct experimental proof of the existence of rigidity in the radial direction is lacking. But the fact is that, if the liquid in the central part of the film is free, as indicated by absence of circumferential rigidity, it does not flow radially under the external load. Hence, there must be some restraining radial force.

It was found that at loads above 350 lb per sq in. with olive oil and above 100 lb per sq in. with DTE light, no constant low-torque conditions could be established at 11.06 rpm. It would be interesting to investigate the effect of speed on these limiting loads. In fact, an explanation of the breaking up of films under loads too great to permit constant low-torque conditions would be most interesting indeed.

Liquids other than lubricants should be investigated for film-forming characteristics. Olive oil and DTE light should be highly filtered and the experiments repeated. Filtering should be done by the methods employed by Bulkley and also by centrifugal methods, since there is a possibility that Bulkley's methods might remove the components responsible for rigidity. It would also be interesting to discover if the diameter of the plane surfaces has any influence on final film thickness.

Finally, a study of polished metal surfaces would undoubtedly result in improved methods of producing smoother and truer finishes.

Acknowledgments

This study was made possible through Dr. Kingsbury's generous interest in the fundamental problems of lubrication. The author feels himself most fortunate in having his advice and encouragement during the entire course of the study, as well as his active leadership during its earlier stages. Constructive criticism of the manuscript by Mayo D. Hersey is also greatly appreciated.

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Discussion

C. H. Bierbaum.[4] After reading the paper, the thought seems to persist that the oils tested were oxidized, at least so in a limited degree and that, therefore, the researches of Langmuir[5] seem to have a direct bearing. His findings are that the oil molecules are columnar in form and that, in the oxidizing process, one end remains hydrophobic and the other end to which the oxygen is attached becomes hydrophilic or metallophilic, so that these molecules will orient themselves perpendicularly upon a metal surface.

It was the writer's observation that by oxidizing an ordinary motor oil by bubbling pure dry air through it at the temperature of boiling water, for say 2 hr, the ultimate load-carrying capacity of this oil was increased by more than 4000 lb per sq in. of projected bearing area. This effect seems entirely due to the increased tenacity with which the oriented oil molecules attached themselves to the metal surfaces, an adsorbed effect.

Oils oxidized in the foregoing manner have a distinctly corrosive[6] action, in varying degrees, upon the different metals ordinarily used for bearing purposes. It seems safe to assume that the chemical affinity of such modified oils is greater for the more corrosive materials than for the less corrosive; as an illustration, silver, a more noble metal, is not corroded in this manner, which is one reason why in the pure state it has not given a better account of itself for bearing purposes.

Fortunately, it has been found that the full benefit of this oxidizing of oils can be taken advantage of without having the deleterious corrosive effect and, at the same time, be made into a positive inhibitor of corrosion, by the elimination of the corrosive by-product.

It would be interesting to know whether any oxidized oil molecules were present in the tests described in the paper; especially so with the mineral oils, since "the residual film remained fixed in the presence of air and thoroughly wetted the surfaces." Very interesting results can be expected in future tests with different materials for the two bearing surfaces.

The electrical conductivity of the film seems in keeping with the experience in the breaking down of polarized oil in electric transformers, a strong suggestion that oxidized molecules were present.

The fact that the addition of 2 per cent of ester of phosphoric acid had no effect either on the torque or film thickness in the experiments described seems in itself unaccountable, except for the possible presence of an oxidized oil film. The results seem to show that boundary film lubrication may be divorced completely from the properties of the lubricant as a whole.

High-power photomicrographs of the bearing surfaces before and after the tests would have been desirable.

Mr. Needs is certainly to be congratulated upon the accuracy and refinement of his tests. His paper shows distinct progress on the subject of lubrication.

L. J. Bradford.[7] The explanation advanced by the author for the difference between the calculated and measured film

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[5] "Constitution and Fundamental Properties of Solids and Liquids," by Irving Langmuir, *Journal*, American Chemical Society, vol. 39, 1917, p. 1848.

[6] "Corrosion Effects of Lubricants Upon Bearing Surfaces," by Christopher H. Bierbaum, *The Iron Age*, August 31, 1933.

[7] Professor Machine Design, Pennsylvania State College, State College, Pa. Mem. A.S.M.E.

thicknesses shown in Fig. 12 of the paper is attractive, but not wholly convincing. We should expect the extent to which the molecules were detached from their static positions to be dependent upon the force with which they were held, and upon the distorting force applied to them. This latter will in turn be proportional to the rate of shear at the boundaries of the active portion of the film. If we assume that the viscosity of the oil in the active portion is the same as that of the oil in bulk, we may then take the distorting force as inversely proportional to the ordinates of the curves of calculated film thickness. It would be greatest at heavy loads and least at light loads. We should, therefore, expect the static, oriented portions to be thickest at the light loads and thinnest at the heavy.

An examination of the curves shows that this is true. The difference between the measured and calculated thicknesses, which we may take as representing twice the thickness of the oriented, static layer, is greatest at the light loads and least at the heavy. We should, however, expect the orientation to be more vigorous with a substance which is decidedly unsymmetrical in molecular structure, than one with greater symmetry.

The observations recorded in Fig. 12, do not sustain this expectation, for curves for Valvoline, a mineral oil and castor oil are almost identical, while that for Atlantic superheat cylinder oil shows a markedly greater difference between the measured and calculated thicknesses. Apparently viscosity is still associated in some way with these phenomena.

Figs. 8 and 9, showing marked increases in the torque if the apparatus is allowed to stand, suggest that the orientation takes time to form, but that it breaks down quite rapidly.

F. J. Villforth and the writer repeated the stop-and-start experiments, using a tapered-plug viscometer and olive oil. Clearances much greater than those mentioned in the paper were employed, in order to get some idea of the distance to which these surface effects were felt.

With a clearance of 0.0055 in. an increase in torque of about 10 per cent was found when the apparatus was allowed to stand for several hours. When the clearance was reduced to 0.0033 in. the increase was found to be about 20 per cent. A further reduction to 0.00164 in. raised the increase to about 25 per cent. These results were obtained with rates of shear of 786, 978, and 1950 reciprocal sec, respectively. There was also some indication that after a rest the torque first increased and then decreased to a value below that at the start.

If the foregoing results can be validated by subsequent and more extensive experimental work, they would indicate that surface effects extend to distances which are truly enormous in terms of molecular dimensions.

RONALD BULKLEY.[8] The methods of test described in this paper are refreshingly novel and possess great possibilities for the further elucidation of thin-film phenomena. In attacking related problems in the past, several investigators have drawn conclusions regarding the nature of liquid films as close as a few millionths of an inch to a solid wall, but in most cases their actual experiments have been made on films whose thickness has been perhaps 200 times this amount. Now, the author has been able to work with actual films only 0.00002 or 0.00003 in. in thickness and truly this is a great stride forward in the art of thin-fiim experimentation. His conclusions, however, are not in accord either with classical theory or with the conclusions of several previous experimenters in the field.

The concept of long-range forces which increase the viscosity of liquids out to distances of 0.0001 in. or more was championed 15 or 20 years ago by Sir Wm. B. Hardy. In 1922, Wilson and Barnard published experiments purporting to show that oily liquids of polar characteristics, e.g., oleic acid, could be solidified by these forces clear out to distances of 0.005 or 0.006 in. from a solid wall. For 8 or 10 years these results were accepted and widely quoted by many lubrication engineers. Orthodox physicists and chemists pointed out, however, that on the basis of their theoretical calculations, molecular influence can extend outward to not more than about 0.0000005 in.

In 1931, the writer repeated Wilson and Barnard's experiments and discovered that the clogging of capillaries which they attributed to solidification of the liquid was due to lint and foreign matter in the oil. In the same year two of Hardy's own students, Bastow and Bowden, published a paper,[9] giving the same explanation for Hardy's anomalous long-range results, namely, dust and lint in the liquid. In a paper 2 or 3 years later, the same men described viscosity experiments conducted in an apparatus not too different from the author's disk viscometer. These experiments completely confirmed the writer's earlier work, indicating, as he had claimed, that no rigid layer existed on a solid wall out to distances greater than 0.000001 or 0.000002 in.

It is difficult indeed to reconcile these results with those now found by the author. Another difficulty arises from the fact that even Hardy and Wilson and Barnard did not postulate or find any orientating influence for ordinary hydrocarbons or pure petroleum oils, but only for polar materials or materials possessing good oiliness, such as lard oil or oleic acid. The author, however, finds just as pronounced an effect for a mineral oil as for castor oil and olive oil.

Moreover, his conclusions do not seem always to be justified by the data he presents. For example, of the 12 runs he made on 3 oils in the disk viscometer, 5 agreed closely with the theoretical curves. The other 7 were wide of the theoretical curves and the author notes as a general comment that "satisfactory duplication of tests was found impossible." Yet notwithstanding the narrow margin between the number of curves which did and those which did not agree with theory he concludes, contrary to theory, that "the above experiments indicate that there is some limiting film thickness reached by plane surfaces approaching under load." He places the thickness of the limiting film reached under these conditions at about 0.00003 or 0.00004 in.

In the second viscometer, much higher loads are used and the limiting film thickness is, therefore, reached in a much shorter time. The thickness of the film, however, is substantially the same as in the first viscometer, notwithstanding the fact that the loads may be some 4000 times as great. It is hard to imagine what kind of material the oil has been converted into in this relatively thick layer to permit the layer to remain as thick under 800 lb per sq in. attempting to squeeze the oil out as under only 0.2 lb per sq in. The picture the author draws calls for the intensification of viscosity to be greatest at the wall where the forces are greatest and to diminish as the distance from the wall is increased until at last out in mid-stream, 0.000015 or 0.00002 in. away, the viscosity is the normal bulk viscosity. Would we not expect heavy loads to squeeze out more of these partially solidified layers than light loads and thus to give a thinner film? Maybe we could not expect the light load to give a film 4000 times as thick as the load which is 4000 times as great, but should we not expect the film under the light load to be, say, 1000 times as thick or maybe 100 times as thick or at least 10 times as thick? In the author's experiments, the films are of the same thickness regardless of load. This seems to suggest some limitation, not yet discovered, inherent in his apparatus or method

[8] Haddon Heights, N. J.

[9] "On the Contact of Smooth Surfaces," by S. H. Bastow and F. P. Bowden, *Proceedings* of the Royal Society, vol. 134, 1931, pp. 404–413.

of experimentation which leads him to erroneous conclusions.

Failing, to date, to find any such limiting conditions, the author's only other recourse, naturally, has been to invent a new concept to explain his data. This has taken the form of a layer of material some 500 or 1000 molecular lengths in thickness which possesses the surprising property of being a solid in one direction and a simple viscous liquid at the same time in another direction.

Regarding this revolutionary concept of directional rigidity, it should be observed that every investigator has a right to invent new concepts when his data are incapable of explanation in terms of old concepts. The author has found it necessary to invent the concept of directional rigidity in liquids which up till now have been regarded as possessing uniform properties in all directions. Until some one can explain his results in a more simple or more conventional manner his new concept must stand.

J. R. CONNELLY.[10] The first part of this discussion will deal with specific matters mentioned in the paper and the second will be a brief statement concerning some of the larger questions in the field in which this investigation exists. In the course of the writer's comments some questions will be raised that cannot be answered and some techniques of the author questioned where an improvement cannot be suggested at present. The specific discussion follows:

1 The method of indicating contact between the micrometer and the beam is open to question because of the possible unevaluated constant and variable errors. The writer and his associates have struggled with this problem for some years and do not feel they have a satisfactory solution, although this method has been tried.

2 Which results were obtained using normalized low-carbon steel and which using hardened tool steel? It is often contended that a low-carbon steel, work-polished against a soft bearing, will react differently from a hardened tool steel which is not work-polished. It seems that some future investigation by this method should endeavor to compare a work-polished (probably burnished is a better term) low-carbon steel with a hardened tool steel. It might also be worth while to prepare two highly polished flat glass disks, since the surfaces exposed would not furnish recesses for the lubricant.

3 In the section devoted to disk-viscometer experiments the author mentions that "satisfactory duplication of results was found impossible." For the castor oil, the variability of results is explainable and, for the olive oil, the extreme variation of one determination is more of quantity than kind. However, in the case of the DTE light mineral oil, it may be said that agreeing data are definitely in the minority. It raises the question of whether all the variables are under even approximate control.

4 In Fig. 10, the author shows a plot of torque taken during the constant-torque period, against rpm. In discussing this plot, the statement is made that the relation is a straight line passing through the origin, provided the speed range is low enough to avoid frictional heating of the film. Now the writer does not wish to contend whether the relation is a straight line or not but to infer that a rotative speed of 100 rpm does not produce frictional heating of the oil film seems far-fetched indeed. Some other explanation would be more satisfactory.

In discussing the method used in the investigation, the author states that it "is free from the complications and uncertainties introduced by abrasion and wear." This statement is true as far as it goes, but to it could be correctly added, "and gives no information on abrasion and wear."

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It would seem that our knowledge has advanced to a state where we can recognize a number of different classes of conditions in bearing operation, as follows:

Class A dry rubbing contact, resulting in reduction of contact surfaces (abrasion).

Class B rubbing contact in the presence of a lubricant, resulting in reduction of contact surfaces (wear).

Class C relatively thin film separating the surfaces (present paper).

Class D relatively thick film separating the surfaces (hydrodynamic theory).

The term boundary film or boundary lubrication would seem to apply to both classes B and C in the sense that they are intermediate between dry-rubbing and thick-film lubrication. For purposes of study it would seem wise, however, to keep these classes separate in our thinking since the variables give evidence of being quite different.

M. D. HERSEY.[11] While a variety of new phenomena are reported in these experiments, they can be harmonized somewhat by noticing the close analogy which they bear to the more familiar phenomena of plastic deformation in everyday tests of greases, and many other types of plastic material, particularly materials that are strongly thixotropic. This analogy might be made more complete if the author could supply a typical curve for torque against speed, taken during the recovery period after rigidity has set in.

It would seem that the possibility has not been entirely ruled out that the observations may, in part, be explained as due to a slight degree of plasticity caused by suspended particles; not necessarily foreign particles. Such plasticity might be present in commercial oils without becoming noticeable until the film is sufficiently thin.

Two experiments might be suggested for the future: (1) Repetition of some of the rotation tests, substituting annular surfaces to simplify the mathematical analysis; and (2) observations of thixotropy of any convenient plastic material in bulk to see if the properties in one direction are influenced by motion at right angles.

G. B. KARELITZ.[12] The work reported in this paper is so extraordinary that one is tempted to look for signs of experimental error. It seems, however, that none of the measurements or techniques can be objected to. Yet it is difficult to believe that the influence of steel on oil molecules would penetrate to a depth of 0.00002 to 0.00003 in. It would mean that the effect of the contact surface may be felt at a distance of 200 to 500 molecular dimensions from the steel surface. In a number of experimental bearings, such as tested by McKee at the Bureau of Standards, the minimum film thickness was of the order considered in the paper, 40×10^{-6} to 60×10^{-6} in. It appears that these bearings should have shown a high coefficient of friction, but they did not. It may be that the agitation in an actual bearing, together with higher speed, would prevent the stratification of the lubricant into an orientated layer.

If the orientation of the molecules extends that far into the film, the several layers near the metal surface must be lined up rather well. It should then be comparatively easy to detect this quasi-crystalline structure by X rays, if pyrex-glass disks are used instead of steel. Reasonably flat disks could be pressed together, with an oil film trapped between them and a pencil of X rays played through the assembly. Of course, it may be that

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[12] Professor Mechanical Engineering, Columbia University, New York, N. Y. Mem. A.S.M.E.

glass does not produce the same orientation as does steel. A number of observations with techniques different from that used by the author would be required before the existence of directional rigidity could be accepted. This is said not to minimize in any way the importance of the contribution but rather to emphasize the significance of the report.

The low electrical resistance of the film is also very disturbing. It is very unusual that a film 10^{-4} in. thick and only a fraction of a square inch in area, should pass a detectable electric current with an imposed voltage of 25 mv.

One can only hope that this investigation will be followed up by other experimenters who will have the opportunity to study the physics of a thin oil film. The painstaking manner in which the work was performed and reported by the author speaks for itself and adds further credit to the laboratory of Dr. A. Kingsbury.

MELVIN MOONEY.[13] Regardless of what the explanation may be for the formation of a stable thin film of oil, the facts established in the author's experiments are of fundamental importance in problems of lubrication. The explanation given for the facts observed is not very convincing. The data can readily be explained on the assumption of a layer of foreign particles between the parallel plates. Such particles would not be eliminated by centrifuging if they have approximately the same density as the oil. When the upper plate is rotated, the particles roll and form effectively a ball or roller bearing. If they are irregular in shape or slightly compressible, they would tend to increase the separation between the plates when the upper plate is rotated at a fixed pressure. This appears to be in agreement with the experimental facts. (The data on this point are not quite conclusive, for they require an extrapolation of the thickness-pressure curves under rotation back to 0.2 lb per sq in. pressure.) On the other hand, if the strength and rigidity of the final film depend upon a special arrangement of the molecules of the oil, it would be anticipated that the shearing action, when the plates are rotated, would disturb the arrangement and cause a decrease in the thickness of the film. This is contrary to the observed facts. The rigidity to rotation developed on standing may be ascribed to yielding and flattening of the foreign particles.

The difficulty of reconciling the author's explanation of his own experiments with the results obtained by Bulkley with fine capillary tubes is suggested but not discussed by the author.

It is to be hoped that the work with the parallel plates will be continued as planned. There is a variation in the experimental procedure which might be adopted to determine definitely whether the rigidity of the film is due to properties of the oil itself or to fine particles suspended in the oil. The variation here suggested is that one of the test plates be drilled through its center. Then, after a stable film has been produced, it would be possible to test whether oil under pressure can be forced through the central hole and out between the two plates. If so, the rigidity must be due to isolated points of support, presumably foreign particles. If not, the entire film is a rigid structure involving all the oil.

M. MUSKAT AND F. MORGAN.[14] Whether one agrees or disagrees with the observations presented in this paper, the importance of its implications in all problems of the flow of liquids near solid boundaries as well as in boundary lubrication must be recognized. In view of the perhaps almost universal opinion, based largely on the researches of Bulkley and of Bowden and his colleagues, that no such effects exist, the author and Dr. Kingsbury should certainly be commended for their activity in a field in which the possibility of obtaining positive results must have appeared very small.

Before all the previous results are discarded, however, or at least explained on the basis of different experimental conditions, it is perhaps worth asking if the results of this paper can be interpreted in any other way. For instance, the possibility that dust particles may contribute to the experimental results has already been considered, but apparently no great pains have been taken to remove this unknown factor from the experiments. The elaborate precautions necessary to obtain clean surfaces and purified air or liquids relatively free from dust particles have been described by Bastow and Bowden.[9] In spite of the methods used, however, they report that, under the ultramicroscope, purified alcohol still showed occasional dust particles. In fact the presence of some sort of contamination in the author's experiments seems almost to be postulated from the observation in part I that the micrometer zero could only be set by bringing the surfaces together with platinum wires between them, since a false reading would probably be obtained due to surface contamination or adsorbed films if they were brought together without the wires.

In any case, even though the author does feel that dust particles were not present, it is interesting to observe that if we do assume their presence practically all of the experimental results can be explained without invoking the peculiar rigidity properties proposed in the paper. Thus the linearity of the friction-versus-speed curve would simply be the result to be expected of a normal viscous film. As the dust particles would be supporting the load, the surface-tension forces at the edge of the surfaces would more than suffice to keep the liquid in without requiring any rigidity characteristics. The increase in torque observed after the rotation has been stopped might be due to a flattening of the dust particles on standing and subsequent return to approximately spherical shape upon rolling between the rotating disks.

Admittedly, the discrepancy between the directly measured film thicknesses and equivalent film thicknesses calculated from the torque observations must then be ascribed to errors in the pin method of direct thickness measurement. But, until there are other independent confirmations of the accuracy of this method, it is difficult to have sufficient faith in it to compel acceptance of the rigidity hypothesis. As to the lack of wear and scratches, this might be explained if the dust particles act in a manner similar to that of a polishing agent, even though they apparently do not become broken up.

While this picture is presented only as a working hypothesis, the alternative interpretation of the author requires that the film have a tremendous anisotropy. Thus in the direction of rotation the rigidity is equivalent only to a viscosity 5 times its normal value, whereas its radial viscosity is practically infinite. Furthermore, when the disks are at rest, the film apparently has an isotropic rigidity. Moreover, if rotation tends to remove the rigidity, it is surprising that, over the entire range of speeds of 120 rpm practically to zero, the effective viscosity remains so strictly constant.

If these properties are substantiated, the author will certainly have established a most remarkable phenomenon. It is felt, however, that just because this phenomenon would have such a profound bearing upon the general problem of lubrication, it is highly important that all factors which may permit alternative explanations of the experiments be positively eliminated before what are generally believed to be well-established physical concepts are discarded.

B. L. NEWKIRK.[15] In this paper, the observed effects are so

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[14] Gulf Research & Development Company, Pittsburgh, Pa.

[15] Rensselaer Polytechnic Institute, Troy, N. Y. Mem. A.S.M.E.

striking that they call for careful study and further efforts to check them experimentally from new angles if possible. Particularly impressive is the consistency of measurements of the individual curves of Figs. 3, 4, and 5. The departures of individual points from smooth curves are only a few millionths of an inch. In Fig. 3 agreement with theory is close. Figs. 4 and 5 show discrepancies which would be reduced materially if some reason could be found for a change in the zero reading from one test to another. These curves show some apparently real diversities in shape that could not be so explained. For example, the curve marked by crosses in Fig. 5 would not fit the theoretical curve even after displacement. In view of the author's remarks about the difficulty of the zero measurement, it is not surprising that vertical displacements of the curves appear. A wave length of sodium (yellow) light is nearly 0.000024 in. On the other hand, it would certainly not be wise to dismiss these discrepancies in the view that they may be due to zero errors, for it is possible that some important fact would be brought to light if we could find the real cause. If means could be found to check the zero measurements independently, something further of importance might be discovered.

The experiments of part II also turn to a considerable extent on the measurement of distance between the plates. The method used in this case to measure the distance seems admirable but an independent check of a factor of such critical importance would be desirable if it could be managed. The measured distances between the plates are of the order of 400 times the length of the long polar molecules. The apparent fluidity for circular motion and rigidity for radial motion and the rigidity developing during periods of no rotation are very remarkable. If the surfaces had bumps and hollows or if they were distorted by pressure or the mounting, this might account for some of the observations, but the author's statement that the surfaces were flat to better than 0.000001 in. and the care taken in the design and mounting of the disks seem to exclude these possibilities.

K. Terzaghi.[16] The personal experience of the writer concerning the subject of the paper is limited to the physical properties of boundary films in clay-water mixtures. Therefore the writer was much impressed by the similarity between his own experience with water films and that of the author with oil films.

The findings of the writer can be condensed into the following statements: If the average width of the voids in a clay-water mixture is smaller than about 0.1μ the average effective viscosity of the water increases rapidly and, at the same time, the water assumes more and more the characteristics of a solid substance.[17] The increase in viscosity is apparently due to the restraining influence of the proximity of the solid surface on the movement of the molecules of the liquid.[18] The existence of an almost perfectly constant ratio between the normal pressure and the corresponding shearing resistance of the boundary film seems to be due to an orientation of the molecules of the liquid within the boundary film.

In 1933, B. Derjaguin published the results[19] of investigations concerning the elastic properties of water films between an optically plane and a curved surface of glass. The thickness of the film was determined by observing the Newtonian rings around the point of minimum distance between the two surfaces. The data required for computing the modulus of shear of the boundary film were obtained by recording the period of oscillations produced by the application and subsequent release of a torque. The following table contains the results of the investigation, quoted from the paper:

Thickness of film in microns	Modulus of shear in absolute units
0.089	190×10^6
0.093	170×10^6
0.137	4×10^6
0.150	0.0

This table shows that the thickness at which the film began to exhibit a measurable modulus of shear (0.14μ) is of the same order of magnitude as the thickness at which the writer noticed a marked increase of viscosity (0.1μ). However, as soon as the water acquires elastic properties, one can no longer call it a liquid and the physical meaning of the term viscosity changes. The transition from the liquid to the solid state, which takes place if the thickness of the film is decreased, also requires a corresponding progressive departure from Stefan's equation for the relation between the time and the thickness of the film (Equation [6] in the paper). In order to bring this departure more clearly into prominence, it would be advisable to plot the value $h\sqrt{t}$ against $\sqrt{t}$, because according to Equation [6] the value $h\sqrt{t}$ should be a constant, which appears in such a graph as a horizontal line. The departure from a horizontal line is more easily detected than that from a hyperbolic curve.

As h approaches a final constant value h_e the empirical curve approaches an asymptote through the origin $\sqrt{t} = 0$ and the value h_e is determined by the slope of this asymptote. The writer would appreciate it if the author would present such a graph in his final discussion.

The rapid increase of the torque after stopping, as shown in Figs. 8 to 10 of the paper reminds the writer of the thixotropic phenomena in the clay-water mixtures. The increase of stiffness with time under unaltered external conditions, characteristic of thixotropic phenomena seems to be due to the gradual building up of a molecular structure within the boundary films. Such an internal change may also account for the ultimate increase in the thickness of the films in the tests illustrated by Figs. 3 to 6.

As soon as the rotation ceases, the resistance against sliding approaches the value determined by the coefficient of static friction for lubricated surfaces, which is normal pressure per unit of area times coefficient of friction.

The coefficient of static friction for olive oil (Figs. 8 and 9) seems to be of the order of 0.14. For castor oil W. B. Hardy[20] gives a value of 0.10. In this connection, it is interesting to note the extraordinary difference between the rate at which the sliding resistance approaches the static value under a pressure of 40 lb per sq in. (Fig. 9, very rapid) and under 100 lb per sq in. (Fig. 8, very slow). Noticing this difference one feels tempted to consider whether an increase of the pressure may not increase the resistance against re-establishing the molecular structure within that part of the film which is kept by rotation in a liquid state.

Regarding the rigidity of the film during rotation, the writer is inclined to believe that the rigidity existed throughout the films, except within a layer with a thickness equal to the diameter of a few molecules along a plane half way between the solid surfaces. This may also account for the failure of the oil to flow out in a radial direction, in spite of liquid behavior with respect to rotation. Finally it may eliminate the apparent contradiction be-

[16] Harvard University, Cambridge, Mass.

[17] "Versuche über die Viskosität des Wassers in sehr engen Durchgangquerschnitten," by K. Terzaghi, *Zeit. für Angewandte Mathematik und Mechanik*, vol. 4, 1924, pp. 107–113.

[18] "The Mechanics of Adsorption and of the Swelling of Gels," by K. Terzaghi, Fourth Colloid Symposium Monograph, Chemical Catalogue Company, Inc., New York, N. Y., 1926, pp. 508–708.

[19] "Die Formelastizität der dünnen Wasserschichten," by B. Derjaguin, *Zeit. für Physik*, vol. 84, 1933, pp. 657–670.

[20] "Note on Static Friction," by W. B. Hardy and J. K. Hardy, *The Philosophical Magazine*, London, vol. 38, 1919, pp. 32–48.

tween the measured and the calculated values of film thickness (Fig. 12 of the paper), because the calculation seems to be based on assuming that during rotation the film is liquid throughout its thickness. The writer would appreciate an expression of opinion by the author as to whether he considers this hypothesis compatible with his observations. In every other respect his conceptions regarding the structure of the oil films are practically identical with those of the writer, derived from a very different realm of empirical evidence.

Author's Closure

Many interesting points have been brought out in the discussion. Some of these points may be clarified, but in most cases the information is not available at present to answer the questions raised.

Attempts by Professor Bradford and Mr. Villforth to detect indications of film rigidity by means of a tapered-plug viscometer are most interesting. Since the rate of approach of the surfaces appears to be in agreement with theory, until the film thickness is close to its minimum value, it would be indeed surprising if surface influence were detected in such comparatively thick films as 0.005 to 0.006 in.

Dr. Bulkley passed highly filtered fluids through fine capillaries and arrived at the conclusion that there was no surface influence sufficient to produce a rigid layer out to distances greater than 0.000001 or 0.000002 in. from walls of platinum or glass. The author worked with films of oil of the order of 0.00003 in., bounded by two plane steel surfaces, and found unmistakable evidences of unexpected behavior. This behavior he attributed to surface influence. Aside from questions of preparation and purity of the liquids, technique, and others, which become apparent when reading the papers and comparing the findings, the two following important facts stand out: A single surface, such as the wall of a capillary tube, has little or no influence on a contiguous liquid. Two steel surfaces in close proximity exert an appreciable effect on the oil between them. These two findings are not essentially in conflict. All the experiments on which the present paper is based indicated that, when the films were relatively thick, their behavior could be calculated from simple theory. It was only when the two steel surfaces came close to each other that their influences became apparent. It was also found that there appeared to be an appreciable change in electrical resistivity as the films became thinner. This point was not investigated to the extent warranted by its importance, but it was noted that the beginning of the change in resistivity occurred at approximately the same time as the departure of the rate of approach from Stefan's law. Investigation along these lines might be fruitful.

Professor Connelly has pointed out that the work done in shearing the oil film must cause heating of the film. At low speed and low torque, however, the rate at which heat is generated is so small that its influence on the film cannot be detected with the present apparatus. The rate at which heat is generated depends upon work done in shearing the film, and not upon rotative speed alone. For example, in the test shown by Fig. 10 of the paper, the rate at which heat is generated at 100 rpm is about the same as at 30 rpm in the test shown by curve 2 in Fig. 14 of this discussion. Data are available to show that, under these respective conditions, heat is generated at a rate sufficient to cause a 10 per cent drop in torque in 1 min. Only a few seconds are required for a torque observation and, between observations, the speed is reduced to 11 rpm or less where heating of the film is not appreciable. During the constant-torque period in the test shown by Fig. 9 of the paper, the reduction in torque due to shearing of the film is probably not in excess of 2 per cent.

No difference in test results could be detected by using hardened tool steel in place of normalized low-carbon steel. Preference of one steel over another for making the test surfaces was due only to the reasons stated.

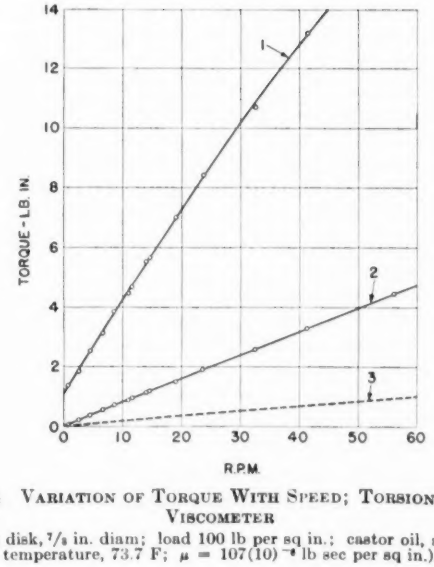


Fig. 14 Variation of Torque With Speed; Torsional Disk Viscometer

(Plane disk, 7/8 in. diam; load 100 lb per sq in.; castor oil, average temperature, 73.7 F; $\mu = 107(10)^{-6}$ lb sec per sq in.)

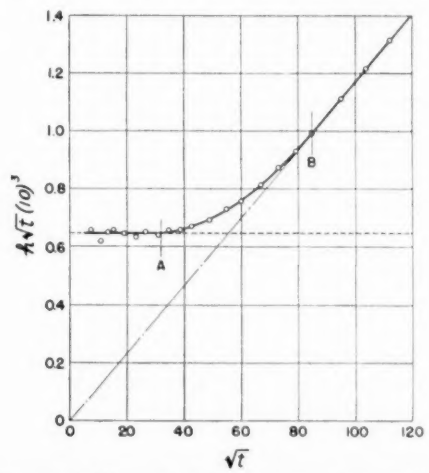


Fig. 15 Terzaghi's Method of Determining Time of Departure From Stefan's Equation; Torsional Disk Viscometer

(Plane disk, 7/8 in. diam; 11.06 rpm; load 40 lb per sq in.; castor oil, average temperature, 71.8 F; $\mu = 117(10)^{-6}$ lb sec per sq in.)

The torque-versus-speed curves requested by Mr. Hersey are shown in Fig. 14 of this discussion. The data were taken with castor oil and a load of 100 lb per sq in. After a continuous run of 8 hr at 11.06 rpm, rotation was stopped and the film was allowed to remain undisturbed for 15¾ hr. Variations of torque with speed were then observed and plotted in curve 1. Rigidity at zero speed is shown by the intercept at about 1.05 lb-in. Continuous rotation was then resumed and when the constant torque of the previous day had been reached, the speed was again varied with the results shown by curve 2. As in all cases when rigidity is not present, the torque is proportional to the speed and the curve is a straight line passing through the origin. From Fig. 12 of the paper, the film thickness with castor oil at 100 lb per sq in. is $38(10)^{-6}$ in. and, for the ordinary bulk viscosity of the oil, the theoretical torque is shown by the broken-line curve 3. Comparison of the slopes of curves 2 and 3 shows that, in this film, surface influence has increased the effective viscosity to nearly 5 times its normal value.

That the data may be readily explained on the assumption of a layer of foreign particles between the parallel plates, as suggested by Drs. Mooney, Muskat, and Morgan, is not quite clear to the author. It is true that no elaborate precautions were taken definitely to exclude dust from the atmosphere during the short time required to apply the oil and start the test, but to visualize a particle that could account for the observed phenomena is very difficult indeed.

Dr. Terzaghi's suggestion that the point of departure from Stefan's Equation [6] may be more clearly seen by replotting the data, has been carried out in Fig. 15 of this discussion. Data therein presented are from a test with castor oil and a load of 40 lb per sq in., with continuous rotation at 11.06 rpm. The horizontal broken line represents the constant value of $h\sqrt{t}$, which in this case is 0.000648 from Equation [6]. The point of departure A is at $\sqrt{t} = 32$. Therefore, during the first 17 min, the surfaces are approaching at the rate calculated for a purely viscous fluid. Torque becomes constant at B, 2 hr after starting, and approach of the surfaces ceases. From the slope of the curve beyond B, the minimum film thickness is $11.7(10)^{-6}$ in. This may also be found from the observed torque and Equation [11]. But from Fig. 12, however, we find the actual minimum film thickness is about $41(10)^{-6}$ in., nearly 4 times the theoretical value.

From Fig. 8 of the paper, it is seen that, with olive oil at 100 lb per sq in., the torque increases from 0.125 to 1.54 lb-in. over the 2-hr period CD. Time, in Fig. 9, is plotted in hours instead of minutes; hence, it is not quite so clear that, at 40 lb per sq in., the torque increases from 0.08 to 0.8 lb-in. in a similar 2-hr period. The rate of torque increase, therefore, is greater at the greater pressure and the final values reached depend upon pressure and time.

With reference to film rigidity and the general behavior of these thin films, there seems little difference between the conclusions which were reached by Dr. Terzaghi and those expressed by the author.

It is evident that further study of the boundary-film problem is desirable and warranted by the promise of additional information. This discussion has been most valuable in that it points out several paths for future research.

High-Speed Lightweight Trains

By C. T. RIPLEY,[1] CHICAGO, ILL.

The purpose of this paper is to outline the changes which have occurred during the last five years in high-speed passenger-train cars and in motive power for hauling them and the economic factors which have brought about these changes. The new designs for passenger cars and the materials used in their construction are discussed. A detailed comparison of steam-locomotive and Diesel electric-locomotive characteristics as they affect the operation of these new high-speed trains is presented. Test data are included to indicate the importance of comparative stress in track produced by two types of power. Reference is made to the steady improvement which has been made in steam-locomotive design, but it is shown that there is a need for some rather extensive experimentation to make this type of power more suitable for this particular class of service. In conclusion, the author presents his views on the general results which have been secured from the operation of these new trains and the probable trend in their future development.

DURING the last twenty years the travel habits of the American people have expanded greatly. The development of the low-priced automobile, and the construction of hard-surfaced roads have been the major factors in this change. In the face of increased travel, railway passenger traffic steadily declined until in 1933 it produced only about one third the gross revenue which it did in 1920. The data in Table 1 show this decrease both in the number of passengers carried and in the rates and revenues.

TABLE 1 PASSENGER-TRAFFIC STATISTICS, CLASS 1 RAILROADS IN UNITED STATES

Year	Passenger revenue, dollars	Revenue per passenger mile, cents	Passengers carried
1921	1,153,792,000	3.09	1,035,496,000
1925	1,057,704,231	2.94	888,267,000
1930	729,470,279	2.72	703,598,000
1933	329,342,000	2.61	432,980,000
1934	346,325,993	1.92	449,775,000
1936	412,379,000	1.84	490,091,000
1937	442,809,000	1.79	497,288,000
1938	405,476,000	1.87	452,808,000

The large decrease in railway passenger traffic was undoubtedly due to the inroads of bus travel, private-automobile travel and, to a lesser degree, airplane travel. Prior to 1930, the railway managements, with properties still fairly prosperous because of good freight traffic, did little to remedy the situation except to reduce rates, as indicated in Table 1. They also eliminated some nonpaying trains and in the case of branch-line operation, where they were usually forced by governing bodies to continue passenger service regardless of losses involved, they substituted gasoline-electric motorcars for steam trains. These cars were definitely successful in reducing operating costs to such an extent that their first cost was paid for in a few years, but there was nothing about their service to attract new passenger business or even to hold what was left. Their usage did influence later developments in that they brought to the attention of railway managements the possibilities of utilization of the internal-combustion motor in railway service. Their success in mechanical performance led the manufacturers to develop the new type of lightweight two-stroke-cycle Diesel motor, which is now playing such an important part in the operation of high-speed streamlined trains.

About 1930 the railroads started air-conditioning passenger cars. This was an instant success with passengers and though the cost was high, about $8000 per car, it became necessary to air-condition practically all main-line cars, with the result that there are now about 11,000 such cars in operation. In some of these cars there were other improvements made for the comfort of the passengers such as more comfortable seats and better lighting. The success of this development, from a traffic viewpoint, convinced the passenger-traffic officers that it might be possible to recapture at least a part of the lost business.

In 1900 the 60-ft wooden passenger car, seating about 80 passengers, weighed about 40 tons or 1000 lb per passenger. In the interests of safety, designs were changed first to include steel underframes, and in about 1910 to all-steel construction. Various parts were also increased in size and weight in order to secure greater strength and durability, until in the 1920's the 80-ft passenger car seating about 70 passengers weighed approximately 80 tons or 2300 lb per passenger. About 1930 still further weight was added, due to the introduction of air-conditioning equipment, weighing about 5 tons. As a result of this continued increase in weight of passenger cars, as well as a gradual increase in the speed of operation, the size of steam motive-power units steadily increased. Track structures and bridges had to be made stronger to carry these heavier locomotives and cars, and thus there was, from all of these angles, an increase in the cost of operating passenger trains, while at the same time the total traffic was diminishing and there was a continual demand for lower rates to save the remaining traffic.

Development of Lightweight Cars

About this same time some new structural materials became available for use by the car builders, namely, aluminum alloys, stainless steel, and various so-called low-alloy high-tensile steels. Because of these developments the railway engineers and the car manufacturers started active work in the design and construction of new trains of a lighter type for main-line operation. The first of these were small trains consisting of three or four cars with 600-hp Diesel engines located in the head car. They were made as light as possible in order to hold down the power requirements and to enable them to compete with buses and private automobiles by giving cheap transportation. The cars were of non-standard height and some were narrower than the standard. They were streamlined to reduce power requirements and also to make an appeal to the public, already educated to such design by automobile developments. The reaction of the traveling public was so favorable that there was an immediate demand for more elaborate and larger trains for longer runs, with sleeping accommodations and other features for maximum comfort. Low first cost was soon lost sight of in this development. Interior decorators designed the most elaborate fittings and the trains were built with as many as 14 cars with power units totaling as

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

much as 5400 hp and auxiliary power for the cars of 1200 hp. A few of these trains were powered with streamlined steam locomotives but there was little change in the design of these locomotives from the old type except that higher driving wheels (84 in.) were used together with higher steam pressures (300 psi) and larger tenders to reduce delay in taking fuel and water. They also had roller bearings and some other refinements in design.

Most of the early trains had articulated cars because it was thought that this design reduced weight and gave better riding conditions. It developed however that neither of these factors proved to be important. There was a definite disadvantage in articulation particularly on long runs, in that standard cars could not be mixed with such cars. In case of difficulty, such as a bearing or wheel failure, a car failure might become a train failure. In the last few years practically all the lightweight cars have been built as individual units with standard floor and coupler heights and standard outside dimensions, so that they can be associated with any standard car. The weights of these cars were reduced about 40 per cent from those of the conventional structural-steel cars and thus four-wheel trucks could be used.

Specification Requirements for Passenger Cars

The only standards for design strength available at the time most of these cars were built were the provisions of the United States Railway Post Office mail-car specifications (1).[2] These specifications were first prepared in 1912 by a group of representatives of the Post Office Department, the Master Car Builders Association, and the car manufacturers. It was required that all postal cars meet these specifications. While other cars were not required to meet them, practically all passenger-train cars were designed at least to these requirements. In July, 1938, they were revised to clarify certain features. These new specifications were written so as to cover practically all types of construction with all types of materials. The major provisions of both the 1912 and 1938 specifications were, first, a minimum strength of center-sill construction based on 400,000 lb buffing static load and a factor of safety of two, and second, a minimum strength of end construction to protect against telescoping action. This latter required minimum section moduli for vertical end members and minimum shear values for their top and bottom connections. There were also numerous other limitations of stress in frame members.

Cars built to meet these specifications have rendered excellent service with a remarkably good record in the protection of passengers under wreck conditions. As a consequence when lightweight construction was started in about 1931 these specifications were used as a minimum requirement, even though there were no rules of the American Railway Association or of the Government setting such limitation. However, a feeling developed that because of the operation of lightweight cars between heavy cars in trains, it was necessary for the Association of American Railroads to set up more detailed specifications to cover the construction of all passenger-train cars built in the future, giving particular attention to prevention of damage from telescoping. A committee of the Association of American Railroads has recently developed these new specifications and they will probably govern all new construction of passenger cars. While these new specifications are generally based on the Railway Post Office specifications, they include some major changes: They provide for a center-sill strength such that an 800,000-lb load applied on line of draft will not produce any permanent deformation. The Railway Post Office specifications contained only a stress limitation and permitted the load application to be divided between the coupler line and the buffer line. This change will probably result in the addition of more material than is now used in center sills of lightweight cars designed with high center sills and buffers, with little compensating benefit from added protection. The new specifications also include increased requirements for strength of couplers, carrier irons, and end construction, all of which mean some increase in weight, but which are apparently justified in view of the importance of maximum protection against telescoping of cars.

Generally speaking, the new lightweight cars are being designed and built just as strong and safe as the older heavy types. Better engineering, stronger materials, and improved methods of fabrication make this weight reduction possible. New welding technique largely eliminates the human element and when properly applied should be preferable to the old riveting practice.

Need for Light Weight

Light weight is almost a necessity for high-speed operation in order to hold down the size and cost of power units, as well as the cost of operation. This is particularly true with Diesel-electric locomotives with their high first cost. Experience indicates that 450 lb per hp (including weight of locomotive) is a proper design ratio for high-speed operation. Some of the new trains have a power ratio as high as 550 lb per hp but the maintenance costs appear to run higher on such trains. In order to secure satisfactory performance on high-speed schedules, it is necessary to have some reserve power in the locomotive because some delays are inevitable and extra power is needed to make up the lost time. While the schedules may indicate that maximum speeds of only about 90 mph are required, actual operation requires at times speeds of 100 mph or even higher. This fact must be taken into consideration when designing locomotives and trains for this type of service.

Car Construction Materials

Several of the early trains were built of aluminum alloys but in recent years the majority of them have been built of stainless steel or low-alloy high-tensile steel. The major claims for the stainless steel are lower weight due to higher physical properties, better welding by the automatic shot-weld process, and high resistance to corrosion. Stainless steel does not have to be given paint protection of any kind, which reduces both first cost and maintenance cost. Its polished surface gives an attractive modernistic appearance to the cars and is easily cleaned. In high-speed operation the sandblast effect of particles from the ballast or right of way damages paint finishes quite rapidly. The low-alloy high-tensile steel advocates claim that they secure equally low weights for lower costs. Tests indicate that these steels have from two to four times the corrosion resistance of ordinary carbon steel. They are made to various formulas and have different trade names. It is claimed that some of them have somewhat better welding qualities than others.

Inasmuch as no exactly similar cars have been built using the different types of steel or aluminum, it is not possible to make accurate weight comparisons. While it is true that the full use of the superior physical properties of stainless steel or the light weight of aluminum is limited to some degree by deflection requirements, it should be possible, by using these materials instead of low-alloy steel, to construct a lighter car of equal strength. A general comparison of the large number of cars of the various types which have been built indicates a weight advantage for stainless steel and aluminum of at least 5 per cent. It would be expected that the construction cost of cars built of these materials would be somewhat higher due to the high cost per pound of such materials, but competition between builders has resulted in about equal bid prices.

It is too early to evaluate fully the different types of lightweight-car construction. Maintenance costs over a longer period

[2] Numbers in parentheses refer to the Bibliography.

TABLE 2 PHYSICAL PROPERTIES AND COMPOSITION OF MATERIALS USED FOR PASSENGER-CAR BODIES

	Structural steel for cars	Typical low-alloy high-tensile steel	Stainless steel Grade 1	Stainless steel Grade 2	Typical aluminum alloy 17ST
Minimum yield point, 10^3 psi	[a]	50	110	120	32
Minimum tensile strength, 10^3 psi	[b]	70	140	150	50
Minimum elongation in 2 in., per cent	25	22	18	18	8 18
Modulus of elasticity, 10^6 psi	28 30	28 30	27	27	10.5
Carbon, per cent	0.30	0.10	0.08 0.20	0.08 0.20	..
Manganese, per cent	0.30 0.60	0.10 0.50	0.70 max	0.70 max	..
Copper, per cent	0.20	0.30	0	0	4.0
Chromium, per cent	0	0.50 1.50	18	18	..
Nickel, per cent	0	0	8	8	..
Approximate cost, cents per lb	$2^1/_4$	$3^1/_2$	35	35	35

[a] One half tensile strength. [b] 50,000 to 60,000 psi.

and performance in accidents will ultimately provide the answer. It appears, however, that a weight of about 100,000 lb for an 80-ft coach is entirely practicable with full safety to passengers. If too much space for extra luxuries is provided, part of the advantage of lightweight construction is lost, as the important factor is the weight per passenger carried. Table 3 shows the major parts of a typical lightweight car in which the weight saving has been made.

TABLE 3 COMPARATIVE WEIGHTS OF CONVENTIONAL AND TYPICAL LIGHTWEIGHT AIR-CONDITIONED PASSENGER COACHES

	All weights in pounds	
	Conventional steel coach	Lightweight stainless-steel coach
Length over end sills, ft	70	79
Seating capacity	58	63
Weight of trucks	52800	34000
Weight of car-body structure	66500	33000
Weight of inside furnishings	7000	5000
Weight of equipment	45400	25000
Total weight	171700	97000
Weight per passenger	2962	1540

Riding Characteristics of Lightweight Cars

Due to higher operating speeds, the design of passenger-car trucks is a most important factor. The former six-wheel trucks were not entirely satisfactory at these increased speeds, and doubt existed in the minds of some engineers as to the possibility of securing good riding qualities with four-wheel trucks. The weights of the new car bodies were so low that only four axles were necessary to carry the load, affording further opportunity for weight reduction by the use of four-wheel trucks. Extensive experiments were made by various railroads and manufacturers, which resulted in the development of many new features of truck design with a decided improvement in riding characteristics.

Among these new features were the use of softer springs with greater deflections, hydraulic shock absorbers to control both lateral motion and vertical spring reactions, multiple-spring systems in triple-bolster trucks, and roll stabilizers. The use of cylindrical tread contours on wheels to reduce the nosing of trucks, and the grinding of treads to secure true rotundity and concentricity, were both helpful in softening the ride. Balancing of the wheels was also practiced to some extent, although a complete check of this feature has not been made due to lack of balancing machines in this country. It is now being given careful check on one railroad, which has installed the necessary machine. In an effort to reduce noise in the cars, rubber or other semiresilient materials have been introduced in various parts of the trucks, such as pedestals, spring seats, and center plates. The mounting of the brake cylinders on the trucks instead of the body has also reduced the noise produced by the brakes. Improved insulation of car floors, as well as the sealing of windows, has also made a marked improvement, but there is still need for further noise elimination. The passengers can still hear the click of the wheels on the rail, the grinding of brake shoes on the wheels, and in some cases generator noise. Longer rails and better track maintenance would help to reduce rail and wheel noise and the use of disk-type brakes would reduce brake noise.

In some of the lightweight cars a certain amount of vertical body vibration was noticeable to passengers. This has been reduced by stiffening the car-body structure. The fore-and-aft vibration of cars which is so noticeable in many of the standard cars, particularly in operation with reciprocating steam locomotives, has been largely eliminated through the use of tight-lock couplers, which have little or no free slack and by softer acting draft gears.

The most unpleasant shocks in these cars are those of lateral acceleration, resulting either from nosing of trucks on tangent track, or the entering or leaving of curves. A novel attack on this problem is being made by one manufacturer, who has developed a pendulum type of car. This method of mounting the car on the truck differs completely from standard practice, in that the car body is virtually suspended from the truck, floating on soft vertical helical springs located well above the center of gravity of the car body. These springs permit through horizontal deflection the necessary truck motion relative to the body, and this motion is positioned and controlled by a pair of horizontal links, which are restrained by rubber disks and shock absorbers acting between the car body and vertical extension of the truck frame at a point above the center of gravity of the body. In going around curves this type of car tends to bank and thus avoids the unpleasant sensation resulting from car-body roll. Preliminary tests of this type of car indicate an improvement in riding characteristics particularly on curves.

In general it appears that there is still need for research in truck design, in order to secure even greater comfort for passengers and to make possible higher operating speeds on sharp curves which, particularly on western roads with much track curvature, are a definitely limiting factor to fast over-all operating schedules.

Improvements in track structure and maintenance are also necessary if still higher speeds are to be attained. Whatever de-

sign of truck is used, satisfactory riding characteristics cannot be secured in high-speed operation over bad track.

Braking

The introduction of high-speed lightweight trains brought many new problems in braking. The stopping of these trains within reasonable distance is a difficult problem. While the car weights are less, individual wheel loads are as high and in some cases (particularly with the articulated type) higher than with the old heavy cars. In order to reduce the stopping distance, there have been a number of changes from former practice, such as truck mounting of brake cylinders which increases rigging efficiency, much higher braking ratios (some as high as 250 per cent), decelakrons to reduce pressures as the train slows down and thus prevent sliding of wheels, automatic car sanders acting only in emergency application, and electropneumatic control. In some cases the very high brake ratios resulted in considerable wheel and brake-shoe trouble. With all of these improvements there is still a question whether emergency stopping distances are as short as they should be. To effect shorter stops without undue punishment to wheels and shoes, new types of disk brakes are being tested. One train so equipped is now in operation. This type of brake has the advantage of a more uniform coefficient of friction throughout the speed range, thus permitting brake pressures which more closely approach the coefficient of adhesion between the rail and the wheel. Unfortunately, this coefficient of adhesion varies widely under different rail conditions. Thus if higher braking pressures are used, there is danger of sliding under unfavorable conditions. To meet this problem, either car sanding equipment or governors to reduce pressures automatically, when sliding begins, may prove to be a necessary complication.

In both steam- and Diesel-powered lightweight trains, the locomotive weight is a large percentage of the train weight, but in most cases locomotives do not do their full share of the braking. The steam locomotive is particularly lacking for three reasons: First, there are often some unbraked wheels on engine trucks for operating safety; second, the large spread between light and loaded weight of tenders makes it necessary to design braking power on the basis of light weight, and third, enginemen have to relieve engine brake pressure frequently to guard against the loosening of driving tires due to heating. The Diesel is better in that all wheels are braked and the spread between light and loaded weight is relatively small, but the small-diameter wheels are so highly loaded that the braking ratio must be limited in order to protect the wheels from damage. It would be advantageous if Diesel-electric locomotives had a supplementary dynamic electric brake such as is used in some electric locomotives as well as in the new experimental turboelectric design.

Comparison of Diesel-Electric and Steam Locomotives

It may be noted from Table 7 that the Diesel-electric locomotive is more commonly used in these new high-speed streamlined trains than is the steam locomotive. It is claimed by some that this is chiefly due to the interest of the public in a new type of power, but the users apparently have good engineering arguments to support their choice. The advantages claimed for the Diesel are: First, high availability; second, rapid acceleration because of high tractive power at lower speeds; third, low maintenance costs; fourth, low fuel cost (usually less than half that of steam); fifth, lower rail stresses. The steam locomotive is less expensive, costing about $37 per hp. However, it is harder on track at high speeds due to the dynamic augment produced by overbalance in the driving wheels. It appears that a radical change in the design of the steam locomotive is necessary to make it capable of fully competing with the Diesel in this service. The important objects of such redesign should be the securing of a more constant torque and the reduction of track punishment. Three outstanding locomotives of new design are under construction or in service, namely, the 5000-hp turboelectric being manufactured by the General Electric Company for the Union Pacific Railroad, the Baltimore and Ohio Railroad's four-cylinder engine, and the Pennsylvania Lines' four-cylinder high-speed engine. All of these new types are experimental, but they indicate a trend toward a type of steam locomotive having more constant torque. A step still further in this direction is involved in a new Baltimore and Ohio design, which includes individual axle drive.

The comparison of typical modern high-speed steam locomotives and Diesel-electric locomotives on a horsepower basis, as shown in Table 4, is open to criticism because of the basic difference in the horsepower curves of the two types and also the difference in method of power transmission. The nominal horsepower of the Diesel-electric locomotive is available over practically the entire speed range, whereas the maximum indicated horsepower of the steam locomotive is available only during a limited speed range. With locomotives of equal horsepower, the Diesel has the advantage of greater power for accelerating at low speeds and greater sustained horsepower at very high speeds. Some difficulties with pistons, heads, and liners have been experienced in services where Diesel-electric locomotives were operated with such loads and schedules as to require almost continuous maximum output of power. Recent improvements in materials and designs of these parts have largely overcome this trouble. Nevertheless, the maintenance cost of any type of locomotive increases as the working-load factor increases. Proper design of a motive-power unit should provide some reserve power for emergency use, but not such an excess as to result in uneconomically high cost of construction.

TABLE 4 COMPARISON OF TYPICAL MODERN HIGH-SPEED STEAM AND DIESEL-ELECTRIC LOCOMOTIVES

	Diesel-electric	Steam 4-6-4	Steam 4-8-4
Nominal hp of Diesel motors	4000	....	....
Maximum indicated hp	....	4300	5750
Weight with fully loaded tender, lb	608000	809000	896000
Weight, lb per hp	152	188	156
Approximate cost per hp	$87.50	$35.00	$34.00

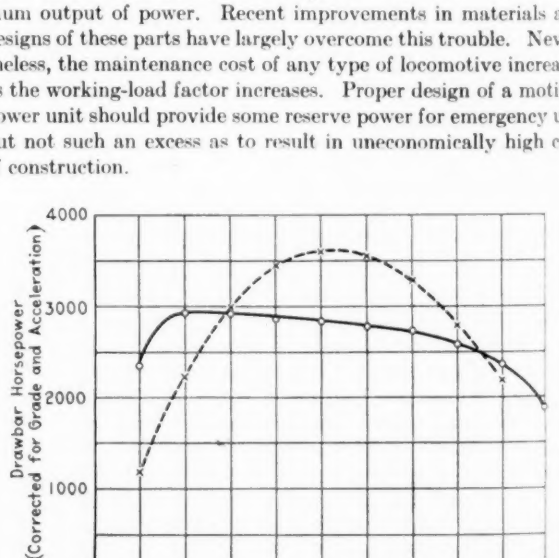


Fig. 1 Comparative Drawbar-Horsepower Curves for Steam and Diesel-Electric Locomotives

(Solid curve. Diesel-electric locomotive, 4000 nominal hp. Dotted curve. 4-6-4 steam locomotive, 4300 max ihp.)

Fig. 1 shows the relative drawbar horsepower of a 4000-hp Diesel-electric locomotive and a modern (1938) high-speed steam locomotive of the 4-6-4 type with 4300 max ihp. These curves are plotted from actual dynamometer-car readings with necessary corrections for grade and acceleration. They show the advantage

of the Diesel-electric locomotive at low speed and very high speed and also the advantage of the steam locomotive at intermediate speeds, such as would exist in ordinary grade working. This advantage in grade climbing is offset to a considerable extent by the 33 per cent greater weight of the steam locomotive.

The sharp dropping off of the horsepower curve for steam locomotives at high speed is due largely to low mean effective pressure in cylinders. To improve this condition, radical changes both in the design of valve and of valve gears appear to be necessary.

In the case of Diesel-electric locomotives, the curve does not drop off as sharply as does that for steam locomotives and further improvement has already been secured in the latest designs by change in the generator voltage control. This change has also increased the power output at low speeds.

In regular operation of the two types the performance of the Diesel-electric locomotive is usually that which is indicated by the drawbar-horsepower curve, but in the case of the steam locomotive there are a number of variables which may result in average performance somewhat below what is indicated by the horsepower curve derived from tests. The skill of the engineer varies and the condition of the locomotive is not always as nearly perfect as it is in the test.

Manufacturers of Diesel-electric locomotives have an advantage in that they produce a practically standardized product and thus can reduce cost through line production. On the other hand almost no two lots of steam locomotives are the same. Each order means a new design with resulting high costs for engineering and production. This also means higher cost for and larger stocks of repair parts. American railroads have made a marked advance in the standardization of freight cars but there has always been strong opposition to any plan for standardization of steam locomotives or passenger cars, due mainly to differences of opinion among different managements. The usual explanation is that operating conditions vary but this is hardly a valid excuse at least when certain railroads are compared.

The use of Diesel-electric locomotives is more extensive on western roads than on those in the East. This is probably due to the following factors: Most of the oil production is in this territory, which results in lower prices, and the traffic in oil and its by-products is a more important source of revenue than that from coal, whereas the reverse is true in the eastern territory. In the Southwest most of the steam locomotives use oil as fuel because of its lower first cost. The usual Diesel-locomotive fuel is about 28-gravity furnace oil costing about four cents per gallon at the refinery. Steam-locomotive fuel oil is a low-grade residuum costing about two cents per gallon at the refinery. The boiler-feedwater supply and treatment is also much more of an expense and problem in at least a large part of the West than it is in the East.

Diesel-electric locomotives provide better vision for the engineer because the cabs are located at the head end. The riding quality of Diesels is better than that of steam locomotives. They also provide smoother-riding trains as their high tractive effort at low speeds enable them to start and accelerate trains more quickly and smoothly.

Steam locomotives have an advantage in that more horsepower can be put in one unit. It may be noted in Table 4 that the increase in horsepower is secured with a lower percentage of increase in first cost and weight, that is, both the cost and weight per horsepower are lower for the 4-8-4 locomotives than for the 4-6-4 type. The maintenance cost per horsepower should also be somewhat lower for the 4-8-4 locomotives. It is true that any number of Diesel units can be coupled and controlled from one cab, but this increase in number of units means increased first cost and increased weight. A 2500-hp Diesel unit would appear to be a desirable size for heavy service, particularly in mountainous territory, but this does not appear practicable at present.

The performance records of Diesels show fewer failures than similar records for steam locomotives, even when the latter are operated in slower-speed service. The multiunit power plant is a helpful feature in the prevention of failures. If something goes wrong in one power plant the locomotive can still go on to its terminal and in many cases the defect can be repaired en route with little or no delay. A major breakage or defect on a steam locomotive means a failure.

If an entire territory were completely Dieselized, major savings could be secured in elimination of water and fuel facilities and also intermediate terminals. With partial Dieselization, it is necessary to provide certain special facilities for water and fuel, but these are not expensive. Diesel-electric locomotives are designed to operate about 700 miles without refueling or watering, whereas steam locomotives with large tenders operate only about one half of this distance for fuel and one third the distance for water. Fewer and shorter stops for fuel and water are essential in making high-speed schedules.

Diesel-electric locomotives have a higher availability than steam locomotives because they need less servicing. Furthermore, individual major parts can be changed in a short time, thus avoiding delay of the unit for repairs. In service they are regularly making $1^1/_2$ to 2 times as much mileage as steam locomotives, though it is only fair to say that part of this may be due to their use on preferred runs. Diesel-electric locomotives are cleaner in their operation, due to lack of smoke. This is particularly advantageous in cities and towns. It also adds to passenger comfort in that the view from the car windows is not obstructed by smoke.

Table 5 gives a breakdown of the weight of a typical 2000-hp Diesel-electric locomotive. The large percentage of the weight involved in the trucks may be noted. This is due to the heavy construction needed to support the traction motors and withstand the shocks of high-speed operation. The trucks are constructed of alloy steel and there appears to be little opportunity of making them lighter. The Diesel motors are almost as light as they can be made to withstand the service, but it may be possible in future development to take more power out of each cylinder, which would result in a reduction of weight per horsepower. The transmission accounts for a considerable percentage of the total weight. There is some possibility of reducing both the cost and weight of this part through the development of suitable hydraulic transmissions, but no large locomotives of this type have as yet been built in this country. The weight of the body structure cannot be reduced to any great extent as alloy steel and welded construction are already used throughout.

TABLE 5 WEIGHT OF STANDARD 2000-HP DIESEL-ELECTRIC LOCOMOTIVE

	Per cent
Two 1000-hp Diesel motors and auxiliary equipment	23
Two electric transmissions with auxiliary equipment	21
One steam boiler with auxiliary equipment	2
Two trucks (less traction motors)	28
One car body	26
Total	100

Total light weight	277000 lb
Weight of oil (1420 gal)	11000 lb
Weight of water (1360 gal)	10800 lb
Weight of sand (16 cu ft)	1600 lb
Total	300400 lb

The center of gravity of the Diesel-electric locomotive is considerably lower than that of steam locomotives and this fact, together with the shorter rigid wheel base makes it possible to operate the Diesel-electric locomotive at about 15 per cent higher speeds on sharp curves. This is an important factor in making high-speed schedules, particularly on runs where there is a great deal of curved track.

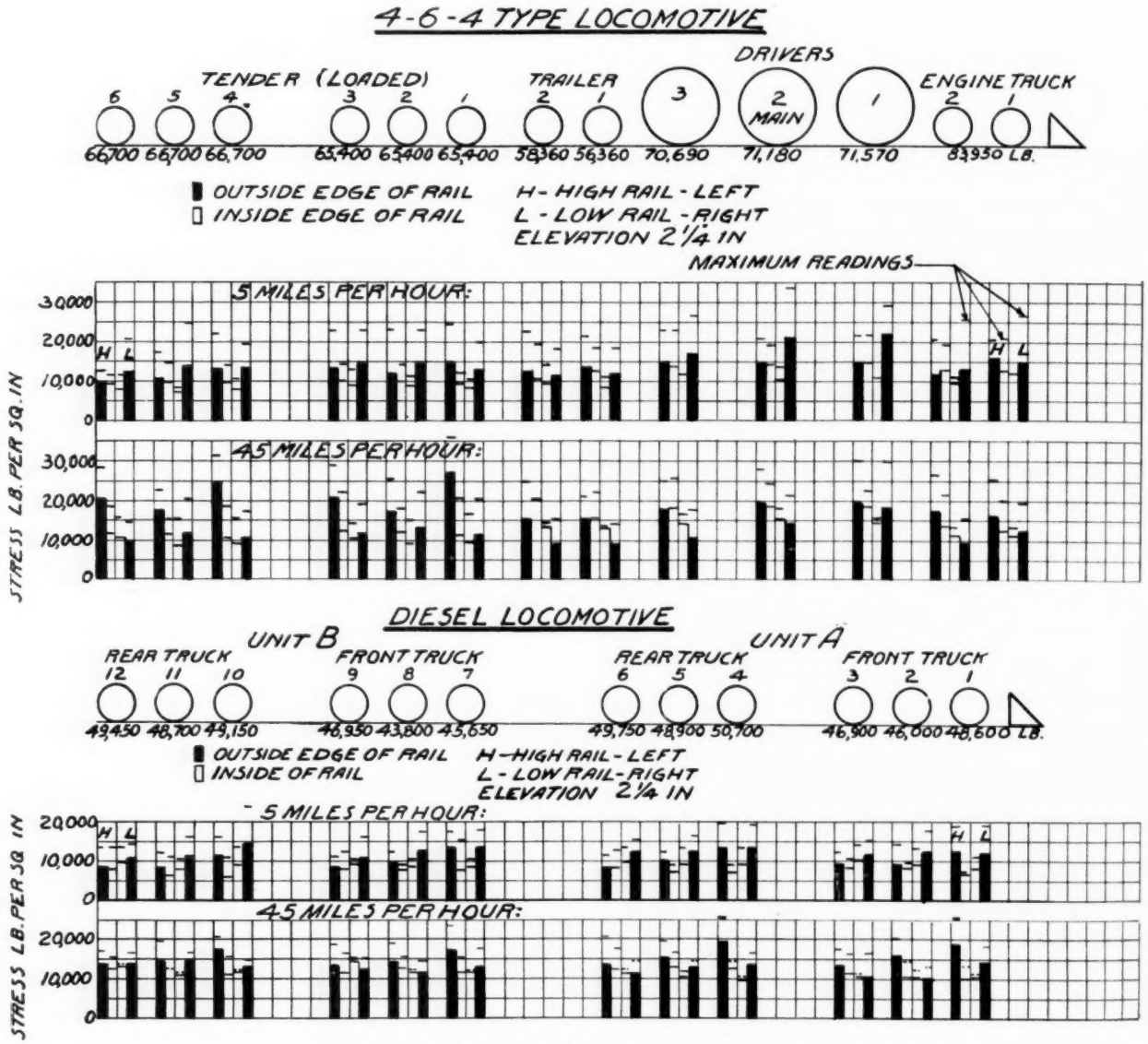


FIG. 2 MEAN STRESSES IN RAIL BASE UNDER EACH WHEEL
(Five-deg curve, 110-lb rail.)

RELATIVE COST OF MAINTENANCE OF DIESEL-ELECTRIC AND STEAM LOCOMOTIVES

There is a great deal of controversy as to the relative cost of maintenance of Diesel-electric locomotives and steam locomotives. It is difficult to secure reliable data from which to make comparisons. The comparison of costs on different railroads is open to question because of difference in operating conditions and accounting practices. The two types of power are not regularly used in the same type of service on the same railroad. Furthermore, the Diesel-electric locomotives are rather new and very few of them have received general repairs. It may be said that the two types are definitely different in regard to general repairs; the steam locomotive because of its boiler, flues, and firebox must be given general overhauling at relatively frequent periods. The Diesel-electric locomotive, on the other hand, is being constantly repaired by the regular replacement of the wearing parts. Some roads have put them in the back shop for general repairs after about 750,000 miles of service. It was originally thought that this might be the life of the major element of the Diesel motor, the crankshaft, but it has developed that the life of this part will normally be greatly in excess of this figure, and some roads are planning on one million or more miles between general shoppings.

The best available data appear to be the five-year-period maintenance-expense figures of a group of Pacific-type locomotives with approximately 3800 max ihp capacity, operating in normal passenger service, and similar figures for a group of 3600-hp Diesel-electric locomotives in high-speed service on the same railroad. These data show that the steam locomotives cost about 20 cents per mile for maintenance, whereas the Diesel-electric locomotives cost approximately 17 cents per mile for maintenance. These particular Diesels have never been given a general overhauling and, therefore, in order to be conservative, it might be fair to add as much as 3 cents per mile to this figure. These data indicate that it may be conservatively claimed that Diesel-electric locomotives can be maintained for at least the same cost as steam locomotives. If this same group of steam locomotives were operated at high speed, the repair costs would

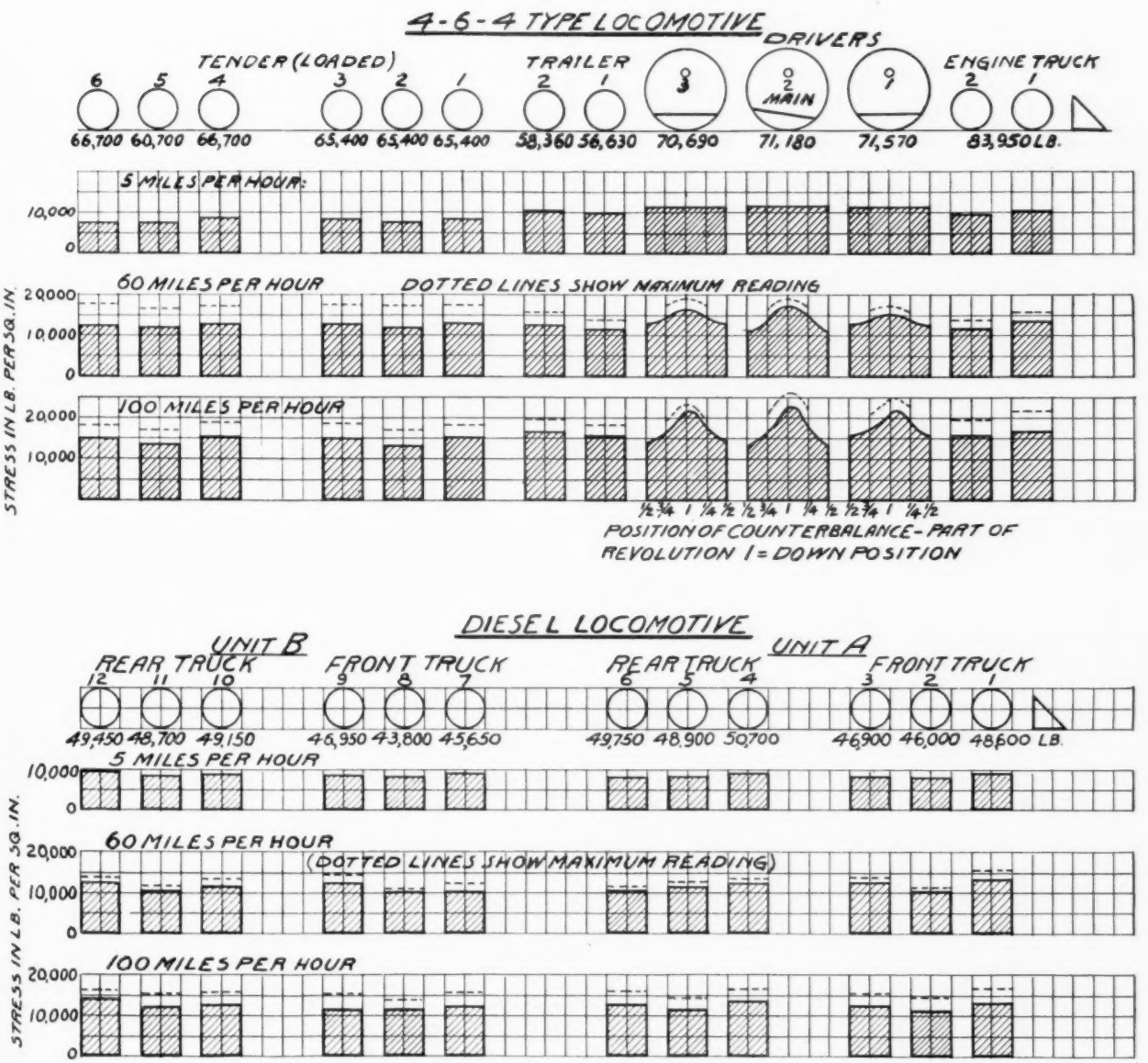


FIG. 3 VERTICAL STRESSES IN BASES OF BOTH RAILS
(Tangent track, 110-lb rail.)

undoubtedly be greater than the 20 cents shown. It is claimed by the builders of steam locomotives that the most modern type will have a lower cost of maintenance than older types, but it can also be claimed that the details of Diesel-electric locomotives are being steadily improved with a resultant reduction in maintenance cost. It must be recognized that costs on different railroads vary widely, particularly those applying to Diesels, since they are new and maintenance forces are not as familiar with them as they are with the steam locomotive. Furthermore, most railroads' repair facilities were designed for steam-locomotive work and are not suitable for efficient handling of Diesel repair work. New facilities are gradually being provided for taking care of Diesel work and their use should result in some reduction in maintenance cost.

TRACK STRESSES

Every steam locomotive must have a certain amount of overbalance in the driving wheels, in order to balance the reciprocating parts. This overbalance results in a dynamic augment which increases as the square of the speed and thus becomes an important factor in producing stresses in the rail and roadbed at high speed. In some cases this effect has been so serious that the main drivers actually lift from the rail at high speed and may produce kinked rail. Improvement in this condition has been made by reducing the weight of the reciprocating parts to a minimum and reducing the percentage of the reciprocating weight balanced to a minimum limited by the production of nosing and fore-and-aft motion of the engine. A still further effort along this line is embodied in the new experimental designs of locomotives having two complete sets of cylinders and motion work. This not only reduces the weight of reciprocating parts but gives another desirable characteristic, that of more constant torque. In order to illustrate the difference in rail stresses produced by a large Diesel-electric locomotive which has no reciprocating parts and a large modern high-speed reciprocating steam locomotive, Figs. 2 and 3 are included. These show the results of

rail-stress measurements on 110-lb rail with a modern 4-6-4 steam locomotive and a 3600-hp Diesel-electric locomotive. Fig. 2 shows stresses on a 5-deg curve at speeds of 5 and 45 mph and Fig. 3 stresses on level tangent track at speeds of 5, 60, and 100 mph. In Fig. 2 the stresses are shown separately for both edges of both rails, and the maximum stress at each location, as represented by the average of three highest readings at each location, is shown in short horizontal lines. In Fig. 3 the average vertical stress in the rail base is shown and the effect of the counterbalance by indicating its relative position. The maximum stress is also an average of three maximum values. The damaging effect of counterbalance is clearly shown in these data which are actual stress measurements and not calculations.

If a steam locomotive slips at high speed, the rotational speed of the driving wheels may be greatly in excess of that involved in these tests, therefore, the dynamic augment and resulting rail stress are greatly increased. The high rail stresses under steam-locomotive tenders due to too-high loads on small-diameter wheels may also be noted. Large tenders are necessary on high-speed locomotives in order to reduce to a minimum the delays for taking fuel and water, but it appears necessary to add two more pairs of wheels or increase the diameter of the wheels, in order to reduce wheel loads to a reasonable figure where damage to both track and wheels is not excessive.

No complete data are available for rail stresses under 4-8-4 type high-speed steam locomotives. However, the tests made indicate that rail stresses under the main drivers of such locomotives are about 25 per cent greater than those shown for the 4-6-4 type. This is due to the fact that the reciprocating parts in these larger and higher-powered engines are heavier and the driving-wheel diameter is usually made smaller to meet heavy-grade operating conditions.

All of these track-stress data point to the need of a radical change in steam-locomotive design for competition with the Diesel-electric locomotive in high-speed passenger-train operation. The four-cylinder design is a step in the right direction. The individual axle drive proposed by the Baltimore and Ohio Railroad is a further step and also the new turboelectric type. Up to the present time the gear-drive turbine locomotive has not been tried out in this country, but it too may have possibilities. The individual axle drive for a steam locomotive appears to have the most interesting possibilities since it involves not only relatively constant torque but also better track protection resulting from complete balance. A lower center of gravity can also be secured since driving wheels of smaller diameter can be used. A special committee appointed by the Association of American Railroads composed of leading railway mechanical engineers and representative engineers of the locomotive builders is making a thorough study of the entire question of design of steam locomotives for high-speed operation. Out of their work there will probably come some concrete and valuable recommendations on this important question.

Until some marked changes are made in steam-locomotive design it appears that the Diesel-electric locomotive will have a definite advantage in so far as effects on rail and roadbed are concerned.

Lightweight passenger cars also have an advantage over conventional cars in the matter of track-maintenance costs. Though their individual wheel loads are just as high, their lower total weight and trucks with shorter wheel base result in lower lateral rail pressures in high-speed operation.

Thermal Efficiencies

The theoretical maximum thermal efficiency of the Diesel-electric locomotive figured on the basis of energy developed at the rail is about 27 per cent, whereas the corresponding figure for a modern steam locomotive seldom exceeds 7 per cent, which would indicate a relative fuel consumption of about 1 to 4. This is somewhat altered for over-all performance by difference in weight of the two types of locomotive and difference in stand-by losses. It is difficult to find accurate comparative fuel-consumption data because the two types of power are not ordinarily used on similar runs. One comparison available is on a main-line fairly high-speed through train where steam power was superseded by Diesel-electric locomotives. The relative round-trip fuel-consumption figures based on a 12-month average on this run are as follows: Steam locomotives used 124 tons of coal; Diesel-electric locomotives used 3950 gal of Diesel oil. Based on the Btu content of the two types of fuel, these data indicate that the thermal efficiency of the Diesel-electric locomotives figured at the drawbar was 5.5 times that of the steam locomotives.

A careful check was made on another railroad to determine the relative amount of Diesel oil consumed by 3600 hp Diesel-electric locomotives and modern Pacific-type oil-burning steam locomotives, when pulling both light and heavy medium-speed main-line passenger trains on a 550-mile run. These data show the steam-locomotive oil consumption to be approximately four and one half times that of the Diesel-electric locomotives.

There appears to be little chance of improving the efficiency of the Diesel-electric locomotive except in the factor of transmission loss. There should be a possibility of marked improvement in steam-engine efficiency because of the many losses now involved. There has been a steady improvement in steam-locomotive thermal efficiency as indicated by the figures of the Association of American Railroads, which show that in 1920 it took 173 lb of coal to produce 1000 gross-ton-miles of freight transportation and 18.8 lb of coal per passenger-car-mile, whereas in 1938 these figures were only 113 lb for freight and 14.7 lb for passenger trains. A large part of this fine showing is undoubtedly due to improvements in steam-locomotive design and the reductions would be even greater if all the locomotives in service were of a modern type. Credit is due the designers for these improvements, which incidentally have been developed without the urge of competition from other types of power. Naturally under noncompetitive conditions the designers have been more or less bound by precedent. Conditions have now been changed. The young virile Diesel locomotive (whose designers were not swayed by precedent and who had at their disposal the vast amount of research data developed by the automotive industry) has reached a remarkable state of development in the short period of five years. This new competition should prove to be a spur and a benefit to the steam-locomotive designers, as is indicated by the numerous new designs which are mentioned elsewhere in this paper.

The Diesel designers had an advantage in connection with high-speed operation in that they started their development of road locomotives on the basis of high speed (top speed 118 mph). On the other hand, steam-locomotive designers focused their attention on the improvement of thermal efficiency and greater power development with top speeds of about 90 mph. Thus when the demand for high-speed operation developed so rapidly, their designs were not entirely adequate. This feature of high speed is the one which is now being given major consideration, and improvement should result.

Electric Locomotives

The modern electric locomotive is also a highly satisfactory type of motive power for high-speed operation, both from the viewpoint of capacity and effect on track structure. Early types had detrimental effects on track due to high lateral forces, but later designs have overcome this. The use of this type of power in this country has been limited, due to the fact that the

density of traffic on most railroads does not warrant the high cost of overhead construction. Where it is used for long-distance high-speed operation, as on the Pennsylvania Railroad, between New York and Washington and Harrisburg, it has shown excellent performance, but there appears to be little probability of electrification of less dense traffic lines, particularly those in the western part of the United States.

Operating Expenses

In 1938 Coverdale and Colpitts (2) issued a comprehensive report covering the results of their studies of operating costs and revenues of a majority of the streamlined trains in operation at that time. The cost data for the various railroads show such wide variations that it is difficult to arrive at a fair average figure for cost of operation of Diesel locomotives as compared with steam. Unfortunately, there are no roads on which comparable trains are operated with steam and Diesel-electric locomotives, so that there is no fair basis for comparison. However, a study of all the data in this report indicates that the cost of maintenance of a 3600-hp Diesel-electric locomotive is no greater and probably less than that of a modern steam locomotive of similar power, handling equal-weight trains on fairly comparable schedules. The fuel cost is approximately one half as much. The cost of crew wages is about the same, though it should be noted that on some roads a maintainer is included in the crew of Diesels for long-distance operation. This should not be necessary when firemen are fully trained in their work on this type of power. Lubrication costs (a lesser factor) are greater for Diesels than for steam locomotives. It is noticeable that the maintenance costs for Diesels are higher when the total load is much above 450 lb per hp.

The outstanding feature of this report is the large margin between gross revenue and net revenue (depreciation charges are not included). Most of the trains show net revenue over 50 per cent of gross revenue and in some cases this is as high as 75 per cent. These figures, are, of course, unduly favorable because of the omission of two important factors, namely, the overhead charges for depreciation and interest, and the item of track maintenance. The overhead in one particular case is shown in Table 6 of this paper. Even with the high mileage made by such trains, the overhead is a major item of expense because the first cost of these trains is high. As to track maintenance, it is well recognized that safe and comfortable high-speed operation necessitates a high standard of track maintenance. Furthermore high-speed operation, particularly with steam locomotives, increases the cost of maintenance, though no definite figures are available to determine such extra cost. It is a fact that large expenditures have been made, particularly on the western railroads to eliminate curvature and alter curves to make high-speed operation successful. As an example of this, it may be noted that one transcontinental railroad spent over four million dollars on such work during the last three years, and similar work is still under way. The total cost of these improvements should not be charged against the operation of the new trains for these changes also benefit the operation of other trains. Nevertheless, it must be admitted that there is a considerable part of this extra expense, which should properly be charged against the new trains to give a full picture of the financial results. If speeds are still further increased, this factor will become of even greater importance as more work of this kind will be necessary to make such schedules.

Coach-Type Streamliners on Long Runs

Table 6 shows a comparison of weights and operating costs between two streamlined trains operating in similar transcontinental service on one railroad. One of these trains is a first-class all-sleeping-car train and the other is an all-coach train. All of the data are prepared on the basis of full load of passengers which, of course, is not always the case. Estimated data are also shown for a train similar to the coach streamliner but with twice the capacity. All operating costs are taken from the Coverdale and Colpitts report (2).

TABLE 6 COMPARISON OF OPERATING COSTS FOR A FIRST-CLASS ALL-SLEEPING-CAR STREAMLINER AND AN ALL-COACH STREAMLINER

Item	First-class train	All-coach train	Estimated 10-car all-coach train
Number of cars	9	5	10
Number of Diesel power units	2	1	2
Horsepower of locomotive	3600	1800	3600
Total weight of cars, tons	482	248	496
Weight of power units, tons	287	147	287
Total weight of train, tons	769	395	783
Length of train, ft	890	473	970
Total salable seats or berths	121	156	312
Out-of-pocket cost per mile of operation[a]	$0.88	$0.68	$0.88
Train length per passenger (full load), ft	7.4	3.0	3.0
Car weight per passenger, tons	4.0	1.6	1.6
Train weight per passenger, tons	6.4	2.5	2.5
Out-of-pocket cost per passenger per mile of operation	$0.0073	$0.0043	$0.0028
Total yearly operation, miles	232,000	232,000	232,000
Total first cost of train	$1,110,000	$580,000	$1,160,000
Overhead cost per mile, based on 10 per cent of first cost	$0.48	$0.25	$0.50
Overhead cost per passenger per mile (full load)	$0.004	$0.0016	$0.0016
Overhead and operating cost per passenger-mile (full load)	$0.0113	$0.0059	$0.0047

[a] This cost does not include sleeping-car employees' wages or maintenance expense for sleeping cars.

It may be noted that the train weight per passenger for the first-class train is 6.4 tons, whereas for the coach train it is only 2.5 tons, which accounts largely for the higher out-of-pocket cost for the former. The overhead cost per passenger is more than twice as great. The total overhead and operating cost per passenger-mile is approximately twice as great for the first-class train as for the coach train. It is thus apparent that much lower rates can be charged on coach trains. This latter type of train has been remarkably successful in attracting passengers for this long run, although it involves their spending two nights on the road.

It is the coach type of train that primarily has the possibility of diverting traffic from private automobiles and buses to the railroads. Because of the low rates it also has the possibility of developing a large amount of new traffic among people in the low- and medium-income group, who are becoming more and more travel-minded and have more and more time available for such travel. There is almost no limit to the possibilities for development of this type of travel. People of this class want low-cost transportation with high speed and comfort. All of these are being provided in such trains.

The Coverdale and Colpitts report shows remarkably successful results from the operation of all of the high-speed streamlined trains. They have undoubtedly brought back to the railroads some of the lost traffic and give promise of developing new traffic.

It appears that these new high-speed trains, particularly those in the western territory are now operating at about the maximum speed, which is possible on the present track structure. The limitation to further increase in speed lies in the large number of rather sharp curves which are unsafe for too high speed and uncomfortable for passengers when taken too fast. The number of unprotected grade crossings is also a limitation, as well as speed restrictions through towns with grade crossings. It would be very expensive to correct all of these track conditions.

Outside of the passenger-traffic gain, these new trains have been warranted by the public interest which they have aroused.

TABLE 7 LIST OF HIGH-SPEED LIGHTWEIGHT STREAMLINED PASSENGER TRAINS

Railroad	Name of train	No. of trains	Service	Type of locomotive	Run	Route, miles	No. of cars
A. T. & S. F.	*Super-Chief*	2	Twice weekly	Diesel 3600 hp	Chicago Los Angeles	2229	9
A. T. & S. F.	*The Chief*	6	Daily	Steam	Chicago Los Angeles	2229	10
A. T. & S. F.	*El Capitan*	2	Twice weekly	Diesel 1800 hp	Chicago Los Angeles	2229	5
A. T. & S. F.	*San Diegan*	1	Twice daily	Diesel 1800 hp	Los Angeles San Diego	126	7
A. T. & S. F.	*Chicagoan Kansas Cityan*	2	Daily	Diesel 1800 hp	Chicago Wichita	678	7
A. T. & S. F.	*Golden Gate*	2	Twice daily	Diesel 1800 hp	Bakersfield San Francisco	316	6
Baltimore & Ohio (Alton)	*Abraham Lincoln*	1	Daily	Diesel 1800 hp	Chicago St. Louis	282	8
Baltimore & Ohio (Alton)	*Ann Rutledge*	1	Daily	Steam	Chicago St. Louis	282	8
Boston & Maine	*Flying Yankee*	1	Daily ex. Sun.	Diesel 600 hp	Boston Portland	114	3
C. B. & Q.	*Pioneer Zephyr*	1	Daily	Diesel 600 hp	Kansas City Omaha-Lincoln	250	4
C. B. & Q.	*Sam Houston Texas Rocket*	2	Twice daily	Diesel 600 hp	Fort Worth Dallas-Houston	283	4
C. B. & Q.	*Mark Twain*	1	Daily	Diesel 600 hp	St. Louis Kansas City	279	3
C. B. & Q.	*General Pershing*	1	Daily	Diesel 1000 hp	St. Louis Kansas City	279	4
C. B. & Q.	*Denver Zephyrs*	2	Daily	Diesel 3000 hp	Chicago Denver	1036	12
C. B. & Q.	*Twin City Zephyrs*	2	Twice daily	Diesel 1800 hp	Chicago Twin Cities	419	8
C. M. & St. P.	*Hiawatha*	4	Twice daily	Steam	Chicago Twin Cities	422	9
C. R. I. & P.	*Rocket*	1	Twice daily	Diesel 1200 hp	Chicago Peoria	161	4
C. R. I. & P.	*Rocket*	1	Daily	Diesel 1200 hp	Chicago Des Moines	358	4
C. R. I. & P.	*Rocket*	2	Daily	Diesel 1200 hp	Kansas City Minneapolis	489	3
C. R. I. & P.	*Rocket*	2	Daily	Diesel 1200 hp	Kansas City Dallas	627	4
Gulf, Mobile & Northern	*Rebel*	2	Daily	Diesel 660 hp	New Orleans Jackson	488	4
Gulf, Mobile & Northern	*Rebel*	1	Daily	Diesel 660 hp	Mobile Union	181	1
Illinois Central	*Green Diamond*	1	Daily	Diesel 1200 hp	Chicago St. Louis	294	4
New York Central	*Mercury*	1	Daily	Steam	Cleveland Detroit	165	4
New York Central	*Twentieth Century*	2	Daily	Electric & steam	New York Chicago	960	9+
N. Y. N. H. & H.	*Comet*	1	6 trips daily	Diesel 400 hp	Providence Boston	44	3
Pennsylvania	*Broadway Limited*	2	Daily	Electric & steam	New York Chicago	907	8+
Reading	*Crusader*	1	Twice daily	Steam	Philadelphia Jersey City	90	5
Seaboard Air Line	*Silver Meteor*	1	Twice weekly	Diesel 2000 hp	N. Y.-Miami N. Y.-St. P.	1389 1247	7
Southern Pacific	*Sunbeam*	1	Daily	Steam	Houston Dallas	264	8
Southern Pacific	*Daylight*	2	Daily	Steam	Los Angeles San Francisco	471	14
Union Pacific	*City of Salina*	1	Daily	Diesel 600 hp	K. C.-Salina K. C.-Topeka	187 68	3
Union Pacific	*City of Portland*	1	5 trips monthly	Diesel 1200 hp	Chicago Portland	2272	6
Union Pacific	*City of Los Angeles*	1	5 trips monthly	Diesel 2400 hp	Chicago Los Angeles	2298	13
Union Pacific	*City of Los Angeles-2*	1	5 trips monthly	Diesel 5400 hp	Chicago Los Angeles	2298	17
Union Pacific	*City of Denver*	2	Daily	Diesel 2400 hp	Chicago Denver	1048	10
Union Pacific	*City of San Francisco*	2	5 trips monthly	Diesel 5400 hp	Chicago San Francisco	2259	14

NOTE: Six more trains for various railroads are under construction at the time of preparation of this tabulation, and also a considerable number of lightweight cars for use in other trains.

Prior to their advent, there was a feeling that the railroads had not kept pace with developments as they should have, and consequently the public had less interest in their welfare. They are now much more favorably inclined and this may have an influence on future legislation.

Large numbers of new lightweight cars are now being added to main-line trains, and there is a gradual replacement of old types. The public demand for such cars is due largely to their many comforts and attractive appearance. There is every indication that there is a real revolution in American passenger trains. Table 7 shows a list of the high-speed streamlined trains now being operated on the railroads of the United States.

CONCLUSIONS

The streamlined high-speed passenger train has been definitely successful on American railroads in recovery of traffic, financial return, and mechanical performance. The number of such trains in service should continue to increase.

Both steam and Diesel-electric locomotives will pull these new trains, but the Diesels will predominate unless new designs of steam locomotives prove themselves better suited for the long high-speed runs than are the present types. There should be a continual improvement in the design details of Diesel-electric locomotives. Radically new designs of steam locomotives will probably be built to meet the high-speed operating conditions.

Schedules faster than those now being made are possible and probable as soon as roadway conditions are further improved by curvature reduction and grade-crossing elimination.

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January 5, 1935, page 3, "New York, New Haven & Hartford Streamlined Coaches."
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May 4, 1935, page 671, "Baltimore & Ohio *Abraham Lincoln*."
May 11, 1935, page 719, "Chicago, Milwaukee, St Paul & Pacific *Hiawatha*."
June 8, 1935, page 875, "Union Pacific *City of Portland*."
June 15, 1935, page 910, "Gulf, Mobile & Northern *Rebels*."
November 2, 1935, page 563, "Chicago, Burlington & Quincy *Mark Twain Zephyr*."
March 28, 1936, page 534, "Illinois Central *Green Diamond*."
May 30, 1936, page 864, "Union Pacific *City of San Francisco* and *City of Los Angeles*."
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March 27, 1937, page 540, "New Haven Installs Streamline Passenger Locomotives."
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Discussion

T. R. Cook.[3] The writer agrees with the author's conclusions that the railroads have and can regain considerable of their lost passenger business by the adoption of better schedules and the provision of more comfortable passenger trains, and there is no question but that lighter construction is the principal fundamental factor involved.

As to whether these trains will be hauled by Diesel or steam power is a question which will be resolved by the economies of the situation. There are over 43,000 steam locomotives in existence in the United States, divided in the order of freight 65 per cent, passenger 17 per cent, and switchers 18 per cent.

It is quite evident that the major activity, that of hauling the freight will alone keep the steam locomotive on the rails for some years to come. A number of the railroads are coal carriers. It would, therefore, seem logical that the roads and the steam-locomotive builders, to say nothing of other investments in property and plants dependent upon steam, should constitute a sufficient urge to force the advancement of steam locomotives. There is no doubt but that there is room for improvement both in engineering and in operation of steam power.

As to the present situation of steam, the writer is doubtful of the superiority of the Diesel as indicated in the paper, particularly when the Diesel locomotive is considered as a prime mover. The author gives a thermal efficiency at the rail of 27 per cent for the Diesel and 7 per cent for the steam locomotive. The 27 per cent is correct for the Diesel, but 7 per cent in the writer's opinion is quite low for the modern steam locomotive.

A more interesting figure, however, would be a statement of the efficiency at the drawbar where the work is done, and not only at full load but at various load factors and at various speeds. We are dealing with types of power having entirely different characteristics. In order to make a comparative statement, the writer has worked out the thermal efficiencies of a modern 4-8-4 and a Diesel locomotive each of 5400 hp, through a range from 30 to 100 mph, and load factors varying from 100 per cent down to 40 per cent. The results indicate that the maximum efficiency of the Diesel is developed at 30 mph with 100 per cent load factor; this efficiency decreases as the speed increases and as the load factor decreases. The steam locomotive, on the other hand, generally increases in efficiency as the load drops and has higher efficiency at 80 per cent load at all speeds than at 100 per cent load; up to about 50 mph this increase in efficiency continues down to a 40 per cent load factor. It may fairly be assumed that, in high-speed passenger service, the load range is primarily between 60 and 80 per cent, at speeds varying from 50 to 70 mph. In the example taken in the ranges cited, the Diesel shows an efficiency of 22 per cent and the steam slightly over 8 per cent. This would indicate that the cost of the fuel in the normal range of unit prices would be about equal.

As to the cost of repairs, it is believed that the age of the equipment must be taken into consideration. The increased cost in repairs of steam locomotives with increasing age has been thoroughly demonstrated. The Diesel switcher has shown a greater rate of increase and, although there are no figures available, it is safe to assume that the Diesel road locomotive will show a similar increase.

The author in Fig. 1 shows comparative drawbar-horsepower curves of a steam versus a Diesel locomotive, stating that the curves are plotted from actual dynamometer-car readings. As a rule, dynamometer cars are placed in a given scheduled train and the readings taken show the resistance of the train under various conditions, but do not show the capacity of the prime mover at various speeds. The writer has found it of interest to compare the theoretical drawbar horsepower with that shown in the author's curves. His calculations indicated that the Diesel was operating below full load up to 80 mph and, thereafter, was working at an overload; while, in the case of steam, the locomotive was operating at full load up to 50 mph and was working considerably under capacity at higher speeds.

In Table 4 of the paper, in reference to steam power, what is

[3] Coverdale and Colpitts, Consulting Engineers, New York, N. Y. Mem. A.S.M.E.

the author's definition of "maximum indicated horsepower?" The indicated horsepower of a steam locomotive increases with speed up to a point where the steam-pipe valve and port resistance prevents the use of all the steam the boiler can evaporate.

Under the section on "Maintenance," it is stated that the cost of repairs of a steam locomotive is 20 cents per mile, and the comparative Diesel costs 17 cents per mile; the writer would like to know the age of both the steam and the Diesel units used in this comparison.

Under the section on "Thermal Efficiency," a comparison is made between Diesel and steam power. In one case, a ratio of 5.5 is shown and in another a ratio of 4.5. In the two instances was the work done the same, i.e., did the Diesel and steam locomotives haul the same-weight trains over the same tracks?

L. B. Jones.[4] The author quite properly calls attention to the attractive weight reduction made possible by the use of lightweight cars. Parallel with the development of lightweight

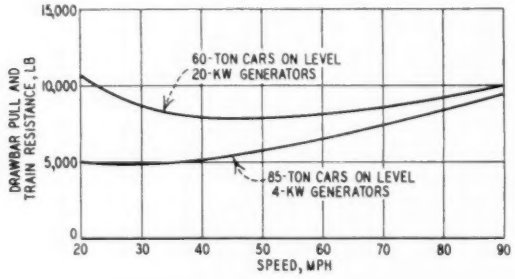


Fig. 4 Drawbar Pull Required for 12-Car Trains (85-ton cars, 1020 tons per train; 60-ton cars, 720 tons per train.)

cars, however, has come the development of mechanically driven air-cooling systems which offset any advantage the locomotive might realize from the substitution of lightweight cars in the train.

In Fig. 4, two 12-car trains have been assumed, one made up of 85-ton cars having 4-kw axle generators working to rated capacity, and another 12-car train of 60-ton cars, with 20-kw generators, generating at rated capacity. It will be noted that the drawbar pull required for the 60-ton cars is greater, up to speeds considerably in excess of 90 mph. The figures are for level track. On an upgrade, the lightweight cars would gain some advantage.

The development of heavy axle-generator loads has, therefore, robbed the locomotive of whatever advantages the lightweight cars offer in the way of drawbar pull. The development of air cooling has been very rapid, and sufficient consideration has not been given to a logical and comprehensive method of supplying the power necessary. As a starting point, it is suggested that a 440-v, 3-phase train line, powered from a small plant on the locomotive, would supply all necessary power for cooling and lighting at an over-all efficiency probably double that of individual axle-generator units. Storage batteries could also be reduced to about 1/4 of their present size, and kept charged through a small automatic rectifier. Stand-by service would be provided in large terminals, as is now done with steam heat.

Reference is made to the lower center of gravity of Diesel-electric locomotives, with higher possible speed on curves. This is no doubt true as regards speeds at which the locomotive will upset, but there have been cases where it was necessary to raise the center of gravity of electric locomotives in order to prevent serious lateral forces which pushed the track out of line or overturned a rail. In Figs. 5 and 6, curves have been plotted showing the maximum possible speed of stability of a locomotive on a 6-deg curve. Fig. 5 shows a curve with 5 in. superelevation and Fig. 6 a curve without superelevation.

While the speed required to upset the locomotive increases rapidly with reduced center of gravity, it will also be noted that the speed required to overturn the rail has the opposite trend, and the two curves cross at 98 mph in Fig. 5. These curves are plotted on the basis of centrifugal force only. When we consider the maximum forces resulting from nosing or lurching of the locomotive, it is evident that too low a center of gravity can create dangerous lateral pressure on the rail, and therefore the advantages of a low center of gravity must be circumscribed.

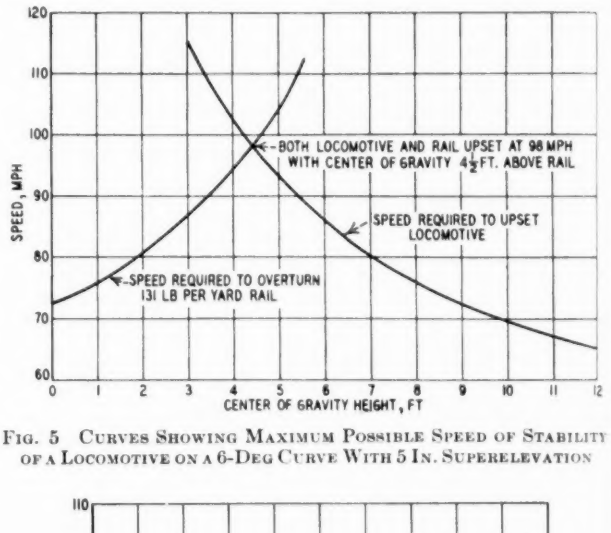


Fig. 5 Curves Showing Maximum Possible Speed of Stability of a Locomotive on a 6-Deg Curve With 5 In. Superelevation

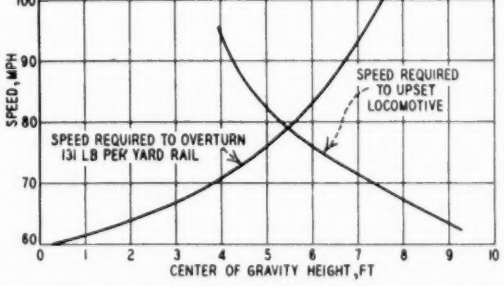


Fig. 6 Curves Showing Maximum Possible Speed of Stability of a Locomotive on a 6-Deg Curve Without Superelevation

Data are presented in the paper showing rail stresses under a steam locomotive in comparison with a Diesel. Omitting the steam-locomotive drivers for the moment, and considering only the engine and tender trucks, which are not subjected to dynamic forces, it will be noted that the rail stresses under the locomotive trucks increase much more rapidly from 5 to 100 mph than the stresses under the Diesel-locomotive trucks. It is not apparent why rail stresses should increase more rapidly with speed under a locomotive-tender truck, for example, than they do under a Diesel-locomotive truck.

Considering now the locomotive driving wheels, it will be noted that the minimum rail stress under the main wheel at 100 mph is only slightly less than the stress due to static loading at 5 mph (it is assumed that at 5 mph no dynamic stresses are noticeable). In most 2-cylinder steam locomotives which have been tested, there is a definite tendency for the main wheels to leave the rail

[4] Engineer of Tests, The Pennsylvania Railroad, Altoona, Pa. Mem. A.S.M.E.

at a speed of 100 mph and, although with well-balanced locomotives no lifting actually occurs, the rail stress at one point of the revolution should be very nearly zero. This raises the question whether the strain gages used were sufficiently sensitive to record the actual stress variations. If average figures were used in plotting the vertical stresses, this might explain the tendency to level off the variations throughout one revolution.

While there can be no challenging the author's statement that future development of the steam locomotive should look toward reduction of the dynamic augment, it seems only fair to observe that the comments on undesirable dynamic behavior have been based upon the performance of a few locomotives, whereas hundreds of other steam locomotives are handling fast schedules every day without any symptom of trouble. We recently conducted a series of tests in which several locomotives were slipped on greased track up to speeds very much in excess of their maximum operating speeds and, while the dynamic effect was definitely noticeable, there was no trace of rail damage in 100-lb rail on cinder ballast, and the rail stresses recorded were not excessive. The only precaution taken in connection with the locomotives tested was that the counterbalances were precisely checked in advance of the tests, as well as the quartering and tramming of the locomotive wheels.

A. I. Lipetz.[5] There is a common but unjustified notion that a Diesel-electric locomotive has the advantage of greater acceleration as compared with a steam locomotive. This is a remnant of the common theory that the electric locomotive provides quicker acceleration than the steam locomotive. In this consideration, the fact is overlooked that the electric locomotive usually draws power from an outside source to about double its nominal power at low speeds. However, this is not true for the Diesel-electric locomotive, in which the electric feature lies only in the transmission, not in the generation of power. The advantage of the Diesel-electric locomotive is in the magnitude of the tractive effort only at low speeds, since the power is always limited by the power of the Diesel engines and by the greater number of drivers which are carried on the locomotive chassis, just as the power of the steam locomotive is limited by that of the boiler on the locomotive frame and its number of drivers. Thus, the author's statement regarding the "rapid acceleration (of the Diesel-electric locomotive) because of the high tractive power at lower speeds" may be literally correct, but can be misleading if the speeds to which this applies are not given. This advantage of a Diesel locomotive is of comparatively short duration and is not of the same order, as in the straight electric locomotive with an outside source of power. In order to evaluate this advantage, a more accurate and detailed calculation is necessary. This has been done and is presented in Fig. 7.

Referring to Table 4 of the paper, the steam locomotive shown in the third column is the one discussed by the author. The maximum indicated horsepower is given as 4300, and the graph, which is shown in Fig. 1, gives the drawbar horsepower, both for a steam and some Diesel locomotives. The steam locomotive is referred to as a modern locomotive of the 4-6-4 type. Regarding the drop of the horsepower shown in the curve, it is explained that this can be improved if a change both in the design of the valve and the valve gear is made. Fig. 7 also refers to the 4-6-4-type steam locomotive of the New York Central of a lighter design but, nevertheless, of 4700 ihp, built for high speed with this very improvement in the valves; the steam curve is taken from actual tests.

The drooping characteristic of the horsepower curve of the

[5] Chief Consulting Engineer, American Locomotive Company, Schenectady, N. Y. Fellow A.S.M.E.

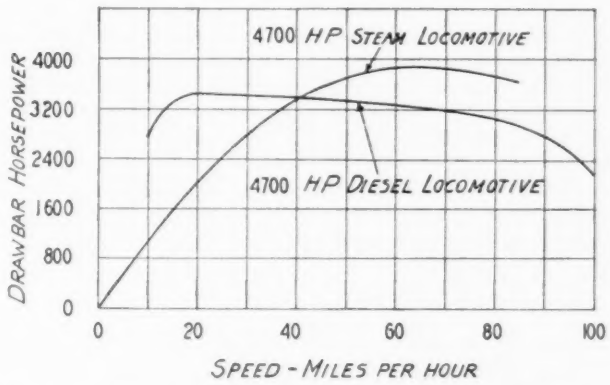


Fig. 7 Comparison of Drawbar Horsepower of Steam and Diesel Locomotives of Identical Indicated Horsepower

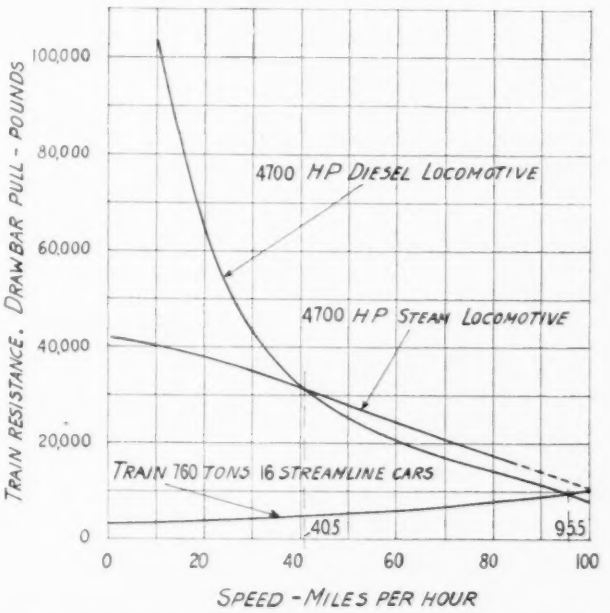


Fig. 8 Tractive Effort in Relation to Speed in Miles Per Hour of Steam and Diesel Locomotives of Identical Horsepower

steam locomotive of Fig. 1 has been corrected in Fig. 7; the part of the curve at high speeds has a flatter appearance. Furthermore, the weight of the locomotive with a fully loaded tender is given in Table 4 of the paper as 809,000 lb. For the 4700-hp steam locomotive it is only 674,300 lb and, therefore, the weight per horsepower, instead of being 188 lb as shown in Table 4 is only 144 lb. Moreover it should be noted that the steam locomotive is not running all the time with its tender fully loaded and, therefore, has the advantage of reducing its dead weight; this remains practically constant in a Diesel locomotive. However, this slight advantage of the steam locomotive will be disregarded and the resistance will be based upon the weight of the locomotive with fully loaded tender as given, namely, 674,300 lb. The maximum drawbar horsepower of the 4700-hp steam locomotive is 3880 hp, as established by very elaborate tests[6] of the New York Central.

[6] "New New York Central Locomotives Show High Power Concentration," *Railway Age*, vol. 104, 1938, p. 601. Also: "The New York Central Receives 50 Powerful 4-6-4 Locomotives," *Railway Mechanical Engineer*, vol. 112, 1938, p. 173.

As to the Diesel-electric locomotive shown in the second column of Table 4, the power of the Diesel engines is only 4000 hp and the maximum drawbar power is 2950 hp, against 3880 for the steam locomotive. In fairness to the Diesel-electric locomotive, which the writer will compare with the steam, the drawbar power given by the author in Fig. 1, from actual dynamometer tests of the 4000-hp Diesel locomotive, should be increased in proportion to the maximum indicated horsepowers of the steam and Diesel locomotives; in other words, for the Diesel-electric locomotive, the maximum drawbar power should be

$$2950 \times \frac{4700}{4000} = 3465 \text{ hp}$$

It is only fair to the Diesel-electric locomotive to assume that the Diesel power is equal to the indicated power of the steam locomotive, since both must be multiplied by the efficiency of power transmission and mechanical efficiency of the chassis, in order to arrive at the drawbar powers. Thus, we are impartially comparing a steam and a Diesel locomotive of identical maximum indicated horsepower, which is 4700 in either case, by studying their corresponding drawbar-horsepower curves.

Fig. 8 shows the tractive efforts in pounds in relation to speed in miles per hour for these two locomotives of identical maximum horsepower. The advantage of the tractive effort of the Diesel locomotive at low speeds is evident from this illustration. It is true that, due to the electric transmission there is an advantage in developing greater tractive effort than with the steam locomotive for all speeds below 40.5 mph, but the difference between these two curves drops very rapidly and is negative at speeds above 40.5 mph.

Also in Fig. 8 is shown the resistance of a 16-car lightweight streamlined train weighing 760 tons, or 47.5 tons per car. This is what the Diesel-electric locomotive would be able to pull on level track at high speed. The balancing speed of the Diesel-electric locomotive is about 95.5 mph on the level. For the steam locomotive it is slightly higher, about 101 mph, because of greater sustained power at high speed.

The acceleration curves of these two trains are shown in Fig. 9. From the comparison, it may be observed that the intersection point of 40.5 mph is reached by steam in 2 min 10 sec within a distance of 0.75 mile, whereas the Diesel will do it in 1 min 16 sec in a distance of 0.51 mile. The Diesel will thus gain 54 sec and 0.24 mile. But, we are concerned here with performances at higher speeds than 40.5 mph and, if we continue beyond 40.5 mph, we shall be gaining time with steam; in fact, the steam locomotive will reach 95.5 mph in 10 min 17 sec from the start, while the Diesel will take 15 min 20 sec to attain this speed. In other words, the Diesel will lose the 54 sec which it gained at the start before reaching the intersection point of 40.5 mph, but will lose an additional 4 min 9 sec, showing a total loss from the start of 5 min 3 sec, just contrary to what might be expected. Every time the train is slowed down (due to speed restriction) to a speed about 40.5 mph and accelerated again, the total gain in time, in the case of the steam train, will be increasing cumulatively.

It is interesting to note that L. Dumas, assistant director of the French National Railway, who has had considerable experience with Dieselized streamlined trains, made a special report,[7] in which he gave a summary of three different reports before the International Railway Congress Association. In this report he also made a statement concerning the advantage of quick acceleration of trains due to Dieselization with electric transmission, as compared with steam locomotives, but confined his remarks to trains with electric-motor coaches, essentially rail cars, and high-speed trains of the German and French types, in which the ratio of Diesel power to the weight of the cars is very high. The acceleration is, therefore, due not to the electric transmission, but to the high power-to-weight ratio, which is lacking in the American high-speed trains with sleeping accommodations.

In connection with the author's remark that the performance of the steam locomotive, especially of modern type, depends largely on the engineer and is subject to increased efficiency and capacity, an interesting incident demonstrating this fact occurred on the Chicago, Milwaukee, St. Paul & Pacific on November 18, 1939. On one of the new class F-7 locomotives of the 4-6-4 type, which is now operating fast heavy trains out of Chicago, something went wrong and the locomotive was cut out at Milwaukee.

A 4-4-2 *Hiawatha* locomotive, which is lighter and less powerful than the 4-6-4 regular locomotive, was put on in its stead and pulled the 10-car train, weighing approximately 1,000,000 lb from Milwaukee to Minneapolis, bringing it in on time and making up almost the entire delay of 22 min; the train arrived at St. Paul only 2 min late. It is interesting to note that originally the *Hiawatha* was designed to pull only 6 cars weighing 700,000 lb, but lately has pulled 8 cars of corresponding weight. A Diesel locomotive would meet its requirements only, and no more; it does not have the flexibility of the steam locomotive.

Referring to the matter of counterbalancing, discussed in the section on "Track Stresses," there can be no doubt but that the steam locomotive is at a disadvantage as compared with any other locomotive without dynamic augment, such as Diesel-elec-

[7] "Methods Used to Speed Up Passenger Trains, and Resulting Expenditure," Special Report by L. Dumas, Bulletin of the International Railway Congress Association (English edition), July, 1939, pp. 661–663.

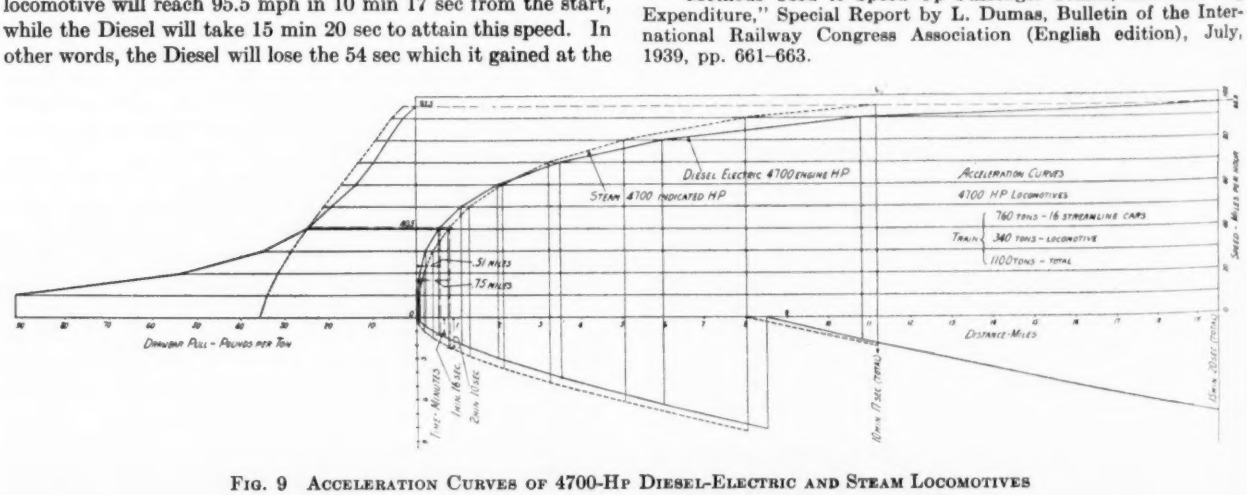


FIG. 9 ACCELERATION CURVES OF 4700-HP DIESEL-ELECTRIC AND STEAM LOCOMOTIVES
(Train composed of 16 streamlined cars, 760 tons; locomotive 340 tons; total 1100 tons.)

tric, turbine, or straight-electric. However, the examples given in the paper are not fully convincing; first, because the steam locomotive in question was heavy, 71,570 lb on the first driving axle, as compared with 50,700 lb on the fourth driving axle of the Diesel-electric. Furthermore, the locomotive, as may be judged from Fig. 3, was dynamically counterbalanced, or as we call it, cross-balanced, only on the main wheel, which was the second driving wheel. The front and back wheels were statically balanced. The percentage of balancing is not given and it is natural that the dynamic stresses in the rail should be greater for the steam locomotive. It is remarkable that, with all the handicaps of this steam locomotive, the difference is so small; the maximum rail stress is 19,000 lb for the steam locomotive at a speed of 100 mph as compared with 17,000 lb for the Diesel-electric at the same speed, or an increase of 12 per cent for the steam; much less than the increase in static weights, which is 41 per cent. For the Diesel-electric locomotive, the maximum at 100 mph is about the same as for the tender of the steam locomotive, although the weight on the tender axle is still about 30 per cent greater. This proves that the dynamic augment of the steam locomotive is not so grave as it is usually represented; it is the static weight which matters most, probably due to the element of time.

The old-time track and bridge engineers were mainly concerned about the static weight per axle. It is only recently, when the balancing of locomotives became better known and understood, that the dynamic stresses in rails began to bother them, especially when improperly evaluated.

The questions of track stresses and of counterbalancing of locomotives had a very peculiar history. Without going into details, it is enough to say that only lately proper attention is being paid to the question of counterbalancing of steam locomotives.

With the old-time counterbalancing, speeds up to 80 mph could be easily obtained. Hammer blows as high as 15,000 lb per wheel and more, which would kink and break rails, were evidently tolerated on a great many railroads. The track was simply built very stiff so the locomotives could run at these speeds. As one engineer expressed it, the "anvil" was strong enough to stand the "hammer blow." However, when the electric and the Diesel-electric locomotives came into being and proved their superiority for track conditions and their maintenance, the steam-locomotive engineer became cognizant of the unfitness of the old-time steam locomotive to modern high-speed conditions. Cross-balancing and lighter reciprocating weights, to which 15 years ago no one would listen, became very popular.

To a great extent the question of adaptability of steam locomotives to high speeds depends upon designing the locomotive so that it may have good tracking conditions at high speeds. American locomotives, especially of the type with front and rear trucks, have been very well suited to good negotiation of curves at high speeds. This partly eliminated the necessity of paying much attention to proper counterbalancing of locomotives. The recent development of lateral-motion and cushioning devices in locomotives was also a great help in this direction.

It is now possible to build locomotives with very small overbalances on wheels and, consequently, small hammer blows. Although it may not yet be possible to bring it down to zero, as was done on the Madras & Southern Mahratta Railway of British India, an improvement in track stresses, nevertheless, would result from the small overbalances, if the percentage of balancing is reduced to a figure close to zero and compatible with the shaking forces of light reciprocating weights in modern steam locomotives.

Wide discussion is taking place concerning the low center of gravity of some trains and Diesel-electric locomotives, as compared with the high center of gravity of steam locomotives, although nothing is pointed out as to what kind of disadvantages may result from the high center of gravity of the steam locomotive. This question was investigated years ago by many locomotive designers. In the 1870's, it was even proved experimentally that the high center of gravity is an advantage because it stabilizes the locomotive at high speeds. Of course, on curves, the centrifugal force may overturn a locomotive, if proper care is not taken in the design, but if the correct track elevation is made and the centrifugal forces are thus neutralized, nothing will happen on a properly designed and maintained track. However, the locomotive will run more smoothly.

It should be remembered that any train, even on first-class track, is subjected to violent vibrations due to the unevenness of the permanent way, its discontinuity due to rail joints, the resiliency of rails, ties, and ballast, and that a certain limit of speed must be set for any train with any locomotive—steam, turbine, Diesel, or electric, on a flexible track.

In a recent paper,[8] F. G. Gurley, vice-president of the Atchison, Topeka & Santa Fe Railway, predicted speeds of 140 to 150 mph and indicated the motive-power equipment which would satisfy this condition. He even saw some possibilities for the steam locomotive, but, of course, had no doubts about the Diesel-electric locomotive at those speeds. As far as the writer is concerned, the figure of 150 mph cannot be accepted for any locomotive on any track until by actual experience the possibility of these high speeds is proved. It is not known that they are wanted for rail transportation.

It is interesting to note that electric locomotives on the Pennsylvania Railroad, which has first-class track, the heaviest in the world, are limited to 90 mph and are permitted to run at this speed only because their weight per axle is comparatively low.

Modern steam locomotives, properly cross-balanced on all axles, with cushioning devices on properly selected axles, can easily run at 100 mph, and have already proved their riding qualities at 115 mph. In fact, abroad, a steam locomotive developed 125 mph in the presence of the London Institution of Locomotive Engineers. References to this trip have been widely published.[9]

Speeds at 100 mph with steam locomotives are now not unusual in this country. There is no doubt in the writer's mind but that the steam locomotive can meet any higher speed, limited only by the speed of cars, obtained by any other type of locomotive.

In discussing the economics of the problem, the author in Table 6 gave us a comparison of operating costs between two streamlined trains in transcontinental service on one railroad. He also referred us to the well-known report prepared by Coverdale and Colpitts (2). He mentioned, however, that interest and depreciation charges have not been taken into account by these analysts.

This comparison has been completed by the writer by adding several more trains to Table 6, and by trying to figure out from various sources the cost of streamlined trains, using for this purpose a paper[10] by E. E. Chapman on the subject. The results of this comparison are given in Table 8. Several representative streamliners, like the *City of San Francisco*, *Denver Zephyr*, and *Hiawatha*, have been added and the table is complemented by the cost of locomotives and cars, fixed charges per year on the basis of 10 per cent of interest and depreciation, the income per train-

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[8] "Performance Limits of Transportation; Range, Speed, and Capacity," by J. G. Gurley, Michigan-Life Conference, University of Michigan, Ann Arbor, Mich., November, 1939.

[9] "New German Streamlined Locomotives," *Railway Gazette*, vol. 62, 1935, pp. 1209–1217. Also: "The Borsig Centenary," *Railway Gazette*, vol. 67, 1937, p. 318.

[10] "Diesel and Steam Locomotives in High-Speed Service," by E. E. Chapman, *Railway Age*, vol. 105, 1938, pp. 733–735.

TABLE 8 OPERATING RESULTS OF HIGH-SPEED STREAMLINED TRAINS

Railroad	Southern Pacific	B. & O.		Union Pacific	Milwaukee	Burlington	Burlington *Zephyrs* (avg, 8 trains)	Santa Fe streamliners (avg, 7 trains)	Rock Island *Rockets* (avg, 6 trains)
Route	San Francisco–Los Angeles	Chicago–St. Louis		Chicago–San Francisco	Chicago–Twin Cities	Chicago–Twin Cities	....	....	....
Train	*Daylight*	*A. Lincoln*	*A. Rutledge*	*City of San Francisco*	*Hiawatha*	*Twin City Zephyr*			
Steam or Diesel-electric	Steam	Diesel-electric	Steam	Diesel-electric	Steam	Diesel-electric	Diesel-electric	Diesel-electric	Diesel-electric
Daily mileage per train	471	564	564	753	422	882	738	630	620
Train-miles per year	172000	204000	149000	275600	153000	322000	269000	229950	226300
Number of cars per train	14	8	8	14	9	7	5.90	6.85	3.33
Revenue (gross) per train-mile, dollars	4.631	3.020	2.585	4.188	4.343	1.902	1.893	2.072	1.343
Total operation cost per train-mile, dollars	1.420	1.038	1.065	1.659	1.126	0.619	0.610	0.801	0.501
Net revenue, dollars	3.211	1.982	1.520	2.529	3.217	1.283	1.283	1.271	0.842
Per cent of net revenue to gross	69.3	65.6	58.8	60.4	74.1	67.5	67.8	61.3	62.7
Cost of locomotive and train, dollars	1133000	711000	630000	1557000	713000	647000	569125	792292	379000
Fixed charges, 10 per cent total cost per yr, dollars	113300	71100	63000	155700	71300	64700	56913	79229	37900
Fixed charges per train-mile, dollars	0.658	0.349	0.422	0.565	0.466	0.201	0.211	0.345	0.187
Income per train-mile, dollars	2.553	1.633	1.098	1.964	2.751	1.082	1.072	0.926	0.655
Return on investment, per cent	38.8	46.9	26.0	34.7	59.0	53.8	50.7	26.9	39.1
Period of amortization in years	2.58	2.17	3.85	2.83	1.69	1.86	1.98	3.72	2.56

mile (net revenue less fixed charges), and amortization of the investment.

A study of Table 8 is illuminating. The number of cars is greatest only in the case of two trains, namely, in that of the *Daylight* on the Southern Pacific (14 cars) and the *City of San Francisco* on the Union Pacific (also 14 cars). The gross revenue of these two long trains amounts to more than $4 per train-mile, irrespective of whether Diesel or steam, thus indicating the obvious fact that the gross revenue depends upon the number of passengers, which naturally is greater on the longer trains. To the same class of large-revenue trains belongs the Milwaukee *Hiawatha* steam train, which has only 9 cars. This is due to its great popularity. All the Diesel trains, except the *City of San Francisco*, earn less gross income per mile.

The percentage of net revenue to gross is about the same in all streamliners (60 to 70 per cent), but the greatest is the *Hiawatha* (74.1 per cent), except the *Denver Zephyr* (included in the Burlington *Zephyrs*) for which it is 75.1 per cent. This is because the mileage of this train is very high, being more than double that of the *Hiawatha*. The return on investment, however, is lower.

The costs of the trains are, of course, different, the greatest being that of the long steam and Diesel-electric trains, especially the Diesel. This, of course, reduces the net revenues by the amount of fixed charges, making the income per train-mile lower for the Diesel trains. Only the steam *Daylight* and the steam *Hiawatha* show high incomes, above $2.50 per mile. The steam *Ann Rutledge* makes only about $1 per train-mile, probably due to traffic conditions. The total income per train-mile is figured, and the total per year is then referred to the investment. It will be seen that again the highest figure is for the *Hiawatha*, the cost of which is paid up in the shortest time, a little over $1^1/_2$ years.

The return on the investment of all these trains, both steam and Diesel-electrics, is not only the result of their utilization, especially the latter, but also a result of the comparatively low costs of the steam locomotives. In some cases the Diesel-electrics show remarkable mileage, as, for instance, the *Denver Zephyr*, which makes 1036 miles daily. However, but few trains, even in this country, have such a long run and can show such a remarkable utilization, which, after all, depends also upon the convenience of schedules, location of cities, meeting points, turnover of locomotives, and other factors. Therefore, the writer does not think the author's conclusion that Diesel trains will predominate is at all obvious, because to a great extent the success of Diesel trains is the result of novelty, streamlining, comfort, air conditioning, special service, etc., all of which are obtainable with steam trains and are matters of competition. In the long run the preference is given on economic grounds, as pointed out by Mr. Gurley,[8] if the engineering of the product is right. There is nothing inherently wrong in the design of the steam locomotive, and this type of prime mover may be further perfected for high speed.

The writer agrees with the second part of the author's conclusion in which he states that radically new designs of steam locomotives will probably be built to meet high-speed operating conditions. He enumerates the possibilities. It should be added that the combustion-gas-turbine locomotive of Brown, Boveri & Company looms now on the horizon, and although it is not strictly a steam locomotive, it can be built with direct drive, which has been spelling the success of the steam locomotive for over a hundred years. Furthermore, it has no dynamic augment. So far, the steam locomotive has been able to meet new conditions in similar circumstances.

Of course, we cannot foresee the future. But there are no indications which would point away from the locomotive with direct drive.

K. F. Nystrom.[11] In the cost of operation of lightweight trains, it is necessary to include the initial cost of the power plant and cars. In the writer's opinion the costs of steam locomotives and passenger-train cars, at the present time, are entirely too high, as the railroads, and particularly the builders, have failed to standardize on designs. It is appreciated that different operating conditions exist. However, both steam locomotives and passenger-train cars might be standardized to a point where the cost would be substantially reduced. If we are to continue making inroads on the private automobile and recapture passenger business for the American railroads, considerably greater economies will have to be made so that railroad fares can be placed at a popular-price level.

Much has been done in regard to reduction in weight of passenger-train cars, however, much remains to be done. Trucks under passenger cars delivered to railroads this year weigh 34,000 to 36,000 lb per carset. This weight, considering the total weight of the car, is too high. At the present time, one railroad is experimenting with a truck which will not weigh more than 25,000 lb per carset and appears to offer some decided advantages.

The author of the paper commented on the change in the Railway Post Office Department specification as follows: "This change will probably result in the addition of more material than is now used in center sills of lightweight cars designed with high center stills and buffers, with little compensating benefit from

[11] Chief Operating Officer, Chicago, Milwaukee, St. Paul & Pacific Railroad Company, Milwaukee, Wis. Mem. A.S.M.E.

added protection." The author, in his modesty, failed to mention that under his supervision a design[12] of passenger-train car was developed by the Federal Co-Ordinator of Transportation. This design was developed after exhaustive studies had been made and met all the requirements of the Railway Post Office Department's specification then existing. The estimated weight of this design was 96,900 lb and in the writer's opinion would have been entirely satisfactory.

Unfortunately, the Association of American Railroads prematurely adopted the so-called "tight-lock coupler" before a satisfactory application was developed, with the result that the car builders abandoned the conventional practice which had been in vogue for more than 20 years and lowered the center sills, taking all the stresses on the coupler instead of on the buffer and coupler. This change created an undesirable condition when new cars were coupled with older-type cars. To correct this condition, the specification for passenger cars was hastily changed and it is the writer's strong conviction that this specification is causing an undue penalty when employing material other than low-carbon or so-called high-tensile steel.

Three passenger cars were recently built for one railroad by three different carbuilders, one employing high-tensile steel, the second, stainless steel, and the third, aluminum. All three cars weighed substantially the same. The writer is satisfied that the reason for practically the same weight was not that one concern applied greater engineering skill than the other, but was solely due to arbitrary deflection requirements in the new specification. The foregoing is, in the writer's belief, substantial proof of the inconsistency in the existing specification and it is hoped that the Association of American Railroads will not stand in the way of progress, but correct this inconsistency.

J. W. Ragsdale.[13] The author cites the case of a 79-ft coach which weighs 97,000 lb. To the uncalculating reader this weight may appear to be at variance with the weights more recently published (6). These figures approximate 104,000 lb. One is apt to overlook the additional statement that they also refer to coaches having a length of 84 ft 8 in. The difference in weight and the difference in length are almost exactly proportional.

The author also refers to the Association of American Railroads specification which emphasizes the strength of the center sill in car construction. He does not mention the novel requirement that deflection may govern instead of ultimate strength. While the writer is absolutely in sympathy with the spirit which inspired the new A.A.R. specification and with all of its other requirements, he has differed on this matter of deflection. It penalizes the use of high-tensile steels such as stainless steel and it penalizes the Budd type of construction. On the basis of strength alone, a stainless center sill of 8 sq in. would be sufficient. We are now using as much as 18 sq in. If we "floated" the center sill instead of making it an integral part of the structure, we could meet the strength requirement with 8 sq in. and have absolutely no deflection of the car body at all. But, this member should be made to serve a general structural purpose rather than to be a mere part of the draft gear. This belief costs us 2200 lb. This weight might be used to better advantage elsewhere.

After all, it is the resiliency of a structure which counts, not its deflection. An 800,000-lb end loading presumes a crash. A resilient structure might better withstand this crash than an absolutely rigid one. The A.A.R. limits the deflection under this loading to 1 in. or less. A truly resilient car structure might hump up in the middle as much as 4 in. and yet suffer no damage other than to the trim. When we have a wreck, trim is of small consideration compared to passenger safety. The writer, therefore, is opposed to the deflection clause of the A.A.R. specification.

Another phase discussed by the author, and again in the interest of safety, relates to braking. He indicates that any increased braking must automatically involve some means of sanding. While this is quite true, the writer wishes to point out that wheel locking results largely from the peaks of the braking-torque curve and that these peaks are far more pronounced with a metal shoe-to-wheel braking than they are with the disk type of braking, such as has been developed by the company with which he is connected. The eventual development may well be not only disk-type braking but automatic sanding and a torque control.

E. W. Test.[14] The writer wishes to discuss several points, particularly the references involving stainless steel in comparison with the low-alloy high-tensile steels in car construction. The author states: "While it is true that the full use of the superior physical properties of stainless steel or the lightweight of aluminum is limited to some degree by deflection requirements, it should be possible, by using these materials instead of low-alloy steel, to construct a lighter car of equal strength."

The writer is prompted to discuss this statement, principally in view of the publicity which has been given stainless steel, in which the virtues of the metal have been widely heralded to the complete exclusion of its eccentricities. It is felt that, in an engineering discussion of this kind, utmost care should be exercised not to foster any questionable ideas which may have been implanted in the minds of readers by an overzealous publicity effort. Most readers do not have the opportunity carefully to study this enormous flow of publicity, hence, the value of such engineering papers as the present one.

The author's statement might be accepted without question by many, if consideration were given only to those physical properties of stainless steel which are superior. Tensile strength is the property which has been generally overemphasized, not only in the case of stainless steel, but of other metals as well.

A study of the tensile properties alone either of stainless steel, high-tensile low-alloy steel, or mild steel, as the basis for the selection of a material for passenger-car construction, is not the proper approach to such an important problem. A material with 4 times the tensile strength is not 4 times as strong. The word "strong" must embrace all the properties of a metal and the quality of each must be such that the metal will not fail under any of the conditions in which it is to serve.

A very important requirement of a metal, if it is to be used in a structure, is that it have high compressive-stability properties. Dr. C. S. Aitchison and Dr. L. B. Tuckerman of the National Bureau of Standards, in the introduction of their report No. 649 to the National Advisory Committee for Aeronautics, clearly expressed this idea as follows:

"During recent years, a remarkable expansion has taken place in the use of thin sheet and thin-wall material in lightweight structures, such as airplane wings and airplane fuselages. The strength of these structures is generally limited by the strength of certain members carrying compressive loads. These members have frequently been designed on the basis of the tensile properties of the material. This is convenient as the tensile test is relatively simple and is widely used. However, it may lead to an unsafe structure, on the one hand, or an uneconomical structure,

[12] "Design of Typical Lightweight Coach," Report of Mechanical Advisory Committee to the Federal Co-Ordinator of Transportation, Washington, D. C., 1935, pp. 597–678.

[13] Chief Engineer, Stainless Steel Division, The Edward G. Budd Manufacturing Company, Philadelphia, Pa.

[14] Assistant to President, Pullman-Standard Car Manufacturing Company, Chicago, Ill. Mem. A.S.M.E.

on the other hand, if the compressive properties of the material differ from the tensile properties. There is an urgent need for a method which makes possible a direct determination of compressive stress-strain graphs for thin-wall material."

Any material offered for lightweight car structures comes in this category and it is essential that the compressive stress-strain ratio of these materials be satisfactory so that sufficient stability may be assured. In case of a wreck, which must always be considered in the design of railway passenger cars, substantially every structural member may be called upon to resist high compressive stresses.

It is a well-known fact, at least for the last several years, that compressive tests on 110,000-lb per sq in. yield strength 18-8 stainless steel show a drop in the modulus of elasticity to 22,000,000 at stresses as low as 44,000 lb per sq in. At 57,000 lb per sq in. stress, the modulus of elasticity has dropped to 19,000,000. In other words, a very important factor with reference to the ability of the metal to stand up as a column has been reduced in the ratio of 29 to 19, as the unit compressive stress was increased from zero to 57,000 lb per sq in. Compressive tests further reveal that, long before the stress has reached the alleged yield point of 110,000 lb per sq in., the modulus of elasticity has dropped to 12,000,000. Incidentally, under increase of tensile stress, test samples also show a reduction of the modulus of elasticity. At a tensile stress of 88,000 lb per sq in. the modulus of elasticity has decreased to 22,000,000. This is the disappointing quality of stainless steel which is revealed by stress-strain graphs. It is a major deficiency and it must be a limiting factor in the design of a structure using this material.

We are not unmindful of the many good qualities of stainless steel but these, including its superior tensile strength, cannot be used to offset the deficiencies when the material is called upon to resist buckling, in which case the low and variable modulus of elasticity becomes the weak link in the otherwise strong chain.

A theoretical conclusion as to the weights of structures, based upon a comparison of the known properties of the metals involved, is at the best a precarious adventure, particularly so in the case of a passenger-car structure. It has been difficult, as the author stated, to make accurate weight comparisons of the different types of structures used in passenger cars because of the dissimilarity in specifications.

We have, however, one comparison available, which probably has been published since the paper was prepared. The following statement is quoted from the article[15] in question:

"Early in January the Pennsylvania ordered 15 dining cars, 5 from the Pullman-Standard Car Manufacturing Company, 5 from the Edward G. Budd Manufacturing Company, and 5 from the American Car & Foundry Company. All of the cars are built to the same general specifications and are believed to represent the first instance in which three dissimilar sets of materials and techniques of construction have been so employed."

In this instance, the stainless-steel car was approximately 5000 lb heavier than a similar car with a structure of low-alloy steel:

Stainless steel: 118,170 lb; designed and built by Edward G. Budd Manufacturing Company. Low-alloy steel: 113,240 lb; designed and built by American Car & Foundry Company.

It is felt that the author has dismissed too lightly the economic aspects of the stainless-steel car. If there have been instances where bids on cars with stainless-steel structures, including the variety of equipment, accessories, and trucks, have by some method been reduced to the total price of other cars having low-alloy-steel structures, it would appear that this total car price is of little significance in an engineering analysis of the relative costs of the two body structures. The cold facts are that stainless steel costs 30 to 40 cents more per lb than the low-alloy steels. If no decrease in weight can be obtained by the use of the high-priced metal, the body structure alone will be penalized for material only, as much as $8000 to $10,000 per car. Recent competitive bids have fully reflected this differential in cost of materials.

It has not been the writer's purpose to advance the opinion that stainless steel is not a suitable material for passenger-car structures. By the recognition of its deficiencies it can be successfully used. We doubt the possibility of producing a car of stainless-steel structure which will weigh less than a car of low-alloy high-tensile-steel structure and which will be of equal strength. Even though high-tensile stainless is nowhere used structurally in any important way in vehicles engaged in major transportation, except in the case of railway passenger cars, it is believed that no one disputes the propriety of such use. The one question in relation to its use is that of economics, namely, as to what it may prove to be able to do toward cheap maintenance in order to justify the substantially higher investment.

J. R. Thompson.[16] The writer has been with the Bureau of Valuation of the Interstate Commerce Commission since the valuation work started in 1914, and has also been assigned to cooperate with the Depreciation Section of the Bureau of Accounts since the depreciation order affecting equipment was put into effect in 1934. In this connection, they have been interested in the service life of all railway equipment in the United States and, in recent years, particularly in the new designs of locomotives and passenger-train cars. Therefore, it will be helpful if the author in his closure will comment further upon the life expectancy which modern Diesel locomotives and lightweight passenger-train cars may attain.

Victor R. Willoughby.[17] The A.A.R. passenger-car specification, wherein additional strength in the end portion of the center sills and the end sills is required, is in the writer's opinion a very necessary requirement, especially where the new lightweight equipment is operated in the same trains with the old heavier equipment having high center sills. It is true that this adds considerable weight to the underframe of the car but this additional weight is in the proper place and is of material value at the time of an accident. The author states that no cars exactly similar in type but of different materials and construction had been built.

Recently there has been built a lot of 15 dining cars;[15] 5 using aluminum, 5 using stainless steel and 5 using low-alloy high-tensile steel. These cars were all constructed to the same strength specifications, practically the same floor plans, and very similar equipment.

The interior design of the stainless-steel car added about 700 lb to its weight, as compared with the low-alloy high-tensile-steel car. The differences in specialties amounted to a saving on the stainless-steel car of about 200 lb, making a net weight credit for the stainless-steel car of 500 lb. The trucks were supposed to be identical. The dry weight of the stainless-steel car was nearly 5000 lb more than the low-alloy high-tensile-steel car, meaning that after adjustment, because of interior equipment and finish, the stainless-steel car weighed about 4500 lb more than the low-alloy high-tensile-steel car.

It is the writer's opinion that the modern passenger car, with

[15] "Pennsylvania Receives Diners From Three Builders," *Railway Age*, vol. 107, 1939, p. 469.

[16] Equipment Engineer, Bureau of Valuation, Interstate Commerce Commission, Washington, D. C. Mem. A.S.M.E.

[17] Vice-President in charge of Engineering, American Car & Foundry Company, New York, N. Y. Mem. A.S.M.E.

an over-all length of about 85 ft, meeting all the strength requirements of the A.A.R. specification, when equipped with 4-wheel trucks, will weigh in the neighborhood of 108,000 lb.

The most important problems, which yet require solution, are better riding trucks and improved braking.

W. E. Woodard.[18] The author mentions the fact that steam-locomotive designers are studying the question of improving their designs, particularly for high-speed work, and out of this will probably come some valuable recommendations. He speaks of this as in the future. The writer will mention some results already accomplished in this direction.

Fortunately, the writer's experience relates to a locomotive which has a little less than 4000 ihp, so the data are comparable with the 4000-hp Diesel mentioned in the paper.

The method of comparing modern locomotive types, as used by the author, is open to question. For the steam locomotives the weights are given with a full load of coal and water. This is an unfair basis of comparison, so far as the steam locomotive is concerned, and makes the figures look extremely favorable for the Diesel. If we use two-thirds coal and water in the tender, the comparison can be made much more nearly on the basis of actual conditions of continuous operation. Moreover, this is the figure used in all tests[19] and comparisons of the Association of American Railroads.

Using this basis, the steam locomotive, to which reference has been made, compares as follows:

	Diesel locomotive	Steam locomotive 4-6-2 type
Nominal hp	4000	.....
Maximum hp	.....	4000
Weight with two-thirds fuel and water, lb	609600[a]	506000
Weight of locomotive only, lb	.....	330800

[a] Latest published weights of 4000-hp Diesel.

Actual drawbar pulls have been secured from this 4000-hp steam locomotive and the curve looks very much different from that given in Fig. 1 of the paper. The steam-locomotive drawbar-pull curve crosses and goes above the Diesel at about 40 mph, touches almost 3000-drawbar hp at about 65 mph, and remains above the Diesel curve all the way to 100 mph. Such a drawbar-horsepower curve gives operating characteristics far better than the steam-locomotive curve in Fig. 1.

The author mentions the accelerating abilities of the Diesel locomotive as compared with steam locomotives. On all runs in passenger service, except strictly local, the ability to reaccelerate from 40 mph up to about 70 or 75 mph is far more important than the rapidity with which the train reaches 40 mph. Almost any division, say of 200 miles in length, has at least five or six slowdowns; it is the ability to reaccelerate at these points which counts. In other words, normally there are five or six slowdowns to one start from zero speed. This is the reason why the increased drawbar-horsepower curve of the 4000 ihp steam locomotive at 40 mph and above is of such vital importance, particularly in high-speed work.

As an example of what a 4000-hp steam locomotive is doing in regular service, some interesting runs recently made with a 13-car train, weighing approximately 900 tons, are mentioned as follows:

Distance traveled 98.3 miles at a speed of 83.07 mph
Distance traveled 78.8 miles at a speed of 84.4 mph
Distance traveled 25.1 miles at a speed of 100.4 mph

These figures are from the official train sheet. The maximum-speed stretch of 25.1 miles was over rolling profile (not all downgrade).

In the case of a 12-car train, consisting of standard Pullmans and diners, weighing about 1100 tons, a maximum speed of 96 mph was attained on the run from Fort Wayne, Ind., to Crestline, O., which was accomplished in 2 hr flat, 5 min less than the "Broadway Limited's" schedule. One stretch of 116.4 miles was negotiated at a speed of 74.3 mph.

These are not mentioned as outstanding, simply because they were high-speed runs. Many such runs have been and are being made. The writer wishes to emphasize the fact that they were accomplished with a 4000-hp steam locomotive, the locomotive itself weighing 330,800 lb. This locomotive did not have any special crew or preparation. The runs were made in regular "pool service."

Author's Closure

Mr. Cook presents theoretical calculations indicating that the cost of fuel for Diesel and steam locomotives should be equal. The figures given in the paper are actual performance data for similar operations and the author feels that such data are more reliable than any estimates which might be made.

There is no basis for the theory that the maintenance expense for Diesel engines increases with the increasing age of the equipment. Thirty years' history with rail motorcars refutes this theory. Furthermore, the comparisons made by the author involve comparatively new steam locomotives as well as new Diesel locomotives. The steam locomotives referred to in the study of maintenance costs were built in 1928 and the figures were taken from records for the years 1930 to 1935. The figures used for the Diesel locomotives were taken from records for their first 4 years of service.

There is a misunderstanding of the methods mentioned as being used in making dynamometer-car tests. In testing a new type of locomotive, various train consists are used on numerous runs, so that the maximum performance can be checked. The steam locomotive referred to was not working below capacity at high speeds, but was being worked to maximum capacity. By maximum indicated horsepower the author refers to the maximum computed from the many hundreds of indicator cards taken during the various tests.

A question is raised regarding the comparison shown for thermal efficiency. The author made the comparison on exactly the same basis, that is, the same trains were handled on the same schedule.

Mr. Jones raises an interesting point as to the extra power required by the use of large axle generators, in connection with air-conditioning equipment. The author feels that the best answer to this problem is in the use of the steam-ejector system of air conditioning, wherein the power is secured from the locomotive boiler. The suggestion for the use of a head-end electric power plant to provide electricity for the cars may be suitable for certain conditions but its disadvantage lies in the fact that nonwired interchange cars could not be used in such trains.

It is undoubtedly true that locomotives with too low a center of gravity may produce serious lateral forces. It is noted from Mr. Jones's curves that the ideal height for the center of gravity is approximately 4 ft 6 in. This is the approximate height for Diesel locomotives, whereas the center of gravity for steam locomotives is considerably higher. The stress curves shown in Fig. 2 of the paper indicate that the Diesel locomotive does not produce any undue rail stress on curves. The question is asked why rail stress increases more rapidly with speed under locomotive tenders than is the case under Diesel locomotives. This is probably due to the fact that there is a greater shifting of wheel load in high-speed operation with steam locomotives than with

[18] Vice-President, Lima Locomotive Works, New York, N. Y. Mem. A.S.M.E.

[19] "A.A.R. Passenger Locomotive Tests," Report of Association of American Railroads, Chicago, Ill., October, 1938.

Diesel locomotives. Most track stress tests with steam locomotives have indicated this shifting of load.

The author does not agree with the inference that stresses developed under driving wheels are not serious unless rails are kinked. Higher stress must result in greater punishment to the rail and more chance of failure. It also results in increased roadbed-maintenance expense.

The curve presented by Mr. Lipetz for drawbar horsepower of a steam locomotive does not go beyond 85 mph and no figures are given to indicate how it droops beyond that point. His calculation of theoretical time for acceleration is interesting but in the dynamometer-car tests referred to in the paper different results were obtained. In mountainous territory, where there is sharp curvature, speeds are often brought below 40 mph. Furthermore, in service the steam locomotives do not appear to produce their theoretical power at all times. Most of the tests are made under the best operating conditions. The human element, the element of weather conditions, and the condition of locomotives all play an important part in steam-locomotive operation, whereas the performance of Diesel locomotives is much more uniform. There is no skill required to apply the power; rail conditions have very little effect because of nonslipping characteristics; and very cold weather is in no way detrimental to the Diesel. Another factor which appears to be a distinct advantage in the Diesel or electric locomotive is the continuity of the torque.

The author does not feel that the lack of cross-balance on other than the main wheels of a steam locomotive has any appreciable effect on track stress. It is not understood how Mr. Lipetz arrives at the figures quoted for maximum rail stress. It appears that there has been a misunderstanding of the diagrams Fig. 3, of the paper. The actual maximum stress at 100 mph is 27,000 lb for a steam locomotive and 17,000 lb for a Diesel locomotive at the same speed. Thus the increase for the steam locomotive is about 60 per cent instead of the 12 per cent referred to. It is undoubtedly true, as Mr. Lipetz states, that some locomotives have been built which may show a better performance than the one referred to in the paper. It is also true that a great many locomotives show poorer performance. It may be noted that the particular steam locomotive therein recorded was a strictly modern high-speed locomotive built in 1938, and therefore, should be fairly representative of present-day power.

As regards permissible speeds of operation on curves, the best indication of the facts is the definitely higher speeds permitted for Diesel locomotives than for steam locomotives on individual railroads. These rules have been based on actual experience.

As regards comparative costs of operation, the author does not believe it possible to compare operation on different railroads, because of the fact that operating conditions, traffic conditions, and equipment-maintenance conditions are different. It is for these reasons the author confined his comparisons to trains operating on one railroad.

The author does not agree that the success of the Diesel has been due to its novelty, steamlining, etc. In various parts of the paper the economic reasons for the success of the Diesel engine have been brought out. Economy of operation is the governing factor in the selection of any type of power.

The comments of Mr. Nystrom, Mr. Ragsdale, Mr. Willoughby, and Mr. Test indicate the wide difference of opinion which exists regarding materials and design best suited for lightweight passenger-car construction. The major parts of a passenger car must be designed to have strength characteristics far in excess of what is needed for ordinary operating conditions, in order to stand up in case of wreck. The best criterion is a study of wrecks which have actually occurred with equipment of various designs built with different types of lightweight materials and also the old-style heavy equipment. The author's observation of these results leads him to believe that the importance of deflection requirements has been exaggerated. What is desired in a passenger car is protection to lives of passengers in case of wreck rather than to minimize the damage to equipment. This feature should be the primary object in designing passenger cars. The author does not have the data upon which to base a comparison of the weights of the diners referred to in the discussion, but it is his understanding that there were certain requirements in the specifications which did not permit of the manufacturers designing the cars to make maximum usage of all the strength properties of the particular materials which they used.

Mr. Thompson requests further comments on service life of Diesel locomotives and lightweight passenger cars. There is no reason why this equipment should not have just as long physical life as the older types. However, there is a question as to how rapid will be the rate of obsolescence. Because of the Diesel power as well as the lightweight car construction being new developments, it is possible that there may be many improvements developed during the next 10 years, which would affect the rate of obsolescence to a greater extent than has been the case in the past with the older-type equipment, when there was much less new development work going on.

Steam-Boiler Performance and a Method of Comparison

By E. G. BAILEY,[1] NEW YORK, N. Y.

This paper presents data from two methods of determining rates of heat absorption in different sections of steam-boiler units. One by direct measurement of evaporation, first published in France in 1874, describing French locomotive-boiler tests, is compared with recent data on modern furnaces using the gas-temperature-weight method. A method of comparing data from all types, kinds, and sizes of boiler units at diverse rates of output is presented. It consists briefly of plotting "heat absorbed" in per cent of total heat available against "equivalent surface area," which results in all points, having the same Btu available per sq ft per hr, being on the same ordinate. Representative data from modern stationary and marine boilers are compared, together with the data from the French locomotive-boiler tests of 65 years' standing. Data on volumes of furnaces and entire boiler units are given together with draft losses and efficiencies.

THE three branches of steam engineering—locomotive, marine, and stationary—have developed their individual terms with respect to rates of boiler output and performance. Some of these terms are becoming less significant due to varieties of fuel, the addition of superheaters, economizers, air heaters, etc. However, there are common bases for comparison which can and should be used in ways that will be universally applicable to all forms of fuel-burning and heat-absorbing equipment. One of the principal purposes of this paper is to present a method which is believed to be of value, not only in comparing boilers of different types, but also in comparing more accurately the relative results from similar boilers.

As a basis for comparison with the modern locomotive boiler, which is being covered in another paper from the A.S.M.E. Railroad Division,[2] data are given in this paper on the performance of present-day stationary boilers. Because of the great diversity of boiler types, a really comprehensive treatment of the subject has not been possible within the space limitations. However, it is believed that the most value can be obtained by giving performance data from a few representative types of modern stationary boilers, and presenting them in a form which may be readily compared with similar data from locomotive or other boilers.

Coal-burning locomotives are fired by hand or stoker and none greatly exceeds an output of 100,000 lb of steam per hr. Many locomotives in the South and West are oil-fired. In order to make the results more directly comparable, the stationary-boiler data presented herewith are taken from boilers of a smaller size than is normally used in central-station practice and with working pressures approaching those now used in locomotives.

In addition to hand, stoker, and oil firing, one unit burning pulverized coal is included, since many stationary boilers are now designed for burning pulverized coal, oil, or gas without any change in the equipment. Some marine-boiler performance data are also given to show wherein conditions aboard ship have influenced the design and performance requirements.

While boiler efficiency is the final basis of comparison, with a complete heat balance to show the distribution of heat losses in unburned fuel, stack gases, excess air, etc., it is of further importance to know the distribution of heat absorbed by the boiler, superheater, economizer, and air heater and, from the surfaces of each, to know the average rates of heat absorption of these component parts. The absorption of radiant heat in the furnace, the effect of slag, cleanliness of heating surfaces, and the absorption of heat in different parts of the boiler sections are also factors of importance in connection with the design of furnaces and boilers, as well as in the operation of the fuel-burning and steam-generating equipment. The latter information is not obtainable from present standard tests.

Methods of Measuring Heat Absorption

An interesting study of the rates of heat absorption in different sections of the firebox and boiler tubes of a small locomotive boiler was made between 1860 and 1864 by the Northern Railway of France,[3] where a boiler divided into five sections was built and the evaporation was determined separately in each section.

Another method of determining the rate of heat absorption in different sections of furnaces and boilers was described by the author in a previous paper[4] before the Society. This method determines heat absorption by measuring the furnace gas temperatures and gas weights at consecutive stages in the travel of the gases, the difference in the heat content of the gases between any two points in the gas travel being the heat absorbed by radiation and convection by any surface between these points.

Results From Heat-Absorption Tests

A diagram of the French locomotive is shown in Fig. 1.[5] The firebox was 3 ft square at the grate, and there were 125 tubes about $1^7/_8$ in. diam and 12 ft 4 in. long. The five divisions of the boiler were arranged for continuous gas flow and separate water levels were maintained. The evaporation from each section was determined by a number of tests at different ratings. Three tests at the highest draft used have been selected for presentation and the essential data are given in Table 1. Tests A-1 and A-2 were made with briquettes and test A-3 with coke. Test A-2 was run with half of the tubes plugged and the total evaporation was about 87 per cent of that obtained in test A-1 or at about 62

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[1] Vice-President, The Babcock & Wilcox Company. Mem. A.S.M.E.

[2] "The Locomotive Boiler," by C. A. Brandt, Trans. A.S.M.E., vol. 62, July, 1940, pp. 379–419.

Contributed by the Power Division and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] "The Practical Physics of the Modern Steam Boiler," by F. J. Rowan, D. Van Nostrand Co., New York, 1903, p. 143.

"Permanent Way Rolling Stock and Technical Working of Railways," by C. H. F. Couche (translated from the French by J. N. Shoolbred and J. E. Wilson), vol. 3, Dulau and Co., London, 1878.

"Vaporisation Decroissante en Progression Geometrique," by Paul Havrez, *Annales du Génie Civil*, vol. 3, 1874, pp. 520–528 and 545–564.

[4] "Modern Boiler Furnaces," by E. G. Bailey, Trans. A.S.M.E., vol. 61, Oct., 1939, pp. 561–569. Discussion, pp. 569–576.

[5] Reproduced from reference (3), by F. J. Rowan, Fig. 69.

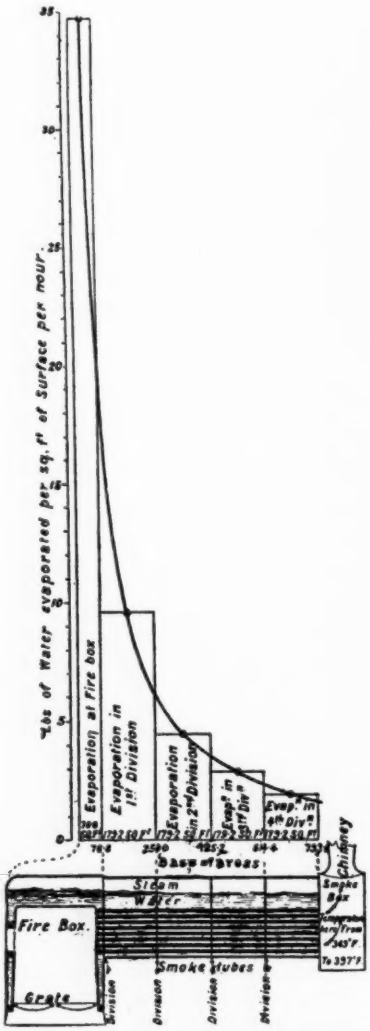


FIG. 1 DIAGRAMMATIC ARRANGEMENT OF LOCOMOTIVE BOILER (NORTHERN RAILWAY OF FRANCE), SHOWING DIVISIONS OF HEATING SURFACE AND RELATIVE EVAPORATION FROM EACH DIVISION

TABLE 1 TEST DATA ON FIVE SECTIONS OF FRENCH LOCOMOTIVE BOILER

		A-1	A-2	A-3
1	TEST			
2	BOILER		FIG 1	
3	RATED STEAM OUTPUT - LB./HR.			
4	STEAM OUTPUT ON TEST - LB./HR.	7934	6886	5830
5	STEAM PRESSURE - LB./SQ.IN. GA.	80	80	80
6	SATURATION TEMPERATURE - °F	324	324	324
7	STEAM TEMPERATURE SUPERHEATER OUTLET - °F	SATURATION		
8	FEED WATER TEMPERATURE - °F	60	60	60
9	AIR TEMPERATURE LEAVING AIR HEATER - °F	–	–	–
10	METHOD OF FIRING	HAND FIRED GRATE		
11	KIND OF FUEL	COAL BRIQUETTES		COKE
12	GRATE AREA OR FURNACE CROSS SECTION - SQ. FT.	9	9	9
13	LB. OF FUEL PER HR. DIVIDED BY ITEM 12	108.7	94.3	85.7
14	HEATING VALUE OF FUEL "AS FIRED" B.T.U./LB. ①	13000	13000	12000
27	TOTAL HEAT AVAILABLE ABOVE 80°F - MILLION B.T.U./HR. ①	11.69	10.14	8.34
28	LIBERATION - ITEM 27 DIVIDED BY ITEM 39 - B.T.U./CU.FT.HR.	354000	307000	253000
29	ITEM 27 DIVIDED BY ITEM 41 - B.T.U./SQ.FT. HR.			
	FIREBOX	152200	153700	108700
	1ST. DIVISION	45700	65300	32600
	2ND. "	26900	41400	19170
	3RD. "	19030	30300	13600
	4TH. "	14750	23900	10530
30	GAS TEMPERATURE LEAVING UNIT - °F	373	373	373
35	HEAT ABSORBED - CUMULATIVE DIVIDED BY ITEM 27 - %			
	FIREBOX	29.5	34.9	25.1
	1ST. DIVISION	53.8	56.3	51.5
	2ND. "	65.9	67.3	65.6
	3RD. "	73.5	73.9	74.1
	4TH. "	78.5	78.6	81.0
39	FURNACE VOLUME - CU. FT.	33	33	33
41	SURFACES - CUMULATIVE - SQ. FT.			
	FIREBOX	76.8	65.9	76.8
	1ST. DIVISION	256.0	155.4	256.0
	2ND. "	435.2	244.9	435.2
	3RD. "	614.4	334.4	614.4
	4TH. "	793.6	423.9	793.6
42	EQUIVALENT SURFACE AREA (500×10⁶ ÷ ITEM 27) × ITEM 41 - SQ.FT.			
	FIREBOX	3285	3240	4610
	1ST. DIVISION	10950	7660	15330
	2ND. "	18670	12070	26100
	3RD. "	26300	16490	36850
	4TH. "	33950	20850	47600
43	VOLUME OF ENTIRE UNIT - CU. FT.	320	320	320
44	ITEM 27 DIVIDED BY ITEM 43 - B.T.U./CU.FT.HR.	36500	31700	26100
45	TOTAL DRAFT LOSS - INCHES OF WATER	3.94	3.94	3.94
47	OVERALL EFFICIENCY - %	72.2	72.3	72.9

① ESTIMATED BY ORROK - COMBUSTION APRIL - 1938

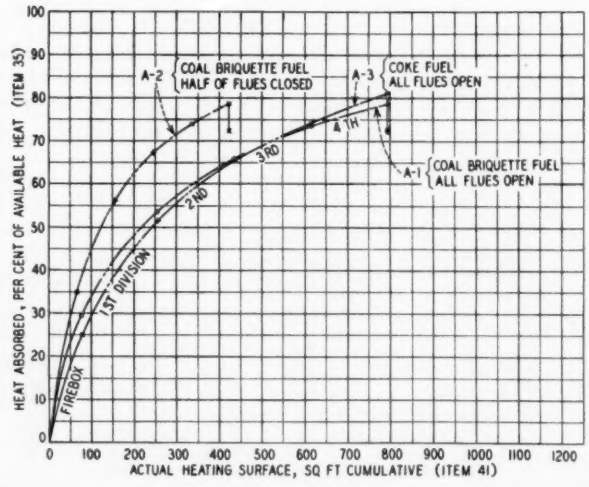


FIG. 2 HEAT ABSORPTION IN FRENCH LOCOMOTIVE BOILER PLOTTED AGAINST ACTUAL HEATING SURFACE
(Boiler of Fig. 1 and Table 1; tests A-1, A-2, and A-3.)

per cent higher rate on a basis of heat available per sq ft of active heating surface. With the same draft the coke gave a lower rate of evaporation. The heat absorbed, expressed as a percentage of the heat available for absorption, item 35, Table 1, is plotted in Fig. 2 against cumulative actual heat-absorbing surfaces, item 41. No useful comparison of these fuels and rates of output can be readily obtained from this plot because there is no way in which the rate of heat available or rate of heat absorption can be graphically indicated.

Tests B-1 and B-2 from Table 2 were obtained from the pulverized-coal-fired boiler of Fig. 3, rated at 100,000 lb of steam per hr. These tests are plotted in Fig. 4, which repeats test A-1, but to a scale suitable for plotting the B series. In this boiler the ash is removed in a dry form by blowing it from the flat water-cooled floor to the hopper at the end away from the burners. A certain amount of dry sponge ash accumulates on the water-cooled furnace walls and roof, which are not normally cleaned; the ash reaches an equilibrium by falling by its own weight from time to time in small amounts. The heat absorptions in the furnace and the different sections of the boiler tube banks, item 32, were obtained by the "gas-temperature-weight" method previously referred to, a brief detailed description of which may be in order. Test data are taken which include items 4 to 27, inclusive, of Table 2, the principal purpose being to determine the total heat input, item 24; the total heat available, item 27; and the total gas weight, item 20.

The total heat available, item 27, is the total heat from the fuel actually burned, plus the heat in the preheated air above 80 F, minus the latent heat in the moisture from the fuel and from the burning of hydrogen. In other words, it is the sensible heat in the gases of combustion which is available for absorption by

radiation and convection by any heating surface over which it may pass.

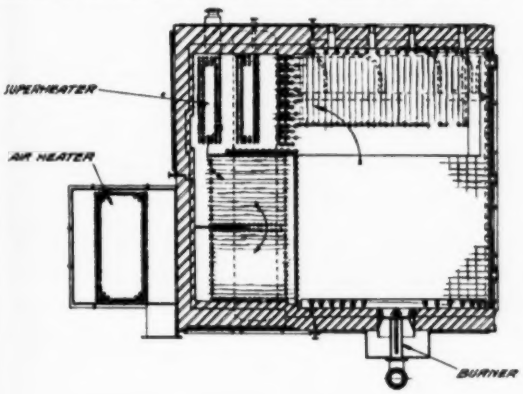
Another very important determination is item 30, wherein the average true gas temperature is obtained at different points in the path of gases by means of the high-velocity thermocouple.[6] From these temperatures and the gas weights, the total heat absorbed in each section, item 32, is determined.

Item 36, Btu absorption per sq ft per hr for the different sections is also very interesting. The actual furnace heating surface is taken as the projected area or envelope of all water-cooled furnace areas, with no factors for cleanliness or different kinds of cooling surfaces except in the case of spaced tubes with refractory behind or between, where Hottel factors[7] are applied. Boiler, superheater, economizer, and air-heater heating surfaces are taken as the full tube circumference, plus any other exposed surfaces in contact with the actively flowing gases of combustion. No factors for tube arrangement, spacing, or mass flow are used because it is thought best to keep all units on an actual area and Btu available basis and to study the comparative results for the effect actually resulting from different forms of surface, fuel and ash characteristics, values of excess air and mass flow, etc., as determined from test data.

Many other data and calculations are included in the complete test, but only the essential ones pertinent to this paper are included in Table 2.

[6] "The Accurate Measurement of High Gas Temperatures," by H. F. Mullikin, *Power*, vol. 78, 1934, p. 565. This high-velocity thermocouple is inserted into the gas stream at any desired point in the furnace and, by an aspirator, gas is drawn across a shielded thermocouple at a sufficiently high velocity to bring the couple close to the true gas temperature, free from surrounding radiating influences.

[7] "Radiant Heat Transmission Between Surfaces Separated by Nonabsorbing Media," by H. C. Hottel, Trans. A.S.M.E., vol. 53, 1931, FSP-53-19b, pp. 265–271. Discussion, pp. 271–273.



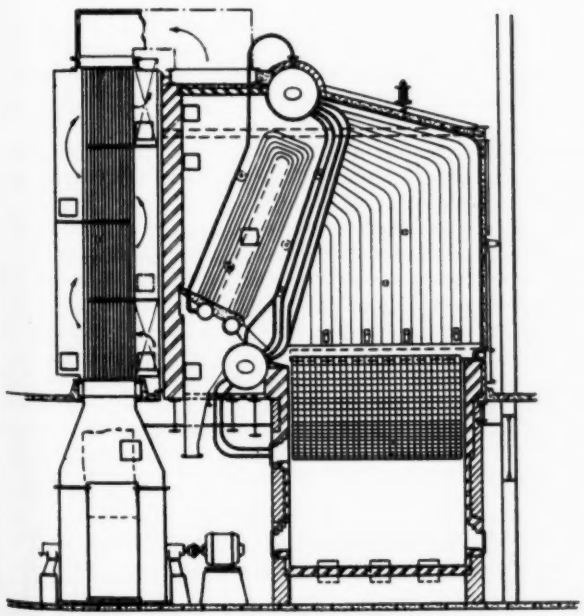


Fig. 3 Pulverized-Coal-Fired Boiler for Stationary Service With Integral Water-Cooled Furnace
(Data given in Table 2.)

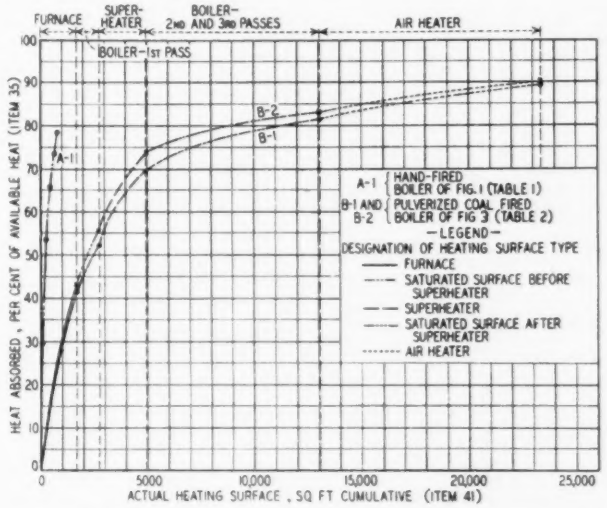


Fig. 4 Heat Absorption in French Locomotive Boiler and in Pulverized-Coal-Fired Boiler Plotted Against Actual Heating Surface
(Boiler of Fig. 3 and Table 2; tests B-1 and B-2, also comparative plot of curve A-1 from Fig. 2.)

During test B-1, the boiler produced 105,900 lb of steam per hr and, during test B-2, 83,300 lb per hr. The higher rating test shows a lower percentage of heat absorbed, but there is no indication from the plots themselves, as given in Fig. 4, as to the relative rates of output or the relative performance. Test A-1 plotted on the same scale of actual surface is still further from being on any useful basis of comparison with the other two curves in Fig. 4, and yet all three curves are plotted on basic units of great significance; viz., actual heating surface and per cent of available heat absorbed. Neither does one obtain any really satisfactory comparison of the relative results from these boilers at the different rates of output by examining all test data relating to them in Tables 1 and 2.

Comparison on Equivalent-Surface-Area Basis

In Fig. 5 the heat-absorption percentages of tests B-1 and B-2 are plotted to "equivalent surface area" instead of actual area. This simple expedient brings into the diagram a rate-of-input factor. The corresponding points of the lower-input test are plotted at higher scale values of equivalent surface areas in order to maintain relationship with tests of greater heat input such that all curve intersections with a given ordinate line shall have the same actual Btu available per sq ft value.

This equivalent surface area is given as item 42 in all tables. It is obtained by using the factor resulting from dividing 500,000,000 by the total Btu available for absorption, item 27, on any test regardless of the size of boiler unit or its rating, and then multiplying this factor by the actual area in sq ft of all cumulative

TABLE 2 TEST DATA ON PULVERIZED-COAL-FIRED AND

1 TEST			B-1					B-2		
2 BOILER			FIG 3					FIG 3		
3 RATED STEAM OUTPUT - LB/HR.			100000					100000		
4 STEAM OUTPUT ON TEST - LB/HR.			105900					83300		
5 STEAM PRESS. AT SUPERHEATER OUTLET - LB/in² GA.			644					643		
6 SATURATION TEMPERATURE - °F			498					497		
7 STEAM TEMPERATURE AT SUPERHEATER OUTLET - °F			825					796		
8 FEEDWATER TEMPERATURE - °F			399					394		
9 AIR TEMPERATURE LEAV. ℄ AIR HEATER - °F			513					497		
10 METHOD OF FIRING		PULVERIZED COAL					PULVERIZED COAL			
11 KIND OF FUEL		SEMI-BITUMINOUS COAL					SEMI-BITUMINOUS COAL			
12 GRATE AREA OR FURNACE CROSS SECTION - SQ.FT.			342					342		
13 LBS. FUEL PER HR. DIVIDED BY ITEM 12			27.8					21.3		
14 HEATING VALUE OF FUEL "AS FIRED" - B.T.U./LB.			13950					13950		
15 SECTION OF UNIT	FURNACE	1ST BOILER BANK	SUPERHEATER	2ND & 3RD BOILER BANKS	AIR HEATER	FURNACE	1ST BOILER BANK	SUPERHEATER	2ND & 3RD BOILER BANKS	AIR HEATER
16	CO_2 % BY VOL.	16.3			12.8	12.9	16.7			13.7	13.7
17 FLUE GAS ANALYSIS - LEAVING SECTION	O_2 " " "	3.7			7.1	7.2	3.2			6.4	6.4
18	CO " " "	0.0			0.0	0.0	0.0			0.0	0.0
19 TOTAL COMBUSTION AIR LEAVING SECTION - %		121	121	121	150	151	117	117	117	143	143
20 TOTAL WEIGHT WET GAS LEAVING SECTION - LB/HR.		130500	130500	130500	159000	160000	97400	97400	97400	117300	117300
21 HEAT AVAILABLE ABOVE 80°F - ENTERING SECTION - BTU/LB GAS		1038	605	495	313	144, Lv-80	1063	607	473	279	133, Lv-74
22 MOISTURE BY WEIGHT IN FLUE GAS - %		4.3	4.3	4.3	3.7	3.7	4.3	4.3	4.3	3.8	3.8
23 ADIABATIC TEMPERATURE - °F		3610					3685				
24 TOTAL HEAT INPUT FROM FUEL - MILLION BTU/HR.		132.3					102.0				
25 TOTAL PREHEAT IN AIR ABOVE 80°F - MILLION BTU/HR.		10.6					7.3				
26 TOTAL LOSSES - LATENT HEAT, CARBON LOSS - MILLION BTU/HR.		7.4					5.7				
27 TOTAL HEAT AVAILABLE ABOVE 80°F - MILLION BTU/HR.		135.5					103.6				
28 LIBERATION - ITEM 27 DIVIDED BY ITEM 39 - BTU/CU.FT.HR.		22500					17180				
29 ITEM 27 DIVIDED BY ITEM 41 - BTU/SQ.FT. HR.		79900	49400	27500	10370	5800	61100	37800	21000	7940	4430
30 GAS TEMPERATURE LEAVING SECTION - °F		2280	1900	1268	647	398	2285	1825	1146	604	378
31 HEAT AVAILABLE IN GAS ENTERING - MILLION BTU/HR.		135.5	79.2	64.7	40.9	24.7	103.6	59.1	46.03	27.13	17.40
32 HEAT ABSORBED : INDIVIDUAL SECTION - MILLION BTU/HR.		56.3	14.5	23.8	16.2	10.6	44.5	13.07	18.90	9.73	7.30
33 HEAT ABSORBED : CUMULATIVE - MILLION BTU/HR.		56.3	70.8	94.6	110.8	121.4	44.5	57.57	76.47	86.20	93.50
34 HEAT ABSORBED : INDIVIDUAL SECTION DIVIDED BY ITEM 27 - %		41.6	10.7	17.5	11.9	7.8	43.0	12.6	18.2	9.4	7.1
35 HEAT ABSORBED : CUMULATIVE DIVIDED BY ITEM 27 - %		41.6	52.3	69.8	81.7	89.5	43.0	55.6	73.8	83.2	90.3
36 HEAT ABSORPTION : INDIVIDUAL SECTION - BTU/SQ.FT. HR.		33200	13900	10850	1995	1026	26200	12500	8610	1198	707
37 HEAT ABSORPTION CUMULATIVE - BTU/SQ.FT. HR.		33200	25900	19200	8480	5190	26200	21000	15500	6600	4000
38 EQUIVALENT HEAT RECEIVING SURFACE TEMPERATURE[3]		2010	1960	1362			2080	1932	1280		
39 FURNACE VOLUME - CU.FT.		6030					6030				
40 SURFACES: INDIVIDUAL SECTION - SQ.FT.		1695	1043	2197	8118	10320	1695	1043	2197	8118	10320
41 SURFACES : CUMULATIVE - SQ.FT.		1695	2738	4935	13053	23373	1695	2738	4935	13053	23373
42 EQUIVALENT SURFACE AREA (500×10⁶ ÷ ITEM 27) × ITEM 41 - SQ.FT.		6250	10100	18170	48200	86200	8190	13200	23800	63000	112600
43 VOLUME OF ENTIRE UNIT - CU.FT.				26000					26000		
44 ITEM 27 DIVIDED BY ITEM 43 - BTU/CU.FT. HR.				5220					3980		
45 DRAFT LOSS - IN. OF WATER		TOTAL TO BOILER OUTLET = 0.83				A.H. = 1.88	TOTAL TO BOILER OUTLET = 0.50				A.H. = 1.31
46 TOTAL DRAFT PLUS AIR PRESSURE - IN. OF WATER						6.6					3.81
47 OVERALL EFFICIENCY - %						83.7					84.4

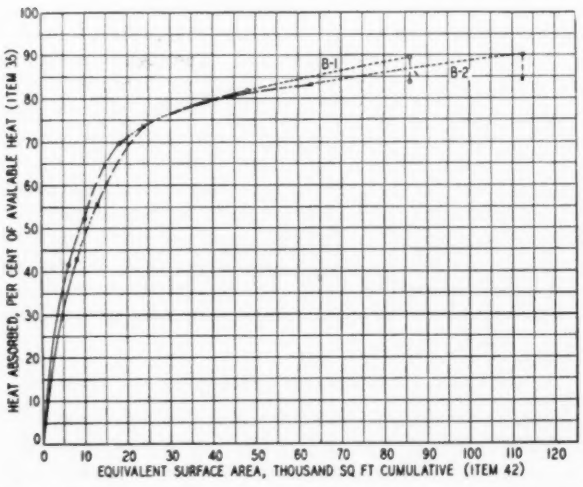


FIG. 5 HEAT ABSORPTION PLOTTED AGAINST EQUIVALENT SURFACE AREA

(Boiler of Fig. 3 and Table 2; tests B-1 and B-2. For legend see Fig. 10.)

surface, item 41, under consideration, including the water-cooled portion of the furnace, and each successive additional portion of heating surface to include boiler tube bank, superheater, economizer, and air heater if present. Each cumulative actual area, item 41, should be multiplied by this factor and plotted as equivalent surface area, item 42, against the total percentage of heat absorbed, item 35, by the corresponding actual surface area.

The arbitrary value of 500,000,000 Btu available per hr was taken after much consideration. It is a nominal value corresponding to many central-station boiler units. Some are much larger and many are smaller. It really makes no essential difference so long as the same unit is used as a standard for all boilers to be compared. It is hoped that others applying this basis of comparison will use this same standard for the sake of uniformity. A unit actually operating at 500,000,000 Btu available input per hr will be plotted with the actual surfaces equal to the equivalent surfaces because the factor is unity.

When comparing tests at different ratings on the same boiler, or any tests on any boiler, it is apparent that all cases at a given value of equivalent surface have identical Btu available per sq ft of the total surface up to that point. If the percentage of heat absorbed is not the same, it may be due to the cleanliness or effectiveness of the surfaces passed over, the temperature, luminosity, or mass flow of the gases or other factors, some of which may not be known or predictable without further study or tests. One may easily determine the Btu available and the Btu absorbed per sq ft per hr for any point on any curve, viz., $\frac{500{,}000{,}000}{\text{Equiv surface}}$ = Btu available per sq ft per hr. This result multiplied by the per cent absorbed gives the average Btu absorbed per sq ft per hr for all surface cumulatively from the fuel bed or burner up to the point selected on the curve. For illustration test B-1, Fig. 5, the entrance to the superheater lies on approximately 10,000 equivalent surface, and therefore corresponds to 50,000 Btu available per sq ft per hr. The percentage absorbed is plotted at 52.3, hence the average rate of heat absorption up to this point is

ON OIL-FIRED STATIONARY BOILERS

C-1					C-2				
SIMILAR TO FIG 3					SIMILAR TO FIG 3				
		160000					160000		
		156000					103000		
		459					451		
		463					461		
		568					555		
		221					218		
		482					438		
MECHANICAL ATOMIZING OIL BURNER					STEAM MECHANICAL ATOMIZING OIL BURNER				
		OIL					OIL		
		425					425		
		26.6					17.0		
		18020					18020		
FURNACE	1ST BOILER BANK	SUPERHEATER	2ND & 3RD BOILER BANKS	AIR HEATER	FURNACE	1ST BOILER BANK	SUPERHEATER	2ND & 3RD BOILER BANKS	AIR HEATER
13.9			13.1	12.8	14.0			13.2	12.9
4.3			5.4	5.6	4.0			5.1	5.1
0.0			0.0	0.0	0.0			0.0	0.0
125	125	125	134	135	123	123	123	131	131
201000	201000	201000	214000	216000	127500	127500	127500	135000	135000
1034	583	448	386	163, Lr-84	1036	515	380	316	134, Lr-62
5.8	5.8	5.8	5.4	5.4	6.1	6.1	6.1	5.8	5.8
3549					3545				
204.0					130.0				
17.4					10.1				
13.4					8.1				
208.0					132.0				
27800					17600				
127200	57400	48100	14380	6720	80800	36300	30500	9120	4270
2175	1725	1510	707	409	1940	1485	1260	600	325
208.0	117.5	90.5	78.2	37.7	132	65.7	48.5	40.34	20.0
90.5	27.0	12.3	40.5	17.4	66.3	17.2	8.16	20.34	10.1
90.5	117.5	129.8	170.3	187.7	66.3	83.5	91.66	112.0	122.1
43.5	13.0	5.9	19.5	8.4	50.2	13.0	6.2	15.4	7.7
43.5	56.5	62.4	81.9	90.3	50.2	63.2	69.4	84.8	92.5
55400	13500	17850	3990	1054	40600	8600	11820	2005	612
55400	32300	30000	11780	6070	40600	23000	21200	7750	3940
1541	1795	1233			1300	1580	990		
7494					7494				
1633	2000	690	10142	16500	1633	2000	690	10142	16500
1633	3633	4323	14465	30965	1633	3633	4323	14465	30965
3920	8740	10390	34780	74400	6180	13780	16400	54800	117300
		29170					29170		
		7140					4530		
TOTAL TO BOILER OUTLET = 1.35				A.H. = 3.14	TOTAL TO BOILER OUTLET = 0.52				A.H. = 1.13
				14.6					6.5
				83.4					86.2

26,150 Btu per sq ft per hr. The slope of the curve at any individual section is a graphic index of the rate of heat absorption in that section.

In Fig. 5, test B-1 shows 89.5 per cent of the available heat absorbed up to the outlet of the air heater which has an equivalent surface area of 86,200 sq ft. Test B-2 shows only 86.8 per cent heat absorbed at the same equivalent area; this corresponds to a point including all surface of the unit up to the entrance of the air heater plus about 46 per cent of the area of the air heater. However, as the gases pass completely through the air heater, the heat absorbed increases to 90.3 per cent, but the equivalent surface area is 112,600 sq ft, although the actual heating surface is the same as on the other test. At the same actual stages in the boiler unit, test B-2 shows higher percentages of heat absorption than the higher rating test B-1. This difference increases from 1.4 per cent leaving the furnace to 3.3 per cent entering the superheater and 4 per cent leaving the superheater, and then diminishes to 1.5 per cent entering the air heater and 0.8 per cent leaving the air heater. These are all taken from item 35 by subtracting the corresponding percentages from the two tests. This indicates that the first boiler tube bank and the superheater are more responsive to increased heat absorption due to increased heat input because of the temperature and mass-flow factors. The adiabatic or theoretical maximum combustion temperature of test B-2 is higher than on test B-1, hence this will account for some of the difference in heat absorbed in the furnace.

The total heat absorbed by the B-1 and B-2 tests is about 90 per cent and may seem unduly high, but it must be remembered that this is on a Btu available basis, which is more nearly like the net or low heating value often used abroad. The over-all efficiency of the unit by the A.S.M.E. method is shown as item 47 and plotted on the curves as a drop point under the last point on the available basis.

Comparison of Results and Boilers

The French locomotive tests are plotted on the equivalent-surface-area basis in Fig. 6. One cannot help but grasp immediately a realization of the difference between tests A-1 and A-2 due to the higher mass flow and better heat absorption resulting from plugging half of the tubes. There may also have been a lower excess air and higher adiabatic temperature in the firebox of test A-1, as it indicates a higher percentage of heat absorbed in the furnace, contrary to the relationship at the corresponding points of Fig. 5.

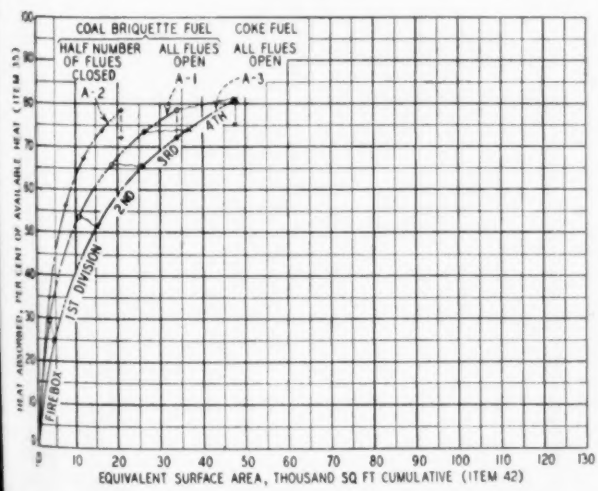


Fig. 6 Heat Absorption Plotted Against Equivalent Surface Area

(Boiler of Fig. 1 and Table 1; tests A-1, A-2, and A-3.)

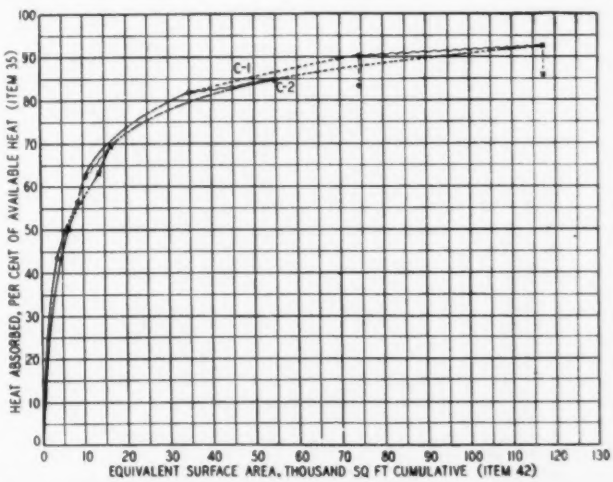


Fig. 7 Heat Absorption Plotted Against Equivalent Surface Area; Oil-Fired Boiler

(Boiler similar to Fig. 3, data given in Table 2; tests C-1 and C-2. For legend see Fig. 10.)

TABLE 3 DATA FROM ELEVEN BOILER TESTS MADE ACCORDING TO

		D-1		D-2		E-1			E-2		
1	TEST										
2	BOILER	SIMILAR TO FIG 8				FIG 8			FIG 8		
3	RATED STEAM OUTPUT - LB/HR.		22000		22000		13000			13000	
4	STEAM OUTPUT ON TEST - LB/HR.		23200		18000		10980			9970	
5	STEAM PRESSURE AT SUPERHEATER OUTLET - LB/SQ.IN.GA.		600		600		401			395	
6	SATURATION TEMPERATURE - °F		494		492		449			449	
7	STEAM TEMPERATURE AT SUPERHEATER OUTLET - °F		872		837		766			750	
8	FEEDWATER TEMPERATURE - °F		350		350		312			313	
9	AIR TEMPERATURE LEAVING AIR HEATER - °F		391		388		305			330	
10	METHOD OF FIRING	MECHANICAL ATOMIZING OIL BURNER				STOKER			STOKER		
11	KIND OF FUEL		OIL		OIL	BITUMINOUS COAL			BITUMINOUS COAL		
12	GRATE AREA OR FURNACE CROSS SECTION - SQ.FT.		57.8		57.8		50			50	
13	LB. OF FUEL PER HOUR, DIVIDED BY ITEM 12		28.4		21.9		23			21	
14	HEATING VALUE OF FUEL "AS FIRED" - BTU/LB.		18400		18400		12127			12288	
15	SECTION OF UNIT	FURN. & S.H.	AIR HEATER	FURN. & S.H.	AIR HEATER	FURN. & S.H.	ECONOMIZER	AIR HEATER	FURN. & S.H.	ECONOMIZER	AIR HEATER
16	CO_2 % BY VOL.	12.8		13.1				12.3			11.7
17	FLUE GAS ANALYSIS - LEAVING SECTION O_2 " " "	4.4		4.7				7.2			7.2
18	CO " " "	0.0		0.0				0.0			0.0
19	TOTAL COMBUSTION AIR LEAVING SECTION - %	125		128				150			150
20	TOTAL WEIGHT WET GAS LEAVING SECTION - LB/HR.	30000		23700							
21	HEAT AVAILABLE ABOVE 80°F ENTERING SECTION - BTU/LB.GA.	1092	136.5	1092	130.8						
22	MOISTURE BY WEIGHT IN FLUE GAS - %	6.75		6.75							
23	ADIABATIC TEMPERATURE - °F	3680		3680							
24	TOTAL HEAT INPUT FROM FUEL MILLION BTU/HR.	30.25		23.35		14.12			12.73		
25	TOTAL PREHEAT IN AIR ABOVE 80°F " " "	1.96		1.49		0.86			0.92		
26	TOTAL LOSSES, LATENT HEAT, UNBURNED CARBON " " "	1.90		1.49		0.92			0.73		
27	TOTAL HEAT AVAILABLE ABOVE 80°F " " "	30.31		23.35		14.08			12.94		
28	LIBERATION - ITEM 27 DIVIDED BY ITEM 39 - BTU/CU.FT.HR.	70000		53800		45400			41800		
29	ITEM 27 DIVIDED BY ITEM 41 - BTU/SQ.FT.HR	7680	4770	5910	3680	6210	4360	2750	5720	4020	2530
30	GAS TEMPERATURE LEAVING SECTION - °F	605	369	586	346	553	428	297	549	451	304
31											
32	HEAT ABSORBED: INDIVIDUAL SECTION - MILLION BTU/HR.		1.9		1.5		0.85	0.87		0.65	0.92
33	HEAT ABSORBED: CUMULATIVE " " "	26.2	28.1	20.3	21.8	11.4	12.25	13.12	10.35	11.0	11.92
34	HEAT ABSORBED: INDIVIDUAL SECTION DIVIDED BY ITEM 27 - %		6.3		6.4		6.0	6.2		5.0	7.1
35	HEAT ABSORBED: CUMULATIVE " " " " %	86.3	92.6	87.0	93.4	81.2	87.2	93.4	79.9	84.9	92.0
36	HEAT ABSORPTION: INDIVIDUAL SECTION - BTU/SQ.FT.HR.		790		623		886	461		678	488
37	HEAT ABSORPTION: CUMULATIVE " " " "	6620	4420	5130	3430	5040	3800	2570	4570	3410	2335
38											
39	FURNACE VOLUME - CU.FT.	433		433		309			309		
40	SURFACES: INDIVIDUAL SECTION - SQ.FT.		2403		2403		959	1885		959	1885
41	SURFACES: CUMULATIVE " "	3951	6354	3951	6354	2263	3222	5107	2263	3222	5107
42	EQUIVALENT SURFACE AREA $(500 \times 10^6 \div \text{ITEM 27}) \div \text{ITEM 41}$ - SQ.FT.	65200	104900	84800	136000	80500	114500	181500	87500	124000	197000
43	VOLUME OF ENTIRE UNIT - CU.FT.		2830		2830		2150			2150	
44	ITEM 27 DIVIDED BY ITEM 43 - BTU/CU.FT.HR.		10729		8250		6530			6020	
45	DRAFT LOSS INCHES OF WATER	0.06	0.24	-0.14	0.30	TOTAL TO OUTLET .05			TOTAL TO OUTLET .05		
46	TOTAL DRAFT LOSS PLUS AIR PRESSURE " " "		5.2		2.90			1.48			1.52
47	OVERALL EFFICIENCY - %		86.3		87.1			86.6			86.1

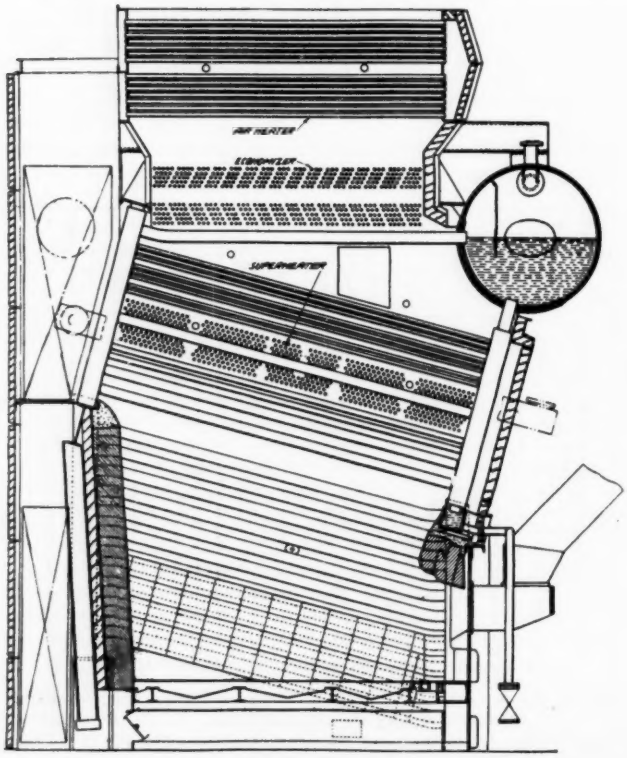


FIG. 8 STOKER-FIRED MARINE BOILER
(Data given in Table 3.)

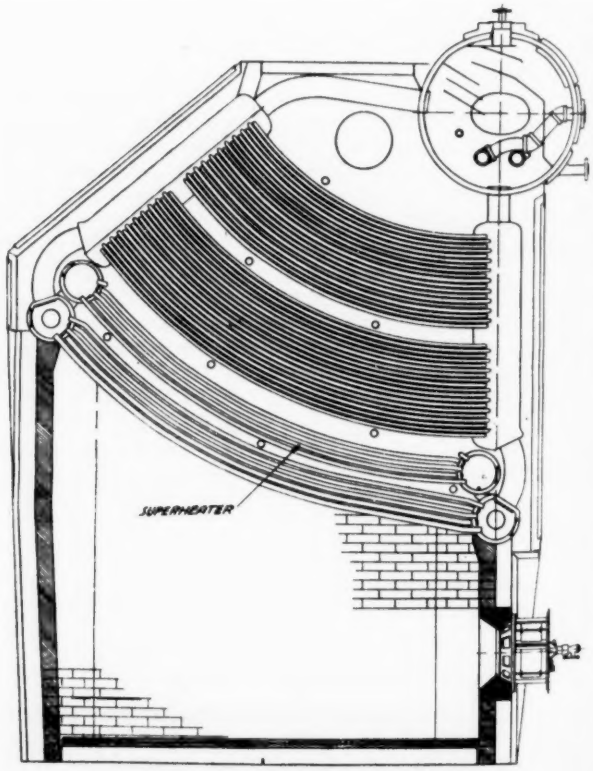


FIG. 9 OIL-FIRED MARINE BOILER
(Data given in Table 3.)

CONVENTIONAL STANDARD TEST METHODS

F-1	F-2	F-3	F-4	G-1		H-1	H-2
FIG 9				BENT TUBE		BENT	TUBE
67000	67000	67000	67000				
66070	50900	34280	17200		16450	27400	19500
309	305	311	301		224	125	121
428	426	426	424		400	354	352
695	679	663	602		536	422	420
211	216	210	211		216	100	94
121	118	113	129		276	74	73
MECHANICAL ATOMIZING OIL BURNER				STOKER		STOKER	
OIL	OIL	OIL	OIL	BITUMINOUS COAL		BITUMINOUS COAL	
71.7	71.7	71.7	71.7		86.5	93.5	93.5
70.3	53.3	30.2	17.5		23.0	37.1	26.9
18625	18625	18500	18625		10756	11883	11795
BOILER & SH				FURN. IN B.B.	AIR HEATER	FURN. IN B.B.	FURN. IN B.B.
12.5	13.3	12.2	10.9		14.2	12.9	13.7
4.1	3.0	4.4	6.3		6.1	6.19	5.2
0.0	0.0	0.0	0.0		0.0	0.02	0.01
123	116	125	141		141	140	132
92200	66100	47600	26100				
960	1007	942	834				
6.3	6.6	6.2	5.8				
3315	3445	3260	2965				
94.1	71.3	47.9	23.4	21.56		41.1	29.6
0.9	0.6	0.3	0.2	0.96			
6.6	5.3	3.4	1.85	1.90		2.8	2.8
88.4	66.6	44.8	21.75	20.62		38.3	27.0
208000	156800	105600	51200				
15800	11900	8020	3880	4550	3050	8500	6000
510	473	445	416				
					0.93		
78.3	59.7	40.2	19.5	18.15	19.08	32.0	22.85
					4.5		
886	896	895	896	87.8	92.3	83.4	84.6
					4.19		
14000	10670	7175	3480	4000	2820	7100	5070
424	424	424	424				
					2220		
5599	5599	5599	5599	4539	6759	4503	4503
31700	42000	62500	129000	110000	164000	58800	83500
2070	2070	2070	2070				
42700	32200	21700	10500				
2.53	1.56	0.98	0.30				
8.52	5.81	3.58	1.47				
83.2	83.8	83.8	83.2		84.1	77.5	77.1

In comparing tests A-1 and A-3, it is noted that, even though the rate of heat available is lower on coke-burning test A-3, the percentage of heat absorbed is less, while normally it would be greater. This again may be due to a lower adiabatic temperature, resulting from either higher excess air or perhaps unburned CO, which might have been very likely when burning coke. Another factor is luminosity, as the coke would have been different from the briquettes. Many desirable data are lacking from these tests.

Tests A-1 and B-1 are shown in Fig. 10 on the equivalent-surface-area basis. It is noted how much more useful is such comparison in contrast to that in Fig. 4, where they are both on the actual area basis.

A boiler similar to that of Fig. 3, but fired with oil, was tested by the gas-temperature-weight method and the essential data are given in Table 2 and plotted in Fig. 7. Characteristics similar to the pulverized-coal-fired tests B-1 and B-2 are noted, i.e., the heat absorbed at the lower rate of input tests C-2 are higher than on C-1 for corresponding locations, but lower on the same equivalent-surface-area basis.

The greater heat absorption in the oil-fired furnace, due largely to cleaner furnace walls, is brought out in Fig. 10, where curves B-1, coal-fired, and C-1, oil-fired, are plotted to the same equivalent-surface-area basis. The greatest difference is shown in the furnace, first boiler tube bank, and superheater. Beyond the superheater, the curves are substantially parallel, indicating about the same rates of heat transfer. The coal-fired boiler is operating at a 14 per cent lower Btu available per sq ft of total surface, and is 0.8 per cent lower in the percentage of available heat absorbed but is 0.3 per cent higher in over-all efficiency.

The French locomotive test A-1 follows the pulverized-coal-fired test B-1 very closely, until the superheater is reached on B-1 where it has a steeper slope and higher percentage rate of heat absorption between the 10,000 and 20,000 equivalent surface area. However, it loses out between the 20,000 and 32,000 equivalent surface area, where evidently the mass flow in the boiler tube bank is not as good as in the last two sections of the French locomotive. It is also noted that the saturation temperature of steam in the French boiler is 324 F, while in B-1 tests the saturation temperature is 498 F, causing an appreciable difference in the rate of heat absorption at these corresponding parts of the units.

The discussion so far has related to two kinds of boilers of three different sizes and with three methods of firing, hand, pulverized coal, and oil. Data from two methods of determining rates of heat absorption in different sections of the heating surface have been compared by a method which seems to have much value.

Table 3 includes data from eleven additional tests, the data being limited to the conventional standard test method. Certain data from these tests are plotted in Fig. 10 to show the percentage of available heat absorbed by the boiler and also by the economizer and air heater separately, where installed, and plotted against equivalent surface areas for comparison with the more

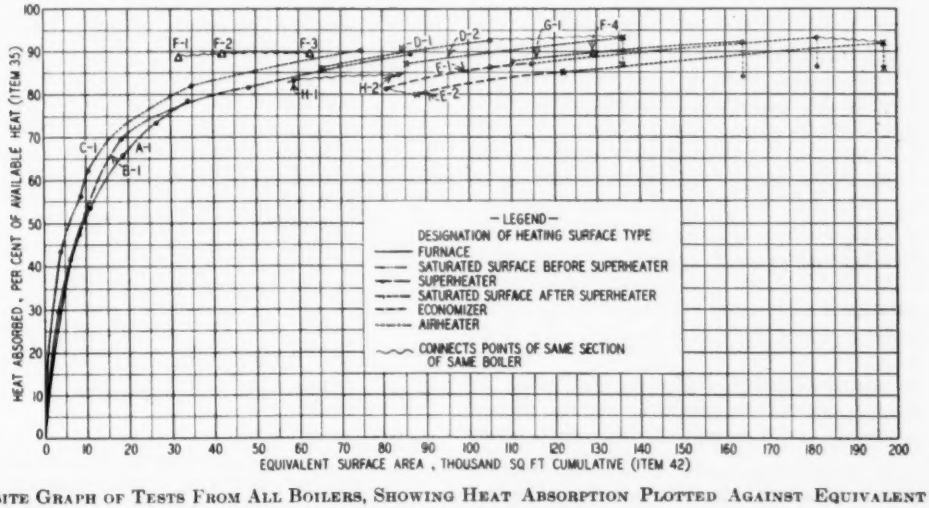


FIG. 10 COMPOSITE GRAPH OF TESTS FROM ALL BOILERS, SHOWING HEAT ABSORPTION PLOTTED AGAINST EQUIVALENT SURFACE AREA
(Data given in Tables 1, 2, and 3.)

complete tests of the A, B, and C series and with each other. This is done because it is believed to be the best known method for obtaining a comprehensive picture of the relative ratings and efficiencies of some representative units in the marine and stationary fields.

Marine Boilers

A typical marine boiler with spreader stoker for coal firing is shown in Fig. 8. It has a rated capacity of 13,000 lb of steam per hr at 400 lb pressure and 750 F. Two tests are given in Table 3 as E-1 and E-2 and are plotted in Fig. 10. While the boiler itself shows relatively low efficiency at lower rates of heat available than others plotted, it is designed with an economizer and air heater to give very good final efficiency.

Tests D-1 and D-2 are from an oil-fired marine boiler similar to Fig. 8, but of 22,000-lb capacity at 600-lb pressure and 850 F steam temperature. It is designed for high efficiency from the boiler itself and has an air heater also. It has less equivalent surface, smaller cubical content, and operates at higher draft loss than the stoker-fired marine boiler, and was operated at lower excess air.

Another marine boiler for Navy use is shown in Fig. 9. Some of the data published by Brierly[8] are reproduced as tests F-1 to F-4 in Table 3. This boiler is rated at 67,000 lb of steam per hr at 600-lb pressure and 700 F, but the tests reported are at 300-lb pressure to be more nearly comparable with present locomotive practice. As no sectional heat absorptions were obtained, only the over-all absorption by the boiler and superheater combined are plotted as triangles in Fig. 10. Even this serves well to show that this high-duty boiler has very good efficiency. It shows about 10 per cent better absorption of available heat than does the French locomotive boiler at the same high relative rating of about 15,000 Btu available per sq ft of total surface per hr. This is undoubtedly due largely to the better arrangement of heating surface and gas-mass flow as indicated by the higher draft loss. It is significant to note the uniformly high efficiency at the wide range of outputs. Tests on the boiler at 600-lb steam pressure show about $1^1/_2$ per cent lower efficiency than those given for 300 lb pressure, due to the difference in the saturation temperature of the heat-receiving surface.

Tests G and H are presented as representative of small stoker-fired stationary boilers of different types, design not shown. Test G-1 has an air heater which shows a comparable rate of heat absorption with other air heaters within the same available heat-rate range.

Comparison of Size, Rating, and Draft Loss

Several items which may be of interest in comparison with locomotive boilers or others are given in the tables but are not specifically discussed in this paper. Coal burned per sq ft of grate is given and, in order to enable a comparison with oil and pulverized fuel to be drawn, the equivalent cross-sectional areas of the respective furnaces are given and also the lb fuel per hr divided by this area.

Btu per cu ft of furnace volume is also given, but it should be remembered that this is a very misleading factor when comparing furnaces of widely different size, even when burning the same kind of fuel. The total cubical contents of the boiler, including the outside dimensions of the casing, economizer, and air heater, may be of interest to some.

Higher draft loss results from higher gas velocities, which are necessary if saving in space and the increasing of rates of heat absorption by convection are to be attained. All arrangements of heating surface are not equally effective, and no conclusions are now attempted from the small amount of data given.

[8] "The Babcock & Wilcox High-Pressure Sectional Express Boiler," by Lieut-Com. R. C. Brierly, U.S.N.R., *Journal of the American Society of Naval Engineers*, vol. 43, no. 4, Nov., 1931, pp. 511–561.

Summary

1 It is very desirable to know the actual rates of heat absorption in different elements of a boiler unit for the purpose of comparing effects of different rates of operation, different values of excess air, kinds of fuel, ash and slag behavior, and kinds of heat-absorbing surface.

2 The direct method of measuring heat absorption by evaporation calls for segregation of heat-absorbing surface which is either very difficult or entirely impossible, due to boiler construction and interference with circulation. The engineers of the Northern Railway of France deserve great credit for having done a very good piece of test work which checks very well with later work by other methods. The heat absorbed in some parts of a boiler unit may readily be determined by the direct method, such as in the case of superheaters, some economizers, and air heaters.

3 A second method is possible, wherein the heat absorbed in a section of the boiler unit, such as the furnace or a tube bank, results from calculation using average gas temperatures and gas weights entering and leaving the section, the difference, allowing for any combustion of fuel taking place within the zone being measured, being the heat absorbed.

4 Data obtained by either method is useful to engineers experienced in calculating boiler performance, but others often find it difficult to grasp an accurate and complete realization of the relative importance and comparisons of different parts of a boiler unit or one with another. A simple method of comparison is presented which consists of plotting a curve showing the relation between the heat absorbed, expressed in per cent of heat available, and the equivalent surface area wherein the heat was absorbed.

5 The data presented have been restricted to relatively small stationary boilers which would be comparable with representative marine boilers, two of which are given, and modern locomotive boilers, data from which are available from another paper.[9] These limited data are not sufficient to draw many conclusions regarding the factors of heat transfer by radiation and convection; however, some well-known facts are clearly demonstrated as being of outstanding significance when presented in this graphic form:

(*a*) Radiant heat produces high rates of heat transfer, as shown by the steep slope of the curve.

(*b*) Excess air and adiabatic temperature affect radiant heat greatly. This point is not fully brought out in this paper, because for the purpose of simplicity, furnace tests were selected with reasonably uniform adiabatic temperatures.

(*c*) High mass flow and high mean temperature difference increase rates of heat transfer. Draft loss, and fan power or its equivalent, are necessary to produce high rates of heat absorption by convection, and high efficiencies with limited surface areas.

(*d*) Surface is the principal requisite for heat absorption. It should be clean for high rates of heat absorption, but sometimes a covering of ash, slag, or other means is a desirable aid toward complete combustion and for protection against too high a rate of heat absorption and to pass higher temperatures on to other surfaces such as superheaters, which will bring the total absorption to a more economical result.

(*e*) Terms like Btu per cu ft per hr of furnace volume, Btu per front ft of furnace width, etc., should be used with caution, because they are not as fundamental in their importance as are some other factors. Pounds of oil per sq ft of

boiler is a basically sound standard, much used in marine engineering, but it becomes less useful with the further use of furnace cooling, economizers, and air heaters. Btu available per sq ft per hr seems to be fundamentally sound, but it needs definition as to its application to furnace alone or to the entire unit. Item 29, Table 2, gives this factor for each cumulative section of the boiler unit. It seems now that this value, including all surface up to the entrance of the superheater, is likely to become more used. It also is useful when applied to the furnace alone.

6 Temperature of gases as measured at different points in the gas path is of great importance regardless of whether these temperatures are used in conjunction with gas weights to obtain heat absorption. A knowledge of temperatures at various points of the entire unit is essential in design and is useful in operation in connection with the behavior of ash and slag. The gas temperature might well be plotted to the equivalent surface area for comparison, but the excess air and air leakage at different points interfere with the same usefulness as is obtained from the heat-absorbed basis herein presented, hence in the interest of brevity for this paper such plots are omitted. Gas temperatures are given in the tables as item 30, and the adiabatic temperature as item 23. The adiabatic or theoretical temperature is useful as an index of the combined effect of excess air, moisture in fuel, and completeness of combustion.

7 Heat absorbed and equivalent surface are undoubtedly the most important basic factors for comparison in studying boiler performance.

Acknowledgment

Acknowledgment is made for valuable data and assistance from stoker manufacturers and from the author's associates.

Discussion

J. C. Hobbs.[9] Mr. Bailey's use of the French tests illustrates clearly the relations between cause and effect. Each part of the steam-generating equipment being set up independently, there can be no question about the heat transfer and the work done.

The curves show clearly not only the relationships indicated, but the slope of the curves, i.e., the increment heat absorbed by the increment heating surface, shows the punishment, if you please, being carried by the heating surfaces comprising different parts of the equipment.

The writer wishes to emphasize by brevity rather than by voluminous discussion, the great value of a uniform distribution of energy absorption over the entire surfaces in so far as it is possible, so that the apparatus may be more reliable and in the long run the operation more successful.

R. S. Julsrud.[10] It would appear from the writer's personal experience, that designers of steam-generating equipment, in their effort to secure a high efficiency in restricted furnace cavities fired by pulverized coal, are apt to lose sight of the physical and chemical limitations of the coal which makes available this heat.

Most coals have certain limitations; these may include low fusion temperature, generally resulting from high sulphur present in the fuel. Again high volatile content may result in coking and possible burning of the tips of pulverized-coal burners and extremely long flame travel with resultant high temperatures entering the convection surfaces. Also, low-volatile coals present the problem of maintaining ignition at low heat-output rates.

Our knowledge of heat-transfer rates to water-cooled surfaces, both bare and refractory-covered when relatively clean, is now fairly well established. However, when such surfaces have become covered with slag, whether fluid, sticky, or spongy, the heat absorption of the surfaces is considerably reduced, and instead of the predicted heat extraction taking place from the gases in the furnace, they enter the convection surface at considerably higher temperatures which may exceed the fusion point of the ash particles present in the gases. The result is a tendency for these sticky ash particles to build up at the heating surfaces. If this occurs on surfaces with restricted gas passages, the result is plugging of the gas passages and eventual outage of the unit. Again the presence of sulphur in coal, resulting in the formation of ferric sulphides when burned, may in furnaces operated at high heat liberation and temperatures result in a reducing atmosphere. It is also possible, but has not as yet been established, that reducing action of these pyrites results in attacking the surfaces exposed to this action. In any event, under high heat liberations these surfaces tend to wash and eventually fail. This action results in boiler outage and increased maintenance costs.

To attempt to reduce these failures, the surfaces are often built up to a greater thickness with a resultant lower heat-transfer rate and the possibility of high gas temperatures entering the generating tubes, as mentioned. Thus in eliminating difficulty in the furnace, it may be transferred to the convection surfaces.

Until such time as we have a better knowledge of the heat absorption of water-cooled surfaces when coated with ash in its various forms, it would appear that a greater degree of conservatism should be exercised in the design of these units.

A reduction of 1 per cent in the CO_2 at the boiler outlet will result in approximately 200 F drop in the furnace-gas temperatures. This may mean the difference between high furnace maintenance and outage as against continuous performance with somewhat lower efficiency. Moreover, high heat releases per cubic foot of furnace volume, which is a measure of furnace temperatures developed, can well be reduced for the same reason.

Such a design would make available to the consumer a wide variety of coals, attractively priced, now handicapped by their physical and chemical limitations.

E. B. Powell.[11] The author has made available an astonishingly simple, logical, and direct method of graphic comparison for evaluating heating-surface effectiveness. The writer would urge that he eliminate at once the only suggestion of complexity, i.e., the factor 500,000,000. Steam boilers of 500,000,000 Btu per hr input rate will soon be outmoded. The method of heating-surface analysis in this paper will serve for years to come. With such repetitive use certainly ahead, why not immediately eliminate all unnecessary effort, even that of dividing by 2, and adopt a more rational, permanently acceptable multiplier, 10^8 or 10^9. Also, to assure instantaneous interpretation of the chart, Fig. 10, the writer suggests that the term "equivalent surface" be defined merely as the square feet of surface per Btu of total heat liberated or made available to the steam-generating unit per hour, with a factor of 10^8 or 10^9 applied to the quotient to secure a number of convenient size. Furthermore, it is suggested that this formula, abbreviated, should be consistently included in the title of the abscissas.

In the closing paragraph, the author, in mentioning a deficiency of data for certain interpretative purposes, would seem to promise a subsequent paper discussing the influence of such factors as draft loss and arrangement of heating surface upon heat-transfer

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[10] Combustion Engineer, Consolidation Coal Company, New York, N. Y.

[11] Consulting Engineer, Stone & Webster Engineering Corporation, Boston, Mass. Mem. A.S.M.E.

effectiveness. The writer trusts he is correct in this inference. The author's clarifying treatment of all factors in steam-generating-unit heat transfer would be a tremendous asset.

In closing, it would be of interest to know whether corrections were applied to observed temperatures in order to allow for possible radiation error of the high-velocity thermocouple.

R. A. SHERMAN.[12] A knowledge of the performace of the various sections of a steam generator is essential to intelligent design for proper distribution of the heating surface to insure that the temperatures in various sections are of the proper order. The rate of heat absorption in the furnace governs the temperature of the gases which, for coal-fired furnaces, fixes the degree of fusion of the ash and, therefore, the type of coal which can be burned in a given furnace or the maximum rate of burning, or minimum excess air that can be maintained with a given furnace.

The Society is indebted to the author for the presentation of the data which he has obtained by gas-temperature measurements, as these permit an analysis of the performance of various sections of a boiler which would otherwise be impossible.

The writer wishes to emphasize the fact brought out by the author that the expression, "Btu per cu ft per hr" is of no significance in comparing the performance of various boilers or of furnace temperatures. Earlier in the year, at Battelle Memorial Institute, we conducted an investigation which indicated that the same expression had no significance in relation to the smoke emission. By a factual survey of 102 power and heating boilers burning up to 1200 lb of coal per hr, and in an experimental investigation of 22 boilers of the same type, no relation was found between the smoke emitted and the rate of heat liberation per cubic foot of furnace volume. Briefly, this may be explained by several facts:

1 It is the time of travel of the gases from the fuel bed to the entrance to the heating surface which determines whether the carbon will be burned before its temperature is lowered below its ignition temperature. Furnaces have volume because they have three dimensions but it is the length of travel which is important. Further, a large part of the volume is frequently not utilized and a large volume is not a definite assurance that the carbon will be burned.

2 Combustion of volatile matter without the liberation of carbon is rapid if the required air is mixed with the volatile and, with good mixing, little time or volume will be required for smokeless combustion. Without good mixing an infinite volume or length of flame travel will not avoid smoke.

3 Still other factors, in addition to the degree of mixing, such as the type of coal, the uniformity of distribution of the coal on the stoker, the excess air, the type of load, and the degree of intelligence and the ability of the operator are of greater importance in the elimination of smoke than the rate of heat liberation.

To return to the author's data, a retabulation of a few of the items given for boilers *A*, *B*, and *C* will emphasize the lack of meaning of the rate of heat liberation per cubic foot of furnace volume. The tabulation is as follows:

Boiler Type	*A* Locomotive	*B* Coal	*C* Oil
Furnace volume, cu ft	33	6030	7494
Furnace heating surface, sq ft	76.8	1695	1633
Ratio, surface/volume	2.33	0.28	0.21
Heat release, Btu per cu ft per hr	354000	22500	27800
Heat release, Btu per sq ft per hr	152200	79900	127200
Heat absorbed, Btu per sq ft per hr	44900	33200	55400

Judged from the relative rates of heat release per cubic foot of furnace volume in the locomotive and in the other two boilers, the temperatures might have been thought excessive in the locomotive boiler but, because of the high ratio of heating surface in the furnace walls to the total volume, the rate of heat absorption was high and the temperatures were probably no higher than in the other boilers. On the basis of the rates of heat liberation per square foot of furnace wall, the differences among the three boilers were much less and the rates of heat absorption per square foot of furnace wall were also of much the same order.

Basing the comparisons of boilers on the ratio of the heating surface to the heat available will bring various boilers more into line but variations will be due, as the author points out, to cleanliness of the surface and to the luminosity and emissivity of the flame. The emissivity of a flame increases as its depth increases owing to greater furnace dimensions, but the emission per square foot of flame surface cannot increase in the same order as the area decreases with respect to the volume.

A spherical furnace would be the least desirable shape of furnace because this gives the lowest ratio of surface to volume. The cubical box-like furnaces which we have in many present-day furnaces are also unfavorable for obtaining a large area of heat-absorbing surface. High, narrow furnaces are more favorable in presentation of surface; this principle can be extended by introduction of division walls of water tubes to obtain greater surface and lower the furnace temperature.

The adoption of the author's method of obtaining data on temperatures and gas weights by many other workers would be desirable. Further studies on the relation of the radiant emission from the flame to the dimensions of the furnace would also be valuable.

The writer agrees with a previous suggestion that the term "equivalent surface area" be replaced by the expression, "ratio of surface area to heat available," multiplied by a simple factor such as 10^6 on the ground that this is a more accurate expression and less likely to introduce confusion. Let us hope that we can mark this day as that on which the interment of the term, "Btu per cu ft of furnace volume," was completed, never to rise again.

JOHN VAN BRUNT.[13] The method suggested by the author for comparing performance of furnaces, boilers, superheaters, economizers, and air heaters, comprising a complete steam-generating unit, is interesting and the selection of a unit of 500,000,000 Btu input for such comparisons serves the purpose perhaps as well as any other arbitrary value.

In the design of large steam-generating units it is necessary to determine the amount of heat absorbed by each portion of the entire unit by using actual fluid quantities and heat-transfer rates, etc. After such a unit is in operation, it is customary to check the calculated performance, and the basic design rates used for its determination.

The amount of heat to be absorbed by each division of the total surface is determined by the performance requirements and the conditions imposed by the purchaser and by the economic factors of each case, plus limitations dictated by the fuel characteristics and factors governing availability of the unit for continuous operation.

Given two units, for example, of 100,000 and 300,000 lb capacity, the smaller for 300 lb per sq in. pressure and 750 F steam temperature with 250 F feedwater, and the larger for 1200 lb pressure, 925 F steam temperature with 380 F feedwater temperature; there can be no purpose in comparing such units on the basis of a 500,000,000-Btu unit, as suggested by the author.

The per cent of heat absorbed in the furnaces in the various tests given in the paper are low, in fact, considerably lower than would be expected in a modern furnace. In tests A-1, A-2, and A-3, the low rate of absorption is probably due to poor combustion conditions and can be compared only with locomotive boilers.

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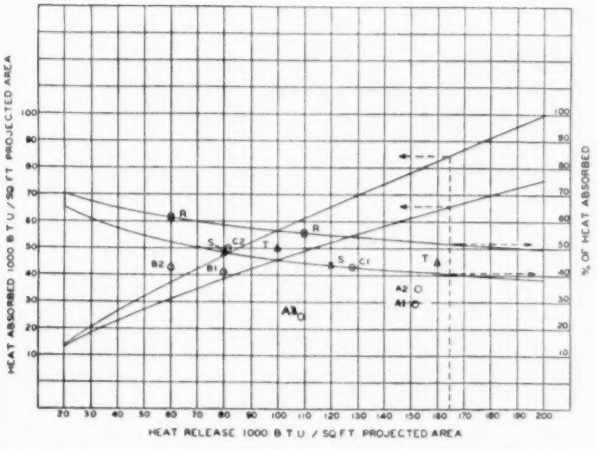


FIG. 11 APPROXIMATE RELATION OF HEAT RELEASED TO HEAT ABSORBED IN A PULVERIZED-COAL WATER-COOLED FURNACE

Fig. 11 of this discussion reproduced from Fig. 9 of the writer's paper,[14] shows the ratio of heat absorbed to heat released, both in Btu and in per cent. The curves sloping upward from the lower left-hand corner are read in Btu per lb per sq ft, to the left, and the curves sloping downward read percentage of heat absorbed or efficiency of furnace absorption, to the right.

The points from the tests in the author's paper are plotted against the percentage of heat absorbed. With the exception of C-1 and C-2, the points are well below the lower of the two efficiency curves. In a modern oil-burning furnace, the points C-1 and C-2 would be expected to fall near the upper efficiency curve. For comparison, the two points *R* are plotted from test results of a tangentially fired pulverized-coal slag-bottom furnace on a unit of 300,000 lb per hr capacity. The points *T* are from a 450,000 lb per hr tangentially fired pulverized-coal slag-bottom furnace and points *S* are from a vertically fired slagging-bottom furnace, pulverized-coal-fired. In all of these units, the furnace walls are composed of bare fin tubes.

The influence of both the amount and character of the ash and of the method of firing on the amount of heat absorbed is marked. It may be seen from the curves and from the points plotted from the tests that the per cent of heat absorbed in any furnace decreases as the rate of release per square foot increases.

Absorption below the lower curves would indicate inefficient use of the furnace wall surface, poor combustion, or walls covered with an excessive amount of ash. Waterwall tubes covered with refractory or refractory coatings will also fall below the lower curves.

AUTHOR'S CLOSURE

The comments of Messrs. Hobbs, Powell, and Sherman are very much appreciated as they fully grasped the importance and significance of this method of comparing the performance of furnaces and heat-absorbing surfaces in steam-boiler units. Mr. Powell has raised a question, in which Mr. Sherman concurs, i.e., whether the equivalent-surface area might be simplified to a factor of square feet of surface per Btu available. Mathematically this would seem to be simpler, but psychologically the author fears it would be more confusing because the logic of the factor is not quite so evident. For instance, assume a unit having 500,000,000 Btu available and, at the point to be plotted 10,000 sq ft of actual surface have been passed over; this factor would be 0.00002. The suggestion of multiplying this by 10^5 would leave it 20. This seems like a very inadequate figure to plot when it actually represents 10,000 sq ft of surface. Even multiplying this by 10^9 or 10^{10} seems in the author's opinion only to add to confusion rather than to simplify the method of comparison used in the paper.

There may well be some question concerning the choice of 500,000,000 Btu when some boilers may go as high as 1,000,000,000 and others may be only $^1/_5$ or $^1/_{10}$ this amount. The value of this entire method comes through standardization and while, as Mr. Powell says, the 500,000,000 may be outmoded, it is greatly to be doubted if the average boiler unit to be considered will vary so much from 500,000,000 Btu available that it cannot still be used as a perfectly logical standard. Certainly 100,000,000 Btu is not a satisfactory standard and 1,000,000,000 is applicable in only a few cases. As a matter of fact, it is felt that the odd number of 500,000,000 has some real value in that it makes one stop to think what it really is, while otherwise it might be an abstract factor and not carry the significance to the mind of the user, which condition now seems to prevail. Another point of significance is that most coordinate papers bring out the values on this basis very nicely, while figures $^1/_5$ or 2 times those used would not be as convenient to plot.

Mr. Julsrud mentions several matters pertaining to furnace design and the characteristics of ash which are really outside the scope of the paper as presented, since the data given were limited purposely to a range of boiler operation permitting a comparison with the sizes prevailing in locomotive practice, and also to bring out the method of comparison rather than to go into the many phases of slag, heat absorption, etc. As indicated by Mr. Powell, there is great opportunity in the future for all these matters to be studied through this method of comparison.

As a matter of fact, Mr. Van Brunt in Fig. 11 of his discussion has already compared some of the data on dry-ash coal-fired and oil-fired furnaces with data on slag-tap furnaces for which he has information.

Before replying to Mr. Van Brunt's discussion, the author checked with him and was assured that his "heat release 1000 Btu per sq ft projected area" is identical with the author's "heat available." It is significant to note that the lower curve of Mr. Van Brunt's Fig. 11 on the percentage basis is substantially identical with Mr. Orrok's curve, which the author reproduced on a similar basis and presented as Fig. 20 in his discussion[15] of Mr. Brandt's paper. We have found that not only do the points C-1 and C-2, which Mr. Van Brunt has plotted in his Fig. 11, fall on this curve of Mr. Orrok's, but they also are in close agreement with many other data obtained on oil-fired boilers having normally clean heat-absorbing surfaces and where the adiabatic temperature is close to 3600 F. Points B-1 and B-2 for the pulverized-coal-fired furnace with dry-ash removal are approximately 20 per cent lower in rate of heat absorption than are those of the Orrok curve on which fall the oil-fired tests. This indicates a certain amount of ash accumulation on the surface of such furnace, which also has been substantiated with many other data. As noted in the author's paper, all of these data, except those concerning the French locomotive, are based upon tests with the high-velocity thermocouple of the single-shield variety, and to answer Mr. Powell's question no correction has been made. It is now known, however, that many of these temperatures are likely to be from 50 to 75 deg low. This would indicate that the actual rates of heat absorption are still lower than those indicated by the *B* and *C* points and, therefore, closer to the French data which Mr. Van Brunt believes to be low. They are in the same region as Goss's locomotive tests of 1912, where heat absorption

[14] "Design of High-Capacity Boiler," by John Van Brunt, Trans. A.S.M.E., vol. 60, 1938, paper FSP-60-17, p. 487.

[15] "The Locomotive Boiler," by C. A. W. Brandt, discussion by E. G. Bailey, published on page 399 of this issue of the Transactions.

in the firebox alone was determined by direct evaporation without depending upon any gas temperatures. These data from Goss's tests are also given in Fig. 20 of the author's discussion[15] of Mr. Brandt's paper.

Mr. Van Brunt has questioned why all of these rates of heat absorption are so low in comparison with his points *R*, *S*, and *T*, taken in furnaces of the slag-tap type where the walls are apparently coated to some extent with slag. However, since Mr. Van Brunt does not give the fluid temperature of the ash or other data, it is beside the point to question them further than to state that any inaccuracies in the measurement of gas temperatures in these portions of boiler units may lead to very erroneous conclusions.

Dr. Mullikin has presented papers[16] before the American Institute of Physics showing comparative results from bare thermocouples of different sizes, single-shield high-velocity thermocouples, and multiple-shield high-velocity thermocouples. A great deal of field data were included. These data emphasize the importance of further studies in gas-temperature measurement.

Further evidence has been obtained showing that the multiple-shielded high-velocity thermocouple is more nearly correct than anything else that has yet been devised. In boiler installations where the heat absorbed in the air heater, economizer, and superheater can be determined separately, the gas temperatures entering the superheater may be determined by calculating back from an accurate measurement of the gas temperature leaving the air heater. Such data check very closely with the multiple-shielded high-velocity thermocouple.

Other confirmation on rates of heat absorption in furnaces has been obtained by a thermal probe wherein the rate of heat absorption has been determined by the rate of flow of a fluid and its rise in temperature through the element when inserted into different parts of furnaces.

Mr. Van Brunt states that there is no advantage gained in comparing a unit of 100,000 lb per hr output at 300 lb per sq in. pressure, 750 F steam temperature, and 250 F feedwater temperature, with a larger unit of 300,000 lb per hr output, 1200 lb per sq in. pressure, 925 F steam temperature, and 380 F feedwater temperature. If the fuel burned and furnace construction are the same, a comparison of heat absorption up to the entrance of the superheater will be well worth-while, for the temperature gradients are not sufficiently different to have any measurable effect upon the rate of heat absorbed up to that point. There will, of course, be a justifiable difference in the shapes of the curves through the superheater, later boiler bank or economizer, and the air heater. However, it is not beyond our experience to learn a great deal from even these comparisons. Of course, the real advantage comes in comparing units in which similar temperature gradients exist, in order to detect the effect of excess air, different kinds of fuel and ash, arrangement of surface, etc. The main purpose of the present paper was to show a simple method of making these comparisons, leaving it to the judgment of those using this method as to the value obtained by the variety of comparisons they might choose to make.

[16] "Gas-Temperature Measurement and the High-Velocity Thermocouple," by H. F. Mullikin, Symposium on Temperature Measurement and Control, American Institute of Physics, New York, N. Y., Nov. 2, 1939.

"Accuracy Tests of the High-Velocity Thermocouple," by H. F. Mullikin and W. J. Osborn, loc. cit.

These papers were published in abstract form in *Glass Industry*, vol. 20, no. 12, December, 1939, pp. 441–442.

The Locomotive Boiler

By C. A. BRANDT,[1] NEW YORK, N. Y.

For many years increased traffic and the demand for high operating economy of the railroads resulted in the development of locomotives of high tractive effort capable of starting trains close to the limit of length for practical handling through yards and terminals. The ever-present demand for still higher transportation capacity of the railroads and competition have compelled the further development of locomotives to handle these long and heavy trains at high speeds. This has meant proportionately larger and more efficient boilers. The author discusses in this paper some of the problems of boiler designs and proportions that affect the efficiency and capacity of the conventional locomotive.

THE PUBLIC, which is served by the railroads, is continuously demanding higher speeds for both freight and passenger trains and this, in conjunction with a more intensive utilization of their motive power, is one of the many important problems confronting the managements of the American railroads today.

Until recent years the steam locomotive had been the principal power unit, but its supremacy is now being challenged by other forms of motive power, particularly the electric- and Diesel-driven locomotives. To meet this challenge the designers of steam locomotives are constantly studying the problem of building boilers of greater steam-generating capacity within the permissible limits of size and providing engines of lowest possible steam consumption per unit of power.

The fact that higher boiler capacity for minimum weight has been an ever-present problem throughout the years of locomotive development in America can best be illustrated by citing the evolution of the eight-coupled locomotive. The first of this design, or the 2-8-0, utilized 90 per cent of the total locomotive weight for adhesion. The demands for higher speeds, requiring greater steam-making capacity, led to the addition of truck axles to carry the heavier boilers, which resulted in the successive development of the 2-8-2, the 4-8-4, and last the 6-4-4-6 type high-speed locomotive, exhibited at the New York World's Fair this year.

The total weight of an early-design 2-8-0 type locomotive with 270,000 lb on the drivers and a tractive power of 67,500 lb was only 300,000 lb. To supply the steam for the high-speed 6-8-6 type locomotive with the same weight on drivers and the same tractive power requires a boiler of such size as to double the total weight of the locomotive to 600,000 lb, as is illustrated in Fig. 1.

As this and other modern locomotives are examples close to the maximum practical size that can be built, the important question is whether greater boiler efficiency is attainable, particularly at high-capacity operation, and what can be done to accomplish this.

The question of reducing the steam consumption of the engines is outside the scope of this paper, but the possibility of reducing the average steam consumption per ihp from an average of 18 lb to 13 lb is possible by the adoption of compound cylinders as reported from tests on French locomotives, as well as by the use of still higher superheat and valve gear adaptable for its utilization.

The paper will be confined to a discussion of those problems of design believed to be most essential in the advancement of the art. To enhance the value of this contribution, Tables 1 to 6 are included which give the principal dimensions and boiler ratios of representative locomotives of various types built in America in recent years. Locomotive test curves are also presented.

The subject matter will be centered on problems affecting the design of the conventional locomotive fire-tube boiler only, not because of any belief that this type of boiler is the final answer to the locomotive steam generator, but because at present most of the locomotives in the world are equipped with this type of boiler. The best solutions to some of the problems encountered in the design of large boilers have not yet been agreed upon principally because of the lack of reliable test data.

A committee of the Association of American Railroads (A.A.R.) submitted a report this year recommending a conventional type of boiler for a proposed 6400-hp high-speed locomotive, which is an acknowledgment that the fundamental principles of this boiler design have proved practical and are the best available at the present time.

TYPE	WHEEL ARRANGEMENT	TRACTIVE POWER LBS	WEIGHT ON DRIVERS LBS	TOTAL WEIGHT OF LOCO LBS	WEIGHT ON DRIVERS PERCENT OF TOTAL	INCREASE IN WEIGHT OF LOCO TO GAIN ADDITIONAL BOILER POWER ACTUAL	PERCENT
2-8-0	4-0-○○○○	67500	270000	300000	90		
2-8-2	4-0-○○○○-0	67500	270000	360000	75	60000	20
4-8-2	4-0-0-○○○○-0	67500	270000	400000	67	100000	33
4-8-4	4-0-0-○○○○-00	67500	270000	450000	60	150000	50
6-8-6	4-0-0-0-○○○○-000	67500	270000	600000	45	300000	100

TABLE SHOWING INCREASE IN THE TOTAL WEIGHT OF AN AVERAGE 8 COUPLED LOCOMOTIVE OF THE SAME WEIGHT ON DRIVERS AND SAME TRACTIVE POWER

FIG. 1 INCREASE IN TOTAL WEIGHT OF EIGHT-COUPLED LOCOMOTIVES DUE TO LARGER BOILER CAPACITY

It is unnecessary to recite in detail the advantages of the conventional boiler; this has been done many times before. The virtues of the completely water-cooled radiant-heat-absorbing furnace, high gas velocities over the convection heating surfaces with forced draft, and high superheat, all originally inherent in the locomotive boiler, are now being recognized as essentials to efficient steam generation in other fields and are being rapidly adopted in stationary-power-plant boilers.

Locomotive boilers of designs radically different from conventional construction have been built in the past, but so far none has proved sufficiently practical for general railroad service. There will soon be placed in service in America a different type of steam generator for operation in conjunction with condensing steam turbines, and still other types are under consideration.

There has been a great increase in the size of locomotive boilers built in recent years in America; many of these have proved very efficient. However, much remains to be done to make possible the burning of more fuel per hour at higher combustion efficiency

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

with lower draft loss, greater heat-absorption efficiency, and higher superheat with less boiler weight per pound of steam produced.

Definition of Boiler Capacity and Existing Formulas

In designing a locomotive, the steam required for a given maximum horsepower capacity can be determined quite accurately from existing knowledge of steam consumption per ihp-hr, when initial and final steam conditions are specified. To this must be added the steam required for auxiliaries, train heating, air conditioning, and other uses. The boiler design must be such as to deliver this maximum steam output at a superheat as high and a back pressure as low as that figured on, or else the steam consumption per ihp will increase and the cylinder power fall below that required. To determine the excellence of a particular boiler design and compare it with others, it is necessary to have a standard measure of comparison.

It would appear logical that the weight of steam produced per pound of total weight of the boiler in service should provide a satisfactory measure when a definite steam pressure, superheat, and draft loss are specified. It is obvious that such a yardstick is of no value, unless a standard method of predetermining the maximum evaporative capacity is found which will satisfy all conditions in a reasonable way.

In reviewing the existing methods of calculating the maximum evaporating capacity of a locomotive boiler and the over-all boiler efficiency, the designer is confronted with the fact that formulas now generally used are inadequate for predetermining the result with accuracy, particularly when large boilers are involved.

In America the theory generally followed is that the steam-generating capacity[2] of a boiler is directly proportional to the amount of evaporating heating surface in square feet.

A set of evaporative values, giving the maximum quantity of steam in pounds per square foot of heating surface per hour, generated by the firebox and flues, was prepared some years ago by F. J. Cole.[3]

These values are generally used by the locomotive builders and railroads today.[2,3,4,5]

Any method of calculation that takes square feet of heating surface only into consideration must be inadequate. It is evident that the arrangement of the heating surfaces, their relationship to the grate area, furnace volume, gas area, firebox heating surface, and hydraulic depth and length of the flues must be considered to give approximately correct results. The reason is that it is these relationships which determine the boiler efficiency, the back pressure required to produce the steam, and the superheat, all of which have a great influence on the efficiency and power output of the cylinders.

Strahl proposed a formula in 1913[6] which takes into consideration the size of the grate area and its ratio to the evaporating heating surface

$$WS = \frac{a}{\frac{S}{R} + 7} \times S$$

where

WS = total steam produced, lb per hr
R = grate area, sq ft
S = evaporating heating surface, sq ft
a = coefficient for superheated locomotive, 778 lb per sq ft

This formula, with the coefficients established by Strahl, comes close to test results on boilers with proportions similar to those which Strahl used in his analysis, but leads to unsatisfactory results on boilers with different ratios. The Strahl formula may be modified to suit different arrangements by changing the coefficients to suit, but this is not an entirely sound procedure.

A method originated by Lawford H. Fry[7] for determining the maximum evaporating capacity of a boiler from the over-all boiler-efficiency curve appears to be a better approach. Fry suggested that, if the over-all boiler efficiency is plotted against pounds of coal or total heat in Btu fired per square foot of grate per hour, the relationship between the efficiency and rate of firing becomes a straight line. The correctness of this theory has been proved on tests examined. The fundamental equation established by Fry is

$$F = m - nG$$

where

F = boiler efficiency in per cent
G = dry coal fired, lb per sq ft of grate per hr
m = coefficient denoting theoretical efficiency at zero firing rate
n = coefficient determining slope of curve

If sufficient test data were available for all kinds of fuel and types of boilers so that the coefficients m and n or the origin and slope of the over-all boiler-efficiency curve could be predetermined with accuracy, together with the superheat and pressure at the outlet of the superheater, then the evaporation for any quantity of coal fired could be determined, as well as the maximum capacity.

It appears to the author that the Fry or other methods proposed do not satisfy the requirement as they stand, because draft loss and back pressure are not part of the picture.

The inadequacies of both the Cole and Strahl methods are apparent when a comparison is made between the data calculated by the different methods, and the actual test results as shown in the following table:

	Evaporation, lb per hr				
Locomotive	Strahl	Cole	Difference, per cent	Actual test	Difference, per cent
P.R.R. K4S No. 5341	48000	52150	+ 8.6	72000	+50
P.R.R. M1A No. 6706	53350[a]	67850[a]	+27.2	99095[a]	+85.7
N.Y.C. J3A	60700[a]	60500[a]	− 0.3	85000[a]	+40

[a] With feedwater heater.

The author called attention to this some twelve years ago.[1] As far as is known, however, no tests have been made of locomotives of greatly varying boiler ratios and types of fuels to establish a satisfactory formula which may be used universally and permit evaluation of the effect that varying proportions have upon the economics of boiler performance and costs.

Boiler Tests

In its 1936 report, the A.A.R. Committee on Locomotive Construction expressed the opinion that the Cole ratios were inadequate and recommended approval of plant tests to obtain neces-

[2] "Locomotive Data," The Baldwin Locomotive Works, eleventh edition, Philadelphia, Pa., 1939, pp. 21–23.

[3] "Locomotive Handbook," American Locomotive Company, Schenectady, N. Y., 1917, p. 58.

[4] "Potential Horsepower Formula Agreed to by American Locomotive Builders," Report of Federal Co-Ordinator of Transportation, Washington, D. C., Nov. 27, 1935, p. 48.

[5] "Horsepower and Tractive Effort of the Steam Locomotive," by A. I. Lipetz, Trans. A.S.M.E., vol. 55, paper RR-55-2, 1933, pp. 5–42.

[6] "Method of Determining the Capacity of Steam Locomotives," by Strahl, *Zeit. V.D.I.*, vol. 57, 1913, pp. 251–257, 326–332, 379–386, and 421–424.

[7] "A Study of the Locomotive Boiler," by Lawford H. Fry, Simmons-Boardman Publishing Corporation, New York, N. Y., 1924.

[8] "The Design and Proportion of Locomotive Boilers and Superheaters," by C. A. Brandt, Proceedings of the Canadian Railway Club, vol. 27, Feb., 1928, pp. 20–64.

sary data. The director of equipment research at that time, L. W. Wallace, made a report and recommended complete locomotive tests, but this program has never been carried out.

Many tests have been made by the Pennsylvania and New York Central railroads and the respective managements deserve the greatest praise for their valuable contributions to the art. Most of these tests, however, have been made on locomotives with relatively small grate areas and with approximately the same ratios of grate areas, gas areas, firebox volumes, etc. There are few data available on the effects of very large grates and large fireboxes.

The tests recommended by the Research Division of the A.A.R. were complete and necessarily expensive. It is believed by the author that adequate information as to the efficiency and maximum capacity of boilers with large grates and furnaces can be obtained with stationary blowdown tests of some three or four boilers with widely different boiler ratios. Such tests should be conducted with oil fuel and also with several different grades of coal, sufficient to establish fundamental data now lacking. With complete data on the quantities, qualities, etc., of the fuel, water, and air used, and the gases and steam produced, all losses could be segregated and closely determined.

This would permit the determination of coefficients m and n in Fry's formula. It is hoped that such a test program will be made possible as this matter is not one merely of academic importance, but is a vital item of railroad economics. The method of standing blowdown tests developed and used by the New York Central Railroad in recent years has proved very effective. Results of such tests are dependable since the boiler testing is separated from and is independent of the locomotive-engine performance. This facilitates the adjustments for setting the required operating conditions for each capacity test and the continuance of each test run for a sufficiently long period of time at constant rate, without interruption by locomotive- and test-plant running conditions.

The New York Central Railroad's method of blowdown test is conducted so that the steam exhausted from the cylinders through the nozzle is desuperheated to a temperature closely agreeing with that actually observed on road tests for equal capacities. The correctness of this test procedure has been proved by the fact that the front-end design and nozzle size established by such test have proved correct for best maximum performance in road service.

Gas Area in Relation to Boiler Efficiency and Capacity

It is well known that the efficiency of the boiler decreases with an increased firing rate. The rapid drop is mainly due to the high losses occurring in the form of unburned fuel escaping with the flue gases. The problem of greater fuel-burning capacity at higher efficiency of combustion is, therefore, the first item which should be considered and involves the arrangement and relative size of the gas area through the boiler, the grate area, combustion volume of the furnace, and the firebox heating surface, also, stoker construction, arrangement of firebrick arches, amount of air opening through the grates, and the introduction of secondary air above the fire bed.

In considering the matter of locomotive-boiler design, as in the case of any other structure, it is well to establish a base from which to start. This is difficult with a locomotive because the problem of its design involves a cycle of successive approximations to obtain maximum power and efficiency within the weight limitations.

In the past either the heating surface or the grate area has been the basis on which the size of the other component parts of the boiler has been determined. This is not the most logical procedure as the dimensions of these parts do not control the limit of size to which the boiler can be built, since they may be increased with the length of the boiler within considerable limits. The diameter and gas area through the boiler constitute the limiting factors because the height, width, and weight are fixed and cannot be exceeded. It is, therefore, more logical that this factor be made the basis on which the other parts are proportioned.

The capacity of the locomotive boiler is limited by its diameter because this determines the gas area of the flues through which all the gases of combustion must pass; in addition it must provide space for the superheater through which all the steam generated must flow. It determines the flue heating surface that can be installed per unit length of flue, the area of the steam-disengaging

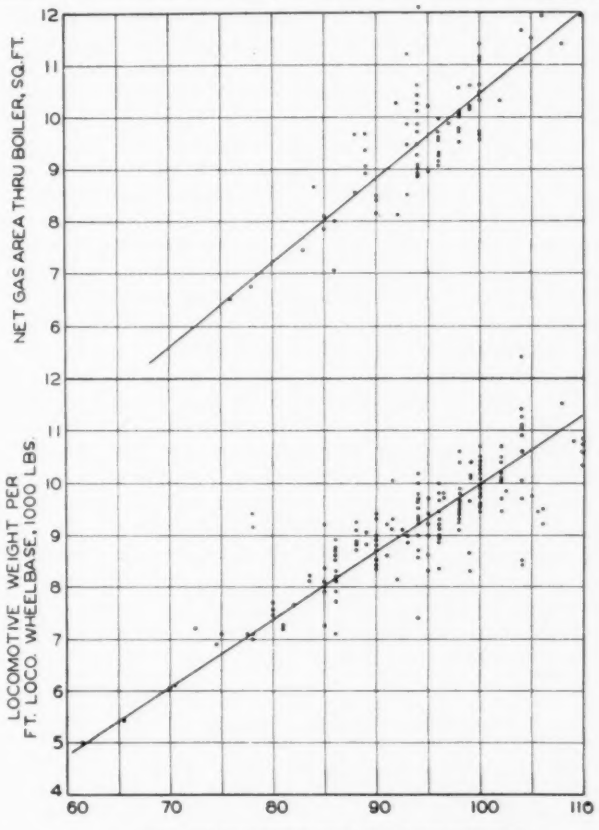


Fig. 2 Relation of Boiler Diameter to Total Weight of Locomotive per Foot of Wheel Base and Also Gas Area

surface, and the steam volume above the water level in the boiler. This last item is of the greatest importance and is, in reality, the limiting factor in high-output operation, as evidenced by the serious difficulties experienced with water carry-over into the superheater and cylinders in many boilers.

Within the clearance, the maximum diameter of the boiler is determined by the weight limits. In order to analyze this problem, the proportions of 165 different designs of modern locomotives have been studied to ascertain the relationship between the diameter of the boiler and the total weight of the locomotive. The curve, Fig. 2, shows this relationship. It is apparent that, while there are many factors which influence the weight, the boiler diameter has been sacrificed for other features of design in many cases. If, as an example, the 104-in. boiler is noted, the total locomotive weight per foot will vary from a minimum of 8400 lb to a maximum of 12,400 lb, or an increase in total locomotive weight of nearly 50 per cent for the same boiler diameter. The chart indicates that there is a uniform increase of 133 lb per ft of total locomotive weight for every 1-in. increase in the boiler diameter. This curve may facilitate studies in efforts to obtain

the greatest possible diameter and gas area for allowable weights.

A relatively large gas area through the boiler in relation to the grate area and heating surface is not generally followed, as noted from a study of the ratios Nos. 30, 31, and 32, Tables 1 to 6.

That the gas area through the boiler is one of the most important items affecting the boiler capacity is not generally appreciated, but may be better understood if actual test figures are analyzed. From the test of the Pennsylvania Railroad, class M1A, locomotive No. 6706, column 10, Table 4, on which sufficient data are available to permit determination of the weight of the gases passing through the boiler, the data shown in Fig. 3 have been compiled. It is interesting to note that, at high firing rates, the gas velocity through the tubes near the back tube sheet reaches the high figure of 204 miles per hour. The great importance of eliminating unnecessary restrictions in the gas passages and the superheater units is apparent.

As the gas area is increased, the velocity and frictional resistance of the gases passing into and through the boiler flues are lowered. This will reduce the draft requirement and in turn the cylinder back pressure, enlarging the power output of the locomotive in direct proportion to the resultant increase in the mep. This is important when high power output is desired at high speed as the reduction of 1 lb of cylinder back pressure gives as much increase in power as a 4-lb gain in pressure on the steam inlet or admission side.

The importance of a large gas area through the boiler was recognized years ago. C. D. Young[9] pointed out in 1915 that from his experience the maximum evaporative capacity of a boiler was in direct ratio to the total gas area through the flues and was apparently limited to an amount of 7000 lb actual or 9100 lb equivalent evaporation per hr per sq ft of gas area for the engines tested up to that time. This has been bettered considerably on modern locomotives with type-E superheaters and more powerful draft arrangements to about 11,000 lb equivalent evaporation per sq ft of gas area. The test results given in the following table confirm this point:

Railroad	Class engine	Engine no.	Max equiv evap per hr, lb	Total net gas area through boiler, sq ft	Max lb of steam per hr equivalent evap per sq ft gas area
P.R.R.	K4S	1737	87,414	9.10	9605
P.R.R.	M1A	6706	123,870	9.71	12755
B.L.W.	3 cyl	60000	84,186	9.37	8985
P.R.R.	I1S	790	89,235	9.9	9015
N.Y.C.	J3A	5408	89,500	8.9	10055

To illustrate the limiting effect that the gas area has upon the evaporative capacity of a locomotive boiler, the test results from several locomotives have been plotted in Fig. 4. The relation between equivalent evaporation and back pressure is illustrated in Fig. 5.

A study of the relationship between the gas area and the boiler diameter of a number of modern boilers has been plotted in Fig. 2; it will be noted that full advantage has not been taken in many cases in providing the largest possible gas area. For some boiler diameters the gas area is as much as 31½ per cent smaller than the maximum possible. This is perhaps due to a difference of opinion as to the most suitable distance between the top of the crown sheet and the inside of the boiler shell at the top; as to the water space between the combustion chamber and boiler shell, or the clearance between tube holes and tube-sheet flanges; and as to tube spacing. The type of combustion chamber, the thickness of the flues, and the superheater design also influence the gas area. Offhand, it may seem that these details are not of great consequence, but really they are very important.

The distance between the top of the crown sheet and the boiler shell determines the steam space and steam disengaging surface. It is obvious that the smaller this distance is made the greater will

[9] Test Bulletin, No. 28, Pennsylvania Railroad, 1915, p. 35.

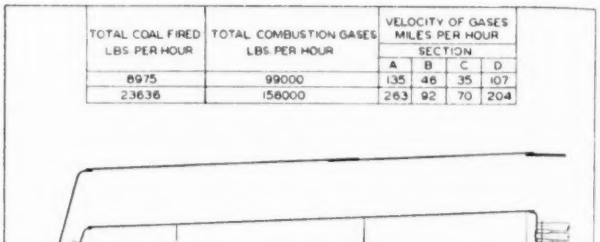


FIG. 3 GAS VELOCITIES THROUGH LOCOMOTIVE FIREBOX AND FLUES

be the tube-sheet area. Information as to the most satisfactory relationship between the height of the crown sheet and other dimensions of the boiler is lacking, but this question deserves most careful consideration.

A slight increase in the water space between the combustion chamber and boiler shell decreases the available gas area considerably. On some boilers, this water space has been made as large as 8 in., while very many large boilers are operating successfully with a water space of only 5 in., giving a large percentage increase in gas area. It appears that the smaller water space imparts a higher velocity to the water circulation at this point, minimizing mud collection which the large water space was supposed to eliminate.

The important influence that the superheater construction has upon the gas area is apparent and will be discussed in more detail under the subject of superheater design.

FIREBOX VOLUME IN RELATION TO COMBUSTION EFFICIENCY

Earlier in this paper, it was pointed out that the firebox volume is not subject to such limitations as the gas area. There are limitations to the width and height of the firebox which are important, but the length of the grate and firebox may be extended considerably beyond present general practice. This is fortunate as the most serious matter confronting locomotive designers today is the problem of improving the efficiency of combustion in the firebox, particularly at high-capacity operation. Incomplete combustion of fuel causes heavy losses in unburned fuel, high maintenance cost due to cinder cutting of firebox sheets, stay-bolt heads, tubes, and superheater. Damage due to fires along the right of way set by sparks, loss in good will to the general public due to smoke near railroad terminals, and loss of passenger traffic for the same cause constitute a serious challenge to designers and operators of steam locomotives.

Improvement in the combustion process of the locomotive furnace is a problem of primary importance and the proportions of the furnace should be such as not only to make possible complete combustion of the fuel with the elimination of smoke and cinders, but also to absorb sufficient radiant heat to cool the furnace gases to a temperature low enough to prevent slagging at the flue sheet and on the superheater units.

One would assume that sufficient data were available today to determine the correct volume of the firebox in relation to the grate area and heating surface. A study of Tables 1 to 6 indicates, however, that there is little agreement on this subject; great variations exist in these basic ratios. To cite an example: The important ratio of firebox volume to grate area ranges from a minimum of 3.70 to a maximum of 8.28 cu ft per sq ft of grate

EQUIVALENT EVAPORATION PER HOUR, 1000 LBS.

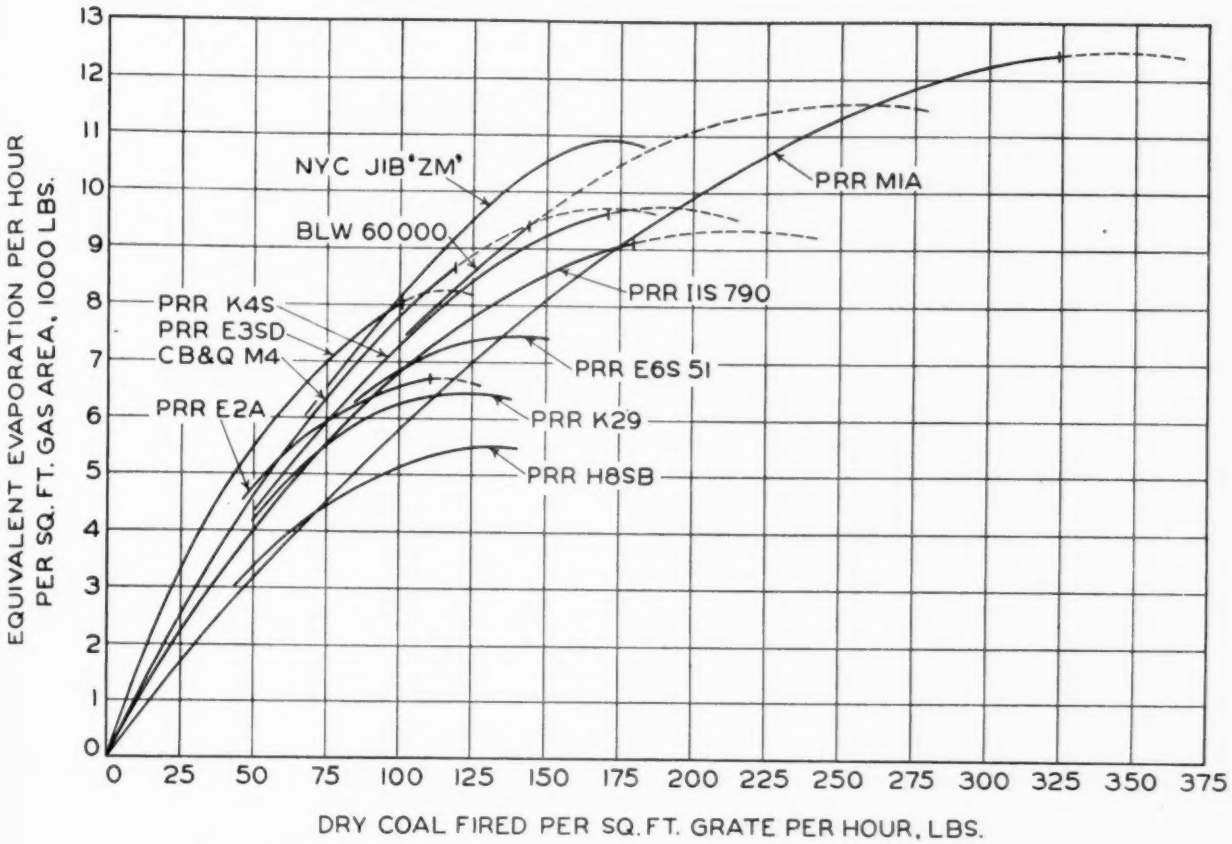


FIG. 4 EQUIVALENT EVAPORATION PER SQUARE FOOT OF GAS AREA VERSUS COAL PER SQUARE FOOT OF GRATE

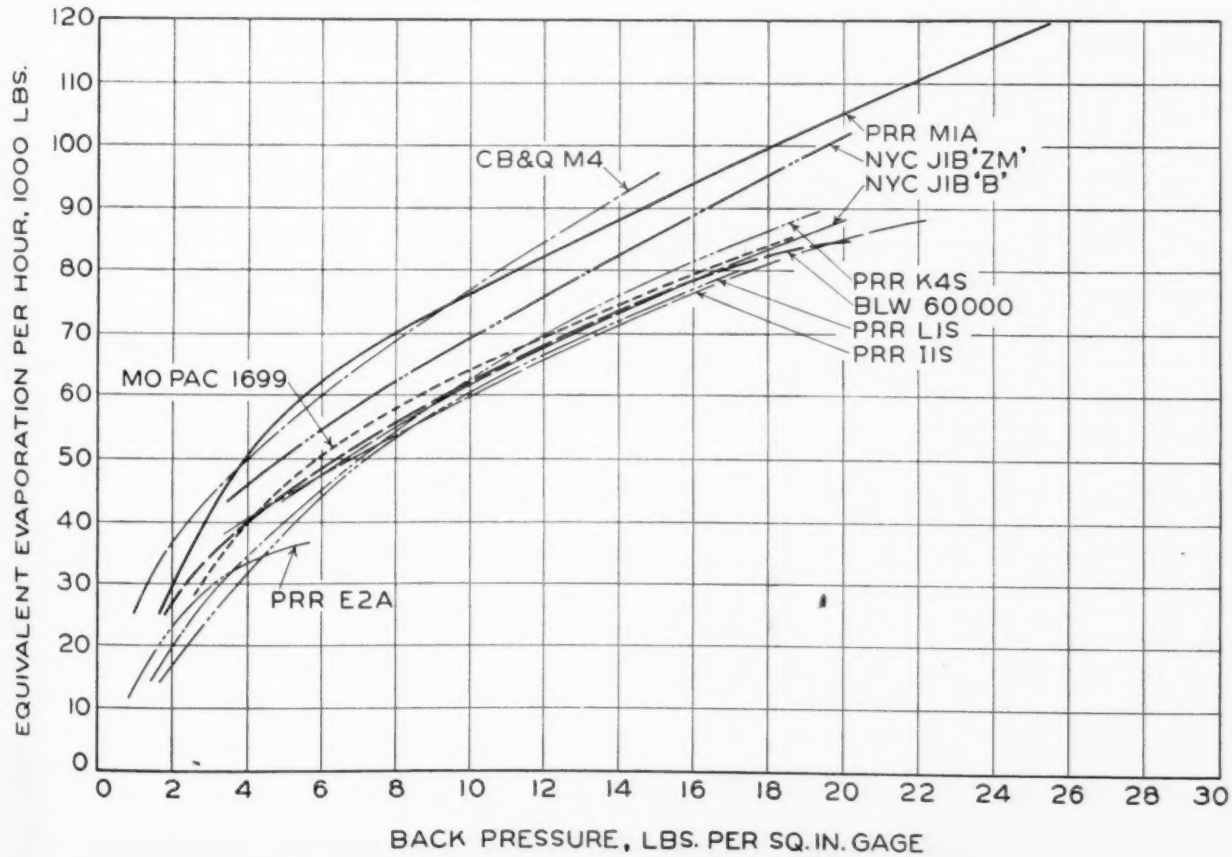


FIG. 5 EQUIVALENT EVAPORATION VERSUS BACK PRESSURE

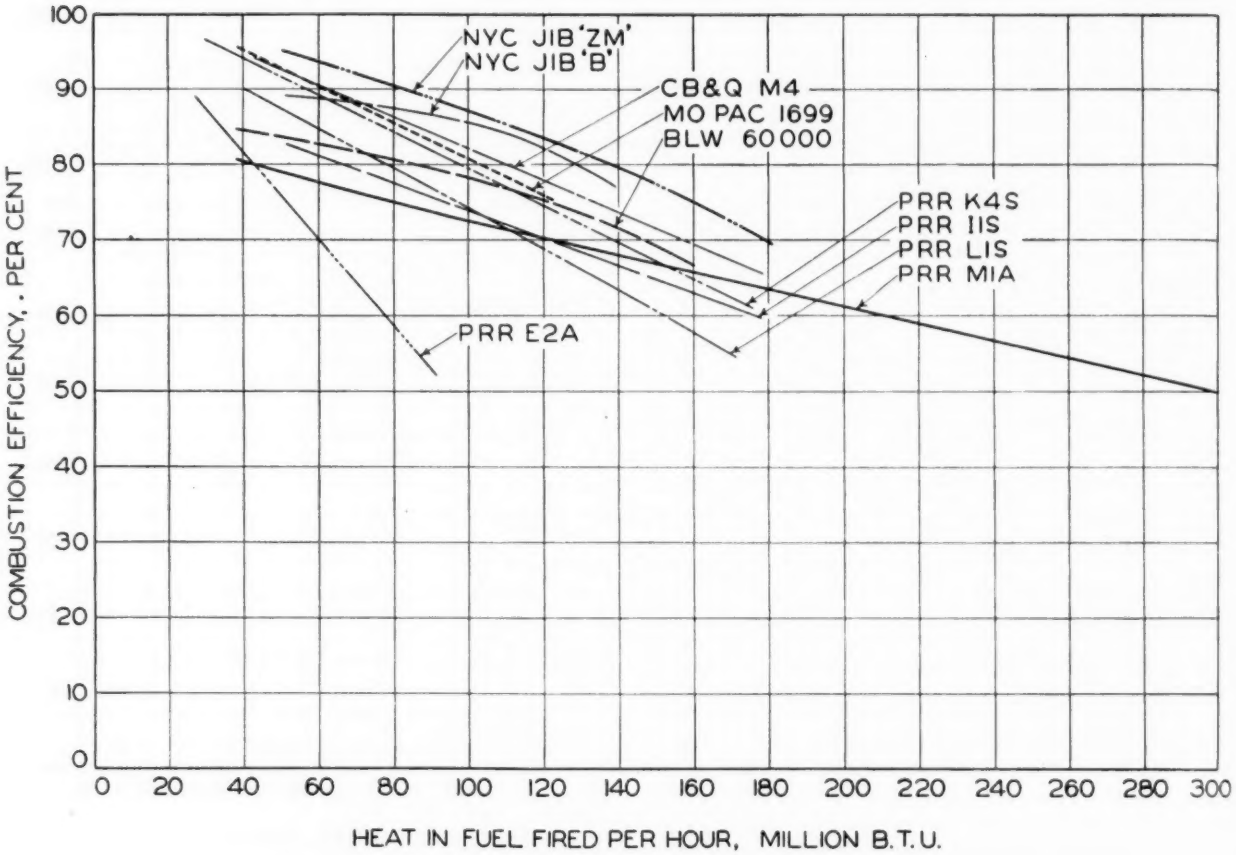


FIG. 6 COMBUSTION EFFICIENCY VERSUS HEAT IN FUEL FIRED PER HOUR

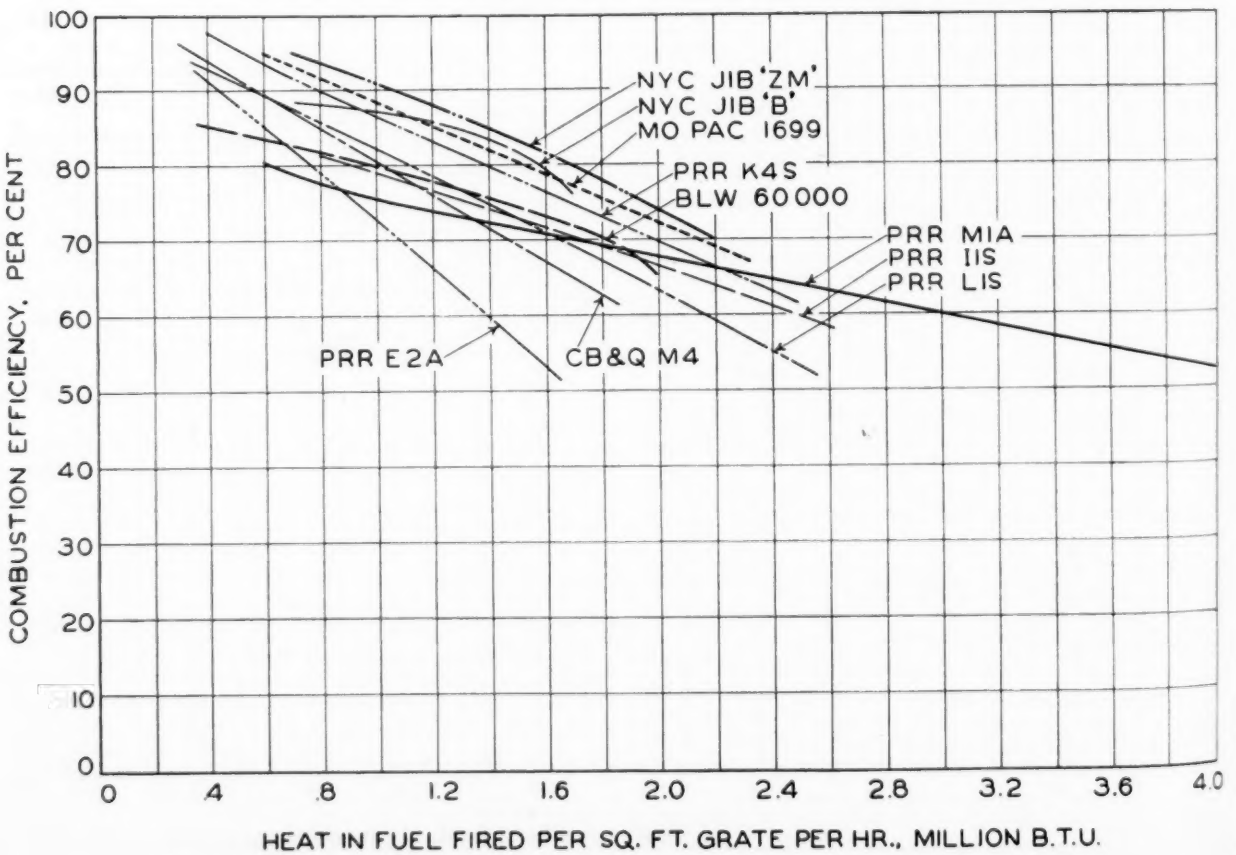


FIG. 7 COMBUSTION EFFICIENCY VERSUS HEAT IN FUEL FIRED PER SQUARE FOOT OF GRATE PER HOUR

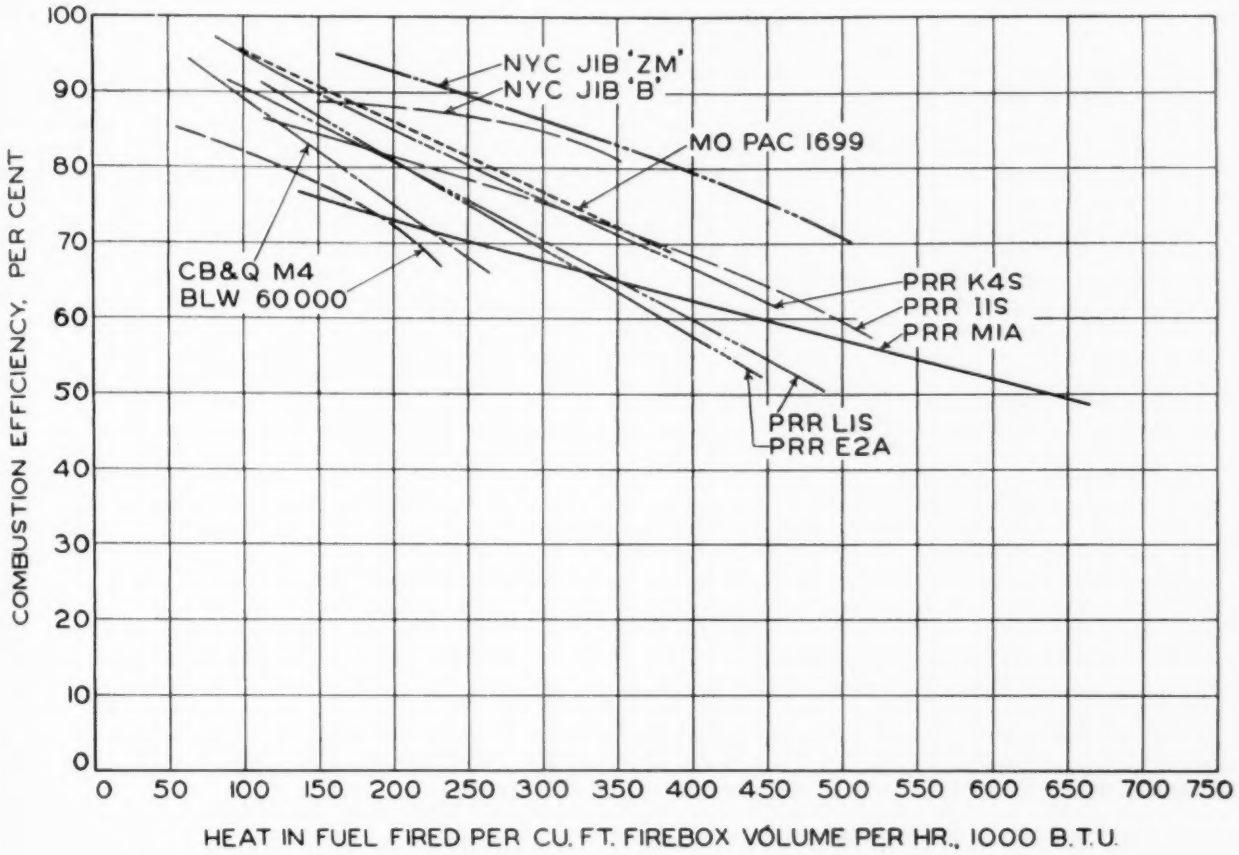


Fig. 8 Combustion Efficiency Versus Heat in Fuel Fired per Cubic Foot of Firebox Volume per Hour

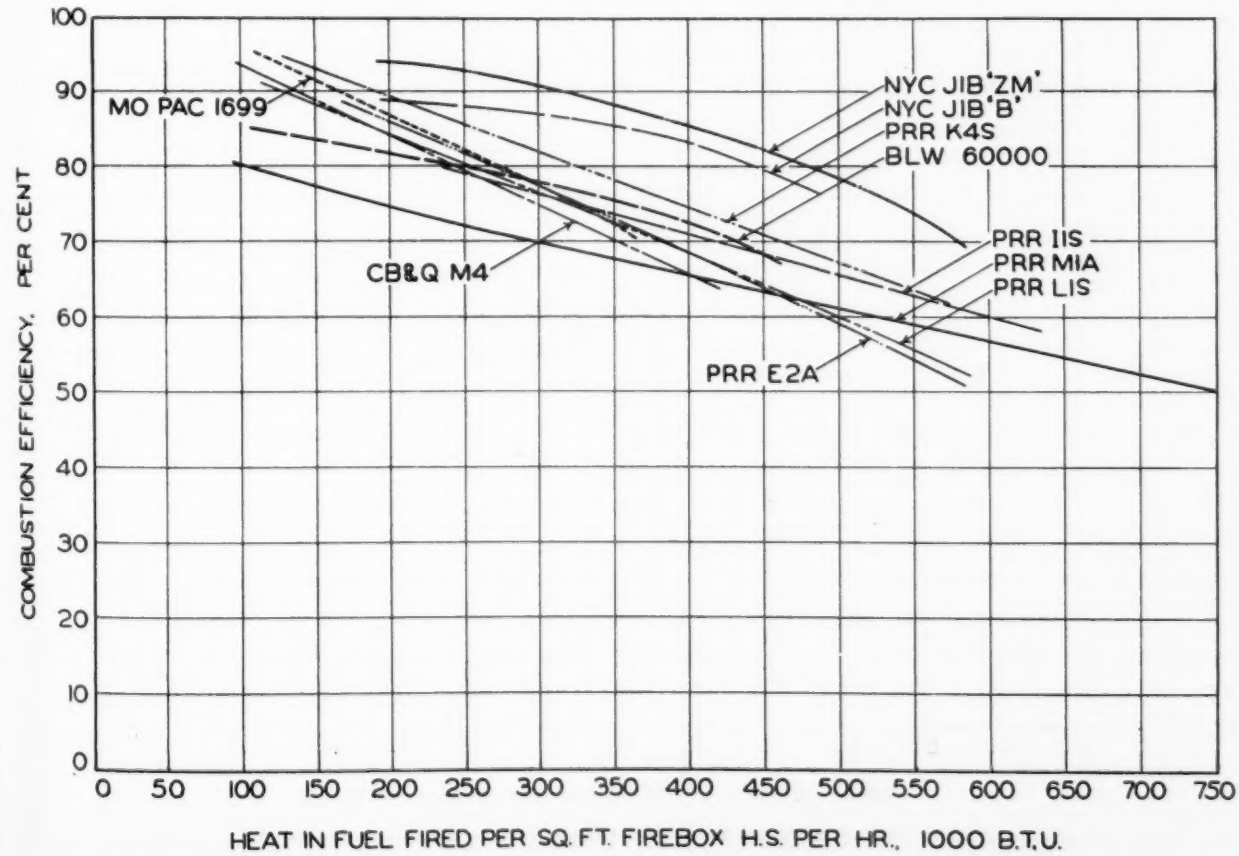


Fig. 9 Combustion Efficiency Versus Heat in Fuel Fired per Square Foot of Firebox Heating Surface per Hour

area. Compared on the basis of tractive power the furnace volume varies from a minimum of 4.04 to a maximum of 11.67 cu ft per 1000 lb of tractive power.

The difference in opinion among locomotive designers as to the correct size of firebox and combustion chamber is due to the absence of pertinent facts and test data to permit a scientific evaluation of the various factors. It is known that the first cost of the heating surfaces in the firebox and combustion chamber is very high as is also the cost of the maintenance and inspection when compared with the tube heating surface. What the relative cost of firebox and tube maintenance amounts to is unknown. Little information is available as to the cost of boiler maintenance, since few American railroads keep accounts to permit such a study. In the report of the Federal Co-Ordinator of Transportation, November, 1935, the estimated average cost of boiler maintenance is given as 30 per cent of the total cost of locomotive repairs. The firebox maintenance is the greatest part of this cost. As a large modern boiler will have from 5000 to 8000 stay bolts in the firebox which must be tested at regular intervals, the size of the firebox is a very important item when the question of intensive utilization of the locomotive is considered.

Against these items of cost should be balanced the great improvements in boiler efficiency and capacity which are generally believed to be obtained by the use of large fireboxes. The heat absorption of the firebox heating surfaces is from six to ten times as great as the average tube heating surface. Another important point not usually considered is that the radiant-heat absorption by the firebox is accomplished without the expenditure of any energy, while the heat absorption by convection in the tubes consumes considerable power to create the high draft required. The energy required to move the gases increases with the fourth power of the weight velocity. Only a few tests have been made which supply data permitting the calculation of combustion efficiency. These tests have been mainly on engines, the boiler ratios of which are very much alike, and thus the effect of greatly different boiler ratios is not clearly discernible from available data.

To illustrate the effect which different boiler ratios have upon combustion efficiency, the data from all tests available have been plotted in Figs. 6 to 9. The combustion efficiency is plotted against various factors as follows:

Fig. 6.... Heat in fuel fired per hr
Fig. 7.... Heat in fuel fired per hr per sq ft of grate area
Fig. 8.... Heat in fuel fired per cu ft firebox volume per hr
Fig. 9.... Heat in fuel fired per sq ft firebox heating surface per hr

It is, however, becoming more generally recognized that the larger the furnace volume of a locomotive, the greater is the opportunity for completing the processes of combustion. This is true only if the larger furnace volume is obtained in conjunction with maximum boiler diameter, furnace width and depth, and not by furnace length alone.

An examination of a typical boiler diagram, Fig. 3, shows the gas velocities at several points in the firebox, an explanation being given why the largest possible cross-sectional area through the firebox is very important. The velocity of the furnace gases in this case between the top of the arch and the crown sheet is as high as 260 miles per hour. It is remarkable that the small particles of coal being blown by the stoker jets into the center of a hurricane of such velocity are able to settle on the grate or have time to burn at all.

A particle of fine coal caught in the gas stream will pass through

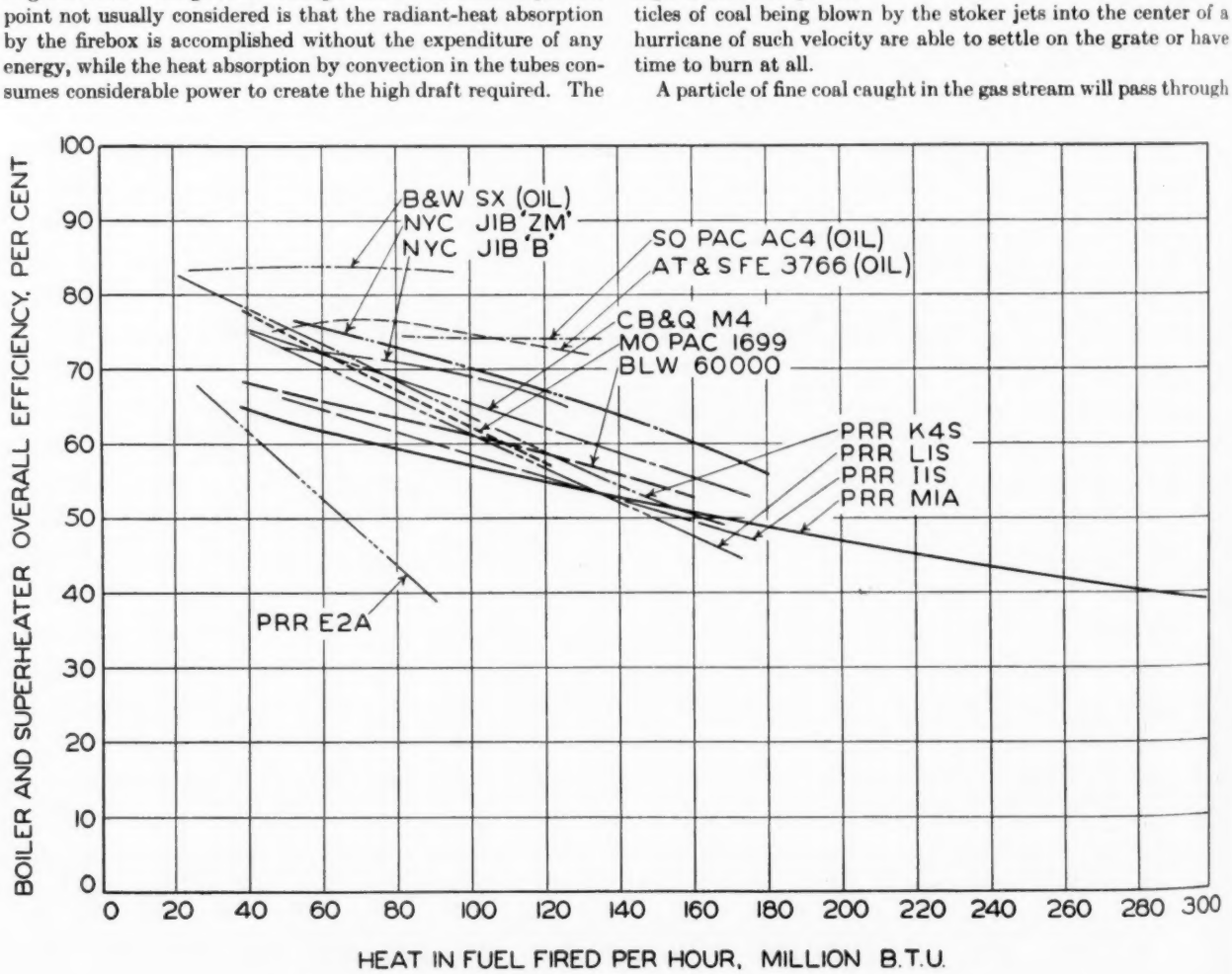


Fig. 10 Over-All Boiler Efficiency Versus Heat in Fuel Fired per Hour

this furnace in 0.1 sec and this is one of the reasons for the great loss of carbon at high-capacity operation. As a comparison, in stationary practice the velocity of the gases just before entering the tube bank is about 10 miles per hour at a maximum. A study of the designs of many recent boilers indicates that the width of the firebox, as well as the depth, could have been increased to give the advantage of a larger combustion volume under the firebrick arch, as well as a larger gas area between the top of the arch and the crown sheet.

A few modern boilers have been built with a firebox ring as low as 53 in. from the rail which with a horizontal grate would give a good depth of the firebox at the rear. On many locomotives with the grate sloping up toward the back end, the distance from the rail to the firebox ring is 72 in., or more, and thus there is a loss of 18 in. in the depth of the firebox. The author has not been able to get a satisfactory reason why sloping grates are still continued, especially since so many boilers have been built with horizontal grates and have given satisfactory service.

While discussing the subject of furnace volume, attention is called to the boiler-efficiency curves plotted in Fig. 10. Three efficiency curves from oil-fired boilers are shown. Two are locomotive boilers and one is a water-tube boiler, tested by the United States Navy.[10]

The following table gives certain pertinent data on maximum-capacity operation:

User	Santa Fe	Southern Pacific	U. S. Navy
Type	4-8-4	AC4	Marine
Engine number	3766	4108	Type 8X
Furnace volume, cu ft	795	970	423.6
Gas area through boiler at furnace, sq ft	10.3	11.98	30.06
Maximum oil burned, lb per hr	7000	6800	5051
Maximum oil burned per cu ft furnace volume, lb per hr	8.8	7.0	11.9
Maximum oil burned per sq ft gas area	680	558	168
Maximum coal equivalent, lb per hr	10800	10500	7850

It is interesting to note that it was possible to burn 35 per cent more oil per cu ft of furnace volume with 8 per cent higher boiler efficiency in the Navy boiler than was attainable in the locomotive boiler. The gas area in the Navy boiler at the gas exit of the furnace, however, was three times greater than on the locomotive boiler, which again confirms the importance of gas area.

Absorption Efficiency of the Heating Surfaces

The absorption efficiency of the surfaces should be expressed as the total heat absorbed in per cent of the total heat available for absorption; that is, of the total heat drop from the furnace temperature to the metal temperature of the flues and superheater units at the smokebox. This is termed the true efficiency and is very high, about 90 per cent in a locomotive boiler, due to high gas velocity and cooling of the gases to within 75 F of the temperature of the water at low rating, and 200 F at high capacity.

On locomotives with type E superheaters the smokebox gases are from 100 to 150 deg lower than the steam temperature. The losses due to the sensible heat escaping in smokebox gases range from 19 to 25 per cent of the heat in the coal fired, at high rating, and the only hope of reducing these is by a radical change in the boiler design whereby the average temperature head could be increased by the use of a counterflow economizer. In this manner the smokebox gases could be lowered some 210 F which would give a 7 per cent increase in the over-all boiler efficiency.

The importance of an effective arrangement of the flue heating surfaces is most apparent at high capacity when 70 per cent of the liberated heat is delivered to the flues. The ratio of the net gas area of a flue to its gas-swept perimeter (sometimes called hydraulic depth) determines the gas-carrying capacity of the flues. The larger this ratio becomes the more gas will flow for equal pressure drop. This has been labeled the "flue capacity factor," or F.C.F.

The longer a flue of a given F.C.F. is made, the greater becomes its true efficiency and also its draft loss. Now if the length of the flue in inches is divided by the F.C.F. a value is obtained which expresses the true efficiency of any diameter and length of flue. This factor has been termed the "flue efficiency factor" or F.E.F. The larger it becomes the more efficient a flue is.

In Fig. 11 has been plotted a curve showing the true efficiency that will be obtained on any size and length of flue with a given flue efficiency factor, or F.E.F.; that is, the ratio of length to hydraulic depth.

In Fig. 11 is also shown the true efficiency of several sizes of flues for different lengths. The F.E.F. for a 2-in. inside-diameter flue 18 ft long is 432 and gives a true efficiency of 88 per cent. If this flue is made 25 ft long the F.E.F. becomes 600 and the true efficiency is increased to 94.5 per cent, but the draft loss is increased in the same ratio as the length. In America many boilers have been built with flues 22 ft to 25 ft long, but usually efforts are made to keep the flue lengths down by lengthening the combustion chamber where this can be done. It is evident from Fig. 11 that a larger-diameter flue of a greater length can be used to give the same true efficiency as a smaller flue of shorter length.

The heat-absorption efficiency of locomotive boilers increases with an increase in the flue efficiency factor of the tubes and flues and ranges usually between 78 and 88 per cent. The absorption efficiency of several boilers has been shown in Fig. 12; it is noted that there is a very small decrease in this efficiency as the rate of firing is increased.

The combined absorption and combustion efficiency gives the over-all efficiency of the boiler and superheater and these data are shown in Fig. 10 for the engines previously cited. The general proportions and boiler ratios of all the engines shown in this and previous curves are given in Tables 1 to 6, as follows:

Pennsylvania R.R.	E2A	Table 5	Column 8
Pennsylvania R.R.	K4S	Table 5	Column 6
Pennsylvania R.R.	L1S	Table 5	Column 7
Pennsylvania R.R.	I1S	Table 2	Column 1
Pennsylvania R.R.	M1A	Table 4	Column 10
Chicago, Bur. & Quincy	M4	Table 2	Column 6
Missouri Pacific R.R.	1699	Table 5	Column 9
Baldwin Loco. Works	60000	Table 2	Column 10
New York Central	J1B	Table 1	Column 3
Atchison, Topeka & Santa Fe	3766	Table 5	Column 4
Southern Pacific	AC7	Table 6	Column 10

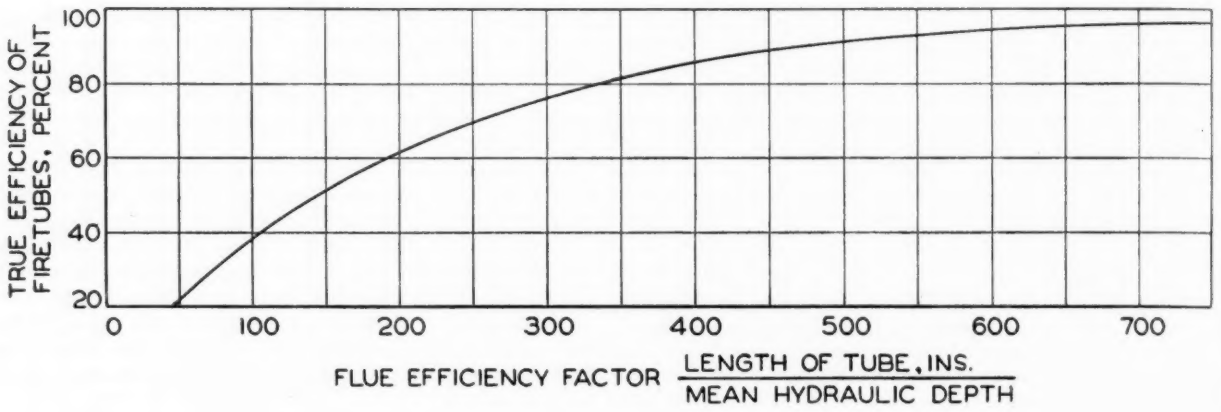
Heat balances showing the heat absorbed by the boiler and superheater together with the various heat losses for two modern locomotives are shown in Figs. 13 and 14.

Before leaving the subject of heat absorption, a few words may be said as to the heat-absorbing capacity of the firebox. Cole proposed 55 lb of steam per sq ft as the maximum based on Professor Goss's test at Coatesville, the only test made on a locomotive where the firebox evaporation was determined separately. This value of evaporation rate has continued to be used up to now even though the size of the locomotive firebox has increased five to six times since the Coatesville tests. It is reasonable to suppose that the same evaporation rates cannot very well apply to a firebox with 800 sq ft of heating surface having siphons, circulators, etc., as to a firebox with only 150 sq ft.

Since the adoption of waterwalls in stationary boilers, the heat absorption due to radiant heat has received much attention and analysis.

The general behavior of radiant energy as expressed and defined in Stefan-Boltzmann's law, Prevost's law, Kirchhoff's

[10] "The B&W High-Pressure Sectional Express Boiler," by R. C. Brierly, *Journal of the American Society of Naval Engineers*, vol. 43, Nov., 1931, pp. 511–561.



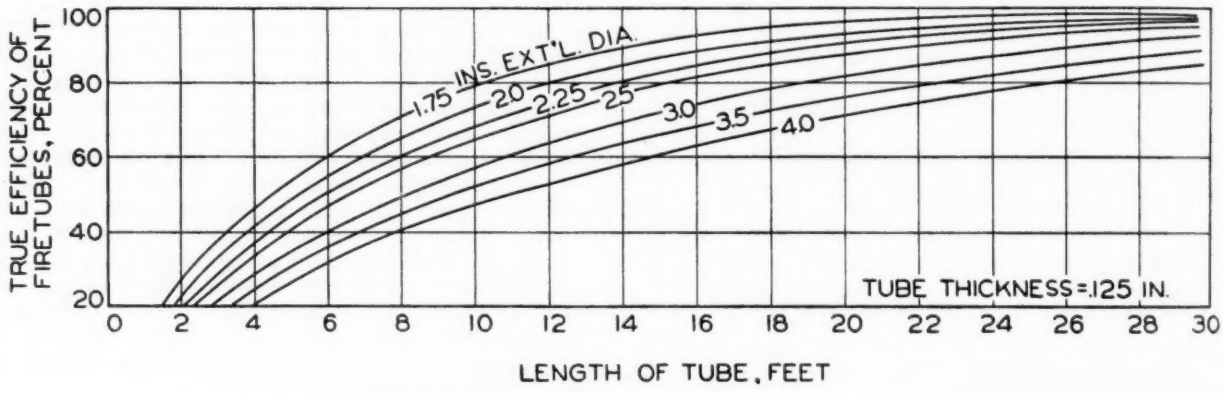


FIG. 11 TRUE ABSORPTION EFFICIENCY OF FLUES

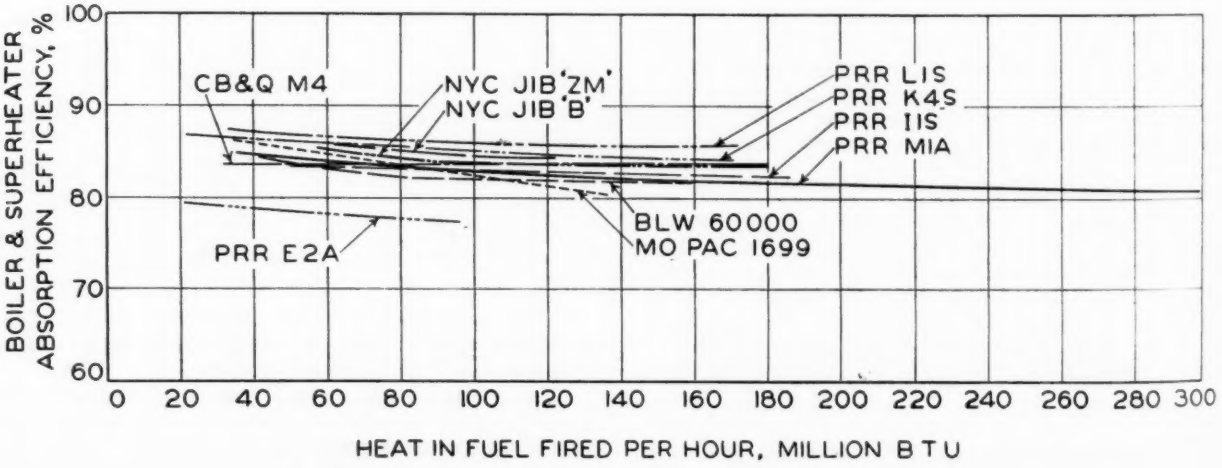


FIG. 12 ABSORPTION EFFICIENCY VERSUS HEAT IN FUEL FIRED PER HOUR

law, Planck's formula, and studies by many other investigators, permits definition and determination of the coefficients of emission, absorption, reflection, and transmission of radiant heat. To apply these laws to locomotive fireboxes, additional test data will be required.

The proportion of the total heat liberated in the furnace, which is absorbed by the firebox heating surfaces has been termed "furnace absorption efficiency," and this determines the temperature of the combustion gases as they leave the firebox. An accurate knowledge of this temperature is important as it determines the design and performance of the superheater.

In tests where complete gas analyses have been made to determine the quantities of air used for combustion, it is possible to calculate the per cent of fuel fired that is actually burned and the total heat liberated in the furnace. A fraction of the liberated heat is absorbed by the firebox and the balance leaves as sensible heat in the combustion gases which, with the total known weight of gases, determines the mean temperature of these gases leaving the furnace. The furnace absorption efficiency can thus be stated in terms of the average temperature of the gases leaving the furnace.

Thus, if

t_1 = temperature of combustion gases leaving firebox, F
C_P = mean specific heat of combustion gases between t and 60 F
H_T = total heat liberated in firebox per lb of combustion gases
H_g = heat in 1 lb of combustion gas leaving furnace
H_F = heat absorbed in firebox in per cent of total liberated

then $$H_g = C_P(t_1 - 60)$$

and the per cent of total heat liberated in the firebox that leaves the furnace in the gas is equal to

$$C_P \frac{(t_1 - 60)}{H_T} \times 100$$

The furnace absorption efficiency, therefore, is

$$H_F = \left[1 - \frac{C_P(t_1 - 60)}{H_T}\right] \times 100$$

The relation between the percentage of heat absorbed by the firebox and the temperature of gases leaving, depends upon the heat liberated per lb of combustion gas, which again depends upon the excess air used for combustion. If, therefore, an accurate measure of the gas temperature leaving the firebox is obtained with a reliable pyrometer the furnace absorption efficiency can be determined.

It has been demonstrated that the radiation from the flames is largely due to incandescent particles of fuel and ash in the flame. The quantity and distribution of this radiating material in the flames, together with the amount of firebox heating surface, determine the coefficient of emission or absorption. This explains why boilers fired with oil give higher superheat; the less luminous flames of the oil fire will give up a smaller percentage of heat to the firebox by radiation, and the gases entering the flues are therefore hotter.

Several formulas have been proposed for the determination of the furnace absorption efficiency. The author has checked a number of the formulas proposed by Hudson, Broido, Wohlenberg, Orrok, Munzinger, Bottomly, and Adloff, but sufficient test data are not available to confirm their methods closely enough for locomotive-firebox conditions. All formulas, however, show that the fraction of the total heat liberated, which is absorbed as radiant heat by the furnace walls, decreases as the total heat liberation is increased. It will range from about 50 to 55 per cent at low rating down to perhaps 25 to 30 per cent at high-capacity operation.

At low rating the heat absorption per square foot of firebox heating surface will be near the figure commonly used, or 55 lb of water evaporated per sq ft but at high capacity the heat absorption will approach a figure of 125 lb of water per sq ft of heating surface. From the studies made of this problem, it is doubtful if the percentage of heat absorption can be increased to much more than 50 to 55 per cent of the heat liberated, by increasing the amount of firebox heating surface, because the amount of radiant energy absorbed is limited by the effective surfaces of the fire bed and the luminosity of the flames.

Grate Area in Relation to Efficiency and Capacity

The size of the grate area should be in proportion to the maximum output expected of the boiler and the quality of the coal to be burned. With the exception of the Dakota and Wyoming lig-

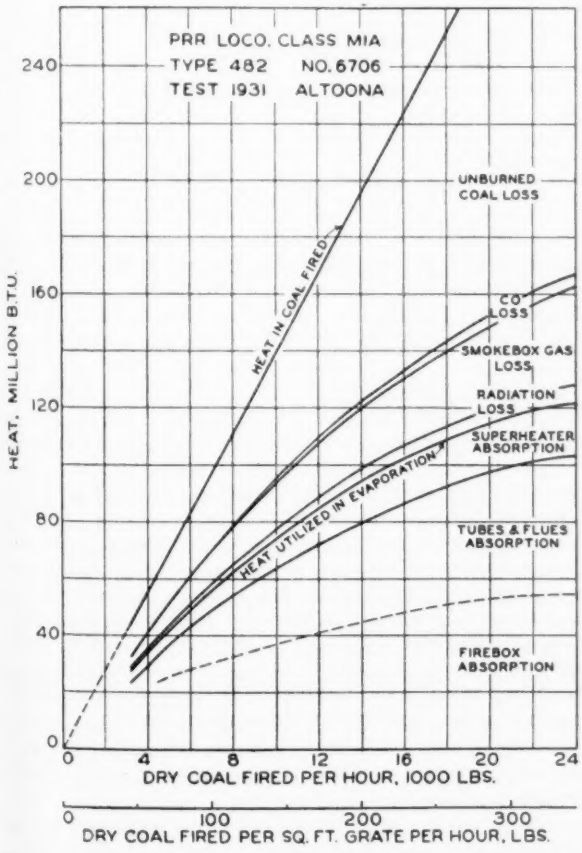


Fig. 13 Heat Balance Versus Dry Coal Fired per Square Foot of Grate per Hour; P.R.R., Class M1A

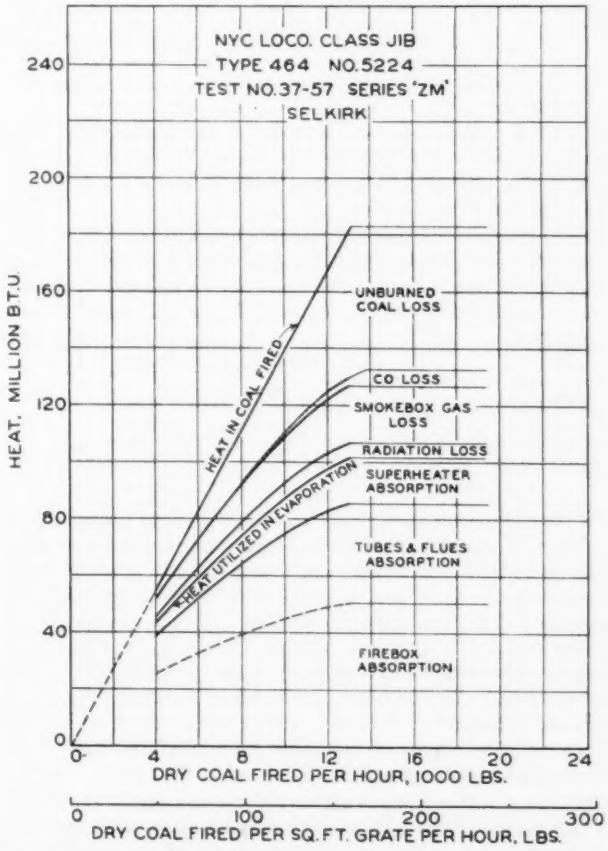


Fig. 14 Heat Balance Versus Dry Coal Fired per Square Foot of Grate per Hour; N.Y.C., Class J1B

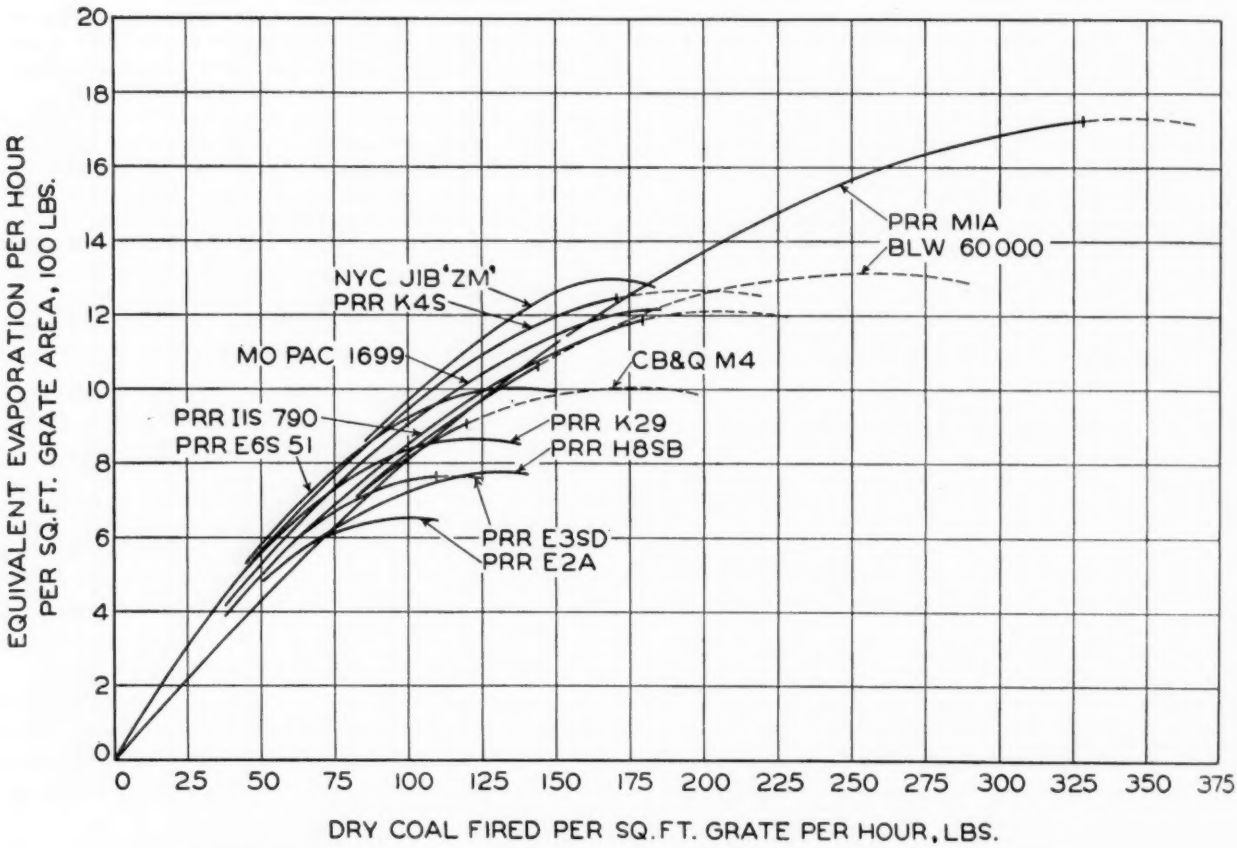


FIG. 15 EQUIVALENT EVAPORATION VERSUS COAL FIRED PER SQUARE FOOT OF GRATE PER HOUR

nites, the average maximum variation in the heating value of the coals mined east of the Mississippi is from 12,000 to 14,000 Btu per lb, and west of the Mississippi from 10,000 to 12,000 Btu per lb, or about 17 per cent for each class of coal.

If the grate-area ratios of modern locomotives, however, are studied it will be noted that they vary to a much greater extent than the heating value of the fuel, as shown in the following table:

	Ratio		Range of difference, per cent
	Minimum	Maximum	
Grate area per 1000 lb tractive effort	0.78	2.32	1:2.97
Grate area per sq ft total heating surface	0.0119	0.0262	1:2.20
Grate area per sq ft firebox heating surface	0.101	0.353	1:3.50
Grate area per sq ft net gas area	7.1	13.8	1:1.94

It would appear that much thought and study must be given to this subject as the proper size of grate area is important in obtaining high furnace efficiency and capacity. If too small a grate is selected, the firing rate will be too high at maximum-capacity operation and, if too large, it is difficult to keep the grate covered at low capacity, which results in bare spots, a high per cent of excess air, reduction of boiler efficiency, and superheat. The service condition and the profile of the road must be carefully analyzed for best results.

The question of the percentage of air openings through the grates has been explored during the last few years. Recent tests on one railroad, where grates with air openings of 10, 24, and 32 per cent were tried, indicated a very small variation in boiler efficiency between the various percentages of air openings, or less than $1^1/_2$ per cent above and below the mean percentage.

The relation between equivalent evaporation per square foot of grate area and coal fired per square foot of grate area per hour has been plotted in Fig. 15.

SECONDARY AIR FOR COMBUSTION

The length of the firebox and the combustion chamber is of importance, but there is a question whether the function of the latter which gives it its name is operative, as there is no air admitted ahead of the arch on most locomotives to permit the carbon to burn completely in its passage through the combustion chamber.

Today, most modern locomotives in America are fired with a stoker which distributes the crushed coal with steam jets through the boiler backhead. The coal is thus thrown into the firebox through and against the gas stream at the point of its highest velocity. The heaviest coal lumps fall on the grate, but many of the fine coal particles are caught in the high-velocity gas stream and partially burned in suspension while traveling at high speed. During the short time the coal particles pass through the furnace, they must be heated to ignition temperature, the moisture driven off, and the volatile gas distilled and burned before the solid carbon is consumed. It is quite possible that most of the carbon now escaping could be completely burned in the large fireboxes if sufficient oxygen were admitted into the firebox and combustion chamber as secondary air. The beneficial effect of admitting secondary air above the fuel bed has been known for many years and recognized by authorities on combustion as very desirable, yet little has been done along this line.

This subject was investigated by Henry Kreisinger[11] as early

[11] "Combustion of Coal and Design of Furnaces," by Henry Kreisinger, C. E. Augustine, and F. K. Ovitz, Bulletin No. 135, United States Bureau of Mines, Washington, D. C., 1917.

as 1917. In these tests it was proved that, with a firing rate up to 185 lb per sq ft of grate and with the thickness of fuel bed varying from 6 to 12 in., all the oxygen in the air which entered through the grate and fuel bed was consumed at the level of approximately 4 in. above the grate.

A sufficient quantity of secondary air admitted above the fuel bed in such a manner as to create a turbulence in the gases would permit of an intimate mixing of the hydrocarbons thrown off by the coal, and provide the necessary oxygen for the complete combustion of volatile gases and carbon. Another very important result from the use of a correct amount of secondary air is that it will help to eliminate smoke which always has been a serious nuisance in locomotive-boiler operation.

Kreisinger states in his report,[11] "the ratio between the weight of air supplied and the weight of fuel gasified remains practically constant at about seven to one so that only about one half of the fifteen pounds of air required to burn the fuel should be supplied through the fuel bed. In order to complete the combustion of all the fuel fed into the furnace, the balance of the air should be admitted above the fuel bed to consume the volatile, as well as fixed carbon present in this zone, or it will escape unburned."

Any scheme to admit secondary air above the fire should be arranged so that the air is controlled in accordance with the demand, in order to keep the excess air at a minimum and not decrease the fuel-burning capacity of the furnace or decrease the over-all boiler efficiency by too much excess air. Several schemes have been prepared to admit secondary air above the fire. The simplest is that of applying air-inlet thimbles in the sides of the firebox and the effectiveness of this should be fully explored. Some studies have been made of the introduction of preheated air by means of blowers. Actual tests have been conducted on one railroad by using blowers without air preheating, but so far no definite advance has been made. It is possible that the cost of such a scheme may be justified from the standpoint of smoke elimination alone.

Front-End Design and Draft

Because of the very high draft required in a locomotive boiler, the proportion of the draft-making equipment in the smokebox has received much attention in recent years and a great many tests have been conducted to discover the most efficient front-end design.

It is well known that most of the large railroads in the United States have conducted many front-end tests in recent years. The best arrangements arrived at differ considerably in design. The important result, however, is that the efficiencies of the draft arrangements have been increased by handling greater quantities of gases with lower back pressure, and that it has been possible to increase the diameter of the stack to a considerable extent without causing the trailing of smoke, which is undesirable from the standpoint of signal observation.

Curves are given in Fig. 5, which show the equivalent evaporation plotted against back pressure. This curve, test *ZM*, shows the improvements gained by the use of an improved front end developed for the New York Central class J1B locomotives as compared with the standard front end.

Superheater

Of all the improvements made on the locomotive, since George Stephenson invented the first one, the superheater holds the first rank. Its great success has been due to the fact that it attacks the heat cycle at a point where the losses are the greatest, i.e., in the cylinders, by the elimination of cylinder condensation during admission and expansion and re-evaporation at exhaust.

Of the many tests made that have proved the beneficial effects of high superheat, those conducted by the Pennsylvania Railroad have been the most complete and conclusive. In his report[12] on the performance of the superheated passenger locomotive E6s, C. D. Young presented a most thorough analysis of the beneficial effects obtained by the use of superheated steam. The following summary is quoted therefrom:

"The application of the superheater to this locomotive increases its economy from a minimum of 23 per cent to a maximum of 46 per cent, the economy increasing with the increased power required of the locomotive.

"It was found that 30 per cent higher capacity was derived from the E6s locomotive when using superheated steam than with the same size and type of locomotive using saturated steam."

This result was obtained with a comparatively low superheat, of 200 to 225 F, as compared with present-day practice. This showed an average increase in economy of 1.15 to 2 per cent for each 10 F increase in superheat. Experience and tests have shown that this rate of increase in economy holds good up to the limits of present acceptable practice of 350 to 400 F superheat. That every effort must be made to maintain highest superheat temperature is apparent.

This problem has become more and more difficult as the available heat for superheating has been reduced because of the high heat absorption in the large fireboxes of modern locomotives. To compensate for this, much larger superheaters have been found necessary and the introduction of the type E superheater was a natural development. This type has been used on most modern locomotives which have been built during the last fifteen years in America.

Because of the great influence that the superheater design has upon the efficiency and capacity of both the cylinders and the boilers, considerably more space could be devoted to the discussion of superheater design than is here available.

A great deal of research work and testing have been done with many different types of superheater units to discover the unit which will give the maximum superheat with the lowest possible draft loss and steam pressure drop, which is at the same time practical and keeps weight, first cost, and maintenance at a minimum.

The improved type E superheater is a very excellent arrangement as it gives a combination of maximum evaporating and superheating surfaces coupled with the greatest possible gas area through the boiler and steam area through the superheater. This assures high sustained superheat at low as well as high rating with minimum draft and pressure loss.

Feedwater Heating

A feature of great value in obtaining sustained capacity of the boiler, which may only be mentioned here, is the beneficial effect of feedwater heating. The increase in economy or power output of a locomotive ranges from 8 to 14 per cent through the recovery of the heat in the exhaust steam that otherwise would be wasted. The feedwater heater is now well recognized as necessary for economical locomotive operation.

Conclusion

In conclusion it may be said that the modern locomotive boiler is a very effective steam generator. Boilers[13] which can produce highly superheated steam at a rate of 1 lb equivalent evaporation per hr per 1.3 to 1.4 lb of total boiler weight, compare very favorably with stationary boilers of about the same capacity which weigh about four times as much, or 6 lb per lb of steam produced not including stack, breeching, brickwork, or draft fans.

[12] Test Bulletin No. 21, Nov. 4, 1912, pp. 82–89, 150–151.

[13] Class M1A, Pennsylvania Railroad; class J3, New York Central Railroad.

The weight of the locomotive boiler cited includes the boiler complete, superheater, smokebox, front end, stack, stoker, grate, and water.

While continued efforts must be made further to improve the boiler, clearly, it is the engine which deserves the most attention, as its thermal efficiency is so low that even a slight improvement is important. This may mean less simple mechanical parts than in the present reciprocating engine, but the mechanical-maintenance forces of the American railroads have proved to their great credit that very difficult mechanical problems, such as those connected with the maintenance of Diesel and electric locomotives, can be handled successfully.

This gives hope that apparatus and equipment proposed for the improvement of the steam locomotive will receive favorable consideration in the future, and that the hazards of added complications will not be considered as formidable as they once seemed.

Acknowledgment

The author wishes to express his appreciation to F. W. Hankins, assistant vice-president, chief of motive power, Pennsylvania Railroad; P. W. Kiefer, chief engineer, motive power and rolling stock, The New York Central System; Frank Russell, mechanical engineer, Southern Pacific Railroad; and E. E. Chapman, mechanical assistant to vice-president, Atchison, Topeka, and Santa Fe Railroad, for their courtesy in supplying the test data used in plotting the curves and for permission to use the data in this paper. Acknowledgment is also made for the assistance rendered by Charles Guarraia in preparation of the curves.

TABLE 1 GENERAL DIMENSIONS AND BOILER RATIOS OF SIX-COUPLED LOCOMOTIVES

	ITEM		UNIT MEASURE	RATIO	1	2	3	4	5	6	7	8	9	10
	TYPE				4-6-4	4-6-4	4-6-4	4-6-4	4-6-4	4-6-4	4-6-4	4-6-4	4-6-4	4-6-4
1	RAILROAD				1 C.P.R.	C.B&Q	N.Y.C.	N.Y.C.	CAN.NAT.	NY.NH&H	B & O	A.T.&S.F.	C.M.ST. P&P	C&N.W.
2	CLASS				H-1-C	S-4	J-1-B	STR. LINE J-3-A	K-5-A	I-5	V-2	ENG.NO. 3460	F-7	E-4
3	YEAR BUILT				1937	1930	1927	1938	1930	1937	1935	1937	1938	1938
4	BOILER PRESSURE		LBS. PER SQ.IN.		275	250	225	275	275	285	350	300	300	300
5	CYLINDER DIA. & STROKE		INCHES		22 X 30	25 X 28	25 X 28	22½X29	23 X 28	22 X 30	19 X 28	23½X29	23½X30	25X29
6	DIA. DRIVERS		INCHES		75	78	79	79	80	80	84	84	84	84
7	WEIGHT ON DRIVERS		POUNDS		186800	207730	184800	201800	188600	193000	156000	211400	216000	216000
8	TOTAL WEIGHT OF ENGINE		POUNDS		354000	391880	348000	365500	356400	365300	294000	420400	415000	412000
9	TRACTIVE POWER		POUNDS		45300	47700	42400	43440	43300	44000	34000	49300	50300	55000
10	BOILER I.D. FIRST COURSE		INCHES		78½	80 3/16	82 7/16	80 5/8	78	82 7/16	72 OD	86 5/16	82½	88 3/16
11	BOILER O.D. LARGE COURSE		INCHES		90⅛	94	87⅝	91½	85⅞	93		91 7/16	94	94
12	LENGTH OVER TUBE SHEETS		FEET & INCHES		18-3	19-0	20-6	19-0	19-1	18-0	25-0	21-0	19-0	19-0
13	COMBUSTION CHAMBER LENGTH		INCHES		27	36		40	31	42	36		44½	31
14	GRATE AREA		SQ.FT.		80.8	87.9	81.5	82.0	73.7	77.1	61.8	98.5	96.5	90.7
15	TUBE & FLUE HEAT. SURF.	WATER SIDE	SQ.FT.		3465	3878	4203	3827	3032	3335	2727	4395	3708	3472
16	FIREBOX HEATING SURFACE	WATER SIDE	SQ.FT.		326	369	281	360	345	480	612	375	458	507
17	TOTAL EVAP. HEAT SURF.	WATER SIDE	SQ.FT.		3791	4247	4484	4187	3377	3815	3339	4770	4166	3979
18	SUPERHEATER SURFACE - STEAM SIDE		SQ.FT.		1542	1830	1965	1745	1492	1042	880	2150	1695	1884
19	FIREBOX VOLUME		CU.FT.		427	494	353	452	362	448		483	503	560
20	GAS AREA THRU BOILER		SQ.FT.		8.38	9.00	9.66	8.90	7.04	8.92	4.61	9.10	9.14	8.07
21	TYPE OF SUPERHEATER				E	E	E	E	E	A	A	E	E	E
22	STEAM AREA THRU SUPERHEATER		SQ.IN.		60.6	65.6	64.2	61.8	52.2	54.7	44.8	71.2	63.6	67.7
23	MAX. EVAP. CALCULATED		LBS. PER HR.		55960	61710	57770	60500	51600	64000	55480	64780	65400	65740
24	MAX. EVAP. ON TEST		LBS. PER HR.				84800							
25	NUMBER & SIZE OF FLUES				171-3½	184-3½	182-3½	183-3½	146-3½	48-5½	27-5½	200-3½	164-3¾	196-3½
26	NUMBER & SIZE OF TUBES				58-2¼	62-2¼	19-3 1/2 37-2 1/4	59-2¼	44-2¼	199-2¼	120-2¼	46-2¼	60-2¼	8-2
27	NUMBER & SIZE OF SUPERHEATER UNITS				86-1 3/16	93-1 3/16	91-1 3/16	93-1 3/16	74-1 3/16	48-1½	27-1¾	101-1 3/16	84-1¼	102-1 3/16
28														
29														

30	GAS AREA PER 1000 LBS. TRACT. POWER	SQ.FT.	20/9	.185	.189	.228	.206	.163	.203	.136	.185	.182	.147
31	GAS AREA PER SQ.FT. OF GRATE	SQ.FT.	20/14	.104	.103	.119	.109	.096	.116	.075	.093	.095	.089
32	GAS AREA PER SQ.FT. OF TUBE AND FLUE H.S.	SQ.FT.	20/15	.00242	.00232	.00230	.00233	.00232	.00268	.00169	.00207	.00246	.00233
33	GRATE PER 1000 LBS. TRACT. POWER	SQ.FT.	14/9	1.79	1.85	1.92	1.89	1.71	1.75	1.82	2.00	1.92	1.65
34	GRATE PER SQ.FT. TOTAL EVAP. H.S.	SQ.FT.	14/17	.0214	.0208	.0181	.0196	.0219	.0202	.0185	.0207	.0232	.0228
35	FIREBOX VOLUME PER 1000 LBS. TRACT. POWER	CU.FT.	19/9	9.43	10.60	8.34	10.40	8.35	10.20		9.80	10.00	10.20
36	FIREBOX VOLUME PER SQ.FT. OF GAS AREA	CU.FT.	19/20	51.0	54.9	36.6	50.8	51.4	50.2		53.1	55.1	69.4
37	FIREBOX VOLUME PER SQ.FT. OF GRATE	CU.FT.	19/14	5.29	5.61	4.34	5.51	4.91	5.81		4.90	5.22	6.16
38	FIREBOX VOLUME PER SQ.FT. TOTAL EVAP. H.S.	CU.FT.	19/17	113.0	116.2	78.5	108.0	107.5	117.5		101.1	121.0	141.0
39	FIREBOX H.S. PER SQ.FT. OF GRATE	SQ.FT.	16/14	4.03	4.20	3.45	4.40	4.68	6.23	9.90	3.81	4.75	5.59
40	FIREBOX H.S. PER SQ.FT. TOTAL EVAP. H.S. ×100	SQ.FT.	16/17 ×100	8.60	8.70	6.25	8.60	10.2	12.60	18.35	7.85	11.0	12.75
41	TOTAL EVAP. H.S. PER 1000 LBS. TRACT. POWER	SQ.FT.	17/9	83.6	89.0	105.9	96.3	77.8	86.6	98.1	96.8	82.9	72.4
42	TOTAL EVAP. H.S. PER SQ.FT. OF GRATE	SQ.FT.	17/14	46.9	48.3	55.1	51.1	45.8	49.5	54.0	48.5	43.3	43.8
43	SUPERHEATER H.S. PER SQ.FT. TOTAL EVAP. H.S.	SQ.FT.	18/17	.407	.431	.438	.417	.442	.274	.264	.450	.406	.474
44	PERCENT GAS AREA THRU FLUES			84.8	85.0	80.1	85.5	86.1	51.0	42.6	89.0	86.1	98.3
45	WEIGHT ON DRIVERS PER 1000 LBS. TRACT. POWER	POUNDS	7/9	4120	4350	4360	4640	4350	4390	4590	4285	4300	3925
46	TOTAL WEIGHT OF ENGINE PER 1000 LBS. TRACT. POWER	POUNDS	8/9	7810	8200	8210	8425	8245	8300	8640	8550	8250	7490

TABLE 2 GENERAL DIMENSIONS AND BOILER RATIOS OF TEN-COUPLED LOCOMOTIVES

	ITEM		UNIT MEASURE	RATIO	1	2	3	4	5	6	7	8	9	10
	TYPE				2-10-0	2-10-2	2-10-2	2-10-4	2-10-4	2-10-4	2-10-4	2-10-4	2-10-4	4-10-2
1	RAILROAD				P.R.R.	CAN. NAT.	N.Y.N.H.&H.	CENT. VER.	C.P.R.	C.B.&Q.	C.&O.	K.C.S.	A.T.&S.F.	B.L.W.
2	CLASS				*790 I 1 S	T 4 B	L 1 C	T 3 A	T 1 B	*6320 M 4	T 1	J	5001	60000
3	YEAR BUILT				1922	1930	1929	1928	1938	1927	1930	1937	1938	1926
4	BOILER PRESSURE		LBS. PER SQ. IN.		250	275	200	250	285	250	260	310	310	350
5	CYLINDER DIA. & STROKE		INCHES		30½ X 32	24 X 28	30 X 32	27 X 32	25 X 32	31 X 32	29 X 34	27 X 34	30 X 34	1HP-2LP 27 X 32
6	DIA. DRIVERS		INCHES		62	57	63	60	63	64	69	70	74	63½
7	WEIGHT ON DRIVERS		POUNDS		352500	261040	301800	285000	309900	353820	373000	350000	371680	338400
8	TOTAL WEIGHT OF ENGINE		POUNDS		386100	344170	363325	419000	447000	512110	566000	509000	545260	457500
9	TRACTIVE POWER		POUNDS		90024	61600	77800	76800	76905	90000	91584	93300	93000	82500
10	BOILER I.D. FIRST COURSE		INCHES		82	74⅝	88	84½	82½	90	98	90	92¼	81⅜
11	BOILER O.D. LARGE COURSE		INCHES		93	86	92 13/16	94	96½	104	108	102	104	94
12	LENGTH OVER TUBE SHEETS		FEET & INCHES		19-1	19-3	18-0	22-0	21-0	21-6	21-0	21-0	21-0	23-0
13	COMBUSTION CHAMBER LENGTH		INCHES		42¼	37	—	48	54	49½	66	75	72	
14	GRATE AREA		SQ. FT.		70.0	66.7	82.0	84.4	93.5	106.5	121.7	107.0	121.5	82.5
15	TUBE & FLUE HEAT. SURF.	WATER SIDE	SQ. FT.		4303	3059	3951	4280	4642	5455	5990	4654	5443	4420
16	FIREBOX HEATING SURFACE	WATER SIDE	SQ. FT.		287	347	454	423	412	449	645	500	632	342
17	TOTAL EVAP. HEAT. SURF.	WATER SIDE	SQ. FT.		4590	3406	4405	4703	5054	5904	6635	5154	6075	4762
18	SUPERHEATER SURFACE - STEAM SIDE		SQ. FT.		1575	1500	1945	2220	2032	2487	3030	2075	2675	1357
19	FIREBOX VOLUME		CU. FT.		364	386	422	458		670	826	722	745	683
20	GAS AREA THRU BOILER		SQ. FT.		9.90	7.05	9.95	8.70	9.75	11.10	12.75	10.35	11.55	9.37
21	TYPE OF SUPERHEATER				E	E	E	E	E	E	E	E	E	A
22	STEAM AREA THRU SUPERHEATER		SQ. IN.		490	522	719	691	651	783	973	704	896	569
23	MAX. EVAP. CALCULATED		LBS. PER HR.		56660	51750	69000	65110	69120	78520	96210	74680	89140	60410
24	MAX. EVAP. ON TEST		LBS. PER HR.		65257		65750			78710				69695
25	NUMBER & SIZE OF PLUES				170-3½	146-3½	204-3½	192-3½	196-3½	222-3½	275-3½	183-3¾	249-3½	50-5½
26	NUMBER & SIZE OF TUBES				120-2¼	44-2¼	57-2¼	33-2¼	72-2¼	87-2¼	59-2¼	73-2¼	56-2¼	206-2¼
27	NUMBER & SIZE OF SUPERHEATER UNITS				85-1⅛	74-1 3/16	102-1 3/16	98-1 3/16	98-1 3/16	111-1 3/16	138-1 3/16	93-1¼	127-1 3/16	50-1½
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30	GAS AREA PER 1000 LBS. TRACT. POWER	SQ. FT.	20/9	.110	.114	.128	.113	.127	.123	.139	.111	.124	.114
31	GAS AREA PER SQ. FT. OF GRATE	SQ. FT.	20/14	.141	.106	.122	.103	.104	.104	.105	.097	.095	.114
32	GAS AREA PER SQ. FT. OF TUBE AND FLUE H.S.	SQ. FT.	20/15	.00230	.00231	.00252	.00203	.00211	.00204	.00213	.00222	.00212	.00212
33	GRATE PER 1000 LBS. TRACT. POWER	SQ. FT.	14/9	.78	1.08	1.06	1.10	1.22	1.18	1.33	1.15	1.31	1.00
34	GRATE PER SQ. FT. TOTAL EVAP. H.S.	SQ. FT.	14/17	.0153	.0196	.0186	.0179	.0185	.0181	.0184	.0207	.0200	.0173
35	FIREBOX VOLUME PER 1000 LBS. TRACT. POWER	CU. FT.	19/9	4.04	6.26	5.42	5.96		7.44	9.02	7.75	8.01	8.29
36	FIREBOX VOLUME PER SQ. FT. OF GAS AREA	CU. FT.	19/20	36.8	54.8	42.4	52.6		60.3	64.9	69.8	64.5	73.0
37	FIREBOX VOLUME PER SQ. FT. OF GRATE	CU. FT.	19/14	5.20	5.78	5.15	5.43		6.28	6.79	6.75	6.13	8.28
38	FIREBOX VOLUME PER SQ. FT. TOTAL EVAP. H.S.	CU. FT.	19/17	7.93	11.35	9.58	9.74		11.35	12.47	14.00	11.25	14.40
39	FIREBOX H.S. PER SQ. FT. OF GRATE	SQ. FT.	16/14	4.10	5.20	5.54	5.02	4.41	4.22	5.31	4.67	5.20	4.15
40	FIREBOX H.S. PER SQ. FT. TOTAL EVAP. H.S. ×100	SQ. FT.	16/17×100	6.25	10.20	10.30	8.98	8.13	7.61	9.72	9.70	10.40	7.18
41	TOTAL EVAP. H.S. PER 1000 LBS. TRACT. POWER	SQ. FT.	17/9	51.0	55.3	56.7	61.2	65.8	65.6	72.4	55.3	65.4	57.7
42	TOTAL EVAP. H.S. PER SQ. FT. OF GRATE	SQ. FT.	17/14	65.5	51.1	53.7	55.8	54.1	55.4	54.5	48.2	50.0	57.8
43	SUPERHEATER H.S. PER SQ. FT. TOTAL EVAP. H.S.	SQ. FT.	18/17	.343	.440	.441	.472	.401	.422	.457	.402	.440	.285
44	PERCENT GAS AREA THRU FLUES			73.2	86.1		91.7	83.7	83.2	89.6	84.8	89.5	51.8
45	WEIGHT ON DRIVERS PER 1000 LBS. TRACT. POWER	POUNDS	7/9	3920	4240	3870	3710	4020	3925	4070	3775	3985	4100
46	TOTAL WEIGHT OF ENGINE PER 1000 LBS. TRACT. POWER	POUNDS	8/9	4290	5290	4670	*5460	5814	5690	6185	5460	5870	5540

TABLE 3 GENERAL DIMENSIONS AND BOILER RATIOS OF EIGHT-COUPLED LOCOMOTIVES

	ITEM		UNIT MEASURE	RATIO	1	2	3	4	5	6	7	8	9	10
	TYPE				4-8-4	4-8-4	4-8-4	4-8-4	4-8-4	4-8-4	4-8-4	4-8-4	4-8-4	4-8-4
1	RAILROAD				N.C.&ST.L.	D.&R.G.W.	ST.L.-S.W.	L.V.	WABASH	C.&O.	CAN.NAT.	SOU.PAC.	GT. NOR.	D.&R.G.W.
2	CLASS				J 2	M 64	L 1	T 2	O 1	J 3	U 2 D	G S 2	S 1	M 68
3	YEAR BUILT				1930	1929	1937	1931	1930	1935	1936	1936	1929	1937
4	BOILER PRESSURE		LBS. PER SQ. IN.		250	240	250	255	250	250	250	250	250	285
5	CYLINDER DIA. & STROKE		INCHES		25 x 30	27 x 30	26 x 30	26 x 32	27 x 32	27 1/2 x 30	25 1/2 x 30	27 x 30	28 x 30	26 x 30
6	DIA. DRIVERS		INCHES		70	70	70	70	70	72	73	73 1/2	73	73
7	WEIGHT ON DRIVERS		POUNDS		220000	252000	248000	268000	274100	273000	237600	266500	273700	279172
8	TOTAL WEIGHT OF ENGINE		POUNDS		381000	408500	425500	422000	454090	477000	390000	448400	472120	479360
9	TRACTIVE POWER		POUNDS		57000	63700	61500	66700	70750	66960	56800	62200	67000	67200
10	BOILER I.D. FIRST COURSE		INCHES		79	82 15/16	84 1/2	84 1/4	86 1/2	90	80 7/8	84 1/8	86 1/16	90 3/16
11	BOILER O.D. LARGE COURSE		INCHES		92	96	98	98	100	100	90	96	98	100
12	LENGTH OVER TUBE SHEETS		FEET & INCHES		20-6	22-0	20-0	21-6	21-0	21-0	21-6	21-6	22-0	21-0
13	COMBUSTION CHAMBER LENGTH		INCHES		54	42	54	54	50	54	48 1/2	60	52	72
14	GRATE AREA		SQ. FT.		77.3	88.0	88.3	88.3	98.2	100.0	84.3	90.4	102.0	108.0
15	TUBE & FLUE HEAT. SURF.	WATER SIDE	SQ. FT.		3751	4473	4259	4933	4694	5013	3805	4502	5004	4952
16	FIREBOX HEATING SURFACE	WATER SIDE	SQ. FT.		444	446	469	508	495	525	415	350	401	555
17	TOTAL EVAP. HEAT. SURF.	WATER SIDE	SQ. FT.		4195	4919	4728	5441	5189	5538	4220	4852	5405	5507
18	SUPERHEATER SURFACE - STEAM SIDE		SQ. FT.		1837	2229	1962	2256	2360	2342	1760	2086	2420	2336
19	FIREBOX VOLUME		CU. FT.		478	402		567		642	408	559		690
20	GAS AREA THRU BOILER		SQ. FT.		8.11	9.04	9.24	10.10	9.70	10.80	7.86	9.13	10.08	10.43
21	TYPE OF SUPERHEATER				E	E	E	E	E	E	E	E	E	E
22	STEAM AREA THRU SUPERHEATER		SQ. IN.		63.5	69.8	67.7	71.9	75.4	74.4	56.4	67.1	74.0	74.4
23	MAX. EVAP. CALCULATED		LBS. PER HR.		63040	68280	70100	78500	74780	79640	60810	63575	70570	80800
24	MAX. EVAP. ON TEST		LBS. PER HR.											
25	NUMBER & SIZE OF PLUES				169-3 1/2	195-3 1/2	200-3 1/2	202-3 1/2	214-3 1/2	220-3 1/2	167-3 1/2	198-3 1/2	210-3 1/2	222-3 1/2
26	NUMBER & SIZE OF TUBES				49-2 1/4	43-2 1/4	52-2 1/4	77-2 1/4	49-2 1/4	65-2 1/4	42-2 1/4	49-2 1/4	61-2 1/4	57-2 1/4
27	NUMBER & SIZE OF SUPERHEATER UNITS				86-1 3/16	99-1 3/16	102-1 3/16	102-1 3/16	107-1 3/16	112-1 3/16	85-1 3/16	101-1 3/16	105-1 3/16	112-1 3/16
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30	GAS AREA PER 1000 LBS. TRACT. POWER	SQ. FT.	20/9	.142	.142	.151	.151	.137	.158	.139	.147	.150	.155
31	GAS AREA PER SQ. FT. OF GRATE	SQ. FT.	20/14	.105	.103	.105	.114	.101	.106	.093	.101	.099	.099
32	GAS AREA PER SQ. FT. OF TUBE AND FLUE H.S.	SQ. FT.	20/15	.00216	.00202	.00217	.00205	.00207	.00211	.00207	.00203	.00201	.00211
33	GRATE PER 1000 LBS. TRACT. POWER	SQ. FT.	14/9	1.36	1.38	1.44	1.33	1.36	1.50	1.48	1.45	1.52	1.58
34	GRATE PER SQ. FT. TOTAL EVAP. H.S.	SQ. FT.	14/17	.0184	.0179	.0187	.0163	.0186	.0188	.0200	.0186	.0188	.0192
35	FIREBOX VOLUME PER 1000 LBS. TRACT. POWER	CU. FT.	19/9	8.38	6.31		8.50		9.80	7.15	9.00		10.30
36	FIREBOX VOLUME PER SQ. FT. OF GAS AREA	CU. FT.	19/20	58.9	44.5		56.1		60.6	51.7	61.3		66.1
37	FIREBOX VOLUME PER SQ. FT. OF GRATE	CU. FT.	19/14	6.18	4.57		6.43		6.42	4.83	6.19		6.50
38	FIREBOX VOLUME PER SQ. FT. TOTAL EVAP. H.S.	CU. FT.	19/17	114.1	81.8		104.1		116.0	96.4	115.2		125.2
39	FIREBOX H.S. PER SQ. FT. OF GRATE	SQ. FT.	16/14	5.74	5.07	5.33	5.75	5.13	5.25	4.93	3.88	3.93	5.24
40	FIREBOX H.S. PER SQ. FT. TOTAL EVAP. H.S. x100	SQ. FT.	16/17 x100	10.60	9.05	9.92	9.33	9.52	9.50	9.84	7.20	7.40	10.07
41	TOTAL EVAP. H.S. PER 1000 LBS. TRACT. POWER	SQ. FT.	17/9	73.6	77.3	76.9	81.5	73.3	82.7	74.3	78.2	80.8	81.9
42	TOTAL EVAP. H.S. PER SQ. FT. OF GRATE	SQ. FT.	17/14	54.3	55.9	53.5	61.6	54.0	55.4	50.1	53.8	53.0	51.9
43	SUPERHEATER H.S. PER SQ. FT. TOTAL EVAP. H.S.	SQ. FT.	18/17	.438	.453	.415	.414	.455	.424	.417	.429	.448	.424
44	PERCENT GAS AREA THRU FLUES			87.0	85.4	88.0	83.2	89.3	86.4	88.0	88.0	86.5	88.3
45	WEIGHT ON DRIVERS PER 1000 LBS. TRACT. POWER	POUNDS	7/9	3860	3950	4030	4020	3880	4077	4180	4280	4080	4150
46	TOTAL WEIGHT OF ENGINE PER 1000 LBS. TRACT. POWER	POUNDS	8/9	6680	6420	6920	6325	6420	6220	6865	7220	7050	7140

TABLE 4 GENERAL DIMENSIONS AND BOILER RATIOS OF EIGHT-COUPLED LOCOMOTIVES

	ITEM		UNIT MEASURE	RATIO	1	2	3	4	5	6	7	8	9	10
	TYPE				4-8-4	4-8-4	4-8-4	4-8-4	4-8-4	4-8-4	4-8-4	4-8-4	4-8-4	4-8-2
1	RAILROAD				D.L.&W.	C.M.ST. P.&P.	C.B.&Q.	C.M.ST P.&P.	CAN.PAC.	L. V.	R.F.&P.	NOR.PAC.	U. P.	P.R.R.
2	CLASS				1631-1650	S1	O5	S2	K1	T3 5129	551-555	A 3	800-819	M1A 6708
3	YEAR BUILT				1934	1930	1938	1937	1928	1935	1936	1938	1937	1931
4	BOILER PRESSURE		LBS. PER SQ.IN.		250	230	250	285	275	275	275	260	300	250
5	CYLINDER DIA. & STROKE		INCHES		28X32	28X30	28X30	26X32	25½X30	27X30	27X30	28X31	24½X32	27X30
6	DIA. DRIVERS		INCHES		74	74	74	74	75	77	77	77	77	72
7	WEIGHT ON DRIVERS		POUNDS		274000	258814	282700	282320	249000	270100	277245	294000	270000	271000
8	TOTAL WEIGHT OF ENGINE		POUNDS		447000	450838	465300	490450	423000	441440	466040	491800	465000	390000
9	TRACTIVE POWER		POUNDS		72000	60000	67500	70800	60800	66500	66500	69800	63600	64550
10	BOILER I.D. FIRST COURSE		INCHES		84¼	86 3/16	86½	90 3/16	82½	84 9/16	86	86⅜	86 3/16	82
11	BOILER O.D. LARGE COURSE		INCHES		95	100	100	100	96½	98¼	98¼	99	100	96
12	LENGTH OVER TUBE SHEETS		FEET & INCHES		21-6	21-0	21-0	21-0	20-6	21-6	21-0	19-6	20-6	19-1
13	COMBUSTION CHAMBER LENGTH		INCHES		66 1/16	60	40	72	60	48	78	90½	72	98
14	GRATE AREA		SQ.FT.		88.2	103.0	106.5	106.0	93.5	96.5	96.3	115.0	100.2	70
15	TUBE & FLUE HEAT. SURF.	WATER SIDE	SQ.FT.		4992	4860	4804	4931	4509	4932	4827	4202	4118	4303
16	FIREBOX HEATING SURFACE	WATER SIDE	SQ.FT.		496	540	433	578	422	507	551	553	479	403
17	TOTAL EVAP. HEAT. SURF.	WATER SIDE	SQ.FT.		5488	5400	5237	5509	4931	5439	5378	4755	4597	4706
18	SUPERHEATER SURFACE - STEAM SIDE		SQ.FT.		2180	2403	2300	2236	2112	2056	2130	2026	1458	1630
19	FIREBOX VOLUME		CU.FT.		534	630	650	700	620	540	640	812	675	475
20	GAS AREA THRU BOILER		SQ.FT.		10.20	10.30	10.15	11.10	9.86	10.05	10.05	10.18	9.67	9.71
21	TYPE OF SUPERHEATER				E	E	E	E	E	E	E	E	A	E
22	STEAM AREA THRU SUPERHEATER		SQ.IN		67.7	76.9	72.4	77.3	69.0	67.7	69.1	75.3	66.0	56.4
23	MAX EVAP. CALCULATED		LBS. PER HR.		76905	79050	72160	82000	69140	76990	79385	75280	68700	67850
24	MAX EVAP. ON TEST		LBS. PER HR.											99095
25	NUMBER & SIZE OF FLUES				202-3½	218-3½	218-3½	201-3¾	196-3½	202-3½	205-3½	186-3¾	58-5½	170-3½
26	NUMBER & SIZE OF TUBES				82-2¼	56-2¼	51-2¼	66-2¼	59-2 1/4 7-3 1/2	77-2¼	73-2¼	58-2¼	201-2¼	120-2¼
27	NUMBER & SIZE OF SUPERHEATER UNITS				102-1 3/16	109-1 3/16	109-1 3/16	102-1¼	98-1 3/16	102-1 3/16	104-1 3/16	94-1¼	58-1½	85-1 3/16
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30	GAS AREA PER 1000 LBS. TRACT. POWER		SQ.FT.	20/9	.142	.172	.151	.157	.162	.151	.151	.145	.152	.151
31	GAS AREA PER SQ.FT. OF GRATE		SQ.FT.	20/14	.115	.100	.095	.105	.105	.104	.104	.089	.097	.139
32	GAS AREA PER SQ.FT. OF TUBE AND FLUE H.S.		SQ.FT.	20/15	.00204	.00212	.00211	.00225	.00219	.00204	.00208	.00242	.00235	.00226
33	GRATE PER 1000 LBS. TRACT. POWER		SQ.FT.	14/9	1.23	1.72	1.58	1.50	1.54	1.45	1.45	1.65	1.58	1.09
34	GRATE PER SQ.FT. TOTAL EVAP. H.S.		SQ.FT.	14/17	.0161	.0191	.0204	.0192	.0190	.0177	.0179	.0242	.0218	.0149
35	FIREBOX VOLUME PER 1000 LBS. TRACT. POWER		CU.FT.	19/9	7.42	10.50	9.63	9.90	10.20	8.12	9.63	11.67	10.60	7.35
36	FIREBOX VOLUME PER SQ.FT. OF GAS AREA		CU.FT.	19/20	52.4	61.1	64.0	63.1	62.8	53.8	63.7	79.5	69.8	49.0
37	FIREBOX VOLUME PER SQ.FT. OF GRATE		CU.FT.	19/14	6.05	6.11	6.10	6.60	6.63	5.60	6.67	7.06	6.75	6.80
38	FIREBOX VOLUME PER SQ.FT. TOTAL EVAP. H.S.		CU.FT.	19/17	97.2	116.0	124.0	127.1	126.0	99.3	119.0	171.0	147.0	101.0
39	FIREBOX H.S. PER SQ.FT. OF GRATE		SQ.FT.	16/14	5.64	5.24	4.06	5.46	4.52	5.26	5.72	4.80	4.79	5.75
40	FIREBOX H.S. PER SQ.FT. TOTAL EVAP. H.S. ×100		SQ.FT.	16/17 ×100	9.03	10.00	8.27	10.50	8.56	9.34	10.25	11.60	10.42	8.55
41	TOTAL EVAP. H.S. PER 1000 LBS. TRACT. POWER		SQ.FT.	17/9	76.2	90.0	77.5	77.8	81.1	81.6	81.0	68.1	72.3	72.8
42	TOTAL EVAP. H.S. PER SQ.FT. OF GRATE		SQ.FT.	17/14	62.2	52.4	49.2	52.0	52.7	56.4	56.0	41.4	46.0	67.2
43	SUPERHEATER H.S. PER SQ.FT. TOTAL EVAP. H.S.		SQ.FT.	18/17	.398	.445	.440	.406	.427	.378	.396	.422	.317	.346
44	PERCENT GAS AREA THRU FLUES				82.2	88.0	89.2	87.0	82.8	83.2	84.0	87.8	55.5	73.0
45	WEIGHT ON DRIVERS PER 1000 LBS. TRACT. POWER		POUNDS	7/9	3800	4315	4185	3990	4090	4065	4170	4220	4250	4200
46	TOTAL WEIGHT OF ENGINE PER 1000 LBS. TRACT. POWER		POUNDS	8/9	6210	7515	6890	6930	6960	6650	7010	7040	7320	6040

TABLE 5 GENERAL DIMENSIONS AND BOILER RATIOS OF EIGHT-COUPLED LOCOMOTIVES

	ITEM		UNIT MEASURE	RATIO	1	2	3	4	5	6	7	8	9	10
	TYPE				4-8-4	4-8-4	4-8-4	4-8-4	6-4-4-6	4-6-2	2-8-2	4-4-2	2-8-2	4-8-4
1	RAILROAD				GT. NOR	SOU. PAC.	A.C.L.	A.T.&S.F.	P.R.R.	P.R.R.	P.R.R.	P.R.R.	MO. PAC.	U.P.
2	CLASS				S-2	G-S-3	R-1	3765	S-1	#1737 K-4-S	#1752 L-1-S	#5266 E-2-A	ENG. NO. 1699	820-834
3	YEAR BUILT				1930	1937	1938	1938	1939	1914	1914	1904	1925	1939
4	BOILER PRESSURE		LBS. PER SQ. IN.		225	280	275	300	300	205	205	205	200	300
5	CYLINDER DIA. & STROKE		INCHES		29X29	26 X32	27X30	28X32	4 CYL. 22X26	27X28	27 X30	20½X26	2-23X32 1-23X28	25X32
6	DIA. DRIVERS		INCHES		80	80	80	80	84	80	62	80	63	80
7	WEIGHT ON DRIVERS		POUNDS		247300	267300	263127	286890	281440	202880	235800	110001	244500	270000
8	TOTAL WEIGHT OF ENGINE		POUNDS		420900	460000	460270	499600	608170	309140	315600	184167	340000	483000
9	TRACTIVE POWER		POUNDS		58300	62800	63900	66000	76400	44460	61500	23880	65700	63800
10	BOILER I.D. FIRST COURSE		INCHES		82 11/16	84 1/4	84 1/2	88 1/4	91 1/16	76 5/8	76 5/8	65 5/8	88	86 3/16
11	BOILER O.D. LARGE COURSE		INCHES		94	96	98 1/4	102	102	89	89	73	92 15/16	100
12	LENGTH OVER TUBE SHEETS		FEET & INCHES		22-0	21-6	21-0	21-0	22-0	19-1	19-1	15-1	19-0	19-0
13	COMBUSTION CHAMBER LENGTH		INCHES		60	80	72	64	113	36	36	—	33	90
14	GRATE AREA		SQ. FT.		97.7	90.4	97.8	108	132	69.3	70.0	55.5	66.3	100.2
15	TUBE & FLUE HEAT. SURF.	WATER SIDE	SQ. FT.		4402	4502	4181	4851	5001	3729	3716	2471	3437	3971
16	FIREBOX HEATING SURFACE	WATER SIDE	SQ. FT.		379	385	568	552	660	304	299	157	363	499
17	TOTAL EVAP. HEAT. SURF.	WATER SIDE	SQ. FT.		4781	4887	4749	5403	5661	4033	4015	2628	3800	4470
18	SUPERHEATER SURFACE - STEAM SIDE		SQ. FT.		2265	2086	1497	2366	2085	908	908	—	1051	1900
19	FIREBOX VOLUME		CU. FT.			600	613	712	844	380	380	205	390	689
20	GAS AREA THRU BOILER		SQ. FT.		8.94	9.04	9.75	10.30	11.39	9.10	9.10	5.26	8.81	9.90
21	TYPE OF SUPERHEATER				E	E	A	E	A	A	A	—	A	E
22	STEAM AREA THRU SUPERHEATER		SQ. IN.		69.8	67.1	66.0	78.3	157.1	45.6	45.6	—	51.3	70.4
23	MAX. EVAP. CALCULATED		LBS. PER HR.		63640	65650	74155	79675	85930	52150	51750	35570	52620	70380
24	MAX. EVAP. ON TEST		LBS. PER HR.							65400	59085	30721	61680	
25	NUMBER & SIZE OF FLUES				195-3½	198-3½	58-5½	220-3½	69-5½	40-5½	40-5½	—	45-5½	184-3¾
26	NUMBER & SIZE OF TUBES				38-2¼	49-2¼	198-2¼	52-2¼	219-2¼	237-2¼	237-2¼	315-2	199-2¼	50-2¼
27	NUMBER & SIZE OF SUPERHEATER UNITS				99-1 3/16	101-1 3/16	58-1½	111-1 3/16	138-1½	40-1½	40-1½	—	45-1½	93-1¼
28														
29														

30	GAS AREA PER 1000 LBS. TRACT. POWER		SQ. FT.	20/9	.154	.144	.152	.156	.149	.204	.148	.220	.134	.155
31	GAS AREA PER SQ. FT. OF GRATE		SQ. FT.	20/14	.091	.100	.099	.095	.086	.132	.131	.095	.133	.099
32	GAS AREA PER SQ. FT. OF TUBE AND FLUE H.S.		SQ. FT.	20/15	.00203	.00201	.00233	.00212	.00228	.00242	.00246	.00213	.00256	.0025
33	GRATE PER 1000 LBS. TRACT. POWER		SQ. FT.	14/9	1.68	1.44	1.53	1.64	1.73	1.56	1.14	2.32	1.01	1.57
34	GRATE PER SQ. FT. TOTAL EVAP. H.S.		SQ. FT.	14/17	.0205	.0185	.0206	.0199	.0233	.0172	.0175	.0211	.0175	.0224
35	FIREBOX VOLUME PER 1000 LBS. TRACT. POWER		CU. FT.	19/9	9.90	9.55	9.60	10.80	11.05	8.50	6.18	8.56	5.93	10.80
36	FIREBOX VOLUME PER SQ. FT. OF GAS AREA		CU. FT.	19/20		66.4	59.7	69.1	73.0	41.8	41.8	39.0	44.2	69.7
37	FIREBOX VOLUME PER SQ. FT. OF GRATE		CU. FT.	19/14		6.64	5.90	6.60	6.40	5.50	5.44	3.70	5.88	6.89
38	FIREBOX VOLUME PER SQ. FT. TOTAL EVAP. H.S.		CU. FT.	19/17		123.0	129.0	131.8	149.0	94.4	94.5	78.0	103.0	154.0
39	FIREBOX H.S. PER SQ. FT. OF GRATE		SQ. FT.	16/14		4.26	5.82	5.12	5.00	4.39	4.27	2.83	5.47	4.99
40	FIREBOX H.S. PER SQ. FT. TOTAL EVAP. H.S. x100		SQ. FT.	16/17 x100	7.93	7.87	12.00	10.20	11.67	7.53	7.45	5.97	9.55	11.19
41	TOTAL EVAP. H.S. PER 1000 LBS. TRACT. POWER		SQ. FT.	17/9	82.0	77.9	74.3	81.9	74.0	90.7	67.0	110.0	57.7	70.2
42	TOTAL EVAP. H.S. PER SQ. FT. OF GRATE		SQ. FT.	17/14	49.0	54.1	48.5	50.0	42.9	58.3	58.8	47.4	57.3	44.7
43	SUPERHEATER H.S. PER SQ. FT. TOTAL EVAP. H.S.		SQ. FT.	18/17	.473	.427	.316	.438	.369	.226	.227	—	.277	.425
44	PERCENT GAS AREA THRU FLUES				90.8	88.7	56.5	88.7	57.5	43.5	43.5	—	50.3	89.0
45	WEIGHT ON DRIVERS PER 1000 LBS. TRACT. POWER		POUNDS	7/9	4240	4250	4120	4350	3680	4560	3830	4590	3720	4230
46	TOTAL WEIGHT OF ENGINE PER 1000 LBS. TRACT. POWER		POUNDS	8/9	7210	7330	7200	7570	7960	6950	5125	7690	5170	7560

TABLE 6 GENERAL DIMENSIONS AND BOILER RATIOS OF ARTICULATED LOCOMOTIVES

	ITEM	UNIT MEASURE	RATIO	1	2	3	4	5	6	7	8	9	10
	TYPE			2-6-6-2	2-6-6-4	2-6-6-4	4-6-6-4	4-6-6-4	4-6-6-4	2-8-8-2	2-8-8-2	2-8-8-4	4-8-8-2
1	RAILROAD			C.&O.	S.A.L.	N&W.	U.P.	NOR.PAC.	D&R.G.W.	GT.NOR.	N&W.	NOR.PAC.	SOU.PAC.
2	CLASS			H4A	R2	A	3900	Z6	L105	R2	Y6	Z5	AC7
3	YEAR BUILT			1927	1937	1937	1936	1936	1938	1929	1936	1930	1937
4	BOILER PRESSURE	LBS. PER SQ.IN.		210	230	275	255	250	255	240	300	250	250
5	CYLINDER DIA. & STROKE	INCHES		20X32	22X30	24X30	22X32	23X32	23X32	28X32	25 X32 39 X32	28X32	24X32
6	DIA. DRIVERS	INCHES		56 1/4	69	70	69	69	70	63	57	63	63 1/2
7	WEIGHT ON DRIVERS	POUNDS		379000	330000	430100	386000	435000	437939	544000	522850	558900	514800
8	TOTAL WEIGHT OF ENGINE	POUNDS		450500	460000	570000	566000	624500	641900	630750	582900	723400	639800
9	TRACTIVE POWER	POUNDS		81240	82300	104500	97400	104500	105000	146000	126838	140000	123400
10	BOILER I.D. FIRST COURSE	INCHES		87	82 5/16	91	96 11/16	96 3/8	92 3/4	98	95 9/16	103 1/4	91 7/8
11	BOILER O.D. LARGE COURSE	INCHES		93 3/4	96	105 1/2	102	102	102	111 1/8	104 1/4	110 1/4	106 1/8
12	LENGTH OVER TUBE SHEETS	FEET & INCHES		24-0	24-0	24-1	22-0	23-0	22-0	24-0	24-0 3/8	22-0	22-0
13	COMBUSTION CHAMBER LENGTH	INCHES		78	72	115 1/2	86	89	109 1/2	84 1/16	36 5/8	72 1/2	68
14	GRATE AREA	SQ.FT.		72.2	96.3	122.0	108.2	152.3	136.5	126.0	106.2	182.0	139.0
15	TUBE & FLUE HEAT. SURF. (WATER SIDE)	SQ.FT.		4807	4914	6062	4756	4993	5563	7395	5207	6800	5990
16	FIREBOX HEATING SURFACE (WATER SIDE)	SQ.FT.		368	515	588	625	839	778	481	430	866	478
17	TOTAL EVAP. HEAT. SURF. (WATER SIDE)	SQ.FT.		5175	5429	6650	5381	5832	6341	7876	5637	7666	6468
18	SUPERHEATER SURFACE - STEAM SIDE	SQ.FT.		2246	2380	2703	1630	2090	2628	3515	1775	3224	2601
19	FIREBOX VOLUME	CU.FT.			532	843	862	1013	1060		630	1215	
20	GAS AREA THRU BOILER	SQ.FT.		9.36	9.13	10.95	10.46	11.05	11.20	12.50	10.72	13.61	11.75
21	TYPE OF SUPERHEATER			E	E	E	A	A	E	E	A	E	E
22	STEAM AREA THRU SUPERHEATER	SQ.IN.		63.5	68.4	80.3	68.3	83.2	79.7	99.4	68.3	98.7	79.7
23	MAX. EVAP. CALCULATED	LBS. PER HR.		64450	74000	88610	81550	95000	98180	94060	71660	115070	84400
24	MAX. EVAP. ON TEST	LBS. PER HR.											
25	NUMBER & SIZE OF FLUES			180-3 1/2	200-3 1/2	239-3 1/2	60-5 1/2	73-5 1/2	238-3 1/2	281-3 1/2	60-5 1/2 220-3 1/2		240-3 1/2
26	NUMBER & SIZE OF TUBES			15-3 1/2 38-2 1/4	38-2 1/4	57-2 1/4	222-2 1/4	192-2 1/4	61-2 1/4	88-2 1/4	223-2 1/4	92-2 1/4	91-2 1/4
27	NUMBER & SIZE OF SUPERHEATER UNITS			90-1 3/16	103-1 3/16	121-1 3/16	60-1 1/2	73-1 1/2	120-1 3/16	141-1 3/16	60-1 1/2	140-1 3/16	120-1 3/16
28													
29													

30	GAS AREA PER 1000 LBS. TRACT. POWER	SQ.FT.	20/9	.115	.111	.105	.107	.106	.107	.086	.085	.097	.095
31	GAS AREA PER SQ.FT. OF GRATE	SQ.FT.	20/14	.130	.095	.090	.097	.073	.082	.099	.101	.075	.084
32	GAS AREA PER SQ.FT. OF TUBE AND FLUE H.S.	SQ.FT.	20/15	.00195	.00186	.00181	.00220	.00222	.00201	.00169	.00206	.00200	.00198
33	GRATE PER 1000 LBS. TRACT. POWER	SQ.FT.	14/9	.89	1.17	1.17	1.11	1.46	1.30	.86	.84	1.30	1.12
34	GRATE PER SQ.FT. TOTAL EVAP. H.S.	SQ.FT.	14/17	.0140	.0178	.0184	.0201	.0262	.0216	.0160	.0119	.0238	.0215
35	FIREBOX VOLUME PER 1000 LBS. TRACT. POWER	CU.FT.	19/9		6.46	8.06	8.86	9.69	10.10		4.97	8.68	
36	FIREBOX VOLUME PER SQ.FT. OF GAS AREA	CU.FT.	19/20		58.3	77.0	82.5	91.6	94.6		58.7	89.3	
37	FIREBOX VOLUME PER SQ.FT. OF GRATE	CU.FT.	19/14		5.53	6.90	7.96	6.64	7.76		5.92	6.67	
38	FIREBOX VOLUME PER SQ.FT. TOTAL EVAP. H.S.	CU.FT.	19/17		98.0	127.0	161.0	173.5	167.2		112.0	158.3	
39	FIREBOX H.S. PER SQ.FT. OF GRATE	SQ.FT.	16/14	5.10	5.35	4.82	5.77	5.51	5.70	3.82	4.05	4.76	3.44
40	FIREBOX H.S. PER SQ.FT. TOTAL EVAP. H.S. x100	SQ.FT.	16/17 x100	7.10	9.48	8.84	11.62	14.40	12.30	6.11	7.63	11.30	7.38
41	TOTAL EVAP. H.S. PER 1000 LBS. TRACT. POWER	SQ.FT.	17/9	63.6	66.0	63.6	55.4	55.8	60.3	54.0	44.4	54.8	52.5
42	TOTAL EVAP. H.S. PER SQ.FT. OF GRATE	SQ.FT.	17/14	71.6	56.4	54.5	49.7	38.2	46.5	62.5	52.9	42.2	46.6
43	SUPERHEATER H.S. PER SQ.FT. TOTAL EVAP. H.S.	SQ.FT.	18/17	.435	.438	.407	.303	.359	.415	.446	.413	.420	.402
44	PERCENT GAS AREA THRU FLUES			81.7	91.2	88.4	54.5	62.8	88.2	86.0	54.0	85.4	82.8
45	WEIGHT ON DRIVERS PER 1000 LBS. TRACT. POWER	POUNDS	7/9	4660	4010	4125	3970	4160	4170	3720	4125	3985	4164
46	TOTAL WEIGHT OF ENGINE PER 1000 LBS. TRACT. POWER	POUNDS	8/9	5550	5835	5455	5820	5975	6100	4310	4600	5160	5180

Discussion

R. W. Anderson.[14] In his paper, the author clearly shows the importance of locomotive design ratios and successfully points out the limitations of formulas advocated by various locomotive-boiler authorities. Such limitations are primarily due to the lack of test data. For modern boilers, this in turn necessitates yet further approximations in the present inadequate formulas which must be used as a basis of design. Locomotive boilers are now being subjected to greater demands than ever before and the performance required will certainly not be lessened to ease the situation.

Weight and clearance dimensions immediately fix limitations which cannot be exceeded and which have the definite effect of hindrance in the design from its inception. However, in spite of these limitations and with the admittedly inadequate design formulas, filled as they are with estimated coefficients, boilers have been and are being built which generate more than the calculated quantity of steam, although the ratios between the grate areas, flue areas, firebox volumes, heating surfaces, etc., on successful boilers, vary within wide limits from the steam-generating viewpoint. However, when such boilers require more than the usual amount of attention considered necessary to keep them in proper condition, they can hardly be classified as being satisfactory for continuous operation.

All these conditions certainly point to the necessity of a central testing plant for such jobs. The experiments on the design formulas and ratios would no doubt take several years. Independent investigations and tests are often unsatisfactory due to lack of adherence to standard test codes and methods of collecting information.

The high draft-tube readings which are obtained in front-end experiments on some railroads lead others to try similar arrangements only to find that the new application could not survive transplanting. It would have been much cheaper to borrow the draft tubes, providing a standard measuring device for testing the existing arrangement.

If a series of boilers of various proportions were developed in such fashion, the major problem, namely, availability, is then only started. Modern locomotives have an exceptionally high first cost. Under present demands, boilers require considerably more than the usual amount of attention in roundhouses and shops. The metallurgist is being drawn further into this problem and it is his responsibility to determine the physical and chemical properties of the boiler steels which will be capable of withstanding present-day loads. He may be the one who will be required to tell the manufacturers to set the side sheets at a certain slope in order to prevent warped plates and leaky stay bolts. Continual firebox-repair work should not be a necessary evil attached to 300-lb-pressure boilers; to eliminate this condition a thorough study should be made of various furnace designs and the proper materials to use in their construction.

The engine portion of the locomotive has always received the greatest amount of attention from the designer's viewpoint. Consequently, the boiler is often changed to accommodate certain other parts which no doubt could have been altered to avoid affecting the boiler design. For instance, the trailer-truck design limits the grate and ashpan arrangement but, if the furnace design were given the attention it should have on modern locomotives and every effort made to have the trailer come within the limits specified, there would undoubtedly be a considerable saving realized in firebox, grate, and ashpan maintenance charges.

The performance of high-speed engines on long runs has created many problems in regard to lubrication and materials which must be solved. It will take a lot of coordinating to accomplish this.

E. G. Bailey.[15] The writer will confine his discussion to one phase of this paper, applying the data therein presented to the method of comparison of steam-boiler performance, covered by his paper[16] on that subject.

Throughout his paper the author has very properly mentioned the question of absorption efficiency and combustion efficiency, or total boiler and superheater over-all efficiency, which is in reality the product of the other two. The data given in his Figs. 13 and 14 are sufficiently complete to show wherein the difference lies between combustion efficiency and absorption efficiency, both relating to coal-fired boilers. However, in Fig. 10 he gives data of an oil-fired marine boiler which happens to be the same as boiler F, Table 3, in the writer's paper,[16] and also two oil-fired locomotive boilers, Southern Pacific AC4 and Santa Fe 3766. The difference between the efficiencies of these and

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[15] Vice-President, The Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

[16] "Steam-Boiler Performance and a Method of Comparison," by E. G. Bailey, Trans. A.S.M.E., vol. 62, July, 1940, pp. 367–378.

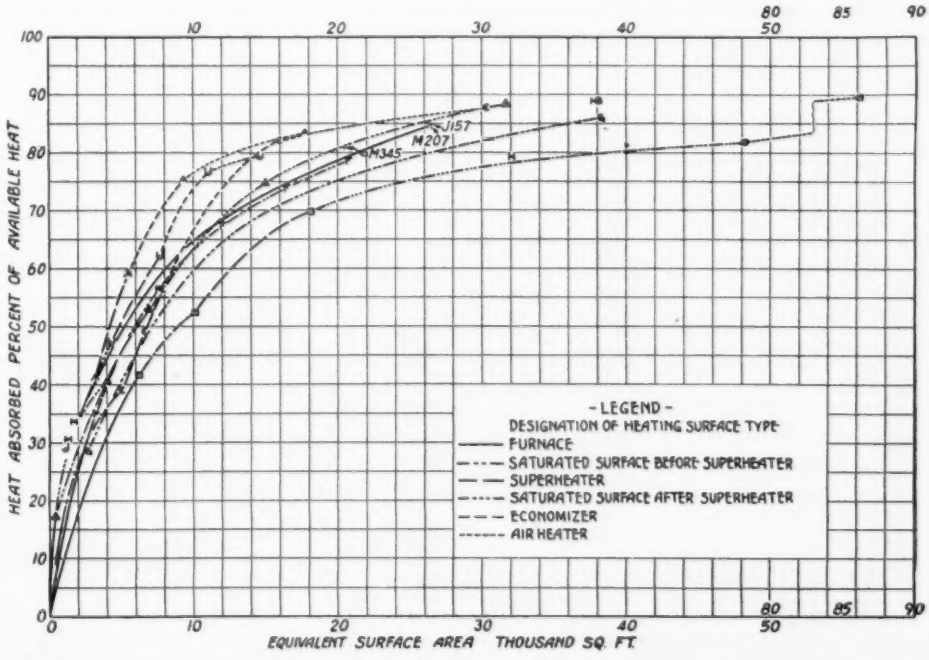


Fig. 16 Boiler Tests Showing Heat Absorption Plotted Against Equivalent-Surface Area

TABLE 7

	SYMBOL	⊡	△					J		M-207		M-345		H	HB
1	TEST	B-1			F-1			N.Y.C. J-1-B		P.R.R. M-1-A		P.R.R. M-1-A		5-9-02	5-20-02
2	BOILER VIEW ⇒	STATIONARY BOILER FIG. 2			MARINE BOILER FIG. 9			LOCOMOTIVE		LOCOMOTIVE		LOCOMOTIVE		HOCKING	VALLEY ER
3	RATED STEAM OUTPUT - LB/HR				61000			57700		67850		67850			
4	STEAM OUTPUT ON TEST - LB/HR				66070			84800		80000		99095		17830	17200
5	STEAM PRESS. AT SUPERHEATER OUTLET - LB/IN² GA				309			225		250		250		166	170
6	SATURATION TEMPERATURE - °F.				428			397		406		406		373	375
7	STEAM TEMPERATURE AT SUPERHEATER OUTLET - °F.				695									SATURATION	
8	FEEDWATER TEMPERATURE - °F.				211									635	724
9	AIR TEMPERATURE LEAVING AIR HEATER - °F.				121										
10	METHOD OF FIRING	PULV. COAL	MECHANICAL ATOMIZING OIL BURNER					STOKER		STOKER		STOKER		HAND FIRED	
11	KIND OF FUEL				OIL			BITUMINOUS COAL		BITUMINOUS COAL		BITUMINOUS COAL		BITUMINOUS COAL	
12	GRATE AREA OR FURNACE CROSS SECTION - SQ. FT.				71.7			81.5		70		70		31.08	30.10
13	LBS. FUEL PER HR. DIVIDED BY ITEM 12				70.3			157		207		345		94.5	84.7
14	HEATING VALUE OF FUEL "AS FIRED" - B.T.U./LB.				18625			13900 (DRY)		13900 (DRY)		13900 (DRY)		11838	12265
15	SECTION OF UNIT		FURNACE	[illegible]	SUPERHEATER	[illegible]	S.H. SECTION	FIREBOX	[illegible]	FIREBOX	[illegible]	FIREBOX	[illegible]	[illegible] BOILER	[illegible] BOILER
16			12.5				12.5							12.2	9.4
17	FLUE GAS ANALYSIS LEAVING SECTION { CO_2 % BY VOL.		4.1				4.1							6.1	9.7
18	{ O_2 " " "		0.0				0.0							0.7	0.01
19	{ CO " " "		12.3				12.3							1.40	1.83
20	TOTAL COMBUSTION AIR LEAVING SECTION - %		92200				92200							36800	38400
21	TOTAL WEIGHT WET GAS LEAVING SECTION - LB/HR.		960												
22	HEAT AVAILABLE ABOVE 80°F. ENTERING SECTION - BTU/LB GAS		6.3				6.3							4.9	4.4
23	MOISTURE BY WEIGHT IN FLUE GAS - %		3315												
24	ADIABATIC TEMPERATURE - °F.		94.1					180.8		208.5		333.6		34.9	31.2
25	TOTAL HEAT INPUT FROM FUEL - MILLION B.T.U./HR.		0.9												
26	TOTAL PREHEAT IN AIR ABOVE 80°F. - MILLION B.T.U./HR.		6.6					61.3		91.4		182.7		8.6	8.8
27	TOTAL LOSSES - LATENT HEAT; CARBON LOSS - MILLION B.T.U./HR.		88.4					119.5		117.1		150.9		26.3	22.4
28	TOTAL HEAT AVAILABLE ABOVE 80°F. - MILLION B.T.U./HR.		208000					340000		246000		318000		179000	155000
29	LIBERATION - ITEM 27 DIVIDED BY ITEM 39 - BTU/CU.FT./HR.		1163000	123100	71600	33200	13780	425000	18800	290500	18950	374000	24400	15500	13200
30	ITEM 27 DIVIDED BY ITEM 41 - BTU/SQ. FT./HR.		2820	2120	1732	1000	510							745	730
31	GAS TEMPERATURE LEAVING SECTION - °F.		88.4	74.75	52.55	41.85	22.50	119.5	84.9	117.1	78.1	150.9	104.9	26.3	22.4
32	HEAT AVAILABLE IN GAS ENTERING - MILLION B.T.U./HR.		15.12	20.7	10.7	19.35	12.4	34.6		39.0		46.0			
33	HEAT ABSORBED: INDIVIDUAL SECTION - MILLION B.T.U./HR.		15.12	35.82	46.55	65.9	78.3	34.6	101	39.0	98.7	46.0	122	20.8	19.9
34	HEAT ABSORBED: CUMULATIVE - MILLION B.T.U./HR.		17.1	23.4	12.1	22.0	14.0	29.0		33.5		30.5			
35	HEAT ABSORBED: INDIVIDUAL SECTION DIVIDED BY ITEM 27 - %		17.1	40.5	52.6	74.6	88.6	29.0	84.7	33.5	84.3	30.5	80.9	79.1	88.9
36	HEAT ABSORBED: CUMULATIVE DIVIDED BY ITEM 27 - %		199000	32300	20700	13550	7240	123000		97000		114000			
37	HEAT ABSORPTION: INDIVIDUAL SECTION - BTU/SQ FT HR.		199000	50200	37700	24800	14000	123000	15900	97000	15900	114000	19700	12300	11700
38	HEAT ABSORPTION: CUMULATIVE - BTU/SQ FT HR.														
39	EQUIVALENT HEAT RECEIVING SURFACE TEMPERATURE - °F.		424					353		475		475		147	144
40	FURNACE VOLUME - CU. FT.		76.0	6.40	5.16	1428	2939	281		403		403		[illegible]	[illegible]
41	SURFACES: INDIVIDUAL SECTION - SQ. FT.		76.0	7.16	1232	2660	5599	281	6356	403	6188	403	6188	1690	1697
42	SURFACES: CUMULATIVE - SQ. FT.		430	4060	6980	15000	31700	1176	26500	1718	26500	1340	20500	32100	37800
43	EQUIVALENT SURFACE AREA (500 A/0⁶ = ITEM 27) ÷ ITEM 41 - SQ. FT.						2070							813	828
44	VOLUME OF ENTIRE UNIT - CU. FT.						42700							32400	27000
45	ITEM 27 DIVIDED BY ITEM 43 - BTU./CU. FT. HR.						2.53							2.03	1.96
46	DRAFT LOSS - INCHES OF WATER						8.52							3.78	4.12
47	TOTAL DRAFT PLUS AIR PRESSURE - INCHES OF WATER														
48	OVERALL EFFICIENCY - %		16.1	38.1	49.5	70.0	83.2	19.1	56.0	16.7	47.3	13.8	34.6	93.6	83.9

DATA GIVEN IN BAILEY PAPER TEST B-1 TABLE-2 FIG. 3

the oil-fired marine boiler is outstanding, and further data as to the breakdown of this efficiency difference would be interesting.

Fig. 16 of this discussion includes a typical pulverized-coal-fired stationary boiler, as represented by test B-1, Table 2 of the paper,[16] and test F-1 for an oil-fired marine boiler from Table 3 of the same source. The latter has been calculated to show the percentage of heat absorbed in the different sections of the unit, although the official efficiency tests cover only the over-all efficiency, as is plotted in Fig. 10.[16] In addition to the data which were plotted from Pennsylvania Railroad class M1A, the author's Fig. 13, and the New York Central class J1B, Fig. 14, the writer has also gone back to the data of Goss's tests at Coatesville in 1912 and selected the one with the most representative maximum rating giving complete results on a coal-fired radial-

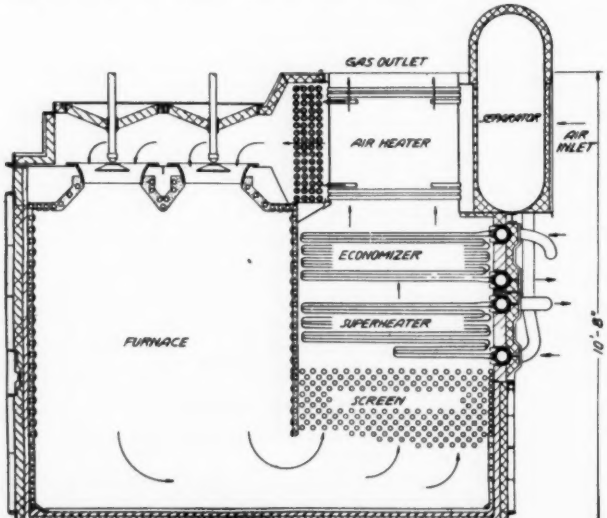


FIG. 17 OIL-FIRED STEAMOTIVE BOILER, SM5L

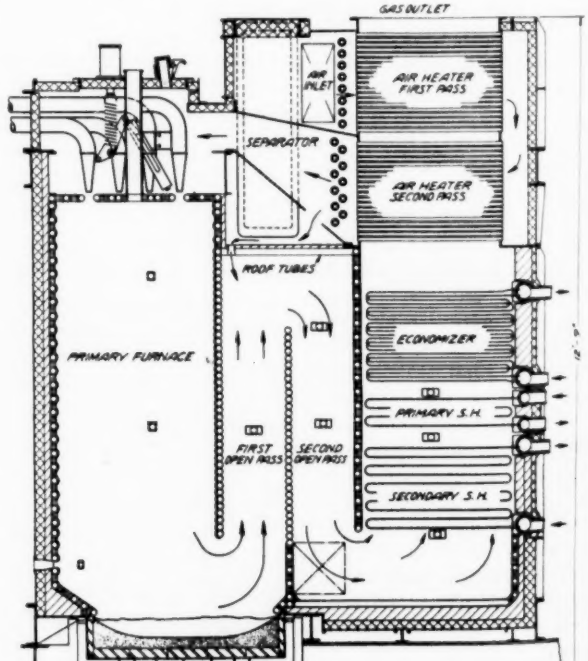


FIG. 18 PULVERIZED-COAL-FIRED STEAMOTIVE BOILER, SM4L

TABLE 7 (*Continued*)

	F	G		A				U				•					G					
	A-2	COATESVILLE A-112R		6-10-35				3-31-38				11-24-39 1ST HOUR OF RUN					11-24-39 16TH HOUR OF RUN					
	NORTHERN RAILWAY OF FRANCE FIG.1	RADIAL STAY LOCO.		STEAMOTIVE AG IL				STEAMOTIVE SM 5L FIG 17				STEAMOTIVE SM 4L FIG 18					STEAMOTIVE SM 4L FIG 18					
1				21000				40000				20000					20000					
2		28547		22050				44350				14100					13700					
3		214		1339				1476				1190					1150					
4		393		583				595				568					563					
5		SATURATION		914				903				869					901					
6		55		346				365				213					211					
7				493				362				533					565					
8		HAND FIRED		STEAM MECHANICAL ATOMIZING OIL BURNER				STEAM MECHANICAL ATOMIZING OIL BURNER				PULVERIZED COAL					PULVERIZED COAL					
9		HAND FIRED BRIQUETTES		BITUMINOUS COAL OIL				OIL				BITUMINOUS COAL					BITUMINOUS COAL					
10		5807		1225				230				65					65					
11		734		149				156				262					262					
12		13065		18470				18570				12965					12965					
13		FIREBOX	BOILER	FURNACE & SCREEN	SUPERHEATER	ECONOMIZER	AIR HEATER	FURNACE & SCREEN	SUPERHEATER	ECONOMIZER	AIR HEATER	FURNACE & SCREEN	SUPERHEATER	ECONOMIZER	AIR HEATER	FURNACE & SCREEN	FURNACE & SCREEN	SUPERHEATER	ECONOMIZER	AIR HEATER		
14			97				13.8				13.2				15.4						15.5	
15			8.8				3.1				3.6				2.8						2.6	
16			0.3				0.0				0.0				0.0						0.0	
17			168	115.7			115.7	120			120	114.5			114.5	113					113	
18	DATA GIVEN IN BAILEY PAPER TEST A-2 TABLE-1 FIG.1	56400	56400	30900				62600	62600	62600	62600	20200			20200	20100					20100	
19				1091				1037	560			1069				1084						
20		3.5		3.5				7.18				5.3			5.3	5.3					5.3	
21				3720				3510				3665				3705						
22		55.8		33.8				66.75				21.9				22.1						
23				2.93				4.03				1.42				1.50						
24		16.8		2.98				5.88				1.72				1.80						
25		39.0		33.75				64.9				21.6				21.8						
26		107700		374000				358000				385000				388000						
27		181600	13100	152600	90750	53400	27900	117200	65000	45300	34300	97700	74700	62700	30900	16320	98600	75400	63200	31200	16470	
28			615	2370	1730	1110	784	2025	1570	1000	891	2230	1900	1650	750	540	2440	2015	1810	815	584	
29		38.0		33.75	19.82			64.9	34.19			21.8	12.10	10.07	8.595	3.555	21.8	13.35	11.12	9.52	3.89	
30		11.1	22.44	13.93	5.94	5.46	2.75	30.71	12.28	6.58	1.89		2.03	1.475	5.04	1.13		2.23	1.6	5.63	1.22	
31		11.1	33.54	13.93	19.87	25.33	28.08	30.71	42.99	49.57	51.46	9.50	11.53	13.005	18.049	19.175	8.45	10.68	12.28	17.91	19.13	
32		28.5	57.5	41.2	17.6	16.2	8.2	47.3	18.9	10.2	2.9		9.4	6.8	23.4	5.2		10.2	7.3	25.9	5.6	
33		28.5	86.0	41.2	58.8	75.0	83.2	47.3	66.2	76.4	79.3	44.0	53.4	60.2	83.6	88.8	38.8	49.0	56.3	82.1	87.7	
34		51700	8120	63000	39400	21000	4760	55500	27600	15250	4090		29700	26650	14280	1806		32600	28900	16000	1950	
35		51700	11200	63000	53400	40000	23200	55500	43100	34600	27100	43000	39800	37800	25900	14480	38300	36900	35700	25700	14470	
36																						
37		362		90.4				181				56.2	(121.2 CU FT TO ENTRANCE OF SH)				56.2					
38		2147	2769	221.2	150.7	2600	5780	5532	4454	4324	4420		684	553	3530	6260		684	553	3530	6260	
39		2147	2984	221.2	371.9	631.9	12099	5532	9986	14310	18930	2210	2894	3447	6977	13237	2210	2894	3447	6977	13237	
40		2755	38300	3250	5470	9300	17800	4270	7700	11030	14600	5120	6700	7980	16150	30620	5070	6640	7900	16000	30380	
41			1340				425				677					5124					5124	
42			29100				79500				95800					42100					42500	
43							248				113					61					89	
44			6.4				44.9				29.7					16.10					16.6	
45		19.9	60.1	37.1	52.9	67.5	74.9	44.3	62.0	71.6	74.3	40.8	49.5	55.9	77.5	82.4	35.8	45.1	52.0	73.9	81.0	

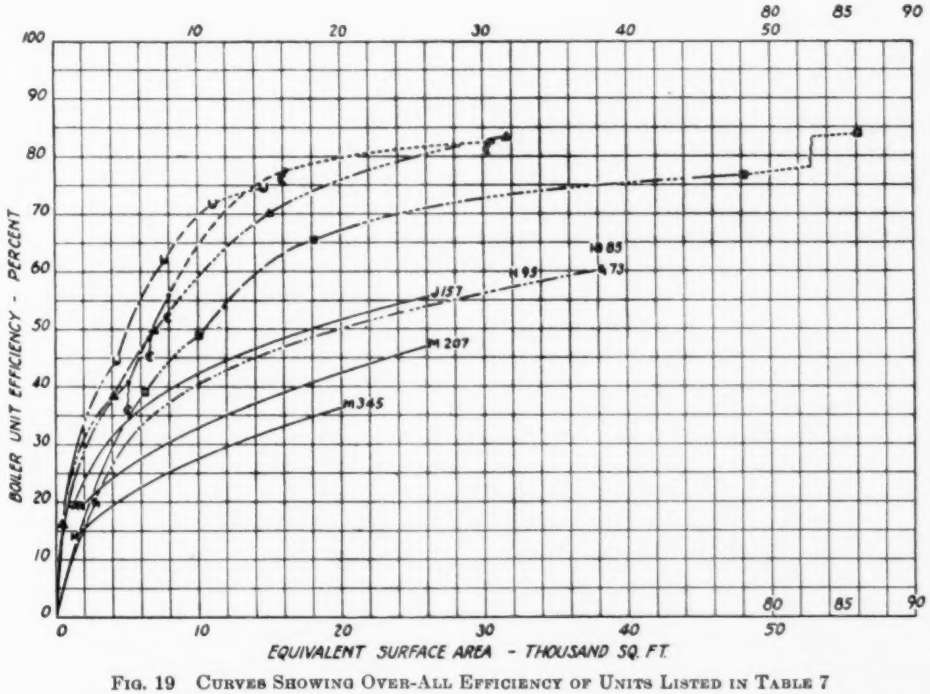


FIG. 19 CURVES SHOWING OVER-ALL EFFICIENCY OF UNITS LISTED IN TABLE 7

stay-bolt firebox, in which Dr. Goss determined the heat absorption in the firebox separately. From Hitchcock,[17] the writer has also selected two road tests on locomotives conducted in 1902, from which sufficient reliable data were obtained to plot the absorption efficiency and the over-all efficiency, but without measured distribution between the firebox and the flues. Both the Goss and Hitchcock tests were without superheaters. Also from the paper,[16] test A-2 of the French locomotive, with one half the tubes plugged, falls directly in line as to rate of output with the Pennsylvania class M1A at 345 lb of coal fired per sq ft of grate per hr. The other point of Pennsylvania class M1A plotted is taken to duplicate exactly the equivalent-surface area of the New

[17] "Road Tests of Consolidation Freight Locomotives," by E. A. Hitchcock, Trans. A.S.M.E., vol. 25, 1904, pp. 550–588.

York Central class J1B at its maximum rate. It is to be noted that they come closely together and are quite consistent, while there is an indication that the tests on the French locomotive with its small firebox showed somewhat less efficiencies.

Superposed on these curves are also data from the oil-fired Steamotive boiler as tested at Schenectady and reported in a paper[18] by E. G. Bailey, A. R. Smith, and P. S. Dickey; from the oil-fired Steamotive boiler on the Union Pacific steam turboelectric locomotive which has been described[19] recently; as well as from tests on pulverized-coal-fired Steamotive boiler, conducted at Erie, Pa., during 1939.

The Steamotive generators are high-pressure high-temperature units, as indicated in Table 7 of this discussion, and the slopes of the curves after the gases enter the superheater and economizer show very high rates of heat absorption due largely to the high mass flow and the suitable arrangement of the tubes with respect to gas velocity and gas temperature. A cross section of the Union Pacific oil-fired Steamotive boiler is shown in Fig. 17 and the pulverized-coal-fired unit in Fig. 18.

Fig. 19 gives data from most of the units listed in Table 7 on the over-all efficiency basis. This includes boiler, superheater, economizer, and air heater.

The pulverized-coal-fired stationary and Steamotive units, as well as the oil-fired marine and Steamotive units show only the normal difference between the absorption efficiency and the total efficiency, but the coal-burning locomotives show a marked diminution in efficiency due largely to cinder loss, which increases in general with the increased rate of firing. However, it is noted that the New York Central at 157 lb of coal fired per sq ft of grate per hr comes within the same range of over-all efficiencies as were obtained from Goss's tests at 73 lb of coal per sq ft, and the Hocking Valley tests at 85 and 95 lb, respectively.

In the author's Figs. 13 and 14, by means of dotted lines, he shows the heat absorbed in the firebox. His treatment of that subject in the text indicates definitely that he has the correct conception of the variation of this figure with rates of heat available, rather than the use of a constant figure of 55 lb of steam per sq ft of firebox per hr, which was evidently based on Goss's tests, as reproduced herewith. Fig. 20 expresses per cent of heat absorbed in the firebox as a function of the Btu available per square foot of actual heating surface. This gives data from Goss and the French locomotive tests as plotted points. It also gives a curve, calculated from the Hudson-Orrok formula[20] for which a basis of 13,900 Btu of coal with 11.8 lb of air per lb of coal, has been selected, which corresponds closely to 19 per cent excess air and 3600 F adiabatic temperature. Broido's average curve also is shown within the range in which it is given in his paper[21] before the Society. It would seem from this that the author's dotted lines of Figs. 13 and 14 may be somewhat high, and the curves of Figs. 16 and 19 of this discussion are plotted on the basis of being taken from the Orrok curve. One could readily plot in Fig. 16 the curves from Brandt's data as shown in Fig. 20. This would indicate a relatively higher rate of heat absorption in the firebox than seems to exist on other types of furnaces and, even at the same rates of Btu available, they are markedly higher than is indicated by Goss and the French locomotive, both of which were based upon actual weights of water evaporated by the firebox surface.

It is doubtful if this question of distribution of heat absorption between the firebox and the flues of a locomotive boiler can be accurately determined by gas-temperature measurement. The difference in the size of the flues, the resistance, gas velocity, and stratification of both quality of gas and temperature make it difficult, even though a device could be used which would get the true gas temperature. The cold surroundings of the tube sheet and flues will influence any thermocouple readings, even with the high-velocity method, to the extent that the data would be questioned. Therefore, it seems that the method used by

[18] "Steamotive," by E. G. Bailey, A. R. Smith, and P. S. Dickey, *Mechanical Engineering*, vol. 58, December, 1936, pp. 771–780.

[19] "Progress in Railway Mechanical Engineering, 1938-1939," Report of Committee RR6, Railroad Division, A.S.M.E., *Mechanical Engineering*, vol. 62, December, 1939, pp. 863–875.

[20] "Estimation of Radiant-Heat Exchange in Boiler Furnaces," by G. A. Orrok and N. C. Artsay, *Combustion*, vol. 9, no. 10, April, 1938, pp. 37–42.

[21] "Radiation in Boiler Furnaces," by B. N. Broido, Transactions A.S.M.E., vol. 47, 1925, pp. 1123–1147.

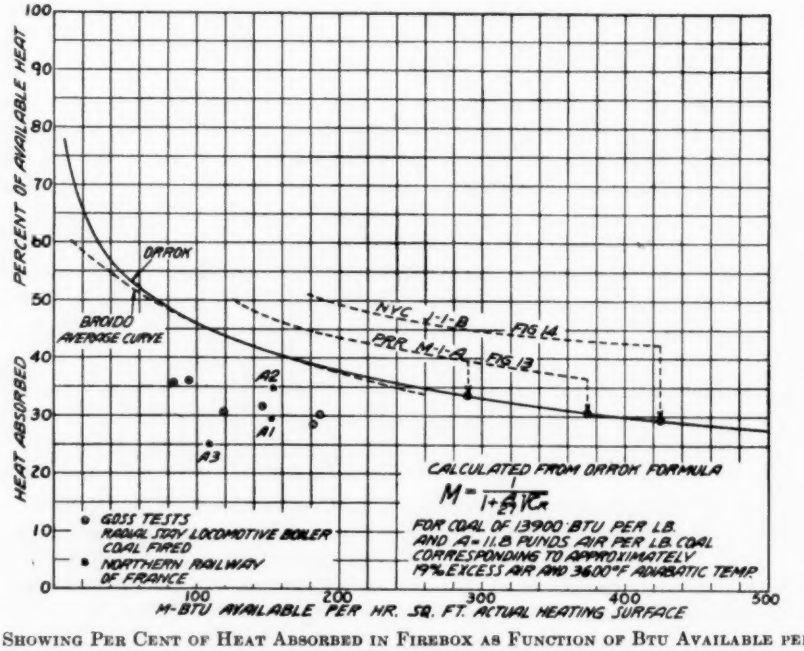


FIG. 20 CURVE SHOWING PER CENT OF HEAT ABSORBED IN FIREBOX AS FUNCTION OF BTU AVAILABLE PER SQUARE FOOT OF ACTUAL HEATING SURFACE

Goss and the French locomotive may be the only one in which such distribution can definitely be determined. Knowledge of this, however, only affects the detailed slope of the curve at the lower portion, when plotted on the equivalent-surface basis. The upper points on both the absorption-efficiency basis and over-all efficiency basis are equally reliable from well-conducted tests and show the important part of the over-all efficiency and the rate of heat input.

The author brings out clearly the limited area for gas flow entering the tubes and flues of the locomotive boiler. On a water-tube arrangement of heating surface, such as used in the Steamotive, this area can be made anything desired within reasonable limits. The designs now operating have more than twice the area on a given capacity basis than is possible on the locomotive units as shown by the author.

A further study of draft loss through the respective units is also of interest. The different tests in the writer's paper[16] give the over-all draft loss including the air heater, if any, burners, or fuel bed, draft loss through boiler settings, superheater, economizer, and air heaters, if any, as the total pressure drop. It would be helpful if in his closure the author could supply the data in a form which might be compared more readily than that as given in Fig. 5, showing back pressure.

J. M. DUNCAN.[22] The author reminds us that, in current locomotive practice, combustion rates and heat releases are far higher than any realized commercially in stationary plants. If industrial and central-station heat-power engineers could approach these rates, they would advance their art by materially reducing capital costs.

Pennsylvania Railroad locomotive class M1A appears to be designed for a heat liberation of 250,000 Btu per cu ft of furnace volume per hr, burning 121 lb of coal per sq ft of grate area per hr. Figs. 8 and 12 of the paper indicate that under these conditions combustion is substantially complete, the dry flue-gas loss being moderate, but the unburned-coal loss is 30 per cent. The furnace heat absorption is 41 per cent of the heat liberated, and the average heat absorption in the furnace is about 87,000 Btu per sq ft of furnace walls per hr. Data on furnace temperatures and excess air are not given. The writer calculates the exit furnace temperature at 1800 F and the excess air at 70 per cent. These figures may be considerably in error as essential data are lacking. Unburned-coal losses for other locomotives are above and below this 30 per cent figure at the same heat-release rate. At this rate, New York Central locomotive J1B has a loss of 12 per cent.

The author's objective is to develop means for increasing locomotive capacity and efficiency. Any increase in efficiency accomplished within existing weight and space limitations will, if present fuel-burning ability is maintained, automatically increase locomotive capacity. Two opportunities for increased efficiency lie in reduction of dry-flue-gas loss and reduction of unburned-coal loss. As pointed out by the author, convection-heat transfer is efficient. Improvements must be sought in furnace conditions.

If the underfeed or traveling-grate stoker can be adapted to locomotive service, will not the unburned-coal loss be automatically almost eliminated? The very fact that such large losses are tolerated indicates that the sprinkler type is accepted as indispensable for the large modern locomotive. But, if these other types can be adapted to this service, important advantages will be realized in addition to reduction of unburned-coal loss. This is principally because with these other types furnace absorption efficiency can be greatly increased.

An increase in furnace-absorption efficiency would reduce dry-gas loss almost proportionally if excess air is unchanged. If the increase is accomplished while reducing excess air, a further gain will be made. The following relations are calculated for a New River coal, assuming complete combustion with no air preheat:

Furnace exit, F	Furnace absorption efficiency, per cent	
	20 per cent excess air	80 per cent excess air
1500	63	46
2000	49	26
2500	34	4

These figures indicate that excess air cannot be materially reduced without incurring slag accumulations in flues with many coals which must be burned on railroads, unless furnace absorption efficiency can be greatly improved. The writer believes that the improvement must be brought about by a radical change in stoker design. This appears to be the author's conclusion when he says:

"From the studies made of this problem it is doubtful if the percentage of heat absorption can be increased to much more than 50 to 55 per cent of the heat liberated, by increasing the amount of firebox heating surface, because the amount of radiant energy absorbed is limited by the effective surface of the fire bed and the luminosity of the flames."

In regard to the effect of smoky flames the author says:

"It has been demonstrated that the radiation from the flames is largely due to incandescent particles of fuel and ash in the flame. The quantity and distribution of this radiating material in the flames, together with the amount of firebox heating surface, determine the coefficient of emission or absorption. This explains why boilers fired with oil give higher superheat; the less luminous flames of the oil fire will give up a smaller percentage of heat to the firebox by radiation, and the gases entering the flues are therefore hotter."

These remarks doubtless apply to a comparison of pulverized-coal and oil flames. But the facts cited tend to divert our attention from the controlling influence of luminous flames in reducing over-all radiant-heat absorption in stoker-fired furnaces wherever the grate is not obscured by an arch. The luminous flame will, it is true, lose more heat than the nonluminous flame (if high turbulence is maintained) but it will at the same time hide the incandescent and much hotter stoker fire from the heat-absorbing walls to a great extent. The author's remarks do apply to the stoker-fired-locomotive furnace shown in Fig. 3, where the luminous flames created by the sprinkler-type stoker are of advantage because that stoker requires an arch to reduce the unburned-coal loss. In such a furnace clear flames would be a disadvantage. Data of Fig. 3 do in fact indicate the presence of highly luminous flames. The velocity at section C is materially lower than at section B although the gas areas are practically equal. If flames were completely transparent, temperatures would be nearly equal throughout this section of the furnace.

The importance of clear furnace atmospheres in underfeed-stoker-fired furnaces was forcefully impressed upon the writer when observing some tests using a high-volatile coal. Operators had been unable to obtain smokeless combustion at moderately high rates. Boiler exit temperatures were higher than they should have been although excess air was less than guaranteed. A test operator then took charge for higher rate tests. While maintaining the same percentage excess air as before, he produced a transparent furnace atmosphere at considerably higher combustion rates and the excess boiler-exit temperatures were reduced. It was obvious that a large increase in the percentage of furnace heat absorption had been obtained. This must have been due to the removal of smoke and luminosity from the flames, thus allowing the fuel bed to "see" the furnace waterwalls and roof.

[22] Petersburg, Va. Mem. A.S.M.E.

During those tests it occurred to the writer that furnace heat absorption and over-all boiler performance could have been increased in spite of smoky furnace atmosphere if the "hydraulic mean radius" of the furnace were reduced by installing tubes spaced, say, on three- or four-foot centers throughout the upper portion of the furnace. (The vertical screen used to divide many modern powdered-coal furnaces, e.g., Fisk Street, appears to be a development of this sort, perhaps made more necessary by the essentially opaque nature of the flames in such furnaces.) The degree to which this could be carried out without prejudice to the furnace function depends upon the effect upon combustion. Has the author data upon the relation between furnace temperature and completeness of combustion—of carbon and hydrocarbons as well as of CO?

The sprinkler-type stoker seems inherently incapable of realizing high radiant-heat absorption. Its fuel bed is probably lower in surface temperature than that of the underfeed or traveling-grate type. Heating of green coal should not be spread over the fuel bed if a hot surface and a smokeless furnace atmosphere are desired. The other types of stokers confine the distillation of green coal to a small section of the fuel bed and so leave a large area of highly incandescent coke to radiate to the furnace walls; they also produce less smoke and consume it more rapidly. Such stokers do not require the arch shown in Fig. 3, which completely obscures the fuel bed from most of the furnace walls even if smoke is not produced.

Furnace-gas turbulence has been recognized as desirable to assist in attaining complete combustion. The writer believes it should be sought also in order to insure maximum radiant heat transfer at all times that an absolutely clear furnace atmosphere is not maintained. Evidently the gas velocities shown in Fig. 3 bring a high degree of turbulence. The temperature drop indicated by velocity drop is several times what could be caused by convection heat transfer. Smoky flames require high turbulence successively to turn new envelopes toward cold furnace walls if high radiant heat transfer is to be realized. The importance of turbulence therefore is great even if combustion (of CO) is complete without it, and especially where small excess-air ratios tend to cause hazy or smoky conditions in the furnace. But the sprinkler type of stoker has some critical rate of furnace-gas turbulence above which unburned-coal losses become prohibitive. Turbulence in furnace gases would not adversely affect the underfeed or traveling-grate stoker in this respect.

Thus the underfeed and traveling-grate stoker appear to be capable of increasing locomotive efficiency, and therefore capacity, by increasing furnace-absorption efficiency and reducing unburned-coal loss. Stationary-plant experience does not yet extend economical coal-burning capacities to the rates evident from an inspection of the author's Tables 1 to 6. However, as the author has noted, furnace-wall area is not the controlling factor in radiant heat transfer. The controlling factor is the area of the hot body, and even more, its temperature. Thus a larger grate area could with advantage be adopted, sacrificing furnace-wall area as necessary to obtain it, provided the furnace wall could see the fuel bed so installed. This larger grate area would also be helpful in reducing such slight tendency as there would be (slight, that is, compared to unburned losses now accepted in locomotive operation) for fuel to blow off the grate. In this connection, it would be interesting to know whether the higher combustion efficiency shown in Fig. 8 for locomotive NYC-J1B is due to the lower rate of coal combustion per square foot of grate area, which is inferred from the data given in Table 1.

It is suggested that a design for high efficiency and capacity should aim at:

1 Feeding coal in such manner as to throw no fines up into the furnace atmosphere

2 Confining distillation of volatile constituents to a small area of the fuel bed.

3 Maintaining a large incandescent coke area within full view of the furnace walls.

4 Operating at high rates with about 20 per cent excess air, with little or no smoke and a nearly nonluminous flame.

5 High turbulence in the furnace, with furnace designed for smallest practicable volume-area ratio to increase furnace heat absorption if smoky flames cannot be avoided.

Such a design should operate with little or no unburned-coal loss and with very low dry-flue-gas loss. If applied to locomotive PRR-M1A a reduction in fuel consumption of over 30 per cent should be realized. Designed capacity would in that event be realized with a fuel-burning rate of about 80 lb per sq ft of grate area per hr. Reduction to this moderate rate would, as shown by the author's data, considerably reduce engine back pressure, still further increasing efficiency and capacity. These possibilities are undoubtedly known to locomotive designers. The writer would appreciate knowing the principal difficulties which have stood in the way of their realization so far.

The author's paper suggests that there is a great opportunity for the application of rational design methods to locomotive-boiler practice. It is evident that a large amount of data awaits systematic analysis, following which the several combinations of furnace, stoker, and boiler indicated by that analysis as worthy, could be tested at moderate cost to achieve a fine commercial design.

R. EKSERGIAN.[22] Not only has the author given a comprehensive contribution on locomotive-boiler characteristics and limitations, but he has pointed out some pertinent factors which have not been clearly recognized before. One outstanding factor is the tremendous influence of increased gas area through the boiler on evaporation capacity and its tendency to sustain fair efficiencies through a wide range of firing rates. The slope of the boiler efficiency against firing rate is particularly significant as to the maximum evaporation capacity. On the basis of a linear variation of boiler efficiency against firing rate, we must necessarily have a parabolic evaporation curve against firing rate. The maximum evaporation occurs, then, at a firing rate at which the boiler efficiency is just one half the efficiency at zero firing rate. This means that, to obtain high firing rates at maximum evaporation, the slope of the efficiency curve against firing rate must be flat. Pennsylvania Railroad locomotive M1A shows this to a predominant degree and, with this characteristic, we note the corresponding large gas area through the boiler as compared with some other types.

One major feature of the advantage of the steam locomotive over all other types of motive-power units is its considerable overload capacity. Locomotives under normal operation do not perform at firing rates corresponding to their maximum evaporative capacity. The normal rating may either be limited to a definite over-all efficiency or to a firing rate corresponding to some percentage of the firing rate at maximum evaporation. In a well-designed boiler, both aspects are compatible. Mr. Fry[7] assumes the boiler efficiency to vary as a straight line against firing rate and arrives at a parabolic curve for the evaporative capacity against firing rate. This assumption is well borne out in this paper, Figs. 4 and 10. On this basis if F is the over-all boiler efficiency and x the firing rate in pounds of fuel per hour

$$F = m - nx$$

and, if H is the heating value per pound of fuel, the total evaporation is

[22] Consulting Engineer, Edward G. Budd Manufacturing Company, Philadelphia, Pa. Fellow A.S.M.E.

$$E = KFHx = KH\,(mx - nx^2)$$

The maximum evaporation occurs at a firing rate x_m such that $\frac{dE}{dx} = 0$; that is, $m - 2nx = 0$

$$\therefore\ x_m = \frac{m}{2n} \quad \text{and} \quad E_{max} = K\,\frac{Hm^2}{4n}$$

If the normal firing rate is taken at some percentage k of the maximum firing rate x_m, then $x = kx_m = \frac{km}{2n}$ and, since the ratio of normal to maximum firing rate is

$$\frac{E}{E_{max}} = \frac{mx - nx^2}{m^2/4n}$$

$$\therefore\ E = (2 - k)kE_{max}$$

With $k = 0.5$ then $E = 0.75\ E_{max}$. If $k = 0.66$, then $E = 0.89\ E_{max}$. This shows that firing rates exceeding 50 per cent of the firing rate, corresponding to maximum evaporation, rapidly approach the maximum capacity of the boiler.

Another limitation of firing rates is dependent upon maintaining a fair over-all boiler efficiency F. In fact, if we limit this over-all efficiency to F_0, then

$$x_0 = \frac{m - F_0}{n}$$

$$\therefore\ k = 2\left(1 - \frac{F_0}{m}\right)$$

which gives the per cent firing rate relative to that at maximum evaporation corresponding to the limited efficiency F_0.

For example referring to the author's Figs. 4 and 10, the K4S locomotive, which is representative of good average performance and, with a boiler efficiency limited to 65 per cent, the corresponding firing rate is roughly 50 per cent of the maximum. Considering this as a normal firing rate, the maximum evaporative capacity is roughly 33 per cent greater. But with this overload the boiler efficiency rapidly falls.

With Pennsylvania Railroad locomotive M1A, the relatively small slope permits a large overload capacity but at the expense of relatively poor efficiencies at the lower firing rates when compared with the K4S, and similar types.

It would seem, therefore, that boiler proportions which permit higher over-all efficiencies than are found in the M1A, but with less slope of the efficiency-firing-rate curve as with the K4S type, are more desirable, thus giving higher efficiencies at normal firing rates with greater overload evaporative capacity.

It is of interest to note that, in normal locomotive operation, the heat in fuel fired per cubic foot of firebox volume is in the order of 250,000 Btu per hr. In the M1A this reaches over 650,000 Btu per hr at a combustion efficiency of 50 per cent. The Velox boiler attains the latter value at a combustion efficiency around 90 per cent, by means of injecting the combustion air under pressure. The pressure in the firebox is in the order of 30 lb per sq in., effected through a compressor driven by an exhaust-gas turbine. It appears that, in the orthodox locomotive boiler, greater combustion efficiency can be obtained through turbulence of the injection air above the grate as suggested by the author.

In the conclusion to the paper, he cites that it is the engine which deserves the most attention as its thermal efficiency is so low that even a slight improvement is important. This is true from two aspects, engines can work on improved cycles, i.e., with higher pressures and corresponding temperatures, provided throttling losses can be reduced by more efficient cylinder design, and by improved cylinder performance at high speeds.

On the other hand, it is important to point out that, based on the availability of steam supplied to the engine at orthodox boiler pressures, engine efficiencies are remarkably high. The following analysis of cylinder performance with orthodox boiler-pressure and temperature limitations is of interest. The data relate to a 4-6-2 type locomotive with cutoff at 25.5 per cent.

The steam conditions are given in the following table:

Pressure, gage, lb per sq in.	Temp, F	Superheat F	Enthalpy	Entropy
212	748	356	$H_1 = 1395$	$\phi = 1.727$
12	361	117	$H_2 = 1220$	$\phi = 1.782$
	$H_1 - H_2 = 175$		$\phi_1 - \phi_2 = 0.055$	

Steam consumption:

$$\frac{2545}{H_1 - H_2} = 14.5 \text{ lb per hr}$$

Isentropic expansion:

$$\left.\begin{matrix} H_1 = 1395 & H_2' = 1175 \\ \phi_1 = 1.727 & \phi_2' = 1.727 \end{matrix}\right\} \quad H_1 - H_2' = 220$$

Thus, the cylinder efficiency based on the available energy supplied is, $E_c = \frac{175}{220} = 80$ per cent; the cylinder losses amount to 20 per cent of the available energy across the engine. However, these losses have a much smaller ratio, when considering the availability of the power plant as a whole, based on the availability of the combustion gases. The largest loss from an availability aspect occurs in the locomotive boiler itself due to the large differences in flame and gas temperatures and steam temperatures in orthodox locomotives. The largest increase of the unavailable energy components is due to heat transfer. Raising the boiler pressure and thereby the steam temperature to pressures exceeding 600 to 700 lb per sq in. reduces this loss, and results in marked improvement in over-all thermal efficiencies.

Thus, cutting down the unavailable heat-transfer losses in the boiler really amounts to increasing the Rankine efficiency of the engine by operating on a higher pressure-temperature cycle. To accomplish this in any marked degree requires a radical change in boiler design and cylinder design as well, such as including compounding and other features, resulting in a greatly modified type of locomotive.

Another opportunity for increasing the over-all availability is in lowering the back pressures at the entrance to the exhaust. The author has pointed out the significance of this feature in connection with the boiler design.

J. R. JACKSON.[24] The performances of the steam boiler as described by the author are predicated on the evaporation of water into steam and involve the factors of fuel and water. In locomotive practice, the control of the grades of fuel supplied is fairly well in hand on most railroads, but the supply of water, as regards the foaming characteristics, is largely as nature supplies it. While it is common practice to treat locomotive-boiler waters chemically to reduce scale and corrosion within the boilers, no practicable method of treatment for the control of foaming within the boiler is known. So-called antifoam compounds aid somewhat in foam control with certain waters, and it is common practice to control foaming by blowing boilers at terminals and on the line. However, in territory west of the Mississippi River, we experience a seasonal foaming condition of the surface boiler-water supply which constitutes a serious operating problem.

[24] Engineer of Tests, Missouri Pacific Railroad Company, St. Louis, Mo. Mem. A.S.M.E.

It was for the purpose of making some studies of the actual conditions of foaming within a locomotive boiler that the Missouri Pacific Railroad in collaboration with the Electro-Chemical Engineering Corporation undertook a series of tests and observations at St. Louis, in August, 1939. For these tests a locomotive boiler was fitted with bull's-eye sight glasses in the dome cover and the interior of the boiler was illuminated with special 1000-w electric lamps to permit making observations and taking motion pictures of the interior of the boiler within and immediately under the dome. A series of standing locomotive tests with different arrangements of steam driers and foam-controlling devices was carried out. The motion-picture record disclosed the following:

(*a*) The relatively rapid movement of the surface water in the boiler in a forward direction, the rate of movement being more or less proportional to the rate of evaporation.

(*b*) The swelling of the surface water with the opening of the main throttle.

(*c*) The development of a spongy or bubbly condition or state of the surface water accompanied progressively by a decrease in the steam space under the roof sheet at the dome entrance; the increase in the velocity of steam movement from the firebox end of the boiler toward the dome; the picking off of the expanded or frothy surface water at the dome restriction; the actual splashing of the water particles and foam carried by the high-velocity steam against the dry-pipe extension and up into the dome against the dome cover; and finally the actual carry-over of water and foam into the dry pipe.

(*d*) The instantaneous subsidence of the expanded and foamy surface water within the boiler with the closure of the main throttle.

(*e*) The control of foaming and prevention of carry-over through the medium of overflow into a foam-collapsing trough, located within the boiler at the surface of the water immediately under the dome and extending along the center line of the boiler toward the rear, and discharge from the trough through an electromatic blowoff connection to the outside of the boiler.

The standing tests were later supplemented by road tests between St. Louis and Kansas City during which additional motion pictures were taken of the interior of the boiler.

L. B. Jones.[25] As a matter of interest, the writer will review the history of the development of the boiler referred to in the paper as the Pennsylvania M1A, which is shown to have exceptionally high evaporation capacity.

The first of these boilers to be built was not successful, so much water being carried over with the steam that no superheat was obtained. After some experiments, it was found that the performance was considerably improved by lowering the water level, the indications being that, if the steam space could be increased sufficiently, the trouble would be overcome. The boiler was accordingly redesigned, with a lower crown sheet and the dome barrel course raised slightly. The steam space was increased from 150 cu ft to 199 cu ft which, expressed in per cent of the total cubical content of the boiler, amounted to an increase from 16.5 to 21 per cent. The total evaporative heating surface was reduced from 4499 sq ft to 4319 sq ft, and the superheating surface was reduced from 2283 sq ft to 2052 sq ft. In spite of these reductions in surface area, the evaporative capacity of the boiler was increased, and a steam temperature of 730 F was maintained at very high rates of evaporation.

The author's comments on the necessity for liberal fire-tube area are deserving of special attention. It is pointed out that the energy required to move the furnace gases through the flues increases with the fourth power of the weight velocity, which involves the expenditure of power in the shape of cylinder back pressure at the exhaust nozzle. It is also pointed out that a reduction of 1 lb back pressure is equivalent to an increase of 4 lb in steam-chest pressure for average rates of working. When we consider the relatively low value of tube heating surface as compared with firebox heating surface, it becomes evident that the locomotive as a whole can be materially benefited by an attack on the space between the front and back flue sheets, in a way which will provide greatly increased flue-gas area, with a possible accompanying increase in superheat area, without doing any damage because of reduced tube evaporating surface, which is not needed.

We are inclined to take issue with the author on one point, namely, that the gas area can be increased at the expense of water space in the water legs of the firebox. While many boilers are operating with water space as low as 5 in., we believe that boiler-maintenance experience, compared with boilers having more liberal water legs, will eloquently support the wider space, with its accompanying longer stay bolts and reduced angularity of flexure. Important as flue-gas area is conceded to be, it should be obtained in other ways than by sacrificing water space along the sides of the firebox.

In Figs. 6 to 10, inclusive, the author has tabulated some interesting data on boiler and combustion efficiencies. Considerable danger is involved in comparing boiler efficiencies from tests made at different times and places, and with different fuels, especially at the high rates of combustion necessary in a locomotive boiler. This is particularly true of the various Pennsylvania locomotives reported. The test data for all these locomotives were obtained at the locomotive test plant, but the tests were made at different times, using different methods of firing, and somewhat different fuels, with the result that the relative boiler efficiencies plotted in the diagrams do not faithfully reflect the true characteristics of the boilers. We have found that, when it is desired to compare conditions involving boiler efficiency, it is necessary to confine the source of fuel to one mine room or entry, and otherwise safeguard the handling of the fuel from the mine to the locomotive firebox. Two fuels of equivalent Btu content may show widely different boiler efficiencies in the same boiler because of texture and friability, as well as other variables.

W. G. Knight.[26] The author's deductions as to the limitation of existing methods for calculating boiler efficiency are certainly correct. The writer feels that Mr. Fry's formula[7] would no doubt be adequate if more data were available for the value of the constants involved.

We have run standing tests on the Bangor and Aroostook Railroad which would bear out the New York Central contention that front-end design and nozzle size can be tested in this manner and properly adjusted for road service.

The curve in Fig. 2 of the paper is interesting and has special value due to its being drawn from data concerning so many engines. Boiler diameter and gas area, of new power especially, could be more clearly considered after a study of data of this kind.

When considering the distance from the top of the crown sheet to the boiler shell, it is advisable to do everything possible to increase heating surface in order to produce maximum boiler capacity and keep within the weight limitations. However, because of the general tendency of engineers to carry high water rather than low, it is the writer's belief that too high a crown sheet will decrease dryness of the steam, lower the superheat, and otherwise affect lubrication and general performance.

We have found on the Bangor and Aroostook that a ratio of

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[26] Mechanical Superintendent, Bangor and Aroostook Railroad Company, Derby, Me.

from 5 to 6, firebox volume to grate area, is decidedly advantageous as the road characteristics demand high boiler capacities for freight service. It is also possible with large fireboxes to adjust the arch to decrease the high velocity of gases at point *A* in Fig. 3 and bring them down to a value equal to or below the velocity in the tubes near the back tube sheet.

During the last 5 years, we have tried several designs and a number of different percentages of air openings in grates, our latest experiment being made on an application with 60 per cent free air opening. As yet, we have not definitely proved its advantage since we are still changing the draft appliances and ashpan arrangement to achieve better combustion efficiency. We feel that a gain will result from using this large percentage opening.

Much has been said in the last year or so on the subject of secondary air for combustion. Whether it would aid the older locomotives without considerable change in their design is a question. It would seem at present that equal efficiency can be obtained by an increased grate area and proper changes in draft appliances.

The general trend of all roads recently has been toward much more efficient draft appliances. We have tried to keep pace with the ever-increasing demand for more horsepower by improving front-end design and decreasing back pressure. The improvements made along these lines have produced satisfactory results.

A. I. Lipetz.[27] The locomotive boiler as part of the steam locomotive has the value advantage of automatic self-regulation; in other words, to a certain limit it adjusts its evaporation to the demand of steam. This automatic feature is actuated by the back pressure from the engine. For a given back pressure the amount of steam exhausted by the engine through the nozzle and the draft of the boiler is fixed. A certain proportion of the combustion gases is then pushed through the flues and tubes of the boiler from the firebox into the front smokebox. This evaporates a certain amount of steam, which in turn is exhausted by the engine. Thus the cycle is completed and then repeated. If the speed and cutoff of the engine are varied, in accordance with the demand upon the locomotive, the back pressure and the evaporation are varied—this is the automatic self-regulation feature.

By assuming a certain back pressure and the resulting draft, and figuring the amount of fuel burned in the firebox, it would be possible to calculate the amount of heat which is thus generated. Being familiar with the laws of heat transmission, and depending upon the gas velocities through the various sections as explained by the author, it would be equally possible to evaluate the amount of steam evaporation. For another back pressure and another fire rate, a different amount of steam evaporation would follow; thus the curve of steam evaporation, as a function of coal firing and back pressure, could be established.

It is evident that this way of figuring, although logical in itself, would involve a great many calculations on the basis of laws of natural phenomena, which are not always known. It is therefore simpler and more reliable to establish these curves by tests with locomotive boilers and to plot the curves, as has been done by the author in Figs. 4 and 5 of the paper. Likewise from tests, the combustion efficiencies can be plotted, the boiler and superheater over-all efficiencies can be figured, and, for further study, depending upon the length of tubes, mean hydraulic depth, etc., the heat absorption and various other factors can be established.

The value of this paper consists of giving us concisely a complete series of curves from reliable tests as material for study. The author did not attempt to draw any conclusions except to say that the present locomotive boiler is a very efficient steam generator, when compared to stationary boilers, especially taking into consideration its simplicity and weight.

However, the writer would like to call attention to the possibilities obtainable from a proper review of the author's curves. From the foregoing statement of the automatic regulation of the boiler cycle, it is obvious that, in application to a locomotive boiler with the same principal fundamentals of design, the efficiency and production of the boiler should be more or less uniform, depending upon certain ratios. These can be established from tests. In analyzing Figs. 7, 9, and 10, especially the latter, it is evident that the efficiency curves are straight lines, as is well known from the investigations of Professor Goss and Mr. Fry.[7] Fig. 12 is still more interesting because it narrows the system of straight lines for a great variety of locomotive boilers tested in Altoona and in actual service on different roads. It also brings out the fact that the Pennsylvania M1A locomotive seems to be represented by a line about the average of all others.

This, together with Tables 1 to 6 given in the paper, permits making a comparison of the various ratios tabulated therein. Thus it can be seen that item 31, which is the gas area per square foot of grate, varies for these locomotives and for these locomotives only, between narrow limits of 1.14 and 1.41 (columns 3 and 4 in Table 1; columns 1, 6, and 10 in Table 2; column 10 in Table 4; columns 6, 7, and 9 in Table 5). For the class M1A locomotive it is 1.39. Further, the ratios of firebox heating surface to the total evaporating heating surface (item 40), of the total evaporating heating surface in relation to the grate area (item 42), and of the superheater heating surface in relation to the total evaporating heating surface (item 43), vary also between limits which represent the best results of American practice.

That these ratios are important and essential has been known for a long time by a great many investigators of American and European locomotives.[8,28,29] They are usually included in articles describing new locomotives. This paper provides an opportunity to study these figures and to establish such ratios as we might consider to be essential. Tables 1 to 6 will permit establishing rules at least for the following boiler ratios: The ratio of the gas area through the boiler to the grate area; the firebox heating surface to the total evaporating heating surface; the total evaporating heating surface to the grate area; the superheater heating surface to total evaporating heating surface.

Of course, variations in these ratios are always necessary, depending upon the grade of fuel and local conditions. However, the writer does not believe that the difference in local conditions justifies the great variations in locomotive design which prevail. Certainly, a certain degree of standardization can be introduced for locomotives on American railroads.

T. C. McBride.[30] This paper indicates the inconsistencies in the proportions of existing boilers and demonstrates the need for further tests to establish the best proportions. The Cole method[3] for determining the evaporation to be expected from locomotive boilers, having the evaporation values based only on heating surface, is condemned as inadequate. Nevertheless, it is stated that "today these values are generally used by the locomotive builders and railroads." With no comparable substitute method in sight, an effort should be made to adapt the old and almost universally used method to present conditions.

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[28] "Die Eisenbahntechnik der Gegenwart," by E. Brückmann, Berlin, 1920, Appendix.

[29] "Handbuch des Dampflokomotivbaues," by Prof. Dr. Igel, Berlin, 1923, pp. 73–86.

[30] Consulting Engineer, Railway Division, Worthington Pump & Machinery Corporation, Harrison, N. J. Mem. A.S.M.E.

It is necessary to reach an understanding of the various terms used throughout the paper to indicate capacity, evaporation, etc. Referring to the first table of the paper, the Strahl formula need not be considered because it is not used in this country. The "Cole evaporation" of this table might be defined as Cole's conception of the maximum evaporation which boilers of about the year 1910 could be depended upon to develop when in operation on the road. Dr. Lipetz, in his 1932 paper[5] on locomotive ratios, endorsed such a definition but applied it to boilers built 20 years later. It is generally conceded now that this evaporation presumed by Cole was very high for the boilers of about 1910 and was not realized until some years later with boilers of better proportions; but today that is immaterial if proper coefficients are applied. The "actual test evaporation" of this table is the ultimate evaporation possible of attainment under the favorable conditions of a standing test of short duration, after elaborate preparations had been made. Often a number of attempts have been made in an endeavor to obtain the highest possible figure. These could not be depended upon for road operation. The Cole evaporation should therefore be lower than the actual test evaporation; the fact that it is lower does not indicate inadequacy but rather confirms its adequacy for the purpose for which it was intended.

The Cole evaporation of the Pennsylvania Railroad K4S locomotive is stated in this table to be 52,150 lb per hr. The actual test evaporation of 65,400 lb given in Table 5, column 6 for the K4S of 1914 should be compared to the Cole value. This was only 25 per cent greater than the Cole, certainly not too great a margin for dependable operation on the road, as implied in the definition of Cole evaporation. The much higher actual test evaporation stated in the table for the Pennsylvania Railroad K4S; the actual test for the Pennsylvania Railroad M1A, 46 per cent higher than the Cole; and the actual test for the New York Central J3A, 45 per cent higher than the Cole, all follow developments and improvements which place these boilers in a later class than was contemplated by Cole. They require the application to Cole evaporation values of a coefficient, such as the "evaporation coefficient" β, of Dr. Lipetz' paper.[5]

However, the maximum evaporation of the greatest importance is that now demanded from modern boilers in operation on the road. A fairly accurate estimate of this evaporation may be obtained by using the pumps of the Worthington heaters as water meters. The pump speed, noted when the locomotive is operating at the highest power output likely to be demanded of it, indicates demands for water up to 115 per cent of the Cole evaporation, that is, an evaporation coefficient of 1.15 for coal burners, with a still higher demand for oil burners.

Twenty years ago, injectors having a capacity 75 per cent of the Cole evaporation were quite usual, at least in the eastern part of the United States. At that time, one 75 per cent injector supplied enough water for most of the operation. Coal burners in the West were operated at a higher evaporation, and oil burners at a still higher rate, but even then the latter did not attain the Cole evaporation. The increase in the evaporation coefficient from not much over 0.75 to 1.15 and the higher pressure and temperature of superheated steam are measures of the progress made in 20 years in feedwater heaters, stokers, better proportions of boilers, and front ends.

Returning to the first table of the paper and applying an evaporation coefficient of about 1.15 to the Cole evaporation of the modern or modernized boilers, it is noted that the Cole value, as modernized by the coefficient, takes its proper position in relation to the actual test evaporation of the table.

While only the proportions of the boiler, principally the firebox proportion, had to be considered, it was not difficult to follow the increasing evaporation coefficient and to predict the evaporation to be expected and the amount of water the boiler would require. But, with the appearance of much more efficient front ends, an additional coefficient, presumably a draft coefficient, is needed.

The author rightly urges more tests to aid in the determination of the best proportions for the boilers of locomotives. However, it is possible that much useful information could be obtained from data of tests already available, if curves were to be drawn showing the relation between boiler efficiency and percentage of Cole evaporation. The boiler showing the highest efficiency at the Cole evaporation would then represent the best proportions, at least as a first approximation. Corrections for differences in draft and other items might be necessary. Comparison of the height and slope of the efficiency curves would indicate which boiler proportion gave the highest efficiency at the higher capacities and therefore promised the highest practical capacity; that is the information desired.

Existing conditions are forcing rapid developments in the boilers of locomotives, especially in capacity. Prompt collection and analysis of all available data are urgently needed. In the writer's opinion useful conclusions can be reached in the shortest time by the determination of the coefficients and the methods reviewed in this discussion.

C. L. MEISTER.[21] Railroad mechanical officials know that in these days of faster train speeds their locomotives must have ample steam capacity to take care of, not only the long-sustained runs, but all the auxiliary demands requiring steam; and careful design of the boiler is the first essential.

This paper gives many timely suggestions: Ample combustion space; reduction at the vital points of the gas flow and the necessity of large gas areas; the relation of boiler diameter to total weight; grate area; flue efficiency factor; front-end design; and others.

A copy of this paper should be in the files of every railroad mechanical officer, and studied not only by the chief, but by his draftsmen as well. It should be carefully studied by locomotive builders, who so often are prone to copy and perpetuate older designs with slight improvements here and there, based on road-performance experience in lieu of scientifically conducted tests.

Although the author has correctly stated: "The superheater holds first rank of all the improvements made on the locomotive since George Stephenson's time," he has refrained from going into detail on this phase of the subject. The writer would like to have further information on the following matters:

1 What is the practical limit to superheat temperatures on the present type of reciprocating locomotive, and what effect has superheat as high as 350 to 400 F, as used in Europe, on the economy and maintenance of locomotives?

2 What are the important problems affecting the design and materials of boilers and superheaters in connection with high steam temperatures?

3 On the basis of final economy would the greatest advantage be obtained by the use of higher boiler pressures or higher superheat, or is there a combination of pressures and superheats which would give the best ultimate economy?

4 In Table 5, column 5, relating to the Pennsylvania Railroad class S1 locomotive, it is noted that this engine is equipped with a type "A" superheater having a large steam area as compared with other types. Is this a special type which permits such a great increase in the steam area, and what should be the advantage, if any, of using such a design?

5 The author points out the desirability and effectiveness of type "E" superheater design in modern boilers, and a review of

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the tables indicates that this type of superheater has generally been applied. It would be interesting to have a concise statement of the principal differences in the various superheater designs and the improvements which can be obtained by the use of the type "E" superheater.

6 Other features on which further enlightenment would be advantageous are the problem of unburned coal, due to large gas velocities, and the location of the stoker. Would not locating the stoker at the front, instead of back, of the firebox be an advantage even if it meant reducing the coal sizes so as to be more easily handled? Would the problem of high gas velocities and resultant losses caused by unburned coal be improved by the use of pulverized coal?

7 In the paper and conclusion, the author refers to the competitive condition of Diesel and electric locomotives. Has a real economic analysis been made of the comparative economics of the modern steam locomotive with that of the Diesel or electric operation, both as to availability, first cost, operating cost, and maintenance?

R. M. OSTERMANN.[32] The author's wide experience in designing superheaters for locomotives, and proportioning them properly for best over-all performance, has perhaps suggested to him that it might be superfluous to discuss the limitations of locomotive-superheater design in a more detailed manner; his discussion on superheaters is brief indeed. And yet, it would seem, since the superheater is such an integral part of the locomotive boiler, and its design is so bound up with the design and performance of the boiler itself, that locomotive designers as well as the users thereof would have been interested in a more thorough exposition of the various factors which not only determine boiler capacity and boiler efficiency, but also the capacity and efficiency of the superheater, the steam pressure loss which it creates, and the added draft requirement which is the result of its installation. The practice of locating the superheater heating surfaces within the flues has evolved from several past trials of other arrangements, and now can be considered a virtual standard wherever steam locomotives are used. The practice has become standard because superheating the steam either in the smokebox or in the firebox was found to be impractical for a number of reasons. Thus, one is confronted with one general limitation of the locomotive superheater in that it is heated by gases, the temperature of which is that available within the locomotive-boiler flues. Such temperatures generally range between 600 and 2000 F.

The author discusses at length the factors which affect the heat absorption of the evaporative heating surfaces of the boiler barrel when in contact with gases of such temperature. These same factors, of course, also influence the superheating capacity because the latter depends upon the mass velocity of the gases flowing over the superheater units, and the temperature difference between the gases and the steam to be superheated, from point to point in the steam path.

There is, however, one essential difference between the mechanics of heat transfer, as between the gases and the water, and as between the gases and steam. The speed of water circulation along the flues does not affect the total heat-transfer resistance on the water side of the flue nearly as much as the speed of the steam moving within the superheater-unit pipes affects the heat transfer between it and the gas. Experiments, covering such heat transfer, are on record and reveal that the specific heat-transfer resistance from the superheater tube to the steam varies inversely with the 0.75 power of the steam velocity. Whenever the velocity of the steam flow is reduced in the presence of an undiminished heating effect of the gases, heat is impounded in the metal of the pipe and its temperature is raised until the adverse effect of the lowered steam velocity upon the heat transfer between the pipe and the steam is compensated by a larger temperature difference between the two. On the other hand, the rise of the metal temperature causes a smaller temperature head between the gas and the metal, and a lessened heat transfer per square foot of superheater heating surface takes place unless compensation is made through a greater mass velocity of the gas flow. This eases up the specific heat transfer from the gas to the metal but, incidentally, also increases the draft loss.

Quite naturally it would be desirable to build superheaters in locomotives which cause as little pressure drop of the steam as possible, for obvious reasons. However, if the physical laws of heat transfer are kept clearly in mind it is recognized that the superheater capacity, the evaporative capacity of the boiler barrel, the draft loss through the boiler barrel, the heat-absorption efficiency, and the pressure loss in the superheater are all interlinked with the size of the available tube-sheet area.

Locomotive superheaters may well be designed for special low pressure drops, but superheating capacity or draft loss or evaporative capacity have to be sacrificed, and such sacrifices may affect the maximum power output of the locomotive more than the pressure drop through the superheater, which, after all, is not a thermal loss. This emphasizes the author's statement that reduction of back pressure is relatively far more essential than a reduction of superheater pressure drop, pound for pound. Resort may be had to the so-called single-loop superheater which halves the steam velocity, and the tubes may be made of special alloy steel so as to operate them safely enough with the resulting higher metal temperature. However, this would result in a lowered superheating capacity. The steam velocity and pressure drop may also be reduced by increasing the diameter of the superheater-unit pipes but, in that event, either evaporative heating surface must be sacrificed or the net gas area through the boiler reduced. Such limitations do not exist in other types of boilers in which the superheater may be placed at will into the gas stream where it is of a higher temperature than the one prevailing in the flues of a locomotive boiler, and, therefore, where the length of the steam path can be correspondingly shortened. At such a point in the gas path, it may also be possible to dimension the net gas area more liberally and without creating any undue draft loss. In such boilers, low-pressure-drop superheaters can then be more easily provided for than in locomotive boilers.

The superheater research to which the author refers, actually produced several types of superheater units usable with $5^1/_2$-in. flues, which are fundamentally different from the well-known double-loop unit, and which, by reason of this difference, are capable of producing, with the maximum gas load through the flue, a superheating effect which is about 20 per cent greater than with the double-loop design of unit. Unfortunately, however, this type of unit is sufficiently more complex in its structure and sufficiently heavier than the double-loop unit as to make it relatively impractical for the highest boiler pressures which are now used on steam locomotives. It is, nevertheless, interesting to mention briefly that the substantially increased heat transfer to the superheater heating surface of the unit mentioned was only partially obtained at the expense of the evaporating capacity of the flue. In other words, in spite of an increased heat transfer to the superheater heating surface, the total heat absorption of the flue-unit combination could be improved; the draft loss was no greater than it had been through the flue with the double-loop unit. This rather gratifying result was obtained by utilizing counterflow between the steam and the gases to a greater extent than is possible with the double-loop-unit construction, and also by special means designed to prevent the temperature of the gases in contact with the superheater heating surface from dropping as

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much as the temperature of the gases in touch with the water-cooled surface of the flue.

With respect to the draft loss through the barrel of the boiler, it should be remembered that it is only a part of the total draft loss through the furnace and boiler as a whole. Thus it must be decided from case to case how much justification there is to decrease the hydraulic depth of the boiler barrel as a whole without running the risk of overloading the draft-making front-end arrangement, and thereby unduly increasing the back pressure of the locomotive. Due to the interaction of the superheater proportions with those of the front end, of the boiler barrel, and of the furnace arrangement—not to speak of differences in fuel—it is impossible to expect that a given standard of superheater design perform in exactly the same manner in locomotives with a wide variety of boiler proportions and operating conditions. Yet, in the interests of maintenance, it is imperative that certain standards of superheater design be adhered to, and it is this phase of the author's subject which the writer particularly wished to bring out.

N. A. Pownall.[33] There is no doubt but that present formulas are inadequate for determining the probable maximum power of a modern locomotive. This is particularly true since the advent of such capacity- and efficiency-increasing appliances as the superheater, feedwater heater, siphon, etc., as well as the effects of variations in exhaust-tip size, stack diameter, front-end appliances, air openings through grates, and various features of boiler design mentioned in the paper. It will be of considerable value if a formula can be developed which will make it possible with fair accuracy to predict the power of a certain design of boiler. It is to be hoped that, by means of the increasingly used standing-test method, such a formula may ultimately be developed, at the same time establishing the best method of boiler design as regards furnace volume, lower gas velocity, greater steam space, etc., as discussed by the author. Research work of this nature will be necessary, for there are certain changes, which are even now being made, that thus far have produced unpredictable effects. For example, opening up the exhaust tip is desirable from the standpoint of reducing energy wasted in back pressure and yet, if this is carried too far, though the engine may still steam well, temperatures may have been lowered so as to affect adversely the superheat, with a resulting negative economy. Any drop in temperature of the superheated steam is attended by a positive rise in fuel consumption.

The steam space above the water level is certainly important. Treated water is now considered a necessity for locomotive boilers, and any antiscale treatment will necessarily increase the foaming tendency of the water, resulting in a greater volume of water bubbles in the steam space, which, because of space limitation, is relatively small for the tremendous steam-producing rate of a modern locomotive. Careful study of the boiler design, in order to increase steam-liberating surface and, by gaining a little here and a little there, to increase steam space above normal water level, will be of considerable value in reducing water carry-over. It is noted that experiments are now under way with a trough or box directly under the dome which increases the steam space at the main-throttle location and offers promise of some relief.

It is astounding to learn that the velocity of gases through the back of flues exceeds 200 mph, and over the arch 260 mph. Therefore, it is not difficult to realize why losses in unburned fuel and stack losses are so high, why stay-bolt heads are cut by cinders, and why there is excessive slagging of flue sheets when the boiler is worked at high rates. As a result, ample gas area is very important, though here again space limitation interferes. Advantage must be taken of small gains here and there; such as, decreasing the space between the combustion chamber and the boiler shell. If water is kept fully treated, and the boiler systematically blown out in service and given thorough washings, this decrease may be made with no bad results.

The cost of boiler maintenance is stated to be 30 per cent of the total cost of locomotive repairs. This is too high. Years ago, the writer had occasion to keep records of the repair costs, divided between the boiler and items not pertaining to the boiler, and found the percentage of boiler repairs to vary from 9 per cent on the best treated water division to 38 per cent on the worst untreated water division. With proper water treatment (and it is prehistoric for any railroad to be without it), this percentage should be kept below 15.

As the author states, at the high combustion rates it is quite important to have maximum possible firebox heating surface and furnace volume, thus reducing the loss of sensible heat up the stack by reducing front-end temperatures; a 210 deg decrease in front-end temperature causes a 7 per cent fuel saving. However, this must not be carried to the point of causing reduction in the temperature of superheat.

Few people realize the high horsepower obtained from this boiler plant on wheels, many 4-8-2 and 4-8-4 type locomotives being capable of developing from 4000 to 5000 hp. We must also pay a tribute to the feedwater heater which heats the water for this 5000 hp up to 210 or 220 F with apparatus of a size that can hardly be seen on the locomotive. Yet under the restricted conditions, we are still striving by intelligent and seemingly insignificant changes in design to develop this high power at even greater efficiency.

Unfortunately trains have to run light, and the engine and boiler do not perform as well as under a heavier loading. The locomotive must be so designed as to operate reliably under all conditions occurring on the territory to which it is assigned. It is preferable and desirable to have a locomotive of general utility but, in view of the many factors entering into the economical development of maximum power, it may be advisable, where it can be done, to adapt design to specific operating conditions.

Our experience bears out the author's remarks concerning front-end design and draft. In this connection, it may be of interest to describe some changes that were made on 4-8-2 class freight locomotives. These engines have 27 × 32-in. cylinders, 245 lb per sq in. boiler pressure, 70-in. drivers, 69,400 lb tractive power, total heating surface 4620 sq ft, grate area 84.2 sq ft, and are equipped with type "E" superheaters, firebox siphons, and feedwater heaters. According to Cole's ratios, their normal expected horsepower was 3214. When received, they developed 3650 ihp, illustrating the inadequacy of this formula for ordinary modern design.

The grates were a finger-bar type with 43 per cent air opening, master mechanic's front end with $6^1/_2$-in-diam round tip, and 19-in-diam stack, and these were changed to 14.9 per cent air opening tuyère-type grates, a Kiesel front end with six-opening 53-in. star-shaped tip, and a $25^1/_2$-in-diam stack. Combustion was better, excess air was considerably reduced which entails a fuel saving, back pressure was reduced, draft was noticeably improved by reason of less impedance to flow of gas in the front end, and the larger stack evidently contributed considerably to greater power. The locomotive developed consistently over 4900 ihp, which confirms the author's feeling that a new formula is needed to measure the power of modern locomotives and the effect of changes in the important parts of the boiler.

We have built locomotives, making use (possibly inadvertently) of correct principles which have resulted in unexpectedly high power; yet now and then we overlook some unrecognized, important feature and are disappointed in the result.

[33] Assistant to Superintendent of Motive Power, Wabash Railway Company, Decatur, Ill.

Mr. Brandt's paper is very comprehensive, should give designers plenty of food for thought, and the writer feels that in the foregoing he has added little, but has tried to confirm Mr. Brandt's reasoning and conclusions.

F. P. Roesch.[34] The writer prefers to leave questions of locomotive design and construction details to those better qualified, and confine his comments to accessories which are not considered as a part of the boiler proper but, nevertheless, have a material effect upon both boiler and locomotive performance.

The Ashpan. During Federal Administration of railroads, a committee from Section 3, Mechanical, American Railway Association (A.R.A.), among other suggestions tending toward fuel economy, reported[35] as follows:

"Many of the older engines are still equipped with ashpans having insufficient air openings, a defect readily detected with a U-tube draft gage. There should be no indication of vacuum in the ashpan. Correction of this condition recently permitted an increase of $^1/_2$ in. in the nozzle diam of several locomotives and not only effected a substantial saving in fuel, but greatly increased the efficiency of the locomotives."

Prior to that time, tests indicated the desirability of free air openings into the ashpan, having a combined clear opening of not less than 14 per cent of the total grate area. This was based on the total air openings through the grates and its relation to total tube area. But apparently these recommendations, as well as those of the American Railway Association committee have either been forgotten or are disregarded for items of presumably greater importance.

Perhaps it may be well to check into the matter to decide the point. A definite amount of air is always required in the combustion of various kinds of coal, as pointed out by the author. Where the area of the supply ports for such air is restricted below requirements, the velocity of supply must be increased. To obtain this velocity we must increase the intensity of the draft. This is usually done by reducing the area of the exhaust-nozzle opening, thereby increasing the cylinder back pressure, so when the cycle is completed, we find that what should be free air has been obtained at the expense of cylinder horsepower.

At this point the writer wishes to make it clear that we are not speaking of test-plant methods in this discussion, but of conditions and methods obtaining in general road service, where a reported engine failure through any cause is a serious matter and any course is often thought justified which will eliminate such reports. At any rate, an engine failure is something real that everybody hears about, while decreased locomotive efficiency is only revealed by the indicator.

However, decreased locomotive efficiency is not the only evil resulting from a restricted air supply. On the contrary, many others such as decreased combustion efficiency, increased stack loss, cinder cutting, etc., all follow in its wake, as will be shown later.

Locomotive Grates. During Federal control a Fuel Conservation Department was organized. One of its aims was to check and reduce waste. In connection with that job, it was noticed when the fire was laid preparatory to firing up a locomotive, that from 500 to 2000 lb of the smaller-sized coals fell through the openings in the grates into the pit over which the locomotive stood. As such coal was afterward covered with ashes falling through the same grate openings, recovery of the unburned coal presented some difficulty and, therefore, was seldom salvaged. To overcome this waste, it was recommended that the grates be covered with perforated sheets of heavy paper before the coal was introduced. This fuel loss received wide publicity. Therefore, when the railroads were returned to corporate operation, many of them applied the logical remedy, where conditions warranted, namely, better-fitted grates with smaller openings.

But little is known of the physicomechanical reactions or behavior of the fuel bed in a locomotive firebox at relatively high rates of combustion. So little in fact, that many of our theories as to what takes place are based on such an insecure foundation that conclusions reached are questionable. The chemical reactions are known and serve as a guide toward improving the combustion of such coal as is actually burned, but this is not what we are concerned about at present. What we need is more definite information covering stack loss, draft currents, both as to direction and velocity, between the fuel bed and the underside of the arch, etc., conditions which can only be observed through small openings cut through the sides, or larger openings in the backhead, covered with some transparent substance. We recommend the latter, as the range of vision through the sides is too limited, although their addition to backhead windows will be helpful.

It has been the writer's privilege to observe some of the physicomechanical behavior of the fuel bed through side holes during tests on a test plant, and later through a large covered opening through the backhead. In the latter case, however, the locomotive was equipped with forced draft. Nevertheless, the behavior of the fire in both instances was quite similar, at least enough so to enable us to set up certain theories, which later practice proved to be sound.

These observations indicated that, while the gases moved in the direction as generally pictured, viz., from front to rear, their velocity above the surface of the fire bed was relatively low until they reached a zone immediately under the rear end of the arch. Here the movement of the particles of coal entrained in the gas currents indicated a point of excessive turbulence and high upward velocity, Fig. 3 of the paper.

This was quite interesting, but the most interesting phenomenon was the behavior of the small lumps of coal lifted above the fire bed by the action of the air jets passing upward through the fire. It was particularly noticeable that in many instances one lump of coal was projected high and far enough to enter the zone of high turbulence, where it disappeared, while another lump of apparently the same size and rising from the fuel bed within 6 or 8 in. from the first one, was only lifted about 1 ft or so and then fell back and was burned. As both were presumably subject to the same pressure or velocity influence, the question excited much interest. Further observations indicated that the same phenomenon occurred at other points. In fact, the fuel bed had the appearance of a group of miniature volcanoes spouting fire and ashes, some far up and others only high enough to overflow the rim.

After the fires were dumped, the reason for all this became clear, viz., difference in size of air openings through the grates. On this observation was based the theory that the lifting action of air jets passing upward through the fire bed was in proportion to the squares of their areas. This theory has been proved and the foregoing observations are largely responsible for the present general use of what are called restricted grates. In these it is not so much a question of the ratio of air openings to total grate area, as a question of the size of the openings, their spacing, and the fit of the bars against each other and against the side and center carrier bars. We do feel, however, that the total air openings need not be much in excess of the total tube area. Or to put it another way, knowing how much air is required to burn 1 lb of coal, air openings can be proportioned accordingly, thus mechani-

[34] Vice-President, The Standard Stoker Company, Chicago, Ill. Mem. A.S.M.E.

[35] "Ash Pans," Report of Committee on Fuel Economy, Proceedings, Section 3, Mechanical, American Railway Association, 1919, p. 481.

cally restricting the supply needed, instead of leaving the matter to the judgment, or lack of judgment, on the part of the fireman.

Needless to say, the general adoption of restricted grates was hailed by locomotive-stoker manufacturers, as it fully supported their contention that stack loss was not a question of coal size as fired, but rather one of air openings through grates and draft influence. Furthermore, stack loss far antedated mechanical stokers and, according to a University of Illinois bulletin,[36] lump coal breaks down during the process of combustion (naturally) and thus accounts for the excess in cinders collected above the amount of fines in the coal as fired. As a matter of fact, a higher heat release is obtained from small-sized coal as fired, other conditions being right, than from lump coal which must be broken down during its combustion.[36]

Draft Streams. In a preceding paragraph, the statement was made that the velocity of the gas streams, moving from the forward part of the firebox to the rear, was relatively slow. In support of this statement, we offer the fact that while the locomotive is in operation it requires a greater distributor-jet pressure to blow a lump of coal from the front of the firebox and presumably with the draft, than it does to blow an equal lump from the rear to the front, and again presumably against the draft. This fact came to light only recently with the introduction of the so-called front-delivery stoker as developed by the Baltimore & Ohio Railroad. However, an approach to the problem with an open mind and no fixed convictions as to what really happens in a locomotive firebox when coal is being burned at relatively high rates, enables one to arrive at a logical solution, namely, the momentum imparted to any mass, in this instance a lump of coal, is proportionate to the velocity or force of the steam jet impinging against it. This should hold true regardless of whether the mass is projected from the rear to the front or from the front to the rear of the firebox. The velocity of the jet is as the pressure. Therefore, it follows, if higher jet pressures are required to blow the coal rearwardly and presumably with the draft stream, it must be subject to other resistance than that of gravity.

This we find the author has clearly indicated in Fig. 3 and subsequent analysis. Reasoning along these indicated lines, we draw the conclusion that the initial momentum is gradually reduced and, when the mass reaches the zone of greatest turbulence and high velocity, it no longer has sufficient momentum to carry it through that zone unless the initial velocity is increased through higher jet pressures. On the other hand, where the coal is introduced at the rear of the firebox and blown forward, the introduction takes place within the high-velocity zone, therefore the mass even at a lower initial velocity still has sufficient momentum to carry it through this zone and deliver it to the forward part of the fire bed.

This theory if accepted, raises another interesting problem. Tests on the Baltimore & Ohio Railroad have definitely shown the advantage of the front-delivery stoker; for instance, a 50 per cent decrease in stack loss, 15.9 per cent reduction in fuel per 1000 gross-ton-miles, and 15.5 per cent increased evaporation per lb of coal, together with the practical elimination of smoke. There is no question as to the correctness of the published data, as the reason for this improved performance is self-evident. However, as the comparisons were made between the front-delivery stoker, which delivers the coal at a point about 14 in. above the normal grate level, and one effecting delivery at a much higher level, there is the possibility that the same performance differences may not obtain where the coal is introduced with a rear-delivery stoker, firing from a level corresponding to that of the front-delivery stoker, for the following reasons:

The statement has been made that the lifting action of the air jets passing upward through the fire bed was as the square of their areas, the velocity remaining the same. In the following, it must be understood that the figures are not at all accurate since, owing to the resistance and varying thicknesses of the fuel bed, accuracy is out of the question, but the comparisons can nevertheless be accepted as valid.

Assuming the author's data as given in Fig. 3 to be based on a grate having an area of 100 sq ft, we find from established tests that an approximate draft equal to $2^1/_2$ in. of water is required to burn coal at the rate of 90 lb per sq ft of grate per hr,[37] and this, neglecting fuel-bed resistance, results in a velocity efflux of 103.30 ft per sec.[38] In contrast, at the higher rates, viz., 236 lb per sq ft of grate, we must have a draft equal to 4 in.,[37] with a corresponding velocity of approximately 130.43 ft per sec.[38] However, in the first instance, due to the expansion of gases, the velocity is increased to 198 ft, and in the second to 386 ft where the gas stream passes over the arch.[8] Therefore, where the fuel is introduced at the lower levels, a lesser amount is caught up by the higher-velocity streams.

Since the velocity of the gas stream, where it passes over the arch, is proportionate to the volume to be moved and the area of the opening between the top of the arch and the crown sheet, it would appear that any increase in this area would decrease correspondingly the velocity at that point. This brings us to a consideration of the setting of the arch.

The Brick Arch. The value of the brick arch is well known and requires no further discussion. Recommended practice[39] suggests a gas area over the arch equal to from 110 to 120 per cent of the minimum net tube and flue area. This is to increase the length of flame travel and reduce the stack loss. However, according to the author, this results in a gas velocity at that point of 198 ft per sec at low rates, and 386 ft at high rates of combustion, while the corresponding velocity through the tubes and flues is 157 and 300 ft per sec, respectively.

The writer believes in a long arch but, as has been shown, the entrainment of the smaller particles of coal is influenced by the velocity of the gas streams. Theoretically, any reduction in such velocity should in turn affect entrainment and, while theory does not always check with practice, it does appear reasonable to assume that some modification in arch application which would tend to equalize gas velocities might be worth our consideration. As a suggestion based on tests conducted by the writer, we propose, beginning at the rear, longitudinal rectangular openings between the arch and both side sheets about 4 in. wide and 36 in. long, leaving the center of the arch to the recommended length. With such an arrangement much of the fines entrained in the gas streams would come directly in contact with the water-cooled firebox sheets, cooling such particles and preventing their adherence to the tube sheet in the form of honeycomb.

We feel this discussion would not be complete without further reference to the mechanical locomotive stoker, as it is only through mechanical firing that the full rating of modern locomotive boilers can be obtained. This being the case, would it not be well when designing such boilers, to give consideration to stoker requirements as to ready and appropriate application?

[36] "Comparative Tests of Six Sizes of Illinois Coal in a Mikado Locomotive," by E. C. Schmidt, J. M. Snodgrass, and O. S. Beyer, Jr., University of Illinois Engineering Experiment Station, Bulletin No. 101, Sept., 1917.

[37] "A Study of the Locomotive Front End, Including Tests of Front-End Model," by E. G. Young, University of Illinois Engineering Experiment Station, Bulletin No. 256, May, 1933.

[38] "Mechanical Draft," by W. B. Snow, B. F. Sturtevant Company, Boston, Mass., 1910.

[39] Report of Committee on Locomotive Construction, Proceedings, American Association of Railroads, 1936, pp. 277–436.

We refer particularly to the type and design as developed by the Baltimore & Ohio Railroad, a type which may tend to revolutionize mechanical firing. This design and its prototype which, however, fires from the rear are the only mechanical stokers satisfying the requirements as to secondary air as discussed by the author under "secondary air for combustion."

No doubt this has escaped general notice, but an examination of either type will reveal this fact. While the method used is more in the nature of a heat exchanger, yet it does supply preheated "top air" and always in direct proportion to combustion requirements. Gas analyses verify this statement. The apparatus simply consists of a vertical grate surrounding that part of the stoker-delivery member extending upward through the locomotive grates.

These vertical grates, termed "protecting grates," extend above the fuel bed and absorb heat from it. This heat is in turn absorbed by the free air contacting the inner surfaces, and is in turn drawn into the firebox and over the fuel bed by the draft through $^1/_2$-in. holes drilled radially and spaced so as to give perfect diffusion. The number of drilled holes is always proportional to the total grate area and to the volume of air supplied through the fuel bed, thereby maintaining a fairly even balance. There can be no question but that this supplementary air so admitted contributes materially to the splendid results obtained on the Baltimore & Ohio as reflected in the boiler and fuel performance.

We have so little factual knowledge of what actually transpires in a locomotive firebox at moderate or high rates of combustion, that, in view of the importance of the subject, it would appear that almost any expenditure for tests would be warranted. Such tests, however, should be made on a plant where road conditions could be simulated. While tests[38] conducted by the University of Illinois indicated no material difference as between constant-flow and pulsating draft, many engineers feel that these are inconclusive, hence the suggestion to simulate road conditions. So unless tests are made, and such tests duplicate as far as possible actual road conditions, we will still find ourselves where we started.

To show to what extent the modern locomotive boiler depends upon the mechanical stoker, we might refer to the large simple "Mallets" built for the Northern Pacific some few years ago. These locomotives burn a subbituminous coal and therefore require a relatively large grate. The proportions as to grate area desired and steam output of the boiler were worked out by the engineering department of the Northern Pacific in collaboration with the locomotive builder. The latter, however, would not agree to conform to the dimensions requested (referring to grate area) until the stoker company agreed to build a stoker with sufficient capacity to meet maximum requirements, and would guarantee to distribute the amount of coal fired evenly over the grate. The maximum requirements were estimated at 45,000 lb of coal per hr. While this rate has never been required, nevertheless there are times when a rate of 30,000 lb of coal per hr is actually reached. The firebox first laid down was 20 ft long and $9^1/_2$ ft wide, or a grate of 190 sq ft. This was later reduced to about 184 sq ft. Needless to say, the requirements were met by the stoker company, otherwise the locomotives would never have been built.

L. A. SHIPMAN.[40] In comparing steam-boiler performance on an efficiency basis, we have two vital factors to consider (1) water and (2) coal.

In the case of locomotive M1A tested by the Pennsylvania Railroad, referred to in the paper, wherein burning rates of 345 lb of coal per sq ft of grate area per hr were obtained, one of the main fuel problems, as indicated by the 50 per cent stack waste, was keeping the fuel on the grate until it was burned. This problem can be solved at least partially by fuel selection and sizing.

In connection with the matter of water circulation and foaming, the importance of the water conditions as they affect the efficiencies of the different tests should receive special attention.

While it has been mentioned that forced circulation of water was used in the Steamotive tests, the fact that practically distilled water was used in both Steamotive tests and in the Navy test, as against the regular water-tank water as used in the New York Central and Pennsylvania Railroad tests, was not mentioned. While this fact certainly affects the efficiencies of the test, it also singles out those who have developed the equipment and its methods of operation so that distilled water is economically possible for special distinction.

Because of the fact that there was a difference in the water, in the water circulation, natural against forced, and in the fact that there was a difference in the percentages of the total fuel consumed because of the high stack loss of the conventional locomotive at high burning rates, the performances are not on an equivalent basis for comparison although the facts presented definitely point out the tremendous improvements possible in fuel economies for which those contributing to the new designs should be highly complimented.

L. K. SILLCOX.[41] The locomotive boiler and its operation can scarcely be discussed without referring to the fact that only a limited number of test data are available. The author is fortunate in having access to new information which heretofore has not been reported. Nevertheless he still is compelled to call attention to the need for vastly more information which can only be derived through further boiler research.

No entirely satisfactory method of defining locomotive capacity has yet been derived which will establish its capacity as limited by the features of furnace, boiler, engines, and rail, acting individually. Boiler calculations are the least exact of all the elements involved, although the investigations of Cole, Fry, Strahl, and others have all contributed in important measure to our total knowledge of the subject.

The author presents a table which shows the failure of Strahl and Cole to estimate the actual evaporative capacity of three locomotive types. In the case of the Pennsylvania Railroad class M1A, Strahl estimates capacity at 54 per cent of test values, Cole at 68 per cent. Perhaps, in justice to Strahl and Cole, it should be pointed out that, when evaporating 99,095 lb of water per hr, the locomotive was fired at the rate of 337 lb of coal per hr per sq ft of grate. No such overload is contemplated in the application of the formulas. At this high coal rate, an efficiency of boiler and furnace of but 37.5 per cent was obtained. Yet the absorption efficiency of the boiler was over 80 per cent. This is illustrative of the mass of operating details which must be explained in every case if test conclusions are to bear all their possible significance.

It has been remarked that, even with the stated excessive and uneconomical firing rate, absorption efficiency did not suffer greatly. This would indicate that high gas velocities may not always be objectionable, in so far as boiler performance is concerned. In fact, whereas a gas velocity of 204 mph in the tubes is quoted as very high, velocities of 445 mph are sought in the Velox steam generator, and blowers are introduced to attain this end. There is a vast difference in function and design, of course, but the Velox generator, one of which is now installed in a loco-

[40] Combustion Engineer, Southern Coal & Coke Company, Inc., Knoxville, Tenn. Mem. A.S.M.E.

[41] Vice-President, The New York Air Brake Company, Watertown, N. Y. Mem. A.S.M.E.

motive of the Paris, Lyons and Mediterranée Railway in France, occupies much less space and represents much less weight than a conventional locomotive boiler of the same capacity. While we are continuing improvements in the conventional firetube locomotive boiler, which no doubt will remain the standard type for general service for many years to come, we cannot afford to confine our attention to this construction to the extent that we are blind to possibilities originating in other fields. Concentration of more boiler capacity within less space and providing this with considerably reduced weight, offers attractive possibilities for investigation by locomotive builders, with available fuel types ever in mind.

It is the limitations imposed by compliance with familiar and proved proportioning of locomotive parts and strict adherence to conventional practices that renders difficult the fitting of required power into the loading restrictions and clearance limits of American railways and calls for a firing rate such as that proposed by the A.A.R. committee appointed to investigate the requirements in a locomotive capable of hauling 1000-ton trains at 100 mph, namely, 152 lb of coal per sq ft of grate per hr under normal conditions. Combustion efficiency would be higher if the unit firing rate were lower, yet weight limitations govern the permissible size of grate which may be practicably employed; approximately 128 sq ft in this case.

The coal-fired steam-locomotive boiler has advanced rapidly and far. It compares phenomenally with central-station boilers in capacity per unit of weight. Present railway-operating practices have taxed its capacity in all practicable sizes yet the demands are unchecked. Perhaps we must search out new approaches to an old problem, replacing well-thumbed pages of past operating manuals and removing the inherent difficulties which have retarded progress in the use of such promising expedients as pulverized coal and the water-tube boiler.

L. W. Wallace.[42] At the outset of this paper, a statement is made to the effect that the public is continually demanding higher speeds for both freight and passenger trains. It may be that such an apparent demand is being taken too literally. Is it higher speed, in terms of miles per hour, that the public wants or is it a reduction in the total elapsed time required to travel between two points? There is a real and significant difference between speed in miles per hour and total elapsed time. The total elapsed time experienced on many runs may be reduced materially without increasing significantly the sustained speed in miles per hour for any major portion of the run. This may be done by reducing the waiting time at stations and elsewhere through more expeditious servicing and handling of trains. It is the waiting period at terminals and elsewhere which causes the largest amount of restlessness and complaint on the part of the traveling public. It would appear that this phase of train operations has not been given the analytical attention which its importance warrants.

The difference between high speeds in miles per hour and a reduction in total elapsed time between two points, realized through more expeditious train servicing and handling, may make a major difference in locomotive design, its first cost, and the cost of maintenance and operation. What this may mean is reflected by the comparison of the 2-8-0 and 6-8-6 locomotive types. This comparison discloses that the weight of the locomotive had to be doubled in order to obtain sufficient boiler capacity. All of which raises the question, "What price is paid for high speeds in miles per hour as contrasted with the price which may be paid for a reduction in total elapsed time?"

The comparison of the locomotives referred to adds emphasis to this significant statement made by the author, ". . . the important question is whether greater boiler efficiency is attainable, particularly at high-capacity operation, and what can be done to accomplish this."

The following pertinent comment also appears in the paper: "The subject matter will be centered on problems affecting the design of the conventional locomotive fire-tube boiler only, not because of any belief that this type of boiler is the final answer to the locomotive steam generator. . . ."

The writer was delighted that the author disavowed any belief that the typical locomotive boiler is the final answer. In my judgment it is not. It may well be that if a broadly conceived and executed line of creative research were undertaken, the final result would be some form of boiler closely akin to the flash type. It seems evident that something different from the present typical locomotive boiler must come, as it is not sound economics to double the weight of a locomotive in order to obtain adequate steam capacity.

Again to quote: "However, much remains to be done to make possible the burning of more fuel per hour at higher combustion efficiency with lower draft loss, greater heat-absorption efficiency, and higher superheat with less boiler weight per pound of steam produced." This is a farseeing, correct, and timely statement of the problem confronting the railroad industry. It is fairly obvious that one approach to relieving the present situation is to supply the steam required for auxiliaries, train heating, air conditioning, and for other purposes through a steam-generating unit other than the locomotive boiler. However, this alone will not give the final answer.

The author states: "The method of standing blowdown tests developed and used by the New York Central Railroad in recent years has proved very effective." This is undoubtedly true. Moreover, it is the writer's thought that considerable valuable work in certain directions can be accomplished by the use of scale models. This means the application of the law of similitude which has been so successfully used in connection with research programs covering a wide range of difficult technical problems. There is no apparent reason why the method cannot be applied successfully to the solution of many railroad problems. The method has the advantage of affording a more definite control of test conditions than for full-scale operations and, furthermore, a material reduction in cost of experimental work may be realized.

The locomotive boiler presents a difficult problem. A comprehensive approach to its solution has been too long delayed. The solution lies in the direction of broadly conceived lines of research coupled with a daring and courageous attitude.

Author's Closure

It is an honor for the author to have the privilege of replying to the many valuable contributions made to his paper, "The Locomotive Boiler."

The interest that this subject has aroused among those responsible for the design and operation of steam locomotives testifies to the importance of further improvements to the coal-burning steam locomotive.

Since the preparation of this paper the European war has started, reminding us that the development of the steam locomotive must not be neglected, and it is pertinent that this subject be again emphasized.

It is often heard today that the railroads, and particularly the coal-burning steam locomotive, are out of date, and will be replaced by airplanes and oil-driven power units on railroads and highways.

Such opinion, if shared by responsible men in government and business, may have a paralyzing effect upon efforts to improve the steam locomotive. Those who are able to see beyond the

[42] Director, Division of Engineering and Research, Crane Company, Chicago, Ill. Mem. A.S.M.E.

present phase in the development of transportation, and are devoting efforts and thought to the perfection of the coal-burning steam locomotive should be encouraged in their efforts.

The struggle now raging in Europe is being fought on land, air, and water with engines of war driven by gasoline and oil power, and one of the major problems of this war is the control of adequate supply of petroleum.

That the world's supply of oil is definitely limited is believed to be true by many eminent authorities. A late estimate of the available oil supply in continental United States, prepared by the Committee on Petroleum Reserve, of the American Petroleum Institute, gives the total available supply as 17,348,146,000 barrels in 1939. The annual production in 1938 was 1,213,186,000 barrels per year which would indicate that our petroleum supply in the United States may approach exhaustion in fourteen years.

That this definitely limited supply of petroleum, absolutely necessary for operation of naval vessels, airplanes, tanks, and trucks, will have to be conserved for our national defense in the future appears reasonable.

The use of oil for the production of power, in places where coal, abundantly available, can be utilized to good advantage, appears unwise from the viewpoint of future national welfare.

The coal-burning steam locomotive has hauled the bulk of the country's traffic in the past. Its gradual replacement now taking place, at a rapidly accelerating pace, with oil-using Diesel locomotives and gasoline power units on highways and in the air, may find this country only a few years hence in a position of having a transportation machine that has cost billions of dollars, but without suitable, or too costly fuel to drive it.

A review of the many discussions reveals a general agreement with the author's thesis, that there is room for improvement and necessity for coordination and clarification of principles with further research and tests.

The opportunity to present a paper on the same occasion as Mr. Bailey is an honor that is very much appreciated by the author. Mr. Bailey's paper on steam-boiler performance[16] is of great fundamental value, and the discussion that he presented on the subject of locomotive boilers is an expression of his theory on "methods of comparison of boiler performance." His discussion, read in conjunction with his own paper just referred to, is so very complete that there is little that can be added for further clarification.

The author in his paper referred to a new type of locomotive boiler soon to be tried out. The boiler referred to is that illustrated and analyzed by Mr. Bailey in his discussion. The results obtained with this boiler, as to efficiency, capacity, and weight, may point the way to further developments on coal-burning steam locomotives.

The author regrets his inability to supply more detailed information on the oil-fired tests of the Southern Pacific, as requested, as these data are not obtainable.

Mr. Bailey suggests that the author supply additional data on the subject of draft loss through various portions of locomotive boilers under different ratings so that the relationship between output and efficiency versus draft loss could be analyzed. It is regretted that sufficient data are not available at the present time to furnish this information.

Mr. Duncan's discussion reveals his interest and insight into the problems under consideration, and deserves a more detailed answer than space will permit at this time. He suggests that if an underfeed or traveling stoker could be adapted to locomotive service a great improvement would result in furnace efficiency. It may be noted here that several designs of both of these types of stokers have been made.

The Crawford underfeed mechanical stoker was built and tested out quite extensively on the Pennsylvania Railroad in 1913 and 1914, and many engines were so equipped. It is believed that there are no underfeed stokers on locomotives now in service. The author has no knowledge of any traveling-grate stokers actually tested, although at least one design was made.

During some thirty years of experiments with different types of stokers the sprinkler type, due to its simplicity and reliability, is the only one that has survived the test of service.

The reason that underfeed and traveling-grate stokers have not been perfected and adapted for locomotive service is due to the fact that they require more room, and weigh more than the existing locomotive limitations permit.

Mr. Eksergian's discussion has amplified and supported the author in several important observations made and it would be superfluous to add anything further to the points touched upon by him, except to express appreciation for his carefully thought out comments.

Any contribution made by Mr. Jones, engineer of test and chief of the locomotive test plant, Pennsylvania Railroad, where most of the tests quoted by the author were made, is of real interest to all. He is eminently qualified to discuss this subject, and his reference to the M1A locomotive with enlarged steam space, emphasizing the improvement possible even with a small change in design, is valued highly.

While on this subject it should be noted that most of the test-plant tests are run with particular attention to keeping the water level below a full glass, or three gages of water, and also without foaming condition of the water. In road service it is known that many times the water level is carried above the highest gage cock and badly foaming water is encountered.

In some cases, on large and long boilers, it has been found that the distance from the water level to the top of the boiler barrel at the dome is as small as ten inches with the water level at the top gage cock. When ascending a heavy grade, say 2 per cent, with engine working at maximum capacity, the water level is usually carried to a full glass of water. When engine is tipping over the grade the water level near the dome will rise as much as 8 in., thus reducing the steam space to nearly zero.

This condition indicates the importance of the use of steam separators in the dome.

The carefully conducted tests of locomotive performance that are frequently being made by Mr. Knight on his railroad are an example of the alertness with which present-day railroad management explores every avenue to improve railway efficiency. His comments based on actual experiences are, therefore, of much interest.

The information pertaining to fuel tests with different grades of coal, furnished by Mr. Knight and quoted elsewhere in this summary, is very valuable. His reference to tests made with different percentages of air openings through the grates is most interesting, particularly the trial with grates having 60 per cent free air opening. Further information on these tests will be of interest.

Mr. Lipetz' contribution is of particular value because of his thorough knowledge of the fundamentals involved. He calls attention to the possibilities obtainable from the proper study of the curves of the test data presented in conjunction with the boiler ratios given in the table.

The data shown represent the results of all the pertinent and complete tests made. Many road tests have been analyzed by the author, but most of them usually lack sufficient test data to permit an accurate analysis of the performance of the locomotive under different conditions of load and speed.

Some road tests analyzed have accurate data on the cylinder performance, drawbar pull, and horsepower, but lack data on the fuel and water consumption or general boiler performance. Other tests have been made with accurate data on coal and water con-

sumption, but without dynamometer or indicator cards to measure the work performed.

The reason for the many incomplete tests run in the past is due to the desire to get a quick picture of the general performance rather than a critical analysis of design; also, the question of the cost of tests is important, particularly when one railroad must bear such cost alone.

The advantage of coordinated research is appreciated by all concerned, and perhaps some day this may be brought about.

Mr. McBride suggests that a prompt collection and analysis of all available data are urgently needed in connection with the problem of proportioning locomotives. The author agrees with this and it was the primary purpose of his paper to make available all the pertinent and accurate test data available just for such analysis and study.

However, the author disagrees with Mr. McBride as to the accuracy of methods suggested by him as they use arbitrarily determined coefficients which apply to empirical formulas, the fundamental sufficiency of which is in question.

We need not be concerned here with a rule-of-thumb method for determining the approximate maximum evaporation of a boiler for the purpose of setting the size of the stoker, injector, etc., but rather a critical analysis of the effect that definite changes in the proportions of various component parts of the boiler have upon boiler efficiency, capacity, first cost, and cost of maintenance.

When this is determined, accurate formulas for calculating the capacity and efficiency of the boiler will follow.

The several questions raised by Mr. Meister, regarding the influence of superheater design upon locomotive economy, are important, and the author will endeavor to answer them.

The first question pertains to the practical limit of steam temperatures on present type of reciprocating steam locomotive; also, its effect on economy and maintenance.

Modern locomotives in the United States, Canada, and Europe operate with steam temperatures of 750 F, and on many locomotives total steam temperature of 800 F, at maximum rate, is not uncommon. With 250 lb boiler pressure this total temperature is equal to a superheat of 350 to 400 deg.

This steam temperature will give a metal temperature of the superheater units of from 875 to 900 F, and units made of open-hearth, low-carbon steel have proved satisfactory for these temperatures. Heat-resisting low-alloy steels are also available to give additional life if desired.

The foregoing statement is predicated on the basis that the units are kept clean from scale on the inside, as it is well known that even a thin layer of scale will decrease the heat transfer from metal to steam and raise the metal temperature.

As far as other parts of the locomotive in contact with superheated steam are concerned the problem of high steam temperatures has been successfully solved by the use of cast steel for the superheater header, throttle, steam pipes, and cylinders. The single-seated multiple-disk valve throttle has proved successful for this service. The lubrication problem has also been satisfactorily solved by the use of the modern high-temperature lubricating oils and forced-feed lubricators.

The question of economy gained by the use of high superheat is answered on page 391 of the author's paper. A reduction of as much as 46 per cent in steam consumption was gained with superheat up to 225 F as compared with saturated-steam operation.

It has been definitely shown on many tests that for each 10 F increase in superheat there would be a reduction in steam consumption of not less than 1 per cent up to the limit of the present practice of 800 F total temperature. This is due to the raising of the total heat content of the steam at admission, and increasing the total heat drop, or the Rankine-cycle efficiency.

The second question raised is a rather broad one and is perhaps best answered by the replies to the other more detailed questions in Mr. Meister's discussion.

The third question, inquiring as to what combination of pressures and superheat will give the best ultimate economy, can best be answered by stating that from a thermal standpoint the higher the pressure and superheat become, the greater will be the Rankine-cycle efficiency of the prime mover. From a practical standpoint it appears that 300 lb gage is the highest to which the conventional stayed locomotive boiler should be built considering material and practice of today. An increase above this pressure would not be of any advantage from an economy standpoint when used in a single-expansion reciprocating noncondensing engine with dimensions of present-day locomotives equipped with piston valves and Walschaerts or similar type of valve gears. With water-tube boilers of types now on the horizon it appears entirely practical to raise the boiler pressure considerably above 300 lb per sq in. and maintain up to 800 F steam temperature.

With the advent of a practical cylinder design having poppet valves and cam-operated valve gear, like the Franklin gear, referred to by Mr. Jones in his remarks at this meeting, it appears that a reciprocating engine has arrived that is not only well suited for handling high pressure and superheat from a practical point of view, but also capable of utilizing the increased amount of energy supplied with each pound of steam with greater efficiency.

The superheater on the 4-4-4-4 type locomotive, referred to in question 4 of Mr. Meister's discussion, is a large-flue superheater with the same size of superheater flue and arrangement as the type "A." The only difference is that each unit consists of two single loops joined to one set of header connections instead of one double-loop unit as with the type "A."

The heating surface of the flue, and superheater, and gas area in each flue, is the same as the type "A," but the steam area through each unit is twice as large. This results in a reduction of the steam velocity in the unit to one half of that of the double loop for any given rating. The pressure drop through the superheater is reduced thereby, but because the lower steam velocity reduces the rate of heat transfer the superheat will also be less.

In the early days of the development of the locomotive fire-tube superheater in the United States, some thirty years ago, a number of locomotives were equipped with single-loop superheaters having two single loops of $1^1/_2$ in. pipe in $5^1/_2$ in. flue.

In view of the desire to keep pressure drop down to a minimum interest in single-loop superheaters has been renewed.

The problem to be solved with the single-loop superheater is the relatively greater differential expansion of the pipes—this is receiving careful study.

On the basis of an increase of 350 deg in the steam temperature from the inlet side of the superheater unit to the outlet side there would be a difference in the average temperature of the metal in the pipes of each unit loop of 175 F.

As the two ends of each unit loop are fixed the large difference in the expansion of the unit pipes sets up a heavy strain on the unit joints that may cause them to leak and finally break.

The Superheater Company has experimented and tested many different types of single-loop superheater units, most of them based upon the principle of placing the bulk of the superheating surface of the unit so that the steam flows in counterflow with the gases, and only one pipe is used for the return of the steam to the front end in uniflow with the gases.

Example of such units are the "H" type, 5P, 6P, and 7P units. All of these types of units are subjected to the same differential temperature stresses. In the "HA" type unit this has been compensated by the use of a short section of double-loop pipe which acts like an expansion joint thus taking care of the differential expansion.

In the standard double-loop superheaters, both type "A" and type "E," there is sufficient flexibility to distribute and absorb this differential expansion without setting up undue stresses that cause trouble.

In view of the desirability of improving the tractive power of steam locomotives at high speeds the question of low pressure drop is important, and careful attention is given to the design of superheater, dry pipes, throttles, steam pipes, and cylinders to gain this end.

The question of reducing the pressure drop through the dry pipe, superheater header, throttle, steam pipe, and valves presents no difficulty from the standpoint of design as these steam passages can be increased considerably above present practice without interfering with the fundamental design of the locomotive, or reducing the useful life of these parts.

An undue increase in the steam area through the superheater units, however, may under some conditions adversely affect the power output of the locomotive as well as the life of the units.

To reduce pressure drop to a minimum and at the same time maintain high superheat for maximum economy in the use of steam involves not only the steam passages from the boiler through the superheater to the steam chest, the valves, and exhaust passages but also the correct proportions of the cylinders to the weight on drivers.

Experience with superheater engines during the last thirty years has shown that a factor of adhesion of 4 is about the minimum for satisfactory locomotive operation with superheaters of normal pressure drop, and it is only under difficult rail conditions that such locomotives have been found to slip at high speeds.

If locomotives with factor of adhesion near the lower limit have the superheater changed to single loop, cutting the pressure drop in half, and thereby raising the mean effective pressure at high speed, such engines might prove very slippery at high speeds when using ordinary valve gears.

Under such conditions the engine must be throttled, the cylinder diameters reduced, or the initial boiler pressure cut, and this will cancel the improvement that was originally sought to be gained by the lower pressure drop at high output, and one is back where he started.

For locomotives that are undercylindered, and have a factor of adhesion larger than 4, a considerable improvement may be gained in decreased pressure drop through the use of the single-loop superheater.

Engines designed fifteen or more years ago were usually equipped with superheaters smaller than is present practice. Many of these older locomotives have been rebuilt and equipped with more efficient and powerful front ends, stokers, better grates, and valve gears. This has permitted such modernized engines to be forced to much higher rates of evaporation than originally possible and, logically, the superheater should be enlarged correspondingly both as to surface and steam area.

The greatest gain in increased boiler efficiency, higher superheat and low pressure will be obtained by installing type "E" superheaters. Whenever this is not expedient a good improvement can be obtained by the installation of a superheater unit like the "HA," which gives some 50 F higher superheat, without any increase in the pressure drop.

Another solution may be that of using a single-loop type "A" superheater. This would give less superheat, but only one third of the pressure drop of the double-loop type "A" with the same number of units.

In his fifth question Mr. Meister observes that the majority of modern locomotives have been designed with type "E" superheaters, and inquires regarding the improvements that will result from the use of this design as compared with the older type "A" superheater, or other types of superheater and boiler designs.

The type "E" superheater is a fire-tube superheater with a header in the front end of the same general design as the type "A." The difference lies in the arrangement, also the size of the superheater flues and units. The superheater flues in the type "E" superheater may vary in size from $3^1/_4$ in. OD to 4 in. OD depending upon the length of the flues. The size of the flue is selected so as to give the maximum efficiency with minimum draft loss.

Each unit is a double loop with one loop of superheater pipe in each of two superheater flues. The unit pipes are of such size as to give the highest superheat with the lowest pressure drop and draft loss. There are a number of open small tubes that do not contain superheater units.

Due to the use of the smaller size of superheater flues in the type "E" superheater as compared with the larger flues used in the type "A" it is possible to gain from 8 to 10 per cent in the total flue evaporating heating surface of the boiler for the same length of flues; this in addition to a gain of from 3 to 6 per cent in the total free gas area through the boiler.

The heating surface of the type "E" superheater is increased from 35 to 50 per cent over the type "A" superheater thus giving from 60 to 80 degrees higher superheat at maximum capacity for equal conditions of operation, and an even higher difference at the lower rates of working because of the greater surface and the more efficient disposition of same.

The steam area through the superheater is usually greater than the type "A," and is ample for normal low pressure drop.

The foregoing physical improvements in the amount and effectiveness of the heating surfaces result in a decided improvement in boiler efficiency and superheat with a resultant improvement in the over-all efficiency and capacity of the locomotive of upward of 15 per cent as compared with an old-style large-flue superheater.

With a properly proportioned grate and firebox the type "E" superheater-equipped locomotive boiler represents the most efficient concentration of boiler power that has so far been proposed for use in the conventional boiler and has proved practical in every respect.

In addition to the improvements in efficiency and capacity resulting from the use of the type "E" design there are several other practical advantages, such as a reduction in the number of unit header connecting joints to keep tight, and easier locomotive to draft because of the more even distribution of the same size of flues over the tube sheets, a larger space between the flues at the back tube sheet providing more water space and better circulation, which is of considerable importance in flue and tube-sheet maintenance.

The relative small-diameter flues permit the use of thinner flues as compared with the large flue superheater. This, together with the uniform size and thickness of the flues, equalizes the expansion strains reducing flue leakage and flue-sheet cracking.

Regarding Mr. Meister's sixth question, whether better results would be obtained with the stoker located in the front of the firebox. This question is very completely answered by Mr. Roesch's discussion on this paper; and the question of burning pulverized coal is touched upon in another place.

Mr. Meister's seventh question is of great interest to railway management. Many studies have been made on this subject, but it is apparent that sufficient experience has not yet been had to permit an accurate analysis, as suggested by Mr. Meister, from which definite conclusions could be made. This subject is of such large scope that it alone would require a separate treatise.

Mr. Ostermann's discussion on the high economy that results from the use of high superheat is valuable, and brings out the

great importance that correct principles of superheater design have upon the satisfactory operation of locomotives.

The record of locomotive construction throughout the world, during the last thirty years, amply sustains the conclusion that the fire-tube superheater has proved the only practical design for the conventional locomotive boiler so far devised to obtain high superheat. Many other types have been tried, but without success.

This record, however, although gratifying, has not stopped the research work of The Superheater Company as they for many years have been engaged in a systematic and complete program of superheater design.

Every practical form of superheater unit so far conceived has been tested by The Superheater Company to determine the form of unit that would give the maximum superheat and highest evaporation, combined with minimum draft loss, minimum pressure drop through the superheater, low weight and cost, and, at the same time, be easy to maintain.

The author's replies to several of the questions raised by Mr. Meister in his discussion will answer some of the points raised by Mr. Ostermann.

Mr. Pownall adds a valuable contribution. Based on years of experience in this field he has confirmed the importance of testing, also modification of designs to meet specific operating conditions. His work in the development and improvement of the locomotive front-end construction is well known.

Mr. Pownall suggests that the cost of boiler repairs, stated by the author to be about 30 per cent of the total cost of repairs, should not exceed 15 per cent.

The 30 per cent figure was quoted from the report of the Federal Co-Ordinator of Transportation on comparative cost of steam-locomotive repairs, dated November 7, 1935, page 82, and being of great interest is quoted below:

Boiler Repair Costs

In response to the question: "Do you have data separated so as to show what proportion of the total costs of repairs is properly chargeable to locomotive boilers? If the answer is 'yes' state the percentage," eight roads stated that they kept the data separated and showed it varied from 19.2 to 37.4 per cent of the total cost of locomotive repairs; one hundred and ten roads did not have the separation, and ninety seven made estimates that ran from one to sixty-five per cent. A summarization of their estimates showed:

Number of roads	Estimated proportion of total cost of locomotive repairs due to boilers, per cent
17	25
15	20
13	30
8	15
6	35
5	40
5	29
3	10
3	19
3	60
2	18
2	27
2	31
2	65
1	1
1	6
1	12
1	13.5
1	16
1	22
1	26
1	33
1	44
1	49
1	62
97	

Of the roads making an estimate, 68 of them, aggregating 70 per cent of their estimates, ranged from 20 to 40 per cent, so it is probably fair to assume that a mean between these extremes, or 30 per cent, would be a fair general approximation. An effort was made to develop to what extent the treatment of water had affected the cost of locomotive repairs. However, since there is such a limited knowledge with respect to what the boiler repairs cost, it was found impracticable to get much of value in this connection.

From the foregoing it will be noted that only five class 1 railroads out of 165 reporting had a boiler cost below 15 per cent of the total cost.

Mr. Roesch has submitted an interesting discussion on the subject of the great importance to locomotive-boiler operation of the problems and phenomenon of burning fuel in firebox. In the past this subject has been given much thought and study, and there is no question but that further research is necessary, also, that available test data should be published and interpreted.

There is one point in Mr. Roesch's discussion upon which further thought may be given. He states, "Needless to say, the general adoption of restricted grates was hailed by locomotive-stoker manufacturers as it fully supported their contention that stack loss was not a question of coal size as fired but rather one of air openings through the grates and draft influence."

The author feels that this statement cannot be accepted as it stands without further explanations and qualifications as it is contrary to his own experience as to the effect that the size or per cent of fineness of coal has upon stack loss or unburned fuel loss.

The author has personally conducted many fuel tests which showed that the greater the percentage of fineness in the coal as fired, the greater the stack loss, based on exactly identical locomotives, heating value of fuel, and method of firing. Many tests conducted, some within recent years, have proved this to be so.

With the permission of W. G. Knight, mechanical superintendent, Bangor and Aroostook Railroad Company, the following abstract is quoted from the test report of a recent test most carefully conducted on this road.

"In 1939 a thorough investigation was made on the Bangor and Aroostook in connection with noted increase in coal consumption.

"Every cargo of coal that this Company buys is analyzed, and a careful study made indicated that the chemical composition of the coal was good, and it was, therefore, necessary to look elsewhere for the cause.

"Stack losses were known to be excessive and tests with dynamometer cars were made under the following conditions:

"Two carloads of coal of approximately 80 tons were screened and the amount of fines removed from the coal would be everything below the 1/2" screen mesh. This coal was run through the stoker and rescreened again. From this test approximately 28 per cent fines would pass through the 1/2 in. screen.

"Two engines, identically of the same construction, one being hand-fired, and the other stoker-fired, were used for the test.

"A coal car was used behind the locomotive to catch all cinders that could be obtained for the purpose of getting samples to analyze their heat value.

"The results showed among other things that by using screened coal containing 1.24 per cent slack the saving obtained when compared with the same coal having different percentage of slack was found to be as follows:

Slack, per cent	Fuel saving lb per M.G.T.M. in per cent
12.0	8
23.8	12.2
28.0	16.0

The conditions of the locomotive and test were identical in all comparisons.

"It was further found from these experiments that the difference from the coal consumption of run-of-mine coal having 25 per cent to 30 per cent fines as compared with the 15 per cent fines amounted to approximately 10 per cent.

"Results from operation extending over a year on coal having 15 per cent or less slack have even shown on average operation slightly better than 10 per cent saving."

The foregoing quoted test results on the Bangor and Aroostook

are confirmed by recently conducted fuel tests on the New York Central Railroad.

These tests were run for the specific purpose of determining the qualities of different grades of fuel, under identical conditions, of a modern 4-6-4 type locomotive, on standing blowdown tests.

Permission has been granted to quote the following conclusions relating to the subject of the effect of fineness and slack in fuel.

"Preliminary results of tests on fuel from six different mines in which the percentage of slack in the coal varied from straight egg coal with no slack (what is commonly known as 100 per cent egg) to 40 per cent egg with 60 per cent screenings or 60 per cent slack.

"The test showed that where the slack content has been increased the coal rate was correspondingly increased for identical rates of evaporation for practically all the fuel fired.

"Coals of practically the same chemical composition and some slight difference in the B.t.u. value will result in an inferior boiler performance as the use and percentage of screenings is increased.

"The use of increased percentage of screenings, resulting in a lower boiler evaporative capacity, will affect the locomotive performance. This decrease in the boiler performance may be of sufficient magnitude to reduce the locomotive performance required for heavy trains on fast schedules 10 per cent."

From the foregoing it would appear that this question is of great importance and is worthy of the very careful study that it is now receiving at the hands of the various railway administrations.

Mr. Sillcox's comments are of great value because of the prominent part that he has taken in the development of the steam locomotive. His reference to a new approach to the use of pulverized coal and water-tube boilers is particularly timely.

Many studies have been made of this, and different types of water-tube boilers have been built in this country and in Europe. So far only a few locomotive tube boilers have operated successfully. These have been confined to good feedwater territories.

For high availability, that is, long periods of continuous service that we strive for today, it is necessary, no matter what the construction of the water-tube boiler may be, to use only fully treated water; or in conjunction with condensing operation of the engine when pure scale-free boiler feedwater will be assured.

Mr. Sillcox's reference to the use of pulverized fuel is also important. Based on various tests with burning pulverized fuel it appears practical to design a locomotive firebox for the conventional standard boiler, or water-tube boiler, in such a manner as to satisfactorily burn pulverized coal at rates that will satisfy and exceed any demands so far placed on the modern locomotive.

To do so it will necessitate pulverizing the coal very finely as this will increase the speed of the combustion process by a more rapid contact between the molecules of carbon and oxygen.

At maximum rate it should appear possible to obtain a combustion efficiency of 95 instead of 60 per cent, or less, with the present standard method of firing. This increase in the combustion efficiency, in addition to the other advantages that will accrue from burning of pulverized fuel, must pay for the total cost of pulverizing the coal.

The discussion by Mr. Wallace is welcomed indeed in view of the intimate knowledge which he has had of research problems affecting locomotives and railway operation. His comments on the necessity of reducing the delay at terminals are pertinent as every minute saved on stops and slow orders will decrease the elapsed time between terminals.

Mr. Wallace again stresses the necessity of a coordinated program of test and research for the further improvement of the steam locomotive, and that the development of a more efficient and lighter boiler must go hand in hand with the development of the prime mover as the steam consumption of the engine must be reduced substantially below the present practice.

Whether this new prime mover will take the form of a reciprocating or rotating steam engine, or turbine, can only be answered by future developments, but, in any case, it is apparent that condensing operation is a very desirable thing not only to reduce the specific steam consumption of the prime mover, but also to furnish pure condensate for the boiler feedwater.

The most recent practical development now in sight is the perfection of the Franklin improved cylinder arrangement with poppet-type steam and exhaust valves driven with a cam valve gear.

Such an arrangement is now being successfully used in the United States, and has shown a remarkable improvement in power development at high speeds.

Design Factors Controlling the Dynamic Performance of Instruments

By C. S. DRAPER,[1] CAMBRIDGE, MASS., and G. P. BENTLEY,[2] WALTHAM, MASS.

This paper presents a generalized treatment of instruments which have a single movable index, controlled by the magnitude of a single actuating quantity. Such an instrument acts as a system with one degree of freedom and is studied by the well-known methods developed for such systems. Instruments which include servomechanisms cannot be treated as systems with one degree of freedom and are therefore excluded from consideration. From the standpoint of static indications, it is shown that the essential properties of an instrument can be specified in terms of range, scale sensitivity, calibration error, environmental error, and uncertainty. The definition and use of each of these quantities is discussed.

It is shown that the response of an instrument to rapid changes in the actuating quantity can be calculated if two parameters called instrument characteristics are known. One of these parameters has the dimension of time and is called the characteristic time. The second parameter is nondimensional and is called the damping ratio. Nondimensional curves, showing instrument responses to three typical variations in the actuating quantity, are plotted for a series of values of the damping ratio. These curves are used in an illustrative example to show how the values of the instrument characteristics, required to keep dynamic errors below a given limit, can be determined in a practical case.

INSTRUMENTS must be designed to meet many types of service. Although individual instances will often present problems which call for particular construction details or special materials, certain aspects of instrumentation can be generalized to include the requirements of many practical cases. Such generalizations are useful not only in writing specifications for equipment but also in the analysis and discussion of instrument problems. The system which is outlined herewith has been developed for teaching and design purposes at the Massachusetts Institute of Technology. Emphasis is placed on the points of similarity between instrument problems rather than on the differences which appear when a comprehensive scheme of terminology is applied. For a classification of instruments from the standpoint of their use in industry the reader is referred to the works of Behar (1),[3] and Smith and Fairchild (2, 3).

Reduced to the simplest terms, an instrument is so often an arrangement in which the position of a pointer or index is controlled by some physical quantity, that instruments of other types may be considered as special cases. In the operating range of an instrument, it is usually intended that a continuous motion of the index will follow a continuous change in the actuating quantity which controls the instrument. A device in which this result has been achieved by design falls into the category of systems with 1 deg of freedom and will be called an "elementary instrument." Unless otherwise specified, the term "instrument" will always refer to an elementary instrument. One of the authors and G. V. Schliestett (4) have discussed in generalized form the mathematical analysis of elementary instruments. The analysis divides naturally into two cases, depending upon the way in which the actuating quantity changes. If the actuating quantity is constant, the index will become stationary and the instrument subjected to "static conditions." On the other hand, if the actuating quantity is varying so rapidly that the index does not follow the changes exactly, the instrument is operating under "dynamic conditions." Practical problems deal with the properties required in an instrument to meet certain static requirements and to perform satisfactorily under given dynamic conditions. The present paper suggests a method of specifying the essential features of an instrument in terms of characteristics which can be conveniently applied either for design purposes or for determining the suitability of an instrument for use in a given situation.

Static Characteristics

The essential external features of an elementary instrument are shown in Fig. 1. From the standpoint of performance, the

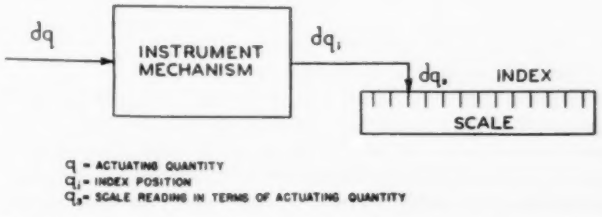


Fig. 1 Essential Features of an Elementary Instrument

mechanical details inside the case of an instrument are unimportant. It is sufficient to know that the instrument contains a mechanism of some sort which produces a change in an index when a change occurs in the actuating quantity. If the instrument belongs to the general class called "meters," a scale is attached to the structure which directly associates each position of the index with some measure of the actuating quantity. Other instruments may not include a scale and so would be disqualified as meters. However, there is always the possibility of interpreting index positions in terms of the actuating quantity by means of tables or calibration curves. For purposes of discussion, any means capable of performing the function of identifying some state of the actuating quantity with each position of the index will be called a "scale."

The general nature of an instrument mechanism is determined when the actuating quantity is given. The requirements of static performance for the mechanism and the scale may be specified in terms of the range and the scale sensitivity. Range is the difference between the highest and lowest magnitudes of the actuating quantity which the instrument is to measure.

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[2] Consulting Engineer, Waltham Watch Company.

[3] Numbers in parentheses refer to the Bibliography.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Scale sensitivity is the ratio of a small change in scale reading to the corresponding small change in index position, i.e.

$$\text{Scale sensitivity} = S_s = dq_s/dq_i \quad [1]$$

In general S_s may be different for different parts of the scale or it may be constant throughout the range. A vapor-pressure thermometer, using a pressure gage of uniform sensitivity as the indicating system, will have a temperature scale of varying scale sensitivity.

For static conditions, an instrument with a scale perfectly suited to the mechanism, with no friction or hysteresis in the mechanism and not affected by changes in physical quantities other than the actuating quantity, would be completely specified by the range and scale sensitivity. Such an instrument would be perfectly accurate in the sense that the scale reading would always be equal to the actual value of the actuating quantity. This condition may not be fulfilled but, in any case, a small change in the scale reading may be related to a small change in the actuating quantity by the equation

$$dq_s = C dq \quad [2]$$

where C is the calibration coefficient. Obviously the condition for an ideal direct-reading instrument is for the calibration coefficient to be unity. Keeping in mind that C may vary over the range in practical cases, the scale reading corresponding to a given magnitude of q can be found by integrating Equation [2] to obtain

$$q_s = \overline{C}q + K \quad [3]$$

where K is the constant of integration and $\overline{C}$ is a proper average value of the calibration coefficient. If the zero adjustment of the instrument is set so that the scale reading is correct ($q_s = q$) at some part of the scale, the constant of integration can be eliminated from Equation [3]. The actual instrument performance can then be expressed by means of a plot of

$$\overline{C} = q_s/q \quad [4]$$

as a function of the scale reading.

In practice the variations of the calibration coefficient over the range of an instrument are so small that a plot of static corrections is more convenient for studying performance than a plot of the calibration coefficient itself. If the static correction is defined as the difference between the actual magnitude of the actuating quantity and the scale reading, Equation [3] gives

$$\text{Static correction} = q_{corr} = q - q_s = q_s\left(\frac{1-\overline{C}}{\overline{C}}\right) \quad [5]$$

or, expressed in terms of the fractional static correction

$$\text{Fractional static correction} = q_{corr}/q_s = (1-\overline{C})/\overline{C} \quad [6]$$

The fractional static correction expressed as a percentage is a measure of the inaccuracy of the scale reading in any particular case, but it is inconvenient for generalized discussions, since it varies with both the correction and the scale reading. For specification purposes, it is better to express the correction as a percentage of the highest reading on the scale. In this form the correction will be called the full-scale fractional static correction and will be given the symbol Δq_s, where

$$\Delta q_s = q_{corr}/Q_s \quad [7]$$

and Q_s is the difference between highest and lowest scale readings or total scale range.

Analysis of Static Corrections

Many instrument mechanisms are affected to some extent by changes in physical quantities other than the actuating quantity. These physical quantities together make up the environment of an instrument. The process of marking off the scale to read directly in terms of the actuating quantity is called calibration and must be carried out under some particular state of the environment which establishes the calibration conditions.

Static corrections can be broken down on the basis of the source responsible for each component. These components and a corresponding symbol in terms of full-scale fractional static correction are as follows:

1 Calibration error (Δq_c) is due to improper fitting of the scale to the instrument under calibration conditons.

2 Environmental error (Δq_e) is due to variations of the environment from calibration conditions.

3 Uncertainty (Δq_u) is due to imperfections in the instrument mechanism such as rubbing friction between solid parts, hysteresis in elastic members, etc.

Calibration errors often appear when instrument mechanisms are attached to independently graduated scales without special adjustments. The slow changes due to wear and aging effects may also produce calibration errors. Such errors may be found by experiments carried out under calibration conditions. Environmental error can sometimes be calculated but can always be determined by experiment. For any particular reading of a given instrument, the calibration error and the environmental error can be combined into a correction which removes both these errors when applied to the reading. In addition to the method of correction, environmental errors can be eliminated by designing the instrument mechanism to be unaffected by changes in the environment. An instrument constructed in this fashion is said to be compensated.

Uncertainties differ from the errors just considered in that they cannot be exactly determined for a particular scale reading and applied as a correction. Uncertainty appears when a scale reading is made from a pointer of finite width. Rubbing friction between solid parts in the mechanism is a second typical cause of uncertainty. Due to this friction, some finite change in the actuating quantity must occur before a detectable motion of the index appears. From this standpoint, uncertainty, as the word is used here, is similar to ultimate sensitivity as this term is accepted by Behar (1).

In general, it is only possible to set an upper limit for the uncertainty associated with a particular scale reading. Smith (3) has applied the term "dead zone" to a similar concept. According to Smith: "The dead zone is the extreme range of values of the variable possible without moving the indicator from a given position, in the absence of abnormal vibration; the dead zone is ordinarily twice the frictional error." Actually the uncertainty in any particular scale reading may be anything from zero to a maximum value determined by friction or the pointer width. The exact value of the uncertainty in a given instance will depend upon the previous history of the scale reading condition of the instrument mechanism, vibration, etc. Although it might be possible to estimate the uncertainty in particular cases, the usual procedure is to accept the maximum uncertainty as a fundamental limitation on the accuracy of an instrument.

Dynamic Errors

Rapid changes in the actuating quantity are accompanied by differences between instantaneous scale readings and the true values of the actuating quantity which occur at the same time. For any particular case, the static correction may be responsible for part of such differences but there will, in general, be another component peculiarly due to the fact that the actuating quantity

is changing. This latter component will be called the "dynamic error." Fig. 2 graphically illustrates the concepts of dynamic error and the corresponding time lag for a case with negligible static corrections. From Fig. 2, dynamic error is the difference between q and q_s at a given instant, while time lag is the time interval between the instant the actuating quantity passes through a certain value and the instant at which the scale reading reaches this value.

Many authors have studied the problem of dynamic errors. In the field of industrial instruments, Behar (1), and Smith and Fairchild (3) have summarized the methods of attack and have given typical results. The work of Draper and Schliestett (4), in reducing the analysis of dynamic errors to generalized nondimensional forms suited for engineering use has already been mentioned. The discussion which follows deals with a scheme for analyzing the dynamic errors of an elementary instrument

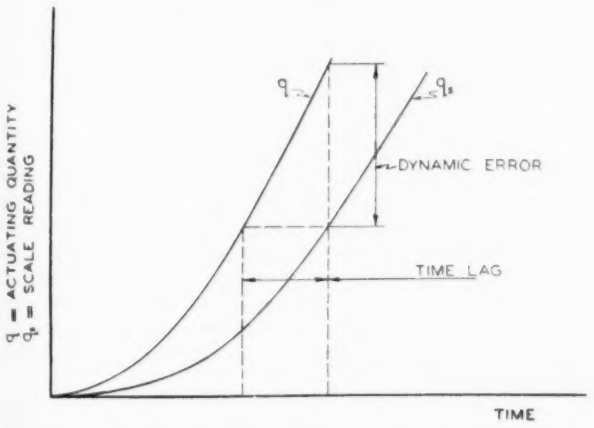


FIG. 2 DYNAMIC ERROR AND TIME LAG

in terms of two characteristic parameters. One of these parameters has the dimensions of time and will be called the "characteristic time." The other parameter is dimensionless and may be either the damping ratio or the characteristic damping number. In certain cases, the undamped natural frequency may conveniently be used as the time parameter. Methods for determining these parameters in actual cases have been described (4). The treatment which is outlined in this reference will be reviewed briefly in the present paper and extended to show how the characteristic parameters may be applied to engineering problems.

For any given instance, the dynamic error will vary with time in a manner dependent upon the characteristics of the instrument used and upon the way in which the actuating quantity changes. This means that each set of circumstances will require special treatment for an exact determination of dynamic errors. The amount of work necessary can be reduced by approximating actual variations in the actuating quantity by means of changes falling into three basic types. In the first of these types, the actuating quantity changes so rapidly between two constant values that the change is completed before the instrument reading has changed appreciably. A change of this sort is called a "step function." The second type occurs when the actuating quantity varies directly with time and will be called a "linear change." The third type is a continuous sinusoidal variation which may actually be one component of a more complicated periodic change. It will be shown how the instrument characteristics can be simply applied to predicting the dynamic errors which occur with each type of variation in the actuating quantity.

GENERALIZED EQUATION OF MOTION

Motion of an instrument index is determined by the balance between effects of four types which, for purposes of discussion, will be considered as generalized forces, as follows:

1 Restoring forces which tend to move the index toward some equilibrium position with a force magnitude proportional to the divergence of the index from that position.

2 Damping forces which resist motion of the index with a resultant magnitude proportional to the index velocity. This assumption yields useful results for many cases in which it represents an approximation.

3 Inertia forces which are proportional to acceleration of the index.

4 Forces which depend upon the magnitude of the actuating quantity. These forces are responsible for initiating changes in the index position, and for this reason, the mathematical expression which specifies the actuating quantity as a function of time will be called the "forcing function."

It has been shown (4), that the first three of these forces may be expressed in terms of the scale reading to give the equation of motion in the form

$$(a_2)d^2q_s/dt^2 + (a_1)dq_s/dt + (a_0)q_s = a_0q \ldots\ldots\ldots [8]$$

This equation can be solved by well-known methods (5) for the case in which the coefficients a_0, a_1, and a_2 are constants. When these coefficients vary either with time or with the scale reading, a solution is usually so difficult to handle that the assumption of constant coefficients is made even for cases where it is not true. The resulting solutions will not be exactly correct but will often give a picture sufficiently close to physical facts for engineering purposes. These solutions will show the way in which the scale reading varies with time for a particular instrument under given conditions. The actual value of the actuating quantity at any instant can be found from the forcing function so the dynamic error is completely determined as the difference $(q - q_s)$ or on a fractional basis as $(1 - q_s/q)$.

Two types of problems involving dynamic errors appear in practice. In the first type the coefficients associated with a given instrument are known and it is desired to find the dynamic errors existing when a particular forcing function is applied. Cases of this nature are simple in that it is only necessary to carry through straightforward solutions of Equation [8]. Problems of the second type start with requirements on the magnitude of dynamic errors accompanying certain forcing functions and lead to values of the coefficients (instrument design constants) a_0, a_1, and a_2 as the desired results. The object of generalizing the analysis of dynamic errors is to reduce the amount of effort required for solution of either type of dynamic-error problem. The method of attack is to divide Equation [8] through by a_2, so that the coefficient of the first-derivative term has the dimensions of reciprocal time, and the coefficient of the term in q_s has the dimensions of reciprocal time squared. The coefficient of the first-derivative term can be used to establish a characteristic time for an instrument. It will be shown that one additional parameter in the form of a dimensionless ratio will completely specify a given instrument. The dimensions of the actuating quantity in any particular case are immaterial since time is the only dimension involved in the characteristics. The characteristics completely define the equation of motion so it follows that all instruments with the same characteristics will have the same dynamic errors under identical forcing functions and initial conditions. Because of this fact, it is possible to plot families of nondimensional curves for the dynamic response of instruments to the three forcing-function types which have been chosen as representative practical cases. With these

curves available, it is possible to solve a wide range of dynamic-error problems by a simple inspection of the curves. The mathematical background necessary for proper use of the instrument characteristics is outlined and the use of nondimensional performance curves is illustrated by examples.

Solution of Equation of Motion in Terms of Instrument Characteristics

Equation [8] has solutions which give the variation of the scale reading q_s with time. These solutions have the general form

$$q_s = q_{s(tr)} + q_{s(ss)} \quad [9]$$

The first term on the right-hand side is called the "transient" and is the solution of Equation [8] with the forcing function taken as zero. The second term is any solution of the complete equation for a given forcing function and is called the "steady-state" solution. Physically, the steady-state solution gives the behavior of the instrument after a particular forcing function has continued indefinitely. The transient represents the period of accommodation required for the instrument to change from one steady-state condition to another.

Mathematical details required to solve Equation [8] are discussed in any course on the differential calculus but the solutions thus obtained are usually left in a form which is unnecessarily clumsy for engineering use. The treatment which follows is intended to reduce the work required to obtain quantitative results in dynamic-error problems. It is found that the transient component of the solution may be either oscillatory or nonoscillatory, depending upon the relative magnitudes of the coefficients in Equation [8]. From the standpoint of analysis, a knowledge of the quantities chosen for characteristics should make it possible to determine the dynamic behavior of an instrument directly from generalized nondimensional curves for either oscillatory or nonoscillatory transients. Reasons for the choice of characteristic time and the damping ratio, as these parameters are defined, will appear as the analysis is developed.

Instrument Transients

The nature of the characteristics can best be explained from a study of the transient component in the solutions of Equation [8]. If the forcing function is placed equal to zero and each term is divided by a_2, Equation [8] becomes

$$d^2q_s/dt^2 + (a_1/a_2)dq_s/dt + (a_0/a_2)q_s = 0 \quad [10]$$

The exponential function, the form of which is unchanged by differentiation, is particularly suited for the solution of Equation [10]. Two arbitrary constants must appear in the solution, since the original equation is of the second order. It follows that the solution may be written in the form

$$q_s = Ae^{-t/\tau_1} + Be^{-t/\tau_2} \quad [11]$$

The symbols τ_1 and τ_2 must represent quantities with the dimension of time, since the exponents must be without dimensions. Substitution of the expression given by Equation [11] into Equation [10] shows that τ_1 and τ_2 must be expressible in terms of the coefficients a_0, a_1, and a_2 by the relationships

$$1/\tau_1 = (a_1/2a_2) - \sqrt{(a_1/2a_2)^2 - (a_0/a_2)} \quad [12]$$

$$1/\tau_2 = (a_1/2a_2) + \sqrt{(a_1/2a_2)^2 - (a_0/a_2)} \quad [13]$$

The first term on the right-hand side of these equations is always real, but the quantity under the radical may be either real or imaginary, depending upon the magnitude of $(a_1/2a_2)^2$ with respect to the magnitude of (a_0/a_2). If the second of these quantities is greater than the first, the radical terms become imaginary and the corresponding motion for the transient part of the expression for q_s is oscillatory. In this case Equation [11] is usually changed from the exponential form to a trigonometric form which is more convenient for practical use. If $(a_1/2a_2)^2$ is greater than (a_0/a_2), the radical term is real and Equation [11] can be used as written. A third case exists which represents the transition condition between oscillatory and nonoscillatory transients.

For maximum usefulness the quantities chosen as instrument characteristics should be easily applied to any case whether the transient is oscillatory or nonoscillatory. The undamped natural period of an instrument is suitable for use when the transient is oscillatory or the forcing function is sinusoidal, but is difficult to interpret directly when the transient is nonoscillatory or forcing functions are not sinusoidal. On the other hand, the quantity $(2a_2/a_1)$ has the dimension of time and is so intimately associated both with the damping of transients and steady-state dynamic errors that it is particularly suitable for use as one of the instrument characteristics. For this reason the characteristic time will be defined as

$$\tau = (2a_2/a_1) \quad [14]$$

To specify completely the behavior of an instrument under dynamic conditions, another parameter is required in addition to τ. This parameter is the second instrument characteristic and can always be chosen as a dimensionless ratio. The ratio between τ_1 and τ_2, as these quantities are defined by Equations [12] and [13], is the most obvious choice for a dimensionless instrument characteristic. This ratio is especially useful for nonoscillatory transients and will be called the "characteristic damping number."

$$\nu = \tau_1/\tau_2 (\tau_1 > \tau_2) \quad [15]$$

When the transient becomes oscillatory, the characteristic damping number becomes a complex quantity which is difficult to interpret for practical purposes. A dimensionless ratio which can be conveniently applied in any case is the damping ratio. This ratio is defined as the ratio of the coefficient a_1 which actually exists to the coefficient a_{1c} which must exist for the condition which divides oscillatory from nonoscillatory transients. Mathematically this critically aperiodic condition is reached when

$$(a_1/2a_2)^2 = a_0/a_2 \quad [16]$$

For this case

$$\tau_1 = \tau_2 = \tau \quad [17]$$

and

$$\text{Damping ratio} = \zeta = a_1/a_{1c} = \nu = \tau_1/\tau_2 = 1 \quad [18]$$

It follows that when ζ is less than unity, the transient will be oscillatory, while the transient will be nonoscillatory for values of ζ greater than unity. The characteristic damping number will be unity when the transient is critically aperiodic, greater than unity for nonoscillatory transients, and complex for oscillatory transients.

In order to solve dynamic-error problems, certain definitions, in addition to those already discussed, are convenient. The "circular natural frequency" is defined as

$$\text{Circular natural frequency} = \omega = \sqrt{a_0/a_2 - (a_1/2a_2)^2} \quad [19]$$

$$\omega = 2\pi n = 2\pi/T \quad [20]$$

In Equation [20] n is the natural frequency and T is the natural period. When the coefficient a_1 is zero, the quantities thus

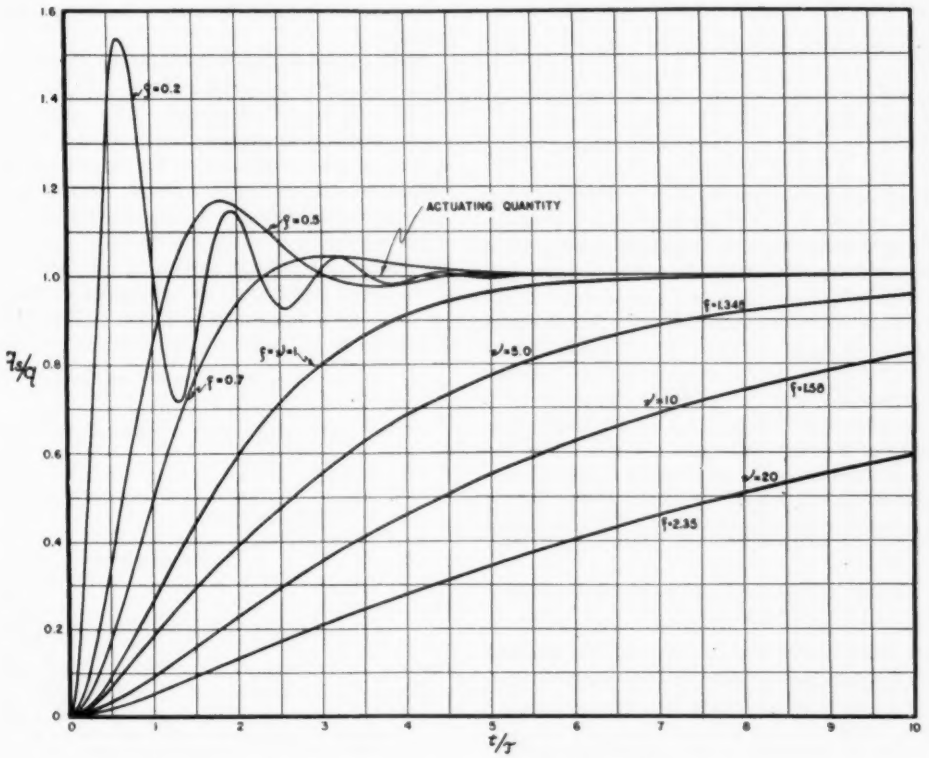


FIG. 3 INSTRUMENT RESPONSE TO A STEP FUNCTION

defined become the undamped natural circular frequency, the undamped natural frequency, and the undamped natural period, respectively. The undamped condition is indicated by a subscript n on each of the quantities so that

$$\omega_n = \sqrt{a_0/a_2} = 2\pi n_n = 2\pi/T_n \quad [21]$$

and

$$\omega = \omega_n \sqrt{1 - \zeta^2} \quad [22]$$

A convenient way of remembering the relationships between the instrument characteristics and the other essential quantities involved in the solution of dynamic-error problems is to write the original differential equation of motion in terms of the derived quantities. The result of this procedure is as follows

$$d^2q_s/dt^2 + (a_1/a_2)dq_s/dt + (a_0/a_2)q_s = (a_0/a_2)q \quad [23]$$

$$d^2q_s/dt^2 + 2\zeta\omega_n(dq_s/dt) + \omega_n^2 q_s = \omega_n^2 q \quad [24]$$

$$d^2q_s/dt^2 + (2/\tau)dq_s/dt + 1/(\zeta\tau)^2 q_s = 1/(\zeta\tau)^2 q \quad [25]$$

$$d^2q_s/dt^2 + (\nu + 1)/\tau_1(dq_s/dt) + (\nu/\tau_1^2)q_s = (\nu/\tau_1^2)q \quad [26]$$

Written in terms of the instrument characteristics, the transient can be conveniently expressed in three forms, depending upon the value of ζ.

Case 1 Oscillatory transient, ζ is less than unity, ν is complex

$$q_s = e^{-t/\tau}\left\{A \sin\left[\left\{\sqrt{1-\zeta^2}/\zeta\right\}(t/\tau)\right] + B\cos\left[\left\{\sqrt{1-\zeta^2}/\zeta\right\}(t/\tau)\right]\right\} \quad [27]$$

or

$$q_s = Ae^{-t/\tau}\cos\left[\left\{\sqrt{1-\zeta^2}/\zeta\right\}(t/\tau) + \varphi\right] \quad [28]$$

Case 2 Critically aperiodic, $\zeta = \nu = 1$

$$q_s = Ae^{-t/\tau} + Bte^{-t/\tau} \quad [29]$$

Case 3 Nonoscillatory, $\zeta > 1$, $\nu > 1$

$$q_s = Ae^{-[2/(\nu+1)]t/\tau} + Be^{-[2\nu/(\nu+1)]t/\tau} \quad [30]$$

where

$$\zeta = (\nu + 1)/2\sqrt{\nu} \quad [31]$$

In Equations [27], [28], [29], and [30], the symbols A, B, and φ represent "fitting constants" to be used for fitting the transient equations to combinations of particular steady-state solutions and given initial conditions.

DYNAMIC ERRORS PRODUCED BY A STEP FUNCTION

The step function will be treated as the first typical forcing function. For analytical purposes, a step function can be considered a case in which the scale reading is constant at zero under a zero magnitude of the actuating quantity, before the instant taken as the initial instant, followed by an instantaneous change of the forcing function to some constant value at the initial instant. Mathematically these circumstances can be expressed as

$$t = 0 \quad q = q_s = 0 \quad dq/dt = dq_s/dt = 0$$
$$t > 0 \quad q = q_s \quad dq/dt = 0$$

When the fitting constants of Equations [27] to [30] are adjusted so that the general transient equations meet the particular initial conditions of the step function, the results are

$\zeta < 1$

$$q_s/q_s = 1 - e^{-t/\tau}\left\{\left(\zeta/\sqrt{1-\zeta^2}\right)\sin\left[\left\{\sqrt{1-\zeta^2}/\zeta\right\}(t/\tau)\right] + \cos\left[\left\{\sqrt{1-\zeta^2}/\zeta\right\}(t/\tau)\right]\right\} \quad [32]$$

$$q_s/q_s = 1 - e^{-t/\tau}\cos\left[\left\{\sqrt{1-\zeta^2}/\zeta\right\}(t/\tau) - \sin^{-1}\zeta\right] \quad [33]$$

$\zeta = \nu = 1$

$$q_s/q_a = 1 - e^{-t/\tau}\,(1 + t/\tau) \quad \ldots\ldots\ldots\ldots [34]$$

$\zeta > 1,\ \nu > 1$

$$q_s/q_a = 1 - [\nu/(\nu-1)]e^{-[2/(\nu+1)](t/\tau)} + [1/(\nu-1)]e^{-[2\nu/(\nu+1)](t/\tau)} \quad \ldots\ldots [35]$$

The general nature of these transients is so well known that it is unnecessary to include a detailed mathematical discussion. From an engineering standpoint, the important point is that a family of curves can be plotted with t/τ as the independent variable and q_s/q_a as the dependent variable, each curve of the family being based on a constant value of ζ. Fig. 3 shows such a family of curves for values of ζ between 0.2 and 2.4. These curves can be applied to instruments of any type which obey the original differential equation. A useful generalization which is apparent from an inspection of Fig. 3 is that the transient is substantially over in a time interval equal to three times the characteristic time, if the damping ratio is in the range between 0.2 and 1. An example showing how the curves of Fig. 3 may be applied to practical engineering problems will be discussed later.

Dynamic Errors Due to a Linear Change of the Actuating Quantity

In many practical cases, the change in the actuating quantity can be approximated as a linear increase or decrease with time. Mathematically this is equivalent to using an expression

$$q = Mt \quad \ldots\ldots\ldots\ldots\ldots\ldots [36]$$

as the forcing function. M is the constant which can be adjusted to make Equation [36] fit the physical facts as closely as possible. The complete solution for q_s, as a function of time, depends not only upon the instrument and the forcing function but also requires that the initial conditions be specified. Out of an infinite number of possible choices, the case in which the reading is constant at zero, when the linear forcing function is applied, is most useful for generalized discussions of instrument performance. The case in which the initial reading is not zero but is constant at some other value may obviously be handled by a simple substitution of $q - q_0$ and $q_s - q_{s0}$ as variables to replace q and q_s. The equations resulting from use of the forcing function $q = Mt$ and $q_s = 0$, $dq_s/dt = 0$ when $t = 0$ are

$\zeta < 1$

$$q_s/M\tau = t/\tau - 2\zeta^2 + e^{-t/\tau}\left\{2\zeta^2\cos\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)(t/\tau) + \frac{\zeta(2\zeta^2-1)}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)(t/\tau)\right\} \quad \ldots\ldots [37]$$

$\zeta = \nu = 1$

$$q_s/M\tau = t/\tau - 2 + e^{-t/\tau}(2 + t/\tau) \quad \ldots\ldots\ldots [38]$$

$\zeta > 1, \quad \nu > 1$

$$q_s/M\tau = t/\tau - (\nu+1)^2/2\nu + [\nu(\nu+1)/2(\nu-1)]e^{-[2/(\nu+1)](t/\tau)} - \left[\frac{\nu+1}{2\nu(\nu-1)}\right]e^{-[2\nu/(\nu+1)](t/\tau)} \quad \ldots\ldots [39]$$

These equations have been made nondimensional by using the amount that the actuating quantity changes in one characteristic time as the unit of scale reading. By this procedure, the equations are simplified and a single family of curves will take care of all possible cases, involving a linear forcing function acting on an instrument with a constant reading at the initial instant.

With τ used as the time unit, the time scale is not uniform as in the normal case when the damping is varied for a system which is otherwise unchanged. This effect is liable to be confusing unless it is properly interpreted. Therefore, the curves of Fig. 4 have been plotted for an instrument with constant undamped natural frequency and varying damping. By using the undamped natural period as the unit of time and the change of the actuating quantity occurring in a time interval equal to the undamped natural period, the distance associated with a given time interval or a given change in scale reading is the same for each one of the family of curves. This makes interpretation of the curves in terms of the physical facts somewhat simpler

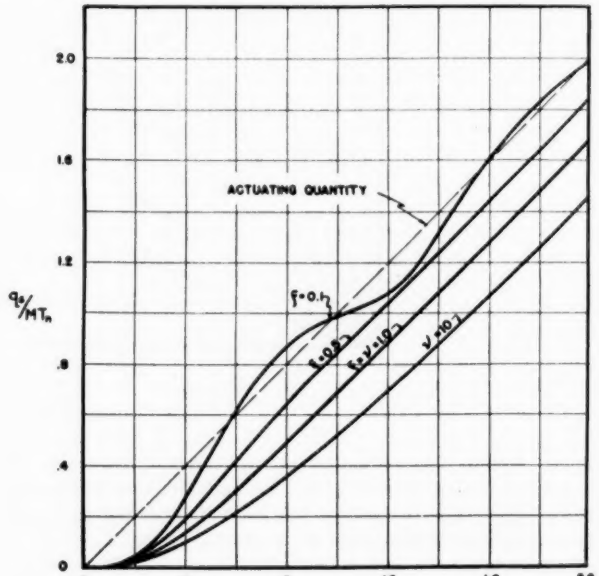


Fig. 4 Instrument Response to a Linear Forcing Function

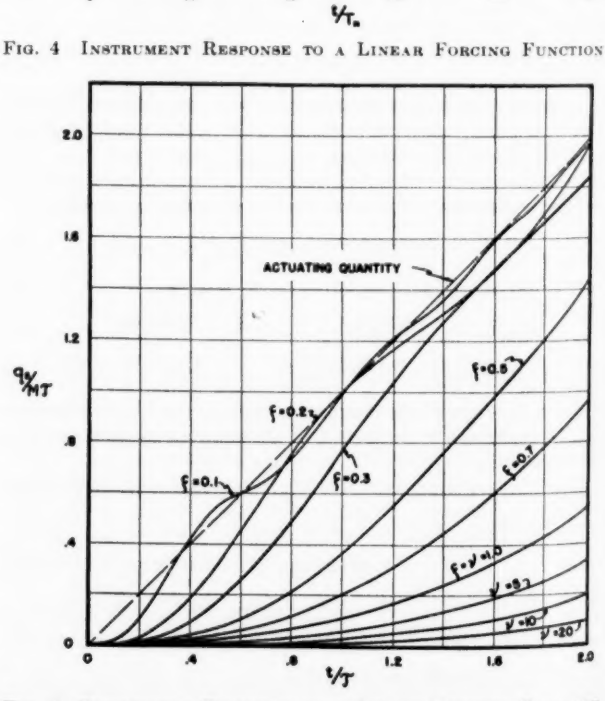


Fig. 5 Instrument Response to a Linear Forcing Function

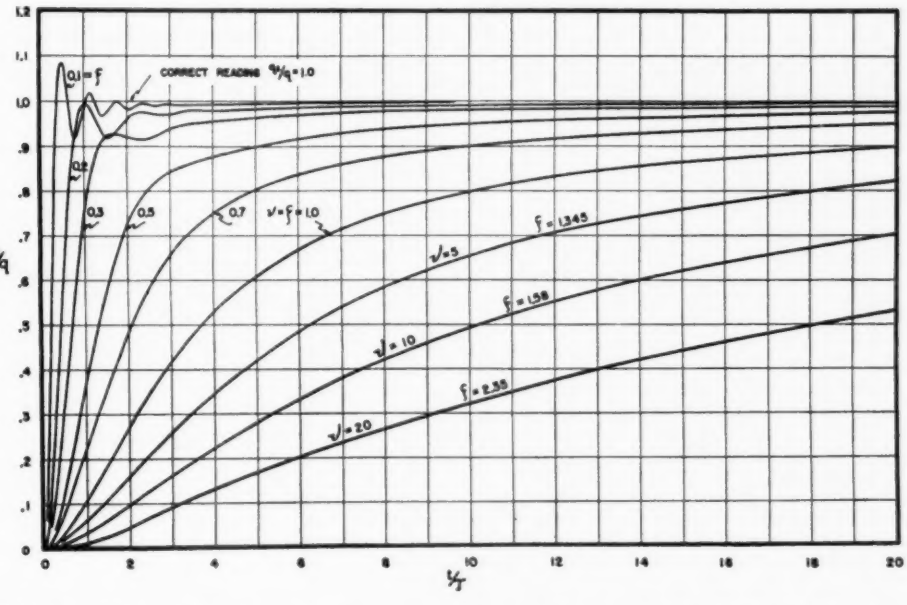


FIG. 6 LINEAR FORCING FUNCTION; VARIATION WITH TIME OF ACTUAL READING AS FRACTION OF TRUE VALUE OF ACTUATING QUANTITY

than if Equations [37], [38], and [39] had been plotted directly. The curves of Fig. 4 show that, as the transient dies out, the instrument reading differs from the true instantaneous value of the actuating quantity by an amount which tends to a constant value of $2\zeta^2 M\tau$ for oscillatory transients, $2M\tau$ for the critically aperiodic case and $[(\nu + 1)^2/2\nu]M\tau$ for the nonoscillatory case. This is an example of a steady-state dynamic error which is constant and corresponds to constant time lag in terms of T_n units. Fig. 5 shows the curves of Fig. 4 plotted directly from Equations [37], [38], and [39]. The effect of damping on the characteristic time is shown by the expanded abscissa scale as the damping ratio is increased.

Instrument engineering problems are usually concerned with the dynamic error rather than with the actual instrument reading at any instant. For generality, it is convenient to express the actual reading in terms of the correct reading at any instant. The correct reading will always be the instantaneous value of the actuating quantity and in terms of $M\tau$ units is equal to t/τ. It follows that

$$q_a/q = (q_a/M\tau)[1/(t/\tau)] \quad \ldots\ldots [40]$$

The instantaneous value of the dynamic error, expressed as a fraction of the true value of the actuating quantity, will be

$$\text{Dynamic error} = 1 - q_a/q \quad \ldots\ldots [41]$$

Curves showing the fractional reading and the dynamic error as a function of t/τ for values of ζ from 0.1 to 2.35 are plotted in Fig. 6. The use of this family of curves will be illustrated later.

STEADY-STATE DYNAMIC ERRORS DUE TO A SINUSOIDAL VARIATION OF THE ACTUATING QUANTITY

Many practical situations may be represented in terms of periodic changes in the actuating quantity. In cases where the disturbance exactly repeats itself, a Fourier series of sines or cosines will accurately describe the physical facts. Even if

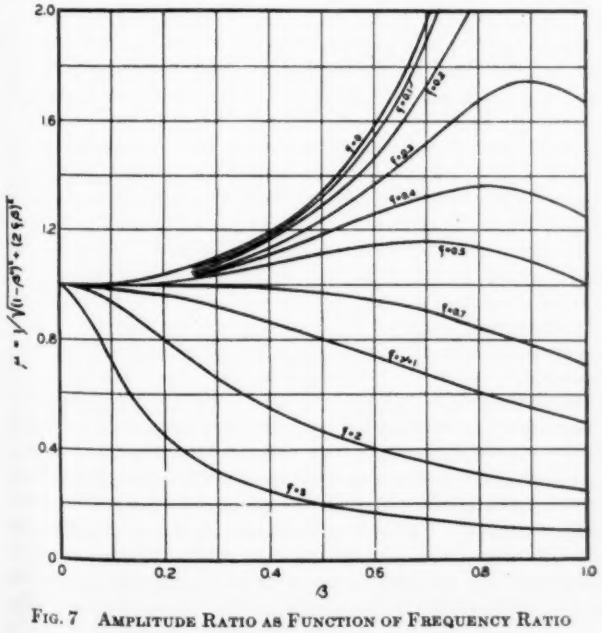


FIG. 7 AMPLITUDE RATIO AS FUNCTION OF FREQUENCY RATIO

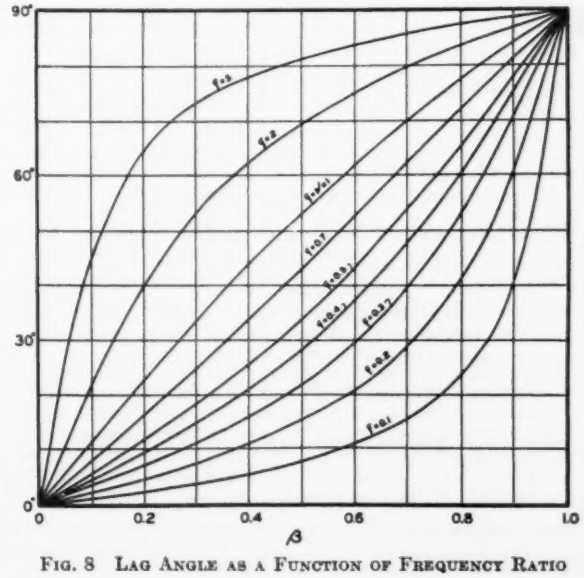


FIG. 8 LAG ANGLE AS A FUNCTION OF FREQUENCY RATIO

this condition is not precisely fulfilled, satisfactory results can often be obtained by the use of a sinusoidal forcing function to find the corresponding steady-state solution of Equation [24]. Thus if the forcing function is

$$q = q_a \sin \omega_f t \quad [42]$$

the steady-state solution as found by conventional methods (5) can be written as

$$q_s/q_a = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} \sin(\omega_f t - \varphi) \quad [43]$$

In Equation [43]

$$\omega_f = 2\pi n_f = 2\pi/T_f = \text{circular forcing frequency} \quad [44]$$

$$\beta = \omega_f/\omega_n = \text{frequency ratio} \quad [45]$$

$$\varphi = \tan^{-1}(2\zeta\beta)/(1-\beta^2) = \text{lag angle} \quad [46]$$

The expression

$$(q_s/q_a)_a = \mu = 1/\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2} = \text{amplitude ratio} \quad [47]$$

is very useful in connection with instrument problems involving periodic changes. A family of curves of μ against β is given in Fig. 7. Practical instrument problems usually involve values of the frequency ratio less than unity so only this range has been covered in Fig. 7. A family of curves showing the lag angle as a function of β for the same range as that of Fig. 7 is given in Fig. 8.

General Design of an Instrument to Meet Given Specifications

The ideas previously discussed are particularly useful for translating performance specifications for an instrument into terms of mechanical details. In practice every instance will present problems requiring special treatment, but the general procedure can be illustrated by an example which might reasonably occur as an actual case. This example will deal with the preliminary design of a recording pressure indicator for slow-speed steam engines or air pumps. Specifications start with the requirement that the instrument must be able to operate for reasonable periods of time under the conditions of pressure, temperature, vibration, etc., which exist in service. Beyond this necessity of satisfactory mechanical operation, the desired properties of the indicator can be defined in terms of static specifications and dynamic specifications. The specifications assumed for the present example are listed as follows:

Static specifications:

- Range—20 lb per sq in. below atmospheric pressure to 160 lb per sq in. above atmospheric pressure
- Scale sensitivity—50 lb per sq in. per in. (constant over range)
- Calibration error—2 lb per sq in. (maximum)
- Environmental error—1 per cent of range per 10 C temperature change
- Uncertainty—1 per cent of range (maximum).

Dynamic specifications:

- Step function—Indicator must give reading within 10 per cent of actual pressure, 0.05 sec after sudden pressure change (q_s/q_a between 0.90 and 1.10; $t = 0.05$ sec)
- Linear change—Gage must follow pressure increasing linearly with time with dynamic error of less than 10 per cent at end of range, when pressure change is such that range would be covered in 0.07 sec (q_s/q_a between 0.90 and 1.10; $t = 0.07$ sec)
- Sinusoidal change—Gage must follow sinusoidal pressure changes with maximum frequency of 4 cycles per sec within an amplitude error of less than 10 per cent and a phase-angle lag of less than 10 deg (μ between 0.90 and 1.10; φ less than 10 deg).

Static specifications determine several of the indicator properties. The range of 180 lb per sq in. and scale sensitivity of 50 lb per sq in. per in. require a scale length of 180/50 = 3.6 in. The pressure-sensitive element and the indicating system must operate over the range and have a spring-stiffness constant within 2/180 = 1.1 per cent if the calibration-error specification is not to be exceeded.

The idea of using sensitivities associated with the component systems of an elementary instrument, as developed (4), is useful in laying out preliminary designs. In the present case of a pressure indicator, the converter can be taken as the system which receives pressure changes and produces changes in the position of some mechanical part which can be used to operate the indicating system. The corresponding sensitivities are

$$\text{Converter sensitivity} = S_c = \frac{\text{small change in position of converter output element}}{\text{small change in applied pressure}} \quad [48]$$

$$\text{Indicating-system sensitivity} = S_i = \frac{\text{small change in index position}}{\text{small change in element connected to converter}} \quad [49]$$

From (4) for a direct-reading instrument

$$S_c S_i S_s = 1 \quad [50]$$

The converter of the indicator could be a spring-and-piston system, working in a cylinder, a metallic bellows, a diaphragm, a Bourdon tube, or any other means for converting pressure changes into mechanical displacements. If a spring-and-piston system is chosen as the converter with a converter sensitivity of 0.001 in. per lb per sq in., the necessary indicating-system sensitivity can be found by use of Equation [50], and the specified scale sensitivity as

$$S_i = 1/S_c S_s = 1/(0.001)(50) = 20 \text{ in. per in.} \quad [51]$$

Uncertainty is the result of imperfections in the indicator mechanism and the width of the recorded line. In the present case, the recording point must produce a line so fine and the mechanical details be so well worked out that the maximum uncertainty is plus or minus 1.8 lb per sq in.

The difficulty of meeting the environmental-error specification can be reduced by establishing calibration conditions as near actual service conditions as possible. In case the indicator temperature varies widely in service, it might be necessary to introduce differential expansion elements to reduce the effect of temperature upon the instrument readings. A problem of this nature must usually be handled by special methods to suit given cases.

Dynamic specifications determine the instrument characteristics τ and ζ. Theoretically, there is an infinite number of characteristic combinations which will satisfy the dynamic specifications. Practical mechanical considerations will determine the particular combination to be selected. For an engine indicator, as in many simple instruments, no benefit is to be gained by using a damping ratio in excess of unity ($\zeta = 1$). On the other hand, values of the damping ratio less than 0.1 ($\zeta < 0.1$) will permit excessive natural-frequency components in the index motion.

With these facts in mind, it is reasonable to survey the range of possibilities by selecting damping-ratio values of 0.2, 0.5, 0.7, and 1 for trial purposes. The dynamic specifications will then place limiting values on the ordinates of Figs. 3, 6, 7, and 8, so that critical values of t/τ can be found for each value of the damping ratio. The spring stiffness of the mechanism has already been determined by the value of S_c. From the standpoint of design it is often desired to find the maximum permissible inertia or mass of the instrument parts. For a constant value

of the spring stiffness, this means that the natural undamped period of the system will be a maximum. For a given damping ratio, the characteristic time is proportional to the undamped natural period, so the general problem can be resolved into finding the minimum value of t/τ for which the dynamic-error specification is not exceeded. For steady-state sinusoidal changes, a minimum value of the frequency ratio β corresponds to a minimum value of t/τ for the other cases.

Table 1 gives a summary of the numerical work required to solve the problem discussed in the preceding paragraphs. For low values of ζ where oscillation is noticeable, the minimum value of t/τ must be selected so that the specified ordinate limits are not exceeded for any greater value of t/τ. For example, in Fig. 3 the curve for $\zeta = 0.2$ crosses the 10 per cent error line several times, but the critical value is the last crossing at $t/\tau = 2.1$.

TABLE 1 SUMMARY OF NUMERICAL WORK USED TO SOLVE TYPICAL PROBLEM

(a) Step-function analysis (Fig. 3)
$t = 0.05$ $q_0/q_a = 0.90 - 1.10$

ζ	$(t/\tau)_{crit}$	$\tau_{crit} = \frac{0.05}{t/\tau\ crit}$
0.2	2.1	0.0238
0.5	2.3	0.0217
0.7	1.8	0.0278
1.0	3.8	0.0131

(b) Linear-pressure-increase analysis (Fig. 6)
$t = 0.07$ $q_0/q_a = 0.9 - 1.10$

ζ	$(t/\tau)_{crit}$	$\tau_{crit} = \frac{0.07}{t/\tau\ crit}$
0.2	0.7	0.1000
0.5	5.0	0.0140
0.7	10.0	0.0070
1.0	20.0	0.0035

(c) Frequency-response analysis (Figs. 7 and 8)
$\omega_f = 4 \times 2\pi$ $T_f = 0.25$ sec $\mu = 0.90 - 1.10$ $\varphi < 10$ deg

ζ	Frequency criterion, β_{crit}	Phase criterion, β_{crit}	Critical, T_n	τ_{crit}
0.2	0.32	0.375	0.0800	0.0638
0.5	0.47	0.175	0.0440	0.0140
0.7	0.72	0.125	0.0308	0.0070
1.0	0.33	0.082	0.0207	0.0033

TABLE 2 SUMMARY OF POSSIBLE INSTRUMENT DESIGN CONSTANTS

(Determined by minimum value of τ from *a*, *b*, *c*, Table 1)

ζ	τ_{max}	$T_{n_{max}}$	$N_{N(min)}$ (cycles per sec)	Critical specification
0.2	0.0238	0.0299	33.5	Step function (*a*)
0.5	0.0140	0.0440	22.7	Phase angle (*c*)
0.7	0.0070	0.0308	32.5	Linear change (*b*)
1.0	0.0033	0.0207	48.3	Phase angle (*c*)

Values of ζ and τ, which meet the dynamic specifications are listed in Table 1. In order to satisfy all the conditions, the shortest characteristic time required for any of the three forcing functions with a given value of ζ must be chosen. Table 2 is a summary of the critical conditions established in Table 1. The critical values of the characteristic time have been converted into values of undamped natural frequency and undamped natural period by the relation

$$T_n = 2\pi\zeta\tau = 1/n_n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots [52]$$

From Table 2, it is seen that, for low values of the damping ratio, the step function establishes the critical condition, since the low damping permits several natural-frequency oscillations before the reading will remain below the specified error limit. As the damping ratio is increased, the transient component of the step-function response becomes of reduced importance, while the steady-state time lag for the other two forcing functions is increased. In the present case, the steady-state time lag, during a linear change, and the phase-angle lag for a sinusoidal change introduce limitations which are almost identical.

In general as the steady-state-lag specifications are made more severe (specified lags are made smaller), the required undamped natural frequency is increased for a given value of ζ, or the damping ratio must be decreased for a given natural frequency. For the step function, the reverse is true, since the response to such a change is more accurate, as ζ is increased to about 0.7. It follows that the characteristics selected for an instrument must be a compromise between conflicting trends. In many cases, it is mechanically easier to construct an instrument with a low rather than a high undamped natural frequency. This will probably be true for a spring-and-piston engine indicator like that of the present discussion, so the results summarized in Table 2, which are based on the assumption of a critical lower limit to the undamped natural frequency, represent at least one reasonable engineering solution to the original problem. It follows that an indicator with a damping ratio of 0.5 and a characteristic time of 0.0140 sec will be satisfactory. The corresponding undamped natural frequency will be 23 cycles per sec.

After an analysis of dynamic performance has been carried out along the lines suggested in the foregoing discussion, it may be found very difficult to obtain the required characteristics in practice. For example, an engine indicator would probably not have a damping ratio of 0.5 without including some special damping system in the mechanism. In such a case, the original specifications must be relaxed or the expense of building a more than usually complicated instrument accepted. It is evident that the dynamic-error specifications, which can actually be fulfilled by a given indicator, may be found by using the dynamic-performance curves in a process which reverses the method described.

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Discussion

K. J. DeJuhasz.[4] This paper represents a noteworthy endeavor to coordinate the scattered information on the dynamic behavior of instruments into a coherent whole and make it more readily available and understandable for students and practicing engineers. In view of the magnitude of the task, it is no detraction from the value of this paper to state that this objective has been only partially attained. The following remarks are intended to complement the paper in some respects and to call attention to some aspects in which further elucidation would be desirable.

The title of the paper suggests a broader scope than is actually the case, as the treatment does not include discussion of seismographic-type instruments which depend for their function on a low natural period of the actuating member, such as seismographs, vibrographs, and torsiographs. A more detailed treatment of Coulomb friction and backlash also would be desirable, as these enter into the functioning of most engineering instruments.

[4] Professor of Engineering Research, The Pennsylvania State College, State College, Pa. Mem. A.S.M.E.

The writer questions the usefulness of the concept of "time lag" when dealing with transient changes. The "lag angle" in measuring a sinusoidally varying actuating quantity is a well-established and useful concept. The time lag of a transient change has no meaning in cases in which the scale reading cannot follow rapidly enough the increase and decrease of the actuating quantity, hence, some values of the actuating quantity are not indicated at all by the scale reading. Furthermore, it may happen that a scale reading is shown before the actuating quantity attains the corresponding value, i.e., the time lag is negative. This would be tantamount to saying that the instrument knew beforehand, and predicted what was going to occur in the future, which is of course absurd. Therefore the writer suggests that the concept of time lag be dropped altogether when dealing with transient changes, and the performance of the instrument be characterized solely by the "dynamic error," i.e., the difference between the actuating quantity and the scale reading at a given instant, which has a real meaning in all cases. Attention should be focused on the concept of "maximum dynamic error," i.e., the maximum difference between actuating quantity and scale reading.

It would be highly desirable to elucidate the various abstract concepts.

As examples, the writer suggests:

(*a*) The undamped natural period of an instrument is equal to the period of a simple pendulum having a mass equal to that of the instrument, and a length equal to its static deflection under its weight.

(*b*) The maximum dynamic error decreases with the decrease of the natural period.

(*c*) The critical damping is brought about by a damping force which is just sufficient to dissipate the potential energy of the instrument.

Possibly these definitions could be further elucidated or improved, in order to reduce to the minimum the mental effort necessary for their understanding.

The writer cannot call to mind any instrument in which the concept of "error after the elapse of a given time" would be of advantage, or could be stipulated in advance. In most cases the requirements are as follows:

(*a*) That the natural period and damping be as low as possible. This is the case with engine indicators, in which the "maximum error" is proportional to the natural period, and the superimposed waves can be allowed for by simple inspection, or more accurately, deducted from the curve by graphical differentiation. An uncertain amount of damping (or friction) is far more difficult to take into account.

(*b*) That the damping be the lowest causing the instrument to "dead-beat," i.e., critical damping. This is the case with quick-weighing scales, in which the overshooting of the pointer is prevented by a dashpot. In such cases it is farfetched to stipulate, and hardly feasible to design in advance, a definite damping ratio. Such dashpots are designed by a rough advance calculation, relying on the fortunate fact that the final adjustment can be readily made by means of a needle valve.

In view of the ever-increasing importance of instrumentation, and corresponding attention given to the theory and design of instruments, further investigations in this field are greatly to be desired, and the authors' paper is welcomed for stimulating thought in these problems.

E. S. Smith.[5] Figs. 7 and 8 and their context, dealing with "steady-state dynamic errors due to a sinusoidal variation of the actuating quantity," offhand appear to this writer to comprise the novel portions of the paper and as such should be useful to engineers working in this art. Nothing that appears anywhere in this discussion is to be taken as minimizing the value of the authors' contribution which unquestionably involved the expenditure of a considerable amount of effort with a corresponding reduction in the work which others in this special field must do.

The treatment is not in fact general, as stated in the summary, since it is not applicable to common servo-operated measuring and controlling instruments which move at constant speed from one position to the next or are intermittently actuated; instead, it seems to be limited to a special (lineal-law) sort of self-operated meter. The treatment does not fit even the ordinary flowmeter which responds to a differential pressure which increases with the square of the velocity of flow.

The terminology of the paper unfortunately differs radically from any of those in common use in this country. The American Society of Mechanical Engineers' Committee on Industrial Instruments and Regulators is endeavoring to develop an authoritative and generally acceptable terminology which should do much to lessen the effort required in digesting papers such as this one, in which the points of possible originality may be obscured by unusual expressions.

As to discussion in the present paper, of papers (2, 3) by Fairchild and the writer and the use of the term "dead zone" in paper (3), it is regretted that such discussion was not made as such, at the time the papers in question were presented, so that the usual closure could have covered it adequately. In view of such belated discussion, the following brief answer may be in order: The papers were presented with controlling (or regulating) as well as measuring in mind, and the treatment of measuring instruments was intended to be suitable for the metering portions of controlling (or regulating) instruments. They were presented to cover the basic theory involved in an introductory manner and with the use of the unavoidable minimum of mathematics; however, such papers are believed to be more than mere "classifications" and to cover much of the material in the present paper. In this paper, the use of numerous Greek letters and the presentation of lengthy mathematical restatements of generally known material are objectionable, in so far as they exceed a reasonable requirement of completeness. However, such repetition may be defensible as making such material more readily available to mechanical engineers.

The term "dead zone" was intended by Fairchild and the writer to cover the maximum disturbance of the measured variable which cannot affect the meter and, hence, alter the position of a controller, instead of referring to the error of a particular reading. Such a definition of this expression still appears to be substantially correct, with usual ratios of starting friction to running friction, although an attempt was made to attain clarity rather than to sponsor an unassailable definition.

The DeJuhasz treatment of the engine indicator in his book[6] on that subject appears to the writer to be essentially similar to that of the present authors on the linear change of the actuating quantity, although it includes a more limited treatment of the amplitude of the response with sinusoidal variations with, offhand, no treatment of lag in such case.

The authors might well test the usefulness of their two criteria on the design of a commercially needed, reliable, and accurate pressure indicator for hydraulic transmissions of the multiple-piston type using, e.g., a working pressure of about 1500 lb per sq in., a frequency of 12,000 cycles per min, and an unsymmetrical wave form (rounded tops and V-troughs).

[5] Patent Agent, C. J. Tagliabue Mfg. Co., Brooklyn, N. Y. Mem. A.S.M.E.

[6] "The Engine Indicator," by J. K. DeJuhasz, Instruments Publishing Company, Pittsburgh, Pa., 1934.

Authors' Closure

Professor DeJuhasz and Mr. Smith have based their major criticism upon their opinions that the principles discussed in the paper could not be applied to instrument problems of certain types and have advanced examples to prove their claim. The authors feel that this criticism is due to a lack of understanding of the manner in which generalized reasoning in terms of nondimensional variables can be applied to specific problems. When such a method is used, a large part of the mathematical work can be carried out once and for all to produce results in the form of curves which may be applied to any physical case described by the differential equation used as the basis for the analysis. To use such curves intelligently and to be able to handle cases which may fall outside the range of available plots, it is only necessary for an engineer to have a general understanding of the steps followed in order to obtain the final results. For this reason, the analysis has been presented in some detail at the risk of "lengthy mathematical restatements of generally known material." It was the authors' intention to offer the possibility of a saving in time and effort on the part of workers in the general field of instrumentation.

In considering an instrument problem, it is only necessary for the practicing engineer to satisfy himself that the behavior of a given instrument can be described by a second-order differential equation with constant coefficients. When this requirement is fulfilled, ability to use the curves of the present paper will provide an engineer with specific information directly applicable to many problems which could be duplicated from papers similar to references (2, 3), cited by Mr. Smith, only after many hours of detail work. Mr. Smith shows some recognition of this fact in his statement, ". . . such repetition may be defensible as making such material more readily available to mechanical engineers."

The particular objection of Professor DeJuhasz that the method of analysis described in the paper is not general, because it does not apply to the seismographic type of instruments for measuring vibration, is definitely unjustified. As a matter of fact, the method of analysis as presented was largely worked out during the design of a line of vibration-measuring equipment for aircraft which has been commercially available for some time. A detailed discussion of the manner in which the methods explained in the present paper can be applied to vibration-measuring instruments was given by C. S. Draper and Walter Wrigley at the 1940 Annual Meeting of the Institute of the Aeronautical Sciences.[7]

Mr. Smith states that the method of analysis discussed in the present paper does not apply to the case of an instrument such as a flowmeter which involves a square law between the actuating quantity and the reading of an indicator which responds to pressure changes. As a matter of fact, the authors anticipated this objection by defining the sensitivities in terms of differential changes. Where nonlinear laws are required to express the relationship between the actuating quantity and the index position, the coefficients of Equation [8] are actually differential coefficients which can be treated as constants over small ranges of operation. The sentences, which follow Equation [8] in the paper, discuss the case in which the analysis will not produce accurate results due to variations of the coefficients with scale position. In practice, the analysis has been found adequate to handle the case of dynamic errors in air-speed meters for aircraft when relatively small fractional changes in speed are involved. The assumptions are similar to those used by electrical engineers in treating circuits containing vacuum tubes and are subject to the same limitation, namely, that the results will have increasing errors as the magnitude of the changes is increased.

The contention that the analysis does not include the case of systems which contain servomechanisms is only partially correct. Actually, the courses on theory and practice of servomechanisms taught by Professors Hazen and Brown at the Massachusetts Institute of Technology use the same general method of nondimensional analysis as that described in the present paper. For analytical purposes it is convenient to consider any servomechanism as made up of a series of systems with 1 deg of freedom. Frequently, the analysis of the performance of a complete servosystem involves only a second-order differential equation. Even where equations of higher order than the second are required, it has been found helpful to use the ideas of damping ratio and undamped natural frequency. The example of a servosystem of the relay type, as suggested by Mr. Smith, may be analyzed in a particularly simple fashion by use of the ideas discussed in the present case, but the method requires the application of a sequence of forcing functions acting upon the individual 1-deg-of-freedom systems (elementary instruments) which make up the servomechanism.

The criticism of the authors for failing to use a system of terminology which as yet has not been published cannot be given much weight. Surely it is granted that, until a system is unanimously adopted, the opportunity to present the arguments in favor of one which has been evolved through many exhaustive tests should not be denied. The terminology used in this paper has been developed over a period of about 10 years by the students and teachers interested in instrumentation at the Massachusetts Institute of Technology and is certainly familiar to and endorsed by a considerable number of engineers who at present are working in the field of aircraft instruments and vibration analysis. The reason underlying development of the terminology has been the lack of a generally accepted system which could be applied to dynamic problems of any type. The authors' system has facilitated the solution of many problems. Furthermore, the time and effort required to teach or to acquire a knowledge of instrument analysis have been reduced, as compared with the work required by an approach based entirely upon the contemporary treatments available in the literature.

The authors feel that the use of Greek letters in the list of symbols is a definite advantage in forming a systematic nomenclature. The general scheme is to use Greek letters for nondimensional quantities and quantities with the dimensions of time, small Roman letters for variables or directly measured quantities, and capital Roman letters for undetermined coefficients. This system has certainly justified its use by reducing the time required for an engineer to understand the results of the analysis from an unfamiliar project.

The authors disagree with Professor DeJuhasz that the quantity defined as "time lag" is unimportant. Time lag is the essential design variable in any instrument which must disregard small instantaneous variations of a quantity and indicate an average value. For example, the artificial horizon used as an aircraft instrument must have a time lag of such magnitude that it will substantially disregard deviations of the resultant-force vector from the true vertical. The requirement of a short time lag in a rate-of-climb meter used for maintaining level flight is obvious. Another application of the concept of time lag occurs when an elementary instrument is used to detect changes in the physical quantity actuating an automatic control system, short time lag being essential in order to start the system functioning soon after a change has occurred. The importance of "error after the elapse of a given time" is also apparent in this case. The existence of a negative time lag, as suggested by Professor

[7] "Instruments for Measuring Low-Frequency Accelerations in Flight," by C. S. Draper and Walter Wrigley. Presented at the Meeting of the Institute of the Aeronautical Sciences, Jan. 25, 1940. To be published in an early issue of the *Journal of the Aeronautical Sciences*.

DeJuhasz, is a physical possibility and is merely an indication that the elementary instrument is oscillating.

The concept of time lag, as the term was used in the paper, is useful and should certainly be retained in any comprehensive system of instrument analysis. On the other hand, the term "time lag" appears to be unsatisfactory and should probably be replaced by other words to describe the same concept. Professor Brown of the Massachusetts Institute of Technology has suggested the term "response delay" to replace time lag. This term is descriptive of the concept involved and will be adopted by the authors for future work.

A discussion of Coulomb friction and backlash effects as suggested by Professor DeJuhasz should certainly be included in any complete instrument analysis. One of the authors has included these effects in teaching instrumentation for some years. A reasonable generalized treatment would have been too long for inclusion in the paper. A study of Coulomb friction is included by Den Hartog in his book (5).

The use of analogies has not been found particularly helpful. It is better to allow an individual to apply theoretical results to whatever electrical, mechanical, acoustical, hydraulic, or other system which he judges to be most readily understandable. For this reason, the authors have omitted any attempt to make particular applications of the generalized results in a fashion which might be useful in explaining the theory to a given group of listeners. Mechanical analogies are readily accessible in books on mechanical vibrations, while electrical analogies are discussed in many electrical-engineering texts.

The authors have had some difficulty in recognizing the usefulness of the pendulum analogy suggested by Professor DeJuhasz, inasmuch as the mass of an instrument is largely case structure, which is not the mass of interest in this discussion. Furthermore the period of a pendulum is independent of the pendulum mass. Of course, any pendulum of a length equal to the static deflection of a 1-deg-of-freedom system will have the same period as the system, but the advantage of this type of analogy is open to question.

The maximum dynamic error normally decreases with decrease of the natural period, but it is possible to show that this statement is not general. For example, if an instrument period is originally longer than the fundamental forcing frequency, reducing the natural period will cause the system to pass through resonance when the dynamic errors increase.

The statement that the natural period should be as low as possible and the damping near critical is unnecessarily restricting. One of the particular advantages of a generalized set of curves is that, when mechanical considerations limit the natural frequency which may be realized in practice, and hence limit β, a satisfactory instrument may be obtained by a suitable choice of the damping ratio. These considerations are clearly brought out by a study of Fig. 7 of the paper, and further by the following discussion of the instrument problem proposed by Mr. Smith. As noted in the paper, there is usually an infinite variety of combinations of β and ζ which will satisfy specified instrument performance conditions. Therefore, it is the designer's responsibility to select that combination leading to the simplest and most reliable mechanical construction.

The hydraulic pressure indicator, proposed by Mr. Smith as a design problem, cannot be analyzed by any methods without a full performance specification. The word "accurate" is not an engineering specification per se, it must include the acceptable numerical tolerance range. It is this lack of reduction of instrument specifications to numerical terms that leads to many of the instrument designer's headaches. It is also not quite clear whether Mr. Smith intended the quantity to be measured accurately as the mean steady working pressure or the instantaneous variations of pressure throughout the cycle. If the latter were the problem in mind, the unsymmetrical wave form must be broken down into a Fourier series. The predominant term will be the fundamental frequency of 200 cycles per sec. To indicate the pressure variations to any degree of accuracy whatsoever, an instrument must respond to frequencies as high as the fundamental, with an accuracy at least equal to the numerically specified tolerance. When higher frequency components are present in the forcing function, the frequency range for linear response must extend to higher frequencies, if the response is to be a true picture of the forcing function.

In the particular problem at hand, where higher harmonics will be present in the instrument forcing function and no numerical accuracy has been specified, a fundamental response accuracy of at least 5 per cent might reasonably be assumed. Furthermore, to avoid resonant magnification by higher frequency components, μ (Fig. 7) must be kept reasonably close to unity by selection of a ζ of approximately 0.6. From Fig. 7, a β of 0.25 and a ζ of 0.6 will fulfill the accuracy requirements for the fundamental up to the fourth harmonic and will record within 15 per cent accuracy. These harmonic components will undergo a phase shift which is linear with frequency within the frequency range over which the magnitude response is satisfactory (Fig. 8). This means that the first four harmonic terms of the Fourier series will be reasonably well recorded with respect to the magnitude, and the shift along the time scale, due to phase shift, will be the same for all these components, thereby retaining the true wave form of the forcing function. For $\beta = 0.25$ a natural frequency of the indicator of 800 cycles per sec is necessary. It should be noted that the use of critical damping in this case would reduce the accuracy of amplitude recording, Fig. 2, of the paper.

If the desired indicator were to measure the mean pressure only, then β must have some value in excess of unity. This is similar to the case of a seismograph and has been covered in earlier papers.[8] For an indicator reading to oscillate less than 5 per cent, any value of ζ can be used if β is greater than 5 (natural undamped frequency is less than 40 cycles per sec).

The methods of this paper permit the critical instrument constants β and ζ to be evaluated quickly. The problem of achieving these constants in the instrument is the responsibility of the mechanical designer, but it is certain that the desired response cannot be obtained unless the graphically obtained constants β and ζ are actually realized in the completed indicator.

[8] For example, reference (4), Fig. 5.

Thermostatic Bimetals

By S. G. ESKIN,[1] PITTSBURGH, PA., and J. R. FRITZE,[2] CHICAGO, ILL.

The use of thermostatic bimetals has increased rapidly during the last fifteen years. In this period, considerable study has been given the subject. The present paper reviews the available information dealing with the theory and practice in the application of bimetals. A lack of essential data concerning various pertinent properties of the material is indicated. Basic principles are thoroughly considered, as in the analysis of the mechanics by Professor Timoshenko and in the properties of nickel-iron alloys. However, a considerable fund of useful data is also presented in the form of curves and tables which give the properties of representative bimetals.

Introduction

DURING the last few years, the demand for automatic methods of control in regulating devices has expanded in proportion to the ease and comfort enjoyed by mankind. All concerned have increasingly realized the importance and convenience of automatic regulation and the complete satisfaction with which it may be used. Many automatic regulating devices, in which temperature can be used for controlling the regulation, depend for their successful operation on thermostatic bimetal.

In general, automatic control effects considerable savings in fuel cost. It also acts to safeguard the equipment on which it is used. Such equipment normally is left to operate unattended with full confidence that nothing is likely to happen to it. Whether in factory or home, whether to engineer or housewife, automatic control creates a peace of mind in our complex civilization which cannot otherwise be obtained.

Since the use of thermostatic bimetal has so greatly increased in recent years and is so vital to much automatic regulation, considerable work has been done on its theory and application. In this paper, the authors present a review of the available information on thermostatic bimetals which at present is scattered throughout numerous publications.

Definition of Thermostatic Bimetal

Thermostatic bimetal is a metallurgical product made from two metals which have widely different coefficients of thermal expansion and which have been firmly bonded at their face of contact. When subjected to a change in temperature, the composite material will change shape, which change of shape is utilized for control purposes. In addition to having widely different coefficients of expansion, the components must have mechanical properties suitable to minimize hysteresis and provide adequate strength. For example, lead cannot be used as a component. The two component metals are usually of equal thickness and joined together in a firm bond by brazing or welding.

History and Manufacture

Thermostatic bimetals were known for many years as laboratory curiosities. Their first known application was in balance-wheel compensators, supposed to have been used as early as 1775. As far back as 1858, Wilson obtained U. S. patent No. 24,896 covering a thermostatic bimetal. In 1863, in France, Villarceau published an analysis of thermostatic bimetal in the Annales de L'Observatoire Imperial de Paris. Wilson's patent covered a bimetal consisting of brass and steel and, since the difference in the coefficients of expansion of the two metals was not very large, the movement of the bimetal was so limited that its practical value was small. With the discovery by C. E. Guillaume in 1898–1899 of the low-expanding alloy known as "invar," consisting of 36 per cent nickel and the remainder iron, a new impetus was given the work on bimetals.

At the present time, the manufacture of bimetals is still an art rather than a science and the processes involved constitute closely guarded secrets. In one plant thermostatic bimetals were made by brazing two blocks of the component metals in a furnace in which a reducing atmosphere was maintained to prevent oxidation of the surfaces which were to be joined together. Sometimes blocks were used which were $^1/_2$ in. thick, 4 in. wide, and 12 in. long. Prior to brazing, one of the blocks was machined on both surfaces to a thickness of exactly $^1/_2$ in., while the other block was machined only on one surface. While the blocks were being brazed in the furnace, they were subjected to pressure by means of a hydraulic press to obtain a sound joint. It was very important to keep the metal surfaces clean before brazing. The blocks were usually kept in carbon paper, which prevented their oxidizing.

After the brazing operation, the exposed unfinished side of one of the blocks was machined to produce a total thickness of exactly 1 in., thereby assuring that each component was of equal thickness before reduction by rolling. The blocks were then rolled down to the desired thickness, annealing being required at a number of stages. Special rolling mills were used for this work and those in the last stages needed to be extremely accurate to finish the product to within very close tolerances.

In general, the thickness variation of bimetals cannot exceed plus or minus 0.001 in. for thickness above 0.040 in. and plus or minus $2^1/_2$ per cent below this thickness. These close tolerances are necessary to obtain uniform performance since the thickness enters into the deflection equation.

It is interesting to note that, in most cases, the final thickness of each component is very close to $^1/_2$ of the total thickness, even though in some bimetals the hardness of the separate components differs materially.

During the rolling processes, it is necessary to anneal the strips a number of times which it has been found most convenient to do by heating the strips electrically in the flat condition.

Properties of Invar and Related Alloys

As defined, thermostatic bimetals consist of two metals, having widely different coefficients of thermal expansion, which are joined together at their surfaces. The effectiveness of a given bimetal will depend upon the difference in thermal expansivity of the two components. Although numerous alloys now on the market have varying degrees of high expansivity, there are only a few alloys with thermal expansion sufficiently low for commercial use in bimetals.

In making a study of the iron-nickel series, Guillaume discovered that some of these alloys possessed low expansivity at

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

ordinary temperatures, the composition corresponding to minimum expansivity being in the vicinity of 36 per cent nickel. This alloy maintained its dimensions so nearly constant with ordinary variations in atmospheric temperature that he gave it the appropriate name "invar."

Since Guillaume's work, the entire iron-nickel series has been so explored that the engineer can now use an alloy with any desired coefficient of linear expansion between zero and that of nickel. With such a range of low-expansion alloys available, the development of bimetals followed as a matter of course.

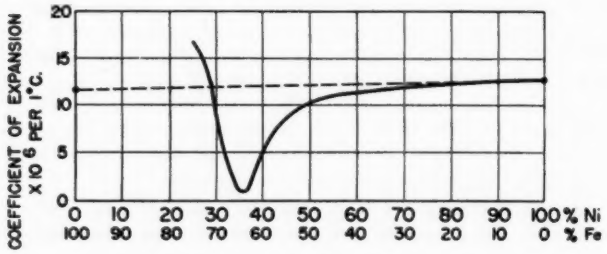


FIG. 1 TRUE COEFFICIENT OF LINEAR EXPANSION AT 20 C OF TYPICAL NICKEL-IRON ALLOYS, CONTAINING 0.4 PER CENT MANGANESE AND 0.1 PER CENT CARBON (GUILLAUME)

In Fig. 1, the coefficient of linear expansion at room temperature of iron-nickel alloys is plotted against nickel content (1).[3] The composition corresponding to the minimum point on the curve is that of invar. As the nickel content varies from the invar point the expansion coefficient increases continuously but not uniformly. Fig. 1 is representative of typical alloys containing 0.4 per cent manganese and 0.1 per cent carbon which have been cooled in air after being hot-rolled. The point on the left edge of the grid shows the value of the coefficient of expansion of pure iron as 11.9×10^{-6} per C, while on the right edge is the point giving the value of pure nickel as 13×10^{-6} per C. The dotted line on the curve connecting the values of the coefficients for pure iron and pure nickel, represents the theoretical results obtained from the law of mixture in a condition stable at low temperatures. However, the true coefficients of the various mixtures are far from following this law, as the curve shows.

The point of low expansivity shown in Fig. 1 holds true only in an alloy in which the proportion of elements other than iron and nickel are also fixed, as has been stated. The position of the minimum is changed considerably when other elements are present.

Fig. 2 shows the change in the expansivity of nickel steels produced by the addition of 1 per cent manganese or chromium.[4] The solid curves give the change in the expansivity due to the addition of the foregoing elements, while the dotted curve gives the actual expansivity of nickel steels. The effect of these additions is to increase the expansivity of invar very considerably.

All thermal or mechanical treatment given such an alloy changes its expansivity. When such an alloy has been heated and then slowly cooled, its expansivity rises. However, its expansivity falls if the cooling has been rapid. When such an alloy is cold-rolled or drawn, its expansivity will decrease still more. By first quenching and then drawing a rod to the limit, it is possible to reduce the expansivity to such an extent as to reach a negative value. Reheating at 100 C for several hours brings the value up to substantially zero. However in processing invar for bimetal applications, advantage is not taken of this property, since reheating to higher temperatures will cause its expansivity to revert to the higher minimum.

[3] Numbers in parentheses refer to the Bibliography.

[4] Bibliography (1), Fig. 24, p. 38.

In Fig. 3 are reproduced curves showing the variation of the true coefficient of linear expansion with temperature for nickel steels varying in composition from 33.8 per cent nickel to 64.8 per cent nickel.[5] The values of the coefficient of expansion are plotted against temperature. The curves show invar (35.4 per cent nickel) to be the most desirable for temperatures from 100 to 300 F. From this point up, the coefficient of expansion greatly increases, thus making such an alloy useless for bimetal applications at higher temperatures if uniform rates of movement are desired over the entire range. Fig. 3 also shows that the alloy of 42 per cent nickel, although having a considerably higher coefficient of thermal expansion than some other alloys at room temperatures, develops only a slight increase in the coefficient with an increase in the temperature and maintains a uniform coefficient over a rather wide range of temperature. It has been found that alloys of 40 and 42 per cent nickel are particularly suitable for bimetals operating in the medium and high temperature ranges and they are used extensively for this purpose.

In an attempt to explain the expansion anomaly of invar several theories were evolved. Some of the earlier views attributed the nonexpansion to an inner molecular change. Another theory expressed the view that the nonexpansibility was caused by the high nickel content bringing the temperature of the A_3 transformation to the vicinity of room temperature, which made the

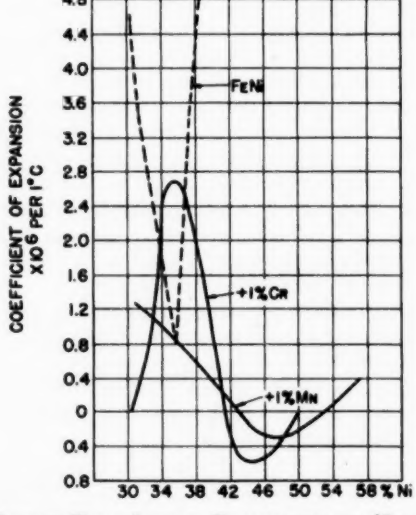


FIG. 2 EFFECT UPON LINEAR EXPANSIVITY OF NICKEL STEELS PRODUCED BY ADDING 1 PER CENT MANGANESE OR CHROMIUM (GUILLAUME)

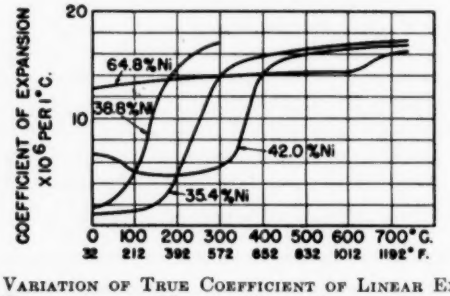


FIG. 3 VARIATION OF TRUE COEFFICIENT OF LINEAR EXPANSION WITH TEMPERATURE IN VARIOUS NICKEL STEELS (CHEVENARD)

[5] Bibliography (1), Fig. 28, p. 44.

transformation of γ into α progressive. However, it was shown that the position of Ac_3 for 36 per cent nickel is about +400 C and that of Ar_3 about —100 C, thus making impossible an equilibrium at ordinary temperature and the invar would be in a decidedly unstable condition.

Benedicks (2) in developing a theory for the expansion of invar, reasoned that the alloy was in reality a two-phase system instead of being homogeneous. He proved his theory by a careful microstudy of the alloy which led him to four conclusions:

1 Strictly local, deep-etching spots occurred regularly throughout the metal.

2 These spots were often more concentrated at definite places, at which points considerably more iron goes into solution.

3 In an unetched surface, a darker, finely divided structure could be made visible in red light.

4 The deep-etching spots of invar presented the same general characteristics as those observed in a larger proportion in a 30.5 per cent nickel alloy where a two-phase microstructure was already known to occur.

From the foregoing evidence, Benedicks concluded that invar possessed a duplex structure. As he was certain that invar contained two phases, he reasoned that, since every transformation between two phases requires a definite time for its completion, by heating invar quickly, sufficient time would not elapse for the transformation between the two phases to take place. In that event, the initial expansion would be a normal one and the coefficient would lie between that of iron (12×10^{-6}) and that of nickel (13×10^{-6}). However, when sufficient time is allowed,

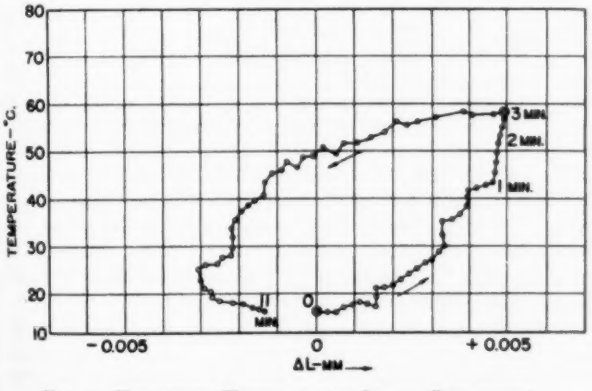


FIG. 4 EXPANSION HYSTERESIS OF INVAR (BENEDICKS)

the change caused by the two-phase transformation will become noticeable. In other words, since invar is a two-phase alloy, quick heating should show a thermal hysteresis, while if invar were a one-phase alloy, no thermal hysteresis would be detected. Consequently, the high-expansion coefficient should be expected to occur at quick temperature changes if the alloy had two phases.

To prove his theory, Benedicks conducted an experiment in which the thermal expansion of invar was measured as a function of time. He developed special equipment whereby measurements of expansions were made with an apparatus which amplified the movement 14,000 times, time being measured simultaneously.

Fig. 4 shows the results of Benedicks' experiments.[6] As expected, the thermal hysteresis was very pronounced. For a given temperature change (42 C), a sample of invar wire had an initial expansion coefficient of 13×10^{-6} or about the normal value for iron or nickel. The coefficient then diminished continually until, after 3 min heating, a considerable contraction had set in. Due to this, the resulting length diminished so much after a lapse of about 11 min as to give an expansion coefficient of 0.2×10^{-6} to 0.6×10^{-6}. This is the same magnitude established for invar by previous workers (0.4×10^{-6}).

In other words, invar upon heating first expands the same average amount as the separate constituents would expand for the same temperature change and then, after a lapse of a few minutes, contracts to show a coefficient of expansion considerably less than that of the two elements.

In applications of thermostatic bimetals where rapid rates of heating or cooling are encountered, this temperature-time hysteresis of invar may affect the deflection of bimetals by an appreciable amount. The present methods of testing bimetals result in data giving the "static" (constant-temperature-deflection) properties only. In order to detect this hysteresis effect, a "dynamic" (temperature-time-deflection) test must be used.

SUMMARY OF PROPERTIES OF INVAR

Invar is a nickel steel made either in the open hearth, crucible, or electric furnace, containing about 36 per cent nickel, 0.1 per cent carbon, and 0.5 per cent or less of manganese with negligible quantities of sulphur, phosphorus, and other elements.

It can be forged, rolled, turned, filed, and drawn into wire. It is very strong and ductile and takes a high polish, giving an excellent surface on which extremely fine lines may be ruled. It is generally ferromagnetic but becomes paramagnetic in the vicinity of 165 C.

Some of the physical properties of invar in the hot-rolled or forged conditions are given in Table 1 (3).

TABLE 1[a] PHYSICAL PROPERTIES OF INVAR

Melting point	1425 C (2600 F) (melts sharply)
Density	8 g per cc
Tensile strength	65000–85000 lb per sq in.
Yield point	40000–60000 lb per sq in.
Elastic limit	20000–30000 lb per sq in.
Elongation	30–45 per cent
Reduction in area	55–70 per cent
Scleroscope hardness	19
Brinell hardness	160
Modulus of elasticity in tension	21400000 lb per sq in.
Thermoelastic coefficient[b] (thermal coefficient of Young's modulus)	500×10^{-6} per C
Electrical resistance	75–85 microhm-cm
Thermal coefficient of electrical resistance	1.2×10^{-3} per C
Specific heat between 25–100 C	0.123 cal per g per C
Thermal conductivity 20–100 C	0.025 cal per sec per cm² per cm thickness per C temperature difference
Linear coefficient of expansion (annealed)	1.7×10^{-6} per deg C

[a] Bibliography (3).
[b] W. G. Brombacher (21) gives 480×10^{-6} per C, in range —50 to 50 C.

The amounts of carbon, manganese, and chromium present appear to exercise considerable influence on the expansion. Above 200 C, the expansion of invar is nearly that of steel.

MATERIALS FOR HIGH-EXPANDING SIDE

It has been pointed out that commercially useful bimetals must consist of two metals having widely different coefficients of thermal expansion. In the previous section, the properties of low-expansion alloys have been described in detail. Materials for the high-expansion side, in addition to having a high coefficient of expansion, should have good brazing or welding properties, should develop high elastic properties as a result of cold-working, and should have high heat-resisting properties.

Initially, brass used in combination with invar resulted in a satisfactory bimetal for low-temperature applications up to approximately 300 F. Further development led to the adoption of monel for high-temperature bimetals, but the deflection constant when using monel was rather low. To meet the requirements for high-temperature, high-deflection bimetals of good constancy,

[6] Bibliography (2), Fig. 63, p. 141.

various nickel-chrome alloys have been adopted for the high-expanding side. Representative analyses contained from 18 to 27 per cent nickel and from 3 to 11 per cent chromium.

Developments in high-temperature bimetals have been mainly in connection with the high-expanding side. Most of the patents taken out recently on bimetals of improved properties cover modifications of the high-expanding side made to obtain the desired performance.

Mechanics of Thermostatic Bimetals

Efficient and economical design of elements of mechanisms requires, first, the knowledge of stresses to which these elements will be subjected under various conditions of service; and second, data giving the physical properties of materials which may be used for these elements. This is particularly the case when controlling or indicating devices are designed, employing thermostatic bimetals, since thermal and residual stresses are present in bimetals in addition to the mechanically induced stresses. Where cost may be a primary consideration, the size of bimetal elements has to be considered, since bimetals are relatively expensive due to the involved fabrication processes. Consequently, a knowledge of the properties of bimetals and an understanding of the stress conditions in sensitive elements made of this material are essential to the most advantageous use thereof.

The first experimental investigation known to the authors of the thermal properties of bimetal strips was made in the laboratories of the General Electric Company. The results of this investigation, published in 1920 (4), were that:

(*a*) For strips, the deflection with change in temperature is given by the relation

$$D \approx \frac{(T_2 - T_1)\, L^2}{t} \qquad [1]$$

where D = deflection
$T_2 - T_1$ = change in temperature of the strip
L = length of strip
t = thickness of strip

The relation given by Equation [1] is in agreement with more recent investigations and theory.

(*b*) The deflection is not affected by changes in width of the strip. Later investigations showed this to be generally true if the width of the strip is less than 20 times its thickness. If the width of the strip is greater than this ratio, cross buckling may take place, reducing the deflection of the strip.

(*c*) The force exerted by the strip varies as the square of the temperature difference and is not affected by changes in length, a conclusion which is erroneous. A simple analysis of the mechanics involved, using the relation of Equation [1], requires that the force vary directly as the change in temperature, inversely as the strip length, directly as the square of the thickness, and directly as the width.

In 1925, S. Timoshenko published a complete analysis (5) of the mechanics of thermostatic bimetals. In this analysis, he developed, in a clear and concise manner, the equations giving the change in curvature of a bimetal strip upon a rise in temperature. The relative effects of variation in the thickness of the two components of the bimetal strip and the difference in the values of their moduli of elasticity were shown; equations being derived for thermal stresses in heated bimetallic strips. In 1934, T. A. Rich presented a similar analysis (6), using a well-planned group of diagrams for visualization of the stress conditions which produce a change in curvature of a bimetallic strip upon an increase in its temperature. The following analysis of the mechanics of bimetals follows closely the analysis of Timoshenko (5).

The method of visualization (6) introduced by Rich has been used in a modified form in Fig. 5 to show the effect of expansion stresses in the two components of an invar-brass bimetal strip on its curvature when heated from a temperature T_1 to a temperature T_2. Assuming that the two separate components of the strip are of the same length at some temperature T_1, the lengths will increase in both cases when the temperature is raised to T_2. The invar will increase in length only slightly, while the brass will expand considerably. Next, the lengths of the two components are equalized by elongating the invar strip with the tensile forces P_1, and compressing the brass strip with oppositely applied compression forces P_2. While under the action of these two sets of forces, the two separate strips are joined over their surfaces by either brazing or welding. The bimetal strip thus

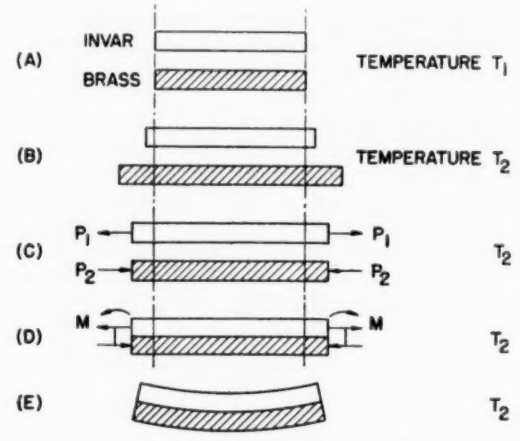


Fig. 5 Diagrams of Forces Which Produce Change in Shape of Bimetal Strip Upon Heating

formed will be straight as long as it remains under the action of the forces P_1 and P_2 applied at the ends. Upon the removal of forces P_1 and P_2, the strip will assume the form of an arc of a circle due to the action of internal moments M, which, in the straight strip, were balanced by external moments of the same magnitude, resulting from the forces P_1 and P_2.

The expression for the radius of curvature of a bimetallic strip subjected to a temperature rise of $T_2 - T_1$ deg can be obtained from the condition that, at the joint of the two components of the strip, the unit elongation is equal in each of the components. The following derivation will be carried through on the basis that the thicknesses of the two components are not equal, which makes it possible to determine the effect of differences in the relative thickness of the components. The width of the strip will be taken equal to unity, since, within certain limits, the width does not affect the deflection. The following notation will be used:

P = force
M = moment of force
α_1 = coefficient of expansion of component having low expansion
α_2 = coefficient of expansion of component having high expansion
a_1 = thickness of the low-expansion component
a_2 = thickness of the high-expansion component
E_1 = modulus of elasticity of the low-expansion component
E_2 = modulus of elasticity of the high-expansion component
I_1 = moment of inertia of the low-expansion component
I_2 = moment of inertia of the high-expansion component
t = total thickness of strip
ρ = radius of curvature, due to temperature rise $T_2 - T_1$

m = ratio of thicknesses of low- to high-expansion components

n = ratio of moduli of elasticity of low- to high-expansion components

Since no external forces are present in a freely supported heated bimetallic strip, all internal forces must be in equilibrium at every section of the strip. This condition makes it possible to establish the relations

$$P_1 = P_2 = P \quad [2]$$

and

$$M = M_1 + M_2 = \frac{Pt}{2} \quad [3]$$

Since

$$\rho = \frac{E_1 I_1}{M_1} = \frac{E_2 I_2}{M_2} \quad [4]$$

$$\frac{Pt}{2} = \frac{E_1 I_1 + E_2 I_2}{\rho} \quad [5]$$

The force P can now be eliminated from Equation [5] by determining its value from the condition that unit elongation of the two components is equal at the joint. This results in the equality

$$\alpha_1(T_2 - T_1) + \frac{P_1}{E_1 a_1} + \frac{a_1}{2\rho} = \alpha_2(T_2 - T_1) - \frac{P_2}{E_2 a_2} - \frac{a_2}{2\rho} \quad [6]$$

By substituting the value of $P_1 = P_2 = P$ from Equation [6] in Equation [5], the expression for the radius of curvature is obtained

$$\frac{1}{\rho} = \frac{(\alpha_2 - \alpha_1)(T_2 - T_1)}{\frac{t}{2} + \frac{2(E_1 I_1 + E_2 I_2)}{t}\left(\frac{1}{E_1 a_1} + \frac{1}{E_2 a_2}\right)} \quad [7]$$

Replacing moments of inertia I_1 and I_2 with their respective values $\frac{a_1^3}{12}$ and $\frac{a_2^3}{12}$, considering unit width, and taking the ratio of thicknesses equal to m and ratio of moduli of elasticity equal to n, Equation [7] reduces to the form

$$\frac{1}{\rho} = \frac{6(\alpha_2 - \alpha_1)(T_2 - T_1)(1 + m)^2}{t\left[3(1 + m)^2 + (1 + mn)\left(m^2 + \frac{1}{mn}\right)\right]} \quad [8]$$

From Equation [8], the effect of the variation in the thickness of the two components of the strip may be determined. Generally the thicknesses of the two components are the same so that $m = 1$. Equation [8] thus simplifies into more useful form

$$\frac{1}{\rho} = \frac{24(\alpha_2 - \alpha_1)(T_2 - T_1)}{t\left(14 + n + \frac{1}{n}\right)} \quad [9]$$

Equation [9] shows that the ratio n of the moduli of elasticity can vary over a rather wide range without affecting to any extent the radius of curvature of a heated bimetallic strip. In most bimetals, the moduli of elasticity of the two components are of the same order of magnitude so that the ratio n can be taken equal to unity. Equation [9] with this approximation reduces to its final form

$$\frac{1}{\rho} = \frac{3}{2}\frac{(\alpha_2 - \alpha_1)(T_2 - T_1)}{t} \quad [10]$$

From Equation [10] it is possible to determine the deflection of strips of various forms supported in a number of different manners. The case of a narrow strip supported on two knife-edges will be illustrated. Referring to Fig. 6, for the case of small deflections

$$1/\rho = 8D/L_2 \quad [11]$$

Eliminating ρ from Equations [10] and [11], the deflection at the center of a freely supported strip will be

$$D = \frac{3}{16}\frac{(\alpha_2 - \alpha_1)(T_2 - T_1)L^2}{t} \quad [12]$$

Usually the difference of the coefficients of expansion for a given pair of metals formed into a thermostatic bimetal is determined experimentally and combined with the $^3/_{16}$ factor into the deflection constant for the bimetal. The common form of the equation for the deflection of a bimetal strip is

$$D = \frac{k(T_2 - T_1)L^2}{t} \quad [13]$$

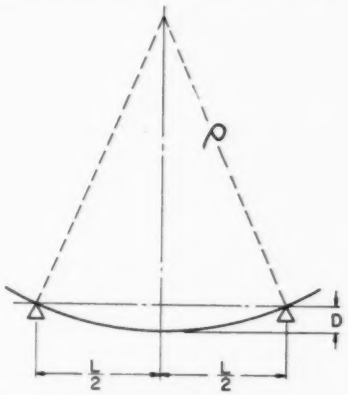


FIG. 6 RELATION OF RADIUS OF CURVATURE TO DEFLECTION OF BIMETAL STRIP

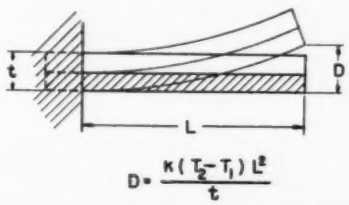


FIG. 7 DEFLECTION OF BIMETAL STRIP MOUNTED AS CANTILEVER BEAM

The deflection constant k is given in this paper for the case of a narrow strip clamped at one end and free to deflect at the other end, as shown in Fig. 7. The force necessary to return a heated bimetallic strip to its initial position is obtained by substituting in Equation [13] the value for the deflection of a cantilever beam with the force concentrated at the free end. The following relation gives the force in terms of the temperature change

$$F = \frac{c(T_2 - T_1)t^2 w}{L} \quad [14]$$

The constant c is known as the force constant and is usually determined experimentally.

STRESSES IN THERMOSTATIC BIMETALS

Bimetals free to deflect upon heating are subjected to internal stresses which at high temperatures may assume large values. If, in addition to internal stresses, bimetals are subjected to stresses due to restraints, loads, etc., their performance may be critically



FIG. 8 THERMAL STRESSES IN BIMETAL STRIP

affected if these stresses exceed the elastic limit of the material at high temperatures. Thus, it is necessary to evaluate stresses for applications involving high temperatures and loadings or restraints. In this paper, only the analysis of internal stresses due to heating will be presented. Stresses due to external forces will vary, depending upon the application. A few simple cases of combinations of internal stresses and stresses due to external forces have been analyzed by Timoshenko (5) and Rich (6).

The strains produced in a freely supported heated bimetallic strip are given in Equation [6] and consist of elongation due to thermal expansion, strains due to axial forces and strains due to bending. The stress at the joint will thus be given by the expression

$$p = P/a + Ea/2\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots [15]$$

assuming equal thicknesses of the two components of the bimetallic strip and equal moduli of elasticity. For this case the value of the force P from Equation [5] is

$$P = \frac{4EI}{t\rho} = \frac{Et^2}{24\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots [16]$$

Substituting this value of the force into Equation [15] and remembering that $2a = t$ we obtain for the stress

$$p = \frac{Et}{12\rho} + \frac{Et}{4\rho} = \frac{1}{3}\frac{Et}{\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots [17]$$

The value of the radius of curvature ρ is given by Equation [10] so that the stress in terms of the difference in coefficients of expansion of the two components of the bimetal and the temperature rise will be

$$p = E/2\,(\alpha_2 - \alpha_1)\,(T_2 - T_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots [18]$$

The maximum stress will occur at the joint of the two components as shown in Fig. 8. The stresses at the surfaces will be half as high as those at the joint.

For a temperature rise of 300 C (540 F) the maximum stress will have approximately the following value

$$p = \frac{20 \times 10^6}{2}\,(15 - 5) \times 10^{-6} \times 300$$

$$= 30000 \text{ lb per sq in}$$

THERMAL PROPERTIES OF THERMOSTATIC BIMETALS

The primary function of thermostatic bimetals is to produce a deflection or a rotation when heated. In many applications, such as thermometers, the bimetal does not have to do any work, except that of moving an indicating pointer, the bearings of which may have some friction. But there are other numerous applications where the bimetal is called on to do considerable work such as lifting parts of mechanisms or overcoming stored energy in spring or magnetic systems to obtain snap action. In the case of those applications where the function of the bimetal is mainly to provide an indication of the temperature change only, the deflection constant has to be considered. On the other hand, in cases where the bimetal is expected to do work, the force constant is of importance and has to be investigated to obtain a design of maximum efficiency.

The important thermal properties of a number of representative bimetals are illustrated in Fig. 9, the data for which were obtained from catalogs. The bimetals selected for this chart characterize the three general groups into which a number of commercially available bimetals may be classified, depending upon the temperature range of application, low, medium, and high. The bimetals selected for this illustration are made by the three largest manufacturers of this material in the United States: The H. A. Wilson Company, W. M. Chace Company, and General Plate Company. In the upper part of the figure, deflection and force constants are given as defined in Equations [13] and [14]. The lower part of the figure shows the temperature ranges for which the deflection is directly proportional to the temperature change and temperature ranges for which these bimetals have a maximum useful deflection. As will be noted, the three largest manufacturers of bimetals produced materials of practically identical thermal properties. Fig. 9 simplifies the problem of the selection of a bimetal for any specific purpose. By extending this chart to include all the available bimetals, it should be of value to designers of devices employing thermostatic bimetals. In addition to the bimetals selected for illustrative purposes in Fig. 9, a large number of other bimetals is available having widely different properties. Manufacturers' literature contains complete descriptions of these bimetals and gives their complete properties in full detail.

In order to prevent changes in calibration of devices using bimetals upon initial heating, it is necessary to anneal the bimetal carefully. During the rolling operation stresses are set up which can be relieved by a proper annealing procedure. High-temperature thermostatic metals should be annealed by, e.g., heating at 600 F for 1 hr if the material is to be used for any temperature below 550 F. In cases where the bimetals are to be used above 550 F, annealing should be done at a temperature 50 deg higher than the maximum temperature encountered in service. Low-temperature bimetals should be annealed at 350 F.

The deflection constant of bimetals changes as a result of annealing. Generally, different bimetals will show a varying degree of change in constant due to annealing. Manufacturers provide data on the effect of annealing on their respective products.

PHYSICAL PROPERTIES OF BIMETALS

In many applications the actuating bimetallic element is used

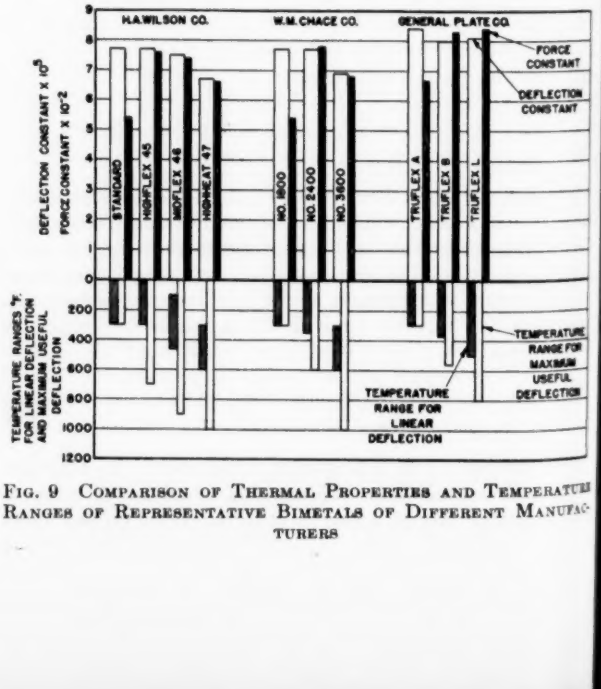


FIG. 9 COMPARISON OF THERMAL PROPERTIES AND TEMPERATURE RANGES OF REPRESENTATIVE BIMETALS OF DIFFERENT MANUFACTURERS

in a manner similar to that of a spring; it is subjected to large deflections or rotations. It is expected, and usually it is essential, that the element should return to its initial position upon the removal of the restraining forces. The design of bimetallic elements, where large deflections occur and permanency of calibration is necessary, requires the knowledge of the stresses involved and the tensile properties of the material. The authors were unable to find in the available literature on thermostatic bimetals any data on the tensile properties of bimetals. Neither have the manufacturers of bimetals published information regarding the tensile properties of their products. In some cases, the moduli of elasticity have been determined and given in the catalogs.

One of the authors, S. G. Eskin, during a study of bimetals eleven years ago, measured the tensile strength and the elastic limit of metals available at that time. The results of these measurements are given in Table 2. Bimetals available at present probably have higher tensile properties than those shown in Table 2. With the exception of the brass-invar type of bimetals, all other bimetals can be subjected to tensile stresses of the order of 70,000 lb per sq in. at room temperatures. Manufacturers generally recommend stresses not exceeding 30,000 lb per sq in. which value allows a very wide margin of safety and makes necessary the use of more bimetal than with a less conservative value. Stresses must be considerably lowered due to "creep" which occurs at high temperatures. Table 3 shows the "safe working" stresses recommended by two manufacturers at various temperatures.

For the calculation of forces exerted by bimetallic strips when subjected to deflections, the moduli of elasticity must be known. The brass-invar type of bimetal has a modulus of elasticity approximately 17,500,000 lb per sq in. and most of the medium- and high-temperature bimetals have moduli of elasticity close to 25,000,000 lb per sq in. These values are for room temperatures; at elevated temperatures, the moduli of elasticity will be lowered from 10 to 20 per cent, depending upon the metal and the temperature rise.

Hardness of bimetals varies considerably with analysis, degree of rolling, and the heat-treatment. In general, bimetals are relatively hard, the low-expansion side varying from 205 to 260 Vickers or Brinell, while the high-expansion side of medium- and high-temperature metals varies from 230 to 300 Vickers or Brinell. These hardnesses are of the same order as those of medium-hard cold-rolled spring materials.

TABLE 2 TENSILE PROPERTIES OF THERMOSTATIC METALS

Metal and treatment	Tensile strength, lb per sq in.	Elastic limit, lb per sq in.	Elongation,[a] per cent
Wilson standard, as rolled	86000	75000	5.0
Wilson standard, annealed at 350 F	86000	75000	11.1
Wilson high-temp. constant, as rolled	110000	95000	3.5
Wilson high-temp constant, annealed at 750 F	110000	85500	12.9
Chace No. 2600, annealed	109000	94000	1.1
Chace No. 2800, annealed	105000	91000	4.0

[a] Gage length = 8 in.

TABLE 3 STRESSES FOR BIMETALS RECOMMENDED BY MANUFACTURERS

Temperature, F	W. M. Chace Co., lb per sq in.	General Plate Co., lb per sq in.
100	25000	...
150	...	30000
200	23000	...
300	...	25000
400	20000	...
500	...	20000
600	15000	...
700	...	15000
800	12000	...
900	...	10000
1000	7000	...
1200	3000	3000

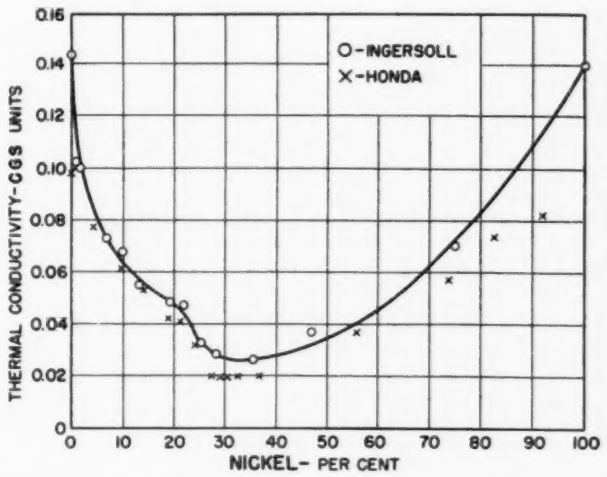


FIG. 10 THERMAL CONDUCTIVITY OF PURE NICKEL-IRON ALLOYS (INGERSOLL)

With the exception of brass-invar and some special bimetals, most high-temperature, high-deflection constant bimetals have very low thermal conductivity. This is due, first, to the fact that invar is a very poor conductor of heat. Fig. 10[7] shows that the lowest thermal conductivity of the nickel-iron alloys is in the range of their lowest thermal expansion, this latter property making them of particular value for bimetal applications. Second, heat-resisting chrome-nickel steels and similar alloys are also very poor conductors of heat. Since this is the case, most high-temperature bimetals are very poor conductors of heat. This should be borne in mind by designers of devices in which the bimetal is to be heated by conduction.

METHODS OF TEST

All manufacturers and most large users of bimetals have developed methods of test which meet their own particular requirements. While there may have been some similarity in the general test procedure, there usually existed enough difference to cause discrepancies in results when testing identical materials. To eliminate this difficulty, the American Society for Testing Materials formed Subcommittee VII, under Committee B4, to study bimetals and set up methods of test. Their latest work may be found in Tentative Standard B106–39T entitled "Tentative Method of Test for Flexivity of Thermostat Metals."

APPLICATION OF THERMOSTATIC BIMETALS

Space limitations do not allow enumeration of the very extensive uses of thermostatic bimetals. Suffice it to state that one manufacturer of bimetals lists in his catalog 60 general industrial applications and 15 automotive, aviation, and marine applications. A number of articles have been written on the application of bimetals, showing the more common forms in which bimetal elements are used; i.e., spirals, helices, disks, flat and U-shaped strips, and also special shapes to meet various unusual requirements. These references are listed in the Bibliography.

Recently two new bimetal units have been introduced which are shown in Figs. 11 and 12. The triple helix shown in Fig. 11 was developed by the Weston Electrical Instrument Corporation and is being used extensively in dial thermometers. By this method of winding a very sensitive element requiring small space is obtained. The double-helix coil shown in Fig. 12 is a product of the General Plate Company. This coil is made by winding a relatively long small diameter helix into a much shorter

[7] Bibliography (1) Fig. 40, p. 61.

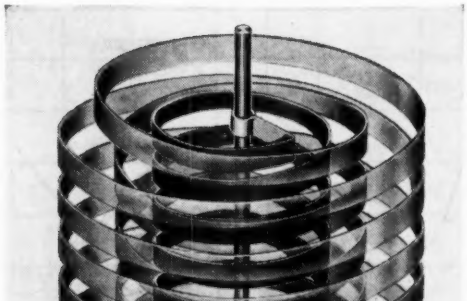


FIG. 11 TRIPLE HELIX FOR DIAL THERMOMETERS MADE BY WESTON ELECTRICAL INSTRUMENT CORPORATION

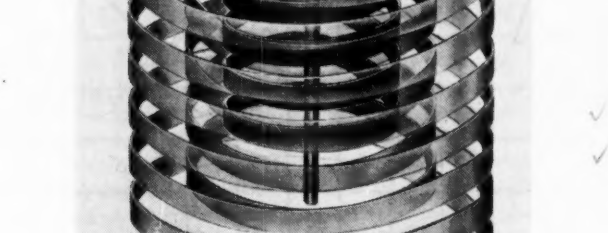


FIG. 12 DOUBLE-HELIX COIL PRODUCING LINEAR DEFLECTION WHEN HEATED; MADE BY GENERAL PLATE COMPANY

helix of a considerably larger diameter. Upon heating, this helix produces a linear deflection along the main axis of the coil.

ACKNOWLEDGMENTS

The authors are indebted to W. M. Chace Company, General Plate Company, and The H. A. Wilson Company, for valuable information in regard to bimetals and their properties. They are grateful to members of the A.S.M.E. Committee on Industrial Instruments and Regulators for many helpful suggestions in connection with this paper. Their appreciation also extends to J. C. Sharp, chief engineer of the Edison General Electric Appliance Company for making the preparation of this paper possible.

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Discussion

J. B. DOWDEN.[8] Fig. 11 of the paper shows a Weston triple-helix bimetal coil and a short discussion of its characteristics may be informative. These multiple-helix coils are made with 2 to 4 helices. A very stable self-supporting mechanical form is obtained by this construction so a long length of bimetal may be wound in a small space. A bimetal strip 25 in. long can be readily wound in a cylindrical form 0.32 in. diam × 0.50 in. long without any one convolution touching another. The combined axial expansion of the helices is practically canceled out since alternate helices are wound in opposite directions. Also there is no side thrust as in a spiral. The maximum accuracy in measuring the temperature of a small portion of a temperature gradient is secured by the most concentrated temperature-sensitive element as in this multiple-helix coil.

With regard to the expansion of invar, Benedicks (2) showed that invar has a two-phase structure and a hysteresis loop of thermal expansion if one considers the changes which occur during the heating cycle. His final experimental proof was obtained by taking many points rapidly on the temperature-expansion curve of an invar bar 80.05 mm (3.15 in.) long as it varied over the temperature range 16 to 58 C. He explained that the length of the invar changed as the material changed from one phase to the other and this counteracted the normal temperature-expansion change which was assumed to be that of iron or nickel, and thus produced an extremely small net linear-expansion coefficient.

Benedicks' experiment was chosen as being most suitable for demonstrating his theory regarding the expansion anomaly. To be directly applicable to the practical use of bimetals, similar tests would have to be made with a variety of heating rates and differently treated invar samples. It would seem from our experience with invar that the size and shape of the hysteresis loop are affected by the rate of heating (or cooling) and the condition of the invar as regards its treatment and composition. We find that bimetal coils with a small heat capacity, having one element of invar, when suddenly changed from one well-stirred liquid bath to another differing by 50 to 90 C, will register a maximum and final change in 10 to 25 sec, as nearly as can be read on a practical scale which is readable within 0.25 per cent. This would indicate that the effect of this hysteresis is negligible in practical applications with good material and design.

[8] Thermometer Division, Weston Electrical Instrument Corporation, Newark, N. J.

H. D. MATTHEWS.[9] Caution should be used in connection with the use of the word "annealing" in reference to thermal elements. Annealing is usually associated with softening the material by the use of elevated temperatures. The words "heat-treatment" are more commonly used rather than "annealing," and the heat-treating temperature should be a value slightly above the maximum value to which the material would be subjected while in service. An appreciable softening of the material will occur at temperatures above 1000 F and, if the material is annealed in the usual sense of the word, the strength would be considerably reduced. Heat-treatment relieves stresses which exist in the material, due to the fact that it is finished by cold-rolling. If the finished elements are formed in any way from strip material, additional stresses are set up, which are also relieved by the heat-treatment. Another very important factor in connection with heat-treatment is that the nickel steels which are used for the low expansive side of bimetals have an expansion value the magnitude of which is determined by the temperature history of the material. When bimetal is finished by cold-rolling, the nickel-steel components have minimum expansion values. During the first heating of the material, the expansion value is raised, depending upon the time of heat-treatment, as well as the exact temperature used. The increasing of the expansion coefficient of the nickel-steel side, due to heat-treatment, therefore, reduces the deflection constant of the finished bimetal element. This change in deflection constant can amount to a value as high as 10 per cent in some bimetals and, if an accuracy of 2 or 3 per cent is desired after being placed in service, it is necessary, of course, that the heat-treatment be given to the finished piece before determining the calibrated scale in the finished instrument. In ordinary air furnaces it requires from 2 to 3 hr properly to stabilize the elements rather than 1 hr, as mentioned in the paper.

In connection with the exact value of the deflection constant, this is affected to some extent by the relation of the thickness to the width of the strip. A change in deflection constant may be as great as 10 per cent if the width of the strip is out of proportion to the thickness. Therefore, for strips of unusual width or thickness, it is desirable to determine from tests the actual resulting deflection constant, as general formulas in use do not make any definite reference with respect to width.

The use of the phraseology "low- and high-temperature bimetals" is becoming more or less obsolete. Up to about 1915, the bimetal in general use was made of brass and invar, and was recommended for temperatures up to 300 F. Since that time a great advance has been made in heating elements, so that with the growth of the electrical heating industry, higher-temperature bimetals were necessary, resulting in the use of monel and nickel-chrome alloys for the high-expansive side. Due to our present wide range of temperature calibrations, the words "high- and low-temperature bimetals" are not particularly significant.

In connection with the theoretical deflection of bimetal as published in 1863 by Villarceau, it is interesting to note that, with present manufacturing methods of direct welding of bimetals, no slippage occurs. The actual deflection of the material agrees with the theoretical values.

The authors refer to other qualities of bimetals besides deflection as being of importance. These might be briefly enumerated as follows: Noncorrosive bimetal; bimetals with low and high resistivity; nonmagnetic bimetals; bimetals with small and large temperature coefficients.

Through several years of field experience, the writer has found that the stresses recommended by bimetal manufacturers referred to in Table 3 are extremely favorable for actual use in design work.

[9] Minneapolis-Honeywell Regulator Company, Minneapolis, Minn.

S. R. HOOD.[10] Based on Benedick's experiments (2) with invar, it is assumed that "where rapid rates of heating or cooling are encountered, the temperature-time hysteresis of invar may affect the deflection of bimetals by an appreciable amount." With all the progress in metallurgy since Benedick's work in 1926, there is still no adequate explanation of the thermal expansion anomaly of invar. The conclusions in Benedick's exposition are based on premises which cannot be verified by tests. In 1928, Howard Scott reported[11] no detectable hysteresis and suggested "the effect described by Benedick must therefore be attributed to temperature lag between his specimen and the temperature-measuring device."

An example of the rapidity with which thermostatic metal will respond to a change in temperature is best shown by the 5000 amperes test given by the Underwriters' Laboratory to domestic circuit breakers. One of the thermostatic metals for this application has a high expansive component with an expansion rate slightly lower than that of invar when the latter is heated to temperatures above 500 F. Therefore, at the beginning of a rapid temperature rise, if the initial expansion rate of the invar were high as stated, no deflection would result in the bimetal. However, in this device, the elapsed time between closing the circuit and its subsequent interruption is 0.010 sec. Only 0.003 to 0.004 sec is required for the deflection of the bimetal and the remaining 0.006 to 0.007 sec is consumed by the remainder of the mechanism. Therefore, it is safer to say that there is no temperature-time hysteresis in thermostatic metals but that their deflection depends upon the rate at which they are heated; they respond as any other metal to radiation, conduction, and convection.

It is also stated that, if the width is greater than 20 times the thickness, the resulting cross-buckling reduces the deflection of a strip. Actually, the deflection of a cantilever, either tapered or rectangular, increases with an increase of the width of the base when the length and thickness are held constant. This variation depends upon the width-length ratio, but for commercial sizes is of the order of 5 per cent.

R. G. WALTENBERG.[12] The writer believes this to be the first paper dealing with the properties of thermostatic bimetal presented before a technical society in this country, and the authors are to be commended for this initial step. There are some features of this paper which it appears desirable to amplify and there are some statements which may be questioned.

It should be noted that the data on the low-expanding nickel steels reviewed in this paper deal mainly with steels which have been cold-worked much less than the usual thermostatic bimetal.

It is surprising that anyone who has worked with thermostatic bimetal would accept without question Benedick's conclusions (2) on the expansion hysteresis of invar. There are millions of devices operated by thermostatic bimetal containing 36 per cent nickel steel which must respond to temperature changes in fractions of a second. The statement, that invar, when subjected to a temperature change from 16 to 58 C, expands for a few minutes at about the rate of iron or nickel, can be proved erroneous by plunging a thin strip of any of the common thermostatic bimetals into hot water. The bimetal will change form immediately. No time lag between change in form and change in temperature will be noticed.

We have attempted to determine expansion hysteresis by tests on thermometal composed of Armco iron against a 36 per cent nickel steel. The test specimen was a loosely wound spiral made

[10] Chief Engineer, W. M. Chace Company, Detroit, Mich.

[11] "Low Expansion Nickel Steels," by Howard Scott, Trans. American Society for Steel Treating, May, 1928, pp. 829–847.

[12] The H. A. Wilson Company, Newark, N. J.

from strip 0.006 in. thick, 1/8 in. wide, and 12 in. long. This would rotate 1.12 angular deg per deg F. The spiral was held at 75 F (24 C), then immersed in oil at 200 F (93 C) and the amount of rotation was automatically recorded at intervals of 0.1 sec. The results were as follows:

Time after immersion, sec	Rotation, deg
0.1	22
0.2	32
1.0	90
4.0	140
No further rotation	

During the first 0.1 sec, the coil rotated 22 deg and in 1 sec it had rotated 90 deg. These rotations correspond to temperature changes of 75 to 95 F and 75 to 155 F with the coil in equilibrium at those temperatures. The temperature change of the bimetal could not have been much faster so there could have been little, if any, time lag between the temperature of the 36 per cent nickel steel and the corresponding expansion for equilibrium conditions. Certainly there was no hysteresis of the order discussed by the authors.

The properties of the components which affect the change in curvature of a bimetal may be combined into what is known as a deflection constant and a theoretical derivation of this constant K is given. The numerical value of K can be determined by temperature-deflection observations on a suitable specimen. Consistent results may be obtained on a beam specimen tested in the manner proposed by the American Society for Testing Materials (A.S.T.M.) and cited by the author in the section, "Methods of Test." The values of K observed by this method agree closely with the values derived from a theoretical analysis based upon the properties of the components.

A rate of deflection may also be determined by observations on the temperature rotation of spirals and helices, but the rate of rotation determined in this manner may vary 20 per cent on a given bimetal, because the amount of rotation depends upon the form and relative dimensions of the bimetal strip, particularly the ratio of width to thickness. The latter method of test is mentioned because some of the deflection constants shown in Fig. 9 were obtained in this manner. A comparison with deflection constants obtained by the A.S.T.M. method leads to erroneous conclusions.

This paper contains an analysis of stresses introduced by a change in temperature of a bimetal. This analysis gives changes in stress, not the actual value of the stress. The amount and distribution of the stresses in any part depend upon its previous history. It is possible to distribute stresses advantageously in a bimetal part by a suitable heat-treatment. Heat-treatment of a finished bimetal part is necessary if uniform and unchanging performance is desired. In most applications the treatment described by the authors is sufficient. In a few applications, it is necessary to carry the assembled thermostat over the entire temperature range, above the maximum and below the minimum temperature it will meet in service.

Authors' Closure

The authors regret that due to lack of space they were unable to give a detailed description of Benedicks' ingenious apparatus for determining the manner in which invar expands on fast heating and to describe more clearly and in greater detail results of Benedicks' experiments. Mr. Hood cited the work of Howard Scott[11] in which Scott was unable to check results previously obtained by Benedicks. The authors have studied the method used by Scott and it seems that, having been designed primarily for the purpose of obtaining easily and quickly a large amount of data, Scott's apparatus may not have been particularly suited to determine an effect which requires extremely high magnification. Scott's conclusion to the effect that Benedicks' results were incorrect was based on the premise that temperature lag existed between the test specimen and the platinum rod used as a thermometer. The authors do not agree with this conclusion but, even assuming that a temperature lag may have existed, it could not have changed the general aspects of Benedicks' investigation. The fact that at the start of heating the expansion of the invar rod was of the order of the expansion of either iron or nickel indicates that the temperature-measuring arrangement was satisfactory.

The authors were quite interested in data presented by Messrs. Hood and Waltenberg, showing that bimetals made with commercial invar give a deflection immediately upon heating. It was thus assumed that, since bimetals containing invar perform in a manner which does not follow from the results of Benedicks' researches, his work was wrong. While the authors have no definite data to explain the seemingly contradictory situation which exists, they feel that these discussers have erred in arriving at their conclusions, having approached the problem too much from the commercial point of view. A careful study of Benedicks' work would show the tremendous importance of the analysis of the invar rod on its hysteresis effect. Fig. 65 in Benedicks' book (2) shows that, while an alloy containing 36 per cent nickel will have a hysteresis effect, an alloy containing 36.9 per cent nickel will not show it. This in itself may be indicative that commercial invar may not have a hysteresis effect and, therefore, bimetals with it will respond instantaneously to temperature changes, as the results of Messrs. Hood and Waltenberg's tests indicate. The authors agree with Mr. Dowden that the entire subject requires further investigation, particularly from a purely scientific point of view.

It may be pointed out that the hysteresis effect theoretically would initially reduce the deflection of a piece of bimetal containing pure invar when heated at a fast rate by about 10 per cent. This amount of reduction in deflection requires careful measurements and may not be apparent when investigated by methods described by Messrs. Hood and Waltenberg.

The authors are grateful to Mr. Matthews for amplifying their comments in regard to annealing of thermostatic bimetals. They have used the term "annealing" in a broader sense, as defined by the American Society for Testing Materials (A.S.T.M.) in Standard A119–33.

Mr. Hood brings out the point that a bimetal strip when used as a cantilever will show a greater deflection with increases in the width. While this may be true in one particular case, generally, the contrary will occur. The authors would like to refer him to an A.S.T.M. treatise,[13] which gives the following formula as a correction for cross-curvature of a freely supported strip of thermostatic bimetal, where w is width of strip and L the distance between supports $1/\{1 + (w/L)^2\}$.

Mr. Waltenberg points out that, in Fig. 9 of the paper, the deflection constants of the General Plate Company's bimetals are higher than they should be, due to the fact that the company used coils rather than strips to determine deflection constants. It was pointed out in the paper that the data for this figure were taken from catalogs of manufacturers, assuming that the manufacturers were familiar with methods for determining accurately the deflection constants of their materials. At the request of the authors, the General Plate Company rechecked the deflection constants of their bimetals shown in Fig. 9 using strips, and found them to check very closely with constants given in the figure.

While it is quite possible that erroneous results may be obtained, when deflection constants of coils of certain geometry are recalculated to obtain deflection constants for strips, it is conceivable that a coil can be used of such a size that its deflection constant would be quite comparable with that obtained with a strip.

[13] "Tentative Method of Test for Flexivity of Thermostat Metals," B106–39T, A.S.T.M. Standards, 1939, Part 1—Metals, pp. 1164–1168.

The Significance of, and Suggested Limits for, the Stress in Pipe Lines Due to the Combined Effects of Pressure and Expansion

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This paper has been prepared as a vehicle for discussion of the following basic problems, on which agreement must be reached to establish a satisfactory working-stress basis for pipe lines:

1 Proper allowable stresses for combined pressure and expansion effects.

2 Influence of and limits for localized stresses under static and repeated loading.

3 Capacity of bolted joints to withstand expansion effects without leakage or damage to flanges, bolts, or gaskets.

4 Effect of prespringing, self-springing, creep, and yielding on operating and off-stream stresses.

An attempt has been made to point out various aspects of each issue, rather than limit the presentation to the personal views of the authors.

DURING the last decade, economic considerations and new processes in the power, oil-refinery, and chemical industries have produced a trend toward large-scale units and high operating temperatures and pressures. With the attendant increase in line sizes and wall thicknesses, the subdivision of pipe lines into convenient runs with expansion bends soon proved entirely inadequate; instead, today most piping for severe service is carefully analyzed for forces and stresses, full advantage being taken of the inherent flexibility by minimizing the number of anchors, guides, or other restraints. A natural consequence of this improved accuracy in evaluating thermal effects is the necessity for a review of stress limitations as they apply to expansion stresses alone and in combination with internal pressure. This problem has received the active consideration of Subgroup No. 3 on Expansion and Flexibility, Subcommittee No. 8 on Fabrication Details of the Code for Pressure Piping (ASA B31) who, functioning with the Applied Mechanics and Power Divisions of the A.S.M.E., have sponsored this symposium.

This paper has been prepared as a vehicle for discussion of the following basic problems, on which agreement must be reached to establish a satisfactory working-stress basis for pipe lines. An attempt has been made to point out various aspects of each issue, rather than limit the presentation to the personal views of the authors.

1 Proper allowable stresses for combined pressure and expansion effects.

2 Influence of and limits for localized stresses under static and repeated loading, such as in bends and corrugated pipe.

3 Capacity of bolted joints to withstand expansion effects without leakage or damage to flanges, bolts, or gaskets.

4 Effect of prespringing, self-springing, creep, and yielding on operating and off-stream stresses

Résumé of Simple Stresses Involved

A pipe line is essentially a pressure vessel, the internal pressure causing a radial stress at the inner face which is converted into circumferential stress on the way through the wall, leaving zero radial stress at the outer surface; at any point the sum of the radial and hoop stresses varies inversely as the radius, while their difference is the same at all points. These are the laws of Lamé which involve the assumption of a uniform or zero longitudinal stress. Following this reasoning, in a closed cylinder the longitudinal stress equals the product of the pressure and the internal area divided by the cross-sectional area of the wall, and has the same value throughout the thickness. While most runs of piping are not technically closed cylinders, the same longitudinal stresses occur due to pressure effects on the projections of elbows, there being a complete absence of longitudinal pressure stress only in straight runs between vessels of infinite rigidity and such including frictionless expansion joints.

Under temperature changes with free expansion no stresses are introduced. However, ordinarily, expansion is restrained by the equipment to which the pipe line is attached, as well as anchors, guides, solid hangers, or supports.

In calculating a line for expansion, the ends are commonly assumed completely fixed; this connotes three forces and an equal number of moments in the case of a problem in space, resulting in longitudinal bending stresses in two mutually perpendicular planes and torsional stress about the pipe axis, as well as two shears and a normal stress. With the assumption of hinged ends, the end moments reduce to zero, a condition which would be fully realized only in a frictionless ball-and-socket joint. Actually, an intermediate condition of restraint will prevail in practice due to sympathetic deflections, rotations, or distortions of the equipment to which the pipe is attached. On the other hand, cases will arise where external movements of the ends tend to increase rather than decrease their degree of fixity; in addition, guides, solid or spring supports, and hangers often inhibit free deformation of the line and thus add to the stresses.

To avoid additional complexity, the weight of the piping is commonly neglected in flexibility calculations. In heavy pipe lines spring hangers are often used to balance the dead-load effects when the line is at working temperature, and solid hangers can be similarly employed in neutral locations. Properly designed supports appear to offer the most suitable means for handling this effect, which can then be disregarded in the stress analysis of most lines.

Where piping is not insulated, the heat loss from the exposed surface produces a temperature drop through the metal thickness causing longitudinal and circumferential stresses of equal magnitude which may be evaluated by the formulas of Lorenz (1).[3] Since the outside surface is relatively colder, tensile

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

stresses will result there while the inner face will be in compression. Where conditions are not conducive to rapid heat loss, or where there is insulation even though of low thermal efficiency, these stresses are not of sufficient magnitude to warrant consideration.

Stress Intensifications and Their Significance

The Intensifications. A pipe being a simple cylinder is in itself free of secondary effects if we neglect nonuniform thickness and defects. However, most runs of piping contain joints, intersections, valves, and fittings, or may be fabricated into special shapes, which introduce differences in rigidity or changes in contour with the result that localized bending stresses must occur to preserve the continuity of the structure. It is our purpose to list the sources of stress intensifications, and briefly discuss their significance under static and repeated loadings.

When a pipe is curved into a torus shape (pipe bend or welding fitting), increased flexibility results under moments in the plane of the bend as a result of flattening of the cross section toward the neutral axis. The effect of this ovalization is to produce circumferential bending stresses of a relatively high order, and also to increase the longitudinal stress in proportion to the added flexibility. This distortion of the cross section is resisted at the ends of the bend by the straight tangents which probably tends to reduce the out-of-round shape of the curved pipe in this area and introduce corresponding ovalization into the straight pipe for a short distance.

All intersections or outlets involve bending effects due to the transmission of direct stress around the opening and the directional change at the sharp corners. In consideration of these effects, standard fittings are increased in thickness and branch connections are provided with reinforcement which, while lessening these influences, on the other hand introduces discontinuity effects due to the additional stiffness. The distortion produced by circumferential welds is in effect a corrugation which, with present-day welding technique, is probably so slight as to be of little or no consequence. In welds, as well as hot or cold bends or other fabricated parts, residual stresses are encountered which are probably reduced to an insignificant level where stress-relieved.

To secure added flexibility, straight and curved pipe are sometimes provided with circumferential corrugations which behave analogous to a bellows; as a result, appreciable longitudinal bending stresses are introduced while the circumferential stresses are probably reduced. Creased bends involve somewhat the same consideration. No adequate analytical analysis has been made of the stresses in corrugated or creased-bend pipe although some experimental research on full-size specimens has been done at the Navy Experimental Station at Annapolis by Dennison (2) and this has been supplemented with tests on 2-in. pipe by the M. W. Kellogg Company. Details of these tests and a summary and discussion of the Dennison and Kellogg data are given in Appendix 1.

In recent years the trend in high-pressure piping has been toward the omission of bolted joints wherever possible. On the other hand, such connections have performed satisfactorily in oil-refinery service under extremely high pressures and temperatures, involving in some cases frequent cycles of application. Pressure discontinuity stresses occur at the back of the flange due to the differences in stiffness, and bending stresses result from the bolt load. Due to the importance of this subject on pipe design, a detailed discussion is considered advisable and this has been incorporated into Appendix 2 to avoid overburdening the body of this paper.

Static Loading. It is generally conceded that, under static loading, localized stresses do not necessarily influence the failure load of a structure. Tests on plain pipe with stiffening rings usually result in failure in the pipe beyond the influence of the ring and not in the zones of high bending stress adjacent thereto. Similarly, many tests on unreinforced openings in pressure vessels and pipe have shown a bursting strength comparable with that of the plate or piping material, unless the openings were large with comparable distortion, in which case the effect is no longer local and failure is a function of the shape involved. Many structures have to undergo a considerable degree of cold working in manufacture and nevertheless give satisfactory service in the absence of stress relieving; for instance, the A.S.M.E. Pressure Vessel Code permits unstress-relieved vessels up to $1^1/_2$ in. plate thickness in proportion to the diameter for U-69 construction, and in most of these vessels the plates are formed cold. With the present safety factor of 5, many stress intensifications are countenanced such as the knuckle stress on relatively shallow dished heads and unreinforced openings of limited size. In the API-ASME Unfired Pressure Vessel Code, an attempt has been made to reduce the degree of possible stress intensification in view of the safety factor of 4, but here also these stress raisers are present although to a lesser degree.

In connection with transverse stresses in curved pipe, Hovgaard (3) has made tests which show that for low-carbon steel, with a minimum yield point of 25,000 lb per sq in., appreciable permanent set would not be obtained until the longitudinal stress as calculated with the longitudinal-stress coefficient derived by him exceeded 20,000 lb per sq in.

With bolted joints, the trend at the present time is to permit an overstress in the hub in proportion to the capacity of the flange proper to take additional stress, that is, the mean of the hub and flange stress must not exceed the allowable. Flanges calculated in this manner have given satisfactory performance on hundreds of heat exchangers. There is the additional consideration of maintaining a tight joint and this aspect is covered in Appendix 2. It would appear that with ASA standard flanges the bolt area is sufficient to transmit bending stress of ordinary magnitude without lowering the gasket pressure beyond the minimum required so that the flange and bolting would not be unduly overstressed. At the same time the bolt load is able to withstand appreciable torsion by means of friction between the gasket and flange faces although to a lesser degree than applies to bending, and caution must be exercised where high torsional moments are involved.

In so far as static stresses are concerned, it would, accordingly, seem to be entirely proper to permit principal stresses in the line which closely approach the yield point and to neglect the influence of highly localized stresses. This is particularly true where the distortion due to the stress intensification tends to result in a shape of increased stiffness.

Cyclic Loading. The problem of establishing suitable allowable stresses becomes more involved in the case of cyclic loading. There are some cases where load variations will definitely exist such as in discharge lines from reciprocating pumps or compressors; more often the number of stress cycles and magnitude of alternating stresses during operation are a matter of conjecture. There is little doubt as to the limitations that should be imposed for applications definitely involving alternating stresses. On the other hand, for average applications, the question whether and to what extent fatigue should be considered is not readily answered. This is particularly true in view of the large amount of pipe which is giving satisfactory service in proportion to the relatively few fatigue failures which have been encountered. Alternating stresses may also be a source of fatigue corrosion which might occur as accelerated attack in corrosive media, and is sometimes present in applications where the type of material is considered fully resistant.

In Appendix 1 the available data on fatigue tests of different types of tubular members are given and an attempt has been made to correlate this information to determine what relation, if any, exists between the flexibility factor and the useful life as indicated by the number of cycles to failure. It is regrettable that the information available is exceedingly meager; on the other hand, we believe that the data indicate a definite trend between the resistance to fatigue and the stress intensification, of which the flexibility factor is a rough measure.

Subzero Temperatures. Occasional failures are sometimes encountered in subzero piping and usually occur in an area of localized stress which might be only a flaw or scratch in the pipe. At the present time, resistance of materials to impact is compared on the basis of a Charpy keyhole-notch bar test, although attempts have been made to correlate similar information from tension impact tests. In the absence of a better explanation, failures of this nature are connected entirely with the composition and the physical properties of the material involved, although stress may be found to be a factor when our knowledge of this subject is further expanded.

Stresses Beyond Yield Point and Creep

The stresses in a pipe wall, being the superposition of individual effects following individual laws of distribution, are by no means uniform over the cross section or length of the pipe. The longitudinal pressure stress is assumed constant; the radial and circumferential pressure stresses are a function of the radius; the bending stresses due to expansion are a maximum at the extreme fibers and zero at the neutral axis of the section, and vary over the length of the pipe; the discontinuity stresses at flanges and intersections, the effects of ovalization in plain bends, and the stress concentrations at corrugations follow laws peculiar to themselves.

Local yielding, where overstress occurs in restricted areas, will effect a redistribution to adjacent fibers. When the load is removed, the fibers which have undergone permanent set will assume a negative strain, the net result being an increase in the elastic range of the structure. As an extreme condition, and neglecting increase in yield point due to strain hardening, it is possible to visualize a line carrying a negative strain with stresses at the yield point in the unloaded condition, and deflecting under expansion forces and moments until an equal positive strain is produced, with stresses again at the yield point but of opposite sign. This increase in elastic range could also be obtained by prespringing the bend to the yield point cold and then applying displacements twice as great as the prespringing but of reversed sign. The difference between "self-springing" and "prespringing" structures would be that in the first case the bend has undergone permanent deformation of certain areas which combine to shorten the distance between the ends while in the latter case no yielding was necessary.

Normally the entire elastic range (from the positive to the negative yield point) will not be utilized, and self-sprung bends will operate hot with a yield-point stress, with an off-stream stress somewhat lower. The same bend with 50 per cent prespringing will have equal stresses hot and cold, both below the yield point, or the prespringing could be up to the yield point with a low operating stress.

We next consider the effect of time and temperature on this problem. Prolonged loading at room temperature will result in a slightly lowered yield point, and as we go to higher temperatures the short- and long-time yield points decrease. This fact is the basis of stress relief at temperatures below the critical range, the procedure being to select a corresponding temperature and soaking time to yield all locked-up stresses to a desired level. This effect probably extends down to room temperature although the soaking time rapidly increases below 1000 F for carbon steel. The first effect of time and temperature therefore is to lower the stress-carrying capacity to the level of the long-time yield point at that temperature.

The yielding which occurs during the period just described is so-called "first-stage creep." Subsequently, the structure will continue to elongate under second-stage creep analogous to decreasing load or relaxation tests, the rate diminishing rapidly as the load is reduced. In time, a stress is reached which will be practically maintained over an extended period, which is commonly called the "relaxation strength." Consequently the second effect of time and temperature is to reduce the stress level to the relaxation strength of the material at that temperature.

With this reasoning, an unstressed bend will first assume elastic stresses proportional to the deflections; subsequently, first-stage and second-stage creep will reduce the stress to the relaxation strength, given sufficient time, so that there appears no possibility of more than initial overstress. The strain corresponding to the overstress will reappear when the off-stream condition obtains, so that the bend has self-sprung itself to this degree. For high-temperature piping, the self-springing therefore provides an expansion range from the desired stress cold to a relaxation stress of reversed sign. If the bend is presprung the period of adjustment is reduced and, if the prespringing is 100 per cent, there would be no bending stress due to expansion under operation.

It is evident that prespringing is beneficial and should be encouraged since it maintains the stresses in a more favorable range. Calculations should have for object the determination of the elastic range rather than the maximum stress. The average stress level should be considered in setting the allowable range, a procedure which also recommends itself from fatigue considerations. For high-temperature piping in the creep range the off-stream stress becomes significant as relaxation occurs, so that calculation for stresses at room temperature is indicated, since the maximum high-temperature stress is regulated at the relaxation-stress level for that temperature. This involves the room-temperature elasticity modulus.

Where the thrusts and moments in operation are significant, calculations at operating temperatures are also necessary. For these the reduced moduli of elasticity at the line temperature are used. In all calculations Poisson's ratio is assumed constant since few data are available on its variation.

Methods of Combining Stresses

A pipe line in space is subjected to flexural and torsional stresses as well as normal stresses and shears due to expansion effects in addition to circumferential, longitudinal, and radial stresses caused by pressure. This connotes a triaxial state of stress which must be correlated with the uniaxial failure stress obtained from the standard tensile test. The three better-known criterions of elastic failure (4) are the principal-stress theory (Rankine), the maximum-shear theory (Coulomb-Guest), and the maximum-shear strain-energy theory (Hencky-Mises). The first, although still widely applied on account of its simplicity, has been discredited, while the results of the other two theories agree fairly well among themselves and with elastic-limit tests. The Hencky-Mises theory is generally conceded to predict yielding most closely.

Neglecting, as of little practical significance, such effects as the radial pressure stress, the circumferential bending stress (which occurs in curved members), the normal stresses, and shears due to expansion, and assuming that stress-intensification factors, where applicable, have already been included in the stress components, the simple stresses which remain to be considered as acting in the extreme tensile fiber are

f_L = longitudinal pressure stress plus extreme-fiber stress due to bending
f_C = circumferential pressure stress
f_S = shear stress due to torsion

The resulting principal stresses can be written as

$$f_1 = \tfrac{1}{2}\{f_L + f_C + \sqrt{[4f_S^2 + (f_L - f_C)^2]}\} \quad [1]$$

$$f_2 = \tfrac{1}{2}\{f_L + f_C - \sqrt{[4f_S^2 + (f_L - f_C)^2]}\} \quad [2]$$

$$f_3 = 0$$

In the principal-stress theory, f_1 is directly compared with the allowable stress f and the two lower principal stresses are disregarded.

In the maximum-shear theory, the difference between the two extreme stresses governs. If both f_1 and f_2 are positive, the same criterion is obtained as for the principal-stress theory. If, on the other hand f_2 is negative, a different limitation applies. Accordingly, the following two conditions must be satisfied

$$f_1 = \tfrac{1}{2}\{f_L + f_C + \sqrt{[4f_S^2 + (f_L - f_C)^2]}\} \leqq f \quad [3]$$

$$f_1 - f_2 = \sqrt{[4f_S^2 + (f_L - f_C)^2]} \leqq f \quad [4]$$

Finally, for the Hencky-Mises theory, the failure criterion can be written as

$$\sqrt{(f_1^2 + f_2^2 - f_1 f_2)} = \sqrt{(3f_S^2 + f_L^2 + f_L^2 - f_L f_C)} \leqq f \quad [5]$$

With steady creep, the maximum-shear theory is commonly used, although Bailey (5) has developed a formula analogous to the Hencky-Mises equation, involving constants which are evaluated from the creep properties of each material. Before selecting a theory, some thought should be directed at criteria of failure. The stress theories mentioned predict yielding, not ultimate failure and, with extensive yielding or creep, the elastic theory no longer controls the stress distribution. In thick cylinders under pressure the Hencky-Mises theory would no doubt predict the yielding of the innermost fibers, but general yielding of the cylinder is probably best predicted by the simple stress theory, and the same is true of failure.

Where stresses are calculated according to the elastic theory, the principal-stress theory could be safely continued in use since it is in step with present-day safety factors. In fact, the Barlow or outside-radius formula of the Piping Code already penalizes piping in comparison with pressure vessels (A.S.M.E. and API-ASME Codes both use the Lamé formula which follows Rankine's theory for thick pressure vessels). If stresses are to be evaluated more closely and the safety factor reduced, then the Hencky-Mises theory should be used if yielding destroys the utility of the structure. If some yielding is not considered detrimental, then further studies and tests are desirable to correlate combined loads with rupture.

Where stresses are calculated under conditions of steady creep rather than the elastic theory, the maximum-shear theory is used by Bailey (5). Norton (6) has made creep tests on tubular specimens which indicate similar rates where the average simple circumferential stress and the simple tensile stress in the cylinder and test bar were the same, although with longer tests there are indications that the creep rate would decrease for the cylinders. Bailey establishes the ratio of diametral to tensile creep as $\frac{1}{2}$ to $\frac{1}{3}$ for the same shear stress. Norton's tests would indicate that the use of the average stress from the elastic theory (Lamé—mean of outside- and inside-diameter stresses) is entirely safe for a hydrostatic loading.

Suggested Design Assumptions and Stress Limits

Up to this point, we have been concerned with the presentation and discussion of the data and reasoning affecting the problem of pipe lines under expansion, and have found the subject to be quite complex. It is obvious that considerable simplification must be introduced in order to establish rules suitable for general application. In the following, basic assumptions for use in evaluating stresses and limits for the combined stresses resulting from pressure and expansion are suggested.

Evaluation of Expansion Thrusts and Stresses. The calculation of the line for expansion serves the purpose of establishing the range over which stresses vary rather than the maxima and minima, and accordingly consideration of the amount of prespringing required can be deferred to the end of the calculation. Since the relaxation stress definitely limits the hot condition, both prespringing and self-springing will tend to make the room-temperature condition the controlling one and calculations accordingly should be based on the elasticity modulus at room temperature rather than the reduced value in operation. Where a close estimate of thrusts in operation is desired, a simple conversion by the ratio of the hot to the cold elasticity moduli will give the required results.

Simple Stresses to Be Considered. In arriving at the principal stresses, consideration should be given in all problems to the following stresses at the outside surface of the pipe:

Simple pressure stresses:
- Circumferential
- Longitudinal

Line expansion stresses:
- Longitudinal bending (two planes)
- Torsion

Local stresses:
- Longitudinal bending in curved pipe and creased ells
- Longitudinal bending in corrugated pipe.

The radial pressure stress is zero at the outer surface. Direct stress due to the axial force and shearing stresses caused by transverse forces need be calculated only where the piping is extremely stiff. The local stresses are evaluated conveniently by using stress-intensification factors which are discussed later. We believe that the circumferential bending stress (curved pipe) and other local stress raisers, such as branch connections, may be neglected, although the use of unreinforced openings and reinforcement details should be more closely controlled where service conditions involve distinctly cyclic loading. Bolted joints will be commented on as a separate item.

Stress-Intensification Factors—Fatigue. In addition to the ordinary usage of the term "stress-intensification factor," it will here also be applied as a correction factor reducing the elastic range of stresses where cyclic loading is involved. The values given in Table 1 are suggested as a working basis.

TABLE 1 SUGGESTED STRESS-INTENSIFICATION FACTORS

Shape	Stress-intensification factor: Noncyclic stresses	Stress-intensification factor: Cyclic stresses
Plain tangents	1	1
Plain bends and creased bends	β	k
Corrugated tangents and bends	$2\frac{1}{2}$	5

Cyclic stresses would be considered to obtain where service conditions involve pulsating loads or frequent temperature changes; a closer definition should be considered, although it cannot be expected to relieve the designer entirely from a consideration of the merits of each case.

The factors for plain tangents require no comment. In the case of plain bends, k and β are the familiar Kármán or flexibility factor and Hovgaard's longitudinal stress-intensification factor, respectively. Both are applied to flexure stresses in the plane of the bend only. Creased bends are realized to occupy an inter-

mediate position between plain and corrugated pipe, but are usually classed with the former in the absence of reliable data. For straight and curved corrugated pipe, stress calculations are based on the moment of inertia of the pipe before corrugating, and the stress-intensification factor is applied to the longitudinal pressure stresses and the bending stresses. It is realized that the moment transverse to the plane of a bend can vary from pure bending to pure torsion with an intermediate range where both effects are present, and that the true state of stress is not evaluated by present methods; however, this and kindred problems can be considered as future refinements as our knowledge of this subject expands.

For noncyclic stresses, we have used the endurance properties of 20,000 cycles as a rough guide; for cyclic loading, 500,000 stress reversals have been considered as fairly representative of the endurance limit.

Combination of Stresses. If failure is associated with yielding, the Hencky-Mises theory most accurately predicts the elastic limit under combined stresses, although the maximum-shear theory offers sufficiently close results and is somewhat easier to apply. On the other hand, present-day safety factors for pressure stress are predicated on the principal stresses (Rankine theory) and, since local yielding results in stress redistribution with no loss in utility, average stresses are probably the best over-all strength index. For the present, we would favor the continued use of the principal-stress theory for calculations based on the elastic theory. Where stresses under steady creep (5) are being considered, the maximum-shear theory may be used.

Suggested Stress Limits. Using as a basis the allowable stress established in the codes for calculating pipe wall thickness under pressure, the following limits for the available stress range (already discussed as a suitable criterion for stress-deflection comparison) are suggested:

For no prespring

$$\text{Allowable stress range} = 0.75\ (S_A + S_A') \quad [6]$$

and for 50 per cent prespring

$$\text{Allowable stress range} = S_A + S_A' \quad [7]$$

where S_A = allowable stress at room temperature and S_A' = allowable stress at operating temperature, both to be introduced with their algebraic values so that they will always be numerically additive.

As an illustration, the allowable ranges obtained for combined pressure and expansion stresses in accordance with formulas [6] and [7] are listed in Table 2 for various line temperatures, as based on allowable pressure stresses for seamless low-carbon steel piping (48,000 lb per sq in. minimum ultimate tensile strength) to A.S.T.M. Specification A-106 as proposed for the ASA Code for Pressure Piping.

TABLE 2 ALLOWABLE RANGES FOR COMBINED PRESSURE AND EXPANSION STRESSES AS OBTAINED WITH FORMULAS [6] AND [7]

Line temperature, F	Allowable stress range			
	Oil-piping section		Power-piping section	
	No prespring	50% prespring	No prespring	50% prespring
Up to 650	18000	24000	14400	19200
700	17663	23550	14100	18840
750	17175	22900	13740	18300
800	16500	22000	13200	17600
850	15413	20550	12330	16440
900	14250	19000	11400	15200
950	12563	16750	10050	13400
1000	10875	14500	8700	11600

For any intermediate degree of prespring, the relation

$$\text{Allowable stress range} = (0.75 + s/2)\ (S_A + S_A') \quad [8]$$

could be used, where s = percentage of prespring expressed as a fraction = 0.50 maximum.

A higher degree of prespring than 50 per cent is beneficial at elevated temperatures in establishing the piping nearer equilibrium, thus reducing the amount of permanent set necessary to bring stresses to the relaxation limit. However, this will not affect long-time conditions, and is therefore not considered to warrant an increase in the elastic range.

Investigation of Bolted Flanged Joints. The rules suggested in the preceding apply to pipe and do not consider joints. These are properly investigated after the completion of the line calculations. We have already indicated that ASA standard flanges are probably sufficiently strong to take any bending stresses ordinarily encountered and, at least with ring-type gaskets, possess a fair resistance to torsion. In doubtful cases, and where special flanges are used, an investigation analogous to that carried out for two flanges in Appendix 2 will be indicated. The ASA flanges should probably be explored more thoroughly, and definite limits imposed on the allowable bending and torsion moments depending on the type of gasket used.

Appendix 1

TESTS OF PLAIN, CORRUGATED, AND CREASED-BEND PIPE UNDER CYCLIC VARIATION OF STRESS

Few, if any, pipes break in service due to the application of a steady load or but a few repetitions. Since the common pipe materials are quite ductile, under steady stress a small amount of yielding ordinarily will relieve highly localized overstress and effect a redistribution without hazard of failure, provided the average stress does not already exceed the yield point. Where, on the other hand, repeated stress variations are incurred due to temperature or pressure changes, or vibration, an entirely different situation arises. The application of a large number of stress cycles of sufficient amplitude will cause fatigue failure, which is accelerated by flaws in the material or surface irregularities.

The fatigue properties of a metal are characterized by the endurance limit which is the stress below which no fracture will occur even after an indefinite number of stress reversals, and is usually derived from cyclic flexure tests in which the extreme fibers are alternately subjected to maximum tensile and compressive stresses of equal magnitude. It has been found that the ratio of endurance limit to ultimate strength, or so-called endurance ratio, ranges between 40 and 50 per cent. Where stresses are not completely reversed, but vary about a definite mean, the limiting stress is raised. For a variation from zero to a maximum, an approximately 50 per cent higher limit is found comparing to between 60 and 75 per cent of the ultimate strength. A general formula covering any arbitrary stress range has been suggested by Moore and Kommers (7), as

$$S_{max} = \frac{3S_L}{2 \pm (S_{min}/S_{max})} \quad [9]$$

where S_L is the endurance limit for complete reversal, and the sign of the ratio of minimum to maximum stress is negative where both stresses act in the same sense, and positive where one is tension and the other compression. In the absence of corrosion, cycle frequency appears to have no influence on fatigue-resisting properties, the number of cycles alone controlling.

TESTS OF CORRUGATED AND CREASED-BEND PIPING BY DENNISON (2)

Realizing the advantages offered by corrugated and creased-bend piping, particularly in confined spaces, and at the same time noting the lack of concise information on the characteristics of such piping, the Bureau of Engineering of the Navy Department

authorized large-scale flexibility and fatigue tests on which Dennison (2) reports. The information contained in this valuable paper is summarized briefly below.

Standard commercial products were used, the specimens including tangents, square and offset quarter bends, and expansion U-bends of 6-in. standard-pipe-size seamless tubing of carbon steel to A.S.T.M. Specification A106–33T, with different heat-treatments. The ultimate tensile strength of the various pieces ranged from 51,500 to 72,600 lb per sq in.; the yield point, from 33,400 to 46,000 lb per sq in.; the elongation in 2 in. from 25.5

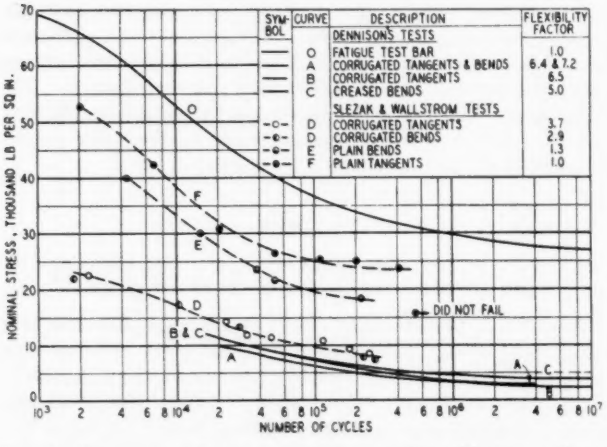


FIG. 1 ENDURANCE LIMITS FOR PIPE AND PIPE MATERIAL FROM DENNISON'S AND M. W. KELLOGG COMPANY'S DATA

to 48.0 per cent; and the reduction in area from 33.5 to 75.1 per cent.

The corrugations showed a $2^1/_2$-in. pitch and $^3/_4$-in. height. The creased bends were made on a 15-in. radius, the creases being on approximately 11 deg 15 min pitch and extending around two thirds of the circumference. For the corrugated tangents, a flexibility factor of 6.4 was established, and for the corrugated arcs, a value of 7.2 (as compared with 2.8 for plain pipe bent to the same radius); for the creased bends the flexibility factor was 5.0 (as compared with a theoretical value of 3.8 for plain pipe of the same radius).

Two types of fatigue-testing equipment were used—the one, a rotating-beam machine, producing a specific stress in the specimen while the other applied a definite deflection through a crank mounted on an eccentric. Tests made with complete stress reversal were correlated with tests where the stress ranged from zero to a maximum by application of formula [9] Moore and Kommers (7); this simply requires reducing the stresses for the second form of loading in the ratio of 2 to 3 before plotting.

Dennison's results are reproduced in the three lower curves on the fatigue graph shown in Fig. 1. Curve *A* includes corrugated arcs and tangents. Curve *B* applies to corrugated tangents of slightly different thickness and shape, and curve *C* to creased bends. Curve *O* gives the results of a standard fatigue test on the material itself which had an ultimate strength of 63,000 lb per sq in. and an endurance limit of 27,000 lb per sq in. or an endurance ratio of 0.43.

The presentation used is the customary semilogarithmic graph in which the logarithmic scale of abscissas measures the number of cycles to failure; it makes little difference whether this is defined as the ultimate fracture or the point at which slightly eccentric rotation heralds the end of a test, since the intervening interval is extremely brief. The vertical scale reads the "nominal" stress calculated for the point of fracture in accordance with the common or Rankine theory based on the pipe dimensions before the corrugations or creases are formed; in other words, the "nominal" stress equals the bending moment divided by the section modulus of the original pipe.

Space does not permit a detailed discussion of the interesting analysis and conclusions offered in Dennison's paper (2). Instead, we will conclude with a few notes on the type of failure experienced. With a single exception the corrugated pipe always failed at the crest of a corrugation, the cracking progressing from the inside out. On the other hand, the creased pipe invariably failed through the trough between two creases, with the cracks starting at the outer surface and proceeding inward; the same was true of the corrugated pipe cited previously as an exception, which was bent to a short radius so that it assumed an intermediate shape between corrugation and creasing. Dennison points out that the failure in all instances starts at the surface of sharpest curvature which is the inside of the crest for corrugated pipe and the outside of the trough for creased pipe.

TESTS OF PLAIN AND CORRUGATED PIPING BY E. J. SLEZAK AND H. WALLSTROM OF THE M. W. KELLOGG COMPANY

While Dennison has admirably achieved his immediate objective of developing information on the endurance of corrugated and creased-bend piping and correlating this with the endurance limit of the parent metal as obtained from a standard fatigue-test specimen, the authors have been reluctant to accept fully the conclusions he derives therefrom in the absence of a parallel investigation on plain piping with which a wealth of service experience is associated. It is felt that comparison with plain pipe would furnish a better yardstick for the appraisal of the limits within which corrugated or creased pipe should be stressed in actual installation. In order to supply such a basis, a simple series of tests were projected which have recently been completed by E. J. Slezak and H. Wallstrom under the authors' supervision. A description of the tests follows:

A 24-in. lathe was transformed into a rotating fatigue-testing machine of the specific-strain variety. The size of the model bends was restricted by available clearance; also, by the force applied, due to the construction of the lathe, in particular the feedscrew. Since the tests were intended to give results representative of commercial pipe and the corrugating processes used for the larger pipe cannot be properly extended to too small a diameter, 2-in. nominal-size standard-weight seamless pipe was used throughout. In setting up for a test, the model was rigidly attached to the chuck of the lathe at one end after the outstanding leg had been carefully aligned to revolve on the dead center of the tailstock. A self-aligning roller bearing attached to the cross-feed mechanism was then advanced over the end of the pipe which, owing to the difference between the size of the bearing and the pipe, was fitted with a reducing collar. Next, a predetermined amount of deflection was applied by means of the transverse feedscrew; as an additional precaution, this was checked against a dial gage mounted on the lathe bed. In order to be able to dispense with an accurate determination of the moment arm and obviate the necessity of complicated deflection calculations (made uncertain by variations in stiffness introduced by the reducing collar as well as the varying flexibility of the corrugated sections), the stress introduced by the applied deflection was directly gaged by means of an accurately calibrated Huggenberger extensometer attached to the plain pipe tangent provided at the chuck end. Both the dial and extensometer were, of course, removed before the lathe was started up. The number of revolutions was recorded by a mechanical counter attached to the shaft. The test arrangement is shown in Figs. 2, 3, and 4.

The material for the 2-in. seamless pipe was taken from stock and was manufactured to A.S.T.M. Specification A53-36. After fabrication, the pipe received a stress relief at 1150 F with

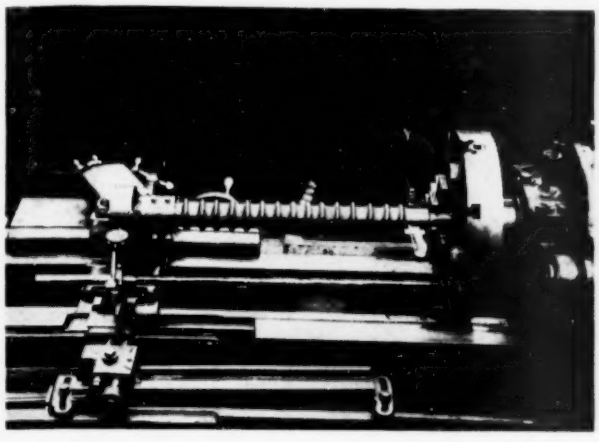


FIG. 2 CORRUGATED TANGENT MOUNTED IN THE LATHE FOR GAGING STRESS AND DEFLECTION PRIOR TO FATIGUE TEST

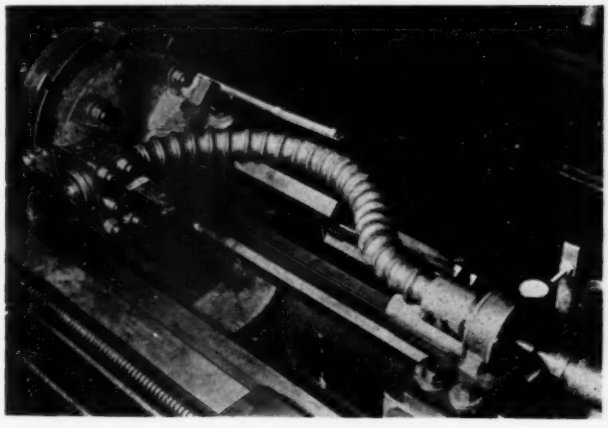


FIG. 4 CORRUGATED OFFSET QUARTER BEND MOUNTED IN A LATHE FOR GAGING STRESS AND DEFLECTION PRIOR TO FATIGUE TEST

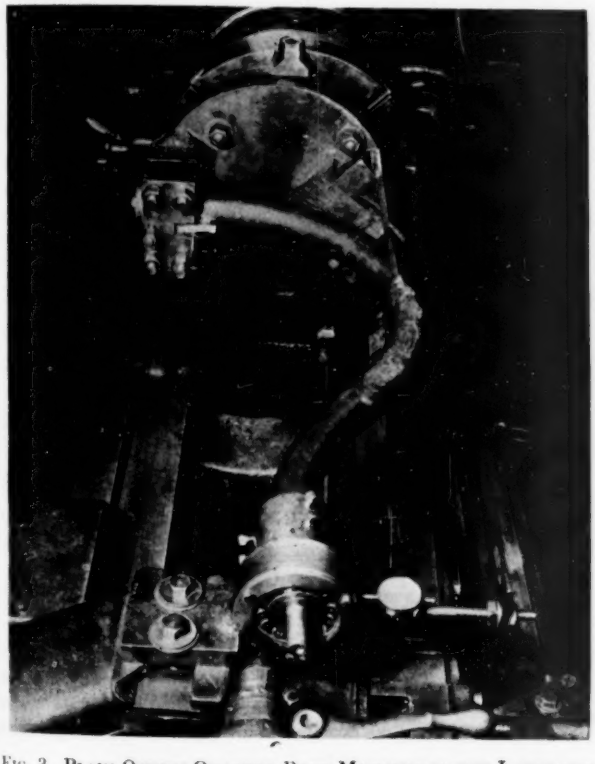


FIG. 3 PLAIN OFFSET QUARTER BEND MOUNTED IN THE LATHE FOR GAGING STRESS AND DEFLECTION PRIOR TO FATIGUE TEST



FIG. 5 MICROGRAPH OF PIPE MATERIAL USED IN M. W. KELLOGG COMPANY'S TESTS

subsequent furnace cooling to 600 F. A tensile test gave an ultimate strength of 59,400 lb per sq in., a yield point of 36,700 lb per sq in. (as obtained by the drop of the beam), and an elongation of 45.6 per cent in $1^1/_4$ in. gage length. Satisfactory bend and flattening tests were made. A micrograph is shown in Fig. 5 which reveals the pearlite nearly completely dispersed, with cementite particles well spheroidized.

A number of tests were run with each of the four types of models shown on the right-hand margin of Fig. 6; these include tangents and offset quarter bends of plain and corrugated pipe. The corrugations used had an average pitch of $1^9/_{16}$ in. and a depth of slightly over $^3/_8$ in. As was demonstrated by a series of load-deflection tests, this contour does not offer the degree of flexibility common to the larger pipe sizes, averaging 3.7 for the tangents and 2.9 for the arcs. This was the first 2-in. pipe corrugated by our shop; the flexibility probably could be improved by changing the corrugations. For the plain offset quarter bend, calculations give a flexibility factor of 1.3 and a (static) stress-intensification factor of 0.9; following the established theory, the bend actually would be stressed lower than an equivalent tangent.

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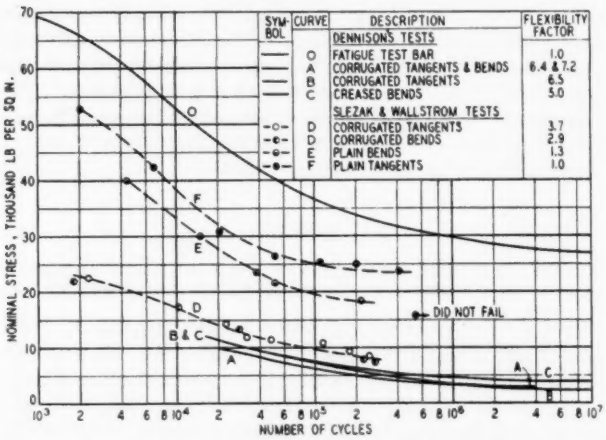


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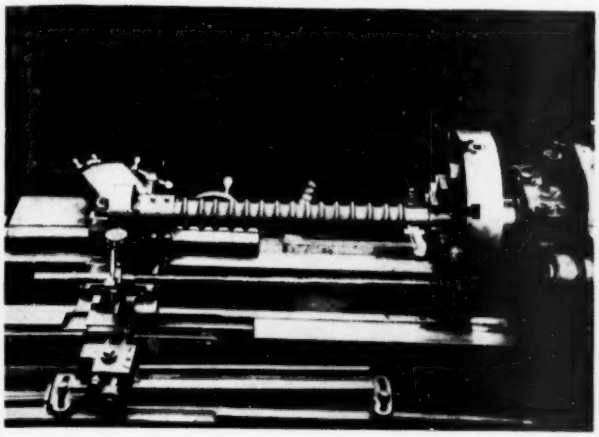


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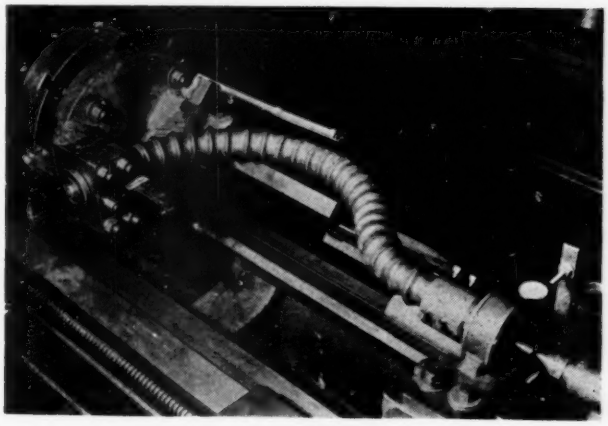


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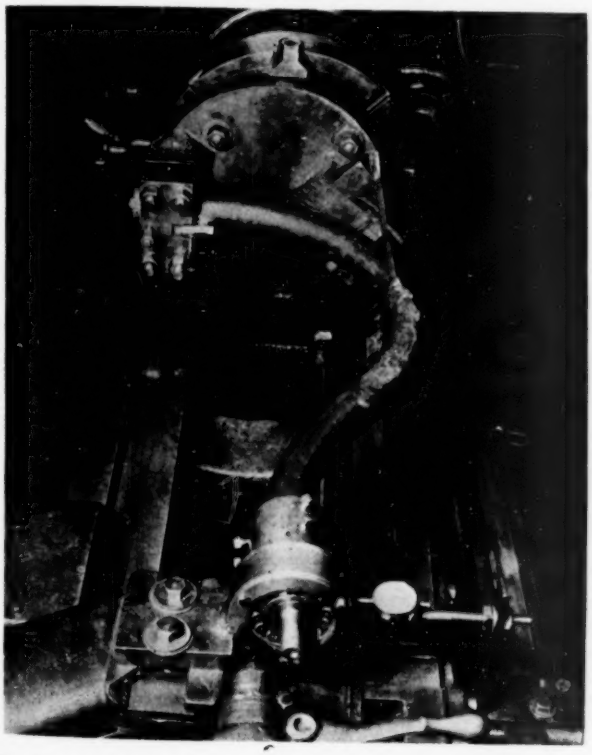


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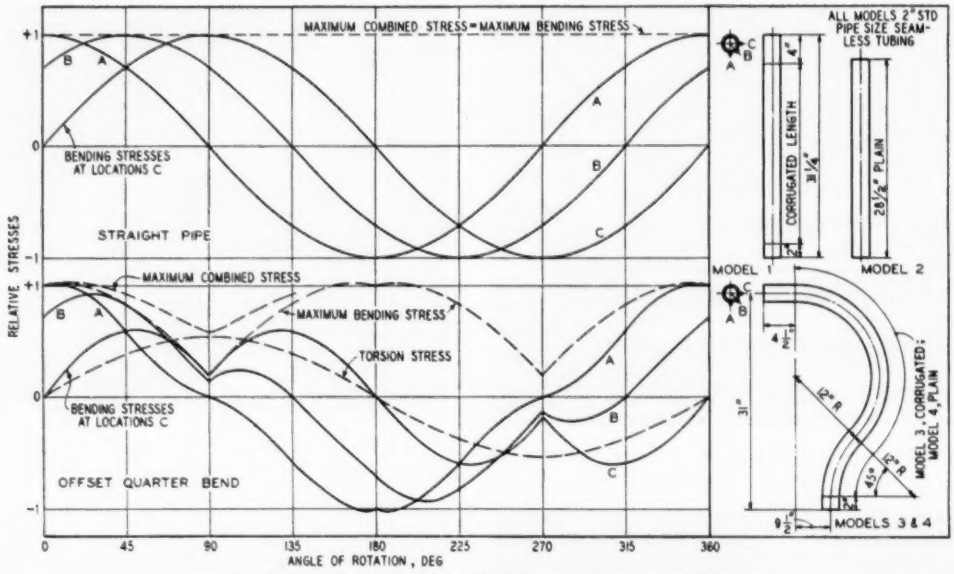


FIG. 6 STRESS CYCLES OF M. W. KELLOGG COMPANY'S MODELS

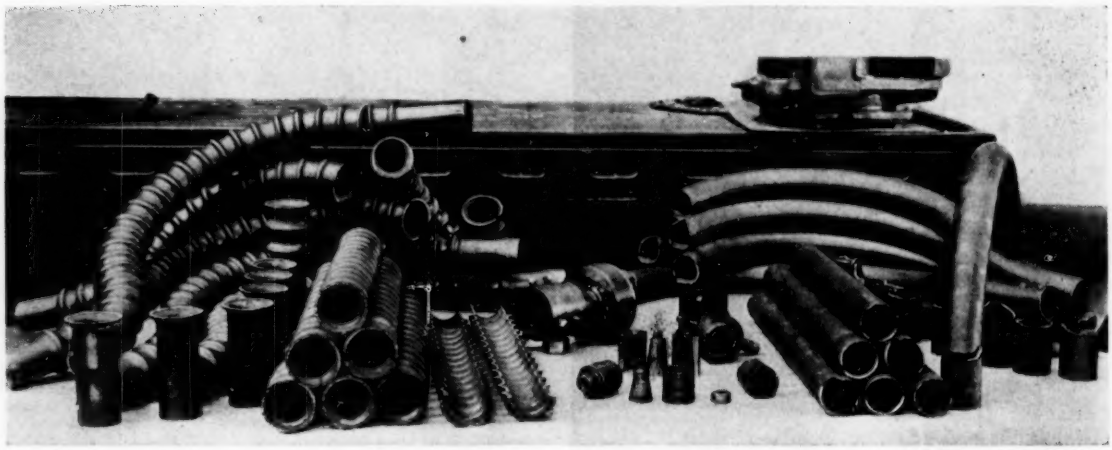


FIG. 7 FAILED SPECIMENS, PARTS OF TESTING APPARATUS, AND CORRUGATED TANGENTS CUT TO ILLUSTRATE THE SHAPE OF THE CORRUGATIONS

series; the arrow attached to the point furthermost to the right denotes a test which was discontinued without failure. Since the results for the corrugated tangents and bends were in close accord, a single curve, marked *D*, was drawn for both series. It should be noted that there is quite a reduction in endurance for the plain tangents (curve *F*) as compared with the metal itself. (Since the ultimate strength of the pipe material in the present tests is close to that obtained for the material in Dennison's tests, it is assumed that curve *O* can serve as a parameter for both investigations.) A further reduction in fatigue properties is found for the plain arcs (curve *E*) despite the reduced theoretical stress.

A short note may be in order on the type of cyclic variation of the stresses applied. While the stress curve for each point on the circumference of the tangents follows a sine wave, this is not true for the offset quarter bends shown in Fig. 6. The curves of both flexural and torsional stress are given at three points on the circumference for the location of maximum stress near the fixed end; point *A* is on the inside of the bend in the plane of its center line, and points *B* and *C*, respectively, are 45 and 90 deg distant therefrom. Only point *A* and a point diametrically opposite are subjected to stress reversal of the maximum amplitude. Despite this fact, and the added complication of a cyclic torsional stress, the corrugated arcs behaved similarly to the tangents, and the conclusion, also implied in Dennison's presentation (2), can be drawn that the highest stressed point controls failure provided average conditions are not too dissimilar.

Examination of the failed specimens of corrugated pipe confirms the observations of Dennison (2). In the case of plain pipe it was natural to expect a more jagged break since there is no sharp concentration of stresses; here surface markings and weak ligaments dictate the path of least resistance. Fig. 7 shows all the failed specimens assembled, with corrugated bends and tangents at the left, and plain bends and tangents at the right; in the center, the roller bearing, reducing collar, and various parts of the clamping devices are shown, together with the two halves of a corrugated tangent cut through longitudinally to illustrate the shape of the corrugations.

DIGEST OF RESULTS OF FATIGUE TESTS

In order to review the results of both the Navy (2) and the Kellogg tests for the purpose of formulating rules suitable for general application to the practical problem of pipe-line design,

TABLE 3 FATIGUE DATA FROM DENNISON'S (2) AND KELLOGG'S TESTS

Test series[a]	Description of models tested	Average flexibility factor, k	Avg ult tensile strength, lb per sq in.	Number of cycles	Limiting stress S, lb per sq in.	S/UTS	S/So	S/S_F
O	Standard fatigue test bar, polished	1.0	63000	20000	46900	0.740	1.00	1.46
				100000	36700	0.580	1.00	1.47
				500000	31100	0.500	1.00	1.32
				2500000	28100	0.450	1.00	..
F	2-In. standard - pipe - size seamless steel tubing, plain tangents	1.0	59400	20000	32000	0.540	0.68	1.00
				100000	25000	0.420	0.68	1.00
				500000	23500	0.400	0.76	1.00
				2500000	...	...	..	..
E	2-In. standard - pipe - size seamless steel tubing, plain offset bends	1.3	59400	20000	27800	0.470	0.59	0.87
				100000	19700	0.330	0.54	0.79
				500000	17600	0.300	0.57	0.75
				2500000	...	...	...	...
D	2-In. standard - pipe - size seamless steel tubing, corrugated tangents and offset quarter bends	3.3	59400	20000	14100	0.240	0.30	0.44
				100000	9800	0.160	0.27	0.39
				500000	7500	0.130	0.24	0.32
				2500000	...	...	..	..
C	6-In. standard - pipe - size seamless steel tubing, creased quarter bends and U-bends	5.0	59000	20000	11400	0.190	0.24	0.36
				100000	7600	0.130	0.21	0.30
				500000	5300	0.090	0.17	0.23
				2500000	4300	0.073	0.15	..
B	6-In. standard - pipe - size seamless steel tubing, corrugated tangents (one creased radius)	6.5	57300	20000	11400	0.200	0.24	0.36
				100000	7600	0.130	0.21	0.30
				500000	4400	0.077	0.14	0.19
				2500000	2800	0.049	0.10	..
A	6-In. standard - pipe - size seamless steel tubing, corrugated tangents and various shapes of bends	6.8	58400	20000	9900	0.170	0.21	0.31
				100000	6600	0.110	0.18	0.26
				500000	4000	0.069	0.13	0.17
				2500000	3000	0.051	0.11	..

[a] Series *A*, *B*, *C*, and *O* are Dennison's tests. Series *D*, *E*, and *F* are Kellogg's tests made by Slezak and Wallstrom.

So = Limiting stress using standard fatigue-test bar, test series *O*.
S_F = Limiting stress using plain straight pipe, test series *F*.
UTS = Ultimate tensile strength.

all the pertinent information on each separate series has been assembled in Table 3. The listing follows the order of increasing flexibility of the significant part of each bend model. Limiting stresses have been read from the original fatigue curves for 20,000, 100,000, 500,000, and 2,500,000 cycles and these are correlated to the ultimate tensile strength in column 7. In column 8, the limiting stresses obtained for the pipe for each number of cycles are compared with the corresponding stresses for the standard fatigue-test bar. In column 9, the limiting stresses for the plain bends, creased bends, and corrugated tangents and corrugated bends are correlated with those applying to the plain pipe tangents.

A relationship can be established between the flexibility factor and the endurance of the material, which appears to be independent of the source of increased flexibility (whether flattening of the cross section, as in the case of plain bends, or partial or complete corrugation, as in the case of creased or corrugated pipe). This will be illustrated by reference to the 20,000-cycle and 500,000-cycle stresses, for which the following formulas permit a reasonably close estimate of the limiting stresses for pipe of varying flexibility. For 20,000 cycles

$$S_{k20} = S_{20}/\sqrt[3]{k^2} \quad \ldots\ldots [10]$$

and for 500,000 cycles

$$S_{k500} = S_{500}/k \quad \ldots\ldots [11]$$

where k is the flexibility factor of the pipe; S_{20} and S_{500} are the "virtual" endurance limits, or limiting stresses for pipe of unit flexibility at 20,000 and 500,000 cycles, respectively; and S_{k20} and S_{k500} are the "virtual" endurance limits for pipe of a flexibility factor k at 20,000 and 500,000 cycles, respectively. Values computed in accordance with formulas [10] and [11] are compared in Table 4 with those obtained by test.

If the exponents of k were changed to 0.65 and 1.04, respectively, for the 20,000- and 500,000-cycle stresses, the difference would be reduced considerably. However, formulas [10] and [11] should be viewed as rough guides in the interpretation and extrapolation of these tests only, and additional refinement appears unwarranted in view of the meager basis on which they are founded.

In the tests, the endurance of the plain tangents was between two thirds and three quarters that of the test bar and, by introducing a factor approximately 0.7, an estimate of the virtual endurance limit for pipe of any flexibility can be derived from the fatigue characteristics of the material.

TABLE 4 ENDURANCE LIMITS COMPUTED FROM FORMULAS [10] AND [11] COMPARED WITH TEST RESULTS

Flexibility factor, k	Virtual endurance limit S_{k20}			Virtual endurance limit S_{k500}		
	Computed	From test	Difference, % of computed stress	Computed	From test	Difference, % of computed stress
1.0	32000	32000	±0	23500	23500	±0
1.3	26900	27800	+ 3.3	18100	17600	— 2.8
3.3	14400	14100	— 2.1	7100	7500	+ 5.6
5.0	11000	11400	+ 3.6	4700	4300	— 8.5
6.5	9200	11400	+23.9	3600	2800	—22.2
6.8	8900	9900	+11.2	3500	3000	—14.3

Appendix 2

THE PROBLEM OF BOLTED FLANGED CONNECTIONS

A study of the effects on bolted flanged joints of eccentric loading introduced by the expansion of a pipe line cannot be properly undertaken without a reasonably correct understanding of the fundamental problem of the action of flange and gasket under the bolt load required to produce a tight seal under internal pressure. While thousands of bolted flanged connections are in continuous successful use in an ever-widening variety of services, surprisingly little precise knowledge of the stresses and deformations of the component parts is available. It is only the recent trend toward higher temperatures and pressures in the steam power plants and chemical industries which has forced a reconsideration of the entire basis of design and lent impetus to research which is already yielding valuable returns. We refer in particular to the work of the British Pipe Flanges Research Committee (8) which operates on a broad basis, to the improved

approaches to flange design published by Waters, Wesstrom, Rossheim, and Williams (9), as well as Bailey's (10) contribution to our understanding of the behavior of flanges under conditions of creep, and the many individual investigations made in recent years here and abroad with reference to the performance of gaskets.

The complexity of the problem involved will be apparent from a consideration of the variables. In general, the shape of a flange does not readily lend itself to accurate mathematical analysis, particularly since the method of its attachment to the pipe, by welding, rolling, screwing, or otherwise, affects the stress distribution. The effects of bolt holes and the gasket groove are difficult to include. Serious uncertainties are introduced by the necessity of making assumptions for the locations of the bolt and gasket reactions which depend on the stiffness of the flange. While a formidable number of test results are available on a wide variety of gasket shapes and materials, no entirely satisfactory correlation of the data has yet been presented and opinions regarding the gasket contact pressures required to seal a joint under varying conditions show considerable divergence. The share of the elastic deformation of the compound system assumed by the bolting depends on the material, size, length, and spacing of the bolts, on the development of the shank and type of thread used, on the material and height of the nut and, to a small extent, even on the compressibility of the washer. Allowable design stresses have to consider the yield strength as well as the fatigue properties and are affected by the manufacture and heat-treatment of the steel.

Conditions are further aggravated in the case of hot lines where the problems of creep and relaxation make it extremely difficult, if at all possible, to develop an accurate analysis of the behavior of a bolted joint. In addition, the different expansions of the component parts must receive consideration and this, in turn, presupposes at least an approximate knowledge of the temperatures assumed by the flange, bolting, and gasket in operation and during starting up and shutting down. The question arises whether flanges should be insulated. If lagging is provided, the metal temperatures can be maintained more nearly uniform and sudden failure due to change in operating or atmospheric conditions or thermal shock is less likely to occur; on the other hand, the life of an uninsulated connection will probably be greater due to the reduced temperatures.

In view of the multiplicity of considerations entering into the design of an ordinary bolted flanged connection it is comforting to be able to state that, in general, a few relatively simple rules supported by experience and good judgment will enable a designer to proportion the parts in such a way that the joint will perform satisfactorily in operation, at any rate under conditions which do not involve excessive temperatures or rapid fluctuations.

Both the A.S.M.E. Code and the API-ASME Code for Unfired Pressure Vessels have adopted the flange formulas developed by Waters and Taylor. An extension of the method to include shear and to cover tapered hubs has been published recently (9); this new derivation also permits the evaluation of the effects of internal pressure by a secondary investigation while the earlier analysis restricted itself entirely to the bolt moment. While the present paper is not concerned with flange design proper, a brief exposition of certain phases will be necessary in order to show how the effects of expansion of a pipe line on the bolted flanged connection can be taken into account.

The following forms of loading must be considered in the design of bolting for a flange:

1 The internal pressure load, which is usually taken over the circular area bounded by the center line of the gasket.

2 The minimum load required to set the gasket in the cold condition. It has been found that, up to certain pressures, the force required to maintain a tight seal is independent of the internal fluid or gas pressure; this limit varies for different materials and shapes of gasket and also depends on the flange facing. The minimum contact load can be interpreted as the load required to establish line contact around the circumference for the harder gaskets; for the softer packings, it probably signals a sufficiently intimate contact over a larger area so that the gap between the gasket and flange faces is reduced to such minute dimensions that crossflow cannot occur.

3 The minimum load required to maintain the joint tight under operating pressures exceeding the limit mentioned in 2. For such pressures the required gasket force appears to be a direct function of the internal pressure. The value of the gasket-contact ratio is dependent on the type of gasket and facing.

4 A sufficient additional compressive load to allow for the reduction in gasket pressure caused at the joint by direct tensile stresses and tensile bending stresses due to thermal expansion of the line, the fundamental idea being that this pressure under no circumstances can be allowed to drop below the minimum required to preserve a tight joint.

5 A sufficient total load to prevent rotation in the joints due to torsional stresses induced by the line expansion. These are taken in friction which is a direct function of the bolt load.

6 An adequate initial load to compensate for yielding of any part of the assembly due to suddenly applied pressure or temperature loads.

7 For service at elevated temperatures a design load which does not stress the component parts beyond their relaxation strength, so that the required gasket compression will be maintained for the desired period.

While the conservation of a sufficiently high gasket load under all conditions of operation is the primary concern in the design of the bolting, care should be taken to proportion the gasket so that overpulling of the bolts or the application of expansion thrusts or compressive bending stresses will not lead to its failure by crushing. Furthermore, the effect of internal pressure on the gasket in producing excessive hoop stresses and the danger of blowouts must be considered. The flanges of course, must be able to withstand the maximum bolt moment exerted, and, while ordinary allowable stresses need not apply, the proportions must be such as to prevent excessive distortion.

Disregarding the effects of line expansion, cold design stresses between 20,000 and 25,000 lb per sq in. have been used rather widely in this country for alloy-steel bolting; these are stresses maintained in operation, although the initial stresses required to set the gasket are very often much higher. For elevated temperatures where creep controls, stresses producing 1 per cent strain in 100,000 hours are applied; as determined by investigators in this country, such stresses will remain below the relaxation strength. In England, Bailey (10) from a test on carbon-molybdenum steel arrives at the conclusion that the relaxation limit for 100,000 hours corresponds to the stress causing 0.1 per cent creep in that period (11). For the flanges, the applicable code stresses are used except that it is not uncommon to allow a 50 per cent higher limit for the hub stresses which are of a highly local nature and easily relieved.

With the introduction of a more accurate analysis of a stress problem, it is common practice to reconsider safety factors. In the past, flanges have withstood stresses in excess of those calculated considering the bolt load required for tightness only, and accordingly a precedent is established for permitting higher design stresses where the effects of line expansion are included. Before we suggest any new stress limits, it will be in order to form an estimate of the magnitude of the hitherto uncalculated expansion stresses in a bolted flanged connection.

A review of a large number of stress calculations involving

TABLE 5 RESULTS OF INVESTIGATION ON TWO ASA WELDING NECK FLANGES WITH STANDARD RING JOINTS FOR THE CONDITION WHERE THE BENDING MOMENT DUE TO EXPANSION IS HIGH ENOUGH TO PRODUCE A STRESS OF 15000 LB PER SQ IN. IN THE ADJOINING PIPING

	Pipe data:		
1	Line size, in.	4	24
2	Flange series, lb	300	1500
3	Pipe schedule	40	160
4	Bending moment producing a stress of 15000 lb per sq in. in the pipe, in-lb	48200	11730000
	Limits for unit gasket compression based on gross width of ring:		
5	Minimum pressure required to set the gasket with no internal pressure (1/8 ring width × 18000 lb per sq in. yield point), lb per sq in.	2250	2250
6	Minimum pressure required to maintain tightness under operation (1/4 ring width × 5.5 × internal pressure), lb per sq in.	413	2063
7	Maximum pressure to prevent crushing gasket, lb per sq in.	30000	30000
	Bolt stresses for various conditions:		
8	Minimum stress required to set gasket with no internal pressure, lb per sq in.	7500	1900
9	Minimum stress to hold internal pressure plus gasket pressure under operation, lb per sq in.	4750	7950
10	Stress required to withstand moment causing a stress of 15000 lb per sq in. in the pipe, lb per sq in.	10100	8600
11	Minimum stress required for tightness (item 8 or 9, whichever is higher), lb per sq in.	7500	7950
12	Minimum stress required for tightness and bending moment (item 10 + item 11), lb per sq in.	17600	16550
13	Probable limit to which bolt may be pulled without control, lb per sq in.	31800	20000
	Flange stresses for various conditions:		
	For minimum bolt load required for tightness (item 11):		
14	Longitudinal hub stress, lb per sq in.	3300	5300
15	Radial flange stress, lb per sq in.	3700	6500
16	Tangential flange stress, lb per sq in.	2800	6200
	For minimum bolt load for tightness and bending moment (item 12):		
17	Longitudinal hub stress, lb per sq in.	6900	10000
18	Radial flange stress, lb per sq in.	7900	12200
19	Tangential flange stress, lb per sq in.	5900	11700
	For condition where bolts are pulled up to limit (item 13):		
20	Longitudinal hub stress, lb per sq in.	12000	11800
21	Radial flange stress, lb per sq in.	13600	14400
22	Tangential flange stress, lb per sq in.	10300	13900
	Maximum gasket compression on gross width:		
23	For minimum bolt load required for tightness (item 11), lb per sq in.	2250	9500
24	For minimum bolt load for tightness and bending moment (item 12), lb per sq in.	5300	19750
25	For condition where bolts are pulled up to limit (item 13), lb per sq in.	9550	23800

pipe lines connecting to pumps gave the following averages for the end reactions:

Vertical thrust plus weight of pump riser including valves and fittings, lb $= 3.25\ D^3$

Lateral thrust in any direction, lb $= 1.50\ D^3$

Bending or torsional moment, in-lb $= 60\ D^3$

where D equals the outside pipe diameter increased by 3 in. The corresponding bending or torsional stresses in the pipe due to expansion ranged from 1000 to 6000 lb per sq in. It has been our practice to limit the total stresses in pipe lines to 1.5 times the allowable code stresses, one third of which is absorbed by the longitudinal pressure stress; accordingly, some carbon-steel installations below 750 F based on the API-ASME Code with material of 60,000 lb per sq in. ultimate strength must have supported up to 15,000 lb per sq in. bending stress due to expansion. Since the ends of a line usually are locations of maximum stress and normally will be flanged for attachment to a nozzle provided on the vessel, there will probably always be one or two flanges which are subject to the maximum moment.

In order to appraise conditions at a joint under such circumstances, two ASA welding neck flanges with standard ring joints will be investigated for the conditions where the bending moment due to expansion is high enough to produce 15,000 pounds per square inch stress in the adjoining piping; the data are given in Table 5. It will be noted by referring to Tables 6 and 7 that the examples selected are fairly representative of the range of conditions met with in ASA flanges.

TABLE 6 PIPE DATA

Pipe schedule	Nominal size, in.	Area, sq in.	Section modulus, in.[a]
40	4	3.17	3.21
	8	8.40	16.81
	12	15.74	47.09
	16	24.35	91.49
	20	36.15	170.40
	24	50.30	285.10
80	4	4.41	4.27
	8	12.76	24.51
	12	26.04	74.53
	16	40.14	144.50
	20	61.44	277.20
	24	87.17	467.90
160	4	6.62	5.90
	8	21.97	38.47
	12	47.14	122.50
	16	70.85	233.50
	20	109.92	453.50
	24	157.51	780.60

TABLE 7 FLANGE DATA

Flange series, lb	Nominal size, in.	Bolts[a] Root area, sq in.	Bolts[a] Section modulus, in.[3]	Ring gasket[b] Gross area, sq in.	Ring gasket[b] Section modulus, in.[3]
150	4	1.616	3.03	5.77	8.45
	8	2.416	7.10	9.57	23.33
	12	5.028	21.37	14.73	55.24
	16	8.816	46.84	17.55	78.43
	20	14.560	91.00	21.60	118.80
	24	18.580	137.00	26.02	172.38
300	4	2.416	4.76	8.07	11.87
	8	5.028	16.34	14.60	38.78
	12	11.650	51.69	20.62	77.33
	16	18.580	104.51	25.43	117.61
	20	22.300	150.50	36.13	207.75
	24	33.720	269.80	53.51	364.54
400	4	3.350	6.60	8.07	11.87
	8	6.610	21.49	14.60	38.78
	12	14.860	65.94	20.62	77.33
	16	23.100	129.94	25.43	117.61
	20	33.720	227.61	36.13	207.75
	24	47.520	380.16	53.51	364.54
600	4	3.350	7.12	8.07	11.87
	8	8.736	30.03	14.60	38.78
	12	18.580	89.42	20.62	77.33
	16	28.100	166.84	25.43	117.61
	20	40.320	287.28	36.13	207.75
	24	55.300	456.23	53.51	364.54
900	4	5.820	13.46	8.07	11.87
	8	13.860	53.71	14.60	38.78
	12	23.100	121.27	20.62	77.31
	16	33.600	203.70	36.32	167.98
	20	53.040	391.17	54.19	311.59
	24	85.840	761.83	85.61	583.22
1500	4	7.430	17.65	8.76	13.96
	8	20.160	78.12	20.86	55.41
	12	43.430	244.29	41.23	154.61
	16	68.670	476.40	65.38	302.38
	20	101.18	828.41	90.32	519.34
	24	140.000	1365.00	117.71	801.90
4000[c]	4	7.430	17.65	11.49	14.00
	8	23.760	94.30	28.27	56.54
	12	51.500	291.30	48.60	136.69

[a] Root areas of bolts are computed for V-threads with 8 threads per inch, minimum. In calculating the section modulus of the bolting, it has been considered permissible to assume the bolt area uniformly distributed over the bolt circle.

[b] The approximate formulas $A = 2\pi rw$ and $SM = \pi r^2 w$ have been used in calculating the ring-gasket areas and section moduli, respectively, where r = radius of the pitch circle and w = gross width of the ring.

[c] Proposed.

TABLE 8 INVESTIGATION OF ASA FLANGES (TABLE 5) EXTENDED TO COVER TORSION EQUAL IN MAGNITUDE TO THE BENDING MOMENTS INVESTIGATED[a]

Line size, in.	4	24
Flange series, lb	300	1500
Pipe schedule	40	160
Torsion moment, in-lb	48200	11730000
Vertical thrust required $P = M/(f \times$ gasket radius), lb	82500	4290000
Corresponding bolt stress = (P/bolt area), lb per sq in.	34000	30600
Required minimum gasket compression = P/gasket area, lb per sq in.	10200	36500

[a] A friction factor of $f = 0.20$ is assumed.

This investigation would tend to show that the ASA flanges, at least where used with ring joints, have a sufficient margin of strength in all parts to take considerable bending moments without danger of overstress. In both examples, the check against overpulling of bolts, to which the authors subject each flange as a routine precaution, gives higher values than the computation including an unusually high bending moment. It is a well-

known fact that ASA flanges are overbolted in order to allow for high gasket contact ratios required on some forms of gaskets. This is no longer true of the proposed 4000-lb standard, which is intended for ring joints only or alternate gaskets requiring a low sealing force. The 4-in. 4000-lb size actually has the same bolting as the 4-in. 1500-lb size, and correspondingly the margin for bending stresses is considerably reduced; also, the gross ring area is by no means in proportion to the rating, and accordingly caution must be exercised in the use of such flanges where line stresses are high.

Where torsion moments are applied to a joint and no mechanical means for absorbing them are provided, these must be transmitted through friction between the gasket and the flange faces. Our study of representative ASA flanges was extended to cover torsion equal in magnitude to the bending moments investigated; the data are given in Table 8. A friction factor of $f = 0.20$ is assumed.

Neither flange can be considered adequate to withstand the torsional moments applied. While it is true that the assumptions used are extremes, the investigation clearly indicates the need for caution where torsion is concerned. This is even more true for gaskets other than metal rings with a lower degree of rigidity particularly such that suffer from abrasion.

Similar lines of reasoning can also be applied to shear effects and pipe thrusts, although it is believed that neither ordinarily will be significant.

Concerning the calculation of flanges under the conditions of creep and relaxation obtaining at elevated temperatures, the reader is referred to an interesting article by Bailey (10). Unfortunately, a large degree of uncertainty attaches to the creep constants and temperature assumptions which must be made, and present experience at high temperatures is insufficient to permit of correlating theory and practice. As far as superimposed stresses due to line expansion are concerned, Bailey suggests that these tend to diminish rapidly and need not be considered in connection with the ultimate steam tightness of the joint.

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Discussion

WILLIAM HOVGAARD.[4] This paper is very suggestive and affords a good basis for a discussion of the many difficult questions which arise in the design of pipe lines. The problem presented may be summarized as that of the determination of factors of safety for pipe stresses.

It is necessary to distinguish between the states of stress which exist locally and those which exist generally throughout the pipe. In the former class belong the longitudinal stresses denoted by f_L in the paper and stresses localized near the ends of the pipe. These stresses combine with hoop stresses and torsional stresses, creating a state of stress which should not be allowed to approach the condition of plasticity. If flow does occur locally, a permanent set is liable to occur, although the safety of the pipe may not be endangered. It is believed that this state of stress is best measured by the Hencky-Mises formula, given as Equation [5] in the paper. The expression on the left side of the equation does not represent an actually existing stress, although it has the dimensions of a stress. It is simply a function of the component stresses, so constructed that when it reaches the value of the yield point, flow of the material will occur. This function the writer has called the "equivalent stress" in various papers, and has denoted it by f_{eq}. It is suggested that the equivalent stress should not exceed one half of the stress at the yield point; that is, the factor of safety, in case of the stresses here under consideration, should be at least 2 relative to the yield point. It is found that then with all ordinary values of the circumferential or hoop stress, f_C, the longitudinal stress, f_L, can be from 1000 to 2000 lb per sq in. higher than the equivalent stress.

In the latter class of stresses, which are general over the entire pipe, belongs the hoop stress, f_C, caused by the steam pressures. It has a special significance, because it cannot be relieved by a change of form of the pipe or by a readjustment of the strains. When the pressure is associated with high temperatures, creep has to be considered and, due to our incomplete knowledge of this phenomenon, it is advisable to use a relatively high factor of safety. Hitherto, it appears that, in the case of the hoop stress, the factor of safety has been ordinarily related to the ultimate strength. This was quite satisfactory so long as the ratio between the ultimate and the yield strength was fairly constant, but in modern alloy steels, where this ratio may vary widely, it seems safer and more rational to relate this factor to the yield point, which determines incipient breakdown. It is of interest to note that Lloyd's Rules, for instance, prescribe a thickness of the pipe wall, which, including the additional increment of 0.12 in., gives a factor of safety of about 7 relative to a yield point at 500 F of 27,000 lb per sq in. This is obtained, assuming a steam pressure of 400 lb per sq in., a wall thickness of 0.319 in., and a hoop stress of 3750 lb per sq in. An analysis of other rules representing ordinary practice seems to indicate that the factor of safety of the hoop or "bursting" stress relative to the yield point should not fall below 6.

A. E. R. DE JONGE.[5] The authors should be highly commended

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for having undertaken the arduous task of outlining the problems encountered in determining the stresses in pipe lines. As their paper is intended to be a "vehicle for discussion" of these problems to supply material for a new piping code, it offers an occasion to stress the difficulties of this undertaking.

As the total stress can only be arrived at by a combination of the individual stresses, the latter have first to be considered separately before they can be combined.

With regard to internal pressure, the Lamé formula for thick-walled cylinders is probably the best to use as it gives results which agree fairly well with practical measurements. It might be said that an attempt should be made to obtain uniformity in all codes concerning pressure vessels with respect to the use of this formula.

Expansion stresses, on the other hand, provide a very different problem. First, the fundamental assumption made in their determination is that the ends of the pipe line are rigidly fixed. This assumption, however, is rarely if ever correct. It is therefore necessary to begin with the investigation of the rigidity of the various apparatus connected by pipe lines and to classify them with regard to the "degree of fixedness" assigning definite percentage values or percentage limits to them. For instance, a pressure vessel supported on a high gantry will offer but a slight degree of fixedness in the horizontal plane while it must be considered more or less rigid in the vertical direction. This investigation and classification has not only to be extended to linear displacements, but also to rotations.

However, even when disregarding all considerations of partly fixed ends, the problem of calculating stresses in pipe lines in space is by no means solved as yet with anything approaching accuracy. The theoretical values for the deflections and rotations in pipe bends, subjected to torsion, transverse bending, and transverse forces, do not check with the test results, differing in some cases by more than 50 per cent, the error being systematic, i.e., always in the same direction. Even straight pipe lines do not seem to follow the laws of ordinary beams any better. Thus, compound pipe lines in space exhibit quite different flexibilities from those usually assumed in the calculations, although it should be stated that these values could be allowed for in the calculations if they were known with any degree of certainty. Dead weight and local stresses at hangers, anchors, etc., are usually neglected and so are temperature stresses. In heavy pipe lines, however, deadweight should not be neglected. With all this uncertainty, it does not seem to be of much use to lay down fixed rules for maximum stresses.

Fortunately, pipe lines are generally more flexible than assumed, and this factor works in the direction of lower stresses. In addition, should overstressing occur, the usual materials for pipe lines exhibit the useful property of ductility and thus automatically lower the stresses in the pipe line by local yielding. The same holds with respect to creep. Prespringing is another uncertain factor, and so is the fact that load variations or load alternations occur. In combining the stresses obtained for these individual effects, it is clear that the end result will be even more uncertain.

Taking all these factors into consideration, the fixing of stress limits can only be regarded as a very rough approximation which of necessity must prove to be highly controversial. The actual approach to the subject obviously lies in investigating all of the different factors separately and more carefully than has hitherto been done, to fix for each one the limits within which theory and measured results agree, and then to compound these limit factors into a common limit factor. It is apparent that, for this purpose, experimental investigations on the flexibility of actual large-size pipe lines are required, and while such investigations have probably been made by individual firms, very little has been published regarding the results of such experiments. Only by coordinating theoretical calculations and experimental investigations is it possible to arrive at any data which may be regarded as approaching reality and thus be of value. By using such an approach to this complicated problem, the usual safety factor will no longer be a "factor of ignorance" in so far as pipe lines are concerned.

JOSEPH MARIN.[6] Considering the condition of static loading, the authors' Equation [5] includes the stresses due to bending, torsion, and the longitudinal and circumferential stresses due to pressure. It is of interest to study, in addition, the radial stress and to determine the influence of the various forces in producing failure. If the radial stress f_3 is included as shown in Fig. 8 of this

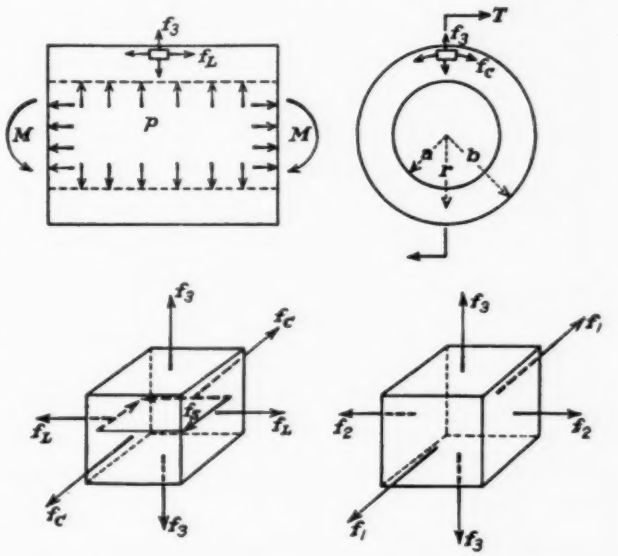


FIG. 8 THREE-DIMENSIONAL ASPECT OF STRESS PROBLEM WITH RADIAL STRESS INCLUDED IN ADDITION TO STRESSES DUE TO BENDING, TORSION, AND PRESSURE

discussion, the problem becomes a three-dimensional one and the distortion-energy theory of failure becomes

$$f_1^2 + f_2^2 + f_3^2 - f_1f_2 - f_2f_3 - f_1f_3 = f^2 \quad [11]$$

where f_1, f_2, and f_3 are the principal stresses.

Designating f_L, f_C, and f_S as the longitudinal stress, circumferential-pressure stress, and shear stress, respectively, the principal stresses f_1 and f_2 are

$$f_1 = \tfrac{1}{2}[f_L + f_C + \sqrt{(f_L - f_C)^2 + 4f_S^2}] \quad [13]$$

and

$$f_2 = \tfrac{1}{2}[f_L + f_C - \sqrt{(f_L - f_C)^2 + 4f_S^2}] \quad [14]$$

Placing values of f_1 and f_2 from Equations [13] and [14] in [12]

$$f_L^2 + f_C^2 - f_Lf_C + 3f_S^2 + f_3(f_3 - f_L - f_C) = f^2 \quad [15]$$

Referring to Fig. 8 the components of stress are

$$f_S = k_1Tr, \quad f_L = k_2p + 2k_1Mr, \quad f_C = k_2p\left(1 + \frac{b^2}{r^2}\right)$$

$$f_3 = k_2p\left(1 - \frac{b^2}{r^2}\right) \quad [16]$$

[6] Associate Professor of Civil Engineering, Armour Institute of Technology, Chicago, Ill. Mem. A.S.M.E.

where $k_1 = \dfrac{2}{\pi\,(b^4 - a^4)}$ and $k_2 = \left(\dfrac{a^2}{b^2 - a^2}\right)$ . . . . [17]

Substituting values of the stress components from Equation [16] in Equation [15]

$$3k_2^2p^2\frac{b^4}{a^4} + 4k_1^2M^2r^2 + 3k_1^2r^2T^2 = f^2 \text{ . . . . . [18]}$$

The left side of Equation [18] is proportional to the distortion energy for an element at a distance r from the center. The critical element at which failure occurs will be for the maximum value

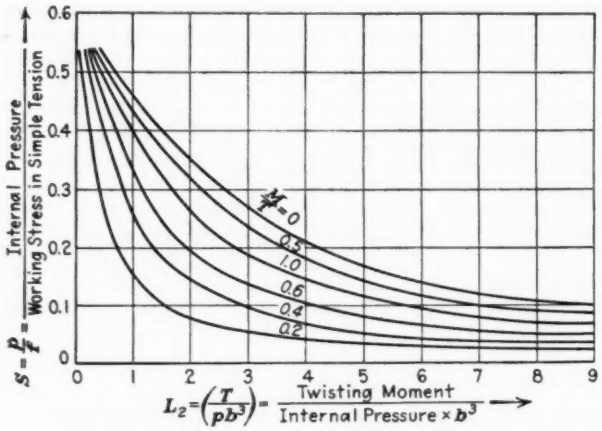


FIG. 9 VALUES OF LOAD RATIO S PLOTTED VERSUS LOAD RATIO L_2 FOR VARIOUS VALUES OF TWISTING MOMENT TO BENDING MOMENT AS A PARAMETER

of the distortion energy or for $r = a$. Then Equation [18] becomes

$$3k_2^2p^2\frac{b^4}{a^4} + 4k_1^2M^2b^2 + 3k_1^2b^2T^2 = f^2 \text{ . . . . . [19]}$$

Substituting values of k_1 and k_2 from Equation [17] in Equation [19]

$$\frac{3p^2}{(1-R^2)^2} + \frac{4(4M^2 + 3T^2)}{\pi^2b^6(1-R^4)^2} = f^2 \text{ . . . . . [20]}$$

where $R = \dfrac{a}{b}$.

Equation [20] gives a relation between the loads, dimensions, and the stress f in simple tension at failure. If the stress f is considered as the allowable stress, Equation [20] can be used for purposes of design. A design chart can be constructed by rewriting Equation [20] thus

$$\frac{3}{(1-R^2)^2} + L_2^2\frac{(4L_1^2 + 3)}{(1-R^4)^2} \times \frac{4}{\pi^2} = \frac{1}{S^2} \text{ . . . . . [21]}$$

where $L_2 = \dfrac{T}{pb^3}$, $L_1 = \dfrac{M}{T}$, and $S = \dfrac{p}{f}$

A plot of Equation [21] is shown in Fig. 9, in which values of the load ratio S are plotted versus the load ratio L_2 for various possible values of the twisting moment to the bending moment as a parameter. Fig. 9 is plotted for a value of $R = \dfrac{a}{b} = 0.25$. In this way the allowable loads can be determined for a given pipe or, by plotting Equation [21] for various values of R, the required size of piping can be selected.

ARTHUR MCCUTCHAN.[7] The authors of this paper have furnished a comprehensive analysis of stress problems involved in the design of pipe lines with a view to finding satisfactory bases for establishing allowable combined bending-plus-pressure stresses. Their lucid account of the factors involved should stimulate discussion of controversial issues and thus aid in reaching a consensus of opinion on how far a safety code such as the Code for Pressure Piping need go in setting limits on stresses other than girth stresses due to internal pressure.

The section on effect of "cold springing," or "prespringing" as the authors have termed it to distinguish from the similar effect resulting from "self-springing," is particularly valuable. Although those who have worked intimately with the subject of flexibility of piping fully recognize that a hot line will relieve itself of excessive bending stress through creep or yielding of the material, there appears to be a rather widespread misconception that creep tends to reduce rather than increase the initial cold spring usually provided in piping for high-temperature service.

The fact that an overstressed line at high temperature tends to relieve itself of any overburden of bending stress through creep or yielding of the component parts is responsible for the statement sometimes heard that: "The Lord takes care of flexibility experts." It is only when excessive bending moments result in leakage of bolted joints, or excessive thrusts or moments displace anchors or cause misalignment of equipment, that there is any untoward manifestation of lack of adequate flexibility. In recognition of the greater significance of stress related to high bending moments at bolted joints, the practice of the writer's company for the last 10 years has been to assign lower values of permissible stress at bolted joints than in curved pipe or straight pipe remote from joints.

In the case of creased bends, and corrugated bends and corrugated straight pipe, lower total combined pressure-plus-bending stresses also have been assigned, but for a different reason. The tests made by Lieutenant Robert Dennison (2), to which the authors refer, indicated rather high stress concentrations in such structures. Under cyclic stress, cracks eventually developed in the creases and corrugations at stresses corresponding to relatively small bending moments.

A study of the variable stress conditions existing in steam piping in central-station power plants, however, indicates that only in the case of a combination of a large constant bending stress and severe vibration of the line would it be possible to produce fatigue failure. Moreover, because of the self-unloading character of bending stress, arising from constraint of thermal elongation of a line, it appears doubtful if even this condition need be feared in practice.

For conditions encountered in power plants, the writer deduced the following stress-intensification factors from Lieutenant Dennison's tests (2) and Professor Moore's repeated pressure tests on boiler drums,[8] as applicable to creased bends and to corrugated pipe and bends:

Creased bends	2 to 2.5
Corrugated pipe and bends	2.5 to 3

These values for corrugated pipe and bends agree surprisingly well with the factor of 2.5 suggested by the authors for noncyclic loading. In the case of creased bends, the authors, in their effort to link such factors with flexibility factors, appear to have overlooked the fact that almost as great stress intensification was found in Lieutenant Dennison's tests of creased bends as in tests of corrugated bends.

[7] Engineering Division, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

[8] "Tests of the Resistance to Repeated Pressure of Forged, Riveted, and Welded Boiler Shells," by H. F. Moore, Trans. A.S.M.E., vol. 53, 1931, paper FSP-53-6, pp. 55–60.

The satisfactory behavior of creased and corrugated pipe under actual conditions of use may be explained on the basis of the self-relieving characteristic of bending stress induced by constraint of thermal elongation. This unloading automatically reduces the bending stresses in the convolutions to those which the material will support at the operating temperature. Since the endurance limit holds up better than the yield strength and the creep strength as the temperature is raised, it is doubtful if a high-temperature line can support enough steady stress for the superposed vibratory stresses to cause fatigue failure.

Since the amount of the unloading will appear as an addition to the cold spring originally provided, rather severe stresses may exist in the convolutions of creased and corrugated pipe in the cold condition. These bending stresses emphasize the desirability of eliminating locked-up stresses by annealing such pieces after fabrication. The number of these major stress cycles, caused by change from a cold to a hot condition in a main steam line of a central station, ordinarily does not exceed a few hundred in the life of a plant. Consequently, failure under such conditions is related to a property of material termed by Professor H. F. Moore as "crackless plasticity."[9] Material used in fabricating creased and corrugated pipe usually is capable of considerable plastic deformation without developing cracks. It can be concluded therefore that the stress-intensification factors given by the authors, with the exception of those for creased bends, will insure proper application of such structures. The writer would suggest a factor of 2 for creased bends under noncyclic stresses and 4 for cyclic stresses.

In the writer's opinion, the maximum-shear theory (4) should be used in combining stresses resulting from effects of bending and pressure rather than the principal-stress theory advocated by the authors, since yielding, not eventual fracture, is the condition of so-called failure which can be anticipated in a structure loaded by constraint of linear expansion. The maximum-shear theory possesses the further advantage of extreme simplicity.

In the absence of torsion, the combination of bending and pressure stresses to obtain the tensile stress equivalent to the maximum-shear stress reduces to the following simple expressions:

1 For side of pipe in tension

$$Seq = S_b + S_l$$

2 For side of pipe in compression

$$Seq = S_b - S_l + S_t$$

in which

Seq = tensile stress equivalent to maximum shear stress
S_b = stress caused by bending moment, tension on one side of pipe, compression on the other
S_l = longitudinal stress due to internal pressure
S_t = transverse or hoop stress due to internal pressure.

Fine distinctions as to which theory to use in combining stresses are not justified, however, in analyzing structures in more than one plane, the flexural characteristics of which can hardly be determined within an assured accuracy of 10 per cent. Furthermore, commercial variations in pipe-wall thickness within the permissible limit of $12^1/_2$ per cent from nominal dimensions may affect the flexibility of the pipe line a corresponding amount. The writer would point out that the paper does not attempt to cover the method of computing expansion stresses, moments, and reactions, since there are a number of available methods which give comparable results, at least for simple structures. Some structures, however, are extremely difficult or impossible to compute by methods approaching rigorous accuracy, and square-corner approximations or other simplified assumptions often have to be resorted to. If too specific flexibility requirements are written into the code, the writer begs leave to ask who can determine whether they have been met in complicated structures? In the case of branch lines, three-ended structures, and the like, the most that can be determined is that certain stresses will not be exceeded.

The idea of establishing a permissible range of total combined stress to limit the stresses in both the hot and cold conditions has the advantage that the numerical value of such stresses can be made the same as the values of allowable stress given in the code for determining pipe-wall thickness and yet be high enough not to interfere with good design from a flexibility standpoint. The definitions of S_A and S_A' given in the paper seem inadequate, however, in that they fail to specify that these terms relate to bursting stress as distinguished from bending stress, or that the so-called allowable stress refers to total combined stress.

The proposed setup allows a conservative designer to ignore, if he so desires, the ameliorating effect of cold spring, whether actually present or not, but still permits him to use a total combined stress of not to exceed 75 per cent of the sum of the allowable bursting stresses at room and at operating temperatures. On the other hand, it allows more latitude in case of need or for those who want it. For instance, at 900 F using carbon-molybdenum pipe, the 75 per cent rule for no cold spring would allow a total combined stress of 15,750 lb per sq in., whereas the 100 per cent rule for 50 per cent cold spring would allow 21,000 lb per sq in.

It is possible that in lieu of these detailed stresses the simple requirement that a line should be designed so that it could be cold sprung 100 per cent of the computed linear expansion without developing stresses exceeding $^2/_3$, or some such fraction, of the minimum specified room-temperature yield strength of the pipe material would be a satisfactory code statement as far as any actual hazard is involved.

The writer's observation of the disturbing effect of bending moments on bolted-flanged joints is at variance with the reassuring conclusions reached by the authors which are to the effect that ASA standard flanges, at least where used with ring joints, can withstand any bending moments commonly encountered. Actual experience with 1000 F experimental flanged joints has indicated that even small bending moments can cause leakage of an otherwise satisfactory ring joint. Calculations also showed the desirability of using 1500-lb alloy-steel flanges on 865-lb per sq in. 910-F service, assuming a $^3/_8$-in-wide metal gasket, even where the bending moment was limited to that corresponding to a total combined stress of only 10,000 lb per sq in. Hence, it would seem desirable that the authors support their conclusions with adequate explanation of how they are arrived at.

Although there are a number of assumptions, regarding initial bolt loadings, necessary residual gasket compressions, and probable residual bending moments existing in steam lines after years of service, to which exception might be taken, the writer feels that a discussion of these points might tend to obscure the real purpose of this symposium, which, as he understands it, concerns how far a safety code need go in prescribing limits for total combined stress in piping systems, together with methods for combining such stresses.

P. E. PENDLETON.[10] The paper is valuable in that it points out the controlling design features for high-pressure and high-temperature pipe lines. Among these are relaxation stress, a proper theory of failure, and the importance of high localized

[9] "An Investigation of the Fatigue of Metals—Series of 1925," by H. F. Moore and T. M. Jasper, University of Illinois, Engineering Experiment Station, Bulletin No. 152, 1925.

[10] Lieutenant-Commander, U. S. N., Bureau of Engineering, Navy Department, Washington, D. C.

stresses in systems subject to repeated loading, as well as their lack of importance under conditions of static loading.

However, no quantitative data are presented in regard to relaxation stress, i.e., at what temperature does it become the controlling factor for a specific material and what are the values of this stress at various temperatures. Accurate data on this question are difficult to obtain, although several laboratories in the United States are actively engaged in this problem.

The importance of keeping the range of stress of a piping system midway between the positive and negative yield strengths is timely. In this connection, it would be interesting to see a development made of the influence of locked-in stresses of a welded-pipe system and their effect in extending the stress range between cold and hot conditions.

At present, the Bureau of Engineering of the Navy Department calculates total combined or equivalent stress by the Hencky-Mises maximum-shear strain-energy theory, specifying a maximum of 15,000 lb per sq in. for this stress. A 50 per cent cold pull-up is required on all main steam lines, but no credit is given for this in calculating the stress in the pipe line when hot. It is thus unlikely that many regions exist in which any localized stresses greatly exceed the yield point of the pipe material. Consequently, there will be but little self-springing in pipe lines designed in this manner. On the other hand, the 50 per cent prespringing will subject the piping to a maximum negative, or cold stress of 8500 lb per sq in. Since the combined working stress of 15,000 lb per sq in. is less than the relaxation stress, the former will control the upper limit of the stress range. This range will then be 23,500 lb per sq in. (giving no credit for reduction in hot stresses) for Navy main steam piping, which is carbon steel for temperatures up to 650 F, and carbon-molybdenum, for temperatures ranging from 650 to 850 F. However, if credit be given for 50 per cent cold pull-up, the stress range will be 16,000 lb per sq in., which compares with a 19,200-lb per sq in. combined-stress range for carbon steel at 650 F as given in the paper. It would be interesting to know what upper limits of combined stress were used in compiling the allowable stress ranges given in Table 2.

Attention drawn by the authors to the limitations of the various theories of failure when applied to piping design suggests that further development of this subject is desirable, especially in the plastic range.

The use of simplified formulas for the determination of thrust and bending moment for particular pipe systems suggests that much could be done in this connection to facilitate design calculations.

The writer wishes to endorse the suggestion often made that the various research problems involved in high-pressure-temperature piping design be allocated by a combination of interested groups to laboratories and technical organizations best-fitted for their solution, so that duplication of effort may be avoided.

In the preparation of this discussion, the writer is indebted to R. Michel, marine engineer, Bureau of Engineering, Navy Department, for his able assistance.

E. L. Robinson.[11] During the last 10 years new installations of high-temperature steam piping have definitely gone beyond the temperature range throughout which elasticity is preserved, until at the present time there are approximately 2,000,000 kw of steam turbines in operation at temperatures where plastic deformation under stress definitely takes place. Furthermore, most new installations are being made at temperatures above that at which elasticity may be expected to persist without plastic effects.

Ten years ago the determination of the elastic reactions and the corresponding stresses due to thermal expansion constituted one of the principal problems in connection with the design of high-temperature piping installations. Most modern installations are being designed for operation at temperatures at which relaxation causes a decrease in elastic stresses as time goes on, rapidly at first and then more slowly. On this account, industry is confronted with problems of design for which there is little substantial background. The authors have made a valuable contribution toward the solution of these problems. They are careful to distinguish between yielding and rupture and, for the most part, their conclusions are so well qualified as to make it difficult to disagree with them.

In a few respects it is possible that the writer can contribute some useful observations.

The authors favor more than 50 per cent prespringing, but it does not seem to the writer that they emphasize the benefits of the greater prespringing sufficiently. The attitude is taken that, since relaxation results in self-springing, the long-time conditions will be the same. However, as they point out, this requires a greater amount of permanent set or creep in order to effect the relaxation.

The results of high-temperature rupture tests conducted during the last 2 years raise a very real question as to whether or not it is wise to ask too much of readjustment by means of creep. Creep, in itself, is beneficial, but when it leads to a brittle break, it does not seem wise to base design upon the expectation that it will occur without any deterioration of the material.

The authors do not discuss the invalidation of elastic analysis by the occurrence of plastic deformation. It is, perhaps, unwelcome to those who have devoted much high-grade analytical work to the determination of elastic stresses in high-temperature piping to realize that the reactions fall off and the stress distribution alters. On the other hand, it might be pointed out that, if the practice of prespringing to the extent of 100 per cent of the thermal expansion should be generally adopted, the elastic analysis would have reality when the piping is cold and strong and when the stresses are not combined with pressure. While at high temperature when the strength is reduced, existing stresses will be due to pressure only and not due to the reactions brought into play by the prevention of thermal expansion. The writer believes that prespringing should certainly be sufficient to assure extremely low rates of creep from the beginning.

Equation [9] of the paper is quoted from Moore and Kommers (7) to represent the fatigue strength where the stresses are not completely reversed. The tests given in the reference were sponsored by the writer's company and were first reported in bulletin form.[12] Originally, in presenting this formula Professor Moore made a number of qualifying remarks. In particular the formula fails as the stress ratio in the denominator approaches unity because the tensile strength is almost never 3 times the endurance limit for complete reversal, and a maximum stress beyond the tensile strength should not be implied as satisfactory. Where test results are not available, the writer would prefer a modified Goodman diagram based upon $^1/_2$ the tensile strength for complete reversal. However, an actual diagram of test results for the conditions of application would be better.

The authors repeatedly refer to the "relaxation limit." As a matter of fact there is no true relaxation limit any more than there is a true creep limit. It all depends upon the time. Thus at high temperature within a few hours all high stresses decrease rapidly to a level at which further decrease is slow. This is what is accomplished during a stress-relief anneal. On the other hand, if the same process is continued for 1000 hr, the gradual reduction of stress proceeds still further, and the residual stress is progres-

[11] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

[12] "An Investigation of the Fatigue of Metals—Series of 1922," by H. F. Moore and T. M. Jasper, University of Illinois, Engineering Experiment Station, Bulletin No. 136, 1923.

sively lower after 10,000 hours and again after 100,000 hours.

The authors quote Bailey's conclusion that the relaxation limit for 100,000 hr corresponds to the stress causing 0.1 per cent creep in that period. The writer doubts the wisdom of such a generalization. It is much better to have a relaxation test than to try to determine bolt performance from a constant-stress test. Several simple methods of conducting relaxation tests, as reported by four different companies, were described in a report of a Subcommittee of the A.S.M.E.-A.S.T.M. Joint Research Committee on the Effect of Temperature on the Properties of Metals made in 1938.[13]

The writer's company has run hundreds of relaxation tests on all varieties of high-temperature steels and prefers such tests for the prediction of bolt performance and uses them instead of constant-stress tests for preliminary evaluations of material for high-temperature service. In many such tests, the residual stress at 10,000 hr corresponds closely to the constant stress causing 0.1 per cent per 100,000 hr, although in so many cases the residual stress is far below this level that it is unwise to take the correspondence for granted. Almost without exception, the residual stress for 100,000 hr is far below the stress corresponding to a creep rate of 0.1 per cent per 100,000 hr.

The authors refer to the "applied stress" for alloy-steel bolting. It is always difficult for the writer to visualize the applied stress in a bolt. It is easier to think of the bolt as having a measured extension when initially assembled, determined by the length of the bolt, the number of threads per inch, and the number of flats the nut is turned in tightening up. Technically, a "strain" is applied rather than a stress, and even this is not easily measured with soft gaskets. The authors suggest applying a stress to produce 1 per cent creep in 100,000 hr. The writer believes that an initial strain, corresponding to a considerably higher elastic stress, is desirable to assure tightness even though relaxation soon reduces it. Bolts may well be taken up 0.1 or 0.15 per cent. However, the effective stress remaining in the bolt to keep the joint tight after a period of time must be determined from the relaxation properties of the material. This will be far less than the elastic stress corresponding to the inital setup, although it is not quite independent of it. A bolt tightened 2 mils per in. initially will be measurably tighter after 10,000 hr than a similar bolt first tightened 1 mil per in.

N. O. Smith-Peterson.[14] The problems involved in such calculations as are carried out in this paper are not simple tension or bending-stress problems to which there can be only a right or a wrong answer. They are complicated and involve many assumptions as well as a choice of different formulas for their solution. In presenting the solutions of these problems in complete detail, with formulas, assumptions, and calculated results, it becomes possible for others to compare their methods with those of the authors. The writer has done so and has obtained results differing somewhat from those listed in Table 5, pertaining to flange stresses. The difference in amount is not nominal and is due to difference in choice of formulas and assumptions made. It is the stated aim of the authors in publishing this very thorough paper to develop just such difference in methods of calculation, and the writer is hereby presenting his methods as they apply to some of the problems.

The writer uses the same formula for flange stresses as the authors, namely, the Waters, Rossheim, Wesstrom, and Williams formula, presented to this society in 1937. The writer's reason for using this formula, among the many flange formulas in existence, is the fact that it is the only formula which permits calculating stresses in the three axial directions, viz., hub, radial, and tangential flange stresses. However, before being able to use these formulas, it is necessary to make assumptions concerning the end-force area, ring contact area (or gasket area), and residual-contact-pressure ratio.

The authors consider the end-force area to extend to the ring pitch diameter, the ring contact area to be equal to $^1/_4$ the annular-ring area, and the residual contact pressure to be $5^1/_2$ to 1.

The writer follows the A.S.M.E. Boiler Code regulation in considering the end-force area to extend to the outside diameter of the ring. The contact area is accurately calculated according to the sloping surfaces of the ring, and the residual contact pressure is taken to be 6 to 1.

This difference in calculations gives approximately 10 per cent higher flange stresses for a 24-in. 1500-lb flange than those listed by the authors in Table 5, items 14, 15, and 16.

These differences are largely in assumptions in that the same flange formulas are used. However, a formula difference also exists between the authors and the writer when it comes to calculating the transmission of pipe-line bending moments by a bolted flanged pipe joint.

The authors consider such bending moments to be transmitted directly to the flange bolts and, hence, use the section modulus of the bolt arrangement in the joint to obtain the total preloading bolt stress required for maintenance of joint tightness.

The writer believes that no bending-moment action can be transmitted to the flange bolts until the contact pressure on the ring has been completely absorbed (leak condition) and that, therefore, for a tight joint, where the existence of a residual contact pressure on the ring is required, the bending moment will affect directly the ring-contact pressure and not the bolt stress; hence, the ring section modulus is involved in obtaining the total preloading bolt stress.

In explanation of this latter theory, which does not take into account elastic effects, consider the universally accepted manner of calculating the required preloading of a ring in order to insure joint tightness when an end force is exerted upon it. The universally accepted way is to assume the end force, a tensile force, to be transmitted directly to the ring contact surfaces and there to reduce the existing contact pressure. The end force is not considered as being transmitted to the flange bolts and, therefore, does not cause any change in the bolt stress. Similarly, a bending moment, which on one side exerts a tensile force, should not be conceived of on that side as affecting the bolt stress, but should be conceived of as here reducing the contact pressure. On the other side, the compression side of the bending moment, the contact pressure is, of course, increased. Therefore, to calculate the amount of such decrease and increase in the contact pressure, the section modulus of the pressure area is required to be used and not the section modulus of the bolt arrangement. After calculating the contact-pressure changes due to a bending moment, the calculation of the required bolt stress is then undertaken.

The writer's method leads to an 18 per cent higher flange stress than those listed under items 17, 18, and 19 of Table 5, for a 24-in. 1500-lb flange.

These percentage differences in calculated stress results are not nominal, and it is to be noted further that the basic flange formulas used by the authors and the writer are the same. Were there any differences in the choice of these formulas, the percentage differences in calculated results would be greater yet. The situation, therefore, seems to call for the adoption, by code-making authorities, of flange formulas and calculating methods in general, as well as allowable stresses. The authors' paper makes this also evident throughout and is an excellent exposition of the whole pipe-line problem.

[13] "The Resistance to Relaxation of Materials at High Temperatures," by E. L. Robinson, Trans. A.S.M.E., August, 1939, pp. 543–550.

[14] Walworth Company, New York, N. Y.

Authors' Closure

The authors acknowledge their debt of gratitude to the many discussers who, by their interesting comments, have materially added to the value of this paper. Considering the relative novelty of the subject dealt with, it is gratifying to note fair agreement on the basic conclusions and, consequently, the closure can be limited to the clarification of certain points brought out in the discussions.

While the discussion of the theory of calculating a pipe line for flexibility has purposely been excluded from this paper, Mr. de Jonge's reminder that much is still to be learned on this subject is not untimely. However, the assumptions usually made are on the side of safety and the available methods of calculation, if applied by competent engineers, are entirely satisfactory for engineering use.

The main issue, that of allowable stresses for pipe-bend calculations, has received wide comment. Professor Marin has contributed a study of the triaxial state of stress and its implications. The Hencky-Mises theory is advocated as a stress criterion by Professor Hovgaard, Lieut-Com. Pendleton, and, in the verbal discussion, by Mr. Soderberg, whereas Mr. McCutchan favors the maximum-shear theory. While it is conceded that a matter of principle is involved, either failure criterion appears adequate considering the present undeveloped state of our knowledge. As pointed out by Mr. Robinson, the authors have avoided more than passing comment on the invalidation of the elastic analysis by the occurrence of plastic deformation. They felt justified in slighting this aspect by the fear of confusing the issue which is to define workable rules for evaluating the flexibility of piping. Again, the redistribution of stresses due to creep and the phenomenon of relaxation are in favor of the structure, and an elastic analysis gives the limits of stress through which the material goes in each cycle of change from the elastic to the semiplastic and ultimately, in extreme cases only, the fully plastic state.

On the other hand, the authors gladly comply with Mr. Robinson's request to lend added emphasis to the injunction to use prespringing in excess of 50 per cent. Appreciation of the benefits obtained from cutting a line short is growing steadily, and we would assume that this will soon be general practice. The authors have used the following rough rule as a guide in setting the desirable amount of prespring s:

$$\frac{S_A/E}{S_A/E + S'_A/E'} > s < \frac{2S_A/E}{2S_A/E + S'_A/E'}$$

In order to understand the significance of this formula, it should be remembered that S_A, the bursting stress specified in the codes, corresponds to 40 to 50 per cent of the yield point cold; and S'_A either corresponds to the same percentage of the hot yield point or to from 80 to 100 per cent of the stress producing 1 per cent creep in 100,000 hr, which can be considered the virtual relaxation limit (in the same sense as Lieut. Dennison defines the "virtual" endurance limit). E and E' are the cold- and hot-elasticity moduli, respectively. Since the allowable stress range for a prespring s in excess of 0.5 is $S_A + S'_A$, the lower limit of s in the formula gives a stress of S_A cold and S'_A hot, or a range between, roughly, one half the cold yield point and one half the hot yield point or, for elevated temperatures, the relaxation limit; this state of stress will also be produced in an unsprung bend (at temperatures involving creep) after a period of operation producing complete relaxation. The upper limit of s, which should be approached in particular for high-temperature applications, places the average stress midway between the yield point at room temperature and the virtual relaxation limit, thus reducing the operating stress. The formula gives presprings between 50 and close to 100 per cent, depending upon the material strengths at room and operating temperatures.

Mr. McCutchan favors reduced stresses at bolted joints. While the authors agree that bolted joints should receive special attention, their experience is that a reduction in allowable stress is not generally necessary with ring-type-gasket joints. Actually, the stress specified at the bolted joint would govern the entire piping design, since maximum moments usually occur at the connections to equipment where bolted joints are normally located. Mr. Smith-Peterson's comments on the detail calculations of the flanges have been found interesting, and the authors agree that it would have been more proper to base their investigations of the effect of bending moments on the ring section modulus in place of the section modulus of the bolting.

Properties and Performance of Plastic Bearing Materials

By L. M. TICHVINSKY,[1] EAST PITTSBURGH, PA.

Bearings made of plastic materials can be used successfully not only for the case of perfect fluid lubrication but also for that of semifluid lubrication. Certain additions, such as graphite, will sometimes permit the application of these materials under conditions of dry friction.

By virtue of good physical properties these bearings find a wide application. Heavy-duty plastic bearings are used in the steel-mill industry. Lubricated and cooled with water, they carry heavy loads at pressures of several thousand pounds per square inch. As guide bearings their performance ranges from small, high-speed spindles, to large ship-propeller shafts. Oil-, water-, and grease-lubricated plastic bearings are used extensively in industrial, marine, and farm machinery.

There are many differences in the behavior of plastic and metal bearing materials by virtue of which the performance is also different. This article intends to point out the most important physical properties, as well as some of the characteristic performances of plastic bearing materials.

THERE are many reasons for the increase in the use and application of various kinds of bearing materials bonded with synthetic resin. The industry in this country became interested in these materials mainly because of some qualities superior for certain applications to those found in bearing metals and alloys. An extensive development and application of plastic bearing materials abroad was and is due primarily to the lack of basic bearing metals.

This article describes the bearing materials bonded with synthetic resin. Rather complete data are given on the laminated materials, including physical properties as well as the results of various bearing performance tests. This information was gathered from various domestic and foreign technical and scientific publications, as well as from testing in the author's laboratory.

Types of Synthetic Resin-Bonded Bearing Materials

All bearing materials bonded with synthetic resin can be divided into three types in accordance with their internal structure.

1 The bearings of the first type are made from various plain or graphitic molding powders. Numerous kinds of bearings for light and medium load applications are made from these materials. Guide bushings for high-speed spindles, or gland materials for sealing purposes, are typical examples of the use of this material; these bearings perform under all types of friction, namely, dry, boundary, and fluid. The application of these materials is rather limited because, as they are very brittle and fragile, impacts of even small magnitude may cause failure. Those made of graphite powder have a rather high heat-transfer coefficient.

2 The second type comprises the most important and widely used bearings. Such bearing material is obtained by molding at high pressure and temperature sheets of woven textile fabric treated with an organic binder. Synthetic resin of the phenol-formaldehyde type is usually used in the manufacture of laminated bearing materials. The molding is done in hydraulic presses (pressure varies from 1200 up to 2200 psi) at temperatures ranging from 120 to 180 C (1).[2] Under the combined action of heat and pressure the resin softens or melts and undergoes a further chemical reaction, resulting in a material which no longer is fusible. By this action the filler (cuttings, laminations, and the like) is permanently bound. The desired thickness for the final product determines the duration of this operation. It may last from 10 minutes to 30 hours.

Laminated bearing material is strong and tough. Bearings made of such materials perform efficiently when lubricated with water in which case the latter serves as lubricant and as a cooling agent. Almost any liquid (except strong alkalies) can be used, and although it might be recommended sometimes to add a lubricant, e.g., tallow or suitable emulsion, to the water, these bearings do not require oil or grease (2).

3 Bearing material of the third type is based on an internal felt-like structure. It is obtained by impregnating a felt of fibrous fillers consisting of cellulose fluff, linters, and similar materials. The resin is generally precipitated from a thin aqueous solution (sodium hydroxide) on and in the felt-forming fibers.

These materials range in their physical properties between the brittle powder moldings and the extremely tough and strong laminated materials.

Properties of Synthetic Resin-Bonded Materials

Laminated materials are the most important in bearing applications as already mentioned, therefore most of the information pertaining to the physical properties will be given in connection with these materials.

(*A*) *Modulus of Elasticity.* The modulus of elasticity, or Young's modulus, gives the relationship between stress and strain in the elastic region. It defines the amount of strain under load and, therefore, enables one to determine the deflection.

In connection with the materials in question, for a long time the value of the modulus of elasticity was found to be equal to 1.0×10^6 psi (2). However, methods were devised through which the use of artificial silk, cotton, and many other fibers resulted in the increase of the value of the modulus of elasticity up to 1.8×10^6 psi (3).

Many of the synthetic bonded materials were developed in connection with their extensive use by the electrical industry. Through a change in the molding pressure alone it is possible to attain a considerable increase in the modulus of elasticity. This is clearly shown in Table 1 (4).

TABLE 1

Molding pressure, psi.....	2300	4600	9200
Modulus in tension, psi....	0.72×10^6	0.85×10^6	1.03×10^6
Modulus in bending, psi...	0.78×10^6	0.91×10^6	1.14×10^6

According to Riechers (4) the values of the modulus of elasticity

[1] Research Laboratories, Westinghouse Electric & Manufacturing Co.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to Bibliography at end of paper.

for molded resinoid materials with various fillers are given in Table 2.

TABLE 2

Filler	Modulus, psi
Wood flour	$(0.8 - 1.17) \times 10^6$
Organic textile	$(1.32 - 2.35) \times 10^6$
Organic filler	$(1.32 - 2.2) \times 10^6$

In the case of Micarta[3] tubing where the fabric during rolling is maintained in tension, the modulus of elasticity reaches a value of about 3.0×10^6 psi.

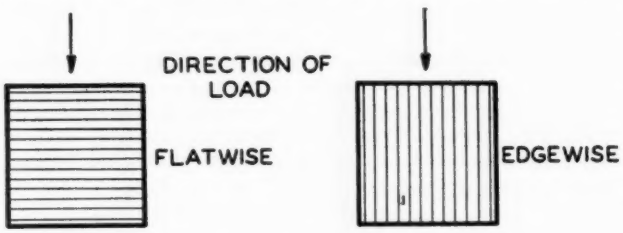
(*B*) *Compressive Strength.* For bearing application the compressive strength of these materials is important.

Compressive strength, as well as such properties as ultimate tensile strength and shear strength, can be increased by improved fabric filler as can be seen from Table 3 (3).

TABLE 3

Lb per sq in.	Ordinary fabric-filled phenol resin	Improved fabric filler
Compressive strength, psi	25,000	27,000
Ultimate tensile strength, psi	11,000	27,000
Shear strength, psi	4000–7000	6000–7000

[The compressive strength is usually given for laminated materials in two directions: right angle to laminae and parallel to laminae, as illustrated by the accompanying sketch.]



In Table 4 data on the compressive strength are given.

TABLE 4 COMPRESSIVE STRENGTH OF LAMINATED MATERIALS

Flatwise, psi	Edgewise, psi	According to
40–45,000	20–24,000	Eyssen (2)
45–50,000	30–35,000	Rochester (5)
30–45,000	.....	Smyth (6)

(*C*) *Heat Conductivity.* In this property all synthetic resin-bonded materials are inferior to white bearing metals. However, some improvements in this property have to be mentioned, especially those achieved during the last year. About a year ago one could say roughly that bearing plastic materials showed a heat-conductivity coefficient which is 100 times smaller than that of a tin-base white metal (0.845 w per sq in. per deg C per in. for babbitt). By reason of improvements made it can now be said that this ratio has decreased about one half.

The thermal conductivity of ordinary Micarta is taken as equal to 0.007 w per sq in. per deg C per in. Micarta for bearing application (I and II) has higher thermal conductivity as shown in Table 5.

TABLE 5

Bearing Micarta I, w per sq in. per deg C per in.	0.0099 –0.01
Bearing Micarta II, w per sq in. per deg C per in.	0.00925–0.01

Eyssen's (2) published data vary between 0.0045 and 0.0059 w per sq in. per deg C per in. for laminated products, and between 0.0077 and 0.0089 w per sq in. per deg C per in. for moldable materials based on felts of impregnated nonoriented fibers.

According to Richardson (7) the thermal conductivity of phenolic-plastic bearings is approximately 0.0072 w per sq in. per deg C per in.

Erk (8) gives a table of the thermal conductivity in a number of the "Kunstharz." In the phenol group there are some with thermal conductivity as high as 0.020, 0.021, and 0.023 w per sq in. per deg C per in.

In an endeavor to increase the thermal conductivity of Micarta and at the same time to improve friction qualities, a new method was developed for the manufacture of graphited Micarta. The thermal conductivity of this material was found to be equal to 0.015 w per sq in. per deg C per in. The heat conductivity of synthetic resin-bonded materials will never be of the same order as that for white bearing alloys. Certain developments in the methods of manufacture may still improve this property without impeding the others.

(*D*) *Other Properties.* The coefficient of expansion for the laminated product consisting of sheets of heavy square-woven duck bonded together by means of a synthetic resin of the phenol-formaldehyde type is 0.00002 per deg C per in. (6).

The specific gravity of the synthetic resin-bonded materials varies between 1.25–1.40. For general calculation it can be assumed that 1 cu in. weighs 0.05 lb. Oil absorption is negligible for practical purposes while water absorption was found to vary from 0.5–0.8 per cent when a test piece ($2 \times 2 \times 1/2$ in.) was submerged in water for 24 hr (2).

The Brinell hardness of synthetic resin-bonded materials was determined by Rochester (5) as 30–42, by Smyth (6) as 35–42.

Performance of Synthetic Resin-Bonded Bearings

The performance of all synthetic resin-bonded bearings is limited, as in the case of bearing metals and alloys, by the maximum allowable temperature of the bearing materials. The heat conductivity of the synthetic resin-bonded materials is lower than that of bearing metals. Therefore, such efficient cooling is required by which maximum amount of heat generated by friction can be removed by the circulating lubricant. An example of this is shown in Fig. 1. Such a bearing being lubricated and cooled with water performs at low speeds and medium loads. The sketch shows the arrangement of cooling and also the location of the high-pressure water inlet. This bearing is not operating in the boundary region at the starting and stopping periods because the high-pressure pump forces water into the bearing clearance.

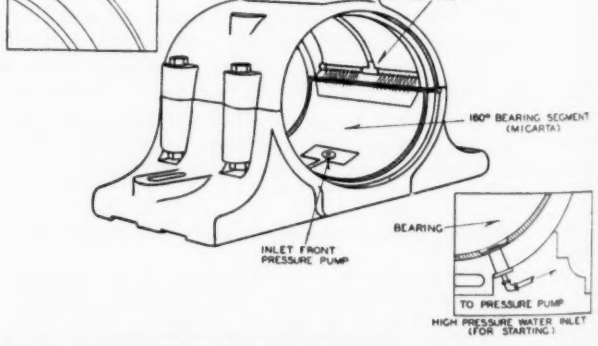


FIG. 1 TYPICAL WATER-LUBRICATED MICARTA BEARING FOR BALL AND ROD MILLS

[3] "Micarta" is a trade name for a plastic material. Other materials of the same general class are Textolite, Formica, etc.

TABLE 6 FINAL BEARING TEMPERATURE C AND COEFFICIENT OF FRICTION MEASURED AFTER 2-HR-20-MIN RUNS

(According to Heidebroek)

Clearance ratio, in. per in.	Bearing pressure = 22 psi — Peripheral velocity, fps				Bearing pressure = 155 psi — Peripheral velocity, fps			
	19.1	32.8	41.3	57.1	19.1	32.8	41.3	57.1
0.00153	55.0	65.0	71.0	....	55.0	67.0	75.0	....
	0.0183	0.0197	0.0206	....	0.0049	0.0049	0.0050	....
0.00175	51.5	60.5	66.5	78.0	55.0	65.5	74.5	85.0
	0.0206	0.0221	0.023	0.0226	0.0051	0.0051	0.0051	0.0051
0.0020	49.5	60.5	66.5	80.5	52.0	63.0	72.0	83.0
	0.0247	0.0267	0.0263	0.0278	0.0058	0.0060	0.0061	0.0062

The softening of the synthetic resin-bonded materials varies with type and grade of the bond. In this respect it is of interest to note some remarks by British authors pertaining to working temperatures of bearings. Smyth (6) indicated a safe working temperature of 100 C and an actual temperature of 150 C at which the material is unaffected. Rochester (5) recommends a maximum working temperature of 60–80 C. He mentions that the maximum allowable running temperature for short periods might reach 250 C. It is self-evident that high, localized temperatures might reach rather high values, not only in the region of dry or boundary friction, but even during perfect fluid lubrication (9). In Table 6 data are given for a series of tests (10) during which the final bearing temperature and the values of the coefficient of friction were measured as a function of peripheral velocity, clearance ratio, and pressures of 22 and 155 psi. The

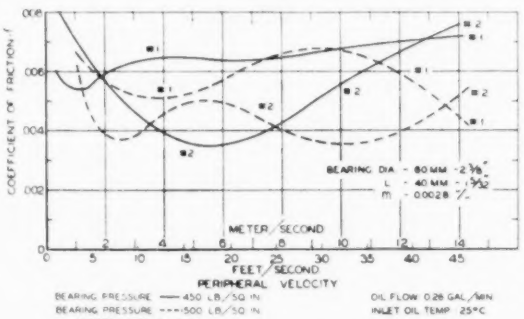


FIG. 2 COEFFICIENT OF FRICTION AS FUNCTION OF PERIPHERAL VELOCITY AT 450 AND 1500 PSI PRESSURE

bearing material was made of resin-impregnated cotton-cloth cuttings. A flow of light turbine oil varied between 0.238–0.715 gpm. The amount of circulating oil showed a tendency to increase with increasing clearance. Results of these tests indicate that the lower the bearing clearance ratio the higher is the running temperature, and the lower is the value of the coefficient of friction. For this reason it is advisable to provide the bearings with chamfered grooves, so that a large amount of lubricant may be admitted not only for lubricating purposes, but also for more efficient cooling. Another investigation was carried on during which the performance of various bearing materials was studied on a specially built test machine (11). The values of the coefficient of friction of two materials are plotted on Fig. 2 as a function of peripheral velocity for pressures of 450 and 1500 psi. The bearing bushings were $3^3/_8$ in. diam, $1^9/_{32}$ in. long, and were machined with a 0.0028 in. per in. clearance ratio. A light oil having 80 Saybolt sec at 100 F and 49 Saybolt sec at 150 F was fed into the bearing housing at 25 C and a rate of 0.26 gpm. The

TABLE 7

Material	Kind of resin	Resin, per cent	Structure	Molding time, min	Molding temp, C	Additional treatment
No. 1	Phenol	50	Cloth cuttings	45	165	10 hr at 80 C
No. 2	Phenol	45	Cloth cuttings	20	165	8 hr at 125 C

two materials tested showed approximately the same performance, the values of the coefficient of friction being slightly higher for material No. 1 than for material No. 2. In Table 7 some of the characteristics of these materials are given.

The performance of wound bushings impregnated by cresol is given in Figs. 3 and 4. The variation of the coefficient of friction with bearing

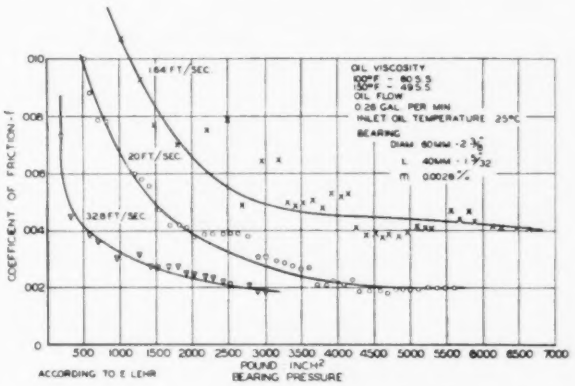


FIG. 3 VARIATION OF THE COEFFICIENT OF FRICTION UPON THE BEARING PRESSURE AND PERIPHERAL VELOCITY

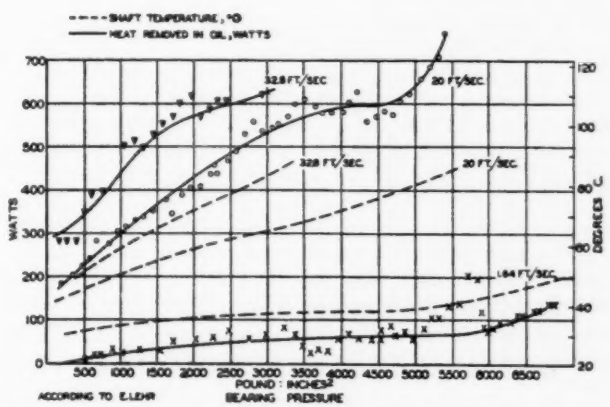


FIG. 4 VARIATION OF HEAT LOSSES AND SHAFT TEMPERATURE UPON THE BEARING PRESSURE AND PERIPHERAL VELOCITY

pressure and peripheral velocity is given in Fig. 3, while the variation of the heat losses and shaft temperature with pressure and velocity is given in Fig. 4.

These bearing tests were made at the same conditions as the tests of bearing materials given in Table 2. From Fig. 3 it is seen that the value of the coefficient of friction shows less variation at high loads at which it approaches some constant value. By diminishing the load, the values of the coefficient of friction rise rapidly. In the region of maximum pressures around 6000 psi the values of the friction coefficient lie between 0.002 and 0.005. More heat is being removed in the oil at higher peripheral velocities than at the lower, as can be seen in Fig. 4. It can also be noted that the slope of the shaft temperature is about the same as that of the heat losses. These shaft temperatures were recorded at steady-state conditions by means of a thermometer inserted in the journal hole specially drilled for this measurement.

All bearing surfaces must be well run in before optimum friction conditions are established. This is especially true for the case of bearings made of plastic material, where the running in

must be done carefully and skillfully. It is found (1) that the best scheme for running in plastic bearings consists in loading the bearing gradually at low speeds until full load is reached and then bringing the journal by steps to full speed. If the load be applied too quickly the excessive heat generated on the sliding surfaces might cause seizure. By frequent loading and unloading at various speeds the plastic bearings finally become insensitive to pressure increase as they become better fitted to the journal.

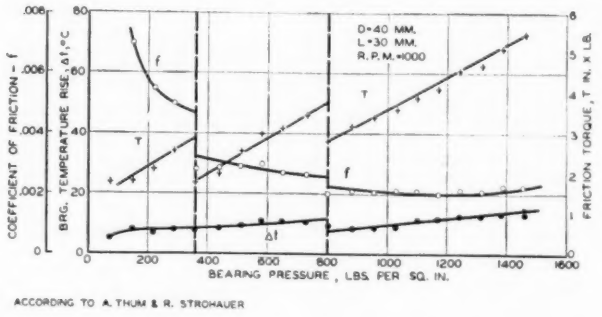


FIG. 5 RUNNING-IN PERFORMANCE OF A LAMINATED BEARING

The bearing can be considered well run in when the originally light-colored and finished areas disappear and form a light-colored, mirror-like, uniform surface. The bearing friction and temperature, which at the beginning were high, decrease considerably at the end of the running-in period. This period usually requires several days and can be substantially reduced by good machining of the sliding surfaces. Such running-in performance is very well represented graphically in Fig. 5. A bearing with textile laminae perpendicular to the axis, impregnated with cresol, was run in on a testing machine with forced-feed oil lubrication during 110 hr. The bearing pressure was increased in steps of 75 psi after steady-state conditions were reached. The friction torque and temperature increased gradually with the loading until at a pressure of 370 psi they rose suddenly and the journal speed dropped rapidly. After a few minutes, however, the torque and temperature readings decreased and their values slowly reached lower readings than previously recorded. By a further similar loading procedure the torque and temperature rose again as may be noticed in Fig. 5. At a pressure of 800 psi the torque and temperature reading went again off scale and the journal slowed down. After a few minutes the torque and temperature decreased again and rose slowly with increased load. The variation of the coefficient of friction may be seen also on the curve.

An article published recently on the performance of railway bearings (12) also points to the importance of running-in performance. This is done in connection with laminated textile material impregnated with cresol resin (65 per cent resin content). This running-in period is artificially accelerated by removing the upper layer of the bearing surface and polishing it by means of a special bushing having the same diameter as the bearing journal. As shown in Fig. 6, such bearing before the surface layer is removed performs with a value of the coefficient of friction which is in the boundary region. At a pressure of slightly over 400 psi the bearing seized. However, the same bearing after the upper bearing surface was removed and polished performed satisfactorily at much higher loads in the region of fluid film lubrication. These tests were carried on several bearings at peripheral velocities of 1.6 and 3.2 fps. Bearing temperatures are also plotted in Fig. 6. Optical measurements of bearing surface finish were made (13) on new and worn bearings; this, it is hoped, will help in further study and improvement of the performance of synthetic resin-bonded materials made for bearings. This is especially so since finish measurements of journal surfaces are now made (14, 15) and the performance of bearings can be elucidated also from this important point of view.

Fig. 7 indicates that the performance of a grease-lubricated Micarta bearing is approximately identical with that of an oil-lubricated babbitt bearing, both in the boundary region. These tests were made at various pressures and peripheral speeds while using a $2^1/_2$-in-diam bearing bushing 2 in. long. Fig. 8 shows the results when another type of Micarta bearing material lubricated with grease was tested at various loads and speeds. The friction properties of synthetic resin-bonded materials can be improved by various additions. The variation of the coef-

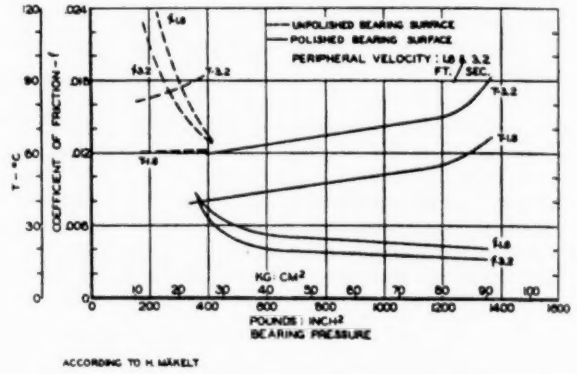


FIG. 6 BEARING PERFORMANCE OF RAILWAY BEARINGS
(Material: laminated textile sheets impregnated with cresol 65 per cent resin.)

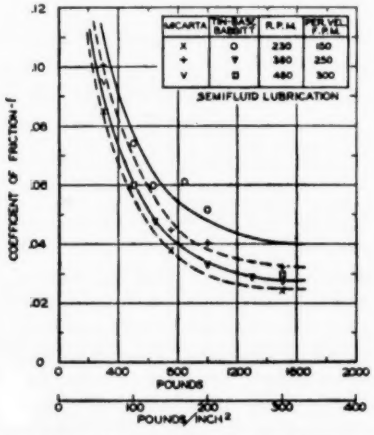


FIG. 7 COMPARISON OF FRICTION OF GREASE-LUBRICATED MICARTA AND OIL-LUBRICATED TIN-BASE BABBITT BEARINGS

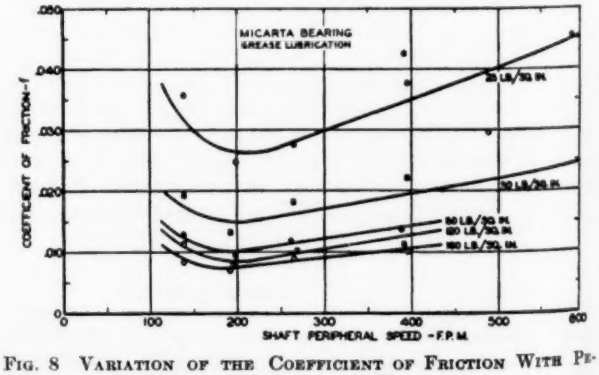


FIG. 8 VARIATION OF THE COEFFICIENT OF FRICTION WITH PERIPHERAL VELOCITY AND PRESSURE

ficient of friction depending upon such additions is shown in Fig. 9. These experiments were made on the machine previously described by the author (14). All bearings tested were run in for a few hours with a small amount of lubricant. Its supply was then shut off and the values of the coefficient of friction and of the temperature were recorded for different pressures. The standard Micarta, as can be seen, performs with a higher friction and temperature. Additions of mica and talc decrease both the friction and the temperature. A marked improvement is obtained when hard graphite is added to Micarta; the values of the coefficient of friction are low and so is the temperature, due to the improved heat conductivity as mentioned previously.

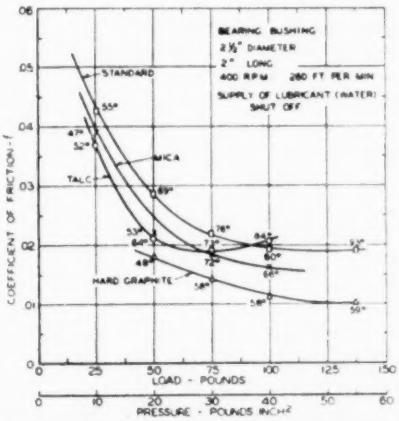


FIG. 9 INFLUENCE OF ADDITIONS TO SYNTHETIC-RESIN BEARING MATERIALS UPON THE VALUE OF THE COEFFICIENT OF FRICTION

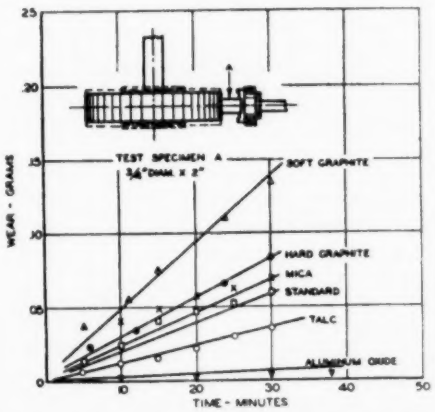


FIG. 10 INFLUENCE OF ADDITIONS UPON THE WEAR

The wear properties of the same materials were also investigated on the wear-testing machine previously described by the author (16). The principle of the machine is sketched in Fig. 10. The rotating test specimen A is pressed on the rotating and oscillating outside surface of a hard disk. A small jet of water is played upon the disk for removing the heat. The loss in weight of the specimen gives a comparative measure of wear. The addition of aluminum oxide greatly improves the wear qualities of Micarta. The rate of wear of the standard Micarta is approximately the same when hard graphite and mica are added to it.

The foregoing data and several performance examples, it is hoped, will assist a designer in his work when selecting and specifying synthetic resin-bonded bearings. Being made of a comparatively new material, the plastic bearings now find more and more application. The continuous development of the material itself and studies of its performance which are carried on by a combined force of chemists, physicists, and engineers will result undoubtedly in many more improvements and in wider application.

ACKNOWLEDGMENT

The writer thanks the Westinghouse Electric and Manufacturing Co. for permission to publish this paper. All the data of experiments performed by the author were made with the assistance of E. G. Fischer, of the Westinghouse Research Laboratories, and C. Jahing, graduate student.

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Discussion

CARL CLAUS.[4] The author mentions that appreciation of plastic bearing materials is not caused by lack of basic bearing materials but by advantages in certain applications. These seem to be confined to installations where it is possible to carry off frictional heat by water or oil which act as coolants and lubricants. There is little objection to this method in rolling mills where operators have long been accustomed to these conditions.

The author states that bearings made from graphitic molding powders show a better heat conductivity, but that they are ex-

[4] Research Engineer, Bound Brook Oil-Less Bearing Company, Bound Brook, N. J.

tremely brittle. A familiar bearing used for several decades consists of a metal shell with graphite inserts. This combination of the conventional metallic-sleeve bearing with the plastic graphite dissipates the heat satisfactorily without a cooling liquid. The metallic structure gives proper support to the graphite. A well-known application is the trolley bushing of streetcars for which no better bushing has as yet been found. The constant jars of the trolley pole do not loosen a well-bonded plastic graphite in this and other applications.

Another bearing somewhat related to plastic bearings is the porous-metal bearing. It is manufactured by the cold pressing of metal powders. Instead of using a resinous binder it is sintered and obtains a strength and ductility not approached by the resin-bonded bearing. Its heat conductivity is excellent and its porosity is used for storing lubricant in its structure.

By using hard graphite in the author's tests with Micarta a "marked improvement" was found in reducing the coefficient of friction and the operating temperature. It would be interesting if the terms "hard graphite" and "soft graphite" were explained, as well as the effect of the different kinds of graphite. Fig. 10 of the paper shows more wear for soft graphite than it does for hard graphite.

The condition of the shaft before and after the test is also of interest. Graphite has a tendency to fill up depressions of the shaft surface and make it smoother. Unless a pure graphite is used it may also act as a polishing agent which will rub off high spots.

Aluminum oxide is reported to have improved the wear qualities of Micarta in a test shown in the testing machine, Fig. 10. This material is an abrasive which may have smoothed the contacting surfaces. It was apparently firmly imbedded in the molded test specimen and may have acted as a lapping compound, which wears down the shaft or disk rather than being worn down by them.

An interesting observation can occasionally be made on oil-impregnated wooden bearings which may wear down a steel shaft when abrasive dust embeds itself into the wood. As a result of such a condition the soft-wood bearing shows no wear but the shaft does.

In conducting the tests the author has found it necessary to run in the bearing surfaces carefully for hours. The designing engineer quite naturally wonders what may happen to a bearing which he specifies and which will assuredly not receive such tender care. Since two bearings are mostly used to support a shaft, the question of misalignment will also have to be considered. Misalignment can easily result in failure at the start when excessive pressure prevails over a small area.

The difficulty with testing machines for bearings seems to be that they do not duplicate operating conditions. Results obtained from them cannot be applied to cases which vary from those of the individual testing machine itself. In the ideal sleeve bearing the load is supposed to be carried by an oil film, and a wearing down of bearing material and shaft should be impossible without contact between them.

As a result of the limitations of the bearing testing machines, most engineers reluctantly turn to comparative tests of bearing materials in commercial applications such as electric motors, automobile engines and their accessories, etc.

Performance tests under operating conditions rather than accelerated breakdown tests give us the only reliable answers to our bearing problems. Even then results are greatly influenced by numerous conditions outside of the bearing material itself.

The hardness and finish of the shaft, the degree of misalignment, and particularly the lubricant between the shaft and the bearing are often of even greater importance than the bearing material itself.

H. A. Everett.[5] It is an axiom in research work that wherever possible the actual points for observed data be shown when curves are presented. The author has been meticulous in this respect in all of the illustrations in his paper except Fig. 2. Will he be kind enough to let us have the observed data points for that figure, as there seems to be a surprising and gregarious change in the coefficient of friction with increasing speed.

E. F. Heer.[6] Limited investigations have been conducted on the graphites used in bearings. While experience thus far has not been sufficiently comprehensive to arrive at definite conclusions, the fact is definitely indicated that for each class of bearing there probably is at least one type out of the several varieties of graphite which is superior to others.

Following the author's division of the subject we shall first discuss bearings made from various plain or graphitic molding powders. Since it is the purpose of the graphite to supply a lubrication factor, it is important to have a complete distribution of the graphite throughout the molding powder in the ratio desired in order to supply a constant coefficient of friction. At the same time, it is equally important to make sure that the structural strength of the bearing is not weakened. This immediately brings up the consideration of—What effect, if any, does the particle size of the graphite have on the performance of the bearing in service and does the particle size influence the internal strength of the bearing? Further consideration must also be given the question as to whether or not the particle size of the graphite influences the molding characteristics of the bearing. What effect does loading have on the finished product?

It is conceivable that the internal strength of a plastic will be affected by the relation of binder to inert material. If such is the case, the embrittlement mentioned by the author may be attributed to overloading with graphite or other inerts (by inerts is meant any nonplastic ingredient).

It would seem to be equally important to study the performance of bearings formed from resin in which dry flake graphite has been used as opposed to bearings formed from resins which have been treated with colloidal graphite in either an aqueous or organic carrier.

As mentioned, some experience along these lines has already been had, sufficient to indicate that there is a relation between the particle size of the molding powder and the particle size of the graphite to be used, if admixture of the graphite and resulting friction properties are to be constant.

A study of the particle size of the molding powder examined under a low-power microscope indicated that the small particles are rod-like crystals about 20 mu in diam and from 80 to 100 mu in length. These are held together in agglomerates with a binder making up particles which range in size up to $^1/_8$ in. diam. It is the writer's opinion that, with molding-powder particles of this nature, it is almost imperative to have the graphite particles of suitable size to maintain proportionate distribution of graphite to produce a homogeneous finished product. If large particles of dry powdered graphite surround the finer particles of molding powder, the proportion of graphite to molding powder will probably be greater than is desired and a weakness in structural strength would probably result. On the other hand, if the largest of the particles of molding powder are to bear their complement of graphite in the form of a coating, it becomes necessary to use a form of graphite other than a dry powder, for obviously it is most difficult to maintain the proper proportion of dry powdered graph-

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ite adjacent to and in contact with these large particles of molding powder. Experience thus far has indicated that graphite, the range of particle size of which lies well below the ordinary fine-mesh powders, when dispersed in a suitable organic fluid, will give much better distribution of the graphite throughout the mass of molding powder and will enable the graphite to be attached to all resin particles regardless of size of the latter. The organic solvent can be disposed of by evaporation which leaves the molding powder in discrete particles not cemented together but at the same time properly coated with graphite.

With respect to the second and third types of bearings mentioned by the author, namely, the laminated type, we would again submit for consideration the selection of the graphite to be used for impregnating the textiles and the methods which should be adopted. A study should be made of the effect of overloading the textile fabrics with graphite as well as a study of impregnating the fabrics with a colloidal dispersion of graphite as compared with fabric which has been treated with powdered graphite. These studies, of course, would not be complete without a further study concerning the variations produced by treating the fabrics with aqueous and organic liquid dispersions of graphite.

Much that has been said with respect to molding powders of course applies to impregnating fabrics, inasmuch as particle size is of prime importance; also distribution of the graphite throughout the fabric to supply properties which will be constant. In dealing with fabrics, as is the case with molding powders, the wetting properties of the graphite dispersions are also of great importance. Some experience has already been had in connection with fabrics. It has been found that dispersions of graphite in aqueous as well as organic liquid dispersions are quite useful. Furthermore, a selection of particle size is important because there must be the proper relation between the particle size and the fibers used in the fabric, also the coarseness of the mesh of the woven fabric. The limited experience thus far indicates that, among the methods which can be used to impregnate the textiles with graphite, the one which appears to hold greatest promise is the immersion of the fabric in a dispersion of graphite in water or suitable organic solvent. The content can be closely controlled by using colloidal dispersions of graphite in these fluids for submerging the fabric for a given length of time. The graphite content together with the time involved in keeping the fabric submerged contributes to a homogeneous result and the proportion of graphite to the fabric can thereby be controlled and, of course, varied according to requirements.

From the paper it may be concluded that the initial run-in period is a predominating factor in the life of the bearing. If that is the case, it is apparent that a study should be made of the behavior of such bearings where water is used as the lubricant. Such study should determine the effect or what can be accomplished by introducing an aqueous lubricant, consisting of a colloidal graphite dispersed in water, into a circulating system for delivery to the bearing during this run-in period. This technique, in the opinion of the writer, should be valuable particularly in locations where the water used to lubricate the bearing is very high in mineral content.

In conclusion the writer should like to emphasize by repeating that experience thus far definitely points to the desirability of using graphite of fine particle size dispersed in suitable fluid carriers in order to produce homogeneous distribution of graphite throughout the bearings, thereby giving the bearings constant friction properties.

W. Trinks.[7] The writer has often wondered why composition bearings are called "plastics," when they are as hard and brittle as a piece of coal. Even less success has attended the determination of the origin of the word "Micarta." It seems to indicate that the bearing material contains, or at some time did contain, mica. Can the author supply any information on these questions?

A great deal of information is given in the paper on strength of Micarta in different directions. This brings to mind the question: Why is Micarta made in layers? The writer is familiar with at least one material in which the shredded cotton fibers are mixed with the uncured phenolformaldehyde and the mass cured as a solid block. If the block of composition is not sufficiently strong, a steel framework is placed in the raw composition and is cooked with it. This design has been successful not only for bearings but also for slippers of universal joints in rolling mills. These slippers act, in a way, like bearings.

Author's Closure

Regarding Mr. Claus's remarks, it was indicated in the paper that it is necessary to differentiate between plastic materials of the first and the other two types. The materials of the first type are made from various plain or graphitic molding powders, while the second and third types are obtained by molding laminated textiles and felt fillers. By virtue of their different internal structures, finished products made according to the three types mentioned have different physical properties, as shown in the paper. Material of type one might be brittle, but laminated bearings are very strong and tough. They are not brittle, and have high bond strength.

Hard and soft graphite Micarta simply designates the way in which graphite is put into the material. In the case of hard Micarta, the graphite is put on the cloth before the resin is applied, while in the case of soft graphite the graphite is put into the resin.

Mr. Heer's discussion is interesting since it indicates a few fine points pertaining to the properties of graphite. The amount of 325-mesh lubricating graphite powder used was evenly distributed in the cloth in such a quantity as not to decrease the bond strength.

Professor Everett correctly indicated the surprising change in the coefficient of friction at increasing speed as shown in Fig. 2 of the paper, taken without alterations from Mr. Lehr's paper.

As far as the writer was able to find a correct answer to Professor Trinks's question, the name "Micarta" (a trade name, used for laminated products only) derives its name from mica, which has a laminar structure and high dielectric strength, and which was originally added in a small quantity to Micarta.

[7] Professor of Mechanical Engineering, Carnegie Institute of Technology, Pittsburgh, Pa. Mem. A.S.M.E.

Neoprene as a Spring Material

By FELIX L. YERZLEY,[1] WILMINGTON, DEL.

In this paper the mechanical properties of neoprene which are important for vibration isolation and damping are evaluated and compared with the analogous properties of rubber. Static and dynamic moduli and damping action are specifically treated.

ADVANCES in engineering design proceed from the development of new materials, from increasing knowledge of available materials, and from the exercise of imagination in the creation of new applications. It must be evident that every increase in the variety of available materials diminishes the number of restrictions on mechanical invention. It must also be true that, as the variety of physical characteristics available in different materials increases, fewer compromises are necessary with imperfectly suited materials. Neoprene has taken its place at the side of rubber as an elastic material for springs. As an addition to the variety of materials available to engineers it has already increased the variety of structures that can be made from them.

The Field for Neoprene Springs

There can be no question of the superiority of metallic springs over those made of the rubber-like materials (1),[2] including rubber for certain applications, and there can be no doubt of the superiority of rubber-like materials in other cases. One needs only to survey the present applications of rubber-like materials in automobiles to verify the latter statement. Many improvements in riding comfort and performance became possible only as the availability and characteristics of rubber were appreciated. Further affirming evidence of the unique advantages of rubber may readily be found in the literature (2, 3) and in the prevailing trend toward further uses of rubber.

Analysis of spring applications reveals at least five outstanding reasons for the use of rubber springs. These are:

1 To reduce noise. The function of rubber in this respect is to avoid transmission of audiofrequencies through metallic supports (4, 5). It is interesting to note, however, that other steps must sometimes be taken simultaneously to eliminate sounds which are air-borne.

2 To impart flexibility. In addition to the technically more interesting applications, involving periodic vibrations which are subject to more or less precise mathematical treatment, there are numerous applications arising from the need for a mild cushioning effect, for example, as supplied by the numerous rubber pads and cushions used in automobile bodies (6). These provide the automobile structure with a necessary degree of over-all flexibility and elastic yield in an economical and simple way.

3 To reduce production cost. The trend in modern machine production is toward lighter construction and higher speeds. In general, this means also that precision in manufacture must be increased, usually at an increase in the cost of machining operations. Rubber in compression will conform readily to irregular surfaces and can, in some cases, be employed to eliminate a machining operation as well as maintenance expense. This is illustrated in Fig. 1. A self-aligning bearing is made by replacing a lubricated ball-and-socket construction with a less expensive assembly involving a rubber-like material. A spherically formed band of neoprene between the bearing and the block not only provides the necessary self-aligning feature, but does so with an appreciable saving in cost.

4 To reduce maintenance costs. Rubber springs frequently eliminate guides and other points of metal-to-metal contact, and thereby reduce maintenance costs (7).

5 To obtain definite damping action. When damping action is required with metallic springs, auxiliary frictional equipment is usually required. Leaf springs provide damping, but the extent of the damping action varies with the lubrication between the leaves and is not subject to practical control. The damping action of a rubber compound is definite and reasonably constant and is obtained without mechanical wear and without need of attention.

The foregoing considerations apply generally, but in varying degrees, to all of the rubber-like materials. In addition, the following distinctive advantages of neoprene over rubber are characteristic:

1 Relative immunity from deterioration in petroleum products. All petroleum solvents cause disintegration of rubber. Although

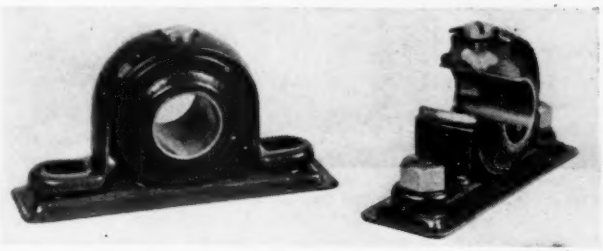


Fig. 1 A Self-Aligning Bearing Constructed With Neoprene Cushion

the rate of deterioration is dependent upon the nature of the petroleum product, the end results are similar. Penetration into the rubber first causes softening and swelling, followed by gradual loss of strength and eventual failure. Neoprene differs from natural rubber, first, by showing much greater resistance to the penetration of petroleum products and, second, by retaining its original physical properties to a much greater extent (8, 9).

2 Greater chemical stability at high temperatures. Rubber and neoprene, which are organic materials, cannot be exposed to the moderately high temperatures at which metallic springs frequently operate without undergoing chemical changes. The changes of rubber and neoprene on continued exposure to high temperatures are quite different. Continuous exposure to elevated temperatures of the best modern heat-resisting rubber compositions, which have low sulphur content, tends to cause reversion. Reverted rubber compositions are characterized by loss of resilience, elastic modulus, and tensile strength. Neoprene compositions do not revert at high temperatures, but gradually become harder with increased modulus and with relatively slow

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

loss of tensile strength. The residual plastic effect, drift, of the vulcanizate is decreased by continued heating, and the modulus increases rather than decreases.

3 Resistance to the action of sunlight and ozone. Sunlight and ozone are grouped together because formation of ozone at the surface of rubber in sunlight is considered the prime cause of the surface deterioration which occurs (10). In many industrial situations, the presence of ozone precludes the use of rubber. Neoprene, on the other hand, is used extensively as a jacketing material for high-voltage cables, where ozone is generated by corona discharge. It is also used in direct sunlight, where rubber does not possess the necessary degree of permanence.

4 Greater damping action. The hysteresis loss and hence the damping action of both rubber and neoprene may be varied considerably by compounding. In general, however, hysteresis loss, with stocks that are also practical in other respects, is greater for neoprene than for rubber. The thermal conductivity of neoprene is also higher than that of rubber and permits more rapid dissipation of heat.

The foregoing statements relate to the preliminary selection of a spring material. It is equally important to have definite knowledge of the various moduli of elasticity in order to determine the size and shape of the required spring. For that reason the balance of the paper will deal almost exclusively with the elastic properties of neoprene.

Elastic Properties of Neoprene

The author has recently described a compact mechanical oscillograph (11), Fig. 2, particularly for the evaluation of rubber-

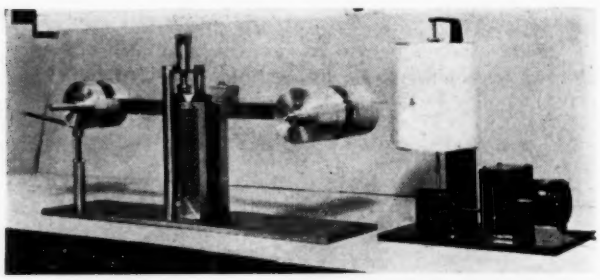


Fig. 2 The Oscillograph: Oscillating Bar and Loading Platform Are Mounted on Knife-Edges
(Machine is shown in static equilibrium position with test piece under compression.)

like materials in compression, both statically and dynamically. This apparatus has been used for the tests reported herein. Small cylindrical test pieces nominally $^3/_4$ in. diam $\times$ $^1/_2$ in. high are used. For static-load-compression relationships, deflections under various dead loads up to 280 lb per sq in. are recorded. For dynamic characteristics, the bar is allowed to oscillate at its natural frequency of vibration under the combined action of gravity and the elastic force of the test piece. The dynamic modulus is then calculated from the frequency and the inertia of the bar by means of the equation, $K = 210\,If^2$. Test frequencies are usually of the order of 2 to 5 cycles per sec. Data are recorded autographically. A typical oscillogram from this series of tests is shown in Fig. 3.

It is well known that rubber and neoprene do not show the same stiffness on the first cycle of loading and unloading as they do on successive cycles. Each of the test pieces was subjected to several conditioning cycles of loading and unloading under an average load of 280 lb per sq in. Static-load-compression data were then obtained, followed by further conditioning and a dynamic test under the average load corresponding to approximately 30 per cent compression. Data obtained in this way were plotted as shown for a rubber and a neoprene compound in Fig. 4. It will be observed, as Haushalter (12) has said, that the curves are approximately straight up to 20 per cent compression but that for greater deformations there is pronounced curvature. The slopes of the dotted lines, shown in Fig. 4, represent the dynamic moduli and were calculated from the dynamic tests under average loads of 120 and 100 lb per sq in. for the neoprene and rubber compounds, respectively. The crosses shown on the dotted lines indicate the points of equilibrium reached at the end of the dynamic test. The difference between the slopes of these lines and the tangents to the static load-compression curve illustrates the difference between the dynamic and static moduli.

It will be noted in Fig. 4, that the general shape of the curves for rubber and neoprene is the same. The neoprene loop is slightly wider, however, indicating greater hysteresis loss or damping action. The increase in modulus of neoprene in the dynamic test is greater than that of rubber. These differences are characteristic of the two materials today. Subsequent chemical developments may change this comparison.

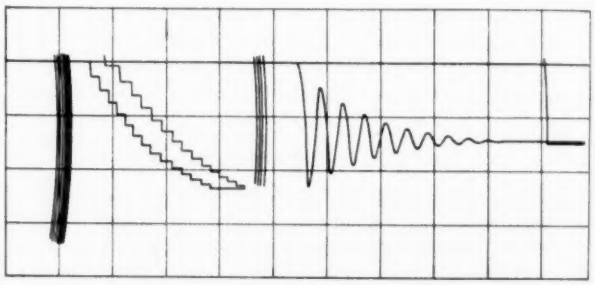


Fig. 3 Typical Oscillogram

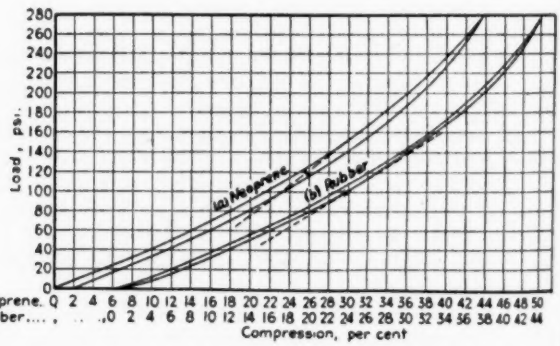


Fig. 4 Static Hysteresis Loops and Dynamic Moduli of Neoprene Test Cylinder and Rubber Test Cylinder in Compression

For the purposes of this paper, it is considered satisfactory to apply the term "compression modulus" to the slope at any point of the load-compression curve. It will be noted, however, that the values of the static modulus are obtained as slopes on the loading branch of the hysteresis loop. Although this is an arbitrary procedure, it appears to be common practice. For certain problems, the slope at points of the unloading curve might be more important. In other cases, an over-all average for successive cycles of loading and unloading is a more important value. Indeed the dynamic modulus reported in this paper is such a value.

During the last few years, correlations have been developed between the elastic moduli of rubber and values of the Shore durometer type A hardness. These correlations, though only approximate, provide a convenient way of determining the elastic moduli near zero load without resort to more extensive and some-

times expensive tests.[3] Conversely it provides the convenient method of specifying moduli by stating the hardness desired, particularly for springs in small quantities. The author intends in the following discussion to establish a similar relationship for neoprene with the hope, however, that some evaluation more precise than hardness data may win preference in the future.

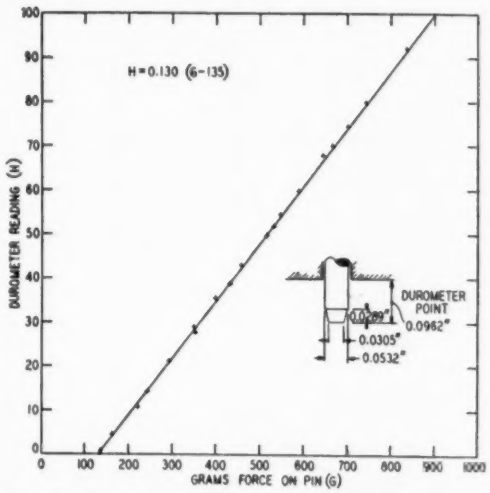


FIG. 5 CALIBRATION OF DUROMETER USED IN TESTS

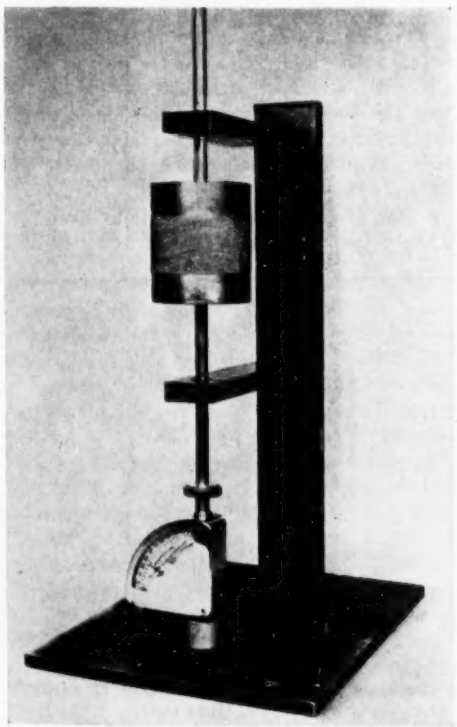


FIG. 6 SPECIAL DUROMETER STAND

There are three apparent reasons for unavoidable errors in correlations between durometer hardness and moduli. In the first place, the calibration of the durometer has not been adequately standardized by the manufacturer. As a consequence, different instruments will lead to different evaluations. In the second place, rubber technologists will readily agree that, unless operators are schooled with extreme care, different operators will obtain quite different durometer readings on the same compound and with the same durometer. In the third place, one cannot expect the same correlation between shear modulus and durometer readings for all types of rubber compositions. In appreciation of the fine work that has been done in establishing this correlation, expressed by Haushalter (12) and others, this criticism is offered only as constructive caution.

To remove doubt regarding the calibration of the durometer employed in this work, durometer readings were correlated with force in grams on the pin. These data are given in Fig. 5. The dimensions of the pin are also shown. Other dimensions are assumed to be standard. To eliminate the personal element as far as possible a stand, Fig. 6, was built. The instrument and the sliding rod together weigh 2 lb which then becomes the total actuating force (13).

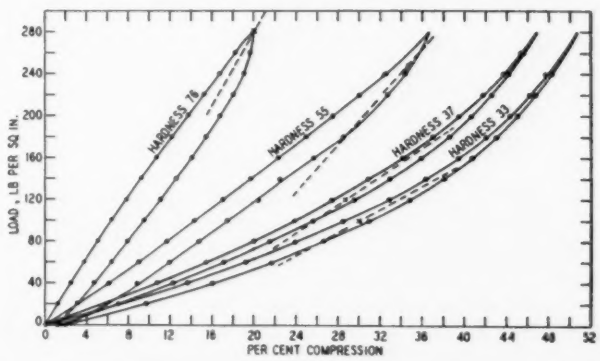


FIG. 7 LOAD-COMPRESSION DATA FOR GROUP OF NEOPRENE TYPE-G VULCANIZATES

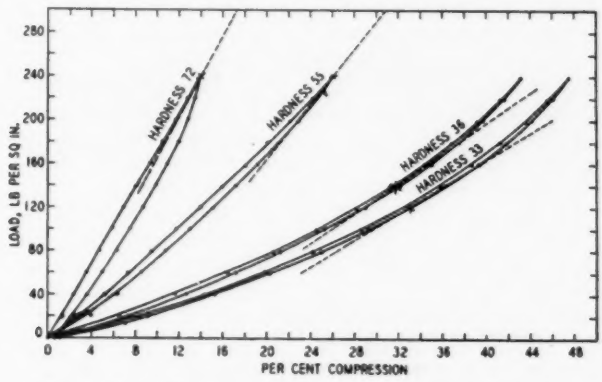


FIG. 8 LOAD-COMPRESSION DATA FOR GROUP OF RUBBER VULCANIZATES

In Fig. 7 are shown load-compression data of a group of neoprene type G compounds having different hardness numbers and, in Fig. 8, a similar group of rubber compounds. It is not feasible to present extensive data in curve form; condensation for the purposes of this paper as well as for practical use is required.

Data for neoprene and rubber compounds in compression are given in Table 1. It will be noted in the table that the static modulus at zero load is in most cases considerably lower than the static modulus under the load used for the dynamic test. As stated previously, the compression curve for rubber, up to 20 per cent, is practically a straight line. This is convenient, since

[3] The oscillograph is intended to provide a direct and inexpensive test for compression moduli and will soon be adapted to direct tests in shear.

TABLE 1 DATA FOR NEOPRENE AND RUBBER COMPOUNDS UNDER COMPRESSION

Compound	Durometer A hardness	Slope of static load-compression curve at zero load, psi	Average compression load in dynamic test, psi	Compression at equilibrium under stated average load, per cent	Slope of static load-compression curve at stated average load, psi	Dynamic modulus calculated from frequency and inertia, psi	Ratio of dynamic to static modulus	Resilience at 30 per cent compression, per cent	Frequency, cycles per min
Neoprene									
302N-42	30	320	100	31.0	440	590	1.34	77	145
						606	1.38	75	212
-41	31	266	100	29.6	440	585	1.33	77	145
						579	1.32	75	206
-40	33	366	100	30.0	440	580	1.32	78	144
						553	1.26	77	202
1522N- 1	33	316	100	28.4	480	542	1.13	78	139
- 3	37	380	120	28.6	550	675	1.23	80	151
-19	49	508	160	30.4	630	1017	1.61	55	185
- 4	52	620	180	30.8	680	982	1.44	65	187
- 5	53	720	200	30.4	740	1069	1.44	71	190
- 6	55	720	220	32.0	820	1142	1.39	68	202
- 7	57	692	200	31.8	770	984	1.28	60	182
-17	61	908	280	31.6	1120	1509	1.35	70	225
-20	62	800	220	31.0	760	1328	1.75	42	218
- 9	62	984	280	31.6	1120	1549	1.38	71	228
-10	62	944	280	32.2	960	1505	1.57	68	225
- 8	63	852	260	31.4	1060	1680	1.58	60	200
-12	73	1264	280	25.4	1360	2110	1.55	51	266
-11	76	1468	280	21.8	1360	1853	1.36	52	250
Rubber									
1333-30	25	150	80	31.4	376	413	1.10	91	126
-31	29	230	80	27.5	403	421	1.04	92	127
-32	33	260	120	33.0	556	614	1.10	92	154
-33	36	340	140	31.6	655	689	1.05	92	169
-34	46	530	200	29.8	872	963	1.10	89	193
-35	55	720	240	26.0	1100	1253	1.14	82	220
-36	64	1100	240	20.6	1352	1580	1.17	74	246
-37	72	1700	240	14.0	1750	1850	1.06	66	266

it may be used in many cases to simplify calculations appreciably. It is true, however, that as a general rule the tangent to the load-compression curve at zero deformation is lower than the tangent at 20 per cent compression. This trend continues and, at 30 per cent compression, a still greater slope exists. The values given in Table 1 for the compression modulus at zero load are the closest values that could be obtained graphically, disregarding any irregular curvatures that were shown by the data for small deformations. In some cases, they correspond to the constant slope of the curve up to 10 per cent compression and, in other cases, they correspond more closely to the particular slope at 10 per cent compression. It will be recognized that errors in obtaining a load-deformation curve for rubber may readily occur at zero because of the difficulty sometimes experienced in determining the zero deformation. This is particularly true of soft stocks which may undergo appreciable deformations with very light loads and it is also true for compositions which have excessive creep or pronounced damping characteristics.

The dynamic tests reported were conducted at 30 per cent average compression, which in terms of usual practice is somewhat abnormal. Various tests conducted by the author have indicated that the relation between static and dynamic moduli is practically independent of the load up to 30 per cent compression. It is also independent of the frequency obtainable by this technique as indicated by the uniformity of the ratio obtained at two frequencies for compounds 302N-40, -41, and -42 in Table 1.

It has frequently been claimed that rubber is considerably stiffer dynamically than it is in static loading. Circulation has even been given to the statement that the dynamic modulus of rubber is 2 or 3 times greater than the static modulus. As the result of his recent work, Kosten (14) has concluded that the ratio of the dynamic modulus to the static modulus lies, as a rule, between 1 and 2 and that the ratio is independent of frequency. The present work, employing a different technique of measurement and a somewhat different frequency range, confirms these conclusions drawn by Kosten. Naunton and Waring (15) on the other hand, have found in tests up to 60,000 cycles per min that the increase in modulus is proportional to frequency. They confirm the finding, however, that the ratio of dynamic to static modulus is between 1 and 2.

The ratio of the dynamic modulus to the static modulus in Table 1 is given in the eighth column and it will be seen that these are in the range 1.04 to 1.75. In the next to the last column, the resilience in per cent, or in other words the elastic efficiency of the compound, is given. These data were calculated from the dynamic records by obtaining the ratio of successive heights of rebound on the oscillograph. If resilience is plotted against the ratio of dynamic modulus to static modulus, as in Fig. 9, it will be noted that there is an increase in the ratio as the resilience of the compounds decreases. It should also be kept in mind, however, that the compounds given in this paper are of a commercial grade and that many complicated factors involved in the state of cure and physical composition have been introduced other than the obvious change in resilience.

Assuming that, under operating conditions likely to be encountered, the dynamic modulus can be as much as 2 times

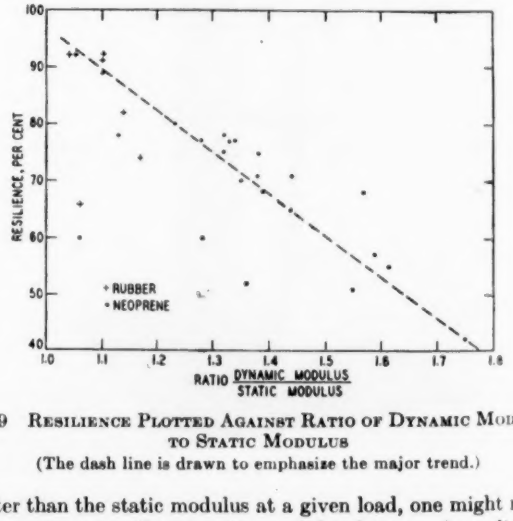


FIG. 9 RESILIENCE PLOTTED AGAINST RATIO OF DYNAMIC MODULUS TO STATIC MODULUS
(The dash line is drawn to emphasize the major trend.)

greater than the static modulus at a given load, one might naturally inquire as to the importance of the change. According to the usual theories of vibration, the frequency at resonance is given by the equation, f = constant $\sqrt{\text{spring constant}}$. In other words, a doubling of the modulus of rubber or neoprene will cause a 41 per cent increase in resonant frequency. This is not a negligible factor, but as this work and that of Kosten (14) indicate, a doubling of the modulus is not apt to occur. The actual change in

resonant frequency will be moderate and in most cases negligible. This is especially true, in view of the usual design practice of allowing a factor of 3 or 4, if possible, between the natural frequency of the supports and the probable frequency of impressed forces.

In his useful summary of the theory of vibration, Rosenzweig (16) has presented a discussion of the importance of damping at resonance. When critical speeds are encountered, some damping action or restraining force is absolutely essential. The data in the next to the last column of Table 1 give values for the resilience of the compounds. For the neoprene compounds selected, resilience ranges from 42 to 80 per cent and the corresponding energy absorption would consequently range from 58 to 20 per cent. The values for rubber are different, ranging from 66 to 92 per cent for resilience and from 34 to 8 per cent for energy absorption in a given cycle of loading and unloading. The art of compounding has not yet progressed to the point where resilience can be varied independently from all other properties, but there are ways of varying the resilience in a limited range for a given modulus.

In Fig. 10 durometer hardness is plotted against compression modulus at zero load as taken from Table 1. Other data of this same form have been published by Haushalter and are shown by the dotted line. This illustration confirms the relationship found by Haushalter (12) between compression modulus and durometer hardness as closely as one might expect. It will be noted, however, that there are points which do not lie on the curve.

Haushalter (12) and Kosten (14) both have emphasized the importance of the ratio between the height of a cylindrical test piece and its diameter. Haushalter has indicated that, if the ratio is approximately 1:1, the compression modulus is 6.5 times the shear modulus. In the present case, the ratio of the height to the diameter is 0.67. Consequently, it is of interest to determine how nearly the test pieces $^1/_2$ in. high and $^3/_4$ in. diam approach the characteristics of test pieces fulfilling the ratio requirement. Fig. 11 shows the changes caused in the load-compression curve by changes in the height of the cylinder from 0.135 in. to 0.995 in., i.e., by varying the ratio from 0.18 to 1.33. It can be seen that the difference in compression between the 0.502-in. and 0.995-in. cylinders is considerably less than 10 per cent of the percentage of compression at all loads.

In Fig. 12, the diameter of the cylinder is varied from 0.433 in. to 0.950 in., i.e., varying the ratio from 1.15 to 0.53. The discrepancy between the curves for the 0.742-in. and 0.433-in. cylinders is less than 10 per cent of the percentage of compression. If the ratio of 6.5 : 1 is valid for rubber, one might reasonably expect approximately the same ratio to apply for neoprene. Smith, on the other hand, has found the theoretically derived ratio of 3 : 1 (17) between compression modulus and shear modulus to be useful. Although the discrepancy between 6.5 and 3 appears offhand to be disturbingly large, further investigation will probably unify the methods of treatment and contribute better understanding. The measurements in shear so far made by the author have been limited and inadequate to establish a clear relationship between the shear and compression moduli. It is known from several comparisons, however, that the shear modulus of neoprene can be varied over the same range as that of rubber; for example, for practical purposes from 50 to 350 lb per sq in.

Whenever damping action is important, due to the unavoidable occurrence of critical speeds, it will be recognized that mountings will be called upon to absorb substantial quantities of energy. The direct result of this absorption of energy will be an increase in the temperature of the mounting. The specific heat of uncompounded and unvulcanized neoprene is 0.52 cal per g and its thermal conductivity 0.00046 cal per sec per cu cm per deg C. Vulcanization seems to have little, if any, effect on the thermal properties of neoprene, although loading ingredients will alter thermal values of vulcanizates in amounts proportional to the volume content and properties of the added ingredients. Since the specific gravity of neoprene compounds is relatively low, being as a rule between 1 and 2, the thermal capacity of neoprene on the volume basis is low. The temperature rise on absorption of energy is, therefore, rapid. In comparison with rubber, however, the thermal conductivity is about 30 per cent higher. Neoprene is, therefore, a better damping material than rubber from the standpoint of its damping action, its chemical stability when

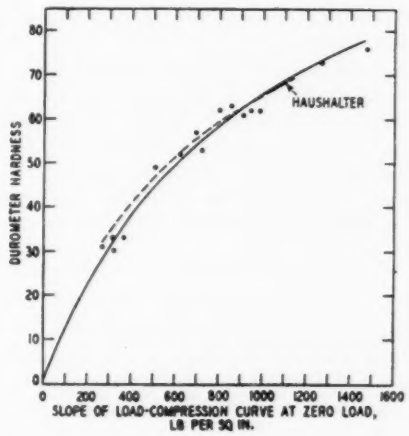


FIG. 10 DUROMETER A HARDNESS PLOTTED AGAINST SLOPE OF LOAD-COMPRESSION CURVE AT ZERO LOAD
(The average obtained by Haushalter for similar data is shown by the dash line.)

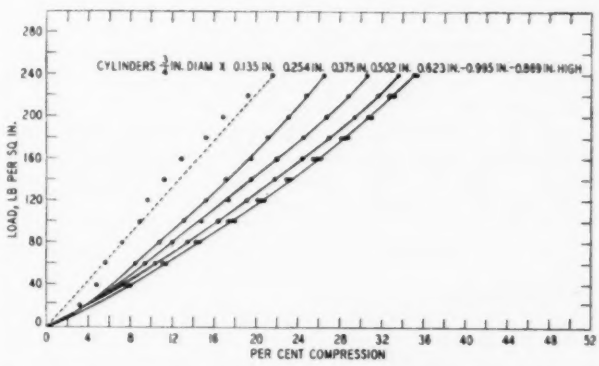


FIG. 11 EFFECT ON LOAD-COMPRESSION CURVE OF TEST CYLINDERS ¾ IN. DIAM, CAUSED BY VARYING HEIGHT

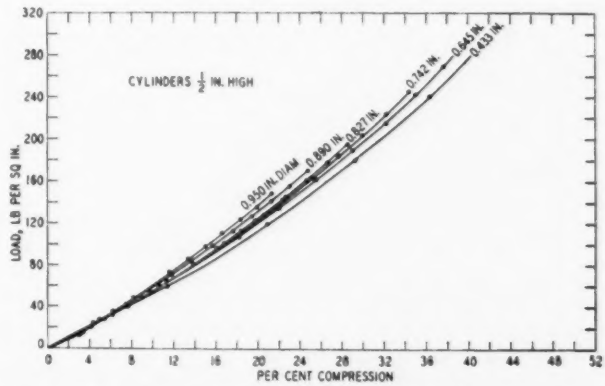


FIG. 12 EFFECT ON LOAD-COMPRESSION CURVE OF TEST CYLINDERS ½ IN. HIGH, CAUSED BY VARYING DIAMETER

heated, and its higher thermal conductivity, permitting more rapid dissipation of heat.

It has been shown that the elastic characteristics of neoprene in which design engineers are interested are similar to those of rubber. Greater damping action and a somewhat higher value for the ratio of dynamic to static moduli are, however, characteristic. The similarity permits the design and application of neoprene springs by the same methods developed for rubber by many investigators (2, 3, 7, 12, 16, 17).

Electrical Properties of Neoprene

Although not related to the main subject of this paper, brief mention of the electrical properties of neoprene is warranted by the occasional demand for combined electrical and mechanical service. The values given in Table 2 are roughly representative of neoprene compounds which contain no fillers and only enough added ingredients to insure proper vulcanization.

TABLE 2 ELECTRICAL PROPERTIES OF NEOPRENE AT 28 C

D-c resistivity	1×10^{12} ohm-cm
Dielectric constant at 1000 cycles per sec	7.5
Power factor at 1000 cycles per sec	3.0 per cent
Dielectric strength, rms at 60 cycles per sec	800 v per mil

Individual compositions will, of course, have different electrical properties. The d-c resistivity, the dielectric constant, and the power factor can be improved somewhat over the figures given in Table 2 by compounding. Compositions containing high proportions of carbon black have lower values of d-c resistivity and dielectric strength than those given in Table 2, and higher values of the dielectric constant and power factor.

Adhesion of Neoprene to Metal

Neoprene compounds can be adhered firmly to brass and brass-plated metals by vulcanization in direct contact with the brass surface. Other commercial methods employ cements to obtain equally strong adhesion. Bond strengths above 1300 lb per sq in. in tension have been obtained under the most favorable conditions.

Performance

Under normal operating conditions, the permanence of neoprene mountings is of a high order. They are not damaged by millions of cycles of loading and unloading and they resist deterioration from oils, chemicals, heat, sunlight, and oxidation. Abnormal conditions may be encountered, however, if mountings are installed or operated incorrectly so that excessive temperature rise occurs in the neoprene as the result of overstressing or excessive frequency of operation. Although no general statement will apply to all conditions, a temperature of 65 C is a reasonable maximum. If operation at higher temperatures is unavoidable, a commensurate reduction should be made in the unit load on the mounting to compensate for loss in spring strength resulting from the abnormal temperature condition.

The functional purpose of neoprene mountings is to isolate and dampen vibrations. The structural requirement is that the mounting shall safely support a given load. If the mounting is operated continuously at excessively elevated temperatures, it is inevitable that the modulus of the composition will gradually increase. The effect of this change will be to alter the characteristics of the mounting, as a vibration isolator, without impairing the ability of the mounting to carry the required load. Hence, although the functional purpose of the mounting may be impaired, the structural safety of the installation ordinarily remains unchanged.

At low temperatures, neoprene temporarily acquires an abnormal hardness. The rate of hardening depends primarily upon the temperature. Preliminary tests, at a temperature of 40 F, have indicated that the modulus of neoprene type-E compositions may increase by a factor of 2 or 3 over a period of 5 days. Furthermore, the change will not occur if the mounting is in normal operation, since the mechanical working of the compound will prevent freezing. If a neoprene mounting has been stiffened by freezing, tests have proved that the mounting will recover its original characteristics quickly upon resumption of operation. It is believed that the rate of freezing increases slightly as the temperature decreases to a value of about 20 F, and that below this temperature the rate of freezing decreases. A study of these points is in progress and more information will be available in the future.

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Discussion

B. S. Cain.[4] It would be helpful if the author could furnish some of the characteristics of neoprene at low temperatures. In particular, is material available for use where some flexibility is required over a wide range of temperatures, including a sub-zero range?

Stuart H. Hahn.[5] This is another excellent presentation of reasons why neoprene can be used for the construction of rubber

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[5] Physical Research Laboratory, The B. F. Goodrich Company, Akron, Ohio. Mem. A.S.M.E.

springs. This material, as well as several other modern synthetic rubber-like materials, has certain desirable properties for this service.

The value of the oil resistance and good aging properties of neoprene is well known and appreciated by rubber technologists. However, the admitted tendency of the material to harden continuously upon exposure to elevated temperatures must be balanced against its age resistance at elevated temperatures when spring service is to be considered.

Many designers of rubber springs will disagree with the author's thesis that damping characteristics greater than those of normal rubber compounds are desirable, particularly since high damping usually is accompanied by increased creep. It is, unfortunately, true that low creep is rarely, if ever, a characteristic of compounds of rubber and other rubber-like substances which have low resilience (high damping). Available information on the creep-resilience relationship for neoprene is even less complete than that for rubber compounds. It is hoped that the author will soon be able to fill the need for such information.

The addition to the literature of data on the dynamic modulus of neoprene and other compounds is welcome. It is unfortunate that the author has not been able to extend his investigations above a frequency of about 4 cycles per sec, which is below the ordinary range of resonance frequencies for the more common rubber-spring designs. The modulus effective in determining the actual working transmissibility of such springs is the modulus at the impressed or forcing frequency, which is usually from 2.5 to 10 times the resonance frequency.

It is interesting to note that the data points, in the author's Fig. 9 which shows the resilience versus dynamic/static-modulus ratio for neoprene and rubber compounds, appear to be grouped distinctly into two categories which seem to distinguish the two materials. Although the rubber data are too limited to be conclusive, a curve giving a good fit to the available points would appear to intercept the 40 per cent resilience axis at a value much lower than the intercept for the line shown, which fits the neoprene data rather well.

One most important property of rubber-like materials for spring service is the safe and economical stress to which they may be subjected in service. The working stresses for compressive use are nearly always dictated by creep considerations so that deflections are ordinarily limited to values of about 20 per cent or less. Failures due to actual fracture or cracking of the rubber are quite rare in such service. The determination of working stress for service in shear is not nearly so simple. Some applications which have proved practical in service have made use of deformations somewhat more than the thickness of the rubber (shear strain $>$ 100 per cent thickness) and static or mean dynamic stresses in the region of 100 lb per sq in. are known to be practical in certain limited classes of use, especially where compressive strain perpendicular to shear adhesion areas is imposed. However, in no case does the actual breaking shear stress in continuous service approach the breaking stress as determined by short-time loading tests. It is important to the designer, who contemplates the use of a material such as neoprene in spring service, to know what safe working loads may be applied to the material if service over a given period at approximately known temperatures is to be rendered without failure. The effect of temperature increase in reducing the strength of neoprene and rubber cannot be ignored.

The author's experience with poor standardization of the durometer (type A) agrees with that of many others. His description of the calibration of his instrument adds to the value of his hardness-versus-modulus curves, even though there are theoretical objections to the method of calibration which he used. It also emphasizes the precautions which must be taken to avoid or compensate for the well-known shortcomings of the instrument. Tests and theoretical studies by another member of this laboratory have indicated very clearly that the most reliable results may be obtained only when the durometer is held in such a way that the lightest possible pressure is obtained between the rubber surface and the foot or disk surrounding the indenter point.

The author seems to be guilty of overemphasis in saying that the slightly higher thermal conductivity of neoprene compounds is sufficiently great to constitute an appreciable advantage of this synthetic over natural rubber. The thermal conductivities of neoprene and rubber compounds suitable for spring service are all so low that any appreciable energy absorption produces sizable temperature rises in both classes of materials. The conductivity of the softer rubber-spring stocks is about 0.95 Btu/(hr) (sq ft) (F/in.) or roughly $^1/_{300}$ that of steel.

An estimate of the temperature which might be produced in mountings made of a neoprene compound and used under conditions of moderately severe vibration has been made for the following hypothetical case:

Machine weight, 2000 lb
Shear stress on mountings, 40 lb per sq in.
Static deflection of mountings, 1 in.
Resilience of neoprene compound used in mountings, 60 per cent
Magnitude of simple harmonic disturbing force, $\pm$20 lb
Frequency of disturbing force, resonant (approximately 190 cycles per min)

For this particular case, the mean rate of temperature rise works out to something like 3 F per min and, since the mountings would necessarily have a thickness of 1 in. or more, the maximum temperature in each mounting would probably attain a value of about 200 F after starting at 80 F and running for about 1 hr, if failure of the material did not occur before that, through the combination of the weakening effects of temperature and the gradually increasing amplitude which would result as the damping coefficient decreased with increasing temperature. Hence, intentional operation at resonant frequency would be impractical, since the comparatively higher conductivity of neoprene stocks is still too low to permit their extensive use as shock absorbers.

In some cases there has been a real need for a cool-running rubber or synthetic compound for use in services where occasional or even rather frequent shocks or dynamic-stress cycles are encountered. However, the solution of the problem in most cases, and probably in every case, has been the use of a compound which possesses relatively high resilience with ordinary values of thermal conductivity, rather than low resilience accompanied by the maximum possible conductivity. It should be emphasized that thermal conductivities of all rubber-like materials are so low that they are unable to dissipate very much energy continuously except in the case of relatively small pieces.

F. L. Haushalter.[6] The writer expected to learn from this paper how high neoprene might be stressed as compared with rubber, without experiencing trouble from creep or drift. If neoprene springs are used to support a machine, what would be a reasonable estimate as to how much lower the machine would be setting in a year's time? Also, what would be the modulus or rate of the spring in a year and, possibly, in 3 or 10 years hence? If the springs are to be used in an atmosphere of elevated temperature, data on creep would be essential before choosing the stress at which the spring will function. Values of dynamic modulus versus static modulus for this material are useful enough, but what are these moduli after creep has taken place at elevated temperatures? We know that for rubber the values are appreci-

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ably higher, and it is reasonable to suppose that for neoprene the values would be still higher because it is the writer's experience that neoprene compounds have greater creep than equivalent rubber compounds.

The more work the writer does with springs made from rubber, or rubber-like materials, the more he is convinced that damping is an unwanted characteristic in the material. About the only place one would require a spring to operate at resonance would be to counteract and neutralize the vibration of a body or member, the critical period of which was at that same frequency. Such applications are rare. About the only instance of this kind which the writer can recall occurred several years ago. In this case, the front wheels of an automobile developed a critical period causing bad wheel shimmy at a certain operating speed when traveling on cobblestone pavement. This vibration, which was in the horizontal plane, was neutralized by attaching a mass on the end of a spring, which in turn was fastened to the tie rod. The spring was designed to give the suspended mass the same resonant frequency as the wheels in the horizontal plane.

In the vast majority of cases, it would certainly be ridiculous to want to isolate a vibrating body with springs and then have the springs function at their point of resonance. If we wish to do any damping in passing through critical speeds, it can be done much more cheaply by using hydraulic shock absorbers. If we do not operate springs in the resonant range, which Hull pointed out should not be less than a frequency ratio of 1.4, damping is a distinct disadvantage, for the energy consumed in damping increases the transmissibility through the mounting.

Furthermore, the more damping there is in the material, the more creep will take place in the spring. The author has a good opportunity to disprove this statement, but if Isaac Newton was right, it is very probable that, as compounds made from neoprene G are developed, which take on less and less creep, we shall find that the hysteresis loss, or damping effect, will diminish accordingly.

As for the argument on durometer and hardness, which seems to be an endless one, as unscientific as this instrument appears to be, the writer dislikes the thought of condemning it to the scrap heap, for when properly used it certainly can pick out small differences in hardnesses of stocks in a comparative way. For the detection of these small changes in hardness, the writer would not use the author's method, but would depend upon the human hand. The writer was once confronted with the task of making a rubber spring slightly stiffer than a previous spring, which meant an increase of about $^1/_2$ point in hardness on the durometer. After using the Olsen hardness tester and the Pusey & Jones plastometer with conflicting results, the only way discovered for accomplishing this was to lay on a table disks of the possible compounds of the same thickness, and go from one to another with the durometer in hand. The stock was selected in this way and, when the spring was made up, the shear-modulus value was found to be exactly that required. It is truly a difficult matter to standardize durometers, but there is no reason why we cannot calibrate one against the other. Undoubtedly, one who works constantly on the compounding of rubber and rubber-like materials soon acquires facility in the use of his own durometer.

J. F. Downie Smith.[7] The material given in this paper will, in the writer's opinion, be welcomed by designers who are interested in the application of rubber and rubber-like materials in industry. It is hoped that more information of this type on other rubber-like materials will be made available in the near future. At present the mechanical engineer feels rather at a loss when he is trying to determine the proper material to use for a specific application.

A short time ago the author gave the writer two samples of neoprene and one of rubber each bonded to metal plates in the form shown in Fig. 13 of this discussion. In order to make a cursory examination of each sample to see if it would be applicable in springs as used in experimental work, each sample was loaded and unloaded quickly a few times. Then a loading curve was taken in $7^1/_2$ min with the unloading curve taken in about the same time. A second set of curves was then made with loading and unloading periods 6 times as great. The results are shown in Fig. 13.

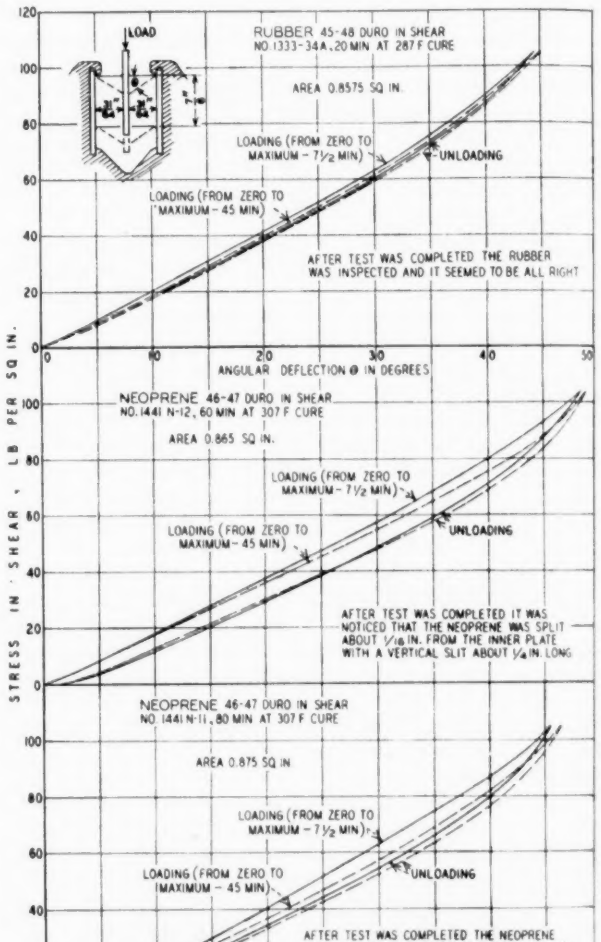


Fig. 13 Loading and Unloading Curves of Neoprene and Rubber Samples Bonded to Metal Plates

One of the neoprene samples, No. 1441 N-12, after the tests were completed, was found to be split, and could no longer be recommended as a spring. The other neoprene sample and the rubber one were apparently satisfactory.

It may be observed that one of the neoprene samples gave a much greater variation between quick and slow loading than did the rubber. At two shear stresses the percentage variations between the two loading curves are as follows:

[7] Executive Engineer of Research, Edward G. Budd Manufacturing Company, Philadelphia, Pa. Mem. A.S.M.E.

		At 40 lb per sq in.	At 60 lb per sq in.
1	For rubber No. 1333-34A	4.4	2.6
2	For neoprene No. 1441 N-12	4.5	4.8
3	For neoprene No. 1441 N-11	9.9	8.9

The writer would like to ask the author if such results would always be expected from typical rubber and neoprene springs, or if these samples are unusual in some way. If results of this type are expected, would he consider them detrimental to the use of neoprene as a spring?

An additional point of concern is that none of the curves comes anywhere near the theoretical shear loading curve which might be expected from previously published data on rubber. This theoretical curve is based on the calibration of definite Shore durometers against known moduli of elasticity in shear for rubber. Now all three of the curves shown in Fig. 13 check one another reasonably well, and one of the samples is of rubber and two are of neoprene so that the change from natural rubber to a synthetic rubber-like material is not the answer. One thought which arises is: "Does the size of the sample have an influence on the rough relationship of durometer hardness and modulus of elasticity in shear?" These particular samples are much smaller than those ordinarily used by the writer, and it occurred to him that this may explain the discrepancy to some extent.

Author's Closure

In this paper, an attempt was made to summarize such portions of the present knowledge of neoprene as might contribute to the design of springs. It is a paper on a spring material, however, and not a detailed paper on springs or on vibration in engineering. The author attempted to cite adequate references, without giving a complete bibliography, on these topics as collateral reading for anyone sufficiently interested. The fact is appreciated by the author that the present knowledge of neoprene has limitations. In some respects, additional research and further years of experience will permit a more definite account in the future than is possible today.

Mr. Cain has asked a question particularly pertinent to outdoor service. A specific answer would require definition of the expression "some flexibility." At — 40 F neoprene compositions of a high-grade mechanical type properly cured approximate the stiffness of hard leather but they are not brittle after exposure for as long a period as 14 days. If this condition is considered to fall within the intended meaning of the question, neoprene is flexible at all low temperatures down to — 40 F for any length of exposure. The change of modulus under these conditions would, of course, drastically alter the characteristics of a neoprene spring and would be against its application in some cases. Stiffness acquired by exposure to low temperatures can be overcome by heating from external sources, or by mechanical working.

A detailed discussion of each of the points raised by Mr. Hahn and, in some cases, again by Mr. Haushalter would be inordinately long and involved. There seems to be peculiar disagreement in the literature and among engineers as to what properties in rubber are desirable for some types of service. Every individual taking part in such technical controversies as this has some real basis for his views but, when the situation is viewed broadly, the arguments in good faith seem to arise from deductions based upon distinctly different types of service. The author has culled from the literature the outstanding reasons given for the use of rubber as a spring material under particular conditions. These reasons are, therefore, advantages possessed by rubber springs over metallic springs and are listed as such in the introduction to the paper. The same literature has pointed out further properties to be desired. Such properties, when possessed by neoprene, thus become advantages of neoprene over natural rubber for particular conditions which have been adequately defined in the early part of the paper. It seems trite to point out that, where these conditions do not exist for either rubber or neoprene, other selections would be made and that the selection of neoprene or rubber would sometimes be based upon one or two outstanding characteristics, in spite of attendant disadvantages.

The relatively high damping rate obtainable with neoprene is an advantage when relatively rapid damping is desirable. In other cases a high rate of damping would be a distinct handicap. The author does not believe that any engineer could specify the properties of an ideal spring system without including in the specification a desirable range of damping rates. Although Rosenzweig (16) has discussed the advantage of damping at critical frequencies, the bulk of his paper is devoted to proof that, outside of the critical range, damping is undesirable in the isolation of vibration. Numerous other references could also be given which cover the same point. The author has merely attempted to show that neoprene increases the range of damping rates available with rubber-like materials beyond the limits feasible with rubber. It is the job of designers to determine how much damping is required in a given case.

Mr. Haushalter's statement that spring systems are not designed to operate at resonance, apparently, is made without considering the present importance of dynamic balancers. The example of front-wheel shimmy on automobiles has been supplanted in industrial importance by a host of harmonic balancers in other applications. Those used on lightweight crankshafts to reduce torsional vibration are particularly interesting. Combinations of frictional and dynamic absorption offer the possibility of effective refinements in design.

Various authorities have suggested the desirability of damping action by rubber springs. Among them Haushalter, in a recent discussion,[8] expressed himself as follows: "It would be very desirable to have a rubber spring with a high damping coefficient but, with a compound giving this, the drift greatly increases. A combination of high damping and low drift in a rubber stock is yet to be found. The two conditions, apparently, are diametrically opposed. Work is still going on, however, and there are some possibilities in this direction with synthetic rubber."

In his experience with neoprene, the author has noted frequent exceptions to the statement that there is a general correspondence between creep and resilience. In a current series of tests, for example, two neoprene compounds, cured for the same length of time and having the same compression moduli, have 78 and 76 per cent resilience, respectively. However, in a shear-drift test, the former compound shows roughly twice the drift rate indicated by the second. This is not an isolated case, but is a repetition of a rather frequent occurrence. If the correlation were strictly true for rubber, as it certainly is not for neoprene, long and expensive tests of creep could be dispensed with in favor of fast and relatively inexpensive tests of resilience. Unfortunately, this is not the case.

Very properly, Mr. Hahn has pointed out that the thermal conductivity of rubber and of neoprene is relatively low. Many mechanical designs of rubber parts have failed because excessive heat was developed in operation. Other things being equal, any increase in thermal conductivity will assist the dissipation of energy, but a 30 per cent increase can only offer help to a minor extent. The relative values of thermal conductivity are quite incidental and, if the author is guilty of overemphasis, the point is well taken.

The use of neoprene commercially as a spring material is a relatively recent development and it is not possible, at the present

[8] "Rubber Cushioning Devices," by C. F. Hirshfeld and E. H. Piron; discussion by F. L. Haushalter, Trans. A.S.M.E., vol. 60, 1938, p. 204.

time, to predict what will happen to a spring in 3 to 10 years. Yet, it is believed to be true that the best rubber compositions have lower drift than the best neoprene compositions. However, the author knows that it is possible to produce neoprene compositions which compare very favorably with some of the rubber compositions commercially used. Unfortunately, tests which are now in progress at the du Pont rubber laboratories have not yet progressed to the point where detailed presentation is permissible.

With reference to the discussion by Dr. Smith, the author considers creep to be a type of dimensional instability and any type of instability in a structural material is undesirable. It is indicated by the tests that neoprene will creep more under dead load during the first hour than a comparable high-quality rubber under similar conditions. Since the neoprene samples under discussion are the best obtainable at the present time from the creep standpoint, it is apparent that neoprene will creep more during the first hour than rubber. The fact that a defective sample appeared to be better than an undamaged specimen is somewhat surprising and is probably an indication that a more reliable comparison between the compositions might be obtained from a more extended test. It has been the author's experience that the maximum discrepancy in rate of creep between properly vulcanized neoprene and rubber compositions occurs during the first 24 hours.

Inasmuch as the discrepancy in theory mentioned by Dr. Smith concerns a correlation of shear modulus with durometer hardness, it does not seem to be a matter of the greatest importance. If we remember that the correlation between elastic moduli and durometer readings is at best an approximation, occasional departures should not be too disturbing. On the other hand, data in the author's personal files indicate that the published values of shear modulus referred to by Dr. Smith are about 30 per cent low and this observation is confirmed by Dr. Smith in this case. It should be more disturbing if the size of the samples has a large effect upon the value of the shear modulus, since this would vastly complicate the design of shear springs by requiring a shape or size factor. The author's experience has been more extensive with small samples than with large ones, but there is inclination to doubt that the size of shear mountings has any great influence upon the modulus of a spring composition, provided each specimen is uniformly cured throughout its entire volume.

It is quite apparent that the introduction of rubber springs to industry, particularly to the automotive industry, preceded the development of an adequate literature and engineering background for general purposes. Even today, data on rubber are extremely meager and without a satisfactory network of correlating theory. It is to be hoped that information on rubber and similar materials will develop more rapidly in the future and that the present mystery surrounding the mechanical properties of these materials will soon no longer exist.

Automatic Integrating Pressure-Traverse Recorder for Study of Flow Phenomena in Steam-Turbine Nozzles and Buckets

By H. KRAFT[1] AND T. M. BERRY,[2] SCHENECTADY, N. Y.

The high efficiency of modern steam turbines can be safely predicted only when the flow phenomena in all passages of the turbine are well understood. The General Electric Company maintains an air-test laboratory where these passages are studied over a wide range of conditions. The principal test is a series of impact-pressure traverses in an air stream simulating flowing steam. The test is a "contour-map survey" of emerging energy, formerly requiring thousands of observations. Recently an instrument has been built which automatically traverses the passage, calculates the corresponding flow and energy quantities, integrates them against the traversed path, and records the integrals in an easily readable manner. The high speed of the instrument increases greatly the range of possible tests. Its inherent accuracy of 0.1 per cent surpasses the best previous performance. The instrument, using mechanical devices for measuring, calculating, and integrating, has all moving parts controlled by electric and photoelectric means.

THE automatic integrating pressure-traverse recorder is an instrument employed by the turbine-engineering department of the authors' company. The air-test laboratory of this department uses it for the following purposes:

The rapid recording of impact-tube pressure traverses run over turbine passages.

The simultaneous evaluation of these readings into flow and energy quantities.

The concurrent integration of these quantities against the impact-tube travel.

Purpose of Turbine Tests

The desirability of a high efficiency in a large steam turbine hardly need be emphasized. A loss of 1 per cent from an output of 50,000 kw represents, in addition to a 1 per cent increase in fuel rate, a loss of 500 kw installed capacity, which is rather an appreciable amount of power. It is necessary, therefore, to be in a position that will allow reliable judgment on all factors contributing to the efficiency of the turbine. As in all similar fields of human endeavor, continuous effort directed toward progress is the only means to assure higher efficiency or, if ever diminishing returns should someday make this impossible, to maintain what has been won.

Published knowledge of the many phenomena, which influence the efficiency of a steam turbine, is very limited. Many years of intensive work, many times too voluminous for publication, have built up a body of information which forms the basis of present-day design. It is imperative to add continuously to this information. Failure to do so would soon lead to questionable extrapolations into unexplored fields. Most surely it would arrest the trend toward ever better efficiencies.

The largest single part of the losses occurring in a steam turbine is caused by phenomena of fluid flow. The behavior of a compressible, viscous fluid, such as steam, is a function of a large number of variables. Every single problem that arises will bring these variables into play in one of a practically infinite number of combinations. No solution so far has been found that would permit attacking the problems by way of theoretical deduction.

On the other hand, it is evident that over-all results, obtained from tests on complete turbines, cannot give much detailed information.

To gain additional design information, the only practical method that is available is to collect a large amount of empirical results from tests on turbine elements, single and in various combinations. If proper consideration is given to unavoidable differences between test conditions and turbine installation, there is every reason to believe that satisfactory results can be obtained.

Of late, the method of model testing has received much attention from engineers. Briefly, it consists of the simulation of the conditions of the actual machine by a convenient model set up in such a manner that a definite comparison with the actual machine is possible. The model may be full-scale or may be merely geometrically similar. Materials and flowing media may differ from machine conditions and so may velocities and forces. Definite rules regarding the similarity of model conditions are available for various applications, e.g., Reynolds', Froude's, Mach's numbers, etc. Probably the model tests owe much of their popularity to the good results obtained from the aircraft engineers' wind-tunnel and hydraulic-laboratory work.

The turbine-engineering department of the General Electric Company has been using this type of model testing continuously during the last 19 years. In 1924, H. L. Wirt presented a paper[3] before the Society, describing this method of testing.

It is a high tribute to the engineers, who instituted and developed this fact-finding method, that this type of test in its essentials is still the same today as it was 19 years ago. Its advantages were so apparent that considerable thought and money have been spent to develop it to still greater usefulness.

Herewith is a short description of the test: Compressed air is passed through the actual turbine element or a scale model of it. Knowing the temperature, the total energy of the entering air is easily ascertained by means of a single impact tube. The test aims at obtaining a complete detailed picture of the flow energy leaving the element. The ratio of the energy integrated over the passage at its exit to the entering energy will furnish its efficiency. The details of the energy "picture" will help toward the explanation and cure of unsatisfactory behavior.

The principal advantages of such a test are:

1 The details of the energy distribution are visible.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] "An Experimental Investigation of Nozzle Efficiency," by H. L. Wirt, Trans. A.S.M.E., vol. 46, 1924, pp. 981–1003.

2 In the case of nozzles and buckets a few single passages can be tested as representative of the whole ring. No additional end conditions need be introduced, as must be done in the case of reaction and impulse tests.

3 The use of air instead of steam facilitates the test very considerably:

(*a*) Free atmosphere provides a known constant pressure in a practically infinite space into which to exhaust and in which the observer can live without special precautions.

(*b*) The temperature is constant enough to eliminate special measurements, at least as long as efficiency ratios close to 1 are obtained. Nor is there any possible condensation to be taken care of.

(*c*) The air temperature is low enough to permit the use of wood as the principal model material. This is important, as the building of models as well as all changes and modifications can be done much faster and cheaper with wood than if metal is used.

Several important characteristics of the test require special consideration:

1 The similarity rules of model testing must be obeyed. When applying air-test results to steam conditions complications can arise. Both steam and air are compressible fluids with low viscosity. So long as the viscosity is neglected, all the forces occurring in the flow of a compressible medium are due to mass actions. The forces acting on every particle of flow are all caused by the mass of the particle, as expressed by its density and its accelerations or decelerations. This includes, of course, all centrifugal actions due to the curvature of the streamlines. The static-pressure forces do not need to be considered. They are in equilibrium with the inertia forces and thus equal and opposed to them. If one set is known, the others are. It can be shown that the mutual relationship of all inertia forces in a flowing medium remains unchanged as long as, at a chosen geometrical position in the flow, the velocity always bears the same relationship to the sound velocity of the medium. Expressed in the terminology of the physicist, this amounts to maintaining a constant Mach number. This condition is easily fulfilled in a model test.

At high efficiencies, however, the viscosity forces enter the picture as the preponderant causes of the flow losses. In this case it is obvious that the ratio of these forces to the inertia forces also must be the same in the test as in the machine, if all forces in the flow are to retain their proper geometrical relationship. The ratio between mass and viscosity forces is expressed by the well-known Reynolds number. It is this Reynolds number which, once the flowing medium is chosen, coordinates one certain velocity to a chosen model scale or vice-versa, i.e., the size of the model passage must be such that the forces, due to speed changes, maintain a proper magnitude with relation to the viscosity forces as given by the medium. Usually it is impossible to find in another flowing medium a condition where simultaneously the same Mach and Reynolds numbers prevail as in the original. This difficulty usually can be taken care of when the behavior of the flow in question is known, both as a function of Mach's as well as of Reynolds' number. This is entirely possible, but calls for a certain expenditure of testing and engineering effort.

2 Means must be provided for recording and integrating rapidly a large number of test readings. For example, the upper curve of Fig. 8, had it been obtained by manual testing methods, would have required the taking, recording, calculating, and integrating of a minimum of 11,000 test readings. This makes the test cumbersome and slow. The more that becomes known about turbine nozzles the more it is apparent that the situation is aggravated by the fact that a systematic, statistical approach seems to be the only reliable method of controlling closely their efficiency. The numerous determining variables are so closely interwoven and related to the geometry of the nozzle as a whole that only a very large number of tests will yield a satisfactory solution.

It is well known how thousands of wind-tunnel tests on various shapes of airplane-wing profiles had to be made before the present knowledge of wing behavior was available. The turbine nozzle presents a much more complicated problem than the single wing, since the close pitching of the partitions, their position on a circle, and the compressibility of the medium introduce additional variables.

Here then, was a definite need for mass production of nozzle tests. The engineers' approach to such a problem was chosen, to wit, the employment of a labor-saving machine.

The conception of such a machine was dormant in many engineering minds. When the need for more and better test results became more and more pressing, the idea began to take form and grew into the instrument that is the subject of this paper.

Design Features of Instrument

The experiences mentioned in the foregoing paragraph crystallized into the following specifications for the instrument:

1 The instrument has to record the pressure difference, measured with impact tubes between the entrance and the exit sides of a row of nozzle or bucket passages. The impact tube at the entrance can remain stationary, as it reads a point value of an equally distributed entrance energy. Since each test naturally is run with the entrance energy remaining constant for one set of readings, this impact tube reads a constant pressure.

The exit impact tube is the traversing impact tube. Low impact pressure at the exit is an indication of the deficiencies of energy that are due to the losses in the nozzle. Set at the angle of the nozzle, the tube is moved over the nozzle exit at a constant clearance, usually equal to the axial nozzle-to-bucket clearance in the turbine. The traverse should include at least three single nozzle passages. The difference between the two pressures should be recorded at a suitable scale.

One set of readings should consist of a number of traverses suitably spaced to give a complete "contour map" of the emerging jet.

2 From the impact readings, the flow and the energy contained in the measured point of the jet should continuously be calculated. The calculation should be made according to the well-known formulas for compressible fluids. ($\gamma = 1.4$ for air.)

It was realized that for most velocities and for most purposes, the simple formulas for incompressible flow would have been sufficient. However, aside from some additional calculation work to be made for the design of the instrument, no additional complication resulted from the application of the accurate flow formulas. With the initial pressure and temperature kept constant, both quantities are functions of a single variable, the pressure difference, and each can be represented by a cam in the instrument.

3 The instrument has to integrate mass flow and energy against the path of the impact-tube travel. This will result in line integrals, i.e., the summed up mass flow and velocity energy along the path of a traverse. The theoretical 100 per cent energy can be found by multiplying the mass flow by the correct factor.

4 The instrument should record the integrals along the impact-pressure curve in such a manner that the integral along any portion of the traverse can be found by taking differences. Thus, it is always possible to evaluate any single section of the readings, wherever special phenomena may warrant it.

5 The instrument should perform these operations with an

accuracy of plus or minus 0.1 per cent of full scale reading or better. A 1 per cent error in the efficiency of a turbine must be considered a rather large figure. It was thought that 0.1 of that amount would constitute the smallest component part that might be of interest. Of course, the result must be so recorded that it is easily possible to read the results to such accuracy.

6 The specified accuracy must be attainable throughout a range of nozzle velocities from 800 to 1200 ft per sec. This covers a large number of turbine-diaphragm conditions. With lower-velocity nozzles, somewhat less accurate results will be expected from an instrument designed for the high velocity range.

7 A high working speed of the instrument is, of course, of great importance. An instrument as complicated as this represents a sizable investment which must be justified at least partially by a saving in labor. Increased accuracy and the possibility of making tests that otherwise never would have been undertaken will compensate for any difference along this line. The speed required of the instrument is such that one normal nozzle opening can be traversed in 1 minute. This represents a speed superiority over the hand-traversing method of between 7 and 10 to 1, exclusive of the simultaneous evaluation.

8 Such speed necessitates one additional specification. The instrument that is expected to use such working speed economically must possess unusual sturdiness. It was foreseen that it would be in practically continuous operation throughout the year. The turbine air test is run on a three-shift basis and the setting-up time of the test, during which the instrument is idle, is comparatively short.

The foregoing specifications were met fully, while in several points the expectations were considerably exceeded.

Description of Instrument

The instrument, as it was built, consists of four main-component sections, i.e., the traversing system, differential-pressure-measuring system, integrating system, and the recording system. Fig. 1 is a diagrammatic representation showing how this device functions.

The traversing system consists of a motor-driven worm A, driving a section of a large gear B, to which is attached the exploring impact tube C. This tube is moved in the arc of a circle across the nozzle opening D. The speed of the motor E, driving this gear, is controlled by means of thyratron tubes (not shown in Fig. 1). The speed-control circuit provides for both hand and automatic control of the traversing speed to give proper integration and recording.

The differential-pressure-measuring system consists of a photoelectric-controlled balancing circuit F, which operates a thyratron-controlled motor G that applies a force, through a steel spring H to a flexible bellows I. This flexible bellows divides a cylindrical compartment into two sections. The initial impact pressure is applied to one section and the impact pressure at the nozzle discharge is applied to the other section. The deflection of the bellows, due to the pressure difference, turns a mirror J and deflects a beam of light which falls on two phototubes. The deflection of the beam causes a change in the division of the light between the two phototubes. This unbalance operates the thyratron motor so that the tension on the spring attached to the bellows increases or decreases to restore the bellows to its normal position. Since the application of force to the bellows by the spring requires an elongation of the spring, which is directly proportional to the applied force, the number of turns of the motor that extends the spring is directly proportional to the total unbalanced force and, hence, proportional to the difference in force per unit area on the two sides of the bellows.

The integrating system consists of two turntables K, turned by a Selsyn receiver L, which is driven by the Selsyn transmitter M. This transmitter, in turn, is driven mechanically from the traversing motor E. Upon the turntables roll two integrating wheels N which are free to turn on their axes, and which may also be moved axially along radii of the turntables.

These integrating wheels are moved radially on the turntables by means of two large cams O, which are positioned by the motor G, balancing the pressure cell.

Since the angular displacements of the turntables are proportional to the angular displacement of the impact tube and, since the integrating wheels are positioned by cams that are cut to represent the proper functions of the differential pressure, the amount the integrating wheels turn is in proportion to the product of the displacement of the impact tube along the arc of the nozzles and the functions of the differential pressure.

In each of the integrating wheels are four holes through which is passed a beam of light. As the integrating wheels turn, light beams are intercepted four times for each revolution. The light beams thus intercepted fall on two phototubes P, which cause electrical impulses to operate counting and recording mechanisms.

The recording system consists of a chart-drive mechanism Q, which moves a paper chart R in proportion to the number of turns of the turntables and, therefore, in proportion to the motion of the impact tube. Recording simultaneously on this chart are three pens S, T, and U, which record the differential pressure and the two integral functions. The differential pressure is re-

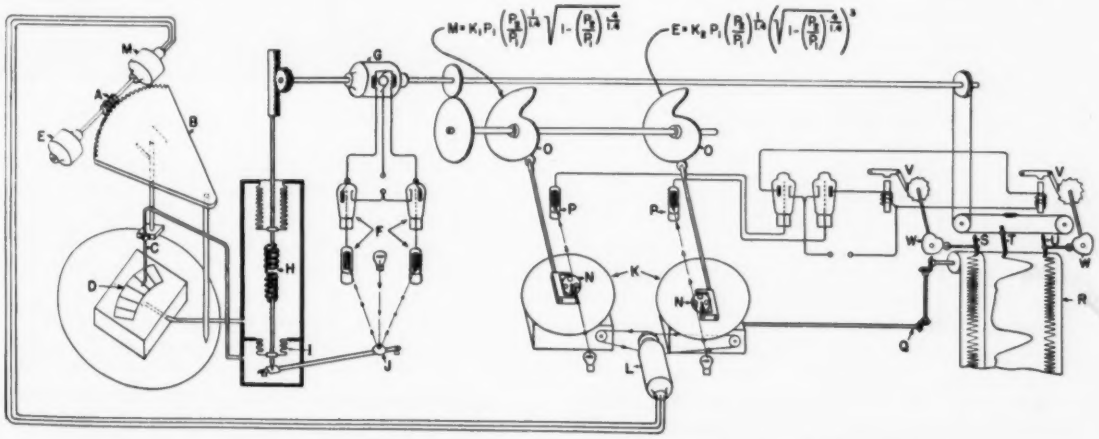


Fig. 1 Diagrammatic Representation of Automatic Integrating Pressure-Traverse Recorder

corded in the center of the chart and the integrals along the two sides. The pen *T*, recording the pressure, is moved by the motor *G*, which drives the pressure-balancing system.

The electrical impulses caused by the rotation of the integrating wheels are used to actuate ratchet-and-pawl mechanisms *V*. Turning with the ratchets are two cardioid cams *W*, which act through their followers to move pens *S* and *U*, in steps, back and forth along the sides of the charts. Fifty electrical impulses are required to turn the ratchets and cams one revolution. By counting the impulses recorded on the sides of the chart, 50 for each cycle, the integrals between any two points on the chart are quickly obtained.

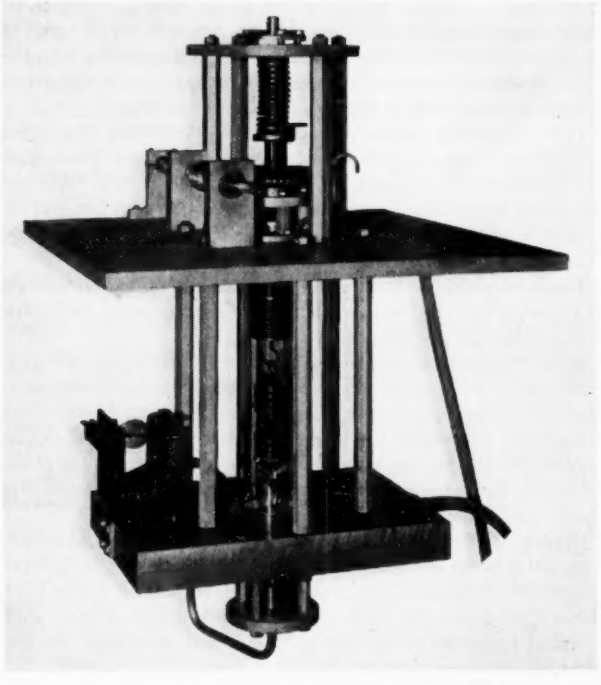


FIG. 2 PRESSURE CELL AND OPTICAL SYSTEM OF RECORDER

Since it was required that the over-all accuracy of the pressure system, integrator, and recorder be within limits of plus or minus 0.1 per cent, special care had to be taken in the mechanical and electrical details for the design and construction of this instrument to make this possible.

In the pressure cell, attention was given to the linearity and permanency of calibration of the steel springs used to balance the differential pressure of the diaphragm. A double helical spring was used, the two halves coiled in opposite directions to eliminate the effect of twisting as the spring extended and contracted.

The volume of air in the low-pressure compartment of the pressure cell was minimized by almost completely filling this compartment with oil. This was done to reduce the time required for the pressure in this compartment to reach its ultimate value when extremely small impact tubes were used. In some cases a modified hypodermic needle is used as an exploring impact tube. It can readily be seen that a large volume of air in this compartment would slow down the response of the pressure cell considerably.

To detect any deflection of the flexible bellows away from its normal position, a pivoted armature is mounted so that a sharp point, attached to the bellows, bears on this armature in such a way that any motion of the bellows causes the armature to rotate on its pivots. Attached to the armature is a small mirror that turns with it and deflects the light beam. The image of the small coiled filament of a lamp falls on a divided mirror, after being reflected from the mirror on the armature. This divided mirror splits the light beam, causing the two parts of it to fall on two phototubes. These phototubes are electrically connected in a bridge circuit. Any change in the division of light falling upon them causes a change in the balance of the bridge circuit. A split-field series motor operated by means of thyratrons in response to the voltage unbalance in the bridge circuit tends to restore the balance of the bridge as previously explained.

The over-all sensitivity of the bellows, optical system, and the electrical circuit is such that the thyratron motor responds to a differential pressure of less than 0.0075 in. of mercury.

Fig. 2 illustrates the pressure cell and optical system.

A differential pressure of 30 in. of mercury causes the two large cams to turn 350 deg. The cams were calculated from the formulas for mass flow and energy of flowing air.

A total of 350 points was calculated for each cam and the cams were cut to an accuracy of plus or minus 0.0002 in.

The integrating wheels were made of tool steel and chromium-plated to prevent corrosion and to give a hard long-wearing surface. Jewel bearings were used for the integrating wheels, in order to minimize friction which might cause slippage between the integrating wheels and the turntables. Plate glass was used for the surface of the turntables, giving a hard smooth surface for the integrating wheels to roll on.

Fig. 3 shows the mechanical arrangement of the parts which constitute the integrating system.

In the recording system, two gear shifts are provided. One shift lever changes the ratio of chart speed to turntable speed, giving three speed ratios.

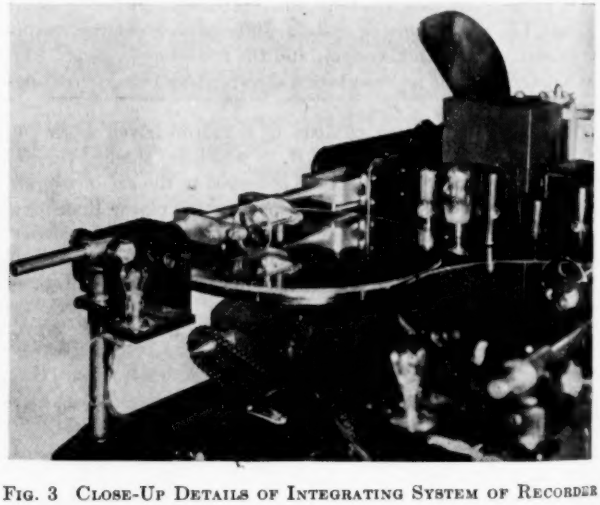


FIG. 3 CLOSE-UP DETAILS OF INTEGRATING SYSTEM OF RECORDER

The other gear-shift lever allows for magnification of the pressure record when low differential pressures are to be recorded. Three ratios are also provided in this gear shift.

A hand crank is used to adjust the calculating cams to the proper position for integration. The position of the cams is indicated by two dials at all times.

Below the recorder compartment are the manual speed-control knob for the traversing motor and two switches. One switch reverses the traversing motor and, at the same time, changes the connections to the Selsyn receiver, driving the turntables so that the turntables and recording chart run in the same direction for both directions of travel of the impact tube.

The other switch changes the speed control from manual to

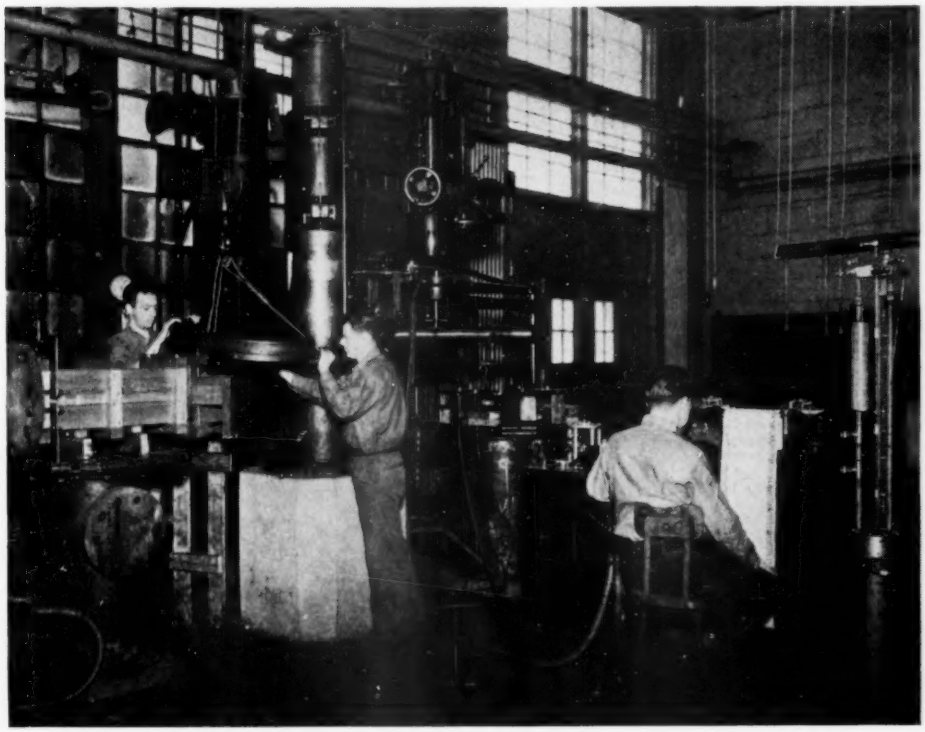


FIG. 4 INTERIOR OF THE AIR-TEST LABORATORY WITH THE RECORDER IN SERVICE

automatic. When the switch is in the automatic position the traversing-motor speed-control circuit is connected to the pressure-cell control circuit so that the speed of the traversing motor is dependent upon the balance of the pressure cell. When the differential pressure is changing rapidly, the traversing motor runs more slowly than when it is constant.

In order that the accuracy of this instrument might be maintained, it was built as sturdily as possible, and a number of safety features were incorporated so that no damage would result from the application of abnormal pressures to the pressure cell. An overdrive mechanism operates when the pressure-recording pen reaches its limit of motion. The pen stays at its maximum or minimum on the chart until the differential pressure returns to a value which again brings the pressure-recording pen on scale.

The large cams are prevented from turning too far by a similar overdrive mechanism which stops them at their limit of motion without damaging the instrument.

A second Selsyn receiver is arranged so that it, instead of the pressure cell, may drive the integrating cams and pressure pen. This Selsyn turns in synchronism with the Selsyn receiver that drives the turntables and is used in checking the accuracy of the integrator and recorder. This accuracy check gives the integration of an equivalent differential pressure which varies uniformly from 0 to 30 in. of mercury in 1600 turns of the traversing motor. This integral has been accurately calculated. The comparison of this theoretical value with the recorded value gives an absolute check of the integrator.

The accuracy of the pressure-measuring system and the integrating system are checked daily. It has been found that the over-all accuracy of the instrument stays well within the required 0.1 per cent.

Fig. 4 shows the instrument in actual use. It was designed by the general engineering laboratory of the General Electric Company in close cooperation with the turbine-engineering department. It was built entirely by the former.

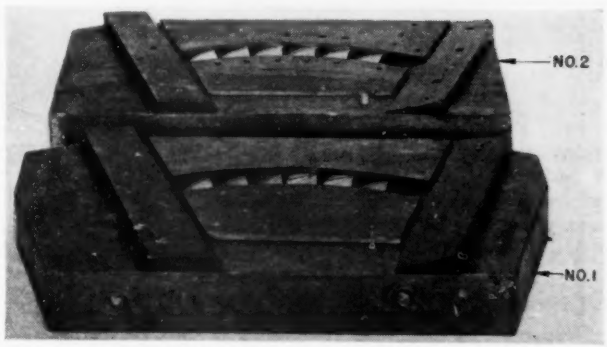


FIG. 5 COMPARISON OF MODELS OF TWO NOZZLES USED FOR TEST

APPLICATION OF INSTRUMENT

The problem to be solved, herewith, to demonstrate the use of the recorder was the design of a nozzle partition for a given set of geometrical conditions with the additional specification of a high strength of the trailing edge against a high steam-pressure difference. Two nozzle models were made having partitions of different design, Fig. 5. The dimensions of the annulus, the exit angle, and the depth of the diaphragm are identical in both cases—so is, of course, the workmanship. The only difference is the contour of the actual passage and that of the partitions, a difference that will easily escape the eye of the nonspecialist. Fig. 8 shows, as the final result, the efficiency of both nozzles plotted against the theoretical air velocity. As can easily be seen, one is superior to the other by a very comfortable margin. A difference of the same order of magnitude would be expected from the nozzle partitions were they put in a turbine. The two curves also show the accuracy that can be obtained with this test, all of the actual test points, which in no way have been corrected, forming a good curve. Deviations from this curve are less than

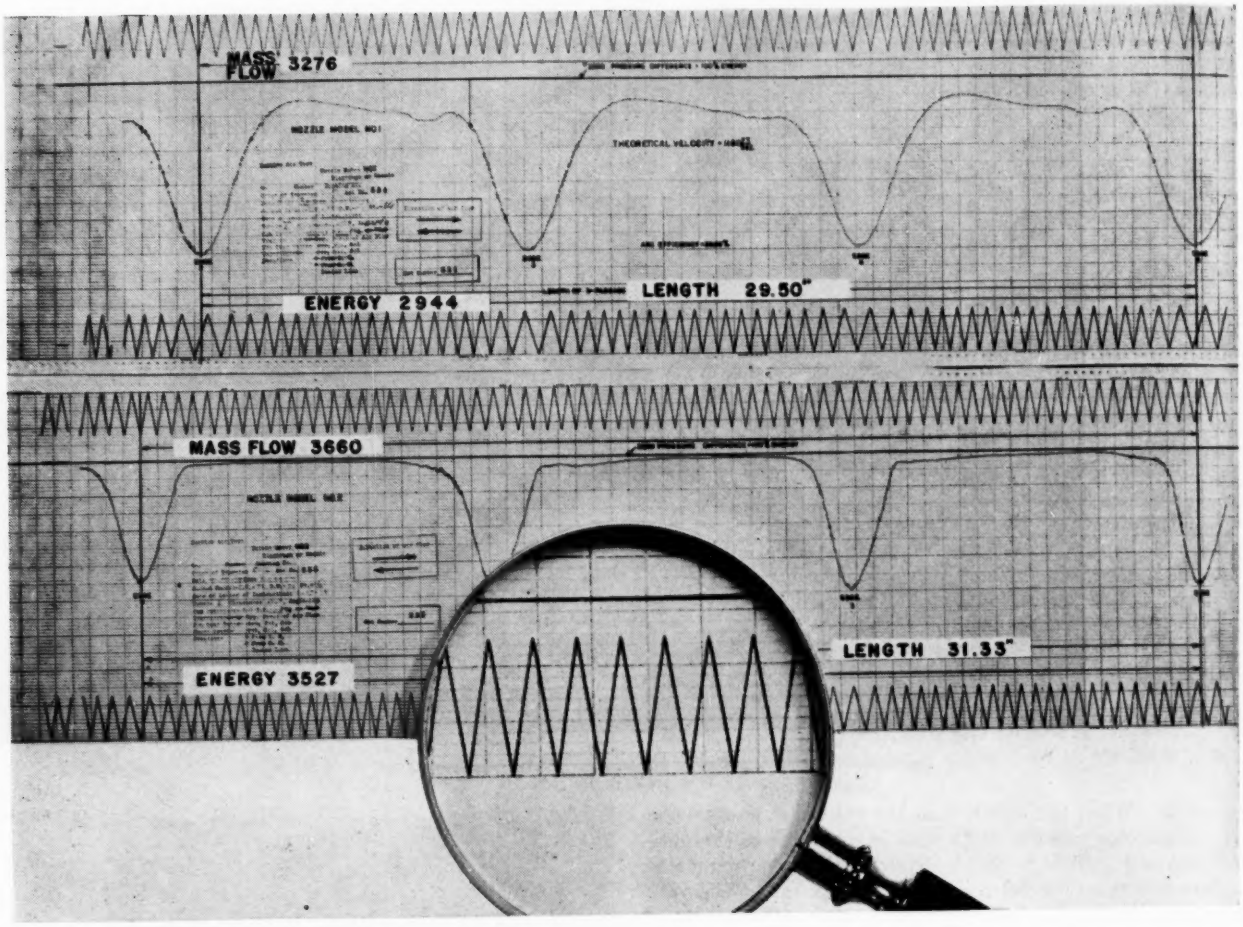
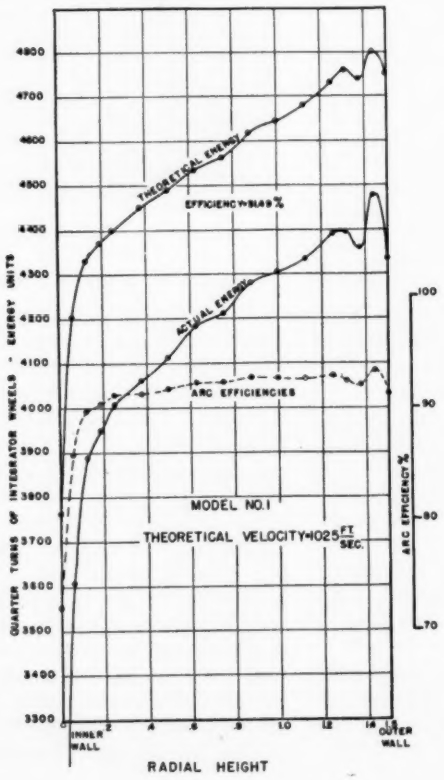


FIG. 6 (ABOVE) INSTRUMENT RECORDS SHOWING COMPARISON OF TWO MODELS



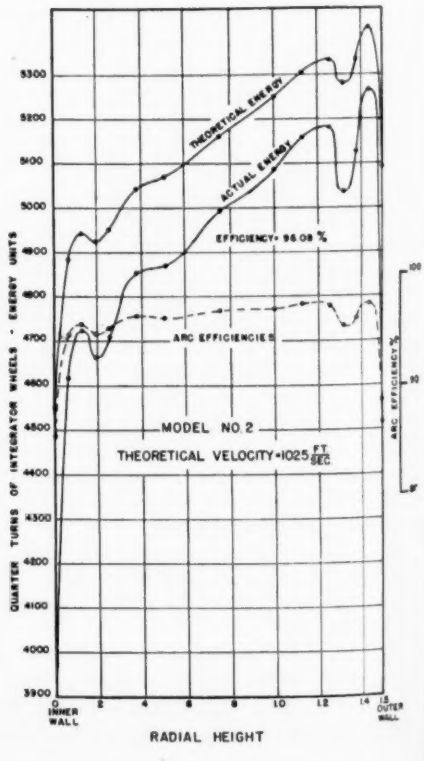


FIG. 7 (RIGHT) ENERGY AND FLOW INTEGRALS PLOTTED AGAINST RADIAL HEIGHT OF THE NOZZLE

0.1 per cent not of full scale reading of the instrument, but of any reading at the tested velocity. It must be remembered that this result is obtained, not from the instrument alone, but that the final cross integration in the radial direction has been made by the manual cross plotting of the line integrals, drawing the curve and planimetering two areas.

Figs. 6, 7, and 8 show the stages of this calculation. Fig. 6 shows one original record per nozzle as obtained from the instrument. It is one of 15 traverses over three nozzle passages at one velocity. For each constant velocity (constant initial pressure) 15 such traverses were made. The more efficient nozzle was tested at 9, the other nozzle at 6 velocities. The curve in the middle of the paper is the curve of impact pressure. The friction and wake loss, caused by the partitions, can readily be seen. The stepped zigzag lines along the two margins give the continuous integration of mass flow and energy. The number of small steps between any two points of the traverse is proportional to the in-

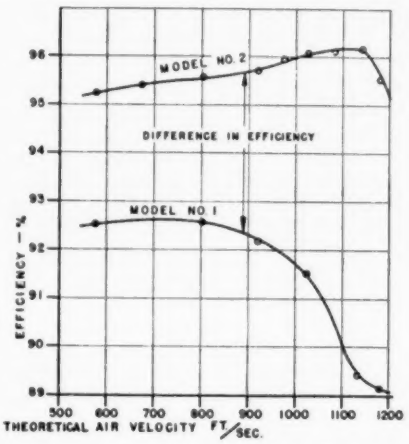


FIG. 8 COMPARISON OF THE TWO NOZZLES SHOWING OVER-ALL EFFICIENCY

tegrated mass flow and energy. Each step represents a $^1/_4$ revolution of a planimeter wheel, which serves as the unit of the integrated energy values. The integral curves are folded over after each 25 steps to allow for a scale large enough so that 0.1 per cent can conveniently be read. The figures shown on the curves are the mass-flow and energy integrals, counted up for the three openings, and also the total length of periphery covered by the flow from the three passages.

In Fig. 7, these energy and flow integrals are shown plotted against radial height of the nozzle. The scale of the ordinate is given in energy units, i.e., quarter turns of the integrator wheels. Since efficiency ratios only are of interest in this test, a conversion into customary energy values is considered unnecessary. The ratio between the areas under the theoretical and actual energy curves is the efficiency. The dotted curve gives the arc efficiencies, i.e., the efficiency of each individual traverse as taken along the traversed arc. It shows the location and size of the deficiencies given by mass flow and energy. For each constant theoretical air velocity one such curve sheet is made. The final result appears in Fig. 8, as previously indicated.

These curves explain, at least to an engineer, better than words the advantages accruing from such a test and also, of course, of the instrument that permits the large-scale application of the test.

No sooner was the instrument built than its advantages were used to the utmost. Many problems that formerly would never have been attacked in an experimental way are now handled in a matter-of-course manner. One very important application of the instrument is the checking of finished nozzle diaphragms before they go into the turbine. It is often desirable to make sure that no variations of design or of finish have reduced the expected efficiency. Automatically such variations are submitted to the instrument and its judgments obeyed.

It hardly need be emphasized that the cooperation of this new member of the turbine-engineering department of the General Electric Company has been greatly welcomed and at present enjoys complete confidence.

Discussion

WINSTON R. NEW.[4] We are indebted to the authors principally for their concurrent emphasis of the general need for recognition of the great number of variables upon which steam-turbine stage efficiency depends. With a background of well over 5 years of successful experience in the use of completely mechanized exploratory traverse equipment, we are gratified at their second statement in the section, "Design Features of Instruments," as well as other statements by independent experts, which so thoroughly justify the type of flow research now a fundamental part of turbine engineering by the writer's company. We join them in paying tribute to the engineers who instituted and partially developed search-tube studies, not 19 years ago, as has been stated, but well over 30 years ago, principally in Europe but also at many places in this country.

Turbine engineers have been quick to realize that a strict adherence to the recognized criteria of dynamic similarity surrounds model testing with difficulties almost as insuperable as those which prevent more exhaustive experiments on full-size, steam-driven stages. Coupled with this, there seems to have been a general failure on the part of many investigators to recognize the important part of the turbine-flow problem from the viewpoint of the designer, or to realize what appears to this writer to be the main strength of air tests. Too much emphasis has been given to fragmentary results established by tests at a few combinations of conditions for which most criteria of dynamic similarity could easily be satisfied. On the other hand, too little emphasis has been accorded to the systematic prosecution of a comprehensive program embracing both the nature and magnitude of the energy losses in representative turbine elements, and their mode of dependence upon key variables involved. Samples of the kind of comparative information needed, selected from a file of thousands of tests, constitute part of a current Westinghouse communication to the profession. When such results are available, improvement in design follows, independent of whether or not the general level of laboratory measurements is optimistic or can be applied without modification to the prototype.

Attempts to magnify an energy loss of 1 per cent to make it sound more impressive bespeaks an inadequate grasp of the magnitude of destructive effects encountered in a general treatment of fluid-flow problems. Further experience in the application of their method will reveal to the authors the wisdom of avoiding needless restriction of search-tube exploration only to the comparatively efficient assemblies. With the multiplicity of variables upon which stage efficiency depends one cannot go far from the old established paths in many directions without encountering the more significant losses. To cite but a single case; excessive pitch of blades with large turning angle almost invariably results in a loss of fluid contact from the passage wall (called "stalling flow" in aeronautics) with complete collapse of performance and not merely a decline of a few per cent in efficiency.

The instrument described by the authors is ingenious but com-

[4] Research Engineer, Westinghouse Electric & Manufacturing Co., Philadelphia, Pa.

plicated by the adherence to photoelectrically controlled mechanical means for both recording and integrating. We have preferred a combination of direct mechanical and electrical means employing the magnetic-strain gage as the sensitive element. A brief exposition of major differences between the two methods and equipment seems appropriate in this discussion.

The cell shown in Fig. 9 of this discussion for converting pressure differentials into small displacements, in content not unlike the one described in the paper, has been used successfully in the South Philadelphia flow laboratory for over 7 years. At this point, however, the similarity ceases, for the displacement in this pressure cell is communicated directly to a magnetic keeper supported at small clearances from the pole faces of two U-cores. High-frequency excitation at constant voltage is furnished by a small inductor alternator to primary windings connected in series on the U-cores. Secondary windings, connected phase-opposed, produce a small voltage linearly proportional to the

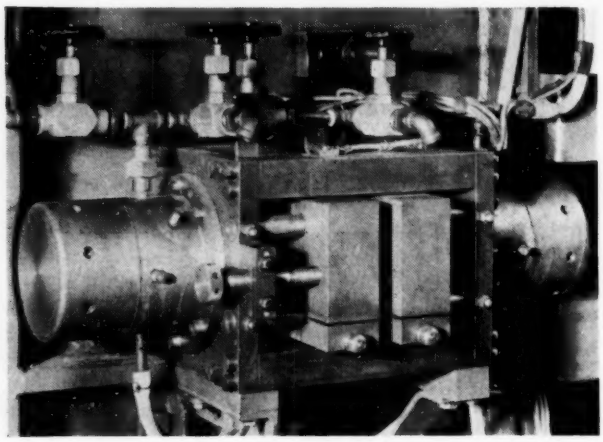


FIG. 9 PRESSURE-DISPLACEMENT CELL

differential pressure on the cell, due to variation in coil reactance with motion of the iron keeper. This voltage is applied to a suitable, high-speed recording voltmeter and to the current coil of a high-precision induction watt-hour meter. The potential coil of the watt-hour meter is fed from the constant voltage of the alternator. The search tube and recorder chart are driven through gears, permitting a selection from among several speeds, by small synchronous motors.

Traverse records from the voltmeter are identical to those produced by the center pen of the instrument described by the joint authors. The watt-hour-meter speeds corresponding to zero and 100 per cent efficiency are known constants which can be readily checked by subjecting the pressure cell to the required maximum and minimum pressure differentials. Integration of a traverse simultaneously with its recording consists merely of counting the watt-hour-meter revolutions for the time of transit of the search tube over an integral number of blade pitches and a small amount of simple arithmetic.

The sensitivity of this system is about the same as the figure of 0.0075 in. of mercury given in the paper. Calibrations systematically repeated over a period of many months likewise exhibit remarkable consistency. Two of the complete assemblies, herein briefly described, and shown in Fig. 10 are still in almost daily operation.

It is recognized, of course, that our integration proceeds from a motion proportional to a pressure difference instead of a combination of odd powers of pressure ratios. A small, experimentally determined, additive compressibility correction to efficiency



FIG. 10 ASSEMBLY VIEW OF WESTINGHOUSE AUTOMATIC-FLOW-EXPLORATION EQUIPMENT

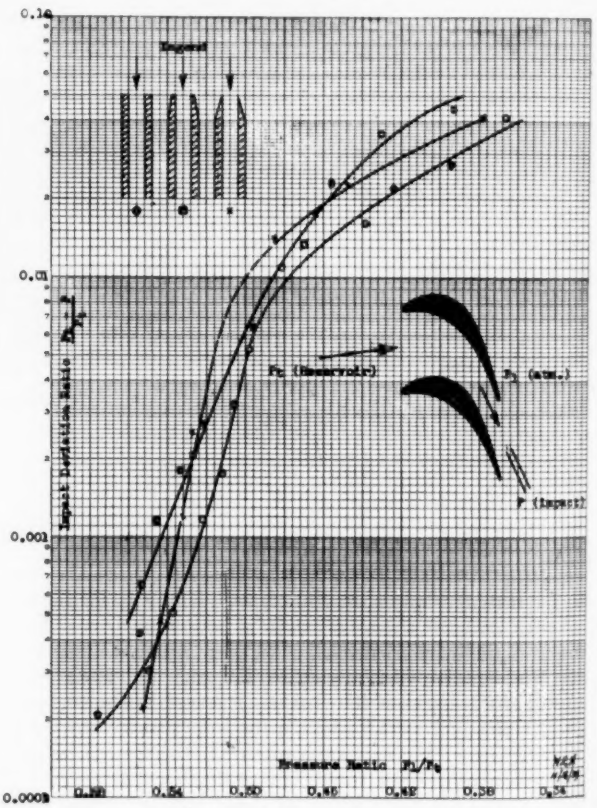


FIG. 11 INFLUENCE OF COMPRESSION SHOCK ON READINGS OF IMPACT TUBES DIFFERING IN LEADING-EDGE THICKNESS
(Tubes located in the substantially loss-free core of an air jet from a reaction-blade grid.)

given as Fig. 8 in a paper[5] by the writer insures that the simplicity of this method involves no sacrifice in accuracy.

The use of segmental packs, as indicated by the authors, instead of rectangular grids with blade edges parallel from base to tip is an unjustified complication. It does not insure closer simu-

[5] "An Investigation of Energy Losses in Steam-Turbine Elements by Impact-Traverse Static Test With Air at Subacoustic Velocities," by Winston R. New, published on page 489 of this issue of the Transactions.

lation of turbine-operating conditions except possibly for the single case of Curtis stage nozzles with a nondirectional stream at the inlet. Furthermore, it always obscures the comparison sought between two or more models by introducing changes in several features of the geometry at the same time. Thus any attempt to specify a dimensionless pitch or gaged opening for the nozzles used as an example in the authors' paper immediately requires designation of the radial position at which the measurements are to be made. This criticism is based upon use of the instrument as a research tool and does not apply for purely comparative work on alternative assemblies of temporary commercial significance.

In concluding these comments attention should be called to an important matter relative to the use of impact tubes at supersonic velocities which seems to have escaped consideration by the authors. Quoting from the section "Design Features of the Instrument:" "The specified accuracy must be attainable throughout a range of nozzle velocities from 800 to 1200 ft per sec." Assuming that the compressed air expands from a room temperature of say 70 F, local acoustic velocity is reached in the stream at 1031 ft per sec for zero energy loss. With reheating of the jet due to losses, the acoustic velocity at any point will be somewhat higher, but by comparatively little, say 1050 ft per sec. The upper limit of test velocities mentioned is, therefore, quite certain to be substantially in excess of the local acoustic velocity. When this supersonic-velocity stream encounters the muzzle of the impact tube, a compression shock occurs. This process involves an abrupt deceleration to a subsonic velocity with increase in entropy. The subsequent isentropic compression to zero velocity at the impact tube fails to restore the initial-state point of the fluid even if there were absolutely no energy losses along that particular filament through the passage. Thus losses attributable solely to the measuring method might be wrongly charged to the assembly on test.

Some idea of the magnitude of losses due to compression shock at the impact tube may be obtained from Fig. 11 of this discussion. Impact tubes with differently shaped leading edges, but each fashioned from a hypodermic needle with 0.025-in. bore and 0.013-in. wall thickness, were properly oriented in the substantially loss-free core of a jet from a blade grid exhausting to the atmosphere. The measured reservoir pressure ahead of the grid was gradually increased and at each step the difference between reservoir pressure and impact pressure was observed for each impact tube. The critical pressure ratio for air is about 0.5283 and the pressure ratio corresponding to a jet velocity of 1200 ft per sec is in the neighborhood of 0.4 with expansion from room temperatures. Of interest are the conclusions:

1 Losses in the central core (remote from blade surfaces) of a subsonic-velocity jet are negligible.

2 The shape of the leading edge of the impact tube is not critical for subsonic-velocity exploration.

3 Shock loss at the impact tube can amount to several per cent of the initial pressure for pressure ratios in the order of 0.4. The importance of not overlooking this where accurate loss measurements are essential seems to require no further comment.

4 The shape of the leading edge of the search tube materially affects measurements at supersonic velocities.

Much thought has recently been given to the matter of search-tube exploration at highly supersonic velocities. Further consideration of this, however, lies beyond the scope of the present discussion.

J. I. YELLOTT.[6] The results, shown by the authors in Fig. 8, indicate the great value of the type of equipment which is so well described in their paper. Without endeavoring to predict analytically the exact performance with steam in an actual turbine, it is undoubtedly true that "what is better with air will be better with steam."

The extremely ingenious devices whereby variations in impact pressure are transformed into efficiency variations deserve particular note. The writer has had the opportunity of seeing this equipment in operation and he has been particularly impressed with the ease and rapidity with which accurate data can be secured.

It would be valuable for the authors to comment briefly on the materials from which the models are made. The relatively low cost of these models is a valuable feature, and the ease and rapidity with which changes can be made is another factor to which attention should be called.

G. B. WARREN.[7] Both of these papers[8] are valuable contributions to the literature dealing with the details of the steam-turbine art. The authors are to be commended for attempting to apply to the steam-turbine problem much of the recent knowledge with respect to fluid flow, most of which has been worked out in connection with aerodynamic research relating to airplanes.

As higher and higher efficiencies are obtained with the turbine elements and as a wider range of conditions of pressure, temperature, vacuum, etc., are encountered, the relationships expressed as Reynolds' number, Mach's number, etc., begin to have greater importance.

On the other hand, it might be worth while to review some of the history relating to the development of this research tool as it has been worked out by the writer's company, through the period when rapid gains were being made on the less efficient turbine elements with this and other tools in their cruder first forms.

Following the rapid development of the turbine through the World War period of 1915 to 1919, it became apparent to those in charge of the General Electric Company turbine work that means would have to be sought to obtain more accurate information as to what was occurring inside the turbine-blading elements.

Two research tools were set up in 1920 to remedy this situation. The first was an 8-stage 2500-kw test turbine. In this machine, a large number of nozzle-and-bucket combinations were tested during the next few years, with steam under actual conditions of operation.

In parallel with this, a second tool was set up in the form of a nozzle-reaction testing machine in which actual nozzle elements could be tested over a wide range of steam conditions. Some of the results of the testing on this second tool were set forth in a paper[9] presented by J. H. Keenan, then of the General Electric Company, turbine engineering department, now professor of mechanical engineering at the Massachusetts Institute of Technology, and the writer. A comprehensive research program, covering scores of nozzle designs, both expanding and nonexpanding and at a range of velocities from 500 ft per sec to 4000 ft per sec was studied.

Early in this program, it became apparent that both of these tools gave over-all answers and did not permit an analysis of the

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[7] Designing Engineer, Turbine Engineering Department, General Electric Company, Schenectady, New York, N. Y. Mem. A.S.M.E.

[8] This discussion applies jointly to the present paper and the paper "An Investigation of Energy Losses in Steam-Turbine Elements by Impact-Traverse Static Test With Air at Subacoustic Velocities," by Winston R. New, which appears on page 489 of this issue of the Transactions.

[9] "A Machine for Testing Steam-Turbine Nozzles by the Reaction Method," by J. H. Keenan and G. B. Warren, Trans. A.S.M.E., vol. 48, 1926, pp. 33–64.

places in the various elements in which losses were occurring.

In order to meet this situation late in 1920 and early in 1921, W. E. Blowney of the turbine engineering department of the General Electric Company started out to build wood models of single-nozzle passageways, picking out certain nozzle passageways in some turbines which did not appear properly designed, particularly those in which a change of diameter occurred between successive wheels. These models were blown with air on a simple test stand and the jet was traversed in a crude way with an impact tube.

It immediately became evident that there was often a "hole" in the discharge of the nozzle, that is, a spot over a considerable portion of the nozzle height in which the velocity was much lower than normal. It could be felt by traversing the nozzle jet with one's finger.

The next step was to make the jet visible, and an attempt was made to introduce a water spray in the air. This showed up certain irregularities in the flow although, because of the inertia of the water, a false picture of the true direction of the jet was obtained.

Next, low-temperature steam was tried mixed with the air and the jets were photographed, with some additional gain in results.

The next step Blowney tried was to put paint dots on the inside of the nozzle passageway. The air was turned on while the paint was wet, and the dots were, of course, streaked out in the direction of the air in the boundary layer at each particular location.

Although crude, all of these methods gave results which were immediately turned into improvements in construction and which resulted in some gains when applied to the 8-stage test turbine described.

H. L. Wirt was then given charge of this development; under his direction it advanced with great strides during the ensuing years. Later the work was continued under the direction of others.

The next development was, of course, the testing of multi-nozzle passageways with much more refined impact tubes and the expansion of Blowney's "paint-dot" practice into a means of getting a "flow cast" of the nozzle, showing the flow in the boundary layers which, to the experienced eye, permits a close approximation to the total flow in the nozzle to be derived. Much of this work was described in a paper[10] written by H. L. Wirt.

This tool was then added to the two previously described, and many points of great inefficiency were discovered in the turbines as then designed which were rather quickly and definitely cleaned up.

The writer has in mind three particular changes in construction, two of which each method of testing showed to give substantially comparable gains. The third modification in construction was not susceptible of analysis on the reaction nozzle-testing machine, but was discovered in connection with the air test, and widely applied to the test turbine and actual turbines in service with resulting improvements in over-all performance.

The nozzle-reaction test was discontinued in 1928, but the "air test" has remained as an ever-expanding tool for the turbine designers, and has continued to give gains in over-all turbine performance year by year. It has been expanded so as to cover not only turbine nozzles and buckets, but the flow through valves, inlet-steam chests, exhaust "hoods," crossover connections, piping connections, and all parts of a turbine where fluid flow with low loss is an important factor.

[10] "An Experimental Investigation of Nozzle Efficiency," by H. L. Wirt, Trans. A.S.M.E., vol. 46, 1924, pp. 981–1003.

Authors' Closure

As Mr. New states, there are some earlier publications on the art of flow testing as applied to steam turbines. They are all mentioned in Professor Stodola's book and are thus readily accessible. Therefore, it does not seem necessary to elaborate further on this matter.

The phenomenon of jet separation or stalling flow, as Mr. New calls it, was correctly explained by L. Prandtl in 1904. It truly is most destructive. Fortunately, this phenomenon is under our control. Except for related phenomena occurring at supersonic conditions, there is little reason to be apprehensive on this account. It appears the part of wisdom to devote the greatest effort toward the improvement of nozzles and buckets already known to be good.

Mr. New has suggested that the testing of nozzles grouped around a circle is an unjustified complication. We cannot agree with this attitude. However, he does admit its value for comparative assemblies of commercial significance. We fully agree with this opinion. In fact, we consider successful commercial application to be our one guiding principle. Therefore, our research must adapt itself to the additional space variable, uncomfortable as that complication may be.

The correction of impact-tube readings under supersonic conditions has been given careful consideration. No fear from wrong readings appears justified down to pressure ratios as low as 0.5. The velocities mentioned in the paper are based on an initial temperature of 185 F, which corresponds to a minimum over-all pressure ratio of 0.5. Thus, the highest velocity corresponds to expansion from double atmospheric pressure.

The instrument described by Mr. New furnishes, as results, velocity coefficients, assuming air to be an incompressible fluid. A compressibility correction is then introduced. Such a correction is based on a particular assumed average form of loss curve.

There can be no argument about the value and practicability of using assumptions and simplifications in connection with engineering problems. They always are present in great number and their existence and limitations must always be borne in mind.

In view of this situation, the authors felt inclined not to introduce, by a measuring method, additional deviations from true conditions which in turn involve additional watchfulness and corrections. This point of view has been held by the research engineers of the General Electric turbine air test since its inception, 19 years ago.

It appeared to be possible to build, without too great an effort, an instrument which is free from all limiting assumptions and correction factors. This course was so attractive that the General Electric Company designed and built its instrument based on this reasoning. It furnishes accurate efficiency directly from the measurements and without any limitations and qualifications.

Mr. New's instrument uses electrical measurements only. The use of purely electrical measuring equipment was, of course, considered during the design of our apparatus. However, its accuracy at the desired speed of operation was not accepted as satisfactory. Hence, the vastly more accurate and speedier photoelectric control of mechanical measuring equipment was chosen.

In the paper the authors mention that wood is used as model material. Following Professor Yellott's suggestion, this fact is herewith emphasized. Where standard partitions are tested, steel nozzle partitions are inserted in the wood. Novel and special partitions are carved from hard wood. Thus, there is great freedom in building new models and such saving in time that the speed of the instrument can be used to best advantage.

No further comment appears necessary on Mr. Warren's vivid description of the early history of the General Electric air tests.

An Investigation of Energy Losses in Steam-Turbine Elements by Impact-Traverse Static Test With Air at Subacoustic Velocities

By WINSTON R. NEW,[1] PHILADELPHIA, PA.

Efficiency of any turbine stage depends upon many variables. Blade pitch, orientation, height, and velocity ratio among purely geometrical quantities, and effects attributable to compressibility and viscosity of the medium all deserve special consideration. Investigation of energy losses as functions of the key variables is accomplished statically on various forms of equipment among which the impact-traverse tester is considered an important type. The necessary and sufficient conditions to be satisfied by the static test, in order that it represent fully dynamic conditions are (*a*) the flow relative to the given static element must be at all points the same in magnitude and direction as it would be if the element were in uniform rotation relative to the flow in a turbine; (*b*) output of the element measured statically must differ from power only by the linear-velocity term eliminated by the static test. The apparatus, procedure, and results attained in meeting these test requirements are explained by the author.

Nomenclature

The following nomenclature is used in this paper:

Symbol	Definition	Unit
	Geometrical Quantities (Refer to Fig. 1)	
α	Efflux angle. Angle which leaving velocity relative to blades makes with exit plane of row	deg
β	Entrance angle. Angle which approach velocity relative to blades makes with inlet plane of row	deg
h	Blade height	in.
l	Blade chord	in.
n	Number of pitches (passages) in test pack	1
o	Minimum opening between blades	in.
r	Mean radius of bladed annulus	ft
s	Blade pitch	in.
u	Peripheral blade velocity at radius r	ft per sec
w	Blade or groove width measured in direction of turbine axis	in.
Δw	Small distance beyond exit edge of blade measured in direction of w	in.
x	Coordinate of position of exploring tube measured in direction of s	in.
y	Coordinate of position of exploring tube measured in direction of h	in.
θ	Angular displacement	1
	General Quantities	
A	Minimum area of a passage	sq ft
c_p	Constant-pressure specific heat of air	Btu per lb per deg F
c_v	Constant-volume specific heat of air	Btu per lb per deg F
g	Acceleration of gravity	ft per sec per sec
ΔH	Enthalpy change at constant entropy	Btu per lb
J	Mechanical equivalent of heat	ft-lb per Btu
k	Exponent of isentropic expansion, c_p/c_v for an ideal gas to which air is assumed to conform	1
K	Constant of proportionality	
N	Useful power output of a stage	Btu per sec
Q	Aggregate energy loss in a stage	Btu per sec
t	Time	sec
W	Actual rate of flow	lb per sec
W_t	Theoretical rate of flow with purely isentropic changes of state	lb per sec
	Pressures, Densities, and Temperatures (Refer to Fig. 3)	
P_t, γ_t, T_t	Initial absolute air pressure, density, and absolute temperature in reservoir	lb per sq ft; lb per cu ft; deg F
P_s, γ_s	Absolute static pressure and air density at any point $(x, y, \Delta w)$ in a plane beyond and parallel to plane of exit edge of blades and distant Δw therefrom	lb per sq ft; lb per cu ft
P, γ	Absolute impact pressure (static plus dynamic) and air density at any point $(x, y, \Delta w)$ in a plane beyond and parallel to plane of exit edges of blades and distant Δw therefrom	lb per sq ft; lb per cu ft
P_1, γ_1, T_1	Barometric pressure, resulting air density, and absolute temperature at any point $(x, y, \Delta w)$ reached by actual expansion from conditions (P_t, T_t) in reservoir to environment	lb per sq ft; lb per cu ft; deg F
γ_1', T_1'	Air density and absolute temperature resulting from isentropic expansion from conditions (P_t, T_t) to pressure P_1	lb per cu ft; deg F
	Velocities and Forces (Refer to Fig. 3)	
C_t, F_t	Velocity and net force on blade pack resulting from isentropic expansion from conditions (P_t, T_t) in reservoir to atmospheric pressure P_1	ft per sec; lb

[1] Research Engineer, Westinghouse Electric & Manufacturing Company.

Contributed by the Power Division and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

C.....Actual velocity at any point (x, y, Δw) in plane beyond and parallel to plane of exit edges of blades and distant Δw therefrom. ft per sec

F.....Integral of elementary forces dF on blade pack resulting from elementary flows $d(W/g)$ at velocities C..... lb

F_1.....Net force on blade pack resulting from an actual flow W/g from conditions (P_t, T_t) in reservoir to atmospheric pressure P_1... lb

C_1.....Fictitious mean velocity for blade pack defined as quotient of actual net force and mass-flow rate, F_1g/W..... ft per sec

a.....Acoustic velocity in air at temperature resulting from isentropic expansion from conditions (P_t, T_t) to atmospheric pressure P_1..... ft per sec

Dimensionless Ratios

η.....Stage efficiency. Quotient of useful energy output and available energy

v.....Velocity ratio u/c_1

σ.....Aggregate energy loss in per cent of available energy

δ.....Energy loss along any particular filament from reservoir to measuring point in per cent of available energy

ξ.....Value of δ when effect of compressibility of fluid may be neglected

ω.....Efficiency of narrow slice dy of passage of height h, neglecting compressibility of fluid

ω_c.....Value of ω corrected for compressibility of fluid

ω_p.....Profile efficiency. Special value of ω_c characteristic of an infinitely long blade

Ω.....Efficiency of a blade passage from impact-traverse tests when evaluated, neglecting effect of compressibility of fluid

Ω_c.....Value of Ω corrected for effect of compressibility of fluid

φ^2.....Square of momentum coefficient for a blade passage defined as $(C_1/C_t)^2$

ϵ.....Exponent employed in formulating velocity distributions in blade passages

χ.....Compressibility factor

Introduction

Recent technical literature appertaining to problems of steam-turbine design contains many articles on the use of air in the study of turbine elements normally steam-driven. By "turbine elements" are meant, collectively, stationary or moving passages accommodating a flowing elastic fluid with enthalpy changes, functioning as generators or absorbers of momentum or both. Air rather than steam is used as a test medium because of practical considerations, such as economy, simplicity in design and operation of the equipment, less danger to observers who work with instruments directly in an exposed high-velocity jet, more precise measurements on a more homogeneous medium, easier control of test conditions, and the applicability of perfect gas laws to simplify analysis.

Similarity Requirements

Adequacy of air tests as a means of indicating performance coefficients for steam-driven apparatus required extensive justification. Some of the articles (1, 2),[1] referred to in the preceding paragraph, deal with the general theory of mechanical similarity applicable to this phase of model testing. It has been shown that, for dynamic similarity of flow of two elastic fluids with no free surfaces, four criteria of similarity must, in general, be satisfied, namely, geometrical similarity, and equality in Mach's, Reynolds', and atomic numbers. Fulfillment of these conditions insures equality in all dimensionless coefficients of which efficiency is one, appertaining to the two flows.

Variables Affecting Passage Efficiency

The turbine designer usually wants to know how passage efficiency depends upon blade shape, pitch, orientation, height, and velocity ratio at which the stage must operate. For any series of blades of different sizes but geometrically similar in cross section, it is possible to define three simple dimensionless ratios which, together with one angle, completely specify geometrical similarity. This is made clear by Fig. 1. If chord dimension l is chosen as a reference length, it is only necessary to know l_1, l_2, l_3, ... etc., to designate a blade of the chosen series, since the blade sections are geometrically similar. Then ratio s/l defines blade spacing or pitch measured circumferentially; ratio o/s defines blade orientation within the row; ratio h/l defines blade height; and angle β defines the velocity field in which the blades work. These parameters are called pitch/chord ratio, gaging, aspect ratio, and entrance angle, respectively, and collectively fix the geometry of the test passage. Efflux angle α

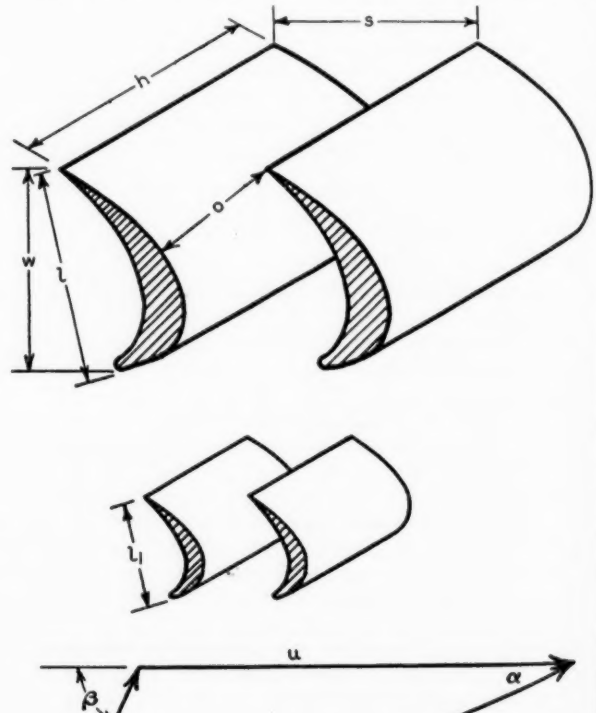


Fig. 1 Geometrically Similar Blade Passages and Conventional Velocity Triangle

$$\frac{u}{C_1} = \cos\alpha - \frac{\sin\alpha}{\tan\beta}$$

[1] Numbers in parentheses refer to the Bibliography at the end of the paper.

cannot be chosen at will. This angle, like the energy losses, is a consequence of the particular flow established. It is a measurable but not predictable quantity.

From the velocity triangle in Fig. 1, the actual velocity ratio of wheel speed to fluid speed is determined by the efflux and entrance angles. With a given pitch and gaging, it has been generally observed that the efflux angle is substantially constant for wide variations in any of the other independent variables. Hence, it is sometimes convenient to use velocity ratio interchangeably with the angles from which it may be calculated. Little generality is lost thereby provided the thought does not grow that α is absolutely invariant and, therefore, that changes in velocity ratio reflect changes in β only.

Experience has verified that the inequality in atomic numbers of air and steam is of no material consequence below acoustic velocity. Reynolds' number and Mach's number are the remaining parameters affecting a determination of energy losses. These ratios, arising from considerations unrelated to the geometry of the passage, indicate the relative magnitudes of forces affecting

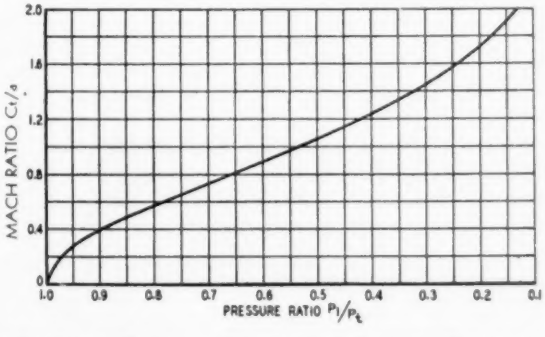


FIG. 2 MACH'S RATIO AS A FUNCTION OF PRESSURE RATIO FOR AIR

$$C_t/a = \sqrt{5\left[(P_t/_1P)^{(k-1)/k} - 1\right]} \quad k = 1.4$$

motion of a submerged fluid particle along a streamline and can be written directly from the terms of the Navier-Stokes equation.

In studies of flow through steam-turbine nozzle and blade passages, the range of velocities and viscosities encountered would theoretically permit of neglecting neither the influences of compressibility nor fluid shear. Prior to 1937, we had in our laboratory no facilities for the independent control of both Mach's and Reynolds' numbers over wide ranges.[3] We are, therefore, not yet in a position to discuss results of any experiments intended to separate these influences, but it has been found possible to obtain a comprehensive picture by considering their combined effect.

The Mach number for a constant value of c_p/c_v is a function of pressure ratio (3). The curve in Fig. 2 shows the Mach number as a function of pressure ratio for air. In our experiments, pressure ratio was obtained directly and, in view of the fact that it was then only necessary to read Fig. 2 to get the Mach number, reference will be made to this single variable. Efficiency measurements presented are characteristic of a certain Reynolds number as well as the particular Mach number. Along with the four parameters mentioned, is the dual parameter herein called Mach's number. These five ratios are key variables affecting efficiency of any blade assembly.

[3] Equipment recently installed provides for variation of back pressure on the test pack, thereby permitting independent control of velocity and kinematic viscosity and, hence, of Mach's and Reynolds' numbers. The new equipment bears the same relation to that described herein as the variable-density wind tunnel does to the atmospheric-pressure wind tunnel.

The variables retained are quantities which change in designing turbines of all sizes for a variety of steam conditions, employing blade sections of a chosen shape. Results which follow may therefore be expected to be of direct interest to turbine builders.

Since efficiency of any stage depends upon no fewer than five variables, there is need for some way other than the conventional economy test for providing information. This fact has greatly stimulated activity in fluid-mechanics laboratories maintained by a number of steam-turbine manufacturers. Investigation of energy losses as functions of the key variables is accomplished statically on various forms of equipment among which the impact-traverse tester herein considered is an important type.

TURBINE TESTS VERSUS STATIC TESTS

The power of any engine may be expressed as the product of torque and angular velocity

$$N = \frac{Fr\theta}{tJ} \quad \text{[1]}$$

If with a flow W lb per sec, an elementary turbine delivers power N Btu per sec at a constant angular velocity, θ/t turbine efficiency, defined as the quotient (energy output) ÷ (energy input), becomes

$$\eta = (N/W)/\Delta H \quad \text{[2]}$$

The value of η could be determined if some type of dynamometer were provided for the measurement of N, the flow rate W were measured by suitable means, and the energy, made available per pound of fluid by isentropic expansion from the initial-state point P_t, T_t to the final pressure P_s, were calculated. If only the energy losses arising from friction in the flow of fluid through closed passages are of interest, the elementary turbine may be further idealized by imposing the restrictions that there shall be no leaving loss or mechanical losses of any kind. Under these conditions the value of η would become the passage efficiency.

Flow, relative to the passages in an ideal elementary turbine, is unaffected by the uniform angular velocity. Theoretically, it would be possible to determine the basic performance of these elements with the wheel held stationary, if means of evaluating output could be substituted for the dynamometer. This is the idea underlying all static testing. The necessary and sufficient conditions to be satisfied by the static test, in order that it fully represent dynamic conditions, may be summarized as follows: (*a*) The flow relative to the given static element must be at all points the same in magnitude and direction as it would be if the element were in uniform rotation relative to the flow in a turbine. (*b*) Output of the element measured statically must differ from power only by the linear-velocity term eliminated by the static test. To comply with these requirements rigidly would be as impossible as to satisfy completely all the demands of dynamic similarity already mentioned. Some degree of arbitrariness must be introduced, and extensive satisfactory experience with the static test must remain the final proof of the adequacy of any procedure adopted.

From the definition of power represented by Equation [1], a comparison of elements statically or definition of their efficiency requires a comparison of forces. The ratio of force actually exerted on the blades per unit of mass flow per second to the corresponding quantity, assuming isentropic energy release, has been selected as a basic index of performance and named velocity coefficient. It is to be interpreted as a measure of the perfection of a passage as a machine for doing work. Velocity coefficient is commonly designed by φ. The extent to which the impact-traverse tests satisfy requirement (*a*) set forth in the preceding paragraph will become apparent from the description following.

Practical limitations have largely shaped the technique. The extent of conformity with requirement (ω) is less obvious and this is, therefore, made the subject of analysis in a separate section.

Impact-Traverse Tests—Description of Apparatus

An impact tube, oriented in the direction of a fluid flow at any point and suitably connected to a U-tube manometer or other differential pressure gage, is an instrument capable of indicating accumulated energy losses to that point from some chosen datum, provided that a satisfactory assumption regarding compression at the muzzle of the impact tube may be made. Having restricted the investigation to subacoustic air velocities, thereby eliminating the possibility of compression shock at the tube muzzle, and with a tube small in proportion to the dimensions of the passage to be studied aligned with the flow so as to receive the impact normally, it is fairly certain that the assumption of isentropic compression is satisfactory.

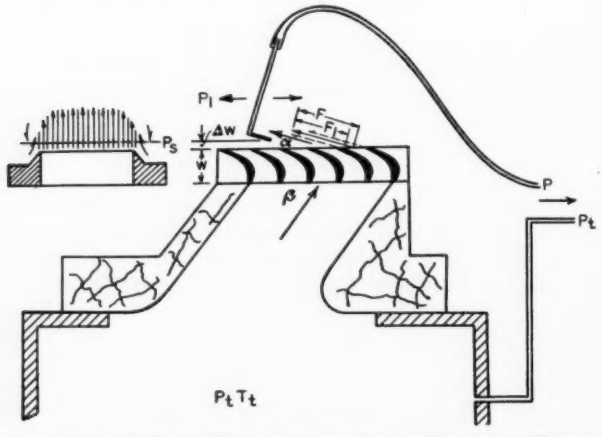


Fig. 3 Blade Pack Mounted for Impact-Traverse Testing

Referring to Figs. 3 and 4, a systematic measurement of total pressure, at many points along a line normal to the blade edges and in a plane distant Δw therefrom, will be called an impact traverse. Precaution against unnecessary disturbance of the flow finally led to an impact tube improvised from a hypodermic needle with 0.025 in. bore and 0.013 in. wall thickness set into a nicely streamlined rod and communicating with the tap through a hole drilled lengthwise through the rod. Smaller needles have been tried with some success, but generally they lack the rigidity required to stand without vibrating in a jet with velocities at times approaching acoustic value.

The impact tube was supported in the jet by a traversing mechanism built on the side of the reservoir from which the blade pack was supplied with air. The reservoir was sufficiently large that approach velocity could be neglected and the air considered flowing from rest at a pressure P_t and absolute temperature T_t. The position of the impact tube in space was controlled by three lead screws which permitted measurement of coordinates (x, y, Δw) from a chosen datum on the pack accurate to 0.001 in. Orientation of the tube on the traverse arm was controlled by two small worms and sectors which permitted rotation of the tube about two axes at right angles. These motions were locked when the apparatus was set for traversing. The tube was aligned with the flow by sighting on a fine silk thread attached to a thin steel rod which the operator held in the jet. The efflux angle α which the jet makes with the exit plane of the blades may be measured to within $\pm 1/2$ deg by aligning the edges of a bevel protractor with the pack and the thread. The search tube is insensitive to changes in velocity direction greater than the limit of accuracy in setting, hence, the accuracy of the total pressure measurements is unimpaired by the rather crude alignment technique.

Blade packs were made in the form of rectangular grids by soldering or doweling the foils to side bars and adding appropriate end pieces to enclose fully the passages. Special jigs and tools have been developed to secure sufficient accuracy in the location of foils. Success in attempts to allocate causes of small differences in efficiency to the many variables involved depends upon precise workmanship in the building of test packs. Flow to the blade packs from the reservoir was through wood approach passages of rectangular cross section with well-rounded entrance.

Energy losses in approach passages can be reduced by good design but can never be eliminated. These losses are not distributed but are localized at the walls. If a grid of several passages be used, and only the interior passages explored, the results will be almost entirely free from inlet losses over extremely wide ranges of the variables. Since only energy losses in the blades are of interest, the possibility afforded by the traverse-type test of selecting for study only those passages unaffected by inlet losses is a great advantage.

Studies of the influence of aspect ratio, gaging, and pitch/chord ratio require alteration of the blade pack for each test point. Variation in the velocity ratio can be accomplished statically by changing the entrance angle β, which is just the angle of the approach passage. The influence of Mach's number or pressure ratio is determined by tests with variation in the tank pressure P_t.

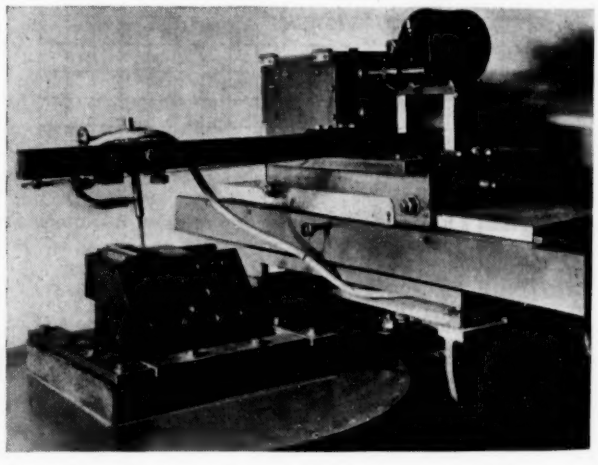


Fig. 4 Blade Pack and Traversing Mechanism Mounted on Air Reservoir

In any axial-flow turbine, the blades are radially disposed and not parallel as in the test packs. Except for the few large low-pressure stages of condensing turbines where blades of nonuniform cross section from base to tip are used, the ratio of blade height to mean diameter of the stage is small (less than 0.2 and usually nearer 0.1) and the flow may, without serious error, be considered two-dimensional.[4] Only cases with a blade of uniform cross section throughout its length are considered here. The work of such blading per pound of steam flow is normally calculated on the basis of velocities at the mean diameter of the row. The impact test should be considered as simulating a stage in which the blading is disposed on the mean diameter as in the test pack, and in which there is a minimum of variation in pitch and gaging from base to tip of the blade.

To measure only energy losses occurring in the element, the impact-tube muzzle should move in the plane of the exit edges of

[4] A three-dimensional flow is considered in calculations for long low-pressure blades.

the blades for which $\Delta w = 0$. This is simply saying that, in measuring friction losses per pound of fluid flowing through a given element, the traverse should exclude the entrainment-velocity field beyond the passage ends or that only those filaments should be included which would move through the element at theoretical velocity if there were no friction. With a finite-size impact tube, it is impossible to traverse in the plane for which $\Delta w = 0$, so we keep Δw a minimum. With the apparatus here described, practical limits of Δw are 5 to 15 per cent of the blade width depending upon the blade size. All that is necessary then is to connect a sensitive differential pressure gage across the search tube and reservoir in Fig. 3 and, with the initial pressure P_t held constant, to measure and to plot as a function of simultaneous values of the position coordinates (x, y), the pressure difference $(P_t - P)$. The datum or zero point of the coordinate system is arbitrary.

If only a few blade packs were to be thus explored, a differential manometer would serve very well as an energy-loss indicator and values of x and y could be measured in terms of the number of turns of the lead screws controlling the motion of the search tube. The labor involved in mapping out the dependence of energy losses upon a number of variables by these primitive means is prohibitive. The success of the undertaking rests entirely upon the adaptation of electrical means of recording and integrating the variation in impact pressure in the traverse plane at no appreciable sacrifice in the new order of precision in efficiencies made possible by direct-loss measurements.

To obtain an accurate continuous measurement of the pressure difference $(P_t - P)$, as the impact tube was driven across several pitches of the blade pack, we used the magnetic-strain gage (5). Developed as an instrument for precise measurement of small displacements, it was only necessary to construct a mechanism to move the keeper of the gage linearly proportional to pressure difference, and to provide suitable meters for recording and integrating the resulting voltage variations. The chart of the recording voltmeter and the impact tube were driven by synchronous motors operated from the same power supply. During the passage of the impact tube over n pitches of the pack, indicated by n cycles on the recorder chart and timed by stop watch, the integrating meter totaled a number of revolutions proportional to n P_s. The constant speeds of the integrator proportional to P_t and P_1 were established by test. To understand how energy-loss measurements can be evaluated so as to yield by summation a coefficient for the whole passage only negligibly different from φ^2 and to see how our instruments reduce the calculation to simple arithmetic, it is necessary to proceed with an analysis of the static test.

Keeping Δw a minimum and traversing only the projected area $h \times s$, it will be assumed that entrainment components are effectively eliminated and that the entire translational flow through the blade pack is included. Another section is reserved for comment on the validity of these assumptions following the analysis.

Analysis of the Static Test

Referring to Fig. 3, the jet as a whole has no rotational-velocity component and, in the plane of the exit edges of the blades for which $\Delta w = 0$, velocity c at angle α and static pressure P_s exist at any point. If dA represents an element of area normal to velocity C, application of the second law of motion leads to

$$\int_0^A \frac{\gamma_s}{g} C^2 dA + \int_0^A (P_s - P_1) dA = F_1 \dots\dots [3]$$

Here F_1 is the net force experienced by the blade pack as a result of a flow W lb per sec from initial conditions (P_t, T_t) to the atmosphere P_1. It will be observed immediately that F_1 differs from the change in momentum by a term composed of a static-pressure difference acting on a projected area A.

The function of any turbine element is to derive a force from a change in the momentum of the motive fluid. Strictly interpreted, therefore, any static test makes possible a determination of efficiency of an element only if means are provided for measuring force F_1, pressure P_s, and area A in Equation [3].

The force F_1 could be measured with fair accuracy in a number of ways that have been frequently described in the literature of static testing. Great precautions are necessary to avoid extraneous forces arising from entrainment of the atmosphere. For the plane for which $\Delta w = 0$, the area A is just the minimum area of the passages. Measurement of the static pressure P_s in such small regions of high velocity, as those encountered behind blade grids, offers difficulties[5] but is possible if we are not too concerned with errors due to turbulence. Fortunately, a very accurate integration of the static pressure in the survey plane is not needed to satisfy the accuracy ordinarily sought in the energy-loss measurements.

The velocity coefficient φ may now be more nicely defined as the ratio of mean momentum per unit mass flow to ideal momentum per unit ideal mass flow and expressed analytically by Equation [4], if ΔF_t represents the second term of the left-hand member of Equation [3], and F_t represents the net force on the blades with isentropic change of state.

$$\varphi = \frac{g(F_1 - \Delta F_t)}{W} \div \frac{g(F_t - \Delta F_t)}{W_t} \dots\dots [4]$$

Equation [4] may be recast in series form and factored as

$$\varphi = \left(\frac{F_1}{W} \div \frac{F_t}{W_t}\right)\left(1 + \frac{\Delta F_t}{F_t} - \frac{\Delta F_t}{F_1} + \frac{\Delta F_t^2}{F_t^2} - \frac{\Delta F_t^2}{F_1 F_t} + \frac{\Delta F_t^3}{F_t^3} - \frac{\Delta F_t^3}{F_1 F_t^2} + \dots\right) \dots [5]$$

The first factor does not involve the static pressure P_s. Using only three terms of the second factor, it is a matter of arithmetic to show that, with a value of the first factor as low as 0.9 and $\Delta F_t/F_t$ as high as 0.05, the first factor can differ from φ by little more than 1 per cent. A value of ΔF_t equal to 5 per cent of the total theoretical force on the blade pack would require an integrated pressure difference $(P_s - P_1)$ over the area A of 5 per cent of the pressure drop $(P_t - P_1)$. In no case have our static-pressure measurements yielded a difference approaching this value.

The commonly accepted practice of disregarding the second factor of Equation [5] and along with it the need for measurement of pressure P_s, and defining φ simply as the first factor of Equation [5] appears justifiable, at least for subacoustic velocity work and will therefore be adopted.

Accordingly

$$\varphi = \frac{F_1}{W} \div \frac{F_t}{W_t} = \frac{C_1}{C_t} \dots\dots [6]$$

where C_1 is a fictitious mean velocity defined as shown by the actual net force on the blade pack and the actual flow rate. It is from this expression that the term φ receives the name velocity coefficient. The more general use in turbine calculations of energy expressions than velocity expressions has caused the square of the velocity coefficient φ^2 to be more often used as a criterion of passage performance than is φ itself. Energy coefficient or efficiency are names frequently given to φ^2. Regardless

[5] Refer to A. G. S. Sandison's comment in discussion of (4) and authors' closure.

of name, it is important to remember that φ^2 is the square of a momentum coefficient which is analytically the basic quantity.

Summarizing, the elimination of the terminal static pressure P_s restricts interpretation of results to the degree of perfection of an indeterminate expansion combined with an isentropic adjustment of pressure and velocity to the environment. To the extent that this latter adjustment is negligible, velocity coefficient is a true barometer of passage losses.

Following Stodola (3) the flux of energy per second through cross section $h \times s$ in the plane of the exit edges of the blades in Fig. 3 may be equated to the available energy less the aggregate friction losses per second.

$$\int_0^A \left[\frac{C^2}{2g} + \int_{P_1}^{P_s} \frac{dp}{\gamma}\right] dW = \int_0^A \left[\int_{P_1}^{P_t} \frac{dp}{\gamma_t}\right] dW - Q \quad [7]$$

Assuming isentropic compression at the impact tube and

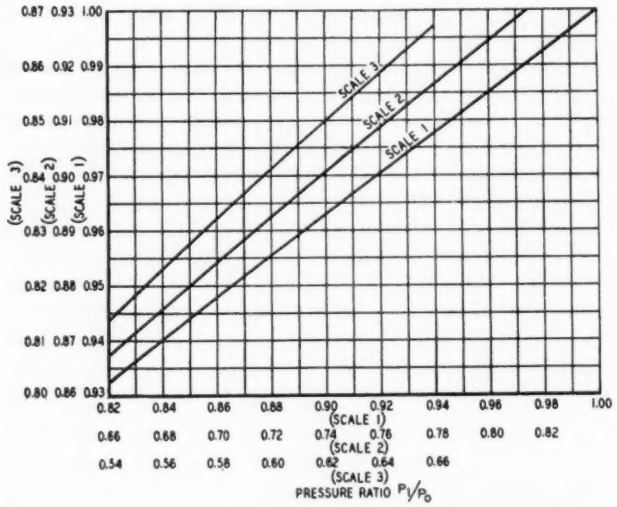


FIG. 5 COMPRESSIBILITY FACTOR AS FUNCTION OF PRESSURE RATIO FOR AIR

$$\left[\left(1-\frac{P_1}{P_0}\right)(k-1)\right] \div k\left[1-\left(\frac{P_1}{P_0}\right)^{(k-1)/k}\right] \quad k = 1.4$$

dividing through by the first term of the right-hand member of Equation [7] we may write

$$\frac{\int_0^A dW\left[\int_{P_1}^{P} \frac{dp}{\gamma}\right]}{\int_0^A dW\left[\int_{P_1}^{P_t} \frac{dp}{\gamma_t}\right]} = 1 - \sigma \quad [8]$$

where σ is aggregate friction loss in per cent of the available energy.

For any elementary filament along which the flow rate is dW, an expression for the efficiency becomes

$$\frac{\int_{P_1}^{P} \frac{dp}{\gamma}}{\int_{P_1}^{P_t} \frac{dp}{\gamma_t}} = 1 - \delta \quad [9]$$

Efficiency has been purposely written (1 — losses). The possibility of direct-loss measurements in the impact test is an advantage not to be overlooked.

The integrations indicated in Equation [9] may be performed by making use of the pressure-volume relationship for isentropic behavior of a perfect gas, yielding Equation [10] in which the only unknown quantity is γ, the density at the muzzle of the impact tube where the air is brought to rest. The quantities χ

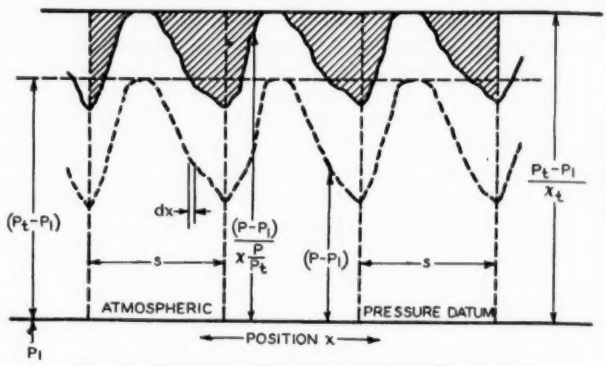


FIG. 6 PITCHWISE TRAVERSE AT SAME POSITION y (Loss area shaded.)

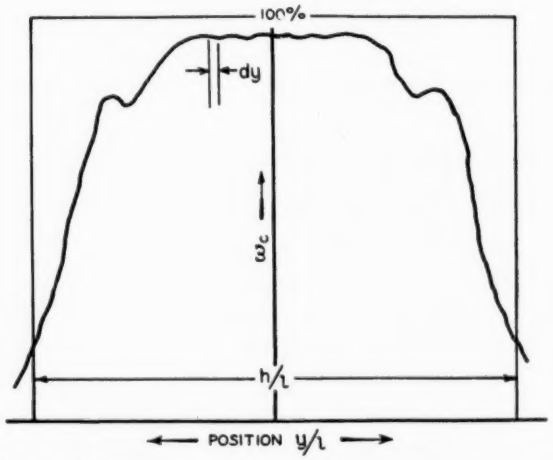


FIG. 7 REPRESENTATIVE EFFICIENCY-DISTRIBUTION DIAGRAM

and χ_t in Equation [10] are compressibility factors defined by Equation [11]. For a gas with a fixed ratio of specific heats, the compressibility factor is a function of pressure ratio and may be evaluated for $k = 1.4$.

$$1 - \delta = \frac{P - P_1}{\chi\gamma} \div \frac{P_t - P_1}{\chi_t\gamma_t} \quad [10]$$

$$\chi_0 = \frac{\left(1 - \frac{P_1}{P_0}\right)(k-1)}{k\left[1 - \left(\frac{P_1}{P_0}\right)^{(k-1)/k}\right]} \quad [11]$$

To eliminate the unknown density γ, it may be observed that, along any filament, the air starts from rest in the reservoir and comes to rest at the impact-tube muzzle without doing any external work. Furthermore it will be assumed that there has been no heat transfer from neighboring filaments. The cycle of expansion and compression may therefore be considered a throttling and, since the medium is nearly a perfect gas, the temperatures at densities γ will be constant and equal to the initial temperature T_i. Equation [12] follows directly

$$\gamma = \gamma_t \frac{P}{P_t} \quad [12]$$

Substituting for γ in Equation [10], gives an expression for

efficiency at a point in terms of measurable pressures and the compressibility factors.

$$1 - \delta = \frac{P - P_1}{\chi \frac{P}{P_t}} \div \frac{P_t - P_1}{\chi_t} \quad \ldots\ldots [13]$$

A pitchwise traverse at some position y along the blade pack would appear as the dotted curve in Fig. 6, where pressures are plotted as functions of positions x with linear scales on both axes. Using the compressibility factors χ and χ_t, and the pressure ratio P/P_t it is possible to correct the pressure traverse, point by point, as has been done in Fig. 6, yielding the full-line curves. The integrations

$$\omega_e = \frac{\int_{-ns/2}^{ns/2} (1 - \delta) dx}{\int_{-ns/2}^{ns/2} dx} \quad \ldots\ldots [14]$$

indicated by Equation [14] may be performed graphically. The value of ω_e so determined is a coefficient of an elementary slice dy of n pitches of the blade pack. Values of ω_e may thus be determined for many slices until the entire height h has been traversed.

An efficiency-distribution diagram, such as Fig. 7, is obtained by plotting ω_e vs. y/l. The result of a second graphic integration Ω_e defined by Equation [15] has been shown in an appendix to

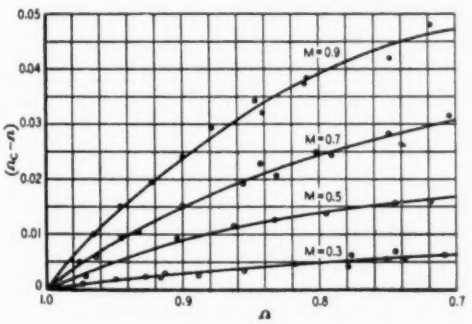


FIG. 8 COMPRESSIBILITY CORRECTION

this paper to be very nearly equal to φ^2 where φ is defined by Equation [6].

$$\Omega_e = \frac{\int_{-h/2}^{h/2} \omega_e dy}{\int_{-h/2}^{h/2} dy} \quad \ldots\ldots [15]$$

LIMITATIONS OF THE METHOD

With aspect ratios in excess of unity, a value of Ω_e, established according to the method described, is substantially invariant with small changes in Δw or with changes in the limits of integration of the numerator of Equation [15] from $\pm h/2$ to $\pm \infty$. Under these conditions, no significant incompatibility with theory arises as a result of arbitrary selection of a test plane and traversing limits, and the measured value of Ω_e has the status of an efficiency measurement in the plane of the exit edges of the blades for which $\Delta w = 0$.

As aspect ratios decrease below unity, impact measurements, intended to represent losses in the passage but made at a small distance Δw above the exit plane, become increasingly inaccurate due to difficulties encountered in eliminating the entrainment-velocity field which becomes an appreciable part of the whole, or in attempting to take it into account. The only satisfactory method that has been devised to handle these cases is to run traverses at decreasing values of Δw, stopping with the minimum physically attainable and, by extrapolation, to determine Ω_e at $\Delta w = 0$. The accuracy of this process is dependent upon how close to the blades the last reading can be obtained.

Attempts have been made to measure Ω_e at some arbitrary value of Δw by traversing beyond the passage ends to values of ω_e near zero as suggested by Tollmien and performed on long thin slots by Förthmann (9). The over-all efficiency established thereby on blade packs appears too high. Reference to the end view of the grid in Fig. 3 shows why this may be so. Secondary flow in the passages to which reference will be made later seems to be a factor in preventing an orderly entrainment of stagnant air

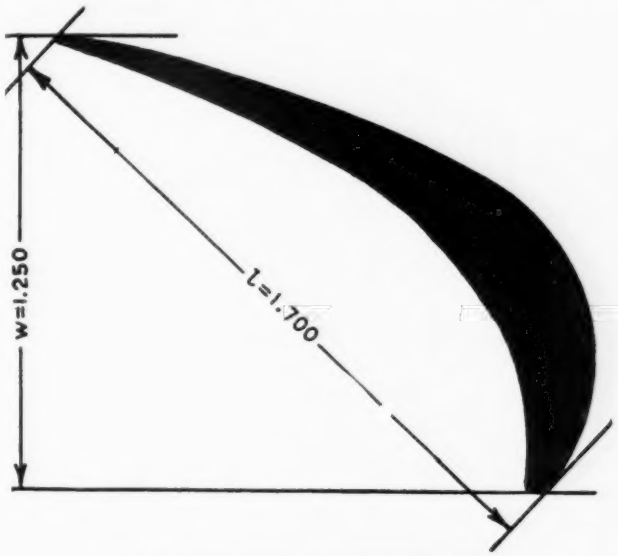


FIG. 9 REACTION BLADE, TWICE FULL SIZE
(These blades were always assembled with the width dimension shown and therefore, may be called constant-orientation blades.)

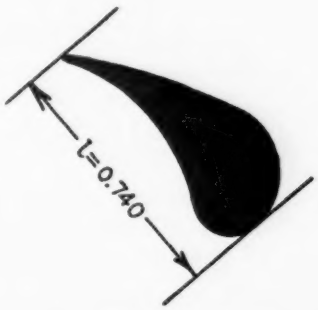


FIG. 10 STREAMLINED REACTION BLADE, TWICE FULL SIZE
(The width dimension was varied in the several assemblies employing these blades, which, therefore, may be called variable-orientation blades.)

by the escaping jet. A fair measurement of impact pressure can be made on those entrained filaments closely adjacent and moving in essentially the same direction as the motive flow. In the arbitrary plane, however, traversing must extend theoretically to infinity and hence include filaments flowing obliquely and frequently backward. A zone of such disturbance cannot be accurately explored with a fixed-tube orientation and, lacking partial compensation for readings obtained in the region of orderly flow, integration of the whole distribution diagram leads to optimistic results. It is for this reason that we have restricted traversing

to the projected area $h \times s$ in a plane at minimum distance Δw from the blade edges.

Mechanization of the Impact-Traverse Test

It was observed that integrations of the pressure traverses alone gave values of ω and Ω differing from ω_e and Ω_e by small amounts dependent only upon the Mach number and the indicated efficiency. By indicated efficiency is meant the integrated result from the pressure record. Thus, from a great number of tests on all kinds of blade sections at various pitches, orientations, heights, entrance angles, and Mach's numbers, it was found possible to plot compressibility corrections as in Fig. 8.

This step permitted an evaluation of Ω_e in any case by just adding a correction to Ω determined by double integration of the pressure traverses and allowed complete mechanization of the traverse test. The expressions for ω and Ω are the simpler forms related to incompressible fluids thus

$$\xi = \frac{P_t - P}{P_t - P_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots [16]$$

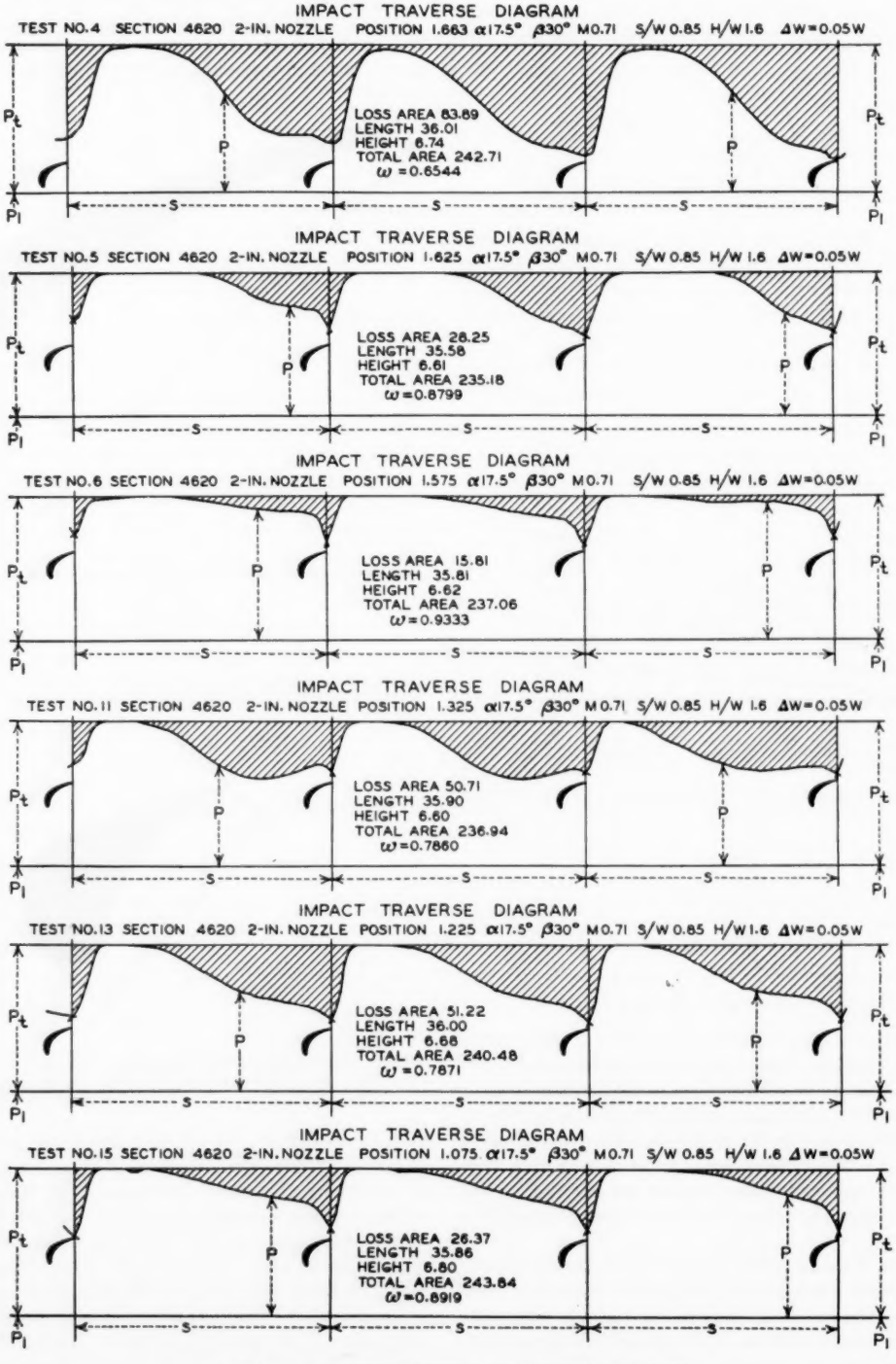


Fig. 11 Traverse Records for Inefficient Flow

$$\omega = 1 - \frac{\int_{-ns/2}^{ns/2} (P_t - P)dx}{(P_t - P_1)\int_{-ns/2}^{ns/2} dx} \quad \ldots\ldots\ldots\ldots [17]$$

$$\Omega = \frac{\int_{-h/2}^{h/2} \omega dy}{\int_{-h/2}^{h/2} dy} \quad \ldots\ldots\ldots\ldots\ldots [18]$$

Equation [16], which follows from Equation [13] by neglecting the denominators of the right-hand member, reveals the simplicity of direct-loss measurements. It was simple to construct a pressure mechanism for actuating the strain gage with displacements proportional to a pressure difference. However, it would not have been so simple had we been forced to work entirely with the terms involving pressure ratios to some power of k, which occur when compressibility of the medium must be considered.

Results, Discussion, and Conclusions

From the foregoing exposition of the measurement and evaluation of impact pressure over the exit plane of a turbine element, it follows that the traverse record taken in the direction of pitch s for different constant positions y along the blade height h is indicative of the energy losses. Its structure is of fundamental origin revealing the extent of friction and, therefore, responds to favorable and detrimental stimuli such as strong acceleration of particles in the direction of flow and loss of fluid contact with walls, respectively. The structure of the traverse record should respond only secondarily to changes in the geometry of the test pack. On the other hand, the efficiency-distribution diagram, which is a plot of the mean efficiencies from traverse records as a function of position along the blade height, is indicative of the extent and localization of the losses. Its structure changes in a characteristic way with the geometry of the pack and test conditions. These two types of plots give exhaustive information on the nature and magnitude of the energy losses. The change required to produce an improvement in efficiency is nearly always evident, although it may sometimes be impractical of attainment.

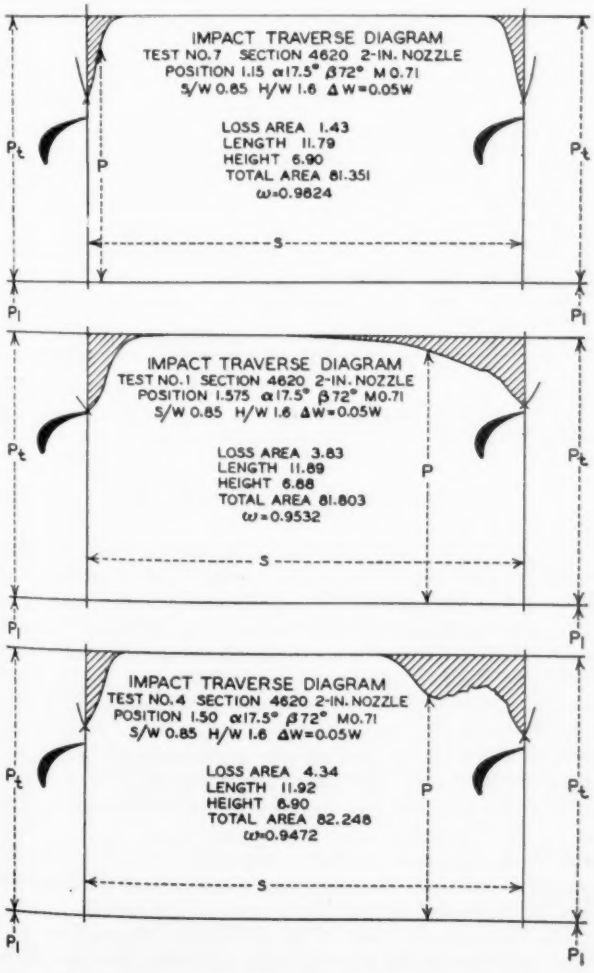


Fig. 12 Traverse Records for Efficient Flow

Cross sections of the foils on which the tests presented were obtained are given as Figs. 9 and 10. These profiles are fairly representative of extremes in mechanical strength required in ordinary reaction stages and will be referred to simply as a reaction blade, Fig. 9, and a streamlined reaction blade, Fig. 10. The former with thin leading edge might be expected to exhibit efficiencies more dependent upon the direction of approach of the fluid than the latter. Fig. 9 could, therefore, be called a fixed-orientation blade, whereas Fig. 10 would be called a variable-orientation blade. Since each of the sections shown is one of a series of geometrically similar shapes, covering a range of sizes, the discussion applies to the entire series rather than to the one blade size.

The traverse records, consisting of a group of 3-pitch traverses in Fig. 11, and a group of single-pitch traverses in Fig. 12, have been selected from a file of several thousand as samples of the type of record obtainable. The group in Fig. 11 represents successive slices of the foil in Fig. 9, assembled as indicated on the records and fed at an entrance angle of 30 deg at Mach's number 0.7. In Fig. 12, the entrance angle has been increased to 72 deg, everything else remaining the same. The smaller entrance angle corresponds to a flow with a large component of impingement on the concave surface of the foil and an eddy field at the convex surface. The larger entrance angle is near the optimum for the blades in question. For a fixed-orientation blade, the width dimension w is chosen as a reference length instead of the chord dimension l for forming the dimensionless ratios s/w, h/w, etc. For a variable-orientation blade, l is used since it is a dimension of the profile itself and is therefore independent of how the profiles are set in the row. Designating the end of the blade pack remote from the search tube as zero the term "position" on the traverse records is simply y/w. Since h/w for the pack was only 1.6, the records for tests Nos. 4 and 5, Fig. 11, were obtained in the entrainment field beyond the pack end.

It will be observed that the chief difference between the traverse records is the rapidity of the impact-pressure recovery at the convex surface of the foil. The loss area associated with the convex surface can vary from a narrow cusp indicative of only a viscous-friction boundary layer to a large irregular pattern emerging from the entire passage with fluctuations attesting the existence of severe eddies. The loss area associated with the concave surface is nearly always of the type attributed to viscous friction. A rational explanation of this basic difference lies in a consideration of the relative stability of streamlines near the two walls. It has been found that the flow conditions prevailing in the boundary layers dictate whether the flow in the entire passage shall be relatively efficient or inefficient (6).

In curved passages centrifugal force causes a pressure increase and velocity decrease at the concave surface of the blade and a pressure drop and velocity rise at the convex surface, compared with the pressure and velocity prevailing at the central filament. Proceeding along a streamline through the passage near the

concave surface, the fluid particles experience a falling pressure. In this boundary layer the pressure gradient provides a motive force on the particles, overcoming the friction drag of the wall, and there is less tendency toward stagnation. The fluid-friction drag transmitted from more rapidly moving particles in the center is in the same direction as the pressure drop. There is no tendency for the flow to leave this wall and the boundary layer is thin and stable. The small loss area on the record reflects these favorable conditions.

Proceeding along a streamline through the passage near the convex surface, the fluid particles may not experience a falling pressure and may even encounter a pressure rise, depending upon the severity of the vortex arising from passage curvature. There may be no motive force on the particles in this boundary layer in the direction of flow other than that transmitted by shear from the body of the stream. Wall friction brings the outside layer of particles to rest, establishing a local reverse-pressure gradient which forms a couple with the drag forces transmitted to these particles from the more rapidly moving adjacent layers. A small

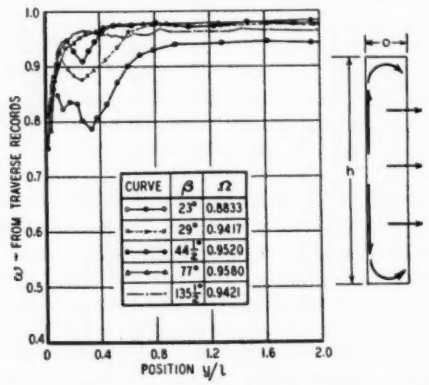


FIG. 13 DISTRIBUTION OF TRAVERSE EFFICIENCY AT VARIOUS ENTRANCE ANGLES FOR STREAMLINED REACTION-BLADE SECTION

$s/l = 0.81$ $h/l = 4$ $o/s = 0.30$
$\alpha = 17$ deg $\Delta w = 0.12w$ $M = 0.30$
$T_1 = 70$ F (approx) $P_1 =$ atmosphere
(Profile Fig. 10)

eddy results; the boundary layer becomes unstable and leaves the wall. Disturbances growing from such elementary losses of fluid contact may fill the entire passage with devastating effect on the energy of translational flow. It is the purpose of the remaining paragraphs and efficiency-distribution diagrams to show the dependence of both ω and Ω on the five important variables selected at the beginning of this paper.

Operation at other than the optimum velocity ratio has the largest effect on passage efficiency of the variables considered. This may be accomplished statically by varying the entrance angle of the flow. In Fig. 13 are shown five distribution diagrams for the highly efficient, heavy-duty type of reaction blade shown in Fig. 10. Owing to symmetry about the mid-height of the blade, the diagrams are shown from the center line to one end only. The spacing and orientation of the blades in the pack and the aspect ratio are average for the section considered. In the turbine, groups of these blades normally operate at a Mach number of 0.3 and this value was accordingly chosen for the air test. The range of entrance angles selected would about cover the extremes encountered in marine practice from maneuvering to full-load propeller speeds. Tests on a pack of very different proportions are represented by the diagrams in Fig. 14. Here the foil with thinner leading edge shown in Fig. 9 has been assembled and tested under conditions simulating its performance in the stationary element of an unsymmetrical stage near the high-pressure end of a turbine where aspect ratios are small. Complete assembly information and integrated results are given on each figure. Each point on every distribution diagram resulted from one traverse record, samples of which are shown in Figs. 11 and 12.

Figs. 13 and 14 reveal a more orderly retardation of the flow heightwise than pitchwise. Particularly in the case of large aspect ratio, but also discernible in Fig. 14, is a tendency for a central region of uniform loss flanked by extreme localized losses at passage ends. With decrease in the entrance angle, end losses become more severe and reach further into the passage until they

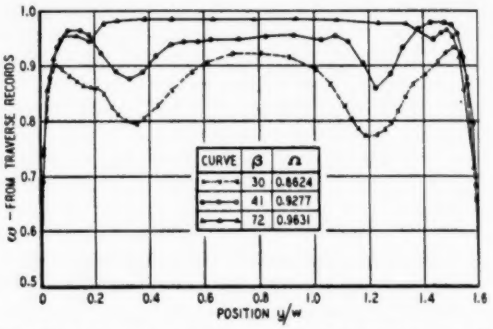


FIG. 14 DISTRIBUTION OF TRAVERSE EFFICIENCY AT VARIOUS ENTRANCE ANGLES FOR REACTION-BLADE SECTION

$s/w = 0.85$ $h/w = 1.6$ $o/s = 0.30$
$\alpha = 17\frac{1}{2}$ deg $\Delta w = 0.05w$ $M = 0.70$
$T_1 = 70$ F (approx) $P_1 =$ atmosphere
(Profile Fig. 9)

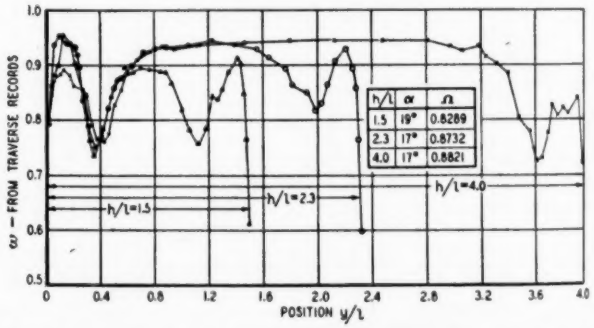


FIG. 15 DISTRIBUTION OF TRAVERSE EFFICIENCY AT VARIOUS ASPECT RATIOS FOR STREAMLINED REACTION-BLADE SECTION

$\beta = 22\frac{1}{2}$ deg $M = 0.30$ $\Delta w = 0.12w$
$s/l = 0.74$ $o/s = 0.30$
$T_1 = 70$ F (approx) $P_1 =$ atmosphere
(Profile Fig. 10)

eventually bridge over, reducing the uniform central region to a point, or even annihilating it altogether. The localization of losses at the passage ends was recognized early in the course of our tests as a manifestation of secondary flow attributable to the same vorticity that augments the losses along the entire convex surface of the foil. In the auxiliary sketch in Fig. 13, if the rectangle represents the minimum area of a passage, the curved arrows indicate the direction of the secondary flow. The pressure built up at the concave surface of a blade diminishes at the passage ends as a result of the heightwise velocity distribution. This pressure gradient along the blade seeks relief in transverse flow at the passage ends to the low-pressure region at the convex surface of the adjacent blade, where it enters substantially normal to the main flow, forcing a partial loss of fluid contact with the wall and attendant destruction of useful energy.

The characteristic shape of all distribution diagrams for passages above a certain limiting value of aspect ratio permits a separation of the losses into two distinct classes. It appears

almost certain that the uniform central portion of the diagram would not improve with indefinite extension of the passage height. This portion may be termed profile efficiency analogous to the profile drag used in aeronautics, and may be considered the efficiency limit with infinitely long foils assembled at the given pitch

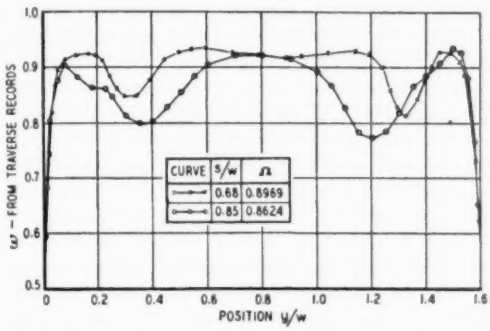


FIG. 16 DISTRIBUTION OF TRAVERSE EFFICIENCY AT VARIOUS PITCH TO WIDTH RATIOS FOR A REACTION-BLADE SECTION

$h/w = 1.6$ $M = 0.70$ $\Delta w = 0.05w$
$s/w = 0.85$ $o/s = 0.30$ $\alpha = 17^1/_2$ deg $\beta = 30$ deg
$s/w = 0.68$ $o/s = 0.275$ $\alpha = 15^1/_2$ deg $\beta = 30$ deg
$T_t = 70$ F (approx) $P_1 =$ atmosphere
(Profile Fig. 9)

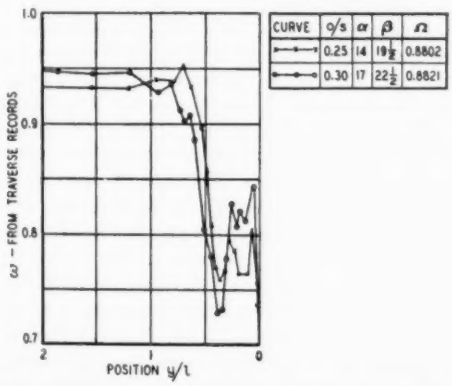


FIG. 17 DISTRIBUTION OF TRAVERSE EFFICIENCY AT VARIOUS GAGINGS FOR A STREAMLINED REACTION-BLADE SECTION

$s/l = 0.74$ $h/l = 4$ $M = 0.30$
$\Delta w = 0.12w$
$T_t = 70$ deg (approx) $P_1 =$ atmosphere
(Profile Fig. 10)

and gaging and operating at the chosen entrance angle and Mach's number. The secondary-flow losses may be treated as a correction factor for finite aspect ratios. The correction factor, as well as the profile efficiency, is a function of the other four variables. It resembles the induced-drag term in aeronautics. For chosen values of the variables, distribution diagrams at various aspect ratios have been superimposed in Fig. 15. For aspect ratios above that exhibiting a profile flow at the center of the pack, a good approximation of Ω_c can be obtained by making use of the fact that the end losses are inversely proportional to the aspect ratio. This is expressed

$$\frac{1 - \frac{\Omega_c'}{\omega_p}}{1 - \frac{\Omega_c}{\omega_p}} = \frac{h/l}{h'/l} \quad \ldots\ldots\ldots\ldots\ldots\ldots [19]$$

Passages with aspect ratio so small that no profile flow develops are of great importance in the high-pressure ends of steam turbines. In such cases, a chaotic merging of the secondary flows takes place with increase of the losses faster than indicated by the linear relationship in Equation [19]. The intended orderly flow is not approached even remotely, and efficiency as a function of the several variables can be determined only by test.

Distribution diagrams showing the effects of variation in pitch and gaging, gaging alone, and Mach's number are shown in Figs. 16 to 18, respectively. Sufficient data are given in each case to identify the test. In general, pitch increases improve the profile efficiency just as increases in aspect ratio improve the over-all

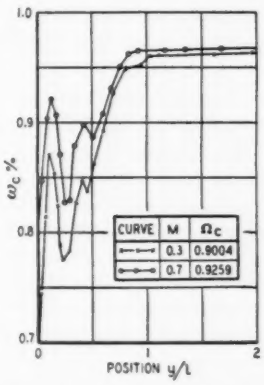


FIG. 18 DISTRIBUTION OF TRAVERSE EFFICIENCY AT VARIOUS MACH'S NUMBERS FOR A STREAMLINED REACTION-BLADE SECTION

$s/l = 0.89$ $h/l = 4$ $o/s = 0.34$
$\Delta w = 0.12w$ $\alpha = 19^1/_2$ deg $\beta = 25$ deg
$T_t = 70$ F (approx) $P_1 =$ atmosphere
(Profile Fig. 10)

efficiency by diluting the loss concentrations at boundary walls with larger regions of potential flow. The term potential flow is used advisedly here. Meticulous observations of the pressure difference $(P_t - P)$ over large portions of the area of efficient packs, using a differential column with light oil as indicator, reveal losses in the reservoir pressure head only in the order of a few hundredths of one per cent. This is entirely negligible and justifies the practice of considering friction losses in turbine elements as boundary conditions surrounding a core of fluid in which energy exchange is isentropic (3). Pitch increases aggravate the end losses and are in this respect similar to a decrease in the entrance angle. Affecting the two basic-loss divisions oppositely, it may be seen that improvement in efficiency can result from increasing or decreasing the blade pitch, depending upon the particular combination of other variables encountered. This conclusion has been repeatedly checked by experiment.

Variation in blade pitch and gaging is frequently an easy way to improve efficiency at little expense. The method is, therefore, very appealing in practice. The qualitative explanation for a large reduction of losses is usually very simple, resting upon some extensive change in the structure of the secondary flow which is apparent on the efficiency-distribution diagram. Thus, in Fig. 16, an improvement in efficiency of more than 4 per cent results from a 20 per cent reduction in the pitch of the foils, together with the 9 per cent reduction in gaging, obtained by simply moving the foils closer together in a groove of uniform width w.

Increase in the Mach number means stronger acceleration of the fluid particles and, hence, less stagnation at any wall, reverse flow, or the building up of eddies. The foregoing exposition of the mechanism of eddy formation contrasted with stable boundary layers indicates an improvement in efficiency with increase in Mach's number. Experiments seem to confirm this reasoning but, until further results have been accumulated in which the influ-

ences of compressibility and viscosity are separated, little more can be said about the dependence of efficiency upon Mach's and Reynolds' numbers.

Little has been done about the extension of traverse tests to superacoustic velocities. The fact that most steam-turbine power is generated at subacoustic velocity justified thorough exploration of this field before attempting interpretation of results complicated by the phenomenon of compression shock. The possibility of efficient utilization of supercritical speeds with attendant reduction in size per kilowatt output is, however, a problem worthy of investigation and a legitimate extension of this project.[6] Regardless of methods employed, the value of some accurate quantitative work in this practically untouched field can scarcely be overestimated, and even fragmentary results will be of interest.

Acknowledgments

The efforts of Carl Deiner, who builds all of the test packs and maintains the equipment herein described in perfect working order, are recognized as a very significant contribution to the success of the entire venture. Much credit for the orderly prosecution of various test programs belongs to J. H. Borton, and the group of laboratory assistants without whose cooperation and painstaking attention to detailed application of the method, results would have been impossible. Finally, the author wishes to accredit the work of Anderhub, Loliger, and others who, as pupils of Stodola, were among the first to recognize the value of the traverse test, and to pay tribute to H. L. Wirt and the joint authors B. Hodkinson and A. E. Devey, who in later years extended application of the method and capably analyzed some results.

An appendix containing derivations to which reference is made in this paper, has been prepared by the author and is available upon request.

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Discussion

Ronald B. Smith.[7] The static-air test, similar to that described by the author and others, is one of the most useful research tools available to the turbine designer. When used intelligently it yields at worst a good approximation to most of the hydrodynamic questions that a turbine builder faces. The degree to which the results are approximate is measured by how faithfully the model tests reproduce the hydrodynamic conditions which exist in the turbine. Because certain conditions of similarity cannot be satisfied statically, the judgment of the designer is, as always, the important consideration.

Deviations from dynamic similarity in tests of this nature are of two types; fundamental deviations which are largely academic in interest, and practical deviations which are probably the ones of greatest importance and yet are the most difficult for the designer to evaluate.

Dynamic similarity for a submerged body exists, providing the model is geometrically similar to the original and providing an equality exists between the model and the original in respect to Reynolds' number, Mach's number, atomic number (as mentioned by the author), and in addition a heat-transfer number. Providing the flow losses are small or the turbulence low, it is reasonable to neglect the dimensionless heat-transfer criterion. With the aid of the variable-density test equipment, the author is able to differentiate between the influence of the Reynolds number and the Mach number but, in regard to the criterion prescribed by the atomic number, the situation is not as clear. Equality of atomic number necessitates that the tests be conducted with a medium having the same ratio of specific heats as in the original turbine. If the tests are made in air and the original operates in steam, it is obviously impossible for the specific-heat ratios to be equal. As far as the writer is aware, the only attempt to evaluate the approximation resulting from a neglect of this criterion has been made by Ackeret and his associates. They conclude that for reasonable velocities, say below about one-half the acoustic, no appreciable discrepancy exists. The author mentions tests extending to velocities which approach the acoustic without apparent consideration of what effect this more or less academic dissimilarity may have.

Dissimilarities of a practical nature which exist between the flow around the blade and that in the turbine arise from a number of causes. If the steam velocity is less than the acoustic, then pressure disturbances are transmitted by waves traveling at the velocities of sound both upstream and downstream from their source. This means that a blade in any location receives pressure impulses originating from disturbances at the blades both upstream and downstream from it, and the flow around any blade is in consequence influenced by these results. This is a condition which is obviously not met in a static assembly and at abnormal velocity ratios is probably one of importance.

Another point of practical dissimilarity arises when the blades are distributed circumferentially around the rotor of a turbine and in the static test they are assembled parallel. Actually, in the flow through the turbine, the pressure drop which exists between a stationary blade and its rotating partner is not uniform. Even though the areas are not laid out for a potential flow with constant circulation, the fluid, adjusting itself to the easiest course, attempts to approximate this condition. There usually exists a greater pressure drop over the tip of the rotating blade than at its base, and a greater pressure drop over the base of the stationary blade than over its tip. These pressure variations must have a marked effect on the secondary flow in the passage; it is certain that they introduce an effect which is difficult to approximate on a static test.

The efficiency of impulse-type blading is affected by the ejector and entrainment losses of the working fluid at the ends of the buckets. This condition arises from the fact that the bucket

[6] As this paper goes to press we have concluded our first year's detailed study of the performance of turbine elements at supersonic velocities. This exploration involved many problems not encountered in the subsonic velocity work.

[7] Development Engineer, Elliott Company, Jeannette, Pa. Mem. A.S.M.E.

overlaps the nozzle. The end losses which exist are difficult to simulate by static test, while their magnitude, at least for buckets not specially sealed at their ends, is for certain velocity ratios as great as the ordinary aspect-ratio correction which the author finds for individual nozzles or reaction blades.

The work delivered by a given turbine blade depends upon the change in momentum in the direction of motion. This momentum change involves not only a knowledge of the efficiency of the passage but a measure of the angle of efflux. The author does not discuss this feature in connection with the test, although it is a consideration to which the designer must devote some attention. These angles cannot be measured with great accuracy; in addition they vary considerably over the discharge section. While it is true that as long as the angle is small, an appreciable error in its determination can be tolerated, since the cosine of the efflux angle is the variable at point, on low-pressure blading and on impulse buckets, the angles are larger and an accurate measurement is of considerable significance. In this connection it might be well to mention the problem of blade pitch. When the aspect ratio is large it is not uncommon to find that the suggestion is made that an increase in the pitch of the blading will improve the efficiency. This suggestion has for its base the mutual interference between blades and has been applied with marked success on the Kaplan runner and on propeller fans. While it is frequently true that an increase in efficiency in the passage is realized with larger pitch, it is also true that this is generally accompanied by an increase in efflux angle so that the final work done in the row is not increased as rapidly as one might at first expect.

In conclusion, it should be emphasized that the author's paper is a worth-while contribution to the art. The remarks of the writer are not intended as a criticism but are merely offered to show that the results derived from static-test equipment, while of real value, must be interpreted with discrimination by the designer because the static test does not always approach conditions which are similar to those met in the final machine.

J. R. Weske.[8] From theoretical investigations by the writer on the interference of successive rows of turbine blading, it appears that for modern high-efficiency blades the dynamic effects are of such magnitude that they cannot be neglected if satisfactory correlation of static-test results and stage efficiency is desired. A brief exposition of the essence of these investigations follows:

When plotting instantaneous-velocity diagrams on the basis of the velocity traverse obtained from static tests, it is seen that the subsequent blades operate in a stream of cyclically varying direction. This produces corresponding variations of circulation around the blades, which in turn cause a sheet of vortexes with axes parallel to the blade to leave the trailing edge of the blade. These vortexes are related to the starting vortex on airfoils and, like the latter, they produce an additional drag on the blade corresponding to the energy consumed by them. The effect of these vortex sheets upon airfoil grids in cascade arrangement has been investigated by Ackeret and Keller.[9]

Another approach to the theoretical determination of interference between successive blade rows can be made through investigation of what is known in aerodynamics as the Katzmayr effect.[10] This effect refers to deviations of the direction of the lift force on an airfoil with change of angle of attack and its effect upon drag or, in the case of turbine blades, upon pressure drop across the row. Calculation of this effect is complicated by phase lag and hysteresis effects at high frequency.

The writer has not been able to check his calculations experimentally. However, he believes that his assumptions will be confirmed in a qualitative way, by photographic investigations with the aid of moving-picture films of two-dimensional flow at low Reynolds' numbers, samples of which were shown in connection with the oral discussion.

J. I. Yellott.[11] This paper is particularly valuable because of the detailed manner in which the author sets forth the theoretical background on which these tests are predicated. It is to be hoped that in his reply to the discussion the author will elaborate somewhat upon the mechanism used for recording and interpreting the impact data. The technique of testing with great rapidity and yet with very high accuracy by means of electromechanical devices is extremely valuable. The taking and interpreting of such data by the more usual methods would be a task of impossible magnitude.

The author does not mention static traverses, and it would be interesting to know whether any such traverses have been made. Although the absolute accuracy of such measurements would be questionable, they would probably reveal the existence of unexpectedly low pressures behind the convex sections of the buckets. At high initial pressures, this reduction of static pressure might well be enough to cause the velocity at these points to be superacoustic. The usefulness of the impact tube for measuring velocities near the acoustic velocity has been discussed,[12] at some length. An earlier investigation[13] in the superacoustic field is that of T. E. Stanton. The Rayleigh theory, commented upon by Stanton, indicates that there should be only a slight error in using the impact tube for Mach's numbers up to 1.3.

Author's Closure

It is a pleasure to acknowledge the value of the discussions, both written and oral, which by their quality, and not mere wordiness, have done much to amplify and improve the author's communication to the profession. To generalize, it seems appropriate to state that a number of the points brought out by the contributors were mentioned, and some were treated at length, in the parts of the original manuscript which were deleted before publication in the interest of brevity.

In the presentation of the paper, more attention than accorded in the text was given to emphasizing the three categories into which known similarity criteria may be grouped, namely, geometrical quantities, fluid-dynamics quantities or properties, and turbulence, which being concerned with temporal as well as spatial variation of velocities seems to deserve a separate rank. The discussers recognize, of course, that the results presented dealt almost entirely with the effects of variation in the geometry of the turbine stage and, therefore, in no wise constituted a complete picture. The geometrical variables are, however, the first concern of the turbine designer.

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Note. A joint discussion by G. B. Warren of this paper and the paper, "Automatic Integrating Pressure-Traverse Recorder for Study of Flow Phenomena in Steam-Turbine Nozzles and Buckets," by H. Kraft and T. M. Berry, appears on page 479 of this issue of the Transactions.

Variable-fluid-density equipment is practically indispensable in separating the influences of viscosity and compressibility. It was pointed out that we are not yet prepared to publish our measurements in this field, but that tests had been in progress for quite some time, and had, in fact, yielded some significant results. Atomic number seems to deserve little more than mention in subsonic-velocity investigations and, with the influence of turbulence, we have only recently begun to work. Aerodynamical physicists in this country have spent the better part of 10 years studying the effects of turbulence on the flow of fluids through passages and around obstacles. Strangely enough, however, it seems safe to say that few steam-turbine engineers are even now remotely acquainted with the meaning of turbulence in the sense in which that term is employed in the more critical analyses of fluid-flow problems.

The author rests his conviction in the value of air tests of the type described on the fact that they make possible a systematic prosecution of a broad program embracing both the nature and magnitude of energy losses in turbine elements and their mode of dependence upon all the key variables involved. Recognition of the dependence of stage efficiency upon from 5 to perhaps 10 variables (depending upon how meticulous we want to be) automatically eliminates the possibility ever of obtaining adequate design information from conventional turbine-economy tests alone. The alternative rather generally adopted consists of orderly examination, by one or more of several exploratory methods available, of the flow through the fundamental elements interspersed by verification tests in an experimental turbine on a workable synthesis of many of the most promising features yielded by the basic investigations. Aeronauticists will quickly recognize that this course is quite parallel to the one they have pursued so profitably. Static tests carefully conducted and skillfully analyzed almost invariably lead to correct comparisons.

It is from the combined study of the experimental turbine performance and the preceding static tests relating thereto that one may partially evaluate the practical deviations of the latter from complete dynamic similarity. Effects totally missing or materially distorted in static tests have been outlined by Mr. Smith. The author strictly avoided implication that over-all turbine-economy tests could be dispensed with. The point of view is rather that their interpretation must rest upon detailed knowledge of the behavior of individual elements as revealed by the more flexible research methods developed.

The results of static-pressure traverses, which play such an important role in the formulation of the method for numerical evaluation of the search-tube studies, had to be condensed into the few statements given in the paper. A brief description of our mechanism for recording and intergrating the impact-tube data, requested by Mr. Yellott, has been made a part of the author's discussion of the paper[14] by H. Kraft and T. M. Berry.

The employment of the so-called "Fales technique," and the method of plaster casts for visualizing velocity distribution in turbine passages, and even the earlier search-tube studies themselves were crude, as Mr. Warren has stated. We seldom trust surface markings now for anything but the roughest kind of check as to whether some change has been beneficial or detrimental.

[14] "Automatic Integrating Pressure-Traverse Recorder for Study of Flow Phenomena in Steam-Turbine Nozzles and Buckets," by H. Kraft and T. M. Berry, appearing on page 479 of this issue of the Transactions.

Photoelastic Analysis of Stresses in a Steam-Turbine Blade Root

By J. J. RYAN[1] and J. T. RETTALIATA,[2] MILWAUKEE, WIS.

The analysis of the stresses in steam-turbine blade roots by photoelastic methods has particular reference to the determination of the loadings on the multiple steps. In this paper, from a comparison of test and calculated values, the stress-concentration factors are obtained for the neck and step sections.

Description of Problem

THE development of large-capacity steam-turbine-generator units has created the necessity for new studies in stress distribution throughout the high-speed rotating elements. This analysis of present-day structures makes possible the new projects in machine construction which are to be developed in the future.

The photoelastic method of analysis is recognized as a reliable means for determining stress values. This method was applied in the determination of stress distribution at the roots of the steam-turbine blade and spindle sections of one of the largest-diameter high-speed steam turbines in service. In the last stage of the low-pressure element of this 3600-rpm turbine, the revolving blading is 18 in. in length and is mounted on a drum $40^1/_2$ in. in diameter. A construction having these proportions approaches the limit of present-day design practice.

Fig. 1 shows a single blade, and Fig. 2 the steam-turbine spindle disk to which it is fastened by means of a multiple-step root. In Fig. 3 is shown the completely bladed low-pressure spindle. The weight of the blade is $4^1/_2$ lb and its centrifugal force at the normal speed of 3600 rpm is about 42,000 lb. As there are 110 blades on the periphery of the rotating drum, the necessity for compact design of the blade roots limits the shape and size of the sections. By calculation and test those sections must be chosen which have the most favorable stress distribution. In general, such a condition requires:

1 A system which is susceptible to calculation and to complete analysis.

2 Adjustment of loading and proportioning of parts to maintain calculated stresses below a given limit.

3 Maximum stress concentrations not exceeding the elastic limit of the material at overspeed operation.

It is the purpose of this paper to indicate how these factors were considered in the design of the last-row blade roots in a modern 35,000-kw 3600-rpm tandem-compound steam turbine.

Description of Apparatus

The model of the cross section of the steam-turbine blade root was made to several times full scale of transparent bakelite. In order to obtain stresses in the spindle portion between the blade roots, three blade models were set in parallel positions, the radial angle between blades in the machine not being sufficient to warrant consideration.

Machining the spindle slots and blade roots in the model was performed by toolmakers who shape the broaches and gages for finishing the turbine rotor and blades. More than ordinary care was observed in matching the respective loading elements of the model, and satisfactory fits were secured as evidenced by the symmetrical fringe patterns obtained. A short specimen bar of the bakelite material was machined for calibration purposes. An illustration of the completed model and the calibration bar is shown in Fig. 4.

The model was mounted in the photoelastic polariscope in the machine-design laboratory of the mechanical-engineering department, Institute of Technology, University of Minnesota. This polariscope,[3] shown schematically in Fig. 5, is the conventional transmission type of prism polarizer but is unusual in detail and size, having a 7-in-diam light field and a length of 18 ft. The light source is a mercury-vapor lamp used in conjunction with a Wratten 77A filter which transmits green light with a wave length of 5460 A. Thus the light emerging from the filter is monochromatic. The water cell is employed to cool the light rays, the heat of which would otherwise affect the polarizing prism. The axes of the polarizer and analyzer prisms are set at right angles to each other so that the plane-polarized light leaving the former will not be transmitted by the latter. If the bakelite model is placed in the light path between the two prisms and stressed, it becomes doubly refracting which causes bands of light to pass through the analyzer prism. Thus the construction of the model and the degree to which it is stressed determine the pattern of the fringes or stress bands from which an analysis of the stresses existing in the prototype can be made.

A triple-lever system, Fig. 6, supported on knife-edges loaded the three blades equally in the stressing frame. The spindle element, in one piece, was fastened to the top of the frame. Rigid supports prevented lateral movement of the sides. A view of the assembly in the polariscope is shown in Fig. 7.

Test Procedure

In order to determine the optical constants of the bakelite, a calibration bar of rectangular cross section was placed in the loading frame of the polariscope and subjected to pure bending. This resulted in the fringe pattern shown in Fig. 8. As the load applied to the calibration bar was known, the stress at its outer edge was easily determined. From Fig. 8, it was observed that there were five fringes from the neutral axis to either edge of the calibration bar. Thus, for the particular bakelite material of which the model was made, the increment stress per fringe was equal to the stress as determined by the foregoing means divided by five. These fringes are lines representing constant difference of the principal stresses $(P - Q)$ at any point.

The principal stresses are of particular importance in the photoelastic analysis of stress in models. Given a rectangular plate of unit thickness oriented in the x and y coordinate directions

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[2] Allis-Chalmers Manufacturing Company.

Contributed by the Power Division and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] "Photoelastic Analysis of T-Tail Stresses," by J. J. Ryan, *Journal of The Franklin Institute*, vol. 223, no. 6, June, 1937, pp. 715–730.

with distributed forces along the sides as shown in Fig. 9 (*a*), the direct stresses P and Q are the maximum and minimum stresses, respectively, that may exist in the plate, and are therefore called the principal stresses. If the plate is rotated an angle θ with respect to the coordinate axes, as shown in Fig. 9 (*b*), other stresses normal to sections along the coordinate axes are found to exist. The resulting stress normal to the x-axis would be N_y, and normal to the y-axis would be N_x. Shearing stresses along both axes would have the value S_{xy}.

The coordinate stresses N_y and N_x change in value during rotation of the plate, as indicated by the dashed lines, approaching stresses P and Q at coincidence of the axes. The shearing stresses S_{xy} are zero in the direction of the principal stresses, Fig. 9 (*a*), and vary as the function sin 2 θ, Fig. 9 (*b*), where for $\theta = 45$ deg, they are maximum and equal to one half the difference of the principal stresses.

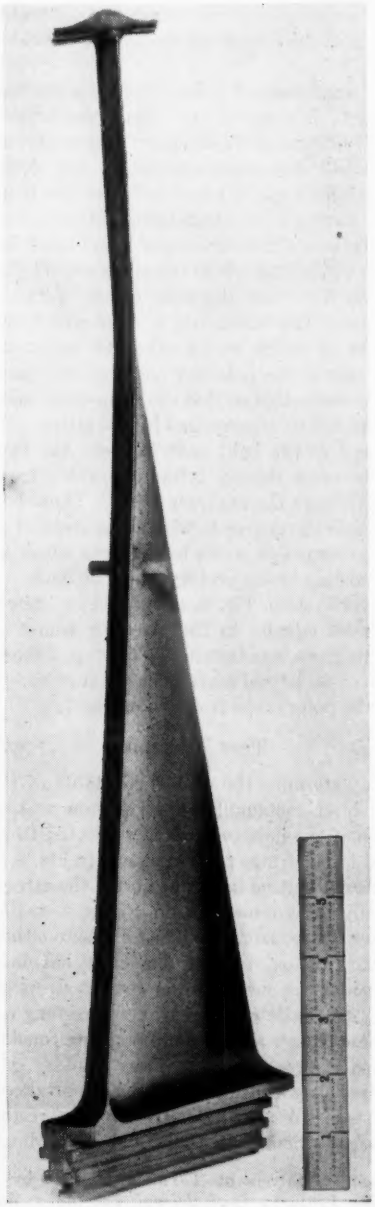


FIG. 1 LOW-PRESSURE BLADE FOR LARGE STEAM TURBINE

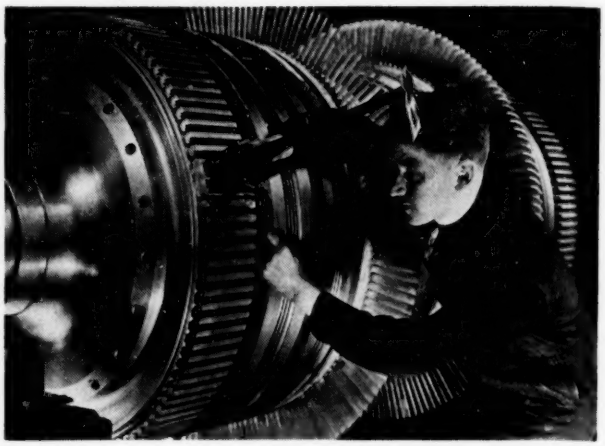


FIG. 2 BLADING LOW-PRESSURE SPINDLE OF 35,000-KW STEAM TURBINE

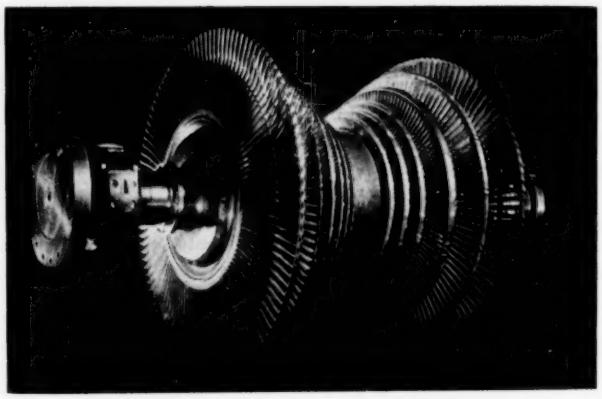


FIG. 3 COMPLETELY BLADED LOW-PRESSURE SPINDLE

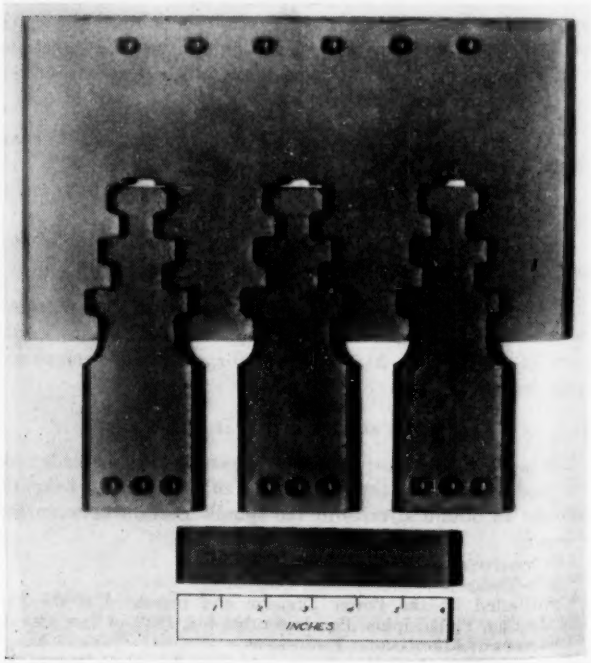


FIG. 4 COMPLETED BAKELITE MODEL AND CALIBRATION BAR OF THREE STEAM-TURBINE BLADE ROOTS

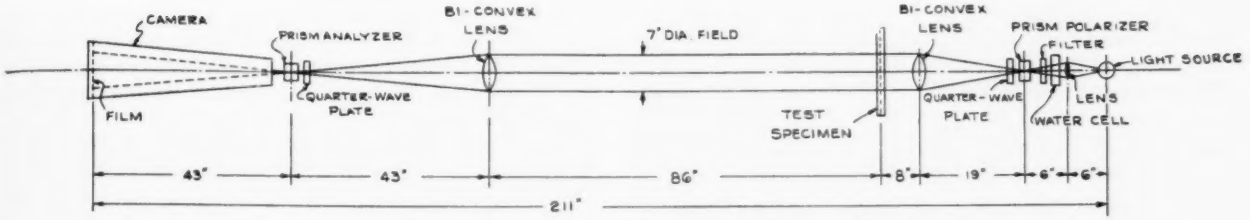


FIG. 5 SCHEMATIC DIAGRAM OF PHOTOELASTIC POLARISCOPE

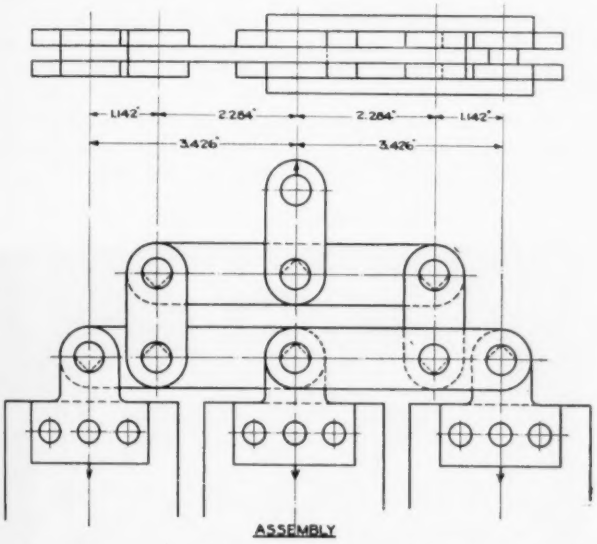


FIG. 6 TRIPLE-LEVER SYSTEM USED TO LOAD BLADES EQUALLY IN STRESSING FRAME

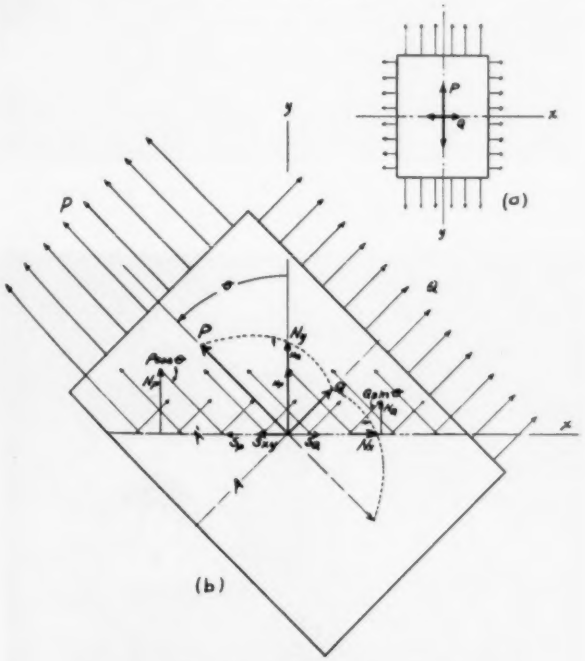


FIG. 9 DEMONSTRATION OF PRINCIPAL STRESSES

Principal stresses exist at every point in a stressed body. In the photoelastic analysis of suitable transparent materials, polarized light establishes interference bands or fringes which are proportional to the difference in value between the principal

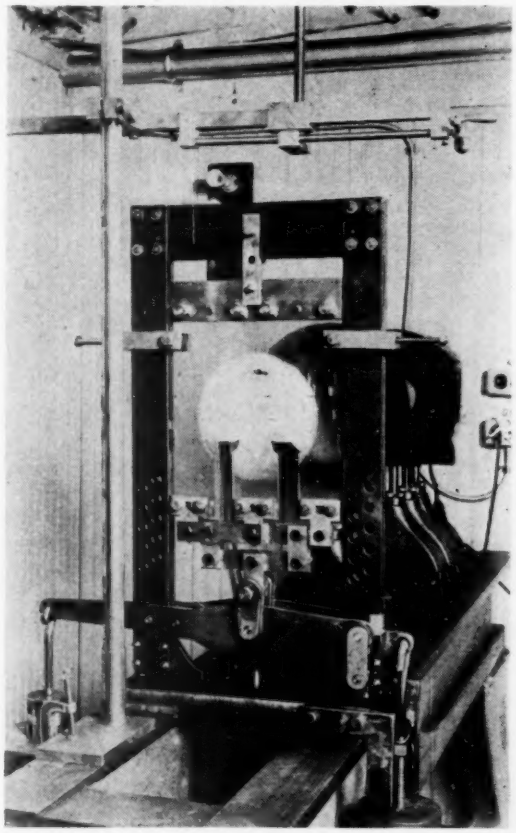


FIG. 7 BLADE-MODEL ASSEMBLY IN FRAME ON POLARISCOPE

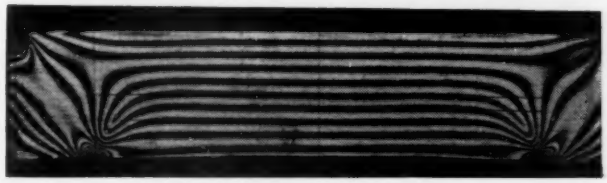


FIG. 8 FRINGE PATTERN RESULTING FROM PURE-BEND TEST ON CALIBRATION BAR

stresses, denoted figuratively as $P - Q$. An adjustment in the polarized condition of the light further obtains values of the angle θ, thus determining directions of the principal stresses.

To calculate the value of the individual principal stresses P and Q, with the difference known at any point, the sum of the principal stresses $P + Q$ must be obtained. This is usually accomplished by measuring the change in thickness at the same points on the photoelastic model, since the sum of the principal stresses is proportional to the change in thickness.

This change in thickness, due to application of load to the cali-

bration bar, was measured by a Huggenberger extensometer.[4] The extensometer, shown in Fig. 10, has a magnification of 1206 and is read to 0.01 unit. Thus the stress at the outer edge of the calibration bar divided by the number of units change in thickness under load represented the stress per unit change in thickness. A compression at a point was measured as negative thickness change, and tension a plus change. From the change-of-thickness readings, the calibration for the sum of the principal stresses $(P + Q)$ was thus determined.

[4] "Fatigue of Shafts at Fitted Members, With a Related Photoelastic Analysis," by R. E. Peterson and A. M. Wahl, *Journal of Applied Mechanics*, Trans. A.S.M.E., March, 1935.

After these necessary measurements on the calibration bar had been made, the actual model was put under a light load to determine the points of contact on the steps of the root elements. It was necessary, first, to obtain symmetrical patterns of stress at the several sections, so that comparative data would be obtained and, second, to eliminate surface roughness at regions where stress concentrations of no general importance existed. As little additional fitting as possible was performed on the model since at higher loads inequalities would tend to adjust themselves.

The model was considered satisfactory when at very light loads indications of stress were shown at all the contact surfaces and at heavy loads the stress pattern was adequately symmetrical.

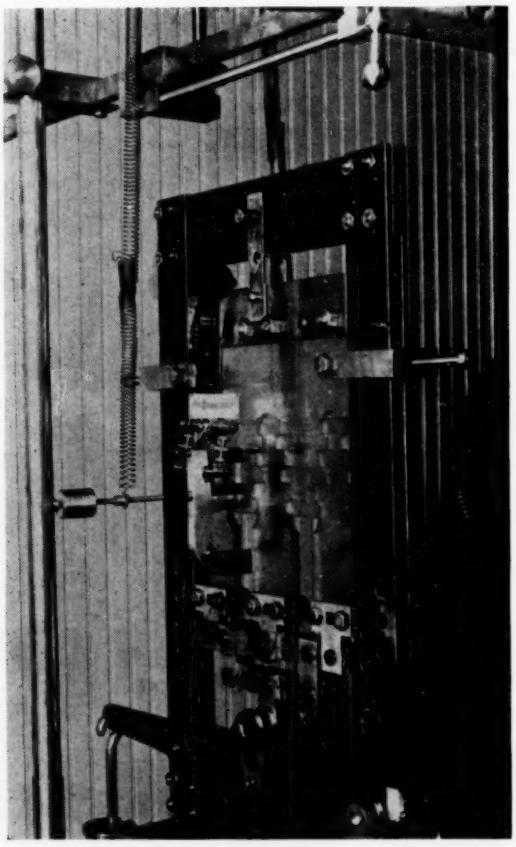


FIG. 10 EXTENSOMETER IN USE ON MODEL



FIG. 12 ISOCHROMATIC PHOTOGRAPH OF STRESS DISTRIBUTION IN MODEL UNDER 780-LB LOAD

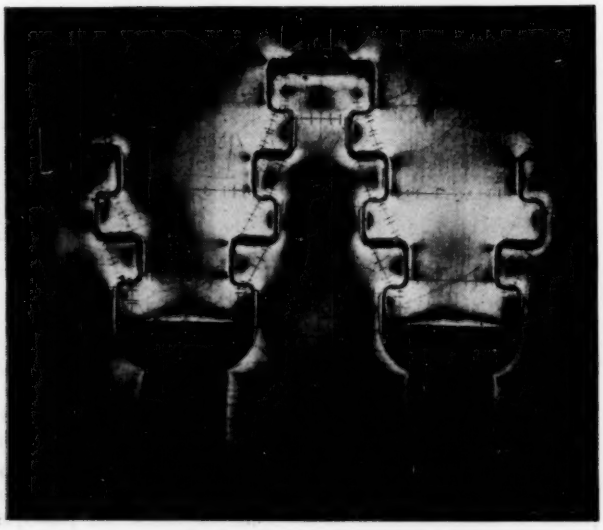


FIG. 11 PHOTOELASTIC PHOTOGRAPH OF MODEL UNDER LIGHT LOAD

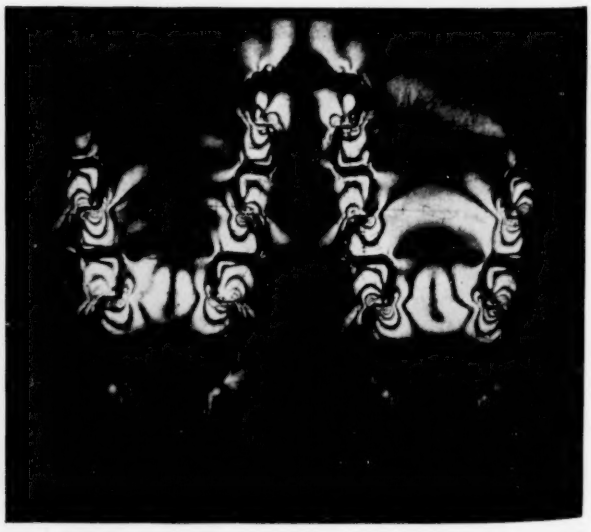


FIG. 13 PHOTOGRAPH OF 0-DEG ISOCLINICS; FULL LOAD

Lines, from points of tangency of the fillets, were scribed on the model at the sections of maximum stress and crossmarks were located at intervals to establish absolute positions.

The stressing frame was adjusted to support freely a live weight of 38.76 lb, the loading mechanism having a magnification of 20. The weight was suspended on a lever for ease in relieving and applying the load during the tests.

The photoelastic or isochromatic photograph of the model under light load is reproduced in Fig. 11. The stress distribution in the model with a force of 780 lb is given on the isochromatic photograph Fig. 12. From this illustration the difference of the principal stresses and the values of maximum shear are determined.

For complete stress analysis it is necessary to determine the isoclinics in the model. An isoclinic is a line which represents the locus of all points in the model having principal stresses in the same directions. To find the isoclinics, the quarter-wave plates were removed from the optical system and the prisms rotated together for given angular intervals. Isoclinic photographs were made for angles of 0, 15, 30, and 45 deg. Fig. 13 shows a photograph of the 0-deg isoclinics.

The method for finding the values of the principal stresses occurring at any point will now be described. To determine the number or order of the fringe, at the particular point where the stresses are desired, the fringes that cross this point as load is applied are counted progressively. This number is multiplied by the stress per fringe as found from the calibration bar, and the value obtained is the difference of the principal stresses $(P - Q)$. The maximum shear at this point is $(P - Q)/2$ and thus can be obtained immediately. The number of units change in thickness at the point, measured by the extensometer as shown in Fig. 10, multiplied by the stress per unit change in thickness, as obtained from the calibration bar, determines the sum of the principal stresses $(P + Q)$. The individual values of P and Q can be found by solving simultaneously. P and Q are principal stresses. To find the coordinate stresses, a Mohr diagram must be plotted as shown in Fig. 14. The angle 2θ is twice the angle which is represented by the isoclinic occurring at the given point in the model.

A sketch of the central root element is given in Fig. 15, showing the sections at which the thickness measurements were made. Complete stress data were then determined for the fourteen sections listed.

TABLE 1 ORIGINAL TEST DATA

Line	$P - Q$ position, 0.1 mm	Band no.	Stress, lb per sq in.	Position, 0.1 mm	$P + Q$ extensometer reading	Stress, lb per sq in.
I	2.5	9	2916	17.4	−0.055	−360
	4.2	8	2592	33.0	−0.02	−130
	6.5	7	2268	65.0	−0.005	− 32
	9.0	6	1944	76.6	.....	0
	12.5	5	1620	108.5	.....	0
	17.4	4	1296	125.0	.....	0
	23.1	3	972	140.8	.....	Edge
	26.7	2	648	...	.....	...
	67.6	1	324	...	.....	...
	110.3	1	324	...	.....	...
	140.8	..	Edge	...	.....	...
X	4.0	4	1296	16.5	0.19	1231
	11.6	3	972	32.5	0.15	971
	25.9	2	648	65.0	0.10	648
	86.5	1	324	97.5	0.095	615
	140.0	1.1	350	130.0	0.065	422
	183.5	1.33	405	183.5	0.08	518
XIV	3.8	7	2268	16	−0.04	−260
	6.6	6	1944	33.5	−0.02	−130
	9.2	5	1620	67.5	−0.01	− 65
	14.6	4	1296	84.0	−0.01	− 65
	27.7	3	972	117.5	−0.025	−162
	43.4	2	648	135.5	−0.03	−194
	90.7	1	324	153.0	.....	Edge
	153.0	..	Edge	...	.....	...

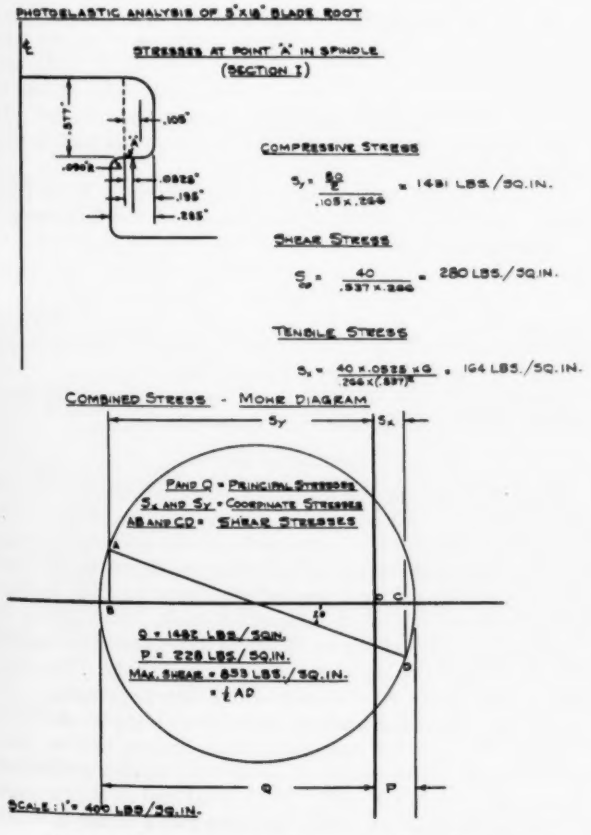


FIG. 14 CALCULATION OF STRESSES IN STEP SECTIONS

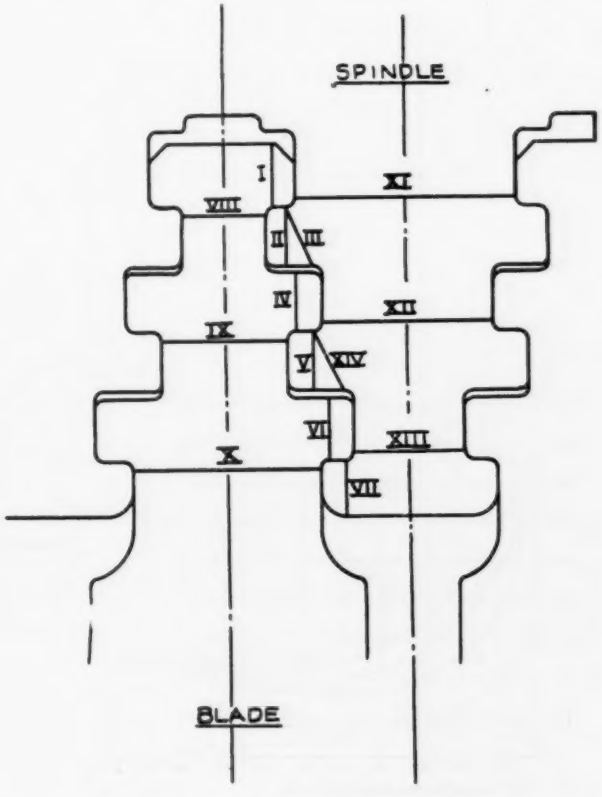


FIG. 15 LOCATION OF ROOT SECTIONS TESTED

Test Data and Curves

The tests of the calibration bar for bakelite material 0.226 in. thick established values of 324 lb per sq in. for each photoelastic band or fringe; and 64.8 lb per sq in. for each 0.01-division change in thickness measured by the extensometer.

The investigation of stress distribution in the model was conducted on three primary cross sections. In Fig. 15, sections I to VII, excluding section III, were analyzed to determine combined stresses on the loaded steps; sections VIII to XIII were considered to ascertain the neck loadings on the step series and the concentrations at the edge fillets; and sections III and XIV were studied to observe the effect of the change in section with respect to the transmitted load.

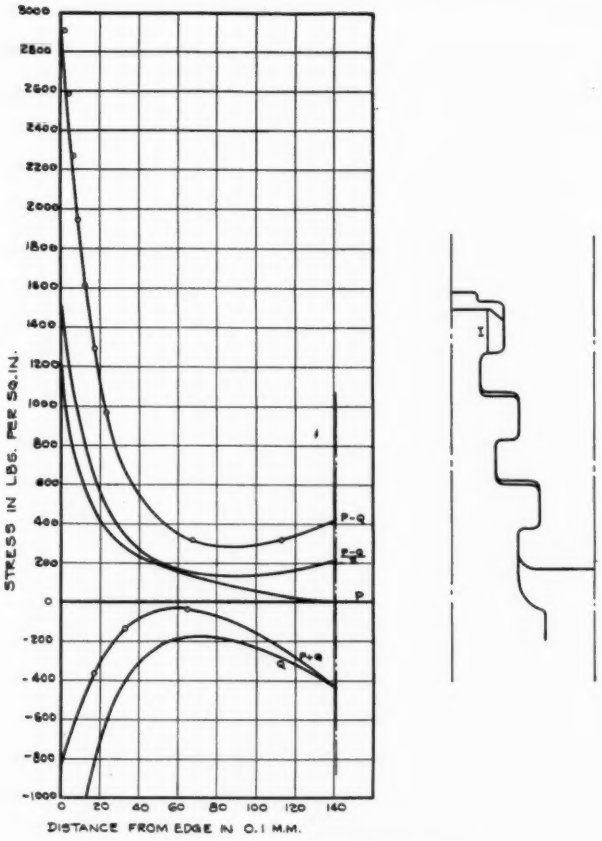


Fig. 16 Stress Curves for Section I

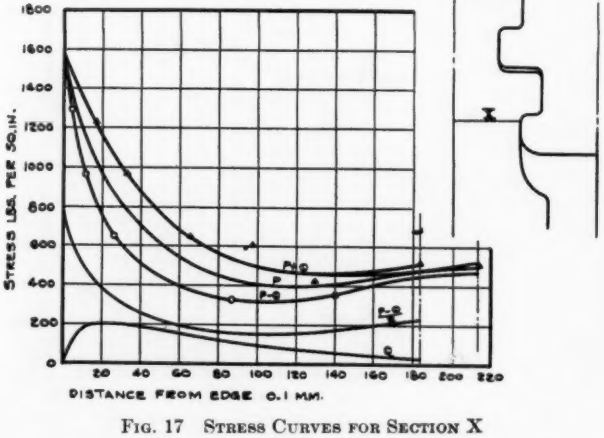


Fig. 17 Stress Curves for Section X

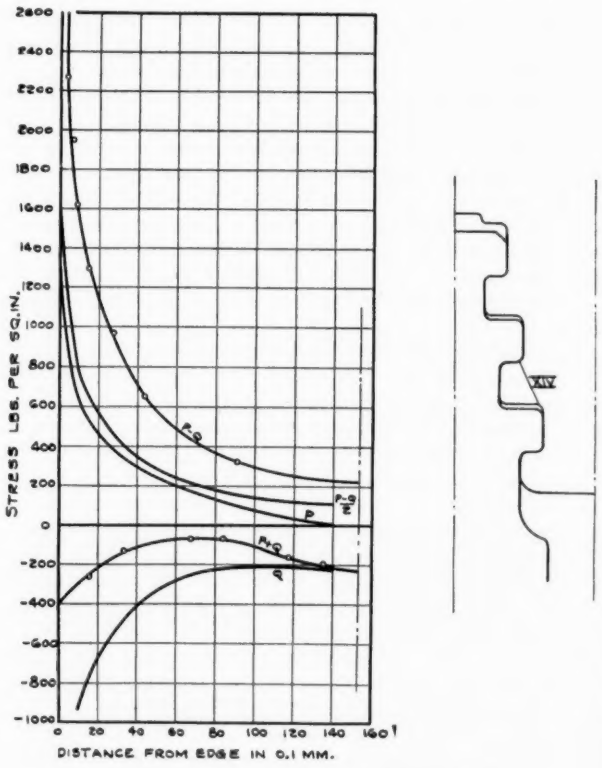


Fig. 18 Stress Curves for Section XIV

Examples of the original test data are given in Table 1 for sections I, X, and XIV. Measurements of the isochromatic photographs were made with a Brinell microscope adjusted to read in 0.1-mm divisions. The photoelastic data give the equivalent stress for the measured number of bands in units of 0.1 mm distance from the active fillets. The changes in thickness data were taken at the cross-marked points and converted into stress, with the same distance units as above.

Curves in Figs. 16, 17, and 18 are representative of the three primary sections I, X, and XIV, respectively.

Results of Tests

The first factor in the analysis of the test results was the investigation of the loads on the step sections in contact. The principal tensile stresses in the necks of the blade and spindle sections, largely in the vertical direction as shown by the zero isoclinics in Fig. 13, were integrated to determine the average stresses and the equivalent transmitted loads. Figs. 19 and 20 present a summary of the tensile-stress distributions in sections VIII to XIII of the blade and spindle, respectively. The areas under these curves were converted into average stresses, total loads, and differential loads at the various steps as shown in Table 2. Further, the maximum tensile stresses, and the ratio of the maximum-to-average stresses and the stress-concentration factors are given in the table. For a total load on the specimen of 780 lb, the load per blade would be 780/3 = 260 lb, which value was approached very closely by the stress analysis.

TABLE 2 FORCES AND TENSILE STRESSES IN NECK SECTIONS

Sec no.	Curve area, sq in.	Total load, lb	Diff. load, lb	Width sec, in.	Area sec, sq in.	Stress avg, lb per sq in.	Stress max, lb per sq in.	Concentration factor ratio, max/avg
VIII	2.145	72	72	0.750	0.199	362	1000	2.76
IX	4.14	139	67	1.125	0.300	463	1210	2.61
X	7.70	258	119	1.680	0.447	576	1600	2.77
XI	5.51	252	78	2.08	0.554	455	1260	2.77
XII	5.19	174	53	1.70	0.452	385	1080	2.80
XIII	3.61	121	121	1.14	0.303	400	1260	3.14
							Avg	2.80

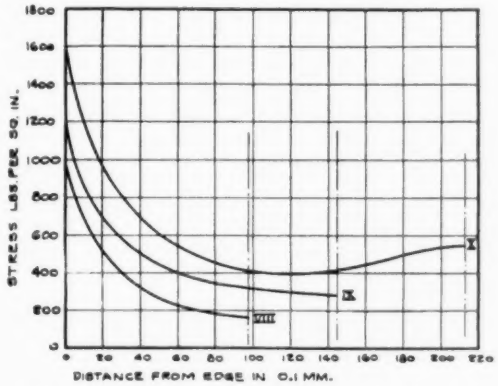


FIG. 19 TENSILE STRESSES IN NECKS OF BLADE

The theoretical calculation of stresses in the blade root and spindle is based upon the assumption of equal elongations between adjacent steps in contact. Thus, in Fig. 21, the elongation between surfaces A and B in the blade root is equal to the corresponding elongation in the spindle. A similar situation is observed between surfaces B and C. Thus, if loads P_A, P_B, and P_C are applied at the contact surfaces of the blade and spindle, certain elongations result. The relative values of the forces which create these elongations may be approximated from the root and slot dimensions. Given

$$e = \frac{PL}{AE} \qquad [1]$$

where e = elongation, in. per in.
P = load, lb
L = length between sections, in.
A = effective area, sq in.
E = modulus of elasticity, lb per sq in.

from Fig. 21, the elongations of the various elements are as follows:

$$\left.\begin{aligned} e_1 &= \frac{(B+C)\,0.561}{1.405E} & e_3 &= \frac{C \times 0.561}{0.938E} \\ e_2 &= \frac{(B+C)\,0.564}{1.125E} & e_4 &= \frac{C \times 0.564}{0.750E} \\ e_7 &= \frac{A \times 0.564}{1.17E} & e_9 &= \frac{(A+B)\,0.564}{1.731E} \\ e_8 &= \frac{A \times 0.561}{1.45E} & e_{10} &= \frac{(A+B)\,0.561}{1.92E} \end{aligned}\right\} \qquad [2]$$

$$\left.\begin{aligned} \text{Setting } e_1 + e_2 &= e_7 + e_8 \\ e_3 + e_4 &= e_9 + e_{10} \end{aligned}\right\} \qquad [3]$$

and solving, the following relationships are found

$$\left.\begin{aligned} B &= 0.351A \\ C &= 0.615A \end{aligned}\right\} \qquad [4]$$

TABLE 3 STEP LOADING

Surface	Calculation: Load, lb	Calculation: Per cent of total	Test: Load (avg), lb	Test: Per cent of total
A	130	50.8	120	47.1
B	45	17.9	60	23.5
C	80	31.3	75	29.4
Totals	255	100.0	255	100.0

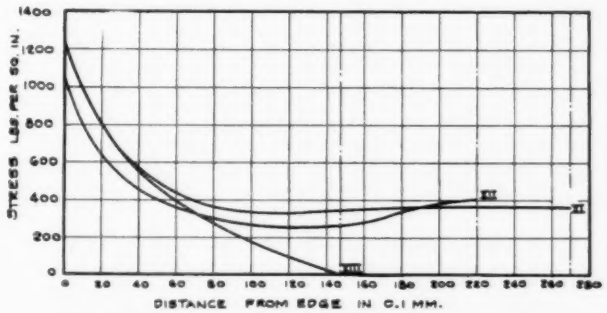


FIG. 20 TENSILE STRESSES IN NECKS OF SPINDLE

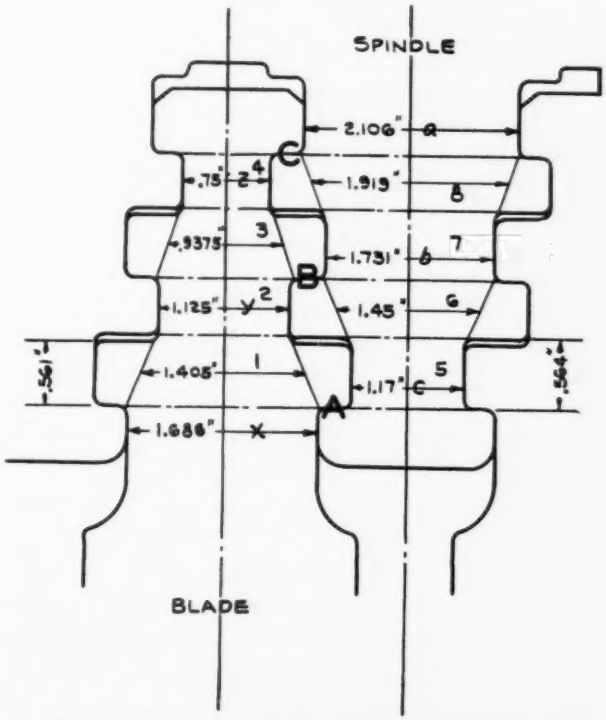


FIG. 21 DRAWING OF ROOT SECTIONS AS CALCULATED

From Table 3, it is observed that the measured average differential load carried by the first step at section X, and in the spindle at section XIII, designated in Fig. 21 as surface A, is 120 lb. Table 3 is then representative of the calculated loading, Equations [4], and the test data, with the percentage of total load carried on each section.

The determination of the loading on surfaces A, B, and C, Fig. 21, makes possible a comparison of calculated stresses with

test stresses on the step sections. Considering the average test loads on the three sections, combined tension, compression, and shear stresses may be calculated by standard equations or Mohr diagrams. A sample calculation for section I is shown in Fig. 14. It is assumed that the bending-moment load is concentrated at the middle of the contact section, while the compression and shear are uniformly distributed over their respective areas. In Table 4, the sections, loads, maximum principal stresses, and shear from test are tabulated with comparative values from calculations. For maximum shear the ratios of the test values and the calculated values determine the stress-concentration factors for the sections.

TABLE 4 STRESSES IN STEP SECTIONS

Sec no.	Data	Diff. load	Stress P	Stress Q	Shear stress	Conc. factor
I	Test	72	1200	−1800	1500	1.76
I	Calc	80	228	−1482	853	..
II	Test	78	1000	−1000	1000	..
III	Test	78	1100	− 900	1000	..
IV	Test	67	1000	−1000	1000	..
V	Test	53	1150	−1650	1400	..
XIV	Test	53	1400	−1800	1600	..
VI	Test	119	1550	−2450	2000	..
VII	Test	121	1750	−2750	2250	..
VII	Calc	130	370	−2410	1385	1.63
					Avg	1.70

The stress distributions for sections III and XIV are useful for comparison with the vertical sections in that the stress characteristics are influenced by the neck stresses at the change in width of the root elements. The maximum stresses are the same as for the vertical sections, but the curves for section XIV, Fig. 18, indicate higher average stresses.

Comparison of Blade and Spindle Stresses

The maximum tensile stresses in the blade root and spindle occur at the sections carrying the total load. At section X in the blade, Fig. 15, the average stress was 576 lb per sq in. (Table 2) and the maximum stress was 1600 lb per sq in.; at section XI in the spindle the average stress was 455 lb per sq in. and the maximum 1260 lb per sq in. Thus, in the model, the blade stress was 27 per cent higher than that in the spindle. The radii of the fillets are uniformly 0.090 in.

However, at the step sections, the greatest force on the contact surfaces occurs at section A. From Table 4, section VI in the blade has principal stresses of 1550 lb per sq in. tension, 2450 lb per sq in. compression, and 2000 lb per sq in. shear; while section VII in the spindle has stresses of 1750 lb per sq in. tension, 2750 lb per sq in. compression, and 2250 lb per sq in. shear. These sections are at the largest part of the blade and the smallest section of the spindle, respectively, and carry a load nearly equal to $^1/_2$ the total load. Thus the maximum stress in the spindle-step section is slightly higher than in the blade at this point.

In general, the maximum principal stresses in tension are the same for the neck and step sections in corresponding regions. However, the compressive stresses on the bearing surfaces of the steps are relatively high and therefore the combined shearing stresses are also high.

For the step sections, II and IV have about the same stress distribution, with sections V, VI, and VII, progressively higher in stress. Sections I and V have similar stresses. The maximum stresses (and loads) are carried in sections VI and VII.

Conclusions

In the model, the dimensions of the step sections for the blade and spindle were the same, whereas the neck depths for the blade were less than those of the spindle. This design was considered in order to take advantage, in the actual steam turbine, of a stronger material in the blades. However, as is observed from a summary of tests and calculations given in Table 5, columns 1 and 2, it is very difficult to favor one element and increase the stresses in the other without an unsymmetrical design with regard to both necks and steps. Thus it was necessary to adopt a design with equal necks and steps, resulting in equal loads on each step.

TABLE 5 COMPARATIVE DESIGNS OF BLADE ROOTS

Column	Test 1	Calculations 2	3	4
	Photo-elastic data	Original data	Equal loads, original step surfaces	Equal loads, increased step surfaces
Load, lb				
A	120	130	85	85
B	60	45	85	85
C	75	80	85	85
Dimensions blade, in. x	1.686	1.686	2.142	2.00
Dimensions blade, in. y	1.125	1.125	1.428	1.34
Dimensions blade, in. z	0.75	0.75	0.714	0.68
Spindle a	2.106	2.106	2.142	2.00
Spindle b	1.731	1.731	1.428	1.34
Spindle c	1.17	1.17	0.714	0.68
$a + z = c + x = b + y$	2.856	2.856	2.856	2.68
Step surface width, in.	0.105	0.105	0.105	0.198
Max principal tensile stress necks, lb per sq in.	1600	1730	1290	1350
Max principal compressive stress on steps, lb per sq in.	2750	2410	1575	860
Max combined shearing stress in steps, lb per sq in.	2250	2250	1540	1250

With equal loads on the step sections A, B, and C, it is necessary that the middle neck sections be equal, and the maximum and minimum sections are $^3/_2$ and $^1/_2$ of the mid-section, respectively. For the same depth of step section as on the model, these dimensions and stresses are computed and shown in Table 5, column 3.

An increase of the surface area at the step contact from 0.105 in. to 0.198 in. would produce smaller neck sections but considerably reduced compression stresses. Equal loads on the steps with balanced neck sections under this condition would give dimensions and stresses shown in Table 5, column 4.

It may be concluded from the foregoing that good agreement exists between calculated and test data. This relationship is established by the concentration factors of 2.80 for the tensile stresses in the neck sections, and 1.70 for the maximum shear stresses in the step sections. These factors agree closely with those of 2.76 and 2.16, respectively, obtained from a photoelastic test of a single T-tail root[3] of an alternator pole connector.

For increased blade loading of future designs, comparative dimensions are obtainable by the calculation methods and concentration factors suggested.

Acknowledgment

This problem was suggested by the steam-turbine department of the Allis-Chalmers Manufacturing Company, and the work was carried on through research and regularly scheduled classes in the mechanical-engineering department with grants from the Graduate School of the University of Minnesota.

Discussion

Stephen Bencze.[5] The authors have presented an interesting analysis of the stresses and stress-concentration factors in one particular steam-turbine blade root of the multiple-step type. However, this analysis is of little value to the turbine-design field, since it deals with one particular case only. Then, too, certain factors influencing the problem have been disregarded.

In Table 2, the authors list stress-concentration factors at various sections of both the blade root and the turbine spindle but make no mention of the conditions for which these factors are valid. It is a fact that stress-concentration factors for re-entrant fillets are influenced by such factors as fillet ratio

[5] Turbine Department, Elliott Company, Jeannette, Pa.

(ratio of fillet radius to neck width), ratio of neck width to flange width, and the ratio of flange width to flange depth. The authors do not give root dimensions and so it is impossible to arrive at the values of geometric ratios for which the reported stress-concentration factors have been determined.

In Fig. 14, a fillet radius of 0.090 in. is shown. Assuming this value to be constant for the various root and spindle sections analyzed, approximate values of fillet ratios for the reported conditions are shown in Table 6 of this discussion. The stress factors, also listed in this table, have been calculated from conditions reported by Timoshenko and Dietz.[6]

TABLE 6 APPROXIMATE VALUES OF FILLET RATIOS

Section no.	Section width, in.	Fillet ratio	Calculated stress-concentration factors
VIII	0.750	0.120	2.73
IX	1.125	0.080	3.45
X	1.680	0.054	4.49
XI	2.080	0.043	5.30
XII	1.700	0.053	4.55
XIII	1.140	0.079	3.48

M. Hetényi[7] reports minimum stress-concentration factors for T-heads for fillet ratios of 0.075 of approximately 3.50, which agrees very well with factors reported by Timoshenko and Dietz. However, factors reported by the authors are considerably lower than any of these.

M. Hetényi.[8] In any particular problem the amount of stress concentration depends upon the geometrical proportions of the sample. For this reason it is customary to discuss concentration factors as functions of the R/d fillet ratios, denoting by R the fillet radius and by d the diameter of the net section under consideration.

The authors, when listing in Table 2 the concentration factors obtained in 6 different sections of the root and spindle, have omitted to indicate the R/d ratios for each of these sections. Since it is stated in the paper that the radii of the fillets were uniformly 0.090 in. and, on the other hand, the d values of the sections are shown in Fig. 21, the R/d ratios can be readily computed. Doing this we find that for sections VIII–XIII the R/d values vary between the limits of 0.043 and 0.120. It is difficult to see what was the authors' justification in taking an average value of the concentration factors obtained in 6 different sections of so widely varying proportions and to suggest this average value $k = 2.80$ for design purposes.

In order to compare the magnitude of the authors' concentration factors with existing data, let us take for example the T-head ending of the spindle (section XIII) where the fillet ratio is $R/d = 0.090/1.170 = 0.0769$. It is known that even a simple tension member with such a fillet ratio can give a concentration factor $k = 2.7$ and this does not yet take into account some particular conditions existing in T-heads, namely, the proximity of the points of supports to the fillets, the shearing and bending action on the flanges of the head, which all tend to increase the concentration of the stress at the fillets. These latter effects naturally will depend upon the geometrical proportions of the head. Assuming that Fig. 15 of the paper was drawn to scale, we find that the proportions of the T-head ending of the spindle are approximately $D/d = 1.5$ and $h/d = 0.5$ where D is the width and h is the depth of the head. For such proportions and for $R/d = 0.075$ the writer obtained a concentration factor of 6.85, referring the maximum fillet stress to the average tensile stress in the neck section. This high value can be traced back to the additional stress sources existing in any T-head fastening.[7] This factor is more than twice as large as the one obtained by the authors for similar proportions. The writer agrees that the stress condition in a simple T-head is not quite identical with the one existing in the T-ending of a multistep spindle. Minor variations, however, could hardly account for the great difference found in this comparison and it is believed that the concentration factors of the authors are too low.

Though it is aside from the main topic of the paper it seems desirable to call attention to the fact that the dashed curve in Fig. 9 should be an ellipse, which is not apparent from the figure.

Authors' Closure

The question of stress concentration is of vital importance in the design of machines, and the authors welcome the discussion contributed by Stephen Bencze and M. Hetényi.

It is quite evident that the stress-concentration factors obtained at the neck sections from the scale model of the machine are not in harmony with the idealized investigation. This is shown in the respective photoelastic illustrations of stress distribution, Figs. 11 and 12 in the paper[7] by Hetényi, and in Fig. 12 of the authors' paper. For the model of the multiple root, the maximum stresses were not found to be concentric about the fillet arc, but were displaced toward the end of the fillet at the contact surface.

The difference between the photoelastic illustrations, exhibited as representative, indicates that the structures or their loadings are not equivalent. In the multiple-root model, all contacting surfaces were of the same material and mating steps were relatively symmetrical. Upon inquiry as to the method of loading and the type of materials in contact for the idealized investigation,[7] Dr. Hetényi advised us that thin cork strips were placed under the supports along the flanges to obtain uniform loading, but did not indicate the construction of the support mechanism.

Based on this information, it may be stated that, in the latter experiments, a particular type of distributed pressure was applied to the T-heads; while in the case of the multiple root, the pressure distribution on the contact surfaces, although not investigated, is presumably determined by the distortion of the mating elements.

The authors are of the opinion that the major part of the apparent discrepancy between the factors given in the paper and those of Hetényi is due to this difference in loading.

It is to be observed, however, that from the complete picture of stress distribution in the complex multiple-root model the stress concentrations which determine the design are those produced by the high contact pressures.

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[6] "Stress Concentration Produced by Holes and Fillets," by S. Timoshenko and W. Dietz, Trans. A.S.M.E., vol. 47, 1925, pp. 199–220.

[7] "Some Applications of Photoelasticity in Turbine-Generator Design," by M. Hetényi, *Journal of Applied Mechanics*, December, 1939, pp. A-151 to A-155.

[8] Research Laboratories, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.

Permanence of the Physical Properties of Plastics

By J. DELMONTE,[1] CHICAGO, ILL.

Plastic materials are subject to certain changes in physical properties during their various applications. Particularly significant are the creep and cold-flow phenomena. In the paper a general equation is developed expressing plastic-deformation rate as a function of a coefficient of viscous resistance and of a coefficient of internal resistance. Data for laminated phenolics, polyvinyl chloride acetate, methyl methacrylate, polystyrene, and cellulose acetate are compared. Experimental evidence is presented to show that after a sufficient length of time plastic deformation assumes a constant rate, continuing indefinitely. The dual nature of the molecular structure of plastics is emphasized, in which true elastic deformation may occur, as well as a time-dependent plastic deformation, even at very low stresses. Further data are presented on the shrinkage of various plastics after exposure to elevated temperatures, and the swelling of these materials after exposure to high humidity or water immersion. Loss in impact strength upon exposure to high temperatures is also disclosed for laminated phenolics.

IN THE examination of the permanence of the physical properties of organic plastic materials, attention should be given to those operating conditions which are likely to produce changes in the materials. When these operating or service conditions have been analyzed, they should serve as the basis of comparison for the various plastics. As engineering materials, one of the most singular characteristics of organic plastics is their response to thermal influences. In fact, they are generally classified as thermoplastic or thermosetting, distinguishing between materials which are profoundly influenced by heat and those which are only slightly affected.

Another characteristic of plastics which has a decided influence upon the design and application of the materials is their behavior under applied stresses. A review of technical literature indicates that with few exceptions, very few factual data useful for engineering purposes are available on the creep and cold-flow characteristics of the various plastics. Variations in test methods; examinations confined to one material; failure to stress the material for a sufficient length of time; and failure to report fully the nature of the molecular structure, composition, or previous history of the material, are some of the reasons detracting from the value of the research. Plastic materials, more so than many other materials of construction, are subject to permanent deformation under stress and, hence, require systematic analysis along these channels.

The media in which the plastic materials are operating may also have a bearing upon their properties. Particularly noteworthy are such influences as the actinic rays of sunlight, high humidity, and the corrosive effects of chemicals, any of which may cause a loss in utility of the material. When the material is to be used in a severe corrosive medium, tests in that specific medium under the most rigid operating conditions are usually conducted by the organization contemplating use of the material. Only in that manner can it be ascertained whether the material will prove acceptable.

Consequently, in examining the permanence of the physical properties of plastics, the following tests may be pursued to evaluate the relative merits of the plastics:

1 Creep and cold-flow tests at various stresses and at various temperatures, for different lengths of time. Recovery upon removal of stress also to be observed.

2 Organic plastics at elevated temperatures:

(*a*) Properties of the plastics at these temperatures.

(*b*) Properties of the plastics after return to room temperature; after exposures for different periods of time.

3 Effect of immersion in water or high humidity upon the properties of organic plastics (or in lieu of water, the use of corrosive chemicals).

Creep and Cold-Flow Characteristics

In this paper, an analysis will be made of the creep and cold-flow characteristics of solid plastic materials, as they are available commercially, in sheet form. The need for data of this nature has been emphasized in recent years (1, 2).[2] Standard compression and tension tests must be modified to take into consideration time and temperature, for organic plastics, besides exhibiting elastic deformation, exhibit a time-dependent plastic deformation. There has been some doubt as to when plastic deformation first begins to make its appearance, when the stress is slowly applied to the material. It has been suggested recently by R. Houwink that for solid organic plastics, every stress will produce a certain amount of plastic deformation (3). The results of the present investigation show this to be the case for a number of plastic materials, even down to very low values of stress.

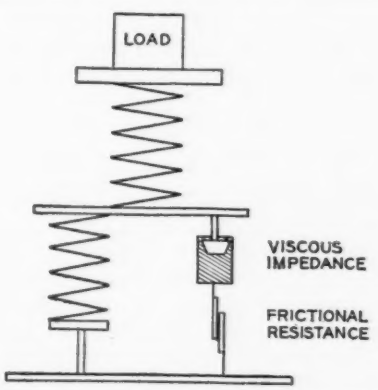


Fig. 1 Mechanical Analogy of Molecular Structure of Plastic Material

[1] Assistant to Chief Engineer of Research, Chicago Flexible Shaft Company. Jun. A.S.M.E.

Contributed by the Rubber and Plastics Subdivision of the Process Industries Division and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

A convenient mechanical analogy of the structure of a typical plastic material is given in Fig. 1, based in part upon conclusions arrived at in this paper. Under constant load, this mechanical analogy presupposes the following to take place:

1 Upon placing the material under stress, there follows immediately a very rapid elastic deformation.

2 This is followed by a plastic deformation which depends upon the stress, temperature, and the time of application.

3 Under the continued application of load, the velocity of this plastic deformation will change with time. The internal structural elements contributing to this deformation characteristic are depicted in the mechanical analogy as possessing internal friction (as sliding friction between adjacent long chain molecules), and as possessing an inertia effect, depending upon the rate of change of deformation velocity (illustrated as a dashpot and cylinder).

4 Upon removal of the load, there is a rapid, immediate elastic recovery, followed by a relaxation period in which further recovery from deformation occurs, depending upon the manner in which the material was stressed.

5 Another condition, not apparent from the mechanical analogy of the internal structure, admits the possibility that the elastic structure can be deformed past its elastic limit, besides the plastic deformation exhibited by those elements not part of the main elastic structure.

The majority of these conditions has been recognized in previous studies (3, 4). A mechanical analogy for plastic materials employing the same elements, shown in Fig. 1, has also been suggested (5), though in an entirely different arrangement.

It is not entirely unexpected that new interpretations are necessary for such constants as elastic limit, yield value, proportional limit, and modulus of elasticity for materials possessing an internal structure capable of elastic and plastic deformation. Most stress formulas for beams and other structural members are developed for materials operating strictly upon the elastic portion of their stress-strain characteristics. It is obvious that these formulas must be modified when the permanency of the physical properties is a function of the material, its temperature, and time duration of stress. The creep and cold-flow data disclosed in this paper have been confined to a study of the influence of time under constant temperature conditions. Further studies are in progress to examine systematically the effects of temperature.

Researches in Plastic Deformation

Method. Many of the researches upon the plastic deformation of amorphous materials have employed methods designed to give uniform tensile or compressive stresses within the test materials, under uniform loading conditions. For example, some tests compare plastic materials on the basis of per cent change in height of a small cube of specimen under a predetermined load at 120 F, over a period of 24 hr (6, 7). This technique is also employed in the mechanical testing of rubber, where dead weights are applied for certain periods of time on rubber samples (8, 9). Other methods quite common employ ball-and-needle penetrometer techniques under certain loads at certain temperatures, for various time intervals. It has been the author's experience that such methods are particularly adaptable to materials which exhibit relatively large deformation for a given tensile or compressive stress. These materials would include those plastics which have been described as possessing rubber-like qualities such as plasticized polyvinyl acetals, plasticized polyvinyl chlorides, rubber-like alkyds, certain acrylic resins, etc. For example, under a constant tensile stress of 41 lb per sq in. at 85 F, the time-deformation characteristics of polyvinyl acetal are shown in Fig. 2. This material has also been examined for changes in its photoelastic qualities under similar test conditions (10). However, when the more solid materials of this investigation were examined under direct tension and compression, the time for appreciable plastic deformation to take place was very long, lessening the accuracy of the readings, and making it difficult to distinguish the various stages of plastic deformation.

Another procedure followed in observing creep phenomena is to load and unload materials within the proportional limit of the stress-strain curve (11, 28), measuring the creep as a function of the elastic-hysteresis loop. Of course this technique is important also in determining Poisson's ratio. However, this method does not permit a study of the effects of time duration of stress which is so highly important to plastic deformation, and is not readily adapted to the study of recovery phenomena, such as the elastic aftereffect.

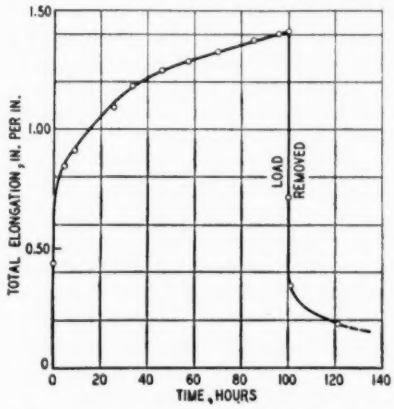


Fig. 2 Time-Deflection Curve for Polyvinyl Acetal

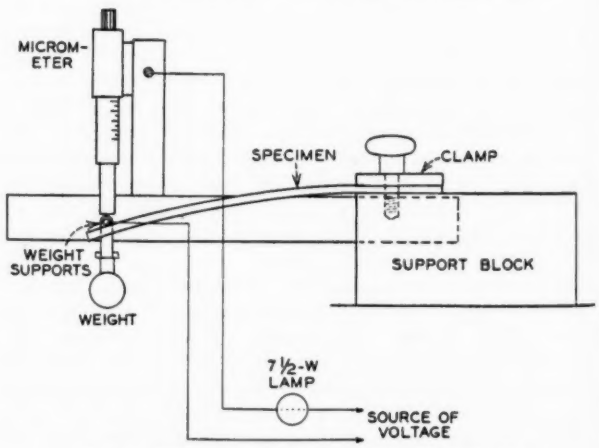


Fig. 3 Schematic Diagram of Test Apparatus

In this investigation samples of the plastic materials were supported as simple cantilever beams and loaded by dead weights at the end, Fig. 3. The apparatus designed by the writer tested four samples simultaneously, making it possible to observe materials under four different maximum fiber stresses during the same period of time. The equipment was enclosed within an insulated chamber, thermostatically controlled with air circulation, to 85 F plus or minus 2 F. Four micrometers attached solidly to the support block measured the end deflection of the specimens by electrical contact with the support for the weights, and completing the circuit with a 7.5 lamp placed in series. The micrometers were operated manually when readings were to be taken. The load applied by spring-actuated dial gages was too great to permit using them in these tests. The supports for the weights

were reduced to a minimum, weighing only 4 g. The micrometers were read when contact was made with the weight support, which was indicated by the lighting of the lamp. The readings were accurate to plus or minus 0.0003 in. The specimens were retained at their ends by 1/8-in. steel pieces, clamped down by two thumbscrews. In making stress calculations, the weights of the supports for the loads and the weight of the specimens were taken into consideration. Electrical connection to the weight supports was made through a fine, B.S. No. 34 enameled wire.

While the values of deformation observed are in fact an integrated value for all stresses up to the maximum fiber stress, a simple cantilever beam affords a simple method of ascertaining the elastic and plastic properties of materials. The simple cantilever beam has an advantage over other types of beams, inasmuch as its moment diagram for loads at the end is a straight-line function of the suspended length, and also possesses a uniform shear diagram. It may be readily seen that the simple cantilever beam by reason of its deflection at the end, acts as an amplifier of stress deformation in its fibers, in a ratio of some function of the length to the thickness of the material. There is much precedence for the use of a deflection of a beam as a measure of flow in solid bodies (12, 13, 14, 15). Trouton investigated flow in rods of pitch under the methods of tension, compression, and bending. His curves for sagging as a function of time show a distinct exponential relationship. In a majority of these earlier instances, tests were conducted upon bituminous substances or concrete, whereby the test specimens were placed upon two horizontal supports and deflection observed in the middle under the weight of the specimen or additional weights.

The deflection of a beam also serves as the basis of comparison for plastic materials in a heat-distortion test in an American Society for Testing Materials specification (16) where the temperature is raised slowly and the temperature noted at which a deflection of 10 mils occurs. In a similar British test specification, a simple cantilever test beam is employed to determine what is referred to as "plastic yield." The plastic yield is described as the specified allowable maximum deflection for a 5 × 5-mm specimen, 150 mm long, at a certain temperature (17).

Materials Tested. The following materials were included in this investigation and a comparison made of their relative deformation as a function of time, for various stresses induced in the materials when they were loaded as simple cantilever beams. The specimen lengths were selected at 5 in., with the exception of polystyrene, which was 2.5 in. long. The results of polystyrene were corrected to a 5-in. length. The widths of the specimens were approximately 1 in. and, while an effort was made to obtain uniform thickness, the laminated phenolic and polystyrene samples differed from the others.

Cellulose-acetate sheet: Specimens were cut from clear transparent sheet stock, approximately 0.089 in. thick; obtained from Fiberloid Corporation (now Monsanto Chemicals Company).

Polyvinyl chloride acetate: Specimens cut from a white, filled stock, 0.094 in. thick; obtained from Carbon and Carbide Chemicals Corporation, series V.

Methyl methacrylate: Specimens were cut from clear, transparent stock 0.09 in. thick; material obtained from Röhm and Haas Company.

Polystyrene: A flat injection-molded test bar 0.069 in. thick served as a test sample; obtained from Dow Chemical Company.

Laminated phenolic sheet, canvas base: Standard N.E.M.A. grade L material, 0.065 in. thick, employed in test.

Theory. Within certain stresses, the initial rapid deflection of the simple cantilever-beam test specimens is a truly elastic deformation, the magnitude of the deflection and the maximum fiber stresses being calculable from the following well-known formulas, which assume validity of Hooke's law and uniform distribution of stress-strain relations in compression and tension about the neutral axis

$$y_e(\max) = WL^3/3\,EI \quad [1]$$

and

$$M = SI/c \quad [2]$$

where y_e = true elastic deflection
W = weight supported at end of beam
L = distance from point of support to weight
E = modulus of elasticity
I = moment of inertia
S = maximum fiber stress
M = maximum bending moment = $W \times L$
c = distance from neutral axis to outermost fiber.

Immediately following the truly elastic deformation, the phenomena of creep and cold flow will commence, increasing the amount of deflection. There are two stages of plastic deformation which were observed for the materials under test:

Stage *A*—rate of change of deformation with time (dy/dt) changes from a maximum to a minimum constant rate.

Stage *B*—in which dy/dt remains constant for an indefinite time for a given fiber stress (or given integrated stress value).

Unlike crystalline materials which may work-harden, whereby dy/dt approaches zero, none of the plastics included in this test behaved that way, even at maximum fiber stresses as low as 500 lb per sq in. For the sake of definition, "cold flow" may be described as that constant rate of change of deformation with time, which a material will exhibit after being subjected to a certain stress for a certain period of time, at a constant temperature. In accordance with earlier suggestions,[3] "creep" may be described as that stage of plastic deformation where the velocity of the deformation rate changes with time (d^2y/dt^2). Creep experiences its greatest value following the elastic deformation, becoming zero when true cold flow finally occurs. From the data presented in this paper, it will be seen that considerable time elapses before creep becomes zero, depending upon the maximum initial fiber stress. In general, it was observed that the lower the maximum fiber stress, the sooner creep was terminated and pure cold flow commenced.

The existence of plastic deformation as a separate phenomenon from elastic deformation is brought out in the mechanical analogy given in Fig. 1. This admits the possibility that part of the stress is carried by nonelastic elements, which may also be in parallel with some of the elastic elements. This last consideration is borne out when one considers that the elastic aftereffect upon removal of load derives its energy from an elastic structure which is retarded in its recovery by the plastic elements.

A general expression for plastic deformation may be developed in the light of creep and cold-flow phenomena. Assuming the two stages of deformation for amorphous materials, we have the following

$$y(\text{total}) = y_e + y_p \quad [3]$$

where y_e = elastic deformation
y_p = plastic deformation

The elastic deformation is defined by Hooke's law within the elastic limit of the material

$$y_e = S/C \quad [4]$$

where C = elastic constant
S = stress required to produce y_e.

[3] Reference (3), p. 188.

Expressing y_p as a function of time and stress, the following formula may be applied

$$S = RV + M\frac{dV}{dt} \quad [5]$$

where $$V = dy_p/dt \quad [6]$$

and M = coefficient of viscous impedance

R = coefficient of internal resistance of molecules.

As a solution to Equation [5], we have the following expression for V, which is the rate of plastic deformation with time

$$V = \frac{S}{R}(1 - e^{-Rt/M}) \quad [7]$$

When t becomes very large, V is equal to the cold flow

$$V = S/R \quad [8]$$

Integrating Equation [7], we obtain an expression for the plastic deformation as a function of time

$$y_p = \frac{St}{R} + \frac{SM}{R^2}(1 + e^{-Rt/M}) \quad [9]$$

From the general solution for plastic deformation as a function of time at constant temperature, it is possible to make the necessary corrections in the calculations of beam deflections. This may be done by assuming that this time-deformation function determines the apparent instantaneous value of the modulus of elasticity. Further, for each material there will be a characteristic coefficient of viscous impedance, M, and a coefficient of internal resistance R. These coefficients will be largely determined by the structural characteristics of the molecules, such as the degree of polymerization; method of polymerization; whether a linear or three-dimensional polymer is present; presence of plasticizing agents; influence of fillers; etc. For further discussion on factors influencing flow characteristics of plastics, there are several papers which may be referred to (18, 19, 20). The coefficients mentioned may be calculated from a time-deformation curve at a stress below the elastic limit of the elastic portion of the structure.

The question would naturally arise, in view of the creep and cold-flow characteristics of the plastics, as to what interpretation should be given to the modulus of elasticity. Strictly speaking, it would be made a function of stress, strain, temperature, and time, inasmuch as the exact location of the elastic limit is open to some question. In the bending of beams beyond the proportional limit, Timoshenko has proposed in the well-known formula $M = EI/p$, that E be replaced by E_r, a variable modulus of elasticity to be determined from the stress-strain relationships in a step-by-step method (21).

In spite of the dual nature of the strain characteristics of the plastic materials, whereby one portion of the molecular structure exhibits true elastic deformation, and the other portion is susceptible to plastic deformation even at the smallest stresses (these effects are evenly distributed throughout the structure), we can with justification employ the straight-line portion of the stress-strain relationship to determine the modulus of elasticity under certain circumstances. These conditions demand a short-time loading of the material, and the existence of an elastic deformation considerably in excess of plastic deformation (which was true for most of the materials tested). On the other hand, when time or permanency is a factor, modifications must be made in accordance with the general relationship established for plastic deformation, which is also verified by experimental results of this paper. The necessity for altering the values of the moduli of elasticity has been recognized by other investigators, one proposing a high initial modulus of elasticity for amorphous materials and a low modulus of elasticity after plastic deformation (22). Better yet, the time dependence of the modulus of elasticity should be determined (23), a sentiment in which the author concurs.

Relaxation. The next point to consider is the relaxation upon removal of stress. If the molecular structure were such that elastic phenomena and plastic phenomena operated independently of one another, a rapid elastic recovery would ensue with no further recovery. However, such a condition does not exist and the energy for the plastic recovery (sometimes referred to as the "elastic aftereffect") is derived from a portion of the elastic structure. The rate of recovery will still depend upon a coefficient of viscous impedance and a coefficient of internal friction, both cooperating to make the rate an exponential function of time. This observation has been made by other investigators (4, 24).

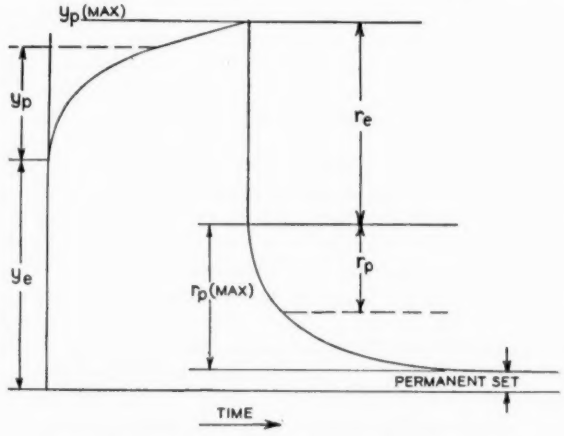


FIG. 4 TYPICAL TIME-DEFORMATION CHARACTERISTIC

Further, when a constant deformation is maintained under loading conditions, we may expect an exponential recovery of stress. Steinberger observed this condition for some cellulose esters, expressing the recovery of stress in an exponential equation (11).

It will also be observed that the recovery from deformation will not be 100 per cent, due to a permanent set which occurs as a result of the loss of stored energy from the elastic structure. Likewise when the stress is great enough to exceed the proportional limit of the elastic structure, the permanent set is very much greater. Thus the permanent set in the material will be a function of the time of stress application, temperature, and the amount of stress to which the material has been subjected. This behavior has been corroborated by the results of this investigation. Houwink also points out the influence of thermo-recovery upon permanent set, wherein a further recovery from deformation ensues when the material is placed at a temperature higher than that at which it was deformed.[4]

If we admit the existence of a true elastic portion of the molecule, how then are we to determine correctly its true elastic limit if the results for the whole structure are susceptible to plastic influences even at low stresses? Grouping the deformation and recovery phenomena, there is the following:

Elastic deformation (y_e) + plastic deformation (y_p) =

elastic recovery (r_e) + plastic recovery (r_p) + permanent set.

If upon the removal of the load, $r_{p\,max}$ exceeds $y_{p\,max}$ we have

[4] Reference (4), p. 7.

then exceeded the true elastic limit of the material, for it is only through a nonelastic deformation of the true elastic structure, resulting in a retardation of the elastic recovery, that this condition would be true, Fig. 4. Thus the true elastic limit of amorphous materials may be defined as follows: When stress is applied such that the elastic deformation y_e exceeds the elastic recovery r_e the applied stress exceeds the elastic limit of the material. This definition is true regardless of how long the stress is applied though the longer the stress is applied the greater is the difference. For example under the conditions of the test, the following results were noted for polyvinyl chloride acetate and methyl methacrylate. Elastic recovery was determined after 50 hr and 100 hr of applied stress, respectively.

Maximum fiber stress, lb per sq in.	Ratio of y_e/r_e	
	Polyvinyl chloride acetate	Methyl methacrylate
500	1.00	1.06
1000	1.00	1.06
2000	1.01	1.20
3000	1.90	1.64

Similar relationships of y_e and r_e were observed for all of the other materials. Thus in applying the definition of elastic limit given in the last paragraph, when y_e begins to exceed r_e at an increasing rate, the elastic limit of the material is exceeded. The longer the stress is applied, the greater will be this distinction, enabling the observer to readily determine the elastic limit of the elastic portion of the molecule. From this tabulation, we find that the elastic limit of polyvinyl chloride acetate lies between 2000 and 3000, and the elastic limit of methyl methacrylate lies

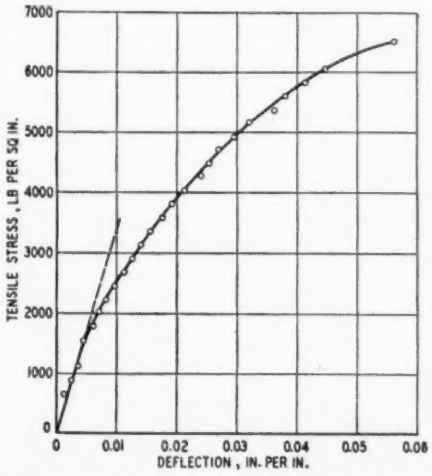


FIG. 5 STRESS-STRAIN CURVE FOR METHYL METHACRYLATE

between 1000 and 2000 lb per sq in. To check this result very careful measurements were made of the direct-tension stress-strain relationships of methyl methacrylate (see Fig. 5), and from these data the elastic limit between 1000 and 2000 pounds per square inch was verified.

Temperature. No results were available at the time of preparing this paper on the characteristics of the materials at different temperatures. Aside from the phenomenon of thermorecovery, already mentioned, also of note are the observations of Hetényi, who pointed out that at temperatures of 110 C, the phenol-aldehydic plastics behaved as true elastic materials. He emphasized also that the speed of deformation is greatly dependent upon the temperature (25). These observations were conducted on test beams supported at two points and under a given stress for a period of 5 hr. The true elastic behavior of plastics at high temperatures was also noted by Houwink (26). Other results on the temperature dependence of plastic deformation are also enlightening (27, 28).

MAKING DEFORMATION TESTS

In preparing the data which are presented in the accompanying set of curves the following procedure was maintained in every case:

Due to very rapid changes in deformation, following the application of the fixed weights serving as the load, a full 60 sec

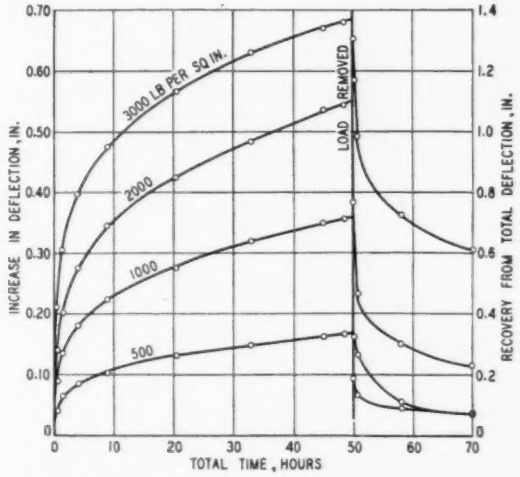


FIG. 6 TIME-DEFORMATION CURVE FOR CELLULOSE ACETATE, 50 HR

were allowed to elapse before the first reading was taken. This value was assumed to be y_e (true elastic deformation). Likewise, very rapid changes in deflection occurred upon removal of load, and again 60 sec were allowed to elapse before the first reading in the recovery curve was taken and r_e (true elastic recovery) measured. Another assumption is the designation of maximum fiber stress above each curve during the loading period, which is correct only for the true initial elastic deformation. This value of maximum fiber stress was obtained from Equations [1] and [2], and naturally will change during time. However, it serves as a comparative indication of how the loads were selected for the materials. The value will be even less significant when the true elastic structure is deformed, which is the case for example, with cellulose acetate at 3000 lb per sq in. maximum fiber stress. Actually the recovery from maximum fiber stress in this one case is so great that there is a smaller separation of deflection ordinates between 2000 and 3000 lb per sq in. than between 1000 and 2000 lb per sq in., Fig. 6.

All values of deflection in the curves are measured at the end of a 5-in. simple cantilever beam which was adopted as a standard

TABLE 1 COLD-FLOW AND CREEP CHARACTERISTICS OF VARIOUS PLASTICS[a]

Material	At maximum fiber stress of 1000 lb per sq in.					
	Total initial elastic deformation (y_e), in.	Total plastic deformation (y_p) — At 1 hr		At 100 hr		Coefficient of resistance to cold flow (R)[b]
		In.	Per cent, y_e	In.	Per cent, y_e	
Polyvinyl chloride acetate	0.407	0.020	4.9	0.106	26.0	1.66×10^4
Polystyrene	0.370	0.012	3.2	0.090	24.0	2.5×10^4
Cellulose acetate	0.727	0.117	16.1	0.480	66.0	0.5×10^4
Laminated phenolic (fabric base)	0.166	0.012	7.2	0.036	22.0	9.08×10^4
Methyl methacrylate	0.446	0.030	6.7	0.065	14.6	4.54×10^4

[a] Data obtained for 5-in. length simple cantilever-beam deflections, corrected for thickness of 0.090 in.
[b] R obtained from slope of curves in Fig. 17.

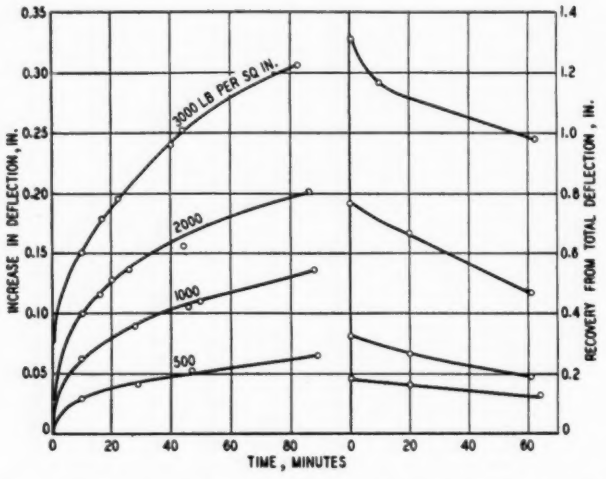


FIG. 7 TIME-DEFORMATION CURVE FOR CELLULOSE ACETATE; 90 MIN

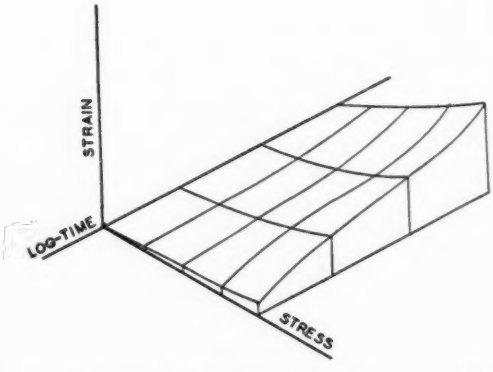


FIG. 8 STRAIN-STRESS-LOG TIME CURVE FOR LAMINATED PHENOLICS

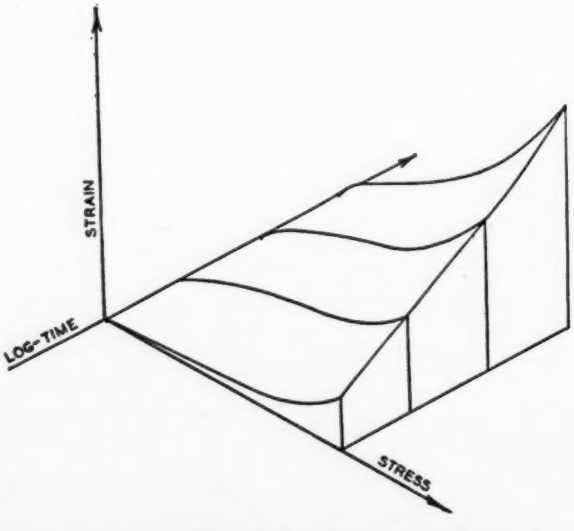


FIG. 9 STRAIN-STRESS-LOG TIME CURVE FOR METHYL METHACRYLATE

for comparison. It is not possible, either, to compare curves of materials with different thicknesses, unless data are presented as per cent of the original deflection. Results are plotted as the "increase in deflection" for various time durations of loads. The increase in deflection over the initial elastic deformation is in fact the plastic deformation Y_p. The elastic deformation was not plotted on the curves, inasmuch as it was desired to compare

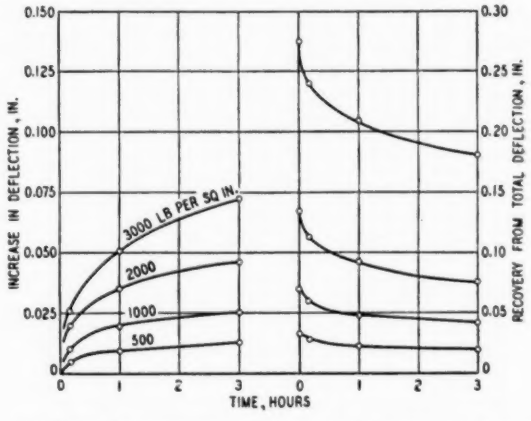


FIG. 10 TIME-DEFORMATION CURVE FOR POLYVINYL CHLORIDE ACETATE, 3 HR

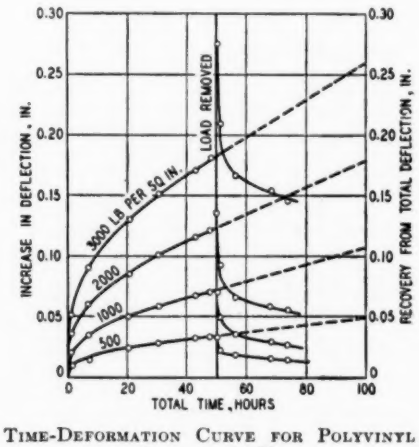


FIG. 11 TIME-DEFORMATION CURVE FOR POLYVINYL CHLORIDE ACETATE, 50 HR

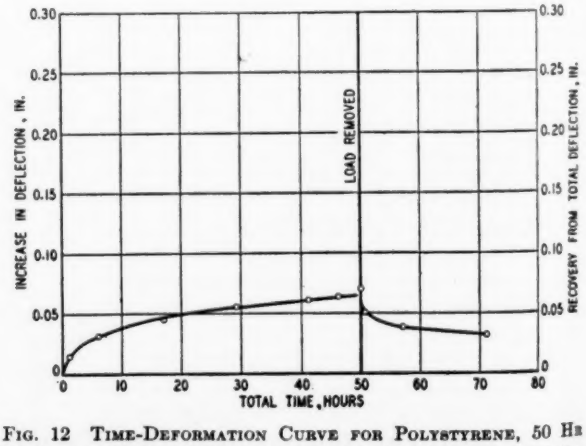


FIG. 12 TIME-DEFORMATION CURVE FOR POLYSTYRENE, 50 HR
(Thickness, 0.068 in.; deflections corrected to 5-in. length; temperature, 85 F; maximum fiber stress 1000 lb per sq in.)

the plastic-deformation properties primarily, though the value may be estimated from Equations [1] and [2] readily. They are also given in Table 1. Further, upon the attainment of various time intervals, 50 or 100 hr, the load was removed and 60 sec allowed to elapse before the first reading in the recovery curve was taken. This reading would, of course, be a measure of true elastic recovery r_e, and further measurements denoted as recovery from total deflection are in fact plastic recovery r_p. To facilitate measurements upon the slopes of the curves at the initial deflection and initial recovery periods, the same data are plotted for shorter time intervals. For further analysis, a typical stress-deflection-log-time curve in three dimensions is plotted for laminated phenolics and methyl methacrylate, Figs. 8 and 9. Both of these curves were plotted according to the same ordinates. The smaller the volume, the more desirable the cold-flow and creep characteristics. The other materials give similar plots, only occupying greater volumes for the same ordinates. All of these data are for a given temperature of 85 F. Figs. 10 to 16 are the deflections in bending, for certain time durations of various stresses for the various materials.

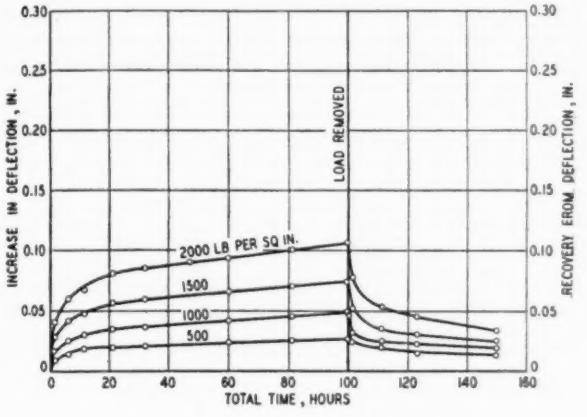


FIG. 13 TIME-DEFORMATION CURVE FOR LAMINATED PHENOLIC PLASTIC, 100 HR
(Thickness 0.065 in.; fine-weave fabric base; temperature 85 F.)

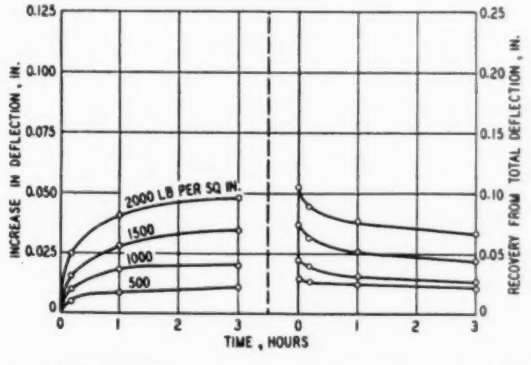


FIG. 14 TIME-DEFORMATION CURVE FOR LAMINATED PHENOLIC PLASTIC, 3 HR
(Thickness 0.065 in.; fine-weave fabric base; temperature 85 F.)

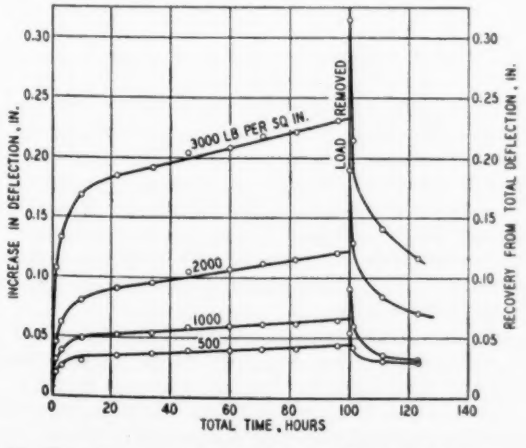


FIG. 15 TIME-DEFORMATION CURVE FOR METHYL METHACRYLATE, 100 HR
(Thickness 0.094 in.; temperature 85 F.)

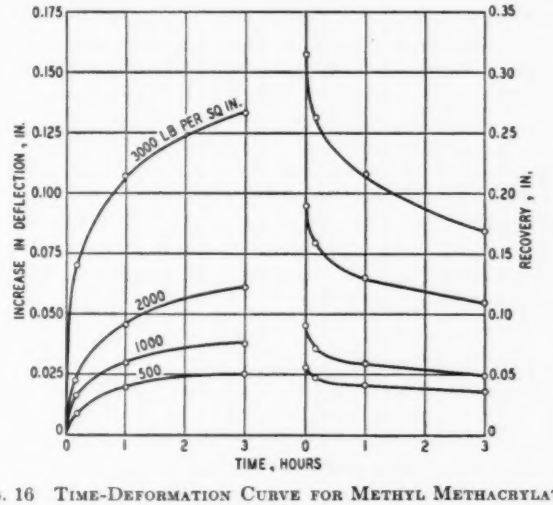


FIG. 16 TIME-DEFORMATION CURVE FOR METHYL METHACRYLATE, 3 HR
(Thickness 0.094 in.; temperature 85 F.)

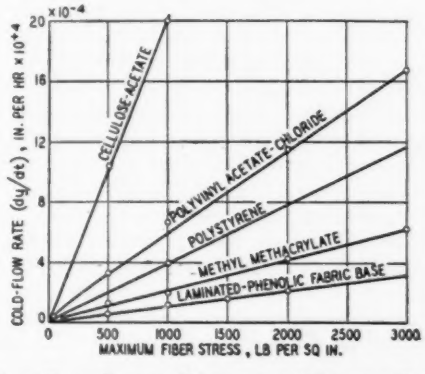


FIG. 17 COLD-FLOW RATE AS FUNCTION OF MAXIMUM FIBER STRESS
(Cold-flow constant dy/dt for given maximum fiber stress, corrected for 0.09-in. thickness, length 5 in., slope of cold-flow characteristics when dy/dt becomes constant at maximum fiber stress.)

For every material the two stages of plastic deformation are quite noticeable, more so in some instances than others, namely, where dy/dt is changing (creep) and where dy/dt becomes constant (cold flow). For higher values of stress, the longer is the time required to attain cold flow. The time required to attain cold flow differs considerably in the various materials. For example, methyl methacrylate attains cold flow at a much earlier period than polyvinyl chloride acetate. The fact that the cold-flow rates of methyl methacrylate are considerably smaller than polyvinyl chloride acetate would indicate that the former is to be preferred for applications involving the imposition of stress

over long periods of time. On the other hand, the smaller initial deflections of polyvinyl chloride acetate occurring during the early stages of plastic deformation, would mark this material as better suited to short-time applications of stress as compared with methyl methacrylate.

The attainment of constant dy/dt may be shown in Equation [8] after long periods of time have elapsed. This condition was observed for all of the materials under test, and is highly significant, for it means that the cold-flow rate is a straight-line function of the stress. The slopes of the time-deflection curves were measured after cold flow commenced, and plotted as a function of the stress to give Fig. 17. Agreement with theory was observed in every case. We may also deduce from Fig. 17 that cold flow occurs even at very low stresses, as the line has its inception at the origin.

In connection with the tests upon methyl methacrylate, it is also interesting to note the results of other creep tests conducted upon the same type of material, which agree with the results of this test (29). These specimens were loaded for 20 min and unloaded for the same length of time, obtaining the time-deformation characteristics. The investigators suggested the following formula for the deformation:

$$y = Ae^{-at} + B \log_e (1 + t)$$

where A and B are functions of the load.

As a basis of comparing the creep and cold-flow characteristics of the materials, Table 1 has been prepared from the data shown in the accompanying curves. For data upon casein plastics, there has been a recent survey of the elastic properties of 60 types of rennet casein (30). In Table 1, low ratio of plastic deformation to elastic deformation may be deemed desirable, as well as high values of the resistance coefficient R.

Effect of Structure. While insufficient types of the same materials were tested to permit a comprehensive analysis of the effect of molecular structure upon cold-flow characteristics, a few results may be reported. For example, when laminated phenolics were baked at 275 F for 24 hr and then tested under cold flow, a marked decrease was observed, indicating of course that further polymerization had removed from the molecular structure some of those elements susceptible to plastic deformation. Tests upon hot-molded phenolics, which were prepared from more reactive phenolic constituents than employed in the preparation of the laminated materials, exhibited less creep and cold flow than the laminated, and considerably less permanent set.

As for the effect of orientation of long chain-like molecules present in polystyrene, for example, a few tests conducted to date indicate less cold flow with the unoriented molecules. Another theory has also been advanced that the shorter the chain length, the less the internal frictional resistance to cold flow (24).

While cold-flow and creep phenomena throw much light upon the relative merits of the commercially available materials, the full answer will not be realized until the effect of molecular weight, molecular orientation, and the effects of extraneous materials such as fillers and plasticizers are fully examined under the same conditions. It is also interesting to note that plastic flow will occur in metals, even at ordinary temperatures at stresses below their elastic limit (31), though the ratio of plastic deformation to elastic deformation is very much smaller than with the plastics.

Shrinkage of Molded Plastics at Elevated Temperatures

Always a factor in determining the permanency of the physical properties of the plastics is the effect of temperature upon the shrinkage of the materials. This aftershrinkage occurs in addition to the normal thermal shrinkage of the material when it is removed from the mold. Failure to take it into consideration has sometimes caused serious discrepancies in design values. A few results are reported in the curves, Figs. 18 and 19, for various plastics. These results are of particular interest to the thermosetting urea and phenolic plastics which are often exposed to higher temperatures than the thermoplastic materials, which are limited by their low heat-distortion point.

It has been generally assumed that further thermal polymerization (with certain temperature ranges) of thermoplastic and thermosetting compounds results in improved physical properties, though some exceptions have been noted (19). In phenolics, for example, the additional polymerization incurred on exposure to elevated temperatures, is accompanied by internal shrinkage in the material as the phenolic constituents attach themselves to the main primary bond structure through methylene linkages.

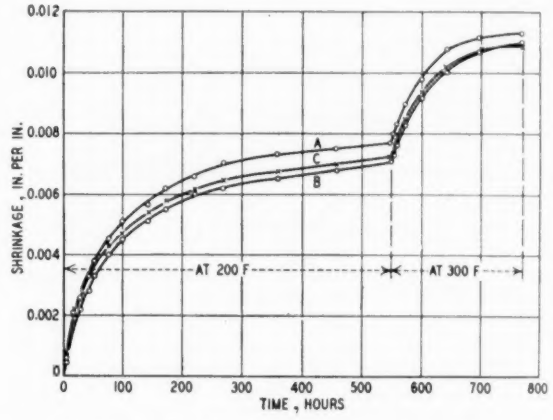


Fig. 18 Shrinkage of Molded Phenolics on High-Temperature Exposure for Different Cures
(*A*, 1 min cure; *B*, 2 min cure; *C*, 10 min cure.)

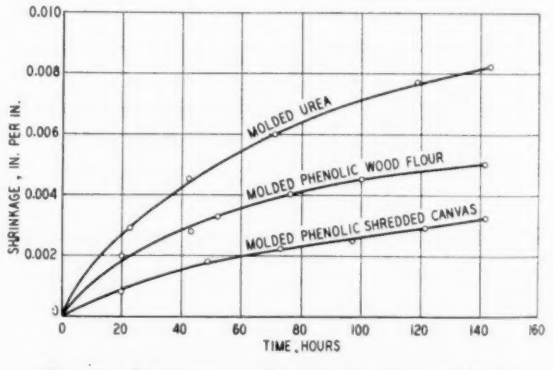


Fig. 19 Shrinkage of Molded Plastics at 200 F

Nevertheless, the results of Fig. 18 are quite interesting, for only slight differences were noted for a molded article with 1-, 2-, and 10-min curing periods in the mold. The measurements were made with a 4- to 5-in. micrometer upon a 4.5-in-diam molded ring and are accurate to plus or minus 0.0001 in. It was expected that the poorly undercured 1-min sample would shrink considerably more than the overcured, whereas only a slight difference was recorded. This observation has been corroborated by other investigators (32).

Application of excessive temperatures of 500 F produced swelling in a very short period of time due to the thermal disintegration of the resin molecule. One must assume that the prime molecular structure of the materials is developed during the period the material is molded under heat and pressure. Further changes in polymerization outside of the mold must result in a reduction of soluble stage "B" resin constituents which Houwink assumes as

being present within a strong, polymerized primary molecular structure (4). Chemical, electrical, and physical properties may be expected to improve through a reduction of these constituents, providing the temperature is not too high to be detrimental to the resin or its filler. For a discussion of other possible effects produced by aging in phenol-formaldehyde products, there are other studies (33).

In summarizing the discussion of the shrinkage of hot-molded phenolics and urea plastics, the significance of time and of temperature must be pointed out again. For example in Fig. 18 (which employed bakelite molding composition No. 8041), a limit in the amount of aftershrinkage was indicated when the material was maintained at 200 F. Continuance of the temperature exposure at 300 F after 550 hr revealed more than a 50 per cent increase in aftershrinkage. After the specimens were removed from test and left to stand in the atmosphere, a slight swelling was observed after several weeks, due to the absorption of moisture, though this new over-all dimension was considerably smaller than the original dimension.

The effects of increasing temperature upon thermoplastic materials are quite marked. If they are under stress, they will distort readily, particularly as the temperature approaches their heat-distortion point. If not under mechanical stress, as for example an electrical insulating coil bobbin, they may be used above their heat-distortion point. It must be pointed out,

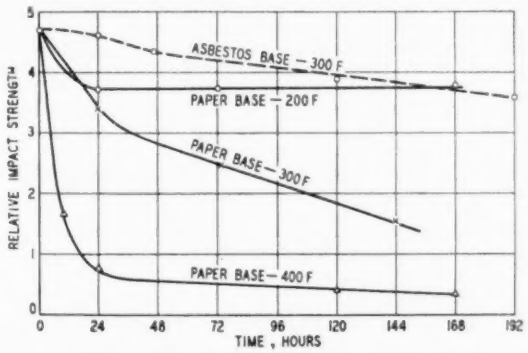


FIG. 20 EFFECT OF TEMPERATURE EXPOSURE ON IMPACT STRENGTH OF LAMINATED PHENOLICS

however, that cellulosic products which are usually highly plasticized, will suffer a loss of plasticizer content and residual solvent and become embrittled if exposed to excessive temperatures for too long a period. Shrinkage in molded cellulose-acetate articles will also be due to loss of moisture content. In comparison with other materials shown in Fig. 19, an injection-molded cellulose-acetate article showed a shrinkage of 0.013 in. per in. in 24 hr, though it does not change much after this. It is noted further that the actual shrinkage was dependent upon which portion of the article measurements were taken, with respect to the injection gate.

Cast products of polystyrene and methyl methacrylate are susceptible to further contraction during aging, due in part to further polymerization. In casting these materials from their monomeric form, there is a 15 to 20 per cent reduction in volume. Tests upon articles cast of methyl methacrylate revealed that as much as 0.020 in. per in. shrinkage took place over a 24-hr period at 200 F.

RETENTION OF PHYSICAL PROPERTIES AFTER HIGH-TEMPERATURE EXPOSURE

What of the physical properties of the plastics after they have returned to room temperature after being exposed to high temperatures for long periods of time? If thermoplastic in nature and they have suffered a loss of volatile materials which contributed to their impact strength, we may of course expect a diminution of the physical properties. For thermosetting materials, much depends upon the temperature in which the material was operating and the time at that temperature. For temperatures up to 275 F, an improvement in physical properties has even been noted for some of the laminated phenolics. In Fig. 20 some interesting data are given to show the effects of high-temperature exposure upon the physical properties of laminated phenolics. Most of the results are for grade-XX paper-base laminated phenolic plastics, though for comparison, the decrease in impact strength of an asbestos-base material at 300 F is included. Considerations, such as those which reveal a marked loss in impact

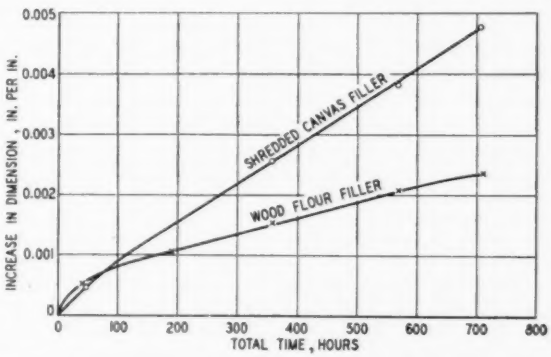


FIG. 21 DIMENSIONAL STABILITY AT HIGH HUMIDITY FOR HOT-MOLDED PHENOLICS

(At 86 per cent relative humidity; 85 F temperature; wall thickness 1/8 in. average.)

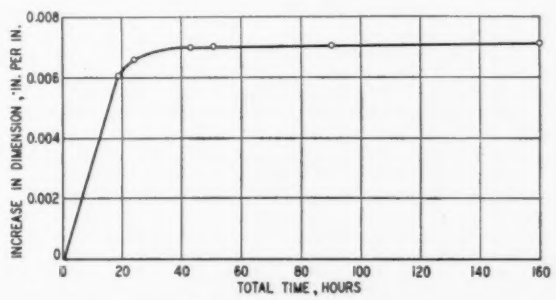


FIG. 22 DIMENSIONAL STABILITY AT HIGH HUMIDITY FOR MOLDED CELLULOSE ACETATE

(At 85 per cent relative humidity; 85 F temperature; wall thickness 0.05 in.)

strength after being placed in service for a certain length of time, necessitate the provision of ample factors of safety. The significance of the previous history of the material has been emphasized by more than one investigator (34).

EFFECT OF MOISTURE ON PERMANENCY OF PHYSICAL PROPERTIES

A few remarks on the effects of moisture on the physical properties will serve to emphasize this important influence on the permanency of the characteristics. Much study has been given to the changes in physical dimensions of various organic plastics upon exposure to moisture (35, 36). This consideration of permanency is of particular import when the molded articles are to operate in high humidity or in water. For example, laminated phenolic bearings operating in water must be designed with adequate shaft clearance to prevent seizure when the materials swell slightly in the course of time. The author has conducted several tests in a highly humid atmosphere to illustrate the effects of high humidity upon dimensional stability of typical molded

articles, Figs. 21 and 22. These data are given primarily for illustrative purposes, as no attempt was made to investigate thoroughly the question of moisture absorption. The designer is cautioned to make adequate allowance for swelling when the plastic parts are operated in water or high humidity. Particularly noteworthy is the more rapid permeation of moisture in the cellulose structure than with the molded phenolics. Under the same conditions of test, polyvinyl chloride acetate and methyl methacrylate showed no measurable change in 48 hr.

In conclusion it must be emphasized once again that there are many factors affecting the permanency of the physical properties of plastics. This paper has attempted to deal with the fundamental aspects of one of the most significant factors, plastic deformation under stress. Further, the reader's attention was called to other significant phenomena such as shrinkage at high temperatures, and swelling in moisture for various molded articles, though the analysis of these conditions was not pursued as actively as for the creep and cold-flow phenomena.

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Discussion

E. H. Hull[5] and A. J. Nerad.[5] It would appear that, in order to account for permanent set in the mechanical analogy given in Fig. 1 of the paper, another friction device should be added which has no spring across it. If the load in Fig. 1 is removed the lower spring will recover completely, since it is spanned by a viscous impedance.

Our experience with rubber and with steels at high temperatures and moderate stress indicate that time-deflection curves do not reach a constant slope within a reasonable length of time.

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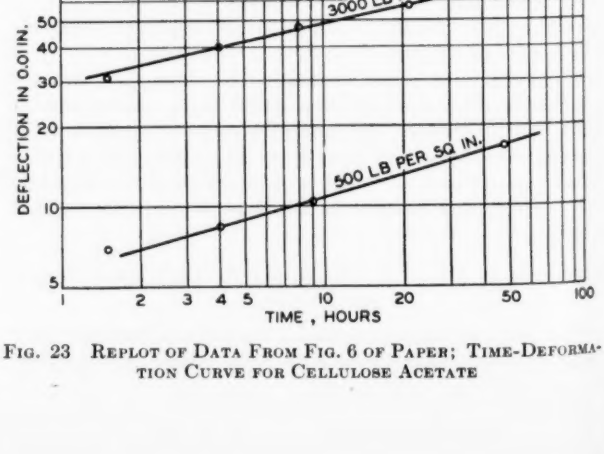


Fig. 23 Replot of Data From Fig. 6 of Paper; Time-Deformation Curve for Cellulose Acetate

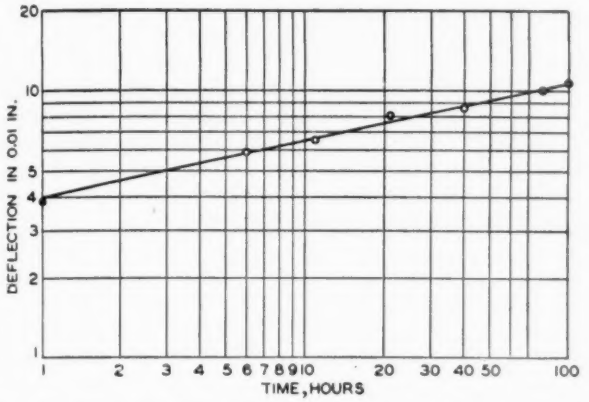


FIG. 24 REPLOT OF DATA FROM FIG. 13 OF PAPER; TIME-DEFORMATION CURVE FOR LAMINATED PHENOLIC PLASTIC

For example, if the data from Figs. 6 and 13 of the paper are plotted on log-log paper as in Figs. 23 and 24, straight lines are obtained with a slope considerably less than unity. An expression of the following form will represent this straight line

$$\gamma = KT^n$$

where γ = deflection, in. at any time T, hr
K = deflection at 1 hr
n = slope of line.

Differentiating this expression we have

$$\frac{d\gamma}{dt} = nKT^{n-1}$$

which will also give a straight line when plotted on log-log paper.

Since n is considerably less than unity, this expression indicates that the change in deflection per unit time decreases with time. This point has been made for steel by Hanson[6] and Zschokke.[7]

C. W. MACGREGOR.[8] The close similarity between the test results for plastics given in the paper and those obtained for the creep of metals is apparent. For the creep curves in Figs. 13 and 15, there is at first a portion of each curve having a diminishing creep rate followed by a region through which the creep rate is constant. In the creep of metals, these regions have been called the primary and secondary stages of creep or primary and secondary creep, respectively. The term "cold flow" used in the paper for the secondary stage of creep appears to be somewhat undesirable not only from the standpoint that this type of creep occurs at different temperatures also, but because the second portion of the creep curve has no distinguishing temperature condition from the initial stage. In metals, it is felt that the constant creep rate in the second stage is due to a balance between the annealing effect which tends to increase the slope and the strain-hardening which has the effect of decreasing the slope. A somewhat similar condition may be present for plastics.

A third stage of creep is obtained for metals if they are subjected to sufficiently high temperatures and stresses. This region is characterized by a rapidly increasing creep rate until fracture occurs. It would seem that a similar stage would be received for plastics if the tests were run for sufficiently long periods of time at higher temperatures than those used in the investigation under discussion.

Metals also show similar creep-recovery phenomena.[9] It is interesting to note that for the higher stressed plastics, after a sufficient time has elapsed, most of the creep has been recovered. From these tests it is evident that creep recovery is a pronounced function of stress at least and that, if the recovery curves were plotted for longer times, they might become nearly asymptotic.

Concerning the method of test, the use of four-point loading or so-called "pure bending," where the bending moment is constant over a considerable length having also zero shear, would seem to have certain advantages over the cantilever method of testing where the bending moment varies linearly and there is a constant shear present. In case four-point loading is used, the deflection of the portion of the beam under a constant bending moment from the inside loading points, i.e., the deflection of the center of the beam from the chord drawn between the inside loading points, could be obtained readily. It would also appear desirable to reduce the width of the specimen from 1 in. to about $^1/_2$ in. or less, since plate action is introduced in the wider cases and the test piece is essentially a cantilever plate, the length-to-side ratio of which is 5 to 1, and not a beam as we ordinarily define it to which Equation [1] in the paper applies.

AUTHOR'S CLOSURE

The differences of opinion which exist as to the attainment of a constant slope in the time-deflection curves of materials under stress are brought out by the two discussions of this paper. Mr. MacGregor presents evidence which agrees with the results disclosed by the paper, while Mr. Hull and Mr. Nerad indicate results to the contrary. It appears to the author that the problem is a fundamental one in materials and that cognizance must be given to the nature of elastic and plastic reactive forces appearing in the material under stress. Direct comparisons of steel which behaves more closely to an elastic body than organic plastics are difficult, primarily because of such phenomena as strain-hardening. By the same token, it would not be propitious to compare directly materials which closely approach true plastic behavior, even though the attainment of a constant slope of load deflection occurs in a very short period of time.[10] Reiterating, it is the author's experience with organic plastics that, under test conditions involving no chemical changes in the material, and under constant stress per unit area loading, for all practical purposes a constant slope in the deflection-time curve will be observed.

Concerning the formulas and replotting of data by Messrs. Hull and Nerad, it should be noted that an actual calculation of n from data presented in the text shows that n is a function of K, and K is dependent upon a reference-time point. A more satisfactory manner of representing the time dependence of the rate of change of deflection with time for true plastic flow is shown in Equation [7] of the text, a substantially constant rate being achieved after long periods of time.

Their comments concerning Fig. 1 of the paper are quite correct, for the mechanical analogy indicated was merely a simple representation of a considerably more complex system. The correct picture would show a friction device between each coil of the lower spring. As in the graphical method of indicating distributed capacitance between the turns of a coil, the friction elements are combined into one illustrative element.

Mr. MacGregor's suggestions for the method of test are inter-

[6] "Creep of Metals," by D. Hanson, Trans. American Institute of Mining and Metallurgical Engineers, vol. 133, 1939, pp. 15–57.

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esting, particularly the desirability of reducing specimen width to 1/2 in., which is slightly more satisfactory. Tests upon polystyrene were conducted with this width, and the author contemplates adopting this size for the others. Regarding the four-point loading, the constant bending moment is in its favor. However, the means of attachment of the loading weights which usually require some hole or notch in the specimen would present a discontinuity at the point of maximum bending moment, whereas, in the case of the simple cantilever beam, the point of attachment is located at the minimum bending moment. Furthermore, shorter specimens may be more easily handled by the cantilever method.

Extensions of the recovery curves show an asymptotic relationship. In fact for given values of applied stress during the loading period, there may be plotted a relationship between permanent set and applied stress, for a given time. Further, an interesting relationship exists between permanent set and time of applied load, for a given stress and temperature.

Laminar-Flow Heat-Transfer Coefficients for Ducts

By R. H. NORRIS,[1] SCHENECTADY, N. Y., AND D. D. STREID,[2] WEST LYNN, MASS.

In this paper, theoretically derived values of the heat-transfer coefficient for flat ducts with constant surface temperature are presented and compared with the corresponding results for round ducts which have been previously published. A few test data for air in flat ducts are compared with the theoretical results. For both flat ducts and round ducts the results are presented both in customary terms and in terms of the logarithmic-mean temperature difference. The latter results provide a consistent basis for comparisons with heat-transfer data for turbulent flow. Some limiting values of the local heat-transfer coefficient for both constant surface temperature and constant heat input per unit length are tabulated, and some equations pertinent to very short ducts are included.

Nomenclature

A = surface area of that part of duct which transfers heat (for a flat duct it includes both sides if both are transferring heat), sq ft
S = cross-sectional area for fluid flow, sq ft
b = minimum distance from center to wall of duct, ft
c = specific heat (at constant pressure), Btu per lb per deg F
C_1 = constant in Equation [12] tabulated in Table 2, dimensionless
D = equivalent diameter = $4LS/A$, ft
g = acceleration of gravity = 4.17×10^8, ft per hr per hr
G = $w/S = \rho V$ = weight velocity, lb per sq ft per hr
G_r = $D^3\rho^2\beta\,\Delta t_a g/\mu^2$ = Grashof number, dimensionless
h = heat-transfer coefficient, based on particular temperature difference indicated by subscript used with h, as defined by Equation [1], Btu per hr per sq ft per deg F
h_a = h on arithmetic-mean-temperature basis (see Equation [1])
h_i = h on inlet-temperature-difference basis (see Equation [1])
h_L = h on logarithmic-mean-temperature basis (see Equation [1])
h_x = h on local-temperature-difference basis (see Equation [1])
j = $\dfrac{h_L}{cG}\left(\dfrac{c\mu}{k}\right)^{2/3}$ = heat-transfer factor, dimensionless
k = thermal conductivity of fluid (subscripts, of same meaning as those for μ), Btu per hr per sq ft per deg F per ft
L = length of duct or plane, or distance from leading edge or entrance of heated portion of duct, ft
q = total heat-transfer rate, Btu per hr
R_D = $\dfrac{\rho VD}{\mu} = \dfrac{GD}{\mu}$ = Reynolds' number, dimensionless
t = fluid temperature, deg F
t_m = transverse mean (mixing-cup) temperature of the fluid (1),[3] deg F
t_1 = t_m at inlet, deg F
t_2 = t_m at outlet, deg F
Δt_a = $t_w - (t_1 + t_2)/2$, deg F
t_w = wall temperature, deg F
t_{w1} = wall temperature at inlet, deg F
t_{w2} = wall temperature at outlet, deg F
V = $w/\rho S$ = mean velocity, ft per hr
w = total weight flow of fluid, lb per hr
x = axial distance from the entrance to any given point in duct, ft
β = coefficient of volumetric expansion of fluid, (deg F)$^{-1}$
μ = viscosity of fluid, lb per ft per hr
μ_f = viscosity at mean film temperature, as defined by Colburn (4), lb per ft per hr
μ_m = viscosity at fluid temperature t_m, lb per ft per hr
μ_w = viscosity at wall temperature t_w, lb per ft per hr
ρ = density of fluid, lb per cu ft
ϕ = empirical correction factor, defined by Equation [24], dimensionless
Φ = cGD^2/kL, dimensionless

Purpose of Heat-Transfer Study

Heat transfer to fluids in laminar flow in ducts is amenable to exact mathematical analysis provided constant fluid properties and reasonably simple flow conditions, concerning temperature distribution along the duct walls and velocity distribution across the duct, may be assumed.

For round ducts, on the basis of these assumptions, a number of expressions (1, 2, 3),[3] derived analytically for the heat transfer, have been available for some years; and empirical correction factors (4, 5) have more recently been developed to correct these equations for the effects of the discrepancies between the assumptions used and the actual conditions in practical applications. Each of the various theoretical expressions is theoretically accurate only within its own appropriate range of the major variable, however, and no convenient combined summary of all these ranges of application seems to be available.

For flat ducts, on the other hand, no theoretical results for assumptions reasonably representative of practical conditions have been available, in spite of the importance of flat-duct problems in such applications as air flow between fins and the flow of cooling oil in narrow annular or flat ducts. Although Nusselt (6) has calculated the heat transfer to a liquid film of constant thickness flowing down a flat plate with the same assumption of velocity distribution (parabolic) which is reasonable for flat ducts, the application of his results to flat ducts has not been made.

This paper has three purposes which are: (*a*) To present for flat ducts theoretical results covering the complete range of variables, based largely on Nusselt's method; (*b*) to present for round ducts similarly complete theoretical results, showing how these are selected from different sources by applying results from each source to a different, specific range of the major variable; and (*c*) to express all these results in terms of alterna-

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

tive bases of correlation, together with certain simple approximations, for convenience for different applications.

For simplicity, the words "flat duct" are used throughout this paper to denote the case of flow between flat parallel plates of infinite extent perpendicular to the flow direction. This represents the limiting case of a duct of rectangular cross section when the ratio of the longer to the shorter side becomes very large.

Theoretical Assumptions

The theoretical results presented in this paper should be used with an understanding of the assumptions on which they are based. Although these assumptions will not be discussed in detail, the effects of the major practical variations from them will be considered later. The assumptions are:

1 The transverse velocity distribution is parabolic with zero velocity at the wall.

2 The duct wall is at constant temperature, unless specified otherwise.

3 The fluid properties (c, ρ, and k) are constant.

4 Heat conduction in the direction of flow is negligible.

5 Temperature conditions do not change with time.

Theoretical Results Shown Graphically and Numerically

One convenient form of correlation of results is in terms of the dimensionless group h_aD/k, as a function of the dimensionless group $\Phi = cGD^2/kL$. For round ducts, many authors have used a different dimensionless group wc/kL, which is related to Φ as follows

$$\Phi = \frac{4}{\pi} \times \frac{wc}{kL}$$

The variable Φ is used in this paper because it is applicable to flat ducts as well as to round ones.

Table 1 presents numerically the theoretical results for this form of correlation, as well as for certain other forms discussed later. Understanding of the sources of the results in Table 1 is facilitated by consideration of Fig. 1, where the results from each source are shown graphically.

Plotted in Fig. 1, for both round and flat ducts, are two theoretically derived solutions for h_aD/k, one of which, the Graetz solution (2) or its numerical equivalent (6), is applicable for low values of Φ, and the other, Lévêque's approximation (3), is applicable for high values of Φ. The range of values of Φ, for which each solution is accurately applicable, is evident from inspection of Fig. 1, and the results in Table 1 were selected accordingly, as explained in the following:

For round ducts, the numerical values of h_aD/k in Table 1 were calculated by the authors from Graetz's original equations (2) for values of Φ below 150, and from Lévêque's approximation (3) for the values of Φ above 2000. For the intermediate values of Φ (150 to 2000), a linear interpolation on the log-log plot was used, as shown by the dashed line in Fig. 1.

For flat ducts, the values of h_aD/k in Table 1 for values of Φ up to 400 were obtained from a numerical solution as calculated by the authors using the method described by Schmidt (7), before they learned that Nusselt (6) had presented the same sort of solution. The authors' results were later found to agree with Nusselt's results over this range of Φ. This agreement should increase confidence in their validity. For values of Φ above 400, the data in Table 1 were calculated from Lévêque's approximation (3), as modified by the authors to make it applicable to flat ducts.

Alternative Temperature Bases

Four alternative temperature differences and the relations between the heat-transfer coefficients based on them are

$$\frac{q}{A} = h_a\left[t_w - \frac{t_2 + t_1}{2}\right] = h_i\,[t_w - t_1]$$

$$= h_L\left\{\frac{(t_{w1} - t_1) - (t_{w2} - t_2)}{\log_e\,[(t_{w1} - t_1)/(t_{w2} - t_2)]}\right\}$$

$$= (1/L)\int_0^L h_x(t_w - t_m)dx \qquad [1]$$

The coefficient h_a based on the arithmetic-mean temperature difference, has heretofore been used widely (5, 8, 9). The coefficient h_i, based on the inlet temperature difference simplifies the evaluation of the total heat q but, as shown later, the coefficient h_i itself cannot be represented by a simple approximate equation over as large a range of values of Φ for round ducts as the coefficient h_a.

The coefficient h_L, based on the logarithmic-mean temperature difference has been previously used in the literature, not for laminar flow, but only for turbulent flow where it is always equal to h_x which does not vary with x (except as changes of mean temperature with x affect fluid properties and thereby affect h_x). Because laminar flow h_x varies with x and its true mean value (1) is represented by neither h_L, h_a, nor h_i, all three of these coefficients may be considered as equally arbitrary choices. Nevertheless, h_L has several advantages over h_a. As shown later, the maximum error introduced by certain convenient approximate equations for h_L and h_a is either smaller for h_L than for h_a, or at least the same, and the direction of this maximum error makes the calculated heat transfer too low instead of too high. Then too, the use of h_L for laminar flow permits comparison with turbulent-flow data on a consistent basis. Finally, for decreasing values of Φ, h_LD/k approaches asymptotically a lower limit, which is not zero but is the same as the lower limit for h_xD/k, whereas h_aD/k decreases linearly with Φ toward zero. This behavior of h_aD/k is likely to be misleading. For example, for values of Φ below 8, a decrease in D will be found to decrease h_a, which may seem at first glance undesirable, whereas actually, it is likely to be desirable, since it will increase h_x and h_L and thereby permit a reduction in the surface area required for a given total heat transfer q.

The local coefficient h_x is useful when the conditions at a particular part of the duct, rather than for the duct as a whole, are to be considered.

TABLE 1 HEAT-TRANSFER RELATIONS FOR PARABOLIC VELOCITY DISTRIBUTION AND CONSTANT WALL TEMPERATURE

	Round ducts			Flat ducts		
$\Phi = \frac{cGD^2}{kL}$	$\frac{h_iD}{k}$	$\frac{h_aD}{k}$	$\frac{h_LD}{k}$	$\frac{h_iD}{k}$	$\frac{h_aD}{k}$	$\frac{h_LD}{k}$
1	0.25	0.50	3.68	0.25	0.50	7.60
2	0.50	0.99	3.76	0.50	1.00	7.60
3	0.75	1.48	3.81	0.75	1.50	7.60
4	0.98	1.92	3.86	1.00	2.00	7.60
5	1.20	2.29	3.91	1.24	2.48	7.62
6	1.39	2.60	3.96	1.49	2.95	7.65
7	1.57	2.86	4.01	1.73	3.42	7.69
8	1.74	3.07	4.06	1.96	3.85	7.74
9	1.89	3.25	4.11	2.18	4.23	7.80
10	2.03	3.41	4.16	2.39	4.58	7.86
12	2.27	3.66	4.26	2.78	5.18	7.92
15	2.60	3.96	4.41	3.30	5.89	7.98
20	3.05	4.38	4.70	4.00	6.67	8.05
25	3.43	4.72	4.97	4.55	7.16	8.15
30	3.76	5.02	5.22	5.00	7.50	8.24
10	4.33	5.52	5.67	5.74	8.04	8.52
60	5.22	6.32	6.48	6.73	8.67	8.93
90	6.23	7.24	7.30	7.87	9.54	9.67
130	7.27	8.18	8.23	8.97	10.4	10.5
200	8.63	9.45	9.45	10.4	11.6	11.6
300	10.1	10.8	10.8	12.0	13.1	13.1
400	11.2	11.9	11.9	13.4	14.4	14.4
600	13.0	13.6	13.6	15.4	16.2	16.2
1000	15.6	16.1	16.1	18.8	19.5	19.5
2000	19.9	20.3	20.3	23.3	23.9	23.9
3000	22.9	23.3	23.3	26.4	26.9	26.9
4000	25.3	25.6	25.6	28.9	29.3	29.3
6000	29.0	29.3	29.3	33.1	33.5	33.5
10000	34.6	34.8	34.8	39.6	39.9	39.9
20000	43.6	43.8	43.8	50.0	50.2	50.2
30000	50.0	50.2	50.2	57.0	57.2	57.2
40000	55.1	55.2	55.2	62.6	62.8	62.8

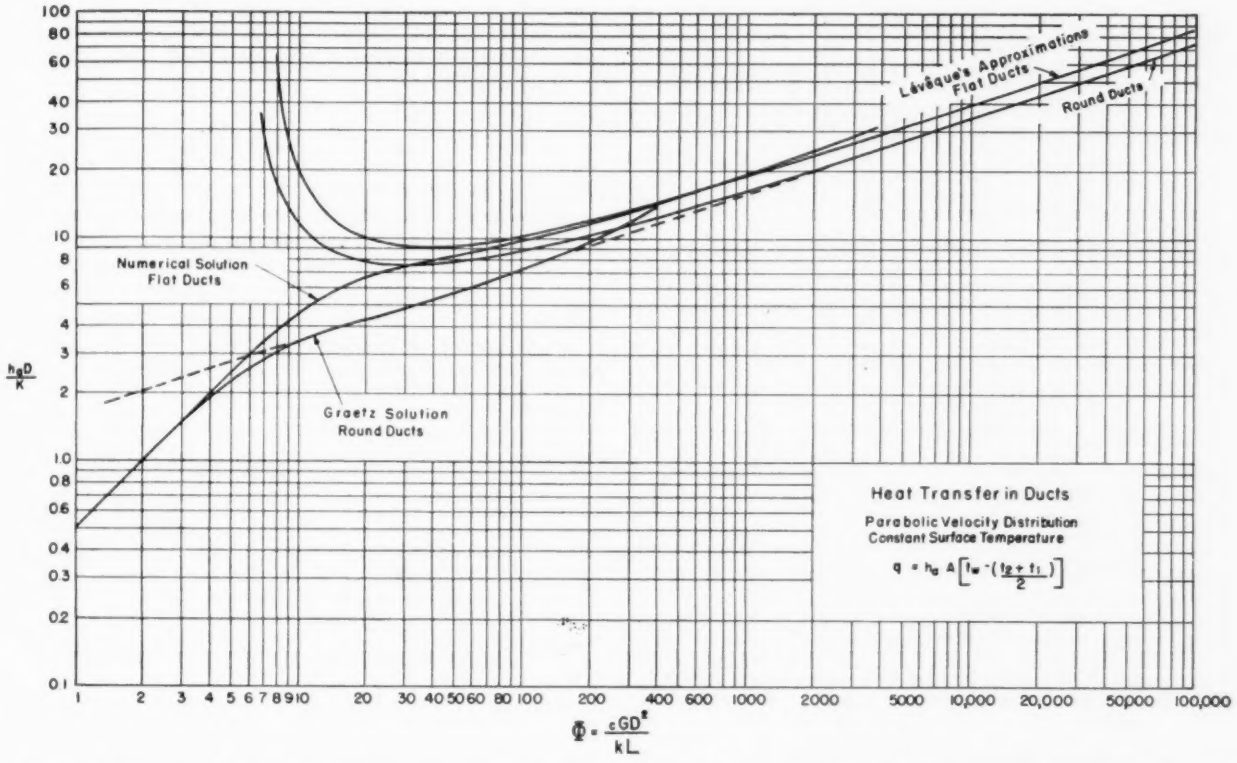


FIG. 1 THEORETICAL HEAT-TRANSFER CORRELATION FOR LAMINAR FLOW ON ARITHMETIC-MEAN TEMPERATURE-DIFFERENCE BASIS, SHOWING RESULTS OF VARIOUS ANALYSES FOR CONSTANT WALL TEMPERATURE AND PARABOLIC VELOCITY DISTRIBUTION

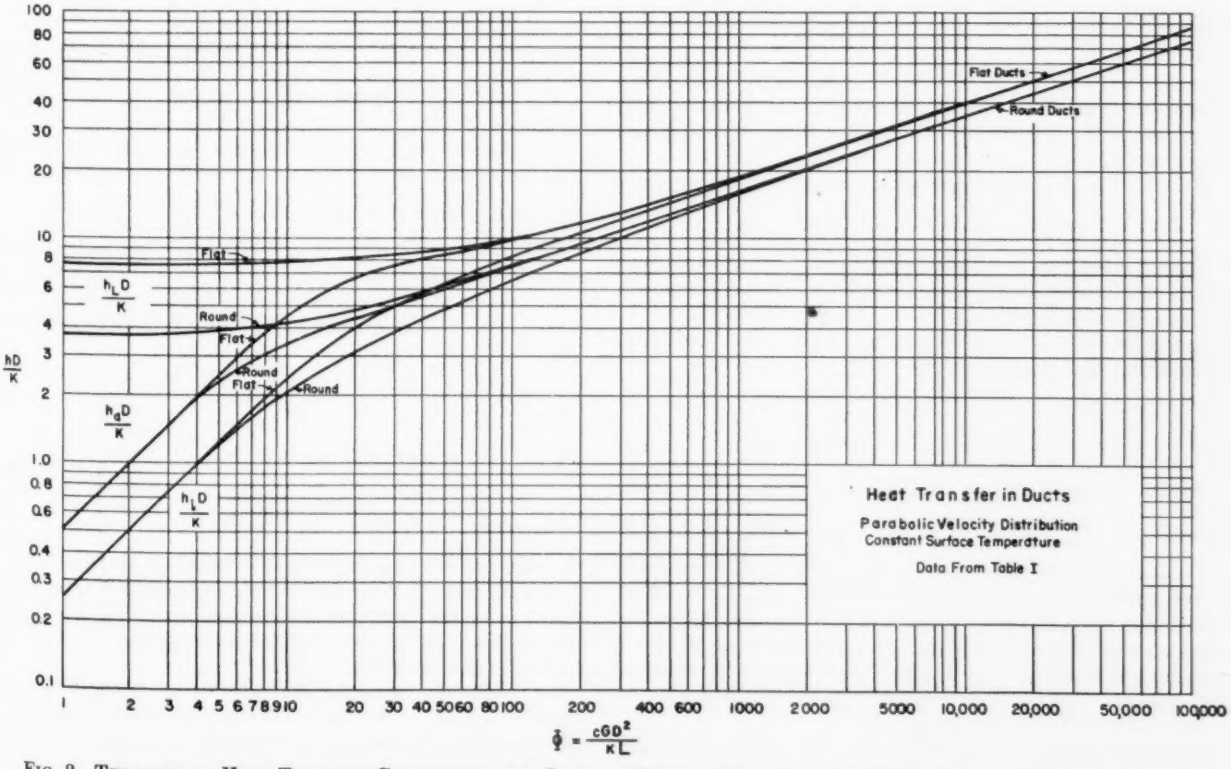


FIG. 2 THEORETICAL HEAT-TRANSFER CORRELATION FOR LAMINAR FLOW IN BOTH FLAT AND ROUND DUCTS; RESULTS SHOWN ON LOGARITHMIC-MEAN, ARITHMETIC-MEAN, AND INLET-TEMPERATURE-DIFFERENCE BASES
(Data from Table 1.)

Table 1 and Fig. 2 (which is based on Table 1) present the theoretical results for h_a, h_i, and h_L, for both flat and round ducts. Results for h_x are discussed later.

The results in this paper do not include explicit values for the ratio of the temperature rise of the fluid to the initial temperature difference, but such values may be obtained from the relation

$$\frac{t_2 - t_1}{t_w - t_1} = 4\,\frac{h_i D/k}{\Phi} \quad \ldots\ldots [2]$$

Approximate Equations for Theoretical Results

The range of applicability specified in the following approximations has been determined by comparison with the complete results in Table 1 and Fig. 2.

Arithmetic-Mean Temperature-Difference Basis. For round ducts, it is a fortunate coincidence that the single simple relation

$$h_a D/k = 1.615\Phi^{1/3} \text{ for } \Phi > 10 \quad \ldots\ldots [3]$$

represents the theoretical results within 5 per cent for the surprisingly large range of Φ indicated. This equation is shown as the dashed line in Fig. 1.

For flat ducts, no simple relation as Equation [3] is available for such a wide range of values of Φ but, for very large values of Φ, the following relation is applicable within 3 per cent

$$h_a D/k = 1.85\Phi^{1/3} \text{ for } \Phi > 2000 \quad \ldots\ldots [4]$$

For sufficiently low values of Φ, the theoretical results practically coincide with their asymptote, so that

$$h_a D/k = \Phi/2 \text{ for } \left\{\begin{matrix} \Phi < 3 \text{ for round ducts} \\ \Phi < 6 \text{ for flat ducts} \end{matrix}\right\} \quad \ldots\ldots [5]$$

which represents conditions existing when all the fluid reaches the wall temperature.

Inlet-Temperature-Difference Basis. For round ducts, Lévêque's approximation (3)

$$h_i D/k = 1.615\Phi^{1/3} \text{ for } \Phi > 2000 \quad \ldots\ldots [6]$$

is applicable but, for lower values of Φ, it deviates from the theoretical results, as indicated in Fig. 1, where it has been expressed in terms of $h_a D/k$.

For flat ducts, Lévêque's method, applied by the authors, gives

$$h_i D/k = 1.85\Phi^{1/3} \text{ for } \Phi > 400 \quad \ldots\ldots [7]$$

Logarithmic-Mean Temperature Difference. The entire range of values of Φ can be covered by the following set of approximations with a maximum error for round ducts of 18 per cent (near $\Phi = 12$), and for flat ducts of 17 per cent (near $\Phi = 70$), and these errors always make h_L too low, not too high

For round ducts $h_L D/k = 1.615\Phi^{1/3}$ for $\Phi > 12$ ..... [8]

and $h_L D/k = 3.66$ for $\Phi < 12$ ..... [9]

For flat ducts $h_L D/k = 1.85\ \Phi^{1/3}$ for $\Phi > 70$ ..... [10]

and $h_L D/k = 7.60$ for $\Phi < 70$ ..... [11]

For the arithmetic-mean temperature difference, the limits of Φ in the approximations of Equations [3], [4], and [5] could be modified to cover the entire range of Φ also, but if so, the maximum error for flat ducts would be 26 per cent instead of 17 per cent, and for round ducts, although the maximum error would be the same, it would make h_a too high, not too low, which is not conservative for design purposes.

Limiting Values for Local Coefficient. The local heat-transfer coefficient in ducts with laminar flow varies greatly with position in the duct (this variation is usually inappreciable in turbulent flow). However, at a sufficiently long distance from the duct inlet (small value of Φ), the local heat-transfer coefficient h_x approaches asymptotically (from above) a limiting value independent of velocity

$$h_x = \frac{k}{b/C_1} \quad \ldots\ldots [12]$$

where b is the distance from the duct wall to the center of the duct and C_1 is a numerical constant, depending upon the shape of the duct and the assumed velocity distribution, as given in Table 2. The physical significance of b/C_1 is that it represents the thickness of a stationary fluid film having a thermal resistance corresponding to the limiting value of h_x. This thickness represents the mean distance the heat has to travel through the fluid. Table 2 shows that this thickness is about one half the distance b, from the wall to the center of the duct.

TABLE 2 VALUES OF CONSTANT C_1 OF EQUATION $h_x = \frac{k}{b/C_1}$ REPRESENTING VALUES OF LOWER LIMIT OF LOCAL HEAT-TRANSFER COEFFICIENT h_x

	Uniform velocity distribution	Parabolic velocity distribution
Round duct; constant wall temperature	2.90[a]	1.83[a]
Round duct; constant heat input per unit length	4.00[b]	2.18[c]
Flat duct; constant wall temperature	2.47[a]	1.90[b]
Flat duct; constant heat input per unit length	3.00[b]	2.06[b]

[a] Reference (1), Bibliography.
[b] Values derived by authors.
[c] Reference (10), Bibliography.

Limiting values of the local heat-transfer coefficient can be obtained by simple analysis not only for constant wall temperature but also for constant heat input per unit length (10). The values for the latter condition are useful in estimating the maximum temperature which will occur on the surface of an electrically heated duct.

The smallness of the range of the values of C_1 shown in Table 2 has practical significance, for it indicates that the values of C_1 for other duct shapes or other surface temperature distributions, not yet analyzed, would not be likely to depart widely from this range. This significance extends also to the coefficient h_L, since the lower limit of h_L can be shown to be the same as that for h_x.

No values of the local coefficient h_x, other than the limiting value, are given here but some will be found in the literature (1).

Comparison Between Laminar and Turbulent Flow

A comparison of heat-transfer coefficients covering both laminar and turbulent flow in terms of the same variables is often desirable. One such comparison was made by Colburn (4), who plotted the dimensionless quantity called the "heat-transfer factor"

$$j = \frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3} \quad \ldots\ldots [13]$$

as a function of Reynolds' number

$$R_D = \frac{GD}{\mu} \quad \ldots\ldots [14]$$

The heat-transfer coefficient h, was based on the logarithmic-mean temperature difference for the turbulent-flow data, but it was based on the arithmetic-mean temperature difference for the laminar-flow data. This change of basis has certain disadvantages to be discussed later.

Fig. 3 for round ducts and Fig. 4 for flat ducts show the heat-transfer factor j, based on the logarithmic-mean temperature difference for both laminar and turbulent flow, as a function of Reynolds' number, as computed from the data elsewhere in this paper.

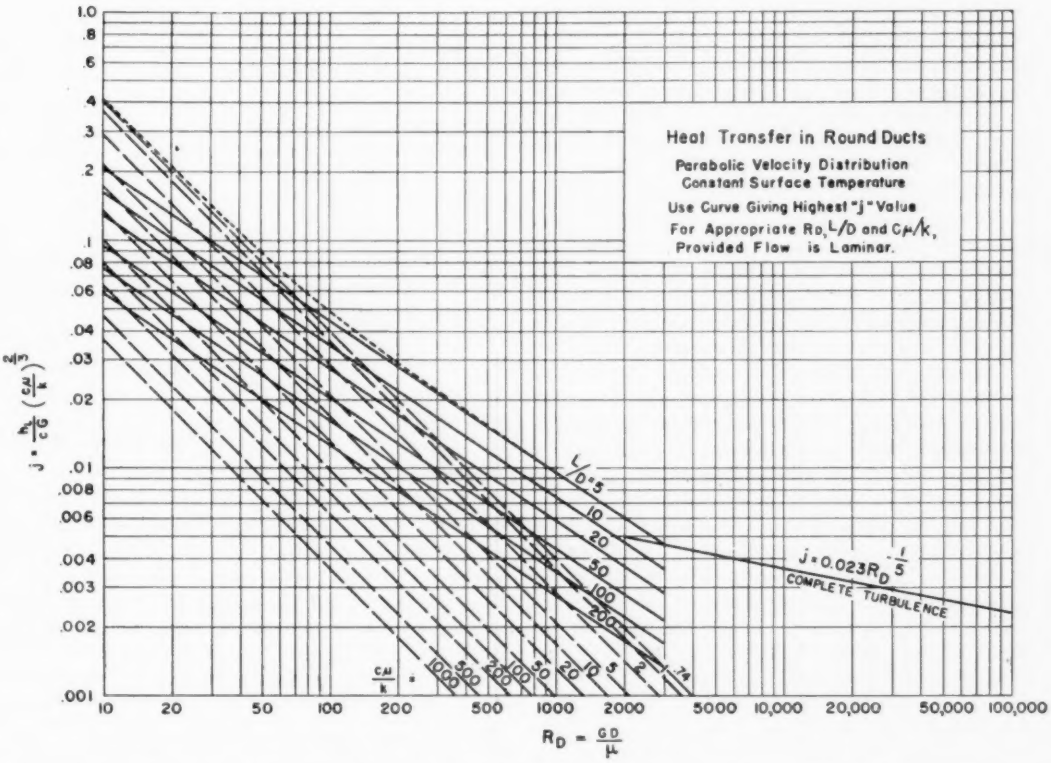


FIG. 3 THEORETICAL CORRELATION OF HEAT-TRANSFER FACTOR AND REYNOLDS' NUMBER FOR ROUND DUCTS, FOR FLOW WHICH IS DEFINITELY LAMINAR OR DEFINITELY TURBULENT

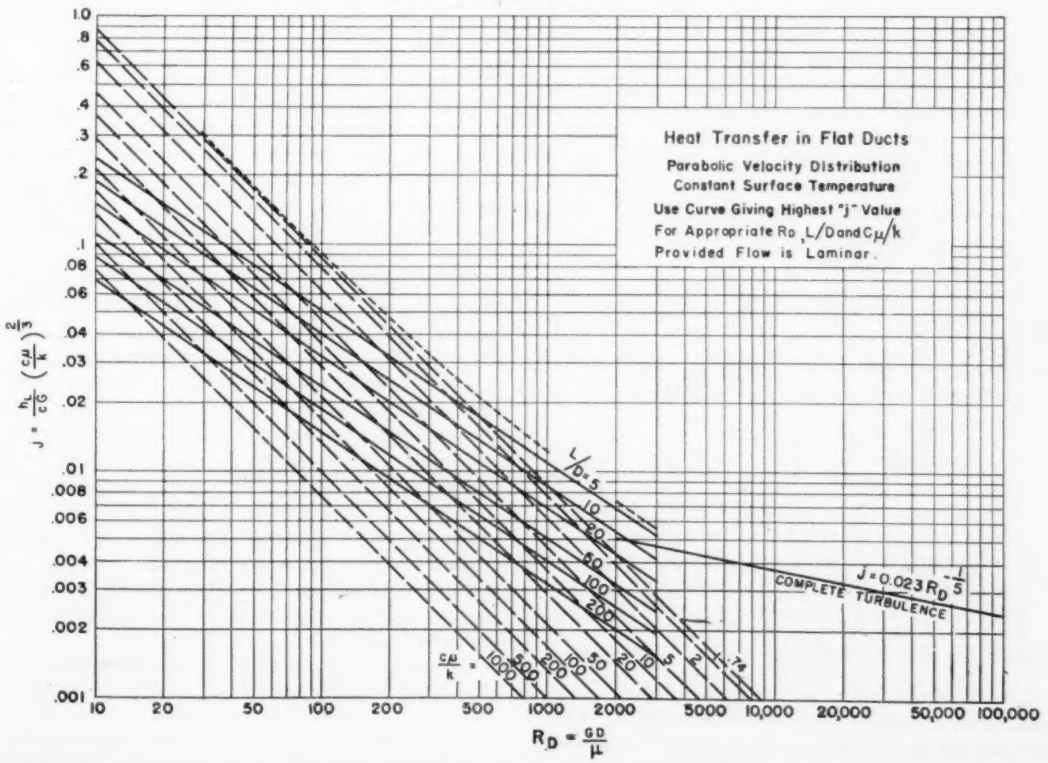


FIG. 4 THEORETICAL CORRELATION OF HEAT-TRANSFER FACTOR AND REYNOLDS' NUMBER FOR FLAT DUCTS FOR FLOW WHICH IS DEFINITELY LAMINAR OR DEFINITELY TURBULENT

When the flow is definitely turbulent, the data for all fluids may be approximated by a single curve, as is well known. This was one of the original reasons for choosing j and R_D as the variables. This single curve is shown in Figs. 3 and 4, for values of Reynolds' numbers above 2000, although often the flow may not be definitely turbulent unless Reynolds' number is much greater than 2000.

For laminar flow, the analytical results in terms of j and R_D are not correlated by a single curve unless two parameters, L/D (a function of the duct proportions) and $c\mu/k$ (a function of the fluid properties) are maintained constant. Families of curved lines would be necessary to show completely the effects of different values of these parameters. Only one of these curved lines is shown in Figs. 3 and 4, namely, the curved line shown dotted for $L/D = 5$, and for $c\mu/k = 0.74$ (the value for air). The curved lines for other values of L/D and of $c\mu/k$ are not shown, but instead are represented, as will be explained, by the two sets of straight lines which they approach asymptotically as limits.

The straight lines of one set, all parallel, approximate the analytically exact curves for sufficiently low values of R_D; the lines of the other set, likewise parallel but of different slope from the first set, are the approximation for sufficiently high values of R_D (provided the flow remains laminar). For values of R_D, where the steeper asymptote represents the curve, the duct proportion L/D has no effect on the relation between j and R_D, so that the location of this asymptote varies only with $c\mu/k$, as indicated in Figs. 3 and 4. The location of the asymptotes of the other set, however, varies with L/D but is unaffected by $c\mu/k$. For any one particular value of L/D and of $c\mu/k$, the curve of j versus R_D is a curved line joining the asymptote of one set to the asymptote of the other. This curved line is identical in shape to the one shown dotted and is merely transposed in position to fit the proper asymptotes.

Fortunately, the maximum error introduced by ignoring the transition curve and using instead, for any particular case, the appropriate asymptote of each set up to the point of intersection of these two asymptotes is only 13 per cent for round ducts and 17 per cent for flat ducts. This error can be reduced to less than 4 per cent by estimating the location of the appropriate transition curve by the use of a pair of dividers.

Both the magnitude and direction of these errors give the logarithmic-mean basis the same advantage over the arithmetic mean for the asymptotic correlations of j versus R_D, as previously mentioned for the asymptotic correlations of $h_L D/k$ versus Φ. The change from one of these pairs of dimensionless variables to the other pair changes neither the magnitude nor direction of the error in the heat-transfer coefficient computed from the asymptotic correlation.

The relations between the variables j and R_D used in Figs. 3 and 4, and $h_L D/k$ and Φ used for other correlations are

$$R_D = \frac{\Phi\left(\frac{L}{D}\right)}{\left(\frac{c\mu}{k}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots [15]$$

$$j = \frac{\left(\frac{h_L D}{k}\right)\left(\frac{c\mu}{k}\right)^{2/3}}{\Phi\left(\frac{L}{D}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots [16]$$

For laminar flow, the equations for those asymptotes in Figs. 3 and 4 which depend on $c\mu/k$ (those applicable at low values of R_D) are obtained from Equations [9] and [11] by the foregoing relations, and are

$$j = \frac{3.66}{R_D\left(\frac{c\mu}{k}\right)^{1/3}} \text{ (round ducts, Fig. 3)} \ldots\ldots\ldots [17]$$

$$j = \frac{7.60}{R_D\left(\frac{c\mu}{k}\right)^{1/3}} \text{ (flat ducts, Fig. 4)} \ldots\ldots\ldots [18]$$

The equations for the asymptotes which depend on L/D (applicable at high values of R_D, provided the flow remains laminar) are obtained from Equations [8] and [10] by the use of Equations [15] and [16], and are

$$j = \frac{1.615}{(R_D)^{2/3}(L/D)^{1/3}} \text{ (round ducts, Fig. 3)} \ldots\ldots [19]$$

$$j = \frac{1.850}{(R_D)^{2/3}(L/D)^{1/3}} \text{ (flat ducts, Fig. 4)} \ldots\ldots [20]$$

The curved lines joining the asymptotes for $c\mu/k$ of 0.74 and L/D of 5, shown dotted in Figs. 3 and 4, were obtained by the use of Equations [15] and [16] and the comparatively exact relation between $h_L D/k$ and Φ shown in Table 1 and Fig. 2. The corresponding curved line joining any other two asymptotes will have the same shape as this dotted curve, but will be transposed, as previously explained.

For completely turbulent conditions, a formula proposed by Colburn (4) on the basis of test data for round ducts from many sources, as a convenient approximation is

$$j = 0.023(R_D)^{-1/5} \ldots\ldots\ldots\ldots\ldots [21]$$

This relation is shown as the single straight line in the right-hand portions of Figs. 3 and 4.

Transition Between Laminar and Turbulent Flow

The flow may, however, be neither laminar nor completely turbulent, but of an intermediate nature, in which case the heat-transfer coefficient will be between the value calculated for laminar flow and the value for turbulent flow. The range of Reynolds' numbers covered by this transition begins not far from a Reynolds number of 2300, but depends also on μ_m/μ_f as explained by Colburn (4), and may extend upward as high as a Reynolds number of 10,000 for large values of L/D, but not as high for smaller values of L/D.

Unfortunately, a set of curves representing the transition from the curves of j versus R_D for laminar flow to the curve for completely turbulent flow are not available for flat ducts, and for round ducts only the tentative empirical transition curves proposed by Colburn (4) are available. The latter curves, although themselves on the logarithmic-mean basis, were chosen to fit the curves for laminar flow based on the arithmetic-mean temperature difference, a procedure which, while useful as a temporary expedient, is not fundamentally sound. Availability of the present correlation on the logarithmic-mean basis gives opportunity for a revised choice of these transition curves on a sounder basis. No further consideration is given this choice here, however, since it is a major problem in itself, requiring both critical study of extensive existing data and experiments to obtain additional test data, particularly for flat ducts.

In practical applications of the foregoing analytic results, deviations from the assumptions used in the analyses require consideration. The likelihood and effect of deviations from constant wall temperature, from constant fluid properties, and from parabolic velocity distribution are considered subsequently. Deviation from the other assumptions are generally negligible.

Variation of Wall Temperature

If the wall temperature is not constant, the data presented here do not apply, in general, but if the variation of the wall temperature is linear, the approximation given by Equation [6] for round ducts remains valid for values of Φ above 2000, according to Lévêque's analysis (3), providing the wall temperature is taken as

$$t_w = t_{w1} + \frac{3}{5}(t_{w2} - t_{w1}) \quad [22]$$

This same expression for t_w should also apply to the approximation Equation [7] for flat ducts, for values of Φ above 400, by reasoning similar to Lévêque's. For lower values of Φ, Equation [22] has sometimes been used to correlate data in the absence of any rational or empirical basis.

If the heat input per unit length is constant, the wall temperature will in general be neither constant nor linear. In the limiting case for very small values of Φ, however, it will become linear, and analysis of the heat-transfer phenomena becomes relatively simple. The results are here expressed, however, not in terms of an equation for the wall-temperature variation, but in terms of the local heat-transfer coefficient, the significance and values of which have already been discussed.

Free Convection and Variation of Fluid Properties

The relations derived previously, assuming parabolic velocity distribution, do not take account of the transverse and longitudinal variation of fluid properties with temperature. The variation of viscosity will tend to produce nonparabolic velocity distribution, and the variation of density will tend to produce free convection currents in large ducts at low velocities of flow. The change in the velocity distribution has recently been analyzed (11), but not its effect on the heat transfer.

To allow both for the effects of free convection in horizontal round ducts and for the transverse nonuniformity of viscosity, Colburn (4) suggested an empirical correction factor ϕ which appears in the approximate Equation [3] as follows[4]

$$\frac{h_a D}{k_f} = 1.615\left(\frac{cGD^2}{k_f L}\phi\right)^{1/3} \quad [23]$$

where

$$\phi = \frac{\mu_m}{\mu_f}(1 + 0.015G_r^{1/3})^3 \quad [24]$$

Later, Sieder and Tate (5) proposed, as an alternative to Equation [23], the empirical equation

$$\frac{h_a D}{k_m} = 1.86\left(\frac{cGD^2}{k_m L}\right)^{1/3}\left(\frac{\mu_m}{\mu_w}\right)^{0.14} \quad [25]$$

which is equivalent to multiplying $h_a D/k_m$ from the approximate Equation [3] by a correction factor of 1.15 $(\mu_m/\mu_w)^{0.14}$. The analytically derived equations modified by these correction factors seem preferable to the empirical equations proposed earlier by King (8) and McAdams (9).

For flat ducts, correction factors similar to those for round ducts are presumably appropriate, provided the proper constants are used. For air, the variation of viscosity with temperature is too small to justify evaluation of these constants from the available test data, which are discussed later. For liquids, some significant data have been published by Bays and McAdams (12) for the heat transfer to a falling oil film which, if they were correlated by the proper theoretical relations given in this paper (Table 1 or Fig. 2) might indicate the proper constants for the correction factors. Until these constants are determined, the use for flat ducts of the correction factors for round ducts is suggested.

[4] The coefficient of Equation [23] as published in Colburn's paper (4) was 1.5, an empirical value, not 1.615, but reference to the original source (3) and to other authors indicates that 1.615 is the theoretically correct value.

Nonparabolic Velocity at Entrance

The analytical solution for parabolic velocity distribution should be applicable, according to the subsequent considerations, even to cases where the velocity distribution at the entrance of the heat-transferring duct is not parabolic but uniform, except when the duct is extremely short.

It is known that the velocity distribution, if initially uniform, gradually changes to parabolic with increasing distance from the entrance. In the initial part of this transition, the heat transfer and velocity distribution can be analyzed by the methods which have been established for a plane surface with fluid flowing parallel thereto (13). The correlation for a plane (14), expressed in terms of Φ is for laminar flow

$$\frac{h_i D}{k} = 0.664(\Phi)^{1/2}\left(\frac{c\mu}{k}\right)^{-1/6} \quad [26]$$

It can also be shown that the heat transfer will in general exceed (or, in the limit, equal), both the value given by the formula for the plane (of the same length as the duct) and the value given by the formula for the duct with parabolic velocity distribution. Hence, whichever of these two formulas gives the higher value should be used, and even this higher value may be somewhat low.

A reasonable choice for the value of L/D to divide short round ducts, where Equation [26] is preferable, from longer ducts, where Equation [3] or [6] is preferable, is the value where Equation [26] and Equation [6] give the same result. It can easily be shown that this choice corresponds to

$$\frac{L}{D} = 4.8\,\frac{R_D}{1000} \text{ (for round ducts)} \quad [27]$$

For substantially complete establishment of parabolic velocity distribution, the minimum value of L/D is only 35 per cent higher (9).

For flat ducts, similar analysis, applied to Equations [7] and [26] gives

$$\frac{L}{D} = 2.1\,\frac{R_D}{1000} \text{ (for flat ducts)} \quad [28]$$

Since R_D cannot usually exceed 2300 with laminar flow, Equation [26] requires consideration only for round ducts where L/D is less than 11, and for flat ducts where L/D is less than 4.8, conditions which are relatively rare.

Test Data for Heat Transfer in Flat Ducts

In connection with another development, the authors had occasion to test the heat transfer for laminar flow of air in relatively flat ducts. The ducts were 0.311 in. × 2 in. in cross section and 6 in. long, formed by a group of parallel flat copper fins maintained at approximately constant temperature. The results of these tests are shown in Fig. 5, R_D being calculated by using the viscosity of air at the mean film temperature. In the test results, $\frac{c\mu}{k}$ was used as 0.74, since air was used, and this value is practically independent of temperature. The L/D for these ducts was 11.1. Although they were not wide enough to be considered perfect "flat ducts," the use of their equivalent diameter in L/D should tend to offset this discrepancy. The air was thoroughly mixed after leaving the duct before the temperature was measured.

Additional data are available from some tests conducted by G. L. Younts of the General Electric Company in 1936. The ducts were 3 in. wide and from 4 in. to 36 in. long, ranging from $^1/_4$-in. to $^1/_{64}$-in. spacing between plates. Particular precaution was taken to eliminate turbulence at the inlet to the ducts by using well-rounded approaches. The results of these tests are shown in Fig. 5 for an L/D ratio of 8.7, 26, and 50. Again R_D was calculated using the viscosity of air at the mean film temperature.

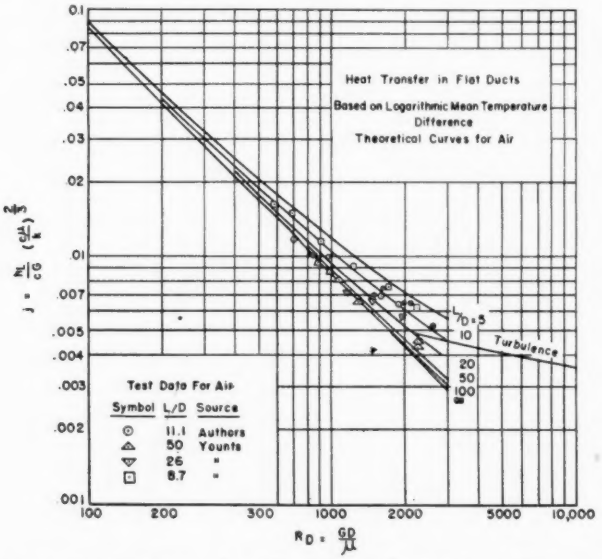


FIG. 5 COMPARISON OF TEST DATA WITH THEORETICAL CORRELATION OF HEAT-TRANSFER FACTOR AND REYNOLDS' NUMBER FOR LAMINAR FLOW OF AIR IN FLAT DUCTS

Although in these tests provision was not made for thorough mixing of the air before measuring its outlet temperature, the trend of the results for the range of the points shown is nevertheless consistent with the theoretical curves.

Some test data for air in flat ducts, by Washington and Marks (15) are in fair agreement with the theoretical curves in Fig. 5 for Reynolds' number above 2000 in the laminar region. However, for lower values of Reynolds' number their data droop below the theoretical curves. Because their technique of measuring outlet-air temperatures at low flows is of questionable accuracy, however, their data are not considered as reliable as the other test data and the theoretical curves given in this paper, and accordingly are not shown in Fig. 5.

Some test data for falling-oil films have been published by Bays and McAdams (12) but require further analysis as already explained to determine the effect of nonparabolic velocity distribution caused by the variation of viscosity with temperature.

The theoretical curves shown in Fig. 5 are the results for laminar flow of air as obtained from analysis as outlined in this paper. The agreement between the test data shown and these theoretical curves seems reasonably good. There is insufficient evidence, however, from these test data to verify completely the analytical curves for flat ducts. More tests, particularly for other fluids than air and other L/D ratios are necessary before these theoretical results can be considered well established for practical conditions.

For round ducts, test data are not considered here, since their agreement with the theoretical results has been extensively reviewed by others (4, 5, 16).

SUMMARY

Heat-transfer data for laminar flow in both round and flat ducts are presented on various bases of correlation, as specified in the following:

The theoretical results, obtained under specified assumptions, expressing hD/k as a function of cGD^2/kL are presented on three different temperature-difference bases (inlet difference, arithmetic mean, and logarithmic mean) as shown graphically in Figs. 1 and 2; numerically in Table 1; and, for restricted ranges of the variables, by Equations [3] through [11]. Convenience in any particular application determines which basis is to be used.

Theoretical lower limits of the local heat-transfer coefficient far from the duct entrance are presented in Table 2, both for constant wall temperature and for constant heat input per unit length.

The theoretical results are also correlated, in Figs. 3 and 4, in terms of the heat-transfer factor j, as defined by Colburn (4), as a function of Reynolds' number, using the logarithmic-mean temperature-difference basis. Figs. 3 and 4 provide a consistent and convenient basis for comparisons between laminar-flow and turbulent-flow heat-transfer data.

Corrections to the theoretical results to account for practical deviations from the assumptions are reviewed. Lévêque's correction (3) for the variation of wall temperature is given by Equation [22]. The corrections for free convection and for the transverse variation of viscosity proposed by Colburn (4) and by Sieder and Tate (5) for round ducts are presented in Equations [23], [24], and [25]; the validity of these for flat ducts is not known. Equation [26] is given for the heat-transfer coefficient close to the duct entrance. This equation is to be used for ducts shorter than specified by Equations [27] and [28] if parabolic velocity distribution is not already established at the entrance.

Test data for laminar flow of air in flat ducts are presented in Fig. 5 and seem to confirm the theoretical results, but the narrow range of variables covered by these test data is insufficient to verify completely the theoretical results.

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Discussion

MAX JAKOB.[8] One item of this valuable paper coincides with a calculation recently published by the writer,[9] i.e., the formula $h_z = C_1 k/b$ for the case of homogeneous heating of a long duct from outside. The writer has determined C_1 for an annular space, the thickness $s = 2b$ of which is so small compared with the radius that the curvature can be neglected, i.e., for an infinitely extended plane duct.

For the case of heating the conduct from one side only, whereas the other side is perfectly insulated, the factor 13/35 in the writer's Equation [17][6] corresponds to $1/2C_1$ in the foregoing formula, so that $C_1 = 35/26 = 1.35$.

For heating from botb sides by analogous calculation instead of 13/35, the factor 17/70 is obtained and from this it follows $C_1 = 35/17 = 2.06$, which is in complete agreement with the last value in Table 2 of the authors' paper.

The authors mention the effect of free convection currents. The writer wishes to point to an analysis[7] of Holden and White's experiments on a viscous liquid, which was heated from outside. Assuming very different distributions of velocity, we calculated the temperature distributions at the end of the tube, and found that the measured values could be explained only by the assumption of radial-flow components in the liquid which were due to local changes of viscosity with temperature and were directed outward after the inlet, and inward close to the end of the tube.

[8] Research Foundation of Armour Institute of Technology, Chicago, Ill. Mem. A.S.M.E.

[9] "Measurements of the True Temperature and the Heat Exchange in a Catalytic Reaction," by Max Jakob, Trans. American Institute of Chemical Engineers, vol. 35, 1939, p. 563.

[7] "Über den Wärmeaustausch bei der Strömung zäher Flüssigkeiten in Röhren," by Max Jakob and H. Eck, *Forschung auf dem Gebiete des Ingenieurwesen*, vol. 3, 1932, p. 121.

AUTHORS' CLOSURE

The value of the constant C_1 supplied by Dr. Jakob for the case of a flat duct heated from only one side, is a useful addition to data provided in the paper. It is interesting to note that the value of 1.35 thus given for C_1 is about 66 per cent of the value 2.06 applicable for heating from both sides, whereas one might expect at first thought that it would be nearer 50 per cent, since the maximum distance the heat has to travel transversely from the heated surface is twice as great when only one side is heated. The excess of 66 per cent over 50 per cent represents the influence of the differences in the velocity distribution, relative to the heated surface, for the two cases.

The existence of radial-flow components produced by free convection, mentioned by Dr. Jakob, is one of the deviations from the theoretical assumptions which the empirical correction factors represented in Equations [24] and [25] are intended to take account of. Although quantitative knowledge of these components is of interest, it seems likely that the distribution of the free convection flow is too complex to justify analytic deviation of the correction factors, so that correction factors obtained empirically from heat-transfer tests will have to continue to be used, although improvements in accuracy may perhaps be obtainable.

The use of the logarithmic-mean temperature-difference basis for laminar flow heat-transfer data has an additional advantage, over the use of the more customary arithmetic-mean basis, which may not be sufficiently obvious in the paper. When the temperature distribution of the surface is not uniform, and, in general, not even known, as is likely to be the case in electrically heated surfaces such as those of oil-cooled transformers, it is helpful to use a temperature difference basis (such as the logarithmic mean) which makes the mean heat-transfer coefficient for the whole surface approach the same limit, and be of the same order of magnitude, as the local heat-transfer coefficient at the hottest part of the surface (at or near the fluid outlet).

Performance Characteristics of Dye Jigs

By W. B. HEINZ,[1] BOUND BROOK, N. J.

Examination of the tension, speed, and power characteristics of regular and "tensionless" dye jigs shows conclusively that the tensionless type maintains more uniform low tension in the cloth with less operator attention. However, it is impracticable to maintain as high a tension as in a regular jig, in spite of the larger motor which must be used. Although the cost of power is at least twice as much as for a regular jig handling the same production the difference is usually not of commercial importance. The cloth speed is not constant. When rolling a 30-in. roll on to a 6-in. beam, the cloth speed more than doubles after the start. The present paper derives formulas which may readily be used for computing the performance of any jig and batch of cloth. Thus are disclosed ways toward improvements in both types of jigs which should overcome their respective disadvantages. Such methods of improvement are believed to be of interest both to manufacturers and to users of dye jigs.

Nomenclature

THE following nomenclature is used in the paper:

- B = width of cloth, ft
- f = total friction torque in each roll, lb-ft
- dF = radial force acting on area Bds, lb
- P = total power output of motor, ft-lb per sec
- q = driving torque, winding roll, lb-ft
- q_b = driving torque, unwinding roll, lb-ft
- Q = main drive torque, lb-ft
- r = instantaneous radius of winding roll, ft
- r_b = instantaneous radius of unwinding roll, ft
- r_1 = radius of beam (empty), ft
- r_2 = radius of roll containing entire batch, ft
- s = circumferential distance along surface of roll, ft
- T = tension in cloth, lb
- U = time per pass (for an end to pass from one roll to the other), sec
- W = energy delivered to jig by motor while an entire end passes from one beam to the other, ft-lb
- β = torque applied by brake to unwinding roll, lb-ft
- θ = angular measurement in cross section of roll, radians
- δ = thickness of cloth on roll, ft
- π = 3.1416 (ratio of circle circumference to its diameter)
- ω = winding-roll speed, radians per sec
- ω_b = unwinding-roll speed, radians per sec
- Ω = main-drive speed, radians per sec
- eu = conventional energy unit
- fu = conventional force unit
- pu = conventional power unit
- tu = conventional cloth-thickness unit

Fundamental Considerations

With few exceptions, dye jigs of today are fundamentally the same as they were in the early days of the textile industry. Most of the improvements which have been incorporated consist almost exclusively in better design and construction of the same simple mechanism, namely: Two sets of shafts and bearings capable of carrying rolls of cloth, a means of driving each of them in turn so that the cloth will wind upon it from the other; and a means of preventing the unwinding roll from overrunning. It might be thought that nothing new could be learned about such a simple arrangement by a theoretical study of its mechanics. As it frequently happens, however, a mathematical analysis has been found useful because it produces quantitative information concerning factors which heretofore have been judged qualitatively by watching machines operate.

Not so simple is the behavior of the so-called "tensionless" jig in which both beams are driven simultaneously through a differential gear. Probably most users of these new machines have contented themselves with qualitative observations of their effect upon cloth speed, tension, power requirements, and final over-all results in quality and rate of production. They may not feel too certain about the actual importance of some of the characteristics as compared with regular jigs, nor of the actual amounts of such differences.

As an early step in the development of certain fundamental improvements to both kinds of jigs, it was found essential to compare their several traits in reliable quantitative terms. The differentially driven jig was analyzed first, with illuminating results. Before these results could be compared with corresponding data for an ordinary jig, it was necessary to apply the same methods to the latter.

The author hopes that the analyses and comparisons of the results given in this paper will stimulate new interest in the finishing industry.

Tensionless Jig

The differentially driven dye jig became known as a tensionless jig because, among its other characteristics, it can run cloth back and forth through the bath with practically zero tension. However, the brakes can be adjusted for considerable cloth tension also, within the practical limitations of motor power and brake cooling capacity.

Although there are certain undeniable advantages in the differential-drive arrangement, a prospective purchaser should not overrate them in comparison with corresponding disadvantages. In the first place the speed is not constant, as claimed. A larger motor is required; and even with the larger motor, it cannot operate at as high tension as a regular jig because the brakes heat too much. Although the user may not notice it, the cost for power to drive tensionless jigs is more than twice what it would be for regular jigs doing the same work.

A mechanical analysis provides means for computing exact information about advantages and disadvantages. How nearly constant is the cloth tension, the brake setting being left always the same? How much does the cloth speed change from beginning to end of each pass? What is the maximum tension that the jig can handle with a certain-size motor and brakes? How much larger will its power bill be than that of a regular jig when operating under prescribed tension? All these questions can be answered definitely by putting numbers into formulas which have been developed. They all follow from simple fundamentals,

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

most of which appear directly from an examination of Fig. 1. Symbols are explained in the nomenclature at the beginning of the paper.

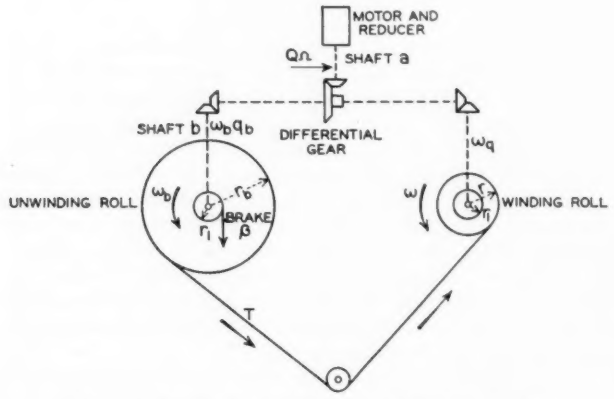


FIG. 1 SCHEMATIC DIAGRAM OF TENSIONLESS JIG

The Differential Gear

The differential gear is exactly the same in principle as that which is used in every automobile. To simplify the analysis, however, most of the speed reduction is assumed to be incorporated with the motor and reducer which drives the shaft a. In the differential gear itself, there is a 2:1 reduction such that the speed of the two driven beams (when turning at the same speed) is half the speed of the driving shaft a. At all beam speeds, their sum is equal to the speed of a.

Symbolically

$$\omega + \omega_b = \Omega \quad [1]$$

This is an inherent characteristic of any differential gear with a 2:1 ratio.

Another inherent characteristic of this gear is that the torques in all three shafts are equal at all times (small differences in friction being neglected).

Thus

$$Q = q = q_b \quad [2]$$

Torques acting on the unwinding beam are the applied torque of the motor, the torque caused by the cloth pulling with tension T at the radius r_b of the unwinding roll, and the opposing torques of bearing and brake friction. Corresponding torques act on the winding beam except that its brake is not in action. These torques on the unwinding and winding beams are related according to Equations [3] and [4], respectively

$$q_b + r_b T - \beta - f = 0 \quad [3]$$

$$q - rT - f = 0 \quad [4]$$

From Equations [2], [3], and [4], we find how the cloth tension depends upon the roll diameters

$$T = \frac{\beta}{r + r_b} \quad [5]$$

The cloth speed $r\omega$ is always equal to the peripheral speed of either or both rolls.

$$r\omega = r_b\omega_b$$

Combining with Equation [1] and solving for $r\omega$, we find that

$$\text{Cloth speed} = r\omega = \frac{\Omega r r_b}{r + r_b} \quad [5a]$$

The torque delivered by the main drive can also be computed from the same equation

$$Q = f + \frac{\beta r}{r + r_b} \quad [6]$$

The power required to drive the jig follows directly, by multiplying the main-drive-shaft torque by its speed.

Thus

$$P = Q\Omega = \frac{\Omega}{r + r_b}\left[\beta r + f(r + r_b)\right] \quad [7]$$

The power computed from Equation [7] represents the instantaneous power delivered by the main drive. It changes continuously as the roll diameters change. It cannot simply be multiplied by the time the jig is in operation to find the energy consumption (kilowatthours) which fixes the power bill. The total energy must be computed by summing up the products of all of the instantaneous power readings, multiplied by the intervals of time they continue. In terms of the calculus, this is done by an integration

$$W = \int_{r=r_1}^{r=r_2} P\,dt \quad [8]$$

In order to express the increment of time dt, in terms of the winding-roll radius r, we must find a relationship showing the rate at which the radius changes as the cloth progresses. This involves the winding-roll speed and the thickness of each wrap[2] on the roll according to Equation [9] since, during each revolution, the radius increases by the thickness of one extra wrap of cloth. The rate of change of radius with respect to time is

$$\frac{dr}{dt} = \frac{\delta\omega}{2\pi} \quad [9]$$

It follows that

$$dt = 2\pi dr/\delta\omega \quad [9a]$$

The energy input of Equation [8] now becomes

$$W = \frac{2\pi}{\delta}\int_{r_1}^{r_2}\frac{P}{\omega}\,dr \quad [10]$$

In order to integrate Equation [10] we must express the speed of the winding roll ω in terms of its radius r. Since the peripheral speeds of the two rolls are always equal (the cloth speed), their rotational speeds are inversely proportional to their radii. Knowing this, we can modify Equation [1] as follows

$$\Omega = \omega\left(\frac{r}{r_b} + 1\right)$$

$$\omega = \frac{\Omega r_b}{r + r_b} \quad [11]$$

Equation [11] expresses the winding-roll speed in terms of the main-drive speed and the two roll radii. However, we still have to convert the radius of the unwinding roll into terms of the winding-roll radius. This additional relationship follows from a consideration of the total cross-sectional areas of the cloth wound on the two rolls. Since the amount of cloth in the batch does not change, the sum of the cross-sectional areas on the two

[2] It has been assumed that changes in tension do not influence the effective thickness of the cloth as it is wound on a roll. Although this is not strictly correct, the error thereby introduced is believed to be unimportant.

rolls is constant and equal to the cross-sectional area of the total batch when it is wound on one roll. This gives us

$$\pi(r_b^2 - r_1^2) + \pi(r^2 - r_1^2) = \pi(r_2^2 - r_1^2) \quad [12]$$

From Equation [12], we obtain the relationship for the unwinding-roll radius which we need, in terms of the fixed radii of the beam and the roll carrying the entire batch, and the instantaneous radius of the winding roll

$$r_b = \sqrt{r_1^2 + r_2^2 - r^2} \quad [13]$$

We can introduce Equations [7], [11], and [13] into Equation [10], and integrate to find the total energy input required to pass an entire batch from one roll to the other. Equation [14] thus gives the total energy input to the jig, the brake setting being left the same throughout the entire pass and the tension changing as it will

$$W = (\beta + 2f)(r_2 - r_1)\frac{2\pi}{\delta} \quad [14]$$

Equation [14] gives the energy input to the jig in foot-pounds. This must be divided by the motor and drive efficiency and by 2,654,000 to give the total kilowatthours input to the motor, from which the cost of the power can be computed.

Time per End

The power and energy requirements are best compared on the basis of equal time per end for the two types of jigs. The time required for a batch to pass completely from one beam to the other can be computed from equations derived as follows:

The cross-sectional area of cloth on the winding roll is

$$A = \pi(r^2 - r_1^2) \quad [15]$$

By integrating Equation [9], we arrive at the following general equation for the winding-roll radius, applicable to both tensionless and regular jigs

$$r = \frac{\delta}{2\pi}\int \omega dt + C \quad [16]$$

For a tensionless jig, on introducing the winding-roll speed of Equation [11], and the unwinding-roll radius of Equation [13], we can integrate Equation [16], arriving at

$$t = \frac{2\pi}{\delta\Omega}(r - \sqrt{r_1^2 + r_2^2 - r^2}) + C \quad [17]$$

This gives the time elapsed from the beginning of an end in terms of the instantaneous winding-roll radius r, its initial and final radii r_1 and r_2, and a constant of integration c.

The constant of integration is determined by considering conditions at the beginning of an end when the winding-roll radius is equal to the radius of the empty beam and the time elapsed is zero. We can thus introduce into Equation [17], the values $t = 0$ and $r = r_1$. Then solving for C, the constant of integration, we find that

$$C = r_2 - r_1$$

Equation [17] now takes its complete form

$$t = \frac{2\pi}{\delta\Omega}(r + r_2 - r_1 - \sqrt{r_1^2 + r_2^2 - r^2}) \quad [18]$$

When the total time per end, U, has passed, the entire batch has been wound upon the winding roll. Its radius is now r_2. On substituting these two symbols into Equation [18], we have the desired formula for the time required for a complete end with a tensionless jig

$$U = \frac{4\pi}{\delta\Omega}(r_2 - r_1) \quad [19]$$

Regular Jig

The regular jig has advantages in low cost, ease and economy in operation and maintenance. It is simple, it has high tension capacity with moderate-size motor and brakes, and its power consumption is low. Although its cloth speed changes widely from beginning to end of each pass, in practice, this is probably of little actual importance. Most of the dyeing is said to take place while the cloth is padded up on the rolls, and changes in the time actually spent in passing through the bath do not have much effect upon the levelness of dyeing.

The machine being so simple, its mechanical analysis is also simple.

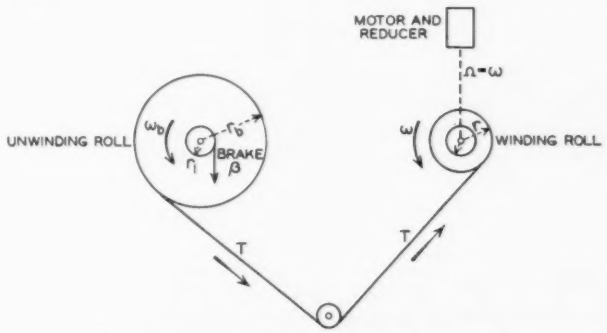


Fig. 2 Schematic Diagram of Regular Jig
(Brake and drive for reverse direction not shown.)

In a regular jig, Fig. 2, the tension in the cloth is that which is necessary to turn the unwinding roll against the friction and the brake torque

$$T = \frac{\beta + f}{r_b} \quad [20]$$

The cloth speed always being the same as the peripheral speed of the rolls, we can write

$$\text{Cloth speed} = r\omega = r\Omega \quad [21]$$

Here the main-drive speed and the winding-roll speed are always equal, all of the gear reduction being assumed to be associated with the motor.

The total power consumed is that absorbed by the brake and miscellaneous friction in the unwinding roll plus the friction in the winding roll. Multiply these torques by their respective roll speeds and add

$$P = (\beta + f)\omega_b + f\omega \quad [22]$$

Since the surface-roll speeds are always equal to the cloth speed and to each other

$$r\omega = r_b\omega_b$$

$$\omega_b = r\omega/r_b \quad [23]$$

Substituting Equation [23] in Equation [22], and noting from Fig. 2 that the winding roll has been assumed to be directly connected to the main drive shaft ($\omega = \Omega$), we now arrive at the power equation

$$P = \frac{\Omega}{r_b}[\beta r + f(r + r_b)] \quad [24]$$

On the assumption that the operator succeeds in adjusting the brake on a regular jig so that the tension is maintained constant throughout the entire pass, the general energy integral develops according to Equation [25]. This equation represents the total mechanical energy consumed per end by a regular jig

$$W = \int_{r=r_1}^{r=r_2} P\,dt$$

$$W = \frac{2\pi}{\delta}\left[f(r_2 - r_1) + \frac{T}{2}(r_2^2 - r_1^2)\right] \quad [25]$$

Time per End

The time required for a complete end in a regular jig can be computed from the general Equation [16] by a procedure similar to that used for a tensionless jig. For a regular jig, however, the winding-roll speed is constant ($\omega = \Omega$). On this basis, the integration of Equation [16] is simple

$$t = \frac{2\pi}{\delta\Omega}(r - r_1) \quad [26]$$

Again, substituting the symbols representing conditions at the completion of an end ($t = U$, $r = r_2$), we quickly find that the time required per end for an ordinary jig is

$$U = \frac{2\pi}{\delta\Omega}(r_2 - r_1) \quad [27]$$

On comparing Equations [19] and [27], the formulas for times per end for tensionless and regular jigs, we note that the first appears to be twice the second. This simply means that if we want a tensionless and a regular jig to handle a certain-size batch in the same time we must make the driving speed Ω, of the tensionless jig twice that of the regular jig. This observation is, of course, based upon the conventions relating to gear ratios which have been set up in Figs. 1 and 2.

With suitable modifications, all these formulas apply equally well to any other gearing arrangement.

Summarizing, comparisons between tensionless and regular jigs can be based upon the foregoing equations, as follows:

Characteristic	Equation no. Tensionless	Equation no. Regular
Tension of cloth	[5]	[20]
Speed of cloth	[5a]	[21]
Time per end	[19]	[27]
Power	[7]	[24]
Energy consumed per end	[14]	[25]

Numerical Examples

In order to visualize the significance of the foregoing equations, jigs have been selected which have beam diameters of 6 in. The batch of cloth being handled makes a roll 30 in. in diam when rolled on one beam. Thus, $r = 3$ in., $r_2 = 15$ in.

In order to simplify the computations and to make the results as general as possible, a group of conventional units has been adopted. They are listed in the nomenclature at the beginning of the paper. For purposes of tension comparison, for example, the cloth tension when the two rolls are of equal diameter has arbitrarily been termed one "force unit," *fu*. When the rolls are equal in size, their radius is

$$r_b = r = 10.81 \text{ in.}$$

computed from Equation [13].

The total frictional torque comprising bearing friction, liquid forces on the cloth passing through the bath, etc., has been assumed to be 0.1, its unit being the product of force units multiplied by inches ($fu \times$ in.). This amount of frictional torque is assumed to be associated with each roll.

The brake torque required to create unit cloth tension under the conditions assumed can be computed from Equations [5] and [13] for a tensionless jig, its value being $\beta = 21.62$.

For a regular jig, the brake torque is computed from Equation [20], giving $\beta = Tr_b - f = 10.71$.

The mechanical power input to a regular jig is defined as one power unit *pu*, when the tension is one force unit and the roll sizes are equal. Thus, the sum of the brake torque on the unwinding roll plus the friction torque on both rolls, multiplied by the roll speed (the same for both) equals unity

$$(\beta + 2f)\Omega = 1$$

Substituting the previously established values for the brake torque and the friction, we find that the main-drive speed for a regular jig must be $\Omega = 0.0917$.

Since it is logical to compare two types of jigs upon the basis of equal total time per end, we note by comparing Equations [19] and [27] that the main-drive speed for a tensionless jig must be twice that for a regular jig. Consequently, for a tensionless jig, $\Omega = 0.1834$.

Cloth Tension

Now we can substitute the foregoing values in the formulas which have been established, arriving at numerical formulas for the cloth tension in terms of the winding-roll radius. Thus, from Equations [5] and [13] we arrive at the following expression for the cloth tension in a tensionless jig

$$T = \frac{21.6}{r + \sqrt{234 - r^2}}$$

Similarly, for a regular jig, the cloth tension is computed from Equations [20] and [13] to be

$$T = \frac{10.8}{\sqrt{234 - r^2}}$$

Curves representing these formulas are plotted in Fig. 3.

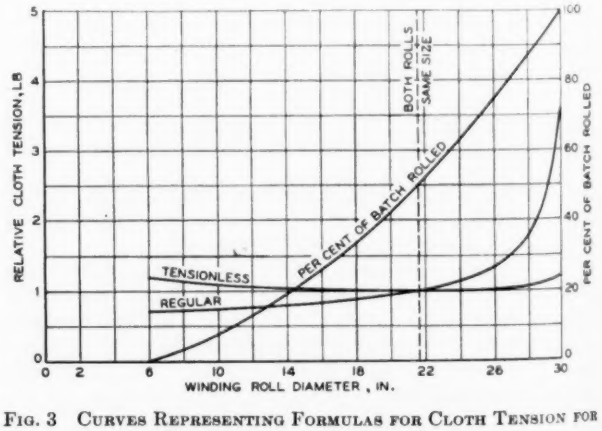


Fig. 3 Curves Representing Formulas for Cloth Tension for Tensionless and Regular Jigs

This illustration shows how rapidly the tension on a regular jig would rise toward the finish of an end if the operator did not loosen the brake. It is evident that no operator can be expected to achieve very uniform tension by periodic brake adjustments. If let alone, the tension would change over a range of nearly 5 to 1; and an operator is expected to keep the tension always the same!

A tensionless jig under the computed conditions keeps the tension always within a total range of only 1.2 to 1.

Cloth Speed

The speed of the cloth is always equal to the peripheral speed of the winding roll. Consequently, for a tensionless jig from Equation [5a] we can write

$$\text{Cloth speed} = r\omega = \frac{0.183\, r\sqrt{234 - r^2}}{r + \sqrt{234 - r^2}}$$

Similarly, for a regular jig, the cloth-speed equation is

$$\text{Cloth speed} = r\omega = r\Omega = 0.092\, r$$

The cloth speeds of various roll diameters are shown in Fig. 4.

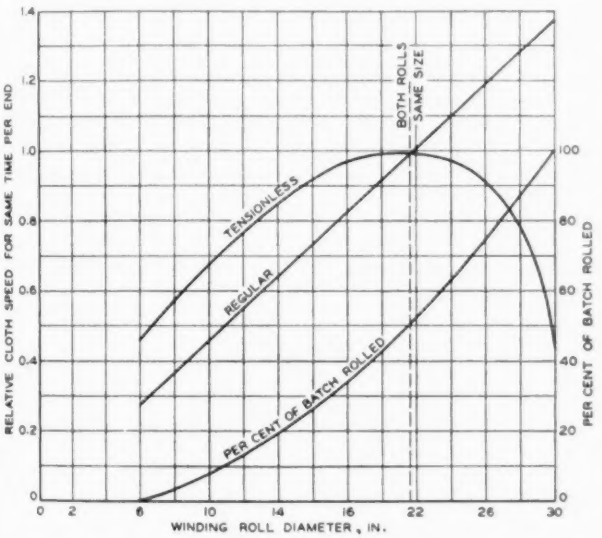


FIG. 4 CLOTH SPEEDS FOR VARIOUS-DIAMETER ROLLS

Evidently, the speed of cloth in a tensionless jig can change during an end through a range of more than 2 to 1. However, a regular jig is worse. Its cloth speed shifts over a range of 5 to 1. This would be an important disadvantage if it were not for the fact that most of the exhaust is said to take place while the cloth is padded up on the rolls, and not while it is actually passing through the bath.

The curves of Fig. 4 were computed for equal-size batches and equal total times required for an end.

Time per End

The total time required for a complete end has, for comparison, been set the same for both tensionless and regular jigs. Thus, from either Equation [19] or Equation [27] we find that $U = 821$. These units of time are neither seconds nor minutes, but instead, certain arbitrary conventional time intervals which involve the conventions which have been established for other units. For the cloth thickness there has been used the dimension $\delta = 1$ cloth thickness unit, *tu*.

Power

From Equations [7] and [13], the following formula for the mechanical power input to a tensionless jig results

$$P = \frac{0.183}{r + \sqrt{234 - r^2}}\left[21.6\, r + 0.1\,(r + \sqrt{234 - r^2})\right]$$

A corresponding formula for power input to a regular jig from Equations [24] and [13] is

$$P = \frac{0.092}{\sqrt{234 - r^2}}\left[10.7r + 0.1(r + \sqrt{234 - r^2})\right]$$

In practice, the operator of a regular jig attempts to keep his brake adjusted so that the cloth tension stays about the same. Otherwise it would increase too much toward the finish of each end. If he were to succeed in maintaining constant tension by adjusting the brake, the power would be substantially proportional to the winding-roll radius. A curve showing the power under these conditions of assumed constant tension accompanies those for the foregoing two equations in Fig. 5.

Here is illustrated the most striking difference between

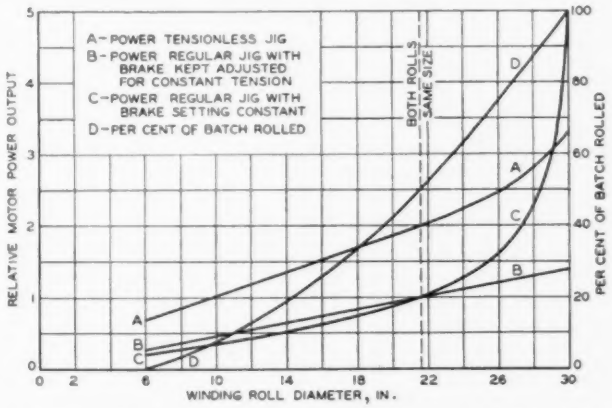


FIG. 5 POWER REQUIRED UNDER CONDITIONS OF ASSUMED CONSTANT TENSION

regular and tensionless jigs. At the time when one half the batch has been rolled (both rolls are the same size), the power required by a tensionless jig is twice that of a regular; and more than twice at other times. As more cloth is rolled, the operator of a regular jig keeps the power down by loosening the brake instead of allowing it to go up fast as the chart shows. With a tensionless jig he cannot do that without dropping off the tension. Consequently, the power for a tensionless jig goes on up at the end of the batch. That is why the brakes on a tensionless jig run so hot if high tension is set on the machine.

Energy Consumption—Cost of Power

The relative amounts of energy consumed per end for tensionless and regular jigs can be computed from Equations [14] and [25], respectively. On introducing our assumed dimensions, we obtain the following

Tensionless

$$W = (21.6 + 0.2)(15 - 3) \times \frac{2\pi}{1} = 1640 \text{ energy units}$$

Regular (constant tension)

$$W = \frac{2\pi}{1}\left[0.1(15 - 3) + \frac{1}{2}(225 - 9)\right] = 685 \text{ energy units}$$

Thus we find that, under the conditions of our numerical example, a tensionless jig requires 1640/685 = 2.4 times as much energy input for the same production output as a regular jig with correctly adjusted tension. Power costs are in substantially the same ratio.

In actual service, the beam and roll diameters are frequently different from those of the example. Corresponding numerical comparisons can quickly be computed for any other set of

conditions. Such further computations will also show that the tensionless jig is notably advantageous in constancy of cloth tension; but that the user pays for it, not only in higher initial cost for the machine, but also by limitations in tension capacity, by higher installed power demand, and by doubling his power bill for jig dyeing.

There evidently is a need for improvement in dye jigs of both types, to overcome their respective limitations.

Discussion

J. D. ROBERTSON.[3] Mr. Heinz's admirable paper gives a thorough mathematical analysis of the variation in cloth speed and cloth tension, among other operating characteristics, during the passage of a lot of cloth from one beam to the other, both on regular jigs and on so-called tensionless jigs.

According to formula [20] of his paper, the cloth tension on regular jigs varies inversely with the instantaneous radius of the unwinding roll. For any given number of yards in a batch and for a constant setting of the brakes, therefore, the larger the diameter of the beams used the smaller will be the variation in cloth tension from end to end of the batch. In the same way Mr. Heinz's formulas show that the variation in cloth speed is kept very low by using large beams.

It may be of interest, therefore, to consider a difficulty which arises when the comparatively simple modification of increasing the diameter of the beams of regular jigs is made for the purpose of securing approximately constant cloth speed and tension.

It is common experience that the larger the diameter of a roll of cloth on a jig beam the greater the tension required to keep it from "coning" or slipping endwise. This effect is independent of the diameter of the beams. When the character of the cloth being dyed does not permit heavy cloth tension large beams cannot be used because of this endwise slipping.

For any given cloth tension it can be shown that the greater the diameter of the roll, the less the radial pressure between the outside layer of cloth and the layer immediately under it. The writer believes from observation that the endwise slippage of cloth on a large-diameter roll of cloth occurs when the radial pressure between adjacent layers near the outside of the roll drops to the point where the water carried up on the cloth fails to be squeezed out between the outside layer of cloth and the layer of cloth under it.

At this stage the water is carried in between the layers of cloth near the outside of the roll until the additional cloth rolling on top of it continues to build up the pressure of water to the degree required to force its way out at one of the ends of the roll. This explanation can be checked by feeling the roll with the hand when it is running under those conditions. The roll feels very spongy because of the water pocketed between the layers of cloth. When the pressure on the water gets high enough it squirts out, most of it at one end, carrying the cloth with it. Probably what happens is that the selvages, stretching less than the rest of the cloth, hold the water in until the pressure is increased to a sufficient degree to move the selvage sidewise by the amount of its width. This tends to open the space between the layers of cloth, permitting the water to rush out, and making the cloth much looser than it was when the water was between the layers. The cloth rapidly moves endwise from this stage.

The foregoing explanation agrees with the observed fact that larger rolls can be run on regular jigs without coning by slowing them down. Less water is carried up with the cloth at the slow speed.

It may also explain why so-called tensionless jigs can handle larger rolls without coning than regular jigs for a given elapsed time per end.

The percentage increase of cloth speed as the winding roll diameter increases on a regular jig of ordinary (6 to 8 in.) beam diameter is very substantial, carrying considerably more water up as the roll gets larger. The cloth speed on a tensionless jig in comparison is practically constant.

[3] Plant Engineer, Mt. Hope Finishing Co., North Dighton, Mass. Mem. A.S.M.E.

AUTHOR'S CLOSURE

Causes of coning are numerous and difficult to isolate. Analysis of the radial pressure exerted by each lap of cloth as it winds onto a roll indicates that variations in this radial pressure might easily be partly responsible. The author would welcome correspondence from anyone whose observations tend either to confirm or disprove the inferences which are drawn.

The most interesting result of this analysis is the demonstration that there is almost certain to be a soft region toward the center of a roll wound by a regular jig. If the brake setting be left constant from beginning to end of a pass, the radial compression starts out high, declines to some minimum value at the mid-point of the pass, and thereafter rises rapidly again to a high compression at the finish. Correct adjustment of brakes by the operator can prevent the compression from going up again, but it is improbable that this is accomplished very satisfactorily in regular practice.

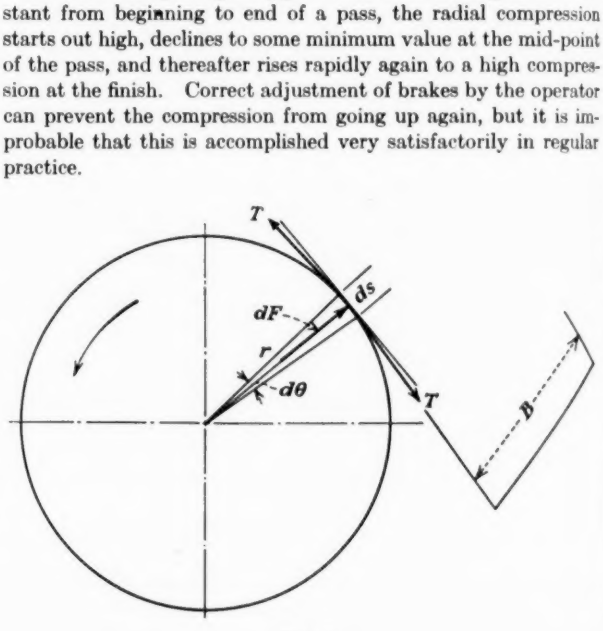


FIG. 6

With reference to Fig. 6, the normal force exerted by an incremental length of cloth ds is

$$dF = 2T \sin \frac{d\theta}{2} = 2T \frac{d\theta}{2} = T\,d\theta, \text{ lb} \quad [28]$$

The radial pressure over the area B in width and ds in length is

$$p = dF/Bds = Td\theta/Brd\theta = T/Br, \text{ lb per sq ft} \quad [29]$$

In a regular jig from Equations [20] and [13] the cloth tension at any instant with constant brake setting is the following

$$T = \frac{\beta + f}{\sqrt{(r_1^2 + r_2^2 - r^2)}}, \text{ lb} \quad [30]$$

Equation [29] consequently takes the following form

$$p = \frac{\beta + f}{Br\sqrt{(r_1^2 + r_2^2 - r^2)}}, \text{ lb per sq ft} \quad [31]$$

If we put into Equations [29] and [31] the same numbers as

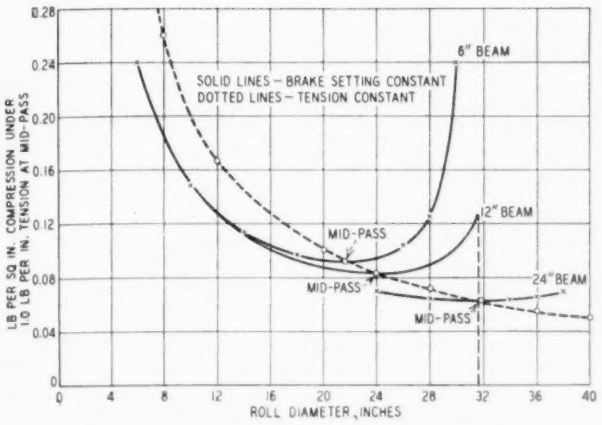


FIG. 7 ROLL COMPRESSION DURING WINDING

were previously used in numerical examples, we obtain curves which appear in Fig. 7. The three solid lines represent the compression created by each lap of cloth as it winds onto a roll from beginning to end of a pass for 6, 12, and 24-in. diameter beams, handling the same size batch of cloth. The brake setting in each case is left constant at such a value as to give the same tension at the mid-point of the batch for all beam sizes.

The dotted line shows the way in which the compression at the surface changes as a pass progresses if the tension in the cloth be always the same. Since this curve depends only upon the external radius of the roll at each instant, it is not influenced by the diameter of the beam within the roll. Consequently, the corresponding sections of this one curve represent conditions for all beam sizes.

The compression at any point within the roll (beneath the surface), of course, depends upon the cumulative effects of instantaneous compression at the surface as the roll builds up, shrinkage within the roll from squeezing out of water, gradual compression, and shrinkage of the fabric itself. These influences are not subject to any simple analysis. Their effects can only be judged by observation or tests in the field.

On examining the curves for the regular jigs, one wonders about the probable effect of faulty tension (ordinary manual control) upon the stability of a roll wound upon normal size beams. The curves show that if the brakes be left unadjusted after the mid-point in a pass, there is a very pronounced minimum compression in the vicinity of the mid-point. The compression rises very steeply thereafter unless the operator promptly and correctly loosens the brakes. As further winding goes on, a soft region is left in the roll. This region is continually subjected to higher and higher shear stresses as the roll radius increases, particularly if the tension be allowed to increase also. It is to be expected that high shear inside the roll might frequently cause slippage in the direction of tighter winding there. Any slippage at all is going to facilitate endwise slippage; and again coning may result.

Successful Mechanical Atomization of Fuel Oil Heavier Than Bunker C

By G. G. MARTINSON,[1] MARCUS HOOK, PA.

The purpose of this paper is to report the difficulties encountered and their successful solution when burning a heavy-petroleum cracking-still residue as a fuel in combination pulverized-coal and fuel-oil burners for the boilers of the American Viscose Company powerhouse at its Marcus Hook plant. Though the problems involved were comparatively simple, the development of a burner arrangement to achieve a solution and the results obtained may be of value to the designers and operators of boilers who are in search of both flexible and efficient combination firing equipment for pulverized coal and fuel oil in boiler plants.

EQUIPMENT of the boiler plant of the American Viscose Company at Marcus Hook consists of three boilers of C.E. type fin-tube waterwall-furnace construction, each having a maximum steam output of 150,000 lb per hr at 425 psi and 700 F total temperature. Fig. 1 shows a side view in cross section of one of these boilers. The furnaces, of the dry-bottom type, have a volume of 7000 cu ft and a waterwall surface of 2300 sq ft. Steam from the boilers is used to drive three 3000-kw turbogenerators and three 2000-kw turbogenerators. Some of the steam is extracted from the turbines at 175 psi and some at 20 psi for plant process duty.

Each boiler is equipped with four Lo-Pul-Co type C.E. combination burners for oil and pulverized-coal firing, the general arrangement being shown in Fig. 2. When firing pulverized coal, the fuel is mixed with tempered primary air at the pulverizers and delivered through the center pipe of the burner assembly, while secondary air is forced through the air preheater by a forced-draft fan and delivered to the burner assembly outside of the pulverized-coal pipe. Control of the secondary air is accomplished by regulating the register shown in Fig. 2 and the speed of the forced-draft fan. Primary-air fans are direct-connected to the bowl-mill motors and, therefore, when using oil as fuel for these furnaces, no primary air is delivered through the coal pipe, combustion depending entirely upon the mixture of the fuel oil and secondary air supplied in the manner described.

Control of the firing rate of each of these units is completely automatic for both types of firing equipment and, after setting the proper air-fuel ratio for the type of fuel used, the controls maintain this ratio automatically within close limits throughout the range of the controls. An increase in steam demand causes the controls to increase the air supplied to the burners before increasing the fuel supply and, with a decrease in load, the fuel supply is decreased before the air supply is reduced. This cycle of operations is very effective in preventing smoke when changes in load occur.

The proximity of the plants of the American Viscose Company and the Sinclair Refining Company made the direct delivery of heavy fuel oil by pipe line from the latter to the former a rather simple problem. Fig. 3 shows a flow diagram of the manner in which this heavy fuel oil is measured and delivered to the consumer's burners without the heat loss attendant upon other methods of fuel-oil delivery, such as tank-car and barge deliveries. The refinery delivers this heavy fuel oil to the supply tank shown in Fig. 3 at a temperature about equal to the optimum temperature at which the oil is most readily atomized. Therefore, in order to have the fuel oil at the correct temperature for atomization at the burners in the Viscose plant, it is only necessary for the consumer to supply the heat lost in the oil delivery line.

Difficulties Encountered in Oil Firing With Original Equipment

The original oil burners and burner assemblies gave unsatisfactory results because the atomized fuel oil and the air required for combustion were not properly mixed. This condition caused the atomized-oil streams to impinge upon the side-wall tubes of the furnace where the oil distilled and the residue formed a coke deposit which kept building up in size until the weight of the mass of coke overcame its adhesive powers and the entire mass fell onto the screen tubes across the bottom of the furnace. Some of these blocks of coke were heavy enough to bow the floor screen tubes slightly.

Fig. 2 shows the combination burner and air register in cross section with the normal angle of atomized-oil spray from the standard mechanical atomizer and illustrates the relative positions of the air and oil streams. In this illustration the oil burner is shown withdrawn to the greatest possible extent without impingement of the oil on the nozzle of the coal pipe. It can be readily noted that the flame in this position fails to fill the throat and, since the stream of air leaving the secondary-air passage is "barreling," there is little mixing of the air and fuel except at the outside of the oil-flame cone. Therefore, a considerable portion of the air introduced into the furnace in this manner is not efficiently utilized in combustion and, although excess air exists in the furnace, smoke and soot fill the firebox and boiler setting.

Steam atomizing oil burners with the tips drilled to give approximately the same included angle of flame cone as that produced by the mechanical atomizing burner tips were tried and produced only slightly better results than the mechanical atomizers. Experimental work was carried out with steam atomizing oil-burner tips with various drillings; it developed that a tip drilled to give a flame included angle of about 120 deg produced the best results. With this tip, the flame could be pulled back into the throat, without impinging on the coal-pipe nozzle, thereby forcing the air into the flame and resulting in good mixture of the fuel oil and the air required for combustion. The amount of steam consumed by the steam atomizers, while probably in line with the steam consumption of other modern steam atomizing oil burners, was considered to be an unnecessary loss. Therefore, a further solution to the problem, using mechanical atomizing oil-burner tips, was sought.

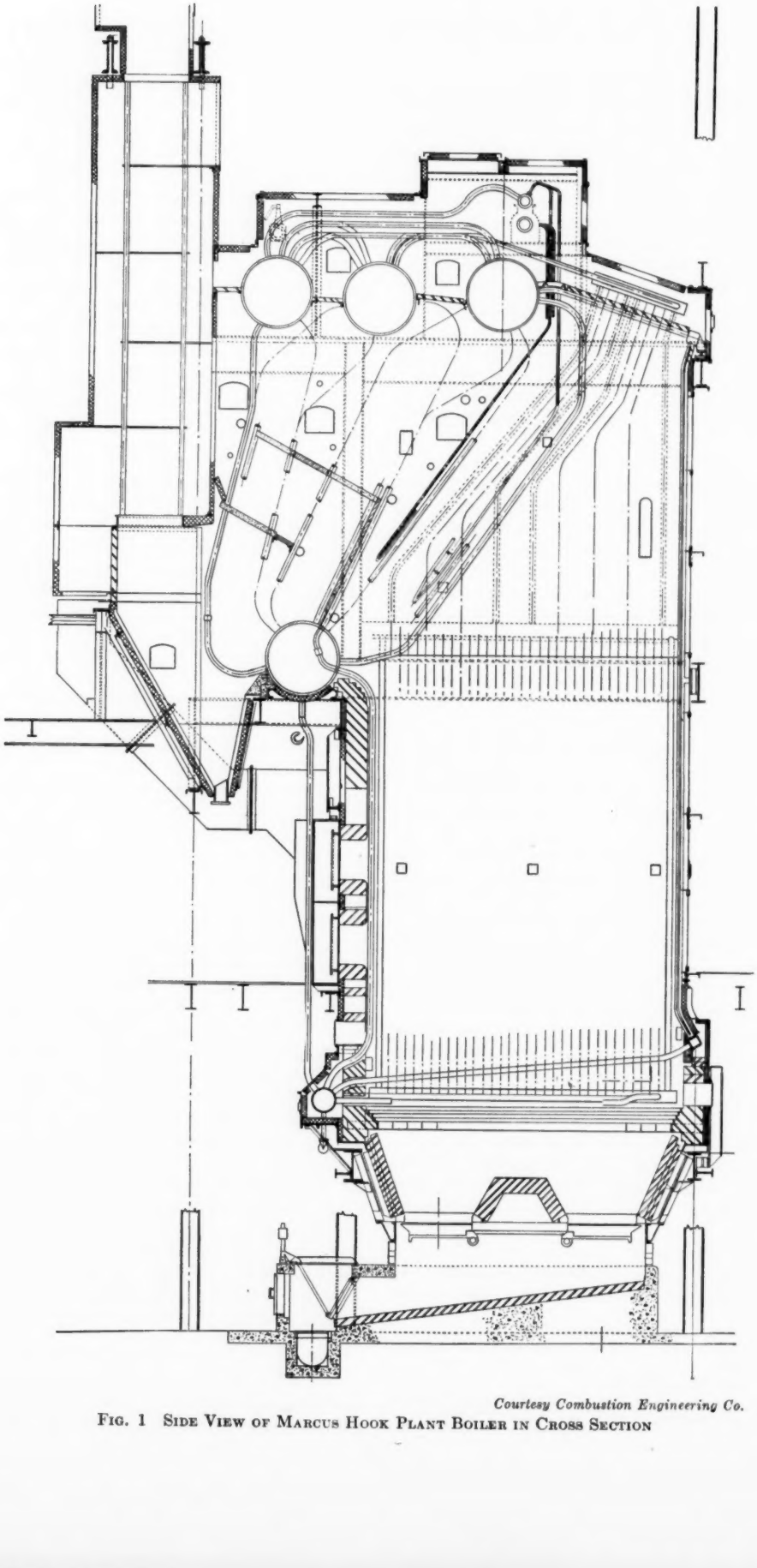
Design, Installation, and Operation of Fixed Air Register

Since the normal mechanical atomizing oil burner gives a flame cone with an included angle of about 75 to 85 deg, which experience with steam atomizing burner tips had proved to be too

[1] Fuel Engineer, Sinclair Refining Company. Jun. A.S.M.E.

Contributed by the Fuels Division and presented at the Spring Meeting, Worcester, Mass., May 1–3, 1940, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

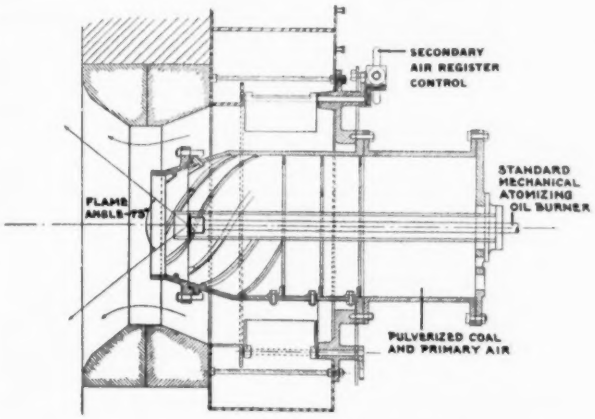


Courtesy Combustion Engineering Co.

FIG. 1 SIDE VIEW OF MARCUS HOOK PLANT BOILER IN CROSS SECTION

narrow, it was deemed advisable to change the manner in which the air was introduced into the furnace in an effort to mix the combustion air properly with the fuel oil. Another important reason for the decision regarding the air introduction was the fact that firing conditions using pulverized coal as the fuel were very good and it was felt that a change in air introduction would have less effect upon coal-burning characteristics than a change in the coal nozzle. The coal nozzle could, of course, be shortened to the extent of permitting the oil burner to be withdrawn far enough to enable its normal flame included angle to fill the throat of the burner. However, this procedure would undoubtedly disrupt the coal-burning characteristics of the burner assembly.

To direct the air stream into the flame produced by the standard mechanical atomizing oil burner, an assembly of 24 fixed



Courtesy, Combustion Engineering Co.

FIG. 2 ORIGINAL BURNER ASSEMBLY WITH STANDARD ATOMIZING OIL BURNER

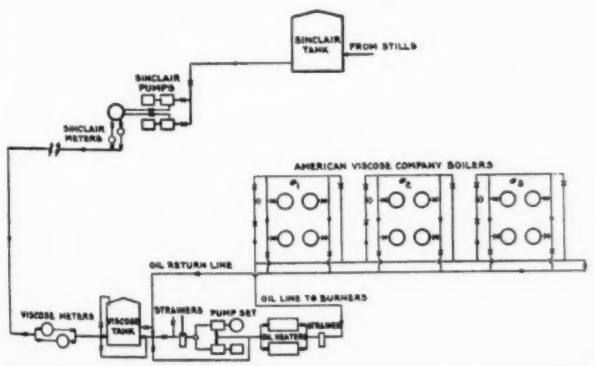


FIG. 3 DIAGRAM OF OIL FLOW FROM REFINERY TO CONSUMER'S BURNERS

vanes was made into a circular register, fitting between the coal nozzle and the refractory throat of the burner assembly, and placed at the furnace inlet of the secondary-air supply. Fig. 4, although it shows a cross section of the burner assembly fitted with a movable-vane register, indicates the position of the registers of both the fixed type and the movable-vane type which was developed later. The vanes of the fixed register were arbitrarily set at an angle of 45 deg to the face of the burner assembly as it was thought that this setting would be sufficiently accurate to determine definitely the feasibility of this method of mixing the air and fuel oil more effectively. The shroud around the outer ends of the vanes was formed to produce a constriction on the air stream by making the outlet diameter smaller than the inlet and, thereby, to assist in obtaining a good mixture of fuel oil and air.

Tests were made, using these fixed-vane registers under actual firing conditions, which indicated that the angle chosen for the vanes had been too great for any normal load on these furnaces. Under light loads, the fire had an excellent appearance, all of the fuel oil being consumed completely within a short distance of the burner tip but, as the load was increased and the fuel-oil and air quantities increased, the flame was torn apart and a "sunflower" appearance of the flame became very noticeable. There was a short solid flame for about 3 ft from the burner tip, then flame propagation was apparently too slow for the speed of the combined air-and-oil-stream mixture and no flame was apparent for the next 4 or 5 ft of travel, after which ignition again took place and combustion was completed.

One of the surprising features of the test was that the flame produced, even though of such unorthodox shape, was remarkably stable and gave no indications of "puffing" or "backfiring." Since the direction and rotation of the air by the fixed-vane register caused the flame to flare out into the sunflower shape, it is

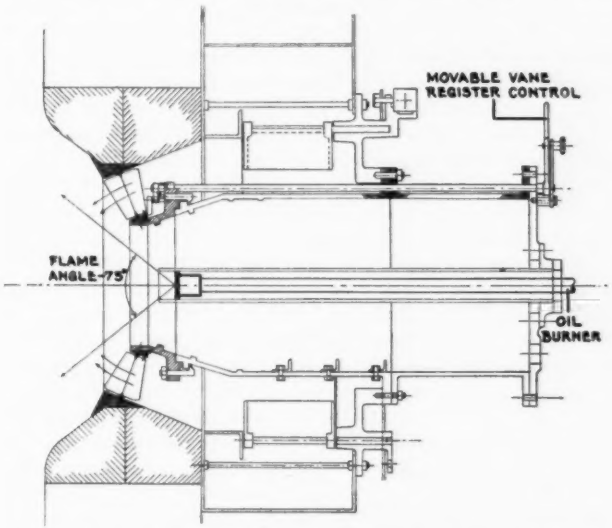


FIG. 4 MOVABLE-VANE AIR REGISTER FITTED TO ORIGINAL BURNER ASSEMBLY

obvious that there was no real length of flame extending into the furnace and most of the heat was concentrated on the front wall.

DEVELOPMENT OF MOVABLE-VANE AIR REGISTER AND ITS OPERATION

The logical procedure from this point was to develop a movable-vane register which could be controlled from the firing floor of the boiler room, varying the position of the vanes with changes in load sufficiently great to require their readjustment. The register shown in Fig. 4 was designed with movable vanes located in about the same position in the burner assembly as the fixed vanes had been in the previous test. Each of the movable vanes was connected to a control ring by a small lug welded to the corner of the vane and riding in a slot in the control ring. This control ring was connected, through suitable linkage, to a rod which extended through the front plate of the burner assembly, terminating in a handle by which the vanes could be adjusted while the burner was in operation and could also be locked in a given position.

After the movable-vane air registers had been constructed and installed, a test was made to determine their characteristics using oil as the fuel and varying the load on the boiler within limits allowed by plant operations. Some difficulties were encountered

in the operation of the movable vanes of the register, chiefly due to the expansion of the 18-8 chrome-nickel alloy from which the vanes had been made. Greater clearances were given to the vanes and their bearing supports and no further problems arose from that source.

Further tests indicated that a position of the vanes, suitable for normal load swings, was about $^{2}/_{3}$ open at about 80 per cent maximum output of the boiler. Decreasing the load on the boiler more than approximately 15 per cent required the closing of the vanes of the register proportionately and vice versa. Firing conditions on oil were considered satisfactory as combustion was complete and the flame shape and length could be controlled by the adjustment of the movable-vane air registers.

The characteristics of the revamped burner assembly had not been determined with pulverized coal and tests were therefore started to investigate the firing conditions with that type of fuel. As previously mentioned, the secondary-air stream from the original burner assembly entered the furnace in approximately a hollow rotating cylinder form or "barrel." This air-stream shape had been disrupted and changed in both direction and shape by the addition of the movable-vane air register at the secondary-air outlet. Tests made with coal as the fuel and the movable-vane register in the air stream were generally unsatisfactory as the register in the wide-open position seemed to take any rotational effect out of the secondary air that had been imparted to it by the register of the original equipment, causing an unstable flame condition. If the movable vanes of the added air register were partially closed, the flame was also unstable and ignition was difficult to maintain.

A proposal was made that deflectors be installed in the pulverized-coal pipe to direct the pulverized coal and primary-air stream toward the secondary-air stream in a manner similar to the shape of the atomized-oil stream. It is believed that this would have overcome the unsatisfactory coal-burning characteristics of the movable-vane register, but the idea was tabled in favor of further consideration of the original equipment using mechanical atomizing oil burners of another make in place of the burners originally supplied.

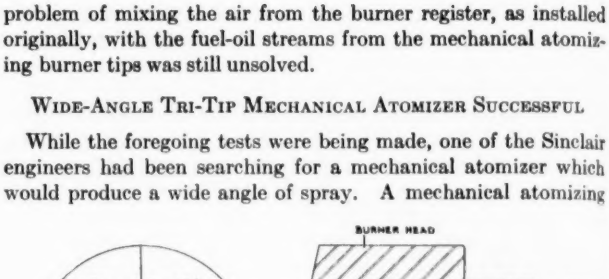
Sinclair engineers had definitely established the fact that the heavy fuel oil supplied by the refinery could be efficiently burned as fuel in the Viscose plant, even though the furnaces were completely water-cooled and the resultant furnace temperatures were comparatively low.

In seeking an adequate solution, another manufacturer was permitted to demonstrate a different type of mechanical atomizer. This burner manufacturer had had considerable experience with mechanical atomization of fuel oil in marine boiler furnaces and it was felt that engineers of the company might suggest a solution to the problem based on previous experience. Tests were made under the direction of two engineers, using this company's mechanical atomizers in the original burner assemblies, both with and without additional deflectors placed on the oil-burner guide pipes. These deflectors could be moved in or out along the oil-burner guide pipes. However, changing the position of the deflectors seemed to have but little effect on the fires, since no primary air passes through the coal pipes on oil firing and the deflectors could have no effect on the secondary-air streams.

Because of the fact that the mechanical atomizers used were more or less of the conventional type with the usual included angle of flame cone of about 75 deg, the results obtained from tests of these burners were similar to those obtained on the original mechanical atomizers. When using the atomizers of the second manufacturer, it was impossible to get more than a slight mix between the fuel oil and air and, consequently, a very smoky unsatisfactory condition existed in the furnace. The basic problem of mixing the air from the burner register, as installed originally, with the fuel-oil streams from the mechanical atomizing burner tips was still unsolved.

Wide-Angle Tri-Tip Mechanical Atomizer Successful

While the foregoing tests were being made, one of the Sinclair engineers had been searching for a mechanical atomizer which would produce a wide angle of spray. A mechanical atomizing



Courtesy Coen Burner Co.

Fig. 5 Tri-Tip Mechanical Atomizing Oil-Burner Head

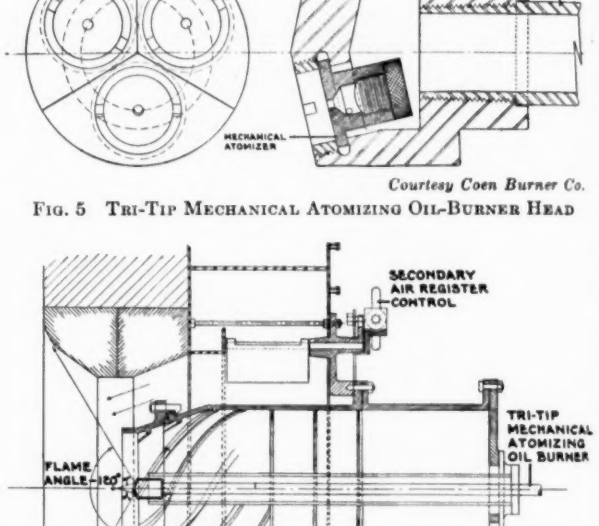


Fig. 6 Original Burner Assembly With Tri-Tip Mechanical Atomizing Oil Burner

burner tip was located which had been developed for steel-mill purposes. This tip consisted of three atomizers set in a single burner head in the manner shown in Fig. 5. The included angle of flame cone that would be produced by this arrangement was apparently about 120 deg. Judging from previous satisfactory results obtained with wide-angle steam atomizing tips, it was assumed that the wide-angle mechanical atomizer would be successful. Four of these burner heads were immediately ordered, each containing three atomizing tips of a size estimated as correct for the normal load conditions.

Preliminary tests of these burners upon their arrival indicated that they would be entirely satisfactory, except for size, as the included angle of spray was such as to produce the proper air-fuel-oil mixture. The atomizers of the original installation were too large for the load requirements, which resulted in a lower pressure on the fuel oil to the burners than that required to give the best atomization. However, the tests that had been made showed sufficient promise to warrant the purchase of additional burner heads of this type to complete the installation on all of the boilers.

Atomizers of several sizes were ordered, the range in size making it possible to have all four burners of the boiler furnace in operation producing a balanced firing condition and an even distribution of heat in the furnace. Normal changes in load are

easily taken care of by the range of the oil pressure to the burner, while the large changes, such as occasionally occur due to starting up or shutting down portions of the plant, can be taken care of in a few minutes by changing the atomizer tips.

In the operation of these burner tips, flame propagation begins within 1 or 2 in. of each of the atomizers but, 6 or 8 in. from the tip, the three streams lose their identity and become one large flame issuing from the burner throat. Because of the wide angle of the combined atomized-oil sprays from these burner tips, the burner can be withdrawn to a point where the fire completely fills the throat without danger of impingement of the oil spray on the coal nozzle of the burner assembly. Thus, the atomized-oil spray is forced into the secondary-air stream, as shown in Fig. 6, and complete combustion takes place with little excess air. Firing conditions are good and the complete operation of these furnaces on heavy fuel oil is entirely satisfactory.

These furnaces have been operated for more than 15 months using heavy oil as fuel, and the costs of steam and power produced during this operating period compare very favorably with those obtained while burning pulverized coal. Additional advantages, not included in the cost statement but important to the operator, are the ease of handling and over-all plant cleanliness of the oil-firing operation.

Automatic Control Satisfactory for Heavy Oil

Very few problems have arisen on the control system used in connection with the firing of heavy fuel oil. The controls vary the amount of fuel oil and air to the burners in response to load changes on the boiler, maintaining the fuel-oil-air ratio set on the master control throughout the range of the control. The fuel-air ratio is manually set for the best combustion on any particular load and the proper ratio is then maintained by the control. A pilot or measuring valve in the fuel-oil line to the boiler burners actuates suitable equipment to damp the action of the control valve in the same line and, thereby, smooths out the flow of oil to the burners. This pilot valve as originally installed was of the piston-type and the heavy-oil fuel had a tendency to stick the piston, making the control sluggish. To remedy this condition, a diaphragm-type valve was substituted.

Physical Properties of Fuel Oil

The heavy fuel oil mentioned in this paper is a residuum product of a petroleum-refinery-cracking process and, as such, the characteristics of the oil may vary slightly from time to time with changes in operation. Representative characteristics of this heavy fuel oil may be listed as follows:

Specific gravity	1.014	Flash point, F	250
B.S.&W., centrifuge, per cent	0.50	Sediment, per cent	0.05
Viscosity, Furol at 122 F	800–1400	Weight, lb per gal	8.45
Pour point, F	80	Btu per lb	18300

A temperature of approximately 275 F has been found satisfactory to reduce the viscosity of the oil to about 20 Furol, or 150 S.S.U. At this viscosity, the oil is readily atomized and its characteristics are such that the viscosity at the firing temperature is about the same, whether the Furol viscosity at 122 F is 800 or 1400. The carbon-hydrogen ratio is greater for the heavy fuel oil than it is for the lighter oils, consequently, the advantage of the heavy fuel lies not only in the additional heat obtained from the carbon but also in the proportionate reduction of loss from the formation of water vapor.

Discussion of Conditions and Results

In general, combination burner assemblies are of incorrect design for the efficient combustion of all the fuels included in the combination. The apparent practice, in the East, has been to design for pulverized-coal firing and to fit in accommodations for some other fuel, such as oil, in the easiest manner possible and with little regard for resultant operating conditions. The true combination burner assembly should handle all fuels intended for it in an equally efficient manner. Such an assembly cannot be designed without a complete knowledge of the characteristics of all the fuels and burner equipment proposed for a particular installation. Since this information may not be available to the designer and since there are many installations of improperly designed combination burner assemblies in operation at the present time, it would be advisable, where one of the fuels used is oil, for the designer or operator to investigate the possible advantages in using a multiple-tip mechanical atomizing oil burner in his original burner assembly.

The number of atomizers per burner head is not limited to three, nor is the included angle of spray from the tip limited to 120 deg. These conditions were found satisfactory for the installation reported in this paper, but another installation might require five or six atomizers, or more, per burner head and a much greater included angle of spray than the 120 deg used in the Viscose installation. It would appear possible, where faulty air distribution around the burner is inherent in the burner assembly, to improve over-all combustion by the selection of proper-sized atomizers and face angles to alleviate this condition to a considerable extent.

Satisfactory combustion of heavy fuel oils, using mechanical atomizing oil burners, can be readily accomplished when the proper equipment is available for heating, atomizing, and mixing the oil and combustion air. The additional cost of this equipment over that required for steam atomizing oil burners can usually be justified on steam saving alone.

Conclusion

The results obtained in this investigation and herein reported have definitely shown that it is not only possible, but practical and economical, to burn heavy petroleum oils in properly designed combination coal-and-oil burners, using mechanical atomization under automatic control, in completely water-cooled furnaces.

Acknowledgment

The advice of O. F. Campbell, R. J. Self, and E. W. Griscom of the Sinclair Refining Company is acknowledged in the successful completion of the investigation and the preparation of this paper. The author wishes to acknowledge, also, the willing cooperation of the American Viscose Company, the Combustion Engineering Company, the Coen Burner Company, and others for their assistance.

Discussion

J. M. Goldstein.[2] Briefly, the paper relates to the operating difficulties encountered in an oil-burning power plant using fuel oil heavier than bunker C. Here particular difficulty was experienced in the building up of coke deposits on the side-wall tubes of the furnace, as after a time these coke deposits fell to the bottom screen tubes. After experimenting with different burner tips and various methods of air introduction, the tri-tip head was used successfully to eliminate the coke deposits building up on the side-wall tubes.

In analyzing the type of oil spray that is obtained with the tri-tip head, it appears that the outside cone consists of a much finer spray than that on the inside of the cone; i.e., the oil particles nearest the center of the head from each tip will meet and the resultant oil particle will become larger and more difficult to

[2] Combustion Service Company, New York, N. Y.

burn completely. Close observation will show decidedly black streaks of unburned oil near the center of the oil fire, with the result that carbon deposits will accumulate on the upper tubes, and there will be excess heat losses due to unburned gaseous hydrocarbons. However, finer atomization on the outside of the flame cone will aid combustion nearest the side-wall tubes, and thus eliminate the difficulty due to coke deposits building up on these tubes.

The use of a specially designed atomizer head to produce a fine oil spray of the desired angle represents, however, only one important factor in obtaining clean, efficient combustion. In order to obtain satisfactory operating results in the burning of heavy fuel oil with mechanical atomizers, other important factors in addition to atomization are oil-air intermixture, combustion process, and combustion atmosphere.

In breaking up the oil into a very fine spray with the mechanical atomizer, the control of the process can be regulated to some extent by adjustment of the oil pressure and temperature. It is desirable that the oil particle, when projected in suspension, should not come in contact with either solid surfaces or other oil particles. However, in the case of some heavier bunker oils, atomization will be poor in spite of the fact that oil temperatures and pressures are adjusted over the entire operating range as recommended for the fuel and oil burner. Inherent characteristics of the fuel itself will affect the quality of atomization. Heavy bunker oils, which are residual fuels, are subject to change in content and characteristics from time to time, and require constant attention to maintain best operation.

A proper oil-air intermixture depends upon the quantity, direction, and velocity of the air supplied in relation to the oil spray. Each oil particle should be completely surrounded by air to promote oxidation of the hydrocarbons. Regulation of the oxidation process will control combustion so that it is neither too rapid nor too slow, thereby greatly diminishing the "cracking" or thermal decomposition of the oil particle.

When combustion takes place due to the oil-air intermixture, a combustion atmosphere is created in the furnace. Since the air used for combustion is subject to change due to temperature, humidity, and contamination, it follows that proper combustion is also a function of a proper quality of air supply.

The common practice of sampling fuel oils in a laboratory for analysis and burning characteristics has proved inadequate. For example, good ignition characteristics and balanced distillation are important factors affecting combustion. Then again, fuel oil, after leaving the refinery, is subject to contamination in transportation, storage, and handling, which also affects atomization and combustion.

Again referring to the paper, it is shown that the fuel oil is transported by pipe line from the refinery to the power plant with small loss in temperature; also that the boilers are of waterwall-furnace construction. These conditions are unusually favorable for oil burning, and are the exception rather than the rule. For instance, in most installations fuel oil is delivered either in tank trucks or barges and, as mentioned, becomes subject to contamination which in turn increases the difficulties in burning. In many cases, refractory material is used in combustion chambers, with and without facilities for cooling. The presence of noncombustibles in the oil, such as natural salts, chemicals used in the refinery operations, scale, and dirt will cause rapid deterioration of the refractory materials, particularly at high combustion-chamber temperature.

The mechanical atomization of heavy bunker oils is a problem which has confronted engineers in charge of every type of installation used for heating and processing. In many plants, high-pressure steam for steam atomization is not available, which automatically eliminates this method of solving combustion problems.

Methods must be provided to control within limits the variable factors which affect atomization and combustion. Recent tests with the use of a chemical oil conditioner to improve the atomization and combustion of light and heavy fuel oils have resulted in the solution of many common difficulties encountered in oil burning.

A recent troublesome installation, similar to the case cited, where carbon deposits formed in the furnace, was in an apartment building in New York City, where a rotary-type oil burner using bunker C oil had been installed in a low-pressure fire-tube boiler several years ago. The combustion chamber was built of standard firebrick, and was too short and too narrow for the size of the boiler. A dutch oven was recommended but not installed.

The oil fire impinged somewhat on the side walls but much more on the rear wall of the combustion chamber. During the several years that the burner was in operation, the quantity of carbon deposits which accumulated on the walls of the chamber and in the fire-tubes over a 30-day period was not excessive or bothersome. However, by the fall of 1939, the carbon deposits on the rear wall of the chamber became excessive and required removal every two hours when the burner was in operation. Oil temperature and pressure were changed, primary- and secondary-air adjustments were regulated, and even the oil supplier was changed, but the carbon deposits kept building up. The oil-burning system was reset for normal operation, using the same bunker oil, and "chemical conditioning" was applied. Now for the last several months, in spite of continued flame impingement on the walls, carbon deposits have been eliminated, and trouble-free, efficient operation has been maintained.

Another interesting case is the conversion of several coal-burning boilers to oil burning by a public-utility company. The bunker oil used is a by-product of the gas-making division of this company, and a standard-make mechanical-atomizing oil-burning system was installed and operated in accordance with the manufacturer's instructions. Difficulties encountered were high furnace temperature and melting of the refractory material in the combustion chamber at normal overloads. There was no flame impingement, fires were sparky, and there were complaints of smoke nuisance when a boiler was put on the line. Changes were made in the type of burner tips, design of the combustion chamber, air delivery, and quality of firebrick, but operation was only slightly improved. As a further remedy, this heavy-residuum bunker oil is now treated so that its specifications are equivalent to or better than a commercial bunker C fuel oil. Further experimentation is being carried on by the company engineers in cooperation with the equipment engineers to burn this residuum by-product successfully, without the necessity of this expensive treatment.

In view of the fact that in recent years revolutionary new catalytic processes have been developed in petroleum refining, and since bunker oils are thereby subject to change, it may prove difficult to design a mechanical atomizer which will function efficiently with all grades of bunker oils under all conditions. When mechanical changes fail to solve oil-burning problems, the chemical-conditioning method may prove economically sound and advantageous. The consumer of fuel oil and his staff should seek the cooperation of the oil supplier and equipment manufacturer, and in addition, the impartial advice of the combustion specialist.

R. C. Vroom.[3] The paper emphasizes the desirability of installing suitable combined burners initially, when it is possible that future requirements may make it necessary to burn a liquid or gaseous fuel instead of a pulverized fuel or vice versa. The writer feels, however, that the paper may have created an impre-

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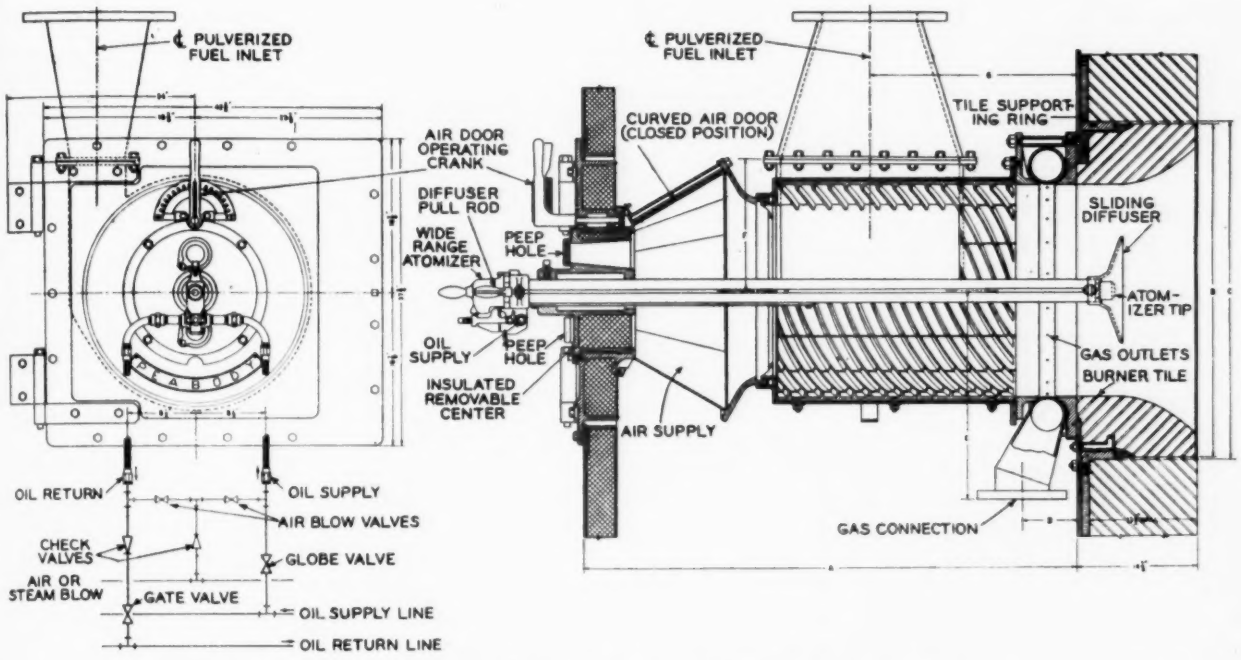


FIG. 7 COMBINATION BURNER DESIGNED TO BURN PULVERIZED FUEL, GAS, AND OIL

sion which surely was not intended, i.e., prior to the experimental program which the author describes, there were not available suitable combined burners, nor atomizers, of the mechanical type which could handle liquid fuels heavier than bunker C.

For nearly 15 years the company with which the writer is connected has been building combined burners for liquid, gaseous, and pulverized fuels varying in capacity from 15 to 150 million Btu per hr liberation each. They have been described in papers presented before the A.S.M.E. and a typical burner is illustrated in Fig. 7 of this discussion. These burners are designed to meet and do meet combustion guarantees which are in every respect competitive, and no difficulty is experienced in burning individually any of the fuels which the burner is designed to handle, nor in changing from one fuel to another without loss in steaming capacity. They are successfully firing, in many of the well-known steam plants in this country, in units varying in capacity from 20,000 to 500,000 lb of steam per hr each.

In connection with mechanical atomizers for heavy liquid fuels, the writer spent about 6 weeks in Louisiana 10 years ago running tests with such atomizers on all sorts of refinery-waste fuels. These included not only neutral fuels but acid and alkaline sludges, which are far from homogeneous, may contain various quantities of water, and tend to precipitate solids. As a result of these tests, mechanical atomizing liquid-fuel burners were installed in a new utility plant which was to use the waste fuels purchased from the refinery and in turn supply the refinery with process steam. In addition to the waste liquid fuels, it was necessary to burn petroleum coke from the refinery and also natural gas to make up the deficiency between the available waste fuels and the total fuel requirements.

The initial installation comprised four boilers, each to produce 350,000 lb of steam at 600 psi and 750 F temperature. Two of these boilers were equipped with 18 burners each, for liquid and gaseous fuels, and two with 15 such burners, and also two burners for handling the pulverized coke. More recent extensions have employed a smaller number of higher-capacity burners per boiler. There are several other plants which over a number of years have successfully employed mechanical atomizers for firing high-capacity boilers with such fuels, and also plants in which heavy residual fuels and tars with melting points of 300 F or over are being handled. In addition, of course, there are hundreds of combined burners in use where the liquid fuel is ordinarily bunker C.

T. H. WALKER.[4] The design of combination burners to fire pulverized coal and oil, or coal, oil, and gas has been given considerable study, since economic conditions or character of plant load often justify their use. As pointed out in this paper, such burners should handle all fuels equally well, i.e., without operating difficulties or smoke nuisance, and with combustion efficiencies corresponding to best practice for each fuel. Usually the change from one fuel to another must be made without shutting down the unit and sometimes two fuels must be burned together; therefore, no device can be added which will improve operation with one fuel at the expense of that with another.

It has been and to some extent still is the practice with oil-and-gas firing to use a number of small burners. This assists in the utilization of all the furnace volume, reduces the flame length, and to some extent simplifies the problem of introducing the correct amount of air at the proper place, since the flames tend to be mutually sustaining and the burners are close enough to each other so that a slight excess of air from one burner may tend to compensate for a deficiency in another. Also, wide-range operation can be obtained by shutting down some burners and operating others at approximately their design velocities.

Pulverized-fuel burners for the same steam delivery must be fewer and larger because no satisfactory means has yet been devised for splitting a coal-air mixture into many small streams at the same pressure and air-coal ratio; the use of many small mills would be complicated and expensive. Thus the individual burners must be capable of handling a wider load range. Thorough mixing of fuel and air as they enter the furnace, a minimum of excess air, and stability of ignition are essential. In combination burners these objectives must be accomplished with fuels of quite different physical characteristics, since the volume of coal-air mixture is greater than that of either oil or gas. The coal is

[4] Combustion Engineering Company, Inc.

carried to the burner by a quantity of air which often is a sizable proportion of that required for combustion and, hence, requires less secondary air. In burners of the type under discussion where furnace turbulence must be obtained by the use of a turbulent air stream, the fuels must be introduced concentrically, and the necessity for a rather large coal-air nozzle, as compared with the size of oil or gas burners, moves the latter further away from this turbulent air stream than would be the case in similar burners designed for oil or gas only.

With these considerations in mind, the solution which the author describes is interesting. The advantage of the tri-tip burner lies in its ability to provide a wide-angle spray which can be directed into the secondary-air stream, thus accomplishing more thorough mixing of fuel and air than the conventional single-jet atomizer.

Author's Closure

Mr. Goldstein's observations regarding the oil spray obtained with the tri-tip burner head apparently did not take into account the action of the turbulent-combustion air stream on the oil spray from this type burner head. An intimate mix is obtained between combustion air and the atomized fuel oil, as reported in the paper, and observation of the fires, from any of the furnace observation ports, reveals no black streaks of unburned oil in the center, or at any other portion of the flame. These boilers were recently shut down for internal inspection and no appreciable deposits were found in the furnaces or flue gas passages. This would appear to be a further indication of complete consumption of the carbon in the fuel oil. This writer's discussion also states that the waterwall-furnace construction of these boilers is unusually favorable for oil burning. It is felt that most authorities agree that fuel oil requires a higher furnace temperature for good combustion conditions than other fuels, i.e., pulverized coal or gas. Furthermore, furnaces designed primarily for fuel-oil firing usually contain more refractory, to maintain a higher temperature, than those furnaces designed primarily for pulverized coal. The furnace walls of the boilers which were described in the paper are almost completely covered with water-tubes and, due to the amount of heat absorbed by these tubes, the furnace temperatures are considered low.

Mr. Goldstein reports a few interesting cases of oil-firing difficulties but they seem to be concerned with lighter fuel oils and are therefore hardly comparable to the installation described in the paper.

Mr. Vroom's discussion is of great interest to the author. Combination burners for liquid, gaseous, and pulverized fuels, as he points out, have been on the market for many years. A great many of these combination burners, however, have left much to be desired when operating on one or more of the fuels for which they were designed. Revision of such an installation, to correct the undesirable condition, becomes a problem of that installation only; the solution found satisfactory in one instance might or might not be successful if applied in another case. There are combination burners in operation with undesirable firing conditions recognized by the operators but accepted by them as unalterable. Consequently, no effort is made to correct the difficulties.

In his discussion, Mr. Walker emphasizes many of the points which the author sought to bring out in the paper. The observations regarding general practice on burner spacing and placement, with his explanation of the reasons for such spacing and placement, tend to clarify the statement of the problem involved in the original installation.

The author wishes to thank all the discussers of the paper for their contributions to the general subject.

The Analogy Between Fluid Friction and Heat Transfer

Discussion of Paper[1] by Th. von Kármán

By BORIS A. BAKHMETEFF,[2] NEW YORK, N. Y.

IN HIS paper[1] Dr. von Kármán has focused light on a fundamental precept, the import of which lately may have become somewhat eclipsed.

The heat-transfer analogy was originally offered by Reynolds as a fascinating hypothesis to be probed and substantiated by observation and experiment. Quantitatively, the interpretation from the very first was in terms of momentum, the heat exchange due to turbulent convection being appraised at

$$q = \frac{c\tau\Theta}{U} \qquad [a]$$

This basic relation, equivalent to Equation [19][3] and leading directly to von Kármán's

$$C_H = C_f/2 \qquad [16]$$

was accepted in all subsequent interpretations and improvements without scrutiny, even if certain physical aspects cast doubt as to the validity of expressing the Reynolds analogy in that particular form. Indeed as Equation [a] applies to the heat exchange in the turbulent zone only, the symbol $\Theta = \Theta_\delta - \Theta_0$ should signify the difference between the temperature of the fluid Θ_0 and a certain temperature Θ_δ at the boundary between the turbulent core and the laminar region. Accordingly Θ in all cases must be less than the full temperature difference $\Theta_0 = \Theta_\infty - \Theta_0$ between the wall and the fluid. In fact one may properly introduce

$$\eta = \frac{\Theta_\delta - \Theta_0}{\Theta_\infty - \Theta_0} = \frac{\Theta}{\Theta_0} \qquad [b]$$

as a "reduction coefficient," featuring the "insulating effect" of the laminar zone.

The practical meaning is, that if Equation [a] were to express correctly the physical facts relating to turbulent heat transfer, as for example in a pipe, then the quantities of heat observed experimentally should under all circumstances be below the figures computed from Equation [a] with $\Theta = \Theta_b$. In other words, observed values should comply with

$$q = \eta \frac{c\tau\Theta_0}{U} \qquad [c]$$

where η is < 1.

Such, however, is not the case. Since the early experiments by Stanton and Pannell,[4] it has been known that, for air and gases for which the Prandtl number $\sigma = \mu\, c/k$ is < 1, measured heat values are in excess of q, computed from Equation [a] with $\Theta = \Theta_b$. Accordingly, the over-all experimental values of the heat-transfer number C_H in Equation [5] exceed $C_f/2$. Mathematically, this discrepancy has been repaired by G. I. Taylor, Prandtl, and von Kármán, who complemented the original Reynolds disclosure by accounting supposedly for the effect of the near-to-the-wall laminar film, across which the transfer of heat is by conduction. To elucidate, one may refer to the prototype Taylor Equation [21], written in the form

$$\left.\begin{aligned} q &= \frac{c\tau\Theta_0}{U}\left[\frac{1}{1 + U_\delta/U\,(\sigma - 1)}\right] \\ C_H &= C_f/2\left[\frac{1}{1 + U_\delta/U\,(\sigma - 1)}\right] \end{aligned}\right\} \qquad [d]$$

Indeed for $\sigma < 1$, the bracket value becomes larger than unity, a feature which, because of the presence in the structure of the group $(\sigma - 1)$, is equally shared by Prandtl's Equation [22] and von Kármán's Equation [34] expressions.

The physical implications devolve from comparing Equation [d] with Equation [c]. The bracket groups are obviously equivalent to the reduction coefficient η, which for $\sigma < 1$ becomes thus larger than unity, meaning that a correction term, ostensibly intended to account for the insulating effect of the laminar zone, results in increasing the heat flow numerically beyond the highest limit which could possibly be achieved in accordance with Equation [a] if the whole of Θ_0 were contributing to turbulent convection. Obviously, the source of the discrepancy must lie in the very form of the basic relation of Equation [a], which seems to understate the actual intensity of the heat transfer due to turbulence.

The writer finds, that the inconsistencies are easily removed if instead of using the "momentum," one were to interpret the Reynolds analogy in terms of "energy."[5] In such a case the second of von Kármán's Equations [5] could be appropriately expressed by the ratio

$$C_H = \frac{q}{\rho c U\Theta} = \frac{\text{Energy lost per unit wall surface}}{\text{Energy of flow per unit cross-section area}} \qquad [e]$$

Using for the numerator Equation [10] in a text[6] by the writer, we obtain

$$C_H = \frac{q}{\rho c U\Theta} = \frac{U\tau}{\rho U U^2/2}$$

from which there follows

$$q = 2\,\frac{c\tau\Theta}{U} \qquad [f]$$

$$C_H = C_f$$

By comparison with Equation [a] and Equation [16], the heat transfer derived from interpreting the Reynolds analogy in terms of energy, is twice as large as when using momentum. Physically, there is no reason why the analogy between heat transfer and friction should not be considered in energy terms. In fact convective heat transfer is a process of diffusion, and the

[1] "The Analogy Between Fluid Friction and Heat Transfer," by Th. von Kármán, Trans. A.S.M.E., vol. 61, 1939, pp. 705–710.

[2] Professor, Columbia University. Mem. A.S.M.E.

[3] Numbers in brackets refer to equations in the original von Kármán paper.[1]

[4] "The Mechanical Properties of Fluids," a collective work, Blackie & Son, Ltd., London, 1925, p. 174.

[5] This method of presentation was first disclosed by the writer in his lectures at Columbia University in 1932–1933.

[6] "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton University Press, 1936, p. 4.

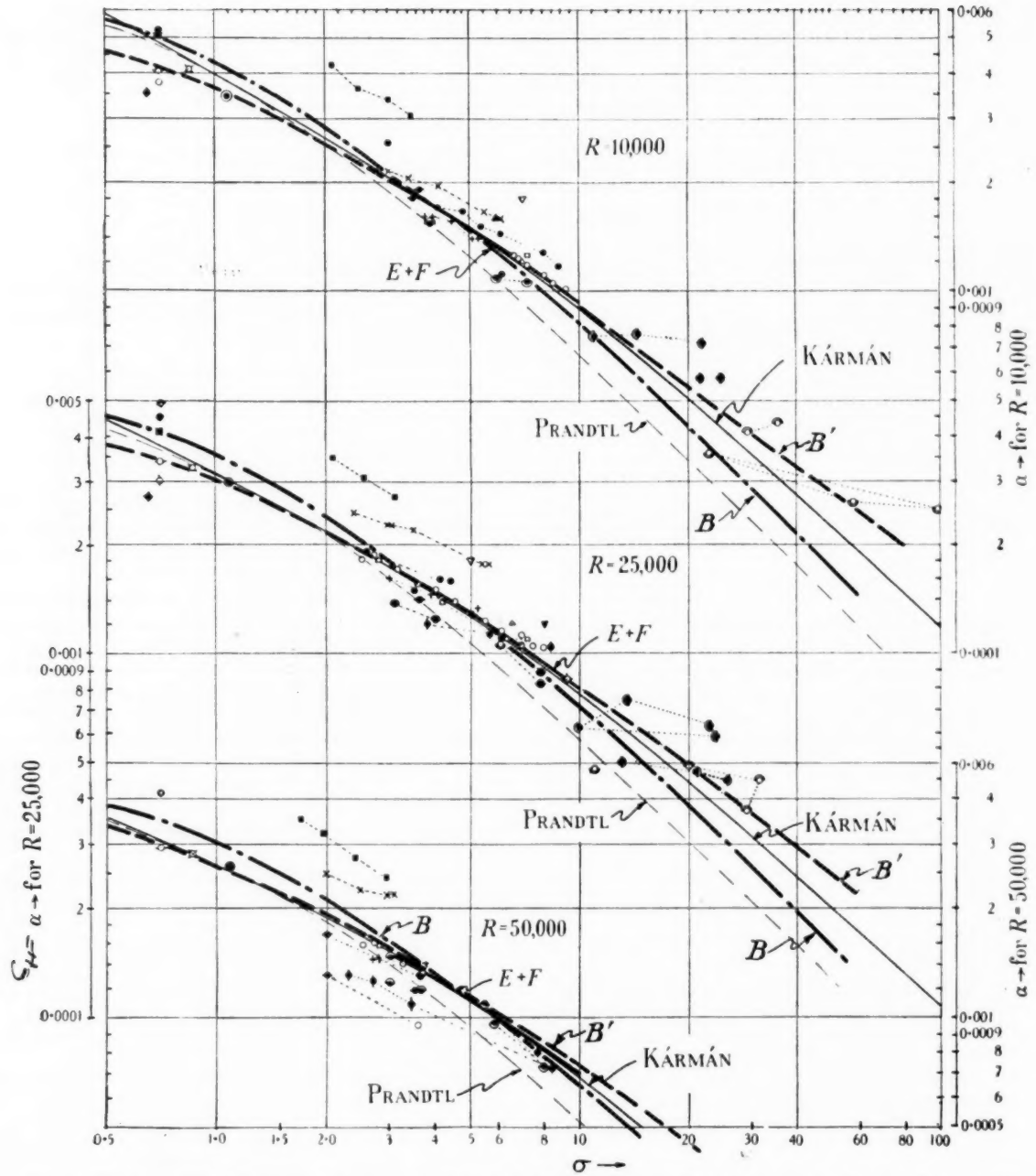


FIG. 1 HEAT-TRANSFER NUMBER IN PIPES, FIG. 4 OF VON KÁRMÁN PAPER,[1] WITH ADDITION OF CURVES REPRESENTING EQUATIONS [h] AND [j]

rate of the latter is appropriately reflected by energy loss.

To arrive at final expression of an equation of the type of Equations [22] and [34] one proceeds by equating the turbulent heat exchange $q = 2c\tau(\Theta\delta - \Theta_h)/U = \eta(2c\tau\Theta_0/U)$ based on Equation [f], with the conduction across the laminar film $q = k(\Theta_w - \Theta\delta)/\delta = (1-\eta)(\Theta_0 k/\delta)$, in which δ is the thickness of the insulating layer, equivalently expressed through $\delta = \mu U_\delta/\tau$. This determines the reduction coefficient as

$$\eta = \frac{1}{1 + 2\sigma U_\delta/U} \qquad \text{[g]}$$

and then with Equations [f] and [c]

$$\left.\begin{aligned} q = \eta 2\,\frac{c\tau\Theta_0}{U} &= \frac{2c\tau\Theta_0}{U}\left[\frac{1}{1 + 2\sigma U_\delta/U}\right] \\ C_H = \eta C_f &= C_f\left[\frac{1}{1 + 2\sigma U_\delta/U}\right] \end{aligned}\right\} \qquad \text{[h]}$$

Assuming finally with von Kármán that, in terms of the dimensionless distance parameter y^* Equation [25], the $U\delta/U$ ratio may be expressed through

$$U_\delta/U = N\sqrt{C_f/2} \qquad \text{[i]}$$

with N a numerical constant, one obtains for η the convenient form of

$$\eta = \frac{1}{1 + N\sigma\sqrt{2C_f}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [j]$$

In first approximation, one single value of N can be assumed to qualify conditions within the whole range of σ and R, the particular number lying ostensibly somewhere near the point, where in Figs. 51 and 53 of Bakhmeteff's text,[6] the u/u_* curves leave the laminar outline. Accordingly in Fig. 1 of this discussion, complementing von Kármán's Fig. 4,[1] the curves, marked B, represent Equations $[h]$ and $[j]$ with N = constant = 7. The practical accord with the experimental points, notwithstanding the crudeness of the assumptions, is quite satisfactory. It does seem, however, that with the increase of σ, the N value, which features the size of the conduction laminar zone, should somewhat decrease. The possible variation could be appraised by a simple exponential expression $N = N_0/\sigma^m$. This possible improvement is introduced into the curves, designated as B', taking N_0 = constant = 9.5, and $m = 0.2$, so that in Equation $[j]$ $N\sigma = 9.5\ \sigma^{0.8}$.

In all computations, the Blasius values of $\lambda = 4C_f$ for smooth pipes were used.

The insulating effect of the laminar film is especially evidenced by the comparative outlines of the η curves which are plotted in Fig. 2 of this discussion. Obviously with σ approaching zero, the insulating effect vanishes, and C_H tends to its limiting value C_f.

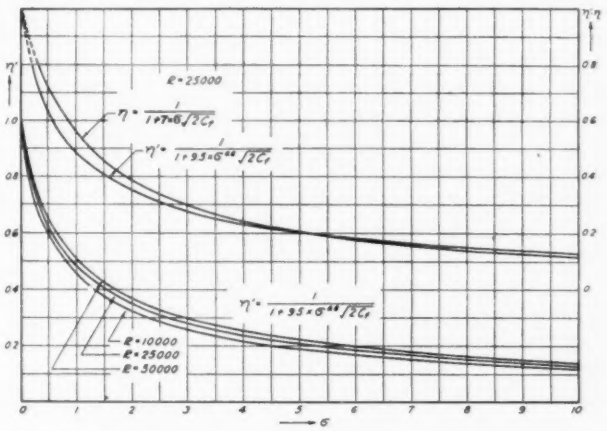


FIG. 2 INSULATING EFFECT OF LAMINAR FLOW EVIDENCED BY COMPARATIVE OUTLINES OF η CURVES

Pump Discharge Valves on the Colorado River Aqueduct

By R. M. PEABODY,[1] LOS ANGELES, CALIF.

This paper outlines the arrangement and physical equipment of the five pumping plants installed on the Colorado River Aqueduct. Studies and estimates indicated that for these aqueduct plants a common delivery pipe with a valve at each pump was more economical than separate delivery lines. Rotating-plug-type pump valves were finally selected for shutoff service at all plants. With proper control of the rate of closure they can be made to limit the pressure rise to a moderate percentage in systems comparable to those of the Metropolitan Water District. Constructed by two manufacturers for application at different plants, the valves, while varying somewhat in detail, give practically the same operating result. Model tests were conducted by the manufacturers and by the hydraulic laboratory at the California Institute of Technology. Details of the design, construction, and operation of the valves, as well as the test results, are included in the paper.

THE Colorado River Aqueduct extends from the Colorado River across the entire width of California and is to supply water for domestic and industrial purposes to the highly developed areas of the south coastal basin of Southern California. The main aqueduct is 242 miles in length. The greater part lies across a barren desert country at a higher elevation than the Colorado River. To cross this elevated region a succession of pumping plants, five in number, are required with a total lift of 1617 ft. The designed capacity of the aqueduct is 1600 cfs. Power for pumping, which for the ultimate development will be about 300,000 kw, is obtained from the power plant at Boulder Dam about 150 miles north of the point of diversion on the Colorado River. Fig. 1 shows the location and the profile of the portion of the aqueduct which includes the pumping plants.

Pumping Plants

Each pumping plant will ultimately contain nine pumping units of 200 sec-ft capacity each, which will allow one spare unit. Three such units are installed in each plant initially and others will be added from time to time as the demand for water increases. Table 1 gives the principal engineering data for each of the plants.

The five pumping plants are almost identical in general design. There are differences in detail made necessary by the variation in head and by the different intake and discharge conditions at the site of each plant. The pumps are all of the single-stage, single-suction, vertical-shaft, volute type, without guide vanes or diffusers. The pumps are set below the minimum inlet water levels, not only to avoid the necessity of priming apparatus, but to give sufficient positive inlet pressure to insure operation at minimum head and maximum discharge without cavitation. The ultimate nine pumps of each plant will be connected, in groups of three, to three main 10-ft diameter, steel delivery pipes.

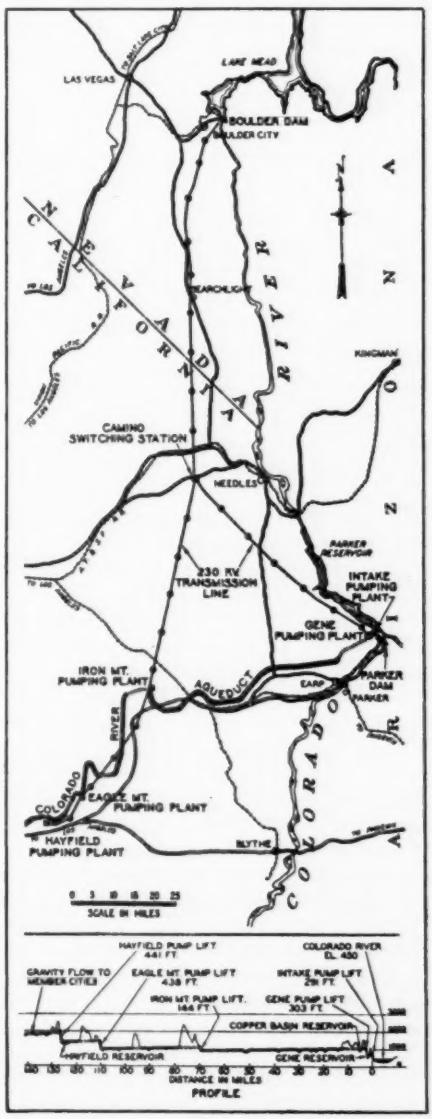


FIG. 1 MAP OF POWER SYSTEM FOR COLORADO RIVER AQUEDUCT AND PROFILE OF SECTION WHICH INCLUDES PUMPING PLANTS

Discharge pipes 6 ft in diameter from each pump converge through a three-branch manifold or wye to the main delivery pipe at a short distance from the plant. The delivery lines at three of the plants, Intake, Gene, and Eagle Mountain, connect at the upper end through surge chambers to pressure conduit or pressure tunnels and reservoirs. At the other two plants, Iron Mountain

[1] Senior Engineer, Metropolitan Water District of Southern California.

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Discussion of this paper was closed September 1, 1939, and is published herewith directly following the paper.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 PRINCIPAL ENGINEERING DATA CONCERNING PUMPING PLANTS

Items	Intake	Gene	Iron Mt.	Eagle Mt.	Hayfield
Location - miles west of Intake	0	2	69	110	126
Static Lift	291	303	144	438	441
Average Operating head	294	310	146	440	444
Elev. hydraulic grade at top of lift	740	1,037	1,047	1,404	1,807
Delivery lines - slope length (center line pumps to center line surge tank)	946	2,186	689	947	1,287
Diameter of main lines	10'-0"	10'-0"	10'-0"	10'-0"	10'-0"
Plate thickness (min. to max.)	3/8"-7/8"	3/8"-7/8"	3/8"-6/8"	3/8"-1-1/8"	3/8"-1-1/8"
Diameter of branch lines	6'-0"	6'-0"	6'-0"	6'-0"	6'-0"
Plate thickness (min. to max.)	9/16"-5/8"	5/8" only	1/2"-9/16"	3/4"-7/8"	3/4"-7/8"
Time for round trip of pressure wave, seconds ($^{2L}/_{a}$)	0.664	1.533	0.486	0.638	0.850
Pumps - Manufacturer	Byron Jackson Co.	Byron Jackson Co.	Allis-Chalmers Mfg.Co.	Worthington Pump & Machinery Corp.	Worthington Pump & Machinery Corp.
Capacity at Rated Head, g.p.m.	90,000	90,000	90,000	90,000	90,000
Speed, (r.p.m.)	400	400	300	450	450
Specific speed (g.p.m. units)	1,690	1,624	2,140	1,403	1,396
Motors - Manufacturer	Gen'l Electric Co.	Gen'l Electric Co.	Allis-Chalmers Mfg. Co.	Westinghouse Elec. & Mfg. Co.	Westinghouse Elec. & Mfg. Co.
Horsepower (each motor)	9,000	9,000	4,300	12,500	12,500
Flywheel Effect, motor and pump (WR^2)	468,000	468,000	339,000	506,000	506,000
Pump Discharge valves, Manufacturer	S. Morgan Smith Co.	S. Morgan Smith Co.	S. Morgan Smith Co.	Pelton Water Wheel Co.	Pelton Water Wheel Co.
Valve size (inlet and outlet dia.)	42" x 58-3/8"	42" x 58-3/8"	48" x 64-1/2"	40-1/2" x 53-7/8"	40-1/2" x 53-7/8"
Inlet Reservoir - useful capacity	Parker Reservoir 387,000 acre-feet	Gene Reservoir 3,000 acre-feet	100 acre-feet	112 acre-feet	Hayfield Reservoir 86,500 acre-feet
Discharge Reservoir - useful capacity	Gene Reservoir 3,000 acre-feet	Copper Basin Reser. 6,000 acre-feet	112 acre-feet at Eagle Mountain	Hayfield Reservoir 86,500 acre-ft.	Cajalco Reservoir 100,000 acre-feet

NOTE: The average submergence of the pump center line below the inlet water level is as follows for the various plants: Intake, 14 ft; Gene, 17 ft; Iron Mt., 15 ft; Eagle Mt., 20 ft; Hayfield, 37 ft.

and Hayfield, the delivery lines connect through simple transition structures to gravity-flow tunnels. A headgate is installed at the upper end of each delivery line so that any main pipe line can be unwatered independently and without draining the reservoir.

It is not considered desirable to sacrifice efficiency by operating the pumps with throttled discharge. The flow in the aqueduct will therefore be in multiples of about 200 sec-ft, the unit pumping capacity. Due to variation in reservoir and canal water levels, the pumps at consecutive plants may operate at heads above or below the design point with a consequent variation in discharge. The reservoirs at Gene draw, Copper basin, Hayfield, and Cajalco are sufficient to hold the entire capacity of the aqueduct between these reservoirs and the next pumping plant upstream. At these plants, variation in pump discharge may be compensated for by starting and stopping one or more pumps at comparatively long intervals, and in power outages of long duration no water will be wasted from the aqueduct. The available regulatory storage at Iron Mountain and at the inlet of Eagle Mountain is small (about 100 acre-feet at each plant) so that starting and stopping of one pump at each of these plants may be necessary at shorter intervals to maintain a balanced discharge. These small reservoirs will provide rejection storage for power outages up to 30 or 45 min duration and, for unanticipated longer outages, spillways provide for wasting the excess flow.

Figs. 2 and 3 show the general arrangement of the Intake plant and appurtenant structures. Fig. 4 shows a cross section of the Intake plant in more detail and is typical of all the plants, except for the arrangement of the pump inlet. In all of the other plants, the inlet is through a branch from a 16-ft-diameter steel manifold. There is a butterfly valve instead of a slide gate to close off the pump inlet.

With the adopted arrangement of three pumps discharging into a common delivery pipe, valves at the outlet of each pump are necessary. Another possible arrangement, which was studied and abandoned because of greater cost, was to have separate delivery lines from each pump all the way to the upper reservoir or conduit with no discharge valves at the pumps, but with a device at the upper end of each pipe line to prevent backflow from the reservoir. With such an arrangement, should a pump be taken out of service, either intentionally or as a result of emergency shutdown, the water would run back through the pump until the delivery pipe was empty. The pump would of course reverse its rotation under these conditions. Studies and estimates showed conclusively that for the aqueduct plants the common delivery pipe with a valve at each pump was more economical than separate delivery lines.

Discharge Valves

The large size of the pumps and the high discharge velocities called for careful consideration of the various available types of valves, which include plain swing-check valves, combinations of check valves, and pressure- or mechanically operated relief valves, butterfly valves, gate valves, needle and plunger valves, and plug-type valves or cocks. On account of the high velocity at the pump discharge, reaching 24 fps at maximum flow, loss of head is an important consideration. Butterfly valves, needle and plunger valves, and certain types of plain gate valves all create a certain amount of obstruction or disturbance in the flow, with more or less head loss. However, two types of valves, the ring-follower gate valve and the rotating-plug-type valve, could be so constructed that at full opening they would offer no more resistance to flow than an equal length of straight or tapered circular pipe. In both types, when fully open, the water passages through the valve body and the valve disk or plug are circular and of uniformly increasing diameter from the pump-discharge flange to the connection of the valve to the delivery pipe.

Of even greater importance than the type of valve is the operating and control mechanism. Considerable power is necessary to operate high-pressure valves of this size, particularly for the rapid valve movement on emergency closure. Electric power is not suitable since failure of power supply would leave the valves inoperative at the very time when emergency operation is necessary. Hydraulic operation, using the delivery-line water pressure, is not wholly desirable, partly because of the variation in the pressure source during closure and even more so because of

the possibility of incrustation and corrosion of the small control valve parts and the resulting irregularity or failure in operation. The most reliable power source, although the most expensive, appeared to be hydraulic cylinders, using oil from air-pressure-accumulator tanks similar to those commonly used for supplying power to actuator-type governors for hydraulic turbines.

Specifications were issued for both ring-follower gate valves and rotating-plug-type valves. Hydraulic-cylinder operation was required using oil pressure. The control mechanism was specified to be flexible enough to give the slow valve travel desired for normal opening and closing, as well as an adjustable fast travel for emergency closure. The bids received showed a lower cost for the rotating-plug-type valves in all plants and contracts for this type were awarded to the S. Morgan Smith Company for the valves at the Intake, Gene, and Iron Mountain plants, and to the Pelton Water Wheel Company for the valves at the Eagle Mountain and Hayfield plants. The hydraulic-cylinder operators for the Eagle Mountain and Hayfield valves were made by the Chapman Valve Company for the Pelton Water Wheel Company. Controls for all valves were made for

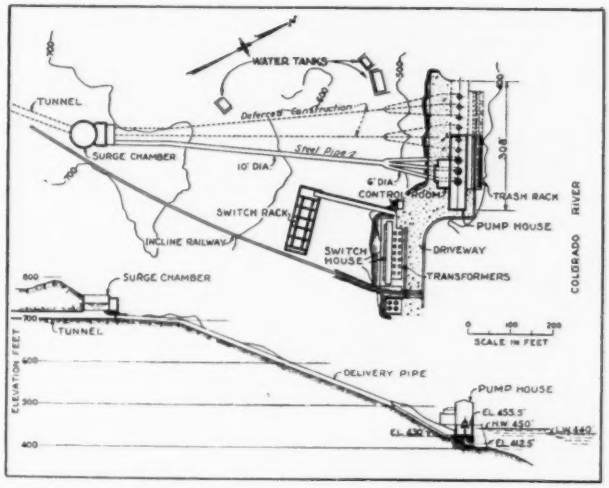


FIG. 2 PLOT PLAN AND PROFILE OF INTAKE PUMPING PLANT



FIG. 3 THE INTAKE PUMPING PLANT ON THE COLORADO RIVER

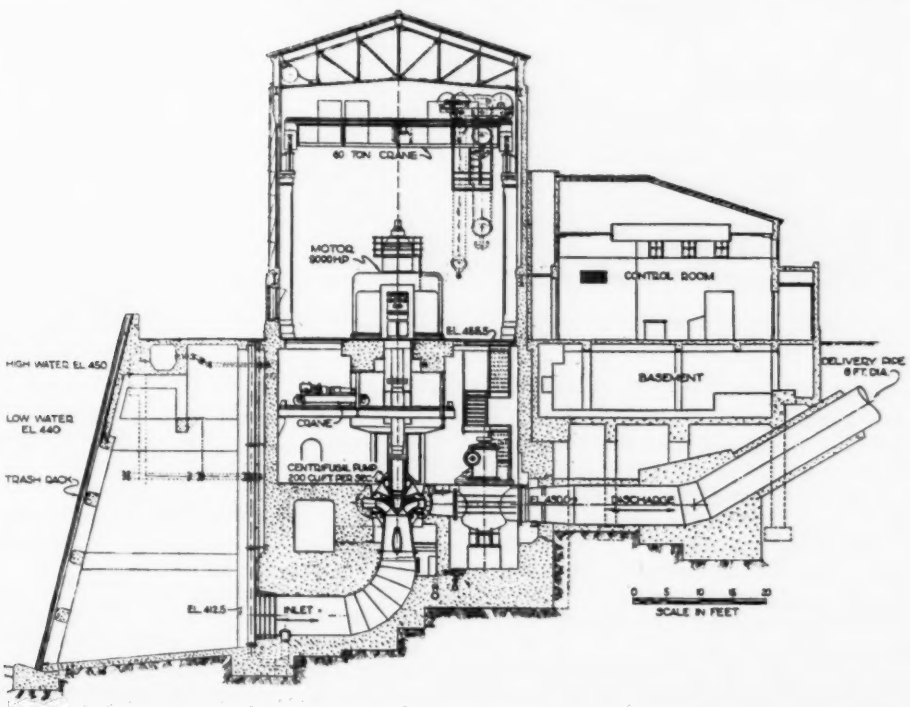


FIG. 4 TRANSVERSE SECTION OF INTAKE PUMPING PLANT

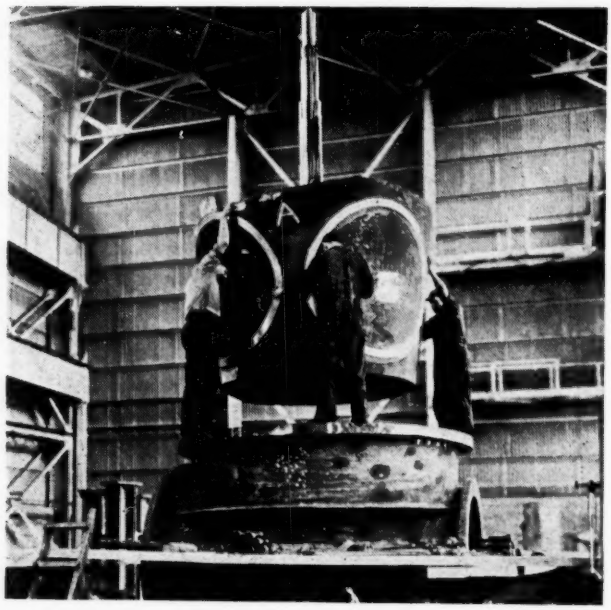


FIG. 5 ASSEMBLING THE BODY AND PLUG OF A 42-IN. BY 58 3/4-IN. TAPERED ROTOVALVE; S. MORGAN SMITH COMPANY

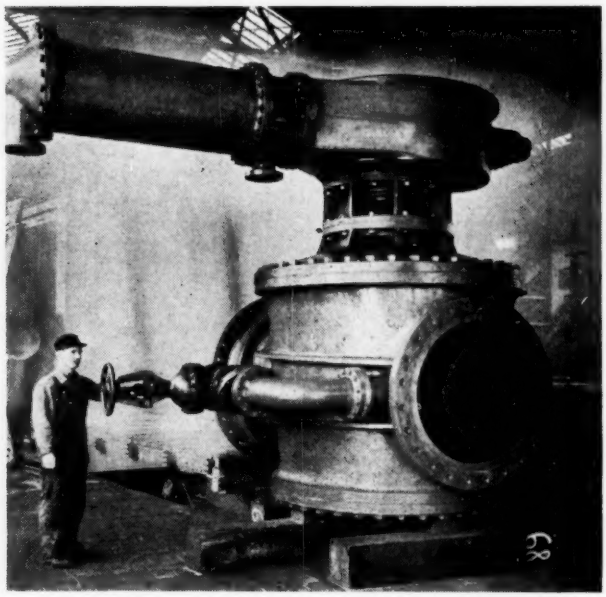


FIG. 6 COMPLETE ASSEMBLY OF A 40 1/2-IN. BY 53 7/8-IN. RESEATING HYDRAULIC CONE VALVE; PELTON WATER WHEEL COMPANY

both principal contractors by the Woodward Governor Company.

The general design of the valves made by the two companies is similar, except for the operating mechanism. In all valves, with the plug in the fully open position, the water passage is circular in cross section, increasing in diameter from the pump to the delivery pipe. The total angle of the taper is about 8 deg. The valve thus forms an extension of the pump diffuser and helps to regain the velocity head at the pump discharge. Close to the periphery of the water passage on both the valve plug and valve body are noncorrosive seats, which are of monel metal on the S. Morgan Smith valves and bronze on the Pelton valves. The plugs are conical and, in both the open and closed position, the plug is pushed down into the valve body so that the seats are in contact. To open or close a valve, the entire plug is first raised sufficiently to separate the seats, then rotated 90 deg, and finally pushed down again to reseat. On the sides of the plug are seats corresponding to those around the water passages. There is thus full seat contact in both the open and closed positions. Fig. 5 is a shop erection view of one of the S. Morgan Smith valves which shows the opening and closing seats on the plug. Fig. 6 is an

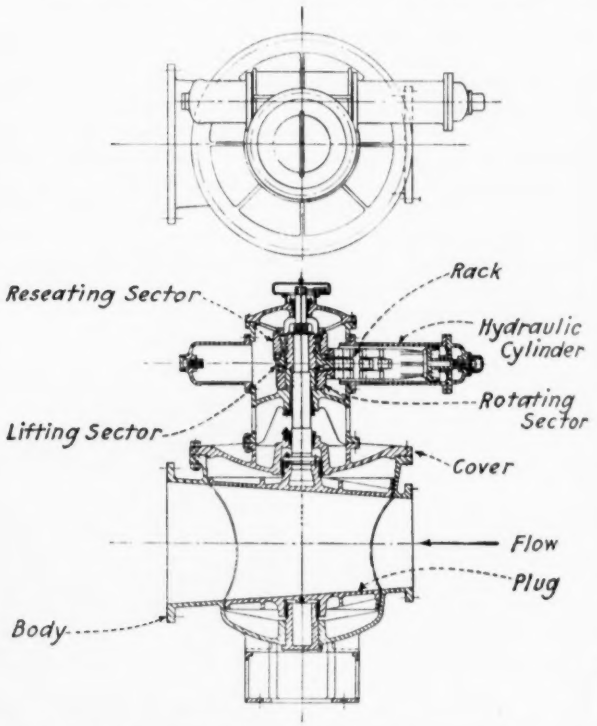


FIG. 7 CROSS SECTION OF 42-IN. BY 58 3/4-IN. TAPERED ROTOVALVE; S. MORGAN SMITH COMPANY

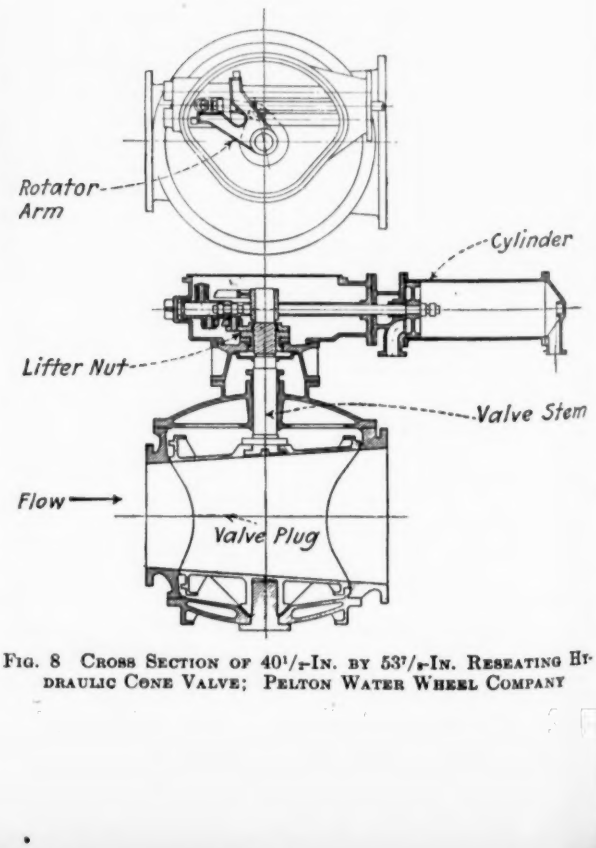


FIG. 8 CROSS SECTION OF 40 1/2-IN. BY 53 7/8-IN. RESEATING HYDRAULIC CONE VALVE; PELTON WATER WHEEL COMPANY

assembled view of one of the Pelton valves. Figs. 7 and 8 show cross sections of the two valves and their operating mechanism.

The valves are all hydraulic-cylinder-operated with oil at 300 lb per sq in. working pressure. The principal difference between the S. Morgan Smith valves and the Pelton valves is in the mechanical arrangement by which the straight-line motion of the hydraulic piston is made to raise, rotate, and then lower the plug.

On the Pelton valves, the operating cylinder is separate from the housing containing the mechanism for rotating the plug. A guided crosshead on the piston rod operates a rotator arm and a lifting nut which are attached to the valve stem. For the first few inches of crosshead travel, the rotator arm is prevented from rotating. Through a linkage the crosshead turns the nut on the threaded portion of the stem in a direction to lift the valve stem and plug. After the crosshead has traveled far enough to lift the plug completely free, the rotator arm is released and a pin on the crosshead enters a slot in the rotator arm. Further travel of the piston rotates the entire assembly of rotator arm, lifting nut, valve stem, and plug. After the required 90-deg rotation, the rotator arm strikes a stop which prevents further angular motion;

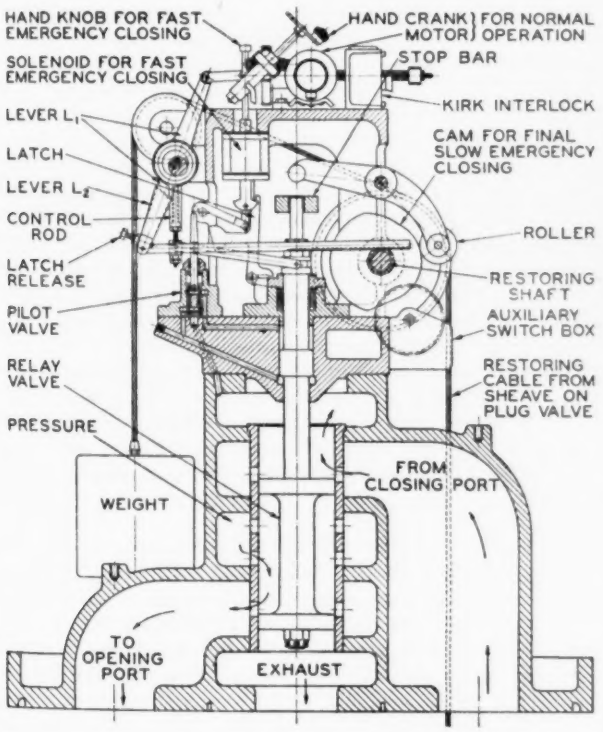


Fig. 9 Cross Section of Control Mechanism Used on All Valves; Woodward Governor Company

the pin on the crosshead comes out of the slot in the rotator arm; and further travel of the piston turns the nut in a direction to lower the valve stem and reseat the plug. The crosshead, rotator arm, and lifting nut operate in a bath of lubricant.

On the S. Morgan Smith valve, the lifting, rotating, and reseating motions are accomplished through an interacting combination of racks and gear segments. The piston acts directly on a rack which has three separate rows of teeth, each row matching with a separate gear sector. The first few inches of piston travel cause rotation of the lifting gear sector. The lifting sector is keyed to a screw which rotates freely on the valve stem but is threaded into a nut which is keyed to the reseating gear sector. During this portion of the stroke, the rotating sector and the reseating sector are both prevented from turning by special teeth which slide on flat surfaces on the rack. When the stem has lifted the required amount, all three gear sectors are engaged by the rack teeth and rotated 90 deg. The rotating sector and the lifting sector then come out of mesh with the rack and are prevented from rotating further by stops in the housing. The reseating sector is still in mesh, however, and the relative motion between the nut and the screw forces the nut downward and

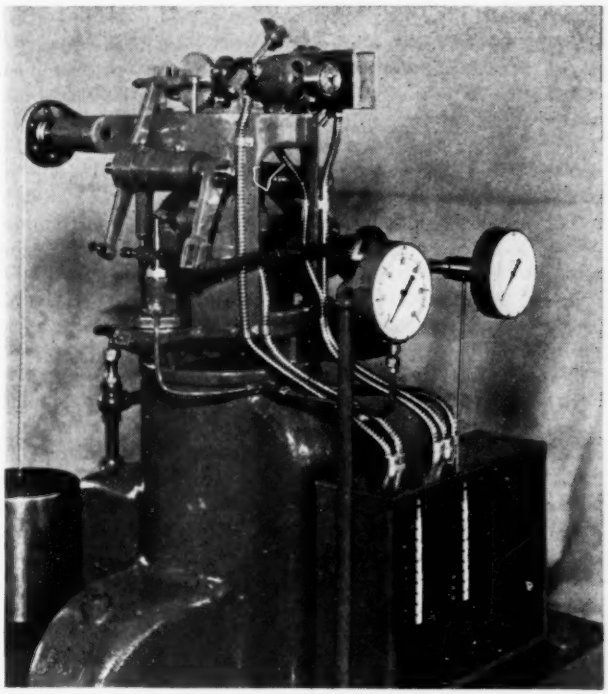


Fig. 10 Assembly of Control Valve, Cover Removed; Woodward Governor Company

reseats the plug. Adjusting screws limit the rack travel for both seating and reseating and can be adjusted to give the proper seat contact.

The principal parts of all the valves are steel castings fully annealed. Shop hydrostatic tests were made on all valves at a pressure approximately two and one-half times the maximum static head. The operating cylinders were tested at 450 lb per sq in. oil pressure. The specifications required the operating mechanism to have sufficient capacity to unseat the valve against an unbalanced pressure of 120 per cent of the maximum static head, assuming a friction coefficient of 0.50 for the seats, and a pressure of 250 lb per sq in. in the operating cylinder.

Control Mechanism

The source of oil pressure for valve operation is an air-pressure-accumulator tank, with return-oil sump tank and pump, very similar to the oil-pressure sets used for actuator-type governors for hydraulic turbines. The proper oil level and pressure are maintained in these tanks automatically. There is an independent pressure set for each valve. The control mechanism, furnished by the Woodward Governor Company for all valves, operates the valves slowly for normal starting and shutting down a pump. On failure of power to a pump motor, this mechanism closes the valve very rapidly to about 5 per cent of the effective opening and then very slowly to full closure. The time of the fast part of the stroke can be varied from about $1^1/_2$ sec to 5 or 6

sec. The point on the stroke where the motion changes from fast to slow can be varied. The slow-closing part of the stroke can be adjusted to any time up to about 60 sec. For normal opening and closing, the time of the full stroke is 40 to 60 sec and the rate is nearly uniform.

Before starting a pump, the inlet valve is fully opened and, since all the pumps are set well below the inlet-water level, the pump casing is completely filled. The discharge valve is closed and normally the discharge pipe above the valve is completely filled. The motor is started, brought up to speed, and synchronized, the pump operating at practically shutoff head. An 8-in. by-pass with a check valve on each of the main plug valves permits sufficient water to be pumped to prevent overheating the water in the pump casing. The by-pass is also used to fill the delivery line if it is empty when the pump is started. The main plug valve is then opened slowly. When taking a pump out of service, the discharge valve is first slowly closed, the motor circuit breaker opened, and the unit decelerates to rest.

Fig. 9 shows a cross-sectional view of the control mechanism. Fig. 10 shows the control valve with the cabinet removed. The relay valve for applying oil pressure to operate the hydraulic cylinder, the pilot valve, the floating lever (in this case a double eccentric), and the control rod are similar in construction and operation to the corresponding parts in a water-wheel governor. The control rod corresponds to the speed rod of the flyballs.

Referring to Fig. 9, the control rod is actuated by a double eccentric. The control rod is journaled on the hub of the outer eccentric and the outer eccentric is journaled on the inner eccentric. The outer eccentric is integral with the lever L_1 and is turned thereby. The lever is operated by the back-geared motor, through gearing and a screw. The inner eccentric is integral with the central shaft, which is turned by the restoring lever, L_2. To the end of lever L_2 is attached a rack bar which meshes with a gear on the restoring shaft. This shaft is connected to the vertical shaft of the main pump discharge valve by a cable and sheaves, the cable and sheaves all being enclosed. Thus lever L_2 moves with and its position corresponds to the angular position of the cone valve plug. The cable is kept taut by a heavy weight. With the levers L_1 and L_2 in line with each other, the control-rod length is adjusted to hold the relay valve in its central position with the relay ports closed. If lever L_1 is moved out of line with L_2, the control rod will be moved up or down, causing the relay valve to be moved down or up and admit oil to the main-valve operating cylinder. The main valve plug then rotates and, through the cable connection, the restoring shaft, and the rack, brings lever L_2 in line again with L_1, moving the pilot valve in a direction to cause the relay valve plunger to close the relay ports and stop the movement of the plug. If L_1 continues to move, then the main valve plug and lever L_2 will continue to move at the same rate.

In Figs. 9 and 10, the levers are shown in the position they assume after the main valve has been fully opened. To close the valve in a normal slow-closing operation, the control motor is started in the direction to cause the upper end of lever L_1 to move to the left, which will cause the main valve plug to unseat and rotate in the closing direction, and lever L_2 to move toward a position in line with L_1. When lever L_1 has been moved to its extreme left position, the main valve plug will have rotated to its fully closed position. A little overtravel of the lever L_1, causes the piston of the hydraulic cylinder to continue on and seat the plug. The control-valve motor is provided with a friction governor to secure a constant speed irrespective of the voltage. The governor is adjusted to give full plug rotation in about 40 sec.

To open the valve the motor is operated in the opposite direction.

In case of motor or current failure, the valve may be operated by the small hand crank shown close to the motor. This is geared for a number of turns calculated to make it convenient to operate at about the same speed as motor operation gives.

Limit switches operated by the motor screw open the motor circuit when the screw has traveled its full movement in either direction.

For emergency closing, the main-valve plug is closed rapidly by energizing the 125-volt, d-c solenoid or by raising the solenoid core by means of the hand knob. As the core is raised, it acts, through the link and lever shown in Fig. 9, to depress the pilot valve. A spring in the control rod permits it to do so. The pilot valve then causes the relay valve to rise, closing the plug rapidly, at a rate determined by the setting of the stop bar, which limits the amount the relay valve can open.

When the solenoid core is raised, the latch shown falls into place and holds it up. Then the plug cannot be moved to open until the latch is released. This will occur when the lever L_1 has been moved to the closed plug position and the lower extension of lever L_1 strikes the latch.

When the plug has been closed rapidly to a predetermined position, the cam, shown on the restoring shaft, engages a roller on the right-hand end of a rocker arm shown just above the cam. The other end of the rocker arm is thereupon depressed and forces the relay valve down to a port opening which will cause the final closing of the plug to be made at the desired slow rate.

The principal adjustments on the control mechanism are as follows: (1) The rate of normal opening or closing can be varied by the governor on the d-c motor; (2) the initial rapid-closing rate can be varied by raising or lowering the stop bar, which limits the travel of the relay valve; (3) the position of the main-valve plug, at which the emergency-closing rate changes from fast to slow, can be varied by rotating the cam with reference to the restoring shaft; (4) the final slow-closing rate can be varied by replacing the roller which is lifted by the cam and using another roller of slightly larger or smaller diameter. Once made these adjustments are locked and are not readily changed.

The entire control mechanism is enclosed in a dustproof cabinet. The top cover of the cabinet is hinged and provided with a lock. When this cover is raised it gives access to the hand control for normal and emergency operation. For access to the adjustments, there are two side openings with dustproof covers, bolted in place from the inside.

Operation

Control of the main-pump motors and of the pump discharge valves is centralized in a control room adjacent to the pumping plant. Normally, the sequence of operations in starting and stopping a pumping unit is automatic. In starting up, the operator throws a switch on the control board which applies full voltage to the main motor. As the speed approaches synchronism, a relay operates to apply field current and the motor pulls into step. Closing the field breaker initiates the opening of the pump discharge valve. For normally shutting down a unit, the operator turns the control switch in the closing direction, which starts the pump discharge valve closing slowly. When the valve plug has rotated past the cutoff position so that the pump discharge is entirely through the 8-in. by-pass, the motor is tripped off by a switch actuated from a cam on the control-mechanism restoring shaft.

On emergency closure, the motor is tripped off and rapid closure of the valve is started at the same time. There is an emergency shutdown switch on the control board in the operator's room, so that the operator can trip the unit off in this manner whenever necessary. Emergency trip of the valve and motor is also initiated by any one of the following abnormal conditions: (1) Failure of lubricating-oil supply to pump or motor bearings.

(2) failure of water supply to the motor coolers; (3) operation of motor overload or differential relays; (4) operation of transformer differential relays; (5) accidental opening of motor or high-voltage circuit breakers; (6) phase or line-to-ground faults on transmission system.

Laboratory Tests

Extensive tests on small gate valves and plug valves and on scale models of the pump discharge valves were made by both of the firms that manufactured the valves and by the Metropolitan Water District. These tests were made in order to estimate closely the power required to operate the valves under the assumed maximum-flow conditions, and to determine the loss in head through the valves at various openings. Tests by the manufacturers were made in their own laboratories. The District's tests were made in the hydraulic-testing laboratory at the California Institute of Technology.[2]

It is not within the scope of this paper to describe in detail the work done in the various laboratories. Preliminary testing by the District was conducted on a 6-in., quarter-turn, S. Morgan Smith plug cock with a circular water passage of uniform diameter and on a 6-in., ring-follower, Yuba gate valve. Tests were confined to determining the loss of head at various openings and rates of flow. Tests by the valve manufacturers were made on scale models of the actual valves as they were to be constructed. The model valves were about one seventh the linear dimensions of the full-sized prototypes. The manufacturers' tests were primarily for determining the torque required to rotate the valve plugs at various openings and rates of flow. However, since the laboratory facilities also permitted head-loss measurements, these were taken for the same openings and rates of flow as the torque. The independent tests made by the S. Morgan Smith Company and the Pelton Water Wheel Company were in reasonably close agreement both as to torque and head loss.

The torque tests showed conclusively that, with the operating mechanism designed to have power enough to unseat the plug under the maximum unbalanced head required by the specifications, there was more than ample torque to rotate the plug and control its rate of rotation at any angular position and under the severest flow conditions reasonably expected, even considering the possibility of free reverse discharge through a ruptured pump casing.

The model valve, made by the Pelton Water Wheel Company and used by that company in the determination of torque and head loss, was afterward made available to the District. Further head-loss tests were carried out on this model in the District's laboratory at the California Institute of Technology. Prior to these tests, seat rings were added to the model in order to make it conform as nearly as possible in dimensional relations with its prototype. However, in the unseated position after lifting the plug and in that portion of plug rotation between cutoff and the full quarter turn, it was not practicable to make the clearances such that the clearance dimensions were in the same ratio to the prototype as the main water-passage dimensions. Consequently, the water that flows past and around the model valve plug, when it is lifted and rotated to the position where the water passage through the plug no longer opens directly into the water passage through the valve body, is not a correct measure of the flow through the corresponding clearances in the full-size valve. Since this so-called leakage could not be segregated and accurately measured and, even if it could be measured, would not represent the correct leakage in the prototype, it was decided to neglect it in water-hammer calculations and to assume that the flow through the valve is entirely shut off when the plug is rotated to the cutoff position. This assumption is on the side of safety, since abruptly cutting off the final flow results in a higher calculated pressure rise.

For both the Pelton and S. Morgan Smith valves, the cutoff position is about 70 deg from the full-open position. During the remainder of the 90 deg of total rotation, the plug is lifted off its seat and there is a considerable flow around the plug and through the clearance between the plug and body seats until the reseating operation has brought the seats into full contact. The effect of neglecting this flow is to make the calculated pressure rise considerably greater than the actual observed pressure rise.

In the first loss-of-head tests made by the District, as well as in similar tests made by the manufacturers, the model valve was placed in a straight run of pipe and the net head loss due to the valve only was measured. Pressures, measured directly downstream of the valve at partial openings, showed a considerable drop, due to the high velocity. Therefore, the net head loss was measured from a point just upstream of the valve to a point sufficiently far downstream (about six pipe diameters) so that recovery of velocity head was no longer observed. In the prototype installation, however, the valve is placed directly at the pump discharge. In reverse flow, the jet from the partially opened valve enters the pump casing before the jet velocity has been fully converted into pressure head. A setup was made in the laboratory to simulate the actual installation. The model valve was placed directly at the discharge of a model pump, the complete head and torque characteristics of which had been determined previously for a wide range of normal and reverse rotation, and direct and reverse flow. From this series of experiments, it was found that the head loss in the valve, as affecting reverse flow through the pump at both normal and reverse rotation, was somewhat less than the head loss when the valve was tested in a straight run of pipe.

The experiments showed that the head loss through the valve can be expressed with sufficient accuracy by the equation

$$H_v = KV^2/2g \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where H_v is the head loss in the valve, in feet; V is the velocity through the valve plug at the center of the plug in feet per second; and K is a coefficient which is a function of the valve opening and is determined experimentally.

Fig. 11 shows the values of K, determined by these experiments for the Intake and Gene valves and plotted as a function of the angle of the plug in degrees from the full-open position. It should be noted that K approaches infinity as the plug is turned to the cutoff position.

Control of Water Hammer

In order to meet the construction program, it was necessary to design and construct the pump-delivery pipe lines before all of the valve investigations had been completed. Preliminary water-hammer studies were based upon the first experiments on 6-in., uniform-diameter plug and gate valves and upon pump characteristics taken from a pump, having a specific speed somewhat near that of the full-sized pumps, but not in any sense a scale model. These studies showed that the most severe combination of abnormal operations reasonably to be expected was for the motor to be tripped off, the valve not starting to close, until full reverse flow had developed and then closing for its full stroke at a very rapid rate without any retardation near the end of the stroke. It was assumed that this condition would occur on only one pump in any group of three, the valves on the other two pumps operating normally. The resulting maximum pressure rise was calculated to be about 50 per cent for the two highest-head plants, Eagle Mountain and Hayfield; about 75 per cent for

[2] "The Hydraulic Machinery Laboratory at the California Institute of Technology," by R. T. Knapp, Trans. A.S.M.E., vol. 58, Nov., 1936, p. 663.

the Intake and Gene plants; and about 125 per cent for the lowest-head plant, Iron Mountain. The pipe lines were designed for the maximum total heads given by these overpressures, using a maximum stress of 20,000 lb per sq in. in the 10-ft-diameter pipe and 15,000 lb per sq in. in the 6-ft pipes leading from the pumps and in the manifold connecting the 6-ft pipes to the 10-ft pipe.

Water hammer in the pump-delivery lines is a transient condition occurring within a very short period of time. It results

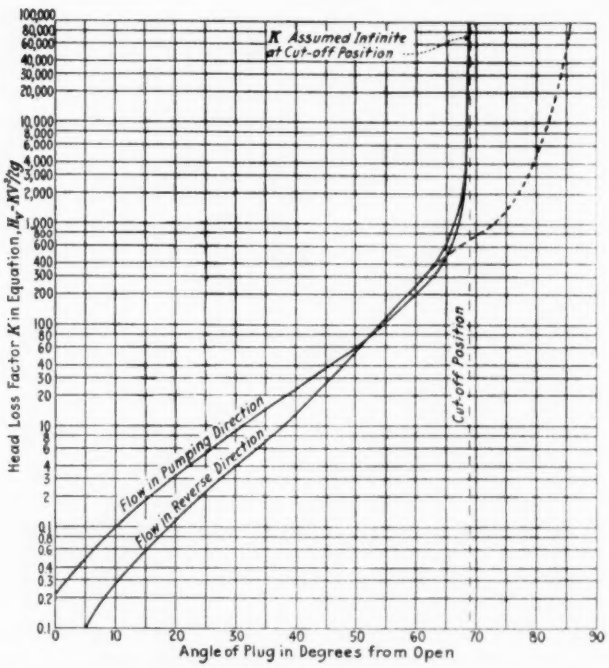


FIG. 11 HEAD-LOSS COEFFICIENT K, PLOTTED AS A FUNCTION OF VALVE OPENING

from rapid variations of the velocity. The slower the changes in velocity, the less the pressure variation. The characteristics of the pipe line, the head loss through the valve as it closes, the variation of pump discharge and torque with changing head and speed, and the combined flywheel effect of the pump and motor, all affect the velocity in the pipe line and hence the magnitude of the pressure variation. The water-hammer pressure produced by an emergency shutdown of a pump is thus the result of interaction between the pump and motor, the pipe line, and the shut-off valve. Analysis to predict water-hammer pressure must take into account the characteristics of all these elements of the pumping system.[3] The motor, pump, and pipe-line characteristics, once selected, are fixed, so that the timing of the valve stroke is the only available means of regulating the velocity changes in the pipe.

The valve timing for emergency closure is adjusted so that closure is very rapid for about 59 deg of plug rotation, reducing the effective valve area to about 5 per cent in about 3 sec. During this period, the pump is rapidly decelerating, the velocity in the pipe line drops, and the pipe-line pressure drops. As shown by the K diagram in Fig. 11, the throttling effect for the first part of the stroke is small. The pressure drop during this period is dependent almost entirely on the pump characteristics, the motor flywheel effect, the length of the pipe line, and the initial velocity. The valve has comparatively little influence since, by the time the valve closes to an appreciable throttling effect, the velocity is low whether forward or reverse. Somewhat before the time the valve reaches the end of the rapid part of its stroke, the pressure beyond the valve has begun to build up due to reflection from the upper end of the pipe line. The speed of the pump in the meantime has dropped to a point where the head developed by the speed of rotation is less than the opposing pressure. The flow reverses, although the pump is still rotating in the pumping direction. The reverse flow through the pump acts as a relief for the overpressure which tends to develop in the pipe line. The very slow final closure of the valve prevents any large pressure rise in shutting off the reverse flow. By adjusting the final slow-closing time, it is found possible to make complete closure before reverse rotation of the pump, and yet without producing overpressure of any considerable magnitude. Fig. 12 shows the valve timing used at the Intake and Gene plants.

Analyses were made for a number of different emergency conditions, such as delayed tripping of the valve until full reverse flow is developed, abnormal initial speed drop of the motor due to electrical trouble, emergency valve closure without tripping off the motor, and other possible abnormal operations. In all cases the calculated maximum pressure rise was well below the maximum

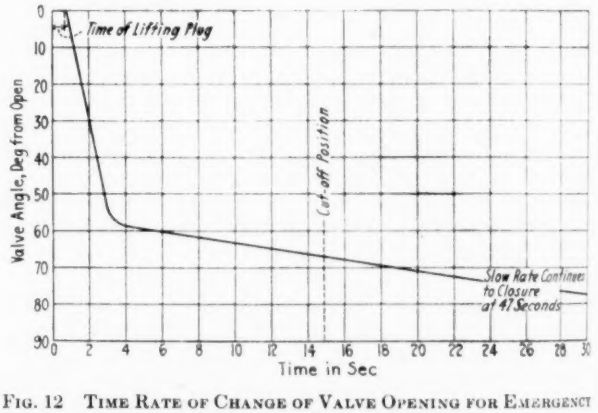


FIG. 12 TIME RATE OF CHANGE OF VALVE OPENING FOR EMERGENCY CLOSURE AT GENE PLANT
(Timing shown is very close to the actual valve timing recorded simultaneously with the test curves in Figs. 13 and 14.)

which the pipe lines and other apparatus are designed to withstand, although it was found that in some cases the pump would reverse rotation and attain considerable speed in the reverse direction.

COMPARISON OF CALCULATIONS WITH FIELD TESTS

At the date of writing (February, 1939), the first two pumping plants, Intake and Gene, had been placed in operation, and a number of observations had been made on the water hammer produced by several different types of emergency shutdowns. Comparison of the observed pressure rise with previous calculations shows that in every case the observed pressure rise is less than that calculated. Figs. 13 and 14 show the comparison for one and three pumps tripped off at the Gene plant, where there is the longest delivery line. One reason for the difference between the observed and calculated results is undoubtedly the assumption made in the calculation that the flow is abruptly cut off when the plug is rotated to cutoff position. Reference to Fig. 12 shows the K curve approaching infinity at cutoff. Probably the K curve in that region is more nearly that shown by the dotted line, indicating that the flow just before cutoff is more than was assumed and that flow continues and is gradually shut off between cutoff position and full rotation.

[3] "Typical Analysis of Water Hammer in a Pumping Plant of the Colorado River Aqueduct," by R. M. Peabody, Trans. A.S.M.E., vol. 61, Feb., 1939, p. 117.

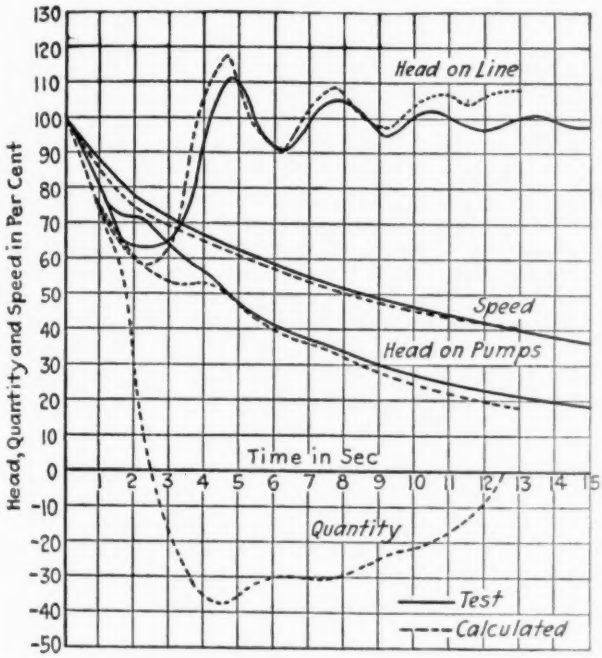


FIG. 13 COMPARISON OF MEASURED PRESSURE AND SPEED VARIATION WITH CALCULATIONS AT THE GENE PLANT; EMERGENCY SHUTDOWN OF ONE PUMP; NO OTHER PUMPS OPERATING

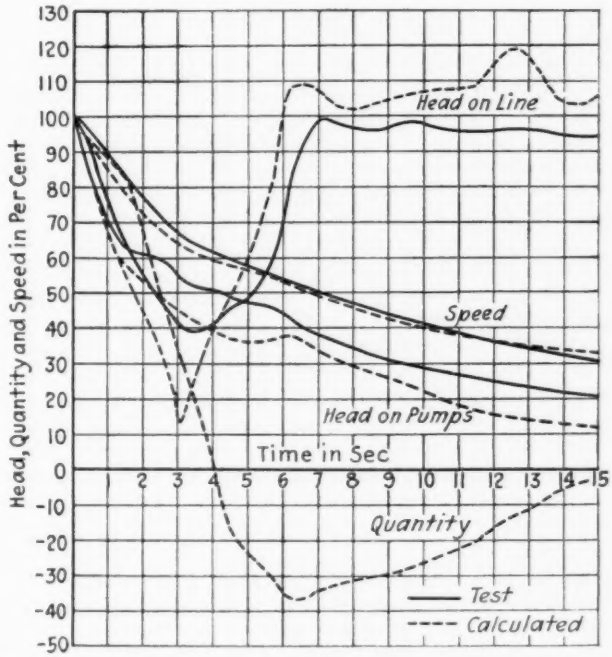


FIG. 14 COMPARISON OF MEASURED PRESSURE AND SPEED VARIATION WITH CALCULATIONS AT THE GENE PLANT; EMERGENCY SHUTDOWN OF THREE PUMPS SIMULTANEOUSLY

It is known from observation that considerable reverse flow continues after cutoff but it has not been possible to measure it. The closing time of the three valves was purposely made slightly different so that all the valves do not fully close at the same instant. This tends to reduce further the pressure rise when two or three pumps are tripped off simultaneously. A careful study of the speed drop at the very start of the transient, which the scale of Figs. 13 and 14 is too small to bring out, shows that, in the first small interval after the start of the transient, the drop in speed is less than given by calculation. This would tend to reduce the initial pressure drop and consequently the subsequent pressure rise. After the first half second or so, the observed and calculated speed drops check fairly well. This indicates a larger flywheel effect at the very start. A suggested explanation is that the rotational inertia of the water in the casing surrounding the impeller tends to increase the total inertia of the pump and motor, and that this effect diminishes rapidly with reduced discharge.

In Fig. 13 the single pump shutdown was made from the emergency switch in the control room. The three-pump shutdown was made by opening the 230-kv circuit breaker close to the plant. The most severe pressure rise observed was when a single pumping unit, no other units operating, was tripped by opening the circuit breaker at the Boulder power plant, with 237 miles of energized 230-kv transmission line between that breaker and the pumping unit. In this case the motor had to act momentarily as a heavily loaded generator supplying charging current to the line. The effect was a rapid drop in the speed of the motor in the short interval before the reverse power relays could act to open the motor circuit breaker and clear the motor from the line. With several motors operating in parallel, each would contribute only a part of the charging current and the speed drop would be less rapid.

Line-to-ground and phase-to-phase short circuits were applied to the transmission system at different distances from the plant, but the resulting pressure rise was less than in the case above cited.

When one pump was tripped off, while either one or both of the other pumps in the plant continued to run, the pressure rise was found to be less than when the pump was tripped while the other two pumps were shut down. This was to be expected since with several pumps running the return-pressure wave, resulting from shutting down a single pump, is relieved through the operating pumps.

Pressure variation in the pumps and pipe line was measured by a fast-motion, strip-chart pressure recorder. The records of valve movement were taken on a fast-motion recorder. Speed variations were observed on a high-speed recording voltmeter, which showed the variation in the voltage of a magneto-tachometer driven from the motor shaft.

CONCLUSION

Rotating-plug-type valves are well adapted for shutoff valves on large pumps. With proper control of the rate of closure they can be made to limit the pressure rise to a moderate percentage in systems comparable to those of the Metropolitan Water District. A marked advantage of this type of valve, which is lifted off its seat while rotating, is that the entire flow is not abruptly shut off when the main water passage is rotated past the cutoff point. The final flow is thus extinguished gradually which tends to cushion the maximum pressure rise.

The magnitude of the pressure rise to be expected can be predicted by well-established methods of calculation, provided that the characteristics of the elements making up the pumping system are known.

All work on the Colorado River Aqueduct is under the direction of F. E. Weymouth, general manager and chief engineer of The Metropolitan Water District of Southern California. The design and construction of the pumping plants are under the direction of J. M. Gaylord, chief electrical engineer.

Discussion

ROBERT W. ANGUS.[4] The writer wishes to commend the author for making available some of the data and results used in connection with the water-hammer study on this very extensive pumping problem. The magnitude of the project is such as to justify experimental investigations of the characteristics of the pumps and also of the valves used, and information is thus available in a somewhat unknown field.

The paper is really part of the one by the same author,[3] where the method of working out the water-hammer pressures is given in some detail. For a proper study of this nature, a complete set of characteristic curves of the pump is necessary, covering the action of the machine (1) as a pump, (2) running forward but with the water running downward through it, (3) running under zero torque with the water passing downward, and (4) operating as a turbine with both water and impeller running in reversed directions. The excellent work[2] of R. T. Knapp, to which the author refers, has given data on a pump in such a complete form that its action on water hammer may be studied. It is hoped that similar information will be made available on a number of other pumps.

Presumably, the valve referred to in the author's earlier paper[3] is similar to that in the present one; in any event, the characteristics of the two are of the same form, and the data in the papers supplement one another. In Fig. 11 of the present paper, the losses in the valve for different plug positions are shown, the loss naturally varying with the direction of flow. The head-loss coefficient K will vary with the reference velocity, which has here been taken as that at the center of the plug, whereas it would seem to be more consistent to use one of the end velocities. For partial openings the plug will not be filled with water and the meaning of the velocity there is obscure; the author apparently means by it the quotient of the discharge divided by the area of circular opening at the center of the plug.

The writer has made a number of experiments[5] on a 3-in. cone valve with a cylindrical opening and, while it is not possible to compare the coefficients on this small valve with those on such a large tapering valve, yet there is in general a very good agreement, particularly for the large openings. In the small valve the cone was not lifted off its seat in any position, and hence the smaller openings would be less in agreement.

Fig. 12 shows that in the Gene plant, for the pipe line of which the value of $2L/a$ is given as 1.533 sec in Table 1, the plug has been turned through about 58 deg in about 3 sec or, if allowance is made for the time of lifting the plug, the latter has been turned through about 58/69 or 85.5 per cent of its total closing movement in about 2.3 sec or 1.5 times $2L/a$ sec, which is very quick. The remaining 14.5 per cent of the movement has been made in about 12 sec and Fig. 14 shows that this causes no pressure trouble when the water is running at full velocity in the delivery line. This further illustrates the point mentioned by the writer in the paper[2] already referred to. Many statements have been made that valves in long pipe lines must be closed very slowly to avoid water hammer, whereas the author shows that a closure in 15 sec causes no appreciable pressure rise.

Provided that it is not less than perhaps 10 times the period of the pipe, the total time of closure of a valve is of far less importance than the way in which the closure is effected. For many types of valves it is scarcely possible to effect the first 70 per cent or so of the closure fast enough to cause any trouble, but the final part of the operation must usually be done with great care and with proper consideration of the characteristics of the valve being used. Great differences exist among valves, in regard to their method of control, and some would cause trouble under similar operating conditions to those herein mentioned. However, the valve described in the paper has good features, and the author has done a real service to the profession in giving information about it in such detail.

The writer feels that the control valve shown in Figs. 9 and 10 would be very expensive and quite impossible for any but the very large plants. There may, however, be ways of effecting a similar movement of the cone in a simpler way and, if so, the writer would like information about them. The writer also thinks that some details of the recording devices used in producing the curves of Figs. 12, 13, and 14 would be of much help to those working on water-hammer problems, and hopes the author will add this information to his paper.

A. HOLLANDER.[6] This paper is a worthy complement to the author's former paper,[3] disclosing the method of analysis and the means of reducing the pressure rise in a large pumping system. It shows that a relatively inexpensive rotating-cone valve, which continues the diffusion of the pump-discharge taper without any additional loss under normal operating conditions, is the only direct regulating means required for such a system. The indirect means are a constant-pressure oil supply and the Woodward actuator-type governor. This is the first application of this turbine governor in a pumping plant, where it serves as a pilot valve for the oil-pressure-operated cone valve. The very complete model experiments included the combined pump-and-valve behavior under transient conditions. In reference to these, the writer would like to ask whether both directions of rotation from open to closed were explored, as they would undoubtedly give different results for reverse flow, due to the different entrances into the volute.

It is interesting to note that the valve coefficient K (should be designated as φ)[7] is increasing from open (0 deg) to closed (68 deg) with an inflection point at around 55 deg. If this coefficient is referred to the gate area actually open, $H_v = \frac{A \text{ gate}}{A} c \frac{v^2}{2g}$ (it should be corrected to read this way in the first paper),[3] then the coefficient c shows a maximum at about 38 deg from open.[8] The two coefficients with the areas at different openings and time rate of opening give a complete picture of the valve effect.

The writer would suggest that in the time-history diagrams[9] the term "quantity" be changed to "flow" or "flow rate" as this is what it means. Furthermore, it should be understood, that this flow, as a function of time, refers only to the flow at the point where the pressure is measured, i.e., to the sections next to the valve, "head on line" at the pipe-line side, and "head on pump" at the pump side of it. For these sections, the flow being proportional with the velocity, a velocity-time diagram in percentage becomes identical with the flow-time curves and therefore could be designated as a velocity-wave curve at these points, in the same manner as the head curves are actually pressure-wave curves at the same sections. For better understanding of the phenomenon, this would be helpful, as the differential of the velocity-time curve gives the acceleration of flow.

The very excellent graphical solution[10] of Professor Bergeron

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[4] Professor of Mechanical Engineering, University of Toronto, Toronto, Canada. Fellow A.S.M.E.

[5] "The Action of Valves in Pipes," by R. W. Angus, *Journal*, American Water Works Association, vol. 30, no. 11, Nov., 1938, p. 1858.

[6] Chief Engineer, Byron Jackson Co., Los Angeles, Calif. Mem. A.S.M.E.

[7] "Symposium on Water Hammer," Recommended Standard Symbols, A.S.M.E., 1933, p. 8.

[8] Ref. 3, Fig. 4.

[9] Ref. 3, Figs. 3, 6, 7, and 8; present paper, Figs. 13 and 14.

[10] "Pompes centrifuges et usines elevatoirs," by M. L. Bergeron, *La Technique Moderne*, March 1, 1935.

reproducing the numerical data in a diagram would help further to illustrate the phenomenon step by step.

The author calls attention to the very short but powerful electrical-braking effect of the motor, during the time interval between the opening of the energized circuit at a far-away point and the opening of the control that takes the motor off the line. The importance of this effect in this case is clearly shown and calls for inclusion of this factor in analyzing similar problems.

The publications describing the great project of the aqueduct of the Metropolitan Water District of Southern California, including the author's two papers, show that they blazed new trails in many fields and by giving all details made notable contributions to the art of designing and building of great waterworks.

RAY S. QUICK.[11] The author and his associates are to be congratulated upon the excellent test results secured in the control of pressure surge and reverse flow in the discharge lines of the pumping plants on the Colorado River Aqueduct of the Metropolitan Water District of Southern California. The prediction of the surge with respect to valve timing shows a satisfactory agreement with the test results and confirms the engineering principles developed and used in the calculations.

The pumps for the Intake and Gene Plants were manufactured in the San Francisco works of The Pelton Water Wheel Company in collaboration with the Byron Jackson Company. The discharge valves for the Eagle Mountain and Hayfield Pumping Plants were manufactured jointly by The Pelton Water Wheel Company in San Francisco and by the Chapman Valve Manufacturing Company, the cover and operating mechanism having been designed and furnished by Chapman. When it is realized that the completed valve, illustrated in Figs. 6 and 8 of the paper, has an approximate weight of 50,000 lb, the problems of manufacture and handling will be appreciated.

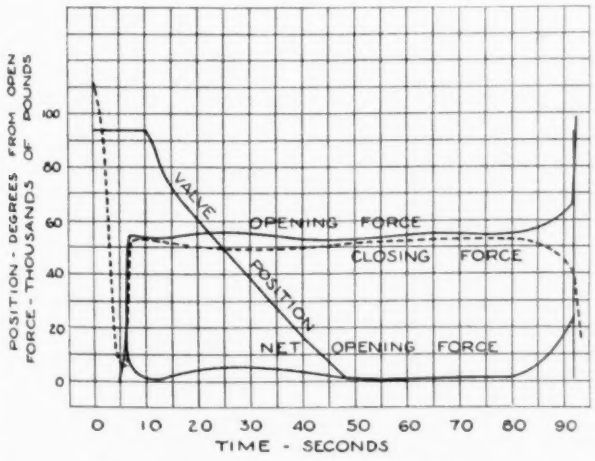


FIG. 15 NORMAL OPENING CYCLE OF VALVE
(Hydraulic cone valve, 40½ in. by 53⅞ in.; Eagle Mountain pumping plant.)

It may be of interest to consider further the performance of the discharge valves as tested at the Eagle Mountain Pumping Plant where the degree of control of pressure surge and reverse flow was equally satisfactory to that at Gene as reported by the author. Three sets of curves, Figs. 15, 16, and 17, are submitted to show the history of the valve action with respect to time for the conditions of normal opening, normal shutdown, and emergency shutdown, respectively. Opening, as illustrated in Fig. 15,

[11] Chief Engineer, The Pelton Water Wheel Company, San Francisco, Calif. Mem. A.S.M.E.

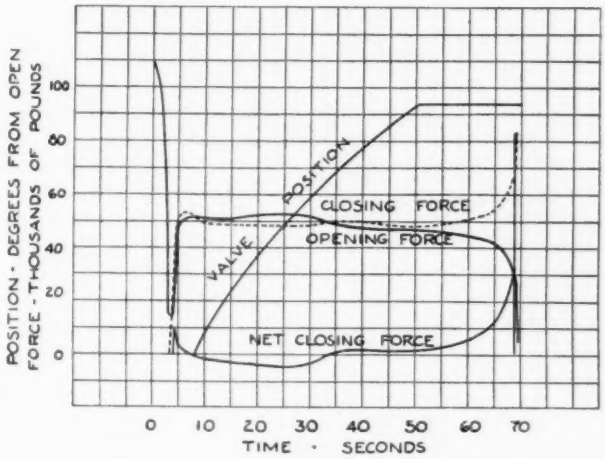


FIG. 16 NORMAL CLOSING CYCLE OF VALVE
(Same as Fig. 15.)

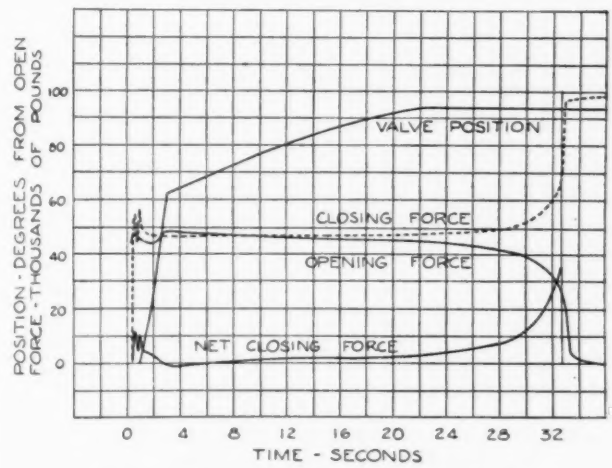


FIG. 17 EMERGENCY CLOSING CYCLE OF VALVE
(Same as Fig. 15.)

takes place only under normal conditions, with the pump running and with pressures substantially equalized on both sides of the valve. Since the rate of opening is controlled by the actuator unit developed and furnished by the Woodward Governor Company, with timed motor control, the forces on the operating mechanism are moderate and there is little or no pressure disturbances. Likewise, normal closing, as shown in Fig. 16, involves forces of moderate magnitude and little or no pressure disturbance, as here again the control motor is used to produce a steady rate of movement. These two cycles of normal opening and normal closing are used under operating conditions as there is no occasion to bring about emergency closure, except in case of power failure or other abnormal operating condition, requiring rapid shutdown.

The performance, shown in Fig. 17, illustrates the condition of emergency closure. It is interesting to note that, even under these conditions, the forces upon the operating piston are moderate and the reserve available at full pressure leaves a factor of safety of a very satisfactory amount. The greatest force is developed when wedging the plug into the seat in the closed position, as here it is desired to have a watertight closure. In the open position, the plug is very lightly wedged, in order that a wide margin of operat-

ing force may be available for emergency closure. When it is considered that the available force in the operating cylinder is about 115,000 lb, the factor of safety even for wedging is on the order of 3.

Laboratory tests conducted on the model cone valve showed the presence of a definite closing torque with hydraulic flow through the partially closed valve. This closing torque, however, is of relatively small magnitude. A study of the figures confirms this evidence and demonstrates that the greatest load on the operating mechanism occurs in the act of wedging the plug into the seat.

During the tests covered by the foregoing figures, the normal pressure in the discharge line was about 196 lb per sq in. On power interruption, the dip in pressure reached a minimum average of 162 and a maximum average of 207 lb per sq in. Thus the rise in pressure was only 5.5 per cent above the pumping head. These tests are well within desired limits and demonstrate conclusively the effective control of surge.

Cavitation of Hydraulic-Turbine Runners

By R. E. B. SHARP,[1] PHILADELPHIA, PA.

This paper contains a brief analytical discussion of runner cavitation. The I. P. Morris cavitation equipment is described, and results with this apparatus are compared with those of the Holtwood laboratory. A calibration gage in the development stage is also described. Stroboscopic apparatus for photographing runners during operation under cavitating conditions is illustrated, and various photographs above and below the sigma break are reproduced and discussed. A comparison of the index test of a prototype with a model at the same value of sigma is submitted.

THE sigma break of any hydraulic-turbine runner is considered as the value of sigma at which the performance characteristics begin to undergo a change due to cavitation. This value increases with the specific speed of the runner, i.e., the greater the specific speed, the greater, in general, must be the allowance for pressure drop $H_b - H_s$ on the back of the blades in the usual Thoma formula (1).[2]

Sigma $= \dfrac{H_b - H_s}{H}$ with H_b as the height of the water barometer and H_s the vertical distance from the center line of the runner; or more strictly, from the local point of lowest pressure on the blade, to the tailwater elevation. The reason for this increasing value of $H_b - H_s$ with increasing specific speeds is due to several factors, namely, the greater pressure differential necessary on the two sides of the blades, the greater absolute velocity, and greater relative velocity at discharge.

In order to present an approximate comparative picture of the increase in differential blade pressure with increasing specific speed, Fig. 1 and Table 1 have been prepared. Runners of three different speeds have been arbitrarily selected, these values being 50, 80, and 135; the two former speeds falling within the usual range of Francis-type runners, and the latter of propeller runners. The proportions and velocities selected correspond with usual practice, and the method employed at arriving at the approximate pressure differential is based on the efficiencies being normal, without excessive losses. Inflow and outflow diagrams are indicated, these being taken in all cases for the sake of simplicity at the runner throat or periphery with the flow axial, i.e., without radial component. In every instance both the runner diameter and the head acting are considered as 1 ft.

The basic-energy relation as set forth by the Eulerian theorem is

$$u_1 Vu_1 - u_2 Vu_2 = gHe \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where u_1 and u_2 = velocity of rotation in feet per second at inflow and outflow, respectively; Vu_1 and Vu_2 = components of the absolute velocity V_1 and V_2 in the direction of rotation; g = acceleration due to gravity; H = head; and e = hydraulic efficiency. With $u_1 = u_2 = u$ and $H = 1$

$$Vu_1 - Vu_2 = \frac{ge}{u} \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

Where $HP = \dfrac{WQe}{550}$, multiplying Equation [2] by $\dfrac{WQu}{g \times 550}$, we have

$$\frac{WQe}{550} = \frac{WQu}{g \times 550}(Vu_1 - Vu_2) = \frac{2\pi FRN}{550} \text{ with } N = \frac{RPM}{60}$$

$$= \frac{u}{\pi D}, D = 1 \text{ ft}, R = \frac{D}{2}, W = 62.4, F = \frac{WQ}{g}(Vu_1 - Vu_2)$$

= tangential force acting on blades at radius R.

Consider as Q the discharge through an annulus of Δr radial depth. Then the area on which F is acting per blade is $L \times \Delta r$, Fig. 1 and Table 1. The force F, however, is the tangential component of F_1 which for this comparison is considered as acting on the angle θ with the tangential, this angle being such that the direction of the force F_1 bisects the angle between v_1 and v_2.

The various steps are indicated in Table 1. Item 21 represents the area plotted between L_a and h_d, the differential blade pressure in feet, projected in the direction of rotation in the annulus Δr to produce each of the specific speeds under consideration. In Fig. 1, these areas are shown at the right-hand portion of the figure. Admittedly the shapes are inaccurate, but the areas serve to indicate the intensity of differential pressure to produce the three specific speeds selected. The minimum pressure is plotted as sigma H, these values in each case being about those usually encountered. Sigma H is made up of the terms, $\dfrac{Vm^2}{2g} \times$ draft-tube efficiency plus the term $K_c H$ (2). The expression $\dfrac{Vm^2}{2g} \times ed$ represents the pressure drop due to head regain in the draft tube. The term $K_c H$ represents the local pressure drop on the blades.

The portion of the blade pressure lower than that corresponding with tailwater level or zero on the head scale is a much greater proportion of the total for the propeller-type runner than for the Francis type of lower specific speeds. A region of relatively low pressure has been shown for the backs of the blades at inflow. This is to some extent the result of actual observation of cavitation (for the propeller type) as discussed later, and is also in line with measured values for airfoils. This same tendency, as indicated by pitting, apparently exists to a minor extent on Francis-type runners. The greater preponderance of positive differential pressure in the Francis runners (above 0) with a relatively small amount below the 0 line is the reason for the difficulty of determining by test, with lower specific speeds, the definite location of the sigma break. In other words, loss of head due to cavitation on the backs of the blades has a relatively small effect on the force causing rotation. The intensity of the differential pressure could, of course, be reduced for the propeller runner by increasing the blade length, but to do so would so increase the friction loss that, with the high relative velocities existing, the value of ϕ at which maximum efficiency occurs would be reduced.

Pitot-tube traverses taken just below a propeller-type runner at a steep blade angle demonstrate that the axial velocities are materially lower at the wall of the tube than further inward and indicate the increasingly severe conditions as regards cavitation for some distance in from the periphery of propeller-type runners when operating at the steeper blade angles. This condition has been taken into account in the diagrams of the propeller-type

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[1] Chief Engineer, I. P. Morris Department, Baldwin Southwark Division, The Baldwin Locomotive Works. Mem. A.S.M.E.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Hydraulic Division, and presented at the Spring Meeting, Worcester, Mass., May 1–3, 1940, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

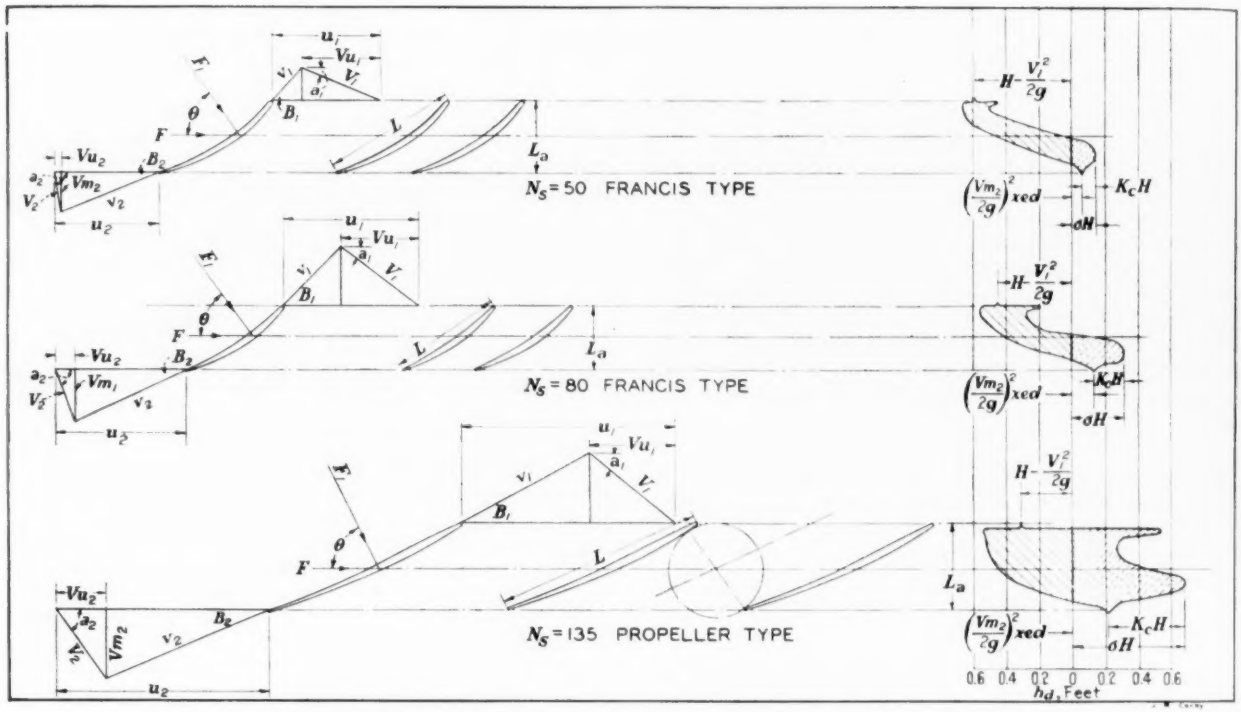


FIG. 1 SHOWING INCREASE IN DIFFERENTIAL BLADE PRESSURE WITH INCREASE IN SPECIFIC SPEED

runner in Fig. 1; that is, the value of Vm_2 in that figure is less than that corresponding with the average flow for this runner. Traverses taken at lower blade angles show that this condition of relatively low velocity at the wall of the tube dies out for lower flows. On this account, and in consideration of other test data, the flow for the two lower specific speeds in Fig. 1 has been considered as uniform across the runner discharge in arriving at the differential areas shown.

The increased cavitation tendency of runners of higher specific speed is also aggravated by the increased relative velocity. The pressure existing at any point on the blade where water tends to leave the surface is a function of the square of the relative velocity. Fig. 1 indicates by the comparative values of v_1 and v_2 for the different specific speeds how much greater this sensitivity is with the increased specific speeds, with resulting need for finishing of the blades true to contour.

That portion of each area, in Fig. 1, representing intensity of differential blade pressure which is below 0 on the head scale in the figure, is a function of Vm_2 and v_2. These values, as has been pointed out, increase with specific speed; and in addition, the ratio of the maximum to the average values of Vm_2 and v_2 undergoes an increase. As a consequence, certain portions of the blade may have a value of sigma H appreciably greater than the sigma H as determined by the change in performance characteristics. This is true even with runners having sharp and well-defined sigma-break curves.

It is not economically practicable to have the margin between the sigma break and the plant sigma sufficient to insure the absence of cavitation on the entire blade surface on this account. It is, therefore, becoming common practice to preweld portions of the runner blades with stainless steel where the operating head is above about 50 ft, in addition to providing a margin between the sigma break and the plant sigma (3).

Fig. 2 is a view of a turbine-runner blade before being welded. It will be noted that the portion prepared for welding extends along the entire periphery with a greater portion near the inflow

TABLE 1 DATA ON RUNNERS USED TO COMPARE INCREASE IN DIFFERENTIAL BLADE PRESSURE WITH INCREASE IN SPECIFIC SPEED

Item[a]	Francis type		Propeller type
(1) $N_s = RPM \sqrt{\frac{HP}{H^{5/4}}}$	50	80	135
(2) $\phi_1 = \phi_2$	0.8	1	1.6
(3) $u_1 = u_2 = \phi \sqrt{2gH}$, where H = 1 ft, fps	6.42	8.02	12.83
(4) a_2 = absolute angle discharging water makes with rotation, deg	80.5	69.5	54
(5) Vu_2 = tangential component at discharge, fps	0.4	1.2	3
(6) $Vu_1 = \frac{gHe}{u} + Vu_2$, fps	4.66	4.61	5.13
(7) $Vu_1 - Vu_2$, fps	4.26	3.41	2.13
(8) Vm_2 = axial component of discharge, fps	2.32	3.25	4.2
(9) Area of annulus, of Δr radial depth with radius r = 6 in., 7 per cent deducted for blade area at discharge, sq ft	0.244 Δr	0.244 Δr	0.244 Δr
(10) Quantity flowing through annulus = (9) × (8), cfs	0.566 Δr	0.793 Δr	1.025 Δr
(11) $F = \frac{WQ}{g}(Vu_1 - Vu_2) = \frac{W}{g}$(10) × (7), lb	4.68 Δr	5.24 Δr	4.25 Δr
(12) B_1 = relative angle at entrance, deg	50	46.5	29
(13) B_2 = relative angle at discharge, deg	21	25.5	23
(14) $\frac{(180 - B_1 + B_2)}{2} - B_2 = \theta$, deg	54.5	54	64
(15) $\frac{F}{\cos\theta} = F_1 = \frac{(11)}{\cos\theta}$, lb	8.05 Δr	8.92 Δr	9.7 Δr
(16) Number runner blades = N	17	16	5
(17) L = distance inflow to outflow edge, in.	4	3.445	6.29
(18) L_a = axial depth of blades, in.	2.125	1.84	2.53
(19) F_1 per blade = $\frac{(15)}{(16)}$, lb (F_1 also = $\frac{\Delta r(L \times h_d)}{2.304}$ where h_d = differential blade pressure in ft of water per sq in.)	0.474 Δr	0.557 Δr	1.94 Δr
(20) $L \times h_d = \frac{F_1 \times 2.304}{\Delta r}$ = area plotted between L and h_d, sq in.	1.09	1.28	4.47
(21) (20) × $\frac{L_a}{L}$ = area projected in direction of rotation, sq in.	0.58	0.685	1.8

[a] Numbers in parentheses in this table are key numbers not to be confused with Bibliography.

edge, and also near the outflow edge. In addition, a strip of rolled stainless steel is welded to the periphery of the blade, the need for which is discussed later. The portion of the blade which is to be prewelded is cast with a depression about 1/4 in. deep, and stainless steel containing essentially 18 per cent chromium, 8 per cent nickel, and with the carbon limited to about 0.08 per cent (3), is welded as indicated, precautions being taken to prevent distortion of the blade during this process.



FIG. 2 TURBINE-RUNNER BLADE BEFORE BEING PREWELDED

CAVITATION LABORATORY

The need for the convenient and prompt determination of the cavitation characteristics of model propeller runners resulted in the construction of the I. P. Morris cavitation equipment (5) in 1938, at Eddystone, Pa., Fig. 3. This involves essentially a closed circulating system with a service pump located about 8 ft below the floor, thus permitting relatively high values of H_s in the turbine draft tube without having as high values at the pump. A calibrated venturi meter measures the discharge. The tests are run under heads up to 35 ft. This equipment is designed primarily for tests of propeller runners of 11 in. throat diam, although it is possible in the case of Francis-type runners to increase this dimension. It is also designed for the cavitation testing of pumps. Variations of sigma are obtained by changing the pressure throughout the system (see Appendix for engineering data and equipment pertaining to cavitation equipment).

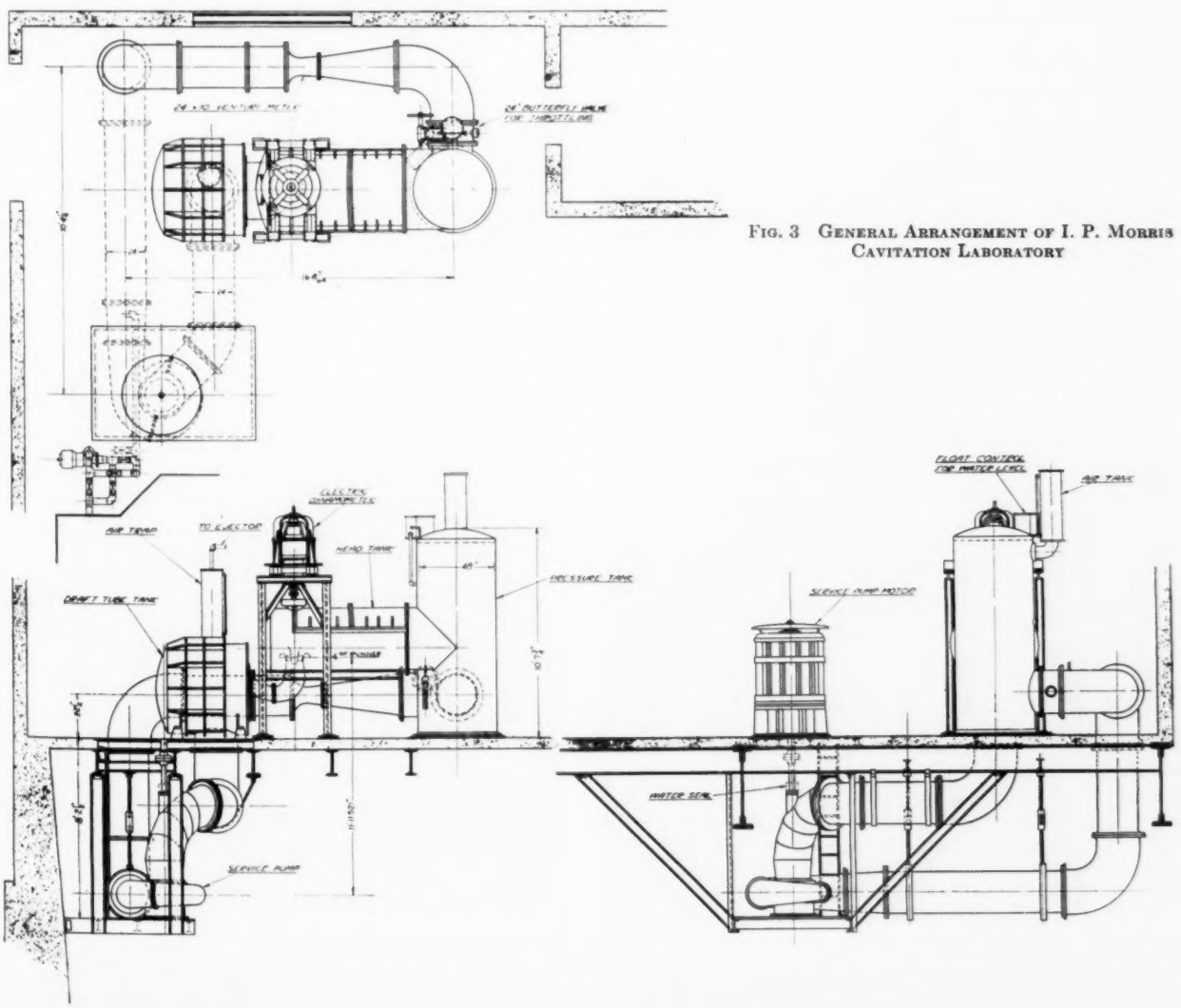


FIG. 3 GENERAL ARRANGEMENT OF I. P. MORRIS CAVITATION LABORATORY

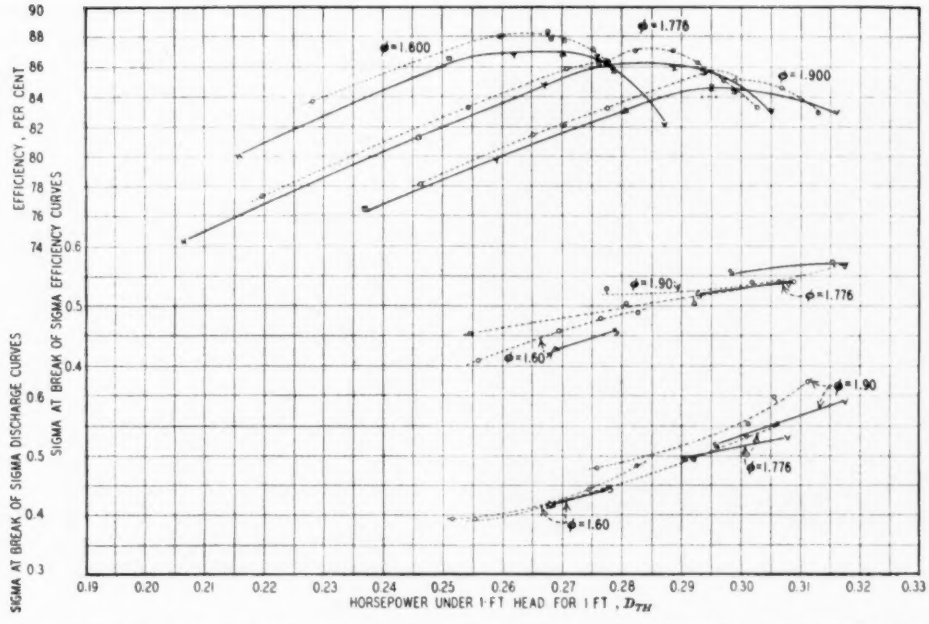


FIG. 4 COMPARISON OF I. P. MORRIS LABORATORY 11-IN. RUNNER MODEL WITH HOLTWOOD LABORATORY 16-IN. PROTOTYPE

This equipment was designed to permit the air to be separated from the water as much as possible, with the recognition that an undue air content would affect the sigma-break values (3, 4). The vertical pressure tank permits the air to rise to the top where it may be drawn off. However, it was later found advisable to add the air trap in the suction tank, and this has been the most effective means of air elimination since most of the air separation takes place in the draft tube. Particular attention has been paid to the possibility of air leaks in the suction tank and at the stuffing box to the service pump, where a water seal has been provided.

To calibrate this equipment as to efficiencies obtained and as to sigma-break values, a comparison was made with the Holtwood laboratory of the Safe Harbor Water Power Corporation by constructing and testing an exactly homologous but smaller (11-in. runner diam) model, for comparison with the Holtwood test of a 16-in. similar model. This particular 11-in. model is used as a standard at the I. P. Morris laboratory for check tests from time to time. Fig. 4 shows the results of a recent (March, 1940) test as compared with the Holtwood test made during June, 1935. The lower efficiencies at Eddystone are consistent with the smaller runner diameter, and the power and shape of the performance curves check closely.

The lower portion of Fig. 4 shows the comparison of sigma breaks for three values of ϕ. The upper set of curves is based on the break in efficiency and the lower set on quantity. While there is somewhat of a difference at 1.9ϕ, particularly on the quantity basis, the remaining curves check fairly well. At least, there is no systematic tendency for the 11-in. Eddystone results to show a higher sigma break due to air content. The tests were made at heads from 27 to 30 ft at Eddystone and under heads of 40 to 60 ft at Holtwood. It has subsequently been found necessary to keep above the 25-ft head at Eddystone to avoid an increase in sigma break due to air content. It is probable that this lower limit could be reduced if this were a noncirculating system.

In an effort to provide a means for determining whether or not or to what extent cavitation exists in a given turbine without recourse to measurement of change of characteristics, the device shown in Fig. 5 has been tried, with partial success. This device as tried out up to this time has been installed in the throat ring of

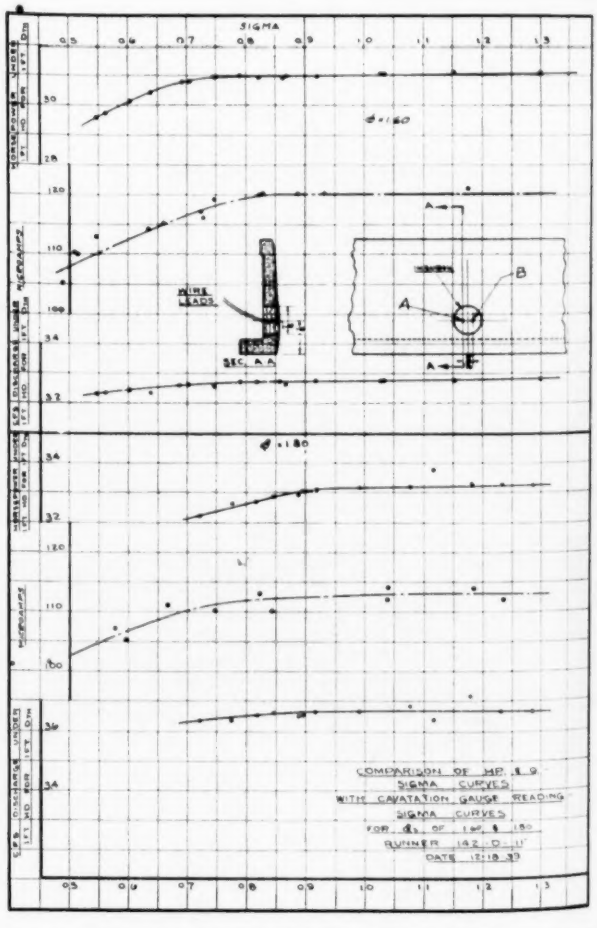


FIG. 5 COMPARISON OF SIGMA-BREAK CURVES WITH CAVITATION-GAGE READING

(Sigma curves for ϕ = 1.60 and 1.80.)

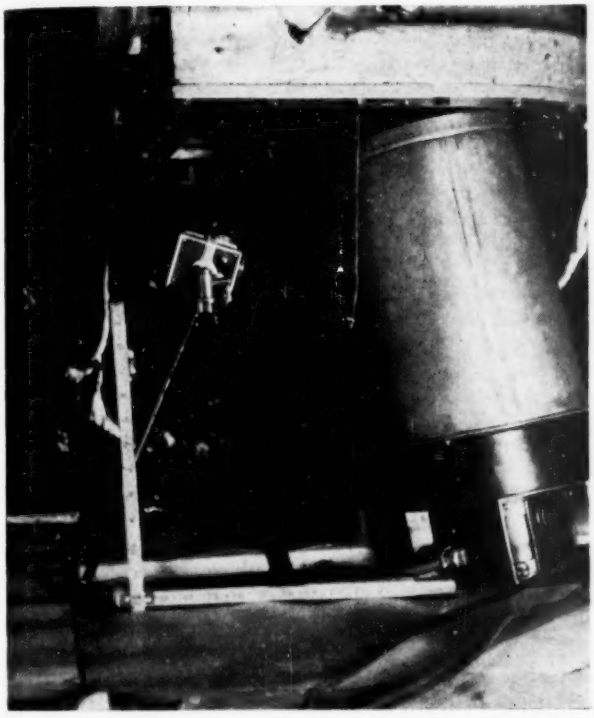


FIG. 6 STROBOSCOPIC EQUIPMENT FOR STUDYING CAVITATION

the turbine. The functioning of the device is based on the change in resistance between the points A and B to an electric current, when operating in cavitation-free water as compared to water with cavitation present. The change in current, in comparison with the measured change in characteristics is indicated in this figure. For $\phi = 1.61$, it may be noted that the change in resistance takes place at about the same value of sigma as that at which the power developed and discharge start to change; whereas, at the higher value of ϕ (1.805), the performance characteristics undergo a change somewhat prior to the change in resistance. A gage of this sort may have possibilities in view of the ease in obtaining readings at any time during the operation of the turbine.

PHOTOGRAPHIC RECORD OF RUNNER TESTS

With a view of having the report of each runner tested include photographs of the location and progress of cavitation for decreasing values of sigma, at different values of ϕ stroboscopic equipment, as indicated in Fig. 6, has been provided in connection with the I. P. Morris cavitation equipment. This too is yet in a state of development.

The elbow portion of the draft tube has been provided with an opening covered with "Plexiglass" which conforms with the curved portion of the tube. Below this is a flat glass opening, the space between these parts being filled with water during operation to eliminate distortion of vision. An Edgerton stroboscope is located on one side of a partition below this opening, with the other side of the partition used for visual study or for photography.

The stroboscope is wired to a commutator geared to the turbine

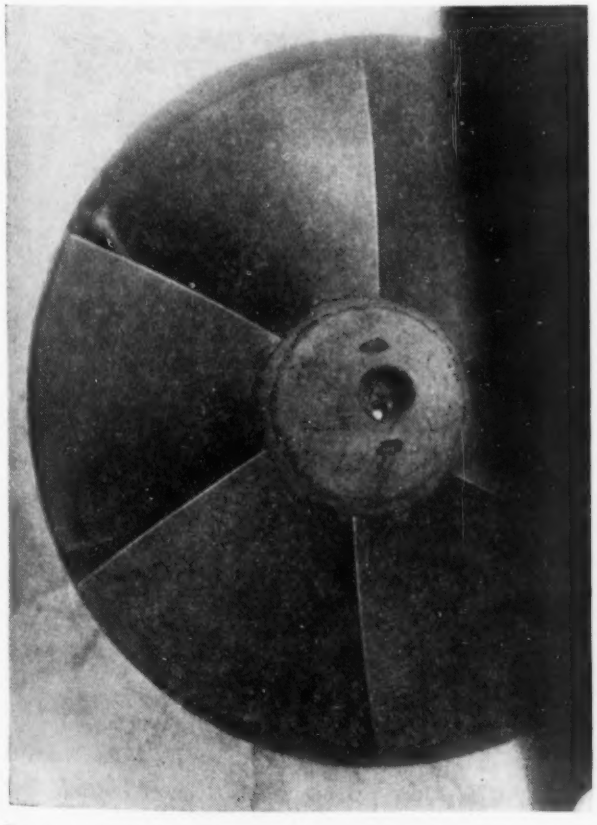


FIG. 7 STROBOSCOPIC VIEW OF PROPELLER-TYPE RUNNER (MODEL 128) WITH FIXED BLADES

(Test No. 617; water temperature = 68 F; head = 32.32 ft; h_s = —0.94 ft; $0.8H_v$ = 5.15; sigma = 1.055; HP_1 = 0.272; Q_1 = 2.76; efficiency = 86.8 per cent; ϕ = 1.60; break at sigma = 0.44.)

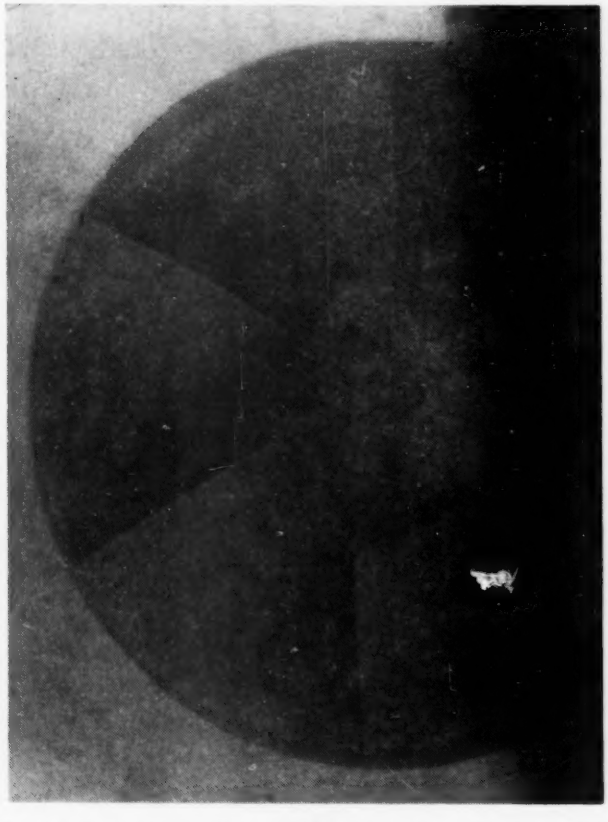


FIG. 8 STROBOSCOPIC VIEW OF PROPELLER-TYPE RUNNER WITH FIXED BLADES WHEN OPERATING JUST ABOVE SIGMA BREAK

(Test No. 617; water temperatures = 68 F; head = 32.22 ft; H_s = +16.21 ft; $0.8\,H_v$ = 5.10; sigma = 0.525; HP_1 = 0.271; Q_1 = 2.75; efficiency = 87 per cent; ϕ = 1.60; break at sigma = 0.44.)

shaft and normally flashes once at each revolution, thus giving a view of each separate blade rather than a composite effect. Time exposures have been found necessary, even with the most sensitive film available, due to the limited amount of light possible for photography. An alternate method has been used sometimes whereby photographs with single flashes were taken, requiring a special condenser for increasing intensity. A lens is provided above the stroboscope to confine light rays as much as possible to the runner blades. The photographs taken with sigma values well above the break, i.e., with relatively high pressures in the draft tube, have been quite successful. However, as the sigma is lowered with consequent reduced draft-tube pressure, the water below the runner becomes cloudy due to air coming out of solution, with resulting interference to the photography. Unfortunately, the photographs reproduced in this paper were taken during the winter months, and it was not practicable to raise the water temperature and thus reduce the amount of air in solution. It is expected that photographs taken during the summer months with water temperatures as high as 85 F will show better results.

Figs. 7 and 8 are stroboscopic views of a propeller-type model runner 128 with fixed blades and with the periphery of the blades turned cylindrical to give minimum clearance throughout their entire length inside of the cylindrical throat ring.

Both photographs were made at ϕ = approximately 1.60. Although best efficiency is at $\phi = 1.55$, it will be noted from Fig. 7 that cavitation starts at the intake edges on the backs of the blades near the periphery, at sigma values considerably higher than the break. This condition is doubtless aggravated at $\phi = 1.55$ where efficiency is maximum. The obvious suggestion for the correction of this condition is to design the blades with a steeper angle of intake where the cavitation takes place. This steeper angle results in excessive blade curvature, and reduces the efficiency and power at higher values of ϕ, which is quite undesirable, particularly where heads below normal are encountered. The efficiency performance of this runner, the unusually low and sharp sigma break, and the occurrence of cavitation at this point on many other runners leads to the belief that this may be a normal condition. This is also the point of minimum pressure on efficient airfoils (8).

Fig. 8 gives a very indistinct idea, due to air separation, of conditions as the sigma break is approached. With the sigma break at 0.44, Fig. 8 indicates the cavitation extending along the blade but still with a greater amount at inflow than at outflow. Some cavitation has appeared near the hub. It may be noted that in neither of these illustrations is there any indication of cavitation at the periphery due to flow through the clearance spaces. This is in sharp contrast with conditions indicated later with Kaplan-type runners, where the clearance at most of the periphery is necessarily greater.

Due to the manner in which the water clouds in these photographs, preventing a clear view of extensive cavitating conditions, it is planned to redesign the equipment in the I. P. Morris laboratory by lowering the turbine so that, by having a portion of the throat ring and upper draft tube transparent, the light can be introduced in a radial direction from the side, with observations and photographs made from adjacent points. This will permit observations of the face side of the blades, with a much shorter distance for the light to travel. The photographs show, however,

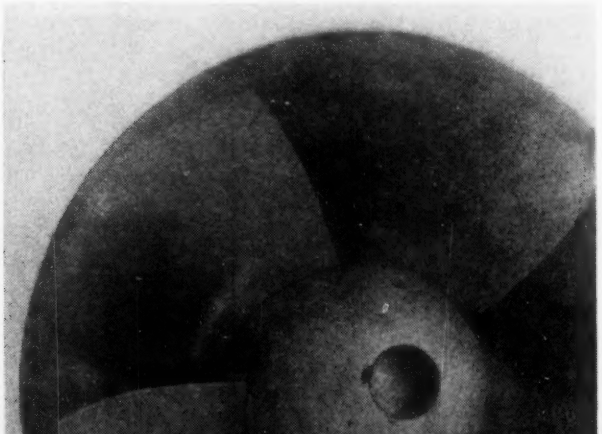


FIG. 9 KAPLAN-TYPE MODEL RUNNER 142-D OPERATING AT SIGMA BREAK

(Test No. 616-II; water temperature = 60 F; head = 30.67 ft; $H_s = +8.11$ ft; $0.8\,H_s = 8.27$; sigma = 0.815; $HP_1 = 0.33$; $Q_1 = 3.53$; efficiency = 82.3 per cent; $\phi = 1.60$; break at sigma = 0.81.)



FIG. 10 KAPLAN-TYPE MODEL RUNNER 142-D OPERATING AT HIGHER SPEED AND BEYOND SIGMA BREAK

(Test No. 616; water temperature = 65 F; head = 29.5 ft; $H_s = +4.16$ ft; sigma = 0.972; $0.8\,H_s = 10.55$; $HP_1 = 0.361$; $Q_1 = 4.06$; efficiency = 78.6 per cent; $\phi = 1.80$; break at sigma = 1.05.)

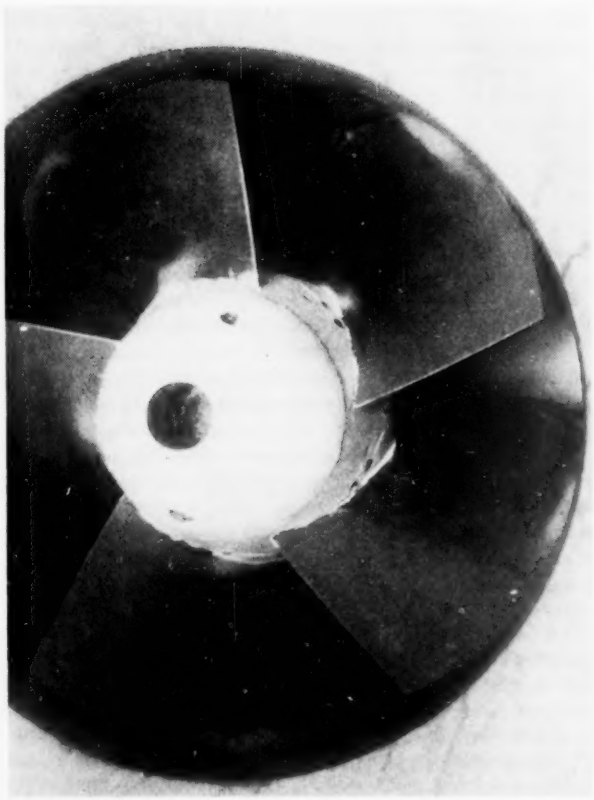


FIG. 11 KAPLAN-TYPE MODEL RUNNER 148 AT SIGMA BREAK, SHOWING CAVITATION AT HUB AND AT PERIPHERY

(Test No. 618; water temperature = 63 F; head = 29.74 ft; H_s = —6.04 ft; sigma = 1.307; HP_1 = 0.359; Q_1 = 3.97; efficiency = 79.8 per cent; ϕ = 1.60; break at sigma = 1.30.)

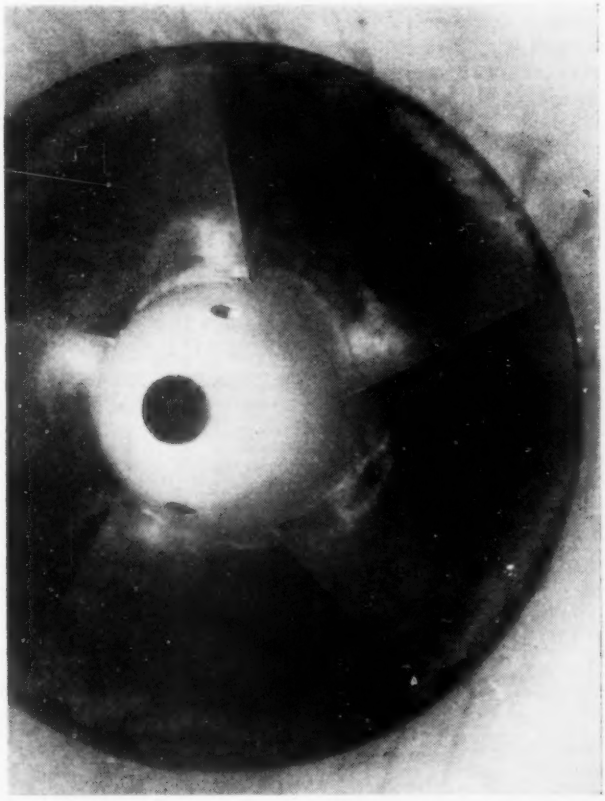


FIG. 12 KAPLAN-TYPE RUNNER MODEL 148, OPERATING FAR BEYOND SIGMA BREAK

(Test No. 618; water temperature 63 F; head = 30.5 ft; H_s = +8.06 ft; sigma = 0.817; HP_1 = 0.330; Q_1 = 3.80; efficiency = 76.6 per cent; ϕ = 1.60; break at sigma = 1.30.)

where cavitation is initiated and this is important as indicating the location of faults in design, or the desired location for prewelding.

Fig. 9 is a view of model No. 142-D which is of the Kaplan type with spherical throat ring. The value ϕ being 1.60 is slightly above that at best efficiency. Sigma is 0.814, the break occurring at 0.81. Cavitation is occurring at the periphery, at the intake, and through the clearance space just upstream and downstream from the blade axis where clearance is a minimum. Cavitation near the hub indicates rather too abrupt blade curvature, about at the blade axis.

At ϕ = 1.8, Fig. 10, sigma 0.972, break 1.05, the disturbance at the hub is accentuated, with indications of outward radial component of flow due to the higher whirl component Vu_2. The cavitation in the clearance space is also aggravated with streamers extending some distance beyond the ends of the blades. Punishment of the throat ring at the blade discharge is indicated. This latter condition exists to a lesser degree at this ϕ (1.8) well above the sigma break.

Figs. 11 and 12 show model No. 148 at ϕ = 1.60 with sigma = 1.307, 0.817, respectively, where the sigma break is at 1.3. The marked increase in cavitation at the hub will be noted. The entire series of photographs taken of this runner at ϕ = 1.4, 1.6, and 1.8 shows an interesting feature, namely, that cavitation at the periphery develops prior to the break in sigma, particularly at low ϕ values. Cavitation near the hub tends to develop later and, at least on this runner, it is the cavitation at this point rather than at the periphery which affects the performance by reducing the output and efficiency. It, therefore, appears that this is the portion of this runner which should be altered for improvement. In general, these studies show the importance of the blade design near the hub, also the need for smoother hub surface between the blades. Cavitation near the hub, however, seldom if ever results in pitting of the prototype due to the low relative velocity v, or more improbably, to the voids at this point on the runner not collapsing on the runner-blade surface.

Model No. 148 was made exactly similar to a former model with the exception that the blade axis was moved further downstream on the blade. The same blade pattern and templates were used, but with a shift in the axis. This change has had the effect of improving the efficiency and sigma break. The effect of this change is to reduce the clearance between the throat ring and the outflow portion of the blade, and the change is consistent with the improvement found with a properly designed spherical throat ring as compared to a cylindrical throat ring.

It is believed that the proper interpretation of stroboscopic photographs of runners is a valuable means of eliminating faults in design. The relation between the progressive change in cavitation with decreasing sigma to the change in performance can be used in deciding on the changes to be made for expected improvement. However, a liberal number of photographs showing the progressive changes are essential. By constructing the blades of a tough, ductile bronze, the same ones may be used for a large number of different blade shapes. Plaster of Paris forms may be kept as a record for each change made. Accurate templates are, of course, necessary for each change, to insure all blades being uniform for each test made.

In connection with the disturbance in the clearance space, Fig.

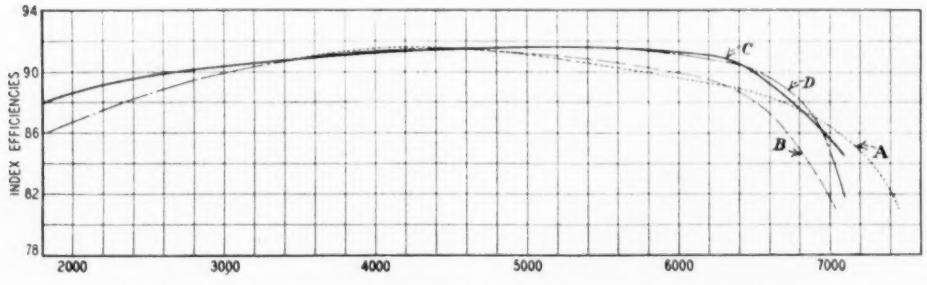


FIG. 14 COMPARISON OF SIGMA BREAK ON MODEL WITH THAT ON PROTOTYPE

(*A* = 16-in. model tested with high sigma; *B* = 11-in. model tested with sigma same as prototype; *C* = 109.5-in. prototype unit No. 1; *D* = 109.5-in. prototype unit No. 2.)

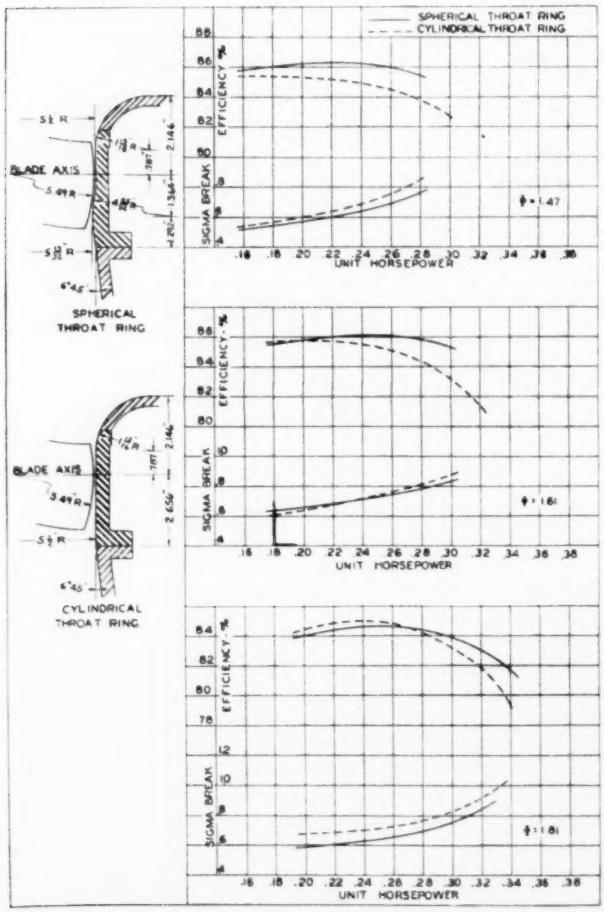


FIG. 13 CURVES SHOWING RELATIVE CAVITATION AND EFFICIENCY PERFORMANCE OF KAPLAN-TYPE RUNNERS WITH SPHERICAL THROAT RING AND STRAIGHT CYLINDRICAL THROAT RING, RESPECTIVELY

13 is introduced to show the relative performance (6, 7) as to efficiency and cavitation of a Kaplan-type runner (*a*) with spherical throat ring the same as used previously, and (*b*) with straight cylindrical throat ring. The former results in less clearance space disturbance but puts a greater burden on the draft tube due to the smaller minimum diameter of the throat ring. The latter is important when it is remembered that draft-tube losses vary with the fourth power thereof. The improved results with the smaller clearance space emphasize the importance of the spherical shape, if properly designed.

SIGMA BREAK ON MODEL AND PROTOTYPE

Fig. 14 is presented as an example of the relation between sigma break on the model and on the prototype. In this instance the model is homologous to the prototype throughout, including intake, casing, and draft tube. The prototype was not tested as to efficiency but the power was recorded accurately, and an index test was made by taking readings of Winter-Kennedy piezometers in the casing and stay ring. The maximum efficiencies shown are those stepped from the 16-in. model runner to a diameter of 109.5 in. by the Moody formula (10). Those of the 11-in. model have been arbitrarily stepped up to give the same maximum value as have those of the prototype.

The 16-in-model test was made at high sigma values, and is, therefore, not affected by cavitation. The 11-in-model test was made at the same sigma as the prototype. Some increase in prototype power is noted over the 11-in. model, with fair agreement in the form of the curves.

The flash photograph of runner No. 148, taken with a special condenser which is necessary to increase the intensity of the light to permit such a photograph, is shown in Fig. 15. This is at blade



FIG. 15 FLASH PHOTOGRAPH OF MODEL RUNNER NO. 148 SHOWING SPIRAL VORTEX

position No. 3 with $\phi = 1.595$, $HP_1 = 1.722$, $Q_1 = 1.82$, efficiency = 86.0, and $\sigma = 0.648$. Sigma break under this condition occurs at 0.4.

This vortex, it will be noted, appears to form in the hole in the center of the hub and is rotating with the runner. A time exposure taken under this condition would merely show a blurred effect even with the stroboscope, due to the rotation of the spiral vortex not being in synchronism with that of the runner. As the σ was reduced to a value nearer the break, this vortex disappeared. The gate opening at which this photograph was taken is such that this would represent an actual operating condition at this value of ϕ when the load on the turbine is reduced to slightly more than half of the normal value.

Acknowledgment is made of the helpful suggestions of L. F. Moody in the preparation of this paper, of the assistance of R. B. Willi, and of K. W. Beattie in connection with the photographs and test data included.

Appendix

I. P. MORRIS CAVITATION LABORATORY

General Data and Equipment

Diameter of runners tested: 11 in.

Head on turbine: 25 to 35 ft.

Variation of sigma: Accomplished by changing pressure in system upstream from turbine from below atmospheric to 20 psi above atmospheric.

Water supply: Closed circulating system with cold water connection automatically added from city main as make-up, and to regulate temperature. Volume of water in system about 500 cu ft.

Service pump: Designed for 25 cfs at 35-ft head; driven by 125-hp slip-ring motor, 220 v, 580 rpm rated speed. Possible speed regulation to one-half normal in eleven steps.

Water measurement: 24-in. by 10-in. venturi meter; calibrated by Professor Pardoe at the University of Pennsylvania.

Dynamometer: Specially built, using 40-hp 1200-rpm d-c motor reinforced for higher speeds. Dynamometer carried by cradle mounted on ball bearings. Torque measured by a Fairbanks beam scale connected through flexible steel tape to dynamometer frame. Electrical output dissipated in heat.

Pressure tank: 48 in. diam, 10 ft high, with supplementary air tank at top. Water level in this tank automatically controlled by float valve.

Head tank: 48 in. diam with float bottom, 7 ft 6 in. long.

Draft-tube discharge tank: 54 in. diam, 4 ft long, with trap for collection and drawing off air.

Gages and instruments: Head and water measurement pot-type mercury gages; rpm dial revolution counter, with electric release controlled by clock; atmospheric-pressure mercury barometer checked at intervals from weather bureau; instrument for measurement of air content in water, as developed at Massachusetts Institute of Technology.

Stroboscope: "Strobulux" controlled by "Strobotac," type 648-A, manufactured by General Radio Corporation.

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Combined Tension-Torsion Tests on a 0.35 Per Cent Carbon Steel

By E. A. DAVIS,[1] SO. PHILADELPHIA, PA.

This paper contains the results of a group of combined tension-torsion tests on both cylindrical and notched bars. The author has tried to keep in mind the close relation between this type of test and the problem of bolting. The test results show the amount of reduction in the ultimate tensile strength when a torque is applied in combination with the tension load. They also show a similar reduction in the torsional strength when a tensile load is added.

IN general practice, it frequently occurs that some members of a structure or machine carry loads in both tension and torsion simultaneously. It is pretty well known or it may be logically deduced that the addition of a torque to a bar already stressed in tension will reduce the tensile load-carrying capacity of the bar. The amount that the tensile strength will be reduced, however, is not so well known, and engineers in general have tried to avoid this type of loading wherever either the tension or the torsional stress is likely to be high.

A typical example of this combined tension-and-torsion loading is a bolt which has been tightened by a wrench. As the nut is being drawn up, the twisting moment of the wrench is applied to the bolt and there is a danger that the bolt may be strained in torsion to the extent that it will not support its load in tension. Because of this danger, methods have been developed for reducing this twisting moment in the tightening of large bolts. It is quite common practice in the assembly of large bolted parts to expand the bolts by heating to such an extent that the nuts can be turned up just enough to give the bolts their proper working stress when the bolts are at the same temperature as their surroundings.

In some large alternating-current generators, the rotors are made up of several forged disks which are bolted together with long axial bolts. These bolts are drawn up to a given load by a "bolt puller" and the nuts are tightened by hand.

Unfortunately not all bolts can be tightened in this manner and often it is necessary to draw them up with a wrench. With this in mind, the present investigation was carried out in an effort to determine just what effect a superimposed load in one direction has on the strength of a bar carrying a second load in another direction. In order to do this, several sets of combined tension-torsion tests were made. In one set, various fixed amounts of tensile load were superimposed upon constant-strain-rate tests in torsion. This set contained one pure torsion test (zero tension). In another set, fixed amounts of torque were maintained while the bars were pulled at a constant strain rate in tension. In this instance, one of the tests was a pure tension test (zero torque). Another test was run in which the strain rates in both tension and torsion were kept constant. Since these tests are closely related to the problem of bolting, it was felt that the effect of notches and threads should be investigated in conjunction with these combined stress tests. Consequently, some of the foregoing tests were repeated on bars containing circumferential notches.

Theoretical Considerations

Probably the best approach to an understanding of the combined tension-torsion test would be to express the test results in terms of shearing stress and strain on some preferred set of planes in the material. For theoretical reasons, the best results would likely be obtained if the octahedral planes were chosen for making this analysis. These planes are the faces of a regular octahedron which is oriented so that its apexes point in the directions of the principal stresses. A. Nádai (1)[2] has made such an analysis of a combined tension-torsion test on a plastic material having a uniform yield stress $\sigma = \sigma_0 =$ constant. The stress distribution under combined tension and torsion in a round bar is reproduced in Fig. 1, in which it is assumed that the material flows plastically over the entire cross section of the bar.

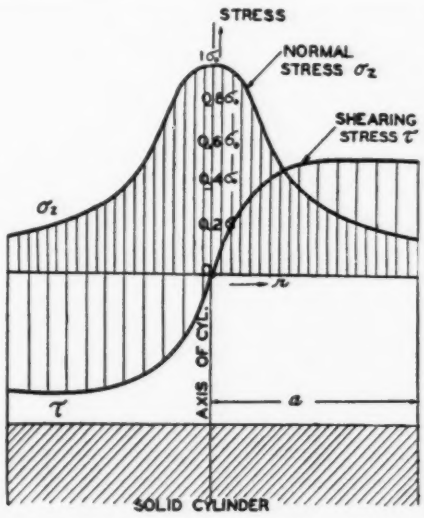


Fig. 1 Stress Distribution Under Plastic Flow in Cylindrical Bar Subjected to Combined Action of Axial Load and Twisting Moment

The distribution of the shearing stresses τ and of the normal stresses σ acting in the axial direction are plotted separately against the radius of the bar. It is to be noted that in the axis of the bar, a state of pure tension exists so that at $r = 0$ the stress in the axial direction is equal to σ_0, the yield stress of the metal in pure tension. Under these conditions, the tensile stress is confined mostly to the inner fibers, while the torque stress is carried by the outside fibers.

---

[1] Land Turbine Engineering, Westinghouse Electric & Manufacturing Company.

Contributed by the Iron and Steel Division and presented at the Spring Meeting, Worcester, Mass., May 1–3, 1940, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

When one tries to apply a similar analysis to a solid round bar of a material which exhibits the property of strain-hardening, certain difficulties arise which make an analysis complicated. With a solid bar in torsion the stress-strain diagram is not directly obtained. The shearing-stress (τ) versus the shearing-strain (γ) diagram is obtained from the moment (M) versus angle of twist (θ) curve by the following relations (2)

$$\tau = \frac{1}{2\pi a^3}\left(\theta\frac{dM}{d\theta} + 3M\right)$$

$$\gamma = a\theta$$

This stress and strain refer to the outside fibers of a bar of radius a. The expression for τ involves the slope of the moment-angle curve and this may lead to difficulty in determining the shearing-stress distribution in a test where the applied moment is held constant.

When tension and torsion are to be compared, another difficulty arises from the fact that the expressions for the strains become

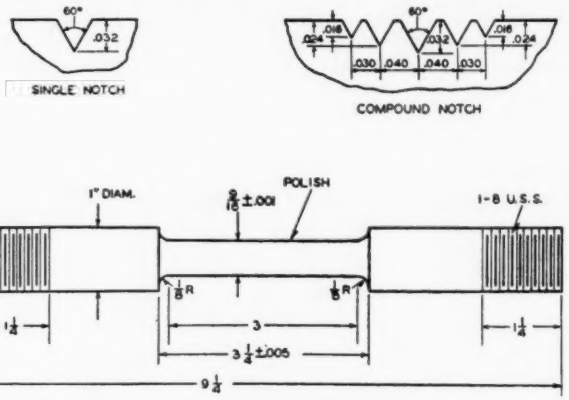


FIG. 2 TENSION-TORSION SPECIMEN

complicated if the strains are large. This is especially true for the strains in the octahedral planes. Also, when a bar extends axially, the diameter decreases in order to satisfy the condition that the volume remain constant. Furthermore, since the shearing stress on the outside fibers depends upon the third power of the radius, a change in the stress will occur even though the applied moment is constant. Because of these conditions, the test results in this paper are given as diagrams in which the tensile stress is plotted as a function of the axial strain and the moment of the applied torque as a function of the angle of twist. The tensile stress that is plotted and referred to throughout the paper is that based upon the original area of the bar $\left(\sigma = \frac{P}{A_0}\right)$.

The tension-torsion test is an example of a state of combined stress, a field in which a considerable amount of work has recently been done. The aim of most of this work has been to check the general conditions under which the ductile metals yield plastically. Among the investigations of this nature which have been made, those by Lode (3) and by Schmidt (4) in Germany; by Taylor and Quinney (5) and by Cook (6) in England; and by Ros and Eichinger (7) in Switzerland may be of interest. A survey of the results of most of these tests was made by Nádai (8).

The author in a former paper (9) attempted to show the relation between tension and torsion tests on a solid bar of copper by expressing the results of both in terms of the shearing stresses and strains on the octahedral planes.

The agreement was good in the region where the strains were small but, for strains greater than those corresponding to about 15 per cent elongation in tension, the curves for the different tests separated somewhat.

The present group of tests differs from most of the previous combined stress tests in that the stress-strain diagrams were obtained up to the point of rupture, while the others were concerned chiefly with the region in initial yielding. Also, in these tests the equipment was so arranged that either the tension or the torsion load could be held constant while the other was allowed to vary to suit the conditions imposed by a constant strain rate.

MATERIALS AND TEST SPECIMENS

In choosing a material for these combined tension-torsion tests, it was felt that certain characteristics would be desirable. If the results were to throw any light on the bolting problem the material should be similar to that generally used in bolts. It should be tough and not excessively ductile; however, a very strong steel would require that the dimensions of the test bar be small due to the limited capacity of the testing machine. Furthermore, the elongation associated with the yield-point region should not be too great, for this phenomenon is not observed to any appreciable extent in the alloy steels commonly used in large bolts. As a compromise among these requirements, a medium-carbon steel (0.35 C) was selected for the tests. With this material a bar with a $^9/_{16}$-in-diam test section could be used in the available equipment.

A sketch of the test specimen is shown in Fig. 2. The 1-in-diam heads were used in order that reasonable torque could be applied through the threaded ends without too greatly tightening the threads in the grips. It was realized that, if the axial extension and the angle of twist were to be obtained for strains up to the ultimate strength, the extensometer would have to be clamped on the shoulders of the bar. This has the weakness that the active gage length is somewhat unknown. For purposes of comparison, however, the actual gage length does not need to be known exactly. In these tests, it has been assumed that the gage length in each case was 3.1 in.

It was felt that it would be desirable to compare the results of tests on bars containing notches and threads with those of the cylindrical bars. Two types of notches, a single and a compound notch as shown in Fig. 2, were designed. The compound notch was included in order to determine whether or not an adjoining notch had any tendency to relieve the stress concentration in the main notch.

As a supplement to the combined stress tests on notched bars, it was decided to make up two mild-steel specimens, as shown in Fig. 3, and to stress them just to the point where yielding starts and then to show up the flow lines by the Fry etching method. The bars were $1^1/_2$ in. in diameter in the central section. One bar had about $1^1/_2$ in. of V threads, 8 threads per in., while the other had a single notch of the same dimensions as the V threads. These tests should show whether or not the flow in a deeply notched bar of mild steel will be confined to the plane at the bottom of the notch where fracture will eventually occur or to what extent it penetrates into the heavier sections of the bar.

TESTING EQUIPMENT

Some years ago at the Westinghouse Research Laboratories on the suggestion of A. Nádai special equipment was designed for running constant-strain-rate tests either in tension or torsion or in a combination of both.[3] The machine is illustrated in Fig. 4. The tensile load was measured on the regular Amsler dial while the torsion load was determined by measuring the deflection of two springs attached to the upper head of the machine. An electrical contact device was used on the dial of the Amsler

[3] For this purpose a 10-ton Amsler testing machine was reconstructed by W. O. Richmond and B. Cametti.

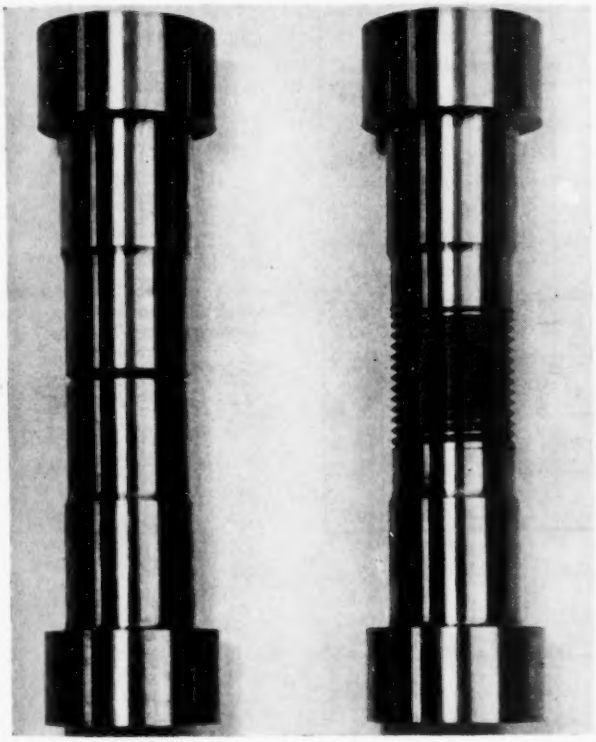


FIG. 3 TEST SPECIMENS FOR SHOWING WHERE INITIAL YIELDING WILL OCCUR

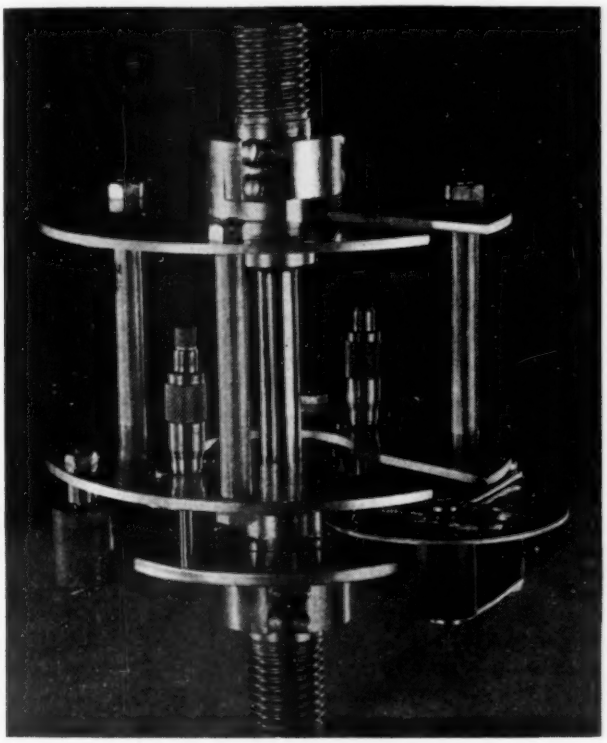


FIG. 5 TENSION-TORSION EXTENSOMETER

load indicator to maintain a constant tension when desired. A similar contact device was arranged on the torsion springs so that a constant torque could be obtained.

The strains were measured by means of the specially designed extensometer shown in Fig. 5. As has already been mentioned, the extensometer was clamped on the shoulders of the test specimen. The extensometer consisted chiefly of a frame which was clamped to the upper shoulder and a plane disk which was clamped to the lower shoulder. The axial strain was determined by two micrometer heads clamped to the frame in such a manner that they could measure the separation between the frame and the plane disk. The angle of twist was measured by a roller attached to the frame which rolled on the periphery of the disk.

TEST RESULTS

Before any combined stress tests were run, the material was tested in pure tension and pure torsion. The pure tension test is curve *J* in Fig. 6. The ultimate strength was 83,000 psi and the elongation at rupture was 27.3 per cent. In the pure torsion test, curve *G* in Fig. 7, the breaking torque was 237 lb-ft and the maximum angle of twist was 584 deg.

In addition to the pure tension test, Fig. 6 shows the constant axial strain-rate curves for bars upon which constant torques of 60, 118, and 199 lb-ft were superimposed. The value of the axial strain and the angle of twist at rupture are listed at the end of each curve. It can be seen readily that the amount of axial strain is considerably reduced when a high torque is applied to the bar. The ultimate tensile strength is also reduced. A torque of 60 lb-ft (one fourth of the breaking torque in pure torsion) reduced the ultimate strength only about 5 per cent, but a torque of 199 lb-ft (five sixths of the breaking torque) reduced the ultimate strength over 40 per cent.

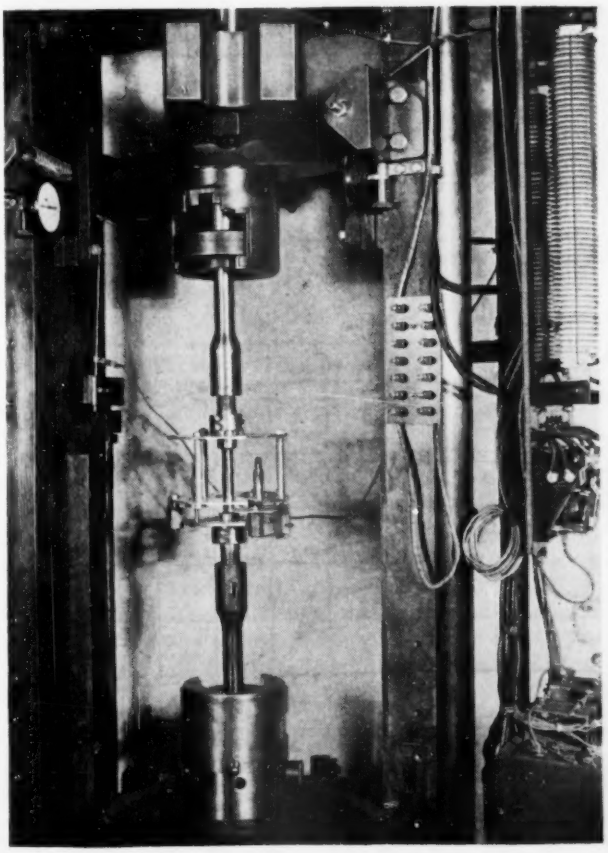


FIG. 4 COMBINED TENSION-TORSION MACHINE

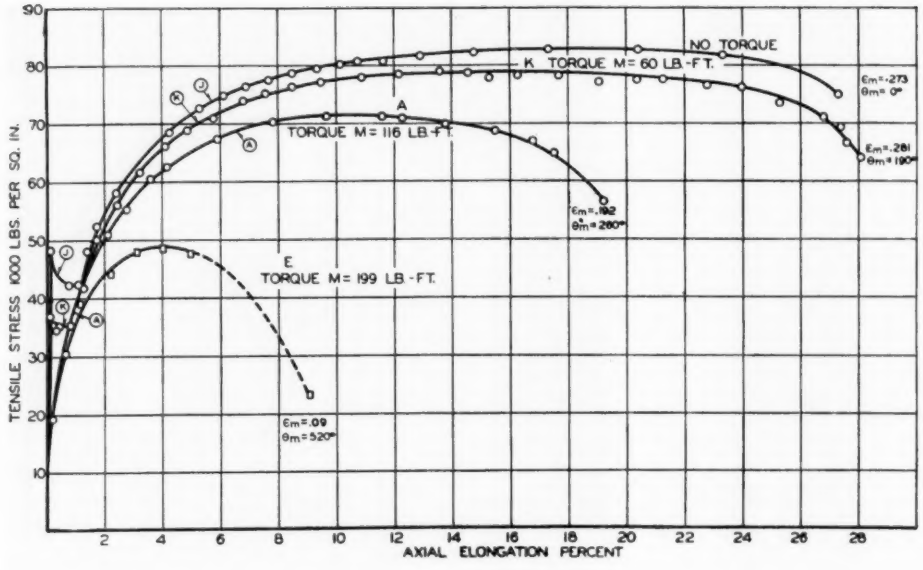


FIG. 6 COMBINED STRESS TESTS; CONSTANT TORQUE PLUS CONSTANT RATE TENSION

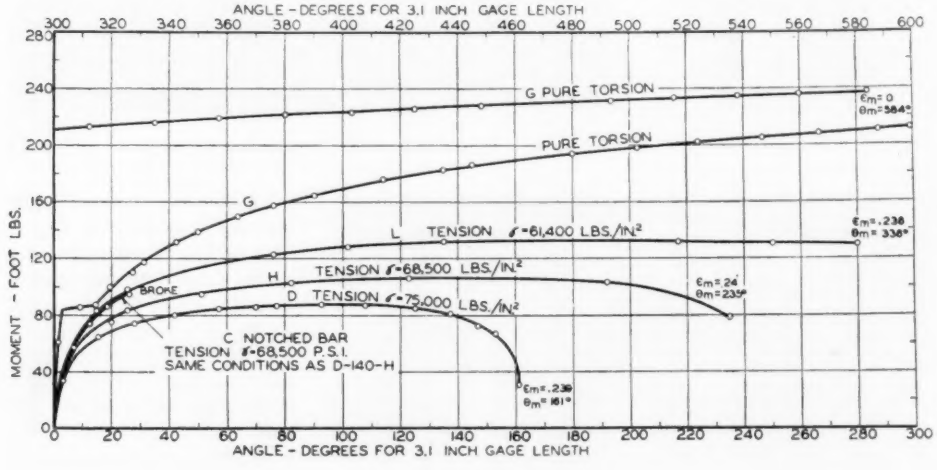


FIG. 7 COMBINED STRESS TESTS; CONSTANT TENSION PLUS CONSTANT RATE TORSION

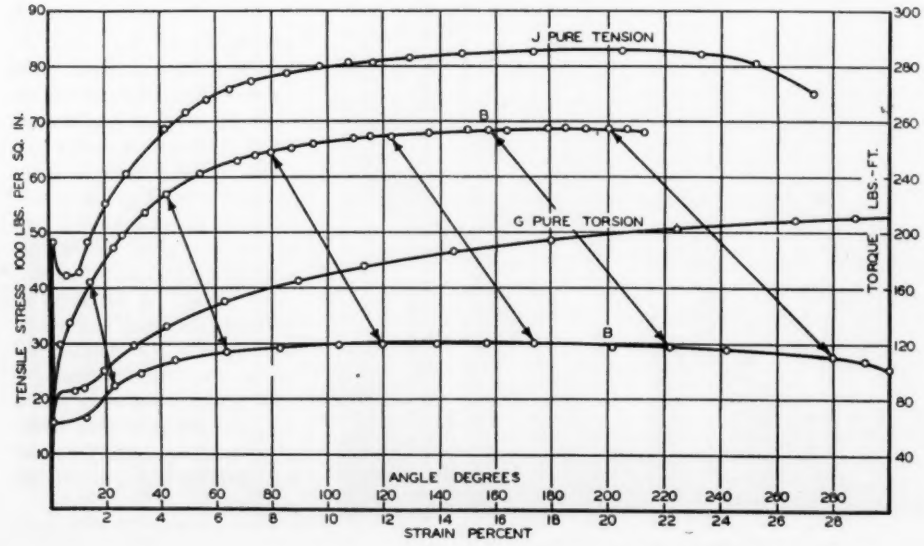


FIG. 8 COMBINED STRESS TESTS; CONSTANT RATE TENSION PLUS CONSTANT RATE TORSION

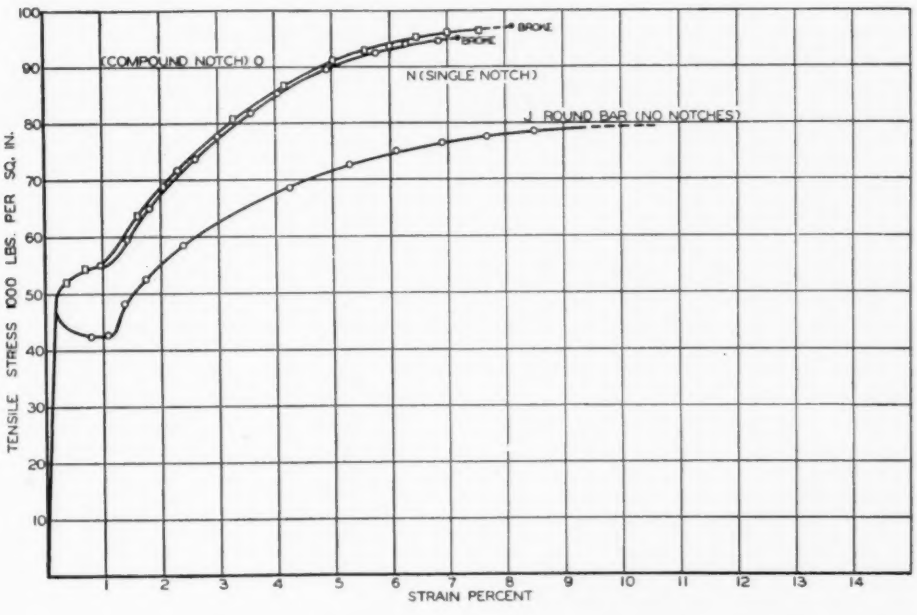


FIG. 9 TENSION TEST ON NOTCHED BARS

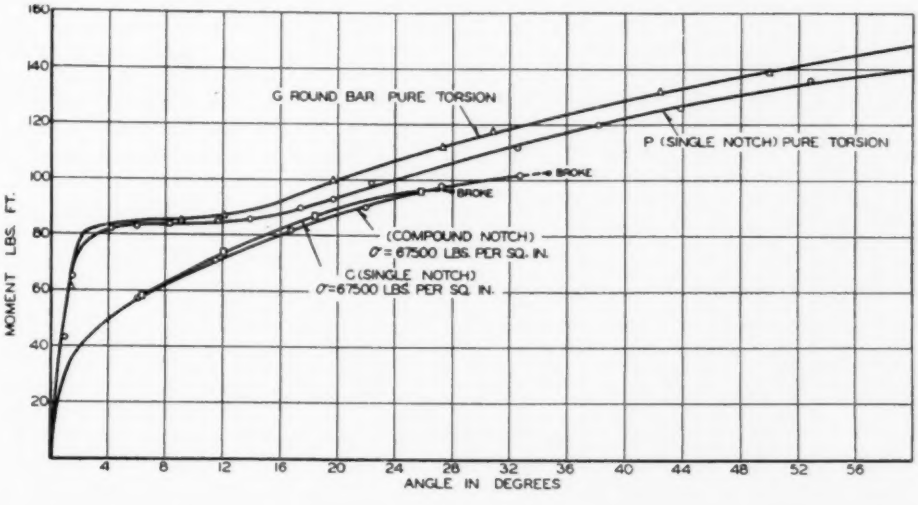


FIG. 10 TORSION TESTS ON NOTCHED BARS

Fig. 7 shows the results of the tests which were run with a constant strain rate in torsion. Curve *G* is the pure torsion test. Curves *L*, *H*, and *D* are torsion tests with superimposed tensile stresses of 61,400, 68,500, and 75,000 psi, respectively. In these tests the tensile loads were fairly high. All were above the yield point in pure tension so that the curves do not show any discontinuity in torsion, such as curve *G* shows. The amount of axial strain did not vary much in these three combined stress tests but the angle of twist at rupture was reduced from 584 deg for the pure torsion test to 161 deg for the bar which also carried a tensile stress of 75,000 psi. Curve *C* shows the results of a test similar to curve *H*. Bar *C* had a single circumferential notch $^1/_{32}$ in. deep at the middle of the gage length. The constant tensile stress based on the area at the bottom of the notch was 68,500 psi. The apparent increase in strength over curve *H* is due partly to the fact that the notch prevents the bar from reducing in diameter and partly to the fact that the actual unit strains in the notch are much greater than shown by the diagram which is based on a 3.1-in. gage length.

In addition to the combined stress tests already mentioned, one test was run with a constant strain rate in both tension and torsion. Curve *B* in Fig. 8 shows this test compared with the pure tension and the pure torsion test. In this figure, the tensile stress and strain coordinates refer to the two upper curves, while the torque and angle coordinates refer to the two lower curves. Bar *B* was being tested in tension and torsion at the same time, hence, there are two curves labeled *B*. The arrows connect points on the two curves which occurred at the same time. For example when bar *B* had a tensile stress of 64,000 psi and an axial strain of 8 per cent, it also was subjected to a torque of 120 lb-ft and had twisted 120 deg in a 3.1-in. gage length.

At this point, an interesting observation can be made concerning the combined tension-torsion tests. In a pure torsion test, such as curve *G* in Fig. 7, the moment-angle diagram has a positive slope until the fracture occurs, and the bar carries its maximum torque just the instant before it fails. In the combined stress tests, where the tensile stress was fairly high, the torque reached a maximum quite some time before failure. This can be seen in curves *H* and *D* in Fig. 7 and in curve *B* in Fig. 8. This may possibly be due to the fact that the diameter of the bar becomes considerably reduced when it is stretched in tension, thus increasing the shear stress without increasing the applied moment.

The results of the tests on notched bars are shown in Figs. 9 and 10. Pure tension tests on the single and the compound notch are compared with that of the cylindrical bar in Fig. 9. Although the bar having the compound notch showed the greatest strength, the difference was but slight. The stress relieving due to the adjoining notches was almost negligible. The notched bars appear to be stronger than the cylindrical bar for the reason already mentioned. Fig. 10 shows the torsion tests on notched bars. Curve *P* is for a single notched bar and can be compared with the pure torsion test (curve *G*). It should be pointed out that these diagrams are moment-angle diagrams and not shear stress-strain diagrams. That explains why the moment curve for the notched bar is lower than the curve for the cylindrical bar. If we should plot the actual stress-strain diagrams, we would probably find that the curve for the notched bar would be higher. Curve *C* from Fig. 7 is replotted here for comparison with curve *M* which is the result of a compound notch tested under the same conditions as the single notched bar *C*. Both had a tensile stress of 67,500 psi superimposed. Here again there is very little difference between the two types of notches.

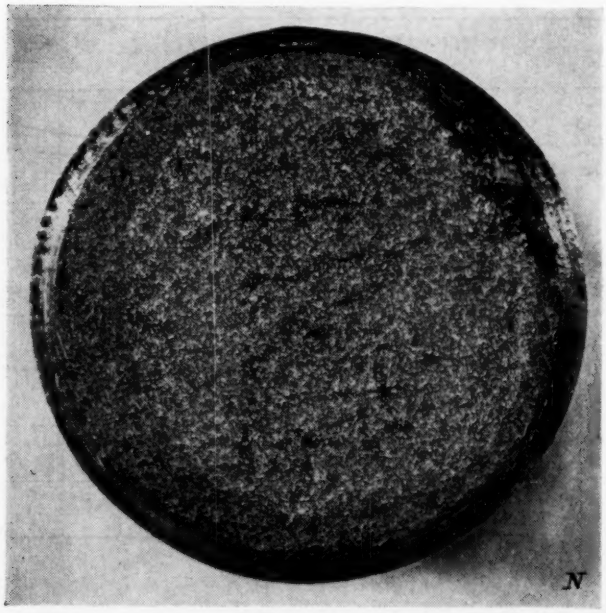
It is quite interesting to note the various types of fracture which occurred in these combined stress tests. Two pure tension fractures are shown in Fig. 11. Bar *J* is the cylindrical bar, while bar *N* had a single circumferential notch. The fracture of bar *J* is a typical tension fracture. In bar *N* two regions in the fracture are discernible. It appears that the bar cracked inward from the root of the notch for some distance and then the bar suddenly pulled apart. The notch prevented any reduction of area and the ruptured surface has the appearance of a brittle fracture.

Bar *G* in Fig. 12 shows how the pure torsion test bar broke. The fracture surface was nearly but not quite perpendicular to the axis of the bar. The upper specimen in Fig. 12 shows the rupture surface of a tension test which had a relatively small torque imposed upon it. This break appears much the same as the pure tension rupture. Fig. 6 shows that its stress-strain diagram was not much lower than the pure tension test.

Two bars which carried a good portion of both tension and torsion loads are shown in Fig. 13. Bar *A* is from a tension test with an added torque, while bar *L* was tested in torsion with an added tensile load. Both broke with a helical fracture; the helix angle, however, was not as great as is encountered in brittle materials in pure torsion.

In Fig. 14, specimen *P* shows the fracture of a notched bar in pure torsion. The rupture surface of bar *M* shown in the lower half of Fig. 14 is probably the most interesting in the group. This was a notched bar tested with combined tension and torsion. The fracture has two distinct regions. Presumably the outer portion where shear stresses predominate failed in shear, while the inner portion which is forced to carry the axial load failed in tension. The phenomenon of the two regions seems to accord with theoretical considerations based upon accepted theories of strength. It is the kind of fracture which might be expected for the case in Fig. 1.

In the tests to show the flow lines in mild steel, the bars were first loaded to the point where yielding had just started. They were then heated to and held at 200 C for 1 hr. Then they were cut along an axial plane through the center of the bar and the exposed sections were polished and etched. The results can be seen in Fig. 15. The portions which have yielded are shown by the dark areas running at about 45 deg to the axis. The dark lines running in the axial direction are probably due to some structure effect or to machining strains and are to be disregarded. In the bar with the single notch, there was some bending stress present and consequently the bar yielded more on one side than on the other. It is evident from this illustration that, for a mild-steel bolt, yielding would start on a conical surface or surfaces. Whether or not a bolt from standard bolting materials at high temperature would start to yield in a similar manner cannot be



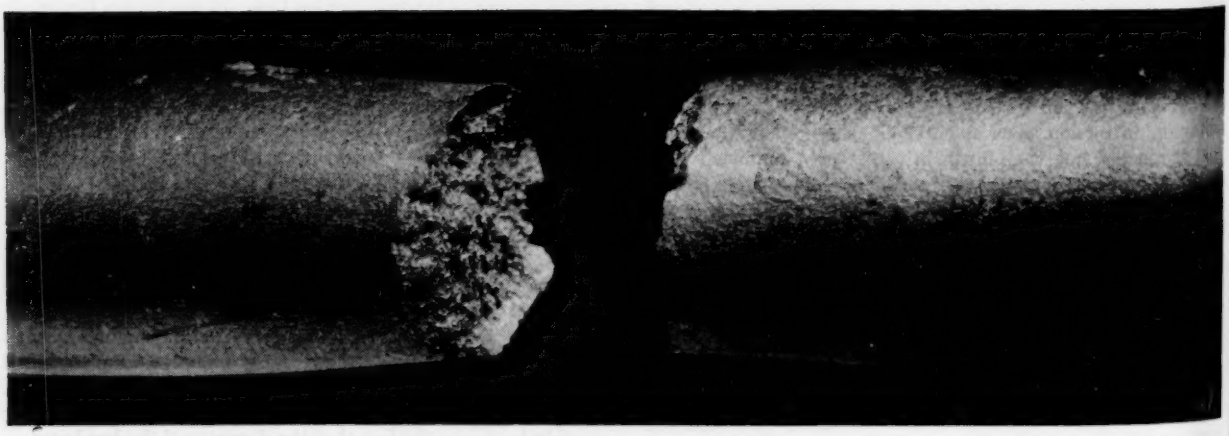


FIG. 11 FRACTURE SURFACES FROM PURE TENSION
(*N* bar with single circumferential notch; *J* cylindrical bar.)

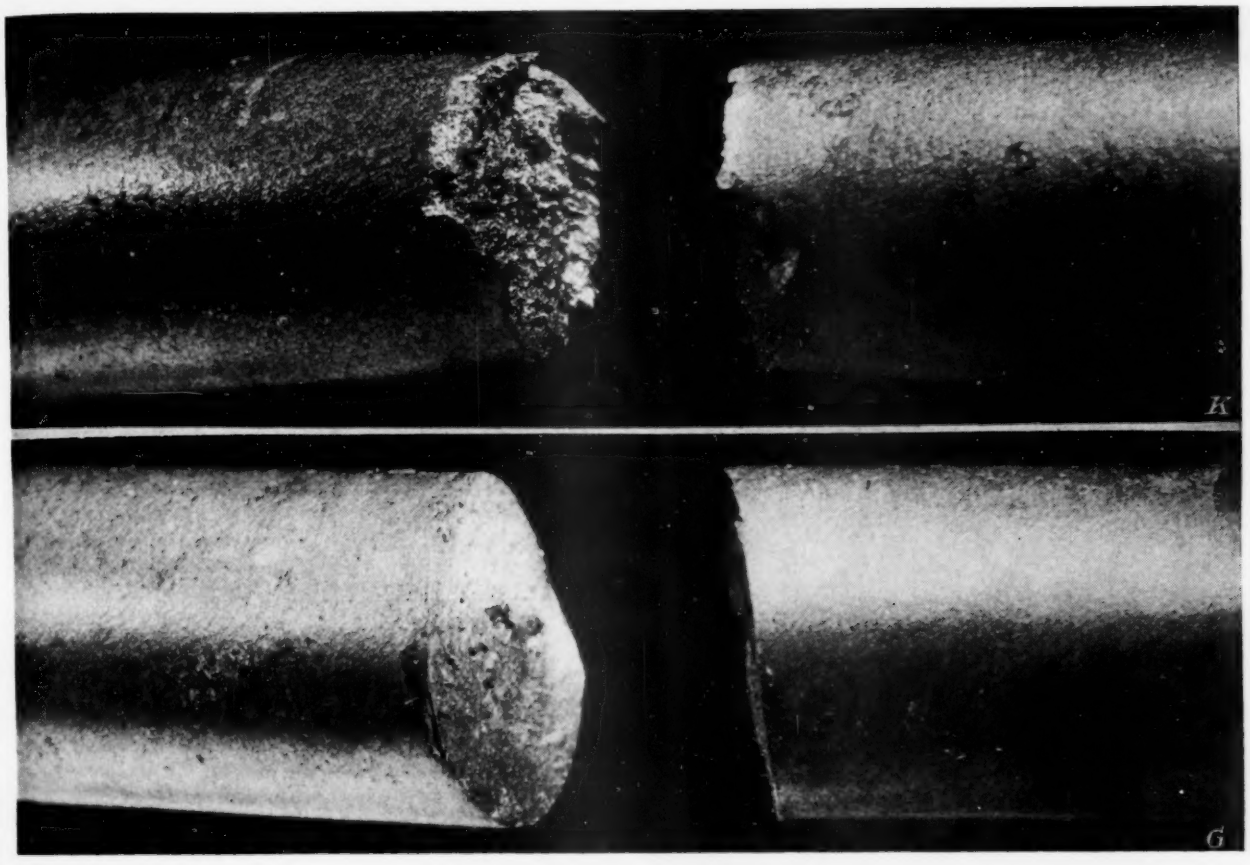


FIG. 12 TYPES OF FRACTURE SURFACES
(Bar *K* shows rupture surface of tension test; bar *G* shows break from pure torsion.)

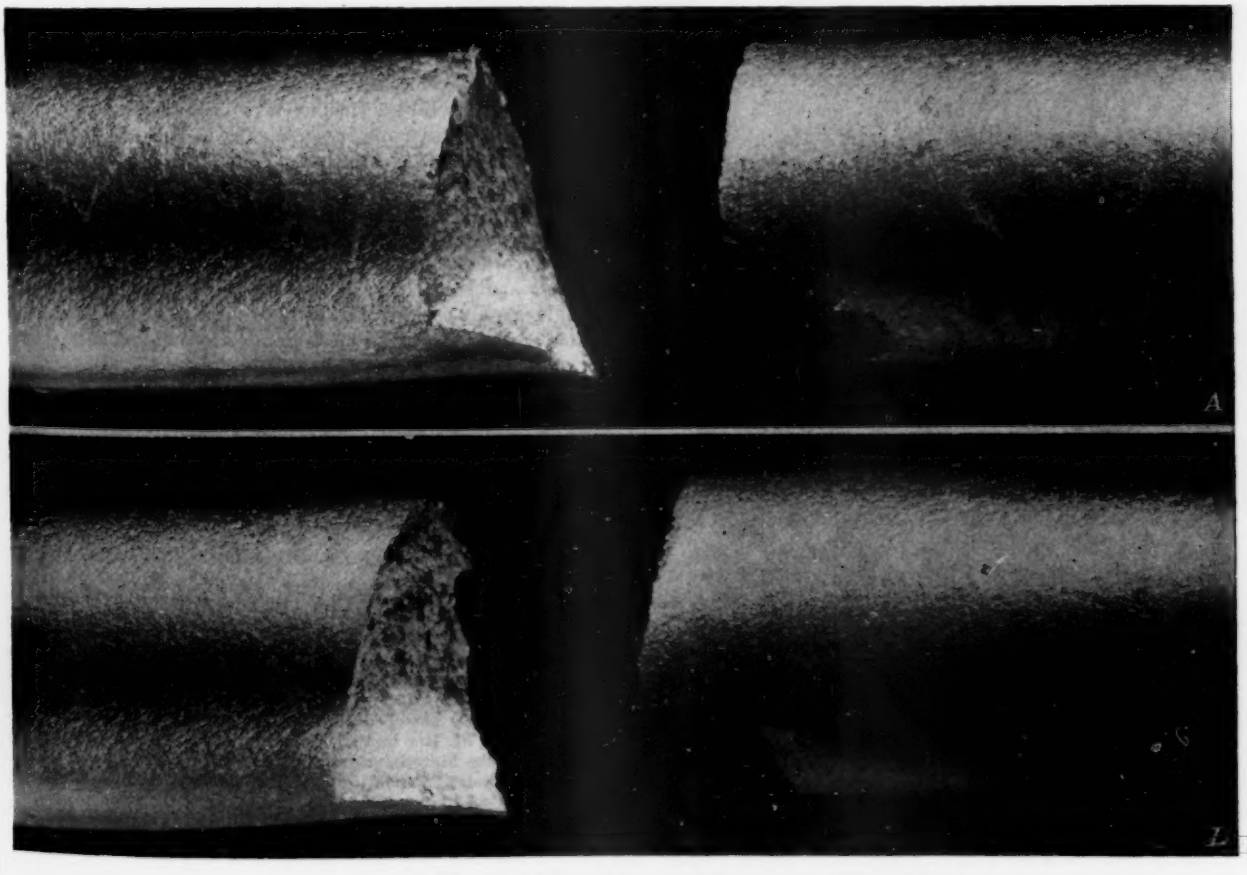


FIG. 13 FRACTURE SURFACES RESULTING FROM TENSION AND TORSION LOADS
(Bar *A* shows rupture from tension test with added torque; bar *L* tested in torsion with added tensile load.)

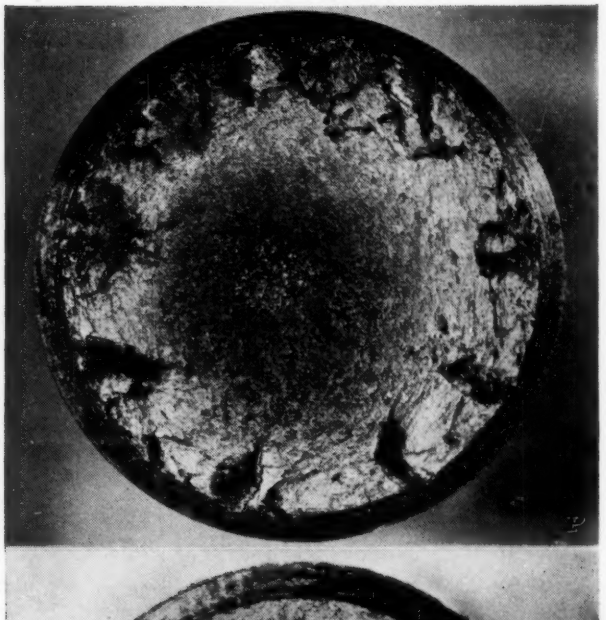




FIG. 14 FRACTURE SURFACES OF NOTCHED BARS
(Bar *P* shows fracture of a notched bar in pure torsion; bar *M* tested with combined tension and torsion.)

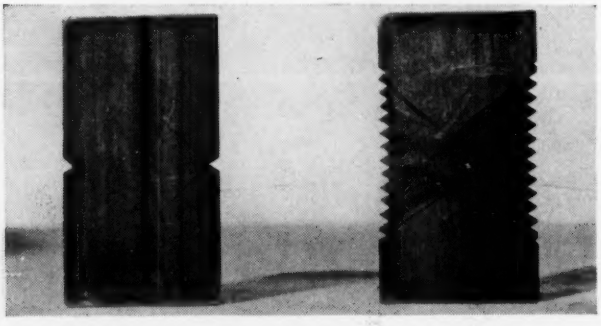


FIG. 15 INITIAL FLOW LINES AT ROOT OF THREAD

determined, for these slip lines can only be made visible on mild steel which exhibits a marked yield point. These flow lines do not show where ultimate failure will occur, but they point out where the stresses are most unfavorable in the elastic state and where the plastic strains will be localized. Ultimate failure would probably occur along a plane surface perpendicular to the axis of the bar and passing through the notch or one of the threads.

CONCLUSIONS

A summary of the test results on the cylindrical bars is shown in Fig. 16. The ratio of the maximum tensile stress in any test to the maximum stress in the pure tension test is plotted as ordinates and a similar ratio of the moments is plotted as abscissas. The test points all fall close to a circle of unity radius passing through the points of the pure tension and pure torsion tests. All points fall outside of a circle the center of which is at the origin and the radius is 0.9.

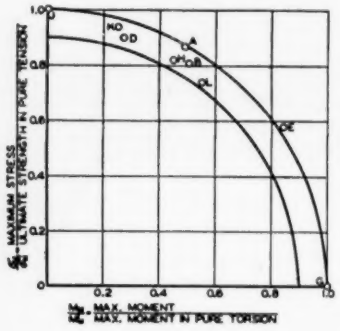


FIG. 16 TEST RESULTS

A sharp V notch in a ductile material does not cause any stress concentration that can be detected in a short-time tension test, but it does seriously affect the amount of strain at rupture. This latter statement is particularly true if torsion loads are present. In the pure tension tests, the maximum strain was reduced from 27 per cent for the cylindrical bar to 7 per cent for the notched bar. In the case of pure torsion, the angle of twist was similarly reduced from 584 deg to about 60 deg.

The fact that the stress-concentration factor for the notch turned out to be unity erased any effect which the compound notch might have shown in a more brittle material.

The amount that the bars necked down was roughly proportional to the amount of ultimate tensile load. When heavy torques were present, the bars failed before any appreciable necking occurred. Initial yielding in a circumferentially notched mild-steel bar occurs on a conical surface and not in the plane where the bar will eventually fail.

ACKNOWLEDGMENTS

The author wishes to acknowledge the help of Dr. A. Nádai who supervised this work. He also wishes to thank Mr. L. W. Chubb and Mr. F. T. Hague of the Westinghouse Elec. & Mfg. Co. for permission to present these results, and Messrs. G. H. Heiser and M. Manjoine for their assistance in making the tests.

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Discussion

C. W. MacGregor.[4] Referring to Fig. 6 of the paper, stress-strain diagrams are included in which the load divided by the original area is plotted against the over-all strain between the heads of the test specimen for various amounts of applied constant torque. It was found that the total axial strain, as usually defined, is considerably reduced when a high torque is applied to the bar. The writer has been interested recently in plotting stress-strain curves for tensile tests in which the average true axial stress P/A is plotted as a function of the true axial strain ϵ or q' obtained from diameter measurements where $q' = \epsilon = \log \frac{A_0}{A}$ (A = actual area; A_0 = original area).

It is realized that the problem of expressing the true stress-strain relationships in the cases treated in the paper is indeed complicated, as mentioned by the author. It would be of interest, however, if the author will include in his closure a comparison of the average true axial breaking stress and strain values as thus defined for the final points in the various curves plotted in Fig. 6. The axial-strain values can be determined from diameter measurements in the constricted portions of the test bars. This may give some additional information as to the effect of torque moment on these quantities.

Joseph Marin.[5] The research reported by the author is particularly important since load-deformation relations for combined stresses are seldom reported beyond the yield point and to rupture. One aspect of the combined-stress problem is the evaluation of working stresses for such a state of stress. In order to do this it is necessary to have some rational theory defining rupture. The author points out that most investigators consider the problem of defining the beginning of yielding rather than final rupture.

Considering the test results reported in the light of theories of failure as applied to rupture, it is of interest to compare tests and theories. In doing this there is encountered the difficulty of evaluating the stresses at rupture. An approximate value of the shearing stress can, however, be obtained by using relations (2), page 578 of the paper for a slope of the torque-twist curve at rupture equal to zero. With this assumption the shear stress is

$$\tau = \frac{3M}{2\pi a^3} \quad [1]$$

where M = torque at rupture and a = radius of specimen.

For tension the stress at rupture is

$$\sigma_x = \frac{P}{\pi a^2} \quad [2]$$

Considering the test results plotted in Fig. 16 of the paper, the equation of the circle representing the test points is

$$(M/M_0)^2 + (P/P_0)^2 = 1 \quad [3]$$

where M_0 = twisting moment in pure torsion at rupture,
and P_0 = axial tension in pure tension at rupture.

Placing values of the stresses from Equations [1] and [2] in [3], the relation between the stresses at rupture is

$$(\tau/\tau_0)^2 + (\sigma_x/\sigma_0)^2 = 1 \quad [4]$$

Using the values of the stresses obtained experimentally, Equation [4] becomes

$$(\sigma_x/\sigma_0)^2 + 1.84(\tau/\sigma_0)^2 = 1 \quad [5]$$

Equation [5] defines the experimental relation between the stress components. The theoretical relation corresponding to this, using the shear theory is

$$(\sigma_x/\sigma_0)^2 + 4(\tau/\sigma_0)^2 = 1 \quad [6]$$

In the same way using the stress theory

$$(\sigma_x/\sigma_0) + (\tau/\sigma_0)^2 = 1 \quad [7]$$

For purposes of comparison Equations [5], [6], and [7], and the test results are shown in Fig. 17 of this discussion. An inspection of this illustration shows that the stress theory agrees best with the test results except for the test in pure torsion. On the other hand, the beginning of yielding for ductile materials is closer to the shear than the stress theory. It is necessary, therefore, to use a different basis in defining working stresses, depending upon whether the yield point or ultimate is considered.

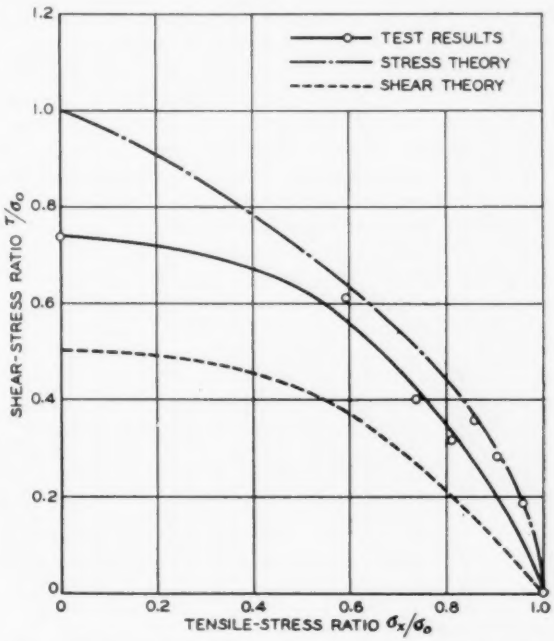


Fig. 17 Comparison of Test Results With Shear and Stress Theories

TABLE 1 ENERGY REQUIRED FOR RUPTURE

Specimen	Tensile-stress ratio σ_x/σ_0	Torque ratio (M/M_0)	Energy in in-lb per cu in. $\div 10^4$
J	1.00	0	2.02
K	0.95	0.25	2.33
D	0.90	0.38	2.12
A	0.85	0.49	2.06
H	0.82	0.44	2.25
L	0.74	0.55	2.37
E	0.59	0.83	3.15
G	0	1.00	3.10

A consideration of the energy required for rupture in the foregoing tests is of interest. Table 1 of this discussion shows the energy in in-lb per cu in. for all of the specimens tested. The values show that there is not a constant value of energy for rupture but an increase from pure tension to pure torsion.

Author's Closure

The writer appreciates the interest shown by Professors MacGregor and Marin in their discussion. The information regarding the dimensions of the necked portions of the plain (those without notches) bars is given in Table 2 of this closure.

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TABLE 2 STRESS RATIOS BASED ON REDUCED DIAMETERS

Bar	Diam at rupture, in.	Tensile stress at rupture, lb per sq in.	Tensile stress ratio	Shear stress[a] at rupture, lb per sq in.	Shear stress ratio
J	0.425	131,000	1.00	0	0
K	0.425	112,000	0.85	35,800	0.58
A	0.472	79,300	0.60	50,500	0.82
E	0.533	26,000	0.20	64,000	1.04
G	0.563	0	0	61,200	1.00
L	0.463	90,200	0.69	61,300	1.00
H	0.439	112,500	0.86	42,200	0.69
D	0.428	129,000	0.98	18,750	0.31
B	0.465	99,000	0.75	46,800	0.76

[a] Based on Equation 1 of discussion.

In regard to Professor Marin's analysis of the shear stresses at rupture it may be well to repeat again that it is difficult to evaluate the stresses and strains at rupture. If Equation [1] of the discussion is used it must be remembered that in several of the curves the slope $dM/d\theta$ is not zero but is actually negative. If, however, this equation is used in conjunction with the reduced diameter at the necked portion and also if the reduced area is used to compute the tensile stress at rupture the stress ratios given in Table 2 are obtained. These results which are plotted in Fig. 18 will agree much more closely with Equation [6] than with Equation [7] of the discussion. This is evident from the fact that the maximum shear stress in the pure torsion test is roughly one half of the maximum tensile stress in the pure tension test and that most of the points lie close to the circle

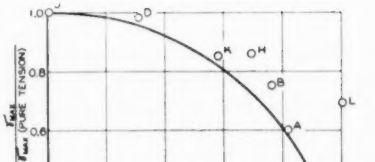


FIG. 18 RUPTURE STRESSES BASED ON REDUCED DIAMETER

drawn in the figure. This is not offered as an ultimate analysis of the stresses at rupture but is merely given to point out the improbability that the actual stresses would agree with a maximum-stress theory of strength.

Since the paper was presented, the carbon and manganese content of the steel used in the tests has been determined. Chemical analysis showed that the steel contained 0.47 per cent carbon and 0.71 per cent manganese.

The Engineering Method in Management

By ANDREW I. PETERSON,[1] NEW YORK, N. Y.

Professional management in modern industry demands competence and vital direction of a high order, approached only with scientific objectivity. Engineering and economics are the basic tools of the profession. The author reduces the problems inherent in applied economics to four types: The selection, replacement, and apportionment of elements of production; the analysis of optimum enterprise economy; the rationalization of the dynamics of specific industries; and the interpretation and guidance of the trends and adjustments in the parent social environment. The paper indicates the necessity for more critical research and broader training in industrial management.

THE transformation of a people in primarily local agricultural pursuits to an industrial society with complex and extensive markets is generally designated as industrial revolution. Since engineering has been a principal factor, it has long had concern for the organization, direction, and economy of such enterprise. But the economic problems created by technological progress have taxed our collective ingenuity with recurrent maladjustments sufficient to jeopardize free private industry. Professional management has been described most frequently as industrial engineering, or more elegantly as scientific management. But many engineers unfortunately have concerned themselves with the techniques of their specialties, allowing their responsibilities to management to be those of technicians and clerks. Truly professional management in modern industry demands competence and vital direction of a high order, approached only with scientific objectivity. We must give constant and critical scrutiny to our methods and qualifications.

Scientific method can briefly be regarded as a way of developing the stability of beliefs by minimizing errors and constantly searching for contradictions. We strive for absolute objectivity in a progressive spiral of hypothesis and test. It must be recognized, however, that in the social sciences it is difficult to exclude thoroughly the subjective or normative point of view. Scientific management, by definition directed to the most efficient performance of industrial enterprise, may involve such norms, particularly with its background of extreme individualism, concern for the welfare of labor, and an increasing responsibility for the rationalization of whole industries. Norms of political origin are not entirely separable from concepts of economic organization.

Competition, an Agency of Selection and Development

Competition has been specified as associating struggle with order; an agency of selection and development, if we recognize a Darwinian scheme of life. History describes the mutations of feudal dominance to the ideals of individual liberty to pursue and retain the yields of effort. If individual incentives are supplanted by abstract group responsibilities, it seems realistic to expect that the performance of the efficient would tend to a popular level. We have observed this in our wage experiences, in the recent Russian land experiments, and in many other cases.

That the competition underlying the petty trade of the days of Adam Smith should have suffered distortion and specialization in the evolution of modern enterprise is obvious. It now exists on many fronts of price, quality, advertising, and intangible services. We have groups of individuals in joint enterprise, still motivated individually by hope of gain but under an imperfect development of corporate mechanisms and an insulation of individual risks from control. Institutional presumptions in administration and finance have often driven the other factors of production to positions of marginal disadvantage. We have concentrated jurisdictions over market price with a logical maximizing of profits at a production volume below capacity and at a price above the social optimum. Overinvestment and rigid capitalization are additional concomitants with low utilization of capacity, high unit overhead, and a downward pressure upon price which, once released by a singular act of competition within an industrial group, leads to ruinous price war and instability.

Fundamentally, we appear to have two alternatives, namely, to discard the economic efficiency of large-scale enterprise by attempting a return to the atomistic competition of another century, or better, to adapt our still imperfect enterprise mechanisms for equitable competition in a broader sense than a ban upon "bigness." Many believe that support can be organized for reasonable but more effective police power which, through professional commissions and the use of the consent decree, will permit more efficient solutions of these problems to originate with the experience of business itself. Objective management and professional competence in both private and public practice are vital to the continuance of private enterprise on an economic scale. Failing this, the alternatives are unpleasant to contemplate.

It has taken many years to identify scientific management with a method rather than with a technique. A review of its development and the work of the truly professional men who have carried on the vision of Taylor, Gantt, and Gilbreth would be repetitious. Some of the procedures of recent years, however, seem predicated upon subjective techniques and many of our engineers appear to enter industry with untoward sophistication. The comments in this paper are therefore offered in examination of a few of the basic tools of professional management.

In classifying the infinite range of responsibilities, we find a convergence upon such derivatives as technical engineering, theoretical and applied economics, accounting, statistical analysis, and psychology. Economics and engineering are the warp and weft of the fabric. With more immediate objectives in production, we might visualize the problems of applied economics concerning the engineer as island families of contours, each containing more specific central issues surrounded by successive levels of environment with increasing complexity and scope. In each one, applied analysis is prone to make predictions of representative constants in place of the dependent variables relating them. The problems might be reduced to four types, namely, the selection, replacement, and apportionment of elements of production, the analysis of optimum enterprise economy, the rationalization of the dynamics of specific industries, and the interpretation and guidance of the trends and adjustments in the parent social environment. Such involution demands competent and professional insight rather than pragmatic philosophy.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

The insularity of specific design problems makes them appear concerned with elementary out-of-pocket economy alone, while in fact highly dependent upon the variables in each circumscribing environment. In selections between new alternatives the simple reckoning of current costs is confused with divergent concepts as to the annual equivalence of depreciable durable assets. Physical lives of general experience are sometimes postulated for probable economic lives in keeping with the obsolescence and actual utility of specific instances. Interest rates upon creditor funds are frequently used in place of anticipated gross rates of return upon capital. Increment- and sunk-cost concepts are imperfectly understood and applied in many situations. In the general employment of minimum-cost-point and break-even-point analyses for multiple alternatives, we have a frequent desire for universal working formulas which lead to mistakes in numerous specific applications. Economic lot-size formulas are frequent offenders in this regard.

Fallacies in Replacement Economy

In replacement economy we appear to have additional fallacies. Each problem strives to compare the significant factors of prospective costs of a new machine, method, or structure with the prospective costs of continuing in service the existing installation. The capital recovery costs in the latter seem properly based upon net realizable value and not the irrelevant book value of accounting, adding of course any modernizing investment necessary to equate reasonably the expected utilities of old and new. The use of book value creates a fiction since the true capital sacrificed by continuance is the realizable value.

There are accessory variations. A few claim that actually recovered capital should be deducted from the first cost of the new alternative in such comparison. This misconception is easily disposed of with the question: How much capital will be actually impounded in the new equipment? Others might profess that these contentions are met by including full book value for the old equipment and, for the new, the old machine's unamortized book value plus first cost. Here the error might arise in equal differences being written off at dissimilar lives in the average case. As to the procedural question concerning the disposition of the discrepancy between book and realizable values, it seems eminently proper to enter it against surplus. The higher book value, born of reluctance to reflect actual current losses in fixed assets, causes overstatement of the operating profit in like degree during the earlier part of the life. Still another form of error lies in the occasional treatment of indirect expense or burden. Being commonly expressed as a rate of some direct cost such as labor, it is often assumed in the case of some labor-saving investment, for example, that an actual reduction in overhead takes place. A realistic approach considers only the absolute changes in each item of indirect expense. The theory and practice of such engineering economy are perhaps nowhere more soundly presented than in the work of E. L. Grant (1).[2] If the degree of rationality which he presents were generally practiced, it would seem that many of the burdensome commitments in plants today would not exist.

In the circumscribing unitary aspects of enterprise we have similar problems, many of them susceptible to advanced quantitative analysis. It is easily established, for example, that the variability of cost functions with output, similar to the natural performance characteristics of machines, is peculiar and stable for every organization and situation. There has been much use of elementary break-even-point analysis, but the apparent lag in extending the introductory method of Rautenstrauch (2), Knoeppel (3), and others is a matter of some curiosity, particularly in view of its success in the management of power-utility industries. Statistical analysis of fixed- and variable-cost performance is vitally significant to finance, budgets, and forecasts. Realism in the internal dynamics of an enterprise is increasingly necessary for flexibility and survival in constantly changing environments.

As our scope proceeds beyond the internal systematics, we embrace the complex ramifications of general applied economics. Even a simple enumeration of the economic and statistical investigations appropriate to modern administration is impossible here. Much of it is of immediate current importance to executives in the timing of purchases, plant investment, or production activity. Other phases relate to the complex significances of change in the general economic environment and the incidence of policies promulgated by enterprises and industries upon the whole economy. We have, however, a variegation of economic doctrine on one hand and on the other a proclamation of half truths and impossible hopes by political extremists. Sincere businessmen are needful of action while scholars procrastinate. It appears inevitable that realistic utility of applied economics must come by way of more advanced economic scholarship in professional management.

Relation of Accounting to Economics

Completely integrated with economic activity is the institution of accounting and its characteristics are pertinent to this discussion. Its principles regard asset values as based upon original cost, except for current assets which are reported at cost or market, whichever is lower. The approach to earning statements is never to anticipate a profit but to provide for all losses. In economics, we fundamentally interpret true earnings or losses as being increases or decreases in actual net wealth. In accounting practice, employing original cost and barring unrealized profit, we have inimical fictions. If assets are reported above their true values, so must be the profits, regardless of the allocation of profits and losses between balance sheet and operating statement. The net worth is out-of-line in either case.

Accountants justify their approaches in various ways. Original costs are often defended upon the grounds of going-concern concepts, undervaluations upon conservatism, and both upon expediency and expense. Going value has had its particular uses but has become a too frequent defense of these discrepancies. In essentially permanent ownerships, a variation in valuation is adjusted by the corresponding long-term level of reported earnings but, with the diverse and short-term ownerships of today, the average stockholder or creditor must rationalize his position with distorted criteria. The claims of conservatism in undervaluation seem equally indefensible upon the same grounds. As for the regular and periodic reappraisal of assets, it has been competently maintained that such procedure need not be impracticable or costly.

Increasing criticism is jeopardizing the future of accounting and the value of much professional engineering research based upon it. It has been stated that accounting must either revise its concepts or be reduced to a level of secondary importance in industry. Some of its spokesmen have attempted to clarify the situation by a sermon of pure conventionalism for the balance sheet and an assignment of predominant accuracy to the operating statement. But there would seem to be an inevitable correlation of their inaccuracies and a futility in such approach. The author is not unmindful of the difficulties illustrated by the conventional requirements of public agencies but the fundamental and necessary realignments are inescapable.

Statistical Method

The third essential, which will be commented upon, is sta-

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

tistical method. We are generally familiar with its importance in all sciences and its powerful modern tools of probability analysis, although some engineering empiricists still popularly consider it as a largely manipulative presentation of data. Its momentous extension to product-quality control in scientific management has evoked a growing realization of its significance in other professional specializations. In time-and-motion studies there are promising speculations of more scientific method. Because of the elementary requisites in equipment and observational skill, and the permitted latitudes of judgment, time-study specialization has grown in popularity with remarkable strides. A good, if not major, proportion of the work done has been viewed as being of questionable value. Managements have in many cases appeared to give insufficient concern to the caliber of young men selected for it. Many of them have been trained in certain techniques of observation but it seems fair to question the number implemented for efficient and objective interpretation. We can easily visualize the potential destruction of managerial control and human values arising from superficial studies.

Time-study observations consist principally of elapsed times for the operating elements of a given job cycle, repeated for reliability from 10 to 50 times. Interpretation aims at establishing a representative time in each standardized element for production planning. The cycle of selected element times is ordinarily adjusted to a good performance level by studies of the concomitant variations of skill and effort. Further transformation produces the long-run performance standard based upon studies of reasonable net productive time per day in each class of work. The final prediction is a basis for wage-rate setting. The variously sponsored techniques are largely empirical and use statistically small samples of observations. It is fully understood of course that the degrees of analysis and accuracy permissible in practice are functions of the size of operations, and that labor's comprehension of the results is a significant factor. The fragmentary speculations raised consider the research that may prove fruitful in establishing more reliable tests and standards of procedure.

Man, like nature, cannot duplicate performance exactly, since many causes operate with unknown frequencies within any one system of conditions. If the latter remains constant, we have some degree of statistical control and establish characteristic limits of variability and credibility to make predictions of repeated performance. Obviously not all systems are found constant or controlled.

In the conventional study of operating elements, we have a row of repeated observations which fluctuate about a central tendency. We face the problem of estimating the underlying universe parameters with reasonable credibility from the corresponding sample statistics, in most cases circumscribed by small-sample errors and some uncertainty as to the form of the distribution function. Care is needed in the selection of sample sizes conforming to the degrees of variation and desired accuracy, and of statistics bearing optimum efficiency in such sample size. The representative time most often used is the mean, sometimes the mode or median.

A few methods attempt to reflect the degree of variation, either to eliminate wild readings or as a part of leveling for continuous performance. The Merrick method, for example, discards extreme readings deviating from their adjacent times by more than approximately 30 per cent of the adjacent times. It also averages the ratios of each mean to its remaining minimum time as a supposedly stable dispersion characteristic of the worker. Superficial tests upon one set of data would appear to suggest that the variation is more peculiar to the operation than the worker. In the description of the Merrick method, there is implied the possibility that minimum times in specific elements are inherently constant for all workers. In view of the increasing fluctuations due to sampling as we go from the median reading to the extreme reading, which are much larger in small samples than in large, it is hazardous to base estimates upon one extreme observation, particularly if arbitrarily selected. If substantiated, this promising hypothesis can be implemented with more reliable statistical tools.

Study of Forms of Time Distribution Needed

The forms of the time distributions also might be more usefully studied. Underlying each operation we have a finite but unknown number of causes contributing time minutiae with individual probabilities of occurrence. The theoretical frequencies of various possible times are those expected in long-run observations under control. Where we have equalized individual cause effects and uniformity in their probabilities of occurrence, the theoretical distribution is based upon the point binomial, positively skewed when the causes are relatively few in number and approaching the bell-shaped symmetrical curve when increased to the limit. In any case it decreases "smoothly" from the mode. When there are predominating causes, these distributions take irregular forms, depending upon the factors present. When we lack a priori knowledge of these causes the distribution smoothness, degree of variability, and symmetry may suggest the degree of control present. Further, the employment of χ^2 against fitted theoretical or graduation functions can give us some test of conformance to control. Do we not have here a promising criterion of the efficiency of motion-study work?

In the establishment of truly representative element times, however, it would seem necessary to make more complete patterns of observation than those taken for short time intervals with one or two operators. The results are dependent upon the time of day, variations from day to day, differences between operators of equal skills and efforts, and actual differences in skill, all in addition to the basic variation within the operation itself. With reasonable care in design, it is possible to take observations over a range of hours, days, operators, and observers by organized sampling, providing complete information for all purposes. Advanced modern methods of statistical analysis presented by R. A. Fisher (4), W. A. Shewhart (5), and others (6, 7)[3] permit practical tests of control, extensive separation of the variables involved, and greater accuracy in determining representative times and variations. Further, the distribution of complete cycle times is a sum function of the element time distributions, where theory establishes their relationships as to mean and variance, depending upon the degrees of correlation between element distributions. With this in mind, the distribution of cycle times has interesting possibilities because of its tendency to greater control. Investigation may support the practical prediction of cycle variance as well as mean time, permitting the use of probability in forecasting performance levels and subsequent operating controls as established in statistical quality-control methods.

It is hoped that these speculations suggest lines of research and a new approach. The statistical competence required for development should not prove unreasonably advanced. As for any misgivings as to the simplicity and economy of such approaches in actual plant applications, the final procedures do not promise any greater involution than other standards resulting from management research.

In general, this paper presents a few thoughts in the direction of more critical research and broader training in industrial management. Engineers in practice, and on college faculties too,

[3] Several working texts for analysis of variance and covariance have recently appeared. The two references cited in the Bibliography are representative.

have a share of human complacence as to their general proficiency in fields outside of immediate experience. Industrial engineering may not have as many of the handicaps of inbred technical specialization, but with three-dimensional responsibilities and opportunities—technology, economics, and scientific management—its success as a profession seems to turn upon a level of fundamental competence which demands the best talent that engineering schools can produce. In management's rapid growth, however, there has been a tendency to conventionalize its practice and research. Rationalization of the rigidities of modern dynamic economy seems to demand a concurrence of experience and scholarship at higher levels than ever before. The marriage of engineering and realistic economics promises a stock that will improve upon the existing parent strains.

The responsibility for development is obviously divided between the engineering schools, industry, and the professional societies. In the first, as recently pointed out by Robert E. Doherty (8), the revision of engineering curricula is hampered by unwarranted insistence upon specialization, vested interest, and rigid viewpoint. To industry would appear to belong the applied specializations in each field, making possible more advanced scholarship in fundamental engineering, economics, and management within the usual span of undergraduate and graduate work. The professional societies bear equal responsibility. It is hoped that the Management Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS can continue to support actively a critical and progressive integration of research and publication, interesting the best minds in our profession so that the rest of us may more efficiently serve our common purpose.

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Discussion

E. L. GRANT.[4] Two comments occur to the writer as pertinent to the author's observations on incorrect methods in economy studies and his interesting summary of critical views on conventional accounting practices:

1 The voices of those who are critical of the misuse of accounting figures have not yet been heard by a great many of the users of those figures. This statement is based in part upon personal observation and in part upon the confusion of thought evidenced in current writings on such matters as replacement economy.

2 There is no way in which accounting practice can be reformed so that accounting figures may always be used without modification as a safe guide to business decisions. The diversity of business alternatives to be compared is too great for their differences to be measurable by any routine systematic process such as accounting necessarily must be.

A recently observed incident may serve as a simple illustration of the latter point. An engineer redesigned a certain part for a technical product to reduce its cost. The new design effected a $2 reduction in the direct material cost of each unit but involved a $1 increase in its direct labor cost. This he believed to be a $1 cost reduction per unit.

However, the cost-accounting system in the plant allocated indirect manufacturing expense in proportion to direct labor cost; the "burden rate" in this department was 175 per cent of direct labor cost. Thus the $1 increase in direct labor cost was responsible for a $1.75 increase in allocated indirect manufacturing expense, making the net result of the new design an increase of 75 cents in the unit cost as shown by the accounts. Despite this indication by the accounts that the new design was uneconomical, a critical consideration of the facts showed that the company would really save money by using it. None of the individual items, e.g., superintendence, shop transportation, cleanup, heat, light, power, depreciation, taxes, and insurance, which entered into indirect manufacturing expense seemed likely to be altered by the proposed change in design.

At first glance one might be inclined to say that the cost-accounting system should be reformed if it seemed to show costs going up as a result of a policy which actually made them go down. However, careful examination of the circumstances indicated that this was not the correct inference. Considering all of the various purposes of cost accounting, this cost system seemed as good a one as could be devised without excessive clerical expense. What was really necessary was a recognition by management that, to compare specific alternatives, it was essential to look beneath the surface of the cost figures to find the true difference between the alternatives. No possible reform of the cost system could eliminate the necessity of doing this in some cases.

P. T. NORTON, JR.[5] In referring to fallacies in replacement economy, the author correctly states that the capital-recovery costs of the existing installation should be based upon its net realizable value and not upon its irrelevant book value. There are so many ways of proving that the book value of an existing installation should not be used in an economy study, unless it happens that this book value is equal to the net realizable value, that the writer is unable to understand why this obvious fallacy should appear so frequently in discussions of replacement economy.

While the author has proved that any such use of the book value of an existing installation is incorrect, it seems to the writer that there is another way of disposing of this fallacy which should be even more readily understood by the average person. It is obvious that one of the principal purposes in setting a depreciation rate is to bring the book value down to the realizable value at the time the asset is displaced. If the book value is not equal to the realizable value at the time the asset is displaced, it is because the book value is in error by the amount of the difference. Replacement-economy studies should be based upon conditions as they will be if the replacement is actually made and not upon what it was estimated some years ago would be the situation today.

J. A. PIACITELLI.[6] In appraising the time-study work, the author points out that a good portion of it has been viewed as being questionable. The writer is in sympathy with this view, for many a study has been rendered incredible by improper methods of observation, inadequate sampling, improper analysis of the

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data, or inappropriate allowances. No matter how much one may do to establish a credible result in any one of these phases, the final answer, or standard, can only be as accurate as the degree of accuracy of each of them. It would be of but slight benefit to work to a high degree of accuracy in the statistical analysis of data when there is doubt as to whether the allowance for fatigue and unavoidable delays should be 10, 15, or 20 per cent.

Viewing the problem as a whole, that is, the development of adequate standards of performance, it is the writer's opinion that attention should be directed toward the standardization of the technique. It appears that we have been too busy standardizing manufacturing operations to get together, examine our own technique, and standardize it. In indicating the desirability of research toward this end, the author has indicated a cause which when removed will contribute greatly toward a more uniform procedure and consistent results.

As for the statistical analysis of data, it is not felt that more accurate methods are needed than the simple ones now at our disposal. Determinations are made for (*a*) the selected minimum, (*b*) lowest third, (*c*) mode or modal average, (*d*) median, (*e*) selected average, and (*f*) the arithmetic average. Which of these is used in the establishment of a standard depends upon the point of view of the analyst. Their use in turn affects the amount of allowance to be applied. For instance, a much higher-percentage allowance must be applied to the selected minimum than the average. What we need is a more common use of the statistical method which would result in the most representative measure of the operator's performance. Investigation by the writer into the relative values of these various statistical methods has led to the adoption of the mode, or modal average, as the simplest means that would yield the most representative results. The modal average, being the value about which most cases recur, is the most reliable method of determining the characteristic or typical performance of an operator. This determination is made possible with relatively few observations as compared with the number that are required when using the method of averaging.

In this connection the writer would like to present for consideration a few facts pointing to the degree of credibility which may be placed upon the modal-average method. What the writer has in mind is the time required to perform an element of a weaving operation in the silk-textile industry. On the basis of slightly over 1000 observations made in approximately 20 silk and rayon mills, the modal average was 0.098 min. Adding to these about 7000 more observations, the modal average was still 0.098, while the arithmetic average dropped from 0.124 to 0.108, or 12.9 per cent. It is interesting to note that the arithmetic average, becoming more representative as the number of observations is increased, approached the modal average to a point within 0.01 min when the quantity of data was increased sevenfold.

Taking at random the observations made on one weaver during one study period, the modal average was 0.116 on the basis of 46 occurrences. When this study was extended to 262 occurrences, the modal average was 0.115 or a change of 0.9 per cent, while the average increased 5.6 per cent. It appears from these data that the modal-average method yields the most consistent results.

The "selected average," determined by the elimination of abnormal readings, is often used as a means of approaching the representative time, but the element of judgment tends to place it on a selective rather than factual basis.

The writer was interested in the author's suggestion pertaining to tests of conformance to control. While it may be desirable to investigate the possibilities of predicting cycle variances "permitting the use of probability in forecasting performance levels and subsequent operating controls," the writer is somewhat pessimistic of the fruitfulness of such an endeavor. A frequency distribution of data is the result of many factors:

1 Degree of standardization of method.
2 Effectiveness of the method (a poor method as well as a good one may be highly standardized).
3 Tools, equipment, and work place.
4 Operator's automaticity in the method.
5 Application or attitude of the operator.

Keeping these factors in mind, the smooth distribution of data, degree of variability, and symmetry may suggest the degree of control present, but it is not believed they will serve as a measure of the efficiency of motion-study work. A symmetrical, smooth frequency distribution with low variability indicates to the writer that the operator is earnestly following a method which is highly standardized. However, it does not suggest that the method is the best which can be devised. Therefore, the writer could not consider the characteristics of the distribution as a criterion of the efficiency of motion-study work. Again, when we have a frequency distribution which is uneven and highly variable, it would appear rather difficult to determine from the array of data whether the operator lacks the required degree of automaticity in the method or is not applying himself, or both.

In closing, may the writer suggest that our immediate attention be directed toward the problem of allowances as one of the steps in the establishment of standards of performance which is far below the recording and analyzing from the point of view of accuracy. A real contribution can be made in supplementing the meager amount of factual data now available to us on this subject. Based as they are largely on judgment and supported by a few scattered actual studies, they tend to nullify the accuracy now possible in our observation and analytical work.

WALTER RAUTENSTRAUCH.[7] The author quite properly emphasizes the need for a more scientific procedure in dealing with the problems of management. It is true, as he says, that we have been concerned more with techniques of management, that is, formulas and rules than we have with the broader concepts of management out of which may be evolved fundamental and basic principles to guide us in the operations of our industries.

The paper states: "Professional management in modern industry demands competence and vital direction of high order, approached only with scientific objectivity." As a matter of fact, how much scientific objectivity is permitted in the management of business enterprise? Those who are engaged as professional consultants in the field of management are expected to show their clients how to meet the competitive conditions of the market place. These clients, being very "practical" people, expect profits to flow from such advice. Frankly, much of the work which has gone under the name of management, particularly that involving revisions in the methods of wage payment, standardization of jobs, etc., has largely resulted in lowering the wage cost of production and in altering the relative claims of capital and labor to the goods produced to unworkable proportions.

Management engineers have never, except in rare instances, examined into the economic consequences of these so-called systems nor taken into account their effect on the national economy as a whole. The writer fails to see how we can have any scientific objectivity on the larger problems of management so long as management is expected only to produce profits for individual enterprise without regard to the effects on the entire national economy. As the author points out, scientific management should be "directed to the most efficient performance of individual

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enterprises." This raises the question of the units in which efficient performance shall be measured. After an extremely rapid growth in so-called efficient performance of industrial enterprise from 1918 to 1929, we suffered a most disastrous collapse in our industrial economy. Was there real efficiency during this period? What was it we did not take into account as managers of industrial enterprises which caused our industries to break down?

Our engineering societies have been lax in addressing themselves to this most important problem. Perhaps this may be due to a misinterpretation of what the author refers to as the "Darwinian scheme of life." The misinterpretation of Darwin's theory has perhaps done more harm to the national economy than most people realize. The world has yet to learn that only by an intelligent integration of the processes of civilization can we hope to attain a high rate of productivity and an increase in the standard of living. It is true that the solution of the problems of industrial enterprise involves continuous struggle but the object of this struggle, in the writer's opinion, has been falsely directed. It has been directed mainly toward competition between the elements of the national economy, which should be operated on a cooperative basis, rather than upon a competitive basis.

Private enterprise was quick to learn, as it did since the beginning of this century, that its growth and development depended upon an intelligent integration of its departments of operation. How long will it take us to learn that the same intelligent integration of the parts of the national plant as a whole must be accomplished before we can stabilize its performance?

The questions of detail of operation within an individual unit of our economy which the author raises are important and should be inquired into more fully as he suggests. We do need to apply scientific principles to many of the problems that we now solve empirically. We also need to re-examine many of the so-called scientific solutions to determine whether or not they are really scientific.

Very often we have gone into elaborate statistical processes in the solution of problems before we have determined fully the functional relations of the elements about which we are endeavoring to obtain statistical interpretation.

The writer presented a paper[8] before the Philadelphia Section of this Society in 1917, in which was pointed out the imperative need for a closer integration of our banking system with our manufacturing enterprises and the need for their integration on a functional basis. In that paper a quotation was made from the records of The Annals of the American Academy of Political and Social Science, concerning the manner in which the Commonwealth of Massachusetts dealt with the problem of farmer and banker relationship, as follows:

"Several of the banks have opened special farm departments and employed men whose business it is to investigate the applications of farmers for credit. The Plymouth Trust Company, of Brockton, has for two years employed two men, graduates of the Massachusetts Agricultural College, to aid the farmers in applying business methods to the business of farming. The object of the directors of the institution was to get acquainted with them so as to make a businesslike application of credits to those engaged in this important industry. This bank has helped the farmers of their vicinity to buy seed, livestock, etc., and stimulate production by offering prizes to the young people on the farm. It is showing the farmer how to keep cost accounts and how to make cost statements; in short, to know his business both from the technical and from the business standpoint. To worthy persons they stand ready to make a small loan to be used for construction work or for improvement under the supervision of the bank's field agent. Every banker will ask himself, "Does it pay?" It has cost the Plymouth Trust Company $4000 a year net to supply this service to farmers in and about Brockton, but as a result of this and similar activities the deposits have increased in the last five years from $400,000 to $3,000,000."

What forces in our national economy have militated against a further extension of such relationships, not only between the bankers and the farmers but also between the bankers and the manufacturing industries. While it is true that we need better scientific methods to solve the internal problems of industry, we should not close our eyes to the fact that by far the most important problems of management lie in the area of our total economy. While it is almost universally believed that Adam Smith developed the principles of the laissez faire economy, and most people are of the opinion that our economy can be operated most effectively according to these beliefs, it may be instructive to read what Adam Smith actually wrote[9] about some of the problems which he saw developing in the economy of the western world:

> The progress of the enormous debts which at present oppress, and will in the long run probably ruin, all the great nations of Europe, has been pretty uniform.
>
> In the reign of King William, and during a great part of that of Queen Anne, before we had become so familiar as we are now with the practice of perpetual funding, the greater part of the new taxes were imposed but for a short period of time (from four to seven years only), and a great part of the grants of every year consisted in loans upon anticipations of the produce of those taxes. The produce being frequently insufficient for paying within the limited term the principal and interest of the money borrowed, deficiencies arose, to make good which it became necessary to prolong the term.
>
> In consequence of those different acts, the greater part of the taxes which before had been anticipated only for a short term of years, were rendered perpetual as a fund for paying, not the capital, but the interest only, of the money which had been borrowed upon them by different successive anticipations.
>
> A sinking fund though instituted for the payment of old, facilitates very much the contracting of new debts. It is a subsidiary fund always at hand to be mortgaged in aid of any other doubtful fund, upon which money is proposed to be raised in any exigency of the State. Whether the sinking fund of Great Britain has been more frequently applied to the one or to the other of these two purposes, will sufficiently appear by-and-by.
>
> In Great Britain, from the time that we had first recourse to the ruinous expedient of perpetual funding, the reduction of the public debt in time of peace has never borne any proportion to its accumulation in time of war.
>
> When funding, besides, has made a certain progress, the multiplication of taxes which it brings along with it sometimes impairs as much the ability of private people to accumulate even in time of peace as the other system would in time of war. The peace revenue of Great Britain amounts at present to more than ten millions a year. If free and unmortgaged it might be sufficient, with proper management and without contracting a shilling of new debt, to carry on the most vigorous war. The private revenue of the inhabitants of Great Britain is at present as much encumbered in time of peace, their ability to accumulate is as much impaired as it would have been in the time of the most expensive war had the pernicious system of funding never been adopted.

When the debt structure of the nation increases at a greater rate than the rate of production, our economy is in peril and the business of every manufacturer is in jeopardy no matter how scientifically its internal problems are managed. Has there been any scientific inquiry into this problem to which Adam Smith called our attention over two hundred and fifty years ago?

If then we are to have a more scientific approach to the prob-

[8] "Manufacturing in Relation to Banking, Research, and Management," by Walter Rautenstrauch, *Journal* of the Engineers' Club of Philadelphia, Philadelphia, Pa., February, 1918.

[9] "Wealth of Nations," by Adam Smith, 2 volume edition, Oxford University Press, New York, N. Y., 1869. Quotations in order presented are from vol. 2, pp. 581, 583, 585, 587, 594, and 600.

lems of management, should we not inquire into the following types of problems:

(*a*) *A Theory of Organization.* It is unfortunately true that, with all the work which has been done on organization, we have never yet developed a rational theory of the principles of an organized procedure. Until we have a frame of reference or a set of guiding principles, by which to judge the adequacy of an organized procedure in relation to objectives and resources, we can never make much progress in solving the problems of organization.

(*b*) *Operating Characteristics of Our National Economy.* Few attempts have been made to describe how our national industrial economy really functions. Until we understand the functional nature of the sum total of our industrial processes, we will never be in a position to suggest how they may be more efficiently managed or how a particular unit of industry should be operated as a part thereof.

When an engineer examines a particular machine the first thing he does is to find out how it functions, that is, how its parts are related to each other and what are the basic principles underlying its operation. When he understands the functional relationships, he is then in a position, and not before, to derive quantitative relationships concerning its operation.

Management which, up to the present, has been almost wholly concerned with the problems of internal operations is now compelled to give increasing attention to the problem of the individual enterprise as a part of the national economy as a whole if our economy is to be preserved.

When the United States Steel Corporation's net earnings dropped from $195,000,000 in 1929, to a net loss of $59,000,000 in 1932, it certainly was not due to the failure of its management to apply scientific principles to its internal operations. What the managers of the steel corporation and the managers of all other corporations failed to do was to lift their eyes from their own financial statements and observe the march of the economic forces bearing down upon them. After 10 years of disastrous depression, how many managers are familiar with the results of the excellent researches of the National Bureau of Economic Research? How many management and engineering societies have given any consideration to the interpretation of these researches?

The present paper is both timely and important and should urge us to give more careful consideration to the scientific aspects of management problems. It is also high time that we select the most important problems to be solved. Perhaps the Management Division of this Society will help us to select these problems. It is not at all unlikely that, in this search for the important problems of management to which scientific procedure may be applied, we may find the very important question: Management for what?

Author's Closure

To the practical difficulties of recasting general accounting techniques precisely to reflect economic reality, the author readily agrees. But a principal issue remains; that the conventionalized record-keeping so vital to business control, like the shadows of the carried lantern, must be interpreted according to the expediencies of each situation. The oracular pronouncements of its disciples in industrial economics are often in conflict with underlying reality, and erudition as to its detail is sometimes mistaken for professional wisdom. The work carried on by Professors Grant and Norton in the applied economics of engineering selections and replacements and the well-known work of Dr. Rautenstrauch in enterprise characteristics, are encouraging signs. We hope that these are the foundations of a new realistic economics, neither at the extremes of record-keeping technique on the one hand nor abstract "cultural" economics on the other. We need an approach resting upon rigidly scrutinized evidence.

As regards the possibilities of advanced statistical analysis in a more scientific approach to time-study work, the author is interested in the views expressed by Mr. Piacitelli. He indirectly suggests an important issue, in that every investigation, no matter what the precision and objectivity employed in basic interpretation, is overlaid with administrative allowances and the opportunistic motivation of management. That standardization of inevitably personal and subjective factors can usefully be employed is readily granted. That standardization of basic measurement technique can be a guarantee of valid interpretation and homogeneous underlying conditions is respectfully denied. Unassignable or chance causes play a leading role outside of the research laboratory.

Statistical analysis is a vital and powerful tool, not to be casually acquired as a convenient set of arithmetic techniques, nor to be approached with the elegant abstraction of the professional mathematician. Th. von Kármán[10] recently observed: "The engineering application of mathematics is not based on purely logical principles alone; its purpose is the correct interpretation of physical phenomena."

In the scientific aspects of time study, we would appear to aim at an objective determination of the actual sustained basic operating time and the degree of variance of the average good operator under statistically constant underlying conditions. If the latter do not obtain, we can neither combine phenomena in similar investigations nor project them as logical expectations. Since any finite set of observations is merely a sample of an underlying universe of values, we must draw inferences cautiously from efficient statistics which minimize the always present chance variations.

If all of the information underlying the data which Mr. Piacitelli presents were available, it would be expected that a critical appraisal might reveal dissociated conditions which explain the otherwise surprising behavior of the statistics reported. Two possibilities suggest themselves, i.e., that the data have not arisen from the same underlying universe; that the modes have been obtained by the usual approximation techniques, where at best the mode is a very uncertain parameter to estimate. In the first instance cited, the mean of the initial sample of 1000 is given as 0.124 and, by the elementary relationship of the sample means to their weighted mean, we find that the mean of the second sample must have been 0.1057. Also we have the elementary large-sample test of significance for the difference between the means of the two samples under the hypothesis that they come from the same universe

$$T = \frac{\overline{X}_1 - \overline{X}_2}{\sigma_p \sqrt{\frac{1}{N_1} + \frac{1}{N_2}}} \quad \text{or} \quad \sigma_p = \frac{\overline{X}_1 - \overline{X}_2}{T \sqrt{\frac{1}{N_1} + \frac{1}{N_2}}}$$

where the standard deviation in the denominator is that of the common population. The value T can be expected to be distributed normally and, taking the most favorable maximum value in standard error units, common to tests of significance as 2.576, we would arrive at a standard deviation estimate of 0.2066 which is the minimum permitting an acceptance of the null hypothesis. It would be a very skewed distribution indeed that had only 0.1057/0.2066 or 0.512 standard deviation units between the second sample mean and its zero origin, and only a fraction of this between its mean and the minimum reading or ob-

[10] "Some Remarks on Mathematics From the Engineer's Viewpoint," by Th. von Kármán, *Mechanical Engineering*, April, 1940, pp. 308–310.

servation.[11] Furthermore, we seem to have a uniquely small difference between the mode and mean of this second sample when associated with such a standard deviation. In fact, the employment of the relations between the mean and mode in known functions that are at least nearly appropriate to this type of distribution gives a skewness measure that seems incompatible with the conditions suggested. If the originally estimated standard deviation is actually smaller, the means are significantly different. Such questions pile up to an extent that makes it not inappropriate to raise the issue as to the propriety of considering the data comparable or of combining the two samples of observations referred to.

Mr. Piacitelli's stressing of the importance of establishing standard allowances to supplement the basic operating times is well taken. The author's comments and interest were only in the research possible as regards operating-time measurements themselves and their interpretation, in the use of more critical tests of stability of the phenomena, and in the suggestion of modern analysis. The author is convinced that the latter will not only improve the objective determination of the sustained and controlled good operating levels but permit the isolation of the component variances. Space does not permit attempting an exposition of well-established statistical theory that can be employed but it is to be hoped that students, who are advanced in their grasp of both fields, will find the opportunity at least to investigate another dimension which has been of proved success in other branches of science.

The author does not presume even to a modest competence in the time study that is so tremendously important in our industrial economy and in which Mr. Piacitelli has deservedly attained such eminence. However, if it is to acquire the highly objective statistical tests of research employed in science, a thorough employment of applied statistical theory is necessary. Simple numerical generalization so common to what may be popularly described as business statistics would merely compound the dangers of purely empirical approaches to professional engineering. The author does not recall the savant who remarked that a tragedy is a beautiful generalization wrecked by a single obstreperous fact.

[11] For purposes of simple illustration no distinction is made between population and sample values of standard deviations since the samples are very large.

Fuels for Diesel Engines in Marine Transportation

By W. F. JOACHIM[1] and H. V. NUTT,[2] ANNAPOLIS, MD.

After reviewing briefly developments in the field of Diesel fuels over the last 30 years, the authors discuss in detail the present-day requirements for marine-Diesel-engine fuels. A summary of engine troubles and casualties caused by unsuitable fuels is also given. The paper includes a series of tables and explanatory matter concerning Diesel-fuel specifications for various types of marine engines.

THIRTY years ago, hot-bulb-type engines, using solid-injection systems with large-orifice, open-type nozzles, and full Diesel air-injection engines, were operating as pumping engines in the oil fields, using fuels varying from raw crude to 43 gravity kerosene. Many of these engines are still in service. Their efficiencies were relatively low for Diesels, their rate of wear was high, and sometimes they were hard to start, but they burned the fuel at hand and produced the power required with relatively small maintenance and other expense, compared to contemporary steam engines. Like stationary land units, the marine engines of that day were making a name for themselves by burning cheap fuel with an efficiency which has as yet been unequaled except by a few high-pressure high-temperature "modern" steam turbines.

The fuel available at that time was no better and little worse than that available today. Cheap fuel was dirty and contained much residuum. However, cracked fuels had not made their appearance, which normally precluded many present-day combustion and maintenance problems. Clean fuels were all straight-run distillates and gave little trouble, except for injection and combustion difficulties, caused by unsuitably high or low viscosities. Typical fuel specifications for that period, one formulated by Dr. Diesel, are given in Table 1.

During the period when "air injection" was used exclusively, little attention was paid to the properties of the fuel, except that the operating engineer knew from experience the approximate degree of purity and viscosity required to produce passable results in his engines. With the advent of the first commercially successful solid-injection system, manufactured by Vickers in 1914, more attention was given to fuel oil, especially from the viscosity standpoint.

Several investigators published papers covering their work on Diesel fuels during the change-over from air to solid injection about 1918 to 1930, and in 1928 members of this Society formed a committee for the investigation of Diesel fuels with the aim of establishing standard specifications. This committee proposed two fuel-specification standards in 1929, one for light high-speed engines, the second for heavy-duty slow-speed engines. Several series of tests were also made to determine the effect of various impurities on engine operation.

As a result of this and other work on Diesel fuels, a combined A.S.M.E., S.A.E., and A.S.T.M. group submitted several "Diesel-Fuel Classifications," one, in 1934, consisting of five fuel grades, in which ignitibility was considered an important factor for medium- and high-speed fuels. These specifications are given in Table 2.

It was found impossible to obtain sufficient funds to carry out the extensive test program outlined by the joint A.S.M.E.-S.A.E. committee, due to the current financial depression, but, in 1933, a group of men formed the Volunteer Group for C. I. Fuel Research and carried out several series of cooperative experiments during succeeding years on the most promising methods of measuring ignition quality (1).[3] The present ignition quality-test method is described in the 1938 transactions of the A.S.T.M.

The original "Volunteer Group" has now become the Automotive Diesel Fuels Division of the Cooperative Fuel Research (C.F.R.) Committee, Society of Automotive Engineers, whose work was broadened in 1939 to include investigations of the effect of fuel volatility, cetane number, viscosity and gravity on full-scale performance, and engine deposits. Other factors such as the effects of sulphur, gums, carbon residue, and impurities on performance, maintenance, and engine wear are to be investigated in the future (2).

MARINE-DIESEL-ENGINE FUEL REQUIREMENTS

The essential requirements for a marine-Diesel-engine fuel are given in Table 3. They are listed according to their order of use rather than their relative importance.

Storage and Handling

A primary consideration of Diesel fuels is that of storage and handling, as the fuel must be safe in storage, it must not deteriorate, and suitable handling methods must be available.

Fire Hazard. Although a Diesel-fuel vapor has a lower autoignition temperature than some of the more volatile hydrocarbons, fire hazard with Diesel fuel is considerably reduced because of its greatly lowered volatility. Tests have proved that Diesel fuel at its flash-point temperature, being run into a tank with sufficient force to produce considerable foam on its surface, will still not burn, only the foam being burned off when ignited by an electric spark. At temperatures below the flash point, only a small portion of the foam on the surface will burn. Therefore, Diesel-fuel fires, caused by electric sparks or open flames in or adjacent to Diesel-fuel storage, are practically nonexistent.

Aging and Deterioration. Tests covering a period of years have shown that, for the distillate fuels investigated, including straight-run and cracked fuels and their blends, no reduction in the more important fuel properties occurred during long-time storage. Experience shows, however, that certain fuels are incompatible, and when mixed will precipitate compounds which affect engine operation by forming sludge, gums, and lacquers.

Corrosive Properties. All Diesel fuels are not chemically neutral, some containing small amounts of acidic materials. Such acids may effect engine wear and corrode injection equip-

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

TABLE 1 PROPERTIES AND SPECIFICATIONS OF EARLY DIESEL FUELS

(Used in large slow-speed air-injection engines)

Property	Date of test 1906	1907	1908	1910[a]	1926[b]	1926[c]
Type of fuel	distillate	crude oil	distillate	...	...	...
Specific gravity	0.852	0.878	0.869	...	...	...
A.P.I. gravity at 60 F	...	...	...	...	26.7	30.2
Cloud point, F	...	...	...	47 Max	...	...
Flash point, F	200	...	150	149 Min	175	175
Viscosity, SSU at 100 F	...	...	...	...	98	50
Water, per cent	...	...	None	1.0 Max	...	...
Water and sediment	...	...	...	...	Trace	Trace
Acidity	...	...	None	...	...	...
Carbon residue, per cent	...	...	...	3.0 Max	3.05	0.4
Asphalt, per cent, Italian tar test	...	...	...	...	45	6
Ash	...	...	...	...	None	None
Insolubles in xylol	...	...	...	Trace	...	...
Total sulphur, per cent	...	0.3	0.17	...	1.34	0.4
Total carbon, per cent	...	87	...	...	...	...
Total hydrogen, per cent	...	12.5	...	...	...	...
Total oxygen, per cent	...	0.23	...	...	...	...
60% distillation temperature, F	...	...	...	572 Max	...	...
Heat value, Btu per lb	19500	19272	19521	16000 Min	19125	19400

[a] Dr. Diesel's recommended specifications.
[b] Great Britain sample.
[c] New York sample.

from 30 sec SU to 200 sec SU, although engine performance will differ markedly with widely varying viscosity fuels.

Pour Point. Pour point is the principal criterion of the fluidity of the fuel at reduced temperatures, although the cloud point should be considered as the minimum temperature permitting certain pumping. Experimental work (4) shows that an exact indication of the pumpability of fuels intended for low-temperature service requires actual low-temperature pumping tests because of the different behavior of the wax structures and inconsistencies in injection failure in relation to fuel cloud and pour points.

Preheated Fuels. Fuels too heavy to pump at existing temperatures may be handled by

TABLE 2 A.S.M.E.-A.S.T.M. DIESEL-FUEL CLASSIFICATIONS

(1929–1934–1938–1940)

Property	A.S.M.E. 1929 Progress Report—High speed	Low speed	1934 1-D	3-D	4-D	5-D	6-D	1938 1-D	3-D	4-D	1940 (Tentative) 1-D	2-D	3-D	4-D
Flash point, cc, min, F	150	150	legal or 115	150	150	150	150	legal or 115	150	150	legal or 100	legal or 110	legal or 110	140
Water and sediment, max, per cent	0.05	1.0	0.05	0.2	0.6	..	..	0.05	0.1	0.6	0.05	0.05	0.1	0.5
Pour point, max, F	..	..	35	35	35	40	40	35	35	35	0	20	35	35
Viscosity, SSU at 100 F, min	45	..	32	32	..	..	..	33	33	..	..	32.6	..	...
Viscosity, centistokes at 100 F, min	..	..	..	..	..	..	..	..	..	..	..	2.0	..	...
Viscosity, SSU at 100 F, max	100	200	50	70	500	..	..	50	70	250	..	45.5	65.0	140.0
Viscosity, centistokes at 100 F max	..	..	..	..	..	..	..	..	..	..	..	6.0	12.0	...
Viscosity, SSF at 122 F, max	..	..	..	..	..	100	300	..	..	..	..	..	..	...
Carbon residue, max, per cent	1.0	5.0	0.2	0.5	3.0	6.0	10.0	0.2	0.5	3.0	..	..	1.0	3.5
Carbon residue, on 10 per cent residue, max, per cent	..	..	..	..	..	..	..	..	..	..	..	0.25	..	...
Ash, max, per cent	0.02	0.08	0.01	0.02	0.04	0.08	0.12	0.02	0.02	0.04	0.01	0.01	0.02	0.05
Sulphur, max, per cent	2.0	3.0	2.0	2.0	2.0	2.0	2.0	1.5	1.5	2.0	0.5	1.0	1.5	2.0
Ignition quality:														
Cetane number, min	..	..	50	40	30	..	..	45	35	30	45	45	35	30
Diesel index number, min	..	..	45	30	20	..	..	45	32	25	..	..	..	...
CCR, max	..	..	8.1	8.8	9.8	..	..	..	..	..	..	..	..	...
90 per cent distillation temperature, F	..	..	..	..	..	..	..	..	..	..	..	675	..	...
Final boiling point, max, F	..	..	..	..	..	..	..	..	..	..	590	..	..	...
Copper-strip corrosion at 122 F	..	..	..	..	..	..	..	..	..	..	Pass	Pass	..	...
Alkali and mineral acid	..	..	..	..	..	..	..	..	..	..	Neutral	Neutral	Neutral	Neutral

TABLE 3 MARINE-DIESEL-ENGINE FUEL REQUIREMENTS

1 Storage and handling
- *A* Minimum fire hazard
- *B* Minimum aging–deterioration
- *C* Minimum separation–settling
- *D* Minimum corrosive properties
- *E* Pumping
  - *a* Adequate fluidity without preheating
  - *b* Minimum sediments and waxes

2 High engine performance
- *A* High efficiency
  - *a* Balanced ignition and combustion
  - *b* Combustion without exhaust smoke
- *B* High ignitibility for cold starting

3 Low maintenance
- *A* Adequate ignitibility minimizing combustion shock, cracked bearings, and engine wear
- *B* Minimum lacquer, gum-, and carbon-forming tendencies
- *C* Minimum lubricating-oil sludging

4 Minimum Wear
- *A* Adequate lubrication for injection pumps and nozzles
- *B* Minimum abrasive compounds
- *C* Minimum corrosive compounds
- *D* Minimum slow, acid combustion

5 Adequate supply—minimum cost
- *A* Production from domestic crude oils
- *B* Refining by commercial processes

ment and storage tanks. Also, increased engine maintenance and casualties may occur through the action of metallic oxides which are not always removed by either centrifuging or filtering.

Pumping. Pumping of the fuel must be considered from several standpoints as follows:

a Pumping under cold-starting conditions, particularly in small-boat service.

b Losses in volumetric efficiency of either transfer pumps or fuel-injection equipment, due to excessively high or low viscosities.

c Preheating of the fuel, either in the storage tank or at the engine.

d Precipitation of sediment and waxes on filters and in transfer lines.

Tests have proved (3) that no difficulty in pumping will normally be encountered with fuels having viscosities ranging

first heating them, thus reducing their viscosities to obtain sufficient fluidity for efficient pumping. Relatively elaborate heating equipment, however, is necessary, together with high rates of flow. Therefore, heavy fuels are usually used only in large installations. Somewhat reduced maneuverability, due to inability to get under way quickly, increased fuel-bunker and engine maintenance, and danger of excessive engine wear and casualties, practically eliminates such fuels for modern, high-specific-power engines in the military services.

Filtering and Pumpability. Some fuels are difficult to pump because of precipitation of waxes at normal temperatures, which clog filter bags, strainers, and pipe lines. These ill effects can only be guarded against by using suitable heating equipment or fuels having pour points from 15 to 20 deg below the lowest atmospheric temperatures encountered.

Engine Performance

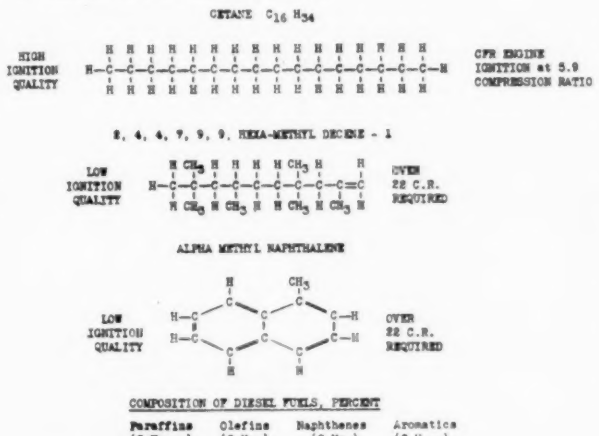
The performance of a Diesel engine is measured by many factors, all interdependent. The effects of various fuel properties on some of these factors have been investigated, while other effects are not yet fully known.

The principal engine-performance characteristics are as follows:

a Specific fuel consumption
b Combustion shock
c Exhaust-gas temperature
d Exhaust-smoke density
e Maximum cylinder pressure
f Cold starting
g Engine wear

The principal fuel properties believed to affect the engine-performance characteristics are as follows:

a Ignition quality
b Viscosity
c Volatility and distillation range
d Carbon residue and impurities



COMPOSITION OF DIESEL FUELS, PERCENT

	Paraffins (C_nH_{2n+2})	Olefins (C_nH_{2n})	Naphthenes (C_nH_{2n})	Aromatics (C_nH_{n-y})
Penna.	70—75	2—5	15—19	4—9
Mid. Con.	50—74	2—14	15—24	3—16
West Coast	40—65	2—18	14—20	10—30

FIG. 1 DIESEL-FUEL CHEMICAL STRUCTURE AND IGNITION QUALITY

Ignition Quality. Ignition quality can be briefly defined as that property of a Diesel fuel which causes it to ignite readily after injection. It is measured by determining the percentage of cetane in a blend of cetane and alpha-methylnaphthalene which produces the same time delay between the beginning of injection and ignition as the fuel under test, a standard C.F.R. Diesel engine being used. An exhaustive treatise on this subject by T. B. Hetzel (5) together with an excellent bibliography has been published.

The factors which govern ignition quality are chiefly chemical:

a Structure of molecule
b Oxidation stability of molecule
c Size of molecule.

Fig. 1 shows the chemical structure of two hydrocarbons, each composed of 16 carbon atoms, one of which has two more hydrogen atoms than the second. The straight-chain saturated compound, cetane $C_{16}H_{34}$, has greater thermal instability than "2,4,4,7,9,9-hexamethyl decene-1," $C_{16}H_{32}$, which is more compact due to the six side chains in its structure. The third molecule, alpha-methylnaphthalene, $C_{11}H_{10}$, is even more compact than the second and, due to its ring structures, will decompose and combine with oxygen only at very high temperatures.

Compounds such as organic nitrates or peroxides are still more unstable in the engine cylinder than cetane, due to free oxygen liberated at comparatively low temperatures. The addition of such compounds in proportions of 1 to 2 per cent will increase the cetane number of an average Diesel fuel by as much as 10 to 30 cetane numbers.

The chemical factors influencing ignition quality cause fuels manufactured from certain crudes to have higher ignition qualities than others. Fig. 1 shows the relative percentages of the four general hydrocarbon families in the three general crude-oil types, which thus provide indications of their true average ignition qualities. The general order of the hydrocarbon families with respect to ignition quality is as follows:

Paraffins—highest ignition quality
Olefins—second highest ignition quality
Naphthenes—third highest ignition quality
Aromatics—lowest ignition quality

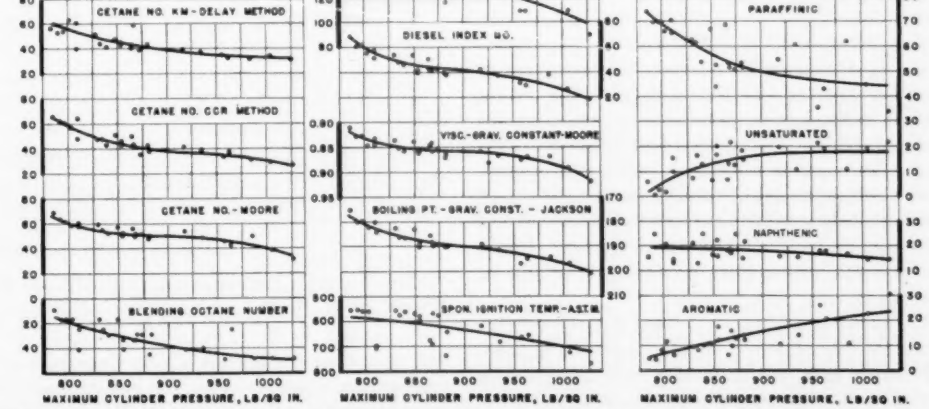


FIG. 2 CORRELATION OF C.F.R. ENGINE PHYSICAL-CHEMICAL AND CHEMICAL-COMPONENT FUEL-IGNITION INDEXES OF 25 DIESEL FUELS WITH THE MAXIMUM CYLINDER PRESSURES DEVELOPED IN A 4-CYCLE DIESEL ENGINE

(Bore and stroke, 4¼ in. by 6 in.; speed, 1200 rpm; 10-bhp single-cylinder engine. Data are for 8 bhp at 1200 rpm.)

Fig. 2 shows the correlation of thirteen methods of measuring ignition quality and the maximum cylinder pressures developed. Inspection of these curves shows that certain of these methods have greater sensitivity per unit of cylinder pressure, and that their data points lie closer to their average curves. They are, therefore, more accurate than the other ignition-quality indexes.

These ignition-quality data were also similarly plotted against the ignition delay of the engine and computed combustion shock.[4] This work indicated that the calculated order of merit of the best seven of these ignition-quality-rating methods was as follows:

1 Boiling point-gravity constant
2 Cetane number, CCR
3 Diesel index number
4 Cetane number (Moore)
5 Viscosity-gravity constant
6 Aniline point
7 Cetane number, KM delay

Since aniline point is included in the now proved, more accurate Diesel index number, and cetane number (Moore) has been

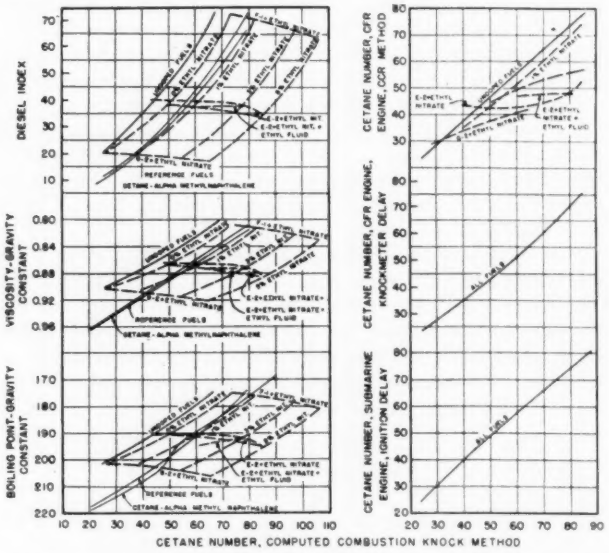


FIG. 3 CORRELATION OF C.F.R. ENGINE AND PHYSICAL-CHEMICAL FUEL-IGNITION INDEXES OF 83 DOPED AND UNDOPED DIESEL FUELS WITH COMPUTED COMBUSTION-KNOCK CETANE NUMBERS DETERMINED IN A 2-CYCLE DIESEL ENGINE

superseded by engine tests using cetane, they were dropped from further consideration.

Further tests using doped fuels, Fig. 3, showed that the physical-chemical indexes, i.e., Diesel index number, viscosity-gravity constant, and boiling point-gravity constant, plus the cetane number by the CCR method, were unreliable in predicting the performance of doped fuels. Thus Diesel-fuel ignition-quality determinations must be made in a running Diesel engine, preferably expressed as that percentage of cetane in a blend of cetane and alpha-methylnaphthalene which performs similarly to the fuel under test as regards ignition, i.e., equal ignition delays, equal critical compression ratios (just producing ignition), or other ignition criteria.

The effect of fuel ignition quality on several engine-performance characteristics for four different engines is shown in Figs. 4, 5, 6, and 7. In every case, the maximum cylinder pressures

[4] Computed combustion shock

$$= \frac{\text{explosion pressure rise} \times \text{burning rate}}{10^4}$$

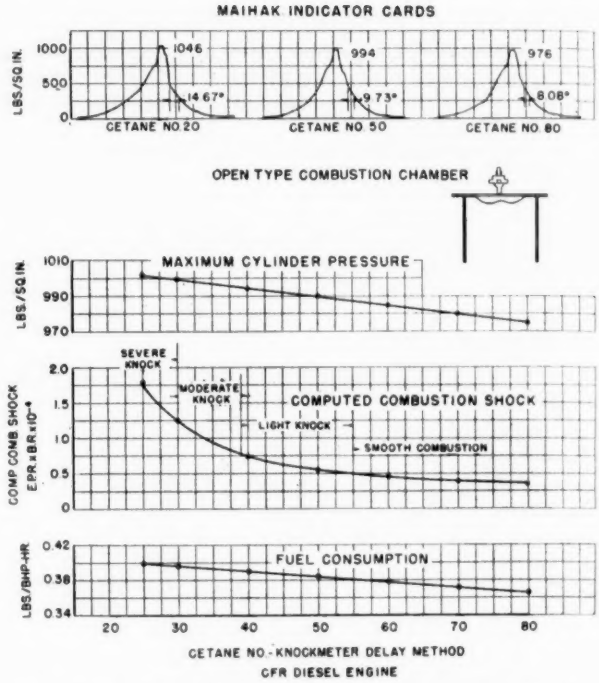


FIG. 4 FUEL PERFORMANCE IN A 2-CYCLE DIESEL ENGINE (Bore and stroke, 8 in. by 10 in.; speed 750 rpm; 75-bhp one-cylinder engine. Data presented are for 83 fuels, including 24 doped fuels, tested at 75 bhp and 750 rpm.)

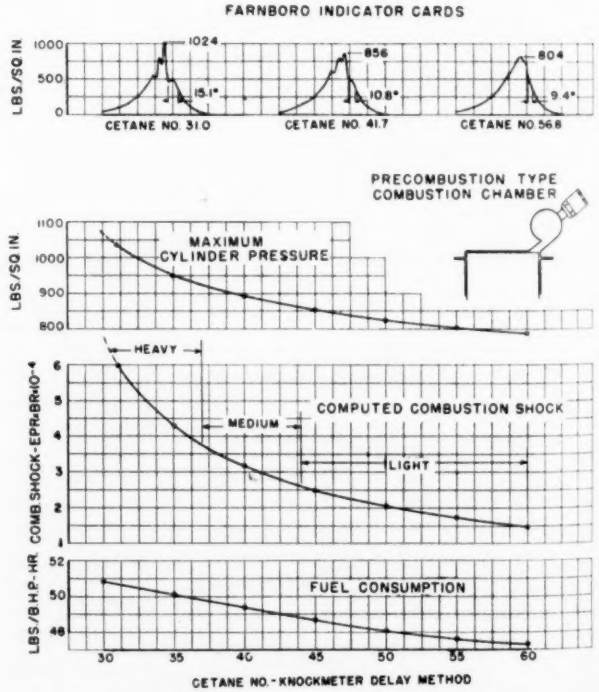


FIG. 5 FUEL PERFORMANCE IN A 4-CYCLE DIESEL ENGINE (Single-cylinder, 4¼-in. by 6-in., 1200-rpm, 10-bhp engine. 25 Diesel fuels tested at 1200 rpm and 8 bhp.)

and computed combustion shocks decreased with an increase in fuel-ignition quality. Also, the specific fuel consumption in the open-type combustion-chamber engine, Fig. 4; the precombustion-type chamber engine, Fig. 5; and the air-cell-type chamber engine, Fig. 7, decreased with higher ignition-quality fuels. This

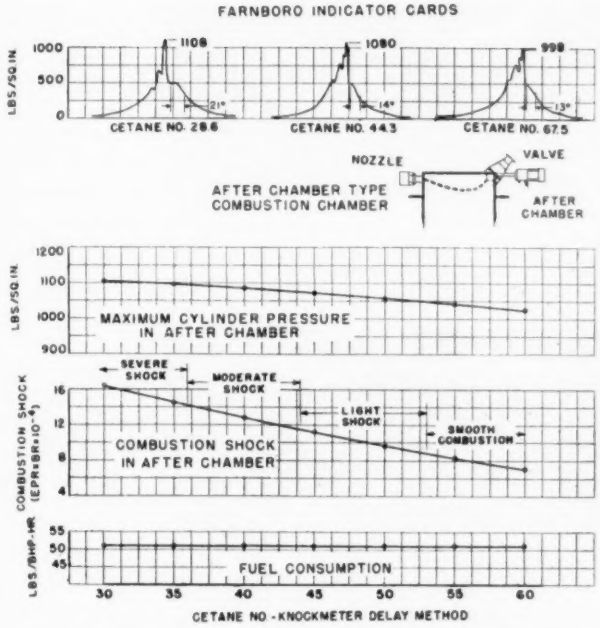


FIG. 6 FUEL PERFORMANCE IN A 4-CYCLE DIESEL ENGINE

(Four-cylinder, 3½-in. by 4½-in., 1400-rpm, 25-bhp engine. Ten fuels tested at 1200 rpm and 15.74 bhp.)

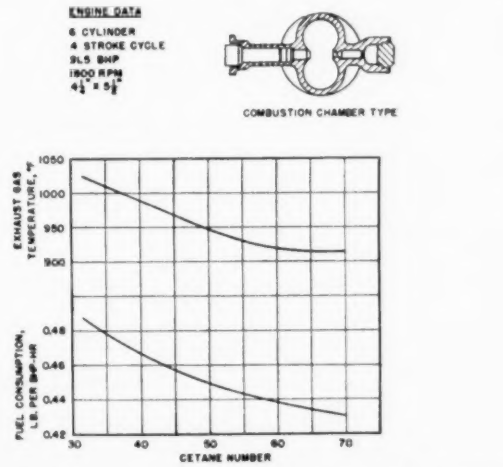


FIG. 7 EFFECT OF CETANE NUMBER ON EXHAUST-GAS TEMPERATURE AND FUEL CONSUMPTION

was not true, however, of an engine using another type of air-cell combustion chamber Fig. 6, as there was no noticeable change in fuel consumption between the limits of 30 and 60 cetane number.

Cold Starting. Fig. 8 shows the results of cold-starting tests on a high-speed marine-Diesel engine. The graph is separated into four regions indicating the relative difficulty of starting the engine. A marked increase in the ease of starting was found as the ignition quality was increased from 20 to about 50 cetane number. Any given cold-starting temperature also required a given minimum cetane number before a start could be obtained. This minimum cetane number decreased as the air temperature was increased.

A wide region of temperatures and cetane numbers was found which produced ignition, but insufficient power due to very late, slow, and irregular combustion to enable the engine to continue running when the starting motor was disengaged. The extreme lower left-hand corner of the graph shows a region where no ignition occurred in any cylinder at any time.

Cetane Number Versus Smoke. Some investigations of Diesel-engine-exhaust smoke (6) show that smoke density is inversely proportional to cetane number, but that other factors have a greater influence on smoke than does ignition quality. This is particularly true of engine design. Fig. 9 shows the effect of fuel ignition quality and engine speed and load on the smoke density of a marine-Diesel engine when using three fuels of 35, 44.4, and 55.4 cetane number which had similar 50 per cent distillation temperatures, i.e., 518, 526, and 525 F, respectively. At full load and speed, 25 bhp and 1400 rpm, the 55.4 cetane-number fuel produced the greatest smoke; the 44.4 cetane-number fuel the next greatest, and the low cetane-number fuel the least smoke. The fuel consumption and exhaust-gas temperature were also minimum at from 30 to 35 cetane number. As the viscosity,

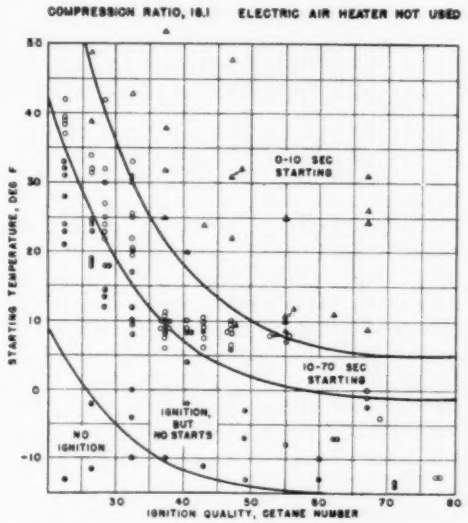


FIG. 8 EFFECT OF FUEL-IGNITION QUALITY ON COLD STARTING OF POWERBOAT DIESEL ENGINE

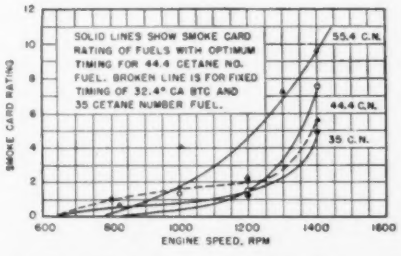


FIG. 9 SMOKE-CARD RATING VERSUS FUEL CETANE NUMBER AND ENGINE SPEED

(Four-cycle powerboat Diesel engine.)

carbon residue, sulphur content, and other properties, except for cetane number, were about the same, the difference in smoke density was largely due to ignition quality.

MAINTENANCE AND WEAR

A severe criticism of Diesel engines has been the high weight-to-horsepower ratio required by the high maximum pressure and shock loading imposed on the working parts. These factors also tend to increase maintenance and replacement of parts. It is recognized that proper fuels can alleviate these conditions considerably.

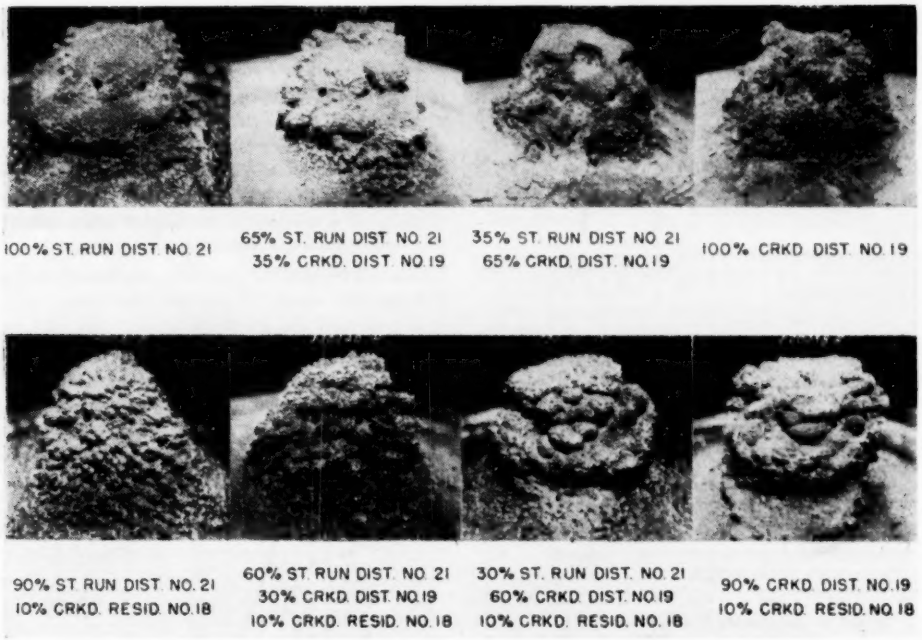


FIG. 10 FUEL-NOZZLE CARBONIZATION WITH VARIOUS FUELS AND FUEL BLENDS

Computed Combustion Shock. An index of the effectiveness of the fuel in reducing the maximum pressures and shock-loading has been developed by one of the authors (7), which includes the sudden pressure rise following ignition and the rate of burning during this combustion. This index also correlates well with cetane number, as indicated by Figs. 4, 5, 6, and 7. These show that, in four different engines, an increase in cetane number results in a corresponding decrease in computed combustion

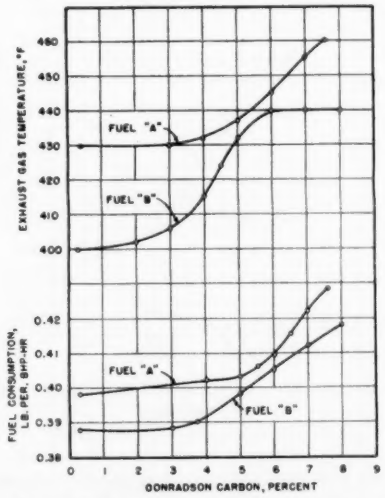


FIG. 11 EFFECT OF CARBON RESIDUE (CONRADSON) ON FUEL CONSUMPTION AND EXHAUST-GAS TEMPERATURE
(14-in. by 17-in. airless-injection, 2-cycle, single-cylinder Diesel engine.)

shock and thus a decrease in maintenance, replacement of parts, and a permissible reduction in specific weight.

Nozzle-Tip Carbonization. Fig. 10 shows the wide difference between fuels in forming objectionable carbon on fuel-nozzle tips. Such crater-like formations tend to distort the fuel spray, which in turn produces poor combustion, high fuel consumption, excessive cylinder-head and piston temperatures, reduced power

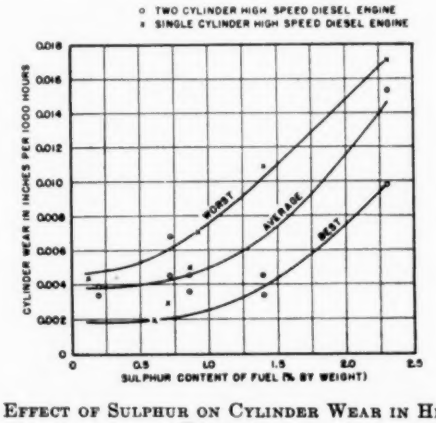


FIG. 12 EFFECT OF SULPHUR ON CYLINDER WEAR IN HIGH-SPEED ENGINES
(Results drawn from 85-hr tests.)

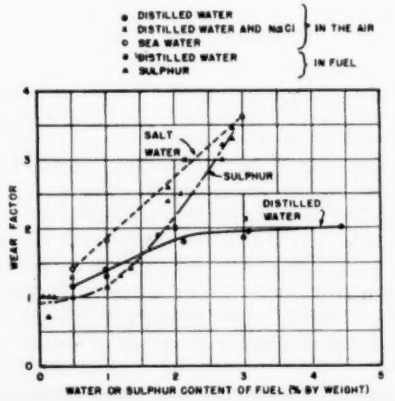


FIG. 13 INFLUENCE OF WATER AND SULPHUR ON RATE OF WEAR
(One-cylinder low-speed air-injection engine. Wear factor = ratio of wear under conditions investigated to standard conditions when using reference fuel having 0.7 per cent sulphur.)

output, and excessive smoke. These factors all influence engine maintenance adversely.

Carbon Residue. The effects of fuels having high carbon residues (Conradson carbon) are shown in Fig. 11. In a large engine of the type tested, a percentage of carbon residue above about 3 per cent resulted in a decided increase in fuel consumption and exhaust-gas temperature. The true effects of carbon residue are not ascertainable in short-time tests, since carbon residue is closely connected with and involves other fuel properties such as high distillation temperatures, cracked and lacquering hydrocarbon products, and impurities, which cause gradual changes in engine performance and increased maintenance costs and wear.

Sulphur. Figs. 12 and 13 have been selected from the literature (8) and are believed to present some of the best results on the effect of sulphur and water on cylinder wear. It may be noted that sulphur above approximately 0.7 per cent causes a marked increase in the wear rate. These investigators also state that such wear is of a corrosive nature due to the presence of sulphur trioxide in the products of combustion rather than sulphur dioxide as has been commonly believed.

Water and Sediment. The wear of cylinders, pistons, and rings is affected by sand, grit, and dust, either from the fuel, lubricating oil, or the air. An unusual incident of dust affecting marine-engine wear is that of a Mississippi work boat, powered by Diesel engines which experienced excessive cylinder wear; requiring replacements of liners, pistons, and rings at an abnormal rate. After long investigations of fuel, lubricating oil, and other common causes of wear, it was ascertained that the boat was required to pass between long stretches of sandy bluffs overlooking the river. The prevailing winds kept the air full of fine silica which caused the damage. The trouble was then easily corrected by the installation of efficient intake-air filters.

Salt water, whether taken into the engine with the fuel or with the air, accelerates wear considerably as shown by Fig. 13. This action is almost wholly corrosive. Small amounts of chemically neutral water do not appear to affect cylinder wear.

Wear Versus Combustion. The manner in which combustion takes place also affects the wear of Diesel engines. Poor combustion can many times be directly traced to unsuitable fuels, i.e.:

1 Viscosity may affect the deposition of fuel on combustion-chamber walls, since it controls fuel-spray penetration, atomization, and distribution.

2 Volatility controls the rate of vaporization, the richness or leanness of the mixture, and distribution of the fuel spray, and thus the progress and completeness of combustion.

3 Ignition quality controls the timing and location of ignition and, therefore, the condition of the fuel-and-air mixture during the initial stages of combustion. This in turn has a marked effect on all subsequent burning, during which the ignition quality further markedly influences the completeness and efficiency of combustion.

4 High distillation-temperature fractions, cracked products, excessive carbon residues, and some impurities all influence the rate and completeness of combustion.

These major factors determine, from fuel standpoints:

1 The dilution of lubricating-oil films on the cylinder walls and their wear.

2 The deposition of fuel carbon and lacquers on valves, pistons, rings, and cylinder walls, and their sticking and abnormal operation, causing combustion-gas blowby, destruction of lubricating-oil films, increased friction, overloading, overheating, and increased or excessive engine maintenance and wear.

3 The types and amounts of corrosive combustion products formed, causing direct, slight, but cumulative corrosion, during each combustion cycle, on dry cylinder-wall areas and thus their corrosive wear.

Engine Troubles and Casualties Caused by Fuels

As soon as arrangements had been completed for presentation of this paper, letters were written to twenty-six selected companies requesting copies of current Diesel-fuel specifications for marine-Diesel engines and records of engine troubles and casualties caused by unsuitable fuels. While some companies claimed to have had no trouble from fuels, others cooperated by submitting comments and records. Too few records were received to permit of classification by fuel properties. However, Table 4 gives a summary of the essential data derived from this survey:

TABLE 4 SUMMARY OF REPORTS ON ENGINE TROUBLES AND CASUALTIES CAUSED BY FUELS

Engine troubles and casualties		Fuel-property causes
1 Excessive carbonization:		
a Exhaust valves and piston rings	(PC)[a]	High carbon residue; hard asphalt; cracked, lacquering compounds; impurities
b Fuel nozzles	(KC)[b]	High carbon residue; hard asphalt; cracked, lacquering compounds; impurities
2 Lacquer formation:		
a Excessive lacquering of engine in few hours prevented operation	(KC)	Fuel was known cause; probably highly cracked fuel or a blend containing residual products
b Stuck fuel pump plungers, prevented engine operation	(KC)	Fuel was known cause; probably contained lacquering gums
3 Corrosion, pitting:		
a Fuel - pump plungers, valves, and fuel nozzles were badly corroded and pitted	(KC)	Fuel contained saline salts
4 Poor combustion:		
a Lubricating trouble; wear	(KC)	Excessive viscosity; probably also high carbon residue; high distillation end point; impurities
b Bearing failures	(KC)	High distillation end point; low cetane number; high sulphur
5 High maintenance:		
a Fuel-injector valve sticking; cracked cylinder heads	(KC)	Incompletely refined, corrosive fuel; plus small amounts of water and silica
b Generally increased maintenance	(PC)	High carbon residue; hard asphalt; cracked, lacquering compounds; impurities
c Generally increased maintenance	(PC)	High carbon residue; hard asphalt; cracked, lacquering compounds; impurities

[a] (PC) = probable cause.
[b] (KC) = known cause.

Since more is often learned from troubles and casualties, if analyzed, than from "clear sailing," it is hoped that a thorough, cooperative survey and classification of troubles and casualties caused in engines of various types by Diesel fuels may be made and correlated with the unsuitable fuel properties, in a future paper.

DIESEL-FUEL PROPERTIES AND SPECIFICATIONS

Major Classifications

There is considerable evidence to support the belief that Diesel fuels can be separated into four major classes, as follows:

1 Fuels requiring heating and centrifuging or other cleaning before use.

2 Fuels for large, slow-speed engines; about 14 in. cylinder diam or over and speeds below about 300 rpm.

3 Fuels for medium-size, medium-speed engines; about 6 to 14 in. cylinder diam and 300 to 1200 rpm.

4 Fuels for small, high-speed engines; below about 6 in. cylinder diam and speeds from about 1200 to around 2600 rpm.

Low cost is the principal reason for that class of fuels requiring heating and cleaning. Such fuels are not usually suitable for Diesel engines without thorough settling, centrifuging, or filter-

ing and heating by the user. There are sound technical as well as economic reasons for the existence of the slow-, medium-, and high-speed-fuel classes:

1 Differences in the time available between various engines for all injection, ignition, and combustion processes.

2 Differences in cylinder sizes and types of combustion chambers.

3 Differences in the tolerations of kinds and amounts of deposits formed on pistons and cylinders and in maintenance and wear.

Time Available. A basic difference between all types and sizes of Diesel engines is speed or rpm. In large, slow-speed engines, each crank-angle degree is equal to a longer period of time than in a higher-speed engine and thus produces many effects on engine performance, viz.:

1 Ignition delay, instead of comprising a period of from 15 to 20 deg of crank angle as in the high-speed engine will only require from 2 to 5 deg, thus greatly reducing combustion shock and maximum cylinder pressure.

2 Lower rates of injection are possible which again tend to reduce maximum cylinder pressures and effect better combustion control.

3 Combustion will be completed much earlier in the power stroke, which produces higher thermal efficiencies, lower exhaust-gas temperatures, and cleaner exhaust gases. It further decreases exposure of portions of the dry areas of the cylinders to the corrosive gases formed as intermediate products of combustion.

4 More complete burning of the fuel is possible, tending to eliminate many of the partial products of combustion which accelerate cylinder wear such as carbon, gum, and organic acids.

These effects greatly reduce the importance of narrow and restricted limitations on fuel ignition quality, viscosity, volatility, and carbon residue.

Cylinder Size and Combustion-Chamber Type. Increases in cylinder sizes result in higher compression temperatures due to reduced heat losses on the compression stroke, better mixing of fuel and air, and less impingement of fuel on cold walls. The ultimate results are, again, shorter ignition delay, lower injection rates, and shorter and more complete combustion. Thus, fuel properties are affected by reducing the importance of narrow and restrictive specifications.

Differences in Classifications

Number and Range of Properties Specified. Because the importance of fuel properties increases with engine speed, the number and range of property values change accordingly. Thus, high-speed-fuel properties are controlled by 12 to 16 property specifications and low-speed and heater-type fuels by 3 to 7 specifications. The spread allowed in fuel-property values is usually very great for slow-speed as compared with high-speed fuels. The range in viscosity for a slow-speed unheated fuel is from 40 to 150 sec SU, as compared to a high-speed range of 35 to 45 sec SU.

Amount of Impurities. The maximum allowable amount of impurities may be increased in slow-speed fuels as compared to high-speed fuels and this is generally done as shown in Table 5 which gives average values from several typical specifications in each class.

Cost. A major consideration when choosing a Diesel fuel is over-all cost, which includes the cost of the fuel and the effects of the fuel on maintenance, replacements, and efficiencies. There is little doubt that a slow-speed fuel will be the most economical to use except where lower costs, including maintenance factors, have been obtained from the use of the heater-type fuels.

TABLE 5 AVERAGE VALUES OF IMPURITIES FROM TYPICAL SPECIFICATIONS

	Fuel Type			
Property	Cleaning required	Slow-speed engine	Medium-speed engine	High-speed engine
Sulphur, per cent	2.4	2.0	1.5	0.75
Water and sediment, per cent	0.5	0.5	0.3	0.1
Ash, per cent	0.2	0.1	0.03	0.01
Carbon residue, per cent	9.0	3.0	1.0	0.1

TABLE 6 PROPERTIES AND SPECIFICATIONS OF DIESEL FUELS REQUIRING HEATING AND CENTRIFUGING OR OTHER CLEANING

Property	Bunker C	Fuel oil No. 6	British B.S.I. fuel oil No. 3	Bunker B
Flash point, F	210	248	150 min	200 max
Viscosity, SSU at 100 F	...	760	840 max	1000 max
Viscosity, SSF at 86 F	160	...	.....	.....
Viscosity, SSF at 122 F	44	39	.....	50 max
Water and sediment, per cent	0.5	...	1.0 max	0.7 max
Specific gravity at 60 F	0.943	1.010	.....	.....
A.P.I. gravity at 60 F	...	...	.....	13 min
Ash, per cent	0.057	0.4	0.1 max	0.05 max
Carbon residue, per cent	6.5	12.9	8.0 max	11.0 max
Sulphur, per cent	1.14	2.39	.....	1.5 max
Heat value, Btu per lb	18563	18275	18250 min	.....
Pour point, F	...	−4	.....	.....
Diesel index number	...	14	.....	.....

TABLE 7 DIESEL-FUEL SPECIFICATIONS FOR LARGE SLOW-SPEED ENGINES

Property	A.S.T.M. 1939	British B.S.I. 1937	Manufacturer 1940	Refiner 1940	User 1940
Flash point, cc, min, F	140	150	150	150	150
Water and sediment, max, per cent	0.5	1.0	0.6	0.5	1.0
Viscosity, SSU at 100 F	140 Max	840 Max	40–150	100 Max	65–130
Carbon residue, max, per cent	2.5	8.0	3.0	1.0	0.75
Ash, max, per cent	0.05	0.10	0.04	0.02	0.1
Pour point, max, F	35	...	35	0	10
Sulphur, max, per cent	2.0	...	2.0	2.5	1.5
Cetane number, min	30	...	35	...	...
Acidity	Neutral	...	Neutral	...	...
Heat value, Btu per lb, min	....	18250	....	...	...
Initial boiling point, min, F	....	...	375	...	...
90 per cent distillation temperature, max, F	....	...	725	...	...
Specific gravity at 60 F, max	....	...	....	0.935	...
A.P.I. gravity, min	....	...	....	19.8	24–28

TABLE 8 DIESEL FUEL SPECIFICATIONS FOR MEDIUM-SPEED, MEDIUM-SIZE ENGINES

Property	A.S.T.M. 1939	British B.S.I. 1937	Foreign navy	Manufacturer	Refiner	User
Cetane number, min	35	...	...	...	....	50
Aniline point, min, F	....	143	...	...	....	....
Viscosity, SSU at 100 F	65 Max	69 Max	60 Max	35–75	80 Max	35–45
Carbon residue, max, per cent	0.5	3.0	0.5	1.5	1.8	....
Carbon residue on 10 per cent residue, max, per cent	....	...	...	...	....	0.25
90 per cent distillation point, max, F	....	...	662	...	....	540–620
10 per cent distillation point, max, F	....	...	...	...	....	425–500
End point, max, F	....	...	...	...	....	675
Flash point, min, F	110	150	140	150	150	150
Pour point, max, F	35	30	14	...	25	−5
Ash, max, per cent	0.02	0.03	0.05	0.05	0.02	0.01
Water and sediment, max, per cent	0.1	0.5	0.1	0.5	....	0.05
Precipitation number	....	...	0.25	...	....	....
Sulphur, max, per cent	1.5	2.0	0.75	2.0	1.75	0.50
Corrosion at 210 F, max	....	...	...	...	....	Tarnish
Acidity	Neutral	...	Neutral	...	....	Neutral
Heat value, Btu per lb, min	....	18750	19000	...	....	....
Specific gravity at 60 F, max	....	...	0.895	...	0.920	....
Specific gravity at 60 F, min	....	...	0.840	...	....	....
A.P.I. gravity, max	....	...	...	...	...	40
A.P.I. gravity, min	....	...	...	...	22.3	32
Water by distillation, max, per cent	....	...	...	...	0.25	....
Sediment by extraction, max, per cent	....	...	...	...	0.1	....

Engine Deposits and Wear. Due to reduced exposure of cylinders to corrosive, intermediate products of combustion, the possibility of more complete burning, the tendency toward elimination of partially burned fuel, and the practical toleration of greater quantities and kinds of deposits and, consequently, wear in large engines, the specifications limiting the maximum amount of impurities become more liberal as the engine size increases and speed decreases.

Availability. It is difficult to make rigid statements with regard to the comparative availability of the four classes of Diesel fuels, as conditions with regard to supply and demand change frequently. In general, the great range in Diesel fuels, from a light kerosene distillate to a heavy cracked residual fuel, greatly increases the availability of Diesel fuels as compared to carburetor-type fuels.

Examples of Diesel-Fuel Specifications

Heater-Type Fuels. Table 6 shows examples of fuels requiring heating and cleaning before use. They are included in the general class of residual or bunker fuels and are characterized by high densities, sometimes heavier than water, high viscosities, 1000 sec SU and above, high carbon residues, high percentages of asphaltic matter, and ash.

Large Slow-Speed-Engine Fuels. Fuel specifications for large slow-speed engines are given in Table 7. These fuels are characterized by large ranges in viscosity, in one case, from 40 to 200 sec SU, large distillation ranges, approximately 375 to 850 F, high percentages of impurities, and ignition qualities down to 30 cetane number. The maximum limits for carbon residue vary considerably from a low value of 0.75 per cent to a high value of 3 per cent. The British fuel shown in Table 7 is of the heater type because of its high viscosity limit and percentages of impurities.

TABLE 9 DIESEL-FUEL SPECIFICATIONS FOR LIGHT HIGH-SPEED ENGINES

Property	A.S.T.M. 1-D proposed 1939	British B.S.I. 1937	Refiner	Manufacturer
Cetane number, min	45	..	...	38–48
Aniline point, min, F	..	140	...	...
Viscosity, SSU at 100 F	..	51 Max	35–45	35–40
Carbon residue, per cent, max	..	0.2	0.1	0.03
Flash point, min, F	100	150	150	150
Sulphur, max, per cent	0.5	1.5	1.0	0.5
65 per cent boiling point, max, F	..	662	...	...
Final boiling point, max, F	590	..	...	700
Ash, max, per cent	0.01	0.01	0.01	0.02
Pour point, max, F	0	20	20	0
Copper-strip corrosion at 122 F	Pass	..	...	...
Alkali and mineral acid	Neutral	..	...	...
Water and sediment, max, per cent	0.05	0.1	0.1	0.2
Heat value, min, Btu per lb gross	..	19000	...	...
A.P.I. gravity at 60 F	..	..	29.3–38	26–29
Color, N.P.A., max	..	..	8	...

TABLE 10 U. S. NAVAL DIESEL-FUEL SPECIFICATIONS

Property	Specification 7-0-2	7-0-2a	7-0-2b	7-0-2c
General fuel type	Hydrocarbon	Distillate	Distillate	Distillate
Cetane number, min	....	....	....	....
Diesel index number, min	....	....	40	45
Viscosity, SSU at 32 F	200 Max	200 Max	....	....
Viscosity, SSU at 100 F	....	....	35 to 55	35 to 45
Pour point, max, F	....	....	0	0
Flash point, closed cup, min, F	150	150	150	150
Carbon residue, max, per cent	....	....	0.5	0.2
Total sulphur, max, per cent	1.50	1.50	1.0	1.0
Ash, max, per cent	0.1	0.1	0.01	0.01
Water and sediment, max, per cent	0.5	0.5	0.05	0.05
Corrosion at 212 F	....	....	....	Negative
90 per cent distillation temperature, F	....	....	....	675
Precipitation number, max	....	....	0.3	....
Color, N.P.A., max	....	....	....	....

Medium-Speed-Engine Fuels. The ranges in viscosity and distillation temperatures for the medium-speed class, Table 8, are definitely less than for the slow-speed classification, ranging from about 35 to 75 sec SU and 350 to 750 F, respectively. The maximum allowable percentage of impurities is reduced and the ignition quality has been raised to a minimum limit of 35 cetane number and above.

High-Speed-Engine Fuels. High-speed-engine fuels range from a light gas oil up into the kerosenes as shown in Table 9. They are typified by narrow ranges in viscosity and distillation temperatures, viz., about 32 to 45 sec SU and approximately 350 to 650 F, respectively. Ash, sulphur, carbon residue, water, and sediment and corrosion are kept as low as is practical from the refining, handling, and cost standpoints. The pour point may be specified at zero, or below, as this type of fuel is often used in services where efficient cold starting and low-temperature pumping are important. The minimum allowable ignition quality has also been increased to an average of about 45 cetane number.

Conclusions

1 The approximate fuel ignition quality, in general, increases with engine speed, viz.:

Heater-type fuel	No limits
Slow-speed fuel	25–35 Cetane number
Medium-speed fuel	35–45 Cetane number
High-speed fuel	45 Cetane number and above

2 The allowable viscosity and distillation ranges become narrower, with lower values, as the size of the engine decreases and its speed increases, viz.:

Type of fuel	Viscosity range, SSU	Distillation range, temp F
Heater-type	200–6000	No limits
Low-speed	40–200	375–850
Medium-speed	35–75	350–750
High-speed	35–45	350–650

3 Carbon residue above approximately 3 per cent may cause marked increases in fuel consumption and exhaust-gas temperatures in relatively large slow-speed-type engines.

4 Salt water, either in the fuel or the intake air, has progressively increased engine wear.

5 Corrosive compounds inherent in the fuel or resulting from refining processes and subsequent incomplete removal have caused corrosion of storage facilities and injection systems resulting in engine failures.

6 Sulphur above approximately 0.7 per cent has caused relatively rapid increases in engine wear.

7 Ash should be held to a practicable minimum in all fuels, but the maximum allowable amount may increase with cylinder size.

8 Sufficient data are not available to show, quantitatively, the effects of volatility, high boiling-point fractions, and carbon residue on the combustion characteristics of a Diesel engine.

9 Sufficient data are not available to make analyses and draw comprehensive conclusions on the effects of sulphur, water, corrosive impurities, ash, incomplete combustion, gum- and lacquer-forming constituents, and cracked products on engine wear.

10 The use of heater-type fuels, i.e., bunker fuel oils, in Diesel engines can be justified, generally, only on the basis of low overall cost, including possible adverse engine performance and increased maintenance, wear, and casualties which, together with somewhat reduced engine availability, practically eliminate their use for modern, high-specific-power engines in the military services.

11 The large variation in fuel-property specifications existing

within each fuel classification is caused by the large differences in present injection-system and combustion-chamber types and design details, which, in turn, prevent practical, fuel-specification standardization.

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Superheat Control and Steam Purity in High-Pressure Boilers

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The author discusses the problem of designing a high-pressure boiler to deliver steam at high temperatures. The problem becomes more difficult as the capacity increases since proper selection and maintenance of boiler materials, design of the separate boiler and furnace units, and maintenance of the latter becomes increasingly more difficult if superheat control and steam purity are to be maintained. The author analyzes these design features and points out the compromises necessary to meet varying operating conditions. He discusses superheater design, maintenance of heat-absorbing surfaces, boiler circulation, relation of furnace-exit temperatures to slagging and boiler performance, and safe metal temperatures in waterwalls and superheaters.

THE problem of designing a high-pressure steam-generating unit to deliver steam at high temperature becomes increasingly difficult as the capacity increases. The most important of the many phases of the problem are:

1 The maintenance of external cleanliness of the heat-absorbing surfaces. (Freedom from slagging.)

2 The maintenance and control of the desired steam temperature over the desired load range.

3 The production of steam of the required purity.

4 The maintenance of safe metal temperatures in the pressure parts of the superheater and boiler.

The first two phases of the problem are difficult to reconcile because with the low furnace-exit temperatures conducive to slag-free operation it becomes difficult, if not impossible, to obtain high steam temperatures over a wide load range with a convection superheater of practical size.

Increases in steam-generator capacities are generally provided for by proportional increases in furnace volume. But slag-free operation with a unit of one size, and coal, does not mean that slag-free operation will follow at the same liberation rate with the same coal if the size and capacity are materially increased, because, at the same rate of heat release in the furnace per cubic foot of furnace volume, the larger the furnace, the higher the furnace-exit temperature. Fig. 1 shows that as the furnace volume increases, there is a rapid decrease in the available furnace perimeter per cubic foot of volume which may be covered with radiant-heat-absorbing surfaces. This ratio may be considered as an index of the cooling capacity of the furnace. For example, if the furnace volume is doubled, the cooling-capacity index decreases 25 to 30 per cent. The furnace-exit temperature, as shown by the upper left-hand curves for a furnace of constant height increases rapidly with furnace volume. Furthermore, if 100 per cent of the available surface perimeter is not covered with heat-absorbing surfaces, or if the surface is installed but made ineffective by slag, the furnace-exit temperature rises rapidly. Thus if 25 per cent of the surface is not effective or not cooled, the furnace-exit temperature will be about 200 F higher than would be the case if 100 per cent of the perimeter is cooled.

If a limit is placed on the maximum permissible furnace temperature, the size of furnace which may be designed to comply is definitely limited. The lower the height of the furnace and the smaller the portion of the furnace perimeter that is cooled, the smaller the size of unit that may be honestly considered to comply, unless (*a*) the heat release is reduced as the capacity is increased in about the same ratio as the furnace-cooling index decreases, or (*b*) the amount of radiant-heat-absorbing surface installed per cubic foot of furnace volume is increased in some suitable manner.

Alternative (*a*) means larger and higher furnaces with lower and lower rates of heat release as the capacity is increased and is the simplest solution if the space is available. If space conditions limit the amount of furnace volume which may be provided, it is possible to subdivide the available furnace volume into two or more furnaces using radiant-heat-absorbing partition walls. Each subdivision adds two such walls with a possible increase of 25 to 30 per cent in the cooled wall surface per cubic foot of furnace volume. This corresponds to a reduction of about 100 F or more in the furnace-exit temperature.

The curves at the right of Fig. 1 indicate the marked increase

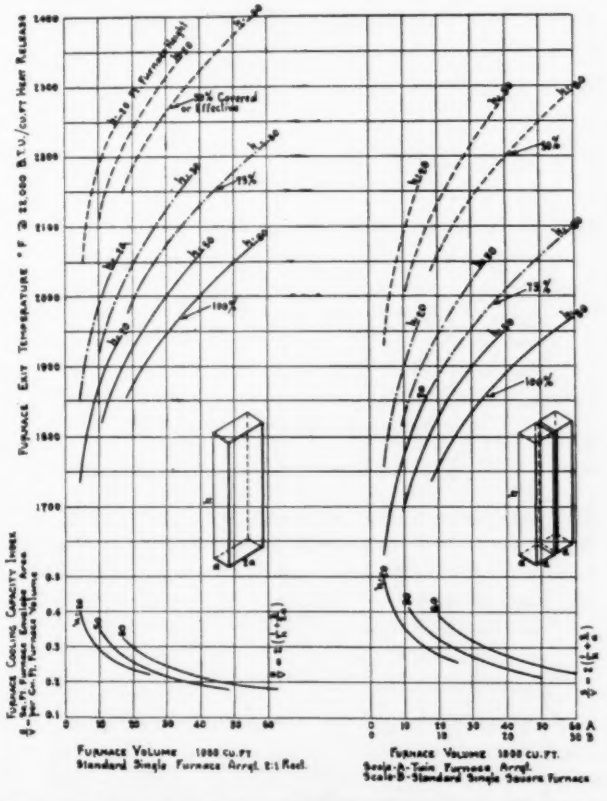
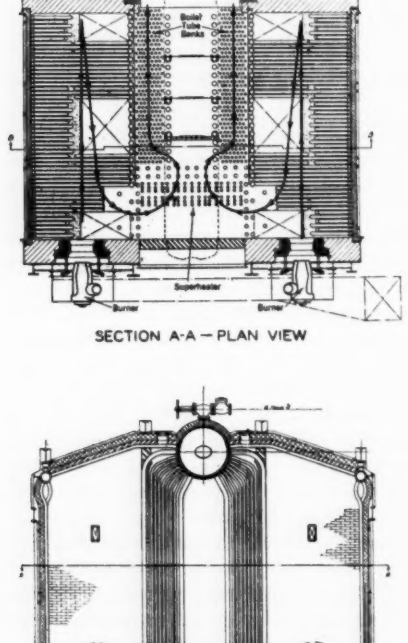


FIG. 1 FURNACE SIZE VERSUS COOLING CAPACITY AND EXIT TEMPERATURE

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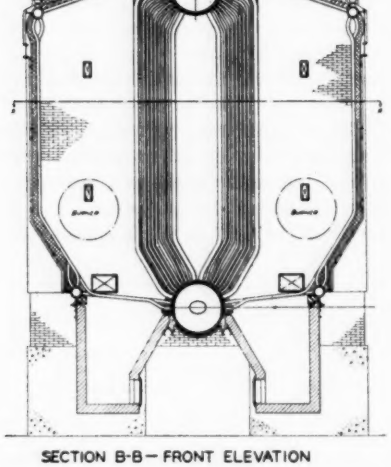
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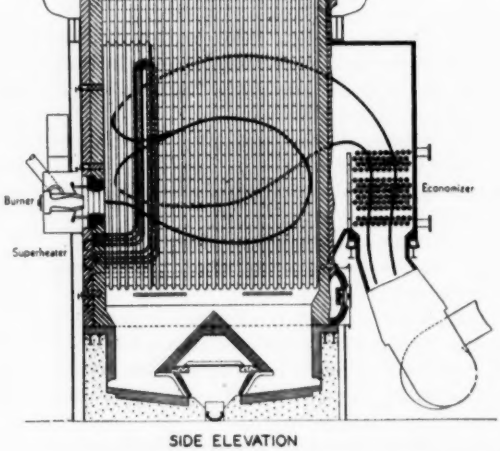


SECTION A-A — PLAN VIEW

FIG. 3 A TWIN-FURNACE UNIT WITH CONVECTION SUPERHEATER, HAVING A CAPACITY OF 100,000 LB OF STEAM PER HR, INSTALLED AT OIL CITY, PA.



SECTION B-B — FRONT ELEVATION



SIDE ELEVATION

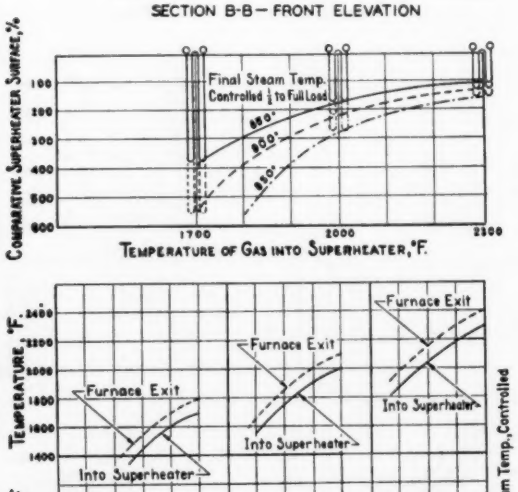


FIG. 2 FURNACE-EXIT TEMPERATURE VERSUS SUPERHEATER SIZE AND CONTROL RANGE

in available furnace-wall area per cubic foot of furnace volume obtainable by subdividing the furnace into two furnaces. There is a correspondingly lower furnace-exit temperature for a given furnace volume for the same heat release per cubic foot. The same low furnace-exit temperature could be attained at the same capacity in a conventional single-furnace unit by reducing the heat release 25 to 30 per cent or for the examples used from 22,000 to between 16,500 and 15,400 Btu per cu ft per hr.

While low furnace-exit temperatures are desirable to prevent slagging of the boiler and superheater, the permissible temperature reduction is limited by the superheater characteristics desired and by the type of superheater used. If a convection type superheater must produce a very high steam temperature at a very low load, its proportions may be impractical, or uneconomical, unless the furnace-exit temperature is so high that slagging of the superheater becomes certain.

For example, the lower curves of Fig. 2 show typical superheater characteristics obtainable with full-load furnace-exit temperatures of 1800, 2100, and 2400 F and corresponding gas inlet temperatures to superheater of 1700, 2000, and 2300 F.

With an 1800-F furnace-exit and a corresponding 1700-F superheater inlet temperature, it is possible to design a superheater for a constant final steam temperature of 850 F at 1350 psi from half load to full load. With a superheater of prohibitive and impractical size it might be possible to obtain the same control range with a final steam temperature of 900 F. However,

it would be impossible to obtain 950 F, except at loads exceeding 70 per cent of full load. If the gas temperature to the superheater is increased to 2000 F, it is possible to obtain 850 F final steam temperature down to about 30 per cent, and 900 F final steam temperature down to 40 per cent of full load. With a gas temperature of 2300 F, it would be possible to obtain 850 F final steam temperature down to 27 per cent, and 900 F final steam temperature down to 40 per cent of full load. The relative cost of superheaters for 850, 900, and 950 F final steam temperature over a load range from one half to full load may be approximated from the upper curves of Fig. 2 which show the comparative surfaces for these various gas temperatures. Assuming as a basis gas at 2300 F and 850 F steam temperature over 50 per cent load range, it may be seen that 60 per cent more surface is required when the superheater inlet gas temperature is 2000 F, and 260 per cent more surface is required when the gas temperature is 1700 F.

To obtain 900 F final steam temperature over a 50 per cent load range, the amount of surface required with a superheater gas-inlet temperature of 1700 F is four times that necessary with 2300 F and 2.6 times the surface adequate at 2000 F.

Obviously, the designer limited to the use of a convection superheater is forced to compromise between low furnace-exit temperature—slag-free operation—and sufficiently high gas temperature for a practical superheater to give the desired steam temperature over the required load range.

Many purchasers who have been, or are, troubled by slagging difficulties arbitrarily specify that new units for them should be designed so that the furnace-exit temperature will be below the lowest expected ash-fusion temperature. In some cases it is even more drastically demanded that the furnace temperature should be less than the initial deformation or agglomeration temperature. For eastern coals this temperature may be as low as 1800 to 1900 F.

Such severe specifications cannot be honestly complied with by conventional units with convection superheaters, especially in large units for high pressures and temperatures if fired with low-grade coals of high ash content and low ash-fusion temperature. If sufficient furnace and radiant-heat-absorbing surfaces can be provided to cool the gases to the desired low value before leaving the furnace, it may be found impossible to obtain the desired final steam temperature over the required load range with any practical convection-superheater arrangement. This difficulty may only be side-stepped by accepting a furnace-exit temperature which may be several hundred degrees higher than originally prescribed, or by using radiant-superheating surface in place of waterwall surface for furnace cooling.

Slagging of superheaters and boiler tubes ahead of them is responsible for a great deal of capacity reduction, outage, and cleaning expense. Periodic or continuous hand lancing is often required to keep such units in service because soot blowers and deslaggers fail to check fouling. Furthermore hand lancing is laborious, disagreeable work, and there may come a time when men willing to do such work will be scarce. It is possible to design steam-generating units in which slag formation may be kept under control and the desired steam-temperature characteristics obtained.

The use of multiple furnaces offers a happy solution of the problem of getting high steam temperatures over a wide load range with low furnace-exit temperatures and comparative freedom from slagging. One of the chief limitations to the use of all superheaters, namely, the difficulty of keeping metal temperatures within safe limits during the starting period, can be eliminated with a multiple furnace arrangement by confining the radiant superheater to one furnace which is fired up only after the boiler starts to generate steam. In such a case, the radiant superheater is never exposed to fire unless steam is flowing through it.

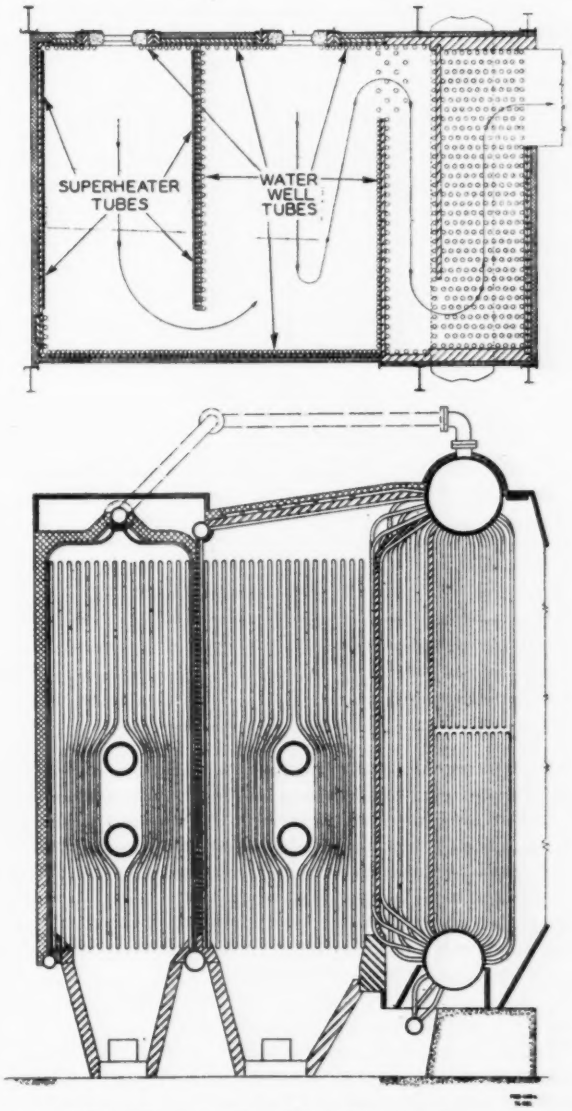


FIG. 4 A TWIN-FURNACE UNIT WITH RADIANT SUPERHEATER IN ONE FURNACE, HAVING A CAPACITY OF 120,000 LB OF STEAM PER HR, INSTALLED AT MOOSE JAW, SASK., CANADA

A number of successful twin-furnace boilers have recently been put into successful operation with and without radiant superheaters. Fig. 3 shows a twin-furnace unit, with only a convection superheater, which has a capacity of 100,000 lb per hr at 725 psi and 750 F final steam temperature. It was installed at Oil City, Pa., in 1935 and was followed by a duplicate order in 1939. Fig. 4 shows a twin-furnace unit at Moose Jaw, Saskatchewan; one furnace is cooled by a radiant superheater while the other is cooled with waterwall tubes only. This unit has a capacity of 120,000 lb of steam per hr at 650 psi and 850 F final steam temperature. It includes no convection superheater. Fig. 5 shows a twin-furnace unit at the Windsor Plant with a radiant superheater in one furnace, in series with a booster convection superheater receiving gases from both furnaces; the capacity is 750,000 lb of steam per hr at 1350 psi and 925 F final steam temperature. The units shown in Figs. 3, 4, and 5 all burn pulverized coal having an ash-fusion temperature often as low as 1900 F.

While the use of multiple furnaces with or without radiant

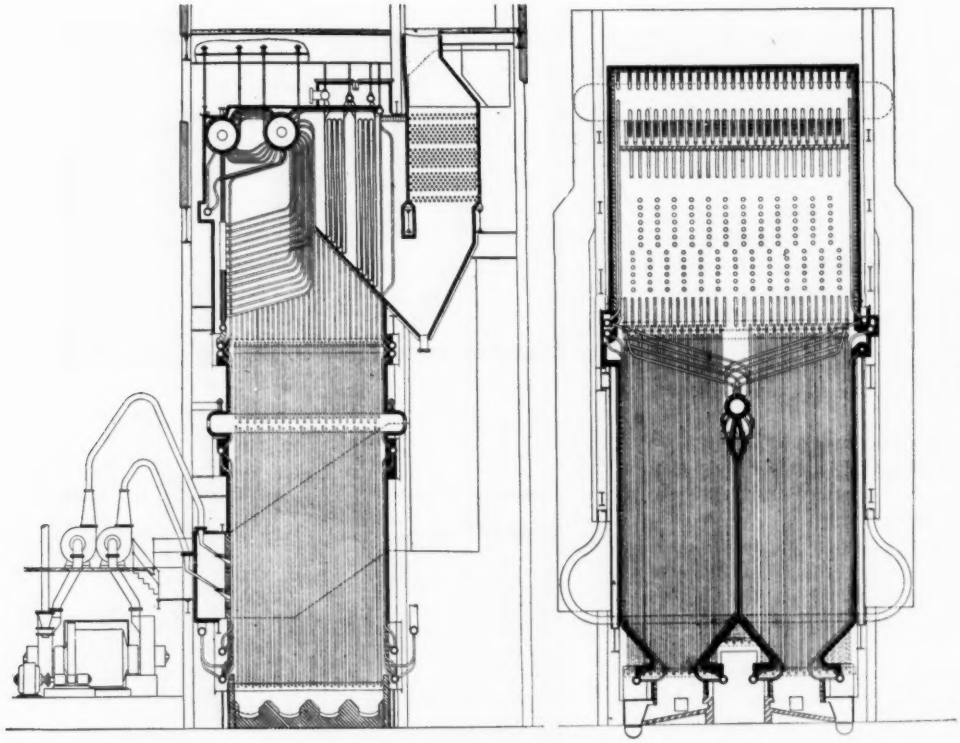


FIG. 5 A TWIN-FURNACE UNIT WITH RADIANT SUPERHEATER IN ONE FURNACE IN SERIES WITH CONVECTION SUPERHEATER RECEIVING GAS FROM BOTH FURNACES. THIS UNIT HAS A CAPACITY OF 750,000 LB OF STEAM PER HR AND IS LOCATED AT POWER, W. VA.

superheaters permits the reconciliation of the conflicting objectives of high steam temperatures and low furnace-exit temperatures, there are sometimes obstacles in the way of applying such designs. Then conventional, single-furnace units, as shown in Fig. 6, must be designed to satisfy the operating conditions as well as possible. This unit, located at New Castle, Pa., is one of seven of the type in operation in various parts of the country with a variety of coals. It generates 400,000 lb of steam per hr at 900 psi and 900 F final steam temperature.

Table 1 shows comparatively the furnace characteristics of the units illustrated in Figs. 3, 4, 5, and 6.

Whether or not it is possible to reduce the furnace-exit temperature as much as desired, the slagging tendency may be minimized by fine pulverization. Fine particles burn faster than coarse particles and may be expected to contain less unburned fuel residues when they pass out of the flame into the clear gas space between the flame and the furnace exit. Such particles can then give up heat by radiation to cold surrounding wall surfaces. If the particle path with respect to the furnace cooling surfaces and the flame is correct, the rate of heat dissipation by radiation from the particle will exceed the rate of heat reception by the particle by radiation from the flame and by convection from the

TABLE 1 FURNACE CHARACTERISTICS OF BOILERS SHOWN IN FIGS. 3, 4, 5, AND 6

Fig. no	3	4	5	6
Capacity, lb per hr	100000	120000	750000	400000
Pressure, lb per sq in	725	650	1555	900
Final steam temperature, F	750	850	925	900
Heat release, Btu per cu ft per hr	21200	20200	23700	24500
Furnace volume V, cu ft	6280	8700	40000	21950
Furnace perimeter S, sq ft	2875	3390	9174	5378
Furnace cooling-capacity index, S/V	0.458	0.389	0.229	0.308
Max cooling surface possible per cu ft, Sm/V	0.720	0.610	0.360	0.490
Actual cooling surface installed per cu ft, Sr/V	0.390	0.560	0.340	0.390
Coverage, per cent	54	92	94	78
Height of furnace, ft	34	34	72	62
Furnace-exit temperature:				
At 100 per cent coverage	1700	1720	1900	1950
At actual coverage	1830	1740	1930	2000
Ash-fusion temperature	1900–2300	2000–2100	1900–3000	2200–2600

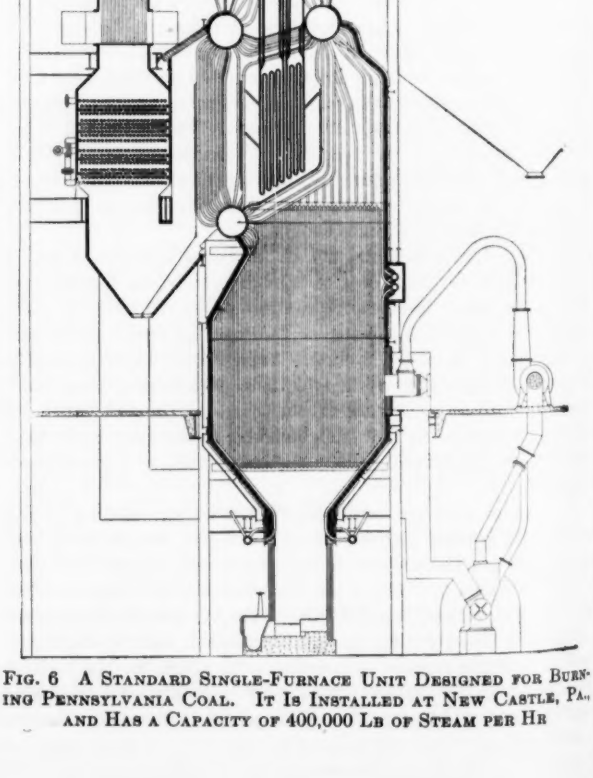


FIG. 6 A STANDARD SINGLE-FURNACE UNIT DESIGNED FOR BURNING PENNSYLVANIA COAL. IT IS INSTALLED AT NEW CASTLE, PA., AND HAS A CAPACITY OF 400,000 LB OF STEAM PER HR

surrounding gases. The smaller the particle, the more certain it is to be frozen throughout by the time it passes into the boiler and superheater.

The additional cost of grinding fine will be partially offset by fuel savings due to reduced carbon loss and more than offset by the reduced cost of boiler cleaning and savings due to fewer outages and resultant loss of production. For example, experience indicates that in the average installation burning Pittsburgh coal one may expect the relationship between fineness, fuel loss due to incomplete combustion, and power consumption and maintenance for a ball-mill job, to be approximately as given in Table 2.

TABLE 2 RELATION BETWEEN FINENESS OF PULVERIZED COAL, FURNACE PERFORMANCE AND MAINTENANCE

Coal fineness, per cent through 200 mesh...	65	70	80	90
Fuel loss due to incomplete combustion, per cent of fuel fired.....	2.0	1.8	1.4	1.0
Fuel loss, lb per ton burned.....	40	36	28	20
Power consumption, kwhr per ton.....	13	15	21	30
Coal equivalent of power at 1.5 lb per kwhr	19.5	22.5	31.5	45
Maintenance cost, cents per ton.....	0.9	1.0	1.4	2.0
Coal equivalent of maintenance cost for coal at $5 per ton.....	3.6	4.0	5.6	8.0
Net coal equivalent of carbon loss + power consumption + maintenance.....	63.1	62.5	65.1	73.0
Per cent of coal fired.....	3.16	3.12	3.26	3.65

Under these conditions the minimum coal consumption occurs at a fineness between 70 and 75 per cent through 200 mesh. If the power consumption or its coal equivalent is lower, or the carbon loss higher, the economical fineness is higher. Actually one may safely conclude that considerable increases in fineness may be made without materially increasing the over-all fuel cost. If the savings due to reduced slagging, reduced boiler-cleaning expense, outage, and loss of production are considered, increased fineness will actually be found to result in lower over-all costs of steam production. The value of fineness should really be evaluated in terms of its effect on slagging, unit reliability, and availability. Fig. 9 shows the effect of fineness on the over-all cost of making steam, in terms of the fuel used for three sizes of plant, if cleaning crews, as indicated, receiving $40 per week per man are used.

While steam-generator reliability and availability depend to a large extent on adequate provisions for slag control, the acceptance of a unit depends also on its ability to deliver steam of extremely high purity.

Steam which is to all intents chemically pure is now required by nearly every steam-generator specification in order to minimize turbine-capacity loss due to blade deposits or hazards due to turbine control-valve fouling. The problem of producing steam of such purity is tied up with boiler circulation, water-level control, and feedwater conditioning. The problem of feedwater conditioning will not be discussed in this paper. Nor can the design problems involved in providing a safe circulating system be completely discussed in the space available. However, it is well to review general principles governing natural circulation. These are as follows:

(*a*) Circulation is easier to maintain in vertical or nearly vertical tubes than in horizontal or slightly inclined tubes.

(*b*) The water flow to be provided per tube in order to maintain it in a furnace at the highest rate of heat input likely to occur is based entirely on experience which indicates that an internally clean tube will not fail if the entering velocity of steam-free water is over 1 fps, and the ratio of water by volume leaving a tube is not less than 5 per cent.

(*c*) The total circulation in a circulating system suitable for high-pressure operation increases as the operating pressure decreases. The system, therefore, may be safely operated at lower pressures up to limiting capacities which vary with the pressure. The superheater pressure drop, rather than circulation, limits the capacity attainable at low pressure for, except at very low pressures, the circulation is good enough to permit operation of the boiler and waterwall at capacities up to or in excess of the maximum capacity at the design pressure.

(*d*) The total circulation decreases slowly with decrease in output, except at extremely low loads, when the decrease becomes more rapid.

(*e*) The total circulation in a given system is a maximum if the water supplied to the circulating system is free of steam. Therefore, it is important to so design the system that the waterwall supply tubes or downtakes entrain as little steam as possible.

Fig. 7 summarizes a circulation analysis of a completely water-cooled steam-generating unit designed to deliver 300,000 lb of steam per hr at 1375 lb per sq in. at the superheater outlet with 1450 lb per sq in. in the drum. Three groups of curves show the characteristics of the waterwall circulation system at the full drum operating pressure of 1450, 850, and 200 lb per sq in. abs, respectively. The dashed curves show the effect of steam entrainment of as much as 25 per cent by volume by the downcomers. This amount of steam entrainment reduces the total circulation materially but not sufficiently, except at a drum pressure of 200 lb per sq in., to limit the safe capacity of the system. A capacity limitation at this low pressure would be advisable because of the reduction in amount of water by volume at the discharge ends of the front wall tubes.

A high-pressure unit would not ordinarily be operated at such low pressure and high capacity because superheater pressure drop would increase rapidly with decrease in superheater outlet pressure. Nor would a good design lack provisions to prevent steam entrainment by the water-supply tubes. Therefore, the real significance of the curves is that the tubes of a properly designed circulating system would be safe under most conceivable operating conditions.

The rate of circulation in the furnace waterwall system has a controlling effect on the behavior of the water levels in the boiler and on the problem of steam purification.

Since the bulk of the steam produced by the steam-generator unit is made by the waterwall system and the first four rows of the boiler, the bulk of the water circulation will be in these elements. Only a small fraction of the total steam produced and a correspondingly small portion of the total circulation will occur in the other boiler tubes. In view of this, if only the steam produced by these relatively inactive tubes is delivered to the rear drum below the water line, the turbulence in this drum from this source will be slight and the density of the water-steam emulsion below the water line relatively high. Therefore, it is usual practice to locate the steam purifier, through which the steam passes on its way to the superheater, in this the most quiet drum *Q* and to limit turbulence in the drum by connecting all waterwall risers or discharge tubes to drum *T* in which the turbulence is naturally greater. This means that the bulk of the circulating water flows into the more turbulent drum *T* first where a preliminary separation of steam and water is induced. The steam flows to the quiet drum *Q* through the steam circulators *SC*. The problem is to return the water to the circulating system. This is accomplished by taking it through the water circulators *WC* into the quiet drum *Q* whence it flows to the lower drum *D* and thence through the downtakes *DT* to the waterwall system.

The head for causing the water to flow through the water circulators is made up of two elements:

a = The head corresponding to the pressure difference between drums *T* and *Q*, and
b = the elevation of the level in *T* above the level in *Q*.

Element *a* increases approximately as some power (about 2) of the load.

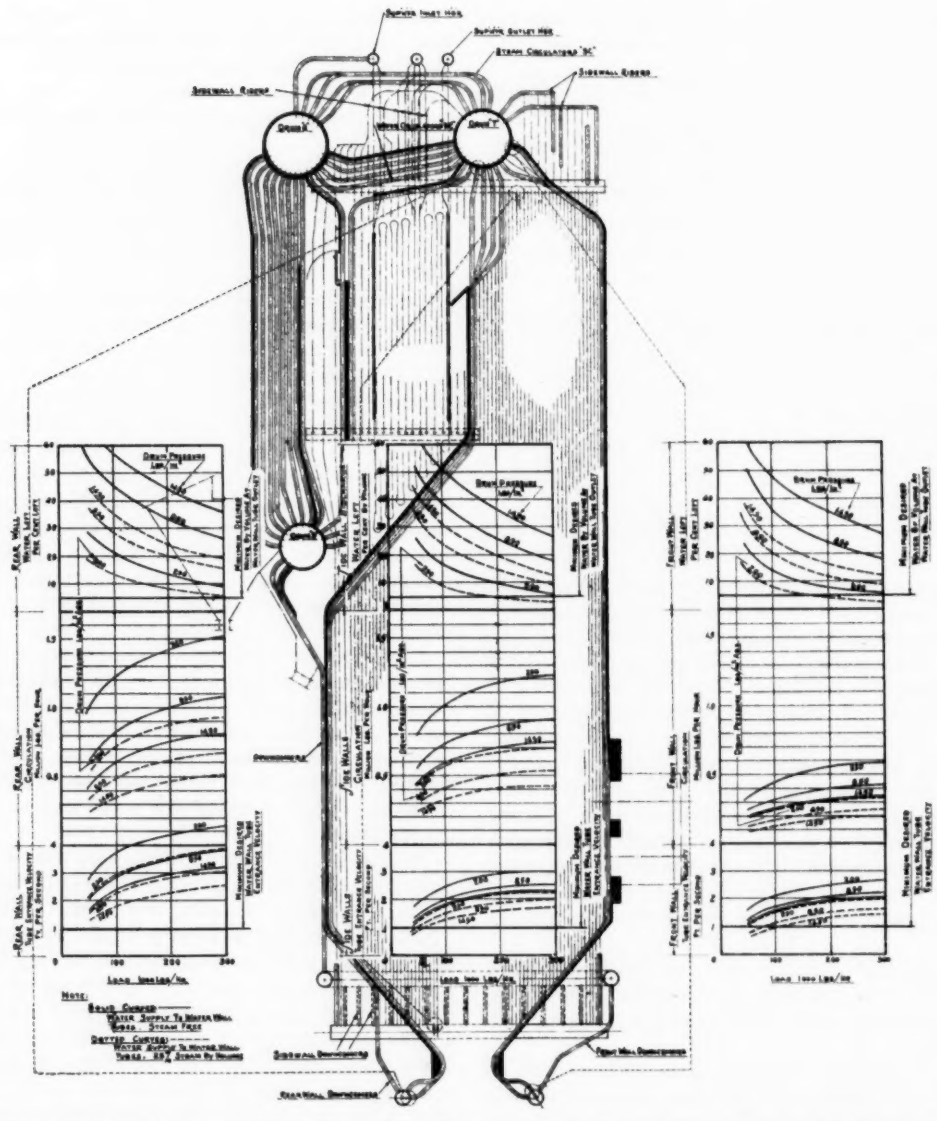


FIG. 7 CIRCULATION ANALYSIS OF A THREE-DRUM HIGH-PRESSURE STEAM GENERATOR AT VARIABLE-PRESSURE OPERATION

Element b depends on the magnitude of a, the density of the water in T and Q, the total amount of water delivered to drum T by the circulatory system and which must be returned to the quiet drum and thence to the circulating system. If the steam circulators were infinite in number and a were zero, b would be independent of a. If the value of b under these conditions is indicated by b_0, then

$$b = b_0 - a$$

The value of b_0 as shown by Fig. 8 changes slowly with load since the total circulation as may be seen by Figs. 7 and 8 does not change very rapidly with load above a certain low load.

Fig. 8 shows approximate relationship between b, b_0, a, and the load.

The elevation of the water in the turbulent drum varies with load as shown by the curve b. Obviously, it is desirable that at no load should the elevation b rise to the level of the lowest steam circulator. This is less important from the point of view of carry-over (if an adequate purifier is installed in the other drum) than from the point of view of having a steady water level. If the level b is allowed to rise above any of the steam circulators, these become water circulators and the number of steam circulators is correspondingly reduced. If one row of steam circulators out of three become water circulators and cease to become steam circulators, the steam velocity in the circulators increases by 50 per cent and a increases approximately 125 per cent. The water-circulator capacity is correspondingly increased and the effect is to instantly depress the level in the turbulent drum. As soon as the level is depressed below the steam circulators the original pressure and head conditions are re-established and the level starts to rise again. This process repeats itself indefinitely and causes the often observed swinging water level. This may be corrected in five ways:

1 Raise the turbulent drum.

2 Lower the controlled level in the quiet drum.

3 Increase the number of water circulators or take some of the water supply for the waterwall system directly from the turbulent drum so that the water circulators will not have to handle it.

4 Decrease the number of steam circulators.

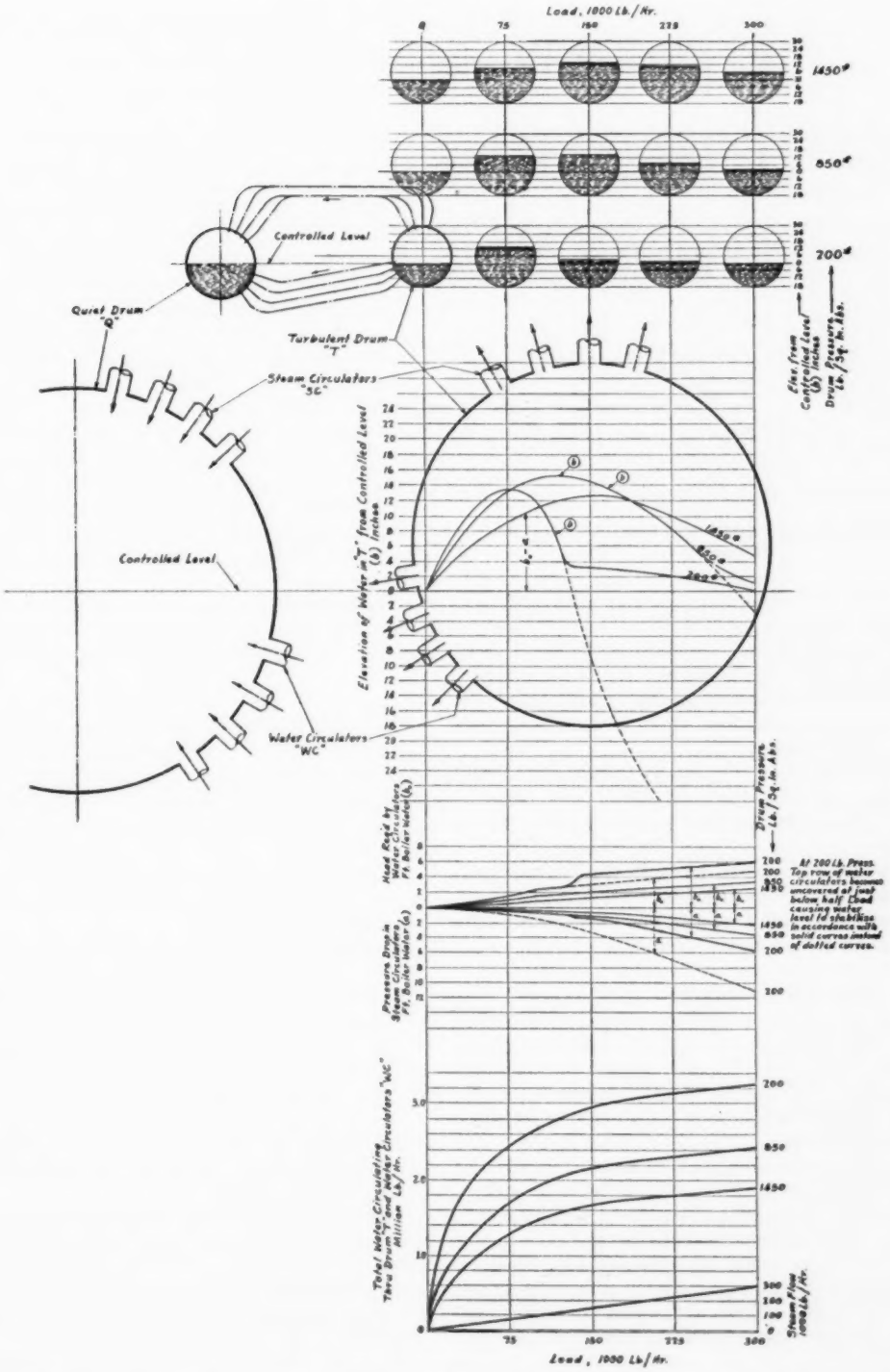


FIG. 8 STUDY OF WATER-LEVEL BEHAVIOR IN A THREE-DRUM HIGH-PRESSURE BOILER AT VARIABLE-PRESSURE OPERATION

5 Arrange the waterwall discharge tubes to have as many as possible discharge above the water line in the turbulent drum, thus making the density of the steam-water mixture below the water line as great as possible.

Of these five correctives, the fourth one does not have as much effect as one might expect. Tests of a number of boilers have borne this out. The reason for this may be seen by referring to Fig. 8 which makes it clear that the effect of increasing the pressure drop between drums by decreasing the number of steam circulators becomes marked only at very high loads. On the other hand the maximum elevation of the level in the turbulent drum above that in the quiet drum occurs near half load when the pressure drop a through the steam circulators is small compared with the head required to cause the circulating water to be transferred from the turbulent drum T to the quiet drum Q through the water circulators WC.

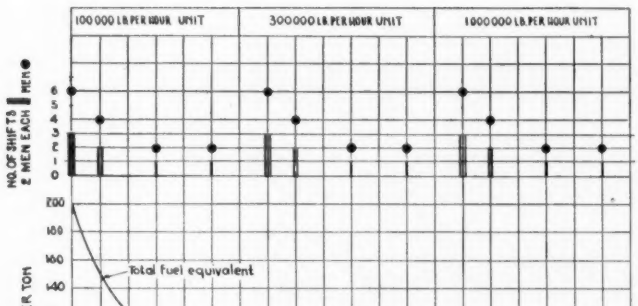


FIG. 9 EFFECT OF FINENESS ON THE OVER-ALL COST OF MAKING STEAM

Most three- and four-drum bent-tube boilers built in the past and still being built are not equipped with an adequate number of water circulators between drums. In this may be found an explanation, at least in part, for the absence of water in the back drum of a four-drum boiler at high loads and for fluctuating water levels and the swamping of the front drum of most three-drum boilers at intermediate loads.

The desirability of elevating the turbulent drum with respect to the quiet drum a reasonable amount and of connecting this drum to the quiet drum with as many water circulators as possible is self-evident.

Steam of uniformly high purity under all conditions is more certainly obtainable if the water levels behave in a predictable manner. Too much emphasis cannot be placed on the necessity of having the waterwall and boiler circulation properly coordinated to this end.

Having taken steps to establish proper water-level behavior, steam of high purity may be obtained by washing or by the use of moisture-eliminating devices which, if the boiler-water concentration is very high, must be nearly 100 per cent effective. With steam washers, since the entrained moisture contains smaller quantities of impurities than the boiler water, the drying of the steam need not be as effective.

Steam washers work on the principle of exchanging for the very impure boiler water, feedwater of much higher purity and then subjecting the mixture of steam and entrained feedwater to a drying operation. One effective exchange method is to bubble the steam through the feedwater at extremely low velocities. The exchange efficiency of this type of washer is between 80 and 90 per cent, and steam with as little as one part per million may be obtained with a boiler-water concentration of 2500 ppm using a feedwater for washing which contains up to 200 ppm of impurities. Certain types of feedwater treatment make the advisability of steam washing questionable because of the tendency of the washer to sludge up too rapidly. Under such conditions the washer should be easily accessible and cleanable so that it may be washed out periodically. It is not advisable to use washers for extremely low pressure because of the difficulty of providing sufficiently low steam velocities through the washer.

When the steam delivered by the boiler is of the purity required by the turbine, safe metal temperatures in the pressure parts of the superheater will be a natural consequence if the superheater design gives sufficiently high steam flow through the tubes. For a 2-in. tube the flow should be in excess of 2000 lb per tube per hr and preferably of the order of 3500 to 4000 lb per hr. This makes it necessary and wise to allow for a reasonably high pressure drop through the superheater. For a 1200 to 1350-lb per sq in. job the pressure drop should certainly not be less than 50 lb per sq in., and if the unit is to be operated at very low loads for considerable periods the pressure drop might better be 75 to 100 lb per sq in. in order to assure good distribution at the lowest load. For example in a unit operating at one-fifth load the superheater would have a pressure drop of only 4 lb per sq in. if the full load pressure is 100 lb per sq in.; at one-tenth load the drop would be 1 lb per sq in.

The metal temperatures in the pressure parts of the boiler and waterwall system will be safely low if the circulation is satisfactory and obeys the criteria previously outlined.

CONCLUSIONS

The chief problem in the design of steam generators of high reliability especially for high-pressure and high-temperature operation in large sizes is the proper reconciliation of the opposed aims, low furnace-exit temperatures with freedom from slagging and high steam temperatures over a wide range with a practical superheater.

High fineness of pulverized coal helps to minimize the slagging problem as does the use of multiple furnaces with radiant or combination superheaters.

Freedom from circulation troubles can be attained by proper design of the circulating system. This involves the use of an adequate number of waterwall supply and discharge tubes so distributed that all parts of the system receive their proper share of the circulating water as free of steam as possible and utilize a proper share of the available pumping head. Internal cleanliness of the tubes must not be overlooked.

Steam of high purity may be obtained more easily if the circulating system is so designed as to assure proper behavior of the water in the boiler drums so that the maximum amount of steam space is always available.

Superheater-metal temperatures will be safe if the steam is pure and the steam flow rate through the superheater elements is sufficiently high. Waterwall-metal temperatures will be safe if internal cleanliness of the tubes is maintained by adequate water treatment and periodic cleaning and if the circulatory system is of proper design.

Properties of Hydrogen Mixtures

By A. W. BRUNOT,[1] LYNN, MASS.

With the increased use of hydrogen as a cooling medium in rotating electrical machines, it has become important to know the properties of hydrogen diluted by small amounts of another gas. Using these properties it is possible to predict the operation of the machine in a hydrogen mixture from the results of tests made in air or pure hydrogen. It is the purpose of this paper to present a summation of data on the properties of hydrogen, air, nitrogen, and carbon dioxide, as well as empirical formulas for the properties of the mixture of hydrogen with any of the other three gases, and to show how accurately the performance of a heat exchanger in hydrogen or hydrogen mixtures may be predicted from tests in air.

Nomenclature

The following nomenclature is used in the paper:

p = pressure, lb per sq in. abs
t = temperature, F
ρ = density, lb per cu ft
c = specific heat at constant pressure, Btu per lb per deg F
μ = viscosity, lb per ft per hr
k = thermal conductivity, Btu per ft per deg F per hr
$(1-y)$ = fraction by volume of hydrogen in a binary mixture
y = fraction by volume of second gas in the binary mixture
h = local gas-side heat-transfer coefficient, Btu per sq ft per deg F per hr
G = weight velocity, lb per sq ft per hr
ΔP = pressure drop, in. water

General Conditions

In this study of the properties of hydrogen diluted by small amounts of another gas, since the pressures encountered in hydrogen-cooled machines are generally less than 2 atm, the effect of pressure on specific heat, viscosity, and conductivity has been neglected. The relations given are for atmospheric pressure but variation in pressure causes little change in properties except density for which a pressure term is included.

In hydrogen-cooled machines the per cent by volume of the second gas is small; therefore, the range of mixtures over which information is desired is 0 to 15 per cent by volume of the second gas. Fig. 1 shows the variation of the properties of mixtures of hydrogen and nitrogen at 70 F as a function of the composition. From the curves it is evident that most of the variation takes place as the first 15 per cent of the second gas is added. The rapidity of the variation, as the heavier gas is added to the hydrogen, may be due to the wide difference in molecular weights but, whatever the cause, the result is that the presence of small amounts of a heavy gas in hydrogen causes a marked change in properties and, hence, a change in the operating characteristics of the fans or heat-transfer equipment.

Due to the scarcity of data on the subject of tertiary mixtures, there has been no attempt made to cover such mixtures in this paper. Instead the paper will deal only with binary mixtures of hydrogen with air, nitrogen, or carbon dioxide.

Density. The density of the individual gases may be calculated directly from the perfect-gas laws. The densities thus computed check very closely the values reported in the International Critical Tables and are the basis for the formulas to be presented.

The density of a mixture of two gases may be calculated directly from Dalton's law. The author has found it more convenient to use, instead of the actual density of the mixture, a density factor, which is merely the ratio of the density of the mixture to the density of pure hydrogen under the same conditions of temperature and pressure.

The density of pure hydrogen is given by the relation

$$\rho_H = 0.188 \frac{p}{t + 460}$$

The density factors are as follows:

Hydrogen-air.....$\rho_m/\rho_H = 1 + 13.36y$
Hydrogen-nitrogen.....$\rho_m/\rho_H = 1 + 12.90y$
Hydrogen-carbon dioxide...$\rho_m/\rho_H = 1 + 20.83y$

Specific Heat. The data on the specific heat of gases at constant pressure are quite plentiful and the results of various investigators check quite closely. The most recent and reliable data have been published by G. B. Taylor (1)[2] and W. M. D. Bryant (2). Their formulas hold throughout a range of 0 to 3500 F with a maximum deviation of 1 per cent for hydrogen and 3 per cent for nitrogen and carbon dioxide. The data for hydrogen, nitrogen, and carbon dioxide were taken directly from these sources, while the data for air were calculated from Bryant's data on nitrogen and oxygen, assuming the air to be 20.9 per cent oxygen and 79.1 per cent nitrogen by volume.

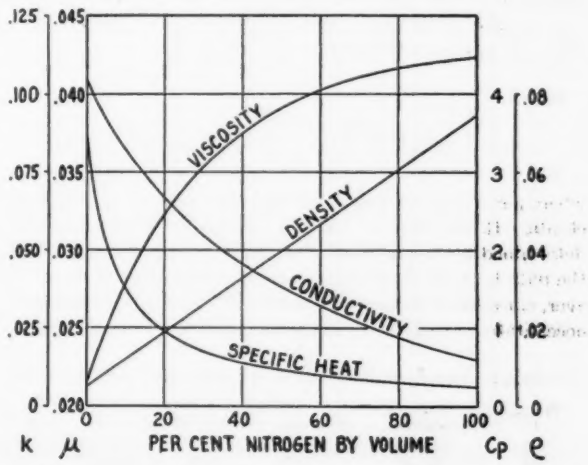


Fig. 1 Properties of Hydrogen-Nitrogen Mixtures at 70 F

The specific heat of a gas mixture appears to obey the well-known law of mixtures, that is

[1] Thomson Laboratory, General Electric Company. Jun. A.S.M.E.

Contributed by the Heat Transfer Group and presented at the Spring Meeting, Worcester, Mass., May 1–3, 1940, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

$$c_m = w_A c_A + w_B c_B$$

where w_A and w_B are the fractions by weight of the two gases present and c_A and c_B their specific heats. The specific heats of the pure gases and the mixtures are given by the following relations:

Hydrogen..... $c_H = 3.430 + 5.75 \times 10^{-5}t + 4.27 \times 10^{-8}t^2$

Air..... $c_{air} = 0.2341 + 3.44 \times 10^{-5}t - 0.463 \times 10^{-8}t^2$

Nitrogen..... $c_N = 0.2407 + 3.26 \times 10^{-5}t - 0.380 \times 10^{-8}t^2$

Carbon dioxide.. $c_{CO_2} = 0.2016 + 9.18 \times 10^{-5}t - 1.736 \times 10^{-8}t^2$

Hydrogen-air.... $c_m = \frac{1}{1 + 13.36y}[3.430 - 0.068y + (5.75 + 43.65y)10^{-5}t + (4.27 - 10.92y)10^{-8}t^2]$

Hydrogen-nitrogen.. $c_m = \frac{1}{1 + 12.90y}[3.430 - 0.084y + (5.75 + 39.56y)10^{-5}t + (4.27 - 9.55y)10^{-8}t^2]$

Hydrogen-carbon dioxide..... $c_m = \frac{1}{1 + 20.83y}[3.430 + 0.971y + (5.75 + 194.65y)10^{-5}t + (4.27 - 42.17y)10^{-8}t^2]$

Viscosity. The available data on the viscosity of pure gases were investigated; the values used as a basis for this paper being selected from the International Critical Tables (3). Formulas to fit these data over the range of 0 to 250 F were then determined.

A search of the literature for a means of calculating the viscosity of a mixture of gases showed that several different formulas had been proposed. Gille (4), working with helium and hydrogen, referred to the work of Puluj and Theisen on the viscosity of mixtures. Trautz (5) proposed a different formula and Schröer (6) and Adzumi (7) give semitheoretical methods of determining the constants in the equation of Theisen. Application of these equations to data on hydrogen-nitrogen mixtures indicated that the equation of the form proposed by Theisen best fitted the data. This equation is of the form

$$\mu_{12} = \frac{\mu_1}{1 + A_1\left(\frac{y}{1-y}\right)} + \frac{\mu_2}{1 + A_2\left(\frac{1-y}{y}\right)}$$

where y is the fraction of volume of gas 2, and A_1 and A_2 are constants. It was found that the constants in the equation as determined by the method of Schröer (6) and Adzumi (7) fitted the data fairly well and included a term for temperature. However, since the relations proposed by Schröer (6) and Adzumi (7) necessitate an empirical determination of one term and since the variations of the constants with temperature are small over the range of 0 to 250 F, the author chose to determine empirically the constants at an intermediate temperature and apply them over this temperature range. Table 1 shows the variation in the values obtained for the viscosity of hydrogen-nitrogen mixtures, using the methods of Puluj, Trautz, Schröer, and the equation of Theisen with empirical constants as determined by the author. The table shows the ratio of the calculated value to the value reported in Tables Annuelles de Constantes et Données Numeriques (8).

TABLE 1 VISCOSITY OF HYDROGEN-NITROGEN MIXTURES

Temp, F	Nitrogen, per cent	Ratio of calculated to reported value — Puluj	Trautz	Schröer	Author
66	19.23	1.0482	1.1454	1.0301	0.9999
66	33.28	1.0329	1.0995	1.0369	1.0034
66	49.47	1.0139	1.0522	1.0320	1.0000
66	79.79	1.0048	1.0140	1.0193	1.0033
212	20.49	1.0586	1.1599	1.0528	1.0136
212	32.81	1.0355	1.1078	1.0319	1.0073
212	49.47	1.0171	1.0571	1.0334	1.0042
212	81.95	1.0012	1.0105	1.0132	1.0003

Rammler and Breitling (9), in an article dealing with the viscosity of flue gases, mention two approximate equations for the viscosity of a mixture but application of these equations to mixtures of hydrogen and nitrogen shows a variation from the data reported in the Tables Annuelles de Constantes et Données Numeriques (8) which is comparable to that obtained using the method of Trautz.

It will be noted from Table 1 that the variation between the calculated and reported values increases as the per cent of nitrogen is decreased in all cases except the author's formulas. Since the region in which we are interested is the region of small amounts of the second gas, the author has in each case determined the constants to be used in the equation by direct substitution of the selected data and these are the values reported.

The data used in determining the empirical constants for the hydrogen-nitrogen mixture were taken from the Tables Annuelles de Constantes et Données Numeriques (8), while those for the hydrogen-carbon-dioxide and hydrogen-air mixtures were taken from the International Critical Tables.[3]

The viscosity of the pure gases and their mixtures are given by the following relations:

Hydrogen..... $\mu_H = 0.02035\left(\frac{t + 459.6}{491.6}\right)^{0.696}$

Air..... $\mu_{air} = 0.04136\left(\frac{707.6}{t + 675.6}\right)\left(\frac{t + 459.6}{491.6}\right)^{3/4}$

Nitrogen..... $\mu_N = 0.04259\left(\frac{732.4}{t + 658.7}\right)\left(\frac{t + 459.6}{533.3}\right)^{3/4}$

Carbon dioxide..... $\mu_{CO_2} = 0.03361\left(\frac{923.6}{t + 891.6}\right)\left(\frac{t + 459.6}{491.6}\right)^{3/4}$

Hydrogen-air.. $\mu_m = \frac{\mu_H}{1 + 1.803\left(\frac{y}{1-y}\right)} + \frac{\mu_{air}}{1 + 0.345\left(\frac{1-y}{y}\right)}$

Hydrogen-nitrogen.. $\mu_m = \frac{\mu_H}{1 + 1.529\left(\frac{y}{1-y}\right)} + \frac{\mu_N}{1 + 0.385\left(\frac{1-y}{y}\right)}$

Hydrogen-carbon dioxide... $\mu_m = \frac{\mu_H}{1 + 1.704\left(\frac{y}{1-y}\right)} + \frac{\mu_{CO_2}}{1 + 0.265\left(\frac{1-y}{y}\right)}$

Fig. 2 shows a comparison of the viscosity as calculated from the foregoing equations and the values reported for hydrogen-nitrogen mixtures in reference (8) and for hydrogen-air and hydrogen-carbon-dioxide mixtures in the International Critical Tables.[3]

Thermal Conductivity. The experimental data on the thermal conductivity of gases were scanty and quite scattered as to value, probably due to the difficulty of measurement.

The available data for the last 20 years or so were plotted and showed a marked increase in the value for any particular gas over this time. The amount of increase was 4 to 5 per cent and was definitely upward for all gases, hence, the most recent data were selected for use in this paper. While it is common to see the formula for thermal conductivity of pure gases written in a form

[3] Bibliography (3), pp. 5 and 6.

using Sutherland's constant, the author could not find enough variation from a straight line to warrant the use of any formula but the straight line $K = k_0(1 + at)$, hence this is the formula used in calculating the conductivity of the pure gas. The equations reported for the pure gases do not strictly follow any particular set of data but rather represent an engineering estimate of the probable value over the range of 0 to 250 F.

There were few data on the conductivity of gas mixtures and no formula was found in the literature to fit the data. However, the same type of equation as used for the viscosity of a mixture was found to fit the data very well if the constants were empirically determined from two test points.

There were available no data on the thermal conductivity of a hydrogen-air mixture. There were, however, data on hydrogen-nitrogen[4] at 32 F and hydrogen-oxygen[5] mixtures at 72 F. Since

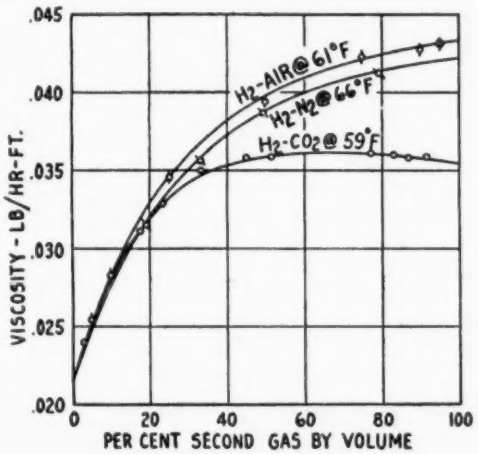


FIG. 2 VISCOSITY OF HYDROGEN MIXTURES; COMPARISON OF CALCULATED AND REPORTED VALUES

air is nearly a 4:1 mixture of nitrogen and oxygen, an attempt was made to predict the conductivity of hydrogen-air mixtures from these data. The constants for mixtures of hydrogen-nitrogen and of hydrogen-oxygen were determined empirically from the data mentioned. Calculated curves for these two mixtures were then drawn at 70 F and the curve for the hydrogen-air mixtures drawn between them. Because of the 4:1 ratio of nitrogen to oxygen in air, the estimated curve for hydrogen-air mixtures was drawn so that the distance from the hydrogen-oxygen curve was $^1/_5$ the distance between the hydrogen-nitrogen and hydrogen-oxygen curves.

The data on the hydrogen-carbon-dioxide mixtures were taken from Physicalish-Chemische Tabellen (10).

The formulas for the thermal conductivity of the pure gases and their mixtures are given as follows:

Hydrogen..... $k_H = 0.0942(1 + 0.00148t)$

Air..... $k_{air} = 0.0133(1 + 0.00170t)$

Nitrogen..... $k_N = 0.0130(1 + 0.00170t)$

Carbon dioxide..... $k_{CO_2} = 0.00783(1 + 0.00266t)$

Hydrogen-air..... $k_m = \dfrac{k_H}{1 + 2.70\left(\dfrac{y}{1-y}\right)} + \dfrac{k_{air}}{1 + 0.243\left(\dfrac{1-y}{y}\right)}$

Hydrogen-nitrogen.. $k_m = \dfrac{k_H}{1 + 2.90\left(\dfrac{y}{1-y}\right)} + \dfrac{k_N}{1 + 0.280\left(\dfrac{1-y}{y}\right)}$

Hydrogen-carbon dioxide.... $k_m = \dfrac{k_H}{1 + 2.85\left(\dfrac{y}{1-y}\right)} + \dfrac{k_{CO_2}}{1 + 0.200\left(\dfrac{1-y}{y}\right)}$

Fig. 3 shows a comparison between the values as calculated using these equations and the values as reported[4] for hydrogen-nitrogen mixtures and for hydrogen-carbon-dioxide mixtures in reference (10).

Prandtl Number. One of the common dimensionless groups

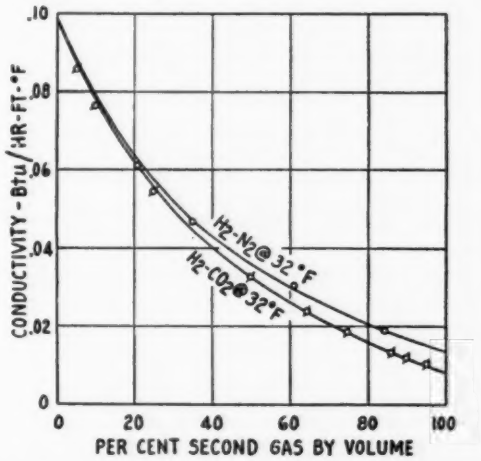


FIG. 3 CONDUCTIVITY OF HYDROGEN MIXTURES; COMPARISON OF CALCULATED AND REPORTED VALUES

used in heat-transfer work is the Prandtl number $c\mu/k$. According to the kinetic theory of gases, this group is a constant for all gases having the same number of atoms per molecule. This is true for ideal and pure gases but there is a variation in the value for impure gases. It was believed that, when two diatomic gases were mixed, this constant remained the same for all compositions, since the value was the same for either of the pure gases alone. Fig. 4 shows a plot of this constant as a function of composition for the three gas mixtures treated in this paper. It will be noted that the "constant" is far from a constant and varies nearly 50 per cent from its value for a pure gas.

The value of the Prandtl number for the pure gases, as determined from the data given in this paper, will vary slightly with temperature instead of remaining constant. This variation is due to the manner in which the equations for the various properties are expressed. The variation over the range of 0 to 250 F is less than 0.5 per cent and for all practical purposes this is negligible.

Application of Data. To illustrate how closely the performance of a heat exchanger in hydrogen can be predicted from the results of tests made in air, the data in the paper were applied to a commercial surface air cooler of the extended-surface type, manufactured by the author's company. This cooler was tested, using air and pure hydrogen and five runs were made with 8.22 per cent nitrogen in the hydrogen.

From the tests using air, the pressure drop and gas-side heat-transfer coefficient were determined. From these values, and the properties of hydrogen and nitrogen, the pressure drop and gas-side coefficient were estimated for pure hydrogen and a mixture of 91.78 per cent hydrogen and 8.22 per cent nitrogen by volume.

The pressure drop ΔP in inches of water is plotted against a modified Reynolds number in Fig. 5. The dotted lines show the estimated pressure drop based on the air tests, while the points show the values as obtained by actual test. It is apparent that

[4] Bibliography (8), p. 60.
[5] Bibliography (3), p. 214.

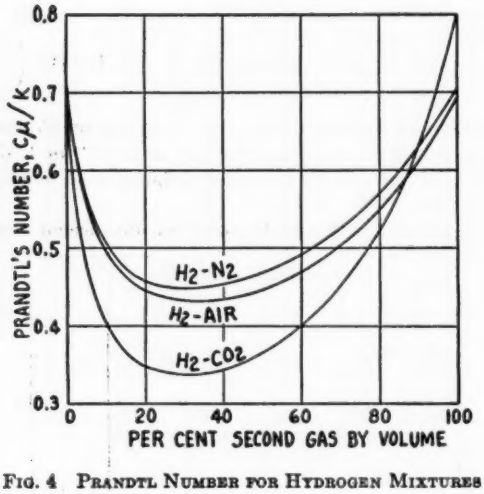


FIG. 4 PRANDTL NUMBER FOR HYDROGEN MIXTURES

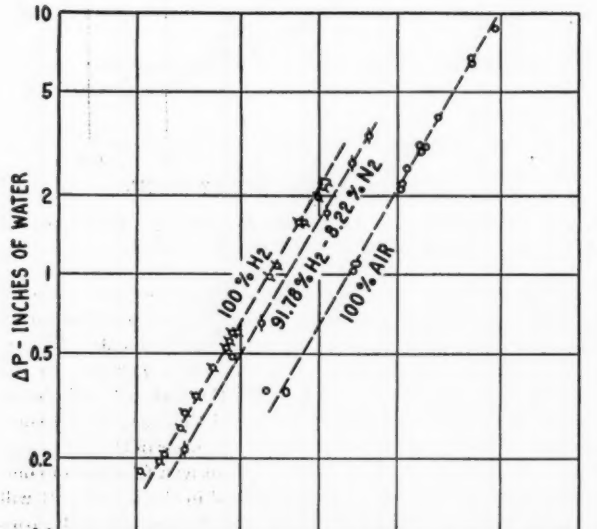


FIG. 5 GAS-SIDE PRESSURE DROP IN HEAT EXCHANGER; COMPARISON OF ESTIMATED AND TEST VALUES

the predicted values are well within the accuracy of the tests and it is possible to predict accurately the pressure drop of a heat exchanger when operating in a medium other than that in which it was tested.

In Fig. 6 is shown the local gas-side heat-transfer coefficient h plotted against a modified Reynolds number. Again, the dotted lines show the estimated values based on the air tests, while the points show the values obtained by actual test. Within the range of Reynolds' numbers covered by the air tests, the test values for the hydrogen lie close to the estimated curve. Below this range, the values depart from the estimated curve but are still within 10 per cent of the estimated value. This variation is due to the fact that the heat-transfer factor $j = \frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3}$ is not a straight line when plotted against Reynolds' number but falls off slightly at the lower Reynolds number.

The points for the mixture of hydrogen and nitrogen are slightly below the estimated curve but, since only 4 points are shown, this cannot be conclusive. The 4 points shown are all within 10 per cent of the estimated value and the accuracy of the data on these particular runs was not very high.

CONCLUSION

The comparison of the estimated pressure drop and heat-transfer coefficient with the test values shows clearly that it is possible to predict the performance of a heat exchanger which is operating in a fluid other than that in which it was tested. Care must be exercised in extrapolating the available data but, within the range of the available test data, the estimated performance will be well within engineering accuracy.

It is the hope of the author that the information presented in this paper will further the application of hydrogen-cooled ma-

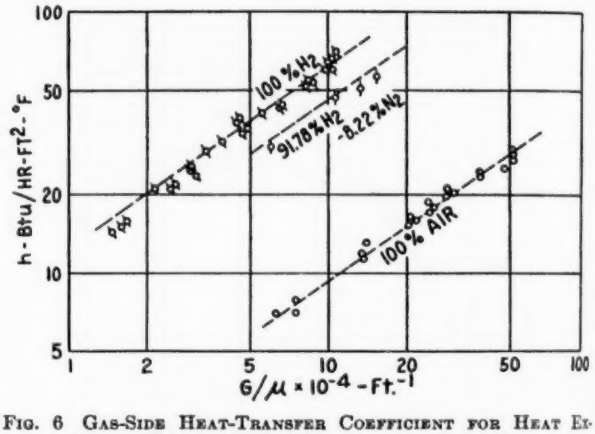


FIG. 6 GAS-SIDE HEAT-TRANSFER COEFFICIENT FOR HEAT EXCHANGER; COMPARISON OF ESTIMATED AND TEST VALUES

chines and will permit the extension of available test data on performance in air to cover the application of hydrogen and hydrogen mixtures.

ACKNOWLEDGMENT

Acknowledgment is made to Stanford Neal and S. L. Jameson, both of the General Electric Company, for assisting in the compilation of these data and for their cooperation and suggestions.

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Discussion

M. W. BEARDSLEY.[6] Specific-heat data have been published since the references cited by the author, and there are now available equations[7] which cover substantially greater temperature ranges and have at least equal accuracy. For the temperature range of interest in this case, calculations can be considerably simplified by using the constant specific-heat values given in Table 2 of this discussion.

TABLE 2 CONSTANT SPECIFIC-HEAT VALUES

Gas	c_p Btu per lb per deg F	Range, deg F	Maximum error, per cent
H_2	3.430	30–250	0.9
Air	0.243	0–250	0.5
N_2	0.249	0–250	0.3
CO_2	0.213	80–250	5.2
H_2-air	$\frac{6.91 + 0.085y}{2.016+26.81\ y}$	30–250 (all proportions)	< 1.0
H_2-N_2	$\frac{6.91 + 0.06y}{2.016+25.98y}$	30–250 (all proportions)	< 1.0
H_2-CO_2	$\frac{6.91 + 2.44y}{2.016+41.98y}$	30–250 (for $y \leq 0.10$)	< 2.0

y = per cent by volume of gas other than H_2

The constant values shown for air and N_2 are actually more accurate over the cited range than the equations given in the paper. The value for H_2 is not as accurate as the equations above 100 F. The value for CO_2 is not recommended for accurate work, but the author's equation for CO_2 is over 3 per cent high for temperatures less than 80 F.

This information on specific heats is offered as data that can be more easily employed in calculations covering a limited temperature range.

W. M. D. BRYANT.[8] The writer was gratified to observe that his gaseous molecular-heat data[9] had been of service in this research. The author has characterized these as "the most recent and reliable data." Neither designation is any longer strictly deserved, for several papers on the subject have appeared since 1933 and some have incorporated refinements of unquestioned value. For example, Murphy[10] has fitted a series of quadratic equations to some of the precision spectroscopic values later used by Dr. Heck. The loci of these equations are in some cases 2 to 3 per cent closer to the true values than those of the writer's equations. Also Sweigert and Beardsley[7] have derived some rather intricate equations which fit the spectroscopic curves with remarkable precision. However, it is doubtful that the use of these more recent data would alter the author's calculations to a significant degree.

H. W. EMMONS.[11] The data used by the author in constructing his formulas appear to be the latest now available. As a convenient reminder, the valid temperature ranges for the various formulas should perhaps have been listed with the formulas rather than being buried in the text.

In general, the formulas chosen are those with some theoretical support, the constants being evaluated from test data. The exception, noted by the author, in the formulas for k is entirely justified from the point of view of available (and reliability of) present data. The Sutherland constant appears in the theoretical formula for k because of its direct appearance in μ and the theoretical prediction that $f = k/\mu c_v$ is constant. Since the Sutherland formula for μ fits the data for gases quite well (the author uses this form for air, nitrogen, and carbon dioxide), while the relation f = constant is only approximately true for actual gases at various temperatures, the thermal conductivity could not be expected to fit a formula with the same Sutherland constant.

The formulas for μ and k of binary mixtures adopted by the author do not agree with the latest theoretical work.[12] However, for the limited data now available, the empirical formulas with the empirical constants as given fit the data very well and are much easier to use than the theoretically more nearly correct equations.

The considerable variation of Prandtl number with composition is interesting and is nicely verified for practical purposes by the accuracy with which the heat-transfer performance for hydrogen-nitrogen mixtures can be predicted from the test data with air.

The experimental checks on the accuracy of calculations made, using the data presented, are excellent. The general value of the paper to those not familiar with the field would have been greatly increased if the author had devoted slightly more space to the dimensionless relations used in predicting performance of heat exchangers from air test data.

F. G. KEYES.[13] The paper is well developed as regards subject matter. A suggestion of possible interest relates to the theoretical work of Enskog who developed the kinetic theory of viscosity and heat conductivity on a very broad basis using the van der Waals molecular-field concept. This leads easily to a theory of mixtures which might find application.

G. W. PENNEY.[14] The effect of small percentages of heavier gases in hydrogen has been of interest ever since the cooling of machines by hydrogen was seriously considered. As plotted in Fig. 6 of the paper, the heat transfer for 91.78 per cent H_2 and 8.22 per cent H_2 appears poorer than that of pure hydrogen. However, in electric machines there are many cases where the velocity may tend to be more or less fixed by peripheral speeds. At a given linear velocity, a small percentage of a heavy gas usually improves the surface heat transfer. This fact made the effect of mixtures of considerable importance when hydrogen was first being considered. In studying this effect, tests were made at the Westinghouse Research Laboratories 12 to 15 years ago. Fig. 7 of this discussion gives the results of a test on a duct which showed a marked improvement in heat transfer when a small percentage of carbon dioxide is added to hydrogen.

The effect of a mixture of CO_2 in hydrogen on the surface heat transfer is shown when the fan speed was held constant in the duct system tested. This closely approximated the condition in many electric machines when the gas composition was varied in a given machine. Under these conditions the gas velocity was 6350 fpm for pure hydrogen and increased to 6460 fpm for 22 per cent CO_2 by volume, and to 7040 fpm for pure CO_2. The heat transfer plotted is the heat dissipated from the external surface of a cylindrical member $1^3/_{16}$ in. outside diam by 18 in. long and located concentrically inside a cylindrical duct of 2 in. inside diam. The

[6] Research Fellow, State Engineering Experiment Station, Georgia School of Technology, Atlanta, Ga.

[7] "Empirical Specific-Heat Equations Based Upon Spectroscopic Data," by R. L. Sweigert and M. W. Beardsley, Bulletin No. 2, State Engineering Experiment Station, Georgia School of Technology, Atlanta, Ga.

[8] Ammonia Department, Chemical Division, Experimental Station, E. I. du Pont de Nemours & Company, Wilmington, Del.

[9] Bibliography of paper (2).

[10] "The Temperature Variation of Some Thermodynamic Quantities," by G. M. Murphy, *Journal of Chemical Physics*, vol. 5, 1937, pp. 637–641.

[11] Assistant Professor, Mechanical Engineering, Towne Scientific School, University of Pennsylvania, Philadelphia, Pa. Jun. A.S.M.E.

[12] "The Mathematical Theory of Non-Uniform Gases," by S. Chapman and T. G. Cowling, Cambridge University Press, London, England, 1939, pp. 230 and 242.

[13] Professor, Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, Cambridge, Mass.

[14] Manager, Electro-Physics Department, Research Laboratories, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.

heat transfer plotted is the watts per square inch divided by the logarithmic-mean temperature difference.

In Fig. 7, the heat transfer is plotted for a gas velocity which varied only slightly. In order to apply such results more generally, tests for a range of velocities are needed, and it is desirable to plot the results on some dimensionless basis. McAdams in his book on heat transmission gives many examples of this. In these

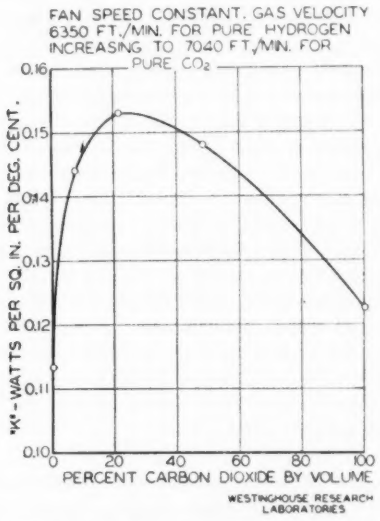


FIG. 7 HEAT-TRANSFER RESULTS OF TEST ON DUCT WITH SMALL PERCENTAGE OF CARBON DIOXIDE ADDED TO HYDROGEN

examples the abscissa is usually related to Reynolds' number and the ordinate usually contains the Prandtl number ($C\mu/K$). By this means the three curves shown in Fig. 6 of the paper can be reduced to one common curve by the proper choice of coordinates. Such curves are valuable, but their usefulness is limited because a given curve applies actually only to geometrically similar surfaces. In the examples given by McAdams, the effect of the Prandlt number varies widely, depending upon the shape of the surface being cooled. For heat interchangers, many different shapes are used so that the experimental data reported are insufficient to cover the various designs being used.

The author has given a concise review of information which was previously scattered through the literature, but he has avoided the question which seems to the writer to be of major importance, namely, the application of these data to the multiplicity of shapes occurring in electric machines. When tests have been made on one particular shape, it is, of course, easy to secure good agreement with calculations on a geometrically similar surface. But it is difficult to predict the effect when a different geometrical shape is used.

It would be of value, if the author in his closure would include a dimensioned sketch of the heat-interchanger surface on which the tests of Fig. 6 were made. Each manufacturer usually has such data for his particular type of surface, but it is difficult to get sufficient data on different shapes.

R. L. SWEIGERT.[15] It is the writer's belief that the author has made a use of specific heats which does not seem quite necessary in dealing with hydrogen mixtures under the conditions under which those mixtures exist when used as a cooling medium.

The specific-heat equations used in the paper do not possess the accuracy stated, within the temperature range which the author gives. The error of the nitrogen and carbon-dioxide equations becomes very large at the higher temperature given in the paper. The error for the hydrogen equation is considerably smaller. The type of equation used is unsatisfactory for a wide temperature range. More recent accurate data and equations[7] on specific heats are available, the new-type equation being $a - \frac{b}{T} + \frac{f}{T^3}$ rather than $a + bT + fT^2$.

For ordinary temperature ranges, relatively close to atmospheric temperatures, older data may vary less than 3 per cent from the new, and the results from such data may come well within the limits of practical accuracy.

In the author's case, constant values of specific heat could have been used just as well, with accuracy equal to that of the equations used. Regardless of the possible inaccuracy of the results for higher temperatures from the specific-heat equations used, the author's results for the temperature range with which he was concerned were not materially affected, as is indicated by the agreement of calculated and experimental results.

AUTHOR'S CLOSURE

The principal criticism of all the discussers appears to be that the specific-heat data used may be 2 to 3 per cent at variance with the latest values. This fact was admitted by the author in his presentation of the paper but it must also be admitted that, at the present time, there is no marked agreement as to the specific heat even of air. In view of the fact that most heat-transfer calculations are excellent if they are within 5 per cent of the true value, this variation of 2 to 3 per cent is not of too great consequence.

In answer to Mr. Penny's question, as to the application of the presented heat-transfer data to "the multiplicity of shapes occurring in electrical machines," it was not the author's purpose to enter into the prediction of the heat-transfer coefficient for a multitude of shapes but rather to demonstrate the accuracy with which the heat-transfer coefficient may be predicted for any gas if the value in air is known.

Mr. Penny's conclusion that the heat transfer was poorer for the hydrogen mixture with 8 per cent N_2 than with pure hydrogen is true if we consider constant values of Reynolds' number. If a

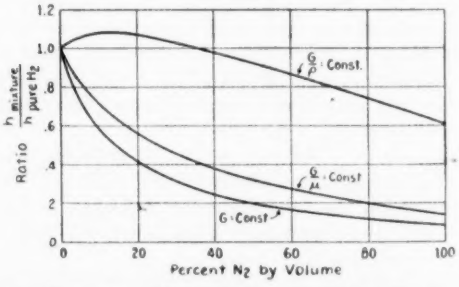


FIG. 8 EFFECT OF MIXTURE RATIO ON HEAT-TRANSFER COEFFICIENT

constant value of linear velocity or a constant value of weight velocity were used, the same would not be true. Assuming that $j = \frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3} \propto \left(\frac{G}{\mu}\right)^{-0.38}$ then for values of constant G/μ constant G and constant linear velocity (or G/ρ), we would obtain three different curves as shown by Fig. 8 of this closure. It may readily be seen that, under conditions of constant linear velocity, the coefficient rises and then falls off in much the same manner as in Mr. Penny's tests. If the Reynolds number or weight velocity are held constant, the heat-transfer coefficient falls steadily as the fraction of impurity rises.

[15] Professor of Mechanical Engineering, Director of Freshman Engineering, Georgia School of Technology, Atlanta, Ga. Mem. A.S.M.E.

In electric machines, the velocity of the gas is nearly constant and is fixed by the machine speed. The heat-transfer coefficient may be improved by adding CO_2 or N_2 to the hydrogen. If this were the only result, this would be the proper procedure but it may be shown that adding nitrogen to 10 per cent of the total volume and maintaining constant velocity will give about 8 per cent increase in heat-transfer coefficient. It will also increase pressure drop by 75 per cent and the power required to circulate the gas by 75 per cent. In many cases, it is desirable to put in 8 per cent more heat-transfer surface in order to save 50 per cent of the power required; this fact must be borne in mind in all problems.

The author wishes to thank all the discussers for their helpful comments and criticisms and trusts that the data as set forth in the paper will prove helpful to those interested in hydrogen-cooled machines.

Experimental Drying

By D. L. COOPER and A. L. WOOD,[1] HALIFAX, N. S.

This paper deals with an investigation of moisture-regain values for salt-fish muscle which show a particularly rapid change in the range between 60 and 75 per cent relative humidity. The determination of experimental drying-rate curves in this range requires study, for much of the commercial drying as now carried out is done within this range. The author describes the system customarily used for this purpose. Dry- and wet-bulb regulators, employing thermionic relays capable of giving control of experimental air conditions in the order of 0.01 F dry and wet bulb, were used in the investigation. Drying-rate curves over a limited range using this method of control are presented.

THE experimental testing of materials, as a preliminary to commercial drier design and fabrication, is now recognized as essential in the development of such equipment. As a result, several types of laboratory equipment have been evolved for this purpose. In general, such equipment is suitable for the majority of requirements; it is only when an odd material is subjected to experiment that a more refined type of equipment is required. Necessary methods of securing satisfactory air flow are well established, but considerable improvement in control is now possible by the use of the newer types of equipment developed during the last decade.

The purpose of this paper is to describe certain refinements in experimental drier control but, in addition, an outline will be presented of the methods and limits of experimental testing as applied to the drying of salt fish, for the controls were developed to satisfy the requirements of this odd material.

Material to Be Dried

The material to be dried may be roughly considered as a biological structure, containing as formative matter 22 per cent salt, 56 per cent water, and 22 per cent protein. It may be further loosely defined as a saturated solution of common salt, given a definite form, and held by a substrate of protein, which was the original muscle of the fish. It has been shown that a more complex combination between salt and protein is formed which has an activity greater than the saturated salt solution itself but, since this could not be detected by equilibrium-moisture measurements, therefore, the first and more simple concept is sufficient for the needs of the drying engineer.

This protein-salt complex is highly hygroscopic, and severe shrinkage of the protein substrate complicates the problem. The formation of a tough, nearly impervious skin during drying under certain conditions is another factor which must be considered.

A further complication is introduced by the heat sensitivity of the system which, in common with all whitefish proteins, cannot be subjected to temperatures higher than approximately 72 F without breakdown. The temperature limit of salt-fish muscle depends to some extent on previous treatments but, in general, 76 F for short periods of time is the upper safe limit.

The problem may therefore be summarized somewhat as follows: To cause evaporation, into ambient air, of water from a saturated solution of salt, forming part of a system in which the remainder is a heat-sensitive, shrinking, skin-forming protein.

The theory of such removal will not be discussed but rather details will be given of the present state of experimentation, designed in a semiempirical manner to set up drying-rate curves without regard to theory, as a basis for drier design immediately required by the industry.

Little work of any value has been done on the drying of salt fish itself. Nor does the literature contain much information of interest on the evaporation from such solutions in protein base.

The complex nature of the material and the scarcity of information on like types made it imperative to disregard any previous work and lay down a program somewhat as follows:

1 Determination of the hygroscopy of the product, that is, determination of the regain values over all ranges likely to be encountered in practice.

2 To determine drying-rate curves for the material in experimental equipment.

3 To check drying rates thus obtained in commercial equipment.

4 To investigate the theory of evaporation from such systems.

So far results in connection with item 1 are complete; apparatus for item 2 has been devised and tested and a few results obtained. Work is proceeding on a commercial drier, designed and put in operation after the results of regain and drying rates were available. The theory is an interesting consideration which yet remains to be undertaken.

Equilibrium-Moisture Contents

A report of this work has been published previously (1).[2] Only sufficient detail will be given here to form a foundation for later discussion.

It is known that moisture regain in this class of material, particularly near the point of equilibrium, is very slow (2). Therefore, any method of determination, employing large samples, is likely to be slow. Unless conditions are controlled more accurately than is usually convenient, a slow method leaves a doubt as to whether the equilibrium recorded is a time equilibrium or one which represents the true equilibrium with vapor at a definite pressure (3). As will be shown later, this is particularly true in the case of salt fish in which the rate of change or regain is very sharp at high relative humidities.

The obvious method of making such measurements is by use of the McBain-Baker adsorption balance so arranged that the air may be removed from the measuring chamber.

In the experiments, the quartz spirals were turned to carry about 0.02 to 0.04 g of fish, and suspended in vacuo at constant temperature until equilibrium had been reached. Moisture at controlled pressure was supplied to these tubes, and an equilibrium read for each pressure of vapor admitted. Both the bath temperatures and vapor pressures could be varied to cover the range desired.

Results obtained in this apparatus are given in Fig. 1. The material was shown to be isohygrometric within the range and accuracy of experimentation, therefore, one curve expresses all the results.

[1] Fisheries Research Bureau.

Contributed by the Process Industries Division and presented at the Spring Meeting, Worcester, Mass., May 1–3, 1940, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Three things require mention: (*a*) The fish system becomes saturated at 75.3 per cent relative humidity, which is the theoretical value of saturated solutions of sodium chloride within the range of temperatures studied; (*b*) the exceedingly rapid increase in water regain at or near this saturation value; and (*c*) the downward and regain values do not agree over the whole range.

The supposition is that the protein system acts as a hygroscopic medium which causes uptake of water until a part of the salt is in free solution. The uptake of water will then be continued until the solution is diluted to give a vapor pressure in equilibrium with that of the surrounding medium. When humidities increased beyond 76 per cent, drops of liquid formed on the sample. These grew until they dropped to the bottom of the measuring tube.

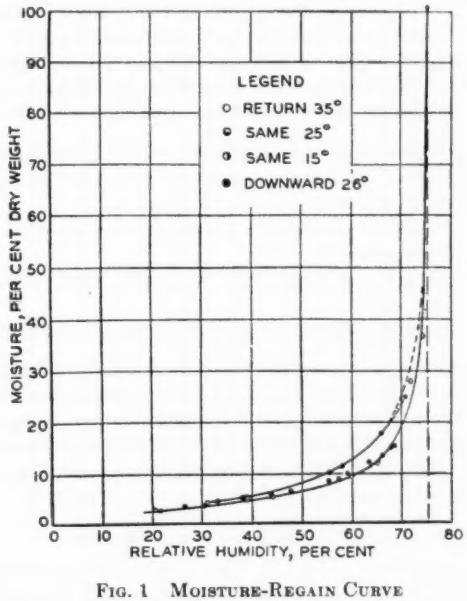


FIG. 1 MOISTURE-REGAIN CURVE

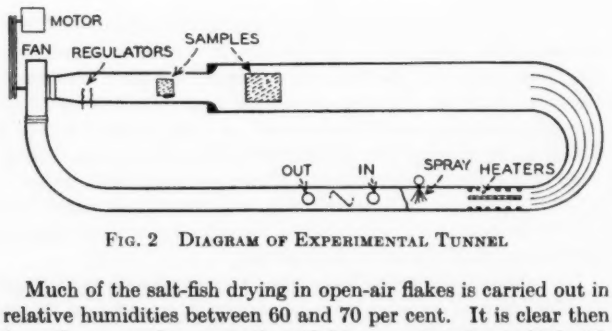


FIG. 2 DIAGRAM OF EXPERIMENTAL TUNNEL

Much of the salt-fish drying in open-air flakes is carried out in relative humidities between 60 and 70 per cent. It is clear then from the curves for regain that, if this range is to be studied with the object of obtaining drying-rate data, the control of experimental drier conditions had to be somewhat better than that usually attempted.

EXPERIMENTAL DRIERS AND APPLICATION OF CONTROLS

A most important consideration in the conduct of the experiments was the application of various types of regulating equipment suitable for the accurate control of conditions in the experimental driers. The systems to be described were all utilized in the experiments.

The drying equipment consisted of a conventional draw-through recirculating tunnel, Fig. 2, the design of which included the usual precautions for obtaining normal turbulent-flow distribution in those sections in which samples were suspended. No provision was made for dehumidifying air, low-humidity experiments having been carried out in winter months. Necessary air exchange to take care of evaporation and to maintain a humidity trend always lower than that desired was easily obtained by suitable adjustment of dampers.

Velocities were controlled at constant fan speeds by damper adjustments, or by varying fan speeds with adjustable drives. Temperatures were maintained with low-heat-capacity electric heaters, and humidities with a water-spray and manual damper control.

CONTROL OF AIR CONDITIONS IN SMALL TUNNELS

The difficulty usually experienced in obtaining accurate control in this type of equipment is caused by the lack of a sink or plenum which takes care of a lag or lead of the commercial type of control. This type will therefore be unsatisfactory except in circumstances in which the heat demand is constant and the supply accurately fixed to meet the demand. Otherwise, the lag together with the heat capacity of the heating elements will cause a hunting action, which becomes more serious if two dependent controls are used, as in the use of wet and dry controls for the regulation of temperatures and relative humidities. The modulating type constitutes an improvement in design. However, no commercial units of this type which were tried would give the required accuracy.

The best method of overcoming the difficulty is to use a combination of high-heat-output, low-heat-capacity heaters, which are operated by an on-off control of several times the required sensitivity.

The cause of failure in the commercial equipment tried appears to be the large currents required across the contacts of a thermoregulator. Even at low voltages and with careful contact design, cumbersome regulators easily disturbed by vibration were required, or regulators enclosed in an inert gas which are also fragile and subject to disturbance. Should this not be done, a change in the condition of the contact will cause excessive variations in the position of the control.

The solution of this difficulty is to use such low currents through the contacts that their resistance becomes negligible. This may be accomplished by several methods; as for example, the use of a very sensitive microrelay, which in turn operates a larger contactor carrying the heater or other load. The difficulties in the use of this system are caused by the sensitivity of the small relay which is usually also responsive to shock, dust, and other conditions. Several very sensitive and satisfactory relays are now available commercially but, as a rule, the second circuit which includes the main contactor must be carefully designed to avoid arcing at the contacts of the small relay. A better method is to use a thermionic tube as the sensitive relay or, if one of sufficient size can be purchased, as the only relay.

The author has tried both methods. The first always caused trouble, but the second has always been trouble-free. Several methods of operating thermionic tubes will be described, and details of others may be found in any standard text on vacuum tubes and their uses (4).

One major effect of systems of this type is to increase the response of common thermoregulators greatly. Thus, the bimetallic spiral regulator, usually sensitive to about $^1/_2$ deg in water and something over 2 deg in air of low velocity, when used with thermionic tubes, will give control of the order of 0.01 F in air.

System 1, shown in Fig. 3, is suitable where direct current is available and there is no objection to using a second relay to accommodate heavy loads. A suitable relay for use in this system is the mercury-in-glass type, tripped by a mechanism operated

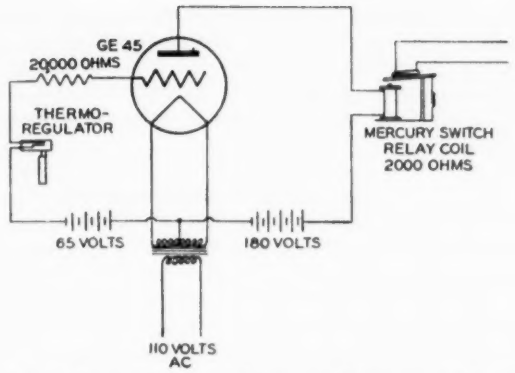


FIG. 3 DIRECT-CURRENT-OPERATED THERMIONIC RELAY

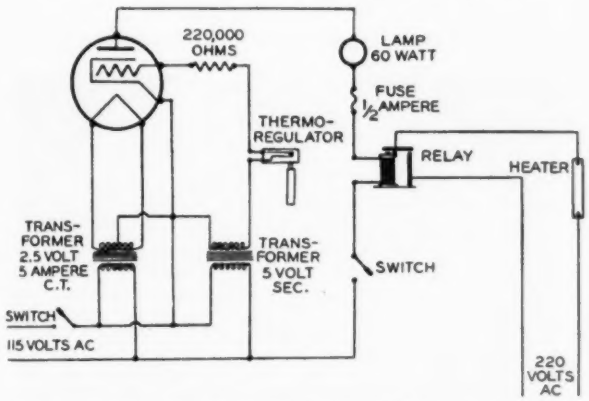


FIG. 4 ALTERNATING-CURRENT-OPERATED THERMIONIC RELAY

by a small magnet. The magnet may be wound to suit the characteristics of the tube used in the circuit.

System 2, shown in Fig. 4, employs the 3 or 4-electrode mercury-vapor-filled tube. One conventional method of use is indicated in Fig. 4, which employs an FG 97 thyratron. This tube is capable of handling small loads of the order of 300 w directly.

DRY-BULB CONTROL

It is clear that, if humidity is to be regulated by wet-wick thermoregulators, the accuracy of control of the dry bulb will affect the precision with which humidities may be maintained.

As previously mentioned, even for the most accurate control, the simple bimetallic type of regulator will be satisfactory if used in one of the systems already described. With suitable auxiliary equipment dry-bulb temperatures have been kept constant to about 0.01 F in this laboratory.

Accuracy of the type specified requires careful design of the auxiliary equipment. Relays, if used, must be of the quick-acting type and operate heaters of low heat capacity, the most suitable of which, for experimental use, is the open-wire electric type. These have very low capacity and no detectable lag. Furthermore, they can be easily adjusted to suit the load for cases where extreme accuracy is desired. A convenient method is to divide the heaters into two sections: (*a*) a variable-control bank, and (*b*) a variable-supply bank. The control bank operates from the thermoregulator through the tube and auxiliary relay (if needed), and the variable-supply bank is easily adjusted by hand, although it may be made automatic if required. The less the load variation, the closer these banks may be set, and the greater the accuracy of control. In practice it was found that, when using open-wire heaters, the supply setting could be varied about 50 per cent without affecting the accuracy materially.

Steam heaters may be used but only at the expense of some precision, since it is nearly, if not quite, impossible to reduce the heat capacity of such heaters to a negligible value for small experimental installations when working below steam temperatures. The thermionic type of control, operating some system of modulation for admission of steam, has been found to help somewhat, but great accuracy is difficult of attainment in spite of refinement of the thermoregulator circuit.

CONTROL OF RELATIVE HUMIDITY

Relative humidities may be controlled by two systems differing in fundamental arrangement: (*a*) a type in which the humidity-regulator setting is influenced by dry-bulb temperatures, and (*b*) a type in which it is not. The first generally employs the combination of wet- and dry-bulb regulators; the second uses a suitable isohygrometric material which is sensitive to changes in relative humidity. Either method has been found to be satisfactory, providing the precautions are such that the sensitivity of the regulator is several times the required amount, and the rate of vapor addition is adjusted approximately to the load.

The actual controls used in this laboratory have always required an addition of vapor to air, for all experiments have been carried out when natural psychrometric conditions permitted this system of control. Similar methods may, however, be applied when dehumidification equipment is available.

THE REGULATOR

Method 1. Almost any type of low-heat-capacity regulator may be used. In practice it has been found that the simple bimetallic-spiral type with the wet wick placed on the spiral gives satisfactory results, providing it is used to operate the control circuit of a vacuum-tube relay, which in turn operates to admit vapor or steam as required.

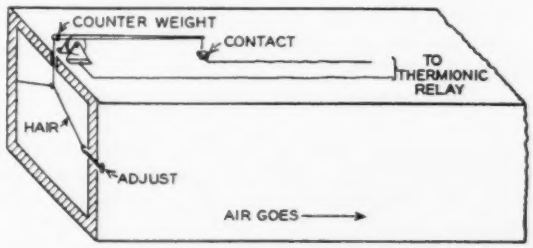


FIG. 5 HUMIDITY CONTROL

The familiar precautions of maintaining the wet-wick regulator in air moving at a velocity of 400 fpm or greater must be taken. In cases of experiments in which very low velocities are used, the regulator may be placed at the downstream side of a shaped nozzle, or fixed in a by-pass, the velocity in which is increased by an auxiliary fan.

Method 2. This method, employing the isohygrometric material, is equally as sensitive as the first, providing arrangements are satisfactory. The material may be either defatted human hair or white Russian horsehair, arranged as shown in Fig. 5. A fine wire, cemented to the long lever arm, ends in a contact which may be made as light as required, since contact resistances are negligible It is essential that the lever fulcrum be as frictionless as possible and that the lever be counterbalanced as shown to take nearly all the strain from the hair. The balance wheel of a cheap clock, mounted in the original frame, has been found to be satisfactory.

This system has the advantage in that it will maintain relative humidities constant, irrespective of changes in dry-bulb temperature. The hair must be reconditioned after each adjustment,

and is, therefore, not suitable when the experiment calls for changes in relative humidities. Further, it loses sensitivity, and usually wanders, when the relative humidity goes above 70 or below 20 per cent.

The use of this system is particularly suitable for a predetermined dry-bulb cycle.

Introducing Vapor. Solenoid valves, operating to regulate the admission of steam or water vapor, are convenient for operation from the controls. Steam is most suitable where the quantity required is small and adds no appreciable heat load to the equipment.

Weight Recorders. It was necessary to continue drying operations for 12 hr or more on light samples, and small interruptions causing slight changes in air conditions affected the rates of loss of water. Therefore, it was desirable to incorporate some automatic method of recording weight changes.

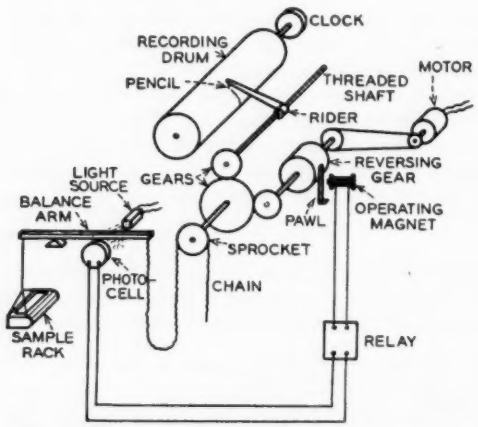


FIG. 6 ARRANGEMENT OF AUTOMATIC BALANCE

Continuous weight recorders have been made in many types from the simple spring-and-stylo, to the various types of weight-dropping balances capable of great accuracy. The simple spring is generally not suitable for investigations on small samples, and the weight-dropping types are too expensive for the average laboratory.

A recording balance, Fig. 6, depending upon the action of a reversing gear, was devised (5) in 1935. It is the most versatile and suitable apparatus for recording loss in weight of a sample being dried.

This type of balance mechanism has the advantage in that it can be made from simple parts such as are supplied by any manufacturer of small gears. The reversing gears in use in this laboratory were made from standard Mecanno parts. These gears may be adapted for use with the finest chemical balances, or the common steelyard capable of measuring several tons. Since they maintain any weighing mechanism in dynamic balance, they increase its sensitivity markedly. A common steelyard loaded with several hundred pounds of rusty iron radiators will record differences in weight, caused by changing relative humidities, with consequent increase or decrease in the amount of absorption on the rusty iron (5).

The equipment consists of two parts: (*a*) the balance proper, and (*b*) the reversing gear with which is incorporated the recording drum. The reversing gear carries a chain, one end of which is attached to the balance, the other to a sprocket on the gear, which is connected to a worm carrying a pencil over the recording drum. The balance works as an automatic chainomatic balance and, if a simple ladder chain be used, the weight of the chain per unit length may be varied to suit the load expected.

The reversing gear is normally motor-turned to keep the sprocket moving forward. When the movement of the gear is interrupted by a pawl, acting on signal from the balance arm, the forward motion of the gear is stopped, the train of gears in the case coming into play and causing the chain sprocket to reverse.

At any position of equilibrium of the balance arm, the gear reverses about 30 times per min, keeping the balance in equilibrium about a point which is determined by the loss in weight of the sample. If the equilibrium is disturbed either by loss or gain in weight of the sample, the balance arm signals the change to the tripping pawl, which causes the gear to rotate in one direction until a new point of equilibrium is reached. This change is recorded on the drum actuated by clockwork and a continuous weight-change record is obtained.

The schematic diagram, Fig. 6, shows the essential actions of the recorder.

Figs. 7 and 8 show recordings of humidity control and weight

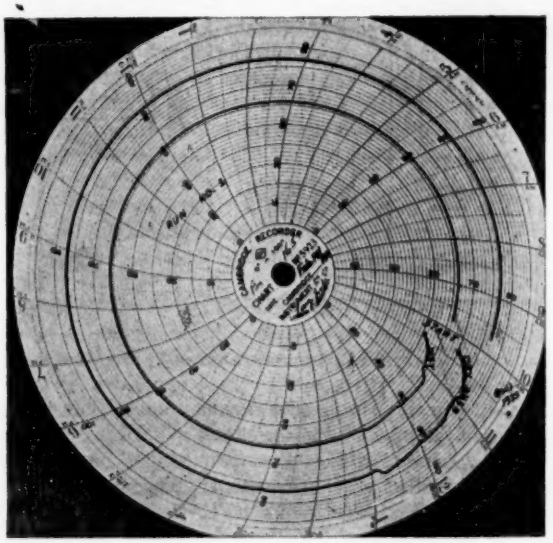


FIG. 7 TEMPERATURE RECORD OF RUN NO. 8

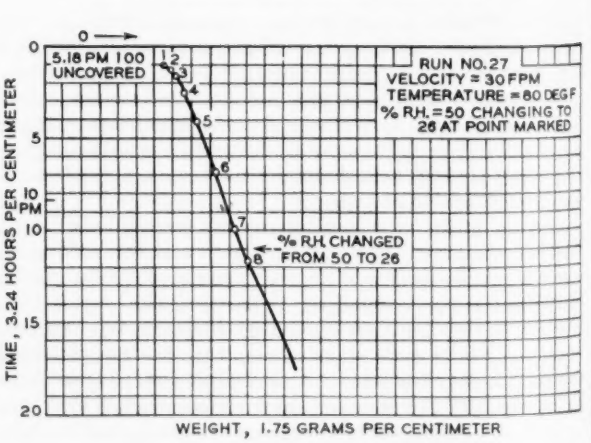


FIG. 8 WEIGHT RECORD OF RUN NO. 27

losses in experiments carried out at the laboratory with which the author is associated.

Fig. 7 was recorded by a Cambridge two-pen mercury-in-steel recording thermometer. The bulbs, being solid, were not particularly suitable for following rapid changes in air temperatures

A mercury-in-glass Beckmann thermometer reading to 0.004 F, placed in the same plane as the recorders, showed variations of the order of 0.01 F. The control in this case was by wet and dry bimetallic spiral regulators acting through thyratron relays to activate (*a*) a self-atomizing spray for humidity control, and (*b*) a contactor which in turn operated electric strip heaters for dry-bulb control.

Fig. 8 is a copy of a typical weight curve taken directly from the graph drawn by the pencil of the automatic recorder. The recorder in this case was made of standard Mecanno parts. The drum was operated by a hand-wound clock. The balance was a common laboratory trip scale which had been in constant use about the laboratory for 4 years and was not in good condition. The weight of the sample and rack averaged 500 g; that of the sample itself about 35 g.

Experimental Results

Experimental drying curves were obtained in the usual manner by suspending the collodion-coated sample, fitted in a streamlined sample holder, from the end of the recording balance. The evaporation occurred from the top of the sample. All samples had a face area of 44 sq cm and were 7.1 mm thick.

TABLE 1 SUMMARY OF EXPERIMENTAL RESULTS

Run	Velocity, fpm	Relative humidity, per cent	Rate (avg), $cm^3/15\ hr \times 10^{-3}$
A	30	65	2.84
B	99	60	5.28
C	200	60	6.85
D	30	50	6.63
E	200	50	6.25
F	600	50	7.25

The results given in Table 1 are insufficient in number to permit conclusions of humidity, velocity, and temperature effects.

The curves shown in Fig. 9 give an idea of the nature of the drying process. The curves for all runs except *A* are similar in form to that of *E*.

One qualitative result of interest was noted as follows: When the average rate was less than approximately 3 to 4 $\times 10^{-3}$ the evaporation caused a heavy deposition of salt crystals on the surface of the sample. Above this range, the sample dried with a smooth nearly salt-free skin. It appears that this is a visual demonstration substantiating the theory of underskin evaporation in processes in which diffusion controls, and the drying potential is high. Measurable limits of undersurface evaporation

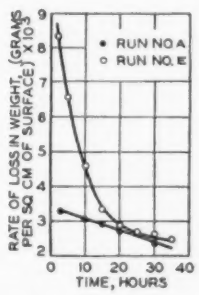


Fig. 9 Curves Indicating Nature of Drying Process

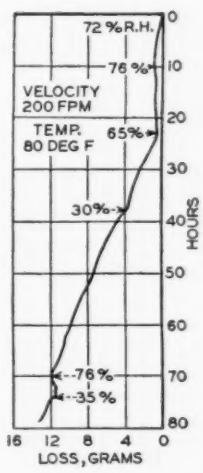


Fig. 10 (Right) Balance Record of a Run at Constant Dry Bulb and Varying Humidities

in many classes of materials should be susceptible of treatment by the use of salt solutions.

A curve of general interest taken from the recording balance is shown in Fig. 10. This was a miscellaneous run completed with the intention of investigating the correctness of the theory of regain as applied to drying. The theory is substantiated by this result.

In general the results obtained on experimental samples are not sufficient in number to enable analyses to be made. A new improved equipment is now in operation in which these results will be checked, and new ones obtained.

The results given herein were used to assist in the design of a commercial drier, now in operation for about 6 months.

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Heat Transfer to Falling-Water Films

By W. H. McADAMS,[1] T. B. DREW,[2] and G. S. BAYS, JR.[3]

Data are presented in this paper for heating water as it flows by gravity in turbulent motion down the inner walls of vertical copper pipes, ranging in height from 0.41 to 6.1 ft. Heat balances check within 5 per cent and the temperature of the inlet water ranges from 38 to 146 F. The data are adequately correlated by the equation: $h = 120\Gamma^{1/3}$, where Γ ranges from 600 to 15,000 lb of water per hr per ft of width of stream. Steam-side and over-all coefficients are given. The rather inadequate data for streamline flow by gravity over nearly horizontal pipes (trombones) are reviewed.

Nomenclature

THE nomenclature which is used throughout this paper is given in the following list:

A = Area of heat-transfer surface, sq ft; A_i for inside, A_o for outside, A_{avg} for wall

C = Specific heat of liquid, Btu per lb per deg F

D = Diameter, ft; D_i for inside, D_o for outside

g_L = Local acceleration due to gravity, approximately 4.18×10^8 ft per hr per hr

h = Individual coefficient between tube surface and water layer, Btu per hr per sq ft per deg F; h_w for inside with turbulent flow, based on logarithmic-mean temperature difference; $h_{a.m.}$ for outside, with streamline flow, based on arithmetic-mean temperature difference

h_{ST} = Individual coefficient between steam and outer wall, Btu per hr per sq ft per deg F, based on saturation temperature of steam and length mean temperature of wall on steam side

k = Thermal conductivity, Btu per hr per sq ft per unit temperature gradient, deg F per ft

L = Distance, ft; L for thickness of tube wall; L_H for height of heating surface; for vertical tubes, L_H is the height; for horizontal tubes, $L_H = \pi D_o/2$

q = Rate of heat transfer, Btu per hr

Re = Reynolds' number $= 4\Gamma/\mu$; Re_m is based on μ_m, Re_f on μ_f

t = Temperature, deg F; t_1 for entering liquid; t_2 for exit; t_p for mean temperature of the surface of the pipe; t_{ST} for steam

U_i = Over-all coefficient of heat transfer, Btu per hr per sq ft of inside surface per deg F logarithmic-mean over-all difference from steam to water

w = Water rate, lb per hr leaving the tube

x = Length of horizontal tube, ft

Γ = Water rate from tube per unit width of stream, lb per hr per ft; for vertical, $\Gamma = w/\pi D_i$; for horizontal, $\Gamma = w/2x$

(Δt) = Temperature difference, deg F; $(\Delta t)_o$, over-all from steam to water; $(\Delta t)_{ST}$, on the steam side; $(\Delta t)_p$, through the pipe wall; $(\Delta t)_i$, on the inside

μ = Absolute viscosity, lb per ft per hr; $\mu_m = (\mu_1 + \mu_2)/2$; μ_f at film temperature; μ_a is taken at $(t_1 + t_2)/2$; μ_s is taken at t_p

π = 3.1416

ρ = Density, lb per cu ft

Introduction

The vertical shell-and-tube condenser, used widely in the refrigeration industry, is cooled by layers of water flowing by gravity down the inner walls of the tubes. Published information from several sources (1, 2, 3)[4] report over-all coefficients of heat transfer from condensing ammonia to water in such apparatus. No corresponding individual heat-transfer coefficients were available, and the work described in this paper was undertaken to fill this need.

Apparatus and Procedure

Three single-tube vertical falling-film heaters were constructed, each being heated by steam condensing on the outer wall of the tube. The three heaters have been described in an article (4), giving results for streamline flow of hydrocarbon oils. Table 1 summarizes the essential dimensions.

TABLE 1 NOTES ON VERTICAL APPARATUS

Heater no.	1	2	3
Heated length, L_H, ft	2.00	0.407	6.08
Inside diam of test section, in.	2.50	1.50	2.50
Outside diam of test section, in.	2.88	2.25	2.88
Metal for wall	Copper	Copper	Copper
Metal on steam side	Chrome plate	Chrome plate	Copper
Promoter on steam side	Oleic acid	Oleic acid	Benzyl mercaptan
Bakelite calming section, ft	0.50	0.75	0.75
Number of pipe-wall couples	10[a]	None[b]	10[c]
Jacket material	Steel	Pyrex glass	Copper-plated steel
Inside diam of jacket, in.	5.05	8	6.07
Filtered steam	Yes	No	Yes

[a] In heater No. 1 the five pairs of pipe-wall thermocouples were 1, 5.25, 12.25, 19.25, and 23.5 in. from the top of the heated section.

[b] In order to determine the average steam-side coefficient, a plot was made of $1/U_i$ vs. $1/\Gamma^{0.3}$, which gave h_{ST} of 10,000.

[c] In heater No. 3 the five pairs of pipe-wall couples were 2 7/16, 18 7/16, 34 7/16, 50 7/16, and 66 7/16 in. from the top of the heated pipe.

The saturation temperature of the steam was used in calculating the logarithmic-mean over-all temperature difference from steam to water $(\Delta t)_o$. The mean temperature of the outer surface of the tube was obtained by graphically averaging the ten temperatures over the length of the tube. The saturation temperature of the steam, less the mean temperature of the tube, gave the mean temperature difference $(\Delta t)_{ST}$ from steam to the outer wall. The mean temperature drop $(\Delta t)_p$ through the copper wall was calculated from q, k, L, and A_{avg}. The mean temperature difference $(\Delta t)_i$ between the inner wall and the cooling water was taken equal to $(\Delta t)_o - (\Delta t)_{ST} - (\Delta t)_p$. The heat balances checked within 5 per cent and the rate of heat transfer q was based on the water rate and the measured temperature

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Contributed by the Heat Transfer Professional Group and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] Numbers in parentheses refer to the Bibliography at the end of the paper.

rise.[5] The following coefficients were calculated and are shown in the tables of data and results

$$U_i = \frac{q}{(A_i)(\Delta t)_o} = \frac{\text{Btu}}{(\text{hr})(\text{sq ft inside})(\text{deg F})}$$

$$h_{ST} = \frac{q}{(A_o)(\Delta t)_{ST}} = \frac{\text{Btu}}{(\text{hr})(\text{sq ft outside})(\text{deg F})}$$

$$h = \frac{q}{(A_i)(\Delta t)_i} = \frac{\text{Btu}}{(\text{hr})(\text{sq ft inside})(\text{deg F})}$$

Tables 2, 3, and 4 show the data and results for one third of the runs, omitting duplicate runs which checked runs included in the tables.

Water-Side Coefficients in Vertical Tubes

It is clear from a study of the data that the water-side coefficient increases with increase in Γ, the water rate per unit width. Except for the runs[6] in which Re_f was less than 2100, the data for all three vertical heaters are plotted on logarithmic paper in Fig. 1 with h as ordinates versus Γ as abscissas. The average line AB corresponds to the equation

$$h = 120\Gamma^{1/3} \quad \ldots\ldots [1]$$

All the data of Fig. 1 lie between the two dotted lines laid off for deviations in h of ± 18 per cent. For the longest tube, the runs with preheated water of series VII, shown by the triangles of Fig. 1, agree reasonably well with those where the water entered without preheat, series I–VI, shown by the circles. It was difficult to employ values of Γ greater than 22,000 due to a tendency of part of the water to arc across the weir and flow down the center of the tube, instead of flowing in a layer down the wall. It is possible that the insertion of a plug in the core of the water feed pipe would have prevented this. From the plot of h versus Γ for turbulent flow, it is concluded that heated length had no substantial effect upon h in the range from 0.41 to 6.1 ft.

Table 3 shows runs in which the temperature of the inlet water was varied substantially while Γ was held constant at various values. Examination shows that h increased somewhat with increase in the inlet temperature of the water. Table 3 shows Re_f based on μ_f and Re_m based on $\mu_m = (\mu_1 + \mu_2)/2$. The ratio $h/\Gamma^{1/3}$ was plotted against both μ_f and μ_m. In neither case was the per cent variation large for $h/\Gamma^{1/3}$, μ_f, and μ_m; the plot involving μ_m gave a somewhat better line than that with μ_f, and indicated

[5] Condensate was not measured for heater No. 2.

[6] Table 3, series L and part of series N.

TABLE 2[a] DATA AND RESULTS FOR NO. 1 HEATER

(L_H = 2 ft; t_{ST} = 227 F)

Run no.		Γ	t_1 deg F	t_2 deg F	$(\Delta t)_{ST}$ deg F	h_{ST}	U_i	h	$4\Gamma/\mu_m$	$h/\sqrt[3]{\Gamma}$
IV-	4[b]	2870	42.0	156.0	9.4	15000	1380	1670	5030	117
	7	3060	42.0	155.0	9.2	16000	1420	1740	5260	120
	10	3120	42.0	153.0	9.1	17000	1430	1720	5430	119
	13	2600	43.0	162.0	9.2	15000	1350	1630	4650	118
	16	2310	44.5	169.0	7.3	17000	1320	1560	4260	118
V-	1	2290	43.0	167.0	9.3	13000	1280	1550	4170	117
	5	1830	44.5	180.0	7.8	14000	1240	1480	3450	121
VI-	2	2250	43.5	172.0	8.7	15000	1380	1660	4160	127
	3	2050	44.5	177.5	8.0	15000	1340	1610	3820	127
VII-	3	2120	46.0	178.0	8.2	14000	1290	1560	3790	122
	6	1930	46.0	180.0	8.2	14000	1290	1530	3700	123
VIII-	1	1760	46.5	181.5	7.3	14000	1220	1440	3400	120
	4	1670	47.0	184.5	7.2	14000	1200	1430	3300	121
	7	1530	46.5	186.0	7.3	13000	1130	1340	3020	116
IX-	1	1700	46.0	182.0	10.0	10000	1180	1440	3280	121
	4	1610	49.0	184.0	8.5	11000	1180	1430	3200	122
	7	1200	52.0	201.0	4.8	16000	1130	1320	2520	125
	10	1470	51.0	192.0	6.5	14000	1190	1410	3060	125
	13	3030	45.0	156.0	11.0	13000	1420	1760	5520	121
X-	2	3360	42.0	149.0	10.4	15000	1450	1790	5800	120
XI-	1	3600	42.0	144.0	13.1	12000	1450	1810	6120	118
	3	4080	41.0	137.0	14.4	12000	1490	1870	6630	118
XIV-	1	3820	60.0	150.0	15.0	10000	1480	1910	5190	122
XV-	1	2120	62.0	173.0	12.9	8000	1180	1490	4820	116
	3	2750	61.0	160.0	11.6	10000	1250	1540	6190	110
XVIII-	1[c]	3820	43.0	141.0	16.5	10000	1450	1860	6460	119
	4	4070	42.0	137.0	14.4	12000	1460	1840	6850	116
	7	4750	41.0	125.0	20.5	9000	1420	1880	7660	121
	11	5620	40.5	117.0	22.3	8000	1480	1980	8830	111
	13	2950	42.5	155.0	13.4	11000	1390	1940	5170	135
XXI-	1	3300	48.0	154.0	12.7	12000	1480	1860	6300	125
	2	3320	60.0	154.0	13.6	10000	1370	1740	7160	117
	5	3320	68.0	159.0	14.0	9000	1410	1820	7890	122
	6	3360	79.0	166.0	11.3	11000	1480	1890	8880	126
	7	3360	90.0	172.0	12.4	10000	1520	2000	9790	134
	8	4490	60.0	142.0	14.9	11000	1520	1950	9450	118
	9	2060	70.5	178.0	10.1	10000	1200	1490	5170	116
	10	2060	77.5	180.0	10.4	9000	1190	1490	5500	116
	11	2060	97.0	190.0	10.8	8000	1300	1690	6660	131
	12	2060	88.5	184.5	10.1	9000	1220	1540	6170	120
	13	2900	72.0	166.0	14.0	8000	1350	1750	7190	122
	14	2550	82.5	175.0	11.5	9000	1300	1660	7080	122
XXII-	1	3350	58.0	153.0	16.6	9000	1470	1930	7500	127
	3	4070	81.0	160.0	16.9	8000	1600	2170	10800	136
XXIV-	1[d]	3130	42.5	105.0			648			
	4	3450	42.0	100.0			660			
	7	4260	41.0	92.0			682			
	10	4830	40.5	87.0			692			
	12	2720	43.0	113.0			648			
	13	2120	44.0	124.5			616			
	15	1380	48.0	143.0			522			
	17	2380	45.0	119.0			620			

[a] References (10, 12).

[b] Just before run IV-1, the inside of the tube was rubbed with a brush. Just prior to run III-1, seven test periods (10 days) before run IV-4 was made, the outside of the tube was swabbed with a cloth and treated with oleic acid.

[c] Just prior to run XVIII-1, the inside of the tube was rubbed with a cloth coated with fine emery powder, and several cubic centimeters of oleic acid were injected into the steam chest.

[d] Water flows upward, pipe running full.

TABLE 3[a,b] DATA AND RESULTS FOR NO. 2 HEATER

(L_H = 0.407 ft)

Run no.		Γ	t_1 deg C	t_2 deg C	t_{ST} deg C	U_i	h	Re_m	Re_f	$h/\Gamma^{1/3}$
F-	1[c]	4660	30.5	38.2	99.7	1360	1800	10500	15900	108
	3	4660	26.5	34.7	99.7	1360	1800	9660	15800	108
	5	4660	22.6	30.9	99.7	1300	1700	8890	15300	102
G-	1	3900	31.4	46.4	99.6	1350	1770	9050	13900	112
	3	3900	26.4	35.8	99.6	1320	1720	8170	13400	110
	5	3900	22.3	32.0	99.6	1210	1560	7550	12900	99
H-	1	3030	33.4	44.3	99.5	1350	1770	7400	11100	123
	4	3030	27.3	38.8	99.5	1310	1710	6580	10600	118
	6	3030	22.8	34.9	99.5	1250	1640	5900	10300	113
I-	1	2140	39.6	51.9	99.6	1200	1520	5940	8380	118
	4	2140	32.6	46.2	99.6	1160	1480	5580	8390	115
	8	2140	27.3	41.6	99.6	1120	1420	4310	7500	110
J-	1	1170	42.0	60.2	99.7	1090	1360	3500	4870	132
	4	1170	34.6	54.6	99.7	1030	1280	3110	4570	124
	8	1170	24.5	46.3	99.7	971	1190	2590	4280	116
K-	1[c]	5130	45.0	51.7	99.8	1630	2300	14900	20100	134
	4	5130	39.3	46.4	99.8	1580	2250	13500	19200	131
	7	5170	34.4	41.9	99.8	1540	2120	12500	18500	123
	10	5170	27.9	35.8	99.8	1460	2000	11000	17700	116
	13	5170	20.5	28.5	99.8	1350	1790	9350	16700	103
L-	1	420	21.0	55.3	99.9	574	640	935	1620[d]	85
	2	420	22.5	60.6	99.9	676	770	985	1640[d]	103
	3	420	23.9	62.9	99.9	710	810	1013	1680[d]	108
	4	420	25.2	64.0	99.9	722	830	1040	1700[d]	111
	5	420	27.0	65.9	99.9	752	870	1070	1710[d]	116
	6	420	29.0	68.2	99.9	790	920	1130	1750[d]	123
	7	420	31.0	69.6	99.9	798	930	1170	1770[d]	124
	8	420	33.2	70.2	99.9	803	940	1200	1770[d]	125
	9	402	36.0	71.3	99.9	750	865	1200	1770[d]	118
	10	391	37.7	72.2	99.9	746	860	1190	1700[d]	117
	11	391	39.1	72.7	99.9	738	850	1220	1700[d]	116
	12	391	46.3	73.1	99.9	643	730	1330	1780[d]	100
N-	1	490	41.7	70.8	100.0	798	930	1560	2130	118
	4	490	38.2	69.5	100.0	817	955	1490	2110	121
	7	490	33.5	68.0	100.0	840	990	1400	2070[d]	125
	10	469	29.8	66.8	100.0	820	960	1260	1940[d]	124
	14	469	22.3	62.0	100.0	780	905	1110	1850[d]	117
P-	1[c]	650	49.7	71.3	100.0	872	1040	2250	2950	120
	4	746	44.0	66.3	100.0	917	1100	2380	3210	121
	7	735	38.3	63.4	100.0	918	1100	2140	3100	122
	10	735	33.3	60.2	100.0	910	1090	2000	3000	121
	14	735	24.4	54.0	100.0	875	1040	1710	2800	116
Q-	1	2650	44.3	54.1	99.9	1280	1670	7840	10700	120
	4	2700	38.0	48.1	99.9	1190	1520	7140	10400	109
	7	2680	33.0	44.1	99.9	1200	1530	6530	10000	110
	10	2690	27.6	39.3	99.9	1170	1480	5920	9670	106
	14	2690	20.9	32.9	99.9	1090	1360	5140	9120	98

[a] References (11, 12).

[b] The inside surface of the tube was swabbed with a wet cloth before each series of runs.

[c] Just prior to series E, K, and P, the outside surface of the tube was polished with rouge and treated with oleic acid.

[d] Runs in which Re_f was below 2100.

TABLE 4[a] DATA AND RESULTS FOR HEATER NO. 3

(L_H = 6.08 ft; D_i = 0.208 ft; D_o = 0.240 ft; A_i = 3.98 sq ft)

(Vertical copper tube, filtered commercial steam, promoted with benzyl mercaptan)

Run no.	$\Gamma = w/\pi D$	t_1 deg F	t_2 deg F	t_{ST} deg F	Δt_{ST} deg F	h_{ST}	U_i	h	$4\Gamma/\mu_a$ A.A.T. (e)	$4\Gamma/\mu_m$ A.A.V. (f)	$h/\Gamma^{1/3}$
I- 1[b]	5300	39.5	181.0	217.5	11.2	10000	1380	1770	14200	9240	101
II- 2	9500	39.7	166.8	217.6	14.6	12000	1940	2680	23700	16200	127
5	10700	39.0	162.5	217.6	19.3	10000	2040	3020	26100	18000	141
9	11400	38.7	153.7	214.7	17.7	11000	1980	2810	26600	18900	125
12	8840	38.7	171.7	216.5	12.3	14000	2000	2720	22600	15000	131
16	8370	38.7	171.0	216.9	19.4	8000	1850	2750	21400	14200	136
III- 1	10300	38.8	154.5	216.0	25.0	7000	1770	2720	23900	17000	125
4	9610	39.0	162.5	216.6	13.6	13000	1880	2530	23400	16100	119
7	7450	39.8	175.0	214.6	9.6	15000	1820	2350	19600	13000	120
10[c]	6490	39.7	187.0	221.1	9.0	15000	1780	2280	18000	11400	123
13	9650	38.6	161.8	218.7	13.2	13000	1830	2420	23400	16200	114
16	14000	37.5	145.5	219.6	16.3	13000	2070	2840	30800	22500	118
19	15800	38.3	137.0	222.0	19.5	12000	2010	2810	33300	25300	112
22	18800	38.0	130.0	223.5	16.7	15000	2130	2900	37900	29500	108
25	15400	38.5	139.0	222.0	18.5	12000	2000	2770	32800	24800	112
IV- 1	4900	39.5	192.0	223.2	11.2	10000	1430	1840	13900	8600	108
5	7170	39.7	180.6	224.4	14.7	10000	1690	2290	19300	12500	119
V- 1	7460	38.8	180.0	225.5	15.3	10000	1680	2330	19600	12500	120
4	7840	38.8	178.3	225.2	17.7	9000	1780	2520	20600	13500	127
6[d]	9050	38.5	173.3	225.2	10.7	16000	1910	2480	23200	15400	119
9	10100	38.5	167.0	225.2	13.7	14000	1930	2570	25200	17000	119
12	12100	38.5	159.8	225.2	13.2	16000	2090	2800	29000	20100	123
15	13600	38.5	152.1	225.2	13.8	16000	2090	2800	31200	22200	118
18	16400	38.5	141.7	225.2	16.2	15000	2200	3020	25300	26400	119
VI- 3	16400	38.6	133.7	222.4	18.6	12000	1960	2680	33900	26000	105
6	17500	38.5	131.0	222.9	19.7	12000	2020	2820	35500	27600	109
9	20000	38.5	123.8	222.9	19.4	13000	2030	2800	38800	31100	103
12	20900	38.5	120.0	222.4	18.4	13000	2000	2720	39500	30200	99
VII- 6	9610	145.7	195.5	220.7	6.7	10000	1730	2390	43000	41700	112
8	12900	111.0	178.8	220.7	10.7	12000	2030	2760	47100	44100	118
10	16500	88.6	158.5	220.7	13.7	12000	2040	2820	50500	46100	111
12	7150	125.7	195.0	220.7	6.4	11000	1540	1980	29600	27900	103
14	7110	101.0	190.5	220.7	8.0	11000	1610	2020	26200	23400	105
VII- 1	12500	87.3	173.0	220.7	9.7	16000	2120	2870	40700	35700	124
2	16500	79.4	157.1	220.7	13.4	14000	2170	3020	48000	42500	119
3	17800	81.9	155.3	220.7	13.2	14000	2180	3040	51800	46700	116
4	21600	71.6	141.9	220.7	15.2	14000	2260	3080	56100	50100	110
5	21700	73.0	143.3	220.7	16.1	14000	2300	3100	57000	51100	111
7	12800	111.0	178.7	220.7	9.7	13000	2010	2760	47000	44000	118
9	16300	89.3	158.7	220.7	13.2	12000	2010	2780	50100	45900	110
11	7170	125.2	195.0	220.7	6.9	10000	1550	2010	29700	28000	104
13	7000	101.7	189.7	220.7	8.0	11000	1540	2000	25800	23100	104

[a] References (9, 12).
[b] Two or three cubic centimeters of benzyl mercaptan were added just prior to run I.
[c] The steam filter was cleaned just prior to run III-10.
[d] Just prior to run V-6, the second addition of mercaptan was made.
[e] The viscosity μ_a corresponds to the arithmetic average of the terminal-water temperatures.
[f] The viscosity μ_m is the arithmetic mean of μ_1 and μ_2 at the inlet- and outlet-water temperatures.

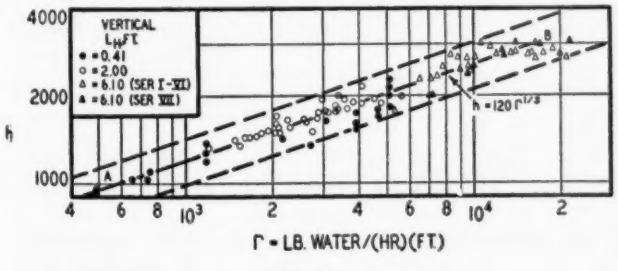


Fig. 1

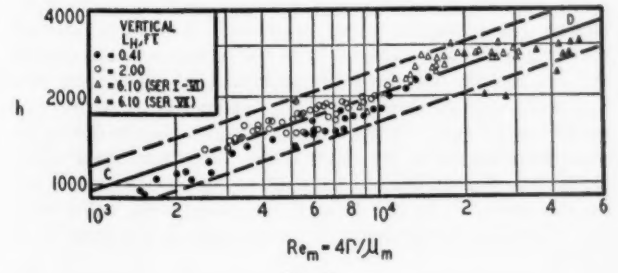


Fig. 2

FIGS. 1 AND 2 WATER-SIDE COEFFICIENTS FOR THREE VERTICAL FALLING-FILM HEATERS

(Re above 2100, plotted as h versus Γ Fig. 1, and as h versus Re_m Fig. 2; the correlation in Fig. 1 is preferred.)

that $h/\Gamma^{1/3}$ varied inversely as $\mu_m^{1/3}$. Consequently the data of Fig. 1 for all three heaters were plotted on logarithmic paper in Fig. 2 as h versus Re_m, and a line CD having a slope of one third was drawn, giving the relation

$$h = 95 Re_m^{1/3} \quad [2]$$

The bulk of the data lie between the two dotted lines which are laid off for deviations of ±20 per cent in h. Although the use of Re_m instead of Γ somewhat improved the correlation of the data for the shortest tube, the high-temperature runs for the longest tube, shown by triangles, show a deviation of nearly 30 per cent, whereas the same data in Fig. 1 did not deviate from the average line by more than 12 per cent. The correlation of Fig. 1 is preferred to that in Fig. 2.

OVER-ALL COEFFICIENTS

Heater No. 2 was also operated with an upflow of water so that the water filled the pipe. The corresponding over-all coefficients are shown in Fig. 3, compared with the data for downflow in layer form. In the range of Γ common to both types of

TABLE 5 RANGE OF HEAT-TRANSFER COEFFICIENTS AND OPERATING CONDITIONS

Heater no.	2	1	3
Heated length, ft	0.407	2.0	6.08
Inlet water, deg F	59–123	41–97	38–146
Outlet water, deg F	83–164	117–201	120–196
Steam temp, deg F	211–212	227	215–225
Γ, lb water per hr per ft width	391–5170	1200–5620	4900–20900
$Re_m = 4\Gamma/\mu_m$	935–14900	2520–10800	8600–51000
h, on water side	640–2300	1320–2170	1770–3100
U_i, over-all coefficient	574–1630	1130–1600	1380–2300
h_{ST}, on steam side	(10000)	8000–17000	8000–16000

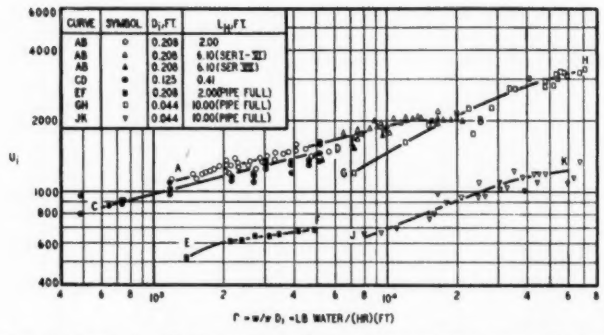


FIG. 3 OVER-ALL COEFFICIENTS FROM STEAM TO WATER FOR THREE VERTICAL FALLING-FILM HEATERS, COMPARED WITH DATA FOR TWO VERTICAL TUBES, RUNNING FULL

(The vertical falling-film heater *AB* gives over-all coefficients 2.2 times those *EF* for the same pipe running full; dropwise condensation on chrome plate, promoted with oleic acid, was used in both cases. For the 10-ft length of standard ½-in. copper pipe running full [Ref. 6], the use of dropwise condensation on chrome plate, promoted with oleic acid *GH* gave over-all coefficients over twice those obtained *JK* without added promoter.* For curves *GH* and *JK*, a value of Γ of 10,000 corresponds to a velocity of 4.06 ft per sec.)

flow (Γ ranging from 1700 to 5000) the falling-film arrangement gave over-all coefficients 2.2 times those for upflow. This increase is due to the fact that the water velocity, for a given water rate, is higher for flow in film form.

STEAM-SIDE COEFFICIENTS

Abstracts of the steam-side coefficients for heaters Nos. 1 and 3 have appeared earlier (5, 6). Tables 2 and 4 show the results. The range of the values of h_{ST} for all three heaters is summarized in Table 5.

HORIZONTAL PIPES—WATER-SIDE COEFFICIENTS

Data (7, 8) for the gravity flow of water over nearly horizontal pipes lie in the region of streamline flow. For these runs it was assumed that half the water flows down each side of the pipe, and hence Γ was taken as equal to $w/2x$, where $w/2$ is the water rate for each stream, and x is the length of the pipe. As for vertical pipes, Re_m was taken as $4\Gamma/\mu$. Since the references (7, 8) did not give the water temperatures, it was necessary to base the coefficients for the horizontal pipes on the arithmetic-mean temperature difference, the type of mean customarily arbitrarily used for runs in streamline flow. Except at the low values of Γ, the resulting coefficients $h_{a.m.}$, are not far different from those which would have been obtained using the logarithmic-mean temperature difference.

For a given Γ, the data for streamline flow over horizontal pipes indicate that $h_{a.m.}$ is an inverse function of the height of the heating surface, $\pi D_o/2$. In the range of the data, for a given Γ, $h_{a.m.}$ varied approximately as $D_o^{-1/3}$; for a given D_o, $h_{a.m.}$ varied approximately as $\Gamma^{1/3}$, for values of Γ ranging from 150 to 400. As shown by Fig. 4, for values of Γ/D_o of 600 and more, the data for horizontal pipes may be approximated within ±25 per cent by the equation

$$h_{a.m.} = 65\left(\frac{w}{2xD_o}\right)^{1/3} = 65\left(\frac{\Gamma}{D_o}\right)^{1/3} \quad \ldots\ldots [3]$$

If the diameter were very large or Γ were very small, the water would be heated to substantially the temperature of the wall, and the final temperature difference, $t_p - t_2$ would approach zero. For the case of negligible evaporation, by definition

$$h_{a.m.} = \frac{(2\Gamma x)(C)(t_2 - t_1)}{\left(\pi D_o x\right)\left(t_p - \frac{t_1 + t_2}{2}\right)} \quad \ldots\ldots [4]$$

The corresponding limit, for $t_p = t_2$ is

$$h_{a.m.} = \frac{4C}{\pi}\left(\frac{\Gamma}{D_o}\right) \quad \ldots\ldots [5]$$

Equation [5] is plotted as the line *AB* on Fig. 4. If there were no evaporation, all points would have to lie to the right of *AB* approaching it asymptotically as Γ/D_o is decreased. At low water rates, the points on Fig. 4 are seen to lie to the left of *AB*, possibly indicating substantial evaporation.[7]

Data (7) for the three horizontal pipes fall within ±25 per cent of the following semitheoretical equation based on stream-

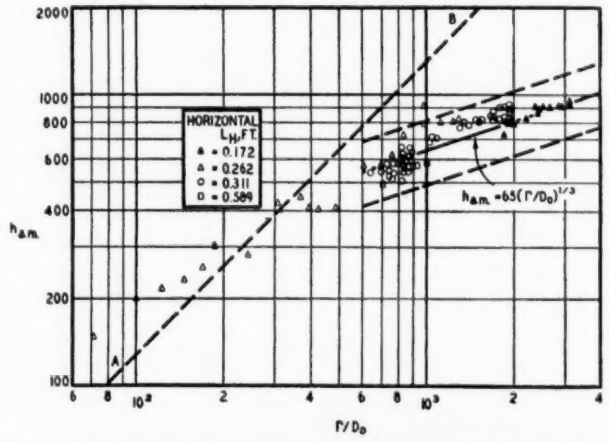


FIG. 4 WATER-SIDE COEFFICIENTS FOR HORIZONTAL (TROMBONE) HEATERS, PLOTTED AS $h_{a.m.}$ VERSUS (Γ/D_o)

(The dotted line *AB* represents the limiting case where the exit water is heated to wall temperature without evaporation, Equation [5]. In the range of Γ/D_o from 600 to 3000, the coefficients for the various sizes of pipe are roughly correlated by the equation $h_{a.m.} = 65\ (\Gamma/D_o)^{1/3}$.)

line flow of oils down the inner walls of the three vertical heaters (4)

$$\frac{h_{a.m.}}{k}\left(\frac{L_H k \mu_a^{1/3}}{C\rho^{4/3} g_L^{2/3}}\right)^{1/3} = 0.67\left(\frac{\mu_a}{\mu_p}\right)^{0.25}\left(\frac{4\Gamma}{\mu_a}\right)^{1/3} \quad \ldots\ldots [6]$$

This relation requires that $h_{a.m.}$ vary inversely as the one-third power of L_H, as shown in Fig. 4. The effect of viscosity on $h_{a.m.}$, predicted by Equation [6], is directly as $\mu_a^{0.025}/\mu_p^{0.25}$. Since in the water runs μ_p varied but little, no substantial effect of viscosity would be expected, and none was found (7, 8) for the water flowing over horizontal pipes. However, Equation [6] predicts that $h_{a.m.}$ varies directly as $\Gamma^{1/3}$, as was found to be the case for the oils, but the data for water on horizontal pipes show that $h_{a.m.}$ varies approximately as $\Gamma^{1/3}$, for Γ above 100. This increased percentage effect of Γ in the streamline region may be due to the presence of ripples in the water layer. The "horizontal" pipes (7, 8) were slightly inclined to facilitate flow of condensate, and the inclination would give an axial component to the water velocity. One (8) of those observers noted ripples in the water layer

* However, as explained in reference 6, "mixed condensation" was obtained in run *JK* due to a small concentration of promoter normally present in the boiler steam used. Data for film-type condensation are given as curve *D*-I of Fig. 1 of reference 6.

[7] The values of $h_{a.m.}$ were apparently based on the condensate rates. Although the rate of flow of effluent water was measured, it is not clear whether Γ was based on the rate of discharge of water or upon the mean of the rates of water feed and discharge. The rise in water temperature was not published, nor was the per cent evaporated given.

and found that the thickness of the water film at the side of the tube varied widely with position along the length.

Summary

Although predictions can be made most closely by using the plots of h versus Γ, and of $h_{a.m.}$ versus Γ/D_o, the following equations are recommended for approximate purposes:

For turbulent flow of water in layer form down vertical tubes, Γ ranging from 600–15,000 lb per hr per ft of stream width

$$h = 120\Gamma^{1/3}$$

with a deviation in h of ±18 per cent.

For the streamline flow of water over nearly horizontal tubes, Γ/D_o ranging from 600 to 3000

$$h_{a.m.} = 65\left(\frac{\Gamma}{D_o}\right)^{1/3}$$

with a deviation in $h_{a.m.}$ of ±25 per cent.

Acknowledgment

The authors wish to acknowledge the work of S. Baum (9), L. M. Blenderman (10), and S. T. Leavitt (11), who participated in the experimental work.

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Discussion

N. C. Ebaugh.[8] The writer would like the authors to answer the following questions:

1 What provision was made in the experimental apparatus for determining the quality or the superheat of the entering steam or, in other words, how was the enthalpy of the entering steam measured? The accuracy of the results has not been questioned since the heat balances were reported to check within 5 per cent. This is merely a request for additional information on the experimental procedure for problems of this type.

2 The over-all coefficient of heat transfer, when dropwise condensation was not promoted appears to be rather high when compared with other available data. Would the authors care to comment on the possible reasons for these high figures?

Authors' Closure

In reply to Professor Ebaugh's first question: High-pressure steam (at pressures usually ranging from 200 to 300 lb gage) from a well-drained line was reduced in pressure through a series of two regulators, giving superheated steam, the temperature and pressure of which were measured at a section just ahead of the jacket on the test section. The saturation temperature of the steam, corresponding to the pressure in the jacket, was used for computing temperature difference, and the enthalpy of the superheated steam was used in the heat balance.

The second question refers to the results (6) shown by curve *JK*, Fig. 3 of the paper. In the preprint, the subtitle to Fig. 3 failed to bring out the fact that a small amount of oleic acid was unavoidably present in the boiler steam used, although this was clearly stated (6). This defect in the subtitle of Fig. 3 was brought out in oral discussion, and the subtitle was revised at that time. The results of curve *GH* of Fig. 3 were obtained when using additional oleic acid above that normally present.

Curve *D-I* of Fig. 1 (6) gives results for a 6-ft × $^5/_8$-in-outside-diam vertical tube of Admiralty metal, operated where it was possible to employ atmospheric-pressure steam sufficiently pure to obtain film condensation, giving U_i of 500 at 6.3 fps.

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Temperature Distribution and Sources in the Conventional Railway Journal Box

By E. S. PEARCE,[1] INDIANAPOLIS, IND.

The author presents the results of an investigation on heat distribution in a conventional journal-box assembly, in the process, outlining the exact test procedure followed. He concludes that the control of heat sources is the first step in avoiding limiting effects of temperature rise in the assembly, and that the condition of the waste, the oil used, and the bearing are the basic factors in controlling these sources.

THE purpose of this paper is to set forth certain data illustrative of the distribution of heat generated in the conventional railway journal-box assembly. These data have been collected collaterally with an extended investigation of heat causes or sources and their control in journal-bearing operation on railroad rolling stock. In a sense, this is an uninterrupted continuation of research and development work described in a previous paper by the author.[2]

The same plant and equipment, as described in that paper, were used for these tests, the amplification of equipment being the application of some 53 thermocouples to a $5^1/_2 \times 10$-in. journal box and contained parts. Fig. 1 shows the box in place on the test plant. Fig. 2 shows diagrammatically the exact location of all the thermocouples and their numerical designation. Fig. 3 shows the method of attachment of thermocouples and their specific location in the bearing and wedge. Fig. 4 is a diagram of the internal arrangement of a thermocouple. Fig. 5 shows the location of the thermocouples in the bearing surface to obtain oil-film temperatures. These are thermocouples Nos. 7, 8, 9, 10, 11, and 12, Fig. 2.

Test Procedure

The preparation for these tests consisted of packing the journal box in the conventional manner with a good grade of wool packing, saturated with oil in the ratio of $3^1/_4$ lb of oil per lb of waste. The car oil used was of a standard brand, having a viscosity of 45 sec Saybolt at 210 F. The combined quantity of waste and oil weighed 8 lb, consisting of $1^7/_8$ lb of waste and $6^1/_8$ lb of oil. Conventional standard Association of American Railroads (A.A.R.) $5^1/_2 \times 10$-in. journal bearings, broached to insure a fixed width of crown, were used. All tests were conducted under a fixed total load of 20,000 lb, giving a unit loading of 890 lb per sq in. of projected actual bearing area.

Tests were run at operating speeds of 20, 40, and 60 mph, equivalent to 268, 536, and 804 ft per min journal surface speed, with rpm of 186, 372, and 558, respectively. A 7-hr continuous run was made at each respective speed, at which time all temperatures recorded were at their stabilized maximum. These temperatures are shown in Fig. 6. Atmospheric temperatures were maintained within practical limits at approximately those of high summer temperatures. Tests were duplicated under conditions of still air and under conditions of moving air, the velocity of the air being proportional to speed, and blowing against one side of the box in the conventional manner obtaining with the journal box under actual service.

For reference purposes, tests were run with the journal submerged in an oil bath to a depth of 1 in., indicated in Fig. 2 as "oil bath only," such tests being designated by this term. The same cycle was used as with the conventional waste pack. A sample comparison, in terms of temperature rise above atmospheric temperature, is shown in Fig. 6 for the temperature at each of the 53 thermocouple locations, as shown by Fig. 2. The temperatures shown in Fig. 6 represent in each case the maximum temperatures obtained after 7 hr continuous running at each respective speed under high atmospheric temperatures.

In general, the center of the journal has the highest temperature; the bearing has the next highest temperature; then, the oil film between the two; and then, the wedge on top of the bearing. The packing temperature is lower than the wedge and lower than the oil film. The temperature of the top of the box is lower than the wedge and the waste pack.

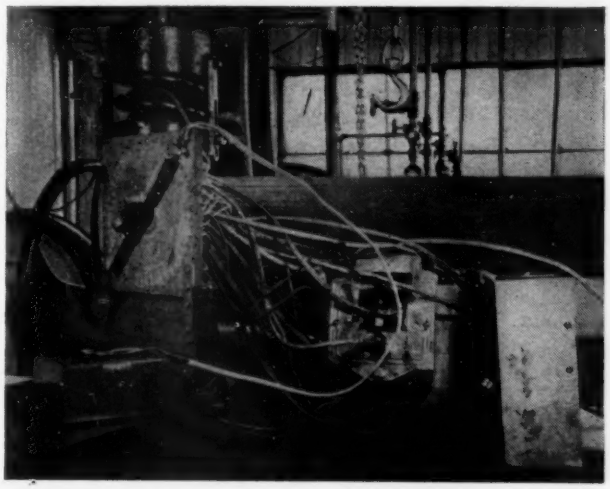


Fig. 1 Box in Place on Test Plant

The comparative temperatures, shown in Fig. 6, are not a measure of the relative proportion of friction losses between perfect bath lubrication and waste-pack lubrication. They have no significance in this respect at all. At 20 mph, the difference in friction loss between bath lubrication and the waste pack was 3.25 per cent; and at 40 mph it was 1.06 per cent, bath lubrication having the lowest loss. At 60 mph, the difference was 1.6 per cent, the waste pack having the lowest loss. No difference in friction could be expected between the two methods of lubrication, due to the fact that the bearing surfaces were receiving all the oil necessary for successful operation and all other conditions were the same.

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[1] President, Railway Service & Supply Corp. Mem. A.S.M.E.

[2] "Locomotive and Car Journal Lubrication," by E. S. Pearce, Trans. A.S.M.E., vol. 58, 1936, pp. 37–45.

Contributed by the Special Research Committee on Lubrication and presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of The American Society of Mechanical Engineers.

Discussion of this paper was accepted until January 10, 1940, and is published herewith directly following the paper.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

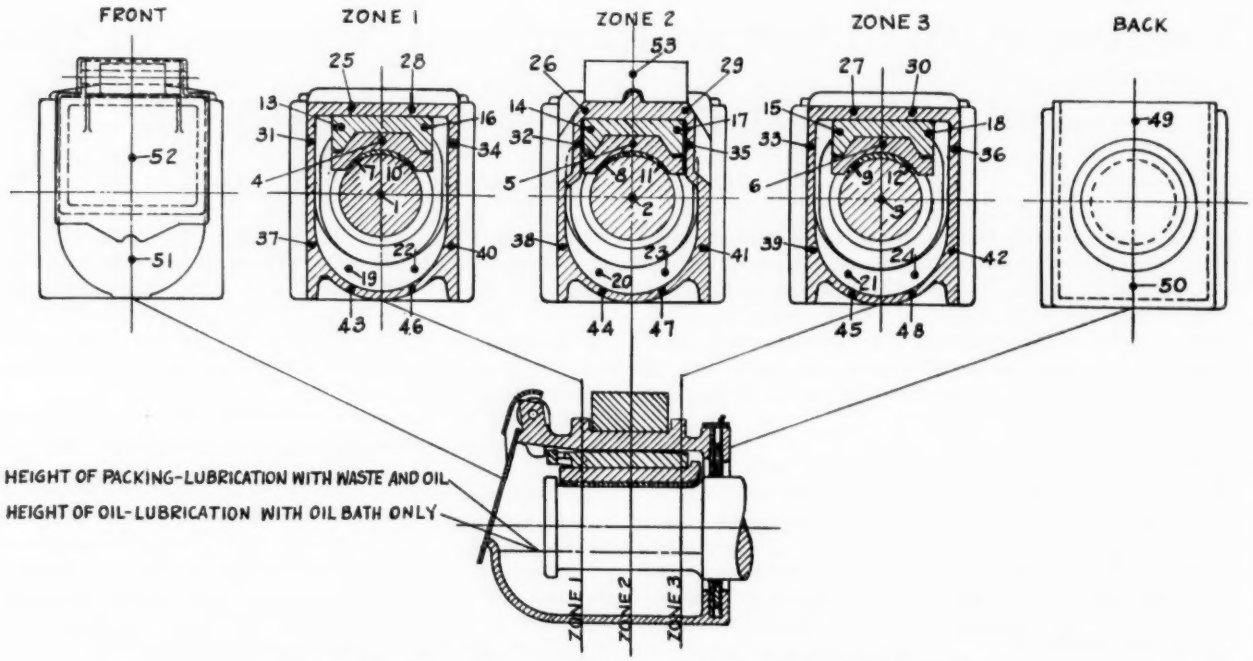


FIG. 2 LOCATION OF THERMOCOUPLES IN STANDARD JOURNAL-BOX ASSEMBLY

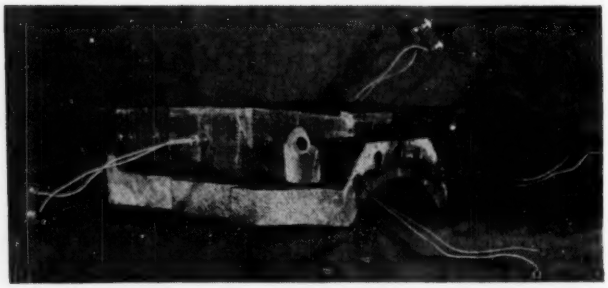


FIG. 3 METHOD OF ATTACHING THERMOCOUPLES AND LOCATION IN BEARING AND WEDGE

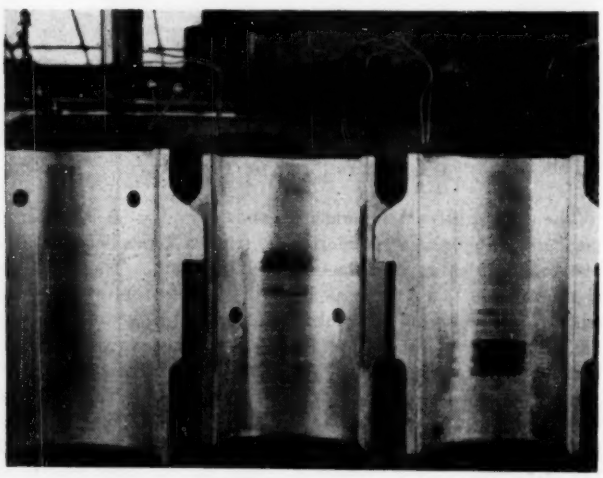


FIG. 5 LOCATION OF THERMOCOUPLES IN BEARING SURFACE

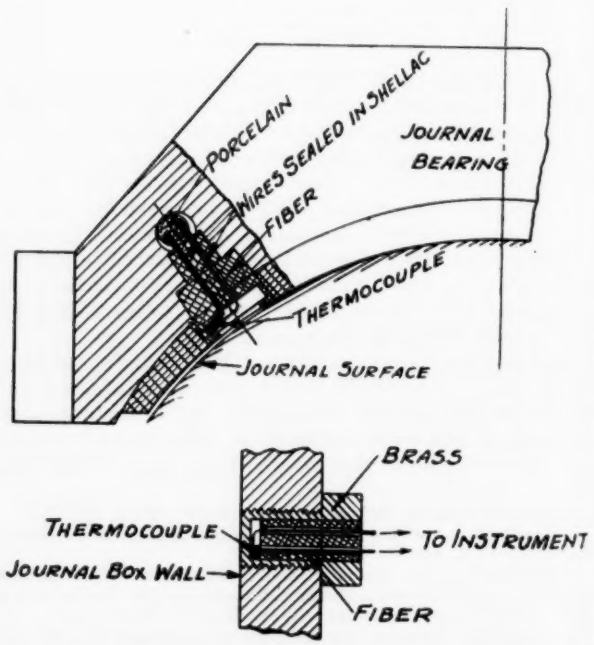


FIG. 4 INTERNAL ARRANGEMENT OF THERMOCOUPLE
(Top: Detail of application of thermocouple registering oil-film temperature. Bottom: Detail of application of thermocouple at various locations in journal box.)

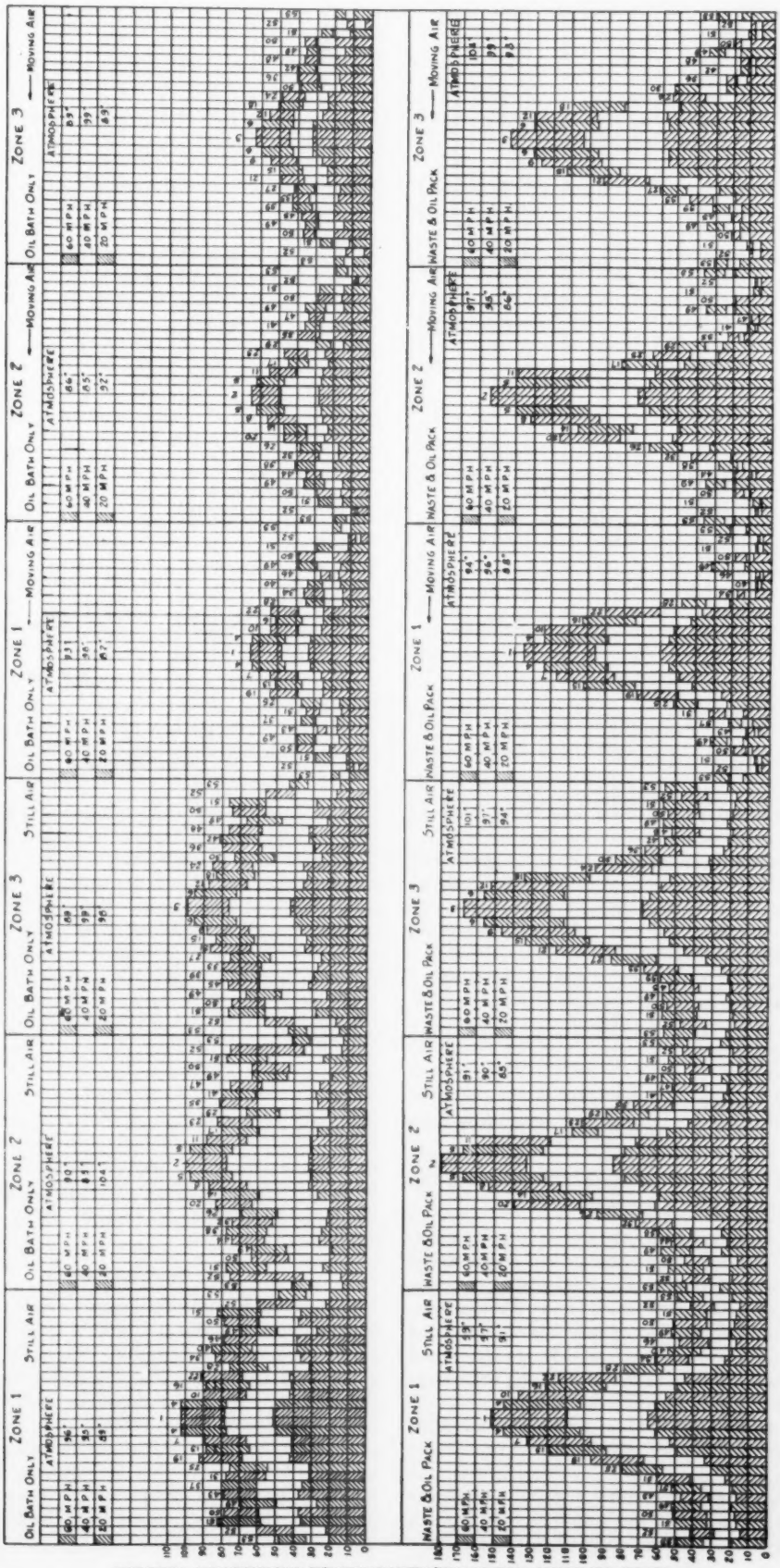


FIG. 6 TEMPERATURE DISTRIBUTION IN STANDARD RAILWAY JOURNAL-BOX ASSEMBLY

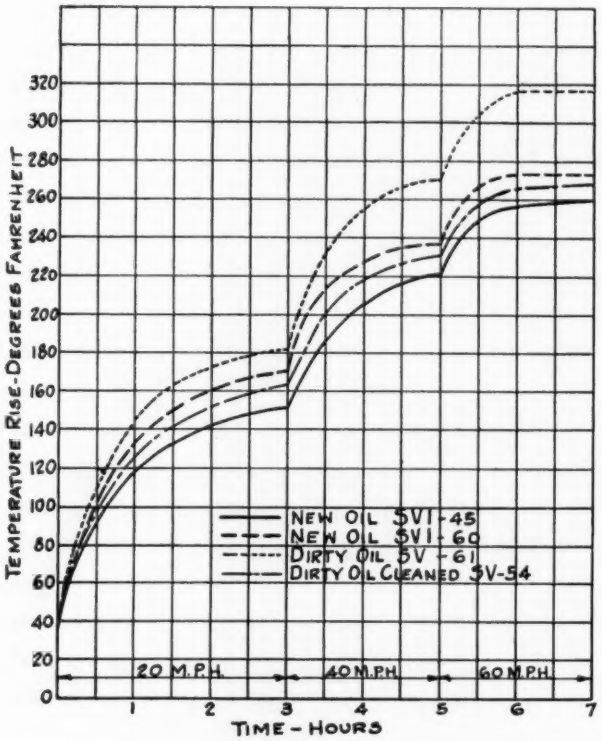


FIG. 7 TEMPERATURE AT CENTER OF JOURNAL UNDER NO LOAD AND WITHOUT BEARING

(Atmospheric temperature constant at 50 F; wool waste at 3.25 to 1 saturation; still air.)

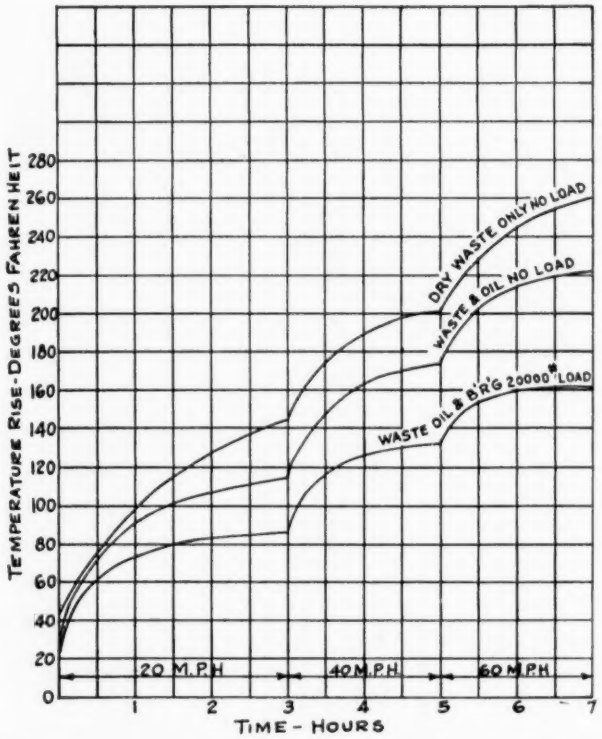


FIG. 8 TEMPERATURE AT CENTER OF JOURNAL UNDER VARIOUS CONDITIONS OF LOAD AND NO LOAD

(SVI-45 sec oil; atmospheric temperature constant at 50 F; cotton waste at 3.25 to 1 saturation; still air.)

Operation in still air and perfect flood lubrication have no practical application in the operation of railroad equipment. It is shown here only to reflect the insulating effect of the waste pack and the effect of air flow against the journal box. Railroad journals to operate at all are immediately subject to air movement. Flood, or bath lubrication, of railroad journals has many mechanical and operating limitations. Due to the fact that railroad journals operate under wide fluctuations of temperature, the insulating effect of waste might be considered an aid to lubrication in its influence on oil viscosity, which would otherwise be controlled by rapid temperature changes, as would exist with flood lubrication. The data reflecting the effect of flood lubrication in still air are to a degree representative of the operating conditions encountered in power-consuming or generating units of a stationary nature.

EFFECT OF VISCOSITY AND CLEANLINESS OF LUBRICATING OIL ON JOURNAL-BOX TEMPERATURES

One common factor which influences the amount of heat generated in a journal-box assembly is the viscosity and cleanliness of the lubricating oil. To illustrate this, Fig. 7 reflects the temperature of the center of the journal under no load and without a bearing, comparing the temperature effect of new oils of 45 sec and 60 sec viscosity at 210 F, dirty oil as removed from service with a viscosity of 61 sec, and this same oil after cleaning with a viscosity of 54 sec at 210 F.

The effect of the bearing under load, as a source of increased journal temperature, is not a factor. It will be seen from Fig. 8 that the bearing serves to reduce the operating temperature of the journal by the fact that it aids materially in the dissipation of heat. This is because it provides the principal path for the transfer of the generated heat to the walls of the journal box. This obtains, however, under the condition of clean journal-box packing and with a bearing in proper mechanical condition. Dirty packing and/or a bearing in improper mechanical condition may be a limiting factor in operation by adding to the heat normally generated in excess above that possible of dissipation through the bearing. The modification of the conventional bearing to compensate further for the effects of temperature and to augment its heat-dissipating capacity has produced outstanding improvements in the actual service operation of bearings.

In Fig. 8 is also shown the temperature of the center of the journal as affected by clean dry waste, and with clean oil-saturated packing, without a bearing or load. Dry packing, or packing with a reduced oil content, approaches the temperature effect of the dry waste shown in proportion to the oil deficiency, which would be further augmented by increasing the dirt content of the packing, as reflected by the combined evidence of Figs. 7 and 8. In actual operation this condition is a source of heating, which is due to the effects of service, producing undersaturated dirty packing.

CONCLUSION

Obviously, the control of heat sources is the first step in avoiding the limiting effects of excessive temperature rise in the conventional journal-box assembly. The conditions of the waste, the oil, and the bearing are the basic factors in controlling temperature sources. There is no economic substitute for clean journal-box packing properly saturated. The proper conditioning of bearings at the time of initial installation, and their modification in design to compensate for the effects of temperature, increase the factor of dependable operation. The extensive reduction to practice of these fundamental principles by several railroads has amply demonstrated their value.

Discussion

G. [illegible] CARELITZ.[3] This comparison of temperatures recorded under several modes of operation and at various locations in the test bearing is rather interesting. In the waste-packed bearing, the temperatures of the journal and brass are considerably higher than those observed with the oil bath, but the wall temperatures are appreciably lower. In other words, the temperatures are more even in the case of the oil bath, while the temperature gradient from journal to wall is much steeper in the waste-packed bearing. This, no doubt, is caused by the oil thrown by the shaft from the bath onto the walls. The oil splash acts effectively in carrying heat from the journal and bearing to the walls. On the other hand, the heat generated in the oil film must travel in the waste-packed bearing by way of the brass, wedge, and bearing top to the bearing walls for final dissipation into the surrounding atmosphere by convection.

The temperature of the oil film was considerably higher in the case of the waste-packed bearing, the respective average temperature rise being 147 deg and 88 deg for waste packing and oil bath (60 mph, still air). The viscosity of the oil was, therefore, considerably higher with the oil bath, and the friction was correspondingly higher. This accounts for the higher wall temperature with the oil bath. The respective average wall-temperature rise was approximately 71 deg and 59 deg, indicating that the losses in the oil-bath bearing were some 20 per cent higher than in the waste-packed. This is at variance with the estimates given by the author. It would be of interest to know how the friction was measured, and to investigate the discrepancy.

The same general condition has developed in the experiments with moving air. However, in the absence of data on the character of draft and its velocity, no comparison can be drawn between the operation in still and in moving air.

Several years ago, the writer, while carrying on work with waste-packed bearings, also had occasion to measure the temperature at the center of the journal by means of a thermometer inserted in the journal. At that time, the temperature was found to be also the highest in the bearing. The heat transfer along the shaft is inappreciable because the heat conductivity of steel is rather low, and the temperature gradient along the shaft is small, due to the presence of a hot supporting bearing in the close neighborhood of the test journal. The center temperature of the journal is, therefore, fairly close to the average temperature of the oil film.

A further analysis of the data published by the author, with a full utilization of the test records and observations, can give a most interesting study in the heat transfer in a railway bearing. This would form a notable contribution to the general subject of temperature distribution in bearings.

C. B. SMITH.[4] The conclusions in this paper merit the careful consideration of all interested in reducing the number of hot boxes occurring on passenger and freight trains.

The comparison of the high temperatures in charts, Figs. 6 and 8, indicates the significance of the journal bearing as a path for the dissipation of heat and directs attention to the proper relation to each other of the surfaces of the journal and of the bearing. Another chart might well have been included in the paper, showing a comparison of temperatures of the bearing with those at the center of the journal. This would illustrate more clearly the data shown in Figs. 6 and 8.

The heat-generating effect of dirty oil and the reduction in

[3] Professor of Mechanical Engineering, Columbia University, New York, N. Y. Mem. A.S.M.E.

[4] Engineer of Tests, Mechanical Engineering Office, Boston & Maine Railroad, No. Billerica, Mass.

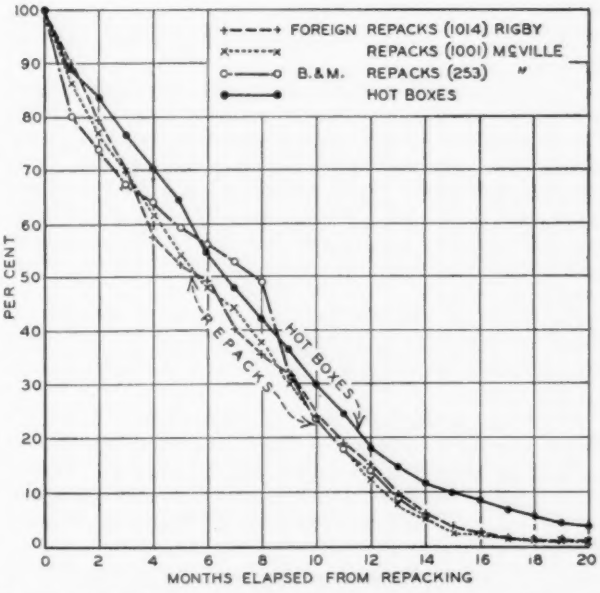


FIG. 9 CHART SHOWING PERCENTAGE OF HOT BOXES IN RELATION TO REPACKS, BASED ON MONTHS ELAPSED FROM DATE OF HOT BOX; BOSTON & MAINE RAILROAD

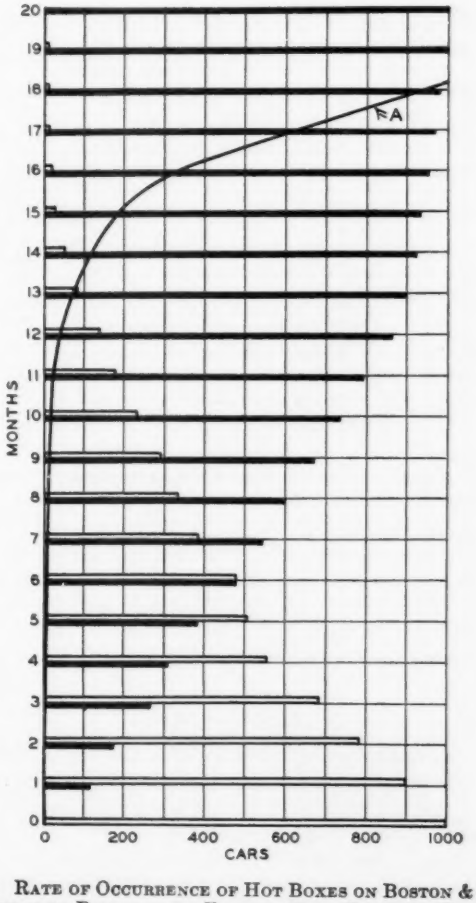


FIG. 10 RATE OF OCCURRENCE OF HOT BOXES ON BOSTON & MAINE RAILROAD IN RELATION TO ELAPSED TIME BETWEEN REPACKS

(Length of open block line indicates cars that have packing dates not less than months shown. Length of solid black line indicates cars that ran hot with packing dates not more than months shown. Curve *A* indicates rate at which hot boxes accumulate as age average of packing in boxes becomes greater.)

temperatures attained after cleaning the dirty oil are impressively revealed in Fig. 7, even when no bearing and, hence, no load is involved.

The effect of dry or undersaturated packing in causing temperature increases ranges from 14 to 26 per cent, as shown in Fig. 8. This justifies free oiling of journal boxes when restoring cars to service which have been idle for a long period, especially in extremely hot weather. Fig. 6 clearly shows that the journal runs hotter than the bearing and emphasizes the maintenance of the best lubricating medium between the journal and the bearing, thus minimizing the heat transferred to the lead lining of the bearing.

The conclusion that the cleanliness of journal packing is economically important should impress those who favor the longer periods between repacking of journal boxes with clean, renovated packing waste.

The Boston & Maine Railroad has made some studies of the relation between the occurrences of hot boxes in freight-train service and the elapsed time between packing renewals. The hot boxes in freight trains on that railroad occur in cases where packing has been in service from 8 to 9 months. Figs. 9 and 10 of this discussion show that beyond 10 months elapsed time, the hot boxes occur at an increasing rate up to 15 months elapsed time and, thereafter, rapidly up to 18 months. A reduction of the repacking period to 9 or 10 months should show justifiable results.

Author's Closure

In the comment by Dr. Karelitz on frictional losses at 60 mph, comparing bath lubrication with waste-pack lubrication, since all conditions were the same on the entire test unit, except the absence of waste in one case and its presence in the other, total input in kilowatts per hour for each of the three 7-hour runs is a true basis of comparison for the determination of frictional losses with the following result:

60 Miles per hr, 7 hr, 20,000 lb load	Average kilowatts per hour — Test run — No. 1	No. 2	No. 3
With oil bath	2.59	2.58	2.61
With waste pack	2.59	2.74	2.70

Obviously the same amount of frictional heat must have been created in both cases. The fact that it was distributed differently to the walls of the box is a phenomenon which has no significance in establishing the quantity at the source.

The points brought out by Mr. Smith are particularly interesting, since they demonstrate that the results of the laboratory findings can be and are reproduced and their effect measured in terms of actual service. To illustrate more clearly, as Mr. Smith points out, the data shown in Figs. 6 and 8, Fig. 11 is included showing the comparison of temperatures of the bearing with those at the center of the journal, for the purpose of giving the effect of oils of approximately the same viscosity; but in the one case clean oil and in the other case dirty oil was used. These charts, taken particularly in connection with Figs. 9 and 10 shown by Mr. Smith, reflect in practical operation the effect of an accumulation of dirt in car-journal oil as it affects journal temperatures.

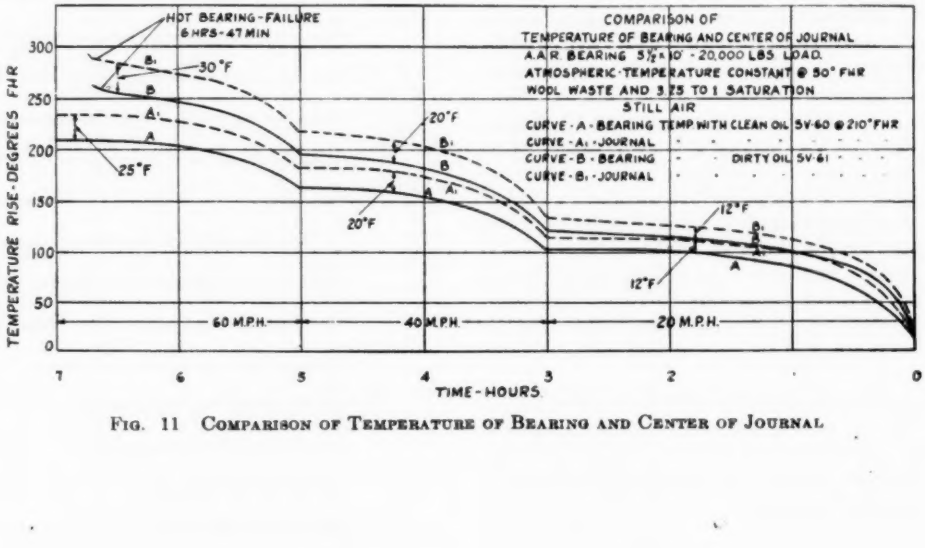


Fig. 11 Comparison of Temperature of Bearing and Center of Journal

Purchase and Use of Fuel

By E. WADSWORTH STONE,[1] THOMPSONVILLE, CONN.

In this paper, the author discusses the location of industry, sources of its fuel supply, available competing fuels and their relative cost, the purchaser's combustion equipment, the method and efficiency of conversion, all of which are factors influencing the cost of power and the fuel necessary to produce it. The history of fuel production, supply, regulation and use, and the substantial improvement in combustion equipment during the last twenty years afford an interesting commentary upon the complicated problem of fuel purchase and supply.

FUEL represents potential energy for release in the form of heat, and its purchase and use involve many interrelated factors.

Consumers evaluate fuel according to the recoverable heat when this is converted in their own equipment and for their own use. Consumer interest will vary to the extent that certainty of fuel supply and fuel cost are important factors in their own production problem.

Producers, to continue in business, must realize in the aggregate their true actual cost of fuel production, preparation, and sale. Producers must also accurately appraise markets, consumer needs, and available conversion equipment in the preparation and merchandising of their fuels successfully to meet competition.

Consuming markets seldom are located at or adjacent to the sources of supply. The cost of transportation, definitely established for whatever means employed, therefore constitutes a necessary, measurable, and usually a major factor in the delivered fuel cost.

The cost of production, distribution, and utilization thus determines the character and extent of the fuel markets, price levels, and the relative values of competing fuels available in any common consuming market.

Regulation, either state or federal, provided it properly recognizes the public, producer, and consumer interests, and fair existing competitive opportunities, may prove helpful. But regulation which ignores the effects of changing economic conditions and disregards the ordinary flow of fuel, in response to normal supply and demand under normal conditions of free and open competition, may cause a shift of markets and a permanent dislocation of production and consumption sufficient to offset any immediate temporary or localized gain.

Integrated valuations of fuel by producer and by consumer, measured one against the other, will determine its fair average value. Any abnormal or arbitrary departure therefrom will adversely affect the producers of fuel and the consuming markets they serve.

Location—Industry and Fuel Supply

This country's growth, commencing at the Atlantic seaboard, has largely been toward the West and Northwest, with industry's movement and location generally determined by the availability of raw materials, consuming markets, and convenient means for the distribution of manufactured products.

Fortunately, acceptable fuels were usually accessible. Fortunately too, excepting a few industries like steel and glass, fuel constitutes but a minor portion of the raw-material requirement. Moreover, the development of power represents but a small, although increasing, part of the cost of production, due to the continued advance in the cost of manual labor. Transportation factors governing commodity movement were also favorable toward the convenient supply of fuels.

Initially, industry, particularly in New England, was located to make use of available water power. Increasing power requirements later necessitated the installation of steam relay stations and, eventually, the use of purchased power.

The gradual movement of industry westward, and its convenient location along rivers, lakes, railroads, and other available means of transportation, has brought about a corresponding redistribution of industrial-fuel markets and, likewise, shifts in the centers of population and domestic-fuel markets.

In each instance, the continued effort to secure lower costs of heat and power has stimulated the use of more efficient combustion equipment and lower grades and less expensive kinds of fuels.

From 1889 to 1929, energy for all uses increased approximately sixfold; from 1909 to 1929, industrial power requirements increased some twofold; and in the last 20 years, the use of electric energy has increased threefold. This increase in the use of power, particularly of electrical energy, has brought about the installation of many large generating stations in and near important industrial centers.

Extensive transmission systems have been provided to interconnect large steam-generating plants and the near-by hydrostations. In spite of this augmented flexibility of generation and supply of electrical energy, the relatively limited storage facilities at the source and at consumers' plants require continued and even greater dependence upon the transportation systems for high-speed service, reliability, and continuity of fuel supply.

Frequently, reference is made to the location of large generating stations at the mine or other source of fuel supply, but the fuel-transportation factor may be offset by the lack of condensing water at the mine and the economics of electrical-energy transmission. Bituminous coal has been and undoubtedly will continue to be the principal source of steam and electrical energy. Competing fuels and hydropower are making substantial increases, although seasonal variations in the latter may necessitate steam relay stations. Calculated at the prevailing fuel rate for electrical energy, the percentage of hydropower to the total energy consumption has approximated 3 to 4 per cent (1, 3)[2] since 1899.

Competing Fuels

Coal deposits, with minor exceptions, are conveniently accessible to all consuming markets. On the other hand, oil is produced principally in the states of California, Oklahoma, and Texas, and to a lesser degree in several other states. The producing areas for natural gas roughly coincide with those for oil.

Bituminous coal, fuel oil, and natural gas not only compete with each other in those markets easily reached or favorably situated, but with domestic anthracite in the East and with manufactured

[1] Research and Consulting Engineer, Bigelow-Sanford Carpet Company, Inc. Mem. A.S.M.E.

Contributed by the Fuels Division and presented at the Spring Meeting, Worcester, Mass., May 1–3, 1940, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

gas and coke in many industrial centers throughout the United States.

New England is a good example of a competitive fuel market. Imported fuels are a negligible factor, but the production of hydropower and the use of fuel oil, anthracite, and bituminous coal, together with the by-product forms of manufactured gas and coke, present to both the industrial and domestic consumer a wide variety of fuels from which to choose.

The importation of oil, although frowned upon by producers of competing fuels, has the effect of preserving our country's rather limited oil reserves. On the other hand, the importation of foreign coal, particularly anthracite along the Atlantic seaboard, does detract from the potential markets for solid fuel, of which there is available several thousand years' supply.

Energy Consumption

TABLE 1 TOTAL ENERGY SUPPLY—ALL FUELS AND WATER POWER[a] (1, 3)

Year	Annual energy supply, Btu (trillions)	Coal equivalent[b] in million tons
1909	14284	545
1918	21706	830
1929	25421	972
1937	24183	921

[a] Refer also to Figs. 1 and 2.
[b] 26.2 million Btu per ton of bituminous coal.

The total annual supply of energy from mineral fuels and water power (at the prevailing central-station heat equivalent) closely approximates 25,000 trillion Btu. As now proportioned, this may be expressed in terms given in Table 2.

Approximately 70 per cent of the total bituminous-coal tonnage is produced in the eastern or Appalachian fields, 18 per cent in the middle-western fields (minimum price areas (2, 10) Nos. 1 and 2, respectively), and the balance in varied and scattered amounts in the South and in the Rocky Mountain and northwestern states.

TABLE 2 ESTIMATED ANNUAL NORMAL PRODUCTION

Bituminous coal	440 to 480 million tons
Anthracite	50 to 60 million tons
Petroleum	1280 to 1300 million bbl
Natural gas	2450000 million cu ft
Hydropower	45000 million kwhr

Note: Coke and manufactured gas involve the consumption of other primary forms of fuel and are not included.

The annual consumption of bituminous coal in the market areas east of the Mississippi River, for industrial and domestic uses, excluding export, railroad, and bunker fuel, approximates $^3/_4$ of the total tonnage of the United States, apportioned (4) as shown in Table 3.

TABLE 3 ANNUAL CONSUMPTION OF BITUMINOUS COAL IN EAST

	Million tons
New England States	17
New York State	23
Pennsylvania, West Virginia, and other North Atlantic States	75
Other states south of the Ohio and east of the Mississippi Rivers	42
States north of the Ohio and east of the Mississippi Rivers	162

Substantially all the anthracite is produced in eastern Pennsylvania and consumed in the North Atlantic States.

The principal consumption of fuel and gas oil is in the producing and adjacent areas and those markets conveniently

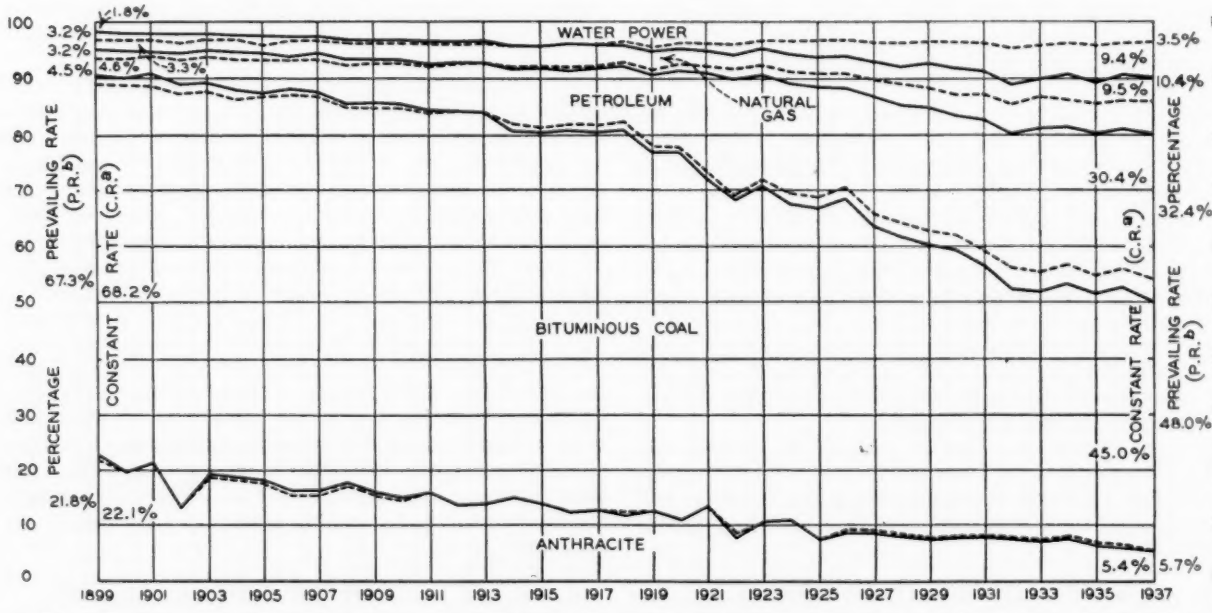


Fig. 1 Percentage of Total Btu Equivalent Contributed by Anthracite, Bituminous Coal, Petroleum, Natural Gas, and Water Power in the United States, 1899–1937

(From supplement to Weekly Coal Report No. 1084 of The National Bituminous Coal Commission.)

Notes on Fig. 1

[a] C.R. Water power at constant rate;———— 1913 rate, 4.02 lb per kwhr
[b] P.R. Water power at prevailing rate; – – – – – – 1918 rate, 3.36 lb per kwhr; 1937 rate, 1.42 lb per kwhr

Unit heat values used

Kind of fuel	Btu value
Anthracite	13600 per lb
Bituminous coal	13100 per lb
Petroleum	6000000 per bbl
Natural gas	1075 per cu ft

Water power equivalent (C.R.) is based on the year 1913 or mid-point of years compiled (1889–1937) and is the constant factor of 4.02 lb of coal per kwhr
Water power equivalent (P.R.) is based on the prevailing factor which varies from 7.05 lb of coal per kwhr in 1889 to 1.42 lb of coal per kwhr in 1937
When production of water power was not available, the horsepower equivalent was used, taken at 20 per cent capacity factor for manufacturing and mines and 40 per cent for utilities

reached via tidewater and pipe line. The total distribution (5) in 1937 approximated:

	Per cent
New England States	9
Middle Atlantic States	29
South Atlantic States	4
North Central States	14
South Central States	20
Rocky Mountain States	1
Pacific Coast States	23
Total	100

The consumption of natural gas likewise is generally confined to the producing areas, except where accessible by pipe line, with the total distribution (5) 1937 divided as follows:

	Per cent
California	14
Louisiana	13
Oklahoma	12
Texas	36
West Virginia	6
Other states	19
Total	100

In 1937, there was produced from fuels 77,348 million kwhr of electrical energy, and from water power, in competition with fuels, some 43,702 million kwhr, a total production of 121,050 million kwhr,[3] apportioned (6) as shown in Table 4.

[3] Approximately 128,000 million kwhr in 1939.

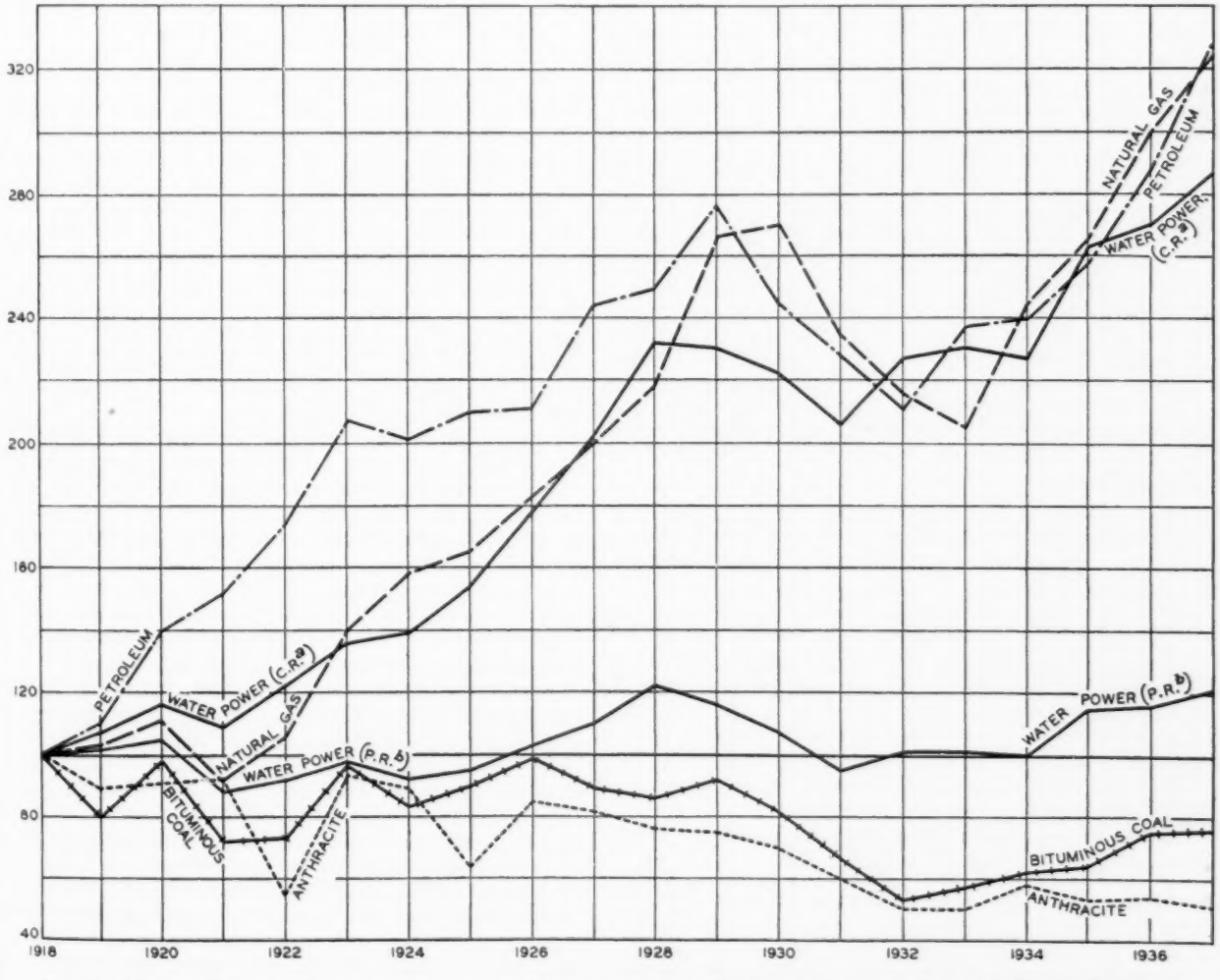


FIG. 2 RELATIVE RATE OF GROWTH OF ANTHRACITE, BITUMINOUS COAL, PETROLEUM, NATURAL GAS, AND WATER POWER IN THE UNITED STATES

(Index numbers—1918 equals 100. Data taken from supplement to Weekly Coal Report No. 1084 of The National Bituminous Coal Commission.)

NOTES ON FIG. 2

[a] C.R. Water power at constant rate;———— 1913 rate, 4.02 lb per kwhr

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When production of water power was not available, the horsepower equivalent was used, taken at 20 per cent capacity factor for manufacturing and mines and 40 per cent for utilities

TABLE 4 APPORTIONMENT OF ELECTRICAL-ENERGY PRODUCTION IN 1937

	Hydroplants, per cent	Fuel plants, per cent	Total, per cent
New England States	7	6	7
Middle Atlantic States	19	29	26
East North Central States	5	34	23
West North Central States	5	7	6
South Atlantic States	16	9	12
East South Central States	8	2	4
West South Central States	1	8	5
Rocky Mountain States	10	2	5
Pacific Coast States	29	3	12
Totals	100	100	100

For comparison, it is interesting to note that the electric-generating capacity of utility and other power and light plants devoted to public use at the end of the year 1937 was reported to be as follows:

10,474,000 kw for hydroplants
26,558,000 kw for those using fuel
37,032,000 kw[4] for entire United States

The total amount and that for the two methods of generation are apportioned (6), as shown in Table 5.

TABLE 5 APPORTIONMENT OF ELECTRIC-GENERATING CAPACITY IN 1937

	Hydroplants, per cent	Fuel plants, per cent	Total, per cent
New England States	8	8	8
Middle Atlantic States	14	29	25
East North Central States	6	28	22
West North Central States	5	9	8
South Atlantic States	19	9	12
East South Central States	10	3	5
West South Central States	1	6	5
Rocky Mountain States	11	2	4
Pacific Coast States	26	6	11
Totals	100	100	100

Those portions of the fuels which compete with each other and with hydropower may be conveniently expressed in terms of "equivalent bituminous-coal tonnage"[5] for a normal year. These values are given in Table 6.

TABLE 6 "EQUIVALENT BITUMINOUS-COAL TONNAGE" FOR COMPETITIVE POWER SOURCES IN NORMAL YEAR

	Million tons	
Bituminous coal:		
Domestic	88 to 96	
Railroad	88 to 96	
Industrial and power use	264 to 288	440 to 480
Anthracite:		
Domestic	33 to 39	
Industrial	17 to 21	50 to 60
Fuel and gas oil:		
Domestic	26	
Power	26	
Railroad	16	
Other	35	103
Natural gas:		
Domestic	20	
Industrial	39	59
Hydropower		31
		683 to 733

Efficiency of Conversion

The improvement in efficiency of converting the potential energy of coal and other fuels into electrical, mechanical, or other useful forms of energy offers an interesting commentary upon the relatively constant total supply of energy from mineral fuels since 1918, except during the depression years of the 1920's and again in the 1930's. The following examples will illustrate this point:

The average number of pounds of coal required per 1000 gross ton-miles for freight service on steam railroads decreased from an average of 170 lb in 1919–1920 to 115[6] in 1938; a reduction of 32 per cent (4, 5, 7).

During the same period, the pounds per passenger-train car-mile decreased from 18.5 lb to 14.9;[7] an improvement of 19½ per cent (4, 5, 7).

In 1918, the production of a gross ton of pig iron required the use of the equivalent of some 3577 lb of coal, whereas, in 1938 this amounted to but 2865 lb; an improvement of nearly 20 per cent (4, 5).

The improvement in the efficiency of production of electrical energy is even more marked. It is reported that in 1889 the average consumption of coal was 7.5 lb per kwhr. This requirement (1, 4, 5, 6, 8, 11) was reduced in

1902 to 6.4 lb per kwhr
1912 to 4.5 lb per kwhr
1919 to 3.22 lb per kwhr
1920 to 3.04 lb per kwhr
1929 to 1.69 lb per kwhr
1938 to 1.41 lb per kwhr

The reduction since 1919 approximates 56 per cent.

Several central stations are now producing electrical energy for 0.75 lb of coal per kwhr, approximately ½ the 1938 average for the entire United States. Many factors have encouraged this improvement in efficiency and undoubtedly will continue to do so, although perhaps not to the extent possible in the past.

Equipment and Changes

The record is replete with coal analyses, boiler- and power-equipment tests, specifications and descriptions in general and in detail, including design, construction, and operation of modern steam-generating stations for the utilization of solid, liquid, or gaseous fuels, singly and in combination, and need not be further amplified in this paper.

Considerable thought has been given to the storage of fuel, its delivery to the furnace, the method of firing, combustion control, the removal of waste, desirable fuel characteristics, boiler design, operating pressures, capacities, and the rate of heat release. Much study has been devoted to size and arrangement of combustion space; gas velocities, to secure maximum rate of heat transfer; furnace temperatures; the use of waterwalls and special refractories to withstand the higher prevailing furnace temperatures; the use of alloy steels and welded construction; pulverizer design and the use of watercooled stokers.

Heretofore, the quantity of ash and of sulphur and the ash-fusion temperatures appeared to be limiting factors in the use of certain types of combustion equipment. Methods for continuous or intermittent ash removal, either in the dry or plastic state, have been improved. We know more about what actually takes place in the combustion of fuel and the behavior of ash on refractory and boiler surfaces. This has indicated that the slagging characteristics of a coal may be dependent upon the fusion temperatures of its component parts rather than an average of all, and that Fe_2O_3, lime, and other components of ash definitely influence slag formation. All this has helped to make possible the combustion of high ash pulverized coal with ash-softening temperatures as low as 2300 F, in a dry-bottom furnace, and, conversely, the use of high-fusion coals with ash-softening temperatures as high as 2550 F, in wet-bottom furnaces, the furnace temperatures in either case being controlling factors.

These numerous experiments, studies, and investigations, individually and collectively, constitute an important contribution to the use, operation, and efficiency of modern steam-generation equipment, and encourage still further improvement. The ex-

[4] Approximately 40 million kw at the end of 1939.
[5] 26,200,000 Btu per net ton.
[6] 112 in 1939.
[7] 14.8 in 1939.

perience and results obtained on the larger units have been and can be applied, with perhaps less refinement, to the smaller and simpler combustion units for industrial use, or for the modification and more efficient utilization of existing equipment.

Large utility and industrial consumers of fuel employ technicians to check fuels and power-plant operation carefully and frequently. To the small consumer, who may question the economic justification of such a check, there are now available fuel technicians and an extensive array of operating data. The larger producers of fuel maintain, for the convenience of their customers, technical staffs thoroughly acquainted with their fuels and frequently experienced in actual plant operation. The National Association of Purchasing Agents (9) has also reminded its membership of the more important factors influencing the use of fuels.

Heretofore, initial cost has frequently been permitted to influence the installation and use of steam-generating equipment, often to the extent of restricting it to a single or a limited number of types or grades or sizes of fuel.

There appears to be a growing realization among equipment manufacturers and, likewise, among the users of all kinds of fuel that, with very little additional effort and expense, steam-generating equipment, having a much wider range of fuel utilization, can now be manufactured and installed.

If this be a correct interpretation of the trend, this increase in the adaptability and range of use of fuel-burning equipment will enable the consumer, in spite of the tendency to regulate both price and source of supply, to purchase fuel at a low average unit cost and effect the same or even greater efficiencies of conversion than have heretofore prevailed.

Likewise, manufacturers may, at little added cost, modify their combustion equipment to make possible the burning of solid, liquid, or gaseous fuel, and the high- and low-grade fuels of any type with substantially equal efficiency.

Thus, the consumer will be enabled to take greater advantage of the potential and changing fuel markets and available methods of transportation, and in his purchases can be governed to a larger degree by the actual fuel value measured by the delivered fuel cost per million Btu.

Market Trends[1]

Natural gas has been marketed in ever-increasing quantity since 1921, similarly fuel oil, with the increased consumption more pronounced for domestic heating than for other uses. At the same time, there has been a definite trend downward in the use of both anthracite and bituminous coal, this trend being more pronounced since 1926.

Throughout the last 20 years, steam-generated electrical energy has increased threefold, with but slight increase in the use of fuel, due to improved efficiency in fuel utilization. Hydro-generated electrical energy has increased in substantially the same proportion, but, if evaluated at the prevailing rate for steam-generated energy, the coal displacement appears to show relatively small change since 1918.

Except for the periods during and immediately following the World War, and during strikes and other abnormal times, all bituminous coal has averaged to realize at the mine between $1.75 and $2 (5) per net ton.

The realization at the mine for Pennsylvania anthracite appears to have declined steadily from an average of $5.35 in 1931 (5) to nearly $4 in 1937. The downward change, which was greater for domestic sizes, was somewhat offset by a gradual increase in the realization for industrial and steam sizes.

If we take the tidewater price of bunker C fuel oil at New York, the average price per barrel for the years 1933 to 1939, inclusive, has ranged from approximately $1 to $1.30 (5), with the actual price, of course, depending somewhat upon the local market conditions, the quantities involved, and subject to greater fluctuation than that of other fuels.

The average annual realization for natural gas, on the other hand, has ranged within relatively narrow limits, from approximately 21½ cents to 24¾ cents per thousand cubic feet at point of consumption for the years 1925 to 1938 (5), inclusive. Although the figure for all natural gas has remained substantially constant, the average for industrial use during the same period appears to have declined from a high of nearly 13 cents to a low of 9.7 cents, whereas, the corresponding rates for domestic consumption appear to have increased from a low of 56 cents to a high of some 69 cents per thousand cubic feet.

These figures, unless otherwise stated, are merely averages for the entire United States. They do not indicate the relative prices at any particular market or destination, where the different transportation costs will substantially modify the actual delivered prices for the available fuels.

The better- and not the lower-grade fuels move to the more distant markets, where transportation is important. From time to time, strikes, changing business conditions, temporary shortages of supply, and other varying factors may serve to modify consumer preference and the relative prices of competing fuels, but the general parallelism between the costs of the different types of fuel (solid, liquid, and gaseous) appears to continue.

Recently, producers of bituminous coal have given more attention to the screening, sizing, cleaning, and possible dust treatment of their coals, the better to meet changes in market conditions and in combustion equipment, realizing that thereby they can substantially improve their opportunities to hold and possibly extend existing markets.

The use of the finer sizes of slack and double-screened coals, respectively, for the industrial and domestic trade has substantially increased. Slack coals, formerly a drug on the market, selling at ridiculously low prices, have encouraged the introduction of pulverized-fuel equipment. The introduction of domestic stokers has made possible and convenient the use of smaller and less expensive double-screened sizes of coal, tending to reduce the supply of slacks available for industrial use.

The price differentials between the so-called "prepared" or double-screened coals, on the one hand, and slack coals, on the other, tend to diminish, with the mean or weighted average approaching but not reaching the mine-run figure. This diminishing differential as to size, coupled with corresponding differentials as to grade, has accelerated the price leveling-out process, as indicated by the generally greater increase proposed under the new price schedules for slacks and industrial coals than for the large double-screened and domestic sizes.

Transportation

Freight rates are not always proportional to the distance traveled, in fact those for long hauls often are relatively lower, to enable newer and more distantly located coal fields to compete in the larger consuming markets with the older near-by fields.

The cost of transporting a ton of coal via rail averaged, for the entire United States, an amount equal to or greater than the average cost of the fuel itself. This cost ranged from $2.15 to $2.27 per net ton during the years 1929 to 1938, inclusive, with the actual transportation charge to many consuming markets several times the actual fuel cost f.o.b. cars at the mine. Approximately 24 per cent of the bituminous-coal production is water-borne wholly or partly via lake, river, or tidewater, and 7 per cent is moved by truck from the mine to near-by destinations and usually at less than rail rates.

[1] Refer to Figs. 1 and 2.

Transportation cost, reliability of the carrier or carrier system, and the continuity of supply are important factors when coal or fuel oil are considered.

The increasing availability of natural gas has encouraged the extension of pipe lines so that now it may be so conveyed thousands of miles from the source of supply to the ultimate consumer.

Anthracite destined for eastern markets generally moves via rail, or rail and tidewater, with a small portion moving via truck.

Selection and Evaluation of Fuel

Purchasers of fuel are anxious to familiarize themselves with the limitations of their own equipment, the operating man's viewpoint, and the characteristics of available fuels. They desire some readily understood measure of fuel evaluation and performance, first, to guide them in its selection and, subsequently, to check actual performance when the fuel is burned in their own plants.

A suitable measure of performance will facilitate comparison between plants and between competing fuels in the same plant, also the effect of variations in preparation and physical characteristics upon the efficiency of utilization.

A vast amount of detailed information and individual plant-operating experience has been accumulated and made available by the engineering profession and by producer and consumer organizations as to available fuels. Nevertheless, the major problem still remains.

A scientific approach to the purchase of fuel is possible. Frequently, however, the power or fuel portion of the total manufacturing cost is not sufficiently large to justify the extensive and detailed evaluation methods used by those who consume millions of tons annually. However, the many chemical and physical characteristics of fuels undoubtedly do have a marked bearing upon the selection of appropriate fuels and, having once determined the desired type or types, the field is considerably narrowed. It then becomes a matter of evaluating fuels on the basis of actual heat recovery rather than on a casual analysis of a salesman.

TABLE 7 SUGGESTED METHOD OF EVALUATING FUELS

	Coal per Ton				Per Barrel		Per M Cu.Ft.	
	Stoker		Pulverizer		Oil		Gas	
Plant:	#1.		#2.		#3.		#4.	
Type of Fuel:	2" x 0 NAS		3/4"x 0 Slack		Bunker "C"		Natural	
Vendor:	X		Z		Y		W	
Average rating:	175%		275%		200%		200%	
Duration of test:	1 month		8 hours		8 hours		3 months	
Steam conditions:								
Pressure:	165# Ga.		175# Ga.		175# Ga.		450# Ga.	
Superheat:	130° F.		150° F.		140° F.		200° F.	
Efficiency overall:	78%		82%		83.9%		80%	
(A) Fuel Cost:								
F.O.B. source:	$1.80		$1.95		$.95		$.05	
Transportation:	4.00	$5.80	2.86	$4.81	.25	$1.20		$.11
(B) Fuel Handling:								
Storage:	.026		.014		.01			
Rehdlg.to Bunker or Tank:	.04		.042		.002			
*Transfer to burner:	.034	.10	.150	.206	.011	.023		.002
(C) Waste disposal:		.04		.10		-		-
(D) Labor, Supervision, Maintenance, etc.								
Grates, Stoker parts, Burners, Pulverizing equipment, etc.	.125		.254***		.004		.001	
Brickwork, Settings, etc.	.04		.050		.035		.006	
Boiler service labor:	.165		.054		.085		.015	
Supervision:	.09	.42	.10	.458	.022	.146	.004	.026
**Total Unit Cost:		$6.36		$5.574		$1.369		$.138
Million B.T.U. absorbed per Ton or unit Fuel Cost:	22.2		22.8		5.2		.86	
Cost per Million B.T.U. absorbed:	$.287		$.244		$.263		$.162	

Notes: * Includes cost of coal pulverization or oil heating.
** Unit for comparison may be Net Ton (Coal), Barrel (Oil), 1,000 cu.ft. (Gas), or cost per Million B.T.U. Inclusion of fixed charges optional.
*** Includes induced draft fans.

Coal is now available in a greater variety of sizes and grades than is generally realized. The producers themselves have learned to better prepare, size, grade, and treat their coals; fuel oils have increased in specific gravity and heat content; refinery by-products and residues have become available; and the extension of natural-gas lines has brought this fuel into many otherwise inaccessible markets. Combustion equipment has been designed or modified to permit high rates of heat release and to burn not only one but several fuels with substantially equal facility. The fuel-selection problem thus has become increasingly complex.

The average consumer is manifestly somewhat bewildered by the kaleidoscopic changes in fuel prices, fuel markets, and factors affecting fuel production, distribution, and use. His interest in the evaluation of fuels leans toward a definite, tangible yardstick or measure of evaluation and a definite departure from the old rule-of-thumb method of purchase from "Smith" or "Brown," because the fireman thought it burned better or merely because the supplier had convinced the fireman that his coal shoveled better than that of his competitor.

The suggested method of evaluation is illustrated in Table 7 for coal, gas, and oil, and for stoker- and pulverized-fuel-fired boilers.

The variable factors, production, transportation, handling, and utilization, may be divided into two fundamental and essential groups as follows:

1 The complete cost of the fuel, including transportation and conversion, per ton, barrel, or million cubic feet.

2 The heat recovery expressed in convenient units, as, for example, million Btu per fuel unit.

Both of these in the average industrial plant may be readily and accurately determined. When compared, they provide a simple and substantially uniform and sufficiently accurate method of evaluation regardless of the fuel, the conversion equipment, the method of operation, plant location, or any other of the many factors which enter into or effect the utilization of fuel.

After the primary determinations, supplemented by a heat balance if desired, fuel evaluations are completed for representative fuels and the usual boiler ratings. Subsequent modification may be quickly approximated to determine the effect of changes in prices, transportation costs, and operating efficiencies. Thereafter, periodic tests and an analysis of the results over extended periods may suggest further changes in fuels and conversion equipment and make possible the widening of the range of fuel application. Thus, the management and operating staff are provided with a yardstick or standard of performance which can be easily and readily applied to a large or a small plant, and which includes all the essential fundamental factors. To illustrate, if we add the following items:

A The delivered cost of fuel, including storage and handling if necessary;

B Cost of handling the fuel into and through the conversion equipment;

C Ash- and waste-removal cost;

D Supervision, labor, maintenance, and other operating expense;

then we can readily determine, from these known and easily secured data, a unit cost for fuel delivered and consumed.

Actual heat recovery may be readily ascertained from the steam produced or an equivalent-use evaluation in other processes. The two predominating and major factors may then be

compared directly to determine the cost per million Btu actually recovered, i.e.

$$\frac{\text{Fuel cost}}{\text{Heat recovered}} = \text{Cost per million Btu recovered}$$

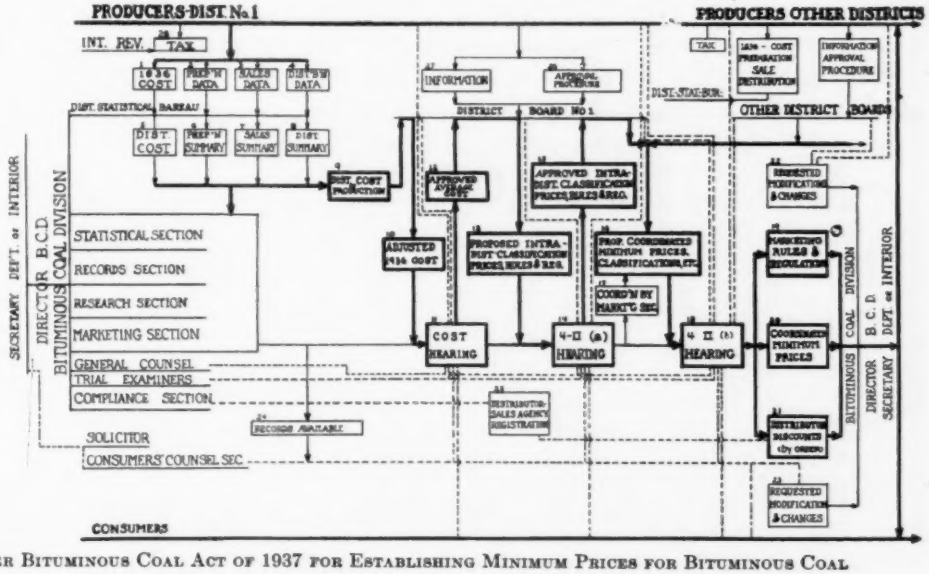
The three cost factors *B*, *C*, and *D*, are the average for the particular plant for the preceding year, weighted (plus or minus) to approximate empirically the probable increase or decrease, as the case may be, for handling the particular type, grade, or size of the fuel in question. The average cost assigned to *B*, *C*, and *D* is controlling and is a definite and known amount for any given plant. Such slight modification as may be required properly to weight these items for any particular fuel is a very small portion of the total cost of the delivered and converted fuel, in fact so small as not to be seriously questioned by either the seller or the consumer of the fuel, and usually small enough to be waived entirely in case of serious argument.

This method of evaluation includes the cumulative effect of many factors not otherwise distinguishable except through detailed analysis. Actual experience and use have shown that it affords a reasonably simple and accurate comparison of the energy-conversion cost for different plants, either large or small, or different usable fuels in the same plant, or even plants other than those used for the generation of steam.

To a particular producer 5 cents per bbl or 10 cents per ton differential in the fuel cost (f.o.b. source) may appear to be a substantial one, and may even mean the loss or gain of his market or customer. To the consumer, transportation is an added and an important element of expense; in fact, frequently, as in New England, transportation cost may average to be twice that of the fuel itself. Thus, small variations in fuel price, although seemingly of major importance to the producer, actually become a minor factor in the consumer's total cost, and particularly so if, for the fuels or the plants being compared, there is a marked difference in the percentage of heat actually recovered.

Minimum Prices For Bituminous Coal

There has been in progress during the last two years a series of investigations, collection of fundamental data, and hearings for the purpose of determining the weighted-average cost of production and establishing minimum prices for bituminous coal in accordance with the Bituminous Coal Act of 1937 (2). The Act authorizes separation of the coal fields of the United States into 23 producing districts and the grouping of these into 10 minimum-price areas. The Act requires that the production of



Notes on Fig. 3
Participation or interest in hearings, procedure, etc., shown by dash lines, thus – – – –
Procedural steps shown thus ⟶ with arrow and numbers 1—23, inclusive, generally indicating direction and sequence

Fig. 3 Procedure Under Bituminous Coal Act of 1937 for Establishing Minimum Prices for Bituminous Coal

Producers' Data to District Statistical Bureau

(1) to (4) *Cost of production:* (1) Direct and indirect expenses plus cost of selling.
Preparation: (2) Kinds, qualities, sizes.
Sales: (3) Selling price, spot orders, invoices, credit memo.
Distribution: (4) Tonnage by kinds, sizes, grades, and use to each market area.

(5) to (9) *District Statistical Bureau to Division Statistical Section:* Summaries of 1936 cost (5), preparation (6), sales (7), and distribution (8) data for each District to statistical section of Bituminous Coal Division; District cost summary (9) to District Boards.

(10) *Adjustment 1936 production cost:* District Boards' recommendations to Division to reflect changes in wages, etc., since 1936.

(11) *Cost hearing:* Regrading determination weighted average cost of production for each District and price area.

(12) *Approved weighted average cost of production* authorized by Division as basis for proposing minimum prices for various kinds, qualities, and sizes of coal produced in each district.

(13) *Intra-district quality and price relationships* to reflect values of the kinds, qualities, and sizes of coal produced by each mine within the district as proposed by each District Board, together with their proposals as to marketing rules and regulations for the sale of coal.

(14) *4-II (a) Hearing:* Regarding proposed intra-district quality and price relationships and marketing rules and regulations.

(15) *Approved intra-district classifications and marketing rules and regulations.*

(16) and (17) *Coordinated minimum prices, coal classifications, marketing rules.* (16). Proposed as result of coordination by the several producing districts for each kind, quality, and size shipped by any of them, in competition into any and every market area, together with coordinated marketing rules and regulations.
In the event of failure of the districts to agree in whole or in part, the Bituminous Coal Division may complete and propose such coordinated minimum prices and marketing rules (17).

(18) *4-II (b) Hearing:* Regarding inter-district coordination, prices, and marketing rules proposed for the shipment of coals into common consuming markets.

(19) Marketing rules and regulations: } Promulgated by Secretary of the
(20) Prices—minimum and if necessary, maximum: } Department of the Interior upon recommendation of the
(21) Discounts to distributors: } Director of the Bituminous Coal Division.

(22) and (23) *Price modifications and changes* found necessary or advisable on the Division's own representations or on request from producer (22) or consumer (23) interests, and after suitable hearing, legal procedure, etc.

(24) Records are available to Consumers' Counsel on request.

(25) Distribution or sales agency, registrations.

(26) and (27) Rules: Procedure and relationship between District Board and producers (26) and necessary assistance and information (27).

(28) *Tax:* None on export shipments or those to federal, state, or municipal purchasers; 1 cent per ton on all other except that produced by noncode members, which shall be 19½ cents per ton.

any district shall realize, as nearly as may be, the weighted-average cost of production of the minimum-price area of which the district forms a part. It also provides for the establishment of Consumers' Counsel as a liaison agency between the Coal Division on the one hand and the consuming public on the other.

The Bituminous Coal Division of the Department of the Interior on July 1, 1939, took over the work of the former National Bituminous-Coal Commission, and since that time has carefully and painstakingly continued the procedure (Fig. 3) of establishing minimum prices, f.o.b. mine, for bituminous coal from all producing districts within the United States and for movement into the many consuming market areas (some 200) within its boundaries. The task proved to be a formidable one; in fact, the final minimum-price hearing, which in a measure comprised an open coordination meeting, has taken 6 months, with the accumulation of some 25,000 pages of testimony and over 1700 exhibits, many of which are replete with data of varied and complicated character. All of this, together with the fundamental data upon which the weighted-average-cost determinations have been predicated, comprises a more extensive accumulation of pertinent data with reference to the production, distribution, consumption, and values of bituminous coal than any heretofore assembled in the history of the industry, and of inestimable value to producers and consumers alike.

The magnitude of the problem and the difficulty of arriving at an equitable solution becomes all the more apparent when one realizes that it involves:

1 The determination of the cost of producing and selling coal.
2 The classification of all coals by size and by grade.
3 The cost of transportation.
4 The establishment of minimum prices for the various kinds, qualities, and sizes of coal, produced in the several districts, to reflect, as nearly as possible, their relative market values at points of delivery in each common consuming-market area.

In addition, the Bituminous Coal Division, in the absence of complete agreement between producing districts, found it necessary to undertake the work of final price coordination.

Production costs for all mines for the year 1936, adjusted to reflect subsequent changes in wages, etc., including a reasonable cost of selling, when summarized, were used to determine the weighted-average cost of production per net ton for each price area for all of the ascertainable tonnage produced therein.

Subject to public hearing and subsequent approval by the Coal Commission, each district board proposed a classification of its coals and differentials as to size and grade. Subsequently, as a part of coordination, the Coal Division determined the base coals of each producing district and related these to the base coals of the other producing districts in their common consuming markets. The remaining coals of each district were then differentially related to its own base coals within the same market.

Throughout, consideration was given to sizing and preparation of coal, methods of cleaning and treatment, coal analyses, and to the possibility of rearranging and regrouping sizes proposed for each producing district, to facilitate the relation of various coals,

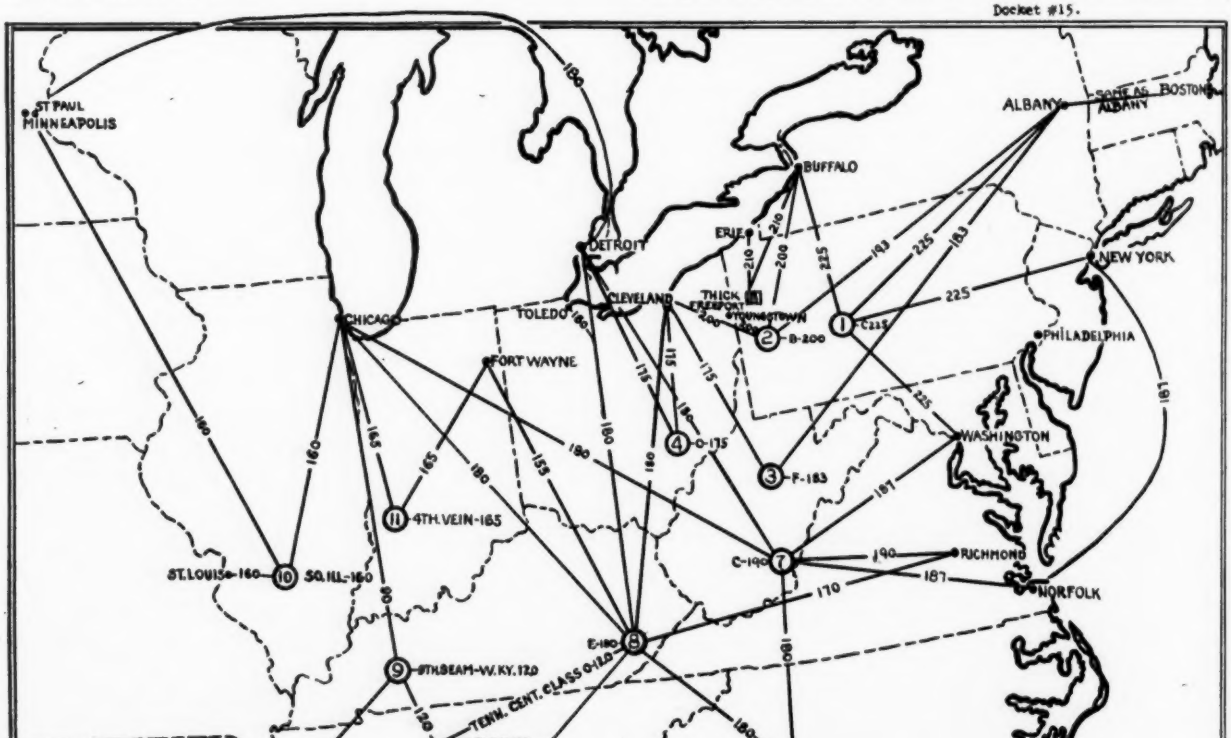


FIG. 4 PRICE RELATIONSHIPS F.O.B. MINE ON SCREENINGS FOR CERTAIN SELECTED DESTINATIONS

and simplify price structure for both producer and consumer.

Transportation tariffs are intricate in relationship and infinite in variety. Transportation cost constitutes a very definite and important factor in determining the f.o.b. mine prices of fuel. This is necessary if coals from the various sources are to compete with other fuels on a basis which will insure fair opportunities for the consumer to purchase and producers to distribute their coal in their natural markets without serious interruption or dislocation of tonnage.

In relating truly competitive coals, substantial freight differentials frequently were reduced by the absorption of a limited portion of such differential by the expedient of lowering the f.o.b. mine price of the coal or coals moving on the long freight rates. Coal for truck delivery was priced at the mine to effect the same delivered price as that for rail movement, an average trucking charge being used for this purpose.

An effort has been made to retain the advantages of low-cost water transportation in some market areas and for those consumers who are privileged to receive fuel on an "alongside" basis. Coal moving ex-dock, ex-river, or ex-lake has been coordinated with competing fuels moving all-rail into the same common consuming market.

TABLE 8 COST AND REALIZATION FOR VARIOUS PRODUCING DISTRICTS AND PRICE AREAS*

Price Area and District			Weighted Avg. Cost of Prod'n.	1937 Production (Net Tons)	Anticipated Realization	Difference: Realization and Cost
Minimum Price Area 1.						
(Eastern Pa.)	Dist.	#1	$2.3887	36,541,359	$2.1685	-$.2202
(Western Pa.)	"	#2	2.2140	65,299,792	2.0560	- .1580
(Northern W. Va.)	"	#3	1.8366	21,419,978	1.8644	+ .0278
(Ohio)	"	#4	1.9356	22,261,907	2.0142	+ .0786
(Michigan)	"	#5	3.6543	502,538	3.8232	+ .1689
(Panhandle W. Va.)	"	#6	1.9775	3,942,168	1.9827	+ .0052
(Pocahontas & New River fields of Va., & W. Va.)	"	#7	2.1940	51,113,582	2.1661	- .0279
(Eastern Tenn., Southern W. Va., & Western Va.)	"	#8	2.0301	86,694,822	2.0887	+ .0586
Price Area 1 average:			2.1284	287,776,191	2.0843	- .0441
Minimum Price Area 2.						
(West Kentucky)	Dist.	#9	1.5805	7,798,272	1.6213	+ .0408
(Illinois)	"	#10	1.7561	49,329,641	1.8442	+ .0881
(Indiana)	"	#11	1.6525	15,594,660	1.7429	+ .0904
(Iowa)	"	#12	2.7636	2,856,813	2.6829	- .0807
Price Area 2 average:			1.7622	75,579,386	1.8320	+ .0698
Minimum Price Area 3.						
(Alabama)	Dist.	#13	2.4382	12,854,838	2.4710	+ .0328
Minimum Price Area 4.						
(Ark. & Okla.)	Dist.	#14	3.6080	1,813,080	3.6447	+ .0367
Minimum Price Area 5.						
(Kans. & Okla.)	Dist.	#15	2.0392	7,391,238	2.0501	+ .0109
Minimum Price Area 6.						
(Northern Colo.)	Dist.	#16	2.5559	2,448,167	2.7344	+ .1785
(Southern Colo.)	"	#17	2.7664	5,285,938	2.6890	- .0774
(New Mexico)	"	#18	3.1519	730,814	2.9185	- .2334
Price Area 6 average:			2.7389	8,464,919	2.7220	- .0169
Minimum Price Area 7.						
(Wyoming)	Dist.	#19	1.9917	5,575,718	1.9936	+ .0019
(Utah)	"	#20	2.4691	3,740,965	2.4427	- .0264
Price Area 7 average:			2.1691	9,316,683	2.1739	+ .0048
Minimum Price Area 9.						
(Montana)	Dist.	#22	1.4851	2,888,850	1.5000	+ .0149
Minimum Price Area 10.						
(Washington)	Dist.	#23	3.2247	2,015,584	3.2496	+ .0249
All Price Areas:			2.0884	408,100,769	2.0729	- .0155

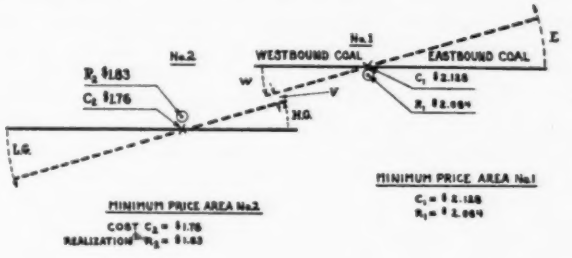
* After recommended adjustments.

Under free and open competition, relative market values in each market area involved price differentials as to sizes and qualities for the coals of each producing district and the level of prices for that market.

Differentials in price are influenced by quality, size, size consist, treatment, use, chemical and physical structure, and the adaptability of a coal or coals to use, markets, transportation, and other factors.

The level of prices in any common consuming market is to a large degree determined by that market, its proximity to the producing field, and the extent to which competing fuels, whether they be gas, oil, other coals, or even hydropower, may affect the movement of bituminous coal into that market.

Fig. 4 is a reproduction of an exhibit (4) introduced by the Coal Division during the final price hearing. On this the Division has noted the principal markets, producing districts, and f.o.b. mine prices for competitive "base" steam coals for "certain selected destinations." This is included here by way of illustration, without in any way implying approval. It seemingly indicates an attempt so to coordinate coal prices as to avoid discrimination



V = VALUE DIFFERENCE, COORDINATION PRICES MIN. PRICE AREAS No.1 and No.2
E = INCREASE EASTBOUND
W = DECREASE WESTBOUND
H.G. = HIGH GRADE - PRICE AREA No.2 COAL
L.G. = LOW GRADE - PRICE AREA No.2 COAL

FIG. 5 COORDINATION BETWEEN MINIMUM PRICE AREAS NOS. 1 AND 2

between classes of consumers, market areas, and industrially competitive sections of the eastern United States.

The proposed prices involved consideration of both levels and differentials, and include all sizes and grades of coals, whether borne by rail, water, or truck, and for the following uses:

a Domestic
b Industrial
c By-product-coke manufacture
d Water-gas and producer-gas manufacture
e Railroad-locomotive fuel
f Vessel and bunker fuel.

The "minimum-price" fixing procedure has reached a point where the Bituminous Coal Division has been able to estimate the realization anticipated from the proposed minimum prices, based on the 1937 tonnage distribution. In some instances, proposed prices are lower and in others considerably higher than the present market, with the average realization exceeding that now prevailing. The anticipated realization, together with the corresponding weighted-average cost for each of the several producing districts and price areas, is included in Table 8 (10).

The weighted-average cost of production and corresponding realization for minimum-price areas Nos. 1 and 2 (comprising the Appalachian and middle-western producing fields, respectively), allowing for subsequent modifications suggested by the Marketing Division, are:

Price area	Weighted average cost of production	Estimated realization	Difference
No. 1	$2.1284	$2.0843	—$0.0441
No. 2	1.7622	1.8320	+ 0.0698

Price area No. 1 underrealizes and price area No. 2 overrealizes their respective costs of production. However, further approach of realization to cost is difficult in view of the Marketing Division's desire to coordinate the prices of those coals produced

within each of the two price areas where they compete in common consuming markets. This may be illustrated diagrammatically by Fig. 5, in which is shown the relative weighted-average production costs and anticipated realizations for price areas Nos. 1 and 2. It so happens that a substantial portion of the tonnage of the southern districts of price area No. 1 moves into the Middle West in competition with the coals of price area No. 2, produced within that consuming area, price area No. 1 coals in both domestic and steam sizes generally being superior in value.

If the relationships between the coals of price areas Nos. 1 and 2, as determined by the Division, are correct, it is obvious that the truly competitive coals from each area must be priced f.o.b. the mine to deliver at their proper relative market value. Such coordination may be illustrated by the indicated rotation of the desired price level or realization lines. The rotation of price area No. 1 line indicates a price decrease for its westbound coals and a corresponding increase of the remaining coals moving eastbound since, only in this manner or by a decrease in realization (R_1), can coordination of price area No. 1 and price area No. 2 coals be effected.

The higher level of prices of those coals of price area No. 2 which actually compete with the westbound coals of price area No. 1 further assist this coordination. However, it is obvious that, for each ton priced above cost, there is a corresponding ton priced below cost in order that price area No. 2 realization may be accomplished.

Actual anticipated realizations indicate that price area No. 1 will underrealize and price area No. 2 overrealize its cost. Therefore, if the coals of these two price areas have been properly related, any further narrowing of the delivered price differentials between their competing coals means an abnormal increase in price area No. 1 coals moving eastbound or an abnormal lowering of prices for the coals of price area No. 2, which are not truly competitive with those of price area No. 1.

Price area No. 2 values cannot approach a point where they are considered "dump" prices, and the eastbound coals of price area No. 1, in turn, cannot be substantially raised without at once throwing an additional abnormal burden upon eastern consuming markets. This would also result in further discrimination, as between the market areas involved and a transfer of the burden of realization to an even greater degree than is appropriate for those coals, principally slacks, moving into market areas comprising New England and the eastern seaboard, which have no alternative but to take the coals of price area No. 1 or turn to competing fuels or forms of energy, such as hydropower, oil, and, in some cases, natural gas. Such competition has already resulted in a substantial increase in the utilization of oil and gas, and a decreased use of bituminous coal and anthracite throughout the eastern markets.

If the relationships between the coals of price areas Nos. 1 and 2, as determined by the Division, are correct, and if a narrowing of the differentials between the competitive coals of these two price areas is attempted in their common consuming markets either by lowering the level of realization (R_1) of price area No. 1 or by raising the level of realization (R_2) of price area No. 2, then the underrealization of price area No. 1 and the overrealization of price area No. 2, with respect to their costs of production, are aggravated to a still greater degree.

Distribution figures indicate and market history confirms the movement, for steam raising or by-product purposes in competition with other price area No. 1 coals, of the bituminous coals of eastern Pennsylvania (district No. 1), western Pennsylvania (district No. 2), northern West Virginia (district No. 3), and the southern low-volatile coals (district No. 7) in substantial quantities into New England, New York State, and other portions of the Atlantic seaboard as far south as Baltimore and Washington.

The southern high-volatile coals of district No. 8 and the low-volatile coals of district No. 7, particularly in the double-screened or prepared sizes, move westward through the Great Lakes and the northwest territory for "alongside" or "ex-dock" delivery in competition with the coals from Indiana, Illinois, western Kentucky, and Iowa (districts Nos. 9, 10, 11, and 12, respectively, comprising minimum price area No. 2). Slack coals from districts Nos. 7 and 8 also move into the industrial markets of Ohio.

Cost per Million Btu

If the average analyses (4) for the several sizes and grades of coal be applied to the delivered prices corresponding to the minimum f.o.b. mine prices proposed by the Bituminous Coal Division, the fuel cost per million Btu delivered is found to range from 9 to 14 cents for the mine run, and 9 to approximately $13^1/_2$ cents for the slack sizes for those destinations and market areas located at or close to the producing districts of the Appalachian coal fields. The delivered slack cost for more distantly located markets may approach 20 cents per million Btu in the eastern markets, 21 cents in the South, 25 and even 30 cents in the Northwest for rail movement, and 15 cents for Great Lakes destinations and for water-borne fuel. The mine-run figure averages 1 to 5 cents higher than for slacks. In any market, the differential in unit delivered cost between the high- and the low-grade fuels is surprisingly small.

Transportation appears to affect the unit cost for steam coals in eastern markets to a greater degree than in southern markets. In the Middle West, rail-transportation charges, both short and cross-haul, and those for water-borne fuel, together with the physical characteristics of the fuel and its adaptability for long-haul movement, evidently combine to make for wide variations in cost per million Btu for apparently small change in actual heat content of the fuel.

A comparison of solid, liquid, and gaseous fuels in those markets where they are actually available and competitive indicates a delivered cost per million Btu of substantially like amounts during periods of free and open competition.

Government Regulation

The World-War period, with its abnormal demand for coal for the manufacture of steel and munitions, and for industry, fostered the opening of many mines and thereby increased the accessible supply. The increased fuel prices prevailing during that period encouraged more efficient utilization and the use of lower-grade and cheaper fuels. These trends continued after the war so that the large potential supply of fuel became a serious burden; the solid-fuel industry, particularly bituminous coal, became demoralized, and the less efficient and higher-cost mines were forced out of business. The closing of mines continued, but not rapidly enough to prevent further demoralization of the fuel industry. Competition was keen and prices were low. These conditions, together with the realization that (*a*) the more accessible eastern coal reserves faced earlier depletion; (*b*) actual fuel recovery warranted substantial improvement; (*c*) improved working conditions and wages were in order throughout the industry, initiated and encouraged governmental interest (11).

This governmental interest continued from the time of the Hammond Commission in 1922 to, and culminated in, the NRA regulated prices. These were established at a sufficiently high level to permit increased wages in the northern and southern fields together with a general unionization of both. The effort to stabilize conditions within the industry, then begun, has continued until the present time. There is now imminent Federal regulation of the industry through the establishment of minimum prices for bituminous coal throughout the United States as prescribed in the Coal Act of 1937 (2).

A recognition of the rather limited petroleum reserves, overproduction of crude oil, and other economic factors has also aroused considerable interest on the part of governmental agencies in an effort to regulate (12, 13) the supply and stabilize prices for both crude and fuel oil and for oil products.

For similar reasons, attention has been directed to the possible control of natural-gas pipe-line installations and a stabilization and regulation of prices for both industrial and domestic use.

At the same time, considerable governmental interest has been shown in the development of hydropower and in the competition of this form of energy with that generated through steam, using solid, liquid, or gaseous fuels.

Although not generally realized and appreciated, this tendency toward governmental regulation of fuel production and price appears to be rapidly approaching a concerted effort to formulate a directed and coordinated program (11) of governmental supervision and control of all our natural resources, including fuel and water power.

This governmental interest apparently involves control of production, allocation of quantity, and source of supply. Its purpose, presumably, is that of conserving our natural resources, preserving the better grades of fuel and those with limited reserves, and encouraging the use of available lower-grade fuels as well as their more efficient production or recovery.

Anthracite producers' efforts parallel those to establish minimum prices for bituminous coal in that field.

Conclusion

The location of industry with reference to its fuel supply, the available competing fuels, and their relative prices in the market, improvements in combustion equipment, and the increasing use of power, all combine to complicate the problem of purchase and efficient utilization of fuel.

The purchase and use of fuel is in a state of evolution, or flux, and the change has been accelerated to a marked degree by governmental effort to regulate both supply and price.

Fuel selection and evaluation is by no means simple. The consumer's efforts may well be directed toward a better understanding of his needs, a consideration of available information, and a knowledge of the contemplated conservation measures (11) and extensive procedure gradually being evolved for the control of both price and supply of our natural resources. A proper interpretation and appraisal of these several factors is of primary importance in the satisfactory solution of his fuelpurchase problem.

Producers have come to realize that the sale of fuel is essentially an engineering problem, and that fuel preparation, to meet market requirements, is as important as its physical and chemical characteristics. Many producers also realize that low rather than high minimum prices are preferable, and that the use of low-cost fuels will increase to the exclusion of those which may be priced higher without possessing a correspondingly greater potential heat value.

There has been suggested a yardstick, or measure, for evaluating fuels within and between plants and for various methods of conversion. A résumé of the procedure for determining the proposed prices for solid fuels is included, with reference to the possible effect of governmental regulation. Reference has also been made to the manner of proper fuel selection and the possibility of cooperation between the combustion-equipment manufacturer and the consumer, in order to make possible a wider choice and use of sizes, grades, and kinds of fuel, to the end that the consumer may more economically purchase and more advantageously utilize available fuels.

The evaluation of fuels represents:

1 An integrated opinion of the producers of fuels which enter a common consuming market.

2 The relative values of available fuels, as determined by the consumer on the basis of conversion in his own plant and equipment and under conditions which govern his actual power production.

The evaluation by both may vary over a considerable range. Nevertheless, the integration of all valuations in each instance, measured one against the other, becomes, as it were, a fair average value of the proper differentials between the various kinds, qualities, and sizes of fuels. The required realization, modified as may be by the price for competing fuels, will determine the final level of prices for a fuel in a common consuming market.

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Discussion

H. K. Dean.[9] The last few years have brought about several changes in fuel-burning equipment and in boiler and furnace design. In so far as coal is concerned, progress in burning is definitely toward pulverized coal, burned in either slag-tap furnaces on the one hand or in dry-ash furnaces on the other. There is less tendency to design the intermediate type, many of which are still giving considerable trouble.

A great deal is yet to be learned regarding the characteristics of ash, since its behavior is different for different methods of burning. The standard ash-fusion-temperature determination has, in the past, been given as of a reducing atmosphere and there is a difference in the fusing temperature in an oxidizing atmosphere; as much as 400 deg with certain types of coal.

[9] Sales Engineer, The Babcock & Wilcox Company, Boston, Mass. Mem. A.S.M.E.

A wide range of fuel utilization may be had, particularly in burning fuels in suspension. A combination of pulverized coal, oil, and gas can be and has been burned successfully in either the slag-tap or dry-ash types of pulverized-coal furnace.

Changes in coal prices and marketing practices may alter the natural movement of fuels as it is now known. Therefore, there appears to be a greater necessity for full investigation and accurate knowledge of the available fuel supply by the purchaser, the engineer, and the manufacturer. Design characteristics of steam-generating equipment and over-all economics will probably place limitations on fuel purchases for some time to come. There will probably always be certain characteristic fuels which will give better over-all performance in certain types of steam-generating equipment.

A. W. Thorson.[10] In view of the proposed minimum prices for bituminous coal, everyone concerned with the purchase or utilization of this type of fuel is vitally interested in and should make an exhaustive study of this paper. Very little engineering information regarding the method of developing prices has been available to date. The paper gives detailed information as to the method of studying markets and production costs together with the correlation of the two to set up the price schedules. If and when these prices become effective, the value of this paper will become more and more apparent.

Has the author prepared a chart similar to that given in Fig. 4, but showing delivered prices at the selected destinations rather than f.o.b. mine prices? It would appear that such an exhibit would be of added value to coal consumers.

Author's Closure

Mr. Dean and Mr. Thorson in their discussions have touched upon two factors, transportation and conversion, which together with the initial fuel cost at its source of supply have a fundamental and important bearing upon available fuels, sources of supply, and the markets which they can economically serve.

As I have indicated, the cost of transporting a ton of bituminous coal by rail averaged, for the entire United States, an amount equal to or greater than the cost of production. Coal, depending upon the mine location and the market destination, may and often does move in whole or in part by other competing forms of transportation, that is, via lake, tidewater, river, and truck. Competition between carriers and, what is perhaps even more important, the actual transportation rate itself may have an important bearing upon the available sources of fuel supply. To a large extent the cost of transportation may and does determine the producer's ability to retain or extend his fuel market, and it is not unreasonable to expect that a reduction in transportation cost might well result in a substantial increase in gross revenue to the coal carrier.

The delivered cost per million Btu, other factors being equal, becomes the consumer's measure of purchased-fuel value, and of this transportation forms a substantial part, in fact in many consuming markets it amounts to several times the actual fuel cost f.o.b. cars at the mine.

Indications are that in many instances the minimum prices now proposed will be revised somewhat in the final price schedules, and that consideration is being given to the modification of transportation rates. For these reasons it would seem more appropriate to defer for the present a graphic illustration of typical delivered fuel prices, those in which the consumer is primarily interested.

In the past, steam-generating equipment generally has been geared to the available fuel supply, its characteristics, prices, and marketing practices. Considerable progress has been made, as Mr. Dean points out, in widening the range of fuel utilization and particularly where fuels are burned in suspension, both singly and in combination. While it is true that design characteristics of steam-generating equipment and over-all economics may place limitations on fuel purchases in many cases and for some time to come, progress in the design of steam-generating equipment, the technique of fuel burning, equipment maintenance, and ash or waste disposal has been sufficiently great in the last few years to indicate that steam-generating equipment having a much wider range of fuel utilization can now be manufactured and installed with very little additional effort and expense.

Thus, through improvements in transportation and utilization of fuel, will the consumer be enabled to take greater advantage of potential and changing fuel markets and be governed in his purchases to a greater degree by the actual fuel value as measured by the delivered cost per million Btu.

The producer, the carrier, the equipment manufacturer, and the consumer, each have a real interest in the contribution of the others in the fair evaluation, economical purchase, and advantageous utilization of available fuels.

The expressed interest of Messrs. Dean and Thorson, as well as that of many others, is encouraging and very much appreciated.

[10] The Chesapeake and Ohio Railway Company, Detroit, Mich. Mem. A.S.M.E.

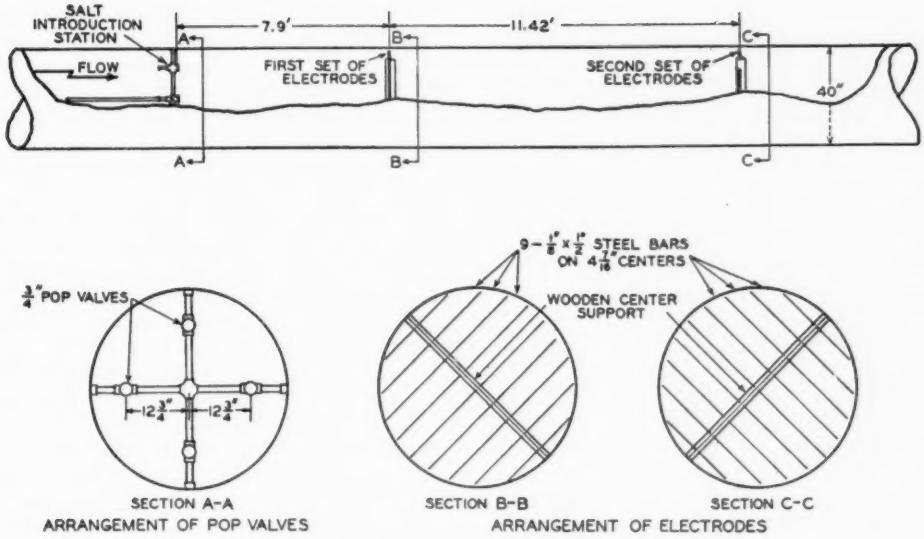


FIG. 1 TEST SECTION IN 40-IN. PIPE

Salt-Velocity Measurements at Low Velocities in Pipes

BY LESLIE J. HOOPER,[1] WORCESTER, MASS.

This paper describes tests made at the Alden Hydraulic Laboratory to determine the performance of the salt-velocity method at low velocities. Three series of tests were made: the first in a 40-in. pipe, the second in a 12-in. vertical pipe in which the test section and its approaches could be readily reversed, and the third in a horizontal 2-in. pipe. The tests show that for flow in a long straight pipe and for velocities lower than those normally found in practice, there exists a critical mixing velocity below which good mixing of the injected brine does not occur. An analysis of the results indicates some of the factors which affect the critical mixing velocity. Finally, it was found that the effect of gravity on the accuracy of the method, when applied to a vertical pipe, is negligible as long as proper mixing is secured.

THE salt-velocity method of water measurement has been primarily used for the measurement of discharge in the determination of pump and water-wheel efficiencies. In these applications, penstock velocities are usually of the order of one pipe diameter per second and no velocity limitations of the method were indicated in this experience. The object of this study was to investigate the sources of error at low velocities in the application of the salt-velocity method to long straight pipes.

DESCRIPTION OF APPARATUS

This investigation was conducted at the Alden Hydraulic Laboratory of the Worcester Polytechnic Institute and the tests were made during 1939 and early 1940.

A straight length of 40-in. pipe was selected for the test section, Fig. 1. Four $^3/_4$-in. pop injection valves were located at the introduction station. The first set of electrodes was located 7.9 ft downstream from the pop valves and the second set of electrodes located 11.42 ft beyond the first set. The brine was injected under 30 to 50 lb per sq in. excess pressure, and the discharge of the valves was controlled by a quick-acting lever valve in the brine supply line. The discharge of the 40-in. pipe passed through a 36 × 16-in. venturi meter, thence through a 100-hp water wheel and, finally, over an 8-ft suppressed weir in the tailrace. The venturi meter and the weir were both calibrated just before the tests by means of a 50,000-lb weighing tank.

The pop valves were built up from $^3/_4$-in. pipe fittings. Each valve had a spring-loaded flat disk operating against a stop. The valves required a pressure of about 5 lb per sq in. to open them. The excess-brine pressure of 30 to 50 lb per sq in. caused all the valves to be operated simultaneously and the spring-loaded disks kept brine leakage at a minimum between shots.

Each electrode in the 40-in. pipe was a grid, consisting of nine parallel $^1/_8 \times {}^1/_2$-in. steel bars, carried on a wooden streamlined center strut which was fixed across one diameter of the penstock, Fig. 1. These steel bars were placed edgewise to the flow to reduce their resistance and were arranged on 4 $^7/_{16}$-in. centers across the pipe, with the first and last bars 2 $^7/_{32}$ in. from the pipe wall. The bars were cut so as to leave an end clearance of $^1/_4$ in. The inside walls of all pipes tested were coated with insulating paint. Alternate bars were connected together and only one pair of leads was brought out from each electrode station.

In order to determine the effect of gravity on the heavier salt solution when injected into a pipe line, the apparatus which is illustrated in Fig. 2 was installed. The 12-in. vertical pipe with its test section and approaches was so arranged that it could be turned

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Contributed by the Hydraulic Division and presented at the Spring Meeting, Worcester, Mass., May 1–3, 1940, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

end for end, thus reversing the effect of gravity on the flow.

The four pop valves were located 18 in. from the flange of the upstream elbow and the electrodes were located 3 ft 7 in. and 18 ft 7 in., respectively, from the pop valves. These injection valves were built of $^1/_4$-in. pipe fittings in the same manner as the $^3/_8$-in. valves described. The electrodes consisted of two

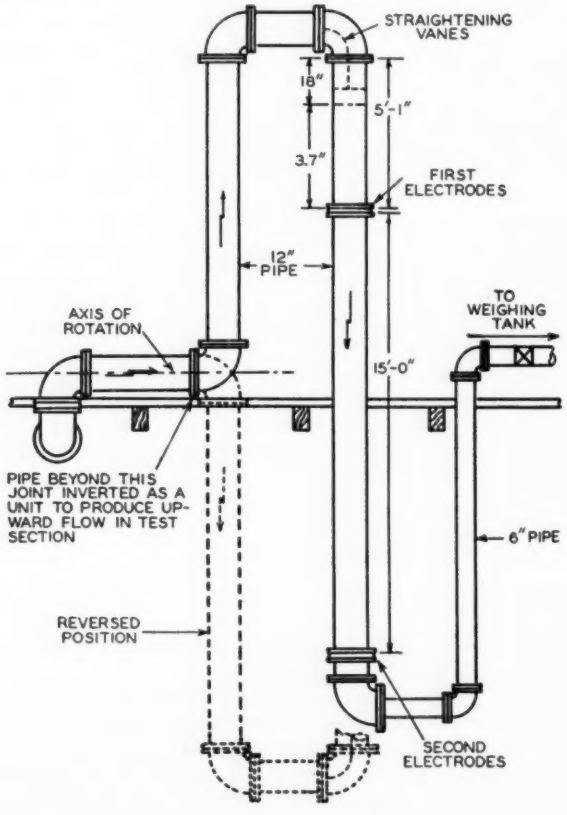


FIG. 2 TEST SECTION IN 12-IN. PIPE

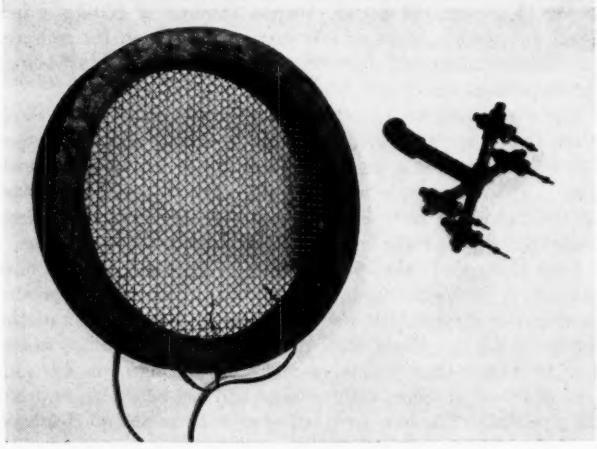


FIG. 3 POP VALVES AND ELECTRODE FOR 12-IN. PIPE

pieces of $^1/_5$-in. mesh galvanized-iron screen wire held between flanges and separated axially from each other $^3/_4$ in. The pop valves and electrodes are shown in Fig. 3. The discharge from the 12-in. pipe was measured directly in a 50,000-lb weighing tank.

The test section in the 2-in. line was located in a straight length of standard pipe. The brine was injected through a $^1/_4$-in. flush nipple located 4.61 ft downstream from a right-angle elbow. The distance from the introduction station to the first electrode was 2.39 ft and to the second electrode 10.48 ft. The electrodes consisted of $^1/_5$-in. mesh galvanized-iron screen wire similar to that used in the 12-in. pipe, except that the distance of separation was $^1/_8$ in. The discharge was measured in a 600-lb weighing tank. A Bristol 5-amp strip-chart ammeter was used to record the passage of the salt solution through the electrodes in the 40-in. and 12-in. line tests. A General Electric strip-chart recording polyphase wattmeter was used in the 2-in. line tests. The power supply for the electrode circuits was taken from a variable-voltage transformer. The electrodes were connected in parallel to the transformer with the recording instrument located in series in the common lead. A time record was made on the recording chart by a chronographic pen actuated from a calibrated timer.

The brine used in these tests was mixed to various specific gravities as determined by means of hydrometers. For most of the tests, solutions of common salt were used but dilute hydrochloric acid was used for the 1.02 specific-gravity tests in the 12-in. line. A hand force pump was used to inject the brine in the 12-in. and 2-in. pipe tests. The acid solution was injected under air pressure with the pop-valve operation controlled by a quick-acting valve.

The volume between the planes of the electrodes was determined by actual measurement of the diameter and length in the 40-in. pipe. Four diameter readings were taken at every foot along the test section together with 18 measurements of its length. From these measurements the test volume was computed.

In the 2-in. and 12-in. lines, the test section was blanked off and filled with water, which was then drained into a weighing tank. This measurement was repeated 8 times and the test volume was computed from the average weight and density of the water.

PROCEDURE

The procedure was essentially the same for all of the tests. The discharge was adjusted to a desired value and from 5 to 10 min were allowed to elapse to secure steady flow conditions. Then from 5 to 15 salt shots were recorded at the same time that the discharge was being measured by the check method. For the 40-in. line the venturi deflection and the head on the weir were read at intervals during each test. For the 2-in. and 12-in. lines, the discharge was measured with the weighing tank.

CALCULATIONS

The passage time for a salt shot was taken as the time in seconds between the centers of gravity of the two electrode curves. The volume of the pipe between the planes of the two electrodes divided by the average passage time of all the salt shots in a run gave the salt-velocity discharge.

The true discharge was that measured by the weighing tank for the 2-in. and 12-in. pipe tests or the average of the venturi and weir discharges for the 40-in. line test.

The difference between the true discharge and that of the salt-velocity measurement, expressed in per cent of the true discharge, was termed the error of the salt-velocity measurement.

The mean velocity was found by dividing the true discharge by the area of the pipe.

RESULTS OF TESTS

The results of these tests are shown in tabular and plotted form.

In a long pipe, where the normal flow pattern has been established, it is impracticable to introduce the salt solution uniformly by means of pop valves or any other known arrangement. In

TABLE 1 LOW-VELOCITY TESTS IN 40-IN. PENSTOCK

Run No.	Test Volume cu.ft.	Passage Time sec.	Salt Velocity Discharge c.f.s.	True Discharge c.f.s.	Error in S.V. %	Mean Velocity ft/sec
1	100.3	9.521	10.53	10.53	0.0	1.20
2	100.3	6.003	16.71	16.74	-0.2	1.91
3	100.3	2.811	35.68	35.70	-0.05	4.06
4	100.3	42.9	2.338	2.529	-7.4	0.288
5	100.3	18.36	5.463	5.488	-0.5	0.625
6	100.3	20.53	4.885	4.957	-1.4	0.565
7	100.3	101.6	0.987	1.283	-23.1	0.146
8	100.3	94.8	1.058	1.182	-10.4	0.135
9	100.3	5.023	19.97	19.98	-0.05	2.27
10	100.3	2.740	36.61	36.50	+0.3	4.16
11	100.3	9.110	11.01	11.03	-0.2	1.255

Water Temperature - 35°F.

Specific Weight of Water - 62.42 lbs. per cubic ft.

TABLE 2 RESULTS OF TESTS IN 12-IN. VERTICAL PIPE

Run No.	Weighing Tank: Net Wt. lbs.	Net Time sec.	True Discharge lbs/sec.	Salt Velocity: Volume lbs.	Passage Time sec.	Discharge lbs/sec.	Error %	Mean Vel. ft/sec.
Brine Specific Gravity 1.10								
1	602.5	262.0	2.299	17.373	7.577	2.292	-.26	1.920
2	502.1	334.3	1.502	17.373	11.55	1.504	+.13	1.247
3	451.9	379.8	1.190	17.373	14.56	1.193	+.25	0.990
4	291.2	382.0	0.762	17.373	25.23	0.689	-9.20	0.633
5	401.7	432.9	0.928	17.373	19.53	0.890	-4.10	0.769
Brine Specific Gravity 1.01								
7	552.3	256.2	2.156	17.373	8.060	2.156	0	1.797
8	451.9	317.6	1.423	17.373	12.21	1.423	0	1.186
9	451.9	454.5	0.994	17.373	17.43	0.997	+.3	0.825
10	401.7	555.4	0.723	17.373	24.04	0.723	0	0.603
11	150.6	386.2	0.390	17.373	52.50	0.331	-15.1	0.325
12	361.3	493.3	0.623	17.373	27.93	0.622	-0.2	0.519
13	401.7	664.3	0.605	17.373	28.72	0.605	0	0.504
14	326.4	615.9	0.530	17.373	33.20	0.523	-1.3	0.442
15	351.5	820.5	0.428	17.373	44.50	0.390	-8.9	0.357
16	276.2	586.9	0.470	17.373	37.60	0.462	-1.8	0.392

Water Temperature 38.2° F.

Specific Weight of Water 62.42 lbs. per cubic foot.

this case, the turbulence existing in the pipe is relied upon to secure uniform mixing of the salt solution with the flowing water before the first electrode station is reached. With this in mind it might be supposed that good mixing would be secured as long as the flow in the pipe was in the turbulent range, as indicated by a Reynolds number of 2500 or more. However, it has been found by von Kármán and others that the turbulence existing in a pipe may be materially reduced by the presence of an unmixed solution either heavier or lighter than water.

To visualize this phenomenon, let us assume the worst case where a lighter liquid is overlying a heavier liquid and they are both traveling along a horizontal pipe together. For the present discussion, chemical or molecular diffusion may be neglected, since these diffusion velocities are negligible when compared to ordinary mixing velocities. If the two liquids are to be mixed, part of the light liquid must move down into the heavier liquid and a corresponding amount of the heavy liquid must be moved upward. Then work is done in submerging particles of light fluid in the heavier fluid and also in lifting the heavy particles into the lighter fluid. Clearly, in the example selected, the axial component of the fluid velocity has no effect upon the mixing

TABLE 3 RESULTS OF TESTS IN 2-IN. HORIZONTAL PIPE

Run No.	Weighing Tank: Net Weight lbs.	Net Time sec.	True Dischg. cfs	Salt Velocity: Volume cu.ft.	Passage Time sec.	Dischg. cfs	Error %	Mean Velocity ft/sec
Brine Specific Gravity 1.20 Downward Flow								
1	10500	2825	.0597	12.04	173.6	.0694	15.2	0.075
2	32000	723.9	.710		17.09	.705	-0.7	0.895
3	11000	449.5	.393		30.30	.397	1.0	0.495
4	12000	1849.7	.1042		113.3	.1063	+2.0	0.132
5	8000	2433.7	.0528	12.04	211.8	.0568	7.6	0.067
Upward Flow								
6	20000	212.1	1.514	12.04	7.925	1.519	+0.3	1.909
7	18000	244.7	1.181		10.19	1.182	+0.1	1.490
8	22000	373.2	.947		12.76	.944	-0.3	1.195
9	17000	428.7	.637		19.14	.629	-1.3	0.803
10	28000	165.3	2.720		4.415	2.727	+0.3	3.430
11	22000	946.9	.373		34.34	.351	-5.9	0.470
12	18600	1484.2	.201	12.04	70.4	.171	-14.9	0.253
Brine Specific Gravity 1.10 Downward Flow								
13	23000	155.7	2.372	12.04	5.078	2.371	0.0	2.990
14	23000	220.8	1.673		7.20	1.672	-0.1	2.110
15	22000	324.3	1.089		11.14	1.081	-0.7	1.370
16	15000	441.0	.546		22.04	.546	0.0	0.688
17	9000	644.0	.224		52.2	.231	+3.1	0.282
18	12100	2400.0	.0809	12.04	133.6	.0901	+11.3	0.102
Upward Flow								
19	27000	164.92	2.629	12.04	4.604	2.615	-0.5	3.31
20	24000	191.8	2.009		5.97	2.017	+0.4	2.53
21	27000	300.2	1.444		8.40	1.433	-0.8	1.82
22	18000	287.1	1.007		12.10	1.002	-0.5	1.27
23	16000	456.7	.563		22.06	.546	-3.0	0.71
24	12500	986.6	.203	12.04	65.3	.184	-9.4	0.256
Acid Specific Gravity 1.020 Downward Flow								
25	13000	163.2	1.279	12.04	9.304	1.294	+1.2	1.615
26	15000	331.1	0.728		16.56	0.727	-0.1	0.917
27	8000	335.0	0.384		30.76	0.391	+1.8	0.484
28	8000	617.8	0.208		56.18	0.214	+2.9	0.262
29	6800	1017.4	0.107		99.10	0.121	+13.1	0.135
30	7100	1913	0.0596		174.6	0.0690	+15.8	0.075
31	6900	7932	0.0140		827	0.0146	+4.3	0.018
32	30000	203.0	2.373	12.04	5.075	2.372	0.0	2.990
Upward Flow								
33	28000	166.1	2.707	12.04	4.452	2.704	-0.1	3.410
34	20000	156.2	2.056		5.80	2.076	+1.0	2.590
35	19000	204.4	1.493		8.02	1.501	+0.5	1.883
36	17000	285.0	0.958		12.61	0.955	-0.3	1.209
37	14000	424.9	0.529		22.86	0.527	-0.4	0.666
38	19500	1438.8	0.218		55.92	0.215	-1.4	0.275
39	12300	1462.8	0.1350		90.1	0.1336	-1.0	0.170
40	7800	1368.4	0.0915		130.7	0.0921	+0.6	0.116
41	5200	1378.7	0.0606	12.04	196.7	0.0612	+1.0	0.764

Specific Weight of Water 62.27 lbs. per cubic ft.

Water Temperature 73.4°F.

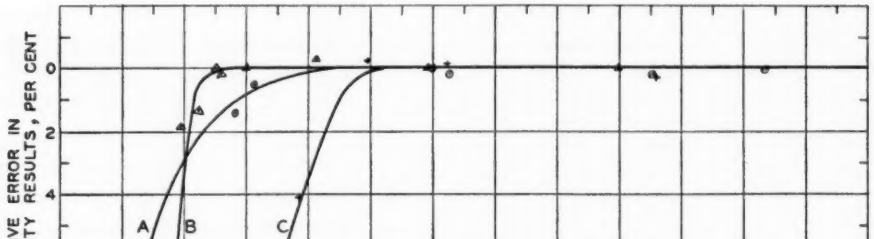


FIG. 4 TEST RESULTS IN HORIZONTAL PIPES
(*A*, 40-in. pipe, brine specific gravity = 1.06;
B, 2-in. pipe, brine specific gravity = 1.01;
C, 2-in. pipe, brine specific gravity = 1.1.)

process and only the radial velocity components are effective in promoting mixture of the two liquids. It naturally follows that the work of mixing the two liquids is accomplished at the expense of the energy available in the radial components of flow. If the work required for mixing is less than the amount of energy available, then the mixing process goes forward rapidly. On the other hand, if the work required for mixing is greater than the turbulent energy present, the process continues until the energy available is used up and then, having reduced the crossflow, the mixing process is checked. Thus, in this second case, the presence of the heavier liquid tends to smother the crossflow in the pipe.

In the example selected, the mixing was accomplished by the radial components of flow but, in the more general case where there is a small isolated mass of heavier liquid, mixing can take place in all directions and it is more correct to say that the mixing process depends upon the energy available in the fluctuating components of the velocity.

From the foregoing, it would be expected that the change from good to poor mixing conditions would take place rapidly. That is, as soon as the heavier liquid starts mixing, its specific gravity is reduced so that the process continues more easily. However, if the heavier liquid is not diluted, it continues to smother the fluctuating component of flow.

The work of mixing is accomplished at the expense of the energy in the fluctuating component. The size of the average fluctuating component is usually expressed as a percentage of the average forward velocity and this is called percentage of turbulence. For instance, 10 per cent turbulence means that the average fluctuating component is $^1/_{10}$ of the average forward velocity at that point. Returning to the mixing process, when a given shot of salt solution is injected into a pipe, a certain amount of energy is required for mixing. Since the energy available for mixing varies as the cube of the fluctuating component and the fluctuating component is proportional to the forward velocity in a given pipe, it follows that the energy available for mixing varies as the cube of the forward velocity. Thus a small change in the forward velocity results in a large change in the energy available for mixing and it would be expected that the change from good to poor mixing conditions would take place suddenly. The test results show this to be true. The curves in Fig. 4 show small experimental or casual errors at high velocities which continue unchanged to some definite low velocity. Then there is a "break" in the error curve showing a sudden departure from the purely experimental type of error. The value of the velocity at the break in the error curve, which is the velocity required for good mixing, is termed the "critical mixing velocity."

FURTHER TESTS WITH BRINE AND HYDROCHLORIC-ACID SOLUTIONS

When brine is injected into water, the excess weight of the brine is proportional to the difference of the specific gravities of the brine and the water. Thus, brine of 1.10 specific gravity has 10 times more weight in water than brine of 1.01 specific gravity. The term "specific gravity minus one" is called the effective specific gravity during the remainder of this discussion.

Tests were then made to determine experimentally the effect of a change in the specific gravity of the injected brine upon the critical mixing velocity. This was done first in the 12-in. pipe and, subsequently, in the 2-in. pipe. In the 12-in. pipe test,

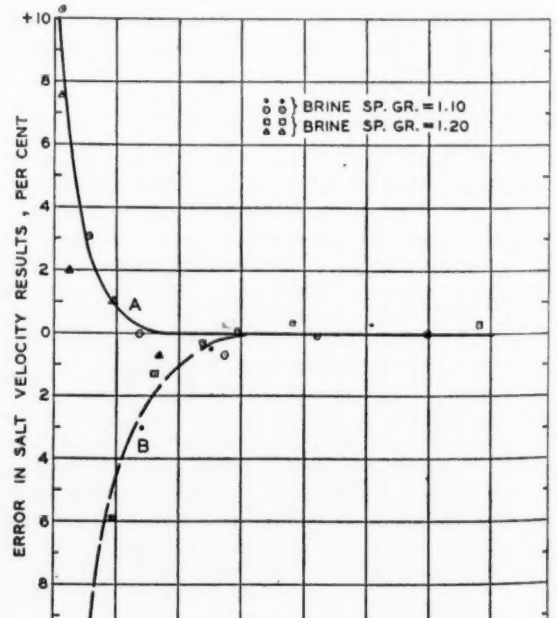


FIG. 5 TEST IN 12-IN. VERTICAL PIPE WITH BRINE SOLUTIONS
(*A*, downward flow; *B*, upward flow.)

brine of 1.2 and 1.1 specific gravity and dilute hydrochloric acid of 1.02 specific gravity were used. The results of the first two tests came out so close together that a single curve represents their average, Fig. 5. The test with the dilute acid, Fig. 6, indicated that the solution was mixed at a lower velocity in the case of upward flow but there was little change with the downward flow. Furthermore, these test points show much more scatter than in any of the other tests with brine. It was believed that the acid probably produced secondary chemical and, therefore, electrical effects at the electrodes which were zinc-covered, with the consequence that the results were not as reliable as the tests made with brine.

The results of tests with brines of 1.01 and 1.1 specific gravity are shown in Fig. 4. Selecting an arbitrary error of —1/2 per cent in the salt-velocity measurement, a velocity of about 0.94 fps is found with brine of 1.1 specific gravity and about 0.45 fps with brine of 1.01 specific gravity. These results are in substantial agreement with the mixing theory stated. On the one hand, the work done varies directly with the "effective specific gravity" of the solution being mixed, while on the other, the energy passing through a unit area in the pipe cross section varies as the cube

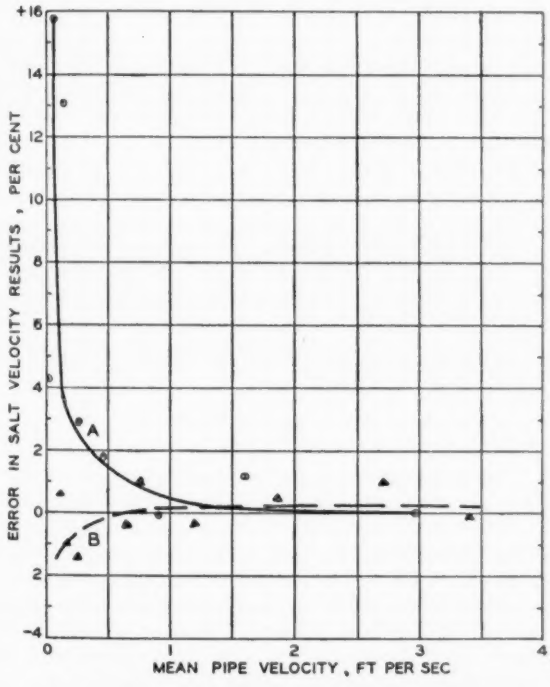


FIG. 6 TESTS IN 12-IN. VERTICAL PIPE WITH DILUTE HYDROCHLORIC ACID

(*A*, downward flow; *B*, upward flow; specific gravity 1.02.)

of the velocity. If the turbulent energy required is reduced to 1/10 of its previous value, the pipe velocity required to produce this energy is only reduced by $(0.1)^{1/3}$ or 0.465 of its previous value. This is practically the change in the critical mixing velocity found in the 2-in. line tests with a 10:1 change in the "effective specific gravity" of the injected salt solution.

The total work done in mixing is also proportional to the quantity of brine injected into a given volume of water flowing in a pipe. The power required for commercial recording instruments makes it difficult to secure sufficient electrode effect in small pipes and thus the quantity of brine is usually increased so that suitable curves will be obtained on the recording chart. For instance, in the 2-in. line tests with the weak brine of 1.01 specific gravity, the ratio of the brine to the discharge was nearly 20 times greater than in larger pipes. Therefore, the critical mixing velocities found in the 2-in. line tests are greater than those necessary in larger pipes with the same flow condition.

The laboratory tests do not indicate directly the effect of wall roughness upon the critical mixing velocity but an idea of its influence may be obtained from theoretical considerations. The previous discussion indicates that a definite value of the average fluctuating component was necessary to produce satisfactory mixing for a given flow condition. This value of the fluctuating component may be obtained with a low percentage of turbulence and a high forward velocity or with a high percentage of turbulence and a correspondingly lower forward velocity. The present theory of turbulence indicates that the percentage of turbulence at a point in a pipe is proportional to the wall roughness. Therefore, the greater the pipe roughness, the lower the pipe velocity necessary to provide good mixing.

EFFECT OF GRAVITY UPON INJECTED BRINE SOLUTIONS

The effect of the force of gravity upon the injected brine or the tendency of the brine to sink through vertically flowing water was studied in the 12-in. pipe tests. At very low velocities when proper mixing is not secured, the errors in the salt-velocity measurements indicate that the unmixed brine masses tend to slip through the flowing water under the influence of gravity. That is, when the flow is upward, the salt masses travel more slowly than the main body of water, resulting in a longer passage time and an indicated discharge which is too small. With downward flow, the brine tends to slip along faster than the water with a resulting discharge which is too large. But a study of the curves, Fig. 5, will show that, when there is proper mixing of the brine, there is no systematic error of the salt-velocity measurements. The importance of this fact cannot be overemphasized.

The test curves made with brine of 1.1 and 1.2 specific gravity in the 12-in. vertical pipe are not symmetrical with respect to the axis of zero error, the upward-flow tests showing a higher critical mixing velocity than the downward-flow tests. It is probable that, in the case of downward flow, the force of gravity, acting upon the unmixed brine masses, tends to give them a slightly increased velocity. This energy is dissipated in eddies and, hence, promotes the mixing process. Conversely, with upward flow, more energy must be supplied from the water flowing in the pipe to lift the unmixed brine in addition to mixing it.

APPLYING SALT-VELOCITY METHOD IN WATER-WHEEL INTAKE

In this discussion of tests at low velocities with the salt-velocity method, only the application of the method to a long straight pipe has been considered. This assumes that normal pipe flow exists at the point where the brine is injected and continues through the measuring section. It might be inferred from these tests that it would be impossible to apply the salt-velocity method in the intake of a large water wheel where pipe flow does not exist and, hence, where turbulence due to upstream wall roughness is lacking. But there are two very important differences to be taken into account when considering flow into an intake. Since pipe flow has not yet started, all of the water is moving forward with very nearly uniform velocity and, hence, the nonuniform distribution of the brine, as it leaves the pop valves, causes an insignificant timing error. On the other hand, turbulence is probably considerably greater than would normally be found at the entrance to a pipe due to the disturbing influence of the racks which are always present in a water-wheel installation. During 20 years of experience with this method, a large number of intake-test installations have been made. In not one of these instances was there any evidence of a lack of proper mixing of the brine, although in some cases the water velocity at

the injection station was below that indicated in these tests as being necessary for proper mixing. Thus, it is believed that the factors concerning the critical mixing velocity which have been discussed only apply to the measurement of flow in long straight pipes with very low velocities.

Summary

The following conclusions may be drawn from these salt-velocity measurements at low pipe velocities:

1 The low-velocity effects discussed in this paper are found at velocities below those normally encountered in practice.

2 The critical mixing velocity is a very definite limiting velocity below which proper mixing of the injected brine is not secured.

3 The force of gravity does not have any influence upon the accuracy of the salt-velocity measurement as long as the injected brine is readily mixed with the water flowing in the pipe.

Acknowledgment

The author gratefully acknowledges the assistance given by Dr. Th. von Kármán who suggested the mechanics of the mixing process and thereby contributed greatly to the value of this investigation.

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4 Theoretical discussion of turbulence: "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Company, Inc., New York, N. Y., 1938.

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Discussion

D. P. Barnes.[2] Referring to this and the paper[3] by O. H. Dodkin, it is begining to be recognized that, for the salt-velocity method to yield ideally correct results, it would be necessary that all the small elementary volumes of salt solution spend statistically equal periods of time in each of the different velocity zones from the center line to the outer radius in passing between electrode stations. This is to say that turbulent mixing should be continuous and complete transversely across the pipe throughout the passage. Any deviation from this continuous symmetry and uniformity of mixing would mean that part of the salt cloud must lag behind the mean flow, the center of gravity of the salt cloud, therefore, with it.

The natural tendency of a dense solution to settle downward and to underflow a lighter liquid as an integral body is now a well-established experimental fact. In the salt-velocity test, if this tendency is not completely countered by the turbulent mixing, it would be expected that in time the salt cloud would become progressively less symmetrical or less uniform with respect to the pipe.

In a sloping pipe, the effect of the tendency for the denser liquid to assume an asymmetrical distribution is compounded with the effect of the component of underflow in the downhill direction. In any case the asymmetry or nonuniformity alone would tend to produce a velocity reading smaller than the true mean, and this effect would be increased in the case of pumps with uphill flow or decreased in the case of turbines with downhill flow by the downhill component of the underflow.

The numerous field checks would appear to establish that the errors introduced by these tendencies are slight and, that where the method is used with full recognition of the possible sources of error and where a proper allowance is made for them, the results may be supposed accurate within perhaps 0.5 per cent, at least for downhill flow. This is a view to be accepted with caution, however, for bends, obstructions, steep slopes, low velocities, or abnormal velocity distributions in the penstocks must continue to introduce uncertainties wherever their effects are not subject to calibration. It would seem that each proposed application of the method should therefore be considered largely on its own merits.

Andrew Fejer[4] and J. W. Daily.[5] During the last few years, it has become more and more desirable to determine large rates of flow with great accuracy. In the case of large-scale field tests the salt-velocity method has been found particularly suited for such measurements because of its simplicity and convenience in application. Very little is known, however, about the performance of the method at low velocities, and this paper furnishes useful experimental information in that range.

In connection with investigations conducted at the hydraulic-machinery laboratory of the California Institute of Technology on the design, operation, and testing of large-capacity pumps and especially in connection with the question of extrapolation of model results to prototype results, it has been of considerable interest for the Institute to subject the salt-velocity method to experimental study. The experimentation was assigned to the writers, who present the following summary of results with the approval of the laboratory directors.

Experimental Work

Following Dr. von Kármán's suggestion that a salt cloud would tend to settle downward in water due to its higher density whenever the turbulent forces were inadequate to keep the cloud in suspension, it was decided to obtain qualitative information about shape and character of salt clouds under different flow conditions. The tests undertaken were made in a straight, transparent, lucite pipe 20 ft long and $5^1/_2$ in. inside diam, mounted in such a way that it was possible to change its inclination from 0 to 13 deg. Solutions of sodium nitrate deeply colored with potassium permanganate were injected into the flow through pop valves under compressed-air pressure, and visual, photographic and cinematographic observations of the behavior of the cloud on its path through the pipe were made. Two types of injection nozzles were used, i.e., a single central pop valve in the pipe axis and a combination of six pop valves distributed uniformly over the cross section. Solutions of five different densities (1.2, 1.15, 1.10, 1.05, 1.025) were injected into flows of velocities, covering a range of 0.1 to 1.5 fps. This range corresponds to a range of 0.5 to 7.5 fps for geometrically similar flow in a pipe 10 ft in diam.

Discussion of Results

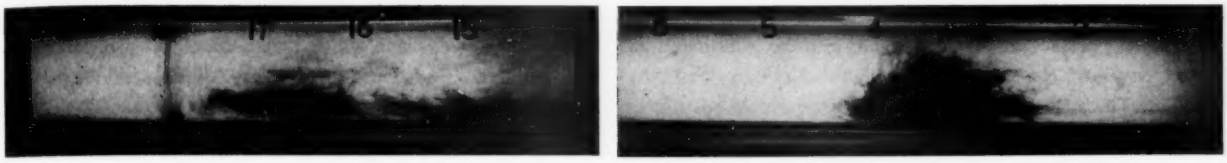
It was found that the salt cloud retained its symmetrical shape only at considerable flow velocities. This can happen only if the

[2] California Institute of Technology, Pasadena, Calif.

[3] This is a joint discussion of the L. J. Hooper paper, and the paper "Field Checks of the Salt-Velocity Method," by O. H. Dodkin, published on page 663 of this issue of the Transactions.

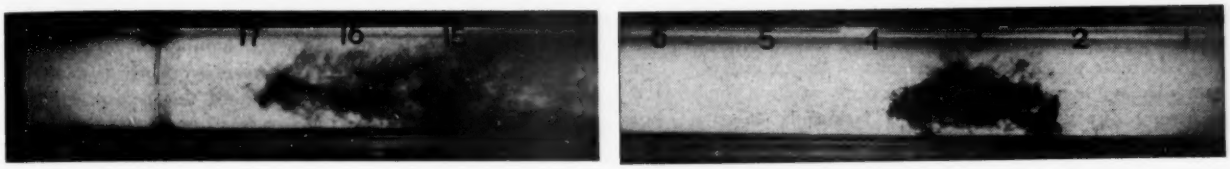
[4] Research Assistant, Hydraulic Machinery Laboratory, California Institute of Technology, Pasadena, Calif.

[5] Research Fellow, Hydraulic Machinery Laboratory, California Institute of Technology, Pasadena, Calif. Jun. A.S.M.E.



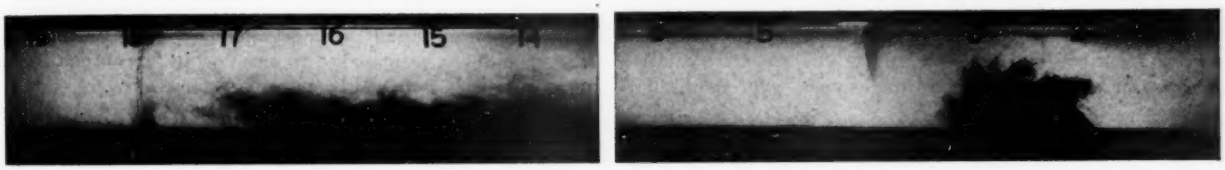
(*b*) (*a*)

FIG. 7 SALT CLOUD; SPECIFIC GRAVITY OF BRINE 1.025; MEAN VELOCITY OF FLOW 0.1 FPS; FROUDE NUMBER 0.03



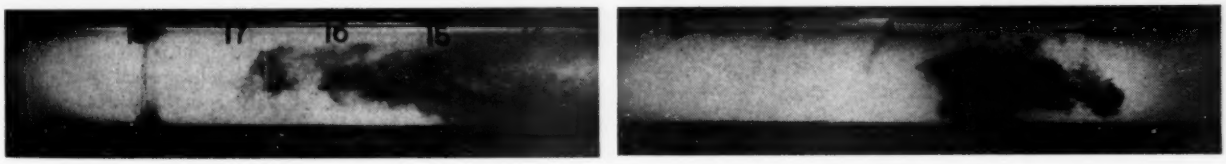
(*b*) (*a*)

FIG. 8 SALT CLOUD; SPECIFIC GRAVITY OF BRINE 1.025; MEAN VELOCITY OF FLOW 0.5 FPS; FROUDE NUMBER 0.68



(*b*) (*a*)

FIG. 9 SALT CLOUD; SPECIFIC GRAVITY OF BRINE 1.15; MEAN VELOCITY OF FLOW 0.3 FPS; FROUDE NUMBER 0.04



(*b*) (*a*)

FIG. 10 SALT CLOUD; SPECIFIC GRAVITY OF BRINE 1.15; MEAN VELOCITY OF FLOW 1.1 FPS; FROUDE NUMBER 0.55

energy of turbulent fluctuation is sufficient to overcome the differential-gravity forces. It may be considered that for a given setup the scale of the turbulence is independent of the rate of flow, while the magnitude of the turbulent-velocity fluctuations is proportional to the mean velocity of flow. This means that the energy of the turbulent fluctuations is proportional to the square of the velocity of flow. For this assumption, the ratio between the energy of turbulence and the work done by differential gravity forces may be expressed by a dimensionless quantity which contains in the numerator the square of the mean velocity and in the denominator the product of the effective gravity and a linear dimension of the setup (say the diameter of the pipe). This quantity, which has the form of a Froude number for a particular setup, is written

$$F = \frac{v^2}{Dg\left(\frac{\rho_2 - \rho_1}{\rho_1}\right)}$$

where v = mean velocity in fps; D = diam of pipe in ft; g = acceleration of gravity in fps^2; ρ_2 = mass density of cloud; ρ_1 = mass density of water; and product $g\left(\frac{\rho_2 - \rho_1}{\rho}\right)$ = effective gravity.

The value of the ratio at which the symmetrical form breaks down can be considered as the critical Froude number. Whenever the density of the injected solution is increased or the velocity of flow decreased so that the Froude number is smaller than the critical, a secondary or density-flow current is set up, the cloud sinks to the bottom of the pipe, and the assumption of the salt-velocity method that the average velocity of flow equals the average velocity of the cloud is no longer valid. Figs. 7 to 10 of this discussion[6] show typical salt clouds within a range of Froude numbers from 0.03 to 0.68. Figs. 8 and 10 show clouds corresponding to the critical Froude number which was found to be reasonably constant over the range of velocities and densities considered. For tests with the single nozzle $F_{crit} = 0.7$, and with the multiple nozzle $F_{crit} = 0.56$. The difference is due to the range in the scale of turbulence caused by the change in the geometry of the flow.

REMARKS ON THE FROUDE NUMBER

The Froude number just discussed should express the ratio of the inertia forces to the effective gravity forces acting on the cloud shortly after the effects of the injection have disappeared. As stated, the term ρ_2 should be the actual "initial cloud density." For practical use of the Froude-number criterion, it is necessary

[6] Figs. 7(*a*) to 10(*a*) show the clouds immediately after the injection, while Figs. 7(*b*) to 10(*b*) illustrate the same clouds some distance downstream. The numbers give the traveled distance from the injection nozzle in pipe diameters.

to correlate this initial density with the known density of the injected brine. Two simple assumptions can be considered as limiting cases:

1 The brine suffers no volume change between the instant of injection and the formation of the cloud; and

2 The injected brine is distributed immediately with a rate of dilution proportional to the velocity of flow.

As Figs. 7(*a*) to 10(*a*) indicate, the volume of the brine cloud does increase immediately after injection. On the other hand, the second assumption is not strictly valid. Rough measurements from photographic records show a definite tendency of the cloud size to decrease with increasing velocity of the main flow. This apparently is due to the fact that the penetration of the brine in the lateral direction decreases with increasing velocity. For high velocities, the cloud has an elongated shape with a small diameter while for small velocities the cloud has a large diameter even though of short axial length. It appears to the observer that because of these facts the two effects approximately balance each other. Consequently, the assumption of a constant "initial cloud size" seems reasonably justified. Since the density of the brine solution as injected is proportional to the "initial cloud density" for a constant injected volume and a constant initial cloud size, it was introduced into the expression for the Froude number.

It should be noted that the critical Froude number is not necessarily the same for different setups. The intensity and scale of turbulence can be different for various setups even in the case of geometric similarity. The type of the injection nozzles, the size of the injected volume, and the pressure used for the injection affect the initial cloud size. However, the experimental results mentioned show that, for a fairly wide range of injection conditions, the values of F_{crit} vary between narrow limits.

Conclusions

Both the author's work and the experiments noted in this discussion appear to indicate conclusively that salt clouds, as used in the salt-velocity method for rate-of-flow measurements, sink by virtue of the differential gravity forces into regions of lower velocities whenever the rate of flow and therewith the turbulent forces decrease beyond a certain minimum value. Below this velocity, which appears to be mainly dependent upon the Froude number of the test, it cannot be assumed that the cloud indicates the average velocity of flow.

It would be interesting to combine the author's method with the type of attack herein mentioned to determine whether the breakdown of the method, as shown by his experiments, coincides with the "critical Froude number" or whether additional sources of error counteract or accentuate the effect of the density currents.

R. T. Knapp.[7] These first published results on one of the auxiliary phenomena accompanying the salt-velocity method of water measurement are of value, especially, since it is possible that the effects of the density difference between the salt cloud and the water may form one of the basic limitations on the accuracy of the method.

The writer considers it rather unfortunate that the author restricted his measurements to horizontal and vertical pipes. The vertical pipe is not a typical case, since it represents a condition of equilibrium between the two fluids. It is true that this equilibrium is unstable, but it is probably very effective in the short time interval which exists in a salt-velocity measurement. Indeed, the existence of this equilibrium has been utilized by some Swedish investigators[8] in the development of a sedimentation type of particle-size analyzer in which a high-density suspension is superimposed upon a body of lower-density liquid. Thus, it seems that it would be unsafe to consider a vertical pipe as an extreme case of an inclined one and to use the results from the vertical flows to predict what would happen in the inclined pipe.

It should be remembered that the effect to be anticipated, as the result of the difference in density of the two flows, is basically a lateral one and not an axial one. In the horizontal or inclined pipe, the salt cloud tends to settle toward the lower side of the pipe, thus effectively lowering the average velocity of the solution. Of course, in the inclined pipe, there will be an axial component as well, which will tend to increase or decrease the velocity, depending upon the direction of flow. However, this axial component is probably of secondary importance.

In conclusion, the writer feels that this phenomenon is a very important one and hopes that the present paper represents only the beginning of the author's investigations on the subject.

M. A. Mason.[9] The suggestion that the possibilities and applicability of the Allen method might be defined by criteria relating the physical and hydraulic characteristics of a proposed test section[10] has been applied to pipe flow by the author, and it seems that at last the Allen method may be thoroughly studied in a logical manner, looking to the complete definition of its place in the art of water measurement.

The writer would, however, disagree with the author's choice of a criterion for good mixing. It is not believed that the "critical mixing velocity" is a sufficient definition of the hydraulic characteristics of a waterway to permit its use as a criterion of the mixing conditions in the test section. A moment's thought will show that, for a particular system in which normal pipe flow has been established, a critical velocity can be found which defines the mixing ability of the flow under the conditions existing. However, as stated by the author, "the percentage of turbulence at a point in a pipe is proportional to the wall roughness. Therefore, the greater the pipe roughness the lower the pipe velocity necessary to provide good mixing." It follows that the mean velocity of flow is not a satisfactory criterion of mixing, except for specified conditions which, in general, will apply only to the system used to evaluate the criterion.

A more important objection to the use of a velocity parameter as a criterion lies in the omission of a time element defining the period during which mixing is accomplished. To be of value in the application of the method, the mixing criterion must define, for a standard injection procedure, the minimum distance along the waterway required for the complete mixing of injected solution and water. Obviously the proposed velocity criterion does not furnish such information and, in fact, defines only the mixing ability of a flow under certain unique conditions.

From consideration of the mechanism of mixing and the character of turbulent flow, one may specify the probable components of a general criterion of mixing. The percentage of turbulence, i.e., the proportion of the forward velocity of flow present as fluctuating turbulence velocities in the flow, defines the ability of the stream to transport solution from the point of injection to other points in a cross section. If the turbulence is isotropic, then the percentage of turbulence is therefore a measure of the

[7] Associate Professor of Hydraulic Engineering, California Institute of Technology, Pasadena, Calif. Mem. A.S.M.E.

[8] "Measurement and Significance of Grain Size Distribution of Cement Particles," by D. Werner and S. Hedstrom, *Zement*, vol. 17, part 1, June 28, 1928, pp. 1002–1005; part 2, July 12, 1928, pp. 1071–1076.

[9] Engineer, Beach Erosion Board, War Department, Washington, D. C. Jun. A.S.M.E.

[10] "Contribution à l'étude de la mesure des débits d'eau par la méthode Allen," by M. A. Mason, *Revue générale de l'Hydraulique*, no. 28, July-Aug., 1939.

rate of diffusion of the solution throughout a cross section due to turbulent mixing. The percentage of turbulence is certainly one of the components of the criterion, and includes not only the velocity, but also the roughness characteristics of the waterway.

Obviously, a size factor of the conduit must also be included in the criterion; for a closed channel the area, and for an open channel, either the wetted area or the hydraulic radius might be considered.

A third factor, time, must be included. Considering percentage turbulence as a measure of the spatial rate of transport of solution throughout the water, it is evident that a certain time must elapse for the transport of a salt particle from, say, the center of the conduit to the wall, or vice versa. The time factor might then be expressed as the ratio of water depth or pipe diameter to the radial component of the turbulence fluctuations. The fractional portion of the linear dimension to be employed in this expression will, of course, depend upon the injection pattern.

If a standard pattern of injection can be adopted the influence of injection procedure may be eliminated, and more important, the time factor may be very considerably reduced by the use of a well-distributed injection pattern.

Assuming the foregoing analysis to be applicable and letting u_t be the mean turbulence fluctuation; A the cross-sectional area of the channel; and T the time factor; the form of the mixing criterion might be

$$M = u_t AT \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

having the dimensions L^3 of a volume.

Because of the practical difficulty of measuring u_t, it may be desirable to substitute for u_t the expression[11] lv, where l is the Prandtl mixing length, and v is the kinematic viscosity of the water. Experiment alone will determine the most satisfactory term to be used as a measure of the turbulent diffusion, the suggestions given herein being intended principally to indicate the nature of the term.

The results of the author's tests to determine the effect of gravity on the operation of the salt-velocity method are a further corroboration of the writer's thesis[1] that turbulence in the stream is the chief factor governing the accuracy of the method.

E. A. Taylor.[12] The author has described a study which gives a laboratory answer to a salt-velocity question which has arisen several times in the field. This question was: "When salt is introduced into a penstock does the higher specific gravity of the salt solution drive the salt through the water and toward the bottom of the penstock, thus changing the time of passage of the salt solution through the test section, and introducing an error in salt-velocity measurements?"

At many of the high-head power plants, where salt-velocity tests have been made, the penstocks have very steep slopes and, at some of them, the penstocks approach the vertical for considerable distances. If this difference in the specific-gravity question were important, then the computations for turbine efficiency at those plants might be appreciably in error.

The results of the laboratory studies described by the author indicate that the effect of gravity on the salt-velocity method of water measurement is negligible, when proper salt mixing and distribution are secured.

In 1939, the salt-velocity method of water measurement was used in making efficiency tests on the Colorado River Aqueduct pumps for the Metropolitan Water District of Southern California. During those tests, a field answer was found to the specific-gravity question. The question was raised at the Iron Mountain Pumping Station and the district engineers agreed to turn that plant into a field laboratory to study the subject.

The regular salt-velocity test section, used for pump-efficiency tests at this plant, was 200 ft long, 100 ft from salt injection to the first set of electrodes, and another 100 ft to the second set of electrodes. For this specific-gravity study, two additional pairs of electrodes were installed in the 10-ft delivery pipe.

These additional electrodes were called spot electrodes and consisted of small steel plates, 4 in. square and spaced 2 in. apart. These spot electrodes were fastened 5 ft beyond the regular second set of electrodes, making the length of test section used for the spot electrodes 105 instead of 100 ft. One pair of spot electrodes was located at the top of the pipe and the other pair at the bottom. During the study, the ammeter connections were alternated, shot by shot, between the two sets of spot electrodes.

A total of 74 salt shots were made and computed in this study. Averaging the results of these shots showed that the passage time to the top spot electrodes was 0.04 sec less than the passage time to the bottom electrodes. This was regarded as a perfect check and a conclusive field answer to the gravity question.

As further assurance of the accuracy of the salt-velocity method of water measurement and confirming the belief that any specific-gravity error was too small to be seriously considered, two volumetric check tests have been made at Pacific Coast plants, one of them being a high-head plant with a steep slope in the penstock.

For those check tests, a 3-acre forebay, 20 ft deep, and a 10-acre reservoir, 10 ft deep, were used as basins for the volumetric measurements. The volumetric results checked salt velocity, by 0.75 per cent at one plant and by 0.5 per cent at the other plant. The latter was the high-head steep-penstock plant where any gravity effect would be expected to be most apparent.

The author's conclusion, from laboratory tests, that "for velocities lower than those normally found in practice, there exists a critical mixing velocity below which good mixing of the injected brine does not occur" is confirmed by some field tests recently made.

Field efficiency tests were made on the Colorado River Aqueduct pumps in May and June, 1939. At Intake, the first station tested, the discharge values for single pumps checked the discharges when two or three pumps were operating together.

At the second station tested, Gene, the results of the preliminary tests, comparing single- and multiple-pump discharges, did not check.

At all five Colorado River Aqueduct pumping stations, one 10-ft delivery pipe carries the discharge from the three pumps now installed at each station. When only one pump is operating, the velocity in the delivery pipe is comparatively slow, about 2 fps. The inside of these delivery pipes is coated with an enamel finish, as smooth as glazed porcelain, and there are no rivet heads or other projections into the inside of the pipe.

This combination of extremely smooth pipe and low velocities created a condition of low turbulence, too low for satisfactory salt mixing, during the passage of a shot. This accounted for the failure of the discharge values to check between single and multiple pumps.

To provide more turbulence, an artificial agitator, called a "turbulator" was installed beyond the salt-injection station. This so-called turbulator was a steel disk, 6 ft in diam, fastened in the center of the delivery pipe and normal to the axis of the pipe. This disk had an 18 × 24-in. hole in the center and was located 40 ft beyond the pop valves. The turbulator was used during the remainder of the pump tests and single-pump-discharge results checked the multiple-pump values.

No artificial turbulator was required at Intake, because the salt-injection station was just above the manifold where the three

[11] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Oxford University Press, New York, N. Y., 1938, chapter 5.

[12] Consulting Engineer, Worcester, Mass.

6-ft pipes enter the one 10-ft delivery pipe. The pop valves were only 25 ft beyond this manifold and the turbulence caused by the bends in the pipes was sufficient to last throughout the salt-velocity test section.

R. M. WATSON.[13] From information now available, it appears that, with normal pipe roughness, mean water velocities, reasonable test sections, and when handled by a competent test crew, the salt-velocity method for determining the rate of flow of water is consistently accurate within 0.5 per cent. The writer belives this to be the limit, at present with few exceptions, to which any evaluation should be made. This accuracy is quite comparable to that by which the other factors, pressure and power, which express the performance of hydraulic machinery, can be determined reliably in most commercial installations.

Among the many factors which affect the accuracy of the salt-velocity method are the velocity of flow and the degree of turbulence in the stream. As shown by the author, a certain minimum turbulence is necessary in order that the salt cloud will travel with the flow without slippage. Apparently this problem had not been encountered in the field until a recent installation which had an exceptional combination of low velocities and smooth pipe walls. Unexplainably low results indicated trouble which was traced to the absence of turbulence. Excessive deviation from the expected results was corrected by the introduction of a "turbulator," the purpose of which was to create high turbulence ahead of the salt-injection nozzles.

In view of the difficulties experienced in the initial tests, opportunity was taken to make a check of the effect of the low velocities at those stations at which pumps manufactured by the writer's company were installed. Each station consisted of three pumps, each pump discharging from a 6-ft pipe into a common 10-ft pipe. The salt-velocity test section was in this 10-ft line, with approximately 4 pipe diameters between the turbulator and the salt-injection nozzles; 10 pipe diameters between the injection valves and the first electrodes, and 10 diameters between the first and second electrodes. The 10-ft discharge pipe at the test section ran upward at an angle of about 25 deg with the horizontal.

The tests were made by the customer's crew of engineers. This test crew had had the experience of previous salt-velocity work, in addition to experience with the use of this method on nine other pumps in three stations at this same large development. In every case the same problem existed; that of measuring by the salt-velocity method the rate of flow with very low mean pipe velocities and with extremely smooth pipe walls. This same crew was present and assisted in the development of a satisfactory technique for the method under the same conditions of low velocities and smooth pipes prevailing at the first three stations, and was therefore well qualified to conduct similar tests under similar discharge-pipe conditions at the two stations in which the writer's pumps were installed.

Piezometer taps were placed at two sections of different areas in the suction cones of all three pumps. The reduction in areas was sufficient to give a differential reading of approximately 6 in. of mercury in the range of flows considered. These venturi meters were calibrated individually by salt-velocity tests made during the operation of each pump singly. The salt-velocity test section was in the 10-ft pipe common to all three pumps. Each pump was operated individually, then in pairs, and finally all three together. Salt-velocity determinations of the flow were made each time. These calibrations were used during the multipump runs to measure the flow delivered by the individual pumps. During the multipump run, the sum of the individual flows thus determined was compared with the total as measured by the salt-velocity method. The individual pump flows varied from approximately 180 cfs to 215 cfs, with corresponding mean pipe velocities varying between 2.3 fps and 2.7 fps. For two and three pumps operating together, the mean velocities in the test section were, respectively, 2 and 3 times the mean velocities in the test section during individual pump operation.

As a result of an analysis of the data by the writer, it appeared that the salt-velocity results obtained with the low mean pipe velocities, existing during the individual pump runs, were low by between 0.3 per cent and 0.6 per cent, as compared to the salt-velocity results obtained with the higher mean pipe velocities existing during the multipump runs. It is interesting to note that the laboratory results presented by the author confirmed the possibility, direction, and order of magnitude of this discrepancy. It is equally interesting to note that the discrepancy between the single-pump and multipump tests was within the 0.5 per cent accuracy obtainable with the salt-velocity method under normally favorable conditions. It was only the peculiar combination of circumstances present at this installation which warranted the attempt for even better accuracy.

In general, the difficulties experienced in the measurement of the flow by the salt-velocity method were traceable, as brought out in the paper, to insufficient turbulence to prevent an actual slippage and retardation of the salt cloud due to its greater density, particularly immediately after the injection. This resulted in an apparent rate of flow which was considerably less than the actual. Improvement was obtained by installing a turbulence-creating construction a short distance ahead of the injection valves. The trend, however, of any error from this effect was shown definitely in these field tests to be negative, as is indicated by the laboratory tests of the author.

Even with the artificially created turbulence, the analysis of the field tests on six of the pumps as made by the writer, indicated that some slippage did occur. In explanation, it is probable that the turbulence resulting from the constriction decreased sufficiently within the test section to permit some slippage of the slightly more dense salt cloud. Certainly such is possible if the rather high turbulent energy in the flow leaving the pump (although this energy would be in small-scale eddies), and the turbulence created at the junction of the 6- and 10-ft pipes during single-pump operation, decreases sufficiently before reaching the injection nozzles to cause the troubles noted.

In opposition to the results of the writer's analysis of field tests and checks and resulting conclusions, conditions at one of the other three stations of this project permitted a volumetric check of the salt-velocity results. By this check the salt-velocity results appeared approximately 0.5 per cent high, after installation of the turbulence-creating construction. However, the writer does not know the full details of this check, and cannot, therefore, attempt to analyze possible differences.

In connection with the field checks of the salt-velocity method, the writer wonders whether or not an examination has disclosed any common characteristics of settings, or other factors, which tend to give high results in some cases, and low results in others. Such an examination should help to predict the direction of any error and possibly ultimately to eliminate some of the causes.

With the increasing size of centrifugal pumps and an expanding knowledge of design, pump efficiencies have now reached and even exceeded 90 per cent. The pump designer is therefore no longer satisfied with the 1 per cent to 2 per cent over-all test accuracy which used to be acceptable. The ability to differentiate between the effects of small design changes requires increasing accuracy in the tests as the efficiency increases. Whereas, at 80 per cent pump efficiency an accuracy of 2 per cent is sufficient to detect

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really significant design differences, this accuracy should be better than 0.5 per cent when the efficiency approaches or exceeds 90 per cent.

Author's Closure

The discussion presented by Messrs. Fejer and Daily is of great interest to the author, especially the suggestion in connection with the Froude number. This number, however, cannot be used as a guide until more tests have been made to determine its critical limits and the factors affecting those limits, because it does not of itself completely define the flow conditions. Furthermore, in addition to the pipe velocity, pipe diameter, and brine specific gravity, which factors make up the Froude number, the roughness of the pipe wall has a direct bearing on the turbulence, as does also the viscosity of the fluid and the quantity of brine injected per second-foot of flow. Finally, there is a modifying factor introduced by the performance of the brine after being injected.

The first three points were mentioned in the paper, but the last may need further clarification. It has been shown by experiment that the same amount of salt solution, injected at a constant rate, provides practically a constant height of electrode curve over a wide range of velocities. Since the duration of the shot remains constant, it follows that for low velocities the salt is confined to a small volume of the penstock water and is distributed over a proportionally larger volume of penstock water at higher velocities. Thus the total energy for mixing becomes the product of the energy per pound and the quantity of water flowing. This indicates experimentally that the energy available for mixing varies approximately as the cube of the velocity rather than the square.

With these thoughts in mind, the critical Froude numbers are presented for those tests where complete information is available, as follows:

Test no.	Pipe diam, ft	Critical velocity, fps	Brine, specific gravity	Factor
1	0.17	0.45	1.10	3.70
2	0.17	0.94	1.10	1.61
3	1.00	0.90	1.10	0.25
4	3.33	0.67	1.06	0.07
5	10.00	2.50	1.18	0.11
Caltech	0.42	..	..	0.56–0.07

The results with the 2-in. pipe seem to be far out of line but it is known in test No. 1 that the quantity of brine introduced per second-foot of flow was nearly 20 times larger than usual. Dividing 3.7 by 20 gives 0.19, which brings the result more in line with the figures obtained for the larger pipes.

The values of critical Froude number obtained by actual salt-velocity measurements are about $^1/_6$ to $^1/_3$ the values given by Fejer and Daily and they extend over a rather wide band. At the start of these tests, all of the factors controlling the mixing process were not appreciated, so that some of the experimental information is approximate. This applies particularly to the amount of salt injected per second-foot of flow. It is hoped that some of these tests can be repeated at an early date so that an experimental verification of this Froude number may be obtained.

Referring to Mr. Mason's discussion, it was not the intention of the author to suggest the use of the "critical mixing velocity" itself as a criterion for good mixing. As stated in the paper, the mixing process is affected by (1) the amount of brine injected; (2) the specific gravity of the brine; (3) the roughness of the pipe wall; (4) the size of the pipe; and (5) the velocity of flow in the pipe. All of these terms affect the mixing process in the first power except the velocity factor which varies as the cube. Therefore, a change in the velocity is more significant than a change in any other factor. From this point of view, the term "critical mixing velocity" was used in a descriptive way and not as a sole criterion of performance.

Furthermore, Mr. Mason states that the time or mixing distance should appear in the mixing criterion. The mixing distance is a factor intimately connected with the design of the pop valves and their installation with respect to the first electrodes; from that point it is most important. But, it was pointed out in the paper that, if the mixing process starts, it will continue; however, if the injected brine smothers the turbulence, the brine falls to the lower part of the pipe and remains segregated. Under these conditions, doubling the mixing distance or time will not improve the mixing. It is believed, therefore, that time or distance should not be represented in the mixing criterion.

With regard to Mr. Watson's discussion, it may be true there actually existed a small error for the flows measured for the single-pump units. Three similar pumping plants were tested in the same fashion by engineers of Professor Allen's office. In each case, the average agreement of the sum of the single pumps, compared with two and three pumps operating together, was to 0.1 per cent. This does not mean that the over-all accuracy of the method is 0.1 per cent on a field test.

In conclusion, it is realized that the work reported in the paper has only been a start in the investigation of the salt-mixing process at low pipe velocities and the constructive discussions which have been offered are appreciated.

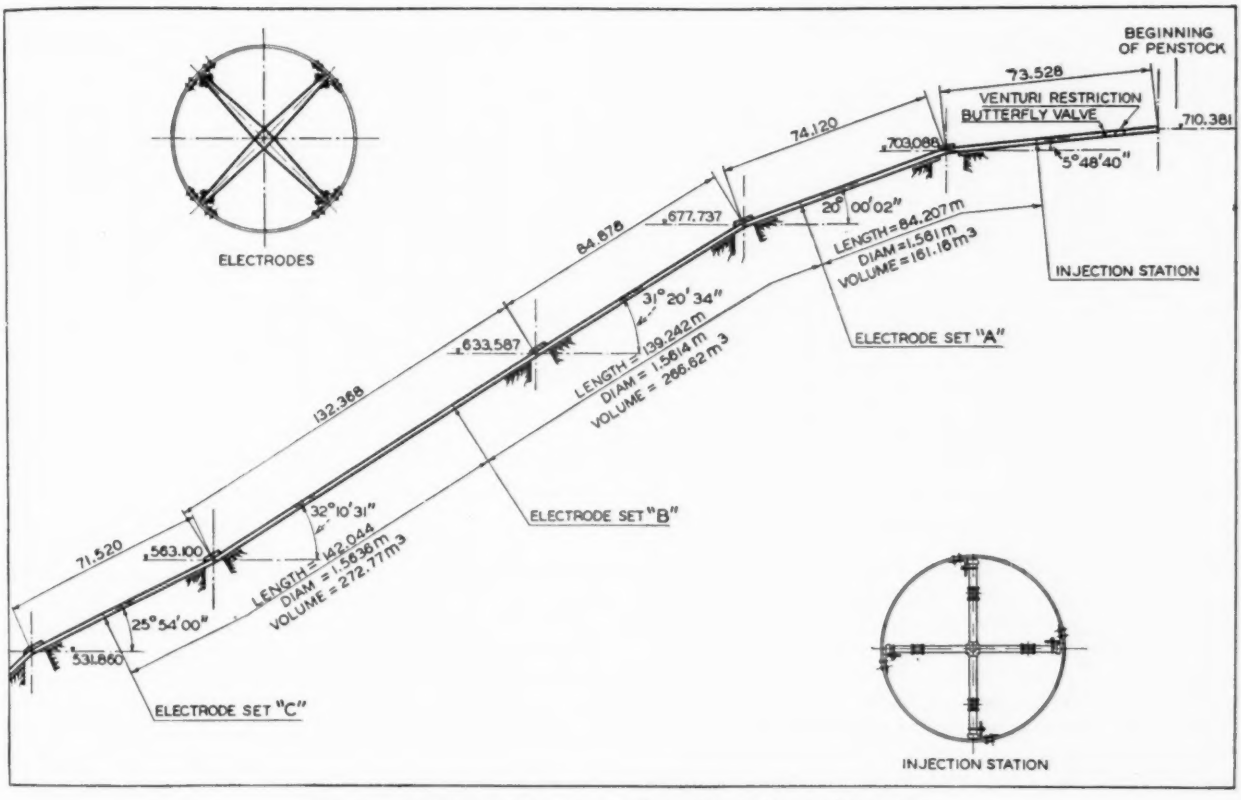


FIG. 1 SALT-VELOCITY LAYOUT AT UPPER END OF SERRA PENSTOCK

Field Checks of the Salt-Velocity Method

BY OSWALD H. DODKIN,[1] SÃO PAULO, BRAZIL

This paper gives the results of some tests made in Brazil on which verification of the reliability of the salt-velocity method of measuring discharge was obtained by cross checks with the method itself, by check tests with the Gibson pressure-time method, and by volumetric tests. These show a highly satisfactory agreement even in set-ups where the testing layout was far from ideal.

THE problem of measuring accurately the large discharges of modern hydraulic turbines is one that has engaged the efforts of many hydraulic engineers. About two decades ago the Gibson pressure-time method[2] and the Allen salt-velocity method[3] were introduced to the profession. Both possessed advantages of simplicity and ease of application as compared with some of the older methods. Laboratory tests and some field tests showed also a very promising degree of accuracy. Since then considerable experience has been obtained with these methods which in general has confirmed the early promises of accuracy of both methods.

Both the Gibson and salt-velocity methods have been used with success in the several plants of the companies subsidiary to the Brazilian Traction, Light and Power Company, Ltd., the former since 1926 and the latter since 1924. The purpose of this paper is to report verifications of both methods obtained in these tests. There will be given two tests in which the accuracy of the salt-velocity method is confirmed by itself with a different apparatus setup, two tests in which the results are confirmed by the Gibson method, and one in which its accuracy is confirmed by volumetric test.

SERRA UNIT 5

Unit 5 of the Serra plant of the São Paulo Tramway Light and Power Company, Ltd., is a double-overhung impulse turbine of which the nominal rating is 84,000 hp at 680 m (2230 ft) net head and 360 rpm; on test it can produce 92,300 hp at 674 m (2210 ft) head. It is fed by a penstock line approximately 1600 m (5250 ft) long with three diameters; 1565 mm (61.61 in.), 1463 mm (58.60 in.), and 1346 mm (52.99 in.). Fig. 1 shows in sketch form the upper end of this penstock and the location of the salt-velocity injection station and the three electrode sets used for testing the unit.

[1] Hydraulic Engineer, The São Paulo Tramway Light and Power Company, Ltd.

[2] "The Gibson Method and Apparatus for Measuring the Flow of Water in Closed Conduits," by N. R. Gibson, Trans. A.S.M.E., vol. 45, 1923, pp. 343–376.

[3] "The Salt-Velocity Method of Water Measurement," by C. M. Allen and E. A. Taylor, Trans. A.S.M.E., vol. 45, 1923, pp. 285–341.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 RESULTS OF COMPARATIVE SALT-VELOCITY TESTS—SERRA UNIT 5

		Section X—A Volume 161.2 m³			Section X—B Volume 427.8 m³			Section X—C Volume 700.6 m³			Section A—B Volume 266.6 m³			Section B—C Volume 272.8 m³			Section A—C Volume 539.4 m³	
Run	No. of shots	T sec	Q m³ per sec	v per cent	T sec	Q m³ per sec	v per cent	T sec	Q m³ per sec	v per cent	T sec	Q m³ per sec	v per cent	T sec	Q m³ per sec	v per cent	T sec	Q m³ per sec
1	1	77.00	2.094	—1.41	202.5	2.113	—0.52	331.00	2.117	—0.33	135.5	2.124	0.00	128.5	2.123	—0.05	254.0	2.124
2	7	13.43	12.003	—1.09	35.37	12.095	—0.33	57.88	12.104	—0.26	21.94	12.151	+0.13	22.51	12.119	—0.13	44.45	12.135
3	4	60.00	2.687	—0.96	158.3	2.702	—0.40	258.80	2.707	—0.22	98.3	2.712	—0.04	100.5	2.714	+0.04	198.8	2.713
4	5	59.90	2.691	—0.37	158.6	2.697	—0.15	259.60	2.699	—0.07	98.7	2.701	0.00	101.0	2.701	0.00	199.7	2.701
5	4	59.60	2.705	—0.26	157.8	2.711	—0.04	258.50	2.710	—0.07	98.2	2.715	+0.11	100.7	2.709	—0.11	198.9	2.712
6	5	45.80	3.520	+0.57	122.0	3.507	+0.20	199.90	3.505	+0.14	76.2	3.499	—0.03	77.9	3.502	+0.06	154.1	3.500
7	5	46.00	3.504	+0.37	122.2	3.501	+0.28	200.50	3.494	—0.09	76.2	3.499	+0.23	78.3	3.484	—0.23	154.5	3.491
8	4	46.21	3.488	—0.60	122.4	3.495	—0.40	199.90	3.505	—0.11	76.16	3.500	—0.26	77.54	3.514	+0.14	153.7	3.509
9	8	15.04	10.718	+0.40	40.01	10.692	+0.16	65.57	10.685	+0.09	24.97	10.677	+0.02	25.56	10.673	—0.02	50.53	10.675
10	7	15.04	10.718	+0.38	40.03	10.687	+0.09	65.56	10.686	+0.10	24.99	10.668	—0.08	25.53	10.685	+0.07	50.52	10.677
11	7	14.91	10.812	+1.00	39.86	10.733	+0.27	65.30	10.729	+0.22	24.95	10.685	—0.19	25.44	10.723	+0.17	50.39	10.705
12	6	39.10	4.123	+0.65	104.1	4.110	+0.34	170.80	4.102	+0.15	65.0	4.102	+0.15	66.7	4.090	—0.15	131.7	4.096
13	5	17.23	9.356	+0.62	45.91	9.318	+0.22	75.24	9.312	+0.15	28.68	9.296	—0.02	29.33	9.301	+0.03	58.01	9.298
14	8	17.42	9.254	—0.73	45.97	9.306	—0.18	75.28	9.307	—0.17	28.55	9.338	+0.16	29.31	9.307	—0.17	57.86	9.323
15	2	17.54	9.190	—1.03	46.39	9.206	—0.86	75.63	9.264	—0.21	28.85	9.241	—0.48	29.24	9.330	+0.47	58.09	9.286
16	6	20.22	7.972	—0.91	53.36	8.017	—0.36	87.25	8.030	—0.20	33.14	8.045	—0.03	33.89	8.050	+0.04	67.04	8.046
17	6	20.03	8.048	—0.17	53.16	8.047	—0.19	86.94	8.058	—0.05	33.13	8.047	—0.19	33.78	8.076	+0.17	66.91	8.062
18	5	19.94	8.104	+0.60	53.05	8.064	+0.10	86.90	8.062	+0.07	33.11	8.052	—0.05	33.85	8.059	+0.04	66.96	8.056
19	6	24.49	6.582	0.00	64.95	6.587	+0.08	106.44	6.582	0.00	40.46	6.589	+0.11	41.49	6.575	—0.09	81.95	6.582
20	7	24.56	6.565	—0.35	65.13	6.568	—0.30	106.43	6.583	—0.07	40.57	6.571	—0.26	41.30	6.605	+0.26	81.87	6.588
21	8	24.44	6.596	+0.30	64.91	6.591	+0.23	106.46	6.580	+0.06	40.47	6.587	+0.17	41.55	6.566	—0.15	82.02	6.576
22	6	30.84	5.227	+0.39	82.00	5.217	+0.19	134.40	5.213	+0.11	51.2	5.207	0.00	52.4	5.206	0.00	103.6	5.207
23	7	30.79	5.235	+0.82	82.19	5.205	+0.25	134.68	5.201	+0.17	51.4	5.187	—0.10	52.5	5.196	+0.08	103.9	5.192
24	6	30.90	5.217	0.00	81.81	5.229	+0.23	134.30	5.217	0.00	50.9	5.238	+0.38	52.5	5.196	—0.40	103.4	5.217

T = average time of salt-cloud passage.
Q = discharge, cu m per sec.
v = variation of discharge from corresponding discharge section A—C.

The injection station consisted of four 2-in. pop valves set in from the pipe wall 38 per cent of the pipe radius. Elementary electrodes were placed at the side of two of the pop valves to determine the time of the salt-solution injection.

Each electrode set consisted of two pairs of electrodes at right angles to each other, extending across the penstock to within $^1/_2$ in. of the penstock walls. These electrodes were bowed to have a center spacing 6.5 times that of the electrode ends.

The volume between the axes of the electrodes was determined from four measurements of diameter in each pipe, approximately one measurement for each 5 ft of penstock, and tape measurements of the lengths, due allowance being made in the volume for the bump joints and rivet heads, from the penstock manufacturer's drawings.

In testing the turbine, a total of 24 runs with an average of 5.6 salt shots per run was made. For each run six different determinations of the discharge were obtained, i.e., from the injection station X to electrode sets A, B, or C; from electrode set A to B, or to C; and from electrode set B to set C.

Table 1 shows the number of salt shots for each run, the time of salt passage for each run and electrode combination, the resulting discharges, and the variations of these discharges from those obtained from the combination of electrode sets A and C.

These variations have been analyzed by computing the probable variation of any one run for the several sections with the formula

$$r = \pm 0.6745 \sqrt{\frac{\Sigma v^2}{n-1}}$$

where r = probable error, per cent
v = variation, per cent
n = number of runs

The probable variations of one run, as percentages, are as follows:

Section	$X-A$	$X-B$	$X-C$	$A-B$	$B-C$
r	±0.47	±0.22	±0.10	±0.12	±0.12

This shows a high degree of consistency of the discharge measurements, the greatest probable deviation of one run being less than 0.5 per cent for section $X-A$ where the testing setup is the poorest. Examination of the variations shows no tendency for consistent positive or negative variations. In the case of section $X-A$, this tendency which is there the greatest, averages —0.07 per cent for the 24 runs, but is meaningless, since the probable deviation of the average is greater, being ±0.10 per cent.

A further indication of the small variation in the discharge is obtained from the calibration of the venturi restriction based on discharges determined in section $A-C$. The least-squares determination of a straight line fitting the observations gives $K = 7.807 + 0.0105\,Q$ in the formula $Q = K\sqrt{H}$, where

Q = discharge, cu m per sec
H = venturi differential head, m of water

The variations of the observations from this line show a probable variation of any one point of ±0.0148 or say ±0.19 per cent. Considering that this variation includes the errors of measuring the differential head by a U tube containing bromoform, as well as the errors of the discharge measurement, it is in line with the variations found in the discharge determinations of section $X-C$, $A-B$, or $B-C$.

Serra Units 3 and 7

Unit 7 of the Serra plant is essentially a duplicate of unit 5 previously described. Unit 3 is similar, but with slightly smaller penstock and nozzle diameters, having a rated output of 72,800 hp and an actual maximum of 78,100 hp. The penstocks for units 7 and 3, especially the upper end, and the salt-velocity setups as applied there, were similar to that for unit 5.

When unit 3 was installed and tested in 1936, its penstock was temporarily connected to the venturi restriction and butterfly valve intended for unit 7. Sixteen calibration runs were obtained giving an average of K in the equation $Q = K\sqrt{H}$ of 7.447 ± 0.006. The probable deviation of one run was r = ±0.32 per cent.

Later in 1939, the penstock of unit 3 was disconnected from this venturi and the penstock of unit 7 connected. The venturi was again calibrated during the tests of unit 7. Twenty-two calibration runs were obtained, giving an average value of K of 7.430 ± 0.005. The probable deviation of any one run was r = ±0.28 per cent.

In either case, the salt-velocity setup was similar to that for unit 5. The discharges were determined from electrode sets A and C. The differential heads were measured by U tubes containing bromoform. The data of these two calibrations are given in Table 2 and are plotted in Fig. 2.

In the case of unit 3, the location of the injection station was

TABLE 2 CALIBRATION OF SERRA VENTURI 7

(Results with discharge measurements in penstock 3 and in penstock 7)

	Discharge measured		Penstock 3	Discharge measured		Penstock 7
	Discharge, m^3 per sec	Dif. head, m	K	Discharge, m^3 per sec	Dif. head, m	K
Run	Q	H	$Q/\sqrt{H}$	Q	H	$Q/\sqrt{H}$
1	10.370	1.9200	7.482	12.020	...	...
2	8.543	1.3050	7.481	10.370	1.964	7.400
3	7.591	1.0270	7.493	10.340	1.935	7.433
4	6.709	0.8040	7.483	9.018	1.480	7.413
5	5.758	0.5960	7.459	9.034	1.476	7.436
6	4.959	0.4395	7.480	7.714	1.074	7.443
7	4.110	0.3075	7.411	7.709	1.071	7.449
8	1.901	0.0675	7.417	6.174	0.686	7.454
9	7.619	1.0400	7.476	6.177	0.686	7.458
10	6.702	0.8160	7.419	4.945	0.440	7.455
11	4.923	0.4445	7.386	12.030	...	...
12	5.834	0.6115	7.460	10.360	...	...
13	8.622	1.3540	7.413	9.103	...	...
14	10.395	1.9460	7.457	7.921	1.139	7.422
15	4.473	0.3655	7.397	6.257	0.711	7.421
16	2.605	0.1225	7.443	4.887	0.436	7.401
17	...	....	...	4.894	0.432	7.446
18	...	....	...	3.821	0.266	7.409
19	...	....	...	3.280	0.193	7.466
20	...	....	...	3.268	0.193	7.439
21	...	....	...	3.282	0.194	7.451
22	...	....	...	2.460	0.111	7.384
23	...	....	...	2.466	0.111	7.402
24	...	....	...	2.466	0.111	7.402
25	...	....	...	1.933	0.069	7.364
26	...	....	...	3.906	0.271	7.503

only 11 m, 7 pipe diameters, below the butterfly valve and venturi restriction, as compared with 24 m, 15.5 pipe diameters, for unit 5. The distorted velocity distribution resulting from the venturi and butterfly valve upset the precision of the salt-velocity discharges determined from the injection station to the electrodes. Such determinations differed from those from electrode sets A to C by as much as —2 per cent for X to A decreasing to —0.35 per cent for X to C. The results between electrode sets A — B and B — C, when compared with those between electrode sets A — C, were satisfactory, the variations ranging from 0.59 per cent to zero, with a probable deviation for any one run of $r = \pm 0.21$ per cent.

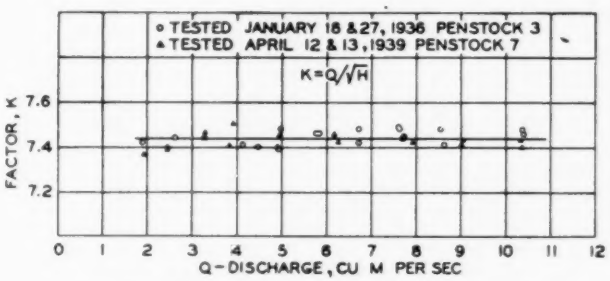


FIG. 2 CALIBRATIONS OF VENTURI; SERRA UNIT 7

The foregoing shows one point in the application of the salt-velocity method which must be emphasized. Where a high degree of accuracy is desired, the timing of the passage of the salt cloud in the waterway must be done between sets of electrodes extending across the entire waterway, not from the injection station to one electrode set, although with care in location of the injection station good results may be obtained thus, as shown by the tests on unit 5.

PARAHYBA UNIT 2

Unit 2 of the Parahyba plant of the Brazilian Hydro-Electric Company, Ltd., is a vertical reaction-type turbine, rated at 28,000 hp at 28.66 m (94 ft) net head or 32,500 hp at 31.7 m (104 ft) head and at 125 rpm. On test it exactly equals its rating.

Two complete efficiency tests were made on this unit December 19 and 21, 1927. On the first test, the discharges were determined

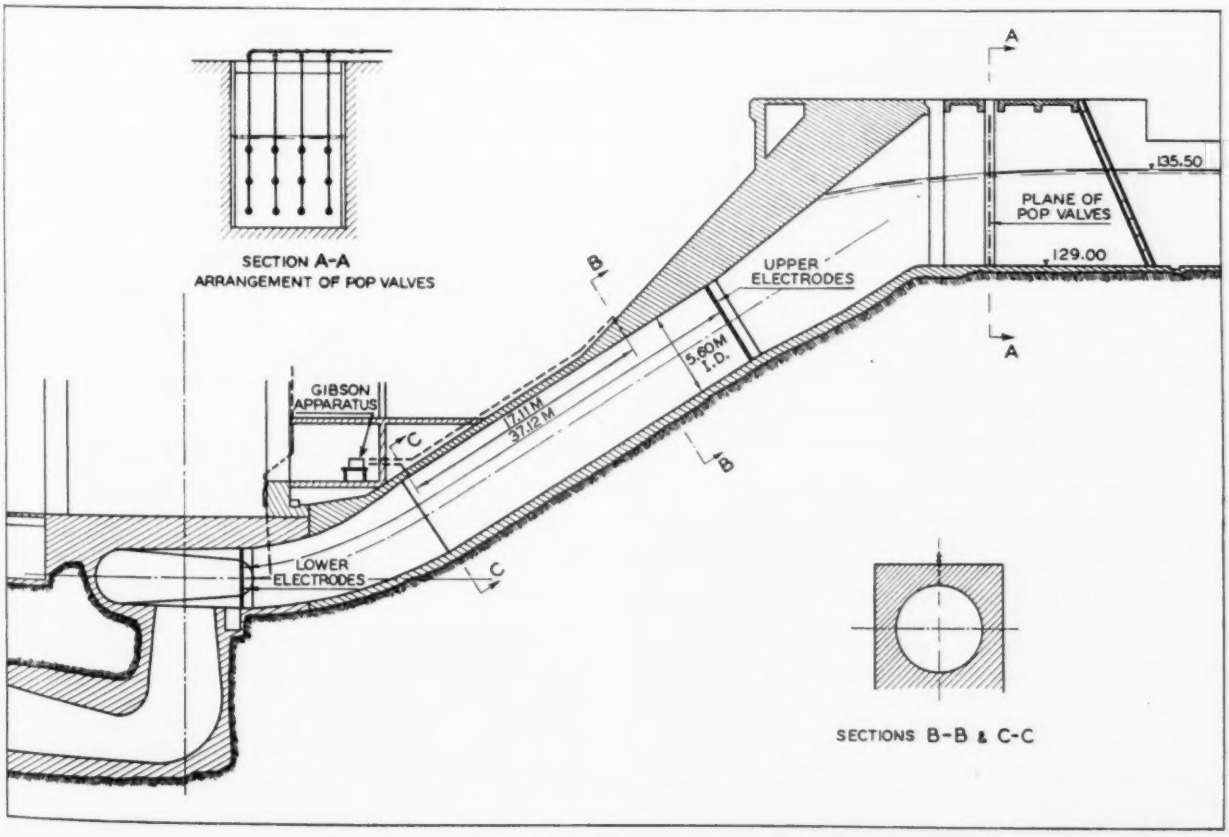


FIG. 3 TESTING LAYOUT AT PARAHYBA UNIT 2

TABLE 3 TEST DISCHARGES BY SALT-VELOCITY AND GIBSON METHODS; PARAHYBA UNIT 2

	Gibson test, Dec. 19, 1927					Salt-velocity test, Dec. 21, 1927				
			Discharge					Discharge		
Run	Guide vanes, per cent G	Net head, m	At test head, m^3 per sec	At 28.66 m, m^3 per sec Q	Q/G	Guide vanes, per cent G	Net head, m	At test head, m^3 per sec	At 28.66 m, m^3 per sec Q	Q/G
1	33.8	29.21	41.37	40.98	1.212	34.7	28.61	41.70	41.74	1.203
2	47.8	29.04	57.10	56.72	1.187	40.6	28.67	48.73	48.72	1.200
3	62.4	28.85	72.49	72.25	1.158	47.1	28.83	56.85	56.68	1.203
4	68.9	28.74	78.67	78.56	1.140	54.0	28.92	65.41	65.12	1.206
5	75.9	28.75	85.83	85.70	1.129	57.1	28.93	67.92	67.60	1.184
6	82.2	28.52	90.17	90.39	1.100	60.0	28.90	70.91	70.62	1.177
7	89.7	28.34	94.67	95.20	1.061	64.0	28.78	74.60	74.44	1.163
8	100.0	27.96	100.61	101.86	1.019	68.1	28.65	77.55	77.56	1.139
9	53.4	28.67	64.54	64.53	1.208	72.0	28.55	81.14	81.30	1.128
10	71.7	28.60	81.85	81.94	1.143	75.3	28.56	84.29	84.44	1.121
11	...	...	...	...	...	82.5	28.51	89.53	89.77	1.088
12	...	...	...	...	...	89.7	28.52	94.78	95.01	1.059
13	...	...	...	...	...	100.0	28.39	102.01	102.49	1.025
14	...	...	...	...	...	55.8	28.88	66.62	66.37	1.189
15	...	...	...	...	...	74.1	28.71	82.88	82.81	1.118

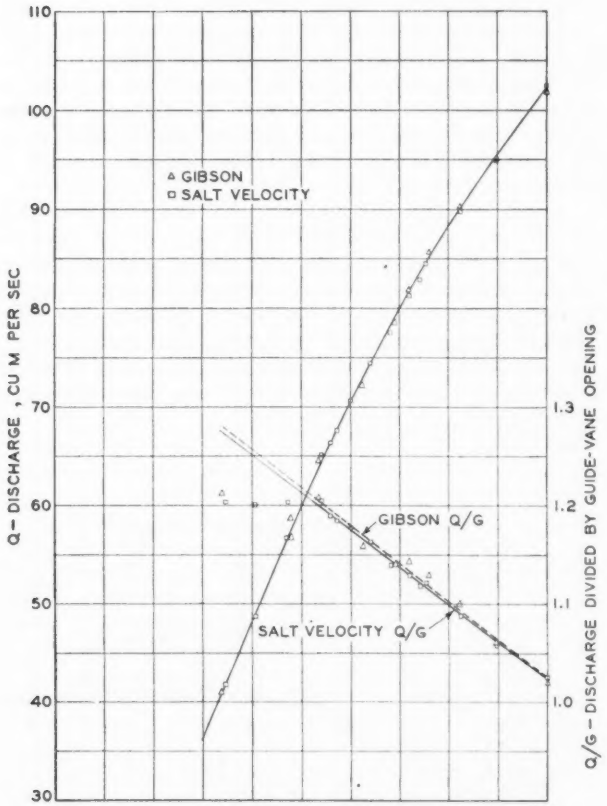


FIG. 4 TEST RESULTS; PARAHYBA UNIT 2

by the Gibson method applied differentially (10 runs). On the second test they were determined by the salt-velocity method (15 runs). Conditions of head, etc., were essentially identical on the two days.

Fig. 3 shows schematically the testing layout. Expressing the lengths in terms of the pipe diameter (average for the part concerned) the salt-velocity mixing length, introduction to upper electrode set, was 2.7 and the test length between electrode sets was 6.82. The length between the Gibson piezometers was 2.85. For the salt velocity, 12 pop valves were used at the stop-log plane. This gave one valve for each 40.5 sq ft of cross section at the injection station or one valve for each 24 sq ft of conduit section at the upper electrode set. The path from the highest pop valves to the upper electrode set was about 17 per cent longer than from lowest pop valves.

It was recognized from the start that, because of the short lengths relative to the diameter, and because the plane of the pop valves was not parallel to that of the upper electrodes, the testing setup was not ideal. Consequently, precautions were taken to insure good results with both methods. Great care was taken to adjust all pop valves for tightness up to 15 lb per sq in., and to have these open uniformly and at the same pressure. The electrodes were carefully shaped to have a center spacing 6.5 times the end spacing. The large electrode area and the natural conductivity of the water required an applied voltage of about 10 v. Consequently, large wires with carefully made joints were used in the electrode wiring.

For the Gibson method, all piping joints were carefully made for tightness. Also, during the performance of tests, the piping was flushed through completely between each run to eliminate any air and to insure that the temperature of the water in the piping was essentially that of the penstock water.

Table 3 presents the data obtained on these two tests. Fig. 4 shows the curve of discharge against guide-vane opening after transferring to a uniform net head of 28.66 m. In order to present graphically and clearly the variations between the discharge determinations by the two methods, and in an attempt to rectify the data for analysis by the method of least squares, each observed discharge at 28.66 m head has been divided by the corresponding guide-vane opening expressed in per cent. These quotients are likewise plotted in Fig. 4.

Examination of the plotting of these quotients shows that they may be approximately represented by straight lines from a gate opening of 53 per cent up to 100 per cent. The least-squares determination of the two equations of such lines gives

$$Q/G = 1.414 - 0.00390\,G$$ for the Gibson data

and

$$Q/G = 1.406 - 0.00385\,G$$ for the salt-velocity data

where

Q = discharge at 28.66 m head in cu m per sec

and

G = guide-vane opening in per cent.

The equation determined by the Gibson data in comparison with that determined by the salt-velocity data gives 0.42 per cent greater discharge at 53 per cent guide-vane opening (lower limit of applicability) and 0.29 per cent greater discharge at full guide-vane opening. The computed values of the probable variation of one run from the equations are $r = \pm 0.48$ per cent for the Gibson method and $r = \pm 0.22$ per cent for the salt-velocity method.

PARAHYBA UNIT 4

Unit 4 of the Parahyba plant is a vertical reaction-type turbine

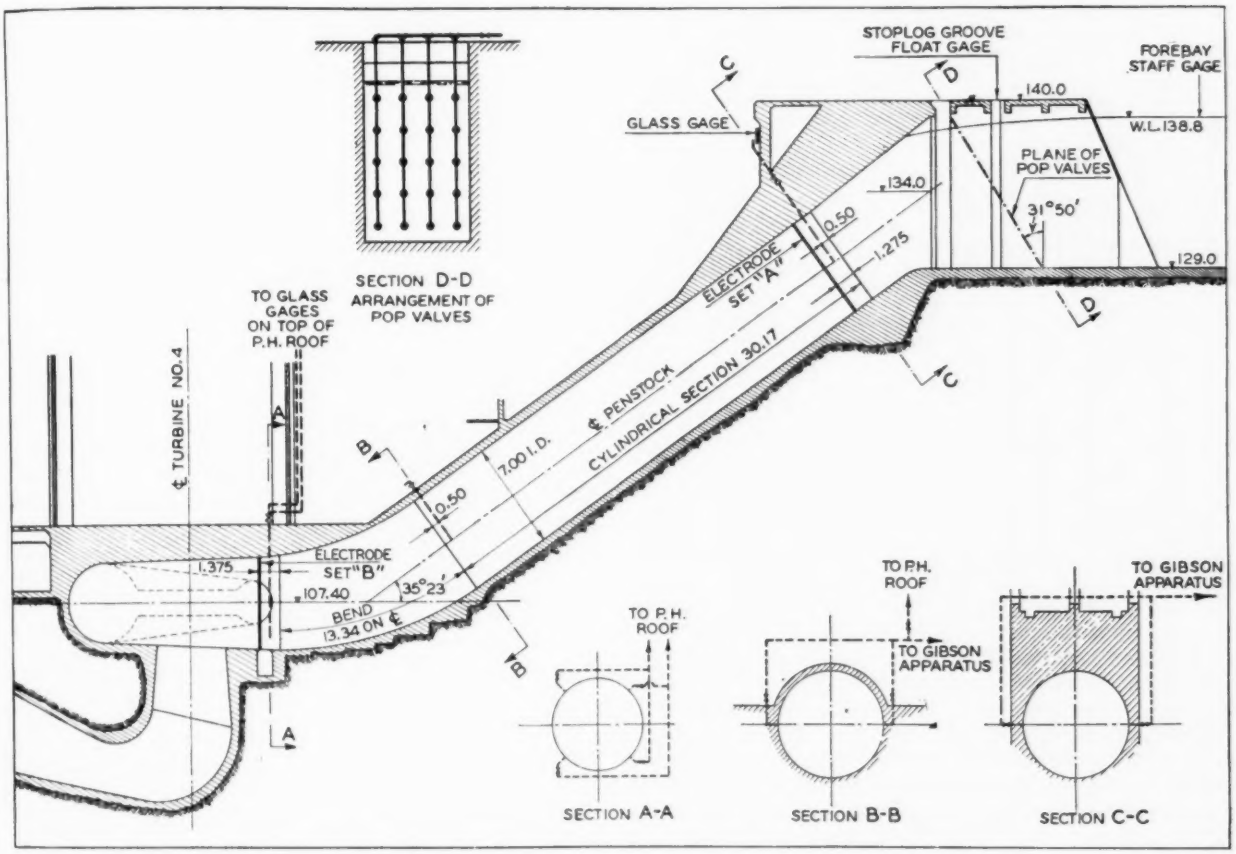


FIG. 5 TESTING LAYOUT, PARAHYBA UNIT 4

rated at 54,000 hp at 31.7 m (104 ft) net head and 115.4 rpm, with a discharge at maximum gate of over 5500 cfs; on test it can produce 57,190 hp at 31.7 m head.

A complete efficiency test of this unit was made in November, 1938. On this test, 28 runs were made using the salt-velocity method and, for 17 of these runs, the Gibson method was used at the end of the 10-min test run.

Fig. 5 shows schematically the testing layout which is generally similar to that for unit 2, except the pop-valve arrangement and the forebay water elevation at the time of the test. The lengths in terms of the pipe diameter are, salt-velocity mixing length 1.6, salt-velocity test length 6.4, and Gibson piezometer length 4.2.

For the salt-velocity method, 20 pop valves were used in a plane crossing the stop-log groove and closely parallel to the plane of the upper electrodes, thus making the paths from all valves of the injection station to the electrodes essentially equal. This gave one valve for each 35.5 sq ft of vertical cross section at the injection station or one valve for each 20.7 sq ft of conduit section at the upper electrode set. The same general precautions for insuring accuracy were followed with both methods as were used for unit 2.

Table 4 presents the data of this test. Fig. 6 shows the curve of discharge against guide-vane opening after transferring to a uniform net head of 31.7 m. As for unit 2, in order to present more clearly the variations of the discharge determinations and in an attempt to rectify the data for analysis by the method of least squares, each observed discharge at 31.7 m head has been divided by the corresponding guide-vane opening in millimeters. These quotients are likewise plotted in Fig. 6.

Examination of the plotting of these quotients shows that they may be approximately represented by straight lines over the range covered jointly by the Gibson and salt-velocity observations, that is, from 46 to 89 per cent guide-vane opening. Thus using all Gibson points and omitting the salt-velocity determinations of runs 1, 4 to 9, and 28, as being outside the range of joint observations, the least-squares determination of the two equations of such lines gives

$$Q/G = 0.5991 - 0.0004455\ G \text{ for the Gibson data}$$

and

$$Q/G = 0.6039 - 0.0004594\ G \text{ for the salt-velocity data}$$

where

Q = discharge at 31.7 m head in cu m per sec

and

G = guide-vane opening, mm.

The equation determined by the Gibson data in comparison with that determined by the salt-velocity data gives 0.42 per cent smaller discharge at 180 mm guide-vane opening (lower limit of applicability) and identical discharge at 345 mm guide-vane opening (upper limit of applicability). The computed values of the probable variation of one run from the equations are $r = \pm 0.0024$ or say ± 0.5 per cent for the Gibson method and $r = \pm 0.0031$ or say ± 0.6 per cent for the salt-velocity method.

SOROCABA UNIT 4

Unit 4 of the Sorocaba plant of the São Paulo Electric Company, Ltd., is a horizontal reaction-type turbine rated at 25,000 hp at 195 m (640 ft) net head at 600 rpm; on test it can produce 26,450 hp at 195 m head. Water is supplied to the unit by a penstock line about 656 m (2150 ft) long, having diameters of

TABLE 4 DISCHARGES BY SALT-VELOCITY AND GIBSON METHODS; PARAHYBA UNIT 4

Run	Guide vanes, mm G	Net head, m	Salt velocity: At test head, m³ per sec	Salt velocity: At 31.7 m, m³ per sec Q	Gibson: At test head, m³ per sec	Gibson: At 31.7 m, m³ per sec Q	Q/G: Salt-velocity	Q/G: Gibson
1	158	31.75	85.14	85.07	...	...	0.5384	....
2	206	31.57	105.33	105.55	...	...	0.5124	....
3	306	31.32	141.23	142.08	...	...	0.4643	....
4	82	32.37	48.22	47.72	...	...	0.5820	....
5	108	32.18	60.21	59.76	...	...	0.5533	....
6	131	32.10	71.77	71.32	...	...	0.5444	....
7	156	32.06	83.62	83.15	...	...	0.5330	....
8	162	32.04	84.60	84.15	...	...	0.5194	....
9	161	31.97	86.14	85.78	...	...	0.5328	....
10	188	31.88	98.12	97.84	97.56	97.28	0.5204	0.5174
11	181	31.87	95.02	94.76	93.68	93.43	0.5235	0.5162
12	185	31.89	95.29	95.00	95.32	95.04	0.5135	0.5137
13	210	31.91	107.29	106.94	106.67	106.32	0.5092	0.5063
14	213	31.95	107.07	106.65	108.39	107.97	0.5007	0.5069
15	213	32.05	107.11	106.52	106.15	105.57	0.5000	0.4956
16	231	32.00	116.50	115.95	...	...	0.5019	....
17	229	31.51	114.85	115.19	113.50	113.84	0.5030	0.4971
18	234	31.53	114.54	114.85	116.87	117.19	0.4908	0.5008
19	252	31.38	122.12	122.74	120.85	121.47	0.4871	0.4820
20	256	31.40	123.56	124.15	124.90	125.50	0.4850	0.4902
21	259	31.43	125.05	125.59	125.51	126.05	0.4849	0.4867
22	294	31.41	136.78	137.41	137.29	137.92	0.4674	0.4691
23	291	31.38	139.74	140.45	137.30	138.00	0.4826	0.4742
24	296	31.38	138.66	139.37	138.67	139.38	0.4708	0.4709
25	337	31.38	150.49	151.26	151.58	152.35	0.4488	0.4521
26	343	31.39	151 95	152.71	149.54	150.84	0.4452	0.4398
27	335	31.41	147.66	148.34	148.56	149.24	0.4428	0.4455
28	390	31.17	162.02	163.40	...	...	0.4190	....

1850 mm (72.83 in.), 1750 mm (68.90 in.), and 1650 mm (64.96 in.).

Fig. 7 shows schematically the layout of the plant from the lower end of the canal to the powerhouse, together with the profile of unit 4 penstock.

A complete turbine-efficiency test of the unit was made in January, 1926. Fourteen runs, measuring the discharge with the salt-velocity method were made. As shown in Fig. 7, the measurements of discharge were made between two sets of electrodes of the customary bowed type. The salt-injection station consisted of 4 pop valves set in from the pipe wall 39 per cent of the pipe radius and was generally similar to the Serra layout in Fig. 1.

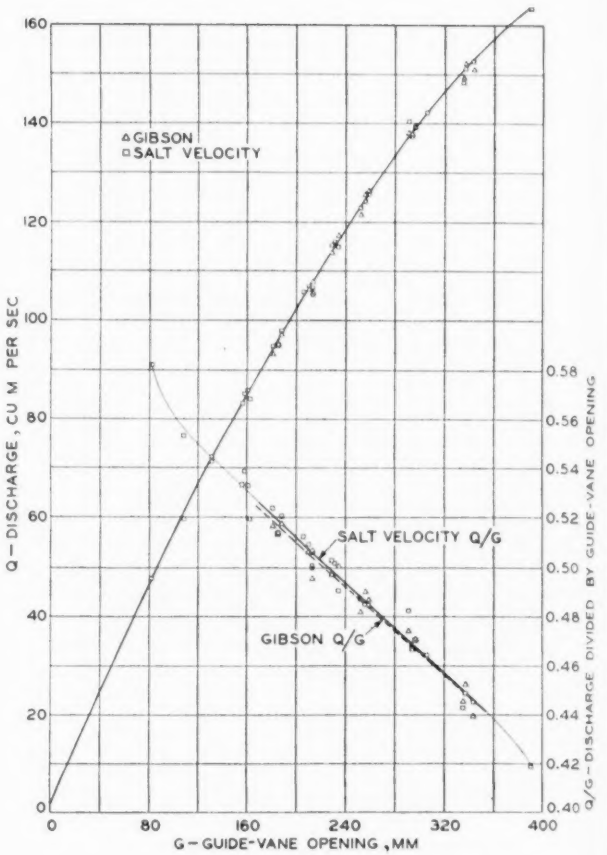


FIG. 6 TEST RESULTS; PARAHYBA UNIT 4

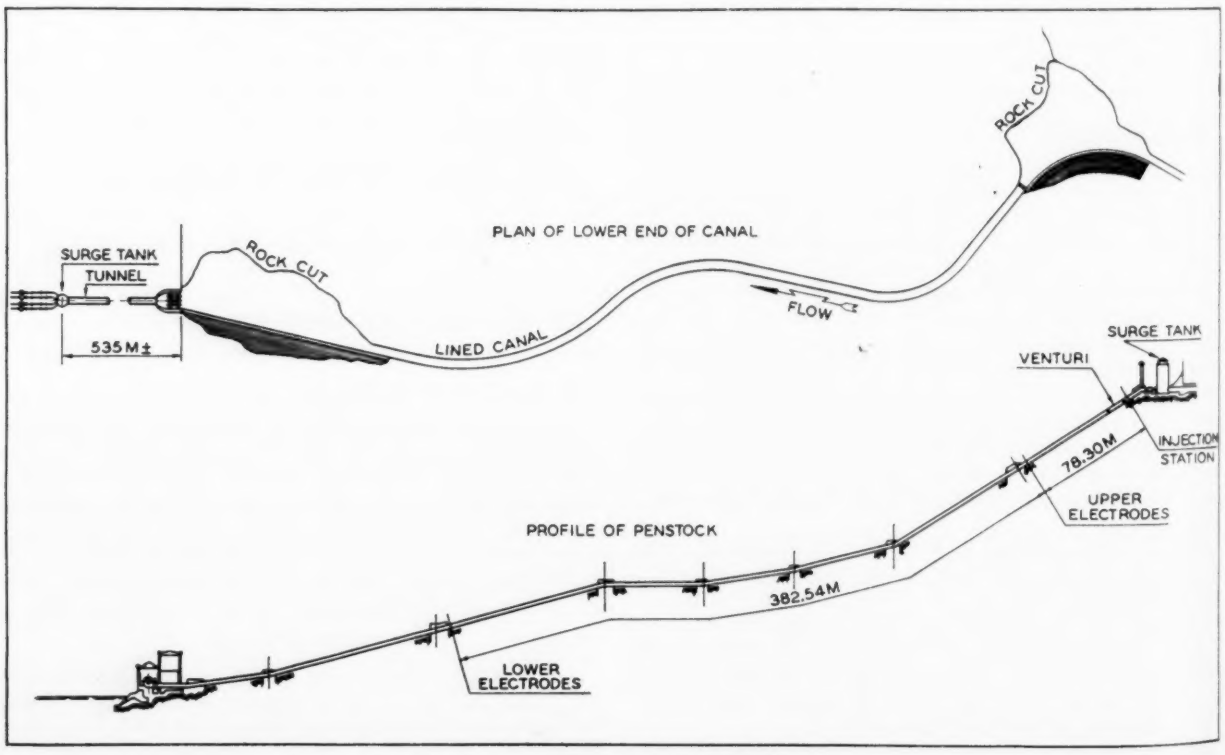


FIG. 7 TESTING LAYOUT; SOROCABA UNIT 4

The results of the test appeared entirely satisfactory, check points at identical gate openings showing a maximum difference of 0.2 per cent. However, the venturi meter, having full and throat diameters of 1850 and 1269 mm, respectively, showed a coefficient about 4.5 per cent lower than the usually expected value of 0.985.

As these tests were made in the early days of the companies' experience with the salt-velocity method, a volumetric check of the discharge was made as this was possible with the plant layout.

The volume of the canal forebay, and surge tank between three elevations, was determined by measurement of the areas. With all inflow shut off except a very small but measured inflow from a spring and no other outflow, this known volume was discharged through penstock 4 by bringing the unit from complete shutdown to a predetermined gate setting in about 2 min, holding this load constant for as long a period as the available water permitted, and shutting down the unit completely in about $1^1/_2$ min. The rate of discharge during the steady-load portion of the run was determined by deducting the percentage of water used during starting and stopping, determined on the basis of the previous constant-load tests from gate openings observed every 10 sec during changes of load. The water used in starting and stopping amounted to 16 per cent of the total on one test and 7 per cent on the other two tests. During the steady-load portion of each run, salt-velocity measurements were made.

TABLE 5 VOLUMETRIC CHECK OF SALT VELOCITY; SOROCABA UNIT 4

Item	Test 1	Test 2	Test 3
Initial level, m	804.688	804.584	804.638
Final level, m	803.858	801.645	801.535
Average area above 803.57, sq m	6130	5640	5700
Draft above 803.57, m	0.830	1.014	1.068
Average area below 803.57, sq m	....	3950	3940
Draft below 803.57, m	....	1.925	2.035
Volume, cu m	5088	13323	14106
Inflow from spring, cu m	20	27	22
Total water used on test, cu m	5108	13350	14128
Water used on steady load, per cent	84.15	93.20	92.70
Water used on steady load, cu m	4298	12442	13097
Time of steady load, sec	720	2100	1700
Average discharge—volumetric, cu m per sec	5.969	5.925	7.704
Average time salt velocity, sec	164.9	167.0	127.9
Average discharge—salt velocity, cu m per sec	5.967	5.892	7.694
Difference salt velocity—volumetric, per cent	—0.03	—0.56	—0.13

Table 5 presents the summary of these tests. It will be seen that the discharges determined volumetrically and from the salt-velocity method agree within 0.25 per cent average for the three tests. It is considered that the penstock volume for the salt-velocity and the canal volume for the volumetric check as determined had each a precision of about ±0.3 per cent. The test agreement is therefore as good as can be expected. At that time no greater refinement in testing results was attempted.

General Comment on Tests

From the period of obtaining the rights to use the salt-velocity and Gibson methods to date, all units of the principal plants of the companies associated under the Brazilian Traction, Light and Power Company, Ltd., have been tested by one or the other of these methods, with the exception of three units which are duplicates of machines tested. In some cases, the tests have been made indirectly by using the Gibson or salt-velocity method to calibrate a venturi restriction, using this calibrated restriction as a measuring device for several units. Eleven units have been directly tested with each method, a total of 16 tests with the salt-velocity and 15 tests with the Gibson method having been made. All of these tests have been satisfactory and, where doubts have occasionally been expressed, repeat tests and checks by other methods have confirmed the accuracy of the tests.

From the experience gained on these tests, the author suggests the following, but by no means comprehensive, precautions regarding the use of the two methods:

Salt-Velocity Method

1 If best results are desired, the timing of the passage of the salt cloud must be done between two sets of electrodes which represent the entire cross section of the waterway.

2 An adequate mixing length between the injection valves for the brine and the first set of electrodes is imperative for best results. The author does not feel competent to place a definite limit on this length, but suggests a lower limit of a length at least equal to the mean transverse dimension of the waterway and preferably at least twice this length. It appears also that the number of pop valves used should depend on this length rather than on the cross-sectional area of the waterway.

Gibson Method

1 The author has found that measuring the leakage with fully closed guide vanes may, at times, be the most troublesome feature of this method. In two cases it appeared conclusively that the leakage through the guide vanes did not vary as the square root of the head on the turbine, thus making it necessary to measure the leakage with the full head. No explanation of the discrepancies has been found, but they may be due, at least in part, to the variation in mechanical deflection of the guide vanes with varying applied pressure.

2 Besides the necessity of absolutely tight piezometer piping, the author considers it necessary to provide adequate provision for flushing the piping between tests to remove air and to maintain the penstock water temperatures if best results are to be obtained.

Acknowledgments

The author wishes to make acknowledgment to A. W. K. Billings, member of this Society, and vice-president of the Brazilian Traction, Light and Power Company, Ltd., for permission to use the companies' data, to C. P. Conrad and L. J. Hooper, also a member of this Society, who made the Sorocaba tests and to A. Santos, Jr., who made some of the many tests carried out by the companies and assisted in the performance of the others.

Discussion

M. A. Mason.[4] The writer had the opportunity, while studying in France as a Freeman scholar, to make an investigation of the possibilities of use of the salt-velocity method in open channels. It was found that previous studies had not resulted in the formulation of any logical conception of the physics of the method which would permit a definition of its limitations. An attempt was made, therefore, to define the limits of applicability of the method by a study of the basic principles governing its operation. The conclusion was reached that the chief requirement for accurate results is the attainment of proper mixing between the water and injected solution before the injected solution reaches the measuring section.[5] It is gratifying that the author arrives at an identical conclusion for the case of flow in pipes.

Concerning the magnitude of the "mixing length" required for accurate results, it was concluded from the writer's studies that this length was probably a function of the velocity of flow, the injection pattern, the form of the channel, the nature of the walls, and some time period. The author presents no evidence to

[4] Engineer, Beach Erosion Board, War Department, Washington, D. C. Jun. A.S.M.E.

[5] "Contribution à l'Étude de la mesure des Débits d'Eau par la Méthode Allen," by M. A. Mason, Thesis, University of Grenoble, France; also *Révue Générale de l'Hydraulique*, No. 28, July-Aug., 1939.

justify the elimination of any of these factors and his conclusion that the "good mixing length" may be specified in terms of only a linear dimension of the conduit system does not seem to be justified.

Professor Hooper, in his paper on salt-velocity measurements of low-velocity flow[6] employs mean velocity of flow as a measure of the mixing of injected solution and water. It is probable that both criteria suggested are involved in the true criterion for good mixing and that neither is sufficient in itself.

Although no concrete evidence of the effect of electrode design is presented in this paper, the writer concurs in the first conclusion to the effect that the electrodes must suitably cover the entire cross section of the waterway. An analysis leading to this conclusion was included in the writer's paper.[6]

E. B. Strowger.[7] The author indicates that he has obtained a satisfactory agreement in the results of tests of two units by the salt-velocity method with those of the differential application of the Gibson method. The writer wishes to submit the results of other tests which show the consistent accuracy of the Gibson method.

Perhaps it should be pointed out that the Gibson method makes use of pressure-time diagrams of either the simple or the differential type. With simple diagrams, the changes of pressure at one piezometer section in the conduit are recorded while with differential diagrams the difference between the changes of pressure at two piezometer sections in the conduit are recorded. Indirect checks of the Gibson method can be made, therefore, when both simple and differential diagrams are used; in tests where two sets of simple diagrams are made, each utilizing a different length of penstock; in tests where two differential applications are made, each utilizing different lengths of pipe; and/or when tests are made using other comparisons.

Some of the material which will be presented herein has been published before but is being included for the purpose of assembling as much of this information in one place as is possible at this time.

Fig. 8 of this discussion shows the cross section of a 70,000-hp hydroelectric unit operating under a head of 217.5 ft. It also shows the piezometer sections used in testing this unit by the Gibson method. Simple diagrams were made on the unit, utilizing approximately 600 ft of penstock, extending from the forebay to the lower piezometer section and differential diagrams were made utilizing about 87 ft of the penstock as shown. The agreement between the two tests is indicated by the two sets of test points which fall on or close to the same power-discharge curve as shown in Fig. 9. The average divergence of the "differential points" from the line established by the "simple points" is within +0.2 per cent and the agreement between the two tests is accordingly within 0.2 per cent.

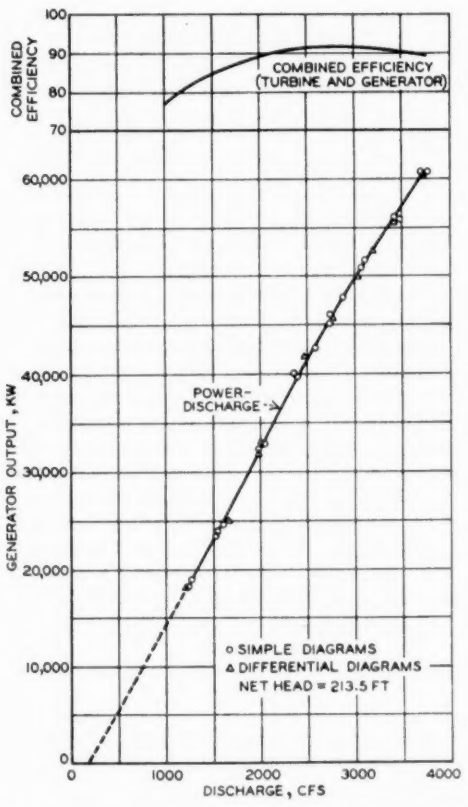


Fig. 9 Power-Discharge Curve Obtained by Gibson Test on Unit Shown in Fig. 8, With Test Points of Both Simple and Differential Diagrams

Fig. 10 shows the plan and cross section of the penstock of a 14,000-hp unit operating under a head of 96 ft. On this unit two tests were made by the Gibson method, one using simple dia-

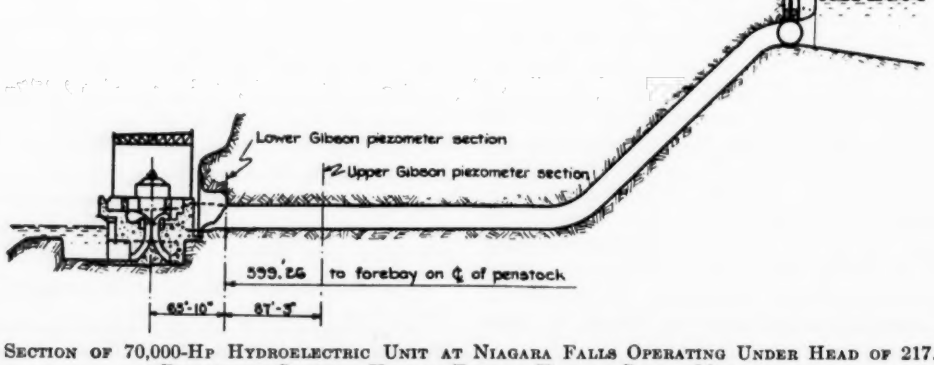


Fig. 8 Cross Section of 70,000-Hp Hydroelectric Unit at Niagara Falls Operating Under Head of 217.5 Ft, Showing Piezometer Sections Used in Testing Unit by Gibson Method

[6] "Salt-Velocity Measurements at Low Velocities in Pipes," by L. J. Hooper, published on page 651 of this issue of the Transactions.

[7] Hydraulic Engineer, The Niagara Falls Power Company, Buffalo, N. Y. Mem. A.S.M.E.

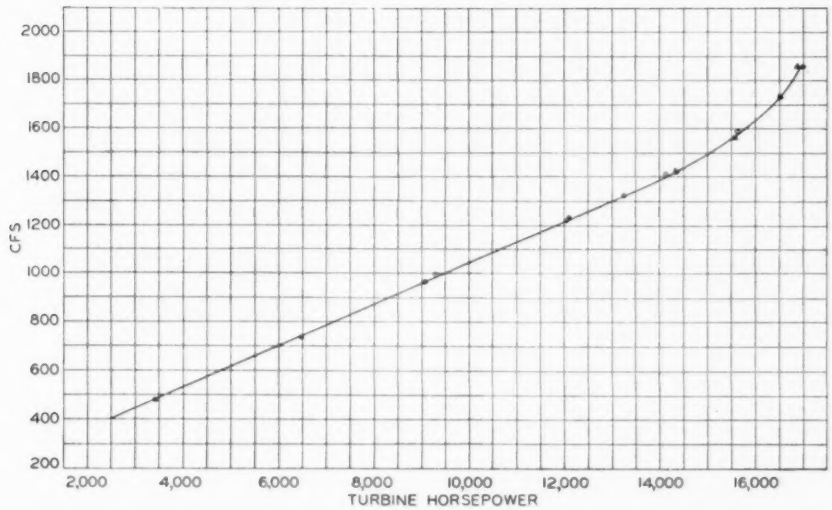


FIG. 11 POWER-DISCHARGE CURVE FROM GIBSON TEST ON 14,000 HP UNIT (PENSTOCK SHOWN IN FIG. 10)
(⊙ Simple diagram. △ Differential diagram.)

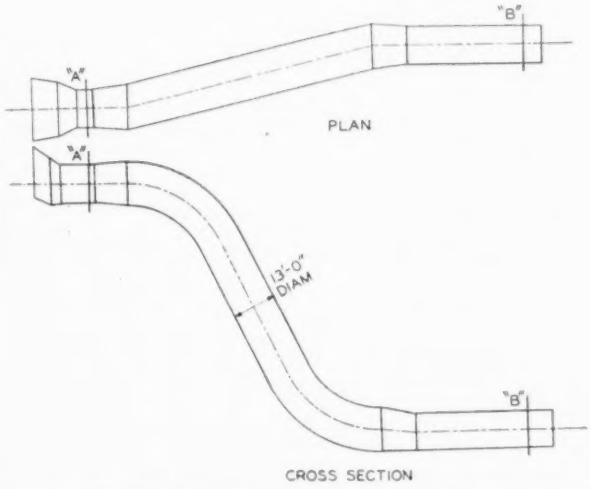


FIG. 10 PLAN AND CROSS SECTION OF PENSTOCK OF 14,000 UNIT, SHOWING PIEZOMETER SECTIONS FOR GIBSON METHOD OF TESTING

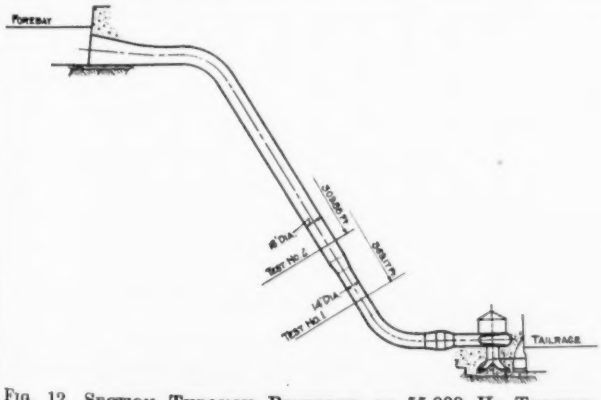


FIG. 12 SECTION THROUGH PENSTOCK OF 55,000 HP TURBINE, SHOWING LOCATION OF PIEZOMETERS FOR GIBSON APPARATUS

grams and the other using differential diagrams. For the simple diagrams, all of the pipe was utilized extending from the forebay to piezometer section *B* and, for the differential diagrams, the section between piezometer sections *A* and *B* was utilized. The

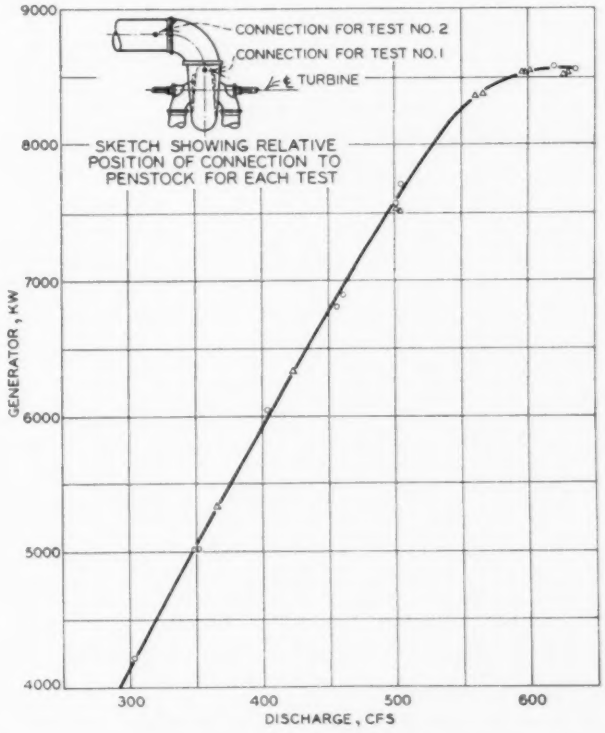


FIG. 13 POWER DISCHARGE CURVE FOR 10,000-HP UNIT; CROSS HEAD 212 FT

(Test No. 1, Oct. 4, 1921, represented by points marked ⊙ made with Gibson apparatus No. 5 attached just below 90-deg elbow. Test No. 2, Dec. 11, 1921, represented by points marked △ made with Gibson apparatus No. 1 attached upstream from 90-deg elbow. In test No. 1, $F = 4.010$, $R = 2.35$, and $K = 118.50$. In test No. 2, $F = 3.672$, $R = 1.528$, $K = 89.35$.)

two sets of test points are plotted in Fig. 11, and determine one curve as shown.

In Fig. 13 is shown the power-discharge curve of a 10,000-hp unit operating under a 212-ft head as determined by two sets of test points, each made with the simple diagram application of the Gibson method but with different piezometer locations. One piezometer section was located upstream from a 9-ft elbow and the other just downstream from the elbow. The two sets

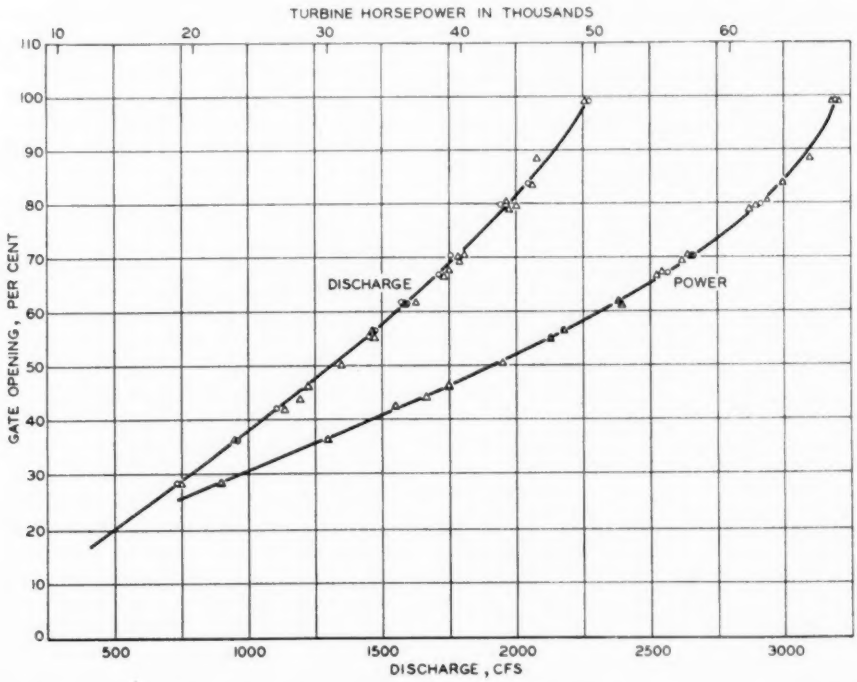


FIG. 14 GATE OPENING DISCHARGE AND GATE OPENING POWER CURVES FOR 55,000 HP TURBINE; NET HEAD 312 FT; BY GIBSON METHOD, DEC. 10, 1922
(In test No. 1 ⊙, $F = 1.742$, $R = 1.802$, $K = 99.45$. In test No. 2 △, $F = 1.385$, $R = 1.515$, $K = 88.98$.)

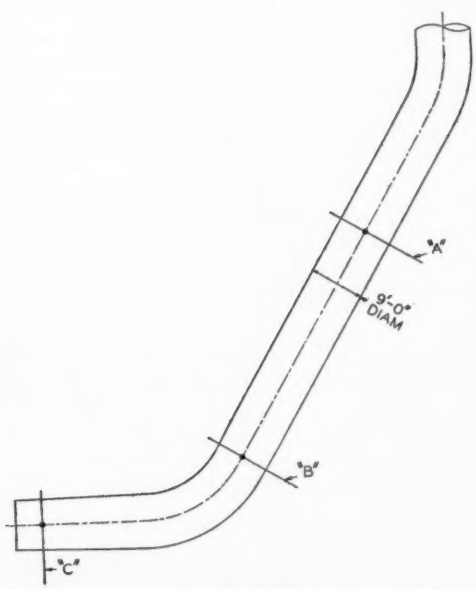


FIG. 15 PIEZOMETER SECTIONS FOR TWO SETS OF DIFFERENTIAL DIAGRAMS, GIBSON METHOD OF TESTING, 10,000-HP UNIT
(Length A to B, 42.10 ft. Length A to C, 79.54 ft.)

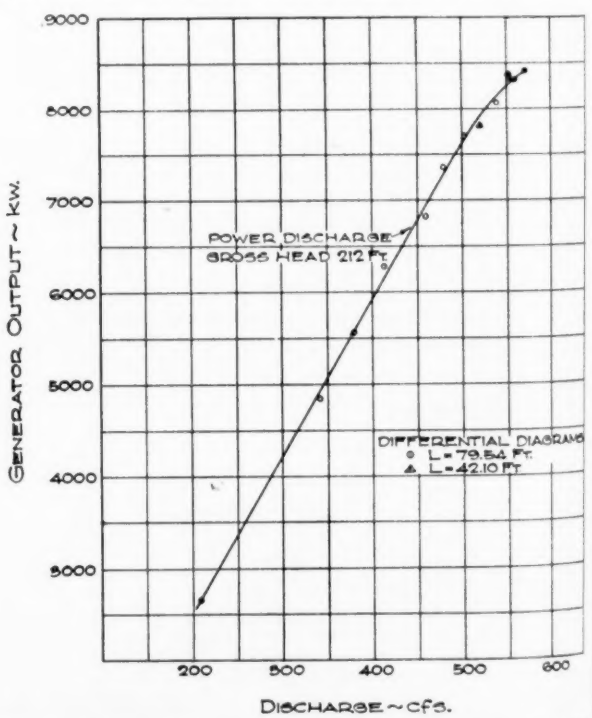


FIG. 16 POWER-DISCHARGE CURVE FROM GIBSON TEST ON 10,000 HP UNIT (PENSTOCK SHOWN IN FIG. 15)

of test points indicate that a common power-discharge curve may readily be drawn.

Another example of two sets of simple diagrams taken on the same penstock is illustrated by the test on a 55,000-hp unit operating under a head of 312 ft, as shown by Figs. 12 and 14. With one set of diagrams approximately 310 ft of penstock was utilized and with the other set approximately 367 ft was used. The diagrams were made simultaneously, i.e., at one closure of the turbine gates, two diagrams were made. The two sets of test points shown by triangles in the one case and circles in the other were determined by two independent testing organizations using different apparatus and working independently.

The penstock shown in Fig. 15 was used for making two sets of differential diagrams, one set using 42.10 ft of length and the

TABLE 6 COMPARISON OF RESULTS OF TESTS MADE IN EUROPE BY GIBSON METHOD WITH THOSE OF OTHER METHODS

No.	Date	Place	Conducted by	Results
1	October, 1928	Heidenheim, Germany	J. M. Voith	Mean variation from weir measurement, + 1.9 per cent
2	July, 1929	Walchensee, Germany	Paul Volkhardt	Mean variation from current meter measurement, —0.3 per cent
3	August, 1929	Eitting, Germany	Paul Volkhardt	Mean variation from current meter measurement, +0.4 per cent
4		Royal School of Engineers, Padova, Italy	Ettore Scimemi	Mean variation from volumetric measurement, —0.2 per cent
5	1930	Technischen Hochschule, München, Germany	Hans Deckel	Mean variation from volumetric measurement, —0.3 per cent
6	1930	Brünnenmuhle in Heidenheim	Hans Deckel	Mean variation from weir measurement, —0.4 per cent

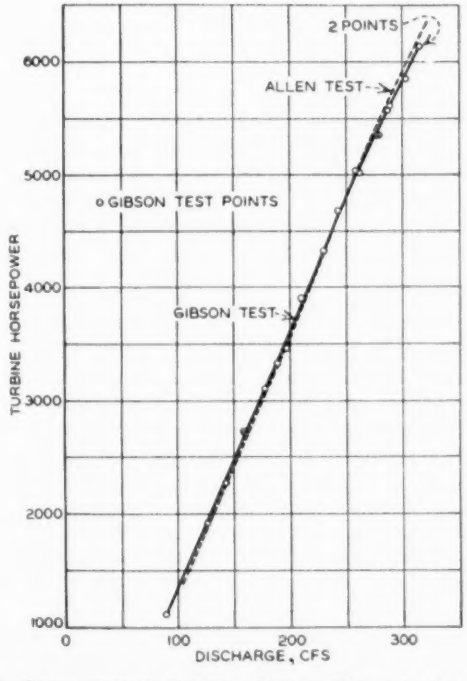


FIG. 17 COMPARISON OF RESULTS, GIBSON TEST AND ALLEN TEST ON 59,000-HP UNIT (UNIT NO. 1) NET HEAD = 196 FT

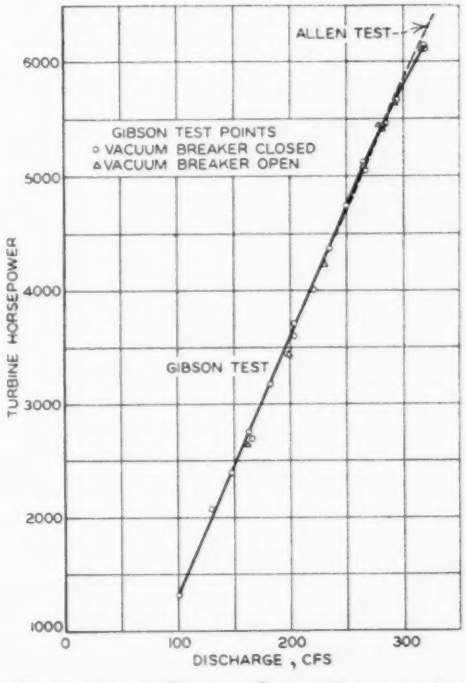


FIG. 18 COMPARISON OF RESULTS, GIBSON TEST AND ALLEN TEST ON 59,000-HP UNIT (UNIT NO. 2) NET HEAD = 196 FT

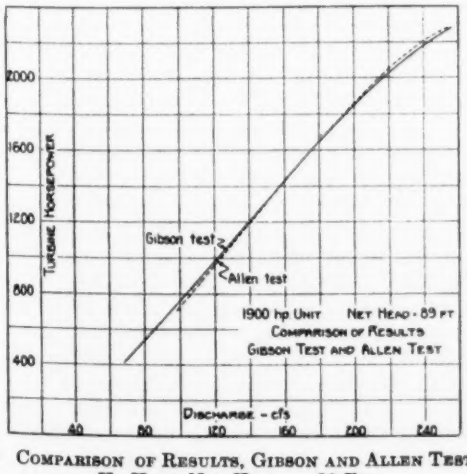


FIG. 19 COMPARISON OF RESULTS, GIBSON AND ALLEN TESTS, 1900 HP UNIT, NET HEAD = 89 FT

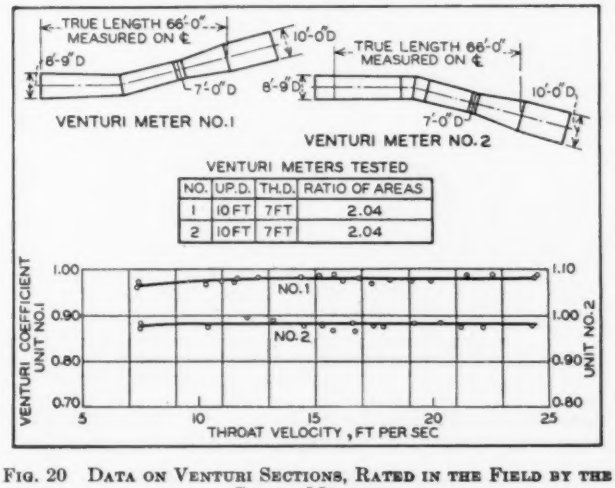


FIG. 20 DATA ON VENTURI SECTIONS, RATED IN THE FIELD BY THE GIBSON METHOD

other set a length of 79.54 ft. The power-discharge curve with the two sets of points is shown in Fig. 16.

Figs. 17, 18, and 19 show the results of testing three units by both the Allen and Gibson methods. The close agreement between the two measurements in each case is apparent.

Fig. 20 shows the coefficient curves of two 7 × 10-ft venturi meters built in the two penstocks of a hydroelectric station and tested by the Gibson method when the acceptance tests of the units were made. It shows the test points plotted as velocity against venturi coefficient. Figs. 21 and 22 show the test points plotted as discharge against manometer deflection on log paper indicating that, with this plotting, the relation between deflection and discharge is a straight line. The slope of the line is 2 and accordingly the discharge varies as the square root of the mercury deflection. This means that the coefficient for these venturis is constant for the range of velocity covered by the test.

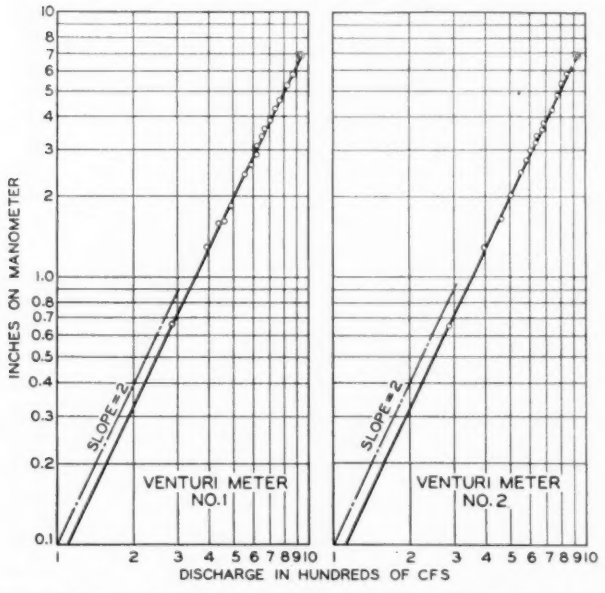


FIG. 21 (LEFT) RATING CURVE MADE BY GIBSON METHOD OF A 120 × 84-IN. VENTURI (METER NO. 1)

FIG. 22 (RIGHT) RATING CURVE MADE BY GIBSON METHOD OF A 120 × 84-IN. VENTURI (METER NO. 2)

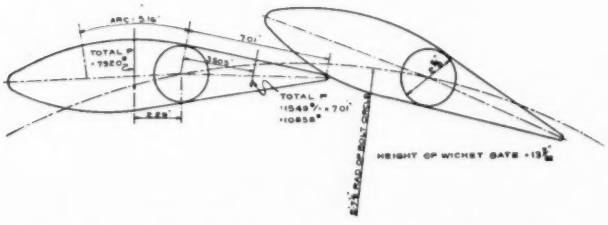


FIG. 23 COMPUTATION OF ANGULAR DEFLECTION OF TURBINE GATE STEM FOR LEAKAGE DETERMINATION

Table 6 of this discussion presents a list of comparative test results using the Gibson method in conjunction with other methods of water measurement as carried out at various places in Europe. The results of these comparisons have been published from time to time in the technical press. With the exception of the first tests at Heidenheim, which were not very good, all these results are within the practical limits of precision in the measurement of flowing water.

The author states he has found at times that measuring the leakage of the turbine with closed guide vanes was the most troublesome feature of the Gibson method and mentions two cases where the leakage did not vary as the square root of the head on the turbine. He suggests that the discrepancy, in part at least, was due to the mechanical deflection of the guide vanes with varying pressure. Since in many cases the turbine gate leakage must be determined under heads which are lower than the normal value and the experimental result must be stepped up to the full head, care must be exercised to see that proper adjustment of the orifice area is made in case deflection of the guide vane stems takes place. If this is done, the writer believes that the author will not consider the measurement as troublesome. Gate-leakage determinations usually can be made at several values of reduced head, especially if the fall in the pressure method of measuring the leakage is used, and the results may be plotted in the form of a leakage-head curve on log paper. If the orifice area changes with head, the relation so obtained is a curve rather than a straight line and may be extrapolated to the full head. The area correction is made in the process of extrapolating the curve.

Another method of determining whether an area correction is needed in stepping up the leakage to the full head is to compare the "squeeze" deflection of the wicket gates under full operating oil pressure with the calculated deflection of the wickets under full head conditions. If the squeeze deflection is greater than the deflection due to casing pressure, no area correction is necessary. An example of such a computation is shown in Fig. 23 for one of the units in a hydroelectric plant in New York State. At this plant the servometer piston travel is limited by a stop which is so placed that the wicket gates are closed with no squeeze when the wheel case is empty. The computed deflection of the guide vanes with full casing pressure was 0.093 in. resulting in an orifice area of 0.233 sq ft and a computed leakage of 28.6 cfs. The actual measured leakage per unit under full head as determined by a tailrace weir was 29.5 cfs. Other cases of leakage determination where the guide vane deflection is a factor in the determination could be cited, although with the method used in assembling and adjusting the gate mechanism of modern hydroelectric units, such deflection is seldom found to be appreciable.

It should be noted also that, as a practical matter, extreme care in measuring the leakage is not as a rule necessary. A relatively large error in the determination of the turbine-gate-leakage quantity affects the turbine efficiency by an insignificant amount. For example, on a recent test, a possible error in the leakage quantity of as much as 20 per cent of the total leakage represented less than 0.1 per cent in turbine efficiency at the point of maximum efficiency.

Unit Pressure, $P = 270 \times 0.434 \times 13.219 = 1549$ lb per in. of gate length

Moments around center line of stem.

Opening M in-lb $= 10{,}858.5$ lb $\times 3.505$ in. $= 38{,}059$

Closing M in-lb $= 7{,}920$ lb $\times 2.29$ in. $= 18{,}058$

Net M (opening) in-lb $= 20{,}001$

α = Angular deflection of stem (radians) $= \dfrac{l\,M}{E_s J}$

l = length of stem = 21.625 in.

M = moment

E_s = modulus of elasticity = 12,000,000

J = polar moment of inertia $= \dfrac{\pi d^4}{32}$

$$\alpha = \frac{21.625 \times 20{,}001}{12{,}000{,}000 \times 4.66} = 0.007735 \text{ radians}$$

Toe of gate moves, $7.01 \times 0.007735 = 0.05422$ in.

Heel of gate moves, $5.06 \times 0.007735 = 0.03914$ in.

Total opening $= 0.09336$ in.

Orifice opening top and bottom of wickets $= \dfrac{0.070}{12} \times 3.1416 \times 5.25 = 0.096$ sq ft

Orifice opening between wickets $= 16 \times 1.104 \times \dfrac{0.09336}{12} = 0.1374$ sq ft

Total orifice opening $= 0.096 + 0.1374 = 0.2334$ sq ft

Gate leakage under 270 ft head $= 0.2334 \times 8.02 \times \sqrt{270} \times 0.93 = 28.6$ cfs

Leakage per unit by tailrace weir = 29.5 cfs

EDWIN A. TAYLOR.[8] The results of three check tests at Sorocaba show that the salt velocity varied from volumetric by 0.03, 0.56, and 0.13 per cent, respectively. These results have a maximum spread for any one run of 0.5 per cent and an average difference between the two methods of 0.25 per cent.

The results of two check tests at Parahyba show that, for the wide range of the tests, salt-velocity and Gibson methods check within less than 0.1 per cent. For each unit, the two curves of discharge on gate opening coincide at full gate and differ only slightly at half gate.

[8] Consulting Engineer, Worcester, Mass.

A check test at Serra No. 5, comparing discharge results, using three different salt-velocity test sections, shows a maximum spread of less than 1 per cent, 0.5 per cent plus and minus. For the whole test, the average difference was less than 0.1 per cent. These comparative tests covered 24 runs and 130 salt-velocity shots. Similar check tests, with equally close comparative results, were made at Serra Nos. 3 and 7.

The writer has made a number of pump- and turbine-efficiency tests, using the salt-velocity method of water measurement. Several times the results by salt velocity have been checked by other engineers, using other methods of water measurement,

TABLE 7 CHECK OF SALT-VELOCITY TEST RESULTS BY VARIOUS METHODS

No.	Location	Date	Salt velocity Runs	Salt velocity Shots	Check method[a]	Average spread for all runs,[b] per cent	Mean difference between methods,[c] per cent
1	Sprite Creek, New York	1924	11	106	Pitometer	0.94	+0.06
2	Walchensee, Germany	1929	4	30	Pitometer	0.77	—0.13
3	Shawinigan, No. 6, Quebec	1922	26	267	Gibson	...[d]	—0.60
4	Moshier, N. Y.	1929	29	140	Gibson	...[d]	—0.10[e]
5	Walchensee, Germany	1929	3	24	Current meter	0.15	—0.13
6	Holtwood, Pa.	1930	23	197	Current meter	...[d]	±0.10[f]
7	Tiger Creek, Calif.	1931	2	24	Volumetric	0.72	+0.45
8	Iron Mountain, Calif.	1939	3	150	Volumetric	0.73	+0.73
						Weighted average	+0.05

[a] When computing differences in results, the check method is assumed to be 100 per cent correct.
[b] Not weighted for signs, plus or minus.
[c] Weighted for signs, plus or minus.
[d] Results of individual runs not available.
[e] Curves showing comparative values for two units appear in a paper[9] by N. R. Gibson and E. B. Strowger.
[f] "The average discrepancy was less than one tenth of one per cent."[10]
NOTE: When the word "test" is used in this discussion, it refers to the whole series of test runs made at any one power plant or pumping station.
When the word "run" is used, it refers to the series of shots and readings taken at any one gate opening or rpm and while all other conditions remain the same.
When the word "shot" is used, it refers to a single salt injection into the pipe or penstock.

[9] "Experimental and Practical Experience With the Gibson Method of Water Measurement," by N. R. Gibson and E. B. Strowger, Trans. A.S.M.E., vol. 57, 1935, p. 224.
[10] "Water Gaging for Low-Head Units of High Capacity," by J. M. Mousson, Trans. A.S.M.E., vol. 57, 1935, pp. 303–316.

TABLE 8 INTERCHECKING WITH SALT VELOCITY

No.	Location	Date	Salt velocity Runs	Salt velocity Shots	Test sections Standard,[a] ft	Test sections Check, ft	Average spread for all runs,[b] per cent	Mean difference between sections,[c] per cent
9	Shawinigan, No. 6, Quebec	1922	6	23	500	various	0.48	—0.02
10	Searsburg, Vt.	1923	4	26	400	250	0.81	−0.18
10A	Searsburg, Vt.	1923	1	2	400	18,600	0.10	0.00
11	Davis Bridge, Vt.	1924	4	66	600	300	0.09	+0.09
11A	Davis Bridge, Vt.	1924	2	58	600	12,900	0.25	—0.25
12	Tepuxtepec, Mexico	1931	15	224	750	300	0.23	0.00
						Average	0.33	—0.06

[a, b, c] These references apply in Table 7.

Test no.	Description of test sections	
9	Standard test section:	500 ft concrete pipe, 20 ft diam
	Check section:	five different channels varying in length from 70 to 140 ft, all concrete
10	Standard test section:	400 ft steel pipe, 6 ft 6 in. diam
	Check section:	250 ft wood-stave pipe, 8 ft diam
10A	Standard test section:	400 ft steel pipe, 6 ft 6 in. diam
	Check section:	18,600 ft wood-stave pipe, 8 ft diam
11	Standard test section:	600 ft steel pipe, 9 ft diam
	Check section:	300 ft steel pipe, 9 ft diam
11A	Standard test section:	600 ft steel pipe, 9 ft diam
	Check section:	12,900 ft concrete tunnel, with horseshoe cross section, 14.5 ft diam
12	Standard test section:	750 ft steel pipe, 7 ft 2 in. diam
	Check section:	300 ft steel pipe, 7 ft 2 in. diam

and frequently the salt-velocity results have been checked by the same method, using different test apparatus and test sections at the same plant.

Tables 7 and 8 show the results of fourteen such check tests on salt velocity, two by current meter, two by pitometer, two by Gibson, two by volumetric, and six by interchecking with various salt-velocity test sections.

Details of tests, apparatus, and procedure for the check tests in Tables 7 and 8 are omitted. The engineers conducting these tests were experienced in their lines and the tests were made in accordance with the latest accepted practice in each method.

A summary of the comparative results, shown in Table 7 of this discussion, indicates a close agreement between salt velocity and current meter, once in Germany and once in Pennsylvania. A comparison of the discharge values by pitometer in Germany and in New York and by Gibson in Quebec and in New York shows a close agreement with the salt-velocity values for discharge at the same plants.

A comparison of salt velocity with the five volumetric check runs in Table 7 shows the results have an average spread for all runs of 0.75 per cent and an average weighted difference for the five runs of 0.6 per cent.

While the values for average difference for the Pacific Coast volumetric checks are plus 0.6 per cent, the Brazilian volumetric checks show an average difference of minus 0.25 per cent.

The average mean difference, weighted for signs, for the eight check tests in Table 7 is +0.05 per cent.

In analyzing and comparing the results in Table 7, the spread and the deviation for runs should be given more weight than the average difference in values for the whole test. This average difference may mean very little alone. For instance, there could be a spread or run deviation in discharge values of 5 or even 10 per cent and the mean difference between methods for the whole test could still be zero. In that case, attaching much importance to the mean difference for the whole test becomes very significant of the accuracy of the method. This is important in field tests for pump and turbine efficiency because, in computing final results, curves are drawn to fit the test points and these curves do graphically what the average differences in Table 7 do arithmetically.

Table 8 of this discussion shows the results of six check tests at four different power plants, using various salt-velocity test sections. These test sections varied in length from 300 ft to 3.5 miles. The average spread for all runs shown in Table 8 was 0.33 per cent. The average mean difference, weighted for signs, for the six check tests shown in Table 8 is minus 0.06 per cent.

The mean difference for all the check tests shown in both Tables 7 and 8, which involve over 100 salt-velocity runs and 500 shots, is zero.

Author's Closure

Messrs. Strowger and Taylor have presented additional data showing verifications of the salt-velocity and Gibson methods. The assembly of such data is valuable in appraising the degree of accuracy which may be expected by these methods. Further testing results, where the agreement was less satisfactory, would also be valuable. Undoubtedly, attempts to extend the field of these methods have been responsible for the majority of tests where agreement was relatively poor. A record of these attempts, and of the experience gained in overcoming difficulties, is desirable.

To present one such case the following is submitted: Prior to the tests of Parahyba unit 4 described in the paper, preliminary tests were made on this unit with both salt-velocity and Gibson methods. The electrode setup for the salt velocity was the same as in the later tests. The injection station consisted, however, of only 12 pop valves placed in the vertical plane of the stop-log grooves. The form of the resulting curves was decidedly poor, each curve showing 3 peaks resulting from the unequal paths from the 3 horizontal rows of 4 pop valves each. The results differed from the final results given in the paper by an amount varying from 0 per cent at 180 mm guide-vane opening to +1.6 per cent at 345-mm guide-vane opening. The use of the salt-velocity method in this manner was an obvious but unwitting abuse. Notwithstanding, there is comfort in the relative closeness of agreement with the later tests made with an improved layout. The preliminary Gibson tests agree very closely with the final ones reported in the paper.

The suggestions given in the paper regarding mixing length resulted principally from thought on conditions of testing at the Parahyba plant. Mr. Mason, in his study[5] and in his discussion of this paper, and Professor Hooper, in his paper[6] have contributed further to this question. The determination of conditions providing a satisfactory mixing and distribution of the salt solution in the water channel between the injection valves and the upper electrodes would be a suitable laboratory study. It is apparent, however, that a requirement relating the number of injection valves to the cross-sectional area of the waterway is entirely inadequate without further stipulations. These probably should include factors for the channel dimensions, form, roughness, and possibly for the velocities expected. For ordinary commercial testing in hydroelectric plants the low velocity limits brought out by Professor Hooper are rarely, if ever, encountered.

The Viscosity of Superheated Steam

By G. A. HAWKINS,[1] H. L. SOLBERG,[2] and A. A. POTTER,[3] LAFAYETTE, IND.

After discussing the disagreement existing in viscosity data for superheated steam as reported by recent investigators, the authors report the results of a new determination of viscosity. A nickel capillary 103.2 ft long was used in this investigation. The differential head across the capillary was determined by radiographing the mercury levels in a steel-tube manometer. Data are reported for 60 calibration tests and for 61 tests on steam at pressures up to 1810 lb per sq in. abs and 1000 F. All of the tests were run under steady-flow conditions for a minimum period of 1 hr. Plank's equation has been modified to express the results over the entire range of pressures and temperatures covered by this investigation.

As a result of additional calibration tests made to determine the effect of curvature on the capillary constant, the results given in this paper supersede those presented at the Annual Meeting of the Society.

IN THE year 1925, Speyerer (1)[4] measured the viscosity of superheated steam by means of a capillary tube at pressures from 1 to 10 atm and at a maximum temperature of 658 F. In 1934, Schiller (2) published data on the viscosity of superheated steam as determined from the flow of steam through a nozzle. He obtained the relation for calculating the viscosity of steam in terms of that of water, by observing the velocity at which the discharge coefficient of a nozzle shows an abrupt increase for water and steam and then equating the two Reynolds numbers. Schiller's curves show one test point at 30 atm, four test points at 25 atm and other points at lower pressures with a maximum temperature of about 540 F. In 1935, the authors (3) published data on the viscosity of saturated and compressed water at temperatures up to the critical and on superheated steam at pressures up to 3500 lb per sq in. abs and temperatures up to 983 F. A modified form of the Lawaczeck (4) viscometer was used in which the time of fall of a metal cylinder in an accurately bored stainless-steel tube was recorded automatically. The results obtained by the authors (3) were in good agreement with those of Schiller and Speyerer.

In 1934, Schougayew (5) published data obtained by measuring the viscosity of steam in a Rankine capillary at pressures up to 93 atm and temperatures up to 400 C. He concluded that, within the limits of accuracy of his apparatus, viscosity was practically independent of pressure. In a discussion of the author's paper, Schougayew (6) presented a curve, showing the difference between the results for the viscosity of saturated steam, as plotted from the author's data, and similar results obtained in Russia by Schougayew and Sorokin.

In 1936, K. Sigwart (7) published data on the viscosity of water and steam as determined by a capillary tube. The results for the viscosity of water are in close agreement with those reported by the authors. However, the values for the viscosity of superheated steam differ widely from the results reported by the authors (3) and, in some cases, disagree by as much as 3 to 1.

A comparison of the results of several different determinations of the viscosity of steam is shown in Fig. 1. H. Speyerer (8) has recently published a discussion of the conflicting results which have been reported by the various investigators. The wide divergence in results has led to a careful examination of the work of Sigwart (7) as well as of the apparatus, the technique, and the results of the authors; also new determinations of the viscosity of superheated steam have been made and are reported in this paper.

Essentially, the method used by the authors in their earlier investigation was to measure the time of fall of a cylindrical body in an accurately bored cylinder containing a fluid of known viscosity and to measure the time of fall of the same body in the same cylinder containing water or steam under a known pressure and temperature. It is necessary that the flow of the fluid in the annular space between the falling body and the cylinder be in the viscous region. This flow was limited in the experiments to a maximum Reynolds number of 2100 based on the calibration data. The essential data required for a viscosity determination by this method are the density of the fluid under test, the density and viscosity of the calibration fluid, and the time of fall. It is not difficult to measure pressure and temperature and thus compute density with reasonable accuracy. The viscosity of the calibration fluids was taken from the International Critical Tables. The time of fall was recorded automatically on the strip chart of an Esterline-Angus recording milliammeter in a manner which eliminated the human element. It is believed that any errors introduced into the results were due not to errors in measuring pressure, temperature, or time, but to incorrect assumptions made in developing the theory of the Lawaczeck viscometer or due to failure to work with pure steam.

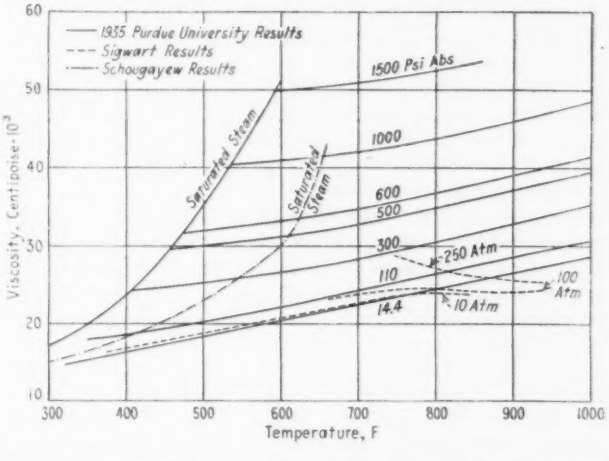


FIG. 1 VISCOSITY OF STEAM AS REPORTED BY RECENT INVESTIGATORS

In developing the theory of the Lawaczeck viscometer, the possibility of flow disturbances in the wake of the fall-body was not considered. Recent findings in the field of hydrodynamics

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[4] Numbers in parentheses refer to the Bibliography at the end of the paper.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

indicate that such flow disturbances may exist. Considerable effort has been expended in a theoretical analysis of the problem of tail turbulence and the magnitude of the error due to the neglect of this factor. Up to date, a satisfactory solution of the problem has not been worked out which would permit a separation of the resistance to falling into viscous drag in the annular space around the fall-body and turbulence at the tail of the fall-body. Also, it may be that, under certain conditions, the motion of the fall-body is not translatory but vibratory.

Sigwart (7) used the capillary-tube method of measuring the viscosity of steam and water. Two capillary tubes were used, each about 14 in. long. The quartz capillary had an internal diameter of 0.375 mm, while the internal diameter of the platinum capillary was 0.547 mm. Steam was generated in a small boiler, superheated, passed through the capillary, throttled, condensed, and weighed. Sigwart states that a brown precipitate formed in the condensate after exposure to the atmosphere for some time. The differential pressure across the capillary was measured by means of a differential-pressure ring balance (9). Sigwart reports only one short table of original steam data. In this table, the maximum and minimum heads, due to the flow of steam in the capillary, were 0.247 and 0.184 in. Hg, respectively. It is obvious that the accuracy and sensitivity of the balance must be of a very high order if dependable results are to be obtained from such small heads.

The differential head, measured by the ring balance, was reported by Sigwart as being due to the equivalent of (*a*) flow through the capillary of the fluid leaving the system, and (*b*) flow of fluid from one leg of the manometer through the capillary and connecting tubing to the other leg of the manometer. The connections from the manometer to the capillary were mounted in a horizontal plane. It is obvious that, when measuring the viscosity of superheated steam, these manometer legs contained superheated steam of varying temperature, density, and viscosity; saturated steam and water of varying temperature, density, and viscosity. Based on certain assumptions, which did not include the variable physical state of the fluid in the manometer legs or the length of tubing occupied by the fluid in these various states, Sigwart derived an equation from which he estimated the time elapsed from the opening of the throttle valve on the discharge side of the capillary until the pressure drop, as measured by his balance, equaled at least 99 per cent of the head loss in the capillary. He then took the manometer reading at this time interval, computed a correction, and used this value to calculate the viscosity. Thus his readings were taken before a steady flow state had been established.

Requirements of a Capillary for Measuring the Viscosity of Superheated Steam

In order to check the results of their earlier investigation by a different method and to compare the new results with those of other investigators, the authors decided to use a capillary tube designed and operated in accordance with the following principles:

1 The capillary tube must have sufficient length to give a head which could be measured accurately.

2 The length of the capillary should be great enough so that the kinetic-energy correction at the ends would be negligible.

3 The tube should be made of noncorrosive material of sufficient strength to prevent distortion at the highest operating pressures and temperatures.

4 A uniform temperature must be maintained along the entire length of the capillary.

5 The manometer must be connected to the capillary in such a way that separation between the liquid and vapor phase of the fluid would be in the horizontal plane of the capillary.

6 In order to eliminate the human element, the manometer must be designed and operated to give a permanent photographic record of the head.

7 No readings should be taken until a steady flow state had been attained and the record of manometer readings must indicate the presence of a steady flow state for a test period of at least 1 hr.

8 The theoretical constant of the capillary should be checked at periodic intervals during the investigation and at the conclusion of the tests by the use of suitable fluids of known viscosity.

Description of Apparatus

In accordance with the above requirements, the capillary was made from 103.2 lin ft of 0.25-in. outside diam × 0.09294-in. inside diam seamless nickel tubing. The random lengths of tubing were connected into one continuous length of uniform internal diameter by facing each end square, butting it tightly against the end of the next piece in a snugly fitting nickel sleeve and welding the sleeve to the tube ends. Each joint was radiographed after welding to make sure that the ends of the tubes were in alignment and fitted against each other to form a smooth internal surface. The capillary was wound into a coil having a diameter of 2.5 ft, covered with a uniformly spaced electric-resistance-heater wire and thoroughly insulated. During the course of the investigation, the capillary was also wound into circle bends with a 1-ft radius, connected by straight runs about 12 ft long, and was later coiled on a 2-ft diam.

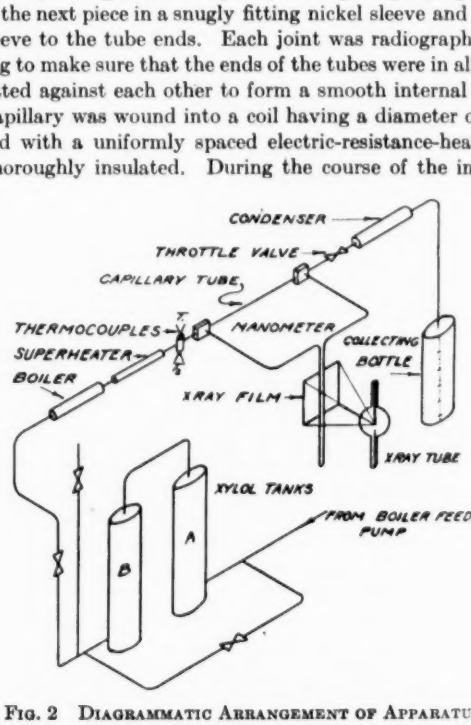


Fig. 2 Diagrammatic Arrangement of Apparatus

The manometer, made of two vertical legs of extra-heavy 1/4-in. iron-pipe-size, seamless steel tubing, was connected to the junction blocks at the ends of the capillary by tubing which was mounted in the horizontal plane of the ends of the capillary, in order to avoid any errors due to the variation in the level of the plane of separation of water and steam between the capillary and the manometer. The manometer was filled half full of mercury with water above the mercury level. Care had to be used to eliminate all air from the manometer legs and connections during the filling process. An X-ray tube was used to radiograph the mercury levels in the manometer and leave a permanent record on the film which was mounted in back of the tube as shown in Fig. 2. Heads up to 5.5 ft could be measured with this apparatus.

The boiler was supplied with deaerated distilled water by placing two tanks, *A* and *B* in Fig. 2, between the feed pump and the boiler. The tanks were connected by pipe at their upper ends

and were filled half full of xylol. This liquid will not mix with water nor contaminate it in any way at low temperatures. Before starting a test, boiling distilled water was supplied to the lower end of tank B and most of the xylol was transferred to tank A. Then during the test, the pressure from the boiler-feed pump was transferred to the feedwater through the xylol. In this way, only pure distilled water entered the boiler. During many of the tests, the xylol tanks were by-passed and the boiler was fed directly from the pump. The results were not affected by this change in the feedwater system.

The boiler consisted of a short length of double-extra-heavy 2-in. seamless steel pipe closed at each end by welding, set on an incline, and fired by a battery of Bunsen burners. Steam was superheated by placing an electric heater around a section of nickel tubing. The input to this heater was controlled automatically by a Wheelco Capacitrol and magnetic switch in order to maintain a constant steam temperature at the entrance to the capillary. Two calibrated thermocouples, welded into the bottom of wells, were placed opposite each other in a thermocouple block immediately ahead of the capillary (T_1 and T_2 in Fig. 2) in order to determine the steam temperature entering the capillary. Twenty thermocouples were provided for reading temperatures along the capillary, and the heater winding was adjusted to maintain a uniform temperature from end to end. The steam leaving the capillary was throttled, condensed in a copper coil, and weighed.

The Viscosity Equation for the Capillary

Poiseuille's law for viscous flow in a straight capillary, with allowance for kinetic-energy losses at entrance and exit, may be stated as follows

$$\mu = \frac{\pi r^4 \Delta P}{8Q(l+\lambda)} - \frac{M\rho Q}{8\pi(l+\lambda)} \quad [1]$$

where μ = viscosity
r = radius of capillary
ΔP = pressure difference between the ends of the capillary
Q = volume of flow per unit of time
l = length of capillary
M = a constant
ρ = density of the fluid
λ = a constant

For long straight capillary tubes in which the end kinetic-energy corrections may be neglected, the equation is reduced to the form

$$\Delta P = \frac{32\mu l V}{D^2} \quad [2]$$

where V = average velocity
D = diam of capillary

Equation [2] may be expressed as

$$\mu = 3.69682 \frac{D^4 h}{LWv} = C\frac{h}{wv} \quad [3]$$

where C = a constant depending upon the dimensions of the capillary
h = in. of mercury measured by the manometer under water
w = weight flow rate
v = specific volume

The theoretical constant C for a straight capillary of the diameter used in this investigation may be evaluated by substituting in Equation [2] the following units:

h = actual head loss as measured under water in in. on the radiograph
l = 103.2 ft
D = 0.09294 in.
w = weight flow rate in lb per sec
v = average specific volume in ft³ per lb
μ = viscosity in centipoises

Then $C = 26.7276 \times 10^{-7}$ or

$$\mu = \frac{26.7276 \times 10^{-7} h}{wv} \quad [4]$$

The capillary, 103.2 ft long, was first coiled on a 2.5-ft diam. Because of serious heater trouble, this arrangement was abandoned and the capillary was formed into circle bends having a 2-ft diam, connected by straight sections 12 ft long. Of the total length, 69.6 per cent was straight and 30.4 per cent curved to a 1-ft radius. This capillary arrangement has been called coil I in this report. Continued heater trouble made it necessary to obtain a new heater and to coil the entire capillary on a 2-ft diam, called coil II in this report.

For a bent capillary, the work of White (10) and Keulegan and Beij (11) indicates that at Reynolds' number above approximately 200, the effect of curvature increases the resistance to flow and the capillary constant then becomes a function of the Reynolds number and the ratio of the diameter of the capillary to the diameter of the coil. Equation [3] for the straight capillary may be modified to account for curvature by introducing a factor φ, for curvature resistance, or

$$\mu = 3.69682 \frac{D^4 h\varphi}{Lwv} = \frac{C\varphi h}{wv} \quad [5]$$

where φ is some function of the Reynolds number and the curvature of the capillary. For coil I, which is a combination of straight and curved sections

$$h = h_s + h_c \quad [6]$$

where h = total head loss in capillary
h_s = head loss in straight sections of capillary
h_c = head loss in the curved sections of the capillary

Substituting Equations [3] and [5] for the respective head losses in the straight and curved sections of the capillary gives

$$h = \frac{\mu wv}{C_s} + \frac{\mu wv}{C_c\varphi} = \mu wv\left(\frac{1}{C_s} + \frac{1}{C_c\varphi}\right) \quad [7]$$

where C_s and $C_c\varphi$ are the factors for the straight and curved lengths of the capillary.

Since $C = \dfrac{3.69682\ D^4}{L}$

then
$$h = \mu wv\left(\frac{1}{\frac{3.69682D^4}{L_s}} + \frac{1}{\frac{3.69682D^4\varphi}{L_c}}\right) \quad [8]$$

The total length of the capillary is 103.2 ft of which 69.6 per cent is straight and 30.4 per cent is curved or

$$L_s = 0.696L \text{ and } L_c = 0.304L$$

or
$$h = \mu wv\left(\frac{1}{\frac{3.69682D^4}{0.696L}} + \frac{1}{\frac{3.69682D^4\varphi}{0.304L}}\right) \quad [9]$$

or
$$h = \mu wv\left(\frac{0.696}{C} + \frac{0.304}{C_\varphi}\right) \quad [10]$$

where D = 0.09294 in.

or $$\mu = \frac{h}{wv} \frac{1}{\left(\frac{0.696}{C} + \frac{0.304}{C_\varphi}\right)} \quad \ldots\ldots\ldots [11]$$

For the straight capillary the value of $C = 26.7276 \times 10^{-7}$

or $$\mu = \frac{h}{wv} \left(\frac{1}{0.0260404 \times 10^7 + \frac{0.304}{C_\varphi}} \right) \quad \ldots\ldots [12]$$

where C_φ has the same value as in Equation [5] for a completely coiled capillary with the same coil diameter and total length as that used in a partly coiled capillary.

Test Procedure

With the throttle valve closed, the capillary was heated electrically to the desired uniform operating temperature and the pressure was raised in the boiler. By adjustment of the boiler and superheater input and the steam throttle valve, desired flow conditions were established in the capillary. All tests were 1 hr in length, as measured with a stop watch. The condensate was collected during this interval in a glass-stoppered bottle, the tare weight of which was known in advance, and weighed on an accurate balance at the conclusion of the test. Radiographs of

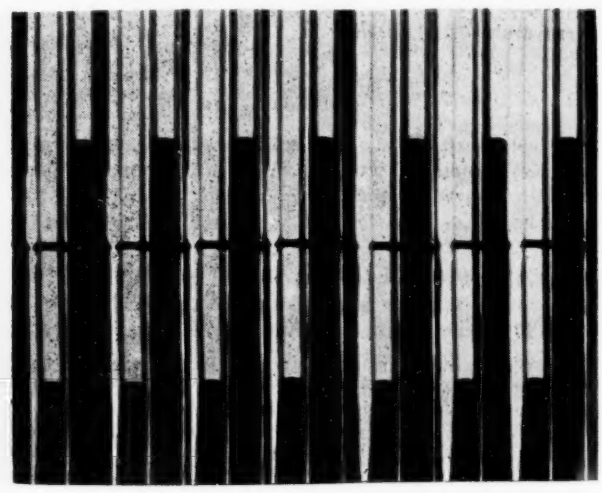


Fig. 3 Typical Radiograph From Acceptable Test

the manometer were taken prior to the start of the flow in order to check the manometer zero. During every 1-hr test, radiographs were taken at 10-min intervals with each exposure requiring from 20 to 30 sec. Fig. 3 shows a typical radiograph from a single acceptable test. Fig. 4 shows a radiograph from a test which was discarded because of unsteady flow conditions. If at any time during a 1-hr test, unsteady flow conditions were encountered, as evidenced by the cloudy meniscus in Fig. 4 or by variations in readings between individual exposures, the test was discarded.

Calibration of the Viscometer

Because of the difficulties encountered in measuring accurately the mean internal diameter of the capillary, the theoretical constant given in Equation [4] was checked by calibration tests. For this purpose nitrogen, dried in a calcium-chloride tower, and water were used for the calibrating fluids. Data for the viscosity of nitrogen were obtained from the International Critical Tables and the "Handbuch der Experimental Physik." Data for the viscosity of water were obtained from the International Critical Tables. The calibration tests for water were carried out strictly in accordance with the test procedure already described. In the nitrogen tests, the flow of the gas was measured by collecting it over water at atmospheric pressure in a burette. During a test about five 2500-cc samples were collected. If the times required for five 2500-cc samples to collect agreed within 1 per cent, the test was considered to be of sufficient length to insure accurate measurements.

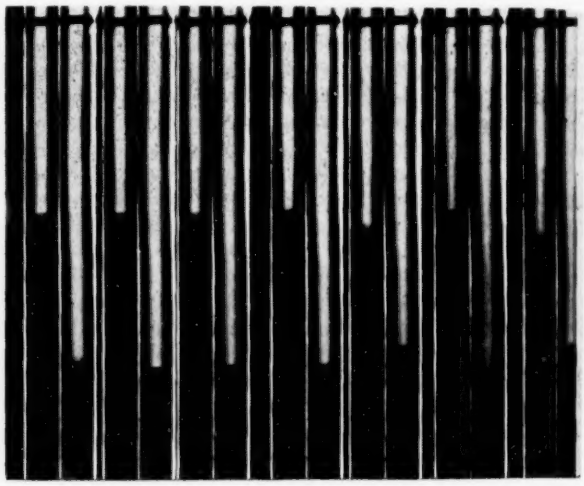


Fig. 4 Radiograph From Test Discarded Because of Unsteady Flow Conditions

Water-calibration data on coils I and II are given in Tables 1 and 2, respectively. Nitrogen-calibration tests on coil II are given in Table 3. The calibration data may be represented by the curves shown in Fig. 5. For a Reynolds number of less than 2000, the theoretical constant of a straight capillary is dependent upon its dimensions only and is 26.7276×10^{-7} for a capillary of the dimensions used in this experiment. For a Reynolds number of less than 180, the average of all data from Table 2 gives a value of 26.69×10^{-7} for the capillary constant. For a Reynolds number of more than 180, the constant must be modified for curvature as shown in Fig. 5 for coils I and II.

The general equation for the flow of a fluid in a pipe may be expressed as

$$h = \lambda \frac{l}{D} \frac{V^2}{2g}$$

where h = pressure difference between the ends of the pipe
g = acceleration of gravity
λ = friction coefficient

For the case of laminar flow in a straight pipe $\lambda = \frac{64}{Re}$. For a curved pipe, the friction coefficient λ is higher at a given Reynolds number than the corresponding value for a straight pipe. Fig. shows a plot of the friction coefficient against Reynolds' number for a straight capillary and for coil II. The solid curve labeled coil II is based upon the data of White (10) and all water-calibration tests are indicated by points.

Test Results

The viscosity of superheated steam was determined at pressures from 72.5 to 1810 lb per sq in. abs and at temperatures up to 1000 F. The essential data and results for the complete series of acceptable tests are given in Table 4. The results are plotted in Fig. 7.

TABLE 1 WATER-CALIBRATION TESTS; COIL I, STRAIGHT RUNS WITH RETURN LOOPS OF 1-FT RADIUS

Date	Weight flow rate, lb per sec $w \times 10^4$	Average fluid temp F	Differential pressure, in. Hg, under water	$\varphi C \times 10^7$	Re
3-1-38	13.2	70	7.77	26.7	330
3-1-38	3.82	70	2.23	26.05	95
3-3-38	14.6	70	8.99	25.56	365

TABLE 2 WATER-CALIBRATION TESTS; COIL II, DIAM, 2 FT

Date	Weight flow rate, lb per sec $w \times 10^4$	Average fluid temp F	Differential pressure, in. Hg, under water	$\varphi C \times 10^7$	Re
6-20-38	9.98	92.5	4.61	25.92	328
6-21-38	20.9	90.0	11.10	23.18	670
7-22-38	11.94	296.0	2.24	18.37	1476
7-23-38	14.4	191.0	3.82	20.20	1091
7-25-38	11.11	475.0	1.76	14.58	2345
7-25-38	5.84	507.0	0.72	18.38	1298
7-26-38	8.29	511.0	1.12	16.72	1860
7-26-38	10.1	400.0	1.56	16.66	1790
7-29-38	15.1	210.0	3.84	19.31	1258
8-1-38	10.4	86.0	5.29	25.30	318
11-26-38	7.99	72.0	4.70	26.10	209
11-29-38	2.78	71.0	1.61	26.80	71
11-29-38	7.91	73.0	4.59	26.00	206
11-29-38	14.1	78.0	8.14	24.58	391
11-30-38	11.3	78.0	6.41	25.10	313
11-30-38	13.4	78.0	7.90	24.05	371
12-10-38	12.1	77.0	6.58	26.45	330
12-13-38	12.3	119.5	4.78	23.50	535
12-14-38	13.8	181.0	3.81	20.88	965
12-15-38	12.9	192.0	3.23	21.25	982
12-16-38	5.25	80.0	2.6	28.99	144
12-20-38	11.6	130.3	4.20	22.90	557
12-21-38	14.56	159.6	4.35	22.00	886
12-22-38	18.7	195.0	5.30	18.44	1451
12-27-38	17.4	200.0	4.90	18.00	1395
1-3-39	11.5	73.0	6.74	25.80	299
1-3-39	18.9	73.0	11.87	24 03	492
1-3-39	3.53	73.0	1.98	26 91	92
1-4-39	14.8	69.0	8.85	26.58	366
4-11-39	1.75	74.5	0.943	27.59	46
4-13-39	4.17	77.5	2.31	28.38	115
4-13-39	5.48	76.0	2.98	26.72	14[illegible]
5-2-39	3.18	78.0	1.69	26.75	88
5-2-39	5.58	78.0	3.03	26.20	154
5-3-39	2.98	83.0	1.57	25.32	88
5-5-39	4.75	86.0	2.43	25.20	145
5-5-39	2.04	83.0	1.06	25.71	60
1-13-40	29.4	61.3	22.31	23.28	651
1-14-40	29.8	65.4	22.32	22.25	701
1-31-40	50.9	72.3	42.65	18.22	1309
2-1-40	24.5	76.4	16.10	22.00	665
2-1-40	38.2	76.1	28.30	19.56	1035
2-1-40	16.65	74.0	10.22	24.30	438
2-2-40	63.2	75.5	53.30	17.40	1690
2-3-40	63.5	70.0	56.70	17.58	1589
2-3-40	2.51	76.7	1.378	26.30	68
2-7-40	17.19	75.5	10.36	24.32	460
2-7-40	26.25	75.7	17.25	22.25	706
2-8-40	29.05	73.0	20.04	21.90	757
2-9-40	44.50	74.5	33.90	19.47	1180
2-9-40	39.55	74.7	29.40	19.88	1050
2-12-40	1.80	74.7	1.024	26.02	48
2-14-40	62.22	69.0	55.0	17.98	1538

At pressures above 1500 lb per sq in. abs, the high pressure drop through the throttle valve and the low specific volume of the steam made it virtually impossible to maintain steady flow conditions for a minimum test period of 1 hr. Inasmuch as the maintenance of a steady flow state for at least 1 hr, as evidenced by successive radiographic exposures of the manometer, was specified in advance as one of the criteria to be fulfilled before any test data would be accepted, no tests are reported in this paper for pressures above 1810 lb per sq in. abs.

In 1933, R. Plank (12) presented theoretical equations for the viscosity of gases and vapors. In the development, he showed that the thermodynamic equation of state may be expressed

$$P = \frac{RT}{v}\left(1 + a_1\frac{b}{v} + a_2\frac{b^2}{v^2} + \ldots.\right) \ldots\ldots\ [13]$$

By analogy, from this equation he states that

$$\mu = \mu_0\left(1 + A_1\frac{b}{v} + A_2\frac{b^2}{v^2} + \ldots.\right) \ldots\ldots\ldots [14]$$

This equation may be written

$$\mu = [A + B(t - C)]\left(1 + \frac{D}{v} + \frac{E}{v^2} + \ldots.\right) \ldots. [15]$$

TABLE 3 NITROGEN-CALIBRATION TESTS; COIL II, DIAM 2 FT

Date	Average volume rate of flow, $Q \times 10^4$ ft³ per sec	Average fluid temp F	Initial pressure of gas, psi abs	Differential pressure, in. Hg under water	$\varphi C \times 10^7$	Re
3-5-40	3.27	70.8	58.46	2.990	19.47	1281
3-9-40	2.45	84.85	58.47	2.135	20.90	917
3-9-40	2.45	85.0	58.47	2.125	21.05	914
3-9-40	3.915	85.0	58.47	3.895	18.34	1446
3-9-40	4.01	77.35	58.47	3.97	18.29	1492
3-10-40	3.98	77.29	58.5	4.02	17.92	1490
3-10-40	3.99	78.0	58.5	4.05	17.84	1492

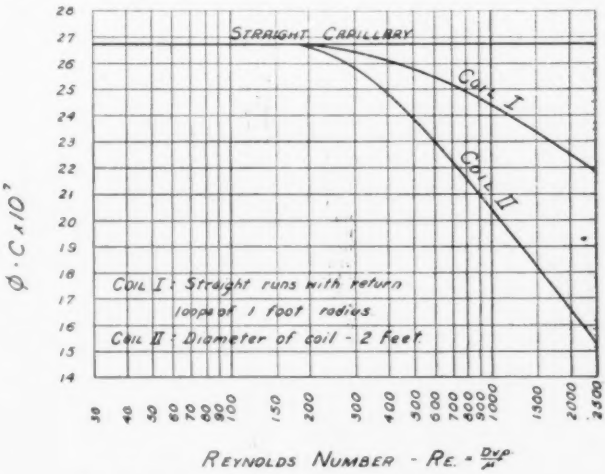


FIG. 5 CALIBRATION CURVES FOR THE CAPILLARY

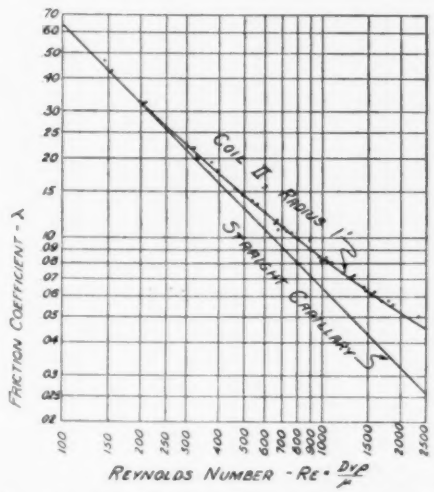


FIG. 6 VARIATION IN FRICTION COEFFICIENTS WITH REYNOLDS' NUMBER FOR STRAIGHT AND CURVED CAPILLARIES

where A, B, C, D, and E are constants

t = temperature of vapor
v = specific volume of vapor

Equation [13] may be fitted to the results of this investigation (for maximum pressures of 1500 lb per sq in. abs) in the following form

$$\mu = (13.421 + 0.0141t)\left(1 + \frac{0.1753}{v} - \frac{0.0116}{v^2}\right)10^{-3} \ldots [16]$$

where μ = viscosity, centipoises
t = temperature F
v = specific volume, cu ft per lb.

TABLE 4 STEAM TESTS

Date	Weight flow rate, lb per sec, $w \times 10^5$	Average temp F	Initial pressure, psi abs	Differential pressure, in. Hg	Viscosity centipoises, $\mu \times 10^3$
3-25-38	5.21	370	72.5	2.57	18.86
3-30-38	11.90	355	94.0	4.44	17.45
3-28-38	6.07	550	105.0	2.86	21.20
3-5-39	1.64	788	100.0	1.20	26.60
5-11-38	15.2	460	215	3.37	18.60
5-3-38	12.6	498	215	2.69	19.70
5-11-38	14.9	503	215	3.40	20.21
5-25-38	15.5	530	215	4.00	22.65
5-16-39	19.9	550	200	7.62	20.91
5-2-38	15.6	552	215	4.17	22.95
3-11-39	5.2	557	200	1.46	22.50
3-11-39	7.1	565	200	1.87	18.65
5-2-38	12.6	606	216	3.48	23.05
3-21-39	19.6	642	200	8.75	22.95
5-2-38	12.1	651	214	3.65	24.04
3-21-39	15.3	657	200	6.47	23.63
5-2-38	12.2	704	217	3.92	24.72
3-12-39	8.4	755	200	3.30	23.40
4-5-39	6.1	798	201	2.36	24.32
5-12-38	16.4	511	415	2.17	24.5
4-19-38	10.6	518	418	1.36	24.95
5-17-38	17.1	549	415	2.46	25.19
3-25-39	5.36	560	400	0.75	23.30
5-17-38	17.4	600	416	2.75	25.70
4-19-38	16.1	604	415	2.40	24.31
4-1-39	7.17	624	400	1.19	24.12
5-26-38	13.9	655	415	2.16	24.30
4-19-38	15.4	700	414	2.76	26.52
4-3-39	9.62	792	412	2.12	26.54
4-4-39	20.00	900	463	5.62	29.15
5-5-38	17.35	554	517	1.96	24.50
5-26-38	15.8	644	524	1.99	25.30
5-13-38	16.6	663	520	2.18	25.40
5-6-38	17.4	670	520	2.33	25.65
4-23-39	9.77	1022	500	2.49	32.84
5-16-38	16.9	522	615	1.69	27.51
6-1-38	15.6	625	618	1.69	27.05
4-20-38	20.2	700	615	2.69	28.97
6-20-38	18.55	860	640	3.41	27.85
4-4-39	9.32	901	605	1.63	29.60
7-5-38	19.75	948	578	4.88	32.03
5-29-38	19.5	542	735	1.65	28.85
5-27-38	20.6	580	818	1.70	30.28
4-21-38	21.4	602	810	1.93	31.04
4-21-38	21.4	650	815	2.08	31.12
6-24-38	24.3	695	827	3.27	30.05
4-21-38	19.8	700	810	1.96	29.80
6-23-38	27.9	690	1022	3.10	30.40
8-14-38	18.6	700	1025	1.85	31.10
4-6-39	20.2	790	992	1.19	28.77
4-4-39	16.0	798	997	1.79	31.40
8-16-38	24.3	818	1020	2.92	28.92
6-20-38	24.9	865	1022	3.21	29.60
4-4-39	12.8	907	1011	1.51	31.67
7-1-38	22.3	920	1030	3.16	33.70
4-25-39	16.8	1000	1000	2.36	32.55
8-12-38	22.6	702	1520	1.55	33.10
8-9-38	30.4	705	1515	2.24	31.30
8-15-38	29.9	800	1520	2.65	33.43
8-9-38	22.6	817	1520	1.91	34.50
7-13-38	21.5	801	1810	1.53	38.30

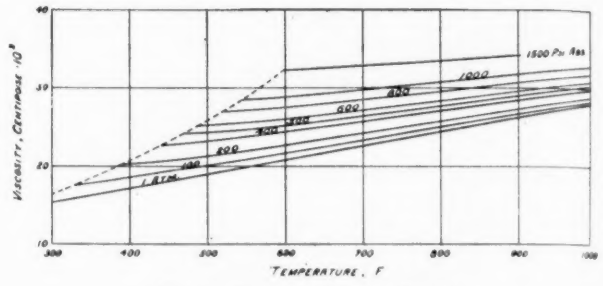


FIG. 7 THE VISCOSITY OF STEAM
(1939 Purdue University values.)

Equation [16] gives results which are in agreement with the smoothed values reported in this paper as shown in Table 5. Except for the 100-lb per sq in. abs pressures, the maximum deviation between the computed and smoothed results is 3 per cent. For the case of 100-lb per sq in. pressure the deviation is less than 3 per cent for temperatures ranging from 700 F to 1000 F but increases to 6.5 per cent at 400 F. The constants of Equation [16] are based upon the smoothed values reported in this paper for a maximum pressure of 1500 lb per sq in. abs, and it is probable that, at higher pressures, the variation in the specific volume of superheated steam is too complex to be expressed by the three terms of the equation.

Values of absolute viscosity have been obtained from Fig. 7 and tabulated in Table 6, together with the computed kinematic viscosity and several convenient conversion factors.

The viscosity data reported in this investigation are considerably lower at the high pressures than those presented by the authors in 1935. The higher 1935 values are probably due to end effects and wake turbulence of the fall body in the Lawaczeck viscometer.

ACKNOWLEDGMENTS

The authors are indebted to Mr. G. H. Van Hengel of The Detroit Edison Company for constructive criticisms and valuable assistance rendered in analyzing and interpreting the data of the investigation, particularly with reference to the effect of curvature

TABLE 5 COMPARISON OF PURDUE UNIVERSITY 1939 SMOOTHED STEAM-VISCOSITY VALUES WITH THOSE OBTAINED BY SUBSTITUTION IN EQUATION [16]

Pressure, lb per sq in. abs, temp F	100		200		400		500		600		800		1000		1500	
	1939	Computed	1939	Computed	1939	Computed	1939	Computed	1939	Computed	1939	Computed	1939	Computed	1939	Computed
400	18.5	19.7	20.2	20.4	..	..	..	..	..	..	..	..	..	..	..	..
500	20.0	21.1	21.2	21.8	23.2	23.1	24.1	24.0	25.1	24.6	..	..	..	..	..	..
600	21.6	22.4	22.6	23.1	24.4	24.2	25.0	25.0	25.9	25.7	27.4	27.0	28.8	28.0	32.3	32.3
700	23.2	23.9	24.1	24.5	25.6	25.7	26.3	26.3	27.1	26.9	28.5	28.3	29.7	29.3	32.7	32.8
800	24.9	25.3	25.7	25.9	27.0	27.0	27.6	27.6	28.3	28.1	29.5	29.3	30.7	30.4	33.4	33.1
900	26.6	26.7	27.2	27.2	28.4	28.3	28.9	28.9	29.6	29.4	30.6	30.5	31.7	31.6	34.2	34.2
1000	28.3	28.0	28.8	28.6	29.8	29.7	30.3	30.2	30.9	30.8	31.8	31.8	32.8	32.9	..	..

TABLE 6 THE DYNAMIC AND KINEMATIC VISCOSITY OF STEAM

Pressure, lb per sq in. abs, temp F	100		200		400		500		600		800		1000		1500	
	μ cP $\times 10^5$	ν ft² per sec $\times 10^5$	μ cP $\times 10^5$	ν ft² per sec $\times 10^5$	μ cP $\times 10^5$	ν ft² per sec $\times 10^5$	μ cP $\times 10^5$	ν ft² per sec $\times 10^5$	μ cP $\times 10^5$	ν ft² per sec $\times 10^5$	μ cP $\times 10^5$	ν ft² per sec $\times 10^5$	μ cP $\times 10^5$	ν ft² per sec $\times 10^5$	μ cP $\times 10^5$	ν ft² per sec $\times 10^5$
400	18.5	6.14	20.2	3.21	..	..	..	..	..	..	..	..	..	..	..	..
450	19.3	6.84	20.6	3.53	22.6	1.79	..	..	..	..	..	..	..	..	..	..
500	20.0	7.53	21.2	3.96	23.2	2.01	24.1	1.61	25.1	1.34	..	..	..	..	..	..
550	20.8	8.27	21.9	4.27	23.7	2.21	24.5	1.78	25.5	1.50	27.0	1.12	28.4	0.866	..	..
600	21.6	9.04	22.6	4.65	24.6	2.44	25.0	1.95	25.9	1.65	27.4	1.25	28.8	0.996	32.3	0.611
650	22.4	9.84	23.4	5.07	25.0	2.63	25.6	2.12	26.5	1.80	28.0	1.38	29.2	1.11	32.5	0.727
700	23.2	10.67	24.1	5.48	25.6	2.84	26.3	2.30	27.1	1.95	28.5	1.50	29.7	1.22	32.7	0.818
750	24.1	11.57	24.9	5.92	26.3	3.07	27.0	2.49	27.7	2.11	29.0	1.62	30.2	1.319	33.1	0.902
800	24.9	12.48	25.7	6.39	27.0	3.30	27.6	2.67	28.3	2.26	29.5	1.74	30.7	1.42	33.4	0.978
850	25.8	13.45	26.4	6.83	27.7	3.52	28.2	2.86	29.0	2.43	30.1	1.86	31.2	1.52	33.8	1.05
900	26.6	14.38	27.2	7.32	28.4	3.78	28.9	3.05	29.6	2.59	30.6	1.98	31.7	1.62	34.2	1.13
950	27.1	15.23	28.0	7.83	29.1	4.02	29.6	3.25	30.2	2.75	31.2	2.10	32.3	1.73	..	..
1000	28.3	16.48	28.8	8.34	29.8	4.28	30.3	3.46	30.9	2.93	31.8	2.24	32.8	1.83	..	..

1 poise = 100 centipoises = 0.0672 lb sec^{-1} ft^{-1} = 0.00209 lb sec ft^{-2} = 1 dyne sec cm^{-2} = 0.0102 kg sec m^{-2} = 0.1 kg sec^{-1} m^{-1}.

of the capillary. Dr. Max Jakob of the Armour Institute of Technology has been most helpful in connection with the solution of certain problems encountered in analyzing the results.

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Discussion

E. F. Leib.[5] For the analytical representation of their data on the viscosity of superheated steam, the authors rely on Equation [14], as recommended by R. Plank. This equation has been derived from the kinetic theory of gases for the viscosity at high densities, where the space occupied by the molecules is of the order of magnitude of the gas volume, and the transfer of momentum occurs at collisions as well as between collisions. Definite values have been derived for all constants in Equation [14] and, due to their significance, these constants are essentially positive.[6] However, the constants in the authors' Equation [16] have nothing in common with these theoretical values and one constant is even negative. Therefore, the latter equation becomes purely empirical. The reason for the failure of Equation [14] is that, in the range investigated by the authors, the state of the vapor is that of moderate density, where the volume of the molecules is of minor importance, and the increase of the viscosity over the corresponding gas condition is mainly due to the forces of attraction between the molecules.

The viscosity of a vapor can, in first approximation, be obtained as follows: From the kinetic theory, we have the fundamental relation for the viscosity

$$\mu = {}^1/_3 \rho l^2 z \qquad [17]$$

where ρ = density, l = mean free path, z = number of collisions.

In the perfect gas condition

$$z = \frac{c_0}{l} \qquad [18]$$

whence the viscosity of the perfect gas, with $l = l_0$ is

$$\mu_0 = {}^1/_3 \rho c_0 l_0 \qquad [19]$$

where c_0 is the mean velocity of the molecules due to the thermal agitation.

The kinetic theory gives

$$c_0^2 = 3gRT \qquad [20]$$

and

$$l_0 = D \frac{v}{2b} \qquad [21]$$

where g = acceleration of gravity, R = gas constant, D = diameter of molecule, b = van der Waals' sphere of exclusion, v = specific volume. Thus, μ_0 depends only upon the temperature T.

Assume the density to be such that only binary encounters between the molecules occur. Consider a molecule midway between two collisions, where it will travel with the velocity c_0. While approaching the object of its next collision, it enters the field of attraction and experiences an acceleration. Thus, the velocity will be greater than c_0, and then, according to Equations [18] and [17] of this discussion, the number of collisions and hence the viscosity will be greater than in the gas condition.

To calculate this acceleration effect, we use a method similar to that by which Sutherland investigated the effect of the attraction on the mean free path. If $u(r)$ is the potential energy of a molecule with respect to another molecule in the distance r, the force of attraction is

$$F(r) = -\frac{du}{dr} \qquad [22]$$

Then the motion of a molecule subjected to this force is given by

$$F(r) = -m \frac{dc}{dt}$$

or

$$\frac{du}{dr} = m \frac{dc}{dt} \qquad [23]$$

where m = mass of the molecule, t = time.

Equation [23] can be written

$$dc \frac{dr}{dt} = \frac{1}{m} du$$

Since also

$$\frac{dr}{dt} = -c \qquad [24]$$

we have

$$2cdc = -\frac{2}{m} du$$

Integration gives the instantaneous velocity of the molecule

$$c^2 = -\frac{2u(r)}{m} + c_0^2 \qquad [25]$$

The time required to travel the distance dr is, from Equation [24]

$$dt = -\frac{dr}{c} = -\frac{dr}{\sqrt{-\frac{2u(r)}{m} + c_0^2}} \qquad [26]$$

Assume the potential energy u to decrease from a maximum ϵ with some inverse power of the distance r

[5] Combustion Engineering Company, Inc., New York, N. Y.

[6] "Mathematical Theory of Non-Uniform Gases," by S. Chapman and T. G. Cowling, Cambridge University Press, London, 1939, p. 288.

$$u = -\epsilon \left(\frac{D}{r}\right)^n \quad \ldots\ldots [27]$$

Then, from Equation [26]

$$dt = -\frac{dr}{c_0 \sqrt{\frac{2\epsilon}{mc_0^2}\left(\frac{D}{r}\right)^n + 1}}$$

where c_0 is given by Equation [20]. If we introduce

$$R = \frac{r}{D}$$

and

$$\Theta = \frac{2\epsilon}{3mgR}$$

we have

$$dt = -\frac{D}{c_0} \cdot \frac{dR}{\sqrt{\frac{\Theta}{T}\frac{1}{R^n} + 1}} \quad \ldots\ldots [28]$$

The kinetic theory gives for the mean free path at moderate densities

$$l = l_0 - \frac{D}{2} \quad \ldots\ldots [29]$$

Then the path from midway between collisions to the next collision is $\frac{l_0}{2} - \frac{D}{4}$. At collision, the distance between the centers of the molecules is D; consequently, we extend the integration from $r = D$ to $r = \frac{1}{2} l_0 + \frac{3}{4} D$, or from $R = 1$ to $R = \frac{1}{2}\frac{l_0}{D} + \frac{3}{4}$. Then we obtain from Equation [28] for the time elapsed between two collisions

$$t = \frac{2D}{c_0} \int_1^{\frac{1}{2}\frac{l_0}{D}+\frac{3}{4}} \frac{dR}{\sqrt{\frac{\Theta}{T}\frac{1}{R^n} + 1}} \quad \ldots\ldots [30]$$

while the number of collisions is given as

$$z = \frac{1}{t} \quad \ldots\ldots [31]$$

Equation [30] permits an exact integration for $n = 2$. Though the actual value of n may be greater, the error introduced by the choice $n = 2$ will be noticeable only at high densities, since it corresponds to a less rapid decrease of the potential energy with the distance between the molecules. Any value of n greater than unity is compatible with the equation of state for vapors, as derived by the writer from statistical methods, due to the particular type of molecular clusters chosen.[7]

The effect of this choice on the viscosity at moderate densities is negligible, and we obtain by integration of Equation [30], between the limits described, for $n = 2$

$$t = \frac{2D}{C_0}\left(\sqrt{\left(\frac{l_0}{2D} + \frac{3}{4}\right)^2 + \frac{\Theta}{T}} - \sqrt{1 + \frac{\Theta}{T}}\right)$$

and from Equation [31]

$$z = \frac{c_0/2D}{\sqrt{\left(\frac{l_0}{2D} + \frac{3}{4}\right)^2 + \frac{\Theta}{T}} - \sqrt{1 + \frac{\Theta}{T}}} \quad \ldots\ldots [32]$$

Substitution of Equation [21] into Equation [29] gives for the mean free path

$$l = D\left(\frac{v}{2b} - \frac{1}{2}\right) \quad \ldots\ldots [33]$$

If we now substitute Equations [32] and [33] into Equation [17], we obtain

$$\mu = \frac{1}{3}\rho c_0 D \frac{v}{2b}\left(1 - \frac{b}{v}\right) \frac{\frac{v}{4b} - \frac{1}{4}}{\sqrt{\left(\frac{v}{4b} + \frac{3}{4}\right)^2 + \frac{\Theta}{T}} - \sqrt{1 + \frac{\Theta}{T}}}$$

or, with the aid of Equations [19] and [21]

$$\mu = \mu_0 \frac{\frac{v}{4b}\left(1 - \frac{b}{v}\right)^2}{\sqrt{\left(\frac{v}{4b} + \frac{3}{4}\right)^2 + \frac{\Theta}{T}} - \sqrt{1 + \frac{\Theta}{T}}} \quad \ldots\ldots [34]$$

Equation [34] is the relation between the viscosity at moderate densities and the viscosity in the gas condition at the same temperature. The quantity b can be identified approximately with the smallest volume which the liquid substance can assume without being compressed; i.e., for H_2O, $b = \sim 0.0160$ cu ft per lb. The quantity Θ was taken as an average calculated from Equation [34] with the 1939 Purdue test data of the viscosity of steam and was obtained as

$$\Theta = 19{,}000 \text{ F}$$

With these constants, Equation [34] represents the test data with a deviation of a few per cent only. Due to the derivation from kinetic conceptions, this equation can be expected to represent the viscosity of steam far beyond the 1500-lb per sq in. limit reached in the 1939 Purdue experiments.

G. H. VAN HENGEL.[8] Recently, Russian results[9] have been published which differ with the results of the authors and other previous investigators. The many differing test results by all the various experimenters indicate the great difficulty in determining the viscosity of steam (8).[10] It seems that the capillary-tube method has inherent difficulties, some of which the writer will here review.

First let us consider the mechanical difficulties of the capillary. The authors have eliminated the complication of the entrance and exit losses of the capillary by taking a long capillary. This did away with the corrections investigated by Schiller.[11]

They introduced, however, some other problems, viz., the effect of the curvature of the coil and the problem of determining the tube diameter. Accurate determination of the diameter is a very important one as $\lambda = 25.103 \frac{\Delta p D^5}{W^2 v}$ for a length of 103.2 ft

[7] "Thermodynamic Properties of Vapors," by E. F. Leib, presented at the Spring Meeting, Worcester, Mass., May 1–3, 1940, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

[8] Mechanical Engineer, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

[9] "Viscosity and Heat Conductivity of Steam for High Temperatures and Pressures," by D. L. Timrot and N. B. Vargaftik (in Russian), *Journal of Technical Physics* U.S.S.R., vol. 9, no. 6, 1939, pp. 461–469.

[10] Numbers in parentheses refer to the Bibliography at the end of the authors' paper, page 683.

[11] "Untersuchungen über Laminare und Turbulente Strömung," by L. Schiller, *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens* heft 248, 1922.

and Re is inversely proportional to D, where λ = a dimensionless resistance coefficient, D = inside diameter in ft, W = weight flow rate in lb per sec, and v = specific volume in cu ft per lb.

The problem of the coil curvature has been investigated (10) and (11) but none had a coil of such a length or of such a heavy wall. Because it forms an oval cross section, the bending of a thick-wall tube may affect the tube diameter and cross-sectional area far more than previous investigators found.[12] This deformation reaches larger proportions at higher internal pressures, an indication of which can be found in an article by Dean.[13] This deformation may even affect the λ vs. Re curve for the coil.[14]

The determination of the inside diameter of the tube presented another problem with such a long coil. The inside diameter actually measured before the bending was 0.091 in. but the inside diameter after the bending could not be measured directly. The internal-volume determination with a liquid gave too large a diameter, due to the filling up of the connections between the different tube lengths. Gas-volume determination did not seem to offer higher accuracy.[15] Thus, finally, the tube diameter of 0.09294 in. was established by using White's formula[16] and the authors' 1940 calibration tests with water shown in Table 2 of the paper under the assumption of a round capillary. Deviations up to 5 per cent occurred for unknown reasons in the λ values.

In the paper, the influence of the temperature on the expansion of the nickel tube has been neglected, although this affects the computation of the viscosity by 3 per cent at the highest temperatures of 1000 F.

Another difficulty is brought out by Keulegan and Beij[17] that a little sag in a straight capillary contributes to large changes in friction coefficients. Here the coil was wound at random and was not at all the case of a coil in a horizontal plane.

The effect of the foregoing deviations, on which the viscosity calculation of steam is so vitally dependent, should have been established with the calibration test before the final steam tests could proceed.

But these mechanical deviations may not be the only items of great importance, because of other difficulties presented by thermodynamic problems. In the first place, among these is the effect of nonisothermal conditions in the viscous flow of incompressible fluids on the friction coefficient.[18,19] The slightest heat transmission will affect the parabolic velocity pattern considerably and thus influence the friction coefficient.

In the case of compressible fluids, there is added to the foregoing problem yet another one, i.e., the knowledge of the proper relationship between the pressure drop and the friction coefficient. This relationship will be different for the isothermal and adiabatic cases.[20]

[12] Bibliography (11), p. 96, and (10), p. 652.

[13] "Distortion of a Curved Tube Due to Internal Pressure," by W. R. Dean, *Philosophical Magazine and Journal of Science*, vol. 28, Oct., 1939, pp. 452–464.

[14] Bibliography (10), Fig. 6, p. 657.

[15] "Determination of the Internal Volume of Steel Capillaries for Measurements With Gases," by J. Kaminsky and B. E. Blaisdell, *Review of Scientific Instruments*, vol. 10, Feb., 1939, pp. 57–58; see also correction, ibid., May, 1939, p. 151.

[16] Bibliography (10), p. 663.

[17] Bibliography (11), final paragraph, p. 98.

[18] "Pressure Drop and Velocity Distribution for Incompressible Viscous Non-Isothermal Flow in the Steady State Through a Pipe," by A. Lee, W. D. Nelson, V. H. Cherry, and L. M. K. Boelter, Proc. Fifth International Congress of Applied Mechanics, John Wiley & Sons, Inc., New York, N. Y., 1939, pp. 571–577.

[19] "Über den Wärmeaustauch bei der Strömung zäher Flüssigkeiten in Röhren," by M. Jakob and H. Eck, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 3, no. 3, 1932, p. 121.

[20] "Steam and Gas Turbines," by A. Stodola and L. C. Loewenstein, vol. 2, McGraw-Hill Book Company, Inc., New York, N. Y., 1927, p. 1026.

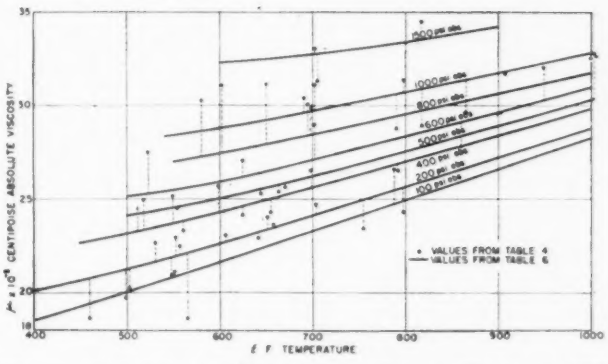


FIG. 8 COMPARISON OF PURDUE TEST VALUES WITH THE 1939 PURDUE SMOOTHED VALUES OF THE ABSOLUTE VISCOSITY OF STEAM

Perhaps one or more of these questions may play an important role in interpreting the results because, if the data of the authors' Table 4 were plotted in Fig. 7 of the paper, as in the case in the writer's curve Fig. 8, it will be noted that several points have a viscosity which deviates more than 10 per cent from the smoothed values given as curves in Fig. 7. This may be partly due to the two different coils that were used, but as long as that effect is not established other causes may exist.

Checking up Plank's formula results with the final smoothed values is only a matter of determining constants and is of no great importance in view of the deviation of the smoothed values from the actual test results. The smoothed values should therefore be used with great reservation.

For the reasons mentioned, the precise method of determining steam viscosities with capillaries must be more fully examined and developed before we can consider the values obtained as final.

Now that the authors are equipped with the experience of their published experiments, let us hope that they will once more go into this problem of fixing the values of the viscosity of steam. For pressure-drop calculations, the authors' values will be useful, but for calculations of nozzle flow, heat transfer, and the like, a higher accuracy is desirable.

F. G. KEYES.[21] Shortly after the publication of the measurements of steam viscosity by the authors (3) over a wide range of temperature and pressure (420 C:240 atm), a paper appeared by Sigwart,[22] reporting measurements considerably lower than those obtained by the American investigators. Recently the first investigators[23] have again measured the viscosity properties of water using the "capillary-flow" method of Poiseuille, whereas, the earlier results were obtained by observing the time of fall of a metal cylinder in a tube filled with steam. Both of these investigations were carried through with care and with the use of ingenious controls. The results are in substantial agreement, albeit the capillary-tube results do not extend over the wide pressure range reported upon in 1935. It is now established that the low results of Sigwart do not represent the true viscosity of steam.

The object of the present comment is to indicate that the entire ensemble of results by the authors may be simply correlated as a temperature function of the low-pressure viscosity of steam plus a function of the pressure only.

There are few data available for the viscosity properties of

[21] Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, Cambridge, Mass.

[22] "Messungen der Fähigkeit von Wasser und Wasserdampf bis ins kritische Gebiet," by K. Sigwart, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 7, 1936, pp. 125–140.

[23] Preprint of this paper as presented at the 1939 Annual Meeting.

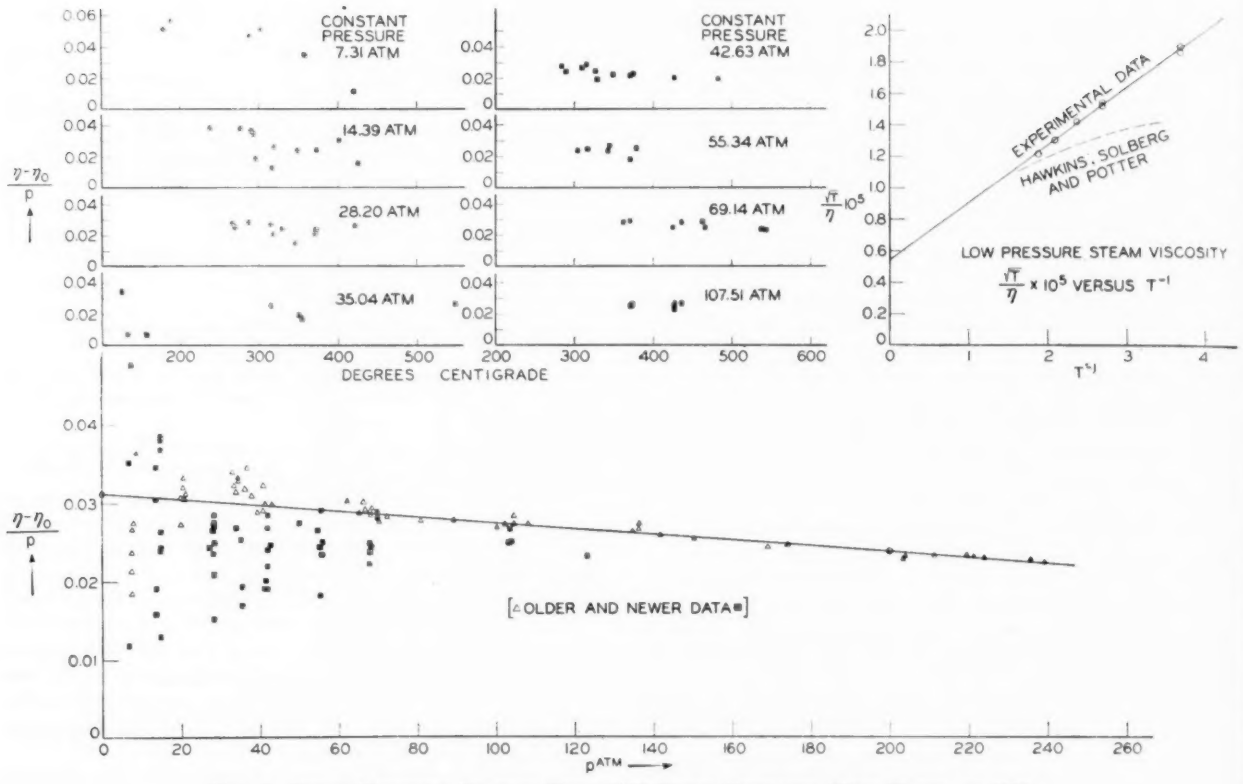


FIG. 9 STEAM VISCOSITY RESULTS FROM 1935 TESTS COMPARED WITH THOSE OF 1939

gases over any considerable pressure range[24] and the theory along classical lines has been mainly developed by Enskog[25] to a first approximation employing the van der Waals concept of the molecular field. The expression obtained for the viscosity η is

$$\eta = \eta_0 \left(\frac{1}{\vartheta} + 4/5\ b/v + 0.7614\ b^2\vartheta/v^2 \right)$$

η_0 is the viscosity as a function of temperatures at low (zero) pressure and $\vartheta = (1 + 5/8\ b/v + 0.2869\ b^2/v^2)$; the symbol b denoting the van der Waals quantity, 4 times the volume of the molecules. The formula was applied successfully to the data for CO_2.

From the computational point of view, a formula in the variables T and v is less convenient for engineering purposes than one in T and p. The writer accordingly attempted in a wholly empirical way to obtain such a formula and it appears that the following equation[26] represents the authors' results

$$\eta = \eta_0 + (0.03103 - 3.65 \times 10^{-5} p) p \times 10^{-4} \quad [35]$$

In Equation [35], the viscosity is expressed in poises and the pressure in atmospheres (14.696 lb-in.$^{-2}$). The quantity η_0 is the viscosity of steam for pressures approaching zero. The quantity η_0 can be represented by the following equation commonly referred to as the Sutherland formula

$$\eta_0 \text{ (poises)} = \frac{1.851 \times 10^{-5} (T)^{1/2}}{1 + 680.1\ T^{-1}} \quad [36]$$

The constants of Equation [36] were obtained by least-square procedure using all available low-pressure data.[27]

Equation [35] may be transformed to the form

$$Z = \frac{\eta - \eta_0}{p} = a + bp \quad [37]$$

It is clear that if Equation [35] is satisfactory, the function Z computed from the data should be linear in the pressure. Fig. 9 of this discussion shows the 1935 data which extend to 240 atm and indicates that the linear relationship is satisfied. There is moreover no detectable temperature influence left unaccounted for.

The new data lie in general below the older results but are confirmatory in general of the older measurements. The upper left portion of Fig. 9 shows the later data Z function plotted for constant pressures with temperature as the abscissas. There is no pronounced trend.

The upper right portion shows the data for low (zero) pressures plotted as follows: Equation [36] may be written in the form $\frac{T^{1/2}}{\eta_0} = \frac{1}{\alpha} + \frac{c}{\alpha} T^{-1}$. If the square root of the absolute temperature is divided by the viscosity, the quantity should vary linearly in reciprocal absolute temperature.[28]

The experimental data are seen to be adequately represented by the Sutherland formula of which the constants in Equation [36] were determined by least-square procedure. The dotted line represents in the same coordinates the low-temperature viscosi-

[24] "The Viscosity of Gases at High Pressures," by R. O. Gibson and H. J. Paris, Amsterdam, 1933.

[25] "Kinetic Theory of Heat Conductivity, Viscosity, and Diffusion in Certain Condensed Gases and Liquids," by D. Enskog, *Kongl. Svenska Vetenskaps-Akademiens Handlingar*, Stockholm, vol. 63, no. 4, 1922, p. 63.

[26] "Thermodynamic Properties of Steam," by J. H. Keenan and F. G. Keyes, John Wiley & Sons, Inc., New York, N. Y., 1936.

[27] "An Experimental Study of the Viscous Properties of Water Vapor," by C. J. Smith, Proceedings of the Royal Society of London, Series A, vol. 106, 1924, pp. 83–96.

[28] "The Sutherland Viscosity Constant and Its Relation to the Molecular Polarization," by F. G. Keyes, *Zeit. für Physikalische Chemie*, Cohen-Festband, vol. 130, 1927, pp. 709–714.

ties computed from the formula $(10.74 + 0.0178t)$ employed by the authors.

Summary

1 The authors' new viscosity data do not indicate that any change in the earlier viscosity Equation [35] is necessary.

2 Equation [35] reproduces the viscosity of steam in relation to all the authors' results within the limits of experimental error.

Comments

The foregoing discussion was prepared on the basis of the viscosity of steam data given in Table 6;[23] these results extend to about 123 atm. It is intended to show that the results are in tolerable agreement with the 1935 data obtained by a method different from that employed in 1939. The 1935 data extend to a pressure of 239 atm or nearly twice the range of the later results.

Recently, corrections have been applied to these data,[23] resulting in a considerable reduction in their magnitude. For example at 801 F and 123.2 atm, 3.83×10^{-4} poises is the corrected 1940 result obtained by correcting the December, 1939, datum, which is 5.34×10^{-4} poises; roughly 40 per cent greater. It is therefore clear, since the uncorrected 1939 results are in fair agreement with the 1935 results, that a serious error affects the 1935 data. The authors note this fact.

An examination of the new data using the form of Equation [35] indicates that the corrected data may be represented within the limits of accuracy. The formula is as follows

$$\eta = \eta_0 + (0.0151 - 5.9 \times 10^{-5} p^{\text{atm}}) p^{\text{atm}} \times 10^{-4} \ldots [38]$$

The formula for η_0 is the same, i.e., Equation [36].

If the viscosity of steam at 239 atm 788 F is computed using Equation [38], the number 2.67×10^{-4} is obtained. The 1935 result is however given as 7.30×10^{-4} which would imply that the Lawaczack method gave a result nearly 190 per cent greater than that which the capillary method might give. The extrapolation using Equation [38] to 239 atm (nearly twice the range upon which this equation is based) may of course be subject to great error.

R. C. H. Heck.[29] The object of this discussion is to set up an argument in terms of simple kinetic theory, for the necessary increase of gas viscosity with pressure and with temperature.

As to the detail of its mechanism, viscous action within a gas is distinctly different from that in a liquid. In the liquid, the shear stress of viscous resistance and the compressive stress of pressure reaction are both phenomena of molecular contact; but, in the gas, they are impact effects of molecules in free translatory motion. Of molecular motions and interactions in the liquid phase we have but vague concepts, knowledge (of over-all kind) coming wholly by observation and empirical determination. The free and simple motions of gas molecules can be reasoned in large degree; at least far enough to account for the major component of behavior that will yet require experiment for close determination.

First comes a brief outline of a part of the course of reasoning that leads to the gas equation $pv = RT$, taking the simplest case of a single gas with molecules all of the same mass m, having the mean velocity u. Imagine a cubic unit of gas enclosed by walls that are perfectly elastic and infinitely fine in structure, so as to be perfectly flat even to molecules. With n molecules in this cube and velocity u in terms of the edge of the cube as linear unit and per second, the number of impacts per second on the square side is $nu/2$. These impacts are in all directions, but the mean normal component velocity is $u/3$. Then it is as if a stream of fluid having the mass velocity $mnu/2$ were flowing squarely against the unit surface with velocity $u/3$ and momentum $mnu^2/6$. This stream is brought to zero velocity by impact and then completely reversed by elastic rebound; an action made possible by the mechanism of real molecular motion, to which the stream concept is merely an over-all equivalent. The effect of impact is now a unit pressure of the value

$$p = mnu^2/3 \ldots [39]$$

Since mn is the same as density ρ, the mass per unit of volume and equal to $1/v$, the relation becomes

$$p = \rho u^2/3 \quad pv = u^2/3 \ldots [40]$$

This pv is magnitudinally equal to $^2/_3$ of the kinetic energy of all the molecules in the unit of mass, but physically it is an inertia-force effect. The final step of making absolute temperature T proportional to u^2 goes beyond the needs of the present discussion. In a gas mixture, molecules of different mass m are put on an equal basis by the fact that mean energy $mu^2/2$ must be the same for all molecules at a given temperature.

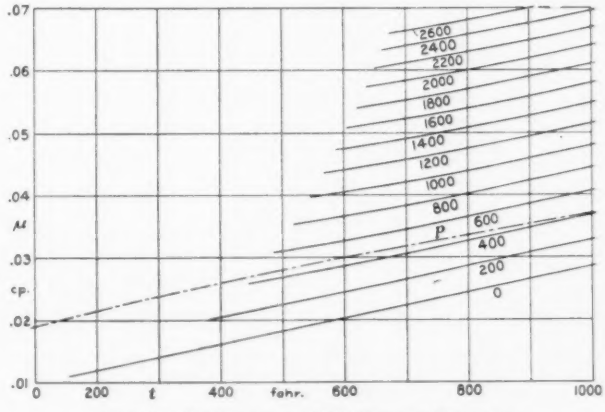


Fig. 10 Chart of the 1935 Purdue Results

Considering gas in motion, i.e., in mass motion, three forms of simple streamline flow are to be distinguished, as follows:

1 Ideal uniform flow, as in a "frictionless" straight channel, with progressive velocity V the same at all places in the current. Completely random molecular velocity continues to be distributed throughout the moving body of gas just as if this were standing still; and the total or absolute velocity of each molecule (with reference to the fixed channel) is the vector sum of the particular u within the current and the general V of the current.

2 Growing velocity of flow, as in current acceleration or jet formation, with V increasing and mean u decreasing as pressure falls. Flow velocity V is uniform over surfaces of like state across the current.

3 Viscous or "laminar" flow, in which V is constant along any line of flow but varies across the current. This is the kind of motion in which we are now interested.

In laminar flow, consider a surface of equal velocity V, which is a cylinder in either of the viscosimeters used at Purdue. This surface is now to be purely geometrical, offering no impedance to the passage of molecules. Across the surface, momentum is being carried at the rate $mnu^2/6$ figured for pressure effect. Also, across the surface (with dimension y in that direction) there is the velocity gradient dV/dy.

Next, consider the cross-stream of molecules from lower to higher V; these must be accelerated in the flow direction, hence, exert an inertia reaction against the higher-velocity stream.

[29] Research Professor of Mechanical Engineering, Rutgers University, New Brunswick, N. J. Mem. A.S.M.E.

Similarly, the cross-stream from higher to lower velocity must be retarded, with a forward pulling inertia effect. The two reactions combine to oppose relative motion of the parallel-flow streams; and the amount of this opposition, due to a deflection of momentum on account of gradient dV/dy, is proportional to the total momentum which manifests itself in pressure p. Since in any rate of streamline flow, velocity V will be very small relative to molecular u, viscosity is a very feeble force action in comparison with gas pressure.

In Fig. 10 of this discussion is shown a chart of absolute viscosity μ, interpolated from Figs. 9 and 10 of the 1935 paper (3). The stage of argument thus far reached, with the conclusion (partial) that μ should be proportional to V, would require that the constant-pressure lines be horizontal and that they be equally spaced vertically. It becomes necessary then to perceive some further influence which will account for the actual behavior seen in Fig. 10 of this discussion; and this is found in the matter of mean length of path of molecule between impacts. The modifying effect of path length is argued thus:

With rise of pressure along the isotherm, the increase of density ρ shortens the mean path; with a shorter path across the flow direction or in the direction of velocity gradient, there is smaller change of the V component of the total molecule velocity; and therefore the increment of μ with p lessens as the pressure is higher.

Along a p-constant line, smaller density with higher temperature lengthens the path of the molecule, hence there is greater change of momentum in the direction V, and μ rises with the temperature.

In Fig. 10 the dot-and-dash curve is the handbook curve of μ for air, at or near atmospheric pressure.

From the simple reasoning thus completed it is not possible to make a quantitative prediction; it is explanatory rather than determinative; and a theory of full quantitative capability would be far more elaborately mathematical. Also, for steam as a superheated vapor or a gas near to saturation, there is the further influence of departure from ideal-gas behavior.

[An additional discussion of the paper, as presented in preprint form at the 1939 Annual Meeting of the Society, was received from J. R. Finniecome of Manchester, England. Since revisions were made in the original paper, it has been impossible to communicate these to him in order that he might have the opportunity to revise his discussion. Therefore, it is with regret that it has been found necessary to omit his contribution at this time. EDITOR.]

AUTHORS' CLOSURE

The discussions of Messrs. Lieb and Keyes are very interesting because different types of equations are developed which closely represent the smoothed values presented in the paper. Dr. Lieb's equation, based on the kinetic theory, should permit extrapolation of the results to pressures much higher than the upper limit of the experimental data reported by the authors.

Mr. Van Hengel has contributed a very thorough and valuable discussion of the difficulties encountered in measuring viscosity by means of a capillary tube. His remarks should be of great assistance to any person interested in experimental work in this field.

Possible deformation of the cross-sectional area of the capillary on bending to a one-foot radius is one of the mechanical difficulties mentioned by Mr. Van Hengel. Five sections have been cut from the capillary coil, the ends have been polished perpendicular to the coil tangent, and enlarged photographs have been made of the tube cross section at a magnification of 100 diameters. The photographs do not show evidence of deformation of the circular cross section at any of the five sections investigated.

Mr. Van Hengel discusses the difficulty of accurately measuring the internal diameter of the capillary and points out that the fifth power of the diameter appears in the general flow equation of Blasius. The constants in the authors' Equations [4], [5], and [12] and the calibration data of Fig. 5 are based on a particular numerical value of the diameter. If the actual diameter of the capillary remains constant and the same value for the diameter is used in all computations involving both calibration and test runs, the final values for the viscosity of steam as computed by the method of the authors are independent of the numerical value used for the diameter.

Mr. Van Hengel mentions the effect of temperature on the diameter of the capillary. This has been considered in computing the Reynolds numbers used in connection with Fig. 5, but its effect upon the curves in Fig. 5 has been neglected. Thus a partial correction has been made and errors due to neglect of expansion of the capillary at high temperatures are considerably less than those indicated by Mr. Van Hengel in his discussion. The use of the general Blasius equation with friction coefficients in the form shown in Fig. 6 would have made the correction for expansion simple and complete but the final results would have depended upon the fifth power of the diameter of the capillary. To summarize, then, the method of computation used by the authors gives viscosity data which are independent of the numerical value assigned to the diameter of the capillary if this diameter is constant, but complete corrections for changes in the actual diameter with temperature, while small, are nevertheless almost impossible to make. The method preferred by Mr. Van Hengel of using the Blasius equation makes it possible to correct completely for changes in capillary diameter with temperature, but the numerical value of the diameter to the fifth power enters directly into the results at all temperatures and therefore necessitates a precise determination of the diameter.

In conclusion, the authors wish to thank all of those who have presented oral or written discussions for their interest, comments and constructive criticisms.

Steam-Turbine Blading

By R. C. ALLEN,[1] MILWAUKEE, WIS.

This paper reviews the blading-design practice associated with modern high-pressure high-temperature steam turbines. The design problems encountered in the development of partial-admission impulse blading for topping units are described, as well as the current engineering practice employed in the manufacture of such blading. The stress analysis used in the construction of full-admission blading is reviewed. The design procedure adopted for high-tip-speed last-row blading and the natural limits in capacity imposed on 3600-rpm turbine construction are also discussed. Materials for turbine blading are considered, as well as the metallurgical problems associated with the fabrication and welding of the high-grade alloy steels now available.

Introduction

NOZZLES and blades for large steam turbines may be classified inclusively as elements incorporating a series of curved passages which control and direct the power-producing phases of the steam flow through the turbine. The general function of the steam passages is twofold: (1) The kinetic energy in the carry-over velocity from the preceding stage must be efficiently turned through a predetermined angle; (2) at the same time, there must be an efficiently carried out expansion through each element by means of which the kinetic energy of the working fluid is increased.

The foregoing general statement may be taken to cover moving or stationary elements or blading of the impulse or reaction type. Reaction-turbine designers often find it desirable to divide the thermal drops unequally between the moving and stationary blades of a stage. Impulse-turbine builders find that the highest efficiency can be achieved by the adoption of a certain amount of pressure drop in the moving blades. The original sharp distinction between reaction and impulse turbines has, therefore, largely disappeared, although there are still notable differences between the two types with respect to mechanical details. The present problem of the blading engineer is rather that of deciding what type of element makes possible the most reliable construction which also will meet the required efficiency level at the lowest manufacturing cost.

The research and engineering aspects of the thermodynamic and fluid-flow problems related to blading design are of the utmost importance. Extensive research has been carried out and is still under way in connection with the determination of the most efficient forms of steam passages. In recent years, attention has been given to the further development of steam-flow problems along lines adopted by aerodynamic engineers in studying the lift and drag of airfoil sections. It is not the purpose of this paper to present the steam-flow phases of the work, but rather to indicate some of the mechanical aspects of blading construction which have been developed in the last few years to meet the demands for high-capacity high-speed machines, designed for maximum pressures and temperatures. Some of the considerations given may seem elementary; others may be open to argument and discussion. In spite of the fact that the steam turbine is not a newcomer in the field of engineering, it must be admitted that recent developments have caused quite radical changes in design reasoning. It may further be said that the design development of machines for the highest pressures and temperatures is still in the process of revision and research.

In this paper, the author has been permitted to supply engineering data from the steam-turbine practice of the Allis-Chalmers Manufacturing Company. The subject matter concerns principally the recent mechanical developments in blading elements.

Blading Materials

Air-Hardening Versus Non-Air-Hardening Stainless Steel

Even the advanced metallurgical practice of the present day, has not produced a blading material in which the most desirable values of all the various physical and chemical properties can be provided in the same alloy. For example, in some materials in which there is high strength at high temperature, the hardness is so great that they cannot be machined. Other materials may have excellent properties, except that the corrosion resistance may not be sufficiently great. Among the materials available, there will be found certain classes of alloys which do not have the highest resistance to stress corrosion, but which do have other essential qualities, i.e., high physical properties at elevated temperatures and weldability without air-hardening. Consideration of all these properties indicates that such materials best meet the requirements for most blading service. In particular, these statements refer to certain of the alloy steels, in which the iron constituent is in the gamma phase, which come under the general classification of austenitic steels.

These materials must be properly processed to avoid undue carbide precipitation or growth of the crystalline grains at the steel mill or during the final-assembly heating operations. Under certain conditions of application, suitably low stresses may of necessity be adopted if impurities are present in the steam which might, in the presence of high stresses, cause stress corrosion. It is the conclusion that the strength at high temperature, the resistance to the various types of corrosion, and appropriate consideration of welding make alloys of this class the most generally useful in steam-turbine-blading practice.

Cyclops No. 17-A Alloy

The standard Allis-Chalmers blading steel is known as Cyclops No. 17-A alloy and is produced by the Universal Cyclops Steel Corporation, Titusville, Pa.

The composition is as follows:

	—Per cent—
Carbon	0.35 to 0.45
Nickel	19 to 20
Chromium	7 to 8
Silicon	0.95 to 1.25
Manganese	0.55 to 0.75

This material has an ultimate strength of 105,000 psi at room temperature in rolled bars $^3/_4 \times 1^1/_2$ in. The corresponding

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Contributed by the Power Division and presented at the Semi-Annual Meeting, Milwaukee, Wis., June 17–20, 1940, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

proof stress at 0.01 per cent plastic yield will be about 50,000 psi. The material has good physical properties at high temperatures.

Recent developments with this material have been in the direction of improved quality and closer control of process work. The austenitic structure of this alloy and its consequent capability of being welded without air-hardening facilitates the blade-assembly processes. During the last year, there has been an extensive study of the rolling-mill processes employed in the production of sections which are rolled to final size. In previous practice, the rolling-mill passes following the last rolling heat were carried out in such a manner that the percentage reduction at various parts of the blade section was not the same. The percentage reduction in the region of the outlet edge was much greater in the last roll passes than the corresponding reduction in the thick part of the blade section; hence, there was a substantial difference in grain shape and size between the material in the edges and the material in the thick parts of the sections. Harold Stein[2] has developed and made effective during the last year a change in the rolling-mill program, whereby the rate of reduction in the passes following the last rolling heat is made essentially the same throughout the widths of the blade sections. This procedure is effective in producing grains substantially equiaxed in form and practically equal in size over the full widths of the sections. The internal stresses set up between the thin and thick portions of the blade sections are reduced considerably by this procedure. It is the present conclusion that certain cases of stress corrosion will be prevented by this change in the method of rolling.

In the rolling of blade sections by the older method, the ends of the bars came out of the rolls tapered to a considerable degree, the bar being several inches longer on the outlet-edge side than on the thicker inlet-edge side. With the new process, the material in the inlet and outlet edges is of approximately the same length. In other words, the bars come out of the rolls nearly square at the ends.

Further improvements have been made at the steel mill with respect to finish-rolling. A cold-finishing pass has been adopted, which gives a higher degree of surface finish to the sections. After the final cold pass, the Cyclops material is stress-relieved at 1425 F.

13-Chrome Stainless Iron

The analysis of 13-chrome stainless iron is as follows:

	Per cent
Carbon	0.06 to 0.13
Chromium	11.5 to 13.0
Silicon	0.50 Max
Manganese	0.25 to 0.80
Nickel	0.50 Max

For the extreme low-pressure end where the operating temperature is low and where the maximum strength is necessary, the 13-chrome stainless iron is considered the best material available. It possesses reasonable corrosion resistance, combined with high strength and good endurance limit. Welding operations must be carefully planned and executed, otherwise the air-hardening characteristics of this material may introduce difficulties. The 13-chrome stainless iron is used by all of the large turbine builders in this country for the highest-stress low-pressure blades. In Allis-Chalmers practice, this material is used for low-pressure blades in which the annular-blade outlet area at 3600 rpm exceeds approximately 21 sq ft.

[2] Director of Research, Chemistry, and Metallurgy, Allis-Chalmers Manufacturing Company, Milwaukee, Wis.

19 Per Cent Chrome, 9 Per Cent Nickel Shroud Material

This is a standard alloy used for reaction-blade-shroud sections of the present standard type which are secured in place by welding. All new central-station machines will have reaction blading so shrouded.

The carbon content is about 0.1 per cent. The columbium content varies from 0.7 to 1 per cent to stabilize the material against carbide precipitation. This material is capable of being easily welded. Standard embrittlement tests show that the material is substantially immune to this type of attack.

25 Per Cent Chrome, 12 Per Cent Nickel Weld Rod

Coated weld rods of this material containing about 0.1 carbon with a stabilizing addition of columbium are made up in sizes as small as $^1/_{32}$ in. in diameter. They are used for welding 19-9 chrome-nickel shrouding on the Cyclops 17-A blading, or the welding together of the integral shrouding on the Cyclops impulse blades.

The welding of austenitic blading materials requires special welding units of the mercury-arc-rectifier type which permit welding with current flows down to about 2 amp. Such welds are formed with a minimum of local heating. Excessive grain growth is prevented by the rapid cooling of the weld by the chill effect of the adjacent material.

Everbrite Base Material

Intermediate blading for moderate pressures, temperatures, and blade speeds is prefabricated in segments in which the ends of the rolled blade sections are joined by casting them into a foundation of Everbrite alloy. Everbrite is a casting alloy of the following composition:

	Per cent
Copper	59.0 to 65.0
Nickel	29.5 to 31.5
Iron	5.0 to 8.0
Manganese	0.60 Max
Carbon	0.25 Max
Silicon	0.60 Max

The process work in connection with this development requires very accurate control of many factors, including extreme cleanliness of the blade ends, a carefully worked out process for the gating and pouring of the blade group castings, and an extremely close control of the temperature of each pot of metal that is poured. With proper control, an excellent bond between the rolled blade material and the cast foundation is obtained.

Summary of Application of Blading Types in Steam Path

1 Impulse blading is milled from bars of the Cyclops alloy with integral base and shroud sections. The shrouding is for the purpose of confining the steam flow to the blade path and also for reinforcing the structure mechanically. Circumferential-groove blading is used up to about 25,000 kw for top-turbine service and 75,000 kw for condensing machines at 3600 rpm. Axial-slot roots are used for higher capacities. An important recent development is the welding of the shrouding in groups of two blades for large machines.

2 High-pressure high-temperature reaction blading is milled from Cyclops bars with integral roots. After assembly in the spindle, the 19-9 chrome-nickel shrouding is welded in place. A characteristic of the high-temperature shrouding is the small number of blades per segment, usually not exceeding 4 to 6 for the highest temperatures.

3 The intermediate blading for moderate temperatures and stresses is prefabricated in segments by casting the rolled sections into foundations of Everbrite or cast iron. The 19-9 chrome-nickel shrouding is welded in place prior to casting.

4 Where stresses make necessary the use of milled low-pressure blading, the Cyclops alloy is used in conjunction with the welded 19-9 chrome-nickel shrouding up to annular areas of 21 sq ft at 3600 rpm.

5 Where the annular-blade outlet area exceeds about 21 sq ft at 3600 rpm, 13-chrome stainless-iron blading is used with roots of the axial-slot type. Shroud sections and lashing wires are of 19-9 chrome-nickel steel, welded in place.

PARTIAL-ADMISSION IMPULSE BLADING FOR HIGH-CAPACITY TOP TURBINES

Review of General Problem

Partial-admission impulse blading has been a standard element of design over a long period. However, the industry has experienced numerous troubles with such blading in the last few years. It may well be asked, what has happened so quickly to upset the design practice established by half a century of practical turbine experience with partial-admission wheels? The answer is believed to be that the magnitudes of the suddenly applied steam forces have been greatly increased beyond the limits of the older practice, largely because of the introduction of high-capacity top turbines at 3600 rpm and, at the same time, the operating temperatures have increased to such an extent that the physical properties of blading materials have been substantially reduced.

The older forms of analysis employed in the design of partial-admission impulse blading were based on static values of the tangential steam driving forces, usually without allowances for stress amplification due to the rapid rate of steam load application and removal, or the effect of the application of the steam driving load in successive revolutions under conditions of resonance. In the earlier partial-admission wheels, the tangential steam driving forces were of much lower magnitude, and the blade heights were also less; hence, the effect of the repeated application of the rapidly applied steam loads did not generally set up destructive stresses. In such machines, the errors introduced by neglect of the stress-amplification factor did not usually cause trouble because of the small magnitudes of the bending stresses. Where troubles developed in the older practice, due to exceeding the safe stress limits accidentally, they were usually corrected by the installation of stronger blades, sometimes without very complete knowledge of the exact cause of failure.

Blade developments for high-capacity top turbines have led to an extensive investigation of the performance of partial-admission impulse blading, with the result that it is now possible to predict with reasonable accuracy the maximum possible stresses in such blading.

Stress Amplification Due to Effect of Successive Impulses

In the following discussion, the successive steps in the development of the theory of dynamic loading will be reviewed. A cursory review of the elementary principles involved because of their importance will also be included.

Fig. 1 illustrates the elementary principle of suddenly applied loads. The spring in the diagram represents the elastic-spring scale of a given blade. The mass of the platform can be taken to represent the mass of the blade. The weight W represents the tangential steam driving force.

At the instant the load W is applied, the full magnitude of the force acts as an accelerating force. After an arbitrary deflection l is reached, the force W is still acting at the constant value established by its sudden application. There has been built up, however, at deflection l, an opposing spring force which is equal to the deflection l times the spring scale P. In other words, at the arbitrary deflection l, the net accelerating force is the difference between the steam force W and the opposing spring force Pl. The elementary theory shows that the triangular area having a horizontal base W and an altitude L_1 represents, to some scale, the energy that is imparted to the blade, which must appear in the mid-position as kinetic energy. Note that at the static deflection L_1 the force W exactly equals the opposing spring force, so that there is no acceleration at the mid-position.

If no damping is present and if the spring is assumed to have a straight-line characteristic, it is evident that the weight will overtravel a distance L_2, which is equal to L_1, in order that the kinetic energy shall be converted into strain energy in the spring. Therefore, the stress in the spring at the maximum extension is twice the stress at the equilibrium position.

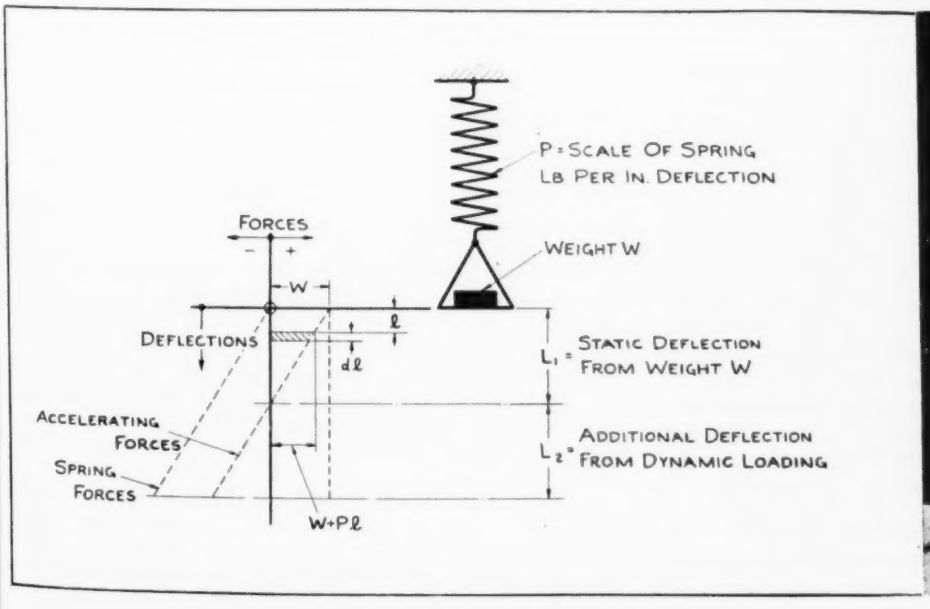


Fig. 1 Diagram Showing Principle of Suddenly Applied Loading

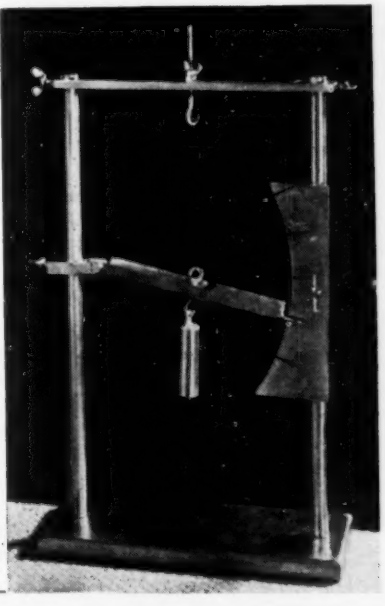


Fig. 2 Model Showing Principle of Suddenly Applied Loading

Fig. 2 shows a model which illustrates the same principle. Without the small weight, the pivoted spring-loaded beam will come to rest in the mid-position. This represents the position of a blade under the action of centrifugal force only. A slow application or removal of the steam driving force will cause the blade to fluctuate between the position of rest and the normal static deflection. This type of loading, the maximum value of which is indicated by the first index mark below the center, represents the operating conditions found in full-admission blades. If the weight representing the steam force is engaged with the hook but just out of contact with it, and is suddenly released, the model blade will deflect a greater amount than represented by the static deflection. In the limit case, as shown by the elementary theory, the maximum deflection will be twice the static value and will be represented by the lower index mark on the model.

Let it be assumed that there is no internal friction in the model and that the sudden application of the weight and the associated vibratory motion continues with no reduction in the amplitude. If the weight is suddenly removed while the blade is at the lower extreme of its motion, the model blade will then vibrate at double the former amplitude. The motion will continue from a positive position corresponding to a tension in the inlet edges of twice the static value to an equal stress in the negative direction. In other words, with a sudden single application of load under conditions of no damping, followed by a sudden removal of load, timed to produce the maximum amplitude, the stress range in the inlet edges will be 4 times that indicated by the static value.

These statements indicate only the beginning of the process. Let it be assumed, in the elementary case, that the load has been suddenly applied and suddenly removed and that the blade is now vibrating through the full amplitude indicated by theory. If, now, the blade enters the admission arc a second time with the motion in the right relation to the action of the steam force imposed by the nozzle, it is obvious that the amplitude will be further increased. The release of the steam force at the correct time will yet further increase the amplitude of vibration. The motion of the blade is thus increased in successive revolutions until, if there is no internal damping of the blade material or other friction effects, the blade would soon vibrate at a destructive amplitude. This condition of timed application and release of the steam forces is called primary resonance.

It is necessary to define primary and secondary resonance of partial-admission blading. Primary resonance is considered to be that state of motion in which the application and release of the steam driving force occurs at the phase relations to the blade motion which promote the maximum amplitude of vibration. This type of resonance is probably unavoidable in steam-turbine practice. It is, therefore, necessary to proportion the design of partial-admission blading so that primary resonance can occur and that the maximum amplitude set up by this condition will not be such as to produce harmful stresses. If, under a given operating condition, the application and release of the steam driving forces are timed to cancel part of the energy input and thus produce small amplitudes of vibration, which is quite possible, this cannot be expected to be the case for all operating temperatures or speeds. Hence, it is necessary to design the blading to run under a condition of primary resonance.

Secondary resonance is the condition which exists when the natural frequency of vibration of the blades is equal to the frequency of the impressed force from the nozzles. This condition is one which can be avoided by design, by proportioning the blades to secure the required mass-stiffness ratio. The periodic variation of the steam driving force, as the blades pass through the admission arc, may set up excessive motion if secondary resonance is not avoided.

In the practical case, in which internal damping is present, a maximum amplitude of the vibratory motion is reached in which the energy dissipated by damping equals the energy input from the intermittent steam forces. This criterion of the conditions regulating the stress-producing forces is capable of theoretical analysis and permits the determination of blade proportions which will survive such service.

It is of interest to note, in passing, that the kinetic energy imparted to an initially nonvibrating blade at the end of the admission arc is 4 times the amount imparted to the blade at the inlet end of the arc. This statement is correct only for one application of the load to a nonvibrating blade and one properly timed release of the tangential driving force in which the application and release each take place in zero time. The system in this example is assumed to have no damping.

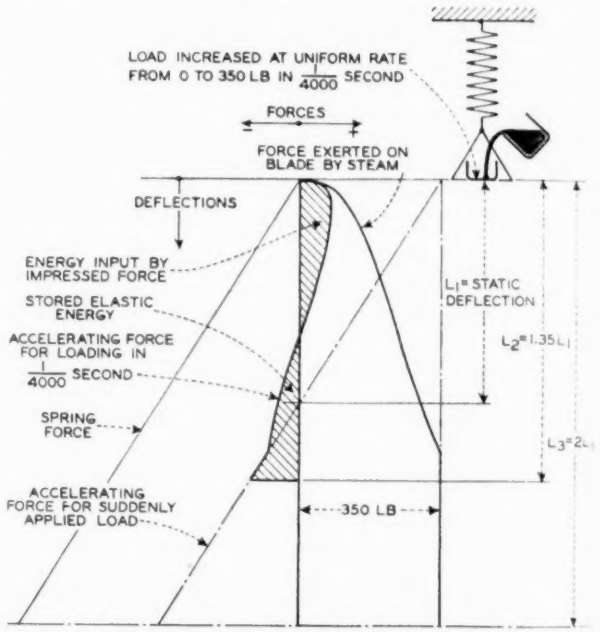


FIG. 3 DIAGRAM SHOWING EFFECT OF ACTUAL TIME OF LOADING ON BLADE MOTION

The foregoing discussion is based on suddenly applied loads. In an impulse wheel for a 50,000-kw top turbine, a standard design employs 66 blades. Each impulse blade will, therefore, receive its loading in approximately $1/66$ of $1/60$ sec, or in approximately $1/4000$ sec. Fig. 3 shows the substantial modification made in the elementary theory by the actual application of the steam load in $1/4000$ sec instead of in zero time.

As the blade enters the admission arc, the steam driving load starts at zero instead of at the maximum value which is established in the case of the suddenly applied load. The tangential driving force on the blade builds up as shown by the curve of steam force on the chart, Fig. 3. At the left of the zero-force line, the straight-line relation of spring force to deflection is shown. The accelerating force at any deflection is the difference between the force exerted by the steam and the opposing elastic force exerted by the blade.

The actual kinetic energy imparted to the blade is proportional to the shaded area at the right of the zero-force line. The energy in this case is about $1/8$ of that imparted to the blade by a suddenly applied load. On this basis, the stress-amplification factor for a single application of the driving force at a uniform rate in $1/4000$ sec would be 1.35 instead of the factor 2, which is found in the case of the suddenly applied load. This analysis

shows the substantial influence on the energy input to a blade under conditions of rapid loading in the short time interval of $^1/_{4000}$ sec, during which the full tangential driving load is built up. As indicated, the loading of such a blade in $^1/_{4000}$ sec results in 80 per cent less energy being imparted to the blade to start it vibrating than in the case of a suddenly applied load.

In the consideration of partial-admission impulse-blade stresses, the first step taken in the development of a rational design procedure was the principle of suddenly applied loads described in the preceding paragraphs. The second step was a process which included consideration of the actual time in which the blade received its load, the maximum amplitude of vibration which could be built up by successive impulses received and removed under conditions of primary resonance, and the principle that the energy input to the blade from the nozzle must be absorbed by the total damping of the system.

The third advance in the analytical reasoning on partial-admission impulse-blade design is divided into three parts. The theoretical work involved is the development of H. D. Emmert.[3] The three divisions are as follows:

a Theoretical solution of the effect of suddenly applied and suddenly removed steam forces on a vibrating blade, including damping.

b Theoretical development which includes correct consideration of the effect of the actual rate at which the steam load is applied and removed on a vibrating blade.

c Development of the complete solution of the practical problem in which the actual rate and time of steam loading of a vibrating blade is taken into account; allowance is made for the variation of steam driving force on the blade as it passes through the arc of admission; and the correct provision is made for the effect of the actual rate and time at which the steam load is removed.

The motion of elastically supported masses such as blades under the action of intermittent forces is a type of problem which

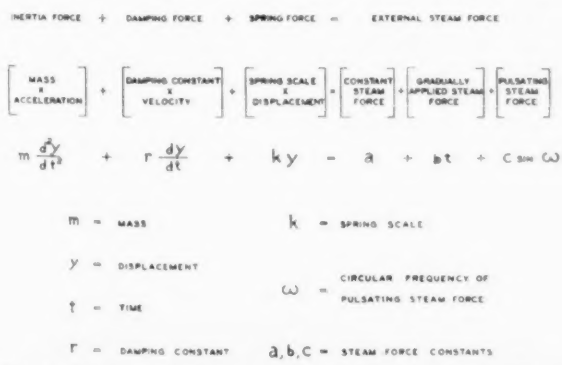


FIG. 4 FUNDAMENTAL EQUATION OF MOTION OF ELASTIC BODIES

has been fundamentally developed on a theoretical basis. The solution of the problem starts with the equation of motion, which, in simple language, is an equation the terms of which represent the various forces acting on the body.

The equation of motion as applied to this problem, referring to Fig. 4, may be expressed as

$$m\frac{d^2y}{dt^2} + r\frac{dy}{dt} + ky = a + bt + c\sin\omega t$$

where

m = mass of vibrating body
y = displacement of body
t = time
ω = circular frequency of impressed force
k = spring constant
r = damping constant

a, b, and c = steam-force constants

In the first term, m represents the equivalent mass of the vibrating body. The remaining part of the term is the second derivative of the displacement with respect to time. This quantity represents the acceleration of the body at any point in its path. In other words, the first term represents the inertia force, which is equal to the mass times the acceleration at the point considered.

The second term represents the damping force. The damping constant r is multiplied by the first derivative of the displacement with respect to time, this quantity representing the velocity. In other words, the damping force equals the product of the damping constant and the velocity.

This assumption represents the conventional procedure and is based upon the damping forces varying directly as the velocity of displacement. The actual damping forces of materials of construction may vary from this relationship. Such variation may be taken into account by experimental determination of the damping curve, and by using the value of the damping constant which is applicable for the actual working condition.

The third term represents the spring force. The term k is the spring scale and y the displacement.

The right-hand member of the equation represents the action of the steam forces on the blade.

The motion of a blade in the inactive part of the circumference is such that there are no steam forces acting, hence each of the constants a, b, and c of the three terms is equal to zero during this part of the motion.

If there is a condition of suddenly applied loading in which the steam load is assumed to reach its full value in zero time, the constant a will have a finite value, while the constants b and c will each be equal to zero.

In the case where the steam load is assumed to be applied as a linear function of time, the constants a and c become zero, and the right-hand member of the equation becomes bt.

If the steam force in the admission arc is assumed to have a harmonic variation which is found to be substantially correct in practice, the constant b of the right-hand member becomes zero, and the complete right-hand member for this case becomes $a + c \sin \omega t$.

The solutions of this type of differential equation of motion are capable of being evaluated by established methods.

IMPORTANCE OF INTERNAL DAMPING IN BLADE MATERIAL

A factor of importance in the practical application of the equations cited is the damping constant. A convenient assumption in damping problems in hard metals is that the damping forces vary as the first power of the velocity. This assumption is relatively accurate over short ranges of operating conditions. On this basis, it can be shown that the amplitudes of consecutive vibrations have a constant ratio. If this law were strictly true, the ratio of the amplitude of vibration at a predetermined time to the next consecutive amplitude would be the same as the ratio of any two consecutive amplitudes. The natural logarithm of the ratio of any one amplitude to the corresponding amplitude of the next cycle, taken as the vibration decays, is called the logarithmic decrement.

The damping constant given in the equation of motion bears the following relation to the logarithmic decrement

$$r = 2mf\delta$$

where

r = damping constant

[3] Steam Turbine Department Allis-Chalmers Manufacturing Company.

m = mass
f = frequency of vibration
δ = logarithmic decrement

Fig. 5 shows a series of damping curves, made by Prof. John T. Norton of The Massachusetts Institute of Technology, on various materials and reported by him in a paper.[4] These curves are determined by means of a torsion-pendulum apparatus, in which the specimen is subjected to alternating torsional stress. The curves are exhibited to show the great differences in the forms of the damping curves of the different materials. Considerable research is now under way in connection with the damping capacity of blading steels.

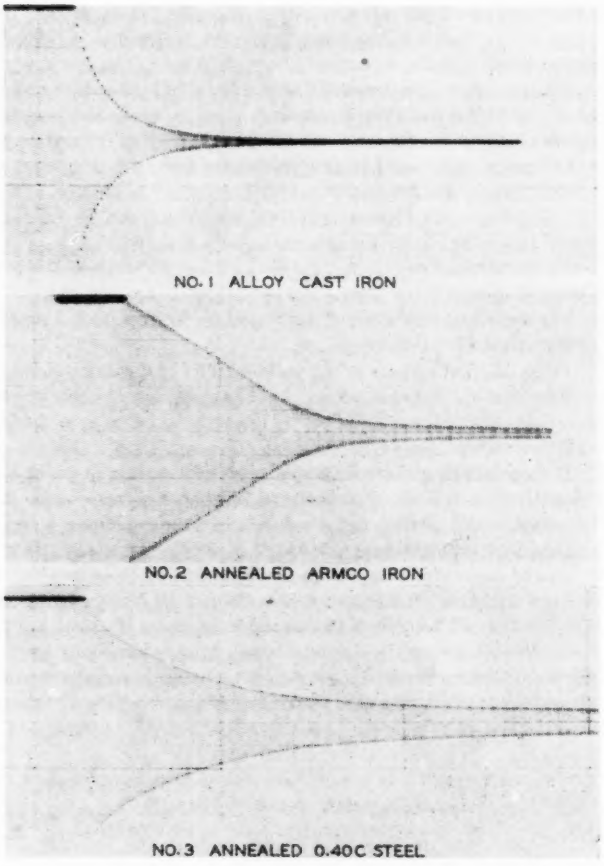


FIG. 5 DAMPING CURVES OF VARIOUS MATERIALS

At present for design purposes, with the Cyclops alloy, a logarithmic decrement of about 0.002 is assumed for blading operating with a maximum skin stress up to 20,000 psi and a temperature of 900 F.

Föppl and others have shown that the damping constant varies with stress. Other investigators have shown that the damping varies with frequency and with temperature. The research now under way will determine more completely the damping capacity of the various blading materials under operating conditions.

Proposals have been made and studied for the introduction of damping devices in the blades themselves, such as a quantity of liquid or powdered material in a cavity in the blade in which is embedded a small weight. While the idea is ingenious, it does not appear necessary to go to this refinement and, furthermore, there are design and manufacturing difficulties associated with this principle.

The first application of the third step in design reasoning is shown in Fig. 6. The length of the abscissa in each case represents the time required for the wheel to make 1 revolution. In the upper curve, the loading is increased from zero to the driving value in zero time and at the end of the arc again reduced to zero in zero time. The lower curve shows the motion of a blade after there has been a sufficient number of revolutions to set up a condition of energy stability. The blade enters the admission arc from the left with an amplitude of vibration which is reduced from the value existing at the end of the previous passage through the admission arc by the damping. The sudden loading increases the amplitude and the corresponding stress as shown. In calculating this case, it is important to note the assumption that the load was applied when the blade under consideration was at the extreme point of the backswing. This, on the chart, refers to the lowest point of the harmonic curve. This is considered a condition of primary resonance. In like manner, it is assumed that the load is released at the time the blade is at the extreme point of the forward swing, which corresponds to the upper point on the harmonic curve at the end of the admission arc. The curve shows the effect of damping in the admission arc and in the idle arc. The reduction of stress amplitude due to damping has been greatly exaggerated for illustrative purposes. By assuming acute primary resonance, as described, the motion builds up in successive revolutions, until, in the steady state of energy stability, the energy imparted to the vibrating blade is equal to that absorbed by damping.

In the case of the 50,000-kw top-turbine impulse blade, for the Northwest Station, which will be described later, application of

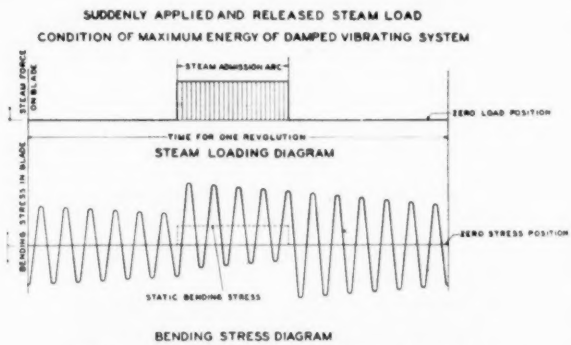


FIG. 6 DIAGRAM SHOWING DAMPED MOTION OF PARTIAL-ADMISSION BLADE WITH SUDDENLY APPLIED AND RELEASED STEAM LOAD

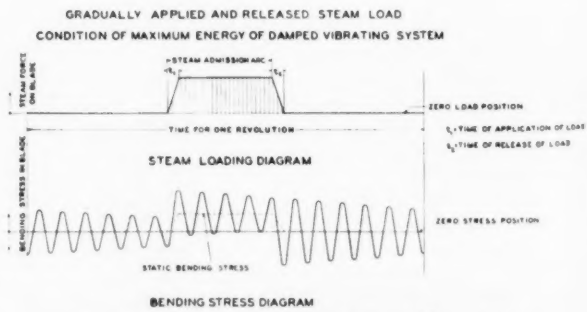


FIG. 7 DIAGRAM SHOWING DAMPED MOTION OF PARTIAL-ADMISSION BLADE WITH ACTUAL RATE OF APPLICATION AND RELEASE OF STEAM LOAD TAKEN INTO ACCOUNT

[4] "Torsion-Pendulum Instrument for Measuring Internal Friction," by J. T. Norton, *Review of Scientific Instruments*, vol. 10, March, 1939, pp. 77–81.

the process just reviewed shows that the maximum bending stress at the end of the first cycle in the admission arc will be about 10 times the static value.

The second application of the equation of motion is shown in Fig. 7. In this diagram, the tangential driving force is assumed to be built up at a uniform rate, which is also assumed for a certain condition of resonance, namely, the beginning of the application of load is at the time when the blade is at the extreme point in the backswing which on the curve is at the lowest point in the motion. For illustrative purposes it is assumed that the load application is completed in $^1/_2$ cycle of blade motion; in other words, at the time the blade reaches the extreme forward point in the swing. This case, which has been adopted for simplicity of illustrative purposes, is not quite that corresponding to exact resonance. In the latter case, the load is applied when the blade is in the position of zero deflection and starting on the backswing. This particular solution involves the assumption that the steam load is constant throughout the admission arc.

The solution further assumes that the release of the steam load starts at the time the blade is in the extreme forward point in the swing, which is at the top of the harmonic curve in Fig. 7. The condition of exact resonance at the release of the load would be that corresponding to the similar state at the point of loading.

The third application of the analysis is illustrated by Fig. 8. In this illustration, the motion of the blade in the idle portion of the arc is treated as a damped free vibration as in the other cases.

The application and release of the steam driving load is assumed to vary with time, this making necessary the treatment of the admission and release phases of the motion as separate calculations. As in the preceding case, the chart shows the load application beginning at the time the blade is in the extreme position of the backswing and ending at the extreme forward limit of motion. The release is taken in the reverse manner.

The steam force imposed on the blade in the admission arc is assumed to vary in the harmonic relation indicated on the chart.

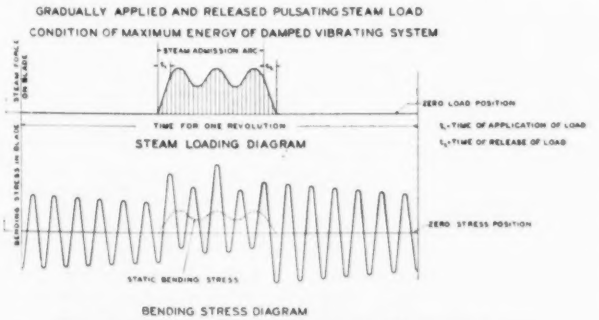


FIG. 8 DIAGRAM SHOWING DAMPED MOTION OF PARTIAL-ADMISSION BLADE WITH ACTUAL RATE OF APPLICATION AND RELEASE OF STEAM LOAD TAKEN INTO ACCOUNT

(The effect of the variation in the intensity of the steam force on the blade at the various points relative to the nozzle is included.)

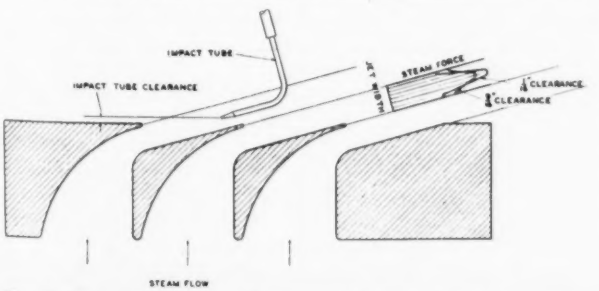


FIG. 9 DIAGRAM SHOWING RESULTS OF IMPACT TESTS OF NOZZLES AT VARIOUS DISTANCES FROM NOZZLE FACE

When this process is applied to the 50,000-kw Northwest turbine blade, the solutions of the equations of motion show that the bending-stress-amplification factor, due to the considerations mentioned, is approximately 3.

Impact-tube tests have been made with standard nozzles in which compressed air is used instead of steam. Fig. 9 shows a summary of the relative results of one set of such tests with the impact-tube-pressure measurements taken at various circumferential positions on the mean diameter of the nozzle.

It is interesting to note that the variation of the pressures per unit area is much greater for the set of readings taken at an axial

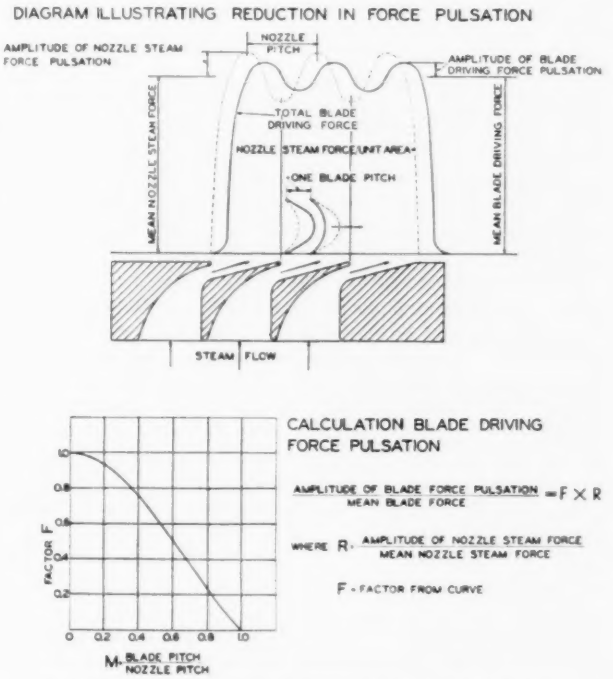


FIG. 10 DIAGRAM SHOWING EFFECT OF NOZZLE PITCH ON VARIATION OF STEAM FORCES ON BLADES AT DIFFERENT POSITIONS ALONG NOZZLE FACE

clearance of $^1/_{16}$ in. from the outlet of the nozzle than in the case of the readings taken at a clearance of $^9/_{16}$ in. This curve indicates the substantial reduction in the variation of the forces per unit area which occurs when a greater axial distance is allowed between the outlet edges of the nozzle vanes and the inlet edges of the blades.

An important factor in the design is the relation of the pitch of the nozzles to the pitch of the moving impulse blades. The upper dotted curve Fig. 10, shows typical steam forces per unit area for various positions along the mean diameter. If the integrated effect of this pressure can be assumed as transmitted directly to the blades with equal efficiency for all positions of the blades, then, theoretically, if the nozzles have the same circumferential pitch as the blades, the tangential driving force imposed on the impulse blades will be constant throughout the admission arc. If, however, the transfer of steam driving load to the blades does not occur with the same blade efficiency for all relative positions of the blades and nozzles, then there will be some periodic variation in driving force which is believed to be the case in practice.

If the other extreme is considered, in which the blades are considered as extremely narrow radial passages, then the driving force on the blades will vary in substantially the same manner as the steam force from the nozzle.

In order to design blades of relatively high frequency, a large circumferential pitch must be used. In the case of the 50,000-kw Northwest turbine blades, this pitch is 1.6 in. on the mean diameter. The Northwest impulse blades are calculated to have a running frequency of 7100 vibrations per sec. The nozzles have been spaced on a pitch of 3.484 in., which gives a frequency of the impressed force of 1837 per sec. The curve, Fig. 10, shows that the theoretical variation of force on the blade is about 0.69 of the variation of the equivalent nozzle steam force. There is, however, a considerable margin between the 7100 per sec frequency of the blade and the 1837 per sec frequency of the impressed force. The magnification of motion from secondary resonance is relatively small under these conditions.

In concluding this section on the stresses in partial-admission blading, it can be stated that the investigations briefly reviewed have shown that it is possible to calculate the effects of rapidly applied loads on partial-admission blades. The only safe ground for progress in matters of this kind is the establishment of a correct background of fundamentals. It is believed that this has been done and that the material constants are sufficiently well known to permit the prediction of safe proportions for this type of blading.

The importance of having the natural frequency of the blade remote from the frequency of the variable impressed force from the nozzle should be further stressed. Theoretically, if the action of the steam on the moving blades occurs at the same efficiency for all relative positions of blades and nozzles, then blades and nozzles of the same circumferential pitch can be used to avoid a periodic pulsation in the steam driving force. Actually, some variation in the efficiency must be expected, as each blade moves in and out of the flows of the various nozzles; hence, there will be some periodic variation in the steam driving force on the blades. If unsuitable design proportions are selected so that the frequency of the impressed force on the blades is equal to their natural frequency, then serious and destructive resonance may take place.

The best procedure is to design the blades so that the natural frequency is substantially higher than the frequency of the impressed force. The higher the frequency of the blades, the greater the amount of energy absorbed by damping in each revolution. The total damping will depend, to some extent, on the variation of the damping constant with stress.

The general procedure for determining the stresses in partial-admission blades can be outlined as follows:

a Determination of natural frequency of blade system including correction for rotation and stiffening effect of shrouding.

b Determination of the frequency of the impressed force exerted by the steam flow from the nozzle. An equivalent-steam-force curve is assumed of simple harmonic form, the amplitude of which must be based on research data.

c Design of blade to place natural frequency of assembled structure substantially above the frequency of the impressed force from the nozzles. Secondary resonance is thus avoided.

d The motion of the blade is calculated and plotted in curve form for the condition of energy stability, in which the energy imparted to a blade during one passage through the nozzle group is dissipated by the total damping of the system in the course of one revolution.

e Care must be taken to determine the most unfavorable phase relations between the free motion of the blade and the time of application and release of the impressed force. The phase relations that will give the maximum amplitude of vibration must be determined. In other words, the blades are designed to withstand primary resonance.

f The correct procedure must be adopted properly to take into account the actual rate of application of the steam load and the corresponding rate of unloading.

g There must be reliable data available on the damping characteristics of the blade material for the operating conditions under consideration.

h The ratio of the maximum amplitude for the condition of energy stability to the static deflection of the blade under the action of the steam driving force taken as a steady value gives the amplification factor to be applied to the bending stress calculated on the static basis.

i The centrifugal stress must be added to the amplified bending stress to determine the maximum resultant stress to which the blade is subjected. All stress concentration factors must be included.

Centrifugal Stability

There is good reason to believe that the initial tightness of high-temperature blading in the spindle grooves is not maintained for a long period of time at normal speed and full operating temperature. This comes about partly due to initial creep in which

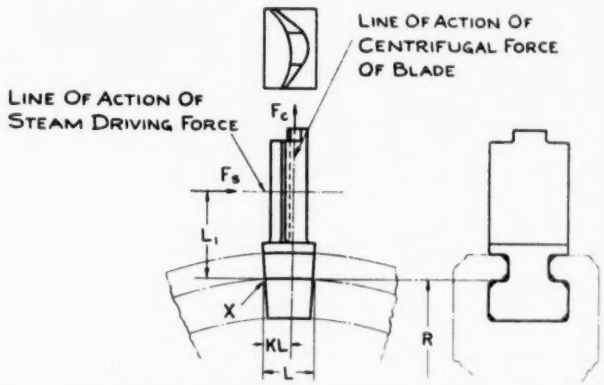


Fig. 11 Diagram Showing Principle of Centrifugal Stability

the minute irregularities of the machined surfaces are crushed until a good bearing surface is secured and partly due to elastic deformation and to the reduction in elastic modulus at high temperature. It is believed that all manufacturers produce excellent initial fits of the blade roots in the spindle grooves. At full speed and temperature, however, the close fit is believed to be lost for the reasons cited. If it be assumed that the blade root is only slightly loose at full speed and full operating temperature, conditions may arise, in which, in partial-admission wheels, the overturning moment set up by the suddenly applied steam load may exceed the stabilizing moment of the centrifugal force on the blade. If this is true, a single blade, or even a group of two or three blades which are joined together by a single shroud, may jump in the groove every time the admission arc is passed. If the blades jump or chatter in their grooves every time the steam forces are imposed on them, local stresses of sufficient magnitude to cause failure may be set up in the blade roots. It is, therefore, necessary to establish a safe ratio between the stabilizing moment created by the centrifugal force and the overturning moment set up by the suddenly applied steam load. Experience indicates that the ratio of the centrifugal moment to the steam driving moment about the base of the blade should, in most cases, be greater than 3 for a single blade, or greater than 6 for a pair of blades tightly shrouded together. In general, it is believed necessary that the stability factor shall be defined as the quotient of the centrifugal stabilizing moment divided by the amplified steam bending moment and that this ratio shall exceed 1.5.

Fig. 11 illustrates the principle of centrifugal stability. The centrifugal moment is the product of the centrifugal force *Fc* and the moment arm *KL*. The steam moment is the product of the



FIG. 12 ASSEMBLED IMPULSE WHEEL; DES MOINES

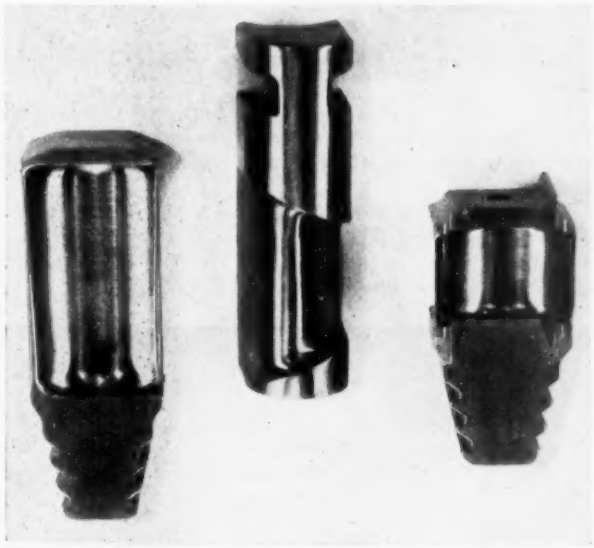


FIG. 13 INDIVIDUAL IMPULSE BLADES; DES MOINES

driving force F_s and the moment arm L_1. The stability factor without amplification is, therefore, expressed as F_cKL/F_sL_1.

LIMITING CAPACITY OF CIRCUMFERENTIAL-GROOVE BLADING

Fig. 12 shows the high-pressure impulse wheel for the 35,000-kw 3600-rpm condensing turbine at Des Moines, Iowa. The first-row blades are $1^1/_2$ in. wide, and the steam-port height is 1 in. The steam load is 238 lb per blade at the $^1/_2$ load point on the turbine. This condition exists when the first two nozzle blocks, to which steam is admitted simultaneously, are operated at full inlet pressure. These blades are of Cyclops 17-A alloy, with the integral shrouding welded together in groups of two blades. The second-row impulse blades are shown welded in groups of three. These second-row blades have since been modified by welding the shrouding in groups of two blades.

Fig. 13 shows a group of standard impulse blades for a two-row wheel of the type used for the high-pressure element of 3600-rpm condensing turbines up to about 75,000 kw or for 3600-rpm top-turbine service up to about 25,000 kw. One of the important features is the adoption of radial planes on the fronts and backs of the root and shroud sections of the moving blades. In the short first-row blades, the center portion of the root and shroud on each side is in a plane passing through the axis of the spindle. The overhanging edges have corresponding projections on the root and shroud to reinforce them. The cooperating fit between pairs of blades is on the central radial-plane surfaces. Note that between the unwelded shroud sections, a clearance of approximately 0.005 in. is allowed during the cold assembly.

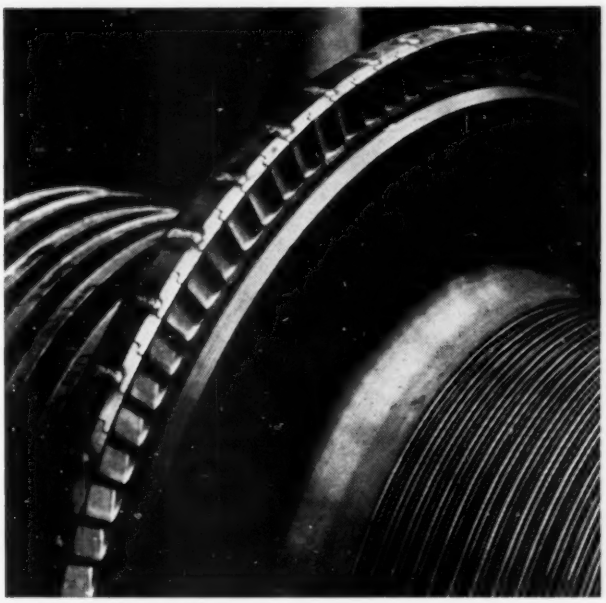


FIG. 14 ASSEMBLED SINGLE-ROW IMPULSE WHEEL

The limiting of the number of blades per group to two for high-temperature service is believed important, as severe bending stresses may be set up if a greater number of blades are joined together. These stresses may be caused by the more rapid heating and cooling of the blades with respect to the massive spindle.

Fig. 14 shows a single-row impulse wheel with the circumferential-groove-type blading. This impulse wheel represents the standard high-pressure element of large Allis-Chalmers high-pressure condensing turbines.

AXIAL-SLOT BLADING FOR HIGHEST-CAPACITY PARTIAL-ADMISSION IMPULSE WHEELS

For maximum-capacity top-turbine service, the design analysis briefly described calls for extremely wide blades. Such blades have a large circumferential pitch, which makes it undesirable to leave out a blade for assembly purposes. The centrifugal force of one of these large blades is such that a practical locking device, whereby the last blade can be secured in a circumferential groove of the conventional design does not appear available. An extensive survey has indicated the necessity for roots of the axial-slot type. Fig. 15 shows the impulse-wheel assembly for the 50,000-kw Northwest top turbine. The blades are inserted in axial slots which are milled and broached in the spindle body. In this way, the full complement of blades is inserted in the wheel without the need of a special entry slot or locking blade.

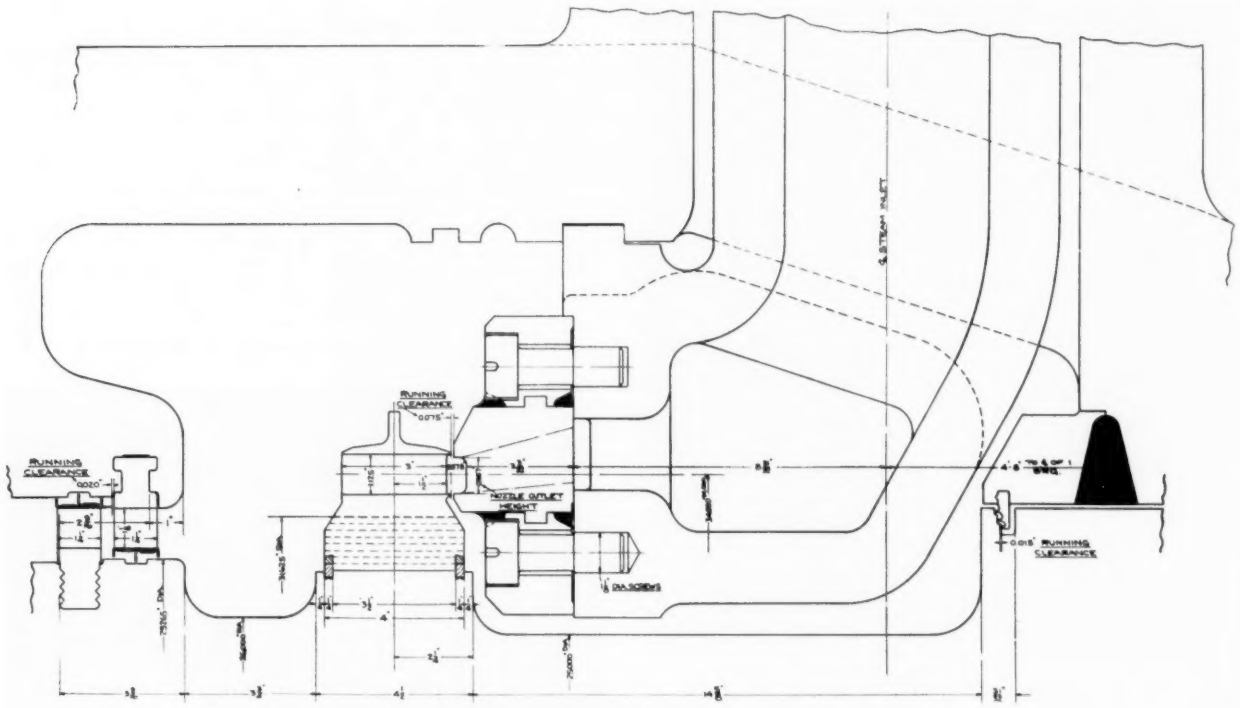


FIG. 15 SINGLE-ROW IMPULSE WHEEL 50,000-KW TOP TURBINE

In this scheme, the blades are arranged to be assembled in the axial slots by entering the complete ring of 66 blades in the spindle at the same time and pressing them home in a single operation. This is necessary because the form of the blade section requires that the edge portions overlap which, therefore, prevents individual assembly. For this purpose, each blade is first tried in its groove independently; each blade is then tried in its groove with the adjacent blade on one side and a third time with the adjacent blade on the other side, and so on around the circumference. By this means, it can be determined that there is no mechanical interference which will prevent complete assembly in one movement.

Figs. 16 and 17 show a model of the Northwest impulse blade. These illustrations show the design of the serrated root and the interlocking shrouding which requires the mass assembly of the complete row of blades.

Note that the axial width of this blade is 3 in. through the port section. The height of the steam port is $1^1/_8$ in. The mean diameter is 34 in., as shown by Fig. 15. The shroud sections are welded in pairs after assembly. The deep radial rib on the

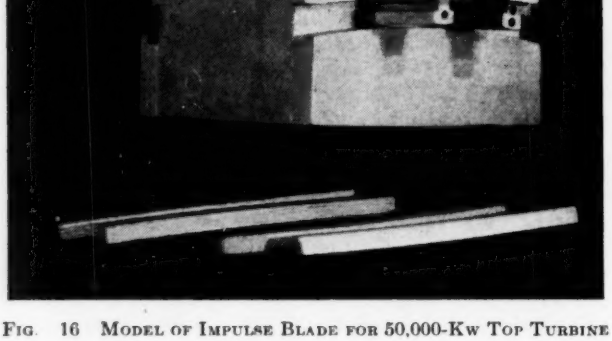


FIG. 16 MODEL OF IMPULSE BLADE FOR 50,000-KW TOP TURBINE

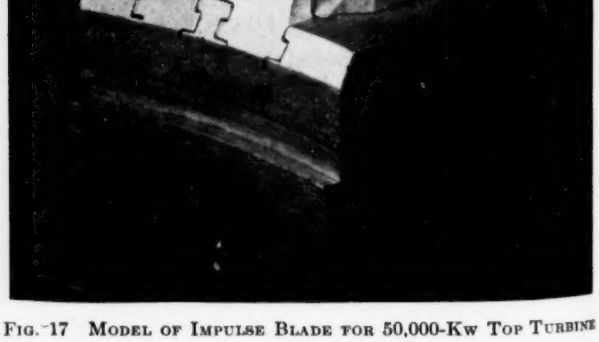


FIG. 17 MODEL OF IMPULSE BLADE FOR 50,000-KW TOP TURBINE

integral shroud is for the purpose of increasing the strength of the welded bonds between pairs of blades. The blades are milled from Cyclops steel bars. The maximum steam load is 350 lb per blade.

Effect of By-Passing the Impulse Wheel on Blade Loading

Some cases of trouble have been reported with top-turbine impulse blades in machines in which all of the steam was supplied through nozzles which acted on the first wheel. In such machines, the first group of nozzles, with which the highest blade loading occurs, occupied a relatively small arc of the circumference. This in turn required relatively high blades with a correspondingly greater length of moment arm on which the steam driving force acted. The small admission arc for the first nozzle group resulted in a correspondingly high driving force per blade. The combination of these factors resulted in extremely high stresses in some instances.

The latest top-turbine practice is to use the impulse wheel up to about $^3/_4$ load, by-passing this element for full load, and further by-passing some of the full-admission stages for maximum load. By this means and by using only two nozzle groups for the impulse wheel for the high-capacity machines, the first group occupying approximately one half the circumference of the wheel, the force per blade is substantially reduced, and, at the same time, the moment arm on which the steam force acts is also reduced greatly below the values found in former practice.

Effect of Avoiding Steam Underexpansion at the Outlet of the First Nozzle Group

In maximum-capacity top-turbine design, it is an accepted practice to design all of the first-stage nozzles of the nonexpanding type; in other words, the steam passages are designed to expand the steam down to the sound velocity. In certain early machines where all of the steam flow passed through the impulse-wheel nozzles, the quarter-load-nozzle group was allowed to expand the steam from 1200 psi down to something considerably less than the critical pressure. This procedure resulted in a large pressure drop through the blading and, in the case of some two-row impulse wheels, a substantial upsetting of the load distribution between the first and second moving rows of blades. With maximum-capacity wheels, the present practice is to arrange the first nozzle group to be fully open at the time that the pressure in the first stage is reached which corresponds to the sound velocity at the nozzle outlet. This practice aids in the reduction of stresses.

Effect of Edge Thickness on Flow Pattern Between Nozzle and Impulse-Blade Edges

It is well known that the maximum efficiency in high-velocity impulse blading requires thin blade and nozzle-vane edges. One concomitant of 1200 psi initial pressure and 925 F initial temperature is thick nozzle-vane-outlet edges and thick impulse-blade edges; at least the edges are relatively thicker than found in turbines of a few years ago. The increased edge thicknesses necessary in high-pressure high-temperature machines have had a small but detrimental effect on the efficiency. The steam-flow pattern opposite each nozzle outlet is also adversely affected to the extent that the alternating forces imposed on the blades as they pass from the stream issuing from one nozzle outlet to the next is believed to be the means whereby the amplitude of vibration of the impulse blades is increased over what it would be with nozzles and blades having thin edges.

Effect of Increasing Axial Distance Between Outlet Edges of Nozzle Vanes and Inlet of Impulse Blades

In the interest of reducing as much as possible the variation in loading on the impulse blades as they pass through the successive flow patterns of individual nozzles, the axial distance between the outlet edges of the nozzle vanes and the inlet edges of the impulse blades is increased to about $^9/_{16}$ in. in the case of the 50,000-kw Northwest top turbine. Taken parallel to the absolute direction of steam flow, the actual distance between the nozzle-vane edges and the blade edges is about 2 in. This is expected to result in some equalizing of the steam-flow pattern at the inlets to the moving blades. It is believed that by this means less total energy will be imparted to each blade in the form of vibratory motion as it passes through the successive flow patterns of the individual nozzles.

DETERMINATION OF STRESSES IN FULL-ADMISSION BLADING

Centrifugal Stresses

Centrifugal stresses in full-admission blading present no particular problem as the determination of forces and areas may be simply calculated from the drawing dimensions.

Bending Stresses

Bending stresses are calculated from the independent steam forces exerted on each blade, and a knowledge of the section properties of the blade section considered as a beam. The tangential driving forces and the axial forces resulting from pressure differences over the blade rows are independently considered.

Ripple Effect

Full-admission blades, especially in high-capacity high-temperature machines, in which relatively thick blade edges must be used, are expected to undergo some periodic variation in the driving forces. This ripple effect probably does not exceed 10 or 15 per cent of the steady driving force. In high-temperature machines, the ripple effect is taken as about 25 per cent of the steady tangential driving force.

Shrouding Correction

The effect of shrouding varies considerably over the blade path. In high-pressure blading, the effect of the shrouding on stress is considerable, although the normal bending-stress reduction may be between 25 and 60 per cent. The effect of the shrouding on bending stress is complex, the calculations on stress reduction being checked by experimental procedure.

Stress Amplification Due to Vibration

The high-pressure and intermediate-pressure blading, in which the natural frequencies of the blades are well over 5 times the rotational frequency, are assumed to be sufficiently remote from a resonant condition arising from periodic steam forces or inertia forces caused by the rotation of the spindle that the stresses are not intensified thereby. In low-pressure blading, in which the frequency of the assembled blades at full speed and at the operating temperature is less than 4 times the rotational frequency, tuning is resorted to in order to avoid resonance under operating conditions. This tuning involves a calculation of each specific blade group, which is followed by a check of the standing frequency when the row is assembled. Correction for temperature and speed is made by calculation. Changes of blade sections, of the rate of taper, or of lashing or shrouding members are sometimes necessary in order to avoid resonance.

Stress Intensification Due to Fillet at Base of Port Section

Milled blading with integral roots is designed with generous radii at the junctions of the port sections and the roots, so that

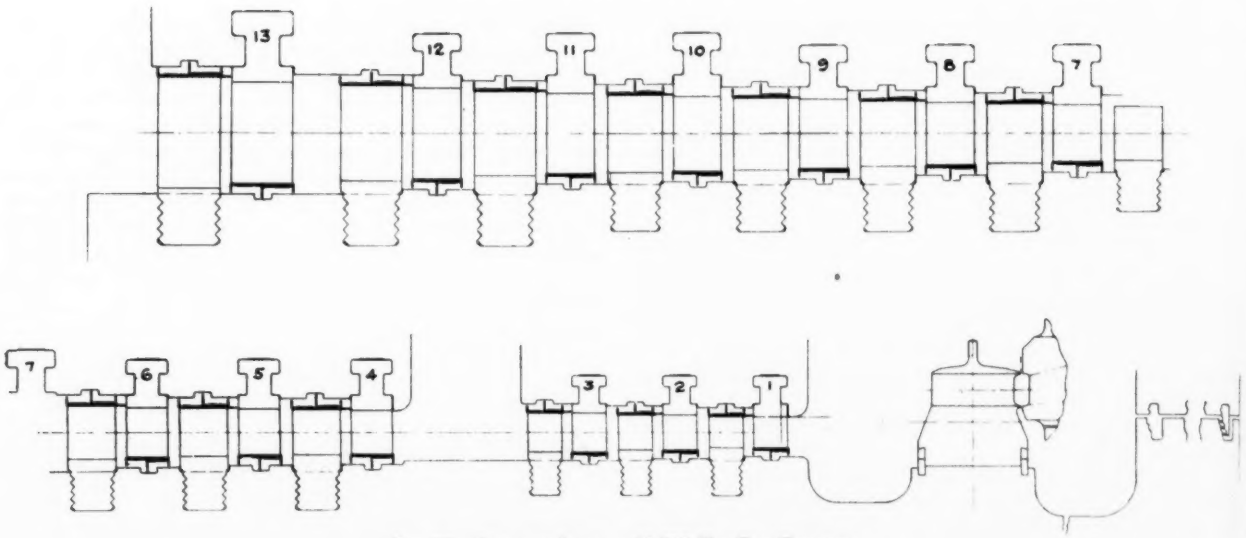


FIG. 18 BLADING LAYOUT 50,000-KW TOP TURBINE

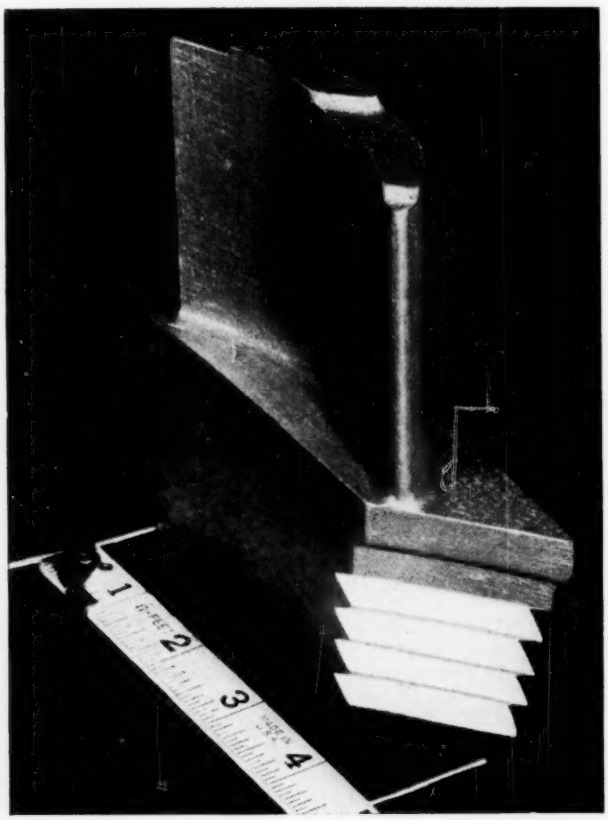


FIG. 19 MODEL BLADE, LAST ROW 50,000-KW TOP TURBINE

the stress-intensification factor can usually be taken at 1.5.

For the intermediate blading with cast foundations, the bending-stress-intensification factor, due to the form of the junction between the rolled blade and the cast base, is taken at 2.

HIGH-PRESSURE REACTION BLADING

BLADES WITH INTEGRAL ROOTS MILLED FROM BAR STOCK

Fig. 18 shows the blading layout for a 50,000-kw top turbine. In this example, all of the blading is of the individual milled type with integral roots. Fig. 19 shows the large size of high-capacity blades of this type. The nominal axial width of the last row of blades, one of which is illustrated in the view of the model, Fig. 19, is $2^1/_2$ in. This blading is milled from bars of Cyclops 17-A alloy. Fig. 20 shows some reaction blades of the milled type for top-turbine service.

DESIGN OF SHROUDING

Referring again to Fig. 18, it will be seen that the shrouding on the stationary blades is made up of two independent shroud sections secured in place by welding. There are three radial-seal strips for each blade row.

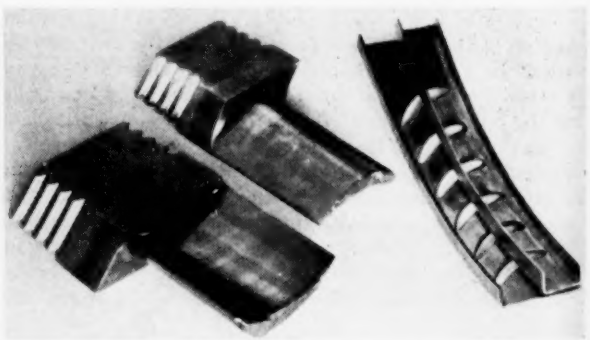


FIG. 20 MILLED REACTION BLADES FOR TOP TURBINE

There is a groove in the spindle to contain the center-seal strip and thereby provide interruption to the leakage flow. The inner axial-seal strip formerly employed is absent in this construction. This is done to avoid the disturbance set up by the high-velocity leakage flow cutting at right angles to the main flow through the blade path.

The shrouding on the spindle blading includes four separate seal elements; the first is of the normal end-tightened type; the remaining three are of the radial type as shown. The shrouding is of welded 19-9 chrome-nickel steel.

Note that the stationary blades of such high-capacity turbines are also of the individual milled type.

WELDING OF SHROUDING

The welding of shrouding has been adopted after considerable research as being the best means for the attachment of such

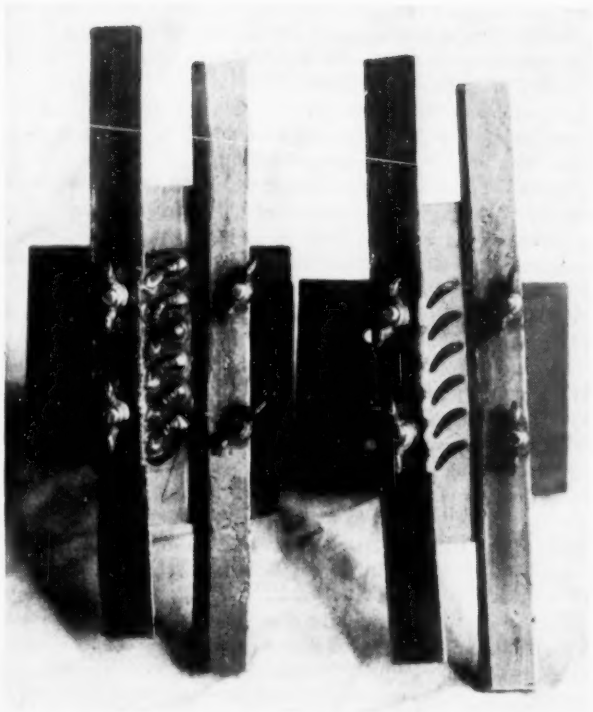


FIG. 21 WELDING OF SHROUDING ON SEGMENTAL BLADING



FIG. 22 WELDED REACTION-BLADE SHROUDING IN FISK TURBINE

members. The stresses set up by riveting are avoided. By the use of austenitic materials, air-hardening does not occur. Fig. 21 shows a group of blades with a section of shrouding ready for welding. A welded shroud section is also shown.

Fig. 22 shows the welded shrouding in the 30,000-kw top turbine at Fisk Station in Chicago.

BLADES WELDED IN SMALL GROUPS

It is important that the shrouding on high-pressure reaction blading include a relatively small number of blades per segment. The bending stresses introduced by rapid expansion and contraction of the blading under conditions of varying temperature may result in excessive stresses and lead to ultimate blade failures unless short segments are used. Fig. 22 shows that the reaction

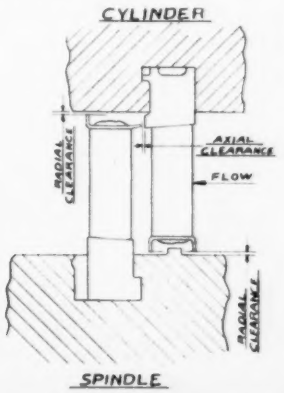


FIG. 23 ASSEMBLY DRAWING OF SEGMENTAL INTERMEDIATE BLADING WITH WELDED SHROUDING

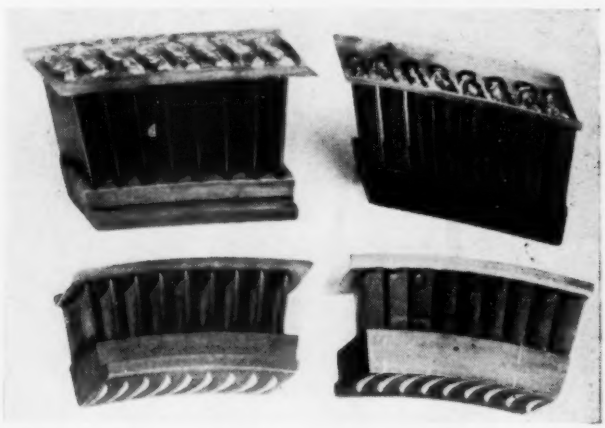


FIG. 24 INTERMEDIATE-BLADING SEGMENTS WITH WELDED SHROUDING

blades in the Fisk turbine have shrouding which contains from four to six blades per segment. It has already been pointed out that the impulse-blade practice is to secure two blades together for the highest-temperature service.

INTERMEDIATE BLADING

BLADES ROLLED TO FINAL SECTIONS

For the intermediate-pressure blading, sections are used which are finished to the final shapes by rolling. These blade sections are of Cyclops 17-A alloy. The material is given a final stress-relieving heat at 1425 F after the rolling operation. Before using the sections, the inlet and outlet edges are carefully polished to remove minute cracks which might have been introduced during the rolling. Fig. 23 shows in section the standard arrangement of intermediate blading. The welded shrouding and the design of the seal strips are shown. Fig. 24 shows a group of intermediate blades of the cast-in type with welded shrouding.

The study of the rolling of the blade sections and the improvements made at the steel mill have been previously described.

Shrouding Design

Blade sections, after being cut to length and after the inlet and outlet edges are polished, are assembled in a special fixture, after which the 19-9 shrouding is welded in place.

Cast Foundation of Everbrite or Cast Iron

After the shrouding is welded in place, the segmental-blade assembly is then placed in a core box, after which the blades and shrouding are encased in core sand, with the exception of the ends of the blades that are to be cast in the foundation. The blades in their assembled core are placed in a suitable mold and the foundation of Everbrite or cast iron poured. Great care is necessary to maintain clean blade roots. It is also necessary to keep the pouring temperature of the base material within close limits in order to secure a good bond with the blade bases without washing the blade-section material away. Optical-pyrometer observations are made on the metal poured in each mold.

Stress-Relieving of Assembled Cast Foundation-Ring Sections

After the foundations are poured, they are then finished on all surfaces, except at the bottom of the steam port, by machining.

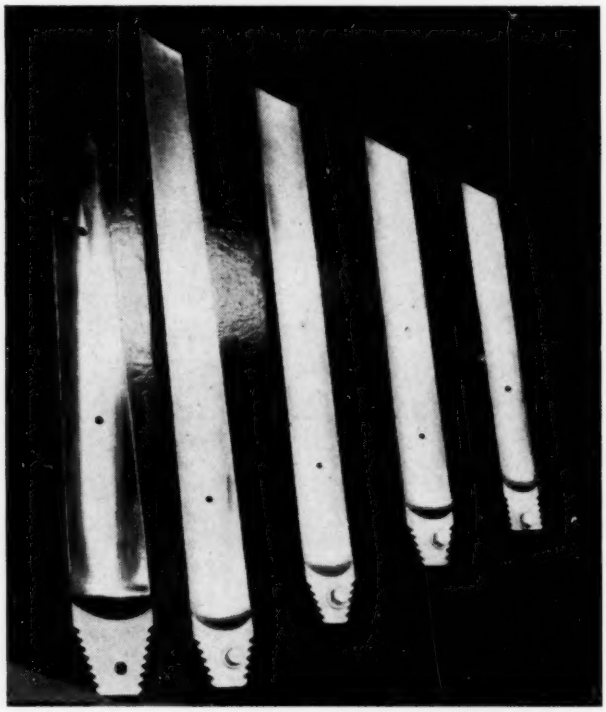


Fig. 25 Milled Low-Pressure Blading

After machining the foundations, the segments are placed in a gas-charged muffle, where they are annealed at 1425 F to remove stresses introduced by the cooling of the foundation metal. By this process, completely finished segmental blading is manufactured for the intermediate portion of the blade path. The blading is practically free from fabrication stresses and in an excellent condition to resist corrosion.

Milled Low-Pressure Reaction Blading

When the blade annulus referred to 3600 rpm exceeds approximately 9.2 sq ft, the blades are milled from bar stock of Cyclops alloy with integral roots, as shown in Fig. 25. Care is exercised in the mill operations to secure a grain size not exceeding 20 per sq in. at 100 diam magnification. The bar stock is stress-relieved at 1425 F after the final rolling.

The milled Cyclops low-pressure blading is reinforced by shrouding of 19-9 chrome-nickel steel which is welded in place as described for the high-pressure blading.

The use of lashing wires is avoided except in the case of low-pressure blades where they are useful for tuning purposes. Lashing wires are not used in blades where the natural frequency of the assembled blade groups under operating conditions is more than 4 times the rotational frequency.

When lashing wires are used in Cyclops blades, the blade is drilled and the edges of the holes suitably rounded. A short section of 19-9 chrome-nickel wire is welded in place in each blade. After welding, the blade is stress-relieved at 1125 F. When the blades are in place in the spindle, the ends of the individual lashing wires are joined in groups by welding. In this way the heating effect of the final-assembly welding does not reach the body of the blade.

LOW-PRESSURE BLADING FOR MAXIMUM TIP SPEEDS

Limiting Capacity of Condensing Machines

At the present time, the limiting exhaust-blade practice is a 20-in. blade with a tip speed somewhat over the sound velocity in steam at 29 in. vacuum. In Allis-Chalmers 3600-rpm practice, 20-in. blades are mounted with a tip diameter of 80.5 in. with a corresponding tip speed of 1264 fps. Regarding the capacities for which these blades can be used, this selection involves a commercial consideration of the actual load requirements and operating charges for a given system, which will in turn indicate the amount of leaving loss which can be allowed. If operating conditions of 1200 psi gage 925 F and 29 in. vacuum be assumed with four stages of feed heating, the leaving loss of a double-flow low-pressure end with 20-in. blades at 3600 rpm will be as given in Table 1 (on the following page) at the respective capacities. These blades are assumed to be mounted with a tip diameter of 80.5 in. and with an opening coefficient of 0.6.

The present outlook does not indicate a radical tendency toward an increase of tip speeds. The length of the Allis-Chalmers blades described is just under $^1/_3$ of the mean diameter. The form of the passage at the tip section must be a compromise with some sacrifice in efficiency in the interest of practical construction. An appreciable increase in blade length with respect to the mean diameter does not seem probable at this time.

The effect of water erosion is sufficiently serious

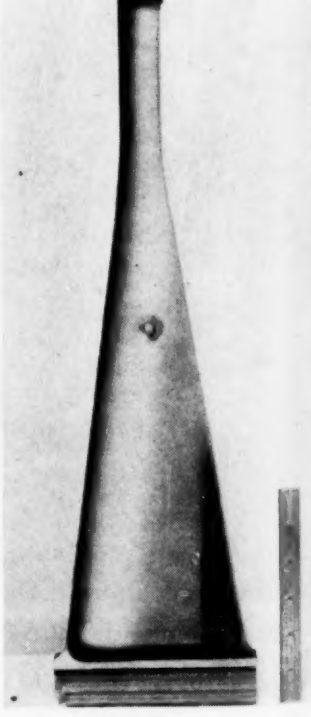


Fig. 26 Axial-Slot Type 18-In. 3600-Rpm Low-Pressure Blade

TABLE 1

Rated load	Leaving loss expressed as a per cent of total adiabatic drop including corrections for feed heating and underexpansion
40000 kw	1.9
50000 kw	2.9
60000 kw	4.0

at the present tip speeds to preclude any strong tendency for increased tip speeds. The present trend of 3600-rpm construction is, therefore, in the direction of multiple-cylinder low-pressure elements where maximum-capacity units are considered.

The present limiting annular area for 3600-rpm low-pressure blades is about 26.4 sq ft. This is the area corresponding to the 20-in. blades mounted with a tip diameter of 80.5 in. previously referred to.

Fig. 26 shows an 18-in. blade of this type with a tip diameter of 76.5 in., with a corresponding annular area of 23 sq ft. Fig. 27 shows an assembled row of these blades in place in the spindle, and Fig. 28 shows the axial-entry spindle slots in which the blades are installed. The design of these blades is based on a cross-sectional-area ratio of 6, i.e., the sectional area of the base of the blade is 6 times the area at the tip. The ratio of mean diameter to length is approximately 3, which means that there is a substantial change in the contour of the steam passage from the base to the tip.

For annular blade-outlet areas between 21 and 26.4 sq ft at 3600 rpm, the 13-chrome stainless iron is used because of its greater strength at low temperature. This material is forged to within about $^1/_{16}$ in. of the finished surfaces and then completely machined to the final sizes.

Shrouding and lashing members are necessary in the last row of blades to reduce bending stresses and to bring up the natural frequency of vibration to the required value. A section of shrouding, one blade pitch in length, is welded to the tip of each low-pressure blade. The shrouding is of the 19-9 chrome-nickel steel. A short stub end of 19-9 chrome-nickel lashing wire is welded to each side of each blade, thereby eliminating the lash-

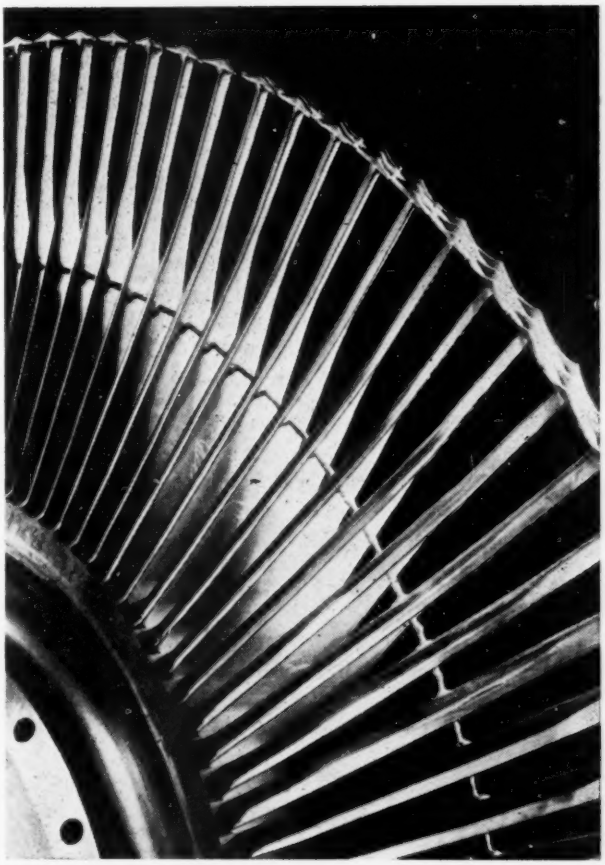


FIG. 27 ASSEMBLY 18-IN. 3600-RPM LOW-PRESSURE BLADES

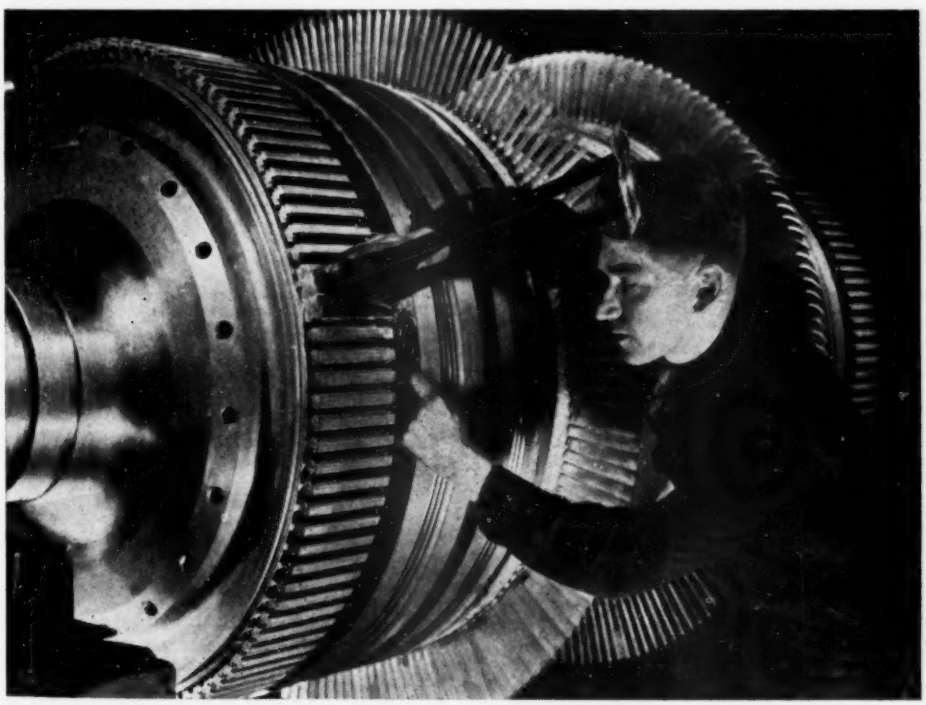


FIG. 28 SPINDLE FOR AXIAL-SLOT BLADES

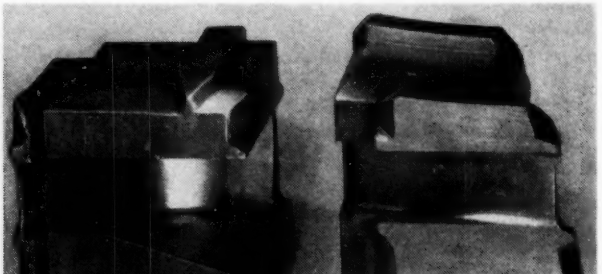


FIG. 29 NOZZLE ELEMENTS

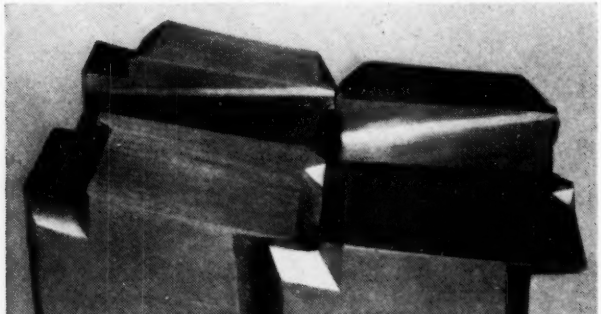


FIG. 30 NOZZLE ELEMENTS

ing-wire hole. A generous coating of stellite is welded on the inlet edge of each blade for about $^2/_3$ of the length from the tip to resist water erosion. The stellite is fused to the blade with an acetylene torch. The blade is preheated to about 900 F for these welding operations.

After the welding is completed, the blade is heat-treated by quenching in oil from 1750 F and drawing at 1125 F. In this way the harmful effect of the air-hardening is greatly reduced.

After the blades are assembled in the spindle, the corresponding shroud and lashing members are joined by welding. The welding after assembly is entirely between non-air-hardening materials. The local heating of the final welding operations does not extend to the body of the blade.

The sectional shrouding is of cantilever form, each section being secured to the blade by a generous welding bead. The extreme overhanging ends of each cantilever are joined by light welds after assembly. The overhanging cantilevers are designed to be stable in the event of accidental breakage of the assembly welds so that failure from centrifugal stress will not occur.

The two machines, equipped with 18-in. low-pressure blades which operate at a tip speed of 1200 fps, are in their second year of service and have thus far disclosed no troubles of any kind in the low-pressure blading. Two rows of these blades are installed in the low-pressure turbine of the 35,000-kw 3600-rpm turbine at Des Moines and one row in the single-flow machine at Springfield, Ill. From this experience, there is good reason to expect that the 20-in. blades, which will be designed on the same engineering principles and stress limits, will also operate satisfactorily.

NOZZLE DESIGN

Figs. 29 and 30 show the type of nozzle elements used in high-pressure high-temperature machines. The nozzle ports are machined in individual stainless-steel elements which, when assembled, form the accurately machined steam ports. The assembly drawing of a similar nozzle is shown in Fig. 31. Each individual steel block in the group of central elements is defined by radial end planes which are carefully fitted together. After

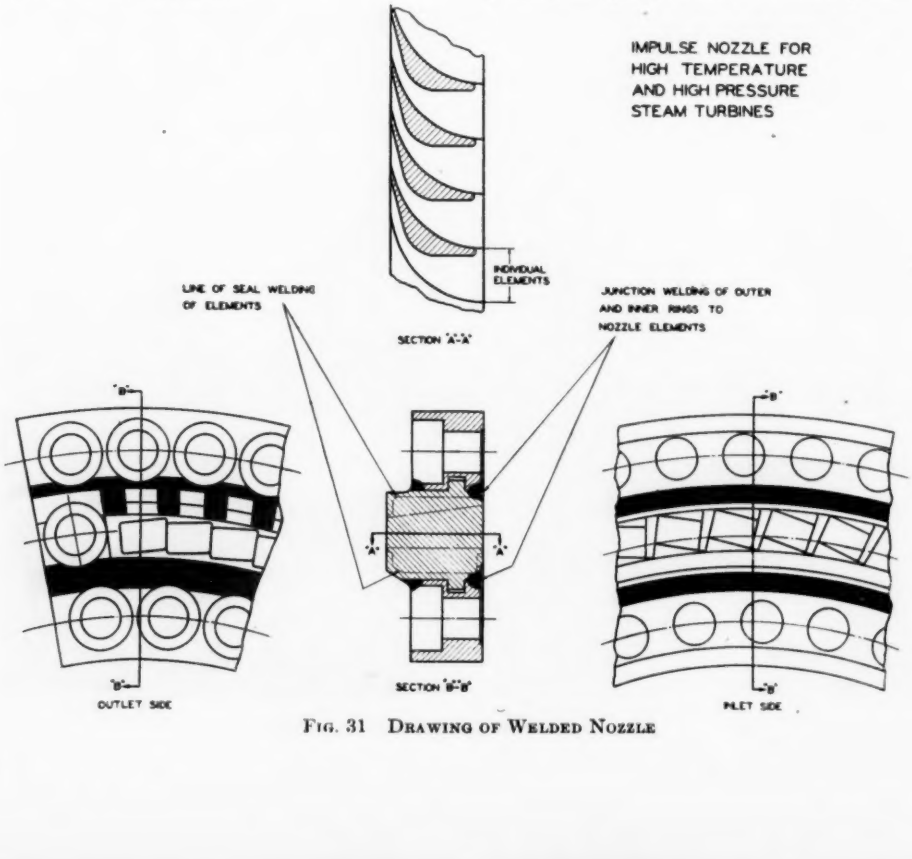


FIG. 31 DRAWING OF WELDED NOZZLE

the central group of elements is assembled, the inner and outer ring segments are fitted over tenons and welded in place.

Acknowledgments

The author wishes to thank the Allis-Chalmers Manufacturing Company for permission to publish the blading-design data included in this paper; Professor J. J. Ryan, of the University of Minnesota, and Dr. J. T. Rettaliata, of the Allis-Chalmers steam-turbine department, for their suggestions during the early development of the partial-admission impulse-blade reasoning; Professor John T. Norton, of the Massachusetts Institute of Technology, for suggestions in connection with damping phenomena and for permission to use Fig. 5; Mr. H. D. Emmert,[5] of the steam-turbine department of Allis-Chalmers, for his assistance in preparing the mathematical solutions of the various phases of the motion of vibrating blades; and to numerous other members of the Allis-Chalmers organization for their help in the preparation of this paper.

Discussion

R. P. Kroon.[6] Air-hardening 13 per cent Cr steel and non-air-hardening nickel-chrome austenitic steels have been described as applied to turbine-blade service. Many years ago the Westinghouse Company adopted 13 per cent Cr blade steel because it was one of the few corrosion-resistant alloys adapted to forging. More recently, the forging process has been abandoned for manufacturing reasons on all except low-pressure blades. We are, therefore, in a position to use any of several high-temperature alloy steels suitable for machining.

A large number of such materials were made into blades for experimental purposes and some have been used in turbines in the field. Among these materials, the 13 per cent Cr steel has distinguished itself because of its superior yield strength and its metallurgical simplicity and uniformity.

We recognize the problem of air-hardening in 13 per cent Cr steel as a result of shroud welding, but have used this process for a number of years under proper control and with but slight difficulty. It has been found recently that the 13 per cent Cr is unique in that it possesses many times the damping capacity of any other high-temperature material known to the art. We continue to regard the 13 per cent Cr stock as that having the most desirable combination of qualities for turbine blading.

We cannot very well agree with the statement that blades having a natural frequency well over 5 times the running frequency are sufficiently remote from resonance so that the stresses are not intensified thereby. It is our experience that, even with full admission, the resonance effect is important at frequencies up to say 12 times the running speed. We are using statistical data to insure a safe design for these medium-size blades.

By this time it should be well recognized that the impulse-blade problem is industry-wide. It behooves us that this fact should be admitted; the problem is there whether we do so or not.

The writer has been particularly interested to find that the assumptions, on which the author has based his calculations, are almost identical with our own original suppositions. However, we have felt very strongly that quite a few of these assumptions are based on such loose grounds as to make necessary the experimental determination of the exact nature of the forces and motions involved in actual operation.

We started a research program using optical equipment to measure actually the blade deflections, first on a small pilot turbine which was placed in operation 9 months ago, then on a full-size Curtis turbine, operating at 1250 lb 900 F. This work was recently described.[7] We are more and more convinced that this investigation is of vital necessity to give those data which cannot be derived theoretically.

There is, for example, the assumption that a blade receives its load gradually as it travels one blade pitch. In the absence of experimental data this assumption is probably as good as any other. But one must not forget that actually the forces on the blade are caused by the reactions of a very complex nonsteady steam flow on either side of the blade. Actually, we find that it may take a distance of several blade pitches before the blade force steadies out.

Instead of a gradually increasing steam load which the author has assumed, we often find that the first shock on the blade entering the steam jet is in a direction opposite to the torque force. This can be explained physically by the fact that, as the steam first enters the blade passage, it tends to push the blades apart. This pressure wave causes a negative rather than a positive torque force on the trailing blade.

Our findings have also indicated that, in general, it is not sufficient to limit consideration to vibration in the circumferential direction only. We formerly believed that the lateral vibrations of the blade were not important, but we now know that, on an impulse-blade group, there are at least three natural modes of vibration which should be considered. The first is a vibration in the plane of rotation, the second is one in which the blades vibrate perpendicularly to the plane of rotation, and the third is a torsional motion of the group as a whole. Under certain conditions, it is necessary to take into account all three modes.

In our opinion, expanding the steam down to more than sound velocity in nozzles of the nonexpanding type is quite feasible, provided that this superacoustic range is explored experimentally to establish safe limits. This we are doing in our developmental turbine. We believe that, with this background, the additional efficiency at low turbine loads can be made available without a risk.

The statement is made in the paper that, with a gradually increasing load, the maximum displacement of the blade will be produced when the initial displacement is zero, as the load is applied. That this is not so can be appreciated when one considers the limiting case of suddenly applied load, for which we know that maximum displacement is produced when the initial velocity, and not the displacement, is zero at the instant the load is applied.

The writer believes the author will be able to check his conclusions that the phase angle for optimum displacement depends upon a relation between loading time and natural period and trusts he will see fit to revise his calculations to take this into account.

G. B. Warren.[8] The author has presented an interesting analysis of the forces on the first-stage partial-arc-admission impulse-blade element, which should have common application in varying degrees to all types of turbines incorporating this construction. Such analyses should be of great value.

The group of engineers with which the writer is associated has followed a somewhat different method in its endeavors to attain the same result, namely, a stable and reliable construction

[5] These solutions in detailed form have been presented by H. D. Emmert in a "Mathematical Section" of this paper, as originally published by the Allis-Chalmers Manufacturing Company, prior to the 1940 Semi-Annual Meeting of the Society. Copies are available on request to that Company.

[6] Manager, Experimental Engineering, Westinghouse Electric & Manufacturing Company, South Philadelphia, Pa. Jun. A.S.M.E.

[7] "Superposed Turbine Blade Research," by F. T. Hague, *Mechanical Engineering*, vol. 62, April, 1940, pp. 275–277.

[8] Designing Engineer, Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

for the heavily loaded buckets of first-stage impulse elements.

Fortunately, we had as background nearly 35 years of experience in building first-stage partial-admission impulse buckets, which included several hundred rows, a few of which had given trouble in service. We became convinced during the course of this experience that the difficulties were due to vibrations produced by the operating conditions. However, it has always been felt that these vibrations and their consequent stresses were so complex and of such varied form as to make any simple rational analysis inexpedient. A statistical study of the conditions under which such troubles occurred, and those under which they were not present, indicated a rather definite function of the ratio between the severity of the conditions and the strength of the bucket and its attachment. Formulating this knowledge into design practice was undertaken about 12 years ago with the result that, except for a few isolated and special cases, difficulties with these elements have practically disappeared. Recent tests appear to indicate that no sacrifice in efficiency seems to result from the use of the wider and, hence, stronger buckets.

The indications from the paper are that the author's analysis is leading him to conclusions of about the same type and magnitude as our own.

As to the details shown, we differ in materials used, in attachments of buckets to rotors, in method or shrouding the buckets, and innumerable details of design and construction. The fact that two or three turbine manufacturers differ in such things is probably the greatest safeguard to the power industry. It insures a minimum average of trouble, and the maximum possible progress.

The design and construction technique shown for the last rows of blading on limiting-capacity turbines is most interesting. From the illustrations, it seems evident that the shroud band and lashing members finally are reduced to one continuous piece. Some additional discussion of this construction would be of interest.

A. L. KIMBALL.[9] This paper brings to focus two things, i.e., the importance of avoiding a condition of resonance, wherein a periodic exciting force is in step with one of the natural frequencies of the elastic bucket system; it brings out the importance of safeguarding possible dangerous resonant-vibration amplitudes by having as much internal damping in the buckets as possible.

The dangerous effect of resonance is well shown from the authors' Equation [19],[10] by setting the exciting frequency equal to the natural frequency so that $\omega = \omega_n$. In this case Ω is seen to be equal to zero and R reduces to the logarithmic decrement δ divided by π or $R = \delta/\pi$. The last two terms of Equation [19] then become

$$y_{max} = \frac{a}{k} + \frac{c}{k}\frac{\pi}{\delta} = \frac{a}{k} + 1570\frac{c}{k} \quad \ldots\ldots [1]$$

$$y_{max} = 158\frac{a}{k} = 0.0055 \text{ in.} \quad \ldots\ldots [2]$$

This is seen to be over 50 times as great as for the case chosen by the authors which gave $y_{max} = 0.000106$ in. Furthermore, the first term of Equation [19] was omitted, which makes the value of Equation [2] of this discussion conservative.

Note also that Equation [1] herewith may be directly derived from the case of a simple linear oscillator,[11] which alone gives us the important part of the picture.

Note that, since the deflection given by the authors equals 0.000035 in. for a steady force of 350 lb at the tip of the bucket, an equivalent force of amplitude 55,000 would be necessary to produce the deflection at resonance given by Equation [1] of this discussion, where the term amplitude in this case, equals the + or — amplitude of the periodic force, the total range of which from + to — is therefore 110,000 lb. This can be seen to produce a maximum stress far beyond the fatigue limit when it is noted that the total volume of this bucket is about 0.6 cu in. of material. Thus, if it is assumed to be of rectangular shape, 1 in. high, 1 in. wide, and 0.6 in. thick, the maximum vibration stress at its base is over 500,000 psi.

The analysis in this paper gives an excellent idea of the importance of impulse excitation of bucket vibration, and shows that its intensity is always far below that arising from resonance produced by even a moderate periodic force, which was taken as 0.1 the average bucket load in the author's case. Furthermore, the analysis also shows that the damping in the buckets is not an important factor in controlling impulse excitation.

The real reason for damping is to control the amplitudes which might arise on resonance as given by Equation [2] of this discussion.

The writer personally dislikes the use of the terms "primary resonance" and "secondary resonance." The former represents a condition of impulse excitation and is not a resonance at all in the sense that the second type is, which is by itself the true resonance.

A complicated elastic system like that of the bucket zone of a turbine wheel may have several modes of high-frequency vibration to each of which a resonant vibration will build up if in step with some periodic exciting force, and some of which may be dangerous. Furthermore, such modes of vibration are not readily predicted by available methods of analysis, so the only sure way is to find what they are from actual test.

Prevention of such failures therefore requires:

1 The definite location of the type and frequency of possible modes of vibration from actual tests, which may be in the range of known high-frequency exciting forces which arise during turbine operation.

2 A thorough study of the damping of bucket materials under operating conditions, the latter being an insurance against danger from the former.

3 Adequate area of cross section for safe performance.

These methods of attack are now being applied by turbine manufacturers who appreciate the danger only too well, and steady progress is being made.

T. C. RATHBONE.[12] The authors[10] are to be commended for the presentation of this valuable and instructive paper and the research behind it on blade-failure problems which have caused some anxiety.

The writer is particularly interested in the reasoning leading to the conclusion that, as some resonance in impulse blading is unavoidable, the blades must be designed to withstand the most unfavorable combination of conditions.

The frequency calculations are based on the assumption that the blades remain tight in the root, that is, the node point is fixed. It was pointed out that looseness in the root with partial admission permits a rocking impact or chatter which may well

[9] Engineering General Department, General Electric Company, Schenectady, N. Y. Fellow A.S.M.E.

[10] This discussion refers not only to the paper proper by R. C. Allen but also to a mathematical section by H. D. Emmert, which was presented at the meeting. Space did not permit publication of this section in the Transactions, but copies of the mathematical treatment may be obtained upon request to the authors.

[11] "Vibration Damping Including Case of Solid Friction," by A. L. Kimball, Trans. A.S.M.E., vol. 51, 1929, paper APM-51-21, p. 230.

[12] Chief Engineer, Turbine and Machinery Division, Fidelity and Casualty Company of New York, New York, N. Y. Mem. A.S.M.E.

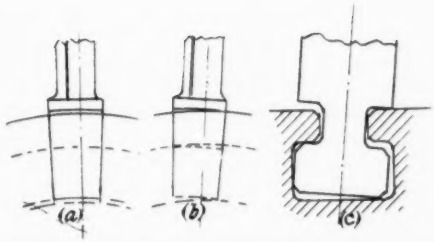


FIG. 32 EFFECT OF LOOSENESS IN BLADE ROOT

(*a*, Blade with loose root fastening, at rest; *b*, position at one extremity of movement; *c*, section through blade root.)

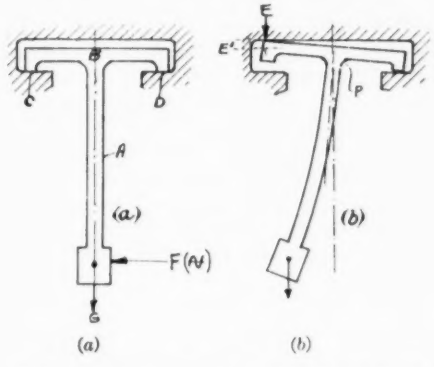


FIG. 33 SCHEMATIC REPRESENTATION OF BLADE-ROOT SYSTEM

have set up excessive repetitive stresses, explaining past failures. By designing for a centrifugal-stability factor over the overturning steam-force moment, the authors insure the nodal fixity at the root necessary for the assumptions in the frequency calculations which follow.

It will be interesting to point out here that, with any appreciable looseness in the root, the natural frequency of the blade becomes a function of the amplitude of vibration as well as of its mass and elastic properties, and thus may undergo wide variations. Fig. 32 (*a*) represents a blade with loose root fastening in repose, and Fig. 32 (*b*) the position at one extremity of movement, having rocked on one bearing shoulder until stopped by the limiting clearance in the groove.

The system can be illustrated schematically by Fig. 33 (*a*), where the blade is represented by a pendulum *A* rigidly attached to a bracket *B*, serving as the root, and supported at the two points *C* and *D* (root shoulders). Gravity (centrifugal force) is pulling downward, attempting to hold the system in repose while the pulsating steam force *F* is tending to rock the system about the bearing point *D*.

If the pendulum is displaced slightly to one side and released, it will oscillate or rock back and forth under the action of gravity alone, at a frequency which increases rapidly as amplitude decreases, precisely like a rubber ball dropped on the floor, until the system comes to rest. So far, neither mass nor elasticity enters the picture; only centrifugal force (gravity).

Assume now that the displacement is great enough for the corner of the bracket or root to strike at *E*. First there will occur an elastic deformation at *E* due to the impact, and also a flexural bending of the pendulum in attempting to carry on in the leftward movement. In addition to the steam and centrifugal forces there, we have both bending and impactive forces at the fillet *P*, with the stress-magnification factor due to impact depending upon the "softness" at the contact points.

The restoring forces to complete the other half of the cycle now depend on:

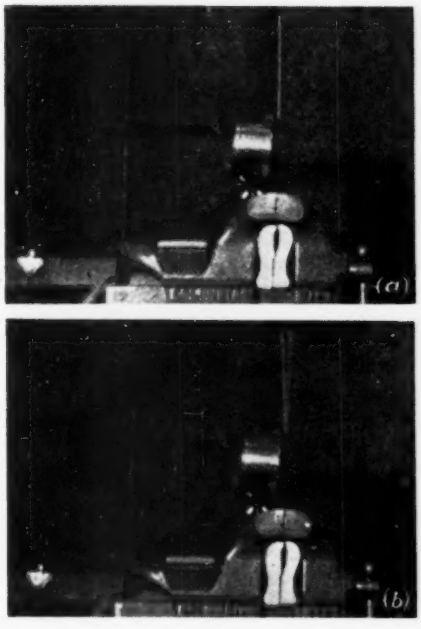


FIG. 34 MODEL ILLUSTRATING TURBINE VIBRATION CAUSED BY IMPACTIVE INTERFERENCE NEAR BLADE ROOT

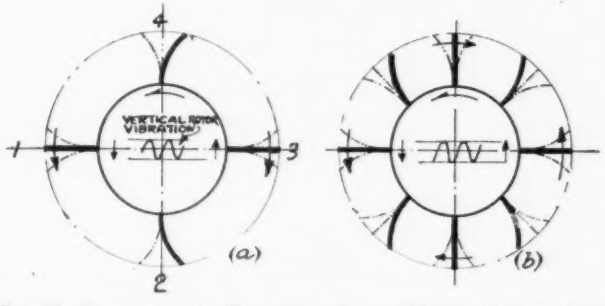


FIG. 35 INSTANTANEOUS POSITION OF BLADE MAKING (*a*) 1 AND (*b*) 2 COMPLETE VIBRATIONS DURING TIME REQUIRED FOR 1 REVOLUTION

1 The elastic "rebound" at the impact areas.

2 The elastic and inertia properties of the pendulum system.

3 The gravity force acting through the clearance space *E* to *E'*.

An intermediate impact also occurs in this simple system as the rocking points *C* and *D* strike their seats.

The frequency of such a system depends not only upon the mass and elastic characteristics of the blade itself, but also on the impactive resilience, and finally on the extent of free movement or amplitude limited by the clearances. The frequency is practically indeterminate. The vibration of such a system can no longer be expressed by linear differential equations, and hence is called "nonlinear" vibration. Professor Timoshenko[13] points out that such systems may often nullify the effect of resonance because, as resonant amplitudes build up, the change in amplitude itself becomes a sort of inhibitor by altering the natural frequency away from resonance.

There have been examples, however, where impactive interference with free vibration caused a great increase in amplitude and hence stresses. These cases seem to involve a critical condition in which the elastic rebounds from impact somehow augment

[13] "Vibration Problems in Engineering," by S. Timoshenko, D. Van Nostrand Company, New York, N. Y., 1937, pp. 117 and 145.

or aggravate critical vibration. Increased turbine vibration due to impactive movement from excess clearance in turbine bearing keys is a familiar example. This is further illustrated by the model Fig. 34. In Fig. 34 (*a*) the cantilever or blade is shown vibrating steadily in response to magnetic impulses; at (*b*), with no change in the magnetic impulses, the vibration has been greatly increased or "drawn out" by impactive interference near the root.

It is pointed out in the paper that, when the natural frequency of the last-row low-pressure blades is less than 4 times the running speed, tuning is resorted to, to avoid resonance. Presumably, the periodic forces from which this blading is to be protected arise from torsional oscillations of the rotor or linear vibration of the rotor due to unbalance, at the frequency of rotation, rather than periodic steam forces.

Primary resonance occurs when the natural frequency of the blade coincides with a torsional oscillation once per revolution, but resonance with linear vibration from unbalance does not occur until the natural frequency of the blade is twice the rotational frequency. Therefore, a blade frequency of 4 times the speed of rotation actually represents a multiple of only 2, as regards fundamental resonance. Likewise, no resonant amplitude should be built up when the blade frequency is just equal to linear vibration at the frequency of rotation.

In Fig. 35 (*a*) are represented the instantaneous positions of a blade making one complete vibration during the time required for one revolution. It is assumed that the blade vibration is caused by the linear vibratory movement of the rotor due to unbalance, acting on the blade through the root fastening. The blade is shown in the neutral position at 1, and at the extreme limit of its vibration in the leading direction at 2, after 90 deg rotation. The blade passes through the neutral position again at 3, and reaches the opposite extreme at 4, in the trailing direction.

But the inertia forces on the blade from the rotor vibration, which caused it to be displaced forward at 1 as shown in the first part of the cycle, would again cause the blade to travel forward at 3 during the second half of the rotor vibration cycle, instead of the backward motion attempted by the free vibration of the blade. In other words, the "return-stroke" exciting force opposes the completion of the blade-vibration cycle, and should discourage building up resonant amplitudes.

In Fig. 35 (*b*) is shown the situation where the natural frequency of the blades is twice the running frequency. Here the natural blade movements are in proper phase with the rotor vibration to produce fundamental primary resonance.

Fortunately, as the author points out, there has been little difficulty with fatigue failures of the last-row blades due to resonance. Curiously, we had all been far more concerned over the possibility of fatigue failures in these large blades than in the rugged shorter blades which actually have given more trouble.

Author's Closure

With reference to Mr. Kroon's discussion concerning the 13 per cent chrome blading steel, the author believes that all turbine builders are in the same position as that assumed by Westinghouse, namely, each is ready to use the material which he considers the best for steam-turbine blading. That the 13 per cent chrome steel has justified its adoption for this service is well demonstrated by the number of turbine builders who make successful use of this material. The author states in his description that this alloy is the best material for low-pressure blading. Many years of development and metallurgical research were required before the 13 per cent chrome steel was produced as a useful blading alloy. It was the author's privilege to be associated with Francis Hodgkinson and Norman Mochel during the early development of this material by the Westinghouse Company.

Paralleling the work of the Westinghouse and General Electric Companies on the 13 per cent chrome steel, the Allis-Chalmers Company has developed the austenitic Cyclops alloy described in the paper. They, too, are in the position of being able to use the blading steel that they consider the best for blading. The conclusion that the Cyclops alloy offers advantages for all but the longest rows of low-pressure blades is again a matter of many years' development and research which was only briefly indicated in the paper. The author recognizes that each material has advantages and disadvantages, so that a careful survey of the merits of the two materials is necessary. The excellent service record of the Cyclops alloy speaks well for its reliable qualities for blading service.

With reference to the difference in damping capacity, this property is still under investigation. It is necessary to point out that, as stated by the author during the discussion of the paper when presented, partial-admission impulse blading, such as illustrated for the Northwest top turbine, is designed so that the structure is safe with a logarithmic damping decrement as low as 0.0002, including a stress-concentration factor of 2.0 due to the fillet at the base of the blade. It is believed that the construction described, in which due allowance is made for amplification of the bending stress from primary and secondary resonance, is based on logical reasoning and that safety is insured if the proportions permit the operation with safe stresses with the low damping decrement mentioned in this paragraph.

Mr. Kroon indicates that he is not in agreement with the statement that full-admission blades having a natural frequency over five times the running frequency are sufficiently remote from resonance so that the stresses are not intensified thereby. He also indicates that he has found the resonance effect to be important at blade frequencies up to about 12 times the running speed. The author's experience is not in agreement with this, although quite a number of cases have arisen in which this was first thought to be the cause. Careful analysis of other effects, such as pressure differences in the blade path or defects in the mechanical construction, were in each instance sufficient to cause the high resonance ratio to be abandoned as a reason for trouble.

Mr. Kroon points out that his company is using statistical data to insure a safe design for blade structures. This practice is today and, so far as the author is aware, always has been the underlying principle employed by all blading designers. Blading stresses and calculation processes have always been checked and compared and the constants evaluated by comparison with practical results taken from field experience or laboratory research.

With respect to Mr. Kroon's comments on the general character of the impulse-blade development, the author agrees that the assumption that a blade receives its load gradually as it travels one blade pitch is much on the safe side as very elementary considerations will serve to show. The author agrees that it may take a distance of several blade pitches before the normal driving force is established on the blade. The statement that, in the experimental turbine built by Westinghouse, the first movement of the blade entering the steam jet is opposite to the torque force is very interesting. All of these effects appear to the author of a nature that does not modify the general validity and safety of the primary assumption of the paper with respect to the time of loading.

The author is in agreement with Mr. Kroon, in that it is necessary to consider the vibration of blade structures axially and torsionally, as well as circumferentially. The study of impulse-blade structures in the paper is limited to the circumferential motion, because, in wide-root blades of the type described, this mode is the only one that need be considered. In long blades

that may be shrouded and lashed together in groups, consideration must be given to the other modes of vibration.

With respect to expansion ratios that correspond to more than sound velocity, the author's statement in this connection applied specifically to high-capacity top-turbine design. In such machines, it is the present practice to arrange the first group of nozzles to be fully opened at the time the pressure in the first stage is reached which corresponds to the sound velocity at the nozzle outlet. In normal condensing turbines, in which the steam flow is less than in top-turbine practice or in top turbines of moderate size, the design is arranged so that the sound velocity is exceeded under light load conditions.

With reference to Mr. Kroon's comments concerning the phase relation of the blade motion with respect to the time of application of the steam load, the mathematical supplement shows that for maximum amplification the steam load is applied at a point where the blade is at zero displacement and starting on the backswing, under conditions where the blade is loaded in an odd number of half cycles. At this time of application the velocity is a maximum in the backward direction. This is not the same as the case of a suddenly applied load. With a high natural frequency of the impulse-blade system, the time required to establish the mean driving load on a blade should always be greater than a half cycle of blade motion. In such cases, the maximum amplification will take place when the mean driving load is established in an odd number of half cycles with the phase relation here mentioned. If, however, the mean driving load is established in less than a half cycle, then the phase angle changes, depending on the exact time relation, until in the limit case, in which the load is applied in zero time, the maximum displacement of the blade will be produced when the initial velocity is zero at the instant the load is applied, as stated by Mr. Kroon. This limit case cannot be realized in practice.

It is desirable to review the conditions that define primary resonance. This summary was given in the abstract of the paper as presented in Milwaukee and also in the mathematical supplement by Mr. Emmert.

(*a*) The start of the application of the steam load takes place at a point where the blade is at zero displacement and starting on the backswing at which time the vibrational velocity is a maximum backward.

(*b*) The full steam driving load is assumed to be reached in an odd number of half cycles of blade motion. If the frequency of the blade is such that there is not an odd number of half cycles in the actual time of load application, the time is adjusted slightly for purposes of calculation.

(*c*) The full steam driving load is assumed to be applied for an even number of cycles of blade motion. The time adopted for calculation purposes is adjusted slightly if necessary.

(*d*) The time at which the release of the load starts is assumed to be at a point where the blade is at zero displacement with respect to the static deflection and when the blade is starting on the forward swing. In other words, the blade is moving at its maximum velocity forward.

(*e*) The removal of load is assumed to be completed in an odd number of half cycles.

(*f*) The idle portion of the arc is assumed to include an even number of cycles.

With reference to Mr. Warren's discussion, he stresses the statistical study of impulse-blade performance from which his company's design practice has been evolved. As indicated in the author's comments on Mr. Kroon's discussion, this same type of statistical analysis has been a necessary standard practice for many years with turbine-blade designers.

Referring to Mr. Warren's comments on the continuous joining of low-pressure blade-shroud bands, this has been the practice of the Allis-Chalmers Company for many years. What the author considers an exceptional record of reliable operation of low-pressure blading justifies this construction. The very few cases where last-row-blade trouble has occurred have been explained by well-recognized causes. In Allis-Chalmers practice, the low-pressure blades are first shrouded in groups leaving, say, ten or more gaps open between sections. The spindle disk, or in 3600-rpm machines the entire spindle, is heated in a suitable oven to about 350 F. The body of the spindle is then protected with insulating material and the blading allowed to cool for a predetermined period, after which the open junctions in the shrouding are joined by welding. When the temperatures in the spindle and the blading are equalized, the shrouding is then in compression. Experience has shown that there is little trouble in the breaking of these joints. Where broken joints between shroud sections have occurred, these have been traced to some recognizable cause, such as the close proximity of ribs in the exhaust chamber, or the unequal draft of steam through a bleeder connection in which the extraction belt was not well proportioned.

Mr. Kimball points out the dangerous effect of resonance of the type classed by the author as secondary resonance. As indicated by Mr. Kimball, resonance between the natural frequency of the blade structure and the periodic force set up by the nozzle flow can produce amplification factors of a high order. In fact, if all terms of Equation [19] are considered for the case reviewed, the amplification factor for true resonance is of the order of 1000.

Mr. Kimball further points out that the intensity of what he terms "impulse excitation of bucket vibration" is responsible for an intensity far below that arising from resonance produced by even a moderate periodic force. This is extremely important, and fortunately a condition that is capable of control by proper design. The author is in full agreement both with Mr. Kimball and Mr. Kroon regarding the desirability of experimental investigation of the vibration of blading, although, in the author's opinion, the real value of such experimentation is not realized unless guided by a sound background of fundamentals.

The author wishes to point out that the steam load of 350 lb is applied at the mean diameter, not at the tip of the blade as assumed by Mr. Kimball.

The cubical contents of the port section of the Northwest blade are over three times the 0.6 cu in. used in Mr. Kimball's discussion.

With reference to Mr. Rathbone's remarks, impulse blading must be designed to withstand primary resonance. Secondary resonance can be avoided by design.

The author is in agreement concerning the important effect of looseness of the blade roots on frequency and the difficulty of predicting the frequency of a blading system in which looseness of the blade roots enters the problem. In past practice, some cases of partial-admission impulse-blade failures have been attributed to undue looseness of the roots.

Where looseness of the roots of partial-admission blades is a cause of failure, it is the author's belief that the blades are not correctly proportioned.

The difficulties associated with securing tightness of the roots in their grooves under conditions of full speed and full operating temperature are stressed in the paper, and the necessity of adopting proportions for the blades and roots that will maintain stability by the effect of centrifugal force is emphasized. If, for example, the stress-amplification factor on the bending moment is found to be 10, the centrifugal stability factor, expressed as the ratio of centrifugal stabilizing moment to the static value of the steam overturning moment, should be at least 15, if stability is to be maintained.

Mr. Rathbone's discussion gives this opportunity to stress

further the importance of maintaining design proportions so that the rocking or chattering of the blades in their grooves due to looseness can never occur.

With reference to the resonant conditions described by Mr. Rathbone in connection with the last-row blades, the author is in agreement with the general principles indicated in Fig. 35. The periodic forces against which the blading must be protected are probably not as much due to the variation in the steam driving force from blade to blade, as to the effects produced when a given blade passes through different pressure zones. The cylinder design is planned to reduce to a minimum such pressure differences. The effect produced when the low-pressure blades move in and out of zones of different pressures is similar to that which would be induced by torsional oscillation of the spindle.

The limiting frequency ratio above which trouble from blade resonance is thought not to occur is in fair agreement with field experience, which indicates that resonant vibration does not cause trouble if there is close agreement with higher multiples of the rotational frequency than 4 or 5.

In closing, the author wishes to express his appreciation to the A.S.M.E. for the privilege of presenting this paper and to the discussers for their valuable contributions.

Discussion of Attack on Steel in High-Capacity Boilers as a Result of Overheating Due to Steam Blanketing

By E. P. PARTRIDGE and R. E. HALL

This paper was presented at the Spring Meeting in New Orleans, La., February 23–25, 1939. Due to the fact that preprints were not available, discussion at the time of presentation was limited. Subsequent to publication in the October, 1939, issue of the Transactions it was decided to continue discussion at the Spring Meeting in Worcester, Mass., May 1–3, 1940. This discussion, together with the authors' closure, appears herewith.

P. B. Place.[1] Although the authors group the given examples of tube failure under a general classification of steam blanketing, there is sufficient difference in the nature of the failures so that they may be arranged in at least two groups, i.e., failures caused or accompanied by cracking, and failures by corrosion without cracking.

The first type of failure is characterized by cracking of the metal in areas that are alternately wet and dry. This type of failure is a form of stress corrosion in which attack is greatly accelerated by alternate heating and cooling and occurs along the bottom of the tube where the water laps the overheated metal. A typical case occurred in a circulator tube between an upper side-wall header and the steam drum. Cracking developed into failure at the bottom of a tube bend which was above the normal water level and normally out of the gas stream. Leakage of gases through a superheater seal near the point of failure caused local overheating, and a surging type of circulation, typical in this part of a boiler, periodically quenched the overheated section.

The second type of failure is characterized by corrosion and loss of metal. Such corroded surfaces have been found along the tops of sloping tubes in the form of a narrow groove and also as deep pits in the bottom of sloping tubes under sediment deposits. Often the corroded surface is covered with soft smutty oxide mixed in some cases with colored chemical deposits. This type of failure appears to be due chiefly to direct chemical attack by steam or water, or by salts which concentrate or deposit locally.

In the analysis of these failures we can perhaps start from the following accepted facts:

1 The failures are not accompanied by a deformation of the tube, except as such deformation may have resulted from excessive heating caused by the insulating effect of deposited corrosion products.

2 The corrosion product is iron oxide which requires a source of oxygen.

3 The protective oxide coating, which normally covers a tube surface, must first be broken down to expose fresh metal before corrosion can develop.

The absence of deformation in a tube, the wall of which is corroded to one half its original thickness over a long period of time, definitely indicates that excessively high temperatures are not a prerequisite of the corrosion. Temperatures in excess of 1200 F are reported, which are believed either to be incidental or a result of accumulation of corrosion products rather than a cause.

There has been built up in the last few years a popular belief that very high metal temperatures are involved in these types of failure. Dissociation of steam has been given in many cases as a direct cause of corrosion and a source of oxygen. It is the writer's belief that dissociation has erroneously been construed to mean a decomposition of water vapor by heat into oxygen and hydrogen with resulting attack on the metal by the liberated oxygen. At temperatures that will not cause deformation of a tube, the amount of dissociation of steam by heat is very small. The oxygen needed for corrosion is not oxygen freed by heat dissociation of water vapor but is extracted from steam, water or oxygen-containing salts by a reduction process. The metal is available for that reduction only after its normal protective oxide coating has been broken down.

The concept of temperature difference in steam-blanketed tubes as given in Fig. 1 of the paper makes no allowance for heat conductance around the tube wall. Any appreciable increase in the temperature difference between the top and bottom of a steam-blanketed tube must result in an increasing proportion of the absorbed heat being conducted through the tube wall around the steam space.

Thermocouple measurements of tube temperatures and metallurgical analyses are given as evidence of high-temperature conditions in conjunction with these failures. Chromel-alumel couples peened into boiler tubes in gas temperatures above 1500 F are quite dependent upon the method of installation for their reliability. The failure of such couples is not due to overheating where they are attached to the tube but due to oxidation and deterioration of the lead wires exposed to the gases. During one investigation of tube failures, we installed platinum thermocouples adjacent to chromel couples on all of the boiler tubes in question. At no time did any of the platinum couples register excessive temperatures although many of the chromel couples did and we found that the maximum period of reliability for chromel couples in gas at 1700 F was 2 days. Though many apparently lasted for weeks, any variations in temperature registered after 2 or 3 days were open to suspicion. Relative to metallurgical analyses, it is common for reports to state that, because of changes in grain structure, it is concluded that the metal has been at temperatures in excess of 1000 to 1200 F for so many hours. In many cases, because of corrosion, this may be quite true. However, in one case reported in this manner, the sample had been taken adjacent to a header located outside of the furnace where such temperatures could not have existed.

The breaking down of the protective coating is slow and difficult in an atmosphere of steam alone. The experience with superheaters, experimental corrosion tests, and the commercial production of hydrogen, all indicate that the action of steam on hot metal is to form the protective coating and that the oxidation slows up as the time of contact increases. Water quenching, chemical attack of salt deposits, or mechanical effect of rapid temperature changes are necessary to break down the coating and allow corrosion to proceed.

[1] Test and Research Department, Combustion Engineering Company, Inc., New York, N. Y. Mem. A.S.M.E.

In dry areas, formed by steam blanketing along the top of sloping tubes, there is little quenching except by water splashed against the top of the tube. However, the evaporation residues from the splashing remain on the tube forming a concentrated salt mat on the dry area. It is believed that the corrosion along the top of screen tubes is chiefly by chemical attack. This attack can be accelerated if the steam pocket periodically vents itself and quenching stresses are added.

A similar concentration of chemical may develop under deposits of loose sludge on steam-generating surfaces with resulting chemical attack and corrosion under the deposit. Boiler water may diffuse into such deposits at a rate sufficient to keep the metal from burning and blistering but not at a rate sufficient to wash away the evaporation residues. Once corrosion is started by the chemical attack, the accumulation of products of corrosion accelerates the action and, unless the tube is cleaned out, the deposit may eventually insulate the metal sufficiently to cause severe overheating and blistering. A failure of this type developed in the bottom row of a sectional-header, high-pressure boiler which was initiated by an excessive phosphate-sludge condition in the boiler.

Periodic quenching of overheated areas prevents the accumulation of chemical deposits and the likelihood of corrosion failure. The quenching, however, sets up very severe local stresses because of the initial rapid absorption of latent heat and the protective coating is continuously cracked, exposing fresh metal to attack. Tube failures under such conditions are crack failures.

The expansion and contraction stresses in a thick-walled high-pressure tube, set up by overheating only 200 to 250 deg and repeatedly cooling by the rapid evaporation of quenching water, are very destructive to the protective oxide film. The metal exposed by cracking of the film reduces the steam by chemical action with liberation of hydrogen.

In conclusion it is believed that the presence of water and/or its associated salts is necessary to the tube failures of the type under discussion and that such failures can be initiated at tube temperatures lower than 800 F. Improvement in boiler design, and water conditioning; elimination of circulating sludge; better distribution of heat absorption; and standardization of conservative heat rates are the indicated trends toward the control and elimination of failures of this type.

H. KREISINGER.[2] The title of the paper implies that the type of corrosion discussed in the paper came into prominence with the use of high steam pressure. Many of the tube failures were accompanied or caused by cracking of the internal surface of the tubes at a point where the internal surface was alternately dry and wet. The stresses produced by repeated quick-cooling result in cracking. The cracks in the stressed metal provide a large and clean surface for the action of boiler water, and the surface corrodes rapidly. The question arises whether the very thick walls of the tubes are not a major contributing factor in the failures? The tubes are selected for a given pressure, with a factor of safety of about 10, whereas the steam drums are designed with a factor of safety of 4.5. Why is the factor of safety for the tubes so high compared with the factor of safety of the drums? Surely, if the drum failed, more damage would be done than by failure of a tube.

Mathematical analysis of the temperature stresses under normal conditions of operation indicate that the thick-walled tube is subjected to severe temperature stresses which would make failure by cracking almost inevitable, if the stresses were not moderated by the flow of metal.[3] Steam drums are shielded from heat because the shell is too thick and might be damaged by heating. At the same time, thick-walled tubes are located where they are subjected to severe heating. When inquiry is made concerning the reasons for the very thick walls of tubes with a factor of safety of about 10, the reason given is that the thickness is somewhat irregular and that the thinnest part of the tube wall after a certain amount of corrosion has taken place must still be strong enough to withstand the pressure. Some of the tubes have been corroded to less than one half their original thickness without any apparent distortion of the corroded area due to pressure. It appears that tubes with much thinner walls would hold the pressure and be less subject to the type of corrosion which is caused or accompanied by cracking of the inner surface of the tube.

The engineer might benefit by the use of thinner-walled boiler tubes. They would cost less and they might reduce appreciably the failures from corrosion. He might also select tubes of smaller diameter and thinner walls.

It is true that there have been some tube failures in low-pressure boilers with thin-walled tubes, but it is probable that the failures would have occurred in much shorter time had thick-walled tubes been used. The failure is due to temperature stresses and the stresses are greater in the thick-walled tube than in a thin-walled tube. Lower stresses must be repeated over a much longer period of time than high stresses before they result in failure. A thin-walled tube may last years, whereas one with a thick wall may fail in a few weeks.

R. T. HANLON.[4] Further tube failures were experienced in the top tubes of the 4-tube-high headers. Thermocouple temperatures at the top of these tubes showed a normal temperature of from 600 to 620 F when the saturation temperature corresponding to pressure was approximately 580 F.

When the boiler was operated under conditions of full rating and pressure, the tube temperatures would suddenly increase to approximately 1040 F at the top of the tube. This was probably considerably lower than that at the bottom of the tube where the damage occurred.

Reduction of drum pressure from 1300 to 1050 psi reduced tube temperatures in all cases to normal values.

It was also observed that elevated tube temperatures increased in frequency, elevation, and duration as the concentration of total dissolved solids in the boiler water was increased.

Various arrangements of the drum baffles and steam washer have been tried. The most effective baffle arrangement found to date takes advantage of the velocity of the water in the drum flowing from the uptake to the downtake tubes. The baffle is so located, as shown in Fig. 1 of this discussion, that it directs the flow of water to the sectional-header downtakes.

The effect of these baffle installations in combination with the installation of restrictor plates upon the upper half of the lower ends of the sectional-header tubes has been to permit the maintenance of approximately 500 ppm total soluble solids in the boiler water. It has been possible to maintain this concentration at full boiler pressure and rating without experiencing elevated tube temperatures or serious carry-over. It had not been possible to maintain concentrations in excess of 100 to 150 ppm under normal operating conditions previous to these installations.

During all of this period the feedwater to these boilers consisted of turbine condensate, the make-up being evaporated in low-pressure boilers. No further failures were experienced in the upper sectional-header tubes.

[2] Engineer in Charge of Research and Development, Combustion Engineering Company, Inc., New York, N. Y. Mem. A.S.M.E.

[3] "Stresses in Boiler Tubes Subjected to High Rates of Heat Absorption," by W. L. de Baufre, Trans. A.S.M.E., vol. 55, 1933, FSP-55-6, pp. 73–98.

[4] Consolidated Edison Co. of New York, Inc., New York, N. Y.

During October, 1939, after two months of operation with externally treated city-water make-up, drifts of loose sludge were observed near the downtake end of the bottom tubes of the sectional-header bank. Subsequently, several tubes failed in operation in this location, as is indicated in Fig. 2, herewith.

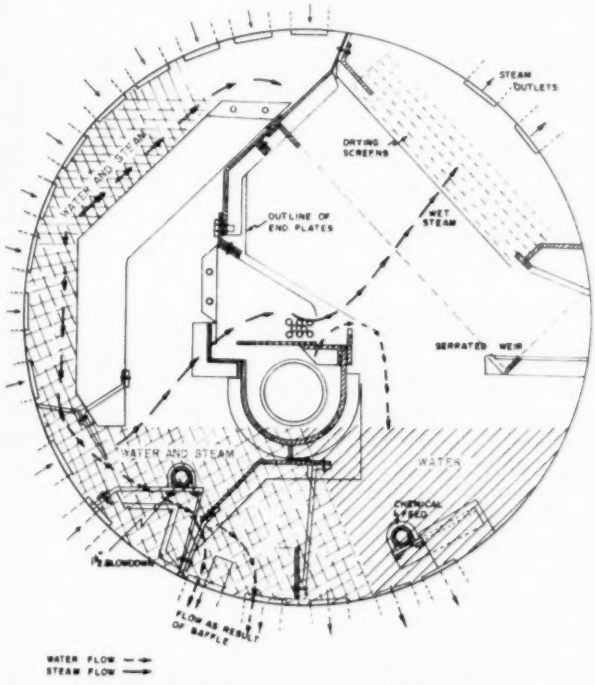


FIG. 1 STEAM WASHER AND BAFFLE ARRANGEMENT TOPPING UNITS BOILERS, WATERSIDE NO. 2 STATION

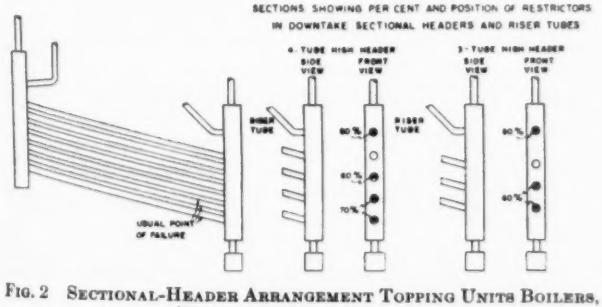
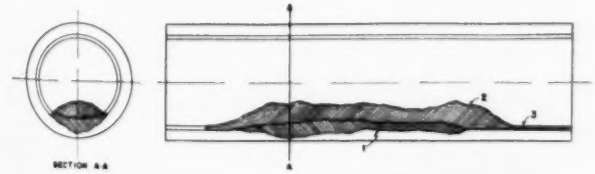


FIG. 2 SECTIONAL-HEADER ARRANGEMENT TOPPING UNITS BOILERS, WATERSIDE NO. 2 STATION



CHEMICAL ANALYSIS OF DEPOSIT
PER CENT BY WEIGHT

	NO. 1	NO. 2	NO. 3 SOLUBLE IN AQUA REGIA	NO. 3 INSOLUBLE IN AQUA REGIA
SILICA AS SiO_2	0.91	1.29	—	11.41
IRON AS Fe_2O_3	80.49	39.84	82.21*	11.06
COPPER AS Cu	9.09	39.38	2.39	—
ALUMINA AS Al_2O_3	—	—	4.37	9.16
CALCIUM AS CaO	2.12	4.17	—	—
MAGNESIUM AS MgO	3.27	5.06	—	—
PHOSPHATE AS P_2O_5	3.10	7.92	—	—

NO. 1 CRUSTY MATERIAL
NO. 2 LOOSE MATERIAL
NO. 3 TOUGH ADHERENT SCALE

*HIGH IRON DUE TO CONTAMINATION OF SAMPLE BY SCRAPING METAL FROM TUBE SURFACE WHILE OBTAINING SAMPLE

FIG. 3 SECTION OF TYPICAL FAILURE NEAR LOW END OF SECTIONAL-HEADER TUBES, BOILER NO. 52, WATERSIDE NO. 2 STATION

In every case of tube failure, large quantities of magnetic iron oxide were observed at or near the point of failure. The appearance and composition of a typical case are given in Fig. 3. Little if any indication of blistering or swelling of the outside of the tubes was observed. The apparent cause of failure was accelerated corrosion on the inside of the tubes at points where deposition had occurred. This deposition apparently caused localized tube overheating due to insufficient heat transfer.

Previous to the time when these failures occurred, continuous blowdown from the drum was employed for control of total solids. Under ordinary conditions of operation, blowdown from the drum is clear, indicating that, while soluble materials are removed from the boiler, insoluble compounds are not. In order to remove sludge from the boiler a procedure was, therefore, instituted by which each boiler was taken out of service every 4 days and blown from the lower waterwall headers and the mud drum.

The average alkalinity of the boiler water was also increased from 15–25 to 20–30 ppm NaOH. This appears to have prevented the further formation of a tough, adherent silica scale upon the tube surfaces.

The amount of calcium-and-magnesium deposit was apparently decreased by shortening the cycle of the external-treatment units. In this manner the total quantity of scale-forming salts introduced into the boilers was decreased and resulted in cleaner tube surfaces.

The over-all effect of the increased boiler blowdown, higher boiler-water alkalinity and the shortening of the cycle of the primary treatment has been to arrest corrosion of the sectional-header tubes due to deposition of sludge and scale. These procedures, however, have resulted in somewhat higher heat losses and carry-over to the turbine, necessitating washing of the blading.

W. L. WEBB.[5] Inasmuch as the authors' description of the Logan ash-screen-corrosion experience was carried only to the end of 1938, experience since that time may be of interest. As the authors indicated, the ash-screen tubes on the west half of the boiler were replaced during the November, 1938, shutdown with 3-in-outside-diam 12.5-deg-slope tubes of the same design as those installed on the east half of the boiler in May, 1938.

Visual examinations of representative screen tubes in February, 1939, showed the usual brownish black areas along the tops of the tubes, some being continuous narrow bands and others discontinuous bands of varying width. Those in the tubes on the west half of the boiler were more clear-cut than those in the east, due possibly to thinner deposits of sludge. Day loads on the boiler during the 3 months prior to this shutdown ranged between 800,000 and 1,000,000 lb of steam per hr.

Although no indication of active corrosion was found in the screen tubes, these markings showed that corrosion might not have been completely stopped. In order to produce more intimate contact of the water with the top surfaces of the tubes, twisted sheet-steel spirals were placed in all screen tubes during the March, 1939, shutdown. These were constructed of pieces of No. 18-gage steel, 5.5 ft long, having a width slightly less than the inside diameter of the tubes and bent to give a complete 360-deg twist in 1.5 ft of spiral. In addition to installing the spirals, the lower burners were tipped upward 2.5 deg, giving a total angle of 4 deg from the horizontal, in order to reduce further the flame impingement on the screen tubes.

No major inspections of ash screen tubes were again made until February, 1940. At that time, spirals were removed from seven representative tubes. All previously affected areas were

[5] American Gas and Electric Service Corporation, New York, N. Y.

evenly covered with a thin reddish colored sludge, no black oxide whatever being evident. The calipering of these tubes indicated no measurable metal loss. Except for a light coating of red oxide, the spirals were clean and smooth and showed no indication of corrosion.

As reported in the paper, hydrogen-evolution measurements in saturated steam were started in March, 1938. The gases were separated from the condensed sample by means of a separator of the boiling-chamber type, similar to that used by Potter, Solberg, and Hawkins in connection with their study of superheater-tube metal and were analyzed with an Orsat. Hydrogen was determined by the heated copper-oxide method. Usually two 2-hr samples were obtained each day. Hydrogen values obtained with this equipment appeared to have no direct relation to corrosion rate. Tests of the gas-removal efficiency of this separator, measured in terms of oxygen removal, showed it to have too poor efficiency to be suitable for measuring accurately the concentrations of hydrogen expected.

An automatically controlled separator for continuous operation, patterned after a tray-type deaerator, was therefore designed and constructed. After completion of preliminary tests, this unit was placed in operation in April, 1939, and has been in almost continuous service since then. Two gas samples are collected per day over 8- and 16-hr periods, respectively. Unfortunately, this new separator was developed too late to be used during the period of severe screen-tube corrosion.

Since hydrogen-evolution measurements are being more extensively made on steam samples from high-pressure boilers, actual values in the Logan saturated steam may be of interest. During the last year the evolution values have had a direct relationship to load and are approximately 18 and 22 l per 1,000,000 lb of steam at half load and full load, respectively. This relationship has been substantially the same when feeding ferrous hydroxide or sodium sulphite in proportion to dissolved oxygen in the feedwater as a scavenger, or when no scavenger was fed. No tests have been made to determine the effect of excess ferrous hydroxide on hydrogen evolution.

Equipment is being provided at both Windsor and Twin Branch for hydrogen-evolution measurements, the latter being equipped with a two-point recorder. Values of hydrogen in the feedwater, sampled at the economizer discharge (in one case equal to one half to three quarters that in saturated steam), indicate the necessity of sampling at both of these points, preferably simultaneously, to show the quantity of hydrogen actually evolved at the steam-generating surfaces.

The Logan boiler water pH is still being maintained between 9.6 and 9.8, since, under present load conditions, it is not considered advisable to conduct experiments at higher pH values and because evaporator carry-over and condenser leakage are at a minimum. It is hoped that the pH ultimately can be carried at about 10.8.

As was indicated by the authors, during the Logan screen-tube corrosion experience, soluble boiler-water chemicals "hid out" during high loads and were redissolved at low loads or when the boiler was taken off the line. At one time when the phosphate as PO_4 in the boiler water was being maintained at 30 to 50 ppm, a total of 350 ppm of PO_4 "hid out." Since the tube corrosion has been stopped, hiding out has ceased.

Experience with the Logan boiler as well as others can lead to no other conclusion than that hiding out is an unfavorable sign, indicating tube starvation or steam blanketing which may be a forerunner of tube corrosion resulting from the reaction between steam or water and the hot metal.

Uncompleted experiments with a 250-lb gage controlled-circulation test boiler, having a single horizontal tube in a gas-fired furnace, indicate that, with heat-absorption rates up to 125,000 Btu per sq ft of circumferential tube area, and with boiler-water pH values of 9.6 to 12, the hydrogen evolution and top-tube temperatures remain normal as long as the steam-water velocity exceeds a critical value. However, as the velocity is decreased below this critical value, the tube-wall temperature rises rapidly. With increasing tube-wall temperatures, hydrogen evolution increases, a higher hydrogen evolution occurring at a boiler water pH of 11 than at a pH of 9.6.

The writer wishes to commend the authors for presenting the case histories and particularly for the accuracy and completeness of the Logan experience. It is evident that too few 1300-lb boilers have escaped the type of tube corrosion which is known not to have been caused by dissolved oxygen in the feedwater and which is apparently beyond correction by chemical control of boiler water. A better understanding of this type of corrosion, both by the boiler manufacturer and the purchaser, should lead to a definite solution of this problem.

L. E. HANKISON.[6] The discussion of any paper presented over a year ago should offer a very excellent opportunity to substantiate or disprove the various observations, theories, and conclusions presented in the paper; and in so far as our experience goes, the findings and theories advanced by the authors have been borne out to an extraordinary degree. Therefore, the writer believes the paper under discussion should be considered as authoritative on the various ideas brought forth.

With regard to the case history reported for Springdale, the observations and causes of the troubles outlined in the paper have been confirmed, and the loss of steam-generating tubes stopped, we believe, by the removal of the cores that were placed in the tubes to promote circulation, and the installation of distributing apparatus in the headers to prevent steam binding. The manufacturer of the steam-generating equipment furnished to Springdale is to be highly commended on the handling of the various problems, the analysis of troubles as they developed, and the application of the proper corrective measures for producing the desired results. As the coal conditions and boiler-water concentrations with which we are now operating are considered much more severe than existed at the time of our troubles, we feel justified in the belief that our present satisfactory conditions will continue.

We have always held to the theory that the steam generator should be so designed as to allow the use of properly treated feedwater, and that undertreating of feedwater in an effort to remedy the shortcomings of boiler design should not be tolerated. The paper suggests that, by the use of inadequate treatment of the boiler water, the detrimental effects of steam binding and the wasting away of boiler metal may be retarded in the dry areas, but that lower alkalinity may possibly cause deterioration throughout the boiler. It seems that metal cracking is likely to take place under these conditions, and that, instead of correcting the trouble, the time is only prolonged until general deterioration takes place. On the theory that proper boiler-water conditions should be maintained, we carried sufficient alkalinity and conditioning chemicals properly to treat the high-pressure boilers at Springdale, even while undergoing tube troubles. Our experience shows that correction of the circulation and steam-binding conditions is all that is required to permit satisfactory operation with properly treated boiler water.

We have had further opportunity at Springdale to confirm the conclusions outlined in the authors' paper. Because of unexpected characteristics of the coal, it was considered advisable to make the walls of the secondary furnace entirely water-cooled by adding extra tubes to the back and side walls. It was thought as

[6] Efficiency Engineer, West Penn Power Company, Pittsburgh, Pa. Mem. A.S.M.E.



FIG. 4 RUPTURED PART OF WATERWALL TUBE AFTER 17 DAYS OF SERVICE

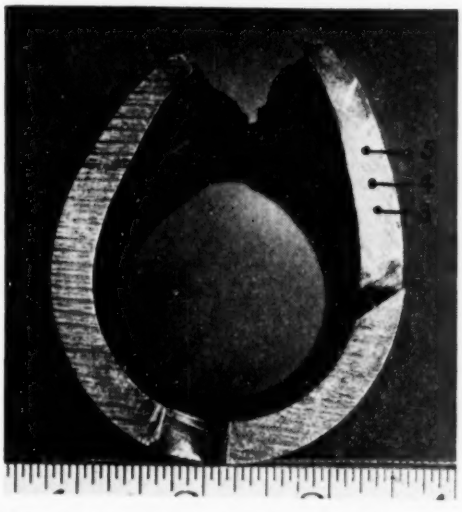


FIG. 5 CROSS SECTION OF TUBE AT RUPTURE MARKED TO SHOW LOCATION OF MICROGRAPHS

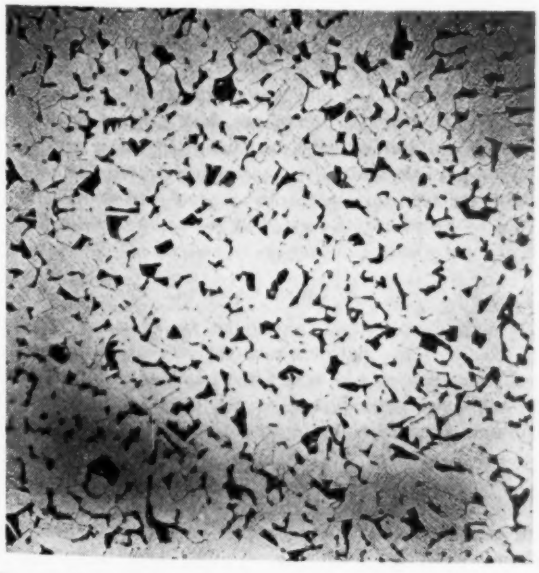


FIG. 6 MICROGRAPH AT POINT 3 (FIG. 5), SHOWING PEARLITIC STRUCTURE OF ORIGINAL TUBE METAL; ×100

the time that the water-supply and steam-relieving conductors already installed were adequate to take care of this extra water-wall surface. After 17 days of operation of the full waterwalls, tube failures were experienced at each end of the back wall, the tubes being the fourth from their respective ends. The ruptured sections were replaced and, after 18 days, one of the replaced tubes failed. Also, one of the side-wall tubes failed at the same time, this tube also being the fourth tube from the cooler end of its header. At a later period, the same tube in the opposite side wall failed, all of which seems to bear out the hypothesis that these failures were caused either by inadequate water supply to the waterwalls or by insufficient steam-relieving tubes from their upper headers.

In an effort to overcome these troubles, the water-supply and

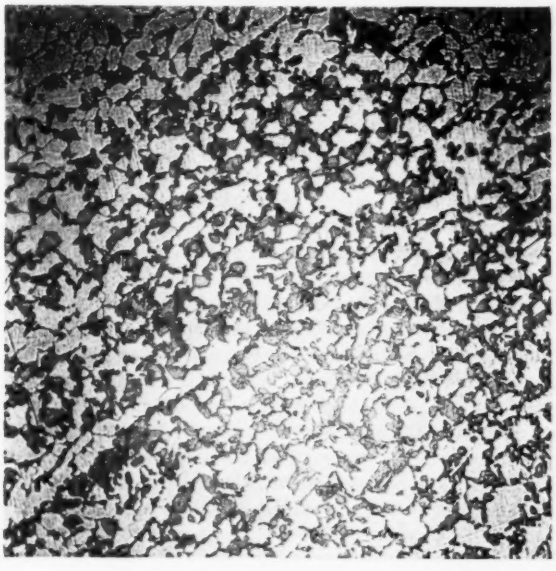


FIG. 7 MICROGRAPH AT POINT 4 (FIG. 5), SHOWING LOCATION OF METAL STRUCTURE CHANGE
(This point is where oxidation and thinning begins to show on inside of tube; ×100.)

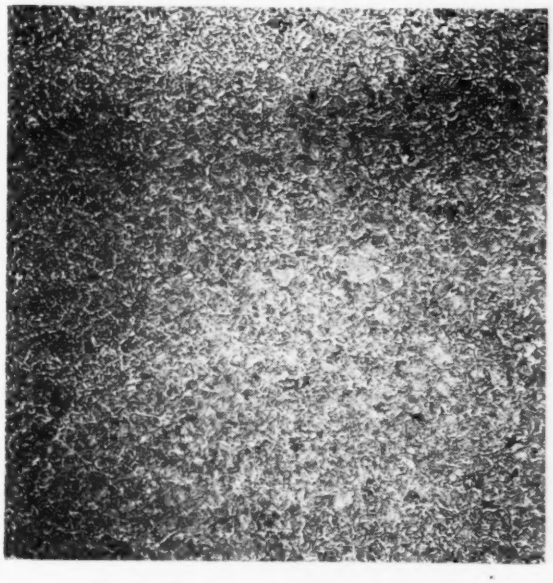


FIG. 8 MICROGRAPH AT POINT 5 (FIG. 5)
(This shows changed metal structure which is typical of metal from this point to rupture; ×100.)

steam-relieving tubes of the headers were increased by approximately 100 per cent. Since these additions, there has been but one failure and this was attributed to oxidation that occurred before the extra water-supply and steam-relieving tubes were installed. These failures were in the nature of metal cracking as described in the authors' paper. As all the tubes were in a vertical position, the conclusion is reached that vertical waterwalls, having inadequate supply and delivery connections, can have certain tubes, presumably the ones in the hottest zone, delivering an upward circulation; others, in the cooler portions of the wall, a downward circulation; while some tubes probably have little or no circulation.

At the time of these failures we were carrying concentrations in the boiler water of 1000 ppm total solids. The standard is now set at 2000 ppm and it is anticipated that this limit will be increased in the near future. The boiler water carries excess phosphate and hydroxide, and from 3 to 10 parts of sodium sulphite. Some of this highly concentrated boiler water is recirculated to the economizer inlet so that the economizer or first heat-absorbing surface of the unit is at all times protected against oxygen corrosion by the alkalinity and sulphite. Boiler feedwater having a pH of about 8 is raised to a pH of 9.5 or 10 by this recirculation. As a constant quantity of boiler water is recirculated, the pH of the water through the economizer naturally varies with the rating on the boiler and the concentration of the boiler water.

Regarding the decomposition of sodium sulphite, we have definite proof of its stability under normal 1300-lb service. During the time when steam binding was taking place, we were able to predict the failure of boiler tubes by the disappearance of sodium sulphite in the boiler under question. While the sulphite content of the other boilers in the plant would remain practically constant, the one suffering deterioration would change the sulphite to sulphate, and the chemical balance of the boiler-water constituents fully accounted for all the sulphite which had formerly been present.

It has been found at Springdale that, by maintaining an excess of sulphite, we are amply protected against the admission of oxygen which seems to appear when adding to or taking off equipment. Even though the Winkler test usually shows zero oxygen, there is a constant slow disappearance of the sulphite and this is particularly noticeable over the week end when severe load changes are experienced.

Further confirming the observations made by the authors in their paper, sections of the vertical waterwall tubes that failed show metal cracking, and micrographs of specimens from the overheated surface emphasize a definite zone where overheating caused the metal structure to change. These pictures are very interesting in showing that the metal structure undergoes a drastic change at the line where thinning by oxidation begins.

Fig. 4 shows the ruptured part of a waterwall tube after 17 days of service. In Fig. 5, the cross section of this tube at the rupture is marked to show the location of the micrographs in Figs. 6, 7, and 8. The micrograph in Fig. 6 exhibits the pearlitic structure of the original tube metal at point 3. At point 4, where oxidation and thinning begin to show on the inside of the tube, the structure shows a change, as indicated in Fig. 7. Fig. 8 illustrates the changed metal structure which is typical of the metal from point 5 to the rupture.

The authors refer to the "hiding out" of boiler-water chemicals and show graphical records of this more or less detrimental factor to boiler-water conditioning. While Springdale has not experienced this "hide-out" in so far as we are aware, we are experiencing it in other plants on the system. A search in one of our high-pressure boilers for this hide-out disclosed a residue of boiler-water solids in the intermediate uptake waterwall header 30 ft below the



FIG. 9 CHEMICAL DEPOSITS IN INTERMEDIATE UPTAKE WATERWALL HEADER, WHICH IS LOCATED 30 FT BELOW CENTER LINE OF BOILER DRUM

normal water level of the boiler. This deposit was found adhering to the surfaces of the header and tubes when the boiler was drained, more than 22 hr after it had stopped steaming. The beautiful frost-like patterns of chemical deposit are shown in Fig. 9.

A very important factor in the satisfactory operation of the Springdale high-pressure boilers is the steam-purifying equipment which was described in a paper by M. D. Baker at the same meeting at which the authors' paper was presented. This equipment is continuing to give first-class performance and shows no deterioration. The clean steam we are able to obtain with high boiler-water concentrations is, in the greatest measure, attributable to this purifying apparatus.

The highly concentrated boiler water, carrying its proportional amount of sulphite, phosphate, and alkalinity that is standard for Springdale, has produced the following beneficial results:

1 Complete economizer protection, as provided by recirculating boiler water through the economizer.

2 Clean metal surfaces as regards scale deposition and corrosion, and reduction of the amount of deposit to a point where this trouble is negligible.

3 A clean, crystalline type of sludge deposit.

4 Reduced blowdown, relieving the load on the evaporated make-up and heat-exchanging equipment.

These desirable conditions are all obtained at a decrease in cost of operation and maintenance, and a paper is being prepared for presentation in the near future in which will be given more detailed information on steam quality, economies gained by higher boiler-water concentration, proof of stability of sodium sulphite, and the benefits derived by recirculation of boiler-drum water through the economizer.

AUTHORS' CLOSURE

The discussion certainly continues the spirit in which the paper was written—that of a cooperative venture in which evidence from many sources was fitted together to produce as rational and consistent a picture as possible. In this same spirit, the authors venture a brief correlation of the preceding comments.

Fundamentally, what has been under discussion is the reaction between iron and water and the effect upon this reaction of dissolved substances and of temperature. As Mr. Place has pointed out, iron is a strong reducing agent. The chemical driving force for iron to take oxygen from water and liberate hydrogen is very great, even at room temperature, but the iron oxide, which is

one product of the reaction, quite literally gets in the way and slows down the reaction to such an extent that it is scarcely noticeable. Anyone can observe the progress of this reaction, however, by allowing a quantity of very finely divided metallic iron to stand in distilled water in a small bottle on his desk. On shaking this bottle once a day, small bubbles of gas will be observed to rise from the powder. This will continue day after day; a test after a sufficiently long time will demonstrate that the gas is hydrogen.

This same oxidation of steel by water, or reduction of water by steel, goes on continuously in every boiler, but it is only when the oxide resulting from it fails to retard the reaction that the boiler operator has a problem on his hands.

The protective oxide film can be at least partially destroyed or rendered more permeable by very high concentrations of sodium hydroxide. Because it is brittle and differs in thermal expansion from steel, it may also be cracked by repeated temperature changes. Anything which causes excessive local concentration of a boiler water containing some caustic soda or repeated overheating and quenching of a tube surface is thus potentially dangerous. To the authors, steam blanketing seems inevitably to tend to produce one or the other or both of these effects.

The factor of caustic attack has not been questioned in the discussion, but Mr. Place has gently indicated his disbelief in the validity of thermocouple measurements or of changes in the microstructure of steel as indications of excessive temperature in the top of a steam-blanketed tube. Base-metal thermocouples admittedly do fail all too rapidly when exposed to the conditions in a boiler furnace, yet recent studies at the Bureau of Standards show a maximum error of only 21 F, and this low rather than high, when chromel-alumel couples were heated for long periods of time in air. This maximum error was found after a couple had been exposed for 200 hr at 2200 F.[7] Failure occurred before 300 hr.

While the authors attach more significance than does Mr. Place to the temperature measurements mentioned in the paper, they feel that the microstructure of the steel is a still more certain criterion of overheating. This is demonstrated particularly well by the photomicrographs presented by Mr. Hankison correlating the change in structure in the tube wall with the zone of damage along the internal surface. Perhaps the sample which inspired Mr. Place's lack of faith in metallographic evidence actually had been overheated at some time in some unrecorded manner.

Mr. Place has argued that, if a tube were seriously overheated, it would deform to a greater extent than has been observed in many cases where grooving along the ceiling produced failure. In a tube of 3.5 in. outside diam with a wall thickness of 0.5 in. subjected to an internal pressure of 1400 psi, the nominal stress tending to rupture the tube is, however, only 3500 psi for metal in the longitudinal section of the wall. That creep of a low-carbon steel subjected to this stress at a temperature of 1000 F is slight has been shown by White, Clark, and Wilson.[8] They observed that SAE 1015 steel held at 1000 F for 16,000 hr under a load of 4000 psi showed an average rate of creep of only 0.043 per cent per 1000 hr.

It must be remembered also that a narrow band of steam along the ceiling of a tube would produce only a narrow band of overheated metal and a narrow groove, as in Fig. 15 of the paper. Deformation in such a case might be limited to a strip not more than 1 in. wide. Where the steam blankets more of the tube ceiling, as in the cross section shown by Mr. Hankison, there, usually is definite stretching of the tube wall prior to failure resulting in longitudinal cracks in the internal and external layers of oxide.

Overheating of a tube is, of course, not a new or unique occurrence. While we have thought for years in terms of the overheating due to deposits of solid on a heat-transfer surface, much less consideration has been given to the insulating effect of a blanket of steam, which was the starting point of the paper.

The experiences at Springdale and at Waterside described in the paper and the discussion by Mr. Hanlon bring us back to the fact that sludge settling out on the bottom of a tube can interfere with the transfer of heat from metal to water just as seriously as steam along its ceiling. The sludge which caused the failures at Waterside, reported by Mr. Hanlon, like that which led to the failures along the bottom of the cored tubes at Springdale was not, however, the relatively light calcium phosphate sludge which results from phosphate conditioning but, instead, a heavy sludge of magnetic iron oxide and metallic copper. Unless powdered metallic iron or ferrous hydroxide is being introduced intentionally into the boiler feed, such a sludge can result only from undesirable attack of water on steel in some region within the boiler, not necessarily the region where the sludge is found.

The initial settling out of the sludge on the bottom of the tubes described by Mr. Hanlon may have been favored by the presence of restrictors at the inlet ends of the lower tubes, as it was apparently by the cores in the tubes at Springdale. Overheating of the portion of the tube covered by the sludge could then have led to progressive oxidation of the tube wall.

Attempts to increase flow in one particular region of a boiler by specifically retarding it in another region seem, in a number of cases, to have transferred trouble rather than to have prevented it. Of the various expedients which have been tried to minimize steam blanketing in tubes through which there was positive circulation, the spirals introduced into the screen tubes at Logan, described by Mr. Webb, seem the most promising.

The suggestion of Mr. Kreisinger that tubes in high-pressure boilers are too thick-walled for their own good merits further study. Reduction in wall thickness will not, however, change the heat input to a tube or minimize the formation of a steam blanket within it. While cracking of tubes subjected to repeated overheating and quenching may be slower with thin than with thick walls, as suggested by Mr. Kreisinger, the paper records extensive damage to the thin-walled tubes of the low-pressure boilers in the Beacon Street Heating Plant in 8 years. The authors believe that grooving along the top of a tube might occur as rapidly in a thin-walled as in a thick-walled tube.

It was first demonstrated at Port Washington that grooving of the ceilings of inclined tubes could be minimized without mechanical changes by eliminating caustic alkalinity from the boiler water. This expedient was adopted at Logan, but the slope of the screen tubes affected was also increased, the spirals were inserted to throw water against the tops of the tubes, and the burners were tilted upward to reduce flame impingement. At Springdale, no change was made in water conditioning, but the difficulties first encountered were eliminated solely by mechanical changes. An appropriate conclusion to the question of the relative importance of the chemical and the mechanical factors has been written into the record by the announcement that, after operation for $2^1/_2$ years with no caustic alkalinity in the boiler water, extensive damage of the original type was recently discovered in the screen tubes at Port Washington.[9]

[7] "Stability of Base-Metal Thermocouples in Air From 800 Degrees to 2200 Degrees Fahrenheit," by A. I. Dahl, Research Paper RP 1278, National Bureau of Standards, *Journal of Research*, vol. 24, 1940, pp. 205–224.

[8] "Influence of Time at 1000 F on the Characteristics of Carbon Steel," by A. E. White, C. L. Clark, and R. L. Wilson, Proceedings of the American Society for Testing Materials, vol. 36, 1936, part 2, pp. 139–156, see Fig. 1, p. 141.

[9] "Port Washington Sustains Its Economy," *Combustion*, vol. 11, no. 7, 1940, pp. 33–34.

Determination of the Purity of Steam by Gravimetric and Spectrographic Methods

By M. C. SCHWARTZ,[1] W. B. GURNEY,[2] AND T. E. CROSSAN[3]

This is the first paper in a series of three covering investigations on the determination of purity of steam. In it the authors discuss the mechanism of steam contamination and methods of measuring steam purity classified according to impurities present. Principally, however, the subject matter is devoted to the apparatus and procedure followed in making gravimetric determinations of the residue from evaporated samples of condensed steam with the results obtained. As a check on these analyses, estimates are made for solids in steam from turbine-blade deposits. The results of spectrographic analyses are also given.

THE purity of steam produced from a boiler is a function of the composition of the boiler water and the boiler feedwater, as well as the mechanical design of the boiler in which it is produced. The production and consumption of huge amounts of steam at high pressures and temperatures have demanded the utmost in purity of the steam leaving the boiler, because of the economic importance of maintaining clean superheaters, turbines, and process equipment. Any changes made to increase the purity of steam are thus quite important and, particularly, since they may involve considerable expense, should be undertaken with due consideration of the methods used to determine the purity of the steam. Investigation of the methods of determining steam purity has been made at the Louisiana Station and has been applied in the control of steam production at this station.

Mechanism of Steam Contamination

Steam becomes contaminated with gases or moisture containing dissolved or suspended solids present in the boiler water or in the feedwater which may be used as a scrubbing medium for the steam. The contamination of steam results from the continuously occurring phenomenon of foaming or from the intermittent and irregular phenomenon of priming. An excellent summary of the factors contributing to foaming and priming has been presented by Powell (1).[4] Under boiler design this author covers such considerations as steam disengaging space, steam storage space, maximum water space, size of steam header, velocity of steam, various obstructions in the steam drum, and boiler pressure. Boiler operation covers such matters as rate of evaporation, load swings, and water level. Water treatment covers such items as composition and concentration of dissolved and suspended solids in the boiler water and in the feedwater, as well as the chemical condition at the metal surfaces exposed to the water.

Methods of Measuring Steam Purity

Methods of measuring steam purity may be classified according to the impurities present. For example, if an appreciable amount of moisture is being determined, the steam calorimeter can be used. If all the soluble electrolytes, arising from the solids or gases in the steam, are being determined in the condensate, then the electrolytic-conductivity method can be used. If specific chemical substances, such as ammonia, carbon dioxide, sodium chloride, and the like, are being determined, then specific chemical methods for these substances can be used. If the total solids present in the steam is being determined, then the residue after evaporation of the condensate can be weighed or, alternatively, it is apparently possible to deposit the solids on a surface and determine them in this manner. It is useful to consider the subject from the viewpoint of the impurity, because it is quite possible that some method, perhaps satisfactory in its own way, serves no really useful purpose in determining why the deposits in the equipment are forming and in what way they may be avoided.

Gravimetric Determination of the Residue From Evaporated Samples of Condensed Steam

The gravimetric determination of the total solids, including thus soluble and insoluble substances, in the steam by weighing the residue after evaporation of a sample of steam condensate can probably be considered the standard procedure. The minute weight of the residue offers no serious drawback since it may be determined readily with a sensitive balance. The problem of evaporating the sample of condensate free from subsequent contamination offers more difficulty. Sufficiently slow evaporation of the sample to avoid loss by spattering results in a serious loss of time and it is this factor which so far has prevented this method from becoming adaptable to routine use.

Although the preparation of pure water has been the subject of numerous researches in the laboratory, the investigators very seldom have determined the residue on evaporation. However, Acree and Fawcett (2) evaporated 30 l of pure water, contained in a quartz flask, in a platinum dish. The maximum residue was never more than 0.15 ppm and was as low as 0.03 to 0.06 ppm. Ulmer (3), using condensed-steam samples from a commercial boiler, determined the residue on evaporation to be 0.19 to 1.01 ppm, depending upon the boiler-water concentration.

A semimicro apparatus has been developed for evaporating 2-l samples of condensed steam. Ulmer (3), Powell (4), and Seyb (5) have considered the problem of evaporating condensed-steam samples. Fig. 1 shows the manner in which the evaporator works. The apparatus was built upon a triple-beam balance, of which Sargent No. 3455 is an example. The knife-edge mechanism, brass lever arm, and transite holder for the Pyrex micro bell jar were built in the University shops. The Pyrex jar is Corning No. 6880.

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[4] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed jointly by the Gulf States Utilities Company, Louisiana Station, Baton Rouge, La., and the Engineering Experiment Station, Louisiana State University, and accepted by the Joint Research Committee on Boiler Feedwater Studies and presented at the Spring Meeting, Boiler Feedwater Sessions, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, Worcester, Mass., May 1–3, 1940.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

The rubber tubing is pure gum. A 2-l Pyrex Erlenmeyer flask, protected from carbon dioxide with a soda-lime-sodium-hydroxide guard tube, contained the sample of condensate. Pyrex tubing was used to convey the water under constant head from the flask to the platinum dish beneath the bell jar. Block-tin tubing was available for use in place of Pyrex tubing if it seemed necessary,

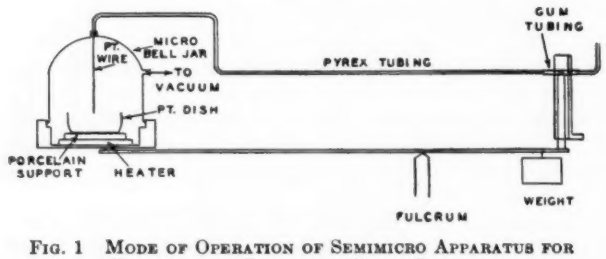


FIG. 1 MODE OF OPERATION OF SEMIMICRO APPARATUS FOR EVAPORATING CONDENSED-STEAM SAMPLES

but the results did not seem to warrant any change. A piece of platinum wire was sealed to the Pyrex tubing so that hot water would not come into contact with the Pyrex-glass tubing. A 200-w $2^1/_2$-in. Chromalox heating unit was used and the heat input was controlled by a rheostat. The platinum dish used, weighing 10 g, was the lightest obtainable commercially for its size. It was 2 in. in diam and 1 in. deep. This dish was supported upon a porcelain ring. The side arm of the Pyrex bell jar was connected to a Fisher air ejector. The trap in this line prevented the possibility of any water being sucked back.

In operation, the lead weight on the balance beam is initially adjusted to balance the weight of the platinum dish and water as well as the Pyrex jar and other accessories. For an actual run, control is made by using small weights on the brass lever arm. A change of about 10 g starts or stops the flow of water. After an initial adjustment of the rate of flow, the evaporation always continues with practically no attention for the time necessary, which is about 48 hr.

The total solids have been determined on condensed-steam samples from the station boilers. Test data are presented in Tables 1 to 3.

From the results of Tables 1, 2, and 3, it will be seen that for this station the totals of solids in the condensate from saturated-

TABLE 1 GRAVIMETRIC DETERMINATION OF TOTAL SOLIDS

No.	Boiler	Identification of condensate[a]	Total solids, ppm	Sample collected, date
		Superheated-steam samples		
1	5	D copper	0.30	12/27/38
2	6	S.S.	0.35	1/13/39
3	6	S.S.	0.40	1/17/39
4(a)[b]	5	D copper	0.35	1/25/39
4(b)[b]	5	S.S.	0.45	1/25/39
5(a)[b]	5	S.S.	0.30	1/31/39
5(b)[b]	5	D copper	0.25	1/31/39

[a] Condensed steam from a copper degasifier, with copper tubing to the degasifier—D copper.
Condensed steam from stainless-steel cooling coil (no degasification)—S.S.
[b] 4(a) and (b) were collected at the same time and similarly 5(a) and (b) were collected together. They were evaporated in the order listed; 2-l samples being evaporated.

TABLE 2 GRAVIMETRIC DETERMINATION OF TOTAL SOLIDS

No.	Boiler	Identification of condensate[a]	Total solids, ppm	Sample collected, date
		Superheated-steam samples		
6(a)	5	D stainless steel	0.25	2/6/39
6(b)	5	S.S.	0.40	2/6/39
		Saturated-steam samples		
7	5	S.S.	0.35	2/28/39
8	5	S.S.	0.30	3/5/39
9	4	S.S.	0.30	9/6/39
10	1	S.S.	0.30	9/11/39

[a] Condensed steam from a copper degasifier, with stainless-steel tubing to the degasifier—D stainless steel.
Condensed steam from stainless-steel cooling coil (no degasification)—S.S.

TABLE 3 SUMMARY OF GRAVIMETRIC DETERMINATIONS OF TOTAL SOLIDS

Superheated-steam samples			Saturated-steam samples
D copper	D stainless steel	S.S.	S.S.
0.30	0.25	0.35	0.35
0.35		0.40	0.30
0.25		0.45	0.30
....		0.30	0.30
....		0.40	....
0.3 avg		0.38 avg	0.31 avg

and superheated-steam samples are practically identical. The copper degasifier, which will be described later and which removes carbon dioxide from the samples, gives results approximating those obtained on samples subjected to no treatment for degasification. In all cases, the samples were collected within several feet of the sampling points.

From previous experience with deposits forming on glass conductivity cells, as well as by the deposits formed in the platinum dish, it was obvious that either copper or iron or both were present in the condensed steam. Tests for copper with diphenylthiocarbazone or dithizone on the condensed steam, and on samples concentrated by evaporation, gave negative tests. However there seems to be no doubt that high steam velocities will cause erosion of copper tubing. Whether or not these minute particles of copper will dissolve seems to depend upon a number of factors, one of the most important of which is the rate of flow of condensed steam through the sampling system. Interesting but old data on the solubility of copper in distillled water are given by Carnelley (6).

Semimicro determination of the iron in the residues from samples Nos. 9 and 10, Table 2, showed 0.23 and 0.25 ppm Fe_2O_3, respectively, out of a total of 0.3 ppm solids. Since these samples were collected in a most satisfactory manner, as far as avoiding possible contamination of iron from the sampling line is concerned, the conclusion seems logical that minute particles of iron are in suspension in the steam and are so finely divided that they show no tendency to settle out in the condensed steam. These iron particles are not visible in the solution but are removed by ultrafiltration. There is no reason to assume that these iron particles will affect the electrolytic conductivity of the solution of condensed steam. Upon the basis of these results of gravimetric determination of the residue from evaporating condensed-steam samples, coupled with determination of iron and of copper in these residues, there is apparently no more than about 0.1 ppm soluble solids in the steam at the Louisiana Station.

ESTIMATION OF SOLIDS IN STEAM FROM TURBINE-BLADE DEPOSITS

The values obtained for the purity of steam by any given method should correspond to the actual practical data observed in the plant. For example, the amount and kind of deposits observed on the turbine blades should be in proportion to the indi-

TABLE 4

Total wash water leaving turbine, lb	8181
Gravimetrically determined total solids in wash water, lb	1.5786
Total steam passed through turbine, lb	340340000
Solids on turbine blades washed off, ppm	0.005

TABLE 5

Total wash water leaving turbine, lb	9148
Gravimetrically determined total solids in wash water, lb	8.34753
Total steam passed through turbine, lb	775483000
Solids on turbine blades washed off, ppm	0.01

TABLE 6

Total wash water leaving turbine, lb	4949
Gravimetrically determined total solids in water, lb	1.4882
Total steam passed through turbine, lb	388975000
Solids on turbine blades washed off, ppm	0.004

cations obtained by any one of the usual methods of determining steam purity. Data of this kind, as useful as they are, do not furnish a continuously available record of steam purity as a guide to efficient operation.

The authors have attempted to determine the solids present in the superheated steam by washing a turbine with condensate, while the machine was rotating at slow speed under no load. Data are presented in Tables 4, 5, and 6 for three different turbines.

Although the solids were removed from the turbine by washing with condensate, all of the deposit so obtained was not soluble. Table 7 shows the relative amounts of insoluble and soluble material.

TABLE 7 RELATIVE AMOUNTS OF INSOLUBLE AND SOLUBLE MATERIAL

Data of	Solids, water soluble, lb	Solids, water insoluble, lb	Total solids, lb
Table 4	1.5462	0.0324	1.5786
Table 5	7.3534	0.9942	8.3476
Table 6	1.2081	0.2801	1.4882

Spectrographic Analysis of Condensed Steam

The spectrographic analysis of condensed steam was made at the spectrographic laboratory of the Massachusetts Institute of Technology. The authors in order to avoid contamination of the sample by the container, selected what was believed to be the most inert substance besides platinum, which was prohibitive in cost. The sample was collected in a 500-ml silver Erlenmeyer flask with a silver stopper. A qualitative analysis of the sample showed the presence of the following elements in diminishing amounts: silver, copper, iron, magnesium, sodium, and silicon. In addition, minute traces of boron, lead, and calcium were found. Phosphorus was not detected even with extreme concentration of the sample; however, this element is one to which the spectrographic method is less sensitive. A quantitative analysis gave the results shown in Table 8.

TABLE 8 QUANTITATIVE ANALYSIS OF CONDENSED-STEAM SAMPLE

Substance	Ppm
Copper	2.4
Iron	0.06
Sodium	0.14
Magnesium	0.056
Silicon	0.015

The presence of such large amounts of silver and copper was entirely inconsistent with the gravimetric results, as well as with the known electrolytic conductivity of this sample. Apparently the silver flask was the source of this unusual contamination. Accordingly another sample was collected for spectrographic analysis, this time in a Pyrex glass-stoppered bottle. Table 9 shows the results obtained.

TABLE 9 QUANTITATIVE ANALYSIS OF ADDITIONAL CONDENSED-STEAM SAMPLE

Substance	Ppm
Copper	0.05
Silicon	0.10
Calcium	less than 0.01

The drop in copper concentration from 2.4 to 0.05 ppm with the change in the container shows contamination from the silver flask, evidently from the silver solder used in fabrication. Correspondingly, the silicon concentration changed from 0.015 to 0.1 ppm, evidently due to the effect of the glass container.

By combining the results of both analyses, we derive the probable composition of the solids present in condensed steam, as shown in Table 10.

On the basis of the results shown in Table 10, one would expect a value for the total solids of condensed steam to be higher than 0.33 ppm, the exact value depending upon the actual negative ions associated with the positive ions shown in the table. It seems somewhat surprising that the results shown by spectrographic analysis should be greater than those obtained by evaporation and subsequent weighing, particularly since it has been assumed that the evaporation might probably be affected by contamination during the long period required. For further investigation using the spectrographic method, it will be desirable to establish the kind and amounts of impurities which may be present in the electrodes which are used in the analysis. That this type of error may exist is quite probable because of the fact that both calcium and magnesium are absent from turbine-blade deposits. In addition, it is well known that the electrodes used in spectrographic analysis are freed from traces of impurities only with difficulty. Traces of boron, silicon, copper, calcium, magnesium, and sodium usually exist even in treated electrodes (7, 8). Hilger H.S. carbon rods of very high purity contain traces of boron, calcium, copper, magnesium, and iron (9).

TABLE 10 RESULTS OF COMBINING ANALYSES OF TABLES 8 AND 9

Substance	Ppm
Sodium	0.140
Calcium	0.010
Magnesium	0.060
Silicon	0.015
Iron	0.060
Copper	0.050

The sensitivity of the spectrographic method, as is evident from the foregoing analyses, makes it an extremely valuable tool with which to attack the problem of steam purity. From data presented by Owens (10) the authors have calculated the sensitivity of the spectrographic method to be as shown in Table 11.

TABLE 11 SENSITIVITY OF SPECTROGRAPHIC METHOD

Sensitivity (inorganic liquid)	Minimum concentration (using 0.1 ml), ppm
Al	0.25
Ca	0.20
Mg	0.15
Si	2.50
Cr	0.10
Cu	0.05
Fe	0.05
Mn	0.01
Na	0.38
Pb	0.10
Sr	0.05

Even with the sensitivity shown in Table 11, some concentration of the condensed-steam samples is required in order to obtain indication of some of the substances present in the sample. In conclusion it should be remembered that spectrographic analysis will give the sum of the soluble as well as insoluble solids present in the condensed steam.

Acknowledgment

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Determination of Ammonia in Condensed Steam

By M. C. SCHWARTZ,[1] W. B. GURNEY,[2] and T. E. CROSSAN[3]

Of the impurities which may affect the electrolytic conductivity and pH of water, the gases carbon dioxide and ammonia are most resistant to removal. Carbon dioxide has previously been given consideration almost to the exclusion of the ammonia-removal problem. This second paper in a series of three on the subject of purity of steam by the authors deals with the preparation of ammonia-free water by various methods, of which phosphorus-pentoxide batch distillation apparently gives the most satisfactory results. The Nessler method of ammonia determination is also thoroughly discussed, as well as the volumetric determination of ammonia with sodium hypobromite and naphthyl red. The authors have used such determinations of ammonia in condensed steam to assist in evaluating a suitable correction factor to compensate for its presence.

THE electrolytic-conductivity method of determining steam purity is the one most frequently used in practice. Fitze (1),[4] Hecht and McKinney (2), Rummel (3), Powell (4, 5), Place (6) have investigated this method. It has the advantages of extreme sensitivity and is susceptible to being made continuously recording. It has the disadvantages of a need for a correlating factor between electrolytic conductivity and soluble solids, as well as correction factors for gases such as ammonia and carbon dioxide which dissolve in the condensed steam to form electrolytes. Rummel (7), Gurney (8), Powell and McChesney (9) have developed apparatus designed to remove carbon dioxide and ammonia from the sample without affecting the solids present but, thus far, ammonia has resisted attempts at complete removal, hence, a correction must be made for its presence. The authors have investigated the determination of ammonia in condensed steam to assist in evaluating the correction to be applied for its presence.

Preparation of Ammonia-Free Water

The preparation of pure water has been investigated frequently in the intervening years since 1894 when Kohlrausch and Heydweiller (10) purified water to the extent that its electrolytic conductivity was 0.04 micromho at 18 C and 0.054 micromho at 25 C.

The methods for the purification of water may be classified as physical and chemical or a combination of both. Hibben (11) classifies the physical methods of purification as falling into four categories, namely, those employing vacuum, those employing heat, those employing both vacuum and heat, and those employing freezing. In his own experimental work (11), Hibben uses the vacuum-sublimation process which includes all the processes mentioned, since it employs heat, vacuum, and freezing. The liquids to be freed from gas are vaporized and frozen in vacuo. By chemical methods we refer to those cases in which chemical substances, such as alkaline permanganate, bromine, phosphoric acid, and similar reagents, are added to cause certain impurities existing in the liquid to be retained in solution or to be made volatile on the application of heat.

The most convenient methods for determining the purity of water are the determination of the electrolytic conductivity and that of the pH. The most resistant impurities to be removed from water, which may affect the electrolytic conductivity and pH of the water, are the gases, carbon dioxide, and ammonia, which, in addition to physical solution in water, undergo chemical reaction to form electrolytes as the bicarbonate and carbonate and ammonium ion. Other gaseous impurities which may undergo similar reaction, as sulphur dioxide, have not been considered in reported work.

For some time carbon dioxide has been given consideration, to the general exclusion of ammonia, as the gaseous impurity in water to be removed. The probable reason for this has been due to the indefiniteness of the electrolytic-conductivity method as an indicator of purity, and to the lack of methods of determining pH without adding electrolytes, which change the pH themselves, as in colorimetric methods. The presence of carbon dioxide in solution tends to lower the pH of pure water. When the glass-electrode method for determining pH was applied to water considered to be free from carbon dioxide, the pH was found to be 7.9 to 8 (12). The increase in pH above that of 7, the value for pure water, was later ascribed to the presence of dissolved ammonia. When definite measures were taken to insure the absence of this impurity, the pH fell to 7 (13, 14).

Presumably the best procedure for preparing ammonia-free water is by distillation, adding acidic substances, as P_2O_5 to retain ammonium compounds in solution, and passing purified air through the distillate (13, 14).

Procedure for Preparing Ammonia-Free Water—Batch Distillation With P_2O_5

Phosphorus pentoxide was added to 1 l of laboratory-distilled water. The solution was distilled at a rate of about 8 ml per min in an all-Pyrex-glass still as shown in Fig. 1. The distillate was collected in the conductivity-cell holder *A* from which it could be collected for use or run to waste. Determinations of ammonia in the distillate, as well as electrolytic-conductivity measurements, indicated that about 200 ml of distillate should be discarded. From then on a receiver could be placed at point *B* and the next 600 ml could be collected. Two such portions were collected and 1 l of this water was then redistilled again, discarding the first 200 ml, and collecting for ammonia-free water the next 600 ml of distillate. The flask from which the solutions were distilled was equipped with a side tube and ground stopper which allowed

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[4] Numbers in parentheses refer to the Bibliography at the end of the paper.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

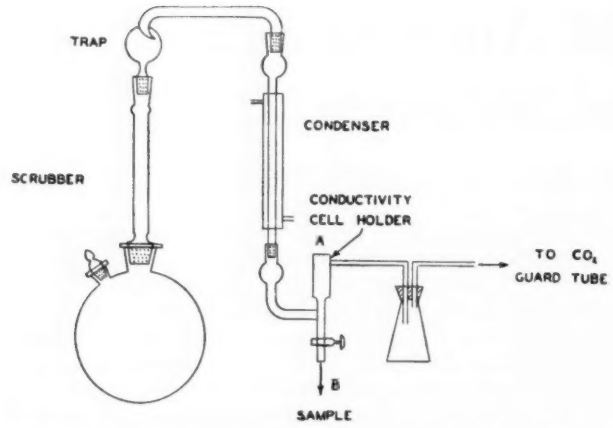


FIG. 1 DISTILLING APPARATUS FOR PREPARATION OF PURE WATER

the vessel to be filled and emptied without dismantling the apparatus. Passing purified air into the distillate was not done in this work. A Leeds and Northrup No. 4866 mho-ohm conductivity indicator with cell constant and temperature compensator was used. The electrodes were platinized platinum.

Tables 1 to 8 and Fig. 2 indicate the comparative effectiveness of various methods of preparing ammonia-free water and seem to confirm the work of Ellis and Kiehl (13) and Cranston and Brown (14) in establishing a phosphoric-acid distillation as the most satisfactory method available.

It is interesting to note, as indicated by the data in Tables 1 to 8 and in other experiments where distillation has been carried practically to completion, that as the distillation proceeds the electrolytic conductivity decreases continuously. On the other hand, in cases where chemicals have been added, the electrolytic conductivity commences to increase when sufficient concentration is reached to start carry-over.

The data of Table 8 indicate that simple distillation will not drive off the ammonia completely, even when some 700 to 800 ml of an original 1 l are distilled.

Considering the simplicity of the still used for preparing ammonia-free water, in comparison with some of those described in the literature, the electrolytic conductivity of the distillate

TABLE 1 DISTILLED WATER REDISTILLED[a]

Distillate collected, ml	Electrolytic conductivity, micromhos	Ammonia nitrogen found, ppm
0	1.45	greater than 0.20
80	4.40	greater than 0.20
80	2.60	greater than 0.20
80	1.60	0.12
80	1.30	..
80	1.04	0.06
80	0.83	..
80	0.69	0.04
80	0.56	0.04

[a] One l distilled water redistilled; 2 ml Nessler reagent used; readings taken after 30 min.

TABLE 2 REDISTILLED "CONDUCTIVITY WATER"[a]

Distillate collected, ml	Electrolytic conductivity, micromhos	Ammonia nitrogen found, ppm
80	2.50	0.17
80	1.40	0.12
80	1.09	0.09
80	1.00	0.07
80	0.85	..
80	0.74	0.05
80	0.60	0.05
80	0.515	0.04
80	0 455	0.03

[a] Collected 1 l of water by redistillation of distilled water collecting only when a value of 1 micromho or better was obtained; 2 ml Nessler reagent used; readings taken after 30 min.

TABLE 3 REDISTILLED GAS-FREE CONDENSATE[a]

Distillate collected, ml	Electrolytic conductivity, micromhos	Ammonia nitrogen found, ppm
0	1.2	..
80	3.5	greater than 0.20
80	1.9	greater than 0.20
80	1.5	0.17
80	1.16	0.12
80	0.88	..
80	0.715	0.07
80	0.59	..
80	0.47	0.06
80	0.37	0.03

[a] Redistilled 1 l degasifier effluent ("gas-free" condensed steam); 2 ml Nessler reagent used; readings taken after 30 min.

TABLE 4 REDISTILLED ALKALINE-PERMANGANATE-TREATED WATER[a]

Distillate collected, ml	Electrolytic conductivity, micromhos	Ammonia nitrogen found, ppm
0	1.9	0.14
80	8.0	greater than 0.20
80	3.7	greater than 0.20
80	1.7	0.09
80	0.72	0.04
80	0.52	..
80	0.50	0.01
80	0.45	0.02
80	0.45	..
80	0.42	..
80	0.465	0.02
80	5.5	..

[a] Redistilled 1 l of distilled water treated with alkaline $KMnO_4$; 2 ml Nessler reagent used; readings taken after 30 min.

TABLE 5 REDISTILLED PHOSPHORUS-PENTOXIDE-TREATED DISTILLED WATER[a]

Distillate collected, ml	Electrolytic conductivity, micromhos	Ammonia nitrogen found, ppm
0	1.9	0.14
80	4.6	0.10
80	2.15	0.03
80	1.1	0.01
80	0.67	0.01
80	0.48	..
80	0.40	0.01
80	0.36	0.01
80	0.325	..
80	0.36	0.01
80	0.585	0.01

[a] Redistilled 1 l of distilled water treated with P_2O_5; 2 ml Nessler reagent used; readings taken after 30 min.

TABLE 6 REDISTILLED BROMINE-TREATED DISTILLED WATER[a]

Distillate collected, ml	Electrolytic conductivity, micromhos	Ammonia nitrogen found, ppm
0	..	..
80	..	..
80	..	..
80	27.0	..
80	17.0	..
80	4.4	0.01
80	1.2	0.01
80	0.67	..
80	0.50	0.01
80	0.55	0.01
80	0.57	0.01

[a] Redistilled 1 l of distilled water treated with bromine; 2 ml Nessler reagent used; readings taken after 30 min.

TABLE 7 REDISTILLED WATER OF TABLE 5[a]

Distillate collected, ml	Electrolytic conductivity, micromhos	Ammonia nitrogen found, ppm
0	..	..
80	1.65	..
80	0.65	..
80	0.46	0.00
80	0.27	0.00
80	0.25	0.00
80	0.22	0.00
80	0.20	0.00
80	..	0.00

[a] Redistilled 1 l of water obtained by redistilling distilled water treated with P_2O_5; 2 ml Nessler reagent used; readings taken after 30 min.

TABLE 8 AMMONIA REMAINING IN SOLUTION AFTER DISTILLATION[a]

Liquid remaining, ml	Electrolytic conductivity, micromhos	Ammonia nitrogen found, ppm	1000 ml sample
230	2.9	0.14	Distilled water
230	3.05	0.20	Distilled water
175	2.8	0.12	Redistilled distilled water
210	2.85	0.04	CO_2-free condensate

[a] Two ml Nessler reagent used; readings taken after 30 min.

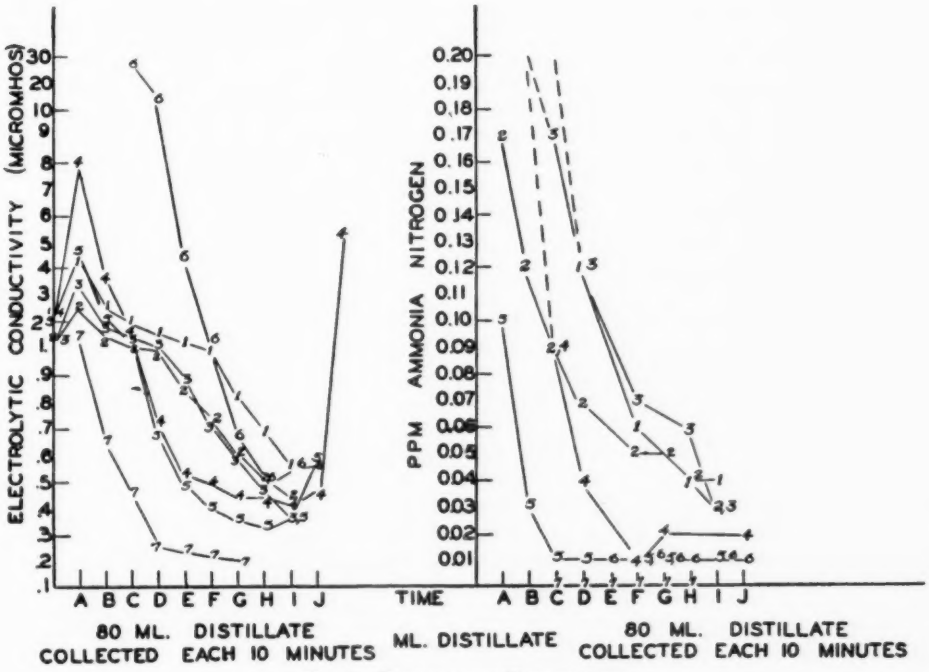


FIG. 2 DISTILLATION EXPERIMENTS

Legend

1 Distilled water redistilled
2 Redistilled "conductivity water"
3 Redistilled "gas-free condensate"
4 Redistilled alkaline $KMnO_4$-treated water
5 Redistilled P_2O_5-treated distilled water
6 Redistilled Br_2-treated distilled water
7 Redistilled No. 5 water

obtained is reasonably good. For comparison of the values obtained with those of other investigators, the data of Table 9 are included.

TABLE 9 CONDUCTIVITY OF "PURE" WATER

Prepared by	Year	Micromhos
Thiessenn, Herrmann (15)	1937	0.065–0.08
Rimattei, Petit (16)	1937	1.2
Straub (17)	1935	0.3–0.5
Best (18)	1929	0.3–0.75
Bengough, Stuart, Lee (19)	1927	0.045
Bencowitz, Hotchkiss (20)	1925	0.06–0.07
Kraus, Dexter (21)	1922	0.05–0.12
Bennett, Dickson (22)	1919	0.4
Clevenger (23)	1919	0.8
Weiland (24)	1918	0.053–0.070
Kendall (25)	1916	0.06–0.09
Bourdillon (26)	1913	0.2
Kohlrausch, Heydweiller (10)	1894	0.05–0.11
(Pure water standing in contact with air)		0.65–0.66

DETERMINATION OF AMMONIA—NESSLER METHOD

Although Nessler proposed a reagent for the direct determination of ammonia in 1856 (27), his method with various modifications apparently remains the most widely used. Naturally, in the intervening time a considerable volume of literature has accumulated, but there are few other methods proposed for the determination of small quantities of ammonia. Bjerrum (28) proposed an acid titration for determining both ammonia and carbon dioxide in distilled water. Makris (29) proposed a test using tannin solution and silver nitrate which he claimed was more sensitive than the Nessler test. Teorell (30) proposed a titrimetric determination for minute amounts of ammonia by using the reaction $2\ NH_3 + 3\ NaBrO = 3\ NaBr + 3\ H_2O + N_2$, the excess NaBrO being determined iodometrically or by means of naphthyl red, which is decolorized by NaBrO.

The most unfortunate feature of the Nessler method for determining ammonia is the variation in the preparation of Nessler's reagent and its resulting varying sensitivity. In addition, the reaction taking place is rather complex, and the Nessler reagent undergoes change with time.

Nichols and Willits (31) indicate the reaction to be

$$2\ (HgI_2.2\ NaI) + NH_3 = (NH_3.HgI_2) + 2\ NaI)$$
$$2\ (NH_3.HgI_2) = NH_2Hg_2I_3 + NH_4I$$
$$2\ (HgI_2.2\ NaI) + 2\ NH_3 = NH_2Hg_2I_3 + 4\ NaI + NH_4I$$

The compound $NH_2Hg_2I_3$ gives the color to Nesslerized ammonia solutions.

AMMONIA IN THE NESSLER REAGENT

In their study of the Nessler reaction, Nichols and Willits (31), working with a solution containing 2 ppm ammonia nitrogen, and adding 5 ml Nessler reagent to a 100-ml sample, found that the apparently clear yellow solution formed could be made clear and colorless by passing it through a microfilter, using 0.06-gage cellophane as the filtering medium, under a pressure of 290 lb of nitrogen. With the exception of one other statement to this effect quoted in Yoe (32), "J. H. Robertson and A. Hisey have recently made an extensive experimental study on the nature of the Nessler color and conclude that it is due to colloidally dispersed particles in suspension and not in true solution (private communication);" it is believed, in general, that an ammonia-free Nessler reagent has a slight yellow-green color definitely distinguishable from distilled water. In fact, the American Public Health Association liquid standards for ammonia nitrogen call for an addition of platinum salt (standard) to make the 0.00 standard. From the previously quoted statements, one would be led to believe that the color in a Nessler reagent is due to the presence of colloidal-like particles resulting from reaction with the ammonia present in the water used to make the reagent.

In the limited time that was available for experimental work on the filtration and ultrafiltration of a Nessler reagent, no success resulted in trying to get a colorless solution, and the filtered Nessler reagent gave results that were only 0.01 ppm lower than those obtained with unfiltered Nessler reagent. Filtrations were made through 12 sheets of quantitative filter paper and through very thick mats of asbestos. Experiments with cellophane in a

silver-plated steel ultrafilter, using nitrogen, resulted in accelerating the formation of a precipitate in the solution, instead of obtaining further clarification. Normally a precipitate forms in a Nessler reagent with the passage of time. Considering the complexities arising from attempting to filter a strongly alkaline liquid by ultrafiltration, it was decided not to attempt further experimentation along this line.

Sensitivity of the Nessler Reagent

Wirth and Robinson (33) point out that a zero blank does not necessarily mean there is no ammonia present but, as they determined experimentally using distilled water and the "standard methods" reagent, that 0.02 ppm or less of ammonia nitrogen is present. However, since they apparently prepared their ammonia-free water by simple distillation, it is quite possible that their minimum of 0.02 is actually somewhat higher.

Experimental Results

For the purpose of simplifying the determination of ammonia by the Nessler method, La Motte liquid standards and reagents were tried. Comparison of solutions was made in a La Motte-Nessler tube comparator. Ammonia-free water, an ammonium-chloride standard, and a Nessler reagent were prepared for comparison of results.

Using 1 ml of Nessler reagent per 50-ml sample and a 10-min reaction time as recommended by the American Public Health Association Standard Methods of Analysis (34), the results seemed definitely to limit the sensitivity of the determination. Using our ammonia-free water, it took 0.05 ppm of added ammonia nitrogen to give any indication of ammonia when compared with the La Motte standards. After considerable testing we decided to use 2 ml Nessler reagent per 50-ml sample and compare after 30 min. Tests were made in Nessler tubes, which were then closed with rubber stoppers. We used ordinary Nessler tubes after we had tried Nessler tubes as advocated by Powell (4) and found that we obtained the same results in both types of tubes. Where the sample to be collected was freely flowing, we caught the solution in an open Nessler tube by allowing it to flow under constant head into the bottom of the tube and then to overflow until the sample was ready to be tested. A flow of about 90-ml per min, for 30 min, was used. For accurate work, we checked and prepared anew, if necessary, the ammonia-nitrogen liquid standards used with a given Nessler reagent. It is quite simple to add ammonia nitrogen to ammonia-free water to make a series of standards and we prefer this procedure.

Electrolytic-Conductivity Correction for the Presence of Ammonia in Condensed Steam

The "ammonia correction" was determined by actual full-plant test. The data are presented in Table 10 and shown graphically in Fig. 3. Ammonia was introduced into the boiler-feedwater suction.

The slope of the line obtained in Fig. 3 gives 1 ppm ammonia nitrogen = 8 micromhos. Extrapolation to zero ammonia gives an extremely low electrolytic conductivity for gas-free condensed steam. From the pH value of the condensate and the known equilibrium existing between the bicarbonate and carbonate ions, it is probable that the ammonia exists in solution in combination with the bicarbonate and carbonate ion.

Determination of Ammonia Volumetrically With Sodium Hypobromite and Naphthyl Red

In order to check the results obtained by the Nessler method for the concentration of ammonia in condensed steam, we have considered other methods of analysis. As a result, we have investigated the oxidation of ammonia by sodium hypobromite, using naphthyl red as an indicator (30, 35). The method has demonstrated excellent possibilities but, as yet, has not been applied to routine testing. The reactions taking place are:

$$2\ NH_3 + 3\ NaBrO = 3\ NaBr + 3\ H_2O + N_2$$

$$NaBrO + \text{naphthyl red} = NaBr + \text{colorless product}$$

The reaction is very sensitive; 1 ml 0.0005 N naphthyl red being equivalent to approximately 1.9 gamma ammonia nitrogen (1 gamma = 0.001 mg).

The procedure as developed by the authors for use in determining ammonia in condensed steam is as follows:

Reagents (use ammonia-free water):

- 0.1 N NaOBr—2.5 g NaOH, 1.25 ml bromine, dilute to 500 ml
- 2 N Na_2CO_3—106 g per l
- 0.001 N NaOBr—dilute 0.1 N NaOBr using 5 ml 2 N Na_2CO_3 per 500 ml
- 4 per cent HBr—24 ml 48 per cent HBr, dilute to 200 ml
- $(NH_4)_2SO_4$—0.4716 g per l; dilute 1 ml to 100 ml; 1 ml = 1 gamma ammonia nitrogen
- 0.0005 N naphthyl red—30 ml glacial acetic acid, 3.75 ml 89 per cent H_3PO_4, 1.9 ml 25 per cent HBr, 10.5 ml freshly prepared 0.1 per cent solution of naphthyl red in water, water to 500 ml.

Procedure:

1. Take 10-ml sample for test in 25-ml Erlenmeyer flask
2. Add 2 ml 0.001 N NaOBr (exact); shake for ½ min
3. Add 0.2 ml 4 per cent HBr and shake
4. Add naphthyl-red solution from a microburette until just visible red color appears while sample is being shaken after addition of reagent; calculate ammonia-nitrogen

TABLE 10 EFFECT OF ADDING AMMONIA TO BOILER FEEDWATER ON THE ELECTROLYTIC CONDUCTIVITY OF CO_2-FREE CONDENSATE[a]

Electrolytic conductivity, micromhos	Ammonia nitrogen found, ppm	Remarks
1.29	0.17	Aug. 31, normal operation
6.60	0.83	Introduced NH_3
2.00	0.25	Discontinued NH_3
1.36	0.15	
4.21	0.50	Introduced NH_3
4.35	0.50	
3.27	0.40	
1.17	0.16[b]	Sept. 1, reflux condenser in
1.20	0.13[b]	
0.86	0.08[c]	Reflux condenser out
0.90	0.10[c]	

[a] Readings taken on the half hour; 2 ml Nessler reagent used; readings taken after 30 min.
[b,c] Different dilutions.

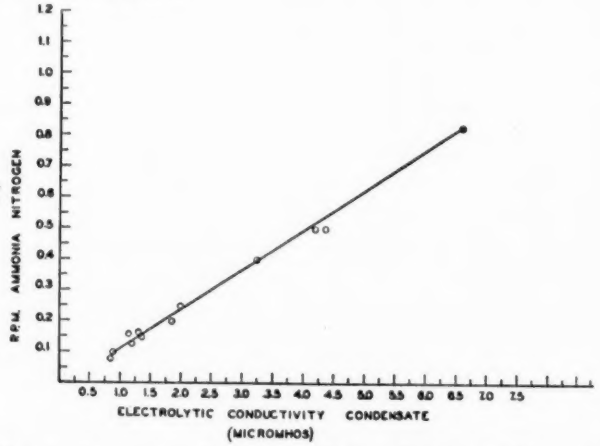


Fig. 3 Results of Full-Plant Test to Determine "Ammonia Correction"

concentration from naphthyl-red factor as obtained under standardization; 1 gamma per 10 ml = 0.1 ppm.

Standardization:

A Take 10-ml sample of ammonia-free water and follow procedure

B To 1 ml ammonia-nitrogen standard add 9 ml ammonia-free water and follow procedure.

Sample calculation:

a 10 ml ammonia-free water require 2.10 ml naphthyl-red solution

b 1 gamma ammonia nitrogen requires 1.60 ml naphthyl-red solution

0.5 ml naphthyl-red solution is equivalent to 1 gamma ammonia nitrogen

1 ml naphthyl-red solution = 2 gamma ammonia nitrogen.

The time required for a titration is only a matter of a few minutes. The standardization of the naphthyl-red-ammonia-nitrogen factor should be made at least every day because the solutions are not stable. Since the standardization is so easily made, our practice has been to make it twice daily if tests are made during an entire day. The constant check obtained by frequent standardization serves a useful purpose. Our results have shown that the method is good in the region 0.1 to 0.3 ppm ammonia nitrogen. In that range results are reproducible to within 0.02 ppm or less, as shown in Table 11. For solutions containing 0.5 gamma or 0.05 ppm ammonia nitrogen, the naphthyl-red factor deviates from the constancy shown at higher concentrations. Krogh (35) states that it is advisable to reduce the volume of hypobromite used when working regularly with amounts of ammonia nitrogen less than 1 gamma.

TABLE 11 TEST RESULTS

Ammonia nitrogen	Ppm	Naphthyl-red-solution factor in gamma ammonia nitrogen	
1 gamma	0.1	1.79, 1.88, 2.00, 1.85	1.88 avg
2 gamma	0.2	1.80, 1.96, 1.90	1.89 avg
3 gamma	0.3	1.96, 1.90, 1.91	1.92 avg

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Determination of Purity of Steam by the Electrolytic-Conductivity Method

By W. B. GURNEY,[1] M. C. SCHWARTZ,[2] and T. E. CROSSAN[3]

In order to assure correct boiler operation and to evaluate the effect of changes in water treatment or boiler design on purity of steam being delivered, it is essential to be able to measure any contamination that may occur. This paper, which is the last of three papers by the authors on the subject of steam purity, gives the procedure for making a practically continuous determination of steam purity by the only satisfactory method thus far available. In this method the electrolytic conductivity of the condensed steam is measured and then converted to a solids basis. The distillation and the gravimetric methods are also discussed by the authors.

THERE are two important and practical reasons for measuring the purity of steam: (*a*) To determine the extent of carry-over or steam contamination during normal operation so that correctness of operation may be maintained at all times and any irregularity detected at once; (*b*) to evaluate the effect of any changes in water treatment or of any changes in the mechanical design of the boiler on the purity of steam being delivered. Therefore, any method of measuring steam purity must be capable of practically continuous determination in order to be of real value. Thus far, the only method that is able to meet this requirement is one in which the electrolytic conductivity of the condensed steam is measured and thereupon converted to a solids basis by means of the proper calculations and corrections.

If either operation, a change in water treatment or a change in the mechanical design of the boiler, is such as to result in priming or a condition approaching that state, the increase in electrolytic conductivity of the condensed steam is sufficiently great to be readily recognized as such. It is only when conditions are such that the steam contamination is quite low, less than 1 ppm, and remains at nearly that value after various changes, that the electrolytic-conductivity method and other methods encounter difficulties in evaluating the results of the changes. It is understandable why there will be difficulty in securing the necessary accuracy when it is observed that condensed steam, containing 0.5 ppm solids, is 99.9995 per cent pure.

Procedure for Determining Steam Purity by Electrolytic Method

The following procedure has been employed for determining steam purity by the electrolytic method:

1 Determine the cell constant of the conductivity cell with a solution of potassium chloride at constant temperature. All electrolytic-conductivity values are referred to 25 C or 77 F by use of the temperature compensator on the conductivity meter.

2 Remove carbon dioxide and some ammonia from the steam sample by passing it through the degasifier. If the pH of the condensed-steam sample rises from the acid to the alkaline side on degasification, removal of carbon dioxide is indicated.

3 Observe the temperature of the condensed-steam sample. Adjust the conductivity meter to correct for the measured cell constant and temperature. Measure the electrolytic conductivity of the condensed steam.

4 Determine the carbon dioxide and ammonia in samples of condensed steam collected at the same time.

5 Compute the conductivity correction due to the presence of ammonia and carbon dioxide, if any; subtract these from the measured conductivity.

6 Subtract the value 0.05 micromho, that of pure water, from the measured conductivity.

7 Compute the total solids in the steam by applying the proper factor to the corrected electrolytic conductivity of the condensed-steam sample.

For measuring the electrolytic conductivity of the condensed-steam samples, Leeds and Northrup Meter No. 4866 is satisfactory as a portable instrument. The instrument is equipped with cell-constant and temperature-compensator control dials. For continuous recording of conductivity measurements, we use a Leeds and Northrup meter with a range of 0 to 2 micromhos in steps of 0.02 micromho. The adjustment for cell constant and temperature, however, is manually controlled. It is possible to compensate for temperature changes automatically by using resistance thermometers, equipped with auxiliary slide-wire drums which can be connected in the bridge circuit of the recording conductivity meter and thus correct for any variation from 77 F. A glass dip-type conductivity cell, having a cell constant not greater than 0.1 is used. The electrodes are platinized platinum.

Potassium-chloride solutions are universally accepted as standard reference solutions for electrolytic measurements. Jones and Bradshaw (1)[4] have prepared potassium chloride as follows: "The purest material available by purchase was recrystallized several times with centrifugal drainage. Qualitative chemical tests for magnesium and sulphate (the most likely impurities) gave negative results. The salt was fused in a platinum crucible, poured into a platinum dish, and transferred to a closed bottle while still hot. The salt showed no tendency to gain in weight by absorption of moisture when allowed to stand on the balance." Jones and Bradshaw (1) have considered the preparation of potassium-chloride reference solutions and, as a result of painstaking work, have recommended the following value for an approximately 0.01 N solution, which is the most dilute solution they considered. The solution is precisely defined as follows: 0.745263 g of KCl per 1000 g of solution in vacuum. At 25 C the conductivity is 1408.77 micromhos. For preparing the standard solution, Jones and Bradshaw recommend using conduc-

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] Numbers in parentheses refer to the Bibliography at the end of the paper.

tivity water prepared in contact with air. Such water will have a specific conductance of approximately 1 micromho at 25 C. The conductivity of the water used should be measured in advance and subtracted from the measured conductivity of the solution. It is desirable to have available a more dilute standard reference solution but, presumably, none has been prepared and measured with the exactness with which Jones and his associates have done for the more concentrated potassium-chloride solutions. Powell (2) states that a 0.0001 N potassium-chloride solution has a specific conductance of 15.27 micromhos at 25 C.

Removing Carbon Dioxide and Ammonia From Condensed Steam

The removal of carbon dioxide and ammonia from condensed steam is difficult. Available methods have been considered by the authors (3). An apparatus, represented in Fig. 1, has been developed for this purpose. So far as the authors have been

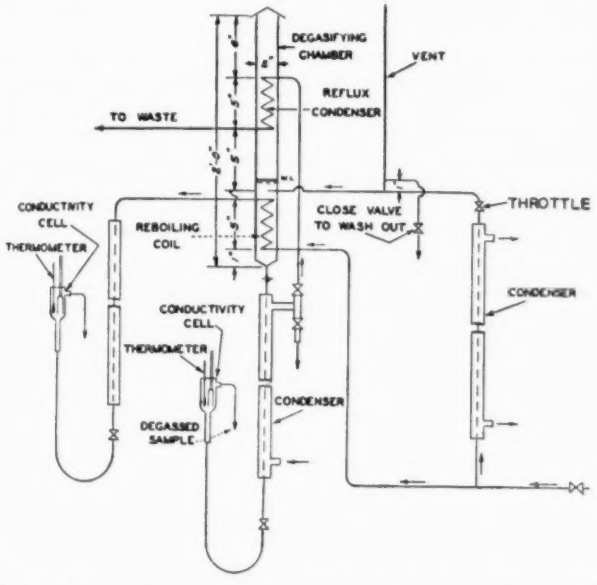


Fig. 1 Arrangement of Apparatus for Removing Carbon Dioxide and Ammonia From Condensed Steam

able to determine by the use of this system, carbon dioxide is removed to the extent that only undetectable amounts remain. Removal of ammonia, however, is incomplete. The pH of the condensed steam, measured with a glass electrode, is on the alkaline side. Steam is sampled with the standard A.S.M.E. sampling nozzle (4) installed in the main steam header close to the boiler. From a steel shutoff valve it is then conveyed in stainless-steel tubing to a tee where part of the steam passes directly to a copper reboiling coil in the copper degasifying chamber. From there the steam passes to a copper condenser. In addition to the dissolved or suspended solids present, this condensed-steam sample contains any soluble gases present in the steam and is referred to as the straight-through sample. It gives a prompt indication of changes in steam composition. The sample passes through a Pyrex-glass holder which contains a thermometer and a conductivity cell.

The remaining portion of steam from the tee passes through a stainless-steel condenser for partial cooling and from there into the copper degasifying chamber. Upon coming in contact with the steam-heated reboiling coil, the condensed steam again boils violently. Dissolved gases are flashed out and steam serves also as a scrubbing medium to assist in removing soluble gases. A copper reflux or vent condenser condenses any steam that might escape along with the dissolved gases and thus eliminates any increase in concentration in the degasifying chamber. A vent and overflow line assist in maintaining a constant water level in the degasifying chamber. The cover on the degasifying chamber fits loosely.

The degassed condensed steam issuing from the degasifying chamber is further cooled by passing it through another stainless-steel condenser from which it passes into a Pyrex-glass holder containing a thermometer and conductivity cell. This sample is referred to as the degassed sample.

The steam flow should be about 280 ml per min. The flow from the reboiling coil should be about 150 ml per min. The flow of degassed condensed steam should be about 80 ml per min. These flows are approximate for 720-F steam. Steam at other temperatures requires variation in flow through the reboiling coil.

The copper reboiling coil is made of $^1/_4$-in. tubing, $10^1/_2$ turns, and $1^1/_2$ in. diam. The copper reflux coil is made the same as the reboiling coil. With respect to the materials suitable for conveying steam and condensate with minimum contamination, it has been our experience that stainless steel is most suitable for high-pressure high-temperature steam, and copper is satisfactory for condensate if a suitable flow is maintained.

In testing for the presence of carbon dioxide in the condensed steam, it is customary to use N/44 or N/50 sodium hydroxide and a sample of at least 250 ml. One or two drops are usually sufficient to give a pink color with phenolphthalein. Under these conditions, the amount of carbon dioxide present is considered negligible. For determining ammonia in condensate, we have used both the colorimetric Nessler method and the volumetric sodium-hypobromite-naphthyl-red method, described in another paper (3). It is important to note that the relative-conductivity corrections to be applied for residual carbon dioxide and ammonia in the condensed steam are approximately in the ratio of 1 to 10 per ppm of each present. It is thus evident that, not only does ammonia affect the results more appreciably, but the residual ammonia must be determined with an accuracy of 0.01 ppm.

Our experience with the degasifier has indicated that no detectable amounts of carbon dioxide remain in the condensed steam, therefore no correction to the measured electrolytic conductivity has been applied in this respect. The value of 1 ppm ammonia nitrogen = 8 micromhos (25 C), which, as we have determined experimentally, agrees well with currently used values (2).

To convert electrolytic-conductivity measurements to ppm dissolved solids, the procedure has been as follows: Small increments of boiler water are added to degassed condensed steam. The total solids in the boiler water are determined gravimetrically. Providing the concentration increments are kept small, the linearity of the relationship between ppm solids and electrolytic conductivity is assured. A typical calibration is shown in Table 1.

TABLE 1 TYPICAL CALIBRATION OF ELECTROLYTIC-CONDUCTIVITY MEASUREMENTS

Boiler water added, ml	Solids added, ppm	Conductivity, micromhos
0.00	0.00	0.82
0.10	0.32	1.49
0.20	0.64	2.09
0.40	1.28	3.38

Boiler water 1601 ppm.

Fig. 2 represents the plotted results. The slope of the line gives the factor for converting micromhos to ppm; in this case it is 1 micromho = 0.5 ppm dissolved solids (25 C).

A typical calculation for degassed condensed steam follows:

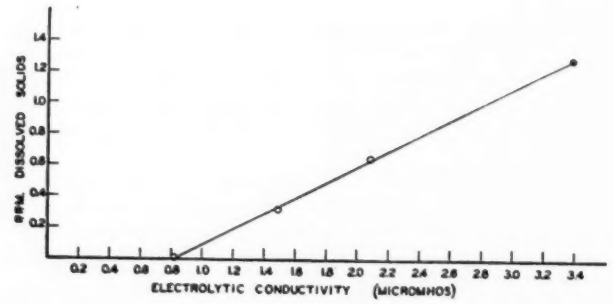


FIG. 2 RELATION BETWEEN PPM DISSOLVED SOLIDS AND ELECTROLYTIC CONDUCTIVITY

1.37 micromhos, measured conductivity; 0.14 ppm ammonia nitrogen; no carbon dioxide; dissolved solids in condensed steam = $(1.37 - 1.12 - 0.05) \times 0.5 = 0.10$ ppm.

DISTILLATION METHOD

Place (5) has recommended distilling a given volume of condensed steam in a glass still and measuring the electrolytic conductivity of the solution remaining in the distilling flask. The portion to be distilled is that sufficient to remove the volatile impurities. Then, providing no dissolved solids have been added from the glassware during the distillation, the remaining liquid will contain only the dissolved solids originally present in the sample. Place, however, considered carbon dioxide as the only gaseous impurity in the solution. As the authors' tests have shown, it is not possible completely to remove ammonia by simple distillation. Ammonia should therefore be determined in the solution remaining in the flask and the proper correction applied for its presence. A typical distillation on a sample of degassed condensate is given in Table 2 (it is not necessary to take a degassed sample for distillation).

TABLE 2 TYPICAL DISTILLATION OF DEGASSED CONDENSATE SAMPLE

Distilled, ml	Conductivity, micromhos	Ammonia nitrogen, ppm
0	0.71	0.07
73	1.15	0.10
71	0.99	0.07
67	0.955	0.07
71	0.875	0.06
67	0.72	0.03
73	0.60	0.01
64	0.50	0.01
65	0.42	0.01
65	0.37	0.01
59	0.35	0.01
Residual volume 83	Residual volume 2.56	Residual volume 0.03 (Total volume 758 ml)

Corrected conductivity of dissolved solids
$= [2.56 - (0.03 \times 8) - 0.05] \times (83 \div 758) = 0.25$

Dissolved solids $= 0.25 \times 0.5 = 0.125$ ppm

GRAVIMETRIC METHOD

In another paper (6), certain values for total solids in condensed steam, obtained by evaporation and weighing, are given. Examination of the residue disclosed appreciable amounts of iron which originally were presumably not in solution but in suspension and hence would not affect the conductance of the sample. More direct evidence was found by adding pure water to the residues in the platinum dish and obtaining the increase in conductivity. On the basis that 1 micromho = 0.5 ppm dissolved solids, the results given in Table 3 were obtained. These values are in line with those derived from more direct electrolytic-conductivity methods.

TABLE 3 VALUES FOR TOTAL SOLIDS IN CONDENSED STEAM BY EVAPORATION AND WEIGHING

From Bibliography (6)	Condensed	Dissolved solids, ppm
Table 2, No. 6(a)	Superheated steam	0.13
Table 2, No. 6(b)	Superheated steam	0.10
Table 2, No. 7	Saturated steam	0.05
Table 2, No. 8	Saturated steam	0.05

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The authors wish to thank H. C. Leonard, vice-president of the Gulf States Utilities Company and Dean L. J. Lassalle, director of the engineering experiment station, Louisiana State University for permission to publish this material. Grateful acknowledgment is made to E. B. Powell, consulting engineer of Stone and Webster Engineering Corporation, and to S. T. Powell, consulting chemical engineer, for their advice, highly valued suggestions, and encouragement throughout the course of the work being reported. Members of the chemical staff of the Gulf States Utilities Company, assisting in this research, were L. Young and J. C. Hill.

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3 "Determination of Ammonia in Condensed Steam," by M. C. Schwartz, W. B. Gurney, and T. E. Crossan, published on page 5 of this preprint.

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5 "Testing Steam Condensate for Its Quality and Purity," by P. B. Place, *Combustion*, vol. 9, no. 9, March, 1938, pp. 25–28.

6 "Determination of the Purity of Steam by Gravimetric and Spectrographic Methods," by M. C. Schwartz, W. B. Gurney, and T. E. Crossan, published on page 719 of this issue.

Discussion[1]

D. S. MCKINNEY[2] AND MAX HECHT.[3] In papers Nos. 1 and 3 the authors have made valuable contributions to the analysis of water of high purity, and to the application of conductivity methods to such water. For the sake of completeness, and for a better understanding of the technique used, some details should be amplified. For example, the drying temperature, which determines the degree of hydration of the weighed residue, is not given. The method of protecting the residue from hydration or carbonation during weighing is not described.

Examination of Table 1, in first paper, indicates that the residue was weighed to 0.1 mg. In view of the small quantity of material, it would seem advisable to weigh the residue to 0.01 mg. McKinney, in a discussion[4] of Ulmer's paper describes a method for securing this sensitivity at relatively low cost with an analytical balance.

DISCUSSION OF THIRD PAPER

Although numerous investigators have measured contamination in steam, few have considered the problem of proper sam-

[1] This is a combined discussion of the series of three papers on the general subject "Determination of Purity of Steam," by M. C. Schwartz, W. B. Gurney, and T. E. Crossan. These papers will be referred to in the discussion as first, second, and third, corresponding to the order in which they are published.

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[3] Pittsburgh, Pa.

[4] "Determination by the Evaporation Method of Small Amounts of Dissolved Solids in Water Such as Condensed Steam From Boilers," by R. C. Ulmer, discussion by D. S. McKinney, Proc. A.S.T.M., vol. 39, 1939, p. 1228.

pling. It should be emphasized that the sample is secured from a heterogeneous system, i.e., steam containing suspended solid or liquid particles. In order to secure a representative sample from such a system, it is essential that the velocity of the steam entering the openings of the sampling nozzle be identical with that of the main body of the steam approaching the openings. If the sample is withdrawn too rapidly, too great a proportion of steam is obtained, if too slowly the proportion of suspended liquid or solid will be too high. It is pertinent to ask if the authors have met this requirement.

The writers wish again to call to the attention of those interested in conductivity measurements, that the relation of total solids to specific conductance is likely to be much more variable than the relation of dissolved salts to specific conductance. The authors apparently do not distinguish between the terms total solids and dissolved salts as indicated by their method of calibration. In Table 2 of the third paper, the authors determine gravimetrically the total solids in one sample of boiler water to obtain the relation between solids in the steam and conductivity. It should be noted that an unknown fraction of the boiler solids are nonconducting (Fe_2O_3, SiO_2, etc). Furthermore, the fractions of nonconducting material may vary considerably during the normal run of a boiler. Yet in Table 3, the same factor is used to calculate the soluble portion of the evaporated residue from steam samples, which was found to be only about one third of the weight of evaporated residue.

It is suggested that the dissolved salts in the boiler water should have been made the basis of the calibration. This point was discussed extensively in an earlier paper[4] by the writers and re-emphasized in the discussion by Hecht of Ulmer's paper.[6]

Various factors for calculating dissolved substances in water from conductivity measurements have been reported in the literature. Reference to Table 9 of the writers' paper[7] shows that, with the exception of H and OH, all common ions in water have specific conductances per ppm between 1.14 micromhos for NO_3 and 2.99 micromhos for Ca^{++}. For OH^- the value is 11.3 and for H^+ 347 micromhos per ppm; all values being referred to 25 C. It is obvious that variations in H and OH concentrations will materially affect the relation of dissolved salts to conductances. It should therefore be emphasized that the factor reported for specific plants applies only to those plants. In general the appropriate factor will depend upon the water supply and its treatment.

E. B. Powell.[8] The authors have demonstrated the practical feasibility of power-plant measurement of solids carry-over in proportions as low as 0.1 ppm, and have described procedures by which a highly creditable approach to precision can be maintained in such measurements continuously. The electrolytic-conductivity method obviously does not give a direct measure of total solids to be encountered in condensed steam. Extensive work on corrections and calibrations, given special impetus by that of Hecht and McKinney,[5] now regarded as a classic in the power-plant field, has been carried on more or less continuously over the last 10 years. Statistical data are not yet available to cover correction factors for all water conditions. However, by securing appropriate calibrations for different water conditions to be met, following the general procedures described by the present authors, it is entirely practicable to set up data from which the individual plant can interpret the conductivity values in terms of actual solids carry-over to the degree of precision mentioned. The ability to measure is indeed an important factor in the ability to control, and the solids in steam are no exception. The authors' contribution will be decidedly welcome to the industry.

S. T. Powell[9] and H. E. Bacon.[10] The steam-quality data presented in this series of papers indicate the unusually low soluble-solids concentration of about 0.1 ppm, yet they are confirmed by conductivity, gravimetric, and spectrographic measurements. The authors have certainly shown to the satisfaction of the most critical reviewers that the total solids in solution in the samples tested were less than 0.2 ppm. This demonstrates that, if currently accepted specifications for steam quality are changed in the direction of higher purity, it is entirely feasible to apply tests which will give meaning to the specifications.

There can be no doubt that the more critical operation of high-pressure boilers requires a specific knowledge of steam quality, which, to be of value, should cover the full range of possible operating conditions. Each of the variables, such as boiler-water concentrations, water levels, and steam flow should be changed one at a time to develop a basis for predicting performance under all conditions. This requires evaluation of the carry-over at frequent intervals, or preferably continuously, which precludes the use of gravimetric determination and leaves no other recourse except estimating solids by conductivity methods. Qualitative measurement, where carry-over is appreciable, presents no problem. The major difficulties, and magnification of all inaccuracies, become increasingly great with improved quality of steam.

Methods for measuring the purity of steam have primary significance in performance tests covering a limited period, but their utility for routine control of operations may be quite valuable. Permanent and continuously recording steam-conductivity apparatus has enabled the operators of one large plant to increase boiler-water concentrations from a tentative limit of 2500 ppm to 3500 ppm as the standard for controlling the plant. The availability of instantaneous indications of carry-over may thus permit the exploration of numerous changes in operation which promise greater economy and efficiency. In such cases, the refinements described by the authors and corrections for dissolved gases are not practicable except for brief periods when highly accurate tests are desired. However, the operators soon learn to discount even large errors from such recognized sources and to correlate the meter charts with actual operating conditions. At the plant mentioned, the conductivity of the boiler blowdown is also recorded to provide a check on boiler-water concentrations.

The experience of the writers has been almost entirely with the vacuum type of steam-sample degasifier. The residual ammonia left by this equipment is of the same order of magnitude as that cited by the authors for the reboiling type of degasifier. The vacuum apparatus, for which the practical application has been described previously,[11] operates for long periods with very little attention. It would be of interest to have the authors suggest the important details in using their apparatus for continuous operation in routine plant control.

In testing condensed steam for ammonia in the atmosphere of most steam-generating stations, it is almost impossible to make and preserve large quantities of ammonia-free water. We have found the use of Folin's ammonia permutit to be satisfactory for preparing water to be used in making up ammonium-chloride

[5] "Electrical-Conductance Measurements of Water and Steam and Applications in Steam Plants," by D. S. McKinney and Max Hecht, Trans. A.S.M.E., vol. 53, 1931, FSP-53-11, pp. 139–151.

[6] Discussion reference 4, p. 1229.

[7] Discussion reference 5, p. 147.

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[9] Baltimore, Md. Mem. A.S.M.E.

[10] Special Representative, Stoker Department, Westinghouse Electric & Manufacturing Company, Philadelphia, Pa. Mem. A.S.M.E.

[11] "Steam Contamination—III. Determination of Steam Quality," by S. T. Powell, *Combustion*, vol. 9, no. 5, Nov., 1937, pp. 25–31.

standards. These are only used, however, in the adjustment of permanent standards for each new batch of Nessler reagents.

We hold no brief for conductivity measurements as providing absolute accuracy, realizing the deficiencies that exist. From our experience, however, we are convinced that steam-station operators now have available a means for closely estimating the performance of the boiler with respect to steam quality. Further improvement in apparatus for such measurements presents a worth-while problem for future study.

ALFRED WATSON.[12] In the application of the electrical-conductivity method to the determination of the purity of steam, the most perplexing problem has been the separation of the effect of dissolved solids from that of the ionized dissolved gases, such as ammonia and carbon dioxide. Various investigators have alternated between the two main lines of attack: (1) Measurement of the concentrations of ammonia and carbon dioxide, calculation of the increment of conductivity due to these concentrations, subtraction of this value from the observed conductivity, and calculation of the residual conductivity into terms of dissolved salts; and (2) physical elimination, in so far as possible, of dissolved gases before measurement of the conductivity, followed by direct calculation of the latter into terms of dissolved salts.

Application of the first method has been hampered by uncertainty as to the corrections to be applied when both ammonia and carbon dioxide were present in the sample of condensate; application of the second by the difficulty of removing all the ammonia by any practical means. The authors have offered the compromise of removing substantially all of the carbon dioxide and part of the ammonia, so that correction need be made only for low residual concentrations of ammonia.

While equipment for degasification of condensed-steam samples may well form part of the permanent testing equipment in a large central station or industrial steam plant, it is inconvenient to transport from plant to plant. The engineer engaged in field tests is predisposed, therefore, to rely upon the first method of leaving the ammonia and carbon dioxide in the sample, and to develop as accurate a means as he can for correcting his observed values for conductivity in the light of his analyses for these gases.

Offhand, it might seem suitable in making correction for carbon dioxide and ammonia to read off from the specific-conductivity curves of each, respectively, that conductivity value corresponding to the determined concentration, to add these two values, and to subtract the total from the observed conductivity of the sample regarding the difference as the conductivity due to dissolved solids.

In a great many instances quite anomalous values have been obtained through the use of this method of correction. The most common anomaly is that of a negative value when the total correction is subtracted from the observed conductivity. Such a value is, of course, an impossibility. Wherein does the error lie?

It is the writer's opinion that the method often followed of making separate corrections for ammonia and for carbon dioxide and then totaling these two values is incorrect, because it neglects the fact that the sample of water can have one and only one pH value and one equilibrium between the various ions for the conditions which existed when the conductivity was measured. Actually, since ammonia is a base and carbon dioxide an acid in solution, with different relative amounts of these two, the relative concentrations of H^+, NH_4^+, OH^-, HCO_3^-, and CO_3^{--} ions in the sample will vary. The specific conductivity of OH^- ion is, however, nearly 3 times, and that of H^+ ion 5 times that of NH_4^+, HCO_3^-, or CO_3^{--} ions. Correction values obtained from the separate specific-conductivity curves of carbon dioxide and ammonia, determined respectively at low and at high pH values, cannot be consistently correlated, since these curves have no community of pH value, either with each other or with the sample to which the correction is to be applied. Therefore, it is not permissible to add such separate corrections and subtract the total from the specific conductivity of the sample.

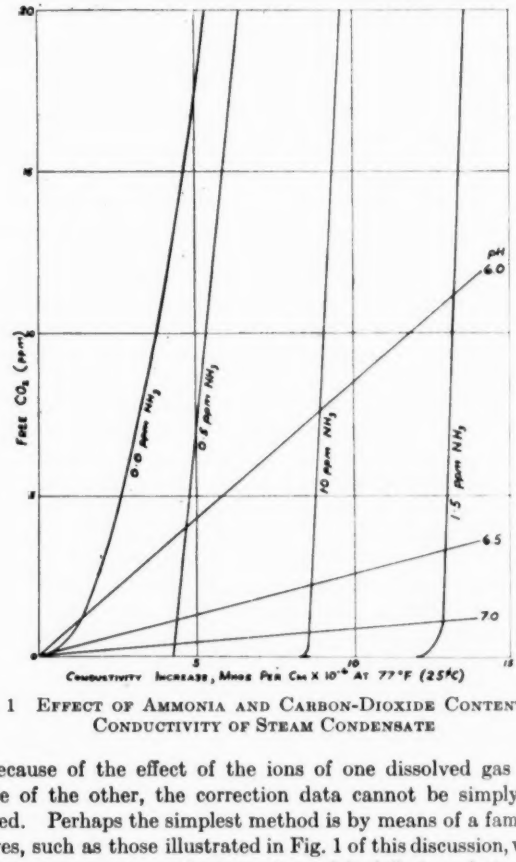


FIG. 1 EFFECT OF AMMONIA AND CARBON-DIOXIDE CONTENTS ON CONDUCTIVITY OF STEAM CONDENSATE

Because of the effect of the ions of one dissolved gas upon those of the other, the correction data cannot be simply presented. Perhaps the simplest method is by means of a family of curves, such as those illustrated in Fig. 1 of this discussion, which give the increase in conductivity for widely differing relationships of carbon dioxide and ammonia in the sample at its resultant pH value. These curves, based on the dissociation constants of carbonic acid and ammonium hydroxide, represent equilibrium conditions.

TABLE 1 WEIRTON STEEL COMPANY CONDUCTIVITY TESTS; 900-LB-PRESSURE BOILERS

				Boiler No. 1				
				Using separate factors for CO_2 and NH_3			Using coordinated factor for CO_2 and NH_3	
Time	CO_2, ppm	NH_3, ppm	Conductivity, mho × 10^{-6}	Correction, mho × 10^{-6}	Dissolved solids, ppm		Correction, mho × 10^{-6}	Dissolved solids, ppm
2:15 p.m.	16	0.4	7.2	7.5	−0.2		5.7	+1.0
2:30 p.m.	16	0.4	7.1	7.5	−0.3		5.7	+1.2
2:45 p.m.	11	0.4	6.0	6.5	−0.3		5.1	+0.6
3:00 p.m.	12	0.4	5.5	6.7	−0.7		5.2	+0.2
3:15 p.m.	16	0.4	5.9	7.5	−1.0		5.7	+0.1
				Boiler No. 2				
11:30 a.m.	6	0.5	5.1	5.9	−0.5		4.9	+0.1
11:45 a.m.	7	0.4	4.9	5.6	−0.5		4.6	+0.2
12:00 noon	7	0.4	5.5	5.6	−0.1		4.6	+0.6
12:15 p.m.	7	0.4	5.4	5.6	−0.1		4.6	+0.5
12:30 p.m.	7	0.3	5.0	5.0	0.0		4.2	+0.5
12:45 p.m.	8	0.4	5.4	5.8	−0.3		4.7	+0.5
1:00 p.m.	8	0.4	5.7	5.8	−0.1		4.7	+0.7
1:15 p.m.	8	0.4	5.3	5.8	−0.3		4.7	+0.4
1:30 p.m.	11	0.3	5.7	5.9	−0.1		4.8	+0.6

[12] Hall Laboratories, Inc., Pittsburgh, Pa.

The curves of Fig. 1 are expressed in terms of ammonia and of "free" rather than total carbon dioxide, as determined by standard methods. The most significant feature is the extremely slight effect of carbon dioxide upon the conductivity when ammonia is also present and the pH value is below 7.

The suggested procedure for testing a sample of steam condensate is as follows:

1 (*a*) The concentrations of "free" carbon dioxide and ammonia in the sample are determined by recognized standard methods.[13]

(*b*) The pH value of the sample may be determined by the glass electrode, as with Coleman electrometer Model 3D, or Leeds and Northrup No. 7661-A1, or equal.

2 In case the content of "free" carbon dioxide is zero or negative, or the pH value exceeds 7, for the most accurate work it will be advisable to adjust the "free" carbon dioxide to an adequate excess or the pH to below 7 by means of pure carbon dioxide, and recheck the ammonia content.

3 The specific conductivity of the sample under these conditions is obtained by means of a conductivity-resistivity indicator, such as Leeds and Northrup No. 4866, or equal.

4 The correction for dissolved gases is read from a chart of the type of Fig. 1 of this discussion, and subtracted from the observed conductivity.

5 The corrected conductivity is converted into ppm of dissolved salts by the use of a suitable factor.

Table 1 with this discussion, presents data on conductivity, ammonia, and carbon dioxide, obtained in a plant test on steam quality. In this case columns 5 and 6 give the correction values and the dissolved solids, respectively, when separate correction factors for carbon dioxide and ammonia were used, neglecting the fact that the sample could only have one pH value. Columns 7 and 8 give the values obtained when the correction value was read off from the curves of Fig. 1. It is notable that in the former case the value for dissolved solids is negative in every case but one—an obvious impossibility. In the latter all values are on the positive side.

The suggestion of adding carbon dioxide to a sample of steam condensate instead of removing it may seem unorthodox, but is as fundamentally sound as the data for the dissociation of carbonic acid and ammonium hydroxide. It should be kept in mind that the curves of Fig. 1 are calculated from these values in the literature, and the job of verifying them completely in practical determinations has only begun. We plan much further work along this line, and, finally, to present this entire subject in more complete form at a later date.

[13] "Standard Methods for the Examination of Water and Sewage," Eighth edition, American Public Health Association, New York, N. Y., 1936.

Authors' Closure

As Messrs. E. B. and S. T. Powell point out in their discussions, it has been demonstrated to be possible to determine, continuously, solids carry-over in steam in proportions as low as 0.1 ppm. The authors feel that they have contributed to this important power-plant problem. The variety of conditions found in different plants and the requirements of different individuals, as pointed out by Mr. Watson, seems to indicate that considerable flexibility will be required in test methods. Under such circumstances, the problem of carry-over measurement must be attacked from all angles.

The drying temperature for gravimetric determination of total solids residue, requested by Messrs. McKinney and Hecht was 110 C. The weight of the residue upon successive weighings, involving the incidental transferring from desiccator to balance, did not change, indicating that hydration and carbonation were not taking place. It is possible to weigh the residue to 0.01 mg but the authors do not believe this accuracy is warranted at present. It is worth-while to point out the fact that the temperature between successive weighings should be practically constant. The question of steam sampling has been considered by the authors and suitable flow maintained to secure proper velocities. This phase of the problem has also been emphasized by S. T. Powell.[11] Messrs. McKinney and Hecht correctly indicate that dissolved solids rather than total solids should be made the basis of calibration for electrolytic conductivity versus dissolved solids. In the case of the Louisiana Station, the dissolved and total solids are substantially the same. The correspondence between gravimetrically determined dissolved solids, after subtracting the weight of nonconducting residue, and independently determined dissolved solids by conductivity measurement bears this out.

Mr. Watson's procedure for computing the correction to be applied to electrolytic-conductivity measurements by determining the ammonia and carbon-dioxide concentration in the condensed steam, adjusted to a pH equal or less than 7 with carbon dioxide, is quite interesting.

In conclusion the authors would again like to repeat: The problem of producing pure steam is of great importance and justifies the attention of all power-plant operators.

A Theoretical Investigation of Plastic Torsion in an I-Beam

By D. G. CHRISTOPHERSON,[1] OXFORD, ENGLAND

The distribution of stress in an I-beam in torsion is investigated by a method of successive approximation, and the regions of initial yielding are determined for several values of the angle of twist. Formulas are given for elaborating the methods of Southwell(1)[2] and of Shortley and Weller (2) in order to facilitate this determination.

The value of the stress-concentration factor in the fillets is found to agree well with results obtained by soap-film tests, and regions of plastic flow approximate closely those observed by MacGregor and Hrones (3) by the use of the etching method.

ACCORDING to the standard theory, the problem of the torsion of a prismatic bar reduces to the determination of a function such that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \phi = -2G\alpha \ldots\ldots\ldots\ldots [1]$$

where G is the rigidity modulus, α the angle of twist per unit length, and ϕ has boundary values everywhere zero.

Many writers have pointed out that an approximate solution to this equation may be obtained by transforming it into an equation of finite differences, in which the value of the stress function at any point is obtained in terms of its value at a number of surrounding points. The most usual procedure has been to cover the area under consideration with a rectangular grid,[3] to compute the value of ϕ at the nodes, and to argue that a surface passing through these points will have nearly the same properties as the actual stress surface, i.e., its contours and gradients will give the directions and magnitudes of the shear stresses, its volume being equal to one half the applied torque. In this procedure the value of ϕ is given in terms of four equidistant values by the equation

$$4\phi_0 = \phi_1 + \phi_2 + \phi_3 + \phi_4 + 2G\alpha h^2 \ldots\ldots\ldots\ldots [2]$$

where h is the length of the side of any square in the mesh.

Means of obtaining a rapid solution of such equations in bulk have recently been given by Southwell (1) and by Shortley and Weller (2), although the latter confine themselves to the solution of Laplace's equation $(\partial^2/\partial x^2 + \partial^2/\partial y^2)\ \phi = 0$

They also give an interpolation function in polynomial form

$$\phi = a_0 + a_1 r \cos\theta + b_1 r \sin\theta + a_2 r^2 \cos 2\theta + b_2 r^2 \sin 2\theta \ldots [3]$$

where r, θ are polar coordinates and ar, br, constant coefficients for use over any single square of the mesh, which affords an excellent basis for the deduction of formulas given later in this paper.

Southwell's solution, on the other hand, was applicable to Poisson's equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \phi = f(x, y)$$

where $f(x, y)$ was any continuous function whatsoever. In making this generalization some rapidity in the convergence of the solution was lost, but in the paper (1) referred to previously a number of problems were solved, including one which referred to plastic torsion.

This present paper was planned with a view to comparing this theoretical approach to the problem of plasticity with the experimental approach recently presented by MacGregor and Hrones (3). However, it has been found desirable to make use of certain formulas not contained in the earlier paper (1), which, it is believed, increase the accuracy obtainable by the use of a mesh of given size, and thus accelerate the convergence.

It is evident that Equation [2] can only be used exclusively when the section under consideration is such that it can be covered by exact squares. For all other sections it is necessary to develop an analogous relation between the ϕ's when the surrounding points are all at different distances h_1, h_2, h_3, and h_4, from the center, as shown in Fig. 1. This relation is

$$\left[\frac{1}{h_1 h_3} + \frac{1}{h_2 h_4}\right]\phi_0 = \frac{1}{h_1 + h_3}\left[\frac{\phi_1}{h_1} + \frac{\phi_3}{h_3}\right] + \frac{1}{h_2 + h_4}\left[\frac{\phi_2}{\phi_4} + \frac{\phi_4}{h_4}\right] + G\alpha \ldots [2a]$$

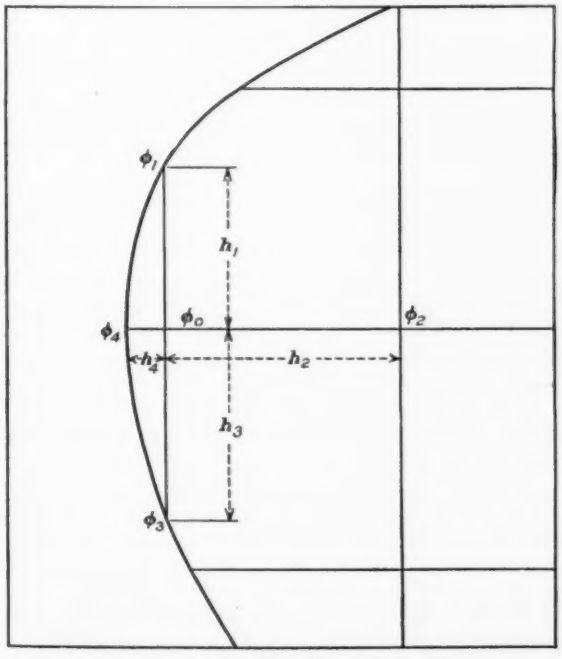


FIG. 1

[1] University College, Oxford University, Oxford, England.

[2] Numbers in parentheses refer to the Bibliography.

[3] An exception is Southwell's (1) triangular mesh.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until April 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

These are the only equations necessary to determine the required values of the stress function in the elastic problem. From these, with the assistance of the interpolation function given previously in Equation [3], the values of the gradients, hence of stresses, may easily be obtained. The volume, hence the torque, comes from the relation

$$\text{Volume of single square} = \left[\frac{\phi_1 + \phi_2 + \phi_3 + \phi_4}{4} + \frac{1}{6}G\alpha h^2\right] h^2$$

When we pass beyond the yield point, however, a different state of affairs obtains. It is first necessary to make a simplifying assumption with regard to the stress-strain curve of the material, and the idealized diagram shown in Fig. 3 is chosen. On this

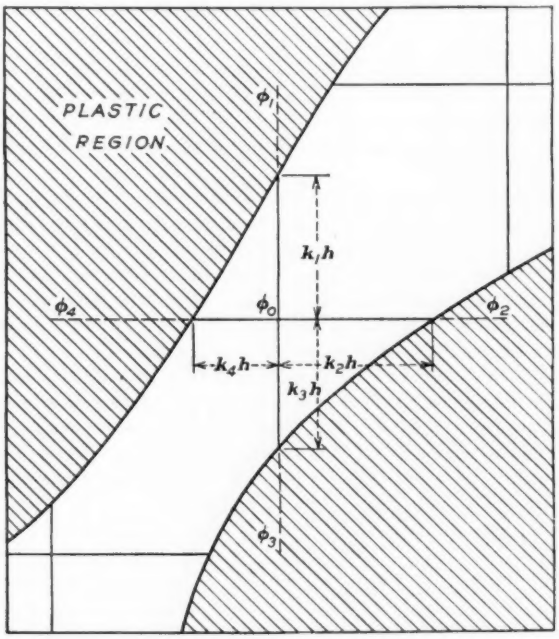


FIG. 2

assumption we may make use of the concept of a roof, or limiting value of the stress function, described by Nádai (4). The contours of this roof run everywhere parallel to the boundary of the section and the gradient is uniform and equal to the limiting shear stress.

On this understanding it is easy to compute the values of the ordinates of the Nádai roof over any section, and to make use of the overriding conditions that ϕ must not exceed these values. All that will then be required will be formulas, which provide relations between the ϕ's when some parts of the square under consideration fall in the plastic zone and some (including the central point) in the elastic.

Let us suppose that the limits of the plastic region lie at distances k_1h, k_2h, k_3h, and k_4h, as shown in Fig. 2 so that when the plastic region does not invade the square under consideration at all

$$k_1 = k_2 = k_3 = k_4 = 1$$

Then the relation which we require is

$$2\left(\frac{1}{K_1 + K_3} + \frac{1}{K_2 + K_4}\right)\phi_0 = \frac{\phi_1 + \phi_3}{K_1 + K_3} + \frac{\phi_2 + \phi_4}{K_3 + K_4} + G\alpha h^2 \quad [4]$$

where $K_1 = 2k_1 - k_1^2$, $K_2 = 2k_2 - k_2^2$, etc.

But for every link crossing the plastic boundary one new variable k will be introduced, so one more equation will be required. This equation is afforded by the fact that at the point of contact the stress surface ϕ is tangent to the roof, which has a known gradient. From this condition we deduce the equation

$$\phi_0[K_1 + K_3 + 2k_1^2] = k_1\phi_3 + (2k_1 + K_3)\phi_1 + (K_1 + K_3)\tau_0 h \sin\psi \quad [5]$$

In this equation ψ is the angle made by the link with the contour line of the roof at the point of contact; τ_0 is the maximum

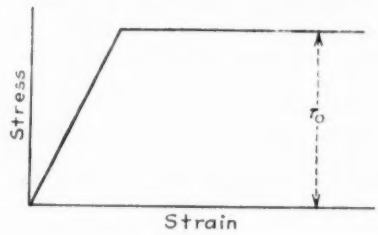


FIG. 3

shear stress as indicated in Fig. 3. There will be one such equation for every link crossing the boundary between the elastic and plastic zones, i.e., for every value of k required.

It is not proposed in this paper to enter into detail with regard to the means of obtaining a rapid solution of these equations. The processes used have been described fully in other papers (1, 2). It is therefore only necessary to consider means of dealing with the additional variables k, and the equations giving them.

Fortunately this presents little difficulty, for, if we bear in mind the condition that the stress function cannot have ordinates in excess of those of the roof, and that, when the elastic equations indicate that it does so, the point considered must fall in the plastic region, we may rapidly obtain an estimate of the boundaries of this region. Applying then equations of the type of Equation [4] only, we shall be able to determine values of ϕ everywhere, using the assumed values for k. Substituting these ϕ's in equations of the type of Equation [5] we can redetermine more exactly the limits of the plastic zone. It will be found that changes in these limits will produce only small or negligible changes in the vertical ordinates of the neighboring points. That this must be so may readily be seen from geometrical considerations. A small and nearly normal displacement of a surface tangent to a given plane produces a large change in the position of the line of contact.

The whole question of the accuracy and convergence of finite difference equations applied in this manner will not be gone into here. It is hoped that the degree of correlation with experimental evidence which has been obtained will be regarded as providing sufficient justification for the procedures used.

We are now at liberty to proceed to the consideration of the specific problem presented in this paper. Evidently, what in fact we need to determine is the extent to which orthodox (simple) theory is erroneous. For this purpose it is desirable to use as standard of comparison the simplest possible theory of the I-beam—that which assumes the beam to be built up of a number of independent rectangles, each of which is long enough in comparison with its breadth for end effects to be negligible.

On this understanding the relation between torque and twist is

$$T = \sum \frac{1}{3} b^3 l G\alpha \quad [6]$$

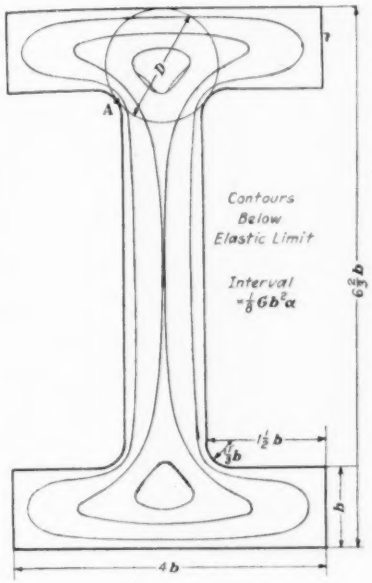


Fig. 4

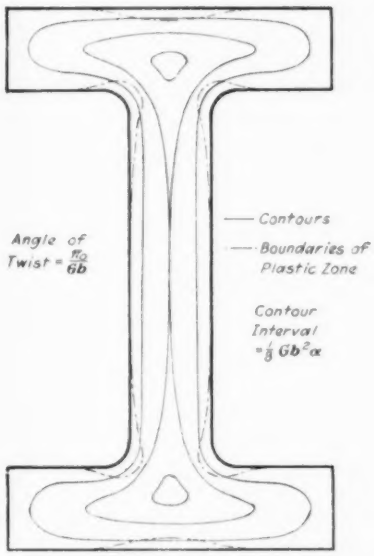


Fig. 5

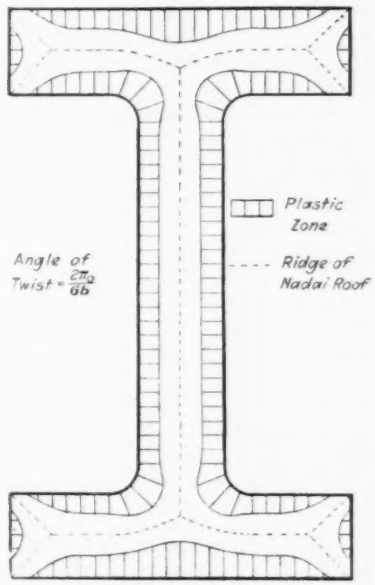


Fig. 6

where l is the length of a given rectangle and b is its breadth. Also the maximum stress in the section is

$$\tau = bG\alpha \quad \text{.....[7]}$$

Using these relations for comparison, we shall determine first, in the elastic region of the stress-strain curve, (*a*) the actual maximum stress in the fillets for a given angle of twist, and (*b*) the actual torsion modulus of the section; then, second, in the region beyond the elastic limit, (*a*) the extent and shape of the plastically deformed areas, for several values of the angle of twist, and (*b*) the torque-twist relationship beyond the elastic limit. The results are presented in this order.

Stress Concentration in Fillets

The maximum stress in the fillets was found to be

$$\tau_{max} = 1.85\, bG\alpha$$

giving a stress-concentration factor of 1.85 when referred to Equation [7]. The greatest stress occurred very near the point of contact of the inscribed circle, that is, at point A in Fig. 4. It is of interest to compare this result with that obtained by experiment on this section. Using the soap-film analogue, Ehasz (5) obtained a curve showing the value 1.90 for the dimensions here analyzed, and claimed an accuracy of 2 per cent. The earlier work of Griffiths and Taylor (6), although it is by no means so complete, appears to indicate a slightly lower value. On the whole, bearing in mind the fact that any errors which exist, either in experimental or in computational technique, are at their maximum in the determination of this quantity, the present author is satisfied with the results obtained.

Torsion Modulus for the Section

With regard to the torsion constant K in the equation

$$\text{Torque} = \alpha GK$$

the elementary theory as expressed in Equation [6] applied to this section gives the results

$$K = 4.222b^4$$

The more complex formula utilized by Lyse and Johnston (7)

$$K = \sum \frac{1}{3} b^3 l + 2\beta D^4 - 0.42016b^4$$

in which the constant β is determined from soap-film experiments gives the value

$$K = 4.748b^4$$

and the present investigation leads to the conclusion

$$K = 4.726b^4$$

While it is difficult to estimate the accuracy which can be guaranteed for the first of these two results it is believed that the difference between them (0.5 per cent approximately) is well within the limits of expected error.

Fig. 4 shows typical stress lines in the elastic region. This figure may be compared to Fig. 9 of a paper by Trayer and March (8) which is a similar contour diagram obtained by soap-film experiment; the similarity between the results of the two processes is very clearly demonstrated.

Regions of Plastic Flow

The boundaries of the plastic region where determined for an angle of twist

$$\alpha = \tau_0/Gb$$

which, according to the elementary theory expressed in Equation [7] would be the twist at the elastic limit, and for angles two thirds as great and twice as great as this. The results for the first and last of these determinations are given in Figs. 5 and 6.

In the second case the plastic area was found to be restricted almost entirely to the fillets, and an additional diagram was not considered necessary. Fig. 5 shows contours identical with those in Fig. 4 and is intended to indicate the effect of the onset of plasticity on the direction of stress in the fillets. It is noticeable in this figure that, while the angle of twist has been almost doubled since the elastic limit was reached, the plastic zone has not yet covered more than 10 per cent of the total area of the section.

Redoubling this angle, however, has the effect of producing a very much larger area of plasticity, as is shown in Fig. 6. This is intended to be compared with Fig. 8, for permission to repro-

duce which I am much indebted to C. W. MacGregor. The dark wedge-shaped marks on Fig. 8 indicate very clearly the regions of plastic flow obtaining in the experiment, as they may be observed by the etching method. It will be seen that the result of such a test agrees well in general shape with the result we have obtained by computation. In order to estimate the quantitative significance of the comparison, we must first consider the torque-twist relation for the section, which is given in Fig. 7.

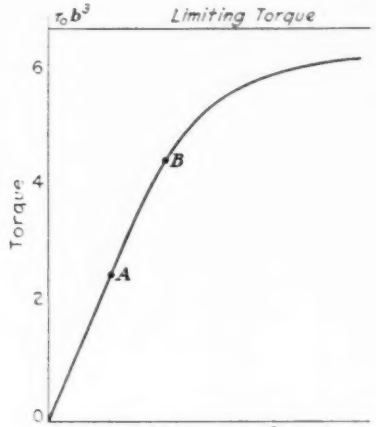


FIG. 7

We have assumed a stress-strain diagram, shown in Fig. 3, which has no true yield point, in the sense that there is no point on the curve at which increasing strain produces a decrease of stress. It is evident that, on this theory, the torque will increase indefinitely to a finite limit, given by the volume of the Nádai roof. The torque-twist curves given by MacGregor and Hrones (3), on the other hand, do not conform to this condition, indicating that for the material of the specimen (S.A.E. 1112 steel) the assumption of Fig. 3 is seriously in error. It is a matter of considerable interest, therefore, if we can demonstrate that in spite of this the position and size of the plastic region is adequately determined. We may best do this by comparing the value of τ_0 the maximum shear stress, observed in the experiment, with the value which must be assumed in the computation in order to obtain, for the same angle of twist, a diagram identical, as far as one can judge, with the experimental picture.

The actual maximum shear strength observed varied between 48,500 and 53,000 lb per sq in. To obtain a diagram identical for identical conditions of twist the assumed value had to be 54,000 lb per sq in., but difficulty in measuring from Fig. 8 may introduce an error of at least 5 per cent in this value. The agreement may therefore be regarded as satisfactory.

In spite of failure to agree with results of experiment, Fig. 7 has some additional interest as describing the properties of a hypothetical material. It is noticeable that while the plastic flow actually begins at the point A, very little falling off from the straight-line relation is visible before the point B, which is the elastic limit under the rudimentary theory of Equation [7], is reached. At this point also the torque-twist ratio differs by only about 4 per cent from the value predicted by this theory. It may therefore be argued that for a material of this type under a state

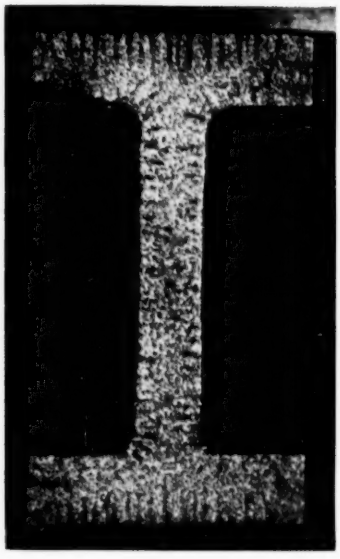


FIG. 8

of stress such that considerations of fatigue could be neglected this very simple theory may be applied with much greater success than could be expected. Evidently the errors resulting from neglecting both end effects and fillets tend to neutralize one another to a great extent.

The author hopes that this paper will have some interest as providing additional corroboration for the results obtained by the soap-film analogue, and as helping to demonstrate that the assumption inherent in the Nádai roof analogy does not lead to erroneous conclusions being reached as to the limits of plastic overstrain.

In conclusion he wishes to express his most grateful thanks to C. W. MacGregor and to R. W. Vose for advice and assistance throughout the preparation of this paper.

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Stress-Concentration Factors Around a Central Circular Hole in a Plate Loaded Through Pin in the Hole

By M. M. FROCHT[1] and H. N. HILL[2]

This paper deals with factors of stress concentrations in plates of finite width with central circular holes loaded through pins or rivets. Two sets of results are presented, one from oversize aluminum specimens in which the stresses were determined from strain-gage measurements, the other, from photoelastic tests with small bakelite models. The two investigations were conceived and executed independently. The tests involving the use of strain gages were made at Aluminum Research Laboratories and the photoelastic tests were made at the Photoelastic Laboratory of the Carnegie Institute of Technology.

While most of the tests involved plates loaded through a single pin in a hole on the longitudinal center line of the plate, several tests were made on plates loaded through two pins symmetrically situated about the center line of the plate. Numerical values for the stress-concentration factor k are given for ratios of hole diameter to width of plate ($2r/D$) ranging from 0.086 to 0.76.

KNOWLEDGE of stress-concentration factors is frequently essential to a proper design, particularly of machine parts and structural members or joints subjected to fatigue stresses. The strength of such elements is greatly affected by concentrations of stress. Riveted and bolted joints or connections are typical examples of such elements. In recent years numerous tests have been made to study the fatigue strength of joints (1, 2).[3] The problem of the determination of the fatigue strength of a joint is of such complexity that it can probably best be solved by a direct experimental approach. A knowledge of the distribution of stresses in such a joint, however, would obviously be of considerable value in the analysis and in the extension of test results.

This paper deals with the problem of determining stress concentrations in a riveted or bolted joint, reduced to its simplest form. While a knowledge of the stress-concentration factors for this simple case is not in itself sufficient for the determination of the maximum stresses in a complicated joint, test results indicate that these stress-concentration factors probably represent upper limits. Although a knowledge of stress-concentration factors might be of rather limited value in a study of the fatigue strength of riveted or bolted joints of mild steel (a material for which the endurance limit is just slightly less than the yield strength), the value might not be so limited when dealing with joints of high-strength alloy steels or lightweight alloys (materials for which the endurance limits are well within the elastic range).

The problem of the determination of the stress distribution in a plate loaded through a pin in a hole in the plate has been treated mathematically, notably by Bickley and Knight (3, 4). Bickley confined his treatment to a plate of infinite width, assuming various expressions for the nature of the distribution of forces between pin and plate. Knight considered only one set of boundary forces on the hole but extended his solution to cover plates of finite width giving specific results for the case of a plate having a width twice as great as the diameter of the hole. Published results of experimental studies of this problem seem to be very meager. Coker (5) reports the results of a photoelastic investigation on one specimen $18 \times 6 \times 0.157$ in. in which the load was applied through a neat-fitting pin of 0.766 in. diameter. The results of an investigation of a specimen of different proportions are credited to Stoltenburg (6).

Tests Involving Strain-Gage Measurements

The tests made using the strain-gage method employed a 1-in-thick plate of high-strength aluminum alloy of the duralumin type and an 8-in-diameter pin. To obtain different ratios of pin diameter to plate width, the plate was decreased in width in successive steps from its original value of 55 in. to a final width of 12 in. Fig. 1 shows the test setup when the plate was 55 in. wide. The original head distance H of 20 in., shown in Fig. 2, was maintained until the final test with the plate 12 in. wide. For this test the head distance was reduced to 12 in.

Strains in the immediate vicinity of the hole were measured with Huggenberger tensometers of $^1/_2$ in. gage length. Strains at other locations in the plate were measured with tensometers of 1 in. gage length. In the vicinity of free boundaries, strains were measured in directions parallel to and normal to the boundary. Elsewhere strains were measured on three or more intersecting gage lines. Because of the finite gage length and width of knife-edges of the instruments, it was impossible to obtain a direct measure of the maximum unit strains occurring at the edge of the hole. These values were obtained by extrapolation of curves constructed from measured values.

In the tests involving the strain-gage method, some tests were made using a steel pin, while in other tests the pin was of the same material as the plate. In most of these tests the pin was a neat fit in the hole, the diameter of the pin and hole being the same within 0.0005 in. In other tests a pin was used which had a clearance of 0.015 in. on the diameter. Several tests were made with neat-fitting pins to study the effect of lubrication of the pin on the factor of stress concentration. A colloidal-graphite grease was used as the lubricant.

Stress-concentration factors have been determined for each test as the ratio of the maximum tensile stress at the edge of the hole to the average stress on the minimum net section of the plate. These factors have been plotted in Fig. 2.

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[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

Presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until April 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Photoelastic Results

In the photoelastic investigations the models, both plate and pin, were of bakelite BT-61-893. The pins or bushings were reinforced with a steel core and the load was applied through a clevis which made it possible to obtain a clear stress pattern of the highly stressed regions around the hole. Fig. 3 shows the test setup in which the clevis used for tests involves two pins. A typical stress pattern is shown in Fig. 4.

The specimens used in the photoelastic investigation had a nominal thickness of $^1/_4$ in. Dimensions of the specimens were chosen so that the influence of the following four factors on the stress-concentration factor could be studied:

1 The ratio of the diameter of the hole to the width of the plate ($2r/D$).

2 The clearance between the pin and the hole.

3 The ratio of the head distance to the width of the plate (H/D).

4 Applying the axial load through two symmetrically placed pins of equal diameters.

Factors of stress concentration have been calculated directly from the data furnished by the stress pattern in the usual manner, using

$$k = \frac{n_{\max} F}{P/A}$$

where k is the factor of stress concentration, $n_{\max}$ is the maximum

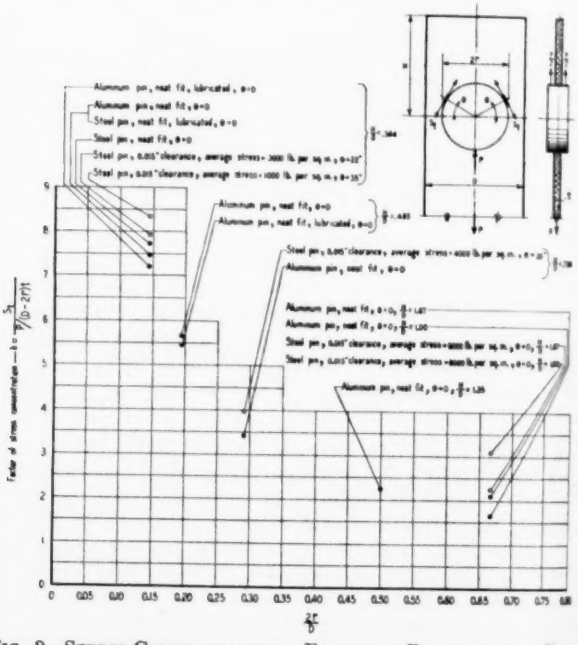


Fig. 2 Stress-Concentration Factors Determined From Strain Measurements on Oversize Aluminum Specimens

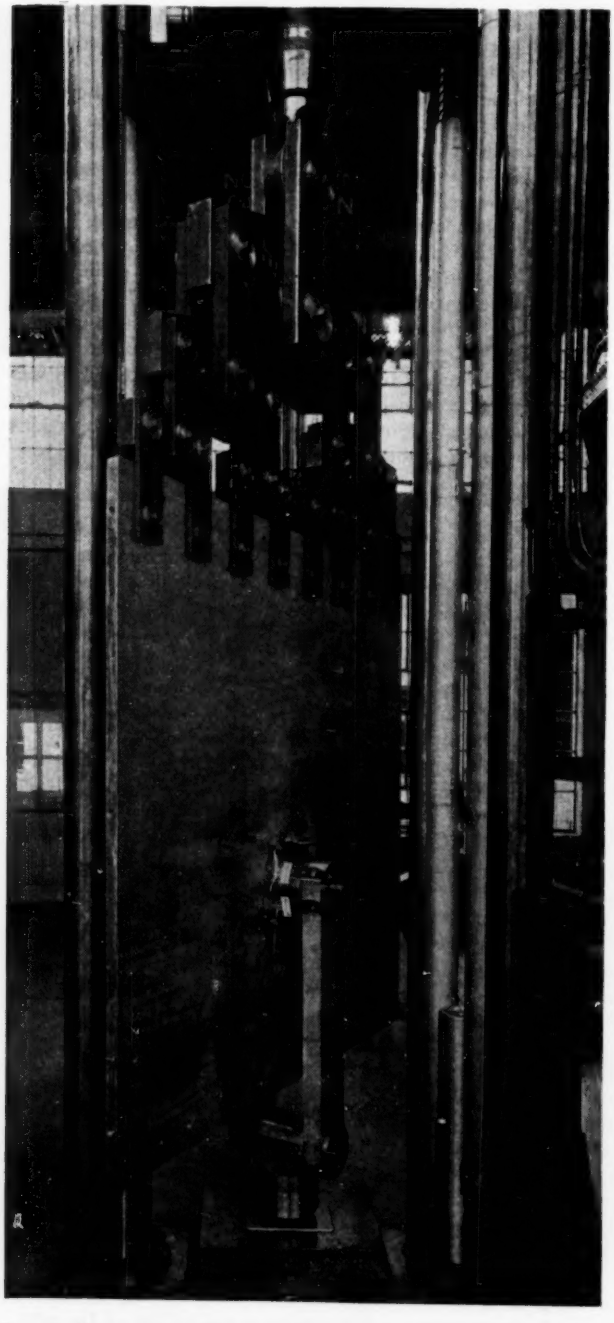


Fig. 1 Setup for Testing Oversize Aluminum Specimens

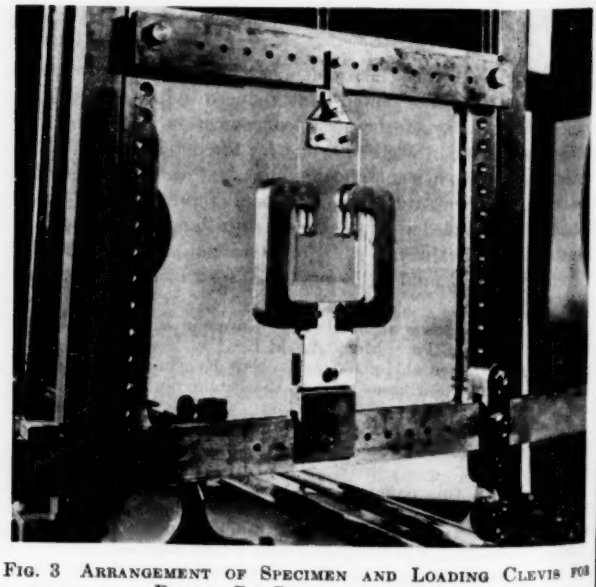


Fig. 3 Arrangement of Specimen and Loading Clevis for Double-Pin Photoelastic Tests

fringe order, F is the model fringe value, P is the applied load, and A is the minimum net area through the hole or holes. Stress-concentration factors thus determined for each of the cases photoelastically investigated are plotted in Fig. 5.

In the test in which the plate was loaded through two pins, the head distances H were great enough so that they did not influence the stress-concentration factors. The diameters of the pins in the double-pin tests were taken equal to the radius of the single pin used for a plate of the same dimensions. The results thus obtained and shown in Fig. 5 provide a comparison between the factors of stress concentration for the single-pin and the double-pin tests under conditions of constant ratio of bearing to mean tensile stresses. The factors shown in Fig. 5 for the double-pin tests have been plotted against ratios of $2r/D$, in which r is the radius of the equivalent single pin for the same ratio of bearing to mean tensile stress. Several tests were made in which the distance between the centers of the pins was varied and this distance was found to have no effect on the factor of stress concentration. Further tests would be necessary, however, in order to determine fully the influence of the distance between the pins on the value of k.

Summary and Discussion of Results

1 *Effect of $2r/D$ Ratio.* As can be seen in Figs. 2 and 5, there is a definite relationship between the stress-concentration factor and the ratio of hole diameter to width of plate. For a $2r/D$ ratio of 0.7, the stress-concentration factors are in the neighborhood of 2. The factors increase as the ratios $2r/D$ decrease, being in the region of 6 to 8 when that ratio is 0.15. The spread in the stress-concentration factors shown for a given ratio of $2r/D$ is largely caused by the effects of the clearance between the pin and the hole, and the head distance.

2 *Effect of Clearance Between Pin and Hole.* Both investigations show that the factors of stress concentration are increased by the presence of clearance between the pin and the hole. This effect was evidenced throughout the whole range of $2r/D$ ratios from 0.086 to 0.76, as can be seen in Figs. 2 and 5. The effect

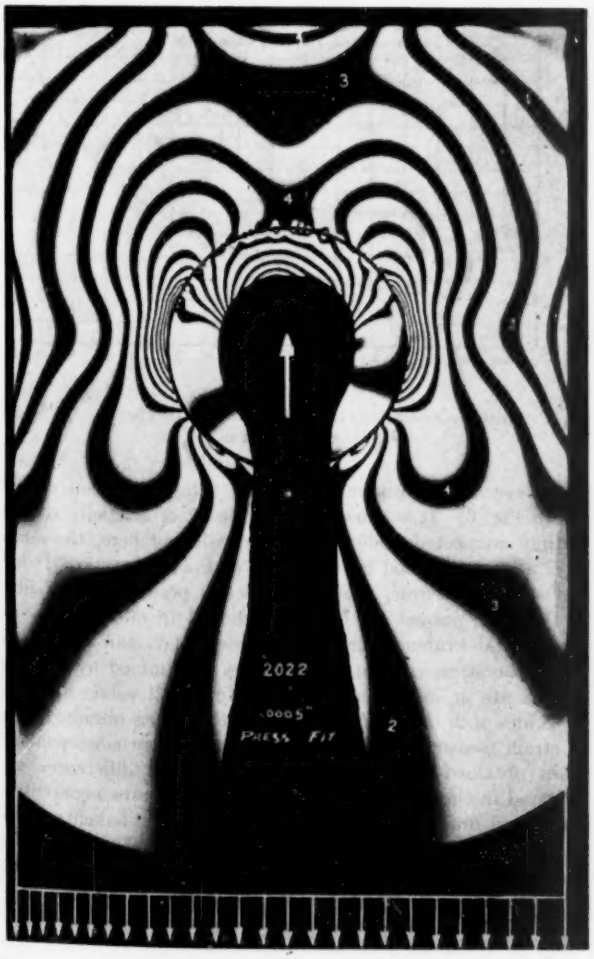


Fig. 4 Typical Stress Pattern From Photoelastic Test

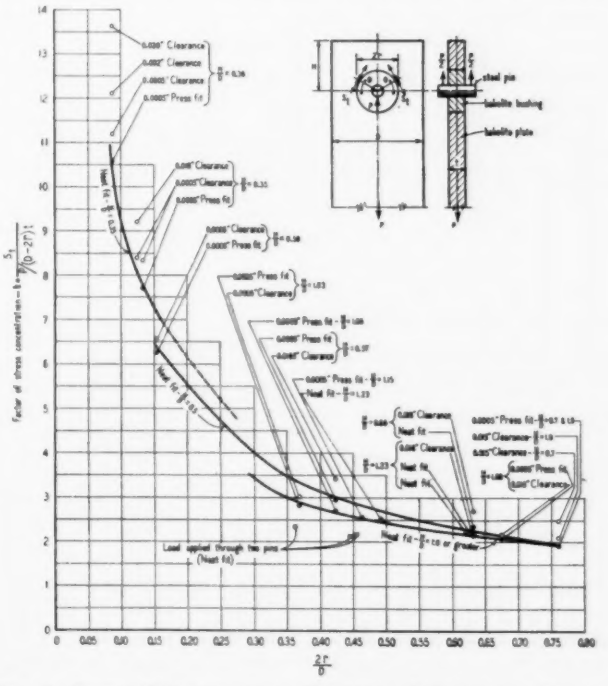


Fig. 5 Stress-Concentration Factors Determined Photoelastically

of the clearance is associated with head distance, being greater for the smaller ratios of H/D. For instance, for a $2r/D$ ratio of 0.76, a stress-concentration factor of 1.9 was determined photoelastically for a neat-fitting pin, both when the head distance ratio (H/D) was 0.7 and 1.9, as shown in Fig. 5. Similarly, in the investigation involving the oversize aluminum specimens at a $2r/D$ ratio of 0.067 in Fig. 2, the introduction of clearance increased the stress-concentration factor about 35 per cent when the H/D ratio was 1.67, while for an H/D ratio of 1.00 the increase was about 50 per cent. The effect of clearance on the maximum stresses has also been observed on previous occasions (7, 8, 9). This effect can be explained on the theory that it permits sharp local changes in the curvature of the circular boundary of the hole.

From the noted description of the points plotted in Fig. 2, it can be seen that for the tests in the investigation involving strain measurements, in which there was clearance between pin and hole, the maximum stresses did not always occur at the ends of the horizontal diameter. However, in every case involving neat-fitting pins, the maximum stresses did occur at the ends of the horizontal diameter. This same effect of the clearance of the pin on the location of the maximum tensile stress at the edge of the hole was noticed in the photoelastic investigation.

As might be expected, while the relation between maximum stress and load was practically linear for the cases involving neat-

fitting pins, for the tests in which the pin was smaller than the hole, the stress-concentration factor was somewhat higher for the smaller loads. This effect is demonstrated by the two values shown in Fig. 2 for a $2r/D$ ratio of 0.145 and the steel pin with 0.015 in. clearance. The same relation between load and maximum stress was encountered in the photoelastic tests. The variations in stress-concentration factors with the load for the cases involving clearance on the pin were small compared to the differences between the factors for the loose pin and the neat-fitting pin.

3 *Effect of Head Distance (Ratio of H/D).* Fig. 6 shows a

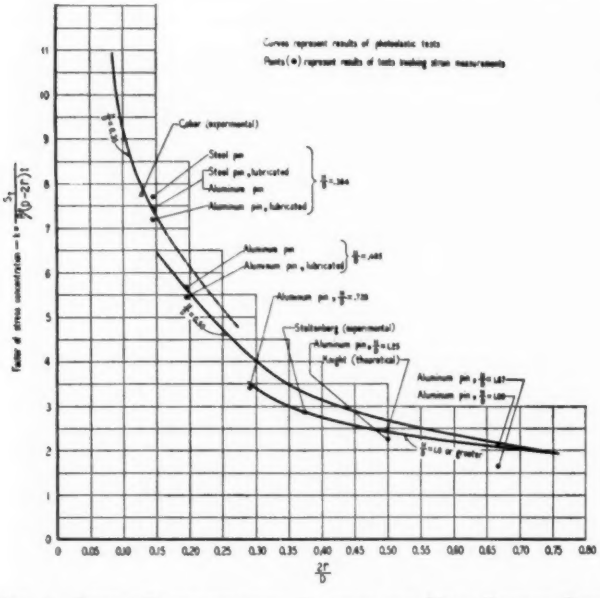


FIG. 6 STRESS-CONCENTRATION FACTORS FOR NEAT-FITTING PINS

composite of the values of k for neat-fitting pins obtained from both investigations, as well as the results of other investigators, both theoretical and experimental. In this figure the effects of clearance have been eliminated but the effect of the head distance on the factors of stress concentration has been emphasized. As the ratio of H/D decreases, the maximum stress and consequently the stress-concentration factor increases. Thus at $2r/D = 0.15$, k equals 6.5 when $H/D = 0.5$ and k equals 7.3 when $H/D = 0.35$. The results of the photoelastic investigation indicate that the effect of the head distance decreases with increasing ratios of $2r/D$. Beyond a certain value of H/D, an increase in head distance has little effect. For a $2r/D$ ratio of 0.76, two specimens were found to have the same factor of stress concentration (for the case of a neat-fitting pin) for an H/D ratio of 1.9, as for an H/D ratio of 0.7. This behavior is not entirely substantiated by the results of the strain-gage measurements on the oversize aluminum specimens. In this investigation, for a neat-fitting pin and a $2r/D$ ratio of 0.667, a stress-concentration factor of 1.65 was determined for an H/D ratio of 1.67, while the corresponding value for an H/D ratio of 1.00 was 2.1, as shown in Fig. 6.

4 *Effect of Number of Pins.* The results of the photoelastic investigation in Fig. 5 show that the stress-concentration factors are smaller when the load is equally distributed between two symmetrically situated pins than when the plate is loaded through a single central pin of a diameter equal to the sum of the diameters of the two pins. The few tests of this type made indicated decreases in the factor of stress concentration of about 20 per cent as compared with corresponding values for plates loaded through a single pin.

5 *Effects of Material of Pin and Lubrication.* The tests made on the aluminum specimens, the results of which are shown in Fig. 2, indicated a very slight increase in the maximum stress when the neat-fitting aluminum pin was replaced by a steel pin having the same fit. Lubrication of the pin produced a small but measurable decrease in the values of k.

6 *Factors of Stress Concentration Based on Average Bearing Stresses.* In Fig. 7 are shown values for a stress-concentration factor k_b defined as the ratio of the maximum tensile stress at the edge of the hole to the average bearing stress on the pin. The same data given in Fig. 6 have been replotted in this manner in Fig. 7. In addition, two points are shown representing the values determined by Bickley (3) and Knight (4) for the case of a hole in an infinite plate. The curve shown in Fig. 7 falls below the plotted points in the region of small $2r/D$ values, because these points represent the results of tests in which the maximum stresses were influenced by the head distance, as shown in Fig. 2. The curve of Fig. 7 indicates that for all cases of a plate loaded through a neat-fitting or loose pin in a hole in the plate, the maximum tensile stresses at the edge of the hole will be greater than the average bearing stress on the pin.

7 *Agreement Between Results of Strain-Gage and Photoelastic Investigation.* Perhaps one of the most important aspects of this paper lies in the agreement between the results of the photoelastic tests, and those obtained from the strain measurements

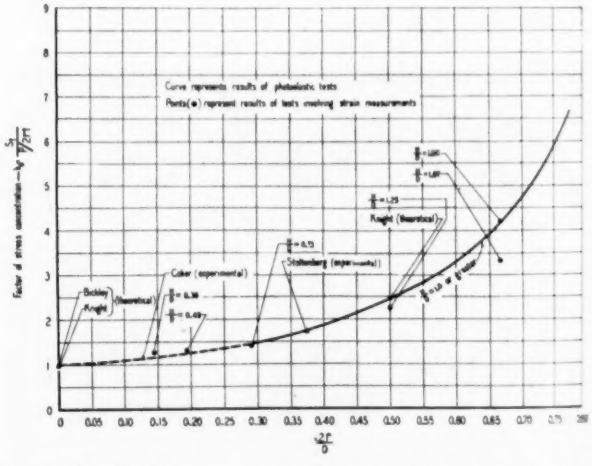


FIG. 7 STRESS-CONCENTRATION FACTORS BASED ON AVERAGE BEARING STRESS

on the oversize aluminum specimens. This agreement can be seen in Fig. 6. It is shown by the theory of elasticity that in multiply connected bodies such as considered here, the stress distribution is affected by the value of Poisson's ratio. It has been stated, however, that this effect is practically negligible (3, 4). The present results are believed to offer convincing experimental evidence for this statement. As can be seen in Fig. 6, the stress-concentration factors determined by the two methods are in very good agreement for small values of $2r/D$. For values of $2r/D$ of 0.5 and greater, the values obtained from the strain measurements are slightly lower than corresponding values obtained photoelastically. The slight differences encountered in these investigations, between the stress-concentration factors obtained for the aluminum and the bakelite specimens, are in the opposite direction to that which theory indicates as the effect of a difference in Poisson's ratio values of the two materials.

The problem of the stress distribution in riveted joints and similar allied problems are becoming increasingly important and merit further study.

Acknowledgment

The tests on the oversize aluminum specimens were conducted under the supervision of R. L. Templin of the Aluminum Research Laboratories by C. Dumont of the same organization. Mr. M. M. Leven, mechanician in the photoelastic laboratory at the Carnegie Institute of Technology assisted in the performance of the photoelastic tests.

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The Distribution of Load on the Threads of Screws[1]

By J. N. GOODIER,[2] ITHACA, N. Y.

The distribution of load on the threads of screws and the types of deformation affecting it have been investigated by means of extensometer measurements made on the outside of the nut. The types of deformation characteristic of concentrations of load on different parts of the thread were found by using a bolt carrying only a single turn of thread. At low loads the deformations of nuts with complete bolts closely resemble the effects of the artificial concentration of load on the base part of the thread, whereas there are marked contrasts with the effects of loads concentrated at the middle or at the free end. At higher loads a more uniform distribution is indicated. Similar conclusions follow from the application of an approximate method for deducing the distribution from the deformation measurements.

It is shown that the distribution is governed by (*a*) stretch and compression in bolt and nut, respectively, which are primarily responsible for concentration at the base; and (*b*) by bending of the thread, circumferential stretch (at the base), and contraction (near the free end) of the nut wall, which have comparable effects in reducing the concentration.

IT IS well known that the thread of a screw connection does not bear its load evenly. The supposition that it does so leads at once to a contradiction. For, in the case of a nut and bolt, the bolt would be everywhere in tension, and would stretch, whereas the nut would be in compression, and would contract. If the threads fitted before loading, they could no longer be in complete contact afterward.

The opinion generally held is that the load is concentrated toward the base of the nut. Evidence has been drawn from modes of failure (1, 6),[3] from the deformation of plane rubber models (1), and from photoelastic observations, also on plane models (2, 3). Of the latter, however, one (2) showed concentration at the free end.

The thread may be regarded as a cantilever "gallery" built into the wall of the nut or the core of the bolt. It will bend under load, and the relative displacements so caused can be roughly estimated by means of beam formulas. They are found to be of the same order of magnitude as those due to simple stretching of the bolt and compression of the nut. It might then be supposed that this bending could take up the gaps due to the stretching and contraction. Den Hartog (3) and Jaquet (1) have given solutions of the problem of elasticity so defined, and both have found severe concentration of load at the base, one case showing a maximum load intensity nine times the average. All but a short base part of the thread is then of course practically free from load. The thread of an actual nut under such conditions would evidently yield at low loads and so modify the distribution.

In view of the rather radical differences between a nut and bolt, and a plane model, and on the other hand of the neglect in the theoretical work of types of deformation other than elastic thread bending, and simple stretch or contraction in bolt core or nut wall, it appeared desirable to make an investigation based on observation of actual nuts and bolts. Since any direct measurement of load intensity along the threads seemed impracticable, external deformations of the nuts were examined, and the character of the load distribution inferred from them. To simplify the relation, the nuts used had their hexagonal corners turned off to render the system approximately symmetrical about its axis.

The Radial Displacement

The curves shown in Figs. 1 to 4, inclusive, represent the measured radial displacements v of points on a generator of the external cylindrical surface, for various $1^1/_4$-in. specimens and loads of 8000, 15,000 (25,000 in Fig. 3), and 50,000 lb. They indicate, therefore, in exaggeration, the actual deformed profile.

The full-line curves shown in Fig. 5 are similar, except that the bolt used carried only a single turn of thread, the rest being turned off. It thus produced an artificial concentration of load wherever it was placed. It was set successively at the free end, middle, and base of the nut. It will be observed that the resulting curves are markedly different. Only that for the base position resembles those of Figs. 1, 2, 3, and 4. It appears then that in actual nuts and bolts there is little load at the free end or middle, but a concentration at the base, the concentration being less marked at the higher loads.

If there were a uniform distribution of load we should expect the curve to resemble the average of the three full-line curves of Fig. 5, which is drawn with a broken line. The influence of the free-end (top) load appears in the curvature, concave to the left, toward the top of the nut. None of the curves obtained with complete bolts show such curvature.

The loads on the middle and top together have the effect of raising the point at which the average curve crosses the axis ($v = 0$). Not all the curves of Figs. 1, 2, 3, and 4 cross the axis, but in those which do, increase of load raises the point of intersection, indicating a more uniform distribution.

The radial displacement at any given point is not proportional to the load. At the base it is less than the proportional value at the higher loads, and this is to be expected if the higher loads are less concentrated at the base. However, the same kind of departure from proportionality occurs for the single-thread bolt, where the cause cannot be redistribution. It seems probable that it is varying friction at the base.

The Axial Displacements

A typical set of curves for three loads is shown in Fig. 6, while the set obtained with the single-thread bolt in three positions is

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[1] This paper was prepared during the author's tenure of a Research Fellowship in Applied Mechanics at the Ontario Research Foundation. Most of the observations were made by A. D. Hogg, then a bursar of the National Research Council of Canada.

[2] Professor of Mechanics, Cornell University. Mem. A.S.M.E.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

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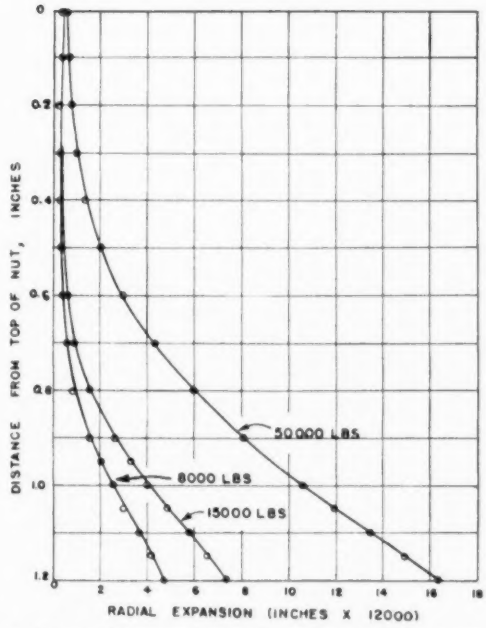


FIG. 1 RADIAL EXPANSION OF A NUT WITH 1¹/₄-IN. AMERICAN STANDARD THREAD, FIRST SPECIMEN

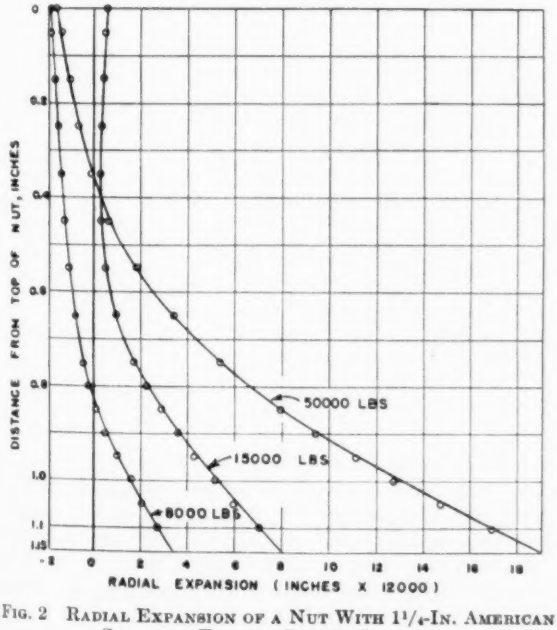


FIG. 2 RADIAL EXPANSION OF A NUT WITH 1¹/₄-IN. AMERICAN STANDARD THREAD, SECOND SPECIMEN

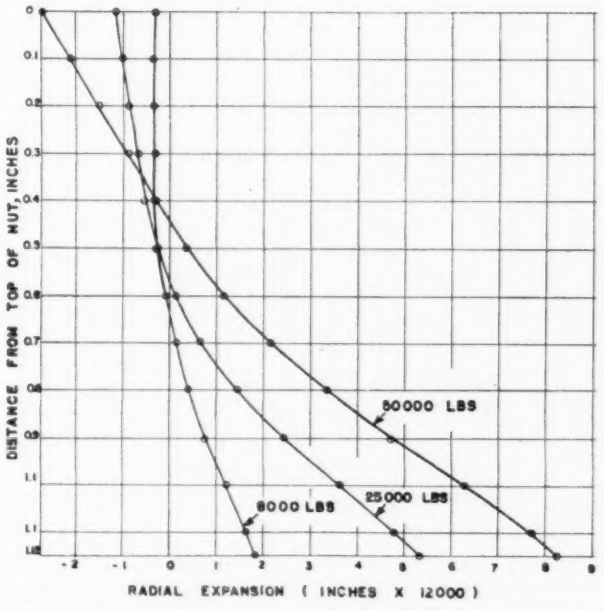


FIG. 3 RADIAL EXPANSION OF A NUT WITH 1¹/₄-IN. AMERICAN STANDARD THREAD, THIRD SPECIMEN

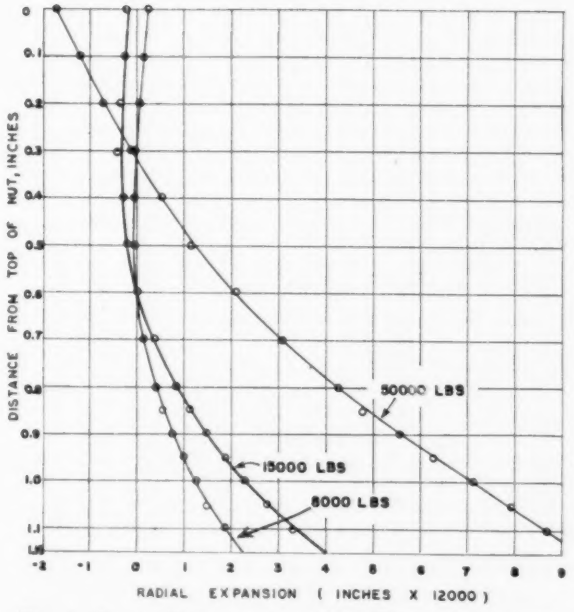


FIG. 4 RADIAL EXPANSION OF A NUT WITH 1¹/₄-IN. ACME THREAD

shown in Fig. 7. It will be observed that there are no such striking differences for these different positions as were found in the case of the radial displacement.

INSTRUMENTS

Measurements of v were made with the mirror extensometer shown in Fig. 8 which was designed for this purpose. It is essentially a modification of E. H. Lamb's (4) roller extensometer for lateral measurements. That instrument has a movable block separated from the "fixed" member by steel balls in grooves and fine rollers the rotation of which is observed optically; the movable block takes the displacement to be measured. In the modification, the movable block B, shown in Fig. 8, is attached to the fixed member by arch springs which simplifies construction. Tilting of the block during displacement, due to inequality of the springs, is allowed for by providing mirrors and their optical levers at each end of the rollers. The rotation of each end is thus measured separately. The rollers were about 5 in. between grips and 0.04 in. diameter, so that any slight difference between the end rotations could easily be accommodated by torsion. The average of the two measurements gives the translation of the block.

The axial displacements u were measured with a Martens extensometer, one end of each comparison strip being mounted on a light dummy cylinder continuing the nut surface, but attached

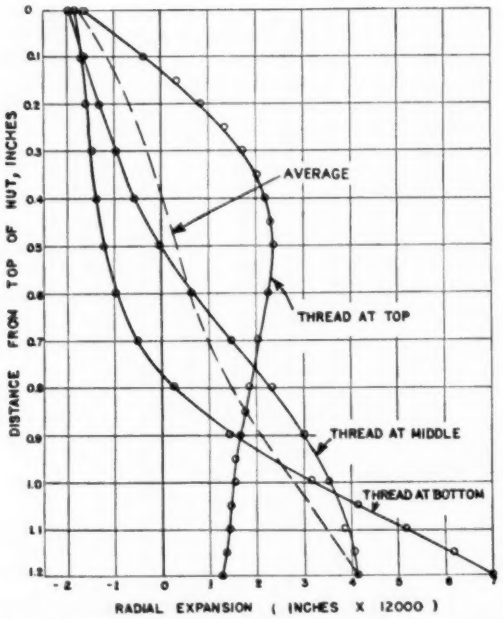


FIG. 5 RADIAL EXPANSION OF A NUT WITH 1¼-IN. AMERICAN STANDARD THREAD, RUN ON A BOLT WITH ONLY A SINGLE TURN OF THREAD

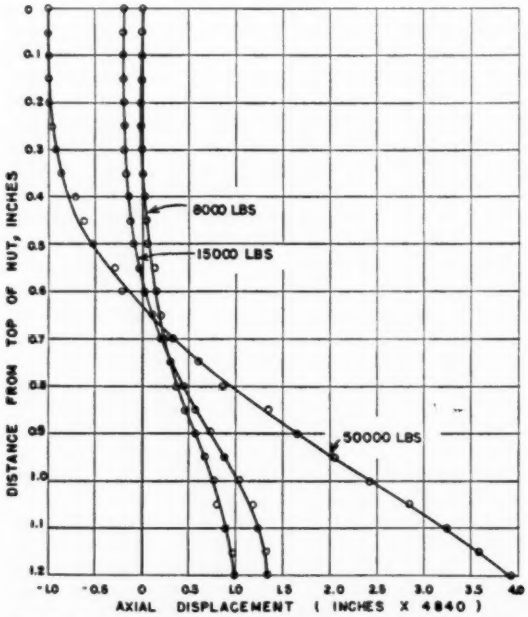


FIG. 6 AXIAL DISPLACEMENTS OF THE SURFACE OF A NUT WITH 1¼-IN. AMERICAN STANDARD THREAD

to the nut at two points only, so that it showed no axial strain within its own length on loading the nut and bolt.

Observations were recorded only after a number of preliminary loadings, sufficient to reach a steady loading-unloading cycle. The instruments were moved from one position to the next, up or down the nut, while a load remained to prevent shifting of the nut. Loading was then repeated three or four times in each position to confirm the steady state.

CALCULATION OF LOAD DISTRIBUTION FROM DEFORMATION

Even when yielding has occurred at the root of the thread, the major part of the nut wall is still elastic. We may therefore

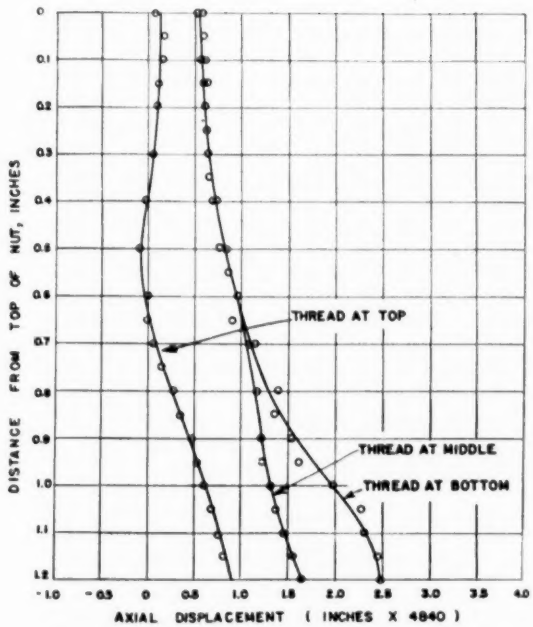


FIG. 7 AXIAL DISPLACEMENTS OF THE SURFACE OF A 1¼-IN. AMERICAN STANDARD NUT RUN ON A SINGLE-THREAD BOLT

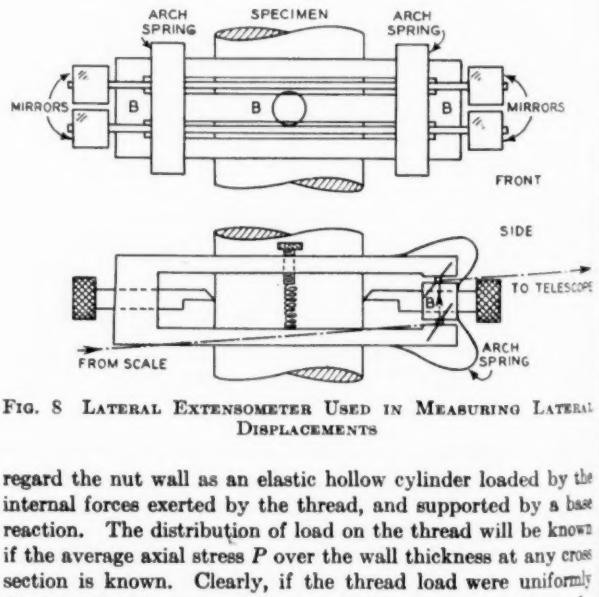


FIG. 8 LATERAL EXTENSOMETER USED IN MEASURING LATERAL DISPLACEMENTS

regard the nut wall as an elastic hollow cylinder loaded by the internal forces exerted by the thread, and supported by a base reaction. The distribution of load on the thread will be known if the average axial stress P over the wall thickness at any cross section is known. Clearly, if the thread load were uniformly distributed, this average stress would increase linearly from the top (where it is zero) to the base.

The expression for P in terms of u and v is

$$P = -\frac{J}{\omega}\left\{ e^{m_1 x}\int_0^x [Sm_1{}^3u + (K - Lm_1{}^2)v]e^{-m_1 x}\,dx \right.$$

$$- e^{m_2 x}\int_0^x [Sm_2{}^3u + (K - Lm_2{}^2)v]e^{-m_2 x}\,dx$$

$$\left. -L\omega v + S(m_1{}^2 - m_2{}^2)u + S\omega(du/dx) + Be^{m_1 x} + Ce^{m_2 x} \right\} \quad \ldots\ldots[1]$$

The derivation, and the values of the coefficients, are given in Appendix 1; B and C are constants of integration to be adjusted

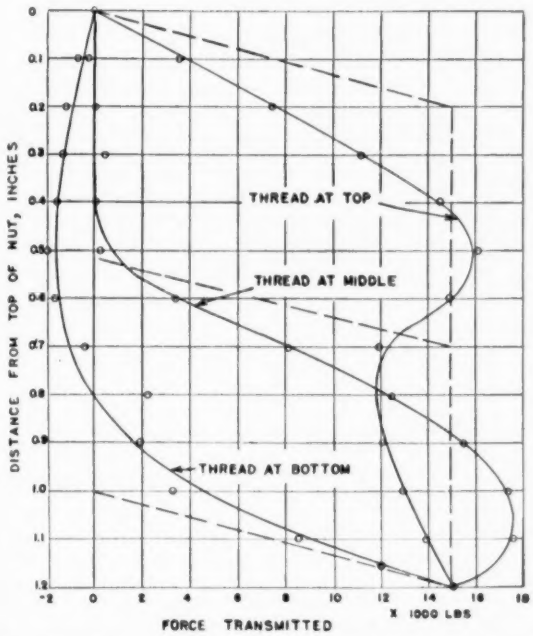


FIG. 9 FORCE TRANSMITTED TO A 1¹/₄-IN. AMERICAN STANDARD NUT AS A FUNCTION OF THE AXIAL COORDINATE, CALCULATED FROM OBSERVED DISPLACEMENTS WHEN THE NUT WAS RUN ON A SINGLE-THREAD BOLT

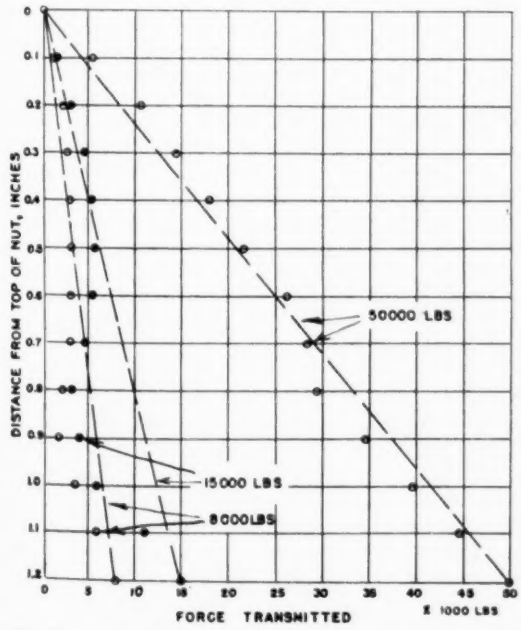


FIG. 10 VALUES OF P FOR 1¹/₄-IN. AMERICAN STANDARD NUT WITH ORDINARY BOLT, CALCULATED FROM FIG. 1

to make P take the required values at the free end (zero) and the base.

This relation was tested by applying it to the u and v curves obtained with the single-thread bolt. In that case the value of P should be zero from the top as far as the loaded thread of the nut, then take within the space of this thread the full value (the load divided by the cross-sectional area of the wall), and retain this value down to the base.

The results actually obtained are shown in Fig. 9. In view of the almost discontinuous character of the loading, and assumptions involved in Equation [1], the indications of Fig. 9 are surprisingly good. Approximate true curves are shown as broken lines.

Curves for several nuts with ordinary bolts, calculated from Figs. 1, 2, 3, and 4 by means of Equation [1], are shown in Figs. 10, 11, 12, and 13. Comparison with Fig. 9 leaves no doubt that there is no concentration of load at the top or middle. While the method is evidently not sufficiently accurate to determine the actual distribution, especially at the lower loads, it indicates substantially uniform loading at 50,000 lb, and a tendency to concentration at the base for the lower loads. The average stress in the bolt core at 50,000 lb is 56,000 lb per sq in., and at 8000 lb it is 9000 lb per sq in. The steel used was Bessemer screw stock S.A.E. 1112 with a yield point at 80,000 lb per sq in.

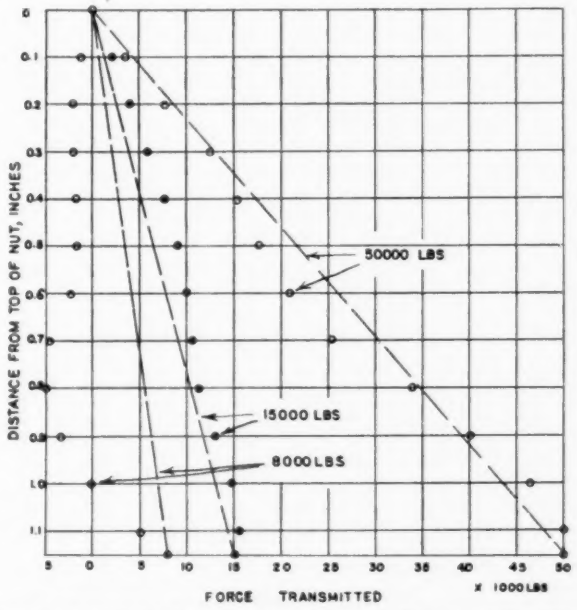


FIG. 11 VALUES OF P FOR 1¹/₄-IN. AMERICAN STANDARD NUT WITH ORDINARY BOLT, CALCULATED FROM FIG. 2

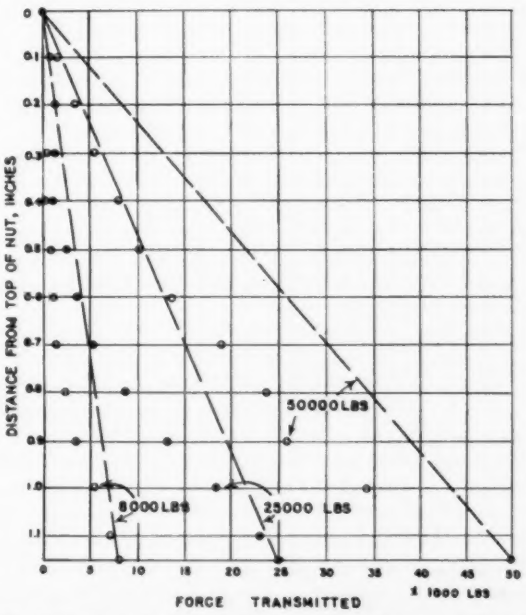


FIG. 12 VALUES OF P FOR 1¹/₄-IN. AMERICAN STANDARD NUT WITH ORDINARY BOLT, CALCULATED FROM FIG. 3

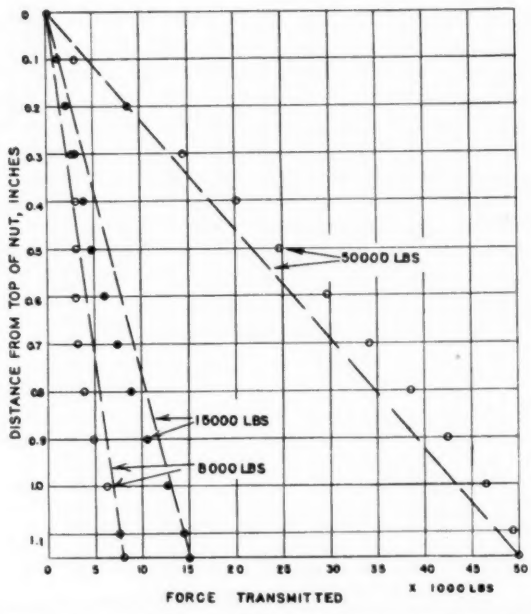


FIG. 13 VALUES OF P FOR $1^1/_4$-IN. ACME THREAD NUT WITH ORDINARY BOLT, CALCULATED FROM FIG. 4

TYPES OF FLEXIBILITY AFFECTING THE LOAD DISTRIBUTION

The deformation as a whole may be resolved into the following three principal types: (*a*) Axial contraction of the nut wall and extension of the bolt core; (*b*) bending of the two threads as cantilevers; and (*c*) circumferential stretch of the nut wall. The curves in Figs. 1, 2, 3, and 4 enable us to estimate the relative importance of these types in governing the load distribution, and thus to judge the probable value of modifications designed to reduce the concentration.

The axial contraction of the nut, and the antagonistic extension of the bolt within the nut, have to be reconciled by means of other types of deformation, since thread contact requires the same deformation of nut and bolt along the bearing surfaces of the two threads. Type (*a*) is thus primarily responsible for the concentration. In fact it may be shown that by itself it would cause complete concentration on an infinitesimal length of thread at the base; or a concentration at *each* end of the thread in the case of the turnbuckle type of screw connection.

The calculations of Den Hartog (3) and Jaquet (1) show that thread bending is adequate to change this to a smooth distribution although still with a severe concentration at the base. Circumferential stretch of the wall, as will appear, has an effect similar to that of thread bending. It creates effectively an *axial* tensile strain at the threads, offsetting the compressive strain in the nut wall. The types (*b*) and (*c*) thus moderate the extreme concentration due to (*a*).

In comparing the effects of these different types of flexibility we may suppose that the thread of the bolt is in contact with that of the nut along the helical middle lines of the active thread surfaces. Let u_b denote the axial displacement of a point of this line on the bolt thread, at a distance x from the base, and let u_n be the displacement of the corresponding point on the nut thread.

The condition of contact is expressed by $du_b/dx = du_n/dx$.

All types of flexibility which contribute significantly to either of these two derivatives, that is, to the relative axial displacements of points on one thread or points on the other, under the actual load distribution, are important in determining the distribution.

Considering the nut only, it is clear that flexibility of type (*a*) contributes to du_n/dx an amount P/E. We may represent the order of magnitude of this by its greatest value.

The bending of the thread as a cantilever depends on the load per unit run of thread. From x to $x + \delta x$ there is a length $2\pi nr\delta x$ where n is the number of turns of thread per unit axial length and r their mean radius. The axial load carried corresponds to the increase in P and is $A(dP/dx)\delta x$, where A is the cross-sectional area of the nut wall. The axial load per unit run of thread is thus $(A/2\pi nr)(dP/dx)$, and the deflection is $\beta(A/2\pi nr)(dP/dx)$, where β is an elastic coefficient depending on the shape and material of the thread. The contribution to du_n/dx is the derivative of this, or $(\beta A/2\pi nr)(d^2P/dx^2)$.

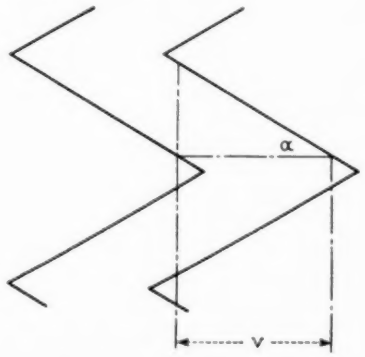


FIG. 14 THREAD RECESSION

In order then to estimate the contribution of type (*b*) directly we should require an estimate of d^2P/dx^2, but such an estimate cannot be made without an accurate knowledge of the load distribution. The contribution can however be estimated indirectly. If the distribution were determined entirely by the flexibilities (*a*) and (*b*), their contributions would necessarily be of the same order of magnitude. Since the actual distribution is certainly more uniform than it would thus be, we shall be overestimating rather than underestimating the effect of the thread bending if we assign to it the same order of magnitude as to axial strain, that is, P/E.

The contribution of circumferential stretch divides into two parts, in the case of V or acme threads. If we imagine a point of contact between the two threads, then allow the nut thread to move radially outward by a distance v, shown in Fig. 14, it is clear that this movement creates a vertical gap $v \tan \alpha$ between the two threads. If v were uniform from top to bottom of the nut, the bolt would merely have to descend bodily the distance $v \tan \alpha$ in order to reestablish contact. But if v should not be uniform, contact could not thus be made along the whole thread. The effect of a nonuniform v is evidently expressed by a contribution $\tan \alpha(dv/dx)$ to du_n/dx. The order of magnitude of this is easily estimated from the v curves of Figs. 1, 2, 3, and 4. It will be referred to as "thread recession." It is partly due to the lateral expansion accompanying the compressive axial stress, and partly to the radial component of the thread load.

The other effect of the circumferential stretch, or radial displacement, comes from the axial curvature involved, a strip of the nut between two neighboring planes through the axis being in the condition of a bent beam with curvature d^2v/dx^2. The contribution to du_n/dx corresponds to the tensile strain, in such a beam, of points at a distance s from the neutral axis, where s may be taken as the distance from the middle of the thread to the middle of the nut wall. Its magnitude is then sd^2v/dx^2.

The curves of Figs. 1, 2, 3, and 4 are sufficiently well defined by the observed points to permit a satisfactory estimate of the order of magnitude of d^2v/dx^2.

Estimating the orders of magnitude of these four contributions to du_n/dx in the ways indicated, for the $1^1/_4$-in. nuts, and for a load of 15,000 lb,[4] we have the values given in Table 1

TABLE 1

Type of flexibility	Order of magnitude of effect on load distribution, measured by contribution to du_n/dx
(*d*) Simple axial strain	30×10^{-5}
(*e*) Thread bending	30×10^{-5}
(*f*) Thread recession	up to 50×10^{-5}
(*g*) Wall bending	up to 40×10^{-5}

The conclusion is therefore that all four effects given in Table 1 are about equally important. The first type of flexibility being the real source of concentration, and the other three being moderating influences, the actual distribution, even without any yielding at the roots of the threads, will be considerably more uniform than that determined by neglecting types (*f*) and (*g*).

Means for Reducing the Concentration of Load

It is evident that any modification which increases any of the types of flexibility (*e*), (*f*), or (*g*) will have a favorable effect.

Any reduction of wall thickness will be effective in this way, but it will at the same time increase the undesirable flexibility of type (*d*). The bending flexibility, however, is likely to be more sensitive to reduction of thickness than the axial compressibility. Jaquet (1) describes a modification of the usual thread, due to J. R. Solt, in which the thread is undercut into the wall, so that the thread depth is increased at the expense of the wall thickness. There is then a gain of flexibility of all four types. Another modification, essentially a reduction of wall thickness at the base, has been tested in fatigue by H. Wiegand (5).

Reduction of friction between thread and thread, and between the base of the nut and its support, should improve the distribution by permitting more radial expansion.

Since the axial bending of the wall must be influenced by the clamping of the base of the nut against its support, there is a possibility of improving the distribution by rounding the base of the nut, so that its contact with the base is over a circle instead of a wide annulus. There would then be an approach to zero bending moment M_1 (see Appendix 1), at the base.

The bending flexibilities of types (*f*) and (*g*) can be somewhat increased, without increasing the axial compressibility (*d*), by making axial cuts into the wall of the nut from the outside.

Increasing the fineness of the thread would appear to decrease the favorable flexibility of type (*e*), for geometrically similar beams have the same deflection per unit width and unit load, and since the finer thread has the greater "width" (helical length), its deflections would be smaller under the same load distribution. In the expression $(\beta A/2\pi nr)d^3P/dx^2$, which measures the contribution of this type of flexibility to du_n/dx, β remains the same, but n, the number of turns per inch, is greater, so that this desirable contribution is reduced.

Appendix 1

The Load on the Thread in Terms of the External Deformation of the Nut

To obtain the relation in question we may consider the condition of an element of the wall of the nut contained between cross-sections x and $x + \delta x$ from the free end, and by two planes through the axis including an angle $\delta\theta$ as shown in Fig. 15. The thread is not included in this element. The forces acting on it are supposed transferred unchanged to the internal wall surface. At the section x there will be an average compressive stress P, so that if A is the cross-sectional area, AP represents the load carried on the thread above the cross section. Then $[d(AP)/dx]\delta x(\delta\theta/2\pi)$ or $(A/2\pi)(dP/dx)\delta x\delta\theta$ represents the vertical component of thread load on the element. The actual force on the thread will also have a horizontal radial component, tan $(\alpha - \phi)$ times this vertical component, if α is the semiangle of the thread and ϕ is the angle of thread friction.

[4] The *relative* orders of magnitude are similar for other loads.

The forces exerted by the thread on the inner wall of the nut are statically equivalent (approximately) to the pressures on the thread. Thus, we may place the thread forces on the element at their actual position as if the thread were there, as is done in Fig. 15.

Let the axial stress form, in addition to the average stress P, a bending moment M_1 per unit length of mean circumference, positive in the sense shown in Fig. 15. There will also be radial shearing force F per unit of mean circumference.

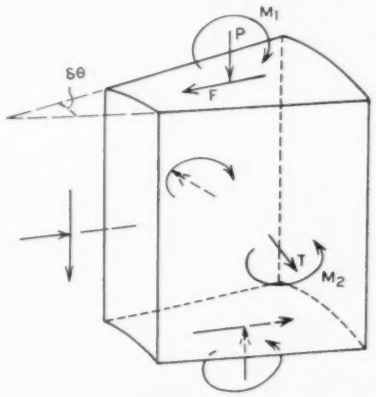


Fig. 15 Element of a Nut Wall

On the faces in the axial planes there will be an average tensile stress T, and a bending moment M_2, positive as shown, per unit axial length. We assume symmetry of the stress and strain about the axis, so that there will be no shearing force.

The equations of equilibrium of the element are then

$$T = \rho \tan (\alpha - \phi) \frac{dP}{dx} + \rho \frac{dF}{dx}$$

$$\rho F = -\frac{\rho}{2h}\frac{dM_1}{dx} - \rho s \frac{dP}{dx}$$

where ρ is the mean radius, $2h$ the wall thickness, and s the radial distance between the load line on the thread, and the middle surface between walls. The neutral axis of bending for the element is taken at the mean radius.

Eliminating F

$$T = \rho \tan (\alpha - \phi) \frac{dP}{dx} - \rho s \frac{d^2P}{dx^2} - \frac{\rho}{2h}\frac{d^2M_1}{dx^2} \quad \ldots\ldots [2]$$

The curvature of generators of the cylindrical surfaces may be ascribed to both bending moment M_1 and shear F. If we consider the complete ring between x and $x + \delta x$, it is clear that the deformation due to F will not alter the circumferential curvature appreciably. If M_2 were wholly due to the prevention of the anticlastic curvature of M_1, we should have $M_2 = \sigma M_1$, where σ is Poisson's ratio, neglecting the difference between the shapes of the elementary areas on which M_1, M_2 are found.

A radial displacement v of the mean radius ρ involves a change v/ρ^2 in the circumferential curvature. But there is a radial displacement due to P alone of amount $\sigma P\rho/E$, where E is Young's modulus, and thus a change of curvature $\sigma P/E\rho$. Thus M_2 must prevent the anticlastic curvature of M_1, and in addition produce curvature $v/\rho^2 - \sigma P/E\rho$. Hence

$$M_2 = \sigma M_1 + \frac{2}{3}Eh^3\left(\frac{v}{\rho^2} - \frac{\sigma P}{E\rho}\right) \quad \ldots\ldots [3]$$

a linear distribution of axial stress across the wall thickness being assumed.

The stress-strain relations for an element on the outside wall are

$$\left.\begin{aligned} P + \frac{3}{2}\frac{M_1}{h^2} &= \frac{E}{1-\sigma^2}\left[\frac{du}{dx} - \sigma\frac{v}{b}\right] \\ T - \frac{3}{2}\frac{M_2}{h^2} &= \frac{E}{1-\sigma^2}\left[\frac{v}{b} - \sigma\frac{du}{dx}\right] \end{aligned}\right\} \quad \ldots\ldots\ldots [4]$$

where u is the axial displacement relative to and toward the base, x being measured upward from the base, and b is the outside radius.

Now, M_1, M_2, and T can be eliminated from Equations [2], [3], and [4], the result being

$$\frac{d^2P}{dx^2} + G\frac{dP}{dx} + HP = f(x) \quad \ldots\ldots\ldots\ldots [5]$$

where

$$f(x) = \frac{1}{\rho\left(s - \frac{1}{3}h\right)}\frac{E}{1-\sigma^2}\left\{\frac{\rho\sigma h}{3b}\frac{d^2v}{dx^2} - (1-\sigma^2)\left(1 + \frac{bh}{\rho^2}\right)\frac{v}{b} - \frac{\rho h}{3}\frac{d^3u}{dx^3}\right\}$$

$$G = -\frac{\tan(\alpha - \phi)}{s - \frac{1}{3}h}$$

and

$$H = -\frac{\sigma(\rho + h)}{\rho^2\left(s - \frac{1}{3}h\right)}$$

The solution of Equation [5] is

$$P = \frac{1}{m_1 - m_2}\left\{e^{m_1 x}\int_0^x e^{-m_1 x} f(x)dx - e^{m_2 x}\int^x e^{-m_2 x} f(x)dx + Be^{m_1 x} + Ce^{m_2 x}\right\} \ldots [6]$$

where m_1 and m_2 are the roots of $r^2 + Gr + H = 0$, and B and C are arbitrary constants. The function $f(x)$ involves the derivatives d^2v/dx^2 and d^3u/dx^3. It would of course be impossible to derive these satisfactorily from the observations, but they can be removed by partial integrations, the final form for P being given by Equation [1].

The constants appearing in that equation are as follows

$$\omega = m_1 - m_2,$$

$$J = \frac{1}{\rho\left(s - \frac{1}{3}h\right)}\cdot\frac{E}{1-\sigma^2} \qquad L = \frac{\rho\sigma h}{3b}$$

$$K = (1-\sigma^2)\left(\frac{1}{b} + \frac{h}{\rho^2}\right) \qquad S = \frac{\rho h}{3}$$

There are no surviving derivatives of u and v except du/dx, which was obtained by graphical differentiation. The integrals were evaluated from the u and v curves by Simpson's rule, and the constants B and C adjusted to make P take the required free end and base values—zero and the full-load value, respectively. The coefficient of friction, involved in m_1 and m_2 was measured by turning the nut or the bolt, under small loads, and found to be about 0.3. It is assumed in the analysis of course that friction has its limiting value throughout.

Appendix 2

Concentration of Load at the Base, and Saint-Venant's Principle

If the base of the nut is taken as fixed, the strain energy of nut and bolt above the base is represented by the work of the stress in the base cross section of the bolt, acting over the displacements in that section. The edges being fixed, through thread contact at the base of the nut, these displacements consist of a deformation of the section.

The stress is of order of magnitude P, the average tensile stress of the bolt. The elastic strains (including those of the threads, since thread strength and bolt strength are comparable) are of order P/E, and consequently the displacements in the base section of the bolt are of order Pr/E where r is the radius. The work done on these displacements will thus be of the order of the resultant $P\pi r^2$ multiplied by Pr/E, or $P^2\pi r^3/E$.

Where the stress is appreciable it is of order P and involves strain energy of order P^2/E per unit volume. The work actually done, $P^2\pi r^3/E$, is then adequate for a volume of order πr^3, or for a distance of order r from the base, whatever the length of the nut.

This argument[5] is substantially an application of Saint Venant's principle to the portion of the nut and bolt above the base, the forces on the complete base section forming a self-equilibrating system.

[5] The Principle of Saint Venant in its usual form may be established in a similar way (*Philosophical Magazine*, April, 1937, and August, 1937).

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Stresses and Deflections of Three-Dimensional Pipe Bends

By H. PORITSKY[1] and H. D. SNIVELY,[2] SCHENECTADY, N. Y.

Many treatments of stresses of three-dimensional pipe bends have appeared in recent times.[3,4] While the calculation of these stresses and deflections is perfectly straightforward, it is still very complicated. It is believed that the analogy to which attention is called in this paper should be of considerable assistance in simplifying these calculations by pointing out their similarity to familiar dynamical concepts involving products and moments of inertia and moments of momentum.

Tractions and Displacements of a Curved Beam Loaded Terminally

CONSIDER a curved beam. Suppose that there is no load on the beam throughout its length, but that it is subjected to tractions over its ends. Let s be the arc length of (the middle line of) the beam and let s_1, s_2 correspond to its two ends P_1, P_2. At a point P corresponding to any value of s, let $\boldsymbol{F}$ be the resultant vector force and $\boldsymbol{M}$ the vector moment of the traction which is exerted by the portion of the beam to the positive side of the section (corresponding to larger s) upon the portion to its negative side. The moment $\boldsymbol{M}$ is calculated with respect to the center of gravity of the cross section of the beam. Applying equilibrium conditions to the beam section between P_1 and P, there results

$$\boldsymbol{F} = \boldsymbol{F}_1,\ \boldsymbol{M} = \boldsymbol{M}_1 + (\boldsymbol{F}_1 \times \boldsymbol{r}) \quad [1]$$

where $\boldsymbol{r}$ is the radius vector P_1P and $\boldsymbol{F}_1$, $\boldsymbol{M}_1$ are the resultant force and moment of the tractions at P_1.

These tractions produce a certain distortion and displacement which result in a motion of one end of the pipe relative to the other one. In fact, it is the displacement of one end relative to the other one that is sometimes the cause of the whole set of forces. In order to calculate this displacement, it is assumed, as is common in the theory of thin beams, that the effect of the compressional or shear components of the resulting traction $\boldsymbol{F}$ is negligible in producing distortions and displacements as compared with the effect of the moment $\boldsymbol{M}$. This moment may be resolved along the tangential direction as well as along the two principal moment directions of the normal section. The first component of the moment is a twisting component which tends to produce torsion in the pipe; the other two components are bending components. If β_1, β_2, β_3 represent the twisting and bending flexibilities of the pipe along its tangential and principal moment normal directions, respectively, and if m_1, m_2, m_3 represent the components of $\boldsymbol{M}$ along these three directions, then the vector

$$\boldsymbol{R} = (\beta_1 m_1,\ \beta_2 m_2,\ \beta_3 m_3) \quad [2]$$

represents the rotation per unit length of pipe; thus, multiplied by ds, it yields the rotation of the section at $s + ds$ relative to the section at s. The sense in which $\boldsymbol{R}ds$ represents the rotation is that the rotation axis points along $\boldsymbol{R}$ while the magnitude of the angular displacement is equal to the length of this vector. In representing rotational displacements of space by means of vectors in this fashion, it must be borne in mind that this is only correct when the displacements are small. This, however, is the usual assumption made in elasticity and it will be adhered to throughout this paper.

Suppose now that the three flexibilities are equal one to the other and are represented by β; then

$$\beta_1 = \beta_2 = \beta_3 = \beta \quad [3]$$

The latter need not be assumed, however, to be constant over the whole pipe length. The flexibilities for a pipe of circular cross section are $\beta_1 = 1/GJ$, $\beta_2 = \beta_3 = 1/EI$, where E is Young's modulus, G is the shear modulus, J is the polar moment of inertia of the cross section, and I is the plane moment of inertia about a diameter. Since

$$G = \frac{E}{2(1+\sigma)},\ J = 2I \quad [4]$$

it will be seen that for a circular pipe

$$\beta_2 = \beta_3 = \frac{\beta_1}{1+\sigma} \quad [5]$$

The assumption of Equation [3] is therefore not too erroneous for a circular cross section.

Under the assumptions of equal flexibilities the rotation vector may be replaced simply by

$$\boldsymbol{R} = \beta \boldsymbol{M} \quad [6]$$

where $\boldsymbol{R}$ is the rotation per unit length. (Compare Equation [6] with the familiar beam equation, $y'' = M/EI$.)

Consider next the displacement of the beam (center line) which occurs as a result of the rotation implied in $\boldsymbol{R}$. If the end s_1 is, for definiteness, held fast, then various sections will suffer both rotational and translational displacements relative to this end section. In particular, the end s_2 will have a net angular rotation relative to s_1 represented by the rotation vector

$$\int_{s_1}^{s_2} \boldsymbol{R}ds = \int_{s_1}^{s_2} \beta \boldsymbol{M}ds \quad [7]$$

since the resultant rotation vector due to several rotations along different axes may be obtained by vector addition of the individual rotation vectors irrespective of where they are located.

The translation of the end s_2 relative to the end s_1 is (again on the assumption of small displacements) given by

$$\boldsymbol{D} = \int \boldsymbol{R} \times (\boldsymbol{r}_2 - \boldsymbol{r})ds \quad [8]$$

[1, 2] Engineering General Department, General Electric Company.

[3] "End Reactions and Stresses in Three-Dimensional Pipe Lines," by G. B. Karelitz and J. H. Marchant, JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 59, June, 1937, p. A-68.

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In Equation [8] the origin of r has again been assumed to be at s_1.

The foregoing formulas allow the computation of the displacement of one end relative to the other one to be carried out in a straightforward manner. While the calculation is simpler than, say, by means of Castigliano's theorem, it is still fairly complex. It is the object of the following portion of this paper to point out an analogy between the foregoing discussion and a simple problem in dynamics of rigid bodies which can be used to simplify the problem and reduce it to more familiar terms.

Analogy Between the Beam Problem and a Dynamical Problem

To formulate this analogy, consider the state of velocities existing at any instant during the general motion of a rigid body, as shown in Fig. 1. In a translation the velocities are everywhere

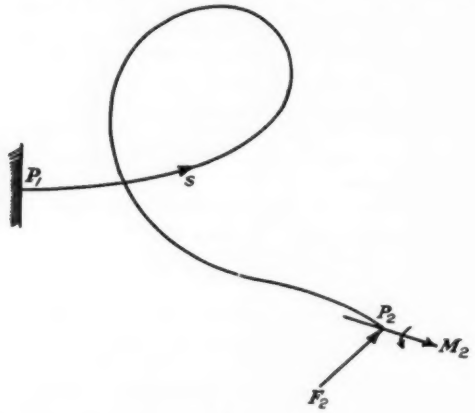


Fig. 1

equal to the same vector which may be used to represent the translation. In a rotation about an axis through the origin, the velocities are given by

$$v = w \times r \quad [9]$$

where w is the "rotation vector," that is, the vector whose length is proportional to the angular velocity of rotation and whose direction is along the axis of rotation and is chosen so that a right-handed screw rotating in the sense of the rotation under consideration will advance along the direction of the vector.

It may be shown that the most general set of velocities of a rigid body may be considered as a superposition of a translation and a rotation about a proper axis with a proper angular velocity. This resolution into a rotation and a translation can be carried out in a great variety of ways. Picking any one point, one may carry out the resolution by using a translational velocity which is equal to the velocity of that point; then, the additional rotation will consist of a rotation about some axis through that point. If different points are used, the translational velocities will, in general, not be the same. However, the rotation vectors which represent the rotational velocities turn out to be the same in magnitude and direction. If the state of velocities has been reduced in the previously described manner with respect to the point P which is taken at the origin, then the velocity of any point of space turns out to be given by

$$v = v_0 + (w \times r) \quad [10]$$

Now turn to the statics of forces acting on a rigid body. As is well known, any system of forces can be reduced to a single force F through a given point and a proper "couple." The latter may be visualized as a pair of equal and opposite forces not along the same line, but it is more conveniently represented by means of its vector moment M about any point of space, since this M turns out to be the same irrespective of the point about which it is taken. Again, any two couples with the same moment are equivalent to each other.

At this stage the familiar analogy between the reduction of velocities of a rigid body and the reduction of equivalent systems of forces becomes clear. A force in the one case is analogous to a rotation vector in the other case, while a couple (or rather, its moment M) is analogous to a translation. The velocity of a point corresponds to the moment of the forces. A system of forces corresponds to a set of rotations which take place simultaneously about various axes, the resultant velocity of any point being the sum of the velocities due to each rotation separately. The fact that two equal and opposite forces acting along parallel lines have a resultant zero and a moment which is constant in magnitude and direction no matter about what point it is taken, now has its analog in the fact that two equal and opposite rotations which take place simultaneously about two parallel lines result in a translation of the rigid body in question.

In the foregoing analogy it will be noticed that forces do not correspond to translations, but rather to rotation vectors, while the moments (of couples) correspond to translations rather than rotations that one would naturally associate with moments of couples or torques. This may be further justified by noting that the point of application of a force could not be moved at random but only along its line of action while the moment of a couple could; similarly, the position of the rotation vectors may be moved along a line of rotation but in no other way, since the rotation about one axis is obviously not equivalent to a similar rotation about a different axis in that it produces different velocities in the body.

Suppose now that the beam is freed at each end and, considered as a rigid body, is given a motion possessing velocities due to a translation M_1 plus velocities due to a rotation about the center of the initial section s_1 represented by the rotation vector F_1. Then the velocity of any centerline point P with arc length s (see Equation [10]) is given by

$$v = M_1 + (F_1 \times r) \quad [11]$$

TABLE 1 SUMMARY OF THE ANALOGY

Stress and deflection in the beam	*Motion of the beam*
M = vector moment of tractions across a section at P.	v = velocity of center point of section at P.
F = resultant force across section at P.	w = rotation vector of the beam motion.
β = flexibility.	β = mass per unit length.
R = rotation vector per unit length of beam.	$R = \beta v$ = momentum per unit s.
$\int_{P_1}^{P_2} Rds$ = vector representing rotation of section at P_2 relative to P_1.	$\int Rds$ = momentum of section between P_1 and P_2.
D = displacement of section at P_2 relative to P_1 when the beam section at P_1 is held fast.	D = moment of momentum of section P_1P_2 about P_1

and if the motion is reduced with respect to s, it will be resolved into a translation of velocity given by Equation [11] and a rotation about P represented by the rotation vector

$$w = F_1 \quad [12]$$

This is an instance of the foregoing analogy and shows that the calculation of the traction across any section, as represented by M and F is completely equivalent to the resolution of the velocities of the beam when properly moving, with respect to any point along its center line.

Examining Equations [7] and [8], it will now be noted that the analogy may be taken further. Indeed Equation [7] is seen to be the total (vector) momentum of the beam moving as previously specified, provided β is taken as the mass per unit length of the central axis. Since $R \times (r_2 - r)ds$ is the moment of the vector Rds about P_2, it follows that D as given by Equation [8] is completely analogous to the moment of momentum of the moving rigid beam about P_2.

The analogy is summarized in Table 1, which appears at the bottom of the preceding page.

Further Discussion of the Analogy

In utilizing the foregoing analogy, it is convenient to recall familiar facts regarding momenta and moments of momenta, such as the following three:

1 The momentum of any system of particles (whether they constitute a rigid body or not) is equal to the momentum obtained by transferring the total mass to the center of gravity and having it move with the velocity of the center of gravity.

2 The moment of momentum of a rigid body moving in any manner, about a point P, is the sum of the moment of momentum about P which results when all the mass is transferred to the center of gravity and allowed to move with its velocity plus the moment of momentum about the instantaneous position of the center of gravity due to rotation of the rigid body relative to the center of gravity. The phrase "motion relative to the center of gravity" refers to the fact that if the velocities of the rigid body be resolved with respect to the center of gravity, they will consist of two parts, namely, a constant translational velocity equal to the velocity of the center of gravity plus a state of velocities due to a proper rotation about some line through the center of gravity. It is the latter component which may be described as the part of the velocity "relative to the center of gravity."

3 The moment of momentum of a rigid body due to a motion of rotation represented by the rotation vector

$$w = w_x i + w_y j + w_z k \quad [13]$$

is given by

$$\begin{aligned}&(w_x I_x - w_y I_{xy} - w_z I_{xz})i \\ &+ (-w_x I_{xy} + w_y I_y - w_z I_{yz})j \\ &+ (-w_x I_{xz} - w_y I_{yz} + w_z I_z)k \quad [14]\end{aligned}$$

where the coefficients of the principal diagonal are the moments of inertia about the x, y, z axes, while the other coefficients are the negatives of the products of inertia about pairs of respective axes.

It must be clearly understood that the moments and products of inertia I_x, I_{xy}, . . . have no relation to the moments of inertia I, J of the beam section which occur in β ($\beta = 1/EI$); they denote the moments of inertia of a mass distribution β per unit length of the beam center line. In the dynamical analog of the statical problem, this mass distribution is allowed to move as a rigid body so that the linear velocity v of each point is equal to M, and so as to possess a rotation represented by $w = F$.

If the axes are properly chosen, namely, if they are made to coincide with the principal axes of inertia, then the products of inertia will vanish and a great simplification results. While theoretically this can always be done, the determination of principal axes of inertia in general requires solving an equation of the third degree. However, in case of plane beams, one of the principal axes of inertia is normal to the plane in which the beam lies so that two products of inertia vanish immediately. The determination of the remaining principal axes now involves only the solution of an equation of the second degree or can be carried out by means of the familiar Mohr circle diagram in which the moment of inertia is plotted along the horizontal axis and the product of inertia along the vertical axis resulting in a circle for the point obtained by taking different sets of axes.

In calculating products and moments of inertia about any point, the calculation of products or moments of inertia can be sometimes simplified by recalling the following theorems: The moment of inertia about any line l is equal to the moment of inertia about a line through the center of gravity parallel to l plus the moment of inertia about l of the whole mass placed at the center of gravity. Similarly, the product of inertia of any mass distribution about two mutually perpendicular axes l_1, l_2 is equal to the sum of the product of inertia about axes parallel to l_1, l_2 and passing through the center of gravity plus the product of inertia about l_1, l_2 of the whole mass placed at the center of gravity.

If the section at P_1 is not held fast, then to the integral Equation [8] must be added the displacement

$$R_1 \times r_2$$

where R_1 is the vector rotation of the end P_1.

Examples

To illustrate this analogy we shall first consider a simple cantilever beam, as shown in Fig. 2, of length l along the positive

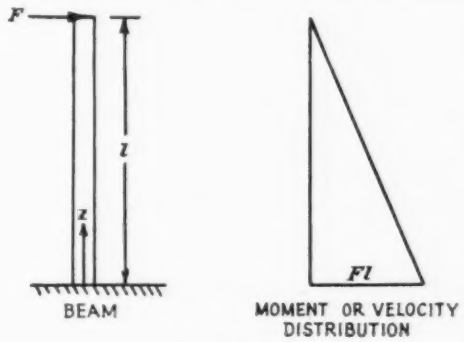


Fig. 2

z-axis with a built-in end P_1 at the origin and loaded at the free end P_2 with the force Fi. The moment distribution is given by

$$M = F(l - z)j$$

and corresponds to the velocity distribution due to a rotation of the cantilever bar about the free end with the rotation vector

$$w = Fi$$

The moment of momentum is simply

$$w_x I_x i = F I_x i$$

Since the radius of gyration of the beam segment about one end is $l/\sqrt{3}$

$$I_x = \beta l \frac{l^2}{3} = \frac{\beta l^3}{3}$$

and the displacement is

$$\beta F \frac{l^3}{3} i$$

The momentum of the moving beam is

$$\frac{Fl}{2} lj = \frac{Fl^2}{2} j$$

hence the angular displacement of the free end is

$$Fl^2/2$$

As another example consider the temperature stresses in a pipe bend forming one horizontal and two vertical sides of a square of length of side l, with built-in ends, as shown in Fig. 3. The

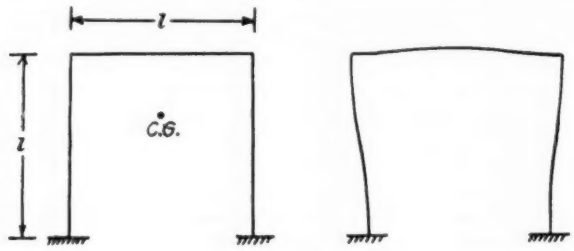


FIG. 3

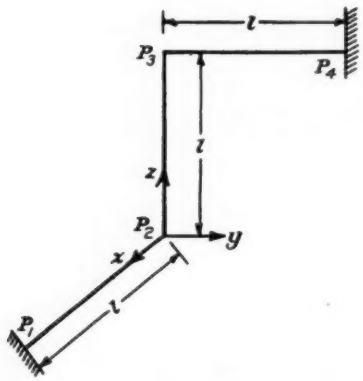


FIG. 4

problem is obviously equivalent to forcing the built-in ends against each other a distance

$$D = -kTl$$

where k is the temperature coefficient of linear expansion, while keeping them parallel. Now $\boldsymbol{D}$ is given, as well as $\int \boldsymbol{R} ds = 0$, and it is required to find the tractions and moments. Recalling the analogy, it will be noted that the total momentum of the motion vanishes; hence, the center of gravity of the beam must be stationary. Therefore, the tractions across any section, if reduced with respect to the center of gravity, are equivalent to a single force passing through it (and a vanishing couple); this force $\boldsymbol{F}$ obviously points in the direction of the horizontal member, which will be taken as that of the x-axis. Moreover, the vector distribution $\boldsymbol{R}ds$, having a zero resultant, will have a constant vector moment about any point of space; hence, in computing $\boldsymbol{D}$ one may obtain the moment of the $\boldsymbol{R}$ distribution, say about the center of gravity rather than about P_1. The center of gravity lies at a distance $l/3$ below the horizontal length. The moment of inertia about the rotation axis through the center of gravity is easily seen to be

$$I_x = (1/3)\beta l^3$$

Substituting in

$$D_x = F_x I_x$$

there results

$$F_x = -\frac{3kT}{\beta l^2}.$$

At the end points P_1, P_2 there is a traction of this amount as well as a bending moment of amount $(2/3)lF_x$ or $2kT/\beta l$. This is also the maximum bending moment.

In the foregoing, the twisting flexibility did not occur. We consider next a problem in temperature stresses of a three-dimensional pipe bend. Assume, as in Equation [3], the same flexibility in twisting as in bending. Let the pipe consist of three straight segments, as shown in Fig. 4, forming adjacent sides of a cube and joining adjacent pairs of the following points

$$P_1 = (l, 0, 0);\ P_2 = (0, 0, 0);\ P_3 = (0, 0, l);\ P_4 = (0, l, l)$$

The stresses developed by an expansion of T degrees are the same as those due to a displacement

$$\boldsymbol{D} = (kTl, -kTl, -kTl)$$

of the end P_4 relative to the end P_1, while at the same time keeping the sections at P_1, P_4 parallel to their original orientation. Again the total momentum $\int \boldsymbol{R} ds$ of the analogous motion vanishes, the center of gravity at $(l/6, l/6, l/2)$ is at rest, and this motion reduces to a rotation $\boldsymbol{F}$ about the center of gravity. The moments and products of inertia about axes through the center of gravity are readily found by first finding them for each segment relative to its center of gravity, then transferring relative to the common center of gravity. There results the following array for the inertia matrix

$$\frac{\beta l^3}{12}\begin{pmatrix} 10 & 1 & 3 \\ 1 & 10 & -3 \\ 3 & -3 & 6 \end{pmatrix}$$

leading to the equations

$$10F_x + F_y + 3F_z = \frac{12kT}{\beta l^2}$$

$$F_x + 10F_y - 3F_z = -\frac{12kT}{\beta l^2}$$

$$3F_x - 3F_y + 6F_z = -\frac{12kT}{\beta l^2}$$

for the force components F_x, F_y, F_z across the beam section. The solution of these equations is

$$F_x = \frac{3kT}{\beta l^2},\quad F_y = -\frac{3kT}{\beta l^2},\quad F_z = -\frac{5kT}{\beta l^2}$$

from which it is readily found that the maximum bending moment occurs at P_1 (or P_4) and is $10kT/3\beta l$. The maximum twisting moment has a value $kT/\beta l$ and exists at each cross section of the pipe from P_2 to P_3.

CASE OF UNEQUAL FLEXIBILITIES

For most engineering computations, the assumption of Equation [3] is satisfactory. If, however, β_1 is not equated to β_2 or β_3, a dynamical analogy can still be built up, though it is rather complex.

Equation [2] is now replaced by

$$\boldsymbol{R} = \beta_1 \boldsymbol{i}_1 m_1 + \beta_2 \boldsymbol{i}_2 m_2 + \beta_3 \boldsymbol{i}_3 m_3$$

where i_1 is the unit tangent vector and i_2, i_3 are unit vectors along the principal directions of the normal section; if

$$\beta_2 = \beta_3 = \beta, \beta_1 \neq \beta$$

then

$$R = \beta M + (\beta_1 - \beta)(M \cdot i_1) i_1$$

Interpreted as a velocity, R/β is seen to consist of two parts: Its previous value M which is like that of a rigid body motion, plus a component

$$\frac{\beta_1 - \beta}{\beta}(M \cdot i_1) i_1$$

along the tangent, corresponding to a sliding along the beam center line and proportional to the projection of M along it. Except for this change, the further analogy to the moment of momentum holds without modification. However, since the mass distribution does not move as a rigid body, the moment of momentum cannot be expressed merely in terms of moments and products of inertia. To the previous expressions for $\int R ds$ and D are to be added

$$(\beta_1 - \beta)\left[M_1 \cdot \int_{s_1}^{s_2} i_1 i_1 ds + F_1 \cdot \int_{s_1}^{s_2} (r \times i_1) i_1 ds \right]$$

$$(\beta_1 - \beta)\{ M_1 \cdot i_1 [i_1 \times (r_2 - r)] ds + F_1 \cdot \int (r \times i_1)(i_1 \times \overline{r_2 - r}) ds \}$$

respectively. The integrals and their integrands are dyadics depending on the geometry of the center line $r = r(s)$ only.

Beam Loaded Along Its Length

If in addition to terminal tractions there are also loads applied along the length of the beam, then the analogy to the simple dynamical problem breaks down further. Suppose, for instance, that there is a single load W applied somewhere at P' between two ends due to some constraining connection at P'. The resultant force F will then no longer be constant throughout the beam section but will change by an amount equal to W at P'. If the dynamical analogy is introduced, it would then be necessary to break up the mass distribution β into two parts at P', whereupon it will be found that the two parts will not move as a single rigid body but that each one will move as a separate rigid body; the velocity of the common end point of the two parts P' is the same, but the angular rotation of one part will differ from the other one.

In case there is a distributed load on the beam, it is advisable to drop the analogy completely and resort directly to the equations

$$\frac{dF}{ds} + W = 0$$

$$\frac{dM}{ds} + F \times i_1 = 0$$

which are analogous to the familiar beam equilibrium equations. The determination of the rotation vector per unit length R and of the displacement D in terms of R is carried out as before.

Comments on the Enthalpy of Saturated Steam From 0 to 100 C and Other Steam Properties

By FREDERICK G. KEYES,[1] CAMBRIDGE, MASS.

A FEW years ago the entire body of available thermodynamic data on water and steam was reviewed and correlated for the purpose of preparing steam tables (1).[2] At the time, precise experimental values of the thermal properties of saturated steam at temperatures below 50 C were not available. This gap, however, has recently been filled by the publication of an extraordinarily painstaking series of measurements by Osborne, Stimson, and Ginnings (2).

At the time referred to, it was known that the values of C_{p_0}, the specific heat of very low-pressure steam, would be slightly changed when more extensive spectroscopic data became available. This deficiency has now been supplied through the recent more extensive measurements on water vapor by Randall, Dennison, Ginsberg, and Weber (3). C. C. Stephenson and H. O. McMahon (4) have recently completed the laborious computations required to deduce the C_{p_0} magnitude from the extended recent spectroscopic data.

The distortion of the water molecule due to centrifugal forces was computed by Wilson (5) leading to a correction in the form $2\rho RT$, where $\rho = 2.04 \times 10^{-5}$, applicable to the earlier computations of C_{p_0} from the older spectroscopic data by A. R. Gordon (6) which had been used in formulating the steam tables. The new computations confirm the magnitude of the Wilson correction in a highly satisfactory way leading to a value of ρ in the mean equal to $\rho = 2.33 \times 10^{-5}$. This value is adopted and the following equation is an expression of our best knowledge of the specific heat of steam corresponding to zero pressure

$$C_{p_0}: \text{Int joule (1.0003)} = 1.4720 + 7.7716 \cdot 10^{-4} T + 47.8365 T^{-1} \quad \ldots\ldots\ldots [1]$$

This is the original correlation formula by Professor Keenan plus the correction term entering the coefficient of T, that is, $7.7716 \cdot 10^{-4}$ instead of the former $7.5566 \cdot 10^{-4}$.

The new formula produces a change in C_{p_0} at 100 C (1.8902, Equation [1], versus 1.8822) amounting to a part in 230. The values computed from Equation [1] are also confirmed by recent direct measurements (7) (1.8930, the mean at 100 C).

Collins and Keyes, besides reporting on directly measured C_{p_0} values, obtain values of $(\partial H/\partial p)$, from which the characteristic-state quantity for water vapor, B_0, from 38 to 135 C could be computed. These essentially confirm the older values extrapolated from higher temperatures. The new values, however, are somewhat larger but not enough to modify the computed properties of water vapor appreciably in the low temperature region. The saturation values of B or B_{sat} are given in the range 0 to 100 C by the equation

$$B_{sat}^{cc/g} = 1.89 - 6777.7\ T^{-1} - 1.0537 \cdot 10^{10}\ T^{-7} \ldots\ldots [2]$$

Used in the equation of state, $v_{sat} = R^* T/p_{sat} + B_{sat}$, the average deviation of the computed saturation volumes from those recently reported by Osborne, Stimson, and Ginnings is a part in 7700 (0 to 100 C, inclusive). The preferred vapor pressure equation (p_{sat}) for the same region based on the Reichsanstalt data is No. 11 of the Steam Research Program, Part IV (8).

The enthalpy equation for saturated vapor based on Equation [1] for C_{p_0} and B_{sat}, using the value of H_1 at 0 C given by Osborne, Stimson, and Ginnings to fix h_1, is

$$H_1^{\text{Int j}} = h_1 + 1.472\,(T - T_0) + 3.8858 \times 10^{-4}\,(T^2 - T_0^2) + 110.1476 \log_{10}(T/T_0) + [0.1915 - 1373.2 \times T^{-1} - 8.539 \cdot 10^{10} \times T^{-7}]p_{sat}^{**}$$

The value of h_1 using the latest enthalpy value of Osborne, Stimson, and Ginnings at 0 C (2500.3 Int j) (9) leads to $h_1 = 2500.78$ and the average deviation of the computed H_1 values at round 10 C from the recent values is a part in 5300. The deviation at 100 C is the largest amounting to a part in 2200. However, the values of the saturation enthalpy in the Steam Tables (Table 1) are also in agreement with the new values to a part in 1600 at 0 C; a part in 2200 at 50 C; and a part in 26,000 at 100 C, the reference point for establishing the steam tables constant of enthalpy.

An important contribution by Hans Eck (10) has also appeared recently on the saturation properties of water from 350 C to the critical temperature. Critical data for heavy water were also obtained.

The critical temperature of water is reported as 374.23. The value reported by Smith and Keyes is 374.11. This is better agreement than is usual in critical temperature measurements by different investigators. The critical pressures are: Eck, 218.258 atm; Smith and Keyes, 218.17—a part in 2500. The new critical volume is 3.0656 cc per gram compared with 3.14 cc of the Steam Tables. The critical magnitudes for heavy water are

$$t = 371.5 \text{ C};\ p_c = 214.387 \text{ atm};\ v_c = 2.74 \text{ cc per gram}$$

The critical-region saturation-liquid specific volumes reported by Eck are lower than those in the Steam Tables above 350, while the vapor saturation volumes are in tolerable agreement.

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

* $R = 4.55504$ per gram of steam; $T = (273.16 + T$ deg C).

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9 Loc. cit. reference (2), Table 2, p. 268.

10 "Untersuchungen mit Sättingungszustand des Wassers von 350°C bis zur kritischen Temperatur," by Hans Eck, *Physikalische Zeitschrift*, vol. 40, 1939, p. 3.

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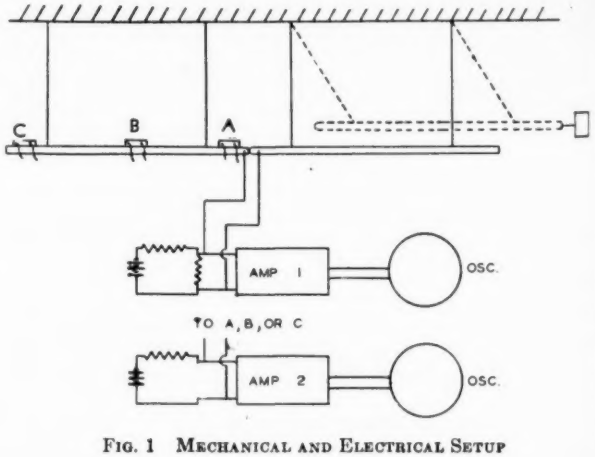


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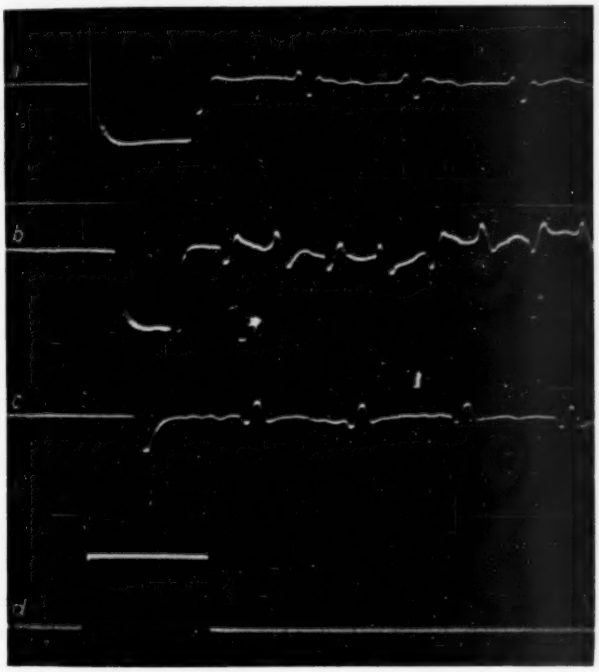


FIG. 2 LONGITUDINAL STRAINS IN LONG BARS

of pressure building up to a practically steady value, strain in the region through which the wave front has passed will maintain a constant level until the return of the reflected wave.

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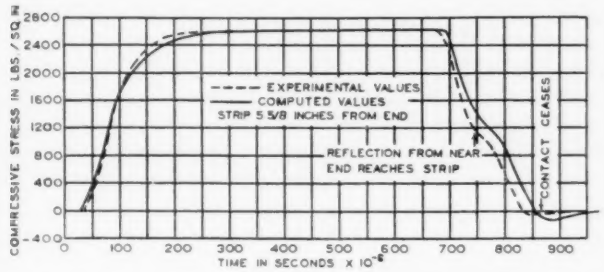


FIG. 3 CALCULATED AND EXPERIMENTAL STRESSES IN 6-FT BARS IN IMPACT

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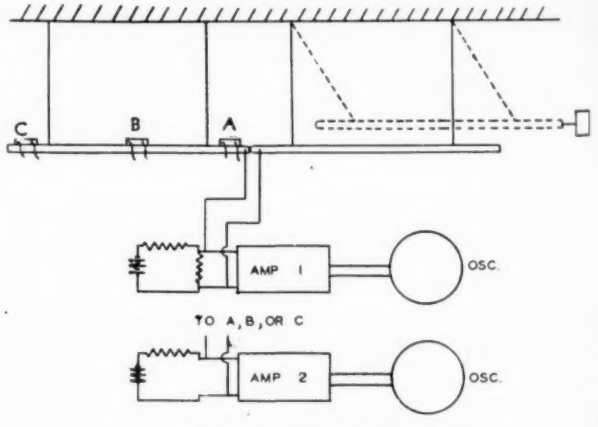


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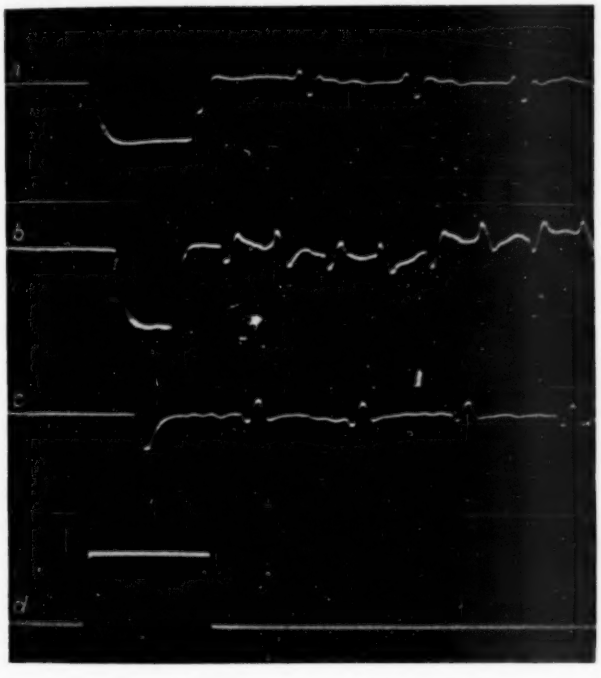


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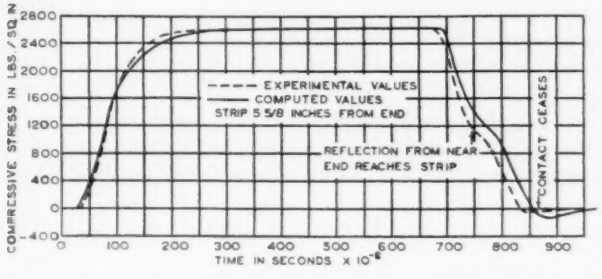


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dicts that the short bar would be free of vibration after separation, which this record shows to be essentially true.

Fig. 5 shows the strain record obtained by striking with another bar the end of a 5-ft bar, the other end of which has been cemented to a heavy iron support. The object of this setup was to obtain a fixed-ended bar. When a wave of strain encounters a free end, it has been seen to reflect as a strain of opposite sign. When a wave meets a fixed end, the result should be a reflection with the same sign, i.e., reflection should double the strain rather than reduce it to zero. The strip was attached near the free, impacting end of the bar. The initial passage of a compression wave was recorded as usual. The reflected wave shows an increase of strain as required by theory. It is not exactly doubled, since the end was not entirely fixed, and there was a momentary dip in the curve, caused probably by the fact that the bar was not firmly in contact with the large mass of iron at first.

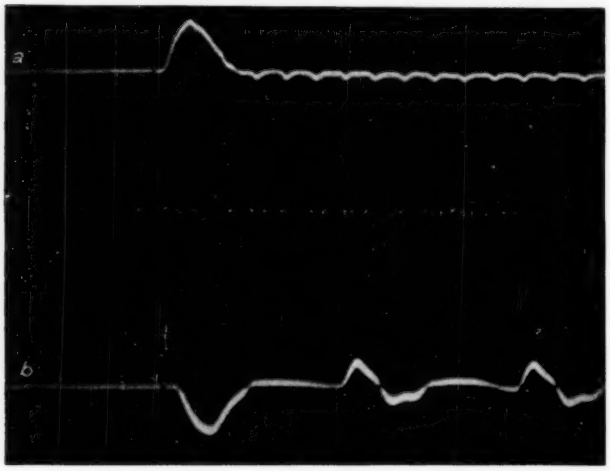


FIG. 4 IMPACT OF A 6-FT BAR AND A 1-FT BAR
[Strain at the near end of the (*a*) 1-ft bar and (*b*) 6-ft bar.]

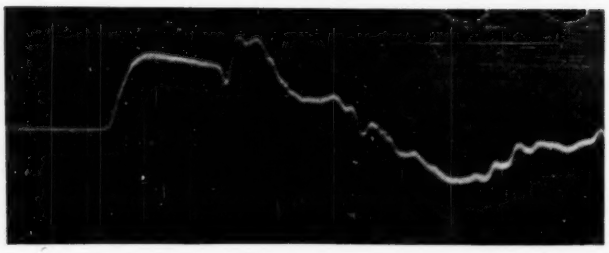


FIG. 5 REFLECTION OF STRAIN FROM A RELATIVELY FIXED END OF A LONG BAR

Records were obtained on bars which had holes of various sizes drilled through them. The object was to determine if a noticeable reflection was produced by such a discontinuity. No appreciable reflection of this sort was found on a single travel of a strain wave once through the bar, even when two holes at right angles were present which reduced the area of cross section of the bar at that point to a quarter of its original value. After the wave had traveled a number of times up and down the bar a slight cumulative effect due to the hole was observable, but still not important. Records were also made of the effect of interposing several layers of tape between the impacting ends of the bars. As was to be expected, the impact was "softened," i.e., the rate of build-up of strain was much lower.

Lateral impact of a compact mass against a beam has been studied recently by Arnold (8), using scratch-recording strain gages. We made a somewhat similar setup, using, however, a bar instead of a compact mass as the striking body, and a light rectangular bar as the struck beam. Fig. 6*a* is a record of strain near the striking end of the bar, and Fig. 6*b* is a record of contact between the bar and the beam. The first pulse on the strain record may be interpreted as due to the impact of the bar with the plug of beam material immediately under the contact end of the bar. This generates a compression pulse which traverses the length of the bar, much as in the case of a long bar striking a short one as shown in Fig. 4*b*. The reflection from the free end of the bar of this pulse momentarily breaks contact upon its return, which sets up further waves in the beam and the bar. These longitudinal vibrations of the bar are of high frequency, and give rise to the closely spaced makes and breaks of contact which are to be seen in Fig. 6. Actually this record shows a second series of contacts commencing about 0.011 sec after the last of the first series, but which is not shown in the portion of the record reproduced here. This longer break in contact, between series of closer spaced contacts, is considered to be due to flexural vibra-

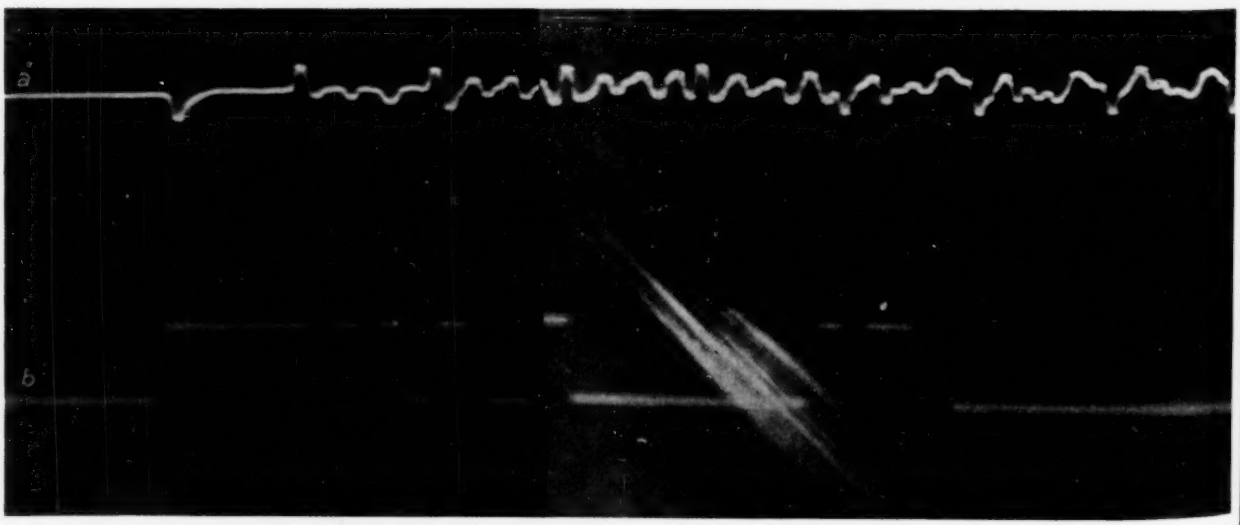


FIG. 6 STRAINS DUE TO THE LATERAL IMPACT OF A BAR AND A BEAM
[(*a*) Strain in striking the bar; (*b*) contact between the bar and beam.]

tions of the beam. It was breaks in contact of this sort which Arnold (8) found and which caused him to describe the phenomena as "multiple impact." In our case we have multiple impacts, each of which is itself a multiple impact due to longitudinal waves in the striking bar.

Conclusions

The longitudinal strains in long bars in impact have been recorded for a number of cases. The phenomena are found to be rather complicated. The case of two bars of equal length, each 6 ft, has been analyzed and the results of theory and experiment shown to agree. As a result of this check, considerable confidence is felt in the results obtained with the electrical-resistance strain gage for impact conditions. The strains obtained in the work reported in this paper were rather small, but further work has shown that the gages work successfully at much higher strains.

Acknowledgment

The experimental part of this work was done by the junior author as an S.M. thesis in mechanical engineering at M.I.T. We wish to acknowledge great indebtedness to Prof. A. V. de Forest in whose laboratory this work was done, for his encouragement and advice, as well as for the materials and equipment which he made available. Professor de Forest has reported this work in part (9).

Appendix

Method of Calculating Strains

The method used for calculating strains is that due to Sears (6). Actually stress is what is plotted in Fig. 3 but here clearly Hooke's law holds, and it was thought that stress figures were more familiar. Referring to Fig. 7, we note that all the force between the bars is transmitted through the small area of contact A. Thus, in the neighborhood of contact, the stress is essentially three-dimensional. However, we know by St. Venant's principle that the stress rapidly spreads out so that at a rather short distance from the contact region the stress, or the strain, is evenly distributed across the cross section.

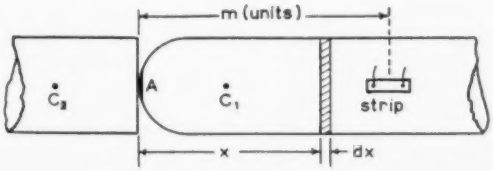


Fig. 7 Impacting Ends of Bars

For any element of the bar of length dx, the net force acting on it is

$$\Delta F = F - \left(F + \frac{\partial F}{\partial x}\,dx\right) = \frac{\partial F}{\partial x}\,dx$$

and now if ρ = density, g = acceleration of gravity, S = area of cross section, and u = displacement of element relative to axes fixed in space, then

$$\frac{\partial F}{\partial x}\,dx - \frac{\rho S}{g}\,\frac{\partial^2 u}{\partial t^2}\,dx = 0 \qquad [1]$$

but

$$F = SE\,\frac{\partial u}{\partial x} \qquad [2]$$

whence

$$E\,\frac{\partial^2 u}{\partial x^2} - \frac{\rho}{g}\,\frac{\partial^2 u}{\partial t^2} = 0$$

or

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{v^2}\,\frac{\partial^2 u}{\partial t^2} = 0 \qquad [3]$$

where $v = \sqrt{(Eg/\rho)}$ = velocity of sound. This equation holds in the interior of the bar. It has the well-known solution

$$u = f_1(t - x/v) + f_2(t + x/v) \qquad [4]$$

Now near the end of the bar, however, the strain is not governed by this one-dimensional-wave equation, but is essentially three-dimensional. Sears (6) shows that this region is small and that points C_1 and C_2 shown in Fig. 7 may be found for which, under static conditions, Hertz's law of deformation holds, i.e.

$$F = k\alpha^{3/2} \qquad [5]$$

where F = force between the bodies, k = a constant $= \dfrac{2E}{3(1-\sigma^2)}\left(\dfrac{r_1 r_2}{r_1 + r_2}\right)^{1/2}$, σ = Poisson's ratio, r_1 = radius of curvature of body 1 at region of contact, and r_2 = radius of curvature of body 2 at region of contact. Furthermore

$$AC_1 = AC_2 = c = a\,\sqrt{(3/2)} + a\pi EB\,\sqrt{(2/3)} \qquad [6]$$

where a = radius of the bar and

$$B = \left(\frac{6E + 5G}{3E + G} + 1\right)\frac{1}{4\pi G}$$

$$G = \frac{E}{2(1+\sigma)} = \text{modulus of rigidity}$$

Now since this region C_1C_2 is small we may neglect the effects of the finite time of propagation of strain through it for the moment and use Equation [5] for the impact case. The force thus found can then be considered to be acting at the very end of the bar, which is now considered as a one-dimensional system throughout for purposes of calculating wave travel. Now let β = displacement of C_1 relative to axes fixed in space, γ = displacement of C_2 relative to axes fixed in space, V_1 = original velocity of C_1, and V_2 = original velocity of C_2, all these being measured in the same direction in space, so $\alpha = \beta - \gamma$.

Now for our case, Equation [4] becomes

$$u = f_1(t - x/v) \qquad [7]$$

i.e., $f_2 = 0$, since it would correspond to a wave starting in the opposite direction in the bar.

Now from Equation [1]

$$\frac{\partial^2 u}{\partial t^2} - \frac{g}{\rho S}\,\frac{\partial F}{\partial x} = 0$$

but

$$F = SE\,\frac{\partial u}{\partial x} = -\frac{SE}{v}\,\frac{\partial u}{\partial t}$$

whence

$$\frac{\partial F}{\partial x} = -\frac{SE}{v}\,\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right) = -\frac{1}{v}\,\frac{\partial}{\partial t}\left(SE\,\frac{\partial u}{\partial x}\right) = -\frac{1}{v}\,\frac{\partial F}{\partial t}$$

whence for the element at C_1, where $u = \beta$

$$\frac{\partial^2 \beta}{\partial t^2} + \frac{g}{\rho v S}\,\frac{\partial F}{\partial t} = 0 \qquad [8]$$

Integrating once

$$\frac{\partial \beta}{\partial t} + \frac{g}{\rho v S} F\left(t - \frac{x}{v}\right) = \text{const} = V_1$$

which we may write finally as

$$\dot{\beta} = V_1 - \frac{g}{\rho v} P\left(t - \frac{x}{v}\right) \quad \ldots\ldots\ldots\ldots\ldots [9]$$

and similarly

$$\dot{\gamma} = V_2 - \frac{g}{\rho v} P\left(t - \frac{x}{v}\right) \quad \ldots\ldots\ldots\ldots [10]$$

where $P = F/S$ = stress.

Now Equations [9] and [10] together with Equation [5] solve our problem. However, a solution in the form $P = p(t)$ is not possible in terms of known functions, so a step-by-step integration is necessary. Equations [9] and [10] hold until reflections return, whereupon further terms of the form $P_{t - \left(2l - \frac{x}{v}\right)}$ are to be added to the right-hand side of each equation.

Now integrating for a small time τ

$$\beta_t - \beta_{t-\tau} = V_1\tau - \frac{g}{\rho v}\int_{t-\tau}^{\tau} P\left(t - \frac{x}{v}\right) dt \ldots\ldots [11]$$

and now if $t = n\tau$, and we denote $\beta_{n\tau}$ as β_n, etc., and put

$$A_n = \frac{g}{\rho v}\int_{(n-1)\tau}^{n\tau} P\left(t - \frac{x}{v}\right) dt$$

$$= \frac{g}{2\rho v}(P_n + P_{n-1}) \text{ since } \tau \text{ is small}$$

$$= C(P_n + P_{n-1})$$

then

$$\beta_n = \beta_{n-1} + V_1\tau - A_{n-1} \ldots\ldots\ldots\ldots [12]$$

and similarly

$$\gamma_n = \gamma_{n-1} + V_2\tau - A_{n-1} \ldots\ldots\ldots\ldots [13]$$

and now we are given in our case V_1 = known velocity of striking bar at impact, $V_2 = 0$, $\beta_0 = \gamma_0 = 0$, $P_0 = P_{-1} = 0$, whence

$$A_0 = C(0) = 0$$

and from Equation [12]

$$\beta_1 = 0 + V_1\tau - 0 = V_1\tau$$

and from Equation [13]

$$\gamma_1 = 0$$

whence

$$\alpha_1 = (\beta_1 - \gamma_1) = V_1\tau$$

and $P_1 = k'\alpha_1^{3/2}$ which is now calculable, $k' = \frac{k}{S}$

we then have

$$A_1 = C(P_1 + P_0) = \frac{g}{2\rho v} P_1$$

and then

$$\beta_2 = \beta_1 + V_1\tau - A_1$$

$$\gamma_2 = \gamma_1 + 0 - A_1$$

of which all the terms on the right are known. Thus α_2 may be found, and then P_2 and so on.

When reflected waves return we have instead of Equation [9]

$$\beta_n = \beta_{n-1} + V_1\tau - A_{n-1} - A_{n-1-2l} \ldots\ldots\ldots\ldots \text{etc.}$$

This gives the stress at the end of the bar, or rather the stress which would exist in an imaginary bar at that point if the stress were distributed uniformly over the cross section right down to the end. However, our measured values were for a strip $5\frac{5}{8}$ in., or approximately 4 intervals of length c from the end. We calculate the stress there; thus:

Let P'_n = stress at time $n\tau$ at a point m units of length c in from the end. Then $P'_n = P_{n-m} - P_{n-(110-m)} + P_{n-(110+m)}$ since the length of the bar down and back is 110 units and P_{n-m} = stress at strip due to direct wave from near end, thus m units late; $P_{n-(110-m)}$ = stress at strip due to wave once reflected from far end (thus the negative sign); and $P_{n-(110+m)}$ = stress due to wave once reflected from far end, then once from near end (thus a positive sign).

For our case $m = 4$ and so

$$P_n' = P_{n-4} - P_{n-106} + P_{n-114}$$

as plotted in Fig. 3.

For this calculation the various constants had the following values: $a = 0.531$ in.; $c = 1.31$ in.; $r_1 = 0.531$ in.; $r_2 = \infty$; $v = 16{,}800$ fps $= 201{,}800$ in. per sec; $\tau = c/v = 6.491 \times 10^{-6}$ sec; $C = g/2\rho v = 2.211 \times 10^{-3}$; $V_1 = 34.75$ in. per sec; and $k' = k/S = 1.554 \times 10^7$.

Note that this value of c is not exactly that given by formula [6] but, as Sears (6) shows, its exact value is not of importance in the calculations, so that it is convenient to take it so that it is contained an integral number of times in the length of the bar. Thus, in our case this value of c makes the bar 55 units of this length long.

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A Rational Representation of the Flow Performance of Reaction Steam-Turbine Blading

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This paper gives a condensed method of representing flow performance of reaction-type turbine blading which is applicable to an extensive range of flow conditions including those widely different from the conditions of maximum efficiency. The method is of particular value in analyzing flow in by-pass blade groups of central-station turbines or of mixed-pressure or bleeder turbines. It is based on dimensional analysis and empirical data are presented in the condensed form suggested by the analysis.

THE blading of modern steam turbines is subjected to operation under very widely divergent flow conditions. Thus, the configuration of the flow in a by-pass blade group of a central-station turbine varies greatly with the load. In mixed-pressure turbines as well as in bleeder turbines complicated flow conditions occur due to externally regulated interstage pressures or externally imposed rates of steam flow. When the throttle valve of a high-pressure superposed turbine accidentally closes and the turbine continues to rotate at full speed as it is driven by the motorized generator, a certain amount of cooling steam must be admitted to prevent dangerous heating of the turbine. A similar situation exists in a marine propulsion turbine where special measures must be taken to avoid excessive heating of the ahead blading when operating astern.

The performance of turbine blading under all these conditions must be computed in advance be it as the basis for the greatly detailed turbine-performance guarantees customary today, be it to assure the safe mechanical operation of the power unit. It therefore has become necessary to develop methods of predicting the turbine flow characteristics under all possible modes of operation. While theoretical analysis of the flow in the blading yields much insight of a fundamental nature, a sufficiently accurate prediction of the performance of a steam turbine cannot be made by this procedure alone. The calculation of the behavior of the turbine is based on tests with an actual turbine.

The number of such tests, because of their cost, however, must be reduced to a minimum which, in turn, calls for the most rational methods of test analysis.

It is the purpose of this paper to show how, with the use of dimensional analysis, the most condensed method of representing turbine-flow performance is found. While this method of attack is applicable to any type or combination of types of turbine elements, the presentation in the paper has been restricted to the reaction type of turbine blading.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

THE SINGLE-STAGE PROBLEM

Consider now a group of reaction-turbine blading. Rotating the spindle with a certain speed while maintaining a certain inlet pressure to the blading, a certain inlet temperature and a certain outlet pressure will, in general, set up a certain configuration of flow of the steam in the blading. A test, in fact, shows that the blading develops a certain definite power and consumes a certain definite amount of steam per unit of time.

In particular one stage out of the group, consisting of a stationary and a rotating blade row, is subjected to the initial pressure p, lb per sq ft; the initial temperature t, deg F; the pressure drop Δp, lb per sq ft; and the blade speed u, fps.

By changing the operating conditions of the blading group, we are able to vary at will any or all of the four stage quantities p, t, Δp, u. Representing furthermore the size of the blading by the characteristic length l, ft, we may consider the five quantities p, t, Δp, u, l the independent test variables. Giving each of them a certain value fixes a certain configuration of flow of the steam in the stage. The turbine designer is interested in certain effects of this flow, such as the work N, ft-lb per sec, delivered to the turbine shaft, the amount of fluid G, lb per sec, passing through the stage. In some instances he may want to know other characteristics of the flow, as, for instance, the temperature drop Δt, deg F, across the stage, and the mean relative efflux angle α of the flow from each blade row. The quantities N, G, Δt, α may then be considered the dependent variables of the reaction-stage problem. Of these dependent variables only two, the mass flow G and the power N, are more or less easily and accurately measurable in a turbine test. The others must be determined by mathematical analysis.

Presently with p, t, Δp, u, l, N, and G as the quantities usually measured in a turbine test, the test performance may be represented by the two functions

$$F_1(N;\ p;\ t;\ \Delta p;\ u;\ l) = 0 \quad \ldots\ldots\ldots\ldots [1a]$$

$$F_2(G;\ p;\ t;\ \Delta p;\ u;\ l) = 0 \quad \ldots\ldots\ldots\ldots [1b]$$

each of which contains six variables.

Plotting the test results in this manner, however, is exceedingly complicated. Allowing only ten values for each of the variables, it requires 2000 curve sheets to represent the two functions graphically. To obtain ten values for each independent test variable, a project of $10^6 = 100{,}000$ tests would have to be conducted.

Is it possible to simplify this situation without losing generality? The answer is obtained from dimensional considerations.

(*a*) *Dimensional Analysis of the Turbine Test.* Suppose we were able to calculate the function Equation [1*a*] analytically. The formula thus derived would, besides the six variables of Equation [1*a*], also contain variables representing the physical properties of the steam. The steam shall for the present purpose be represented by a gas with a constant-specific-heat ratio $k = c_p/c_v$ and a constant viscosity μ, lb sec ft^{-2}. The analytical expression

for the power output of the stage then would be of the general form

$$F(N;\ p;\ \rho;\ \Delta p;\ u;\ l;\ \mu;\ k) = 0 \ldots\ldots\ldots [2]$$

where we have, for convenience, replaced the temperature before the stage by the fluid mass density ρ, lb ft^{-4} sec^2, which is determined by p and t.

Equation [2] contains all the variables that are of importance in determining the power output N. To this equation we can directly apply the rules of dimensional analysis,[1] which are based on the observation that, applied to the present case, the phenomenon of the power developed by the turbine stage does not depend on the size of the units used to measure all the variables involved. It must therefore be possible to rearrange Equation [2] in such a manner that all variables are nondimensional ratios and thus independent of the measuring units.

It is shown in Table 1 that the dimensions of all eight variables involved can be expressed in terms of mass, length, and time, designated by M, L, and T, respectively.

TABLE 1 QUANTITIES INVOLVED IN THE REACTION-STAGE FLOW PROBLEM

Name	Symbol	Dimension expressed in products of powers of the fundamental units of mass, length, and time
Work done per unit time	N	ML^2T^{-3}
Reference length	l	L
Pressure before stage	p	$ML^{-1}T^{-2}$
Mass density before stage	ρ	ML^{-3}
Pressure drop across stage	Δp	$ML^{-1}T^{-2}$
Mean blade speed	u	LT^{-1}
Specific-heat ratio	k	1
Fluid viscosity	μ	$ML^{-1}T^{-1}$

Furthermore Buckingham's π theorem indicates that Equation [2] must be expressible in terms of $8 - 3 = 5$ nondimensional variables, say Ω; Δ; σ; Re; k. Thus, the new form of Equation [2] becomes

$$\Phi_1(\Omega;\ \Delta;\ \sigma;\ \mathrm{Re};\ k) = 0 \ldots\ldots\ldots [2a]$$

The five nondimensional variables, as found by the usual methods of dimensional analysis, result in

$$\Omega = N/l^2p\sqrt{(p/\rho)} = N/l^2p\sqrt{(gpv)} \text{ (mass flow)} \ldots [3a]$$

$$\Delta = \Delta p/p \text{ (pressure drop)} \ldots\ldots [3b]$$

$$\sigma = u/\sqrt{(p/\rho)} = u/\sqrt{(gpv)} \text{ (blade speed)} \ldots\ldots [3c]$$

$$\mathrm{Re} = \rho lu/\mu = lu/gv\mu \text{ (Reynolds' number)} \ldots [3d]$$

$$k = \text{(compressibility)} \ldots\ldots\ldots [3e]$$

with g = gravitational acceleration = 32.17 fps per sec.

A very similar analysis carried out for the mass flow Equation [1b] yields the function of nondimensional variables

$$\Phi_2(\Gamma;\ \Delta;\ \sigma;\ \mathrm{Re};\ k) = 0 \ldots\ldots\ldots [2b]$$

with the new variable representing the mass flow G

$$\Gamma = (G/g)/l^2p\sqrt{(p/\rho)} = Gv/l^2\sqrt{(gpv)} \ldots\ldots [3f]$$

Comparing now the nondimensional functions of Equations [2a] and [2b] with the original Equations [1a] and [1b], we observe that the number of variables has been reduced from six to five. This is the result of the simplifying assumption of constant and independent values μ and k. Actually of a real fluid, such as steam, μ and k are not constant and not independent. For practical reasons we are, however, forced to even further simplify the analysis.

Reaction blading usually operates with fluid velocities well below those of sound. Therefore, the compressibility of the fluid, which is represented by the value k, is generally of little importance in forming the flow through the blading. The variation of k will therefore be neglected. Furthermore, it is easily found that of the reaction stage of a given turbine the Reynolds number Re varies relatively little. Flow changes with Reynolds' number usually being slow, the effect of the variation of Re in the stage of a given turbine can be disregarded.

When comparing turbines of different sizes and widely different steam conditions, the effect of the viscosity is, however, of sufficient importance to be investigated separately.

All following considerations shall be assumed as referring to a range of conditions where the effect of a variation of Re is negligible. The test performance of the reaction stage, then, is represented by the two functions of three nondimensional variables each

$$\Omega = f(\Delta;\ \sigma);\quad \Gamma = f(\Delta;\ \sigma) \ldots\ldots\ldots [4]$$

which compare directly with the two functions expressed by Equations [1a] and [1b] of six variables each. Instead of 2000 curve sheets, we now need two to represent the functions graphically. Instead of 100,000 tests we need but 100 to cover ten values for each dependent variable.

Instead of the seven test variables p, t, Δp, u, l, N, and G we have now the four dimensionless quantities Δ, σ, Ω, and Γ defined in terms of the others by the Equations [3]. We call Δ, σ, Ω, Γ the fundamental test variables.

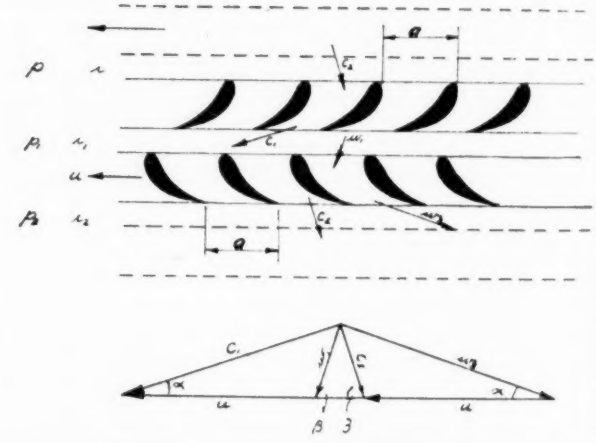


FIG. 1 VELOCITY TRIANGLES OF A SYMMETRIC REACTION STAGE

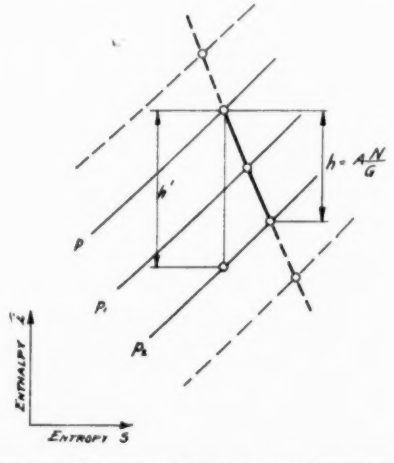


FIG. 2 ENTHALPY-ENTROPY DIAGRAM OF A SYMMETRIC REACTION STAGE

[1] "Dimensional Analysis," by P. W. Bridgman, Yale University Press, New Haven, Conn., 1931.

(*b*) *Characteristics Derived From the Fundamental Test Variables.* The configuration of the flow in the stage is now fixed as soon as certain values are given to two of the four fundamental test variables, say Δ and σ. From the turbine test we obtain directly only the two effects, power and mass flow, now represented by Equations [4]. An example each of a power and a flow characteristic is shown in Figs. 3 and 4 which are based on actual test results.

The turbine designer is interested, however, in a number of other stage characteristics. These must be obtained by analysis.

In the following derivations we will assume that the stage is "symmetric," that is, both blade rows have the same blade profile and the mean absolute velocity approaching the stage is the same in direction and magnitude as the mean absolute velocity leaving the stage. The two velocity triangles make the symmetric picture shown in Fig. 1; c representing velocities as seen by a stationary observer, and w representing velocities as seen by an observer moving with the speed u of the rotating blade row.

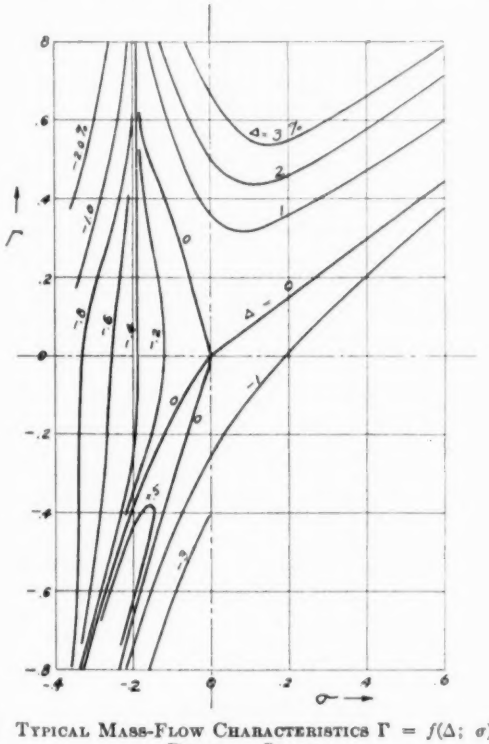


FIG. 3 TYPICAL MASS-FLOW CHARACTERISTICS $\Gamma = f(\Delta;\ \sigma)$ OF A REACTION STAGE

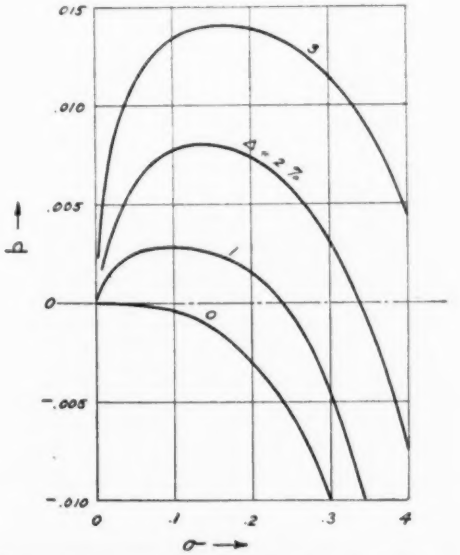


FIG. 4 TYPICAL WORK CHARACTERISTICS $\Omega = f(\Delta;\ \sigma)$ OF A REACTION STAGE

The energy equation for the stage

$$i - i_2 = cp(T - T_2) = AN/G \quad [5]$$

combined with Equations [3*a*] and [3*f*] yields the nondimensional expression τ for the stage temperature drop or stage enthalpy drop

$$\tau = \Omega/\Gamma = (i - i_2)/Apv = \frac{k}{k-1}(T - T_2)/T \quad [6]$$

where A = mechanical equivalent of heat = 778.3 ft-lb per Btu; i; i_2 = enthalpy before and after the stage, respectively, Btu per lb; and T; T_2 = absolute temperature before and after the stage, deg F.

The nondimensional temperature drop τ apparently is a function of Δ and σ; $\tau = f(\Delta;\ \sigma)$.

Denoting with α the relative angle of the flow leaving the row, the condition of continuity may be expressed as

$$G = z(ab/v)c_1 \sin\alpha \quad [7]$$

where a = blade pitch, ft; b = exit blade height of previous rotating blade row = exit blade height of stationary blade row, ft; and z = number of blades in the blade row.

The work done per second in the stage can be calculated by writing down the change of momentum in peripheral direction across the rotating blade row, thus

$$N = (G/g)u(2c_1 \cos\alpha - u) \quad [8]$$

Eliminating c_1 between Equations [7] and [8] and remembering the definitions of Ω, Γ, and σ, an equation for the leaving flow angle is obtained

$$\tan\alpha = (2l^2/zab)/(\sigma/\Gamma)(1 + \Omega/\Gamma\sigma^2) \quad [9]$$

We note that $\alpha = f(\Delta;\ \sigma)$.

Similarly we can derive the expressions for the velocity ratio

$$\nu = u/c_1 = 2\cos\alpha/(1 + \Omega/\Gamma\sigma^2) = f(\Delta;\ \sigma) \quad [10]$$

and the relative flow angle of approach

$$\tan\beta = \sin\alpha/(\cos\alpha - \nu) = f(\Delta;\ \sigma) \quad [11]$$

The blading efficiency which is, referring to the enthalpy-entropy diagram of Fig. 2, defined as

$$\eta = h/h' = AN/Gh' \quad [12]$$

can also be expressed in terms of the fundamental test variables. For the isentropic change of state across the stage we write if the pressure drop is small: heat added $\Delta q = 0 \cong \Delta i - Av\Delta p$, thus, $h' = \Delta i \cong Av\Delta p = Apv\Delta$. Eliminating h' and introducing the nondimensional test variables the blading efficiency becomes simply

$$\eta = \Omega/\Gamma\Delta = f(\Delta;\ \sigma) \quad [13]$$

If the pressure drop Δ is large, the approximation for Δi is not sufficient. The relation between h' and Δ is then found from the steam chart, or with an average value k from the general Bernoulli equation, and the expression for the efficiency becomes

$$\eta = \frac{Apv}{h'}\frac{\Omega}{\Gamma} = \frac{k-1}{k}\,\frac{\Omega/\Gamma}{1-(1-\Delta)^{(k-1)/k}} = (\Omega/\Gamma\Delta)\Big/\left(1 + \frac{\Delta}{2k} + \frac{k+1}{6k^2}\Delta^2 + \ldots\right) \quad [13a]$$

The isentropic velocity ratio $\nu' = u/c'$ often used in turbine calculations with $Ac'^2/2g = h'/2 \cong Av\Delta p/2$ becomes

$$\nu' = \sigma/\sqrt{\Delta} = f(\Delta;\ \sigma) \quad \ldots\ldots\ldots\ldots\ldots\ldots [14]$$

In line with customary procedure in aerodynamics the Mach number $M_a = c'/c_a$, where $c_a = \sqrt{g\,k\,pv}$ = acoustic velocity,

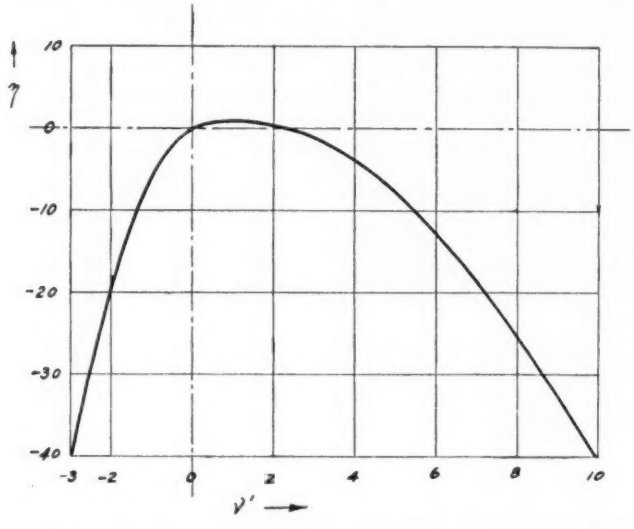


FIG. 5 TYPICAL EFFICIENCY CURVE $\eta = f(\nu')$ OF A REACTION STAGE

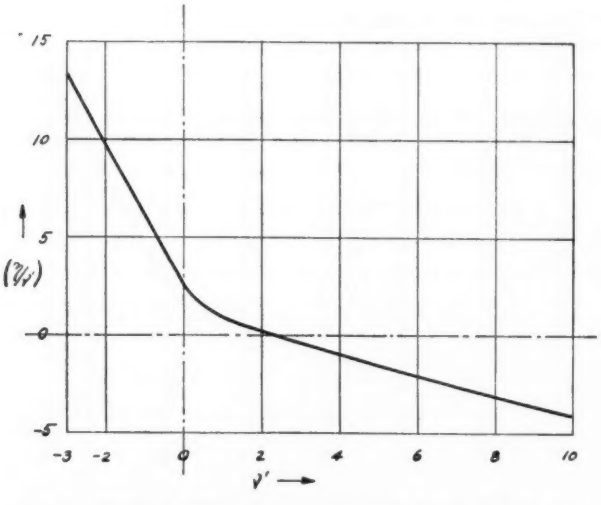


FIG. 6 TYPICAL TORQUE CHARACTERISTIC $(\eta/\nu') = f(\nu')$ OF A REACTION STAGE

is sometimes used as index of the importance of the compressibility of the fluid. It is easily found that M_a is related with Δ through the equation $M_a^2 \cong \Delta/k$. Also Δ is therefore an index of the effect of the compressibility of the fluid.

Summing up the results of our analysis, the flow performance of the reaction stage is described by a series of nondimensional functions; written symbolically

Work done	$\Omega = f(\Delta;\ \sigma)$	
Mass flow	$\Gamma = f(\Delta;\ \sigma)$	.....(4)
Temperature drop	$\tau = f(\Delta;\ \sigma)$	.....(6)
Leaving flow angle	$\alpha = f(\Delta;\ \sigma)$	.....(9)
Approach flow angle	$\beta = f(\Delta;\ \sigma)$	.....(11)
Velocity ratio	$\nu = f(\Delta;\ \sigma)$	.....(10)
Isentropic velocity ratio	$\nu' = f(\Delta;\ \sigma)$	.....(14)
Blading efficiency	$\eta = f(\Delta;\ \sigma)$	.....(13)

It sometimes proves convenient to combine two or more of

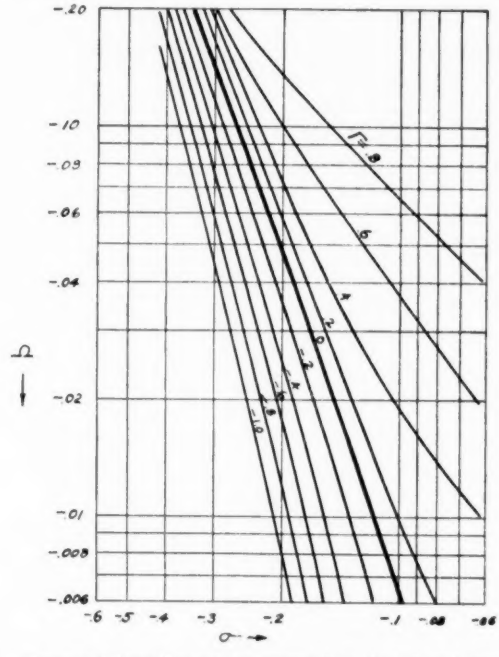


FIG. 7 TYPICAL WORK CHARACTERISTICS $\Omega = f(\sigma;\ \Gamma)$ OF A REACTION STAGE OPERATING IN ASTERN ROTATION

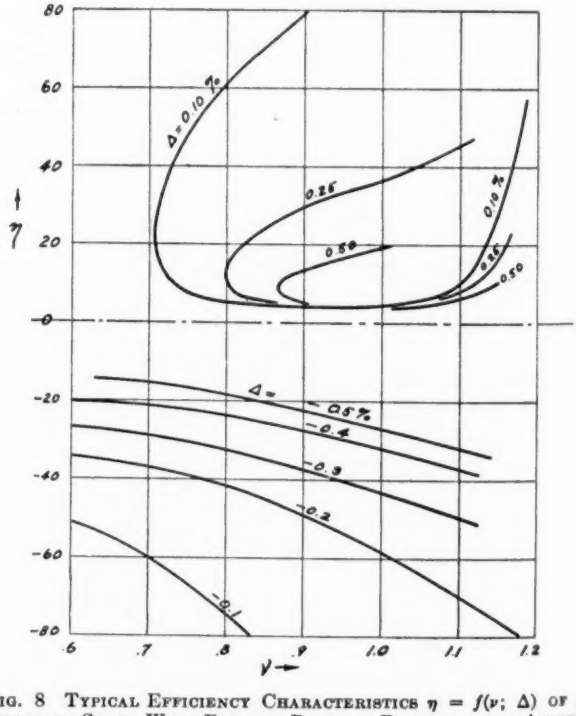


FIG. 8 TYPICAL EFFICIENCY CHARACTERISTICS $\eta = f(\nu;\ \Delta)$ OF A REACTION STAGE WITH FLOW IN REVERSE DIRECTION AND ASTERN ROTATION

these functions and thereby rearrange the variables. Thus, by eliminating σ between Equations [13] and [14] we obtain

$$\eta = f(\nu';\ \Delta) \quad [15]$$

In the normal operating range of reaction blading the influence of Δ on the efficiency has been found negligible. Therefore, $\eta = f(\nu')$ is the type of function usually applied in efficiency calculations, and for which Fig. 5, obtained from tests, is an example. In Fig. 6 the ratio (η/ν') is plotted against ν'; the former, as is easily verified, represents the torque transmitted to the turbine shaft.

Another example of rearranged variables is shown in Fig. 7 showing the power which is necessary to drive a reaction stage in reversed rotation as found from tests.

The test results represented in Fig. 8 show that in the efficiency function $\eta = f(\nu;\ \Delta)$, the influence of Δ is not always negligible.

The Multistage Problem

It is usually not possible to measure the output N of a single stage which is part of a group of stages. The characteristics of the single stage must be calculated by mathematical analysis from the measurements taken in a multistage test.

The single-stage characteristics thus obtained are then the basis for predicting the performance of other blade groups which may be portions of particular turbines.

Both these problems are fundamentally solved if the relationship between the characteristics of a single stage and a group of stages is known. This relationship is obtained in a straightforward manner if we start out with the single-stage characteristics. The details of this analysis are not within the scope of this paper.

Acknowledgment

The author wishes to acknowledge the conscientious work of C. A. Meyer of the Westinghouse Electric and Manufacturing Company in South Philadelphia, who analyzed the turbine tests. The author is grateful to the Westinghouse Electric and Manufacturing Company for permission to publish the results of the tests carried out in their South Philadelphia Works.

Design Data

It is important that the data contained in technical papers be made readily available to designing engineers. In order to satisfy these needs of industry, this section of the JOURNAL will include a concise presentation of data and information drawn chiefly from papers previously published by the Applied Mechanics Division of The American Society of Mechanical Engineers.

The first series dealt with the subject "Strength of Materials" and the present series will cover the subject "Vibration." The data have been prepared by a subcommittee of the Applied Mechanics Division (J. Ormondroyd, chairman), under the general guidance of S. Timoshenko.

The third article, which follows, is by J. Ormondroyd. This considers systems with one degree of freedom.

Vibration Problems

PART III—SYSTEMS WITH ONE DEGREE OF FREEDOM

BY J. ORMONDROYD,[1] ANN ARBOR, MICH.

THE lowest natural frequency of machines and structures is of particular interest to engineers for several reasons. In free vibrations, which occur after structures are distorted from their equilibrium positions by transient forces, it is the lowest-frequency natural vibration which usually has the largest amplitude and which persists in motion the longest. Therefore it is usually the motion at this natural frequency which is the easiest to detect. When periodic forces act on the structure the frequencies of these forces are more likely to coincide with the lowest natural frequency than with the higher modes, since the disturbing frequencies are more often of low frequency. Again a periodic force can act almost anywhere on the structure and excite the lowest mode of vibratory motion. For the higher frequency modes of natural vibration there are locations in the structure (nodes) at which the forces can act without exciting any response. The amplitude of motion at resonance with the lowest mode of motion is usually much larger than the amplitudes of the higher modes of motion, since both velocity and stress damping are generally lower in this mode than in the higher modes. All these possibilities make the lowest natural frequency of great practical importance in many engineering structures. The things of particular interest are the natural frequency itself and the possible amplitude at resonance if resonance occurs. Knowledge of the natural frequency coupled with information on the possible disturbing-force frequencies indicates if resonance is likely to occur while the structure is in use. The resonance amplitudes of the various parts of the structure indicate if the resonance condition may lead to trouble.

The practical problem presented in calculating the motion of any system in its lowest frequency form is to replace the actual system by an equivalent simple system of one degree of freedom. For linear vibrations the equivalent system has the form shown in Fig. 1. From the actual details of the system the equivalent values of k, m, and damping constants must be calculated. The equivalent spring constant k and the equivalent mass m are usually easy to calculate. The damping properties are more difficult to evaluate and frequently they are obtainable only by free-vibration experiments after the structure has been built. The lowest-frequency vibration of a rotor simply supported on two bearings is a problem which frequently confronts designers. This simple case is represented in Fig. 2. For many practical cases the weight of the shaft can be ignored, in which case the value of m to be used in the equivalent system, shown in Fig. 1, is W/g lb sec²/in. The value of the equivalent k is found by calculating the deflection under the load P (any convenient load for calculation purposes) on the beam, abstracted from Fig. 2 and shown in Fig. 3, assuming that the rotor is rigid. The deflection Δ, under the test load P, can be calculated by the energy method or graphical method which were referred to previously.[2] Having found Δ in inches under the load P

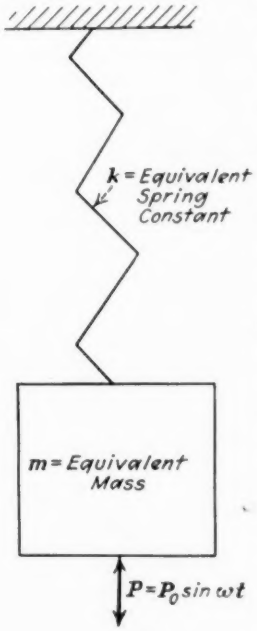


FIG. 1

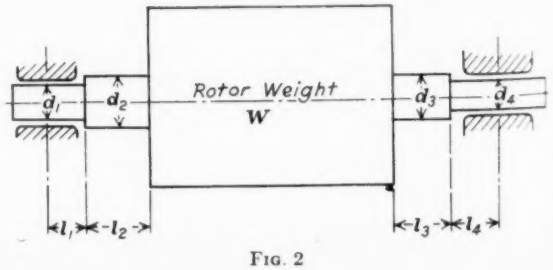


FIG. 2

[1] Professor of Engineering Mechanics, University of Michigan. Mem. A.S.M.E.

NOTE: In part II of these design data on vibration problems, JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 61, September, 1939, the formula for the torsional-spring constant of a truncated cone, given at the top of the right-hand column on p. A-129, should read

$$\frac{3G}{32l}\,\frac{\pi(d_2 - d_1)\,d_1^3 d_2^3}{d_2^3 - d_1^3}$$

[2] "Vibration Problems—Part II," by J. Ormondroyd, JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 61, September, 1939, p. A-127.

given in pounds, the equivalent spring constant $k = P/\Delta$ lb per in.

For torsional vibration the simple equivalent system is shown in Fig. 4 in which the equivalent torsional-spring constant k and the equivalent moment of inertia I are calculated from the actual details. As an example of a system which looks nothing like the one shown in Fig. 4, but which can be reduced to one similar to it for calculation purposes, we might take the suspended sign shown in Fig. 5. The lowest-frequency oscillation of this sign in the vertical plane can occur only by rotations about the pivoted suspension at A. The equivalent moment of inertia I about the axis perpendicular to the plane of the figure through A, including the sign and the horizontal bar AB, is

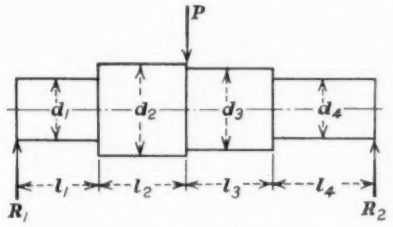


Fig. 3

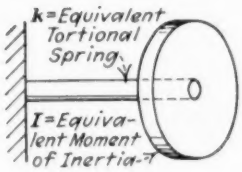


Fig. 4

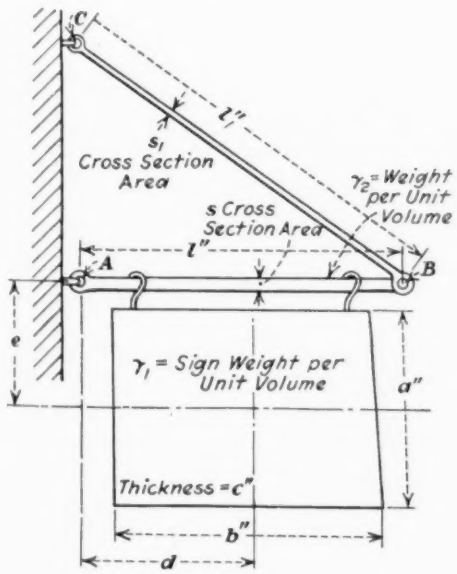


Fig. 5

$$I = \frac{abc\gamma_1}{g}\left(\frac{a^2 + b^2}{12} + d^2 + e^2\right) + \frac{sl\gamma_2}{g}\frac{l^2}{3} \quad \ldots\ldots [1]$$

The equivalent torsional-spring constant about the same center A (including only the elastic effect of the diagonal suspension rod BC) is

$$k = \frac{Es_1l^2}{l_1}\sin^2\theta \quad \ldots\ldots [2]$$

where θ is the angle between the suspension rod and the horizontal bar. In this case again the damping effects would be hard to predict by calculations, but easy to measure by free-vibration tests on the completed construction.

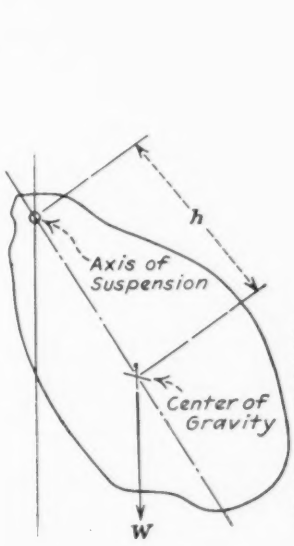


Fig. 6

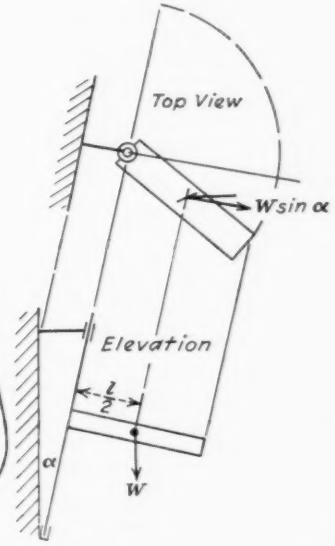


Fig. 7

Fig. 6 shows the simple system to which pendulous structures can be reduced. Here the details of an actual system, such as shown in Fig. 7, provide the data from which the weight, moment of inertia about the suspension point, distance between the center of gravity and the suspension point, and damping characteristics are calculated. In Fig. 7

$$I = \frac{W}{g}\frac{l^2}{3} \quad \ldots\ldots [3]$$

The moment restoring the bar AB to its lowest position is

$$M = Wl\frac{\sin\alpha}{2}\phi \quad \ldots\ldots [4]$$

if the amplitude of oscillation is small.

In the vibration of the system shown in Fig. 1 under the action of P, a simple harmonic periodic force in the direction of its possible motion, the forces acting on the mass m can be recorded in proper relationship by using Newton's second law

$$m\frac{d^2x}{dt^2} = -c\frac{dx}{dt} - kx + P_0\sin\omega t \quad \ldots\ldots [5]$$

x is measured from the position of static equilibrium and the damping from every source is lumped into one equivalent viscous damping. The complete solution of this differential equation is

$$x = e^{-qt}\left[A\cos\sqrt{(p^2 - q^2)}t + B\sin\sqrt{(p^2 - q^2)}t\right] + \frac{P_0}{k}\frac{\sin\left[\omega t - \tan^{-1}\frac{2q}{p}\frac{\omega/p}{1-(\omega^2/p^2)}\right]}{\sqrt{\left[\frac{4q^2}{p^2}\frac{\omega^2}{p^2} + \left(1 - \frac{\omega^2}{p^2}\right)^2\right]}} \quad \ldots\ldots [6]$$

If at $t = 0$, $x = x_0$ and $\dot{x} = \dot{x}_0$, the corresponding values of A and B (the constants of integration) give

$$x = e^{-qt}\left\{\left[x_0 + \frac{P_0}{k}\frac{(2q/p)\,\omega/p}{\frac{4q^2}{p^2}\frac{\omega^2}{p^2} + \left(1-\frac{\omega^2}{p^2}\right)^2}\right]\cos\sqrt{(p^2-q^2)}t + \frac{1}{\sqrt{(p^2-q^2)}}\left[\dot{x}_0 + qx_0 + \left(1-2\frac{1-(\omega^2/p^2)}{4q^2/p^2}\right)q\frac{P_0}{k} \times \frac{(2q/p)\,\omega/p}{\frac{4q^2}{p^2}\frac{\omega^2}{p^2} + \left(1-\frac{\omega^2}{p^2}\right)^2}\right]\sin\sqrt{(p^2-q^2)}t\right\} + \frac{P_0}{k}\frac{1}{\sqrt{\left[\frac{4q^2}{p^2}\frac{\omega^2}{p^2} + \left(1-\frac{\omega^2}{p^2}\right)^2\right]}}\sin\left[\omega t - \tan^{-1}\frac{2q}{p}\frac{\omega/p}{1-(\omega^2/p^2)}\right] \quad \ldots [7]$$

The first term is known as the transient-motion term and dies away in time. Motion represented by this term is brought into existence at any time if the conditions and circumstances of the system change. The second term is the steady-state solution and the motion it represents exists as long as the disturbing force acts.

In Equation [7] $p^2 = k/m$, $q^2 = c^2/4m^2$, and P_0/k is known as the static deflection of the system under a steady load of P_0. Equation [7] contains the solution of motion for free vibrations if P_0 is given the value zero. It then reduces to

$$x = x_0 e^{-qt}\left[\cos\sqrt{(p^2-q^2)}t + \frac{(\dot{x}_0/x_0)+q}{\sqrt{p^2-q^2}}\sin\sqrt{(p^2-q^2)}t\right] \quad \ldots [8]$$

If two phases of the free vibration at time t and time $t+T$ [where $T = 2\pi/\sqrt{(p^2-q^2)}$] are compared, the ratios of the instantaneous excursions of the body from the zero position at these two times will be

$$\frac{x_t}{x_{t+T}} = e^{2\pi(q/\sqrt{(p^2-q^2)}} \quad \ldots [9]$$

The logarithmic decrement of the system is

$$\log_e\frac{x_t}{x_{t+T}} = 2\pi\frac{q}{\sqrt{(p^2-q^2)}} \quad \ldots [10]$$

The natural frequency of the system is $(1/2\pi)\sqrt{(p^2-q^2)}$. When $p = q$, the system is said to have critical damping. Under critical-damping conditions the natural frequency is zero, that is, the disturbed system returns to its position of static equilibrium by a motion which never reverses direction.

Now, $1/q$ is called the "time constant" of the system—it represents the time which is taken for the amplitude of motion of the disturbed system to die down to $1/e$ times the first maximum value which occurs.

In the steady-state motion when $(\omega^2/p^2) = 1-(2q^2/p^2)$, the amplitude of motion has its maximum value $\frac{P_0}{k}\frac{p}{2q}\frac{1}{\sqrt{[1-(q^2/p^2)]}}$. For small damping the maximum amplitude occurs very near to $\omega^2/p^2 = 1$ and has the magnitude $\frac{P_0}{k}\frac{p}{2q}$.

Calculation of the Lowest Natural Frequency

For small values of damping the natural frequency is close to $p/2\pi = (1/2\pi)\sqrt{(k/m)}$. Since most actual structures have very small damping, this last expression is very useful for approximate calculations of natural frequencies.

For the system shown in Fig. 1

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} \quad \ldots [11]$$

where k = equivalent linear spring constant, m = equivalent mass, and Δ = static deflection of system under load mg.

In Fig. 4

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{I}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta'}} \quad \ldots [12]$$

where k = equivalent torsional-spring constant, I = equivalent moment of inertia, and Δ' = static deflection at end of a horizontal radius of gyration considering the whole weight concentrated there.

In Fig. 6

$$f = \frac{1}{2\pi}\sqrt{\left[\frac{g}{h+(k^2/h)}\right]} = \frac{1}{2\pi}\sqrt{\frac{g}{l'}} \quad \ldots [13]$$

where h = distance between axis of suspension and center of gravity of pendulous mass; k = radius of gyration for moment of inertia about axis parallel to suspension axis and passing through center of gravity; and l' is the length of the equivalent mathematical pendulum.

Rayleigh's Approximate Method of Calculating the Lowest Natural Frequencies of Mechanical Systems

In any freely vibrating system of bodies the sum of the various types of mechanical energies is always constant. Neglecting friction losses

$$U + T = \text{const} \quad \ldots [14]$$

where U = potential energy, and T = kinetic energy. When the vibrating system has reached its maximum excursion from the position of static equilibrium, the vibratory velocity is zero and the entire local energy of the system is all potential energy

$$U_{max} = \text{const} \quad \ldots [15]$$

When the system is moving through the position of static equilibrium the entire local energy is all kinetic. Here

$$T_{max} = \text{const} \quad \ldots [16]$$

This indicates that $U_{max} = T_{max}$ $\ldots [17]$

The quantitative values of U_{max} and T_{max} can be calculated for any vibrating system if the relative amplitudes of motion of the various masses in the system are known. Equation [17] gives a means of calculating p^2 for the system.

In elastic systems this principle can be used in two separate, but closely related, ways if the "normal elastic" curve of the system is known. The "normal elastic" curve of any system in the lowest frequency mode of vibratory motion is a space diagram of the system which shows the relative amplitudes of the various masses of the system and the relative distortion of all the elastic elements which connect the masses. It might be considered as a picture of the system just as it reaches its maximum displacement from the position of static equilibrium during the vibratory motion. All the masses in the system are vibrating with the same frequency in the same simple harmonic manner. The normal elastic curve gives a means of calculating the maximum kinetic energy in terms of the unknown frequency and the relative amplitudes of motion. The same picture indicates the data necessary to calculate the potential energy stored in the elastic elements at the point of maximum distortion.

The importance and usefulness of Rayleigh's method for calculating fundamental natural frequencies lies in the fact that the normal elastic curve need not be known accurately. It can

be drawn approximately so as to satisfy the necessary end conditions in the real system. From this approximate normal elastic curve an approximate value of the natural frequency can be calculated. The calculated value will always be too high, but if the arbitrarily chosen normal elastic curve satisfies the real end conditions and is not too badly distorted, the calculated frequency will be close to the actual value.

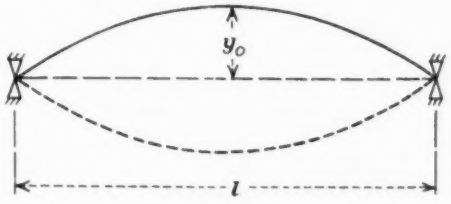


FIG. 8

An elastic beam of constant cross section simply supported on two bearings at its ends can execute an infinite number of modes of free vibration about its position of static equilibrium. The lowest frequency mode of motion will have a normal elastic curve such as shown in Fig. 8. This curve satisfies the conditions that the amplitude of motion at the supports must be zero and it has the general shape of the beam at its maximum excursion from the line of static equilibrium, which is represented here by the straight line between the two supports. Analytically this normal elastic curve could be described quantitatively approximately by several functions. Among them, the following would be possible

$$y' = y_0 \sin \frac{\pi x}{l} \quad [18]$$

The vibratory motion at any point x along the beam then could be given as

$$y = y_0 \sin \frac{\pi x}{l} \sin pt \quad [19]$$

The total maximum kinetic energy of all the particles in the beam will be

$$T_{max} = \frac{1}{2} \frac{\gamma A}{g} \int_0 \left(\frac{\partial y}{\partial t}\right)^2_{max} dx \quad [20]$$

The total maximum potential energy of strain in the entire beam is

$$U_{max} = \frac{1}{2} EI \int_0^l \left(\frac{\partial^2 y}{\partial x^2}\right)^2_{max} dx \quad [21]$$

T_{max} will contain p^2 and U_{max} will not; therefore, the equation $T_{max} = U_{max}$ can be solved for p^2.

For $y = y_0 \sin (\pi x/l) \sin pt$

$$T_{max} = \frac{1}{2} \frac{\gamma A}{g} y_0^2 p^2 \int_0^l \sin^2 \frac{\pi x}{l} dx \quad [22]$$

$$U_{max} = \frac{1}{2} EI y_0^2 \frac{\pi^4}{l^4} \int_0 \sin^2 \frac{\pi x}{l} dx \quad [23]$$

and

$$p^2 = \frac{\pi^4 EIg}{\gamma A l^4} \frac{\int_0^l \sin^2 \frac{\pi x}{l} dx}{\int_0^l \sin^2 \frac{\pi x}{l} dx} = \frac{\pi^4 EIg}{\gamma A l^4} \quad [24]$$

The frequency is seen to be independent of the actual value of y_0.

Static-Deflection Method for Calculating Natural Frequencies

Rayleigh's method can be applied with the static deflection of the system used in the place of the normal elastic curve. Fig. 9

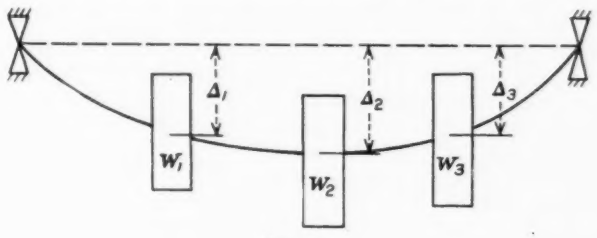


FIG. 9

shows a system in which this procedure can be used. The displacement of each mass is assumed to be

$$y_n = \Delta_n \sin pt \quad [25]$$

$$U_{max} = \Sigma \frac{1}{2} W \Delta_n \quad [26]$$

$$T_{max} = \Sigma \frac{1}{2} \frac{W}{g} \Delta_n^2 p^2 \quad [27]$$

and

$$p^2 = g \frac{\Sigma W_n \Delta_n}{\Sigma W_n \Delta_n^2} \quad [28]$$

In this method the actual values of the static deflection of each body are used for the Δ_n. These are best found by means of graphical integration. For the case of a single body this method gives the same result as Equation [11].

Natural Frequencies of Pendulous Bodies

Rayleigh's method is useful for pendulous bodies also. In these cases the kinetic energy consists of two parts—the linear kinetic energy of the oscillating mass associated with the linear velocity of the center of gravity and the kinetic energy of the body rotating about an axis perpendicular to the plane of motion and passing through the center of gravity. The potential energy depends on the weight of the body and the height of the center of gravity above the lowest possible position it can have.

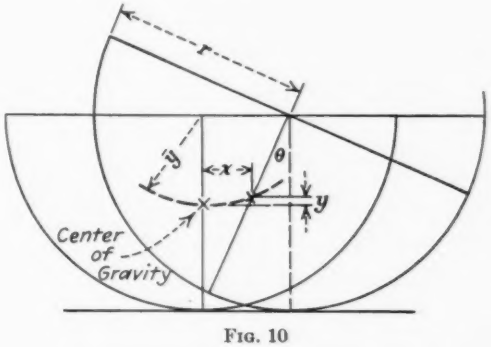


FIG. 10

Fig. 10 shows half of a circular cylinder rolling on its cylindrical surface. The natural frequency of its oscillation can readily be found by the energy method.

The mass of the semicylinder is

$$m = \frac{\pi\gamma l r^2}{2g}; \text{ and } i_0 = \frac{\pi\gamma l r^4}{4g}\left(1 - \frac{32}{9\pi^2}\right)$$

The coordinates of the center of gravity using the position of static equilibrium as the origin are

$$x = r\theta - \bar{y}\sin\theta \quad [29]$$

$$y = \bar{y} - \bar{y}\cos\theta \quad [30]$$

in which $\bar{y}$ is the radial distance between the center of the cylinder and the center of gravity and θ is the angle through which the semicylinder has rolled.

The oscillation of the cylinder is

$$\theta = \theta_0 \sin pt \quad [31]$$

$$\dot{\theta} = \theta_0 p \cos pt \quad [32]$$

$$\dot{x} = r\dot{\theta} - \bar{y}\cos\theta\dot{\theta} \quad [33]$$

$$\dot{y} = \bar{y}\sin\theta\dot{\theta} \quad [34]$$

The linear velocity of the center of gravity is v and

$$v^2 = [(r - \bar{y}\cos\theta)^2 + \bar{y}^2\sin^2\theta]\,\dot{\theta}^2 \quad [35]$$

and

$$v^2_{max} = (r - \bar{y})^2\theta_0^2 p^2 \quad [36]$$

$$T_{max} = \frac{1}{2}m(r - \bar{y})^2\theta_0^2 p^2 + \frac{1}{2}i_0\theta_0^2 p^2 \quad [37]$$

$$U_{max} = mg\bar{y}(1 - \cos\theta_0) = \frac{1}{2}mg\bar{y}\theta_0^2 \quad [38]$$

From this

$$p^2 = \frac{mg\bar{y}}{m(r - \bar{y})^2 + i_0} \quad [39]$$

Amplitude at Resonance—Energy Method

Wherever the energy lost per cycle can be calculated or measured and expressed as a function of the amplitude of motion, it is possible to estimate resonance amplitudes of motion. Whenever a force $P = P_0 \sin \omega t$ acts on a body of one degree of freedom, it causes a motion $x = x_0 \sin(\omega t - \phi)$. The energy put into the moving body during each cycle is $W = \pi P_0 x_0 \sin\phi$. At resonance, if the damping forces are not large, $\sin\phi = 1$. Therefore, the maximum value of energy input per cycle occurs at resonance and is

$$W = \pi P_0 x_0 \quad [40]$$

The energy lost per cycle, in any system that can be disturbed in its lowest-frequency vibration, can be measured from the free vibration record if the original store of potential or kinetic energy can be estimated. If this energy loss per cycle is a function of x_0 such as $W = mx_0^n$, the amplitude at resonance will be

$$x_0 = \left(\frac{\pi P_0}{m}\right)^{\frac{1}{n-1}} \quad [41]$$

This method leads to an indeterminate result when $n = 1$; n has this value in the case of pure Coulomb damping where the output per cycle is $W = 4Fx_0$, in which F is the friction force. If a system could have pure Coulomb friction with no other type of damping present, its amplitude at resonance would approach infinity (mathematically speaking).

For some types of damping, such as that caused by viscous friction forces or internal stress damping, it is possible to calculate the over-all loss of energy per cycle as a function of x_0. In such cases estimates can be made of amplitudes at resonance before the structure leaves the design stage.

Discussion

The Forces Required for Rolling Steel Strip Under Tension[1]

W. O. Clinedinst.[2] The author's solution of the differential equation of rolling by assuming the surface friction to be proportional to the slip velocity presents excellent possibilities. As well as giving distributions of contact pressures which closely approach those obtained experimentally by E. Siebel and W. Lueg,[3] it results in an expression which has the distinct advantage of being a continuous algebraic function from which expressions for the separating force and rolling moment can be obtained by direct integration. Such expressions coordinated to existing rolling-mill test data would certainly be of great value to rolling-mill designers.

Dr. Nádai is to be highly commended for his thorough analysis of the problem of rolling and has contributed much toward its further development.

Creep, Elastic Hysteresis, and Damping in Bakelite Under Torsion*

A. L. Kimball.† It seems to the writer that the agreement between the specific damping capacity in the dynamic and static tests is very good indeed. This brings out more clearly than ever that the damping effect in the case of a vibration is of the same character as the hysteresis phenomena observed by the authors in their static tests. Furthermore, it shows that the speed with which the strain cycles are executed has practically no effect on the frictional loss per cycle.

TABLE 1

Material	Log dec. δ	Specific damping capacity, ψ
Rubber, 90 per cent pure[a]	0.26	0.13
Celluloid	0.045	0.0225
Tin, swaged	0.129	0.0645
Maple wood	0.022	0.011
Zinc, swaged	0.020	0.010
Glass	0.0064	0.0032
Aluminum, cold-rolled	0.0034	0.0017
Brass, cold-rolled	0.0048	0.0024
Copper, cold-rolled	0.0050	0.0025
Tungsten, swaged	0.0165	0.00825
Swedish iron, annealed	0.0079	0.00395
Phosphor bronze, annealed	0.0032	0.0016
Mild steel, cold-rolled	0.0049	0.00245
Molybdenum, swaged	0.0069	0.00345
Nickel, cold-rolled	0.0032	0.0016
Nickel steel, 3½ per cent swaged	0.0023	0.00115
Monel, cold-rolled	0.00143	0.000715
Phosphor bronze, cold-rolled to maximum hardness	0.00037	0.000185

[a] Vibration test.

In Table 1 is given a series of values of the specific damping capacity of a number of materials, as obtained by a special method described in reference (5).

Recent papers bearing on this subject which have interested the writer considerably are referred to at the close of this discussion. The papers by Zener represent the viewpoint of a mathematical physicist and are decidedly analytical in character. Nevertheless, it seems that they deserve careful study by the serious worker, in this field, who is mathematically inclined.

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(1) "The Measurement of Internal Friction in Some Solid Dielectric Materials," by Andrew Gemant, *Philosophical Magazine*, May, 1937.

(2) "The Role of Solid Friction in Synthetic Dielectrics," by Andrew Gemant, *Journal of Applied Physics*, November, 1938.

(3) "Internal Friction in Solids" (5 papers), by Clarence Zener, *Physical Review*, August, 1937; January, 1938; by C. Zener, W. Otis, and R. Nuckolls, January, 1938; April, 1938; June, 1938.

(4) "Damping Capacity, a Factor in Fatigue," by G. R. Brophy, paper presented at 17th annual convention of the A.S.M., Sept.-Oct., 1935.

(5) "Vibration Prevention in Engineering," by A. L. Kimball, John Wiley & Sons, Inc., New York, N. Y., 1932, p. 133.

Numerical and Graphical Method of Solving Two-Dimensional Stress Problems[1]

L. F. Richardson.[2] Besides the work of Thom, to which the authors refer, there is a much earlier paper[3] by the present writer where the same differential equation is treated, though without the conformal mapping used in this paper. Again the problem of the tension member with semicircular notches was treated long ago by a graphical method for drawing the contours of the stress function.[4] The former of these two old papers has been consistently ignored by later writers on similar subjects, who include Thom, as aforesaid, Southwell,[5] and Temple.[6]

Authors' Closure

The authors wish to thank Dr. Richardson for calling their attention to his papers on the use of finite differences in problems involving partial differential equations. The paper by Messrs. R. Weller and G. H. Shortley which appears in the same issue of the Journal of Applied Mechanics (page A-71) refers to

[1] By A. Nádai, published in the June, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-54.

[2] Consulting Mechanical Engineer, Pittsburgh, Pa. Jun. A.S.M.E.

[3] "Untersuchungen über die Spannungsverteilung im Walzpalt," by E. Siebel and W. Lueg, *Mitteilungen aus dem Kaiser-Wilhelm-Institut für Eisenforschung zu Düsseldorf*, vol. 15, 1933, Report No. 218, p. 1.

[1] By H. Poritsky, H. D. Snively, and C. R. Wylie, Jr., published in the June, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-63.

[2] Principal, Paisley Technical College, Paisley, Scotland.

[3] "The Approximate Arithmetical Solution of Finite Differences of Physical Problems Involving Differential Equations, With an Application to the Stresses in a Masonry Dam," by L. F. Richardson, Philosophical Transactions of the Royal Society of London, 1910, vol. 210, pp. 307–357.

[4] "The Approximate Solution of Various Boundary Problems by Surface Integration," by L. F. Richardson, Proceedings of the Physical Society of London, 1911, vol. 23, pp. 75–85.

[5] "Relaxation Methods Applied to Engineering Problems. III. Problems Involving Two Independent Variables," by D. G. Christopherson and R. V. Southwell, Proceedings of the Royal Society of London, series A, vol. 168, p. 317.

[6] "The General Theory of Relaxation Methods Applied to Linear Systems," by G. Temple, Proceedings of the Royal Society of London, series A, vol. 169, pp. 476–500, 1939.

[1] By Herbert Leaderman, published in the June, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-79.

† Engineering General Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

the method as due to H. Liebmann.[7] Dr. Richardson's papers, antedating Liebmann's treatment by some nine years, clearly establish his priority.

A Theory of Flexure for Beams With Nonparallel Extreme Fibers[1]

J. N. Goodier.[2] The stress distributions discussed by the author are of the plane-stress type. They are based on the supposition that the three components of stress (σ_z, τ_{rz}, $\tau_{\theta z}$), which might be found on the plane faces of the uniform plate are zero throughout. This will, as a rule, be reasonably close to the truth only for thin plates. It has been widely supposed that Filon's "generalized plane stress" offers some escape from this restriction, but it can be shown that generalized plane stress has actually no wider range than simple plane stress.[3] This question of thickness arises in the paper in the application to the bend of a rigid frame, with a curved inner flange. The flanged part cannot be regarded as thin. Nevertheless we should not expect the three stresses which are neglected to have appreciable magnitude when the loads merely cause bending, and the agreement of the author's results with test results appears to confirm this point of view. His success with this problem seems to dispose of a structural element which has hitherto been difficult to deal with in a convincing manner.

F. C. Roop.[4] In connection with the author's example of the application of his theory to a plate girder with nonparallel flanges, it is of interest to examine the values of the principal stresses and of the maximum shearing stresses. Since a force applied in any direction at the edge of the wedge produces no shearing stresses on radial and circumferential elements, it is only in the case of loading by a couple that the radial stresses given by the author are not themselves principal stresses.

For couple loading, expressions for the algebraically greatest principal stress and the maximum shearing stress are found by substituting from the author's Equations [20] and [21] (together with $\sigma_{\theta z} = 0$) in the well-known formulas for these quantities. By differentiating these expressions with respect to θ, equating the derivatives to zero, solving the resulting equations, and discarding those solutions which either correspond to minima or give values of θ greater than α, expressions are arrived at which give the value of the coordinate θ at which the greatest values of the quantities in question occur. The algebraically least principal stress for any θ is the negative of the greatest principal stress for $-\theta$, because of the symmetrical nature of the stress distribution about the polar axis.

The analysis, according to the foregoing procedure, of the expression for the greatest principal stress σ_{p3} shows that no solution of $\partial\sigma_{p3}/\partial\theta = 0$ corresponding to a maximum of σ_{p3} can be found which gives θ less than α unless $\alpha > \alpha_c$, where

$$\alpha_c = \frac{1}{4}\tan^{-1}\left(-\frac{2A_f}{tr}\right);\quad 22^1/_2 \text{ deg} \leq \alpha_c \leq 45 \text{ deg} \ldots [1]$$

Consequently, when $\alpha \leq \alpha_c$, the greatest value on any one circular-cylindrical section occurs at $\theta = \alpha$. When $\alpha \geq \alpha_c$, the analysis shows that the value ψ of the coordinate θ at which the maximum of σ_{p3} occurs is given by

$$\psi = \frac{1}{2}\cos^{-1}\frac{\sqrt{1+8s^2}-1}{4s};\quad 22^1/_2 \text{ deg} \leq \psi \leq 45 \text{ deg} \ldots [2]$$

where $s = (2A_f/tr)\sin 2\alpha - \cos 2\alpha$.

Substituting in the expression for σ_{p3} the value of θ at which the maximum of σ_{p3} occurs gives the maximum principal stress $\sigma_{p3\,\max}$. The resulting expressions for $\sigma_{p3\,\max}/\sigma_{r3\,\max} = B$ (see following the author's Equation [20]) are

For $\alpha \leq \alpha_c$

$$B = \frac{1}{2}\left[1 + \sqrt{\left\{1 + \left(\frac{2A_f}{tr}\right)^2\right\}}\right] \ldots [3a]$$

For $\alpha_c \leq \alpha \leq 45$ deg

$$B = \frac{1}{2}\frac{\sin 2\psi}{\sin 2\alpha}(1 - \sec 4\psi) \ldots [3b]$$

For 45 deg $\leq \alpha \leq$ 90 deg

$$B = \frac{1}{2}\sin 2\psi(1 - \sec 4\psi) \ldots [3c]$$

These results are shown in Fig. 1 of this discussion, in which the coordinates α (half wedge angle) and $\frac{A_f}{A_f + tr\alpha}$ (relative flange area) should be regarded as two independent variables. Each point in the figure represents a plate girder having values of α and

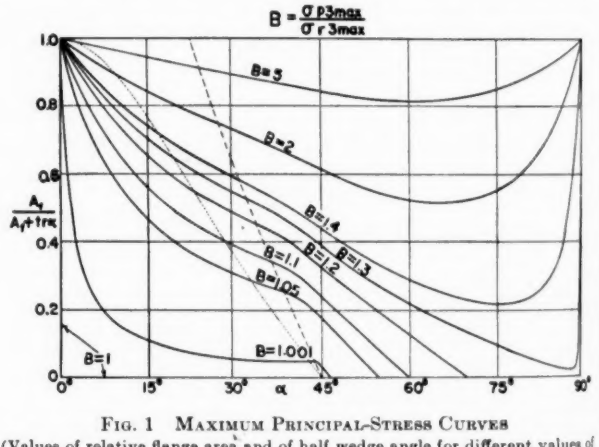


Fig. 1 Maximum Principal-Stress Curves
(Values of relative flange area and of half wedge angle for different values of the ratio of maximum principal stress to maximum radial stress for a wedge-shaped plate girder loaded by a couple.)

$A_f/(A_f + tr\alpha)$ given by the coordinates of the point, and lies on some curve B = const, which gives the value of B for this plate girder. The dashed line represents plate girders for which $\alpha = \alpha_c$ (see Equation [1] of this discussion).

As an example, a plate girder of half wedge angle 22.92 deg ($\alpha = 0.4$), with 55 per cent of its area in the flanges $\left(\frac{A_f}{A_f + tr\alpha} = 0.55\right)$ is seen to have a maximum principal stress 20 per cent in excess of the maximum radial stress ($B = 1.2$); furthermore, this maximum principal stress occurs at $\theta = \alpha$, just inside the junction between the (concentrated) flange and the web, since the representative point for this girder lies to the left of the dashed line.

[7] "Die Angenaherte Ermittelung Harmonischer Funktionen und Konformer Abbildungen," by H. Liebmann, Sitzungberichte der Bayerischen Akademie der Wissenschaften zu München, Mathematische Physikalische Klasse, 1918, p. 385.

[1] By William R. Osgood. Published in the September, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-122.

[2] Professor of Mechanics, Cornell University, Ithaca, N. Y.

[3] "On the Problems of the Beam and the Plate in the Theory of Elasticity," by J. N. Goodier, Transactions of the Royal Society of Canada, vol. 32, 1938, pp. 65–88.

[4] Assistant Physicist, National Bureau of Standards, Washington, D. C.

For $\alpha \geq 45$ deg, the curves B = const in Fig. 1 also represent ψ = const, the correspondence being as follows:

B	ψ, deg	B	ψ, deg
1.001	43.78	1.3	29.98
1.050	36.82	1.4	28.93
1.100	34.30	2.0	26.05
1.200	31.57	5.0	23.63

Dividing the values of $f_3/\sigma_{r3\,max}$ (see third equation following the author's Equation [20]) by B gives values of $f_3/\sigma_{p3\,max}$.

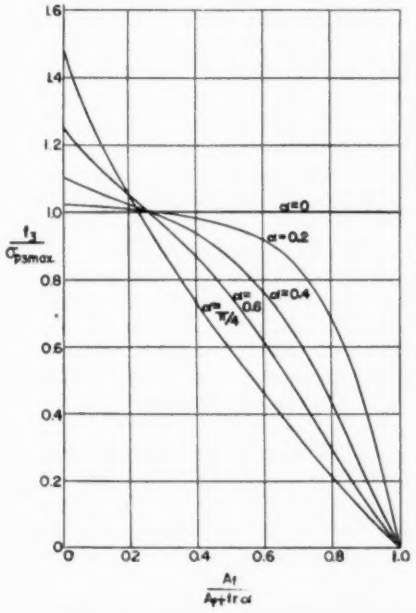


FIG. 2 COUPLE-LOADING CURVES

(Ratio of maximum stress by the ordinary beam theory to maximum principal stress by the wedge theory for all values of relative flange area and for different values of the half wedge angle of a wedge-shaped plate girder loaded by a couple.)

These are plotted in Fig. 2 of this discussion. By taking the same example previously given in this discussion, it is seen that while the maximum fiber stress by the ordinary theory is only 2.2 per cent (see Fig. 4 of the paper) less than the maximum radial stress by the present theory, it is 18.5 per cent less than the maximum principal stress by this theory, shown in Fig. 2 of this discussion. The concurrence of the curves in Fig. 2 at the point (1, 0) is due to the fact that, as the relative flange area approaches 100 per cent, the web becomes vanishingly thin, and the shearing stresses increase without limit.

In the case of the maximum shearing stress τ_{p3}, an analysis similar to the foregoing one in this discussion shows that the greatest value ($\tau_{p3\,max}$) of τ_{p3} on any one circular-cylindrical section occurs at the polar axis ($\theta = 0$) in case $\alpha \geq \alpha_s$, where, for B = const (see Fig. 1 of this discussion), α_s is related to α_c (Equation [1] of this discussion), by the expression

$$\alpha_s = 2\alpha_c - 45 \text{ deg} \quad [4]$$

In this case, the value of $\tau_{p3\,max}$ is equal to $\tau_{r\theta 3\,max}$ (see following the author's Equation [21]), since no normal stresses act at points on the polar axis (Equation [20]). When $\alpha \leq \alpha_s$ the value of $\tau_{p3\,max}$ is given by

$$S = \frac{\tau_{p3\,max}}{\sigma_{p3\,max}} = 1 - \frac{1}{2B} \quad [5]$$

and occurs at $\theta = \alpha$; see Equation [3a] of this discussion.

The dotted line in Fig. 1 of this discussion represents plate girders for which $\alpha = \alpha_s$, and in the region to the left of this line, the curves B = const are also curves for S = const, the correspondence being as follows:

B	S	B	S
1.000	0.500	1.3	0.615
1.001	0.501	1.4	0.643
1.050	0.524	2.0	0.750
1.100	0.545	5.0	0.900
1.200	0.583		

It is interesting to note that when $\alpha = \alpha_s$, τ_{p3} is independent of θ.

It must be remarked that in the case of a plate girder, the load on which is equivalent to forces at the edge of the wedge in addition to a couple, the foregoing results cannot be directly applied. This is true because the principal stresses will not in general act in the same directions in the combined case as in pure couple loading, and consequently they cannot be computed for the combined case by simple addition of their values for the cases which are superposed to produce the combined case.

AUTHOR'S CLOSURE

The author is indebted to Mr. Goodier for pointing out explicitly some of the limitations of the theory of plane stress when applied to cases in which the components of stress in a direction normal to the "plane of stress" are not all zero. These components were considered implicitly, but it would have been well to mention them.

Mr. Roop has gone as far as it is practicable to go in considering maximum normal and shearing stresses in the literal solution. His discussion adds welcomed completeness to the paper.

Clamped Rectangular Plates With a Central Concentrated Load[1]

EVERETT O. WATERS.[2] The author has made a very interesting use of successive-approximation methods in solving a set of linear simultaneous equations. During the last year, the writer was confronted with the task of solving sets of such equations in connection with the general problem of pressure distribution in a journal-bearing oil film, and found that when the set contained more than four equations he could get better accuracy and speed by successive approximation than by the "precise" methods usually taught in algebra textbooks.

TABLE 1 SUCCESSIVE-APPROXIMATION EQUATIONS

FIRST APPROX.	r	+ 12s	+ 13t	+ 14u	+ 15v	+ ..... = 01
SECOND APPROX.	21r	+ s	+ 23t	+ 24u	+ 25v	+ ..... = 02
THIRD APPROX.	31r	+ 32s	+ t	+ 34u	+ 35v	+ ..... = 03
FOURTH APPROX.	41r	+ 42s	+ 43t	+ u	+ 45v	+ ..... = 04
FIFTH APPROX.	51r	+ 52s	+ 53t	+ 54u	+ v	+ ..... = 05
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.

Briefly, the equations had the form given in Table 1 of this discussion. They are characterized by unity coefficients on a major

[1] By Dana Young. Published in the September, 1939, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 61, 1939, p. A-114.

[2] Associate Professor of Mechanical Engineering, Yale University, New Haven, Conn. Mem. A.S.M.E.

diagonal; all other coefficients are less than unity and rapidly approaching zero in directions downward and to the right from the major diagonal.

In these equations, the digits are not numbers in the conventional sense, but are position symbols for numbers that have given values less than unity. For the first approximation, one upper-left term only is used, and $r = 01$; for the second approximation, four upper-left terms are used; for the third, nine upper-left terms, etc. To obtain any approximation after the first, the values of the unknowns from the last preceding approximation are substituted in the lowest equation considered, and this equation is solved for the last unknown occurring in it; then the next higher equation is solved for the next last unknown, and so on until the highest equation and the first unknown are reached. For example, if the second approximation gives $r = r_2$, $s = s_2$, then the third approximations are

$$t_3 = (03) - (31)(r_2) - (32)(s_2)$$
$$s_3 = (02) - (21)(r_2) - (23)(t_3)$$
$$r_3 = (01) - (12)(s_2) - (13)(t_3)$$

After as many approximations have been made as there are unknown quantities, the method may be repeated successively for the full set of equations until the desired refinement of accuracy has been attained.

One great advantage of this method lies in the fact that all division is eliminated; consequently, there is complete avoidance of the accumulation of errors caused by dropping significant figures of doubtful importance, as happens when "precise" methods of solution (other than the unwieldy determinantal method) are used. The rapidity with which the approximations converge—if indeed convergence exists—must of course depend upon the relative values of the coefficients. This involves a mathematical investigation which the writer has not yet had time to make in general terms, although it appears that the only absolute requirement in the special case of two unknowns is

$$1 > (21)(12) > -1$$

It is always possible, and easy, to check any computation by substitution in the left side of each equation and comparison of the resulting sum with the corresponding right side. In every case where the writer has used the successive-approximation method this check has been made, and the comparison has been surprisingly good.

R. L. MOORE.[3] Because of the author's reference to an experimental investigation[4] paralleling certain phases of his analytical study, it would have been of interest if comparisons had been made where possible between the results of the tests and his theory. Such comparisons would have revealed two interesting differences:

1 That the maximum stresses observed in the tests were found directly under the concentrated loads applied and not at the edges of the plates as the author[1] has indicated.

2 That the stresses under the loads were very sensitive to the degree of load concentration, a factor which the author has not considered.

Except in cases where a plate is continuous over a number of panels, similarly loaded, a condition of 100 per cent fixity at the edges is usually difficult to obtain. In the tests referred to,[4] the edges of the aluminum-alloy plate specimens were clamped between pairs of $7 \times 3^1/_2 \times 1$-in. steel angles, thereby providing apparent degrees of edge fixity for the $^1/_2$-in. and 1-in-thick plates of 90 and 65 per cent, respectively. Having measured stresses and deflections under the load for both simply supported and clamped edges, however, it should be possible to estimate by proportion the corresponding values for completely fixed edges.

Table 2 of this discussion gives a summary of maximum measured stresses and deflections for four different sizes of aluminum-alloy plates with central loads distributed over a 2×2-in. area, taken from the experimental investigation previously referred to.[4] Included with these data are estimated values for completely fixed edges and the corresponding computed maximum stresses and deflections obtained from the data given in the author's paper.[1] As will be noted, the computed stresses at the edges of the plates ranged from only 37 to 53 per cent of the values estimated at the center, directly under the load. Obviously the author's solution does not give the maximum bending moments, as he so states. The computed stresses given in Table 2 of this discussion are in fair agreement, however, with those measured near the edges of the plates tested, where the effect of the degree of load concentration is not significant. The center deflections are likewise not sensitive to load concentration and consequently the computed values given in Table 2 are in reasonably close agreement with the measured values.

A. NÁDAI.[5] The solution of the problem of the clamped rectangular plate carrying a single concentrated load in its center was expressed by a sum of three elastic surfaces

$$w = w_1 + w_2 + w_3$$

in which the first term w_1 was identified with the elastic surface of a rectangular plate carrying a concentrated load P in the center and satisfying Navier's boundary conditions $w_1 = 0$ and

$$\Delta w_1 = \frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} = 0$$

along the edges. This first portion of the solution contains a singularity at the center of the plate, where the bending moments become logarithmically infinite, while the terms w_2 and w_3 have no singularities in the interior or along the edges of the plate. It may be of interest to note that w_1 is an important solution of the

[3] Research structural engineer, Aluminum Research Laboratories, Aluminum Company of America, New Kensington, Pa.

[4] "The Behavior of Rectangular Plates Under Concentrated Loads," by R. G. Sturm and R. L. Moore, JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 59, 1937, p. A-75.

[5] Consulting Engineer, Westinghouse Research Laboratories, East Pittsburgh, Pa. Mem. A.S.M.E.

TABLE 2 STRESSES AND DEFLECTIONS IN RECTANGULAR PLATES UNDER CENTER CONCENTRATED LOAD

Plate size, in.	Load,[a] lb	All sides simply supported: Measured[b] stress, lb per sq in.	All sides simply supported: Measured[b] deflection, in.	All sides clamped: Measured[b] stress, lb per sq in.	All sides clamped: Measured[b] deflection, in.	All sides clamped: Measured[c] edge fixity, %	All sides clamped, 100% edge fixity: Estimated[d] stress, lb per sq in.	All sides clamped, 100% edge fixity: Estimated[d] deflection, in.	All sides clamped, 100% edge fixity: Computed[e] stress at edge, lb per sq in.	All sides clamped, 100% edge fixity: Computed[e] deflection under load, in.
1.062 48 × 96	10000	19800	0.308	17800	0.225	65	16700	0.180	8900	0.151
1.062 48 × 72	10000	21500	0.296	19200	0.213	65	18000	0.168	8600	0.147
1.062 × 48 × 48	10000	19600	0.214	18100	0.157	65	17300	0.126	6700	0.117
0.500 × 8 × 48	2000	18900	0.426	16400	0.273	90	16100	0.256	6000	0.225

[a] Load concentrated on 2×2-in. area.
[b] Measured under load.
[c] Determined from slopes measured at edge for simply supported and clamped edges. Zero slope corresponds to 100 per cent fixity.
[d] Estimated by proportion from measured values (under load) for simply supported and clamped edges.
[e] Computed from Equations [13] and [14], using values of α and β given in Table 4 of Young's paper (see footnote No. 1).

theory of flat plates, deserving of being utilized. Bergsträsser[6] has developed exact means for computing not only the values of $w_1(x, y)$ but also of the bending moments corresponding to this elastic surface for any position of the concentrated load. Infinite series representing the moments in the case of a concentrated load are known not to converge well. In the paper quoted,[6] use of a remarkable property of a special case of $w_1(x, y)$ was made. This latter occurs when in the rectangular plate one pair of the sides becomes infinite ($b = \infty$). For such a rectangular plate, that is for an infinitely long strip of the width a which is loaded by a concentrated force P in the interior and which is satisfying the boundary conditions $w_1 = 0$ and $\Delta w_1 = 0$, it is possible to compute the bending moments by finite expressions, because they depend on the surface Δw_1 and on the derivatives of Δw_1. This surface Δw_1 being a harmonic function (Green's function for the infinitely long strip) can be constructed by conformal mapping. Several cases were computed by Bergsträsser.[6]

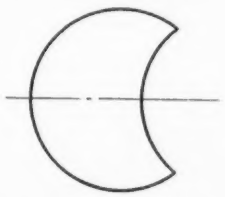


FIG. 1 CLAMPED PLATE HAVING TWO RECTANGULAR CORNERS

The problem of the clamped rectangular plate has been the subject of a series of important investigations during the last decades, both by mathematicians and engineers, and further attempts to express the results of these investigations in a convenient manner for the direct use in engineering applications, as in the paper under discussion, must be particularly commended.

There is another interesting side of the problem of the rectangular plate with clamped edges which, it seems to the writer, would deserve some attention and which possibly is ready for an analytical attack. The mathematical nature of the solution of $\Delta\Delta w = f(x, y)$ around a plate corner having two built-in edges has to the knowledge of the writer never been clarified sufficiently or more fully discussed. I believe that W. Ritz[7] in one of his early studies on clamped plates made the remark that it may not be possible around a corner with clamped edges to develop a solution of $\Delta\Delta w = 0$ or $= f(x, y)$ into an infinite series of terms not unsimilar to those used in the case when along the two edges the conditions $w = 0$ and $\Delta w = 0$ have to be satisfied. Ritz did not use infinite series, but a finite number of terms. In other words, it would be of considerable theoretical interest to know more definitely whether Ritz is right or whether on the contrary a particular solution of $\Delta\Delta w = 0$ or of $\Delta\Delta w = f(x, y)$ can be constructed or found, which would satisfy the conditions $w = 0$ and $\partial w/\partial n = 0$ along perpendicular curves. One suggestion for mathematicians who might possibly become again interested in some of these questions will be mentioned. By searching for an example among the known particular solutions of $\Delta w = 0$ and of $\Delta\Delta w = 0$ when expressing them by means of the curvilinear bicentric coordinates derived from certain transformations used in the theory of conformal mapping, it may perhaps be possible to satisfy the previously mentioned boundary conditions along two circles which are perpendicular to each other. If the elastic surface of a uniformly loaded plate which has two circles for the edges and only two rectangular corners, as shown in Fig. 1 of this discussion, in which the conditions $w = 0$, $\partial w/\partial n = 0$ would have to be satisfied, could be established, the question would be answered. Means which were recently developed by Jeffery,[8] Mindlin,[9] and others could be utilized and seem to be available for this purpose.

[6] "Versuche mit freiaufliegenden rechteckigen Platten unter Einzelkraftbelastung," by M. Bergsträsser, *Forschungs auf dem Gebiete des Ingenieurwesens*, V.D.I., Heft, 302, 1928.

[7] "Gesammelte Werke," by W. Ritz, published for The Société Suisse de Physique, by Gauthier-Villars, Paris, 1911.

AUTHOR'S CLOSURE

In his discussion, Professor Waters has given an interesting illustration of the use of a successive-approximation procedure for solving simultaneous equations. The author agrees that, in cases where the approximations converge rapidly, this method is superior to the so-called "precise" method. Dr. Nádai has made some valuable suggestions regarding further mathematical investigations for clamped plates. It is to be hoped that these may be followed up.

It would appear that Mr. Moore has misunderstood the results given in the paper. The moments given by Equation [14] are maximum *edge* moments and it is *not* stated nor implied that these are greater than the moment at the center under the concentrated load. The stresses in a plate at a concentrated load depend upon the area over which the load is distributed and upon the thickness of the plate. Since this problem has been treated at length by numerous investigators, as for example, Nádai,[10] Timoshenko,[11] Westergaard,[12] Holl,[13] and Woinowsky-Krieger,[14] it was not believed necessary to discuss the problem in this paper. In these references will be found values for the moments or stresses at a concentrated load for a simply supported plate which corresponds to the elastic surface w_1 as defined by Equation [3] in the paper. Then the results for a clamped plate are obtained by adding to the foregoing values the moments at the center corresponding to the functions w_2 and w_3 given by Equations [4] and [5]. Thus, from w_2 we find

$$(M_y)_{\substack{x=0\\y=0}} = -D\left(\frac{\partial^2 w_2}{\partial y^2} + \nu\frac{\partial^2 w_2}{\partial x^2}\right)\Bigg|_{\substack{x=0\\y=0}} = \sum_{m=1,3,5\ldots} A_m(-1)^{\frac{m-1}{2}}\frac{1-\frac{1}{2}(1-\nu)\alpha_m\tanh\alpha_m}{\cosh\alpha_m}$$

$$(M_x)_{\substack{x=0\\y=0}} = -D\left(\frac{\partial^2 w_2}{\partial x^2} + \nu\frac{\partial^2 w_2}{\partial y^2}\right)\Bigg|_{\substack{x=0\\y=0}} = \sum_{m=1,3,5\ldots} A_m(-1)^{\frac{m-1}{2}}\frac{\frac{1}{2}(1-\nu)\alpha_m\tanh\alpha_m-\nu}{\cosh\alpha_m}$$

The corresponding expressions for w_3 are similar. From these expressions can be found the desired center moments. For example, for a square plate, we find that the moment at the center due to the applied edge moments is $-0.0536P$. This is to be

[8] "Plane Stress and Plane Strain in Bipolar Coordinates," by G. B. Jeffery, Philosophical Transactions of the Royal Society of London, series A, vol. 221, November, 1920, pp. 265–293.

[9] "Gravitational Stresses in Bipolar Coordinates," by R. D. Mindlin, Proceedings of the Fifth International Congress for Applied Mechanics, Cambridge, Mass., 1938, pp. 112–116.

[10] "Die elastischen Platten," by A. Nádai, J. Springer, Berlin, 1925, pp. 62 and 308.

[11] "Über die Biegung der allseitig unterstützten rechteckigen Platte unter Wirkung einer Einzellast," by S. P. Timoshenko, *Der Bauingenieur*, vol. 3, 1922, p. 51.

[12] "Computation of Stresses in Bridge Slabs Due to Wheel Loads," by H. M. Westergaard, *Public Roads*, vol. 11, March, 1930, p. 8.

[13] "Analysis of Thin Rectangular Plates Supported on Opposite Edges," by D. L. Holl, Bulletin 129, Iowa Engineering Experiment Station, December, 1936.

[14] "Der Spannungszustand in dicken elastischen Platten," by S. Woinowsky-Krieger, *Ingenieur-Archiv*, vol. 4, 1933, p. 305.

added algebraically to the moment due to the load on a simply supported square plate to get the total moment at the center of a clamped square plate.

It may be of interest to compute the center moment for one of the experimental plates cited in Mr. Moore's discussion. Take for example the $1.062 \times 48 \times 48$-in. plate with a central concentrated load of 10,000 lb. For a simply supported square plate with a central concentrated load distributed over a circle of radius c, Timoshenko,[11] assuming that Poisson's ratio is 0.3, gives the moment at the center as

$$M = P\left(0.1034 \log \frac{a}{2c} + 0.0597\right)$$

in which expression a indicates the length of the side of the plate.

In the experimental plate the load was distributed over an area 2×2 in. square. It seems reasonable to assume that c may be taken as the radius of circle whose area is 4 sq in. Then we find the center moment equals $0.371P$ and the bending stress $\sigma = 6M/h^2 = 19{,}700$ lb per sq in., which compares with the experimental value of 19,600 lb per sq in. Then for the same plate with clamped edges we have the center moment $M = 0.371P - 0.0536P$ and the bending stress $\sigma = 16{,}900$ lb per sq in., which compares with the experimental value of 17,300 lb per sq in.

Book Reviews

A History of Science

A HISTORY OF SCIENCE, TECHNOLOGY AND PHILOSOPHY IN THE EIGHTEENTH CENTURY. By A. Wolf. The Macmillan Company, New York, 1939. Cloth, $6 \times 9^3/_4$ in., 815 pp., 345 illustrations, $8.

REVIEWED BY J. ORMONDROYD[1]

IN 1935 The Macmillan Company published Professor Wolf's "History of Science, Technology and Philosophy in the sixteenth and seventeenth centuries." Professor Wolf stated in the preface to that volume that it was only the first of four similar works to appear. The other three were to cover the same ground for the eighteenth century, the nineteenth century, and the ancient and medieval periods. The subject of this review is the second volume of the promised series. In external and internal appearance it is the exact twin to the first volume; but it contains 122 more pages of printed matter in order to cover the greater wealth of material offered by the eighteenth century. The numerous illustrations, depicting the prominent actors in the scientific drama and their work, along with the enormous number of citations from the original-source papers and publications, will make this completed work a "gold mine" for future historians of science.

The subject matter is arranged in five groups. The first includes mathematics, mechanics, astronomy, and closes with a discussion of astronomical and marine instruments developed during the eighteenth century. Next follow four chapters on the physical sciences of light, sound, heat, electricity, and magnetism. The section on the sciences is concluded by nine chapters which cover meteorology, meteorological instruments, chemistry, geology, geography, botany, zoology, and medicine.

The next seven chapters are of exceptional interest to an engineer, since they deal with the technological advances made during the eighteenth century. The chapters in this group discuss agriculture, textiles, building, transport, power plant, and machinery, the steam engine, mining and metallurgy, industrial chemistry, the making of lenses and specula, mechanical calculators, telegraphy, and other important technical activities which are not named in the chapter headings.

The last five chapters of the book review the eighteenth-century writings on psychology, social sciences, and philosophy.

From the purely scientific point of view, the eighteenth century was an "in-between" century. It was kept busy extending and completing the work which the masters of the previous century had started. It also was preparing the ground for the magnificent generalizations in electricity, thermodynamics, and chemistry which were to appear in the nineteenth century. Men like Copernicus, Kepler, Galileo, and Newton stood out like giants in the sixteenth and seventeenth centuries, their achievements were such that whole chapters had to be devoted to their work in the earlier volume. No single individual in the eighteenth century was important enough by himself to be the subject of a separate chapter in the present book.

If the century was lacking in "stars," it more than made up for that in its awakening "humanism." The eighteenth century saw the beginning of a conscious effort to enlighten everyone as to the results of the new thought. The urge which ultimately led to almost universal education started in this "age of enlightenment and reason." Science and technology began a very conscious cooperation in that century. In that century technology took such a vigorous new start in utilizing natural energy to replace the physical work of man and beast, and in devising mechanisms and processes for manufacturing things useful to man, that we designate it the birth century of the "industrial revolution." Professor Wolf's introductory chapter draws in broad outline the character of that century so full of glorious dreams for a happy future. These dreams were never fully realized and in our own century some of them are farther from realization than they were then.

Perhaps the most interesting section of the book for our profession deals with the emergence of the modern engineer. Most of our early predecessors were mechanics untutored in the schools that existed. From their own native intelligence and experience they built roads, bridges, lighthouses, machinery, and devised processes. They often provided the stones with which nineteenth-century science was to build. Two such men were typical and deserve mention for their importance in introducing the scientific method into practical affairs.

John Smeaton (1724–1792) is mentioned in eleven different places in this volume. "He was at once a craftsman, consulting engineer, and man of science—all of high order." He built devices for measuring the expansion of solids, improved the string hygrometer, anticipated Franklin in constructing the lead-coated glass plate condenser, consulted on bridge construction, and built lighthouses. But he is interesting mainly for his careful experimental work on the efficiency of water wheels, windmills, and pre-Watt steam engines and for his studies of energy storage in flywheels. In his studies of windmills, water wheels, and flywheels

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he made use of the exceedingly modern device of models and anticipated Langley's rotating-arm "wind tunnel." Smeaton's studies permitted him to raise the efficiency of the Newcomen engine from around 0.4 to 1.1 per cent, which was about the best that the pre-Watt engines ever did. He improved the efficiency of both water wheels and windmills also.

The careless modern reader might ask why Smeaton did not deduce his results (at least on water wheels) from the existing Newtonian mechanics. The answer to that is that the Newtonian mechanics of his day were not prepared to give him the information he wanted. It was almost a hundred years after Smeaton's heyday that both mechanic and scientist meant the same thing when they said "work" and "power." The word "efficiency" in its present meaning could not even exist until thermodynamics became a full-fledged science. Even the word "force," so clearly defined by Newton, retained a confused meaning many years after Smeaton's death. As late as 1900 the word "force" can be found in printed literature having the meaning of "energy." Far from having missed an opportunity to apply deductively a well-established science, Smeaton was in reality helping to lay the foundation for the modern range and connotation which the science of mechanics has only recently attained.

Smeaton's younger French contemporary, Charles Augustin Coulomb (1736–1806) was a military engineer whose scientific work all grew out of practical projects. The attempt to improve the magnetic compass led to his quantitative laws of attraction and repulsion for electrostatic charges and for magnetic poles. The measurement of these in turn led to the invention of the torsion balance and the introduction of our modern theory of twisting in beams and rods. Coulomb put the beam-bending theory into its modern form, he founded our accepted theories of elastic arches, correctly worked out a theory of earthwork pressures, and introduced clearly the ideas concerning failure due to shear in materials—all in one study on the use of maximum and minimum ideas in "Statical Problems Applied to Architecture." Strangely enough, Coulomb's work on friction is passed over in silence in this volume.

Smeaton and Coulomb published in the proceedings of the leading scientific societies of their respective countries. They might well be considered as the spiritual ancestors of the Applied Mechanics Division. Many other names and accomplishments could be cited from the wealth of material in the volume.

No engineer who wishes to get an integrated picture of the scientific and technical history of his profession can afford to neglect Wolf's monumental history.

Thermodynamics

INCHLEY'S THEORY OF HEAT ENGINES. Edited and revised by H. Wright Baker, D.Sc. Longmans, Green and Company, New York, 1939. Cloth, 5¼ × 8½ in., xvi and 445 pp., 170 figs., $3.60.

REVIEWED BY JOSEPH H. KEENAN[2]

INCHLEY'S "Theory of Heat Engines" was first published in 1913. It was revised in 1920 by Dr. A. Morley "as a tribute to the memory of a former colleague," the original author having been killed in action in December, 1915. The present edition was prepared by Dr. H. Wright Baker who has made additions at the expense of the omission of the chapters dealing with kinematics and dynamics of engines.

Besides a foundation of thermodynamics the book includes discussions of hot-air engines, air compressors and motors, steam-engine cycles, reciprocating steam engines, nozzles and orifices, steam turbines, internal-combustion engines, refrigeration machinery, heat transfer, combustion, and testing of engines. The treatment of each of these subjects includes a generous number of numerical examples and a collection of problems for the student.

The chapters on reciprocating steam engines and internal-combustion engines are conscientiously detailed, though not exhaustive. The mathematics of heat transfer through solids is presented with unusual punctiliousness for a book of this nature. It would have been better, perhaps, to sacrifice some of it to a treatment of the mechanism of convection.

In the preface to the present edition it is stated that "No rigorous statement of thermodynamics has been attempted . . ." The implication that the author of a text on heat engines might well be excused if he omits a rigorous statement of thermodynamics is entirely defensible; but there is no defense for an author who does not build his work on a rigorous foundation of thermodynamics. In the list of symbols H is "a quantity of heat," while on page 69, H is a coordinate of a diagram of properties. This is the beginning of confusion that makes subsequent explanations long and involved. There are two pages devoted to the calculation of the dryness of steam after isentropic or adiabatic expansion and four and a half pages are devoted to the calculation of the end state after throttling.

The book is based largely on British source material. It includes many references to publications with which American engineers are not familiar and calls attention to test codes and reports by research committees of the British engineering societies.

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Background to Modern Science

BACKGROUND TO MODERN SCIENCE. Edited by Joseph Needham and Walter Pagel. The Macmillan Company, Inc., New York, N. Y., and University Press, Cambridge, England, 1938. Cloth, 5¼ × 8¼ in., xii and 243 pp., $2.

REVIEWED BY J. ORMONDROYD[3]

BACKGROUND to Modern Science" goes over a familiar road, recording part of the history of scientific advances in the forty years between 1895 and 1935. The subject matter is handled in chapters each separately written by different authors. Each chapter is in itself a record of a lecture on the same subject delivered at Cambridge University in the year 1936. These lectures inaugurated an attempt to emphasize the importance of the history of science to the student of scientific subjects at Cambridge. The chief point of interest in the mode of presentation used is that the lecturers themselves were selected from the very group which made a large share of the advances discussed.

The advances in atomic physics are described by Lord Rutherford in two separate lectures which are reproduced in a single chapter. Incidentally, this address was the last delivered publically by Lord Rutherford before his death. F. W. Aston also discusses certain phases of atomic knowledge. W. L. Bragg outlines crystal physics and Sir Arthur Eddington gives the history of the rise of relativity. In these four chapters no philosophy of science is attempted. The contributions merely list the various experiments which turned out to be important historically.

If the chapters listed here were the sole contents of the book, it would not be a book worth reading—important as the contributors are. After all, we have read about these discoveries in a hundred other places, and have seen them more extensively treated. But the last four chapters of the book *are* worthy of attention. They discuss the advances made in the biological sciences—material which has not found its way into the deluge of popular scientific treatises which has poured over

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us in the last few years. For this reason only one of the four lecturers on the biological sciences bears a familiar name. "Physiology and Pathology," by J. A. Ryle, "Parasitology and Tropical Medicine," by G. H. F. Nuttall, "Evolution Theory," by R. C. Punnett, and "Genetics," by J. B. S. Haldane are the subjects and writers of the last four chapters in the book.

The eight chapters which deal with specific subjects in modern science are preceded by two general and introductory chapters. The second entitled "From Aristotle to Galileo" by Sir William C. Dampier outlines briefly the background of modern science, covering material that is treated in many other places. But the first chapter, entitled "Greek Natural Philosophy and Modern Science," written by F. M. Cornford, is very enlightening in spite of the apparent staleness of the subject. Professor Cornford's comparative analysis of the motives, objects, procedures, and methods of the Greek and modern sciences is one of the clearest and most concise to be found in the literature of scientific history.

Buckling of Bars

Die Knickfestigkeit Eines Zentrisch Gedrückten Geraden Stabes im Elastischen und Unelastischen Bereich. By Arvo Ylinen, Technical Director of the State Aircraft Factory, Tampere, Finland. Druckerei-A.G. der Finnischen Literaturgesellschaft, Helsinki, 1938. Paper, 6¼ × 9¾ in., 131 pp., 53 figs., 70 Finnish marks.

Reviewed by S. Timoshenko[4]

IN DISCUSSING lateral buckling of compressed prismatical bars two cases are usually considered (1) buckling within the elastic region, and (2) buckling within the inelastic region. In the first case, the known Euler's formula gives the value of the critical compressive stress; in the second case, the critical value of the stress is usually determined by applying one of the existing empirical formulas. Engesser was the first who indicated that the problem of buckling in both regions can be treated by using the same formula if only the notion of the reduced modulus is introduced. The critical stress then is

$$\sigma_{cr} = \frac{\pi^2 E'}{\lambda^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where $\lambda = l/k$ is the slenderness ratio and $E' = d\sigma/d\epsilon$ is the reduced modulus derived from the tension compression test diagram of the material of the column. Mr. Ylinen shows that for such materials as steel and wood it can be assumed that

$$E' = E\left[1 - \left(\frac{\sigma}{\sigma_{yp}}\right)^n\right] \quad \ldots\ldots\ldots\ldots\ldots [2]$$

in which the constant n must be properly chosen for each kind of material. A table of values of n for several important materials is given in the book. It is shown there also that the effect of strain-hardening and the influence of the shape of the cross section can be taken into consideration by a proper change of the magnitude of the constant n. Substituting the value of E' in formula [1], an expression for critical stress is obtained which is applicable for any value of slenderness ratio. The application of this expression in design of columns is shown and there finally is given a comparison of Equation [1] with various existing formulas. It is shown, for example, that by taking $n = 1$ the known Rankine's formula is obtained. In the last portion of the book the comparison of the formula with test data is given. The known experiments of Kármán, Roš, and Rein with steel columns can be brought in very good agreement with formula [1]. A satisfactory result is also obtained in applying formula [1] to the tests of wooden columns made in Finland.

The book contains several numerical tables simplifying the application of formula [1] in various cases and also diagrams illustrating the difference in the values of λ_{cr} as obtained by the empirical formulas and by the proposed formula [1]. The book will be of interest to engineers dealing with column design.

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Steam Engine

A Short History of the Steam Engine. By H. W. Dickinson. University Press, Cambridge, England; The Macmillan Company, New York, N. Y., 1939. Cloth, 6 × 9¼ in., xii and 255 pp., 78 figs., $3.50.

Reviewed by C. Richard Soderberg[5]

LITERATURE on the history of science and technology has accumulated rapidly in recent years. Nostalgia for past ages, when the belief in the blessings of technological progress was still unshaken, is a natural consequence of the general attitude of hopelessness and despair at the inability of modern man to master his own destiny. It is pleasant to roam in the dusty archives from what now appears the golden age of our era.

Regardless of the impelling motive we are gradually beginning to appreciate the importance of the historical perspective of our technical advance. Much has been said about the failure of the scientist and the engineer to take an active part in the solution of our social and economic problems, and even the engineers themselves have come to regard this state of affairs with misgivings. It is not at all certain that the methods of the exact sciences have direct application to social problems, but if the engineer is to exert an effective influence on the solution of these problems, some of his urge to make history must be converted into intelligent contemplation of past history. This can only be accomplished by an educational process in which the engineer is gradually trained to give attention to the history of his specialty. On this basis the subject of the history of science and technology should be given a prominent position in the curricula of our universities. The lack of suitable literature for such courses has long been an obstacle to this, but is gradually being eliminated by the work of such men as Dickinson, Wolf, and others.

The author of the present treatise has already made a name for himself through his masterly productions on other phases of the history of steam power. The recent books on James Watt, Matthew Boulton, and Richard Trevithick represent a very high standard of performance in this class of literature.

The book under review is divided into two parts—part 1 covering the history of the reciprocating steam engine, and part 2 covering the history of the steam turbine. In the preface the author cites a few simple data which convincingly illustrate the fact that the importance of steam power was never greater than today. This is significant in view of the curious attitude held by many persons that the age of steam is past. The total annual consumption of electric power in the world is of the order of 1700.10^9 kwhr; of this nearly 95 per cent is generated by the combustion of fuel. The purpose of the volume is to trace the development which has taken place since Savery and Newcomen in the latter part of the seventeenth century first produced engines which supplanted the toil of men and animals.

The introductory chapter is a concise sketch of the social and economic conditions of England, out of which the development was precipitated. Here as in the latter chapters Mr. Dickinson has succeeded well in his endeavor to show that the invention of steam power did not arise spontaneously in any inventor's mind;

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it was a logical outgrowth of the needs of the time. For completeness one might wish for some references to the devices that actually were in use earlier, such as the devices in use in the mines of Sweden, but this perhaps belongs more properly to an earlier phase of man's mastery over nature.

In the succeeding three chapters the author deals with the developments that took place in connection with the work of Savery and Newcomen and particularly with the consolidation of position of the atmospheric engine in the period between Newcomen and Watt. It is probably a surprise to many engineers that Savery and Newcomen, although contemporaries, were probably not acquainted with one another, and certainly did not cooperate in their work. Another fact, not usually appreciated, is that the Newcomen engine had existed for about half a century as an important industrial device before Watt arrived on the scene.

The following chapter is devoted to Watt's contribution to the development. This was of such fundamental character that the invention of the steam engine has often been erroneously credited to him. This error is effectively dispelled by the author without any loss to Watt's prestige. A new era was inaugurated through the work of Watt and Boulton, an era which in some respects may be regarded as continuing into the latter part of the nineteenth century, when the steam turbine arrived on the scene.

The remaining chapters of part 1 cover the important developments of the reciprocating steam engine during this period as well as the parallel evolution of land boilers. The concluding chapter is devoted to the "philosophy of the steam engine" an admirable discussion of the seesaw workings of empirical mechanical developments and theoretical advances. The author emphasizes the important fact that the inception of a development like that of the steam engine is necessarily empirical; the resulting advances, although often stumbling and imperfect, give rise to a search for a more rational outlook which gives incentive to the search for more exact knowledge.

The first chapter on the history of the steam turbine gives a historical perspective of the early consideration of this device up to and including the achievement of Parsons. Watt's discussion of the turbine invention by Baron von Kempelen is interesting, not only because of his clear analysis of the turbine problems, but also in illustrating the fact that Watt was led by selfish interest to hope that "without God makes it possible for things to move at 1000 feet p^{r} it cannot do much harm."

The remaining chapters cover the work of other turbine and boiler pioneers down to the present time. His choice of leading pioneers is undoubtedly correct in that the basic turbine inventions were made by Parsons, De Laval, Curtis, Rateau, and Ljungström. It seems somewhat unfair, however, to leave out of this group the name of George Westinghouse who certainly pioneered in the introduction of the steam turbine in the U. S. A. It is true that Westinghouse did not produce any fundamentally new invention on the steam turbine, but his role in the beginning of the present century was, nevertheless, of outstanding significance. Nor does it seem entirely fair to leave out of the discourse such names as Emmett, Junggren, and Hodgkinson, although here he may be justified by the difficulty of encompassing the turbine history in a reasonable space. The excellent papers which were presented in 1936 before the A.S.M.E.[6] on turbine history in the U. S. A. could well have served as a reference for this important phase of turbine history.

In the conclusions the author presents some speculations on the probable future of power generation by entirely new methods. To these speculations he adds the following significant remarks: "The community would be well employed in making better use than it has done hitherto of the gifts that the scientist and the engineer have showered upon it; till then, further gifts would be in the nature of pearls before swine."

It will be evident that the reviewer considers this contribution to the history of steam power of outstanding value and interest.

[6] "The Steam Turbine in the United States:"
"I—Developments by the Westinghouse Machine Company," by Emil E. Keller and Francis Hodgkinson, *Mechanical Engineering*, November, 1936, pp. 683–696.
"II—Early Allis-Chalmers Steam Turbines," by A. G. Christie, *Mechanical Engineering*, February, 1937, pp. 71–82.
"III—Developments by the General Electric Company," by Ernest L. Robinson, *Mechanical Engineering*, April, 1937, pp. 239–256.

Problems in Mechanics

PROBLEMS IN MECHANICS. Based on the original collection of L. V. Mestchersky. By G. B. Karelitz, J. Ormondroyd, and J. M. Garrelts. Engineering Science Series, edited by Earle Raymond Hedrick. The Macmillan Company, New York, 1939. Cloth, 6 × 9¼ in., ix and 271 pp., figs., $2.50.

REVIEWED BY ALVIN SLOANE[7]

THIS is a collection of some eight hundred problems intended for the first courses in engineering mechanics. An appraisal of its value as a manual or collection leads very rapidly to a consideration of the pedagogy of these courses and to the role which this collection, inherently so sound, may play in enhancing the teaching of elementary mechanics.

Engineering pedagogues, like Topsy, "just grow up." Most frequently, untrained in techniques of pedagogy, the engineering teacher is apt to consider formalizations of method as impedimenta, and his course may lack coordination within itself, or in the general educational pattern. While some aspects of formal teaching method may be stultifying, there can be no quarrel with contributions which integrate and coodinate subject matter. When, in his better aspect, the engineering teacher is alert to the accelerated pace of scientific and technological advance, he faces the difficult task of canalizing into the classroom a flow of subject matter adjusted so that it may be compatible with the intellectual level of his student audience; provocative, so that it may stimulate and inspire rise to higher levels; penetrating, so that the basic fundamental may be recognized and its grasp enhanced at each stage; and thorough, that the sure footing of mental discipline may serve in integrating an educational career.

In applied mechanics, the recent advances in explorations toward advanced objectives have received the prominent and primary attention of the leaders. On the educational firing line we find little supporting activity. The classroom too frequently is static in a dynamic, virile, expanding subject, or confused in its efforts to reconcile educational process with research progress.

It is, therefore, encouraging to find leaders of the caliber of Professors Karelitz, Ormondroyd, and Garrelts devoting attention to the necessity of implementing an advancing field with new generations of workers, effectively grounded in fundamentals, and conscientiously trained in methods of attack so that they may serve with distinction as they approach the upper ranks.

Their book, "Problems in Mechanics," is a vital contribution to orientation in the teaching of the undergraduate first courses in statics, kinematics, and dynamics (kinetics). It brings to such undergraduate courses a basis which is intended to serve as a supplement to the expositions offered in the usual textbooks, and to meet the challenge of proper preparation for later stages in the development of perspective and attitude toward engineering mechanics.

The teaching devices employed are conducive to proper and

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vigorous growth—for example, illustrations which are sufficiently complete in detail to acquaint the student with the actual physical aspect of machine or structure, make the transition from abstractions to actualities effective—there has been too much "skeletonizing" of the problems by reduction to lines which psychologically creates a gap between the exposition of principle and the recognition of application of the same principle. Such devices stimulate through presentation of realities, always more intriguing than philosophical speculation in the mind of the embryonic engineer.

The problem material is carefully selected to encourage development; the range of difficulty includes careful treatment of simple, intermediate, and advanced material, and a group of illustrative examples is interspersed to suggest method of attack.

The kinematics section presents more extended treatment than is customary, recognizing the position of this factor in preparation for applications in kinetics. An additional factor of importance is provided in special sections devoted to bearing reactions, vibrations, oscillation, and impact. The inclusion of vibration problems of elementary nature at this educational stage adds a fundamental approach too frequently painstakingly avoided.

The material is coordinated, and a logical sequence employed throughout, so no difficulty of adaptation in classroom can arise.

In addition to the problem material, a résumé of the basic theorems of mechanics forms a first section of the book.

To this reviewer, it seems that this element is the only flaw, not because of its content, which is splendidly presented, but because this book can find its most effective use as a supplement, and not as a substitute for extended exposition of bases, in the case of students who should be equipped with a fuller text.

The space devoted might have been given to an expansion of the philosophy whose surface is but scratched in the preface, e.g., "the expositions of the principles and theorems of mechanics is of little practical value to the student unless he is constantly exercised in their application to actual problems. Only by this means can mechanics become a working tool for the future engineer."

Such an expansion, a discussion of the philosophical and disciplinary possibilities of this subject in encouragement of effective "straight thinking," and a penetrating analysis of the trend of thought in organizing method of attack would, from the pens of these authors, add an element of personal invitation and encouragement which the student too rarely finds in textbooks.

It would be well if this collection of problems could encounter a widespread reception in the classroom, for the orientation of the pedagogy of mechanics in its earliest stages is of vital concern. The textbook is the usual source of problem material, and textbook problems are frequently so intimately associated with the author's exposition that they lack universality. This collection, ramifying no particularized textbook, but written upon premises of universal application, deserves the opportunity of furthering the trend in engineering education, which seeks to establish strength in general fundamentals, and to avoid, in development stages, localization or specialization.

Fluid Mechanics

Mécanique des Fluides Appliquée. (Exercise numériques en vue des applications industrielles.) By A. Tenot. Dunod, Paris, 1939. Paper, $5^1/_2 \times 8^1/_4$ in. Tome III, Hydraulique et Hydrodynamique, viii and 183 pp., 48 figs. Tome IV, Production et Utilisation de la Vapeau d'Eau, viii and 106 pp., 26 figs., 1 plate. Price, each 45 fr.

Reviewed by Hugh L. Dryden[8]

As M. Caquot points out in his brief preface to this series of volumes, "modern theories of fluid mechanics have for some years been the possession of a small number of scientists, but their efficacy in the most diverse fields, such as aviation, ventilation, lubrication, turbine and pump design, have necessitated their use by an increasing number of engineers." M. Tenot brings the knowledge of these theories to engineers by the discussion of selected problems characterized by him as numerical exercises with a view to industrial application. The books are intended for the use of the Écoles Nationales d'Arts et Métiers, advanced technical schools, and industry. Since many American engineers are inexperienced both in the French language and in the use of the metric system, the book will be of primary interest to teachers of fluid mechanics as illustrating a method of instruction and a choice of types of problems to illustrate fundamental principles.

Volume 3 consists of 27 problems in hydraulics and hydrodynamics, as follows: Inclined venturi tube, vertical free jet with axial and spiral flow, sharp-edge weir, influence of compressibility of liquids, emptying of reservoir of variable section through sharp-edged orifice, jet from Pelton injector, diffuser with meridional flow, stationary vortex with and without spiral flow, flow in a spiral volute, spiral flow in a conical diffuser, flow in a diffuser with contravanes, design of a nozzle, siphon, flow in a pump-discharge line, force on an obstacle in a pipe line, functioning of discharge valve, theory of cavitation apparatus, reactions on an elbow from jet discharge, impact of jet on normal plate, deflector, forces on a gate valve, automatically closing gate valve, reaction of flow on walls of channel, pump characteristics, two pumps in parallel, operation of pump on a discharge line of known characteristics, exhaust blower of submarine Diesel engine.

These problems serve to illustrate the use of the equation of continuity, Bernoulli's equation for compressible and incompressible fluids, Pascal's law, the momentum principle, conservation of angular momentum, and so on. The author frequently calls attention to the approximate character of many of the computations and to the reasons for discrepancies between the computed and observed results. Reference is occasionally made to the author's earlier book entitled, "Leçons de mécanique-physique des fluides," for further information on fundamental principles.

Volume 4 contains 13 problems dealing with the production and utilization of steam. These relate to use of Mollier diagram, throttling and superheating, throttling of wet steam, steam turbine, nozzle, high-pressure and high-temperature turbine, low-pressure turbine, interstage superheating, experimental study of nozzle flow, experimental study of momentum loss in moving blades of turbine, reciprocating steam engine, condenser, Claude process for obtaining thermal energy from the ocean.

These books are not books of problems in the sense familiar to many of our teachers. The solutions of the problems are given very completely and there is little demand for effort on the part of the student. If one believes that the student learns faster by doing problems himself rather than by the study of complete solutions of problems by others, this characteristic will make the books unattractive.

The books are, rather, engineering texts in which the discussion of such problems as the performance of a steam turbine is clothed in numerical terms as well as algebraic symbols. A knowledge of fundamental principles is assumed; it is their application that is emphasized. The numerical accompaniment gives a sense of magnitudes and an air of familiarity and concreteness for those to whom symbols and mathematical language are strange and abstract. The statement in terms of problems to be answered helps to make the engineer feel at home. There may possibly be a field for this type of pedagogical approach in our own educational institutions.

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A Convenient Electrical Micrometer and Its Use in Mechanical Measurements

By ROSS GUNN,[1] WASHINGTON, D. C.

A simple electrical micrometer of great mechanical and electrical stability has been developed. The electrical-current output from the micrometer is accurately proportional to the impressed mechanical displacement. The least count of typical micrometers using portable microammeters as indicators and employing no amplification is 5/1,000,000 in. Zero drift, hysteresis, temperature, and pressure variations have been reduced to less than 1 per cent. Special circuits are described which permit the indication of the sums, differences, ratios, or products of mechanical displacements and hence are useful in many mechanical measurements. The micrometer is also convenient for zero and low-frequency vibration studies.

ENGINEERS charged with the research, development, and test of mechanical equipment have been long handicapped in the analysis of their structures by the lack of sufficiently convenient methods for the measurement of such items as displacement, force, torque, and mechanical power. In striving for a direct and satisfactory solution to the difficult problems of mechanical measurement, it seems expedient to adapt as completely as possible, the well-developed science of electrical measurement to the new requirements. To bridge the gap between mechanical and electrical measurement, it is necessary to provide some simple means for converting small mechanical displacements into appropriate electrical quantities like current or voltage. Moreover, it is most essential that the electrical quantity be exactly proportional to the mechanical displacement and not be subject to leakage or other troubles which cause a variation of response with frequency. An electromechanical link, satisfying the imposed requirements, has been developed and since it measures small mechanical displacements, it has been called an electrical micrometer.

The tube element of this micrometer, which serves as the important connecting link between the mechanical quantity to be measured and the resulting electrical current or potential, is shown in Fig. 1. This tube in its simplest form, as shown in Fig. 2, consists of an evacuated glass envelope 29, in which is sealed a hot electron-emissive filament 1, and two insulated plates 3 and 5, mounted on a supporting rod 35. The supporting rod, with its electrodes and necessary electrical connections 7, is carried by an elastic diaphragm 25 suitably sealed to the glass envelope. A small rod 13, spot-welded to the outside of the diaphragm, serves to communicate motion to the insulated-plate assembly 3 and 5. It is clear that a displacement of the arm by a screw 15, for example, will change the relative spacing of the hot filament 1, and the two plates 3 and 5, one plate being moved farther away from the filament while the other is moved closer to it. This change in filament spacing modifies the internal electrical resistance of the tube in accordance with the well-known laws of electronics, and thus changes the current in the associated electrical circuits. The tube shown in Fig. 1 is one type of several that have been developed for the measurement of small mechanical or torsional displacements. The micrometer tubes are designed to have a high mechanical rigidity and a high natural period of oscillation. Even those tubes which are designed to be highly sensitive to exceedingly small forces have mechanical periods exceeding 500 cycles per second, and it is known that they will accurately follow impressed vibrations of a frequency exceeding 60 cycles per second.

The electrical-micrometer tubes just described are used in connection with special electrical circuits which permit the achievement of a high degree of linearity and electrical stability. It is a well-known fact of electronics that the plate resistance of a vaccum tube changes very rapidly with slight or accidental changes in the filament emission or plate potential. In order that these accidental variations may not appear on the electrical meter which measures the small mechanical displacement, an electrical bridge circuit is employed of the type shown in Fig. 2.

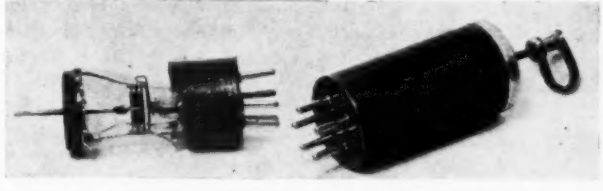


FIG. 1 THE TUBE ELEMENT OF THE ELECTRICAL MICROMETER. EXPLODED TUBE AT THE LEFT AND INTACT TUBE AT THE RIGHT

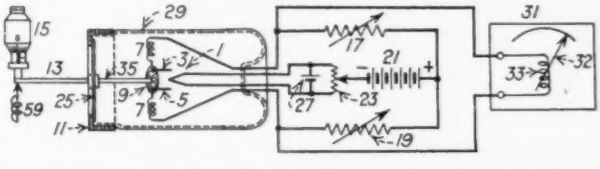


FIG. 2 CIRCUIT DIAGRAM OF THE TUBE FOR THE ELECTRICAL MICROMETER

A storage battery 21, or other source of electrical power which is reasonably stabilized by ballast tubes or other rough voltage-regulating device, supplies heating current to the electron-emissive cathode 1, and a potential of about 60 volts to the bridge circuit. The total power supplied to a typical micrometer tube is 5 watts. The electric current to the two plates 3 and 5, supplied through the highly flexible connections 7, traverses the variable resistances 17 and 19 in such a way that these resistances and the resistance between the filament and the plate 3, and the filament and the plate 5, make up the four arms of a typical bridge circuit. An electrical indicator 33 is connected across one diagonal of the bridge as shown.

The use of an electrical bridge circuit in the manner outlined accomplishes at least two highly desirable objectives. First, it reduces the electrical measurement to a comparison of two chang-

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ing resistances, one of which increases while the other simultaneously decreases. Thus, changes in the filament emission or plate voltage do not affect the indicator to the first order. Such a system is electrically stable and the drift of the electrical zero is not troublesome. Second, the circuital arrangement leads to electrical currents in the indicator circuit which are directly proportional to the impressed mechanical displacement to a high degree of approximation. That is, the response curve is linear.

The underlying reasons for this fortunate state of affairs is readily made apparent by a brief study of the circuital relations and currents in an unbalanced bridge. In Fig. 2, suppose that the resistance between the cathode 1 and plate 3 is N, that between 1 and plate 5 is P, while the resistances of 17 and 19 are M and X, respectively. The electromotive force of the battery 21 is taken as E and its internal resistance as B, while the current in the indicator coil 33, having a resistance G, is represented by I. Under these circumstances, it may readily be shown[2] that the current I is given by

$$I = \frac{E\,(NX - MP)}{\left[B + \frac{(M+N)(X+P)}{M+N+X+P}\right]\left[G(M+N+X+P)+(N+P)(M+X)\right]} \quad \ldots\ldots\ldots\ldots [1]$$

The sensitivity of such a bridge circuit is a maximum when the resistances of all arms are the same so that in a typical case we may assume that the properties of the tube and the associated resistances are so adjusted that at exact balance $M = X = N = P = Q$ and therefore $I = 0$. Now let the plates of the micrometer tube be moved by some small outside mechanical displacement. This changes the resistance of both N and P to new values and a current I flows through coil 33 due to the unbalance. Let the values of resistances in this case be represented by

$$\left.\begin{aligned} M &= X = Q \\ N &= Q + \Delta N \\ P &= Q + \Delta P \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [2]$$

where ΔN and ΔP are the changes in resistance of N and P, respectively, due to the impressed mechanical displacement. Substituting these values in Equation [1], it may be shown that

$$I = \frac{E\,\Delta N}{\frac{BG}{Q}J + BK + \frac{G}{Q}L + \frac{LK}{J}} \quad \ldots\ldots\ldots [3]$$

where $J = 4Q + \Delta N + \Delta P$; $K = 4Q + 2(\Delta N + \Delta P)$; and $L = 4Q^2 + 2Q\,(\Delta N + \Delta P) + \Delta N \Delta P$. This expression reduces to

$$I = \frac{E\,\Delta N}{4Q^2\left(1 + \frac{\Delta N + \Delta P}{2Q}\right)} \quad \ldots\ldots\ldots\ldots [4]$$

if the battery resistance B and the indicator resistance G are both small compared to Q. This is usually the case and therefore since ΔN and ΔP are always small compared to Q, an essential proportionality always exists between the indicator current and the change in bridge resistance or the mechanical displacement of the tube arm 13.

In the exact expression of Equation [3], it is clear that if the second-order term $\Delta N \Delta P$ is kept small, the current (or voltage) output of the bridge I (or IG) is directly proportional to the change in resistance ΔN, if and provided

$$\Delta N + \Delta P = 0 \quad \ldots\ldots\ldots\ldots [5]$$

The foregoing results are of general application to bridge circuits and we proceed to apply them to our special system. The electric current i flowing between a hot cathode and a cold plate in a space-charge-limited vacuum tube is given[3] by

$$i = h\frac{\varphi^{3/2}}{X^2} \quad \ldots\ldots\ldots\ldots [6]$$

where h is a constant depending on the geometry of the tube, φ is the potential applied to the plate and X is the distance between the plate and filament. From Equation [6], we have that the internal resistances between plate and filament are

$$N = \frac{X^2}{h\varphi_N^{1/2}};\quad P = \frac{(d-X)^2}{h\varphi_P^{1/2}} \quad \ldots\ldots\ldots\ldots [7]$$

where d is the total effective separation of the two tube plates; whence

$$\Delta N = \frac{dN}{dX}\Delta X = \frac{2X}{h\varphi_N^{1/2}}\Delta X - \frac{X^2}{2h\varphi_N^{3/2}}\frac{d\varphi_N}{dX}\Delta X \quad \ldots\ldots [8]$$

$$\Delta P = \frac{dP}{dX}\Delta X = -\frac{2(d-X)}{h\varphi_P^{1/2}}\Delta X - \frac{(d-X)^2}{2h\varphi_P^{3/2}}\frac{d\varphi_P}{dX}\Delta X \quad \ldots\ldots [9]$$

therefore

$$\Delta N + \Delta P \doteqdot \left(\frac{4X}{h\varphi_Q^{1/2}} - \frac{2d}{h\varphi_Q^{1/2}}\right)\Delta X - 1/2\left[\frac{X^2}{h\varphi_N^{3/2}}\frac{d\varphi_N}{dX} + \frac{(d-X)^2}{h\varphi_P^{3/2}}\frac{d\varphi_P}{dX}\right]\Delta X \quad \ldots\ldots [10]$$

Since all arms of the bridge are adjusted to the same resistance $\varphi_N \doteqdot \varphi_P$ and $\frac{d\varphi_N}{dX} + \frac{d\varphi_P}{dX} \doteqdot 0$. Thus one sees that Equation [5] is satisfied for finite displacements when $X = d/2$ and this occurs when the filament is exactly centered between the plates in such a way that $N = P$. It is clear therefore that linearity may be established over any selected range by appropriate spacing of the two tube plates. Departures from linearity, even by 1 per cent, are not observed in actual micrometer tubes until ΔX approximates 10 per cent of the cathode-plate spacing. Thus, for all practical purposes the micrometer response relationship is linear.

In constructing the micrometer tube, the plates 3 and 5 are equally spaced from the filament 1 when the arm 13 is undeflected; therefore, the electrical resistance between the filament and each of these two plates is the same. In a typical tube this resistance approximates 10,000 ohms. Therefore, for maximum sensitivity and for a balance of the bridge such that the current in meter 33 is zero, one must adjust the two resistances 17 and 19 until they are approximately the same as the tube resistances. Under this condition of approximate balance, the reading of the meter 33 and hence the stability of the zero reading, is very insensitive to changes in the emission of filament 1, which are brought about by moderate changes in the filament power supply 27. Similarly, due to the balanced nature of the circuits employed, variations in the plate-battery supply 21, cause only second-order changes in the reading of the meter 33. These particular tube structures and special circuits are responsible for the electrical stability which may be attained with the new micrometer. The mechanical stability is also carefully maintained by rigidly mounting the mov-

[2] "Electrical Measurements," by F. A. Laws, McGraw-Hill Book Company, Inc., New York, N. Y., 1938, p. 179.

[3] "Theory of Thermionic Vacuum Tubes," by E. L. Chaffee, McGraw-Hill Book Company, Inc., New York, N. Y., 1933, p. 69.

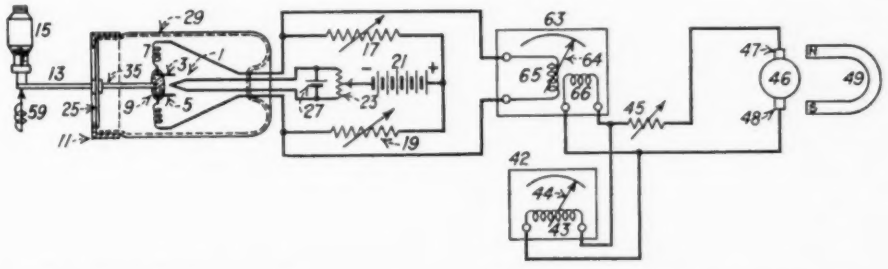


FIG. 4 CIRCUITS OF AN ELECTRICAL TACHOMETER, WATTMETER, AND THE ELECTRICAL MICROMETER COMBINED TO FORM A POWER-AND-TORSION METER FOR MEASURING MECHANICAL HORSEPOWER

ing parts and by the proper selection of material for the diaphragm 25, so that it exhibits vanishingly small mechanical hysteresis. The over-all stability of the device in practice has been found to be very good and will reproduce over periods of weeks to within less than 1 per cent of full scale, either for steady or oscillating mechanical deflections of the rod 13.

The sensitivity of typical micrometers approximates 150 microamperes per mil (0.001 in.) or about $^1/_2$ volt per mil when working into a high impedance. When used with standard rugged-type portable microammeters, the ratio of the meter pointer movement to the movement impressed on the arm 13 is about 10,000. The over-all stability of the device is unusually good if the power supply is reasonably constant, and one may expect the zero to drift less than 3 microamperes in the course of a day. Considerable attention has been devoted to the achievement of mechanical as well as electrical stability. The mechanical hysteresis is less than $^1/_2$ per cent of the maximum deflection even under adverse conditions.



FIG. 3 LIMIT AND TOLERANCE GAGE WHICH EMPLOYS THE ELECTRICAL-MICROMETER PRINCIPLE

Because of the inherent zero stability built into the tubes and associated circuits, this micrometer is especially suitable for the measurement or indication at a remote point of small displacements. These small displacements may originate as a result of tension, compression, or torsion in some stressed member; hence, if the member is elastic and obeys Hooke's law, the indicating electrical meter may be calibrated in terms of strain, stress, force, or torque. A small mass attached to the arm 13 of a micrometer tube converts it into an accelerometer which is readily calibrated in absolute units. By the employment of an elastic bellows, fluid pressures may be read. The device is an excellent scale for weights approximating a fraction of an ounce. By the use of a calibrated proving ring, its range may be expanded indefinitely. It has been applied with considerable satisfaction to limit and tolerance gages, such as shown in Fig. 3, and may be applied to the control of light cutting tools. The foregoing suggests only a few of the possible uses of the micrometer.

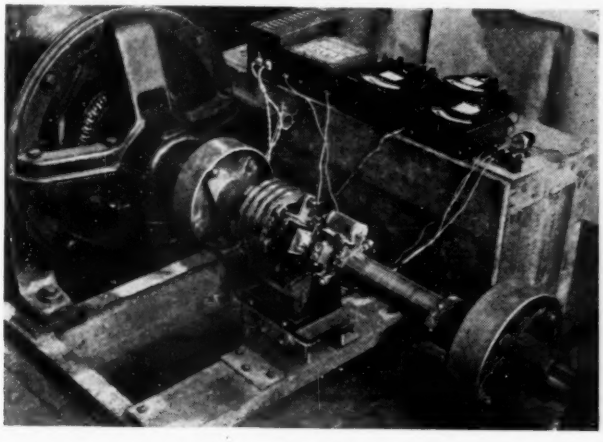


FIG. 5 POWER-AND-TORSION METER, THE CIRCUIT OF WHICH IS SHOWN IN FIG. 4

In the simplest type of application, the meter 33 may be an ordinary microammeter and in such a case mechanical displacements of the arm 13 as small as a wave length of blue light may be readily detected. In a similar way the meter 33 may be an oscillograph with amplifier and an instantaneous trace of the position of the arm 13 may be readily indicated both as to amplitude and wave form.

Clearly, the micrometer is useful for the analysis of low-frequency vibrations and a preliminary investigation in this field has been carried forward. It is found that the micrometer tubes will follow mechanical vibrations up to more than 65 cycles per second and reproduce the amplitude and wave form of the vibration on an oscillograph, at least to the limit of accuracy of measurement of the film. The author is of the opinion that the tubes at present available will follow vibrations accurately up to about 200 cycles per second, and by careful special design this can be raised to about 1000 cycles per second. Because the mi-

crometer can be made quantitatively to measure instantaneous, average, or root-mean-square values of a vibrator displacement, it is ideal for the study of surface finish.

Electrical meters of the wattmeter type are on the market which measure the value of the product of two electrical currents. Thus by the employment of two independent electrical micrometers, it is possible to measure the product of two mechanical displacements or the product of a mechanical displacement and an electrical potential such as that supplied by an electrical tachometer. Thus, it is possible to measure the product of speed and torque or mechanical horsepower. This is accomplished by mounting an electrical-micrometer tube on a drive shaft in such a way that the torsional strain in the shaft produces a mechanical displacement of the tube arm, which is proportional to the torque. The output from this electrical-micrometer tube is supplied to one coil 65 of a wattmeter type of electrical meter as shown in Fig. 4, while voltage from an electrical tachometer is supplied to coil 66. The electrical tachometer, consisting of a permanent magnet 49 and an armature 46 with brushes 47 and 48, provides a current which is proportional to the speed, while the electrical micrometer provides a current which is proportional to torque. Clearly the speed of the shaft may be measured by the current through the meter 43. Therefore, the wattmeter 64 may be calibrated directly in terms of mechanical power. Such a power-and-torsion meter has been built and has given good service. It is illustrated in Fig. 5. An improved model is being applied to the measurement of the mechanical power supplied to the propellers of a destroyer.

Special electrical meters are available on the market which measure the ratio of two electric currents. It is possible, therefore, to indicate on an electrical meter the over-all efficiency of a power plant. For example, the product of speed and torque by the use of a special circuit may be applied to one coil of a ratio meter while the rate at which fuel is supplied to the plant may be provided to the other coil through the agency of a suitable flowmeter employing an electrical micrometer. In such a setup the electrical meter will read the ratio of horsepower output to the rate of fuel supply and therefore can be calibrated directly in terms of over-all plant efficiency.

Because the field of electrical measurement is so well developed, it is possible to measure automatically by the use of this new micrometer almost any type of mechanical quantity. In addition to the quantities mentioned previously, it is possible, by the selection of appropriate electrical measuring devices already available, to measure the average or root-mean-square value of a varying mechanical displacement, its time derivative, its time integral, or the sum or difference of two independent displacements. It is even possible to indicate the value of one mechanical displacement raised to a power proportional to a second mechanical displacement. The electrical equipment is quite simple and even an oscillograph which will depict wave forms as well as the instantaneous values of a mechanical displacement is easy to use.

Space does not permit a detailed description of the applications already made of the new micrometer, but perhaps this article will serve to indicate its general usefulness and its applicability to the important problems of mechanical measurement. Because of the simplicity of the micrometer, its linearity, and its great stability, it has been a most convenient measuring device.

It is a pleasure to acknowledge the material assistance of Wayne C. Hall and Frank I. Louckes in the development and application of this micrometer.

The Influence of Hyperbolic Notches on the Transverse Flexure of Elastic Plates

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This paper[2] considers the solution of the two problems of the infinite plate, with two symmetrically disposed hyperbolic notches, subjected to (*a*) transverse bending and (*b*) twisting. The transverse-bending moments and torsional couples are so applied that the narrow section between the notches transmits the bending moment or the torsional couple. Using the Poisson-Kirchhoff theory, finite expressions were obtained for the deflection and stress in each problem.

THE two problems considered in this paper concern the infinite sheet with two symmetrically disposed hyperbolic notches. In the first problem the plate is transmitting a given total bending moment, as a beam of variable width but of constant thickness, under pure bending. In the second problem the plate is twisted, with OX as the axis of twist, Fig. 1*a*. The results of the two problems are shown graphically in Figs. 3 and 5. The stress concentration, i.e., the ratio of the stress occurring in the plate at the apex of the notch to the average stress over the narrow section between the notches, is plotted as a function of d/ρ where d is the length of the narrow section between the notches and ρ is the radius of curvature at the apex of the notch.

The deflections and stresses are found from the Poisson-Kirchhoff theory of the bending of thin elastic plates (1).[3] The differential equation for the transverse deflection w is $D\nabla^4 w = p$, where p is the pressure per unit area applied to the surface of the plate, and D is the flexural rigidity of the plate. In this instance, since the bending is due to edge loadings only, p is equal to zero; w is then a biharmonic function.

The sign convention and the stress resultants acting over sections normal to the middle plane are given in Fig. 1*b*. According to the Poisson-Kirchhoff theory, the stress resultants, in terms of the deflection w are

$$\left.\begin{aligned} G_x &= -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \\ G_y &= -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \\ H_x &= -H_y = (1-\nu)D\frac{\partial^2 w}{\partial x \partial y} \\ N_x &= -D\frac{\partial}{\partial x}\nabla^2 w \qquad N_y = -D\frac{\partial}{\partial y}\nabla^2 w \end{aligned}\right\} \quad \ldots\ldots\ldots [1]$$

where G_x, G_y are the transverse bending moments, N_x, N_y are the transverse shearing forces, and H_x, H_y are the torsional couples, all per unit length, acting over the sections x = constant and y = constant, respectively; ν is Poisson's ratio. The general conditions for each problem are as follows:

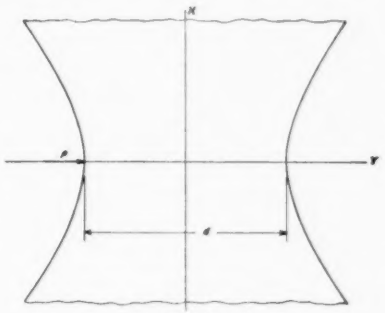


FIG. 1*a*

Bending. The plate must transmit a finite bending moment so that the integral of G over any cross section must yield a finite resultant, therefore, G itself must vanish at infinity. The deflection w must have symmetry with respect to both axes.

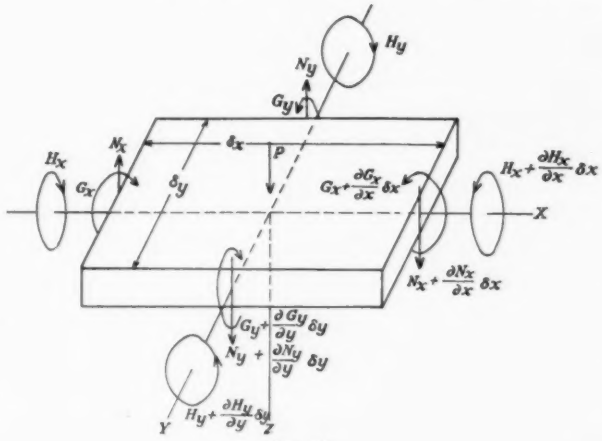


FIG. 1*b*

Torsion. The plate must transmit a finite torsional couple so that the integral of H over any cross section must yield a finite resultant and H itself must vanish at infinity. The deflection must be an odd function of x and of y.

In the orthogonal curvilinear coordinates defined by the conformal transformation $x + iy = f(u + iv)$, Equations [1] are transformed into the following, for actions over the sections v = constant

$$\left.\begin{aligned} J^4 G_v &= -D\left[J^2\left(\frac{\partial^2 w}{\partial v^2} + \nu\frac{\partial^2 w}{\partial u^2}\right) - \frac{1}{2}(1-\nu)\frac{\partial J^2}{\partial v}\frac{\partial w}{\partial v} + \frac{1}{2}(1-\nu)\frac{\partial J^2}{\partial u}\frac{\partial w}{\partial u}\right] \\ J^4 H_v &= (1-\nu)D\left[J^2\frac{\partial^2 w}{\partial u \partial v} - \frac{1}{2}\frac{\partial J^2}{\partial u}\frac{\partial w}{\partial v} - \frac{1}{2}\frac{\partial J^2}{\partial v}\frac{\partial w}{\partial u}\right] \\ J^3 N_v &= -D\left[J^2\frac{\partial}{\partial v} - \frac{\partial J^2}{\partial v}\right]\left[\frac{\partial^2 w}{\partial v^2} + \frac{\partial^2 w}{\partial u^2}\right] \end{aligned}\right\} \quad [2]$$

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[2] From a thesis submitted to Cornell University in partial fulfillment for degree of Doctor of Philosophy.

[3] Numbers in parentheses refer to Bibliography at end of paper.

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where J is the "stretch factor," and is given by the equation

$$J = \left[\left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2\right]^{1/2}$$

For actions over the sections u = constant, u and v are interchanged, except for H_u which is equal to $-H_v$.

The transformation used in the solution of these problems was of the form $x + iy = c \sinh (u + iv)$. From this we obtain the relations $x = c \cos v \sinh u$ and $y = c \sin v \cosh u$. The ranges for the variables u and v are $-\pi/2 < v < +\pi/2$ and $-\infty < u < +\infty$. For v a constant we obtain a family of hyperbolas and for u a constant a family of ellipses.

Over the free boundaries, i.e., the hyperbolas $v = \pm v_0$, the stress resultants must vanish. Kelvin and Tait (1) have pointed out that a distribution of torsional couples over a section is statically equivalent to a distribution of transverse shearing forces equal to $-\partial H/\partial s$ per unit length, where s is the tangent to the section. Then G_v and $N_v - \partial H_v/\partial s$ must both vanish over the free boundary $v = \pm v_0$. The sense of the tangent s is so chosen that together with the outward normal n and the z axis the three form a right-handed coordinate system. Goodier (2) has further modified the boundary conditions as follows: Since $N_v = -D \frac{\partial}{\partial n} \nabla^2 w$; w being biharmonic, $\nabla^2 w$ is harmonic and must possess a conjugate Ω such that $\frac{\partial}{\partial n} \nabla^2 w = \frac{\partial \Omega}{\partial s}$; the latter boundary condition may then be written $D \frac{\partial \Omega}{\partial s} + \frac{\partial H_v}{\partial s} = 0$ Integrating this with respect to the tangent, it follows that $D\Omega + H_v$ equals a constant over the free boundaries $v = \pm v_0$.

The Bending Problem

The solution was found, after examination of numerous simple biharmonic functions having the requisite symmetry, to be of the form[4]

$$w = Av \sin v \cosh u + B \cos v \cosh u + Eu \cos v \sinh u + F$$

In this case, Equation [2] takes the form of Equation [3] below:

$$J^4 G_v = -\frac{Dc^2}{4}\left[\{A[(1+\nu)\cos 3v + (5-\nu)\cos v] - 4B(1-\nu)\cos v + E[2\nu \cos 3v - (1-5\nu)\cos v]\} \cosh u + \{2A + E(1+\nu)\}\cos v \cosh 3u\right] \quad [3]$$

and $D\Omega + H_v$ takes the form

$$D\Omega + H_v = -\frac{Dc^2}{4J^4}\left[\{A[(3-\nu)\sin 3v - (7-3\nu)\sin v] + 4B(1-\nu)\sin v + E[2\sin 3v - (1+3\nu)\sin v]\} \sinh u + \{2A + E(1+\nu)\}\sin v \sinh 3u\right] \quad [4]$$

while the stretch factor J is given by

$$J^2 = \frac{c^2}{2}(\cos 2v + \cosh 2u)$$

Over the free boundary the coefficients of the variable terms of $D\Omega + H_v$ must vanish, while G_v must vanish identically. These conditions give rise to the following simultaneous equations

$$A[(1+\nu)\cos 3v_0 + (5-\nu)\cos v_0] - 4B(1-\nu)\cos v_0 + E[2\nu \cos 3v_0 - (1-5\nu)\cos v_0] = 0 \quad [5]$$

$$2A + E(1+\nu) = 0 \quad [6]$$

$$A[(3-\nu)\sin 3v_0 - (7-3\nu)\sin v_0] + 4B(1-\nu)\sin v_0 + E[2\sin 3v_0 - (1+3\nu)\sin v_0] = 0 \quad [7]$$

$$2A + E(1+\nu) = 0 \quad [8]$$

Solving these equations for B and E in terms of A

$$E = -\frac{2}{1+\nu}A \quad [9]$$

$$B = \frac{3+\nu+(1-\nu)\cos 2v_0}{2(1+\nu)}A \quad [10]$$

It is to be noted that Equations [6] and [8] are identical, and that Equations [5] and [7] are dependent. Imposing the condition that the deflection is zero at the origin ($u = v = 0$), the value of F is

$$F = -\frac{3+\nu+(1-\nu)\cos 2v_0}{2(1+\nu)}A \quad [11]$$

The integral of G_u over the section $u = 0$ ($x = 0$) is equal to the average bending moment per unit length G times the length of the section. This yields the value of A in terms of G

$$A = \frac{1+\nu}{1-\nu}\frac{2Gc^2 \sin v_0}{D\{2(3+\nu)v_0 - (1-\nu)\sin 2v_0\}} \quad [12]$$

The stress resultants then become

$$G_u = \frac{-2G \sin v_0}{(\cos 2v + \cosh 2u)^2[2(3+\nu)v_0 - (1-\nu)\sin 2v_0]}\left[\{-(3+\nu)\cos 3v + [-(5+3\nu) + 2(1-\nu)\cos 2v_0]\cos v\}\cosh u - 2(1+\nu)\cos v \cosh 3u\right] \quad [13]$$

$$G_v = \frac{2(1-\nu)G \sin v_0[(2\cos 2v_0 - 1)\cos v - \cos 3v]\cosh u}{(\cos 2v + \cosh 2u)^2[2(3+\nu)v_0 - (1-\nu)\sin 2v_0]} \quad [14]$$

$$H_u = -H_v = \frac{2G \sin v_0[\{(1+\nu)\sin 3v - [3+\nu-2(1-\nu)\cos 2v_0]\sin v\}\sinh u + 2\sin v \sinh 3u]}{(\cos 2v + \cosh 2u)^2[2(3+\nu)v_0 - (1-\nu)\sin 2v_0]} \quad [15]$$

$$N_u = \frac{8\sqrt{2}\,G \sin^2 v_0[(\cos 3v - 4\cos v)\sinh u - \cos v \sinh 3u]}{d[2(3+\nu)v_0 - (1-\nu)\sin 2v_0](\cos 2v + \cosh 2u)^{3/2}} \quad [16]$$

$$N_v = \frac{8\sqrt{2}\,G \sin^2 v_0[(\sin 3v + 2\sin v)\cosh u - \sin v \cosh 3u]}{d[2(3+\nu)v_0 - (1-\nu)\sin 2v_0](\cos 2v + \cosh 2u)^{3/2}} \quad [17]$$

The deflection of the plate is

$$w = \frac{1+\nu}{1-\nu}\frac{Gd^2}{2D \sin v_0[2(3+\nu)v_0 - (1-\nu)\sin 2v_0]}\left[v \sin v \cosh u + \frac{3+\nu+(1-\nu)\cos 2v_0}{2(1+\nu)}(\cos v \cosh u - 1) - \frac{2}{1+\nu}u \cos v \sinh u\right] \quad [18]$$

In Fig. 2 are plotted G_u, G_v, and H_u in terms of G, over the section $u = 0$ and the free boundary $v = \pm v_0$. The maximum G_u occurs at the apex of the notch and is large in comparison to the maxima of the other stress resultants occurring in the plate

[4] The possibility of solving these problems in this manner was suggested by Neuber's solutions of the corresponding problems of plane stress.

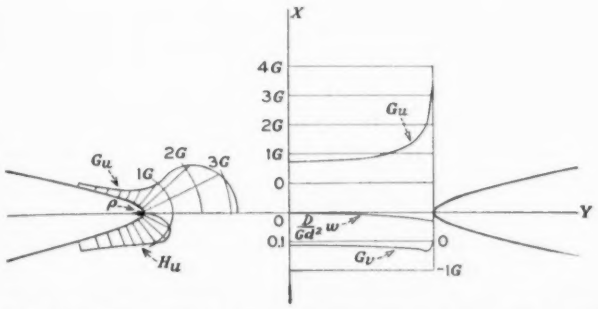


FIG. 2 DISTRIBUTION OF STRESS RESULTANTS IN A NOTCHED PLATE UNDER TRANSVERSE BENDING; $d/\rho = 64.6$

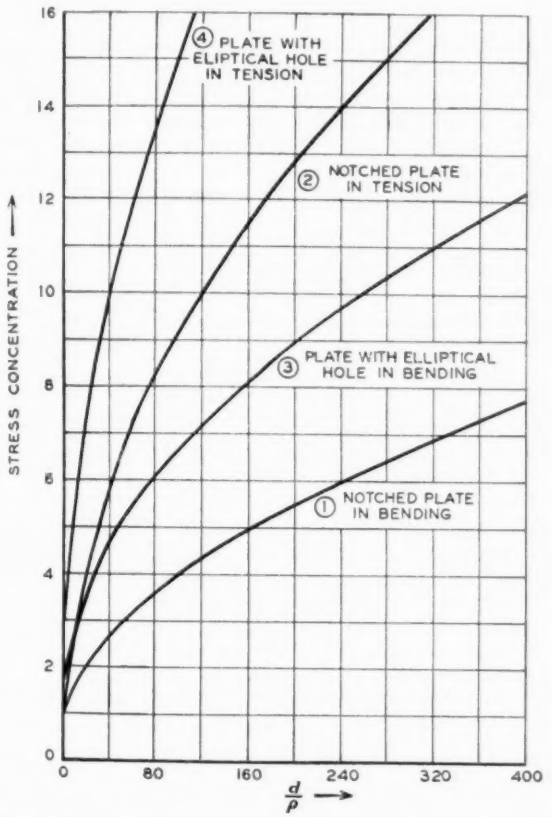


FIG. 3 STRESS CONCENTRATION PLOTTED AS A FUNCTION OF d/ρ

G_u is then the stress resultant that is the governing factor in determining the stress concentration. The ratio G_u/G is plotted against d/ρ, where d is the length of the narrow section between the notches, shown in Fig. 1*a*, and ρ is the radius of curvature of the notch at its apex; for comparison the stress concentration for the notched plate under tension (3) is plotted against d/ρ as curve 2 in Fig. 3. Curves 3 and 4 in Fig. 3 are the stress concentrations for the closely allied problems of the elastic plate with an elliptical hole, major axis parallel to the y axis, subjected to bending (2) and tension (3), respectively. For these latter two curves, the stress concentration is plotted against d/ρ, where d is here the length of the major axis and ρ is the radius of curvature of the ellipse at the end of the major axis. Values of $(D/Gd^2)w$ are plotted along the y axis in Fig. 2.

As a check on the validity of the solution obtained, the deflection should approach that of a strip of uniform width as the notches become very flat, i.e., as $v_0 \to 0$. The region considered must be that for small u; near the axis $u = 0$ the change in section is of secondary importance, but removed from this region, the change in section becomes of prime importance unless $v_0 = 0$, a case for which the present results are not directly applicable. For small u, $\sinh u \approx u \approx x$, $\cosh u \approx 1 + x^2/2$, and $\sin v \approx v \approx y$. Substituting these into the expression for the deflection, we obtain

$$w \to \frac{-G}{2EI}(x^2 - \nu y^2)$$

It will be observed that this limit represents the bending of a strip on the assumption that anticlastic curvature is free to occur. It is well known however that when a strip is appreciably bent, the anticlastic curvature does not develop. The section remains flat, except for a slight curling at the edge. The solution customarily adopted supposes that bending moments are called into play, on sections parallel to the edges of the strip, sufficient to prevent the anticlastic curvature. Near the edge there is a local transition to the edge condition of freedom from stress.

The question arises as to whether the solution for the notched plate should not be such as to give a limiting form for the parallel strip in which the anticlastic curvature does not appear.

The answer appears to be that the solution as given will represent the initial stage of bending, in which no appreciable mean stress develops. For, as appears from Lamb's solution (4, 5) the prevention of anticlastic curvature in the strip is associated with the development of mean stress in the edge zones, and so will not occur in the initial stage.

Since the concentration of stress is localized around the apex of the notch, it is reasonable to expect that the solution will apply with fair approximation to notches in strips of finite width. The corresponding expectations in the case of plane stress have been borne out by the photoelastic measurements of Frocht (6), the calculations having been made by Neuber (3).

THE TORSION PROBLEM

The solution of this problem was found to be of the form

$$w = Av\cos v\sinh u + B\sin v\sinh u + Eu\sin v\cosh u$$

The conditions that G_v and $D\Omega + H_v$ must vanish and equal a constant over a free boundary, respectively, yield the following simultaneous equations

$$[A(3-\nu) - 2E]\cos 3v_0 + [A(7-3\nu) + 4B(1-\nu) - E(1+3\nu)]\cos v_0 = 0 \quad [19]$$

$$2A - E(1+\nu) = 0 \quad [20]$$

$$[-A(1+\nu) + 2E\nu]\sin 3v_0 + [A(5-\nu) + 4B(1-\nu) + E(1-5\nu)]\sin v_0 = 0 \quad [21]$$

$$2A - E(1+\nu) = 0 \quad [22]$$

Solving these equations for B and E in terms of A, we obtain

$$E = \frac{2}{1+\nu}A \quad [23]$$

$$B = \frac{3+\nu-(1-\nu)\cos 2v_0}{2(1+\nu)}A \quad [24]$$

The constant A is evaluated in the same manner as in the first problem: The integral of H_u over the section $u = 0$ is equal to the average torsional couple per unit length H times the length of the section d. The value of A thus obtained is

$$A = \frac{1+\nu}{1-\nu}\,\frac{Hd^2}{2D\sin v_0[2(1+\nu)v_0+\{7+\nu-(1-\nu)\cos 2v_0\}\tan v_0]} \quad [25]$$

The stress resultants are then

$$G_u = \frac{-2H\sin v_0}{[2(1+\nu)v_0+\{7+\nu-(1-\nu)\cos 2v_0\}\tan v_0](\cos 2v+\cosh 2u)^2}$$
$$[\{(3+\nu)\sin 3v+[-(17+7\nu)+2(1-\nu)\cos 2v_0]\sin v\}\sinh u + 2\sin v\sinh 3u] \quad [26]$$

$$G_v = \frac{-4H\sin v_0[-(1-\nu)\sin 3v+\{2(5+\nu)-(1-\nu)\cos 2v_0\}\sin v]\sinh u}{[2(1+\nu)v_0+\{7+\nu-(1-\nu)\cos 2v_0\}\tan v_0](\cos 2v+\cosh 2u)^2} \quad [27]$$

$$H_u = -H_v = \frac{2H\sin v_0}{[2(1+\nu)v_0+\{7+\nu-(1-\nu)\cos 2v_0\}\tan v_0](\cos 2v+\cosh 2u)^2}$$
$$[\{(1+\nu)\cos 3v+[5(3+\nu)-2(1-\nu)\cos 2v_0]\cos v\}\cosh u + 2\cos v\cosh 3u] \quad [28]$$

$$N_u = \frac{8\sqrt{2}\,H\sin^2 v_0[(\sin 3v+2\sin v)\cosh u-\sin v\cosh 3u]}{d[2(1+\nu)v_0+\{7+\nu-(1-\nu)\cos 2v_0\}\tan v_0](\cos 2v+\cosh 2u)^{5/2}} \quad [29]$$

$$N_v = \frac{8\sqrt{2}\,H\sin^2 v_0[(2\cos v-\cos 3v)\sinh u+\cos v\sinh 3u]}{d[2(1+\nu)v_0+\{7+\nu-(1-\nu)\cos 2v_0\}\tan v_0](\cos 2v+\cosh 2u)^{5/2}} \quad [30]$$

and the deflection is

$$w = \frac{1+\nu}{1-\nu}\,\frac{Hd^2}{2D\sin^2 v_0[2(1+\nu)v_0+\{7+\nu-(1-\nu)\cos 2v\}\tan v_0]}$$
$$\left[v\cos v\sinh u+\frac{3+\nu-(1-\nu)\cos 2v_0}{2(1+\nu)}\sin v\sinh u + \frac{2}{1+\nu}u\sin v\cosh u\right] \quad [31]$$

In Fig. 4 is plotted H_u and G_u, in terms of H, over the section $u = 0$ and the free boundaries $v = \pm v_0$. It may be expected,

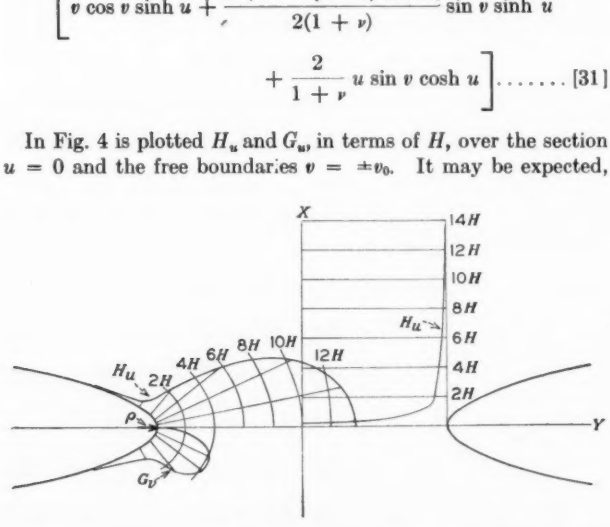


FIG. 4 DISTRIBUTION OF STRESS RESULTANTS IN A NOTCHED PLATE IN TORSION; $d/\rho = 27.8$

as in the bending problem, that these results for the infinite sheet may be applied with fair approximation to notches in strips of finite width. It will be observed that the stress concentration, H_u/H plotted against d/ρ for the present problem, shown in Fig. 5, is considerably higher than G_u/G for the bending problem. As a check on the results of this problem, the expression for the deflection should approach that for the strip of uniform width, as $v_0 \to 0$. For small u and as $v_0 \to 0$, the expression for the deflection approaches the form:

$$w \to \frac{H}{D(1-\nu)}\,xy$$

which is the deflection of the strip of uniform width subjected to torsional couples.

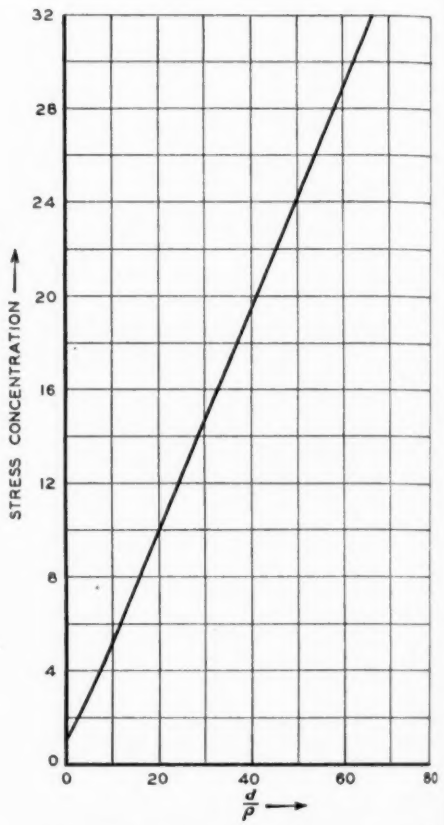


FIG. 5 STRESS CONCENTRATION PLOTTED AS A FUNCTION OF d/ρ FOR THE NOTCHED PLATE IN TORSION

ACKNOWLEDGMENT

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A Photoelastic Study of Stresses in Rotating Disks

By R. E. NEWTON,[1] ST. LOUIS, MO.

The recent experiments of Hetényi and others in developing a technique for "freezing" stresses in bakelite and other photoelastic plastics have broadened tremendously the scope of the photoelastic method of stress analysis. In addition to making possible the solution of three-dimensional problems, the new technique offers itself as a powerful tool in solving problems in which it is inconvenient to study the fringe pattern while the model is actually under load. Problems of centrifugal stresses in constant-speed rotors fall in this class and are readily handled by the stress-freezing method. It is the purpose of this paper to discuss the application of the technique to the determination of the stresses in rotating disks of uniform thickness having symmetrically placed noncentral holes, as shown in Fig. 1 (1, 2).[2]

WE FIRST review the essential phenomenon and theory of the "stress freezing" technique (3). As the models used in the present investigation were made of bakelite BT 61-893, the effect will be described with particular reference to this plastic. The annealing characteristics of bakelite are explained on the assumption that it consists of two principal phases. One of these phases, the A phase, is readily fusible at low temperatures and the other, the C phase, fuses only at quite high temperatures. As the temperature rises, the viscosity of the A phase is rapidly lowered until at about 230 F this phase no longer has appreciable strength. If a load were now applied to the bakelite it would be carried entirely by the remaining three-dimensional lattice—the C phase. Under these conditions the deformation produced by a given load will be about 640 times that which would occur at room temperature. The corresponding photoelastic effect is about 26 times as great as at room temperature. When the temperature is lowered while the bakelite is still under load, the A phase solidifies, filling the interstices of the space lattice formed by the C phase. Then, when the load is removed, the specimen will remain deformed and continue to exhibit substantially the same photoelastic fringe pattern that was produced at the higher temperature under load; however, the specimen actually recovers an amount which is equal to $^1/_{640}$ of the original deformation and the optical effect is reduced by $^1/_{26}$.

Taking advantage of this phenomenon, it is possible to spin a bakelite model of a rotor at constant speed, raise the temperature to 230 F, cool slowly, then stop the rotation, remove the model from the mounting, and examine the resultant photoelastic pattern at leisure. This method avoids the difficulties of a stroboscopic study and makes possible photoelastic investigations of a class of problems of great importance in engineering design. The present problem illustrates these possibilities.

The models used were made of bakelite BT 61-893, as noted previously. The diameter of the model was limited by the size of plate available and the dimensions finally selected, as shown in Fig. 1, were: Outside diameter, $2b = 5.45$ in.; radius of circle containing centers of holes, $a = 1.00$ in.; spacing of holes, 60 deg; diameter of holes, $d = 0.250$ and 0.375 in. for model sets A and B, respectively. Since the speed of rotation was limited by practical considerations to 1800 rpm, it was deemed advisable to use a model thickness of approximately 1 in. to secure a sufficiently high order of isochromatic fringe (about the 9th) at the points of maximum stress. Polished plates of bakelite were not available in thicknesses greater than $^1/_2$ in.; therefore, it was necessary to use two identical models of this thickness and subsequently to examine them together in the polariscope so that the optical effect would be double that secured with one model.

Oversized disks were first cut out of the bakelite plate with a coping saw. The holes were drilled in the disks using a steel faceplate shown in Fig. 2 as a drill guide. The disks were then bolted together to the faceplate and turned to the proper outside diameter.

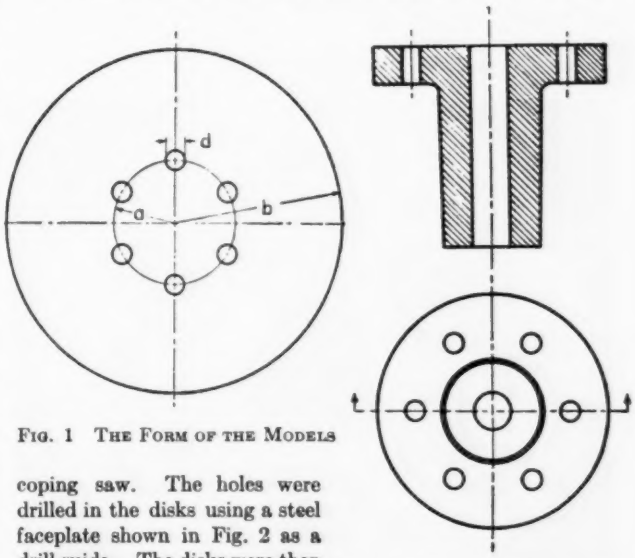


FIG. 1 THE FORM OF THE MODELS

FIG. 2 THE FACEPLATE USED FOR DRILLING AND SPINNING THE MODELS

The mounting system was devised with the dual purpose of keeping the disks accurately centered during annealing and offering a minimum of constraint to their deformation. Since it was not possible to eliminate all constraint, a compromise was made, partially constraining the model by the use of supporting bolts in three of the holes at 120-deg intervals and allowing the other three holes to be entirely free. The faceplate shown in Fig. 2 was used to hold the bolts. To reduce the constraint at the three bolts it was decided to use undersized bolts and surround them with rubber bushings. The use of undersized bolts gave the further advantage that centering could be done by shifting the bolts slightly in the holes of the faceplate. Accurate center-

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

ing was effected by using a dial micrometer to test eccentricity; small errors were corrected by tapping the bolts. A section detail of the method of mounting the models on the bolt is shown in Fig. 3. Rotation was produced by a synchronous motor on whose shaft the faceplate made a tight friction fit.

An oven heated electrically was built around the models and

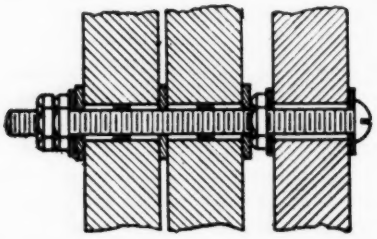


FIG. 3 UNDERSIZE BOLTS FITTED WITH RUBBER BUSHINGS WERE USED TO MOUNT THE DISKS ON THE FACEPLATE FOR SPINNING DURING ANNEALING

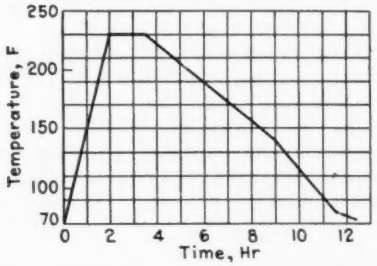


FIG. 4 TEMPERATURE CYCLE FOR ANNEALING THE MODELS

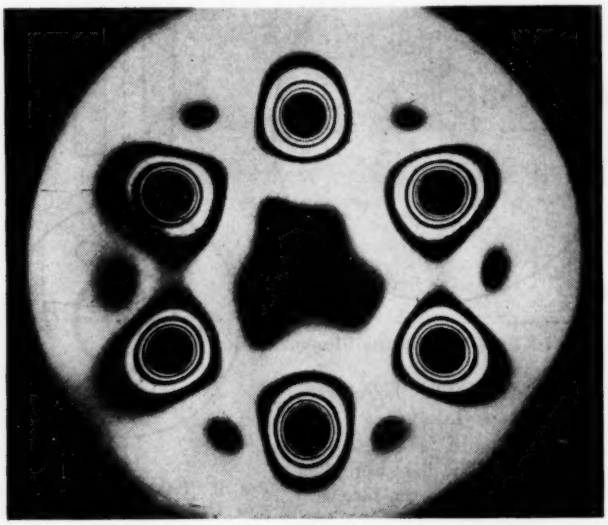


FIG. 5 FRINGE PATTERN OF MODEL *A*, ONE DISK

their mounting, the motor shaft projecting through the oven's side. Immediately after machining, the models were mounted, set in rotation, and the temperature varied in the manner shown in Fig. 4. Upon return to room temperature the models were removed from the oven, demounted, and photographed in the various desired combinations. Like tests were made upon model sets *A* and *B*.

Figs. 5 to 8, inclusive, show the successive stages in the development of the isochromatic fringe pattern for model *A*. The pictures show the fringe pattern corresponding to relative rotative speeds of 1, $\sqrt{2}$, $\sqrt{3}$, and 2, respectively. Actually they were not

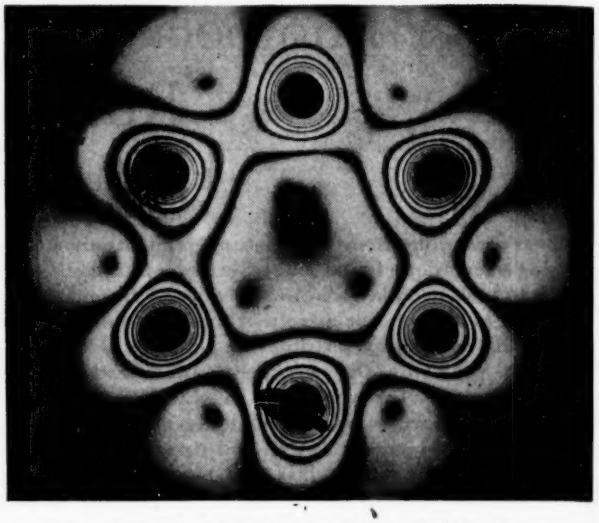


FIG. 6 FRINGE PATTERN OF MODEL *A*, TWO DISKS

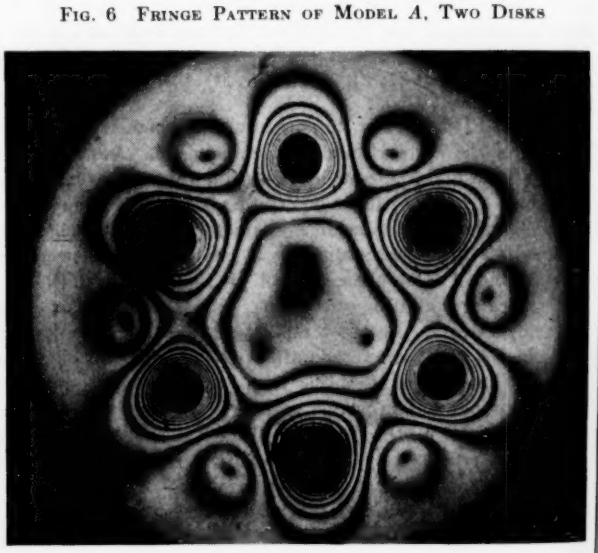


FIG. 7 FRINGE PATTERN OF MODEL *A*, THREE DISKS



FIG. 8 FRINGE PATTERN OF MODEL *A*, FOUR DISKS

obtained by varying the speed of a single model, but by superposing, successively, one, two, three, and four identical disks having the form of model *A*. Isochromatic lines appear first at the points of maximum stress which lie on the edge of each hole, approximately where the lines joining the center of the hole to the centers of the adjacent holes intersect the edge. Beginning as points, the fringes quickly become crescents and then form ovals encircling the holes. Further increase of speed causes the ovals to swell until they meet between the holes and snap apart to form a hexagon in the central portion of the disk and a rosette outside the holes. Shortly thereafter the rosette breaks at points radially beyond the holes and snaps into closed loops around the six outer isotropic points which lie between the holes a short distance beyond the circle of centers. There is also an isotropic point at the center and one radially inward from each of the free holes (the hole at the top of each picture was free). A study of the development of the isochromatic pattern in this manner overcomes one of the characteristic disadvantages of the stress-freezing method which ordinarily does not permit observing the formation of the pattern.

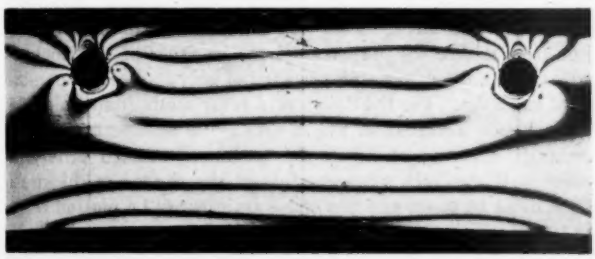


FIG. 9 FRINGE PATTERN OF A BEAM IN PURE BENDING

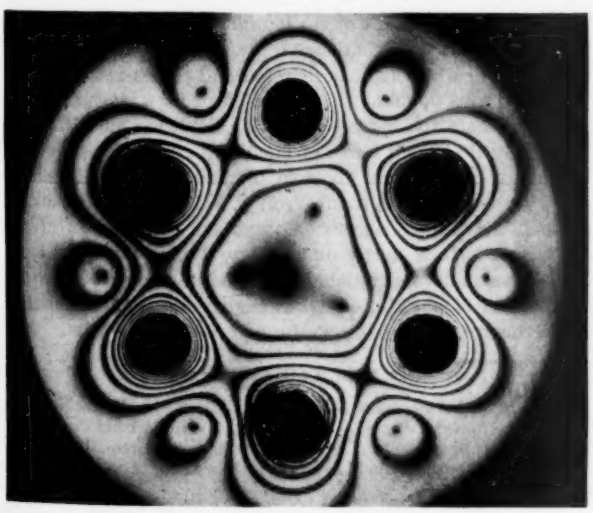


FIG. 10 FRINGE PATTERN OF MODEL *B*, TWO DISKS

Close examination discloses that the patterns around the free holes and those around the holes which held the model do not differ greatly. One may, using Saint Venant's principle, confidently apply the results obtained from a study of the patterns around the free holes without fear that the constraint of the mounting has appreciably affected these results.

An auxiliary test of a beam in pure bending was made to determine the principal stress difference represented by the first-order isochromatic fringe. A beam was made from the same plate as the models and subjected to the same annealing cycle while loaded by a pair of couples of known magnitude. Fig. 9 shows the residual fringe pattern. From elementary theory of strength of materials the stress-optic constant was computed to be 3.73 lb per sq in. per in. per fringe.

Because of the difficulty of accurately lining up four disks, the pattern of two disks, shown in Fig. 6, was used for the quantitative determination of the maximum stress. A similar pattern for model *B* is shown in Fig. 10. The maximum order of line appearing around the three free holes was determined from the original photographic plates by use of a magnifier. It was found to be 9.0 and 9.7 for models *A* and *B*, respectively. The total thickness of the model in each case was 1.046 in., so that the corresponding stresses were 32.2 and 34.6 lb per sq in., respectively.

The effect of increasing the size of hole is to increase markedly the maximum shear stress (shown by the order of the isochromatic fringe) throughout the parts of the disk away from the holes (compare Figs. 6 and 10), whereas the stress around the holes increases only slightly.

APPROXIMATE THEORY

In order to provide some basis for comparison of the photoelastic results with theory, an approximate theoretical solution was constructed. The starting point was the assumption that the thickness of the disk was sufficiently small compared with its diameter that a plane state of stress might be expected. Such a hypothesis is justified by a study of the exact solutions available for a disk of uniform thickness (4) and a disk having the form of a flat ellipsoid of revolution (5). On the basis of this assumption Timoshenko (6) gives the following solution for a solid disk of uniform thickness

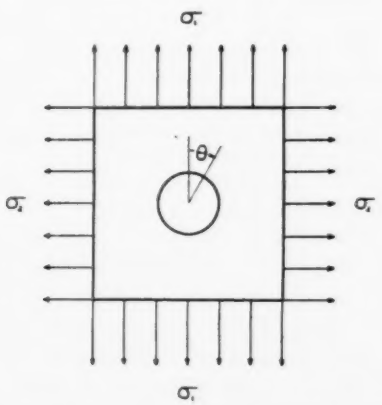


FIG. 11 A BODY, WITH A HOLE IN THE TENSION FIELD, SUBJECTED TO UNIFORM TENSIONS AT RIGHT ANGLES TO EACH OTHER

$$\left.\begin{aligned} \sigma_r &= \frac{3+\nu}{8}\rho\omega^2(b^2-r^2) \\ \sigma_t &= \frac{3+\nu}{8}\rho\omega^2 b^2 - \frac{1+3\nu}{8}\rho\omega^2 r^2 \end{aligned}\right\} \quad \ldots\ldots\ldots [1]$$

where σ_r and σ_t are, respectively, the radial and tangential stresses; ν is Poisson's ratio; ρ is the density of the disk; ω is the angular velocity; b is the radius of the disk; and r is the distance from the center to the point under consideration.

In order to estimate the effect of a hole not at the center we consider a body subjected to uniform tensions σ_1 and σ_2 at right angles, as shown in Fig. 11. If we designate the tangential stress at the edge of the hole by σ, we may compute σ by superposing the results for an infinite plate with a hole under tension and the results for the same plate with the tension at right angles to the first (7). This gives

$$\sigma = \sigma_1 (1 - 2\cos 2\theta) + \sigma_2 \left[1 - 2\cos 2\left(\theta - \frac{\pi}{2}\right)\right]$$

$$= \sigma_1 + \sigma_2 - 2(\sigma_1 - \sigma_2)\cos 2\theta \ldots\ldots\ldots\ldots [2]$$

If we assume that $\sigma_1 > \sigma_2$ and designate the maximum value of σ by σ_m putting $\theta = \pm(\pi/2)$, we obtain

$$\sigma_m = 3\sigma_1 - \sigma_2 \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

We may now obtain an approximate solution for the maximum stress on the periphery of a noncentral hole by neglecting the variation of the tangential and radial stresses in the neighborhood of the hole and replacing σ_1 and σ_2 in Equation [3] by σ_t and σ_r, respectively, from Equation [1]. If the center of the hole is at a distance a from the center of the disk, we have

$$\sigma_m = 3\sigma_t - \sigma_r = \rho\omega^2 \left\{ [(3 + \nu)/4]\, b^2 - \nu a^2 \right\} \ldots\ldots [4]$$

Comparing this result with Equations [1] we see that we shall expect the maximum stress to be found at either the inner or outer edge of the holes. The magnitude of the stress computed by Equation [4] is 27.6 lb per sq in. As noted previously, the maximum stress actually occurs at points on the edges of the holes about 60 deg from the innermost points and has the magnitude 32.2 and 34.6 lb per sq in. for models A and B, respectively. We may explain the quantitative discrepancy by noting that the stress should be greater than that predicted by Equation [4] because the holes reduce the net cross section of the disk around the circle of centers.

We may attempt, in a naïve way, to take account of the effect of this reduction of cross-sectional area by multiplying the value of the maximum stress given by Equation [4] by the ratio of the cross-sectional area along the circle of centers before drilling the holes to the area after drilling. This gives

$$\sigma_m = \rho\omega^2 \left[\frac{3 + \nu}{4} b^2 - \nu a^2 \right] \left(\frac{2\pi a}{2\pi a - 6d} \right) \ldots\ldots [4a]$$

The assumption here is that the form of the stress distribution is unaltered by the reduction of area but that the maximum stresses are in the inverse ratio of the areas. This is clearly not the case and we should expect the stress to redistribute itself in such a way that the maximum value would be less than that predicted by Equation [4a]. Actually this effect occurs and the stresses of 36.3 and 43.0 lb per sq in. for models A and B, respectively, computed by Equation [4a] are higher than the actual stresses.

The appearance of the isotropic points is easily understood. The one at the center needs no comment. In an unpierced disk the tangential stress exceeds the radial stress everywhere except at the center, where they are equal, as seen from Equations [1]. The effect of the holes is to increase both the tangential and radial stress near the holes. Since, however, the isotropic points are ones at which the radial and tangential stresses are equal, it follows that the increase of the radial stress is the greater and this results in the appearance of isotropic points not at the center. The nonappearance of isotropic points radially inward from the holes in which the disks were supported is explained by the fact that the deformation of the disks causes the rubber bushings of the mounting to exert an inward radial pressure at these holes. The effect of this compression was to reduce the radial tension below the tangential-tension value.

Inspection of Equations [1] and [4] shows that the stress distribution depends upon the value of Poisson's ratio. For materials commonly used in rotor construction this value ranges between 0.25 and 0.30. In bakelite at 230 F it is 0.50. This large difference fortunately affects the maximum stress but little. Computing the maximum stress from Equation [4], we find that the value obtained differs by less than 4 per cent when the extreme values 0.25 and 0.50 are used. It is further reassuring to note that the stress in the bakelite will be greater than that in the corresponding metal rotor so that a design based on photoelastic data will certainly be safe. A rather thorough investigation of the effect of the value of Poisson's ratio upon the stress distribution by Coker and Filon (8) also led to the conclusion that the error introduced could safely be neglected.

Conclusions—Acknowledgments

The technique utilized in this problem is applicable to a great variety of other problems involving centrifugal stress. Problems in which gravitational body forces are replaced by centrifugal forces have already been attacked stroboscopically (9). They can now be handled with greater facility by stress freezing. This technique obviates use of elaborate loading devices necessitated by the requirement that photoelastic specimens be observed under load. Hetényi has discussed its applicability to problems of three-dimensional stress (3).

Certain refinements of technique suggest themselves for subsequent experiments. In the present investigation, although the maximum shearing stress was determined throughout the rotor (this is directly proportional to the order of the isochromatic fringe), the direct stress was found only at the free boundaries of the holes. For the design of a rotor made from a ductile material such information would usually be sufficient since the maximum shearing stress is the customary design criterion. However, if auxiliary measurements of the change in thickness of the disks be made and the stress trajectories be plotted from the isoclinic lines, it would be possible to determine completely the state of stress throughout the rotor (2). Because of the small value of the modulus of elasticity (E = 1070 lb per sq in.) and the large value of Poisson's ratio (ν = 0.50) at the elevated temperature, the amount of the lateral deformation is large and easily measured with satisfactory accuracy. The use of a polariscope in which light is reflected and passes twice through the model, thus doubling the optical effect for a given thickness would eliminate the necessity of using duplicate models. It is quite likely that other forms of mounting can be devised which will permit even greater freedom of movement of the model.

The author wishes to thank Prof. F. W. Bubb for his encouragement, his helpful suggestions, and for his criticism of the manuscript. He is deeply indebted to E. H. Sager for innumerable valuable suggestions. He wishes also to thank the departments of physics, electrical engineering, and mechanical engineering for the use of apparatus and research facilities.

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A Rational Definition of Yield Strength

By W. R. OSGOOD,[1] WASHINGTON, D. C.

Explicitly or implicitly the yield strength of a material is often used as a measure of incipient structural damage. With the yield strength determined by conventional methods, however, it cannot be said in general for two structural elements geometrically alike but of different materials that similar loads, producing maximum stresses equal to the yield strengths in the two cases, are simply related to the yield strengths. A definition of yield strength is proposed in this paper which often has the advantage that, for geometrically similar structures of different materials, loads producing maximum stresses equal to the yield strength are proportional to the yield strength.

IT IS generally recognized that most definitions of yield strength are arbitrary. Except in the case of the simplest stress-strain diagram composed of two straight lines, the modulus line and a horizontal (approximated in mild steel), the commonly used definitions of yield strength or yield point have limited physical significance. This difficulty is likely to inhere in any definition of yield strength, but there is an objection to most definitions which can be avoided or mitigated in many cases. When the yield strength is used as a measure of maximum allowable deformation of a structural element, no simple relation exists in general between the damaging deformations obtained with two materials having different stress-strain curves, except under conditions of pure tension or compression. Where it is a question of failure by instability in the plastic range, no simple relation exists in general between the critical loads obtained with two such materials. By defining suitably the yield strength, however, simple relations may be found between the damaging deformations or between the critical loads. These statements become clear as to deformation when, for example, bending is considered.

Suppose that a section of a straight beam is subjected to a bending moment M such that, in the extreme tensile fiber, a yield strength S_1 is just reached. Then from Fig. 1 we may write

$$M = \int_{-c_2}^{c_1} sybdy$$

Since $y = \dfrac{he}{e_1 - e_2}$, this equation may be written

$$M = \frac{h^2}{(e_1 - e_2)^2} \int_{e_2}^{e_1} bsede$$

where e is the strain in the fiber at the distance y from the neutral axis, and e_1 and e_2 are the strains in the extreme tensile and compressive fibers, respectively. Now consider a second beam of the same cross-sectional dimensions but of different material subjected to a moment M' which produces a tensile extreme fiber stress equal to the yield strength S_1' of this material. The relation between bending moment and extreme fiber strains e_1' and e_2' for this beam is

$$M' = \frac{h^2}{(e_1' - e_2')^2} \int_{e_2'}^{e_1'} bs'e'de'$$

The two moments M and M' will be simply related only if a simple relation exists between the stress-strain curves of the materials. If the two stress-strain curves are affinely related, that is, the one curve could be got from the other by the transformation

$$e' = \alpha e \qquad s' = \beta s \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where α and β are constants, then we should have for the second beam

$$M' = \frac{h^2}{\alpha^2(e_1 - e_2)^2} \int_{e_2}^{e_1} b\beta s\alpha^2 ede$$

$$= \frac{\beta h^2}{(e_1 - e_2)^2} \int_{e_2}^{e_1} bsede$$

or

$$M' = \beta M \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

the simplest possible relation between the two bending moments. It is seen from Equation [1] that with S_1 the yield strength of the first material and S_1' the yield strength of the second, it is possible to define yield strength in such a way that $\beta = S_1'/S_1$. It will be explained later how yield strength must be defined in order that β will have this value.

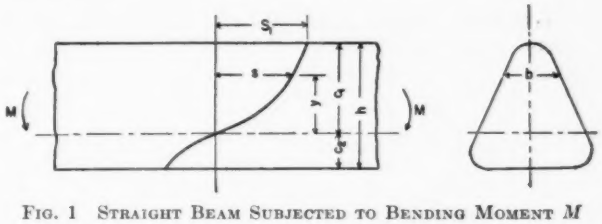


FIG. 1 STRAIGHT BEAM SUBJECTED TO BENDING MOMENT M

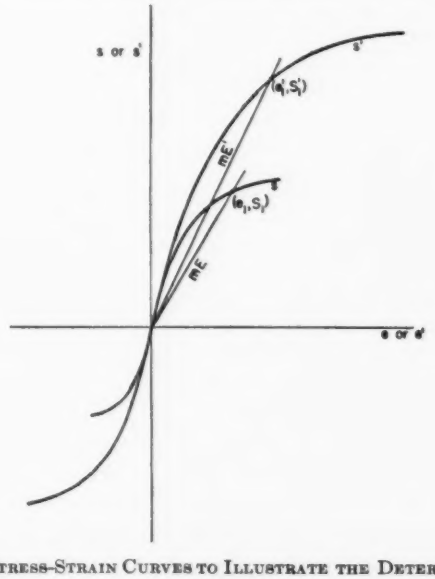


FIG. 2 STRESS-STRAIN CURVES TO ILLUSTRATE THE DETERMINATION OF α AND β

[1] National Bureau of Standards.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

The determination of α and β is illustrated by Fig. 2, which shows two stress-strain curves, related by Equations [1]. We have for any corresponding points

$$\frac{s'}{e'} = \frac{\beta}{\alpha} \times \frac{s}{e} \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

This equation states that β/α is in the ratio of the secant moduli for corresponding points, which in itself is not of much help. At the origin, however, this ratio is E'/E, the ratio of the moduli of elasticity, a known quantity. Since Equation [3] is an identity, in that it holds for all corresponding points, we see now how to identify such points. The secant moduli for corresponding points on the stress-strain curves must be in the ratio of the moduli of elasticity. Corresponding points, therefore, are the points of intersection with the stress-strain curves of lines through the origin (secant-modulus lines) having slopes proportional to the moduli of elasticity, such as the lines in Fig. 2 with the slopes mE and mE', where $0 < m < 1$. The values α and β are now seen immediately to be given by

$$\alpha = \frac{e_1'}{e_1} = \frac{S_1'/(mE')}{S_1/(mE)} = \frac{S_1'E}{S_1E'} \qquad \beta = \frac{S_1'}{S_1} \ldots\ldots\ldots [4]$$

where the points (e_1, S_1) and (e_1', S_1') are the points of intersection mentioned. The values α and β do not depend upon m because any value of m between 0 and 1 determines corresponding points, and by the hypothesis of Equations [1] α and β are constants. If the stress-strain curves are single straight lines, α and β may have any values subject to the condition $\beta/\alpha = E'/E$.

It now becomes evident how yield strengths must be determined if Equation [2] is to hold. By referring to Equation [4] for β, it is clear that the yield strength must be defined as the stress represented by the intersection with the stress-strain curve of a line through the origin, having a slope m times the modulus of elasticity of the material, where $0 < m < 1$.

Examination of a considerable number of (compressive) stress-strain curves of different materials has shown that, for a given material (as supplied commercially, for example), the relations in Equations [1] do often in fact actually hold to a good degree of approximation. In order to determine whether two stress-strain curves are affinely related, as indicated in Equations [1], it is only necessary to draw several secants with slopes m_1E, m_2E, ... m_nE for the one curve and with slopes m_1E', m_2E', ... m_nE' for the other curve. If the two curves are affinely related, it will be found that the ratios of corresponding strains e_1'/e_1, e_2'/e_2, ... e_n'/e_n are all equal, equal to α, let us say; and that ratios of corresponding stresses s_1'/s_1, s_2'/s_2, ... s_n'/s_n are all equal, let us say equal to β.

The choice of m for determining yield strength is largely a practical matter. If m is taken less than $^1/_2$, the stress-strain curve of the material will have to be obtained for greater strains than is now customary. On the other hand, if m is taken near to 1, the yield strength so determined will be a stress below which little plastic action has occurred.

This method of determining yield strength (in compression) has been used by the author for some years to compensate for variations in test results[2] due to variations in the mechanical properties of the materials of the test specimens. It was being used by the Heintz Manufacturing Company, Philadelphia, Pa., in 1936, according to information received by the author at the time from F. M. Lambert of that company.

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Experimental and Theoretical Investigation of a Turbine Foundation

By SERGIUS VESSELOWSKY,[1] DETROIT, MICH.

This paper contains the results of a study made on a turbine foundation for the purpose of elucidating conditions which favor the building up of excessive vibrations when the unit is in operation. The structure studied was a steel framework supporting a concrete table with a turbogenerator erected on it. The experimental part consists of (*a*) dynamical loading of the structure and (*b*) determination of its elastic constants. The theoretical treatment of the problem consists of the determination of the frequencies and the modes of the natural vibrations by analytical methods. A comparison of the theoretical results obtained with the experimental findings is made. The results of the study indicate that vibrations of a turbogenerator on its foundation, for units of conventional design, can be treated as vibratory motion of a deformable body supported by a system of massless springs. Some rules to be followed in designing and means of changing a critical frequency in a desired sense also are included.

The present investigation was undertaken in an effort to eliminate the uncertainty which exists in the design of the foundations for turbogenerators, particularly where a steel substructure is used.

THE first part of the paper consists of a description of tests which were conducted on a foundation supporting a 30,000-kw turbogenerator of conventional design operating at 1800 rpm. The aim of the experimental investigation was (*a*) to locate various critical frequencies at which natural vibrations occur in this type of structure and to find their corresponding modes and (*b*) to determine some of the elastic constants of the structure. This is followed by an analytical treatment of the same problem.

The foundation used for the tests was formed of four steel portals, erected on the principal mat of the Conners Creek powerhouse of The Detroit Edison Company and a heavy concrete table on which the machine was assembled. The mat of the building is a strong plate of reinforced concrete over 6 ft thick and resting on wooden piling of some 25 ft depth. This construction insured a complete immobility of the base of the steel portals supporting the machine. Not even the slightest vibrations were noticeable at these points. The soil under the powerhouse is Detroit blue clay and has permitted a total settlement of the mat from 1924 to 1935 of 2 in.

Figs. 1, 2, and 3 reproduce the structure. Its principal dimensions are: Total length, 47 ft; height of the table over the condenser floor, about 28 ft; width of the concrete table, 28 ft.

The table is a heavy plate of concrete reinforced by a network of steel beams and is mechanically isolated from the rest of the building. On this table is assembled the turbogenerator. The connection between the table and the machine was exceedingly strong. Rigidity of this connection was a very important feature determining the possible deformations of the unit and therefore the modes of the natural vibrations of the system.

Experimental Studies

Natural vibrations of the structure were studied first experimentally by means of a dynamic load. This brought to light critical frequencies of several natural vibrations of the system and revealed their modes, i.e., the configurations through which the system was moving when vibrating. Dynamic loads consisting of harmonic impulses of known direction and frequency were applied at various points while observing the vibrations produced at every frequency in different parts of the structure. These impulses were produced by four vibrators. Each vibrator consisted of a strong frame, supporting between two ball bearings a shaft on which a radial arm was fixed. This arm had two symmetrical openings—one from each extremity. When an unbalance was artificially created, in the form of a bolt inserted into one of the holes of the arm, a rotational radial force due to centrifugal action came into play.

Placing two vibrators, one against the other and letting them run in opposite directions, it was possible to produce only horizontal or only vertical impulses of double intensity.

Each vibrator was rotated by a separate synchronous motor of 7½ hp through a chain drive, which gave an electrically "rigid connection" between different vibrators. All of the motors which operated the vibrators were fed from a common motor generator set, the speed of which could be varied. Thus, it was possible to change the frequency of all the impulses given by the vibrators to the structure without altering their phase relations.

Placing vibrators at different points on the slab and changing the phase difference between individual motors and their vibrators, it was possible to have any desired phase sequence of different impulses in such manner as to give to the slab any desired dynamical loading. Vibrators were bolted very solidly to the table and embedded in about 4 in. of concrete to prevent any local vibrations.

To check the phase sequence of various vibrators already rotating, a stroboscope was used, as well as a special arrangement consisting of sets of contacts attached to the rotating arms of the vibrators so that the correct phase relation of vibrators was made visible at low speed by special signal lights.

The rate of energy consumed by the motor driving the vibrator was measured by wattmeters and their readings plotted against the speed. Noticeable peaks corresponding to critical frequencies were not found on this curve as had been expected.[2] Evidently the greatest part of the energy dissipation was due to

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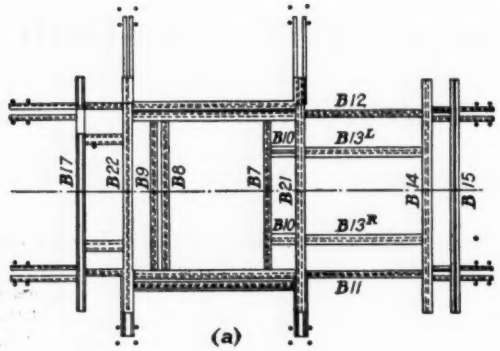
[1] Research Department, The Detroit Edison Company.

Presented at the Fourth National Meeting of the Applied Mechanics Division of The American Society of Mechanical Engineers, Pittsburgh, Pa., June 11–13, 1936. This paper is part of a dissertation submitted in partial fulfillment of the requirements for the degree of doctor of science at the University of Michigan.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until July 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] "Technische Schwingunslehre," by W. Hort, J. Springer, Berlin, Germany, 1922, pp. 193–194.



(*a*) to the "softness" of the structure as compared with the strength of the shaft and (*b*) to the fact that the rotating masses form less than one tenth of the whole vibrating system. This can be checked on the known formulas for the critical frequencies of the turbine on an elastic base.[3]

Elastic reactions of the structure, generated by various deformations, were determined either experimentally or analytically. Reactions of the steel framework caused by horizontal deflections were determined experimentally. For this purpose turnbuckles were attached to the columns of the building on one side and to the foundation on the other, at points *A*, *B*, and *C* shown in Fig. 5, with accurate dynamometers inserted between. By tightening the turnbuckles, it was possible to produce any desired intensity of the force measured by the dynamometers. Deflections were measured by two microscopes rigidly fixed to the table at the

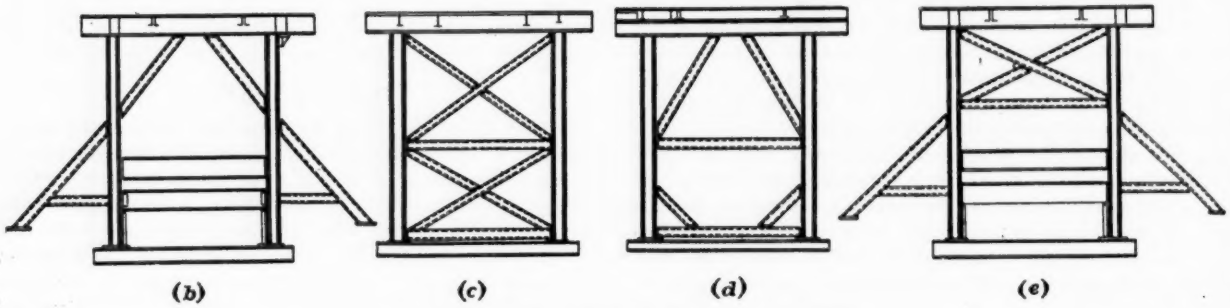


FIG. 1 THE STEEL FRAMEWORK OF THE FOUNDATION

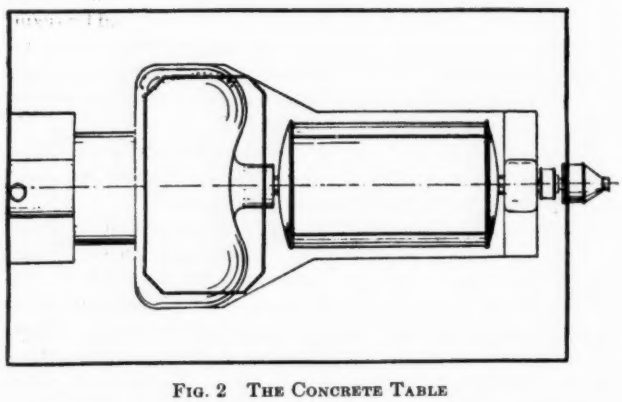


FIG. 2 THE CONCRETE TABLE

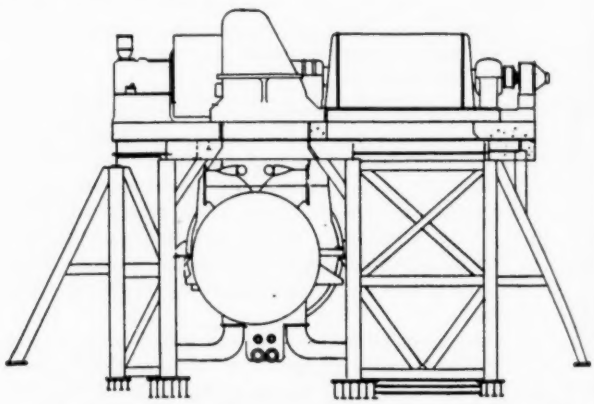


FIG. 3 THE TURBOGENERATOR AND FOUNDATION ASSEMBLY

air resistance of the rotating arms but not due to friction in the structure. The amplitudes of vibrations were measured by a recording Geiger vibrograph.

For every combination of impulses the observed amplitudes were plotted versus the frequency with the setup as described previously. A number of sharp peaks were observed on the curves representing the amplitudes of vibrations (horizontal or vertical) at various points on the slab plotted against the variable frequency of the impulses given to the structure, evidently corresponding to critical frequencies of some modes of vibrations, as shown in Fig. 4.

The results obtained coincided very closely with those obtained by other experimenters in their tests made in other powerhouses of The Detroit Edison Company on various units identical with or similar to that experimented upon by the author. The difference in the experimental setup was that their tests were made, not with the vibrators, but using the unavoidable unbalance of the rotating part, which causes impulses in the bearings of the unit. Evidently the gyroscopic effect of the rotating masses on the whole vibrating system is negligible. This is due

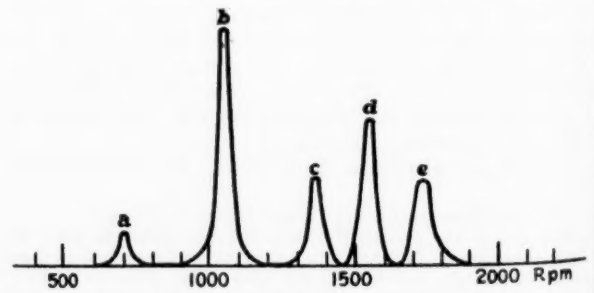


FIG. 4 COMPOSITE FREQUENCY-AMPLITUDE CHARACTERISTIC OF THE UNIT AS OBSERVED

[(*a*) Weak vertical vibrations at the middle of the unit. (*b*) Violent vertical vibrations of the exciter end. (*c*) Vertical vibrations over the entire length of the table with amplitudes increasing away from the axis of the unit. (*d*) Horizontal vibrations especially pronounced in the generator part. (*e*) Vertical vibrations especially pronounced in the turbine part.]

[3] "Steam and Gas Turbines," by A. Stodola, McGraw-Hill Book Co., Inc., New York, N. Y., 1927, pp. 1135–1136.

points D and E, and focused on two immovable scales rigidly attached to special light columns erected on the condenser floor and free from any deflections of the slab. Magnification of the microscopes was sufficient to measure displacements with an accuracy of not less than 0.002 mm. Knowing the displacements of the points D and E, it was possible for every load to determine the deflection of the center of gravity of the total mass and, in the case of the transversal loading, rotation of the longitudinal axis, because the table was not deformed under the action of the applied forces.

Forces in the longitudinal direction were varied from 0 to 20,000 lb, while those in the transverse direction were changed from +10,000 to —10,000 lb.

In most cases when the turbine is directly coupled to the

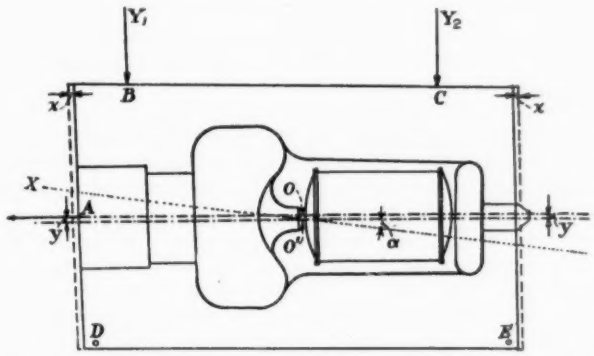


FIG. 5 SETUP OF THE STATIC EXPERIMENTS ON THE UNIT—THE MODE OF LATERAL VIBRATIONS WITH THE TABLE RIGID IS INDICATED

generator, a symmetry exists in the foundation and in the mass distribution of the whole unit with respect to the vertical plane passing through the shaft of the machine. Only the piping could give some asymmetric reactions, but in the present case the asymmetric members were of negligible rigidity. Another feature of the structure studied was the fact that the longitudinal and the vertical deflections of the table were independent of one another. Therefore, the elastic reaction X of the structure in the axial direction could be represented as proportional to the corresponding displacement x. From experiments it was found that

$$X = 5.74 \times 10x \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

wherein X is expressed in pounds per inch.

In the case of a horizontal force Y, applied at the centroid O in the transverse direction, not only would this point be deflected y inches from its position of equilibrium, but simultaneously the whole table would be rotated through an angle α, because the resulting reaction of the structure would not, in the general case, pass through the point O; inversely to any pair of values y and α would correspond a transverse force applied at O and a moment M acting around the vertical axis through O.

The statical tests made it possible to establish relations between these quantities. For the foundation investigated these relations could be represented by two linear equations

$$Y = (3 \times 10^6 y) + (170 \times 10^6 \alpha) \ldots\ldots\ldots\ldots [2]$$

$$M = (170 \times 10^6 y) + (178 \times 10^9 \alpha) \ldots\ldots\ldots\ldots [3]$$

where y is measured in inches and α in radians. Therefore, the potential energy stored in the structure, after a deflection determined by coordinates y and α, was

$$V_s = \left(3 \times 10^6 \frac{y^2}{2}\right) + (170 \times 10^6 y\alpha) + \left(178 \times 10^9 \frac{\alpha^2}{2}\right) \ldots [4]$$

ANALYTICAL DETERMINATION OF THE VERTICAL RIGIDITY OF VARIOUS PARTS OF THE STRUCTURE

The important features of the structure on which the computations were based are the following: (1) The bottom of the steel structure remained immovable, being fixed to the principal mat of the powerhouse. (2) The foundation and the unit were symmetrical with respect to the vertical plane passing through the shaft. (3) The concrete table was assumed rigid if acted upon by forces contained in its plane, therefore, the top ends of the columns could be deflected only as a movement of the whole unit. (4) The forces acting on the structure during the vibrations were symmetrical harmonic impulses of high frequency. (5) The knots of the steel structure are assumed to be perfect joints.

It follows from (2) that under the action of vertical forces symmetrically distributed over the table, no transverse deflection of the unit will take place. Further, it can be concluded from (4) that an axial deflection of the table, acted upon by vertical impulses, will also be impossible because the natural frequency of the longitudinal vibrations of the unit (435 cycles per min) is much lower than that of the impulses taken place during vertical vibrations (1050 or 1735 cycles per min). The top ends of the columns, therefore, undergo deflections only in the vertical direction; they are vertically guided. In this case a distinction is to be made between a "statical" loading of a structure and a "dynamic" one, in which inertia forces may become very important.

Based upon these premises and using familiar methods of the theory of frames, the spring constants of four portals shown in Fig. 1 were computed, assuming every portal independent of the rest of the structure. With the modulus of elasticity of steel equal to 30×10^6 lb per sq in., these constants were found: $K_1 = 11.5 \times 10^6$; $K_2 = K_3 = 8.5 \times 10^6$ lb per in.; and $K_4 = 10.5 \times 10^6$ lb per in.

Masses of different parts of the unit and their moments of inertia with respect to various axes and planes were determined partly by direct weighing of these members and partly by geometric measurements and computations. This was possible on disassembling the unit and demolishing the whole structure when, for example, the density of the concrete of the table was determined by weighing blocks of concrete in air and in water. Centroids of some of the parts were determined by weighing them, using various points of attachment. The greater part of the combined masses and of the total moments of inertia came from the widespread concrete table of the unit. Its simple geometric form made the computations quite accurate. The fact that most of the parts of machinery are bodies of revolution facilitated the computations.

The vibrating system under study was composed of the turbogenerator, rigidly connected with the table, the condenser, and the steel substructure. The weight of the machine and of the table was about 1,100,000 lb; the condenser which participated in the vertical vibrations had a weight of over 250,000 lb. On the other hand, the weight of the substructure was only about 50,000 lb, of which only 17,000 lb were to be added to the vibrating masses. It is evident that the vibratory motion of the unit can be considered as that of a body, rigid or deformable as it may be, on a system of massless springs. Other important features of the structure were the independence of the vertical and the horizontal vibrations from one another and the symmetry of the structure and of the mass distribution with reference to the vertical plane passing through the shaft of the machine.

It follows from this that if the table were rigid the vibratory motion of the unit would have, in the general case, six degrees of freedom, but in the case under consideration this motion can be split into four independent modes of vibrations with no more than two degrees of freedom each. These modes are (1) longitudinal vibrations with just one degree of freedom when the unit vibrates back and forth in the axial direction with purely translatory motion; (2) rolling vibrations also with one degree

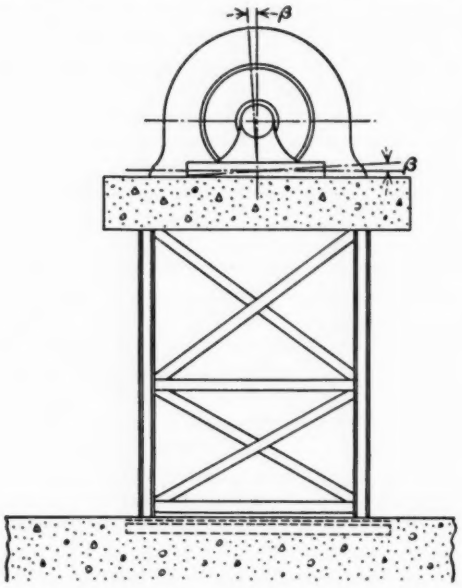


FIG. 6 ROLLING VIBRATIONS OF THE UNIT

of freedom around the horizontal axis passing through the common centroid parallel to the shaft as shown in Fig. 6; (3) horizontal transverse vibrations with two degrees of freedom, given by the variation of the linear transverse deflection y of the common centroid of the unit and the rotation α of the table about the vertical axis passing through the centroid (yawing) as shown in Fig. 5; (4) the vertical vibrations also with two degrees of freedom, determined by the vertical deflections of the centroid in the plane of symmetry and the rotation φ of the table about the horizontal axis passing through the centroid and perpendicular to the shaft (pitching).

The analytical determination of the frequencies corresponding to the modes of vibrations described was done according to the methods established in the theory of vibrations.[4] For the unit under study the frequency of the longitudinal vibrations was determined to be 435 cycles per min, while transverse vibrations were found to have frequencies of 297 and 435 cycles per min. The highest frequency found was that of the "rolling" vibrations. The computations gave a value of 1295 cycles per min; observations during dynamic tests have given the value of 1375 cycles per min.

Experimental verification of these results by a dynamical loading of the structure was not possible, except for the rolling of the unit, because the remaining frequencies for the case of a rigid unit were too low and the impulses of the vibrators were too weak at these speeds. But the observations on three different sizes of turbogenerators of The Detroit Edison Company during the periods of putting them on the line revealed these frequencies and confirmed that in accordance with the analytical findings they are very low for the units of the conventional design, that is, with the machine assembled on a high substructure and with the condenser and the air cooler beneath.

Vertical vibrations of the unit observed during the tests were accompanied by very obvious deformations of the table. Therefore, no computations are given here for the case of a rigid table vibrating vertically. The treatment of a more general case, i.e., of the vertical vibrations of the unit with the table bending in its middle part is discussed later.

CASE OF DEFORMABLE TABLE

Vibration of a turbogenerator with its table rigid, assembled on a system of massless springs, would be a vibratory motion with six degrees of freedom. The dynamic tests showed that the table did not remain rigid either in the horizontal or in the vertical direction. The vibratory motion of a deformable unit of the type studied can be described with sufficient accuracy as having eight degrees of freedom, one additional coordinate each being necessary to describe the configuration of the deformed unit in both horizontal or vertical vibrations.

The fact that the table underwent a noticeable deformation during both vertical and horizontal vibrations does not influence the separation of the general vibratory motion into four mutually independent modes due to symmetry of the structure as described. Thus, the longitudinal vibrations occurred as previously and with no deformation of the table in this mode. The same is to be said about the rolling of the unit. It is important to note that the modes of natural vibrations which can have frequencies approaching the operating speed of the unit (1800 or 3600 rpm) will be only those combined with the deformations of the unit. This makes it imperative to analyze what deformations of the unit are probable.

A characteristic feature of the design of the turbines, used by The Detroit Edison Company, is the great rigidity of the turbine shell and its strong connection with the table. In the case studied by the author the lower half of the turbine shell, weighing over 50,000 lb, was deeply imbedded in the concrete of the table and bolted solidly to its reinforcing beams. Therefore the turbine part of the unit remained rigid during both the vertical and the horizontal vibrations, in spite of the great opening in the table for the exhaust steam. The remaining parts of the system underwent deformations and these were different in the case of the horizontal or the vertical vibrations.

What can be the deformation of the system in the first case under the action of a horizontal force applied at the exciter bearing, as an unbalanced rotor would act? It can be easily seen that only the bedplate of the generator adds to the strength of the table, being immersed in its concrete. The generator itself does not add much to the strength of the corresponding part of the table because the stator, in spite of its impressive weight (130,000 lb), has a very low flexural rigidity against forces acting perpendicular to its axis, being constructed of some 50 bundles of stampings of laminated iron sheets, with as many air ducts in between, and held together by a very light frame. This explains why the table acted upon by a horizontal force applied at the exciter end was not only rotated on the steel substructure but was also deformed in its plane all over the generator length. By the reasons explained later the condenser did not participate in these deformations.

Therefore, our system vibrating horizontally consisted of the machine and the table, and assumed the shape represented in Fig. 7*b*.

The deformations observed during the vertical vibrations of the unit were quite different. As can be seen from Fig. 8*a*, the vibrating system, consisting of the turbine with its condenser, the generator, and the table, is particularly characterized by a great reduction of its flexural rigidity in the indentation between

[4] "Vibration Problems in Engineering," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., second edition, 1937, art. 40.

the turbine shell and the generator. This was due (1) to the very configuration of the turbogenerator representing two different machines coupled together; (2) to the presence of a bolted joint between the lower exhaust shell of the turbine and the bedplate of the generator; and (3) to the discontinuity of the longitudinal steel beams *B10-B13*, which are cut twice by the transverse beams *B21* and *B7*, as shown in Fig. 1*a*. The rigidity of the unit in this indentation, compared with that in other vertical cross sections, was so small that it can be considered as a very flexible joint.

Another cross section, having a very low rigidity was the hood connecting the exhaust shell of the turbine with the condenser. The great inertia reaction of the massive condenser (250,000 lb weight) at the frequencies observed (1050 and 1735 cycles per min) caused a bending of the hood, which is built as a frustum of a pyramid 60 in. in height with very thin walls (1 in.) bolted to the turbine shell and to the condenser. It can be proved easily that the hood was too flexible to transmit horizontal components of the vibrations of the condenser body. Therefore the condenser was not vibrating during horizontal vibrations of the unit. On the contrary, the vertical component of the inertia reaction cannot cause a noticeable deformation of the hood and, as a result, the centroid of the condenser followed the vertical movements of the point B of the turbine shell as shown in Fig. 8*b*.

Finally it can be assumed with a great degree of approximation that the deformations of the unit were plane deformations, since the cross beams of the table were not deformed. The horizontal transverse and the vertical vibrations of a unit with the deformable table are treated in the following manner.

Horizontal Vibrations (Transversal)

On placing the vibrator near the exciter bearing and letting it run so that horizontal impulses perpendicular to the shaft direction were given to the slab, a mode of vibration was found with a critical frequency of nearly 1550 cycles per min having highest amplitudes at the middle part. The observed vibrations were horizontal and were combined with an evident deformation of the table, as represented in Fig. 7*a*.

Transverse vibrations of a rigid table, as explained, occur as motion with two degrees of freedom. The deformation of the table increased the degrees of freedom of the unit by the number of coordinates which were necessary to determine this deformation. Using the approximation suggested in the treatment of similar problems by Lord Rayleigh,[5] the elastic line of the deformed vibrating generator part was assumed to be identical with the elastic line caused by the action of a horizontal transverse force applied to the exterior bearing. An unbalanced rotor would act in this way. Then the deformation of the whole generator part could be determined by just one coordinate, i.e., by the linear deflection d of the end point D from its position on the rigid table. The deflection of any intermediate point (from its position on the rigid table) would be equal to kd, where k is a dimensionless factor fully determined by the (assumed) shape of the elastic line. Therefore, in the case of a deformable table, the transverse vibratory motion of the unit is determined kinematically by the three coordinates, y, α, and d, transverse deflection of the centroid, rotation of the table, and its deformation at the exciter end, respectively.

Placing the zero of the coordinates axes in the centroid O of the system so that the X-axis coincides with the shaft of the machine and the Y-axis is directed as shown in Fig. 7*b* the following relations can be established between the coordinates ξ_i,η_i of the point P at any instant and its coordinates ξ,η in the equilibrium position

$$\xi_i = \xi - \eta\alpha; \quad \eta_i = y + \eta + \xi\alpha + kd$$

By differentiating with respect to time, and squaring and adding, the velocity v of the point P can be expressed as

$$v^2 = \dot{y}^2 + r^2\dot{\alpha}^2 + k^2\dot{d}^2 + 2\xi\dot{y}\dot{\alpha} + 2k\dot{y}\dot{d} + 2\xi k\dot{\alpha}\dot{d} \quad \ldots\ldots [5]$$

where r is the distance of the point P from the centroid; the three coordinates y, α, and d have the same values for all points of the unit; and ξ and k are constant for a transverse strip of the table.

Dividing the generator part into ten transverse strips and knowing the mass m contained in each of them, it is possible to compute the kinetic energy of the vibrating unit

$$T = \frac{1}{2}\Sigma mv^2 = \frac{1}{2}My^2 + \frac{1}{2}I\dot{\alpha}^2 + \frac{\dot{d}^2}{2}\Sigma mk^2 + \dot{y}\dot{d}\Sigma mk + \dot{\alpha}\dot{d}\Sigma mk\xi \quad \ldots\ldots [6]$$

The first two terms in the expression for T are identical with those obtained for a rigid table; the last three, containing the

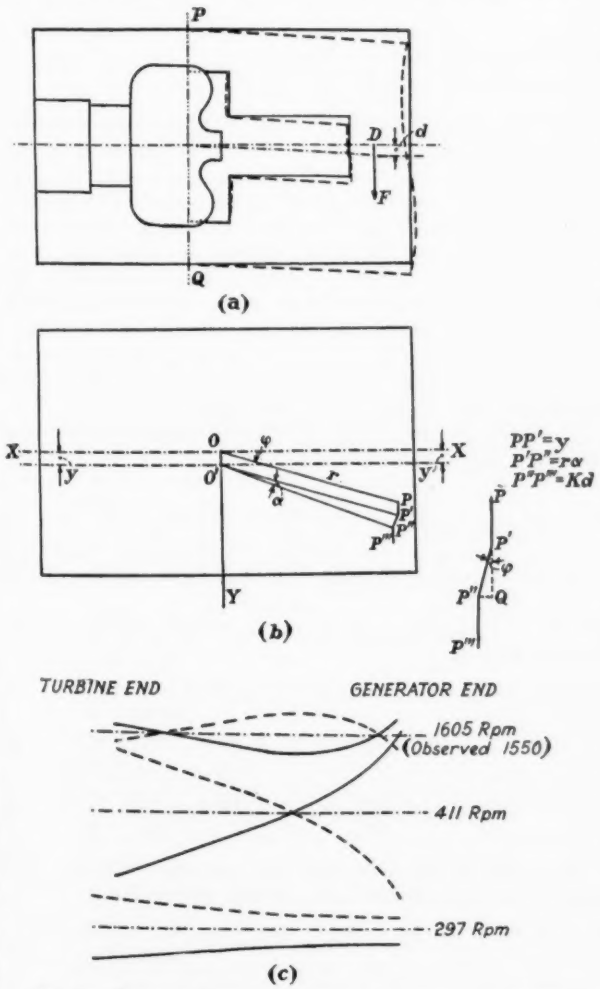
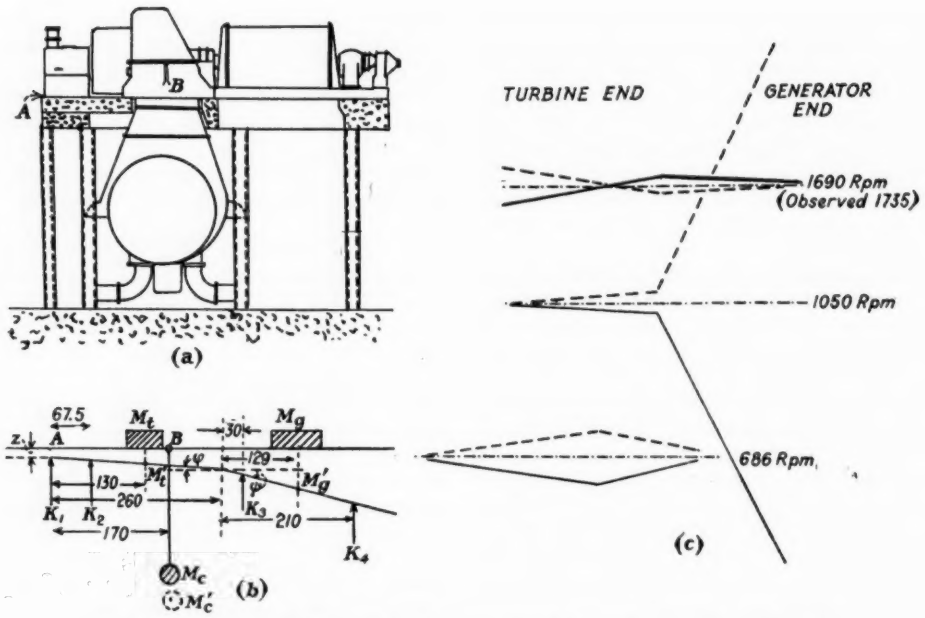


Fig. 7*a* Lateral Vibrations With a Deformable Table

Fig. 7*b* Components of the Deflection During Lateral Vibrations

Fig. 7*c* Modes of the Lateral Vibrations

[5] "Theory of Sound," by Lord Rayleigh, vol. 1, Macmillan and Company, London, 1896.



FIGS. 8a AND 8b THE UNIT AND ITS DEFORMATIONS DURING VERTICAL VIBRATIONS
FIG. 8c MODES OF THE VERTICAL VIBRATIONS

factor k, are the result of the deformation of the generator part. For the rigid turbine part they are equal to zero for all points. The term $\Sigma m\xi$ is equal to zero with the ordinate axis passing through the centroid of the system.

The rigid turbine part of the unit extended up to the cross section PQ, determined by the configuration of the lower very strong shell of the turbine shown in Fig. 7a. The expressions Σmk^2, Σmk, and $\Sigma mk\xi$ for the unit studied were computed to be, respectively, equal to 840, 1000, and 175,000; the dimensions of the first two figures are lb-mass; of the last lb-mass multiplied by inch. The total mass M of the vibrating system was 2760 lb-mass and the moment of inertia I with respect to the vertical axis through the centroid was computed to be 91×10^6 lb-mass $\times$ in.^{2} Thus the kinetic energy T of the vibrating system was equal to

$$T = 2760\frac{\dot{y}^2}{2} + \left(91 \times 10^6 \frac{\dot{\alpha}^2}{2}\right) + 840\frac{\dot{d}^2}{2} + 1000\,\dot{y}\dot{d} + 175{,}000\,\dot{\alpha}\dot{d} \ldots [7]$$

The potential energy of the deformed structure consisted of two parts: (1) The energy V_s stored in the steel structure, as determined by the static tests and corrected for the additional deflection of the fourth portal due to deformation d of the table; and (2) the energy V_c stored in the deformed concrete table. The potential energy stored in the steel substructure after the table had undergone a deflection determined by the coordinates y and α was determined from static tests.

Deformation of the generator part affected only the portal supporting the exciter bearing. The spring constant of this portal in the transverse direction was computed equal to 0.8×10^6 lb per in. It is evident that the deflection (y, α) caused a reaction in the fourth portal equal to $0.8 \times 10^6\,(y + l\alpha)$, where l is the distance of this portal from the centroid (187 in. in this case). A deflection d increases this reaction to the value $0.8 \times 10^6\,(y + l\alpha + d)$. Therefore, the potential energy stored in the portal due to the deflection d was equal to $0.8 \times 10^6\,[y + l\alpha + (d/2)]d$.

The energy V_c stored in the deformed concrete table was determined analytically by computing the deformation of the plane frame as represented by Fig. 7a. To the left of the cross section PQ the table was rigid, at the point D it was acted upon by a horizontal, transverse force. The angle connections were assumed rigid and areas and moments of inertia of various members were computed from the drawings, assuming the ratio of the moduli of elasticity for steel and concrete equal to ten (average for tension and compression). The numerical value of V_c for this table was $1/2 \times 1.1 \times E_c d^2$, where E_c is the modulus of elasticity for the concrete and d is the deflection of the point A; E_c can vary[6] from 2.25×10^6 up to 3.50×10^6 lb per sq in. Thus V_c can be written as $1.1 \times B \times 10^6(d^2/2)$, where B is a numerical factor varying from 2.25 to 3.50.

Therefore, the potential energy V of the system in the position determined by the coordinates y, α, d is

$$V = \left(3 \times 10^6 \frac{y^2}{2}\right) + (170 \times 10^6 y\alpha) + \left(178 \times 10\,\frac{\alpha^2}{2}\right) + \left[0.8 \times 10^6\left(y + 187\alpha + \frac{d}{2}\right)d\right] + \left(1.1 \times 10^6 B\,\frac{d^2}{2}\right) \ldots [8]$$

Forming Lagrange's equations of the vibratory motion, we obtain the system

$$\left.\begin{aligned} &2760\ddot{y} + 1000\ddot{d} + (3 \times 10^6 y) + (170 \times 10^6\alpha) + (0.8 \times 10^6 d) = 0 \\ &(91 \times 10^6\ddot{\alpha}) + 175{,}000\ddot{d} + (170 \times 10^6 y) + (178 \times 10^2\alpha) + (150 \times 10^6 d) = 0 \\ &1000\ddot{y} + 175{,}000\ddot{\alpha} + 840\ddot{d} + (0.8 \times 10^6 y) + (150 \times 10^6\alpha) + 10^6(1.1B + 0.8)d = 0 \end{aligned}\right\} \ldots [9]$$

Proceeding in the usual way[7] and assuming for E_c the value 2.75×10^6, the frequency equation was obtained in the form

$$p^6 - 31{,}081p^4 + (81 \times 344 \times 536p^2) - (50 \times 533 \times 647 \times 960) = 0 \ldots [10]$$

[6] Data about the values of E_c can be found in "Die Druckelastizität und Zugelastizität des Betons," by Otto Graff, *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, No. 227, 1920. Also "Modulus of Elasticity of Concrete," by S. Walker, Bulletin No. 5, Lewis Institute, Chicago, Ill.

[7] Loc. cit., footnote 4, art. 37.

giving for the circular frequency p the values 168; 43 and 31, or 1605; 411 and 297 cycles per min.

The dynamic tests have established only the highest of the computed critical frequencies, i.e., the highest one at 1550 cycles per min. The two others were too low to be noticeable under the impulses of the vibrators used. However, natural vibrations (transverse horizontal) of such low frequencies were observed on several units in the powerhouses of The Detroit Edison Company.

Substituting every one of the values of p^2 in the system of Equations [9], the constant ratios of amplitudes Y, A, D (corresponding to coordinates y, α, d) were found for every p^2. This made possible the establishment of the mode of the vibrations corresponding to this frequency. For $N_1 = 1605$ and for an arbitrary value of Y, the values of A and D are given by the equations

$$A = Y/183 \quad \ldots\ldots [11]$$

$$D = 2.7Y \quad \ldots\ldots [12]$$

The shape of the deformed table as determined by these relations is represented by Fig. 7*c*. Analogously, for $N_2 = 411$, $A = Y/35.5$ and $D = 2.6Y$. Finally, for $N_3 = 297$, indicated in Fig. 7*c*, $A = -y/531$ and $D = 0.04Y$.

As can be seen from Fig. 7*c* for $N_1 = 1605$, the greatest amplitudes were to be expected in the middle of the table. The ends of the table were almost immovable, because at the turbine end rotations almost annihilated the transversal vibrations and at the exciter end the deflections due to flexibility of the table were equal and opposite to the combined deflections caused by the translation and rotation. This explained very well the observations made during dynamic tests (compare part *d* of Fig. 4).

It is interesting to note that the modes of the slower vibrations of 411 and 297 cycles per min were very similar to those of a rigid table.

Vertical Vibrations of the System

The dynamic tests showed that at about 1735 cycles per min the maximum of the amplitudes was observed near the exterior bearing of the turbine with much smaller ones at the exciter. At about 1050 cycles per min, the greatest amplitudes of vibrations took place at the points near the exciter end of the generator, the turbine part up to the middle of the unit remaining almost immovable. Finally, at about 700 cycles per min, the maximum of the amplitudes was observed in the middle of the unit. The last vibrations were very weak due to the insufficiency of the impulses given by the vibrators at this speed. In any case, there was no doubt that a bending of the structure occurred during both vibrations somewhere near the middle of the unit.

The instantaneous position of the vibrating system is fully determined by three coordinates; the linear deflection z of the slab at the point A, shown in Fig. 8*b*, and the two angles φ and ψ formed by each half of the table with the horizontal, shown in Fig. 8*b*. The positions of the three bodies forming the vibrating system, i.e., of the turbine part, of the generator part, and of the condenser part, are readily expressed as functions of these coordinates. Knowing all masses, their centroids, and the moments of inertia, the kinetic energy of the system can be determined in terms of the coordinates chosen.

The elastic elements of the system were formed by the four supporting portals. The potential energy was determined from their computed spring constants and their deflections. The table was assumed perfectly flexible in its middle part.

Referring to the configuration of the deformed system shown in Fig. 8*b* with the dimensions indicated, the potential energy V and the kinetic energy T of the vibrating system can be written as

$$V = {}^1/_2\, k_1 z^2 + k_2(z + 67.5\varphi)^2 + k_3(z + 260\varphi + 30\psi)^2 + k_4(z + 260\varphi + 210\psi)^2 \ldots [13]$$

$$T = {}^1/_2 M_t(\dot{z} + 130\dot{\varphi})^2 + {}^1/_2 I_t\dot{\varphi}^2 + {}^1/_2 M_g(\dot{z} + 260\dot{\varphi} + 129\dot{\psi})^2 + {}^1/_2 I_g\dot{\psi}^2 + {}^1/_2 M_c(\dot{z} + 170\dot{\varphi})^2 \ldots [14]$$

Here M_t denotes the mass of the turbine and the supporting part of the table equal, as computed, to 1260 lb mass; M_g is the mass of the generator with the corresponding part of the table equal to 1512 lb mass; M_c is the mass of the condenser equal to 650 lb mass; I_t denotes the moment of inertia of the mass M_t with respect to the horizontal transverse axis passing through the centroid of M_t, and was computed equal to 12×10^6 lb $\times$ in.^{2}; I_g is the moment of inertia of the mass M_g with respect to the horizontal transverse axis passing through its centroid, and was computed equal to 14×10^6 lb $\times$ in.^{2} With these values of the coefficients, the expressions for V and T become

$$V = \frac{10^6}{2}(39z^2 + 1{,}323{,}000\varphi^2 + 471{,}000\psi^2 + 2z\varphi\ 5500 + 2z\psi\ 2460 + 2\varphi\psi\ 640{,}000) \ldots [13a]$$

$$T = {}^1/_2[3420\dot{z}^2 + 154.3 \times 10^6\dot{\varphi}^2 + 39 \times 10^6\dot{\psi}^2 + 2\dot{z}\dot{\varphi}\ 667{,}000 + 2\dot{z}\dot{\psi}\ 195{,}000 + 2\dot{\varphi}\dot{\psi}\ 51 \times 10^6] \ldots [14a]$$

Knowing V and T, Lagrange's equations for the vibratory motion of the system are readily derived, and for harmonic variations of the coordinates with a circular frequency p, the following system is obtained:

$$\left.\begin{aligned}(39 - 0.00342p^2)Z + (5500 - 0.667p^2)\Phi + (2460 - 0.195p^2)\Psi &= 0\\ (5500 - 0.667p^2)Z + (1{,}323{,}000 - 154.3p^2)\Phi + (640{,}000 - 51p^2)\Psi &= 0\\ (2460 - 0.195p^2)Z + (640{,}000 - 51p^2)\Phi + (471{,}000 - 39p^2)\Psi &= 0\end{aligned}\right\} \ldots [15]$$

Thus, the frequency equation becomes

$$p^6 - 48{,}574p^4 + 603{,}044{,}000p^2 - 1{,}956{,}477{,}030{,}000 = 0 \ldots [16]$$

The three roots of this equation are $p_1^2 = 31{,}309$, $p_2^2 = 12{,}100$, and $p_3^2 = 5164$; therefore, the corresponding critical frequencies are 1690, 1050, and 686 rpm. The values observed during the tests were 1735, 1050, and 700 rpm, respectively. Substituting each one of the found values of p^2 into the system of Equations [15], the corresponding mode of natural vibration was determined. Fig. 8*c* represents the configuration of the longitudinal axis of the unit deformed in the three modes determined in this manner.

Comparison of Fig. 8*c* with the results of the dynamic tests represented in Fig. 4 (peaks *a*, *b*, *e*) shows that in the case of the vertical vibrations the agreement between the theoretical findings and the results of the experiments is very good and better than for horizontal vibrations. This is due principally to more accurate knowledge of the elastic constants of the structure.

Conclusions

The experiments and the analytical study just described show that the vibrations of the turbogenerator on its foundation of the type considered can be treated as a vibratory motion of an elastic body supported by a system of massless springs. The classical methods of analytical mechanics give very satisfactory explanation of the phenomena observed during tests of the unit.

If the steel substructure considered in this case were replaced

by a system of concrete portals, vibratory motion of such a unit can be also treated as has been done in the present study. The same method often can be applied to foundations for other types of machines, such as motor generator sets and electrically driven fans.

It is always possible to design a foundation for a steam turbine so that it will be free from resonance with the operating speed (1800 to 3600 rpm) if the table be built rigid. This is so important an advantage that it is worth while relinquishing the conventional type of table and adding to it some elements of trusses in the vertical planes to increase its rigidity.

If the table is deformable, as it usually is, and if the critical frequency falls near the operating speed, two cases are equally possible: (1) Critical frequency is below the operating speed, and (2) critical frequency is higher than the operating speed. Only in the second case will a strengthening of the structure, so often resorted to by a practical engineer, bring favorable results. In the first case such a measure can make the situation worse by bringing the critical frequency of the structure nearer to the operating speed where instead an increase of dead mass would be indicated.

Analysis of the dynamic properties of a given foundation and determination of the causes and remedies of a roughly running unit can be made only if the design of the structure is as simple as possible. Holes, recesses, and brackets must not complicate the initially simple form of a foundation to such an extent that accurate computation of the rigidity of many members is impossible. A complete consolidation must be achieved between such elements of compound members of the structure as the bed plate of the machine, the concrete slab, and its reinforcing beams.

In solving the problem analytically, satisfactory results can be obtained only by treating the vibratory motion in a most complete and rigorous way. Any attempt to simplify the problem and to treat different modes of vibrations observed as independent vibratory motions of various parts of the unit with others immovable, will, generally speaking, lead to erroneous results.

As a rule, the vibrating system must be analyzed as an integral unit. Frequencies of various modes of vibrations of the unit with their antinodes in various planes of the structure are nothing but different roots of the same "frequency equation," derived from the dynamic relations controlling the vibratory motion of the whole unit.

It would be hopeless to expect great accuracy of the numerical values obtained by an analytical treatment of the problem. They are often affected by many incertitudes, e.g., those inherent to computations of the steel trusses, or those caused either by the variations in the value of the modulus of elasticity of concrete or by the errors in the evaluation of the strength of some machine parts.

It is important to note that the computed modes of vibration do not change much with the variation of the numerical values of critical frequencies. Consequently, conclusions may be drawn, on the basis of the established mode of vibrations, about the effectiveness of any change in the structure.

Analysis of factors controlling the vibratory motion of a turbogenerator on its foundation clearly shows that many recommendations of various authorities on turbine foundations are illogical. Resonance, which ought to be carefully avoided, is determined by the elastic properties of the structure and by the masses and their distribution. Adherence to any arbitrary limiting value of static deflection or stress as prescribed by some authorities is by no means a measure of safety or a condition for success. Other authorities[8] prescribe to design the various portals of a foundation structure in such a way that their individual critical frequencies should be identical. This consideration can be completely omitted. These same experts insist on the separation of a turbine foundation from the common foundation mat of a powerhouse. In the opinion of the author, this separation is also unnecessary and can create great difficulties with piping.

The influence of various details of the machinery can be very great. For instance, if the "rigid connection" between the turbine and the condenser were to be replaced by an expansion joint, usual in Europe, the immediate result would be a substantial rise of the vertical frequency (1735 cycles per min in the case analyzed) due to the elimination from the vibrating system of a very important mass.

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[8] Directions for the Design of Steam Turbine Foundations of Reinforced Concrete (Richtlinien für den Bau von Dampfturbinen Fundamenten in Eisenbeton). Published by the German Society for Building Science, *Der Bauingenieur*, vol. 14, no. 15/16, 1933.

Critical-Speed Behavior of Unsymmetrical Shafts

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The development of large two-pole turbine generators with rotors running above the critical speed, and having long core bodies slotted for field coils, makes desirable a working familiarity with the vibrational characteristics of unsymmetrical shafts. It is the purpose of this paper to describe these characteristics which are distinctly peculiar. This problem has received some attention from previous writers (6–9),[2] who have pointed out an unstable range of operation due to dissymmetry of the shaft, and some of whom made small-scale tests for demonstration. In the present investigation, experimental work on a special rotor of $10^1/_2$ ft span and weighing over 5000 lb was carried out, to test in some detail the indications of a thorough analysis of the problem by the late V. Petrovsky of the Lynn Works of the General Electric Co. This analysis is given in outline in an Appendix.

The first part of the paper deals briefly with the static characteristics of unsymmetrical shafts, as a basis for defining the "degree of dissymmetry." The critical-speed behavior of ordinary round shafts is then reviewed and the theoretical differences due to dissymmetry are described. These are: (*a*) With moderate dissymmetry compared with the degree of damping, a *marked increase in the critical-speed amplitude*, and a *pronounced variation* in the amplitude *with the angular position of the unbalance* of the rotor; (*b*) with greater dissymmetry, beyond a limiting value, an infinite critical-speed amplitude, or rather, two infinite peaks, with unstable operation between them; (*c*) considerable distortion of the phase-angle curves, such that in general the phase angle may be much larger or smaller than 90 deg at the critical speed; (*d*) *double-frequency vibration*, if the shaft is horizontal, with an auxiliary peak, or "subcritical" at half the main critical speed.

Tests on the 5000-lb rotor are described which give general confirmation of these characteristics.

THE effects of an unsymmetrical cross section on the vibrational or "whirling" characteristics of high-speed rotors, such as long two-pole turbine generators, are remarkable—even startling, to those familiar with ordinary critical-speed phenomena but unacquainted with this special subject. Previous writers (6, 7, 8, 9) have pointed out that a rotor with an unsymmetrical shaft may have two peculiarities: (*a*) Unstable operation over a range of speed lying between the two critical speeds corresponding to the principal axes of inertia, and (*b*) double-frequency vibration, with a resonant peak at about half the ordinary critical speed. This second point has also been dealt with by other writers who have omitted all reference to the first.

The purpose of this paper is to present the peculiar theoretical characteristics of unsymmetrical shafts in relatively simple quantitative terms on the basis of an independent analysis; and to describe their general confirmation by shop tests on a special 5300-lb rotor. This subject is of current interest because of the recent development of large two-pole turbine generators having the critical speed of the rotor well below the running speed of 3600 rpm, and with the possibility of an appreciable degree of dissymmetry due to the slotting of the rotor body for the field coils.

Notation

The nomenclature used in this paper is as follows:

a = damping ratio, $= b/\sqrt{cm}$ $(= E/r_s$ for $u = 1)$
b = damping constant, force per unit of velocity
c = stiffness constant of shaft, average, $(c_1 + c_2)/2$
c_1 = stiffness constant of shaft, minimum, along weak axis
c_2 = stiffness constant of shaft, maximum, along stiff axis
d = dissymmetry ratio, defined in Equation [1]
D = damping force
E = eccentricity of center of gravity
F = resultant deflecting force on shaft
h = $(c_2 - c_1)/2$, so that $c_1 = c - h$ and $c_2 = c + h$
m = mass of rotor $= W/g$
r_g = radius of path of center of gravity
r_s = radius of path of shaft center, or deflection
$\bar{r}_s$ = reflection of r_s, as vector, in OX axis (stationary)
$\bar{r}_s'$ = reflection of r_s, as vector, in OX' axis (rotating)
r_y = radius of static-deflection circle, $(y_1 - y_2)/2$
R = radial component of deflecting force
T = tangential component of deflecting force
u = relative speed, running speed/critical speed
W = weight of rotor = mg
x = horizontal component of static deflection
y = vertical component of static deflection
y_1 = maximum static deflection, weak axis vertical
y_2 = minimum static deflection, stiff axis vertical
y_{ave} = average static deflection, $(y_1 + y_2)/2$
α = angle between deflection and weak axis
β = angle between deflecting force and weak axis
δ = angle between unbalance vector and weak axis
ω = speed, radians per sec
ϕ = phase angle between unbalance vector and deflection

NOTE: Bold-face type indicates vector quantities.

Static Characteristics of an Unsymmetrical Shaft

Some of the static characteristics of an unsymmetrical shaft are interesting and have an important bearing on the running characteristics. Fig. 1 illustrates the variation in the static deflection of a horizontal shaft, if it is rolled over slowly, the deflection being a maximum when the "weak axis" is vertical

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

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(position A) and a minimum when the weak axis is horizontal (position C). (Here the "weak axis" means the axis *along* which the stiffness is a minimum.) For intermediate positions (as at S) the shaft center traverses a *circular path* through the extreme points, and makes *two complete circuits* of this path per revolution of the shaft.

It is this variation in the position of equilibrium of a horizontal

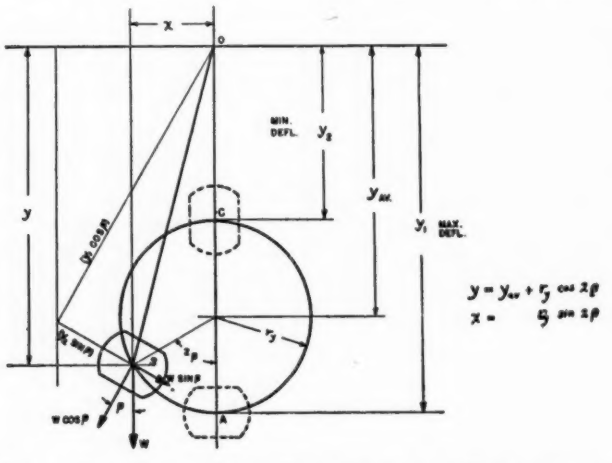


FIG. 1 VARIATION IN STATIC DEFLECTION OF AN UNSYMMETRICAL SHAFT

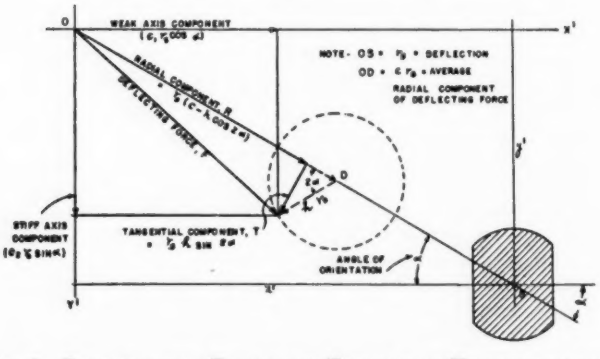


FIG. 2 COMPONENTS OF DEFLECTING FORCE ON AN UNSYMMETRICAL SHAFT, RELATIVE TO DEFLECTION OS

unsymmetrical shaft that is responsible for a disturbance when the shaft is running, substantially like that which would be due to a mechanical unbalance rotating at twice shaft speed, producing double-frequency vibration of the shaft, with a resonant peak at a shaft speed of about half the average fundamental critical speed. An analytical study of this double-frequency shaft vibration is given in the Appendix and test observations on the 5300-lb rotor will be included later.

With regard to the main critical speed, another static characteristic is of perhaps more vital concern, that is, the variation in the force required to impose a constant deflection on an unsymmetrical shaft. This is illustrated in Fig. 2. The force required to produce the deflection OS is obviously a minimum when the weak axis is in line with the deflection ($\alpha = 0$); and a maximum when at right angles ($\alpha = 90$ deg). For an intermediate position, as shown, the deflecting force F may be represented by a vector constructed by the circle diagram indicated in the figure. (See derivation in Appendix.) Note that, in general, *the deflecting force is not in line with the deflection*, therefore it has both radial and tangential components.

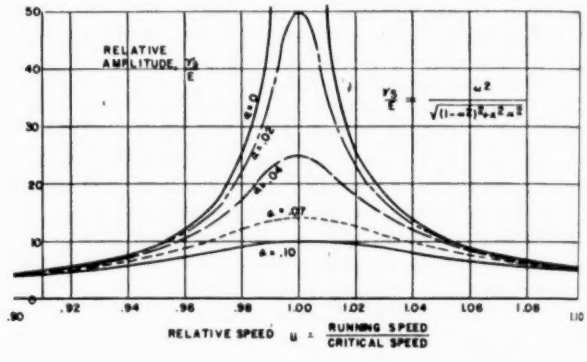
The tangential component of the deflecting force and the double-frequency variation in the static deflection are both unique characteristics of an unsymmetrical shaft. We find it convenient to use them in defining the "degree of dissymmetry" or "dissymmetry ratio," d, as follows

$$d = \frac{h}{c} = \frac{c_2 - c_1}{c_2 + c_1} = \frac{r_y}{y_{av}} = \frac{y_1 - y_2}{y_1 + y_2} \quad \ldots\ldots\ldots [1]$$

These ratios apply strictly to an ideally simple rotor; but the last two, being based on deflections, may be used as an approximate measure of dissymmetry for actual rotors.

THEORETICAL RUNNING CHARACTERISTICS

Normal critical-speed behavior (1–5) for ordinary round shafts is illustrated in Fig. 3, which shows graphs of amplitude and



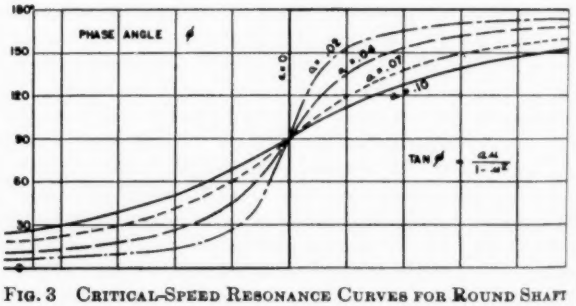


FIG. 3 CRITICAL-SPEED RESONANCE CURVES FOR ROUND SHAFT

phase angle plotted against speed. These are general resonance curves, based on the well-known expressions derived for an ideally simple rotor, slightly unbalanced and subject to linear damping. The amplitude r_y is proportional to the amount of unbalance, and is here plotted in units of the eccentricity E of the center of gravity. Speed u is on a relative basis, with the value 1.0 for the critical speed. The damping factor a, used as a parameter for the two families of curves, is also on a relative basis, as defined in the Appendix. Note that the peak amplitude at critical speed ($u = 1.0$) reaches a value which may be expressed quite simply as E/a; and that the corresponding value of the phase angle ϕ is always 90 deg.

For unsymmetrical shafts, corresponding critical-speed characteristics have been determined by an analysis given in the Appendix. From this, it appears that, subject to a certain condition, *the amplitude of the shaft may become infinite*, in spite of the presence of damping. The condition for infinite amplitude is simply that the degree of dissymmetry should be larger than the degree of damping, or $d > a$. For a shaft with moderate dissymmetry ($d < a$), there is apparently no infinite amplitude, but a number of other peculiarities. Thus we have two classes of unsymmetrical shafts to consider: One with moderate dissymmetry ($d < a$); the other with pronounced dissymmetry

($d > a$), for which the analysis indicates shaft amplitudes going to infinity.

Taking first the case of moderate dissymmetry, a set of typical resonance curves is given in Fig. 4. In contrast to Fig. 3, these are drawn for only a single value of the damping constant ($a = 0.07$). Curves for the corresponding round-shaft case are in short-dash lines for comparison. Note the importance of a new variable, the orientation angle (δ) of the center of gravity. The peak amplitude of vibration apparently varies by five to one on a rotor running with the same amount of unbalance over the same speed range, with the same damping and the same degree of dissymmetry and with changes only in the angular position of the unbalance. The highest peak appears when the unbalance is 45 deg ahead of the weak axis, in the direction of rotation; and the lowest peak, when the unbalance is 45 deg

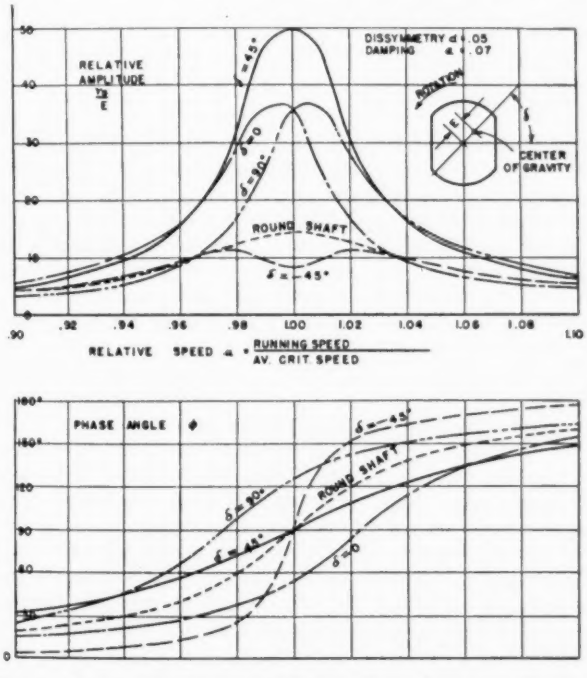


FIG. 4 RESONANCE CURVES FOR SHAFT WITH MODERATE DISSYMMETRY

behind this axis ($\delta = -45$ deg). This suggests the use of a polar diagram, Fig. 5, to show the variation of the peak amplitude with the angle of unbalance. This amplitude, it appears, may vary considerably with the direction of rotation since the most sensitive point for unbalance (45 deg ahead of the weak axis) becomes the least sensitive (45 deg trailing) if the rotation is reversed. Going back to Fig. 4 for a moment, some other peculiar effects of dissymmetry may be pointed out. The amplitude curves for the cases where the unbalance comes directly on the weak axis or the stiff axis ($\delta = 0$ or 90 deg) are somewhat distorted. It will be noted that in these cases the phase angle is materially different from 90 deg at the peaks of the amplitude curves.

Now, let us consider the case of pronounced dissymmetry ($d > a$) for which infinite peaks of amplitude are indicated. Fig. 6 is similar to Fig. 4, except for the increased dissymmetry ($d = 0.09$). It will be noted that the amplitude curves all show infinite peaks at two speeds. For zero damping, these peaks would correspond to the natural frequencies of transverse vibration in the planes of the weak and stiff axes, respectively; but with damping, they are moved closer together, until they merge into one infinite peak when $a \doteq d$. Operation between the two peaks has been shown to be unstable. The phase-angle curves show much the same peculiarities as in Fig. 5, except that in the case of unbalance at 45 deg behind the weak axis (the position of least sensitivity for the region of finite peaks)

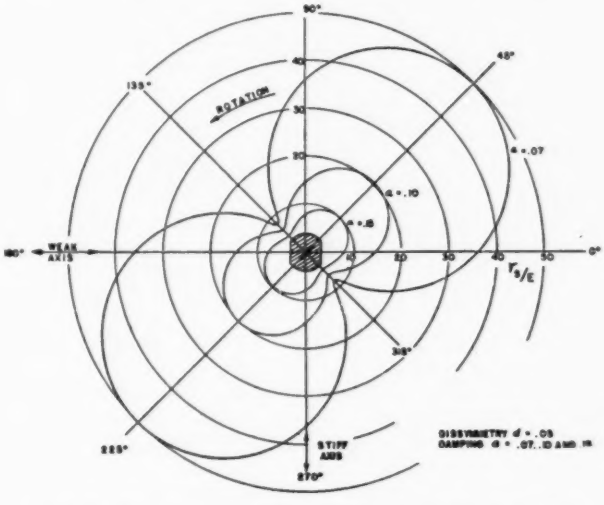


FIG. 5 AMPLITUDE VS. ANGULAR POSITION OF UNBALANCE

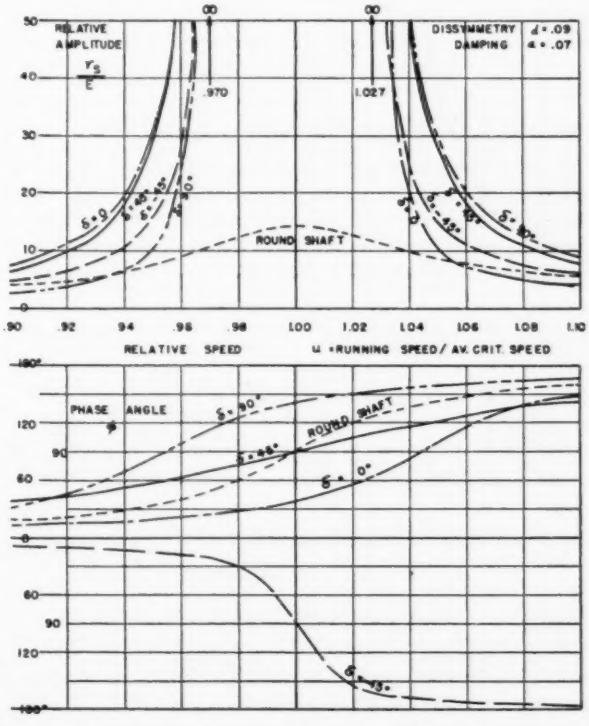


FIG. 6 RESONANCE CURVES, RELATIVELY LARGE DISSYMMETRY

the phase angle seems to be negative. That is, the deflection would seem to lead the unbalance instead of lagging behind it.

From a practical point of view, the theoretical effects of dissymmetry on the amplitude/speed curve of an ideally simple rotor may be simply shown as in Fig. 7. Here the lower dotted curve ($d = 0$) applies to a round shaft slightly out of balance, with moderate damping, over a speed range from 0 to 20 per cent above critical. The dot-dash curve ($d = 0.05$) shows the

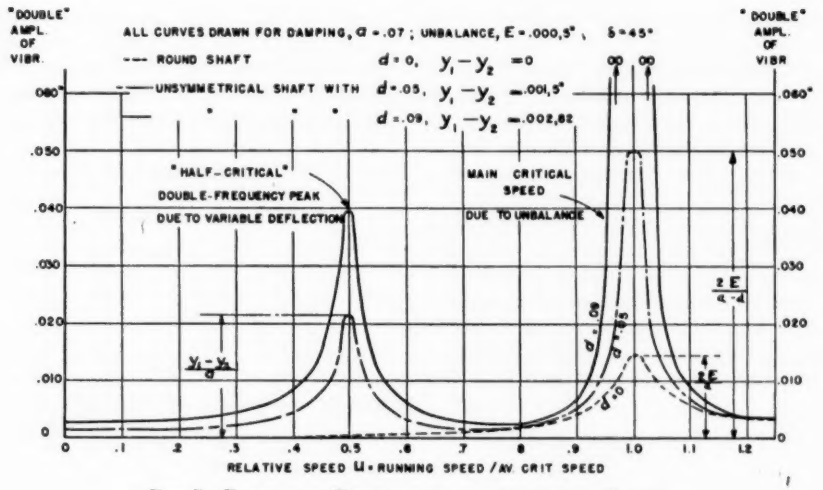


FIG. 7 EFFECTS OF DISSYMMETRY ON AMPLITUDE CURVE

effects of moderate dissymmetry, and the solid curve ($d = 0.09$) those of rather large dissymmetry, both with the unbalance located at the worst angular position ($\delta = 45$ deg). The critical-speed peak is increased materially by the moderate dissymmetry, and opens out to infinity with the larger dissymmetry. A double-frequency disturbance is also present (horizontal shaft assumed) with a resonant peak at half the critical speed, as previously mentioned. The amplitude of this vibration is determined by the variation in the static deflection of the shaft, just as the critical-speed vibration is proportional to the unbalance eccentricity E.

DESCRIPTION OF TEST SETUP

These theoretical indications were tested recently in the General Electric shops by means of a special setup with a rotor weighing 5300 lb, arranged as shown in Fig. 8. The span of this rotor was 10 ft 6 in. from center to center of bearings, and the main part of the shaft was $7^3/_4$ in. diameter. The bearings were

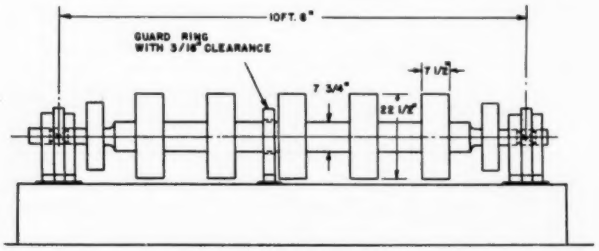


FIG. 8 TEST ROTOR SETUP

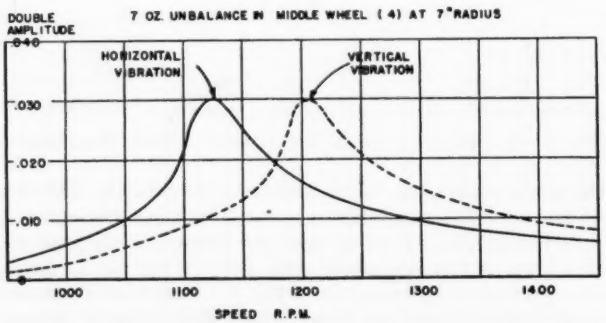


FIG. 9 AMPLITUDE CURVES OF THE TEST ROTOR WITH ROUND SHAFT

4 in. diameter by 5.25 in. long, ball-seated, and mounted in sturdy steel pedestals which were well anchored in a concrete block poured directly on the iron testing floor. The calculated critical speed (with round shaft) was 1200 rpm. For the sake of safety a guard ring was provided consisting of a dummy pedestal lined with a ring of leaded bronze, bored to a diametral clearance of $^3/_{16}$ in. over the shaft. The drive was by means of a flexible coupling from a dynamometer rated 200 kw at 4000 rpm, with Ward-Leonard speed control from a motor generator set. Vibration-amplitude readings were made with a weighted Starrett dial indicator held manually against a wooden "shaft stick" used to pick up the vibration from the shaft.

Preliminary tests were made with the shaft round to establish normal characteristics and particularly to determine the damping constant. This was needed so that the main test program could be planned to include two degrees of dissymmetry—one a little less than the damping, the other a little greater. The first step, of course, was balancing, which was carried out to a high degree of precision, such that the rotor would run gradually through critical speed with not more than 0.001 in. amplitude on the shaft.

Runs were then made with the rotor thrown out of balance by definite amounts, giving data such as shown in Fig. 9. It is interesting to note that the vertical peak checks the calculated critical speed, 1200 rpm, whereas the horizontal one comes at a lower speed, 1125 rpm. This, of course, is common experience with heavy rotors, even when set up solidly, as this one was. From these data, it was concluded that the effective damping constant a for the test setup was about 0.07. Space limitations do not permit giving the details of this determination. For the main test program, it was decided to "flat" the shaft sufficiently to make the degree of dissymmetry about 0.05 as a first step, and 0.09 as a second. Flats measuring $6^3/_8$ in. between faces were therefore cut on the $7^3/_4$ in. portion of the shaft. for the first step; and later these flats were cut down to $6^1/_4$ in. The resulting variation in the static deflection was in each case measured as a check, which agreed closely with the values anticipated.

TESTS WITH UNSYMMETRICAL SHAFT

Results from critical-speed tests carried out with the first step of "flatting," giving an apparent dissymmetry ratio of about 0.05, are shown in Figs. 10, 11, and 12. The first of these shows curves of vibration amplitude versus speed for several angular orientations of the unbalance with respect to the

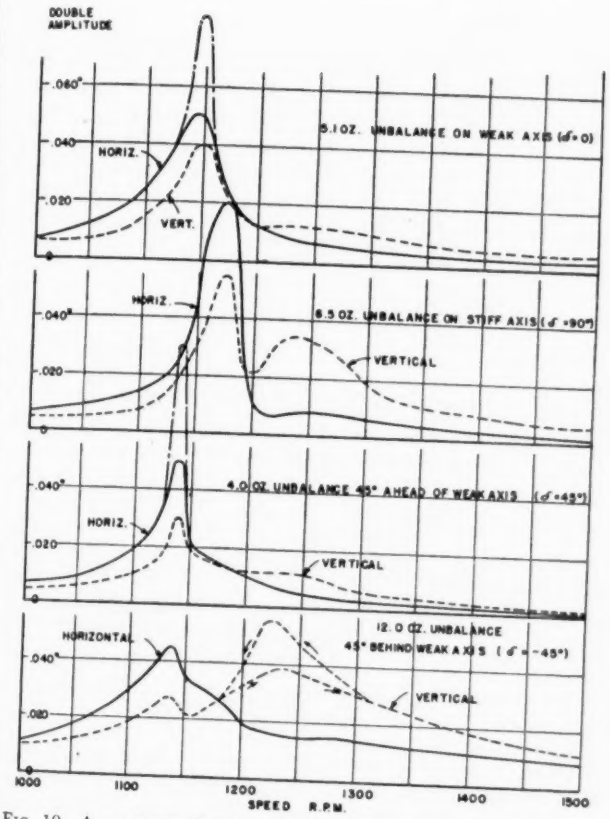


FIG. 10 AMPLITUDE CURVES OF THE TEST ROTOR WITH 6⅝-IN. FLATS ON SHAFT

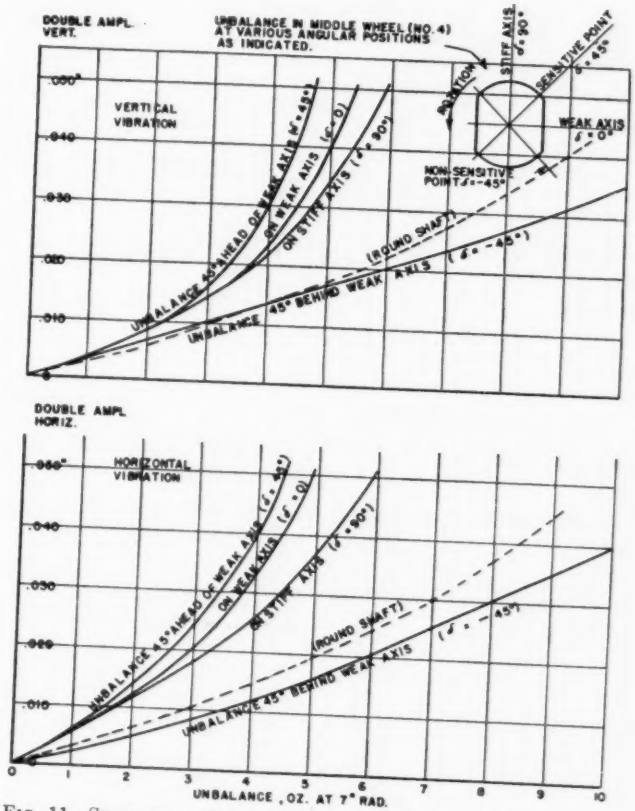


FIG. 11 SENSITIVITY CURVES OF THE TEST ROTOR WITH 6⅝-IN. FLATS ON SHAFT

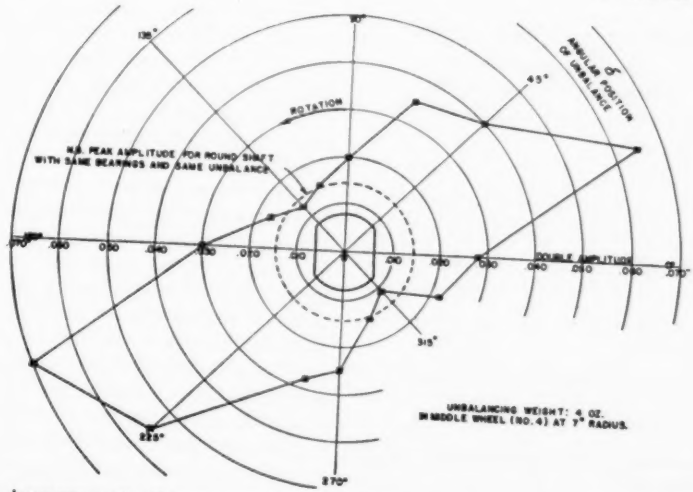


FIG. 12 PEAK AMPLITUDE VS. POSITION OF UNBALANCE FOR TEST ROTOR WITH 6⅝-IN. FLATS ON SHAFT

weak axis. In each case, the amount of the unbalance was enough to give fairly large amplitudes. For each of these angular positions, various sizes of unbalancing weights were tried in a series of runs. The peak amplitudes are plotted against the amount of unbalance in Fig. 11, as "sensitivity curves." The dotted curves here apply to the original round shaft. It is seen that three of the curves show greater sensitivity than for the round shaft—the fourth slightly less. The maximum sensitivity is apparently associated with a position about 45 deg ahead of the weak axis, and the minimum, about 45 deg behind, in accordance with theoretical indications. This is shown more clearly in Fig. 12, which represents in polar form the peak amplitudes obtained from another series of runs in which a 4-oz unbalancing weight was tried successively in each of 16 angular positions equally spaced around the middle wheel. Fig. 12 may be compared with the outside curve in Fig. 5. It may be noted that the shapes are not exactly similar, and yet the test rotor does actually show a 5/1 variation in sensitivity, depending only on the angular position of the unbalance. Also the positions of maximum and minimum sensitivity agree very well

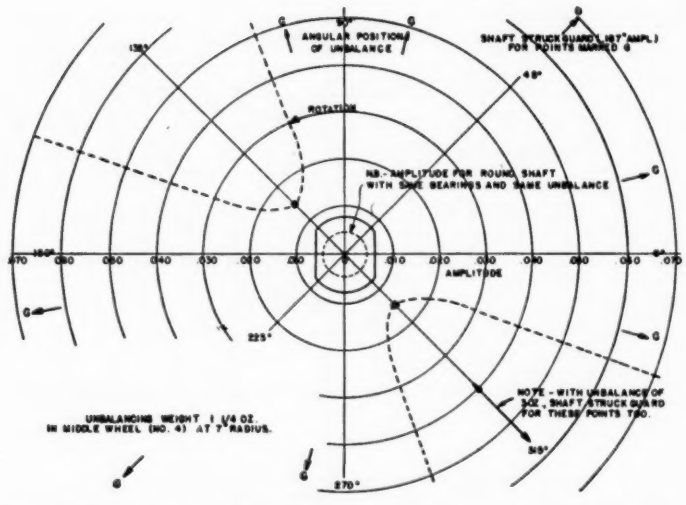


FIG. 13 PEAK AMPLITUDE VS. POSITION OF UNBALANCE FOR TEST ROTOR WITH 6¼-IN. FLATS ON SHAFT

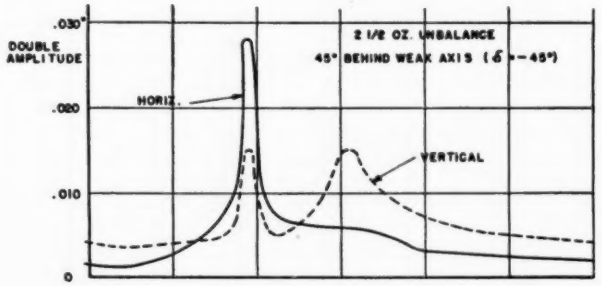


FIG. 14 AMPLITUDE CURVES FOR TEST ROTOR WITH 6¼-IN. FLATS ON SHAFT

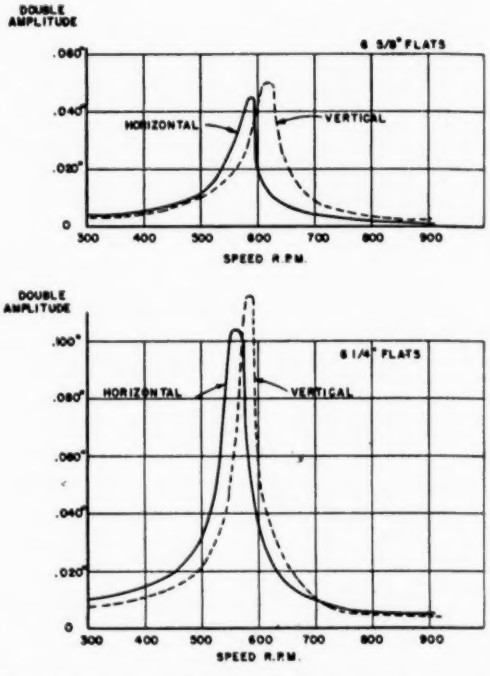


FIG. 15 DOUBLE-FREQUENCY PEAK OF VIBRATION AT HALF OF CRITICAL SPEED

with those theoretically indicated. As a matter of interest, reversed-rotation runs were made with unbalance at one of the 45-deg positions; they demonstrated an equally striking variation in sensitivity with the direction of rotation.

Considerable difficulty was encountered in securing the fine degree of balance prerequisite to the foregoing results. The good balance previously obtained for the round shaft had been spoiled in machining the flats, and rebalancing was necessary. The method used was systematic trial and error, based on critical-speed amplitude. Results were irregular, and it was finally necessary to make a great many runs with minor variations in amount and position of the weight in order to obtain a really close balance. The reason for the difficulty was of course that the critical-speed amplitude was no longer a good measure of the amount of "resultant" unbalance, but depended to a large extent on its angular position also.

When the shaft was flatted to 6¼ in. for the second step, to give $d = 0.09$, rebalancing was again necessary, and this time gave even more trouble than before. In fact, it was very nearly given up as hopeless, but after long-continued effort with very small changes, the rotor would finally run through the critical speed gradually with only 2 or 3 mils maximum amplitude.

The rotor was now so sensitive to a small unbalance that as little as 1.25 oz put on at any but the least sensitive positions (45 deg behind the weak axis) would cause the amplitude to build up at critical speed until the shaft actually rubbed on the guard ring. This building up of amplitude was at first gradual as the speed was increased little by little, but then more and more rapid until at a certain speed the shaft seemed to "shoot out" until it hit the guard. When this happened, the speed was immediately reduced, and the vibration would quiet down. Fig. 13 shows the results of trying this 1.25-oz weight at various angular positions. The shaft hit the guard, at 0.187 in. double amplitude, in every run except two—those in which the weight was located 45 deg behind the weak axis. The little circle at 0.0045 in. double amplitude shows the amplitude caused by the same unbalance on the same rotor before the shaft was flatted.

Greater unbalance was tried at these nonsensitive points, and it was found that 3 oz were sufficient to cause the shaft to build up and hit the guard ring even there. The maximum unbalance which could be safely carried at these points seemed to be about 2.5 oz. Amplitude-speed curves for this condition of unbalance are shown at the top of Fig. 14. The lower curves in the same figure show the results of an attempt to get corresponding curves for a 2.5-oz unbalance at one of the most sensitive positions, 45 deg ahead of the weak axis, by taking data as usual up to the point where the amplitude became excessive, and

then reducing speed enough to have the shaft quiet down again, with the idea that by using full power we could get rapid acceleration and possibly get past the critical-speed range before the vibration should become prohibitively rough; the purpose being then to take data above the critical range, and finally shut down again with rapid deceleration. This scheme did not work, as on rapid acceleration the vibration built up almost instantly to guard amplitude, and before the speed could be brought down again the shaft had rubbed heavily, producing much smoke and considerable heat distortion of the shaft. In spite of the fact that we had successfully run through the critical speed with rapid acceleration a few times with 1.25 oz of unbalance at the same position, it was felt that this was rather dangerous and it was not attempted again.

The double-frequency vibration associated with variable static deflection was conspicuous in all of these tests at speeds well below the main critical and, as expected, exhibited resonant peaks both horizontally and vertically at about half the corresponding main critical speeds. These are shown in Fig. 15 where the upper curves apply to the first step of flatting (d = 0.05) and the lower ones to the second step (d = 0.09). This double-frequency vibration became less and less noticeable at speeds above the range shown, being of no consequence at 1200 rpm and absolutely negligible at 2000 rpm and higher. The peak amplitudes for this "half critical" were not affected by changes in balance.

Summary and Conclusions

The peculiar vibrational characteristics of unsymmetrical shafts first pointed out by earlier writers have been further investigated both by an independent analysis and by tests on a special rotor representing a very large two-pole turbine-generator. Analysis and tests have agreed, even as to quantitative details, that shaft dissymmetry produces two important effects. First, a double-frequency disturbance is introduced, due to the variation in static deflection, which produces a peak of resonant vibration at about half the regular critical speed. Second, the normal behavior at the main critical speed is entirely upset, the sensitivity to unbalance being greatly increased and made a function of the angle at which the unbalance happens to lie on the rotor. In case of extreme dissymmetry, it appears that a rotor might prove to be unmanageable at critical speed.

As far as the writer knows, no difficulty of the second type has so far been experienced on commercial turbine generators, even of the largest sizes. And the double-frequency stator vibration noticeable on many of them disappears when excitation is removed, so that its source is magnetic rather than mechanical. The possibility of these mechanical difficulties seems very real, however, and full acquaintance with them should be of value to both designers and users. The obvious safeguard, since the damping factor is in general unknown, is to build high-speed machinery with a minimum of shaft dissymmetry.

Appendix

This appendix includes (*a*) an elementary derivation of useful expressions for the radial and tangential components of the force required to impress a given deflection on an unsymmetrical shaft; and (*b*) an analysis, based on the work of the late V. Petrovsky, for the motion of an ideally simple rotor consisting of a single heavy disk wheel mounted centrally on an unsymmetrical shaft, and running in rigidly supported bearings.

Radial and Tangential Components of Deflecting Force

Referring to Fig. 2, and the context of the paper, note that on resolving the deflection, $OS = r_s$, into components parallel and perpendicular to the weak axis, the corresponding components of the deflecting force are $c_1 r_s \cos \alpha$ and $c_2 r_s \sin \alpha$, as shown. Each of these may be resolved into radial and tangential components, which when added respectively give the total radial force

$$R = r_s (c_1 \cos^2 \alpha + c_2 \sin^2 \alpha) \quad [2]$$

and the total tangential force

$$T = r_s (c_2 - c_1) \sin \alpha \cos \alpha \quad [3]$$

Now introducing $c = (c_2 + c_1)/2$, and $h = (c_2 - c_1)/2$ we have

$$R = r_s (c - h \cos 2\alpha) \quad [4]$$

and

$$T = r_s h \sin 2\alpha \quad [5]$$

A similar derivation may be made for the gravity deflection diagram of Fig. 1, with results as shown.

Derivation of Equations for the Motion of an Ideally Simple Rotor With Unsymmetrical Shaft

Referring to Fig. 16, a section of the shaft adjacent to its mid-point is shown; the center point of this section is S, and

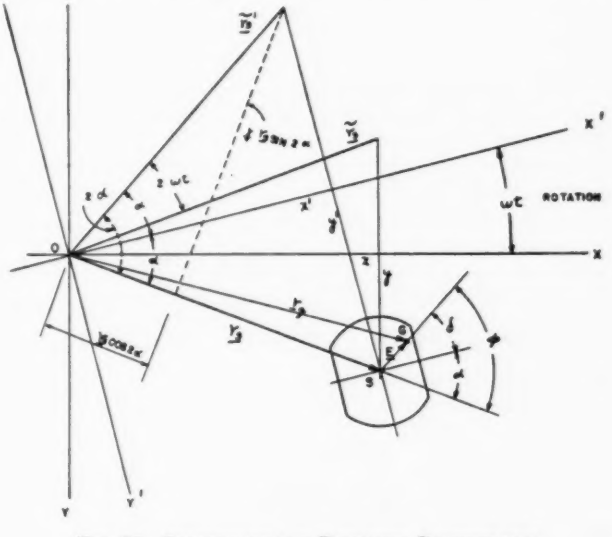


Fig. 16 Stationary and Rotating Coordinates

at the particular instant the lateral deflection of the shaft from the line of bearing centers O is represented by the radius vector, r_s. Rotation is indicated as counterclockwise at a constant speed ω. The unbalanced condition of the rotor is indicated by the fact that the center of gravity G is somewhat displaced from the shaft center S, the distance SG (or vector $\boldsymbol{E}$) representing the eccentricity. Two coordinate systems are shown: OX and OY are fixed coordinates; OX' and OY' are rotating coordinates such that OX' is always parallel to the weak axis of the unsymmetrical shaft. The orientation angles α, δ, and ϕ should be especially noted, α having the same significance as in Fig. 2, δ giving the fixed position of the unbalance with reference to the weak axis, and ϕ being the phase angle or "angle of lag." As such ϕ is usually measured backward from SG (CW), while it has been the author's choice to measure α and δ as positive when forward from the weak axis (CCW). Linear damping is assumed, equivalent to a single force opposing the motion of point S.

The first step toward setting up the equation of motion is the derivation of a vector expression for the deflecting force on the shaft, referred to the stationary coordinates. Note in Fig. 16, that the reflection of $\boldsymbol{r}_s$ in the rotating axis OX' is shown as $\bar{\boldsymbol{r}}_s'$, leading $\boldsymbol{r}_s$ by the angle 2α. Thus, $\bar{\boldsymbol{r}}_s'$ is parallel but opposite in direction to the vector shown as $h\boldsymbol{r}_s$ in Fig. 2. The deflecting force as a vector in rotating coordinates is therefore $(c\boldsymbol{r}_s - h\bar{\boldsymbol{r}}_s')$, where $c\boldsymbol{r}_s$ is the average radial component. Now note that the reflection of $\boldsymbol{r}_s$ in the stationary axis OX, indicated as $\bar{\boldsymbol{r}}_s$, is the same as $\bar{\boldsymbol{r}}_s'$ except for an angular difference of $2\omega t$. Hence, we may write $\bar{\boldsymbol{r}}_s' = \bar{\boldsymbol{r}}_s e^{j2\omega t}$, and the deflecting force becomes $(c\boldsymbol{r}_s - h\bar{\boldsymbol{r}}_s e^{j2\omega t})$. Actually, instead of the force exerted on the shaft, we shall use the opposite force $\boldsymbol{F}$ exerted by the shaft on the wheel, which is

$$\boldsymbol{F} = -c\boldsymbol{r}_s + h\bar{\boldsymbol{r}}_s e^{j2\omega t} \quad \ldots\ldots [6]$$

From this point on, the analysis may be simplified by dividing it into two parts, the first assuming the shaft to be vertical to eliminate the gravity effect which produces double-frequency vibration, and the second assuming the shaft horizontal but in perfect balance to eliminate the single-frequency vibration in the critical-speed range. In an actual horizontal shaft, of course, both forms of motion occur.

With the shaft vertical, we assume, in addition to the spring-constant force $\boldsymbol{F}$ given by Equation [6], a damping force $\boldsymbol{D}$ given by

$$\boldsymbol{D} = -b\dot{\boldsymbol{r}}_s \quad \ldots\ldots [7]$$

where $\dot{\boldsymbol{r}}_s$ represents the linear velocity of the shaft center S, and b is the ordinary type of damping constant, expressed as force per unit of velocity.

The mass of the rotor being taken as m, the equation of motion, in vector terms, is

$$\boldsymbol{F} + \boldsymbol{D} = m\ddot{\boldsymbol{r}}_g \quad \ldots\ldots [8]$$

Noting that $\ddot{\boldsymbol{r}}_g = \ddot{\boldsymbol{r}}_s + \ddot{\boldsymbol{E}} = \ddot{\boldsymbol{r}}_s - E\omega^2 e^{j(\delta+\omega t)}$
and substituting from Equations [6] and [7], there results

$$m\ddot{\boldsymbol{r}}_s + b\dot{\boldsymbol{r}}_s + c\boldsymbol{r}_s - h\bar{\boldsymbol{r}}_s e^{j2\omega t} = mE\omega^2 e^{j(\delta+\omega t)} \quad \ldots\ldots [9]$$

which is the differential equation to be solved.

A tentative solution is assumed, of the form

$$\left.\begin{aligned} \boldsymbol{r}_s &= \boldsymbol{Q}e^{j\omega t} \\ \text{with} \quad \boldsymbol{Q} &= X' + jY' \\ \text{and} \quad \widetilde{\boldsymbol{Q}} &= X' - jY' \end{aligned}\right\} \quad \ldots\ldots [10]$$

where X' and Y' are constants (for a constant speed) to be evaluated by substitution in Equation [9] for $\boldsymbol{r}_s$, $\dot{\boldsymbol{r}}_s$, etc. Expressions for X' and Y' are then found as functions of m, b, c, h, ω, E, and δ, which may be simplified by introducing the ratios u, a, and d for speed, damping, and dissymmetry by the following substitutions

$$\left.\begin{aligned} \omega &= u\sqrt{(c/m)} \\ b &= a\sqrt{(cm)} \\ h &= dc \end{aligned}\right\} \quad \ldots\ldots [11]$$

The scalar value of the deflection or half amplitude r_s is then found as $\sqrt{(X'^2 + Y'^2)}$ from which

$$\frac{r_s}{E} = \frac{u^2\sqrt{[(1-u^2)^2 + a^2u^2 + d^2 + 2d(1-u^2)\cos 2\delta + 2aud\sin 2\delta]}}{(1-u^2)^2 + a^2u^2 - d^2} \quad \ldots\ldots [12]$$

This is the desired general expression for amplitude in units of the eccentricity E.

The phase angle ϕ may also be determined from the values of X' and Y', on the basis that

$$\tan\alpha = \tan(\phi - \delta) = \frac{Y'}{X'} = \frac{(1-u^2-d)\sin\delta - au\cos\delta}{(1-u^2+d)\cos\delta + au\sin\delta} \quad \ldots\ldots [13]$$

These equations for amplitude and phase angle reduce to simpler forms for specific values of the unbalance angle δ such as 0, 45, 90, or 135 deg; and for zero dissymmetry ($d = 0$), reduce to the ordinary equations for the round shaft (see Fig 3).

Note that infinite amplitude is indicated if the denominator of Equation [12] is set equal to zero, which condition reduces to

$$u = \sqrt{\left[\left(1 - \frac{a^2}{2}\right) \pm \sqrt{\{d^2 - a^2 + (a^4/4)\}}\right]} \quad \ldots\ldots [14]$$

This indicates two speeds with infinite amplitude, if the expression under the second radical is positive; one such speed if it is zero; and unreal speeds if negative. The condition for having no infinite amplitude at any speed is then

$$d^2 - a^2 + (a^4/4) < 0$$

or

$$d < a\sqrt{[1 - (a^2/4)]} \quad \ldots\ldots [15]$$

With ordinary values of the damping constant a of the order of 0.10, it may be seen that the value of the radical may be taken as 1, with negligible error, so that to a close degree of approximation the condition for having no infinite amplitude becomes $d < a$.

* * * *

The double-frequency motion due to gravity will now be dealt with, on the assumption that the shaft is horizontal and in perfect balance, so that $E = 0$ (Fig. 16), point G coincides with S, and $\boldsymbol{r}_g = \boldsymbol{r}_s$. The force of gravity $\boldsymbol{W} = -j\boldsymbol{mg}$, must be taken into account, so that the equation of motion, in simplest terms, becomes

$$\boldsymbol{F} + \boldsymbol{D} + \boldsymbol{W} = m\ddot{\boldsymbol{r}}_g = m\ddot{\boldsymbol{r}}_s \quad \ldots\ldots [16]$$

from which

$$m\ddot{\boldsymbol{r}}_s + b\dot{\boldsymbol{r}}_s + c\boldsymbol{r}_s - h\bar{\boldsymbol{r}}_s e^{j2\omega t} + j\boldsymbol{mg} = 0 \quad \ldots\ldots [17]$$

For a tentative solution, it is assumed that, for any constant speed, $\boldsymbol{r}_s$ consists of a stationary vector $\boldsymbol{C}$, plus a rotating double-frequency vector $\boldsymbol{R}$, or

$$\left.\begin{aligned} \boldsymbol{r}_s &= \boldsymbol{C} + \boldsymbol{R}e^{j2\omega t} \\ \text{where} \quad \boldsymbol{C} &= X_1 + jY_1 \\ \text{and} \quad \boldsymbol{R} &= X_2 + jY_2 \end{aligned}\right\} \quad \ldots\ldots [18]$$

By substitution in Equation [17] for $\boldsymbol{r}_s$, etc., the values of X_1, Y_1, X_2, and Y_2 are determined. The amplitude of the double-frequency vector $\boldsymbol{R}$ is then found as $\sqrt{(X_2^2 + Y_2^2)}$.

Using the ratio substitutions given by Equation [11], we have

$$R = \frac{y_1 - y_2}{2}\,\frac{1 - d^2}{\sqrt{[(1-4u^2-d^2)^2 + 4a^2u^2]}} \quad \ldots\ldots [19]$$

For zero speed ($u = 0$), this expression reduces to $R = (y_1 - y_2)/2$, which is the "static whirl" illustrated in Fig. 1. The second term is a resonance factor, indicating that as the speed increases

the amplitude R builds up to a resonant peak when $(1 - 4u^2 - d^2) = 0$, or

$$u = 0.5\sqrt{(1 - d^2)} \quad [20]$$

Again, the radical may be neglected with ordinary values of d, of the order of 0.1, so that the double-frequency peak is found at about one half the critical speed as computed for the mean stiffness of the shaft.

Finally, the peak amplitude for the "half critical," with u as given by Equation [20] becomes

$$R_{peak} = \frac{(y_1 - y_2)/2}{a}\sqrt{(1 - d^2)} \quad [21]$$

in which again the radical term may ordinarily be neglected. This expression is remarkably similar to that for the peak amplitude at the main critical with no dissymmetry, $r_s = E/a$ (obtained from Equation [12], with $u = 1$ and $d = 0$). Equation [21] was used to check the value of the damping constant a during the tests described in the body of the paper, on the basis of the observed values of $(y_1 - y_2)$ and the amplitude at the half critical.

Acknowledgment

The writer is indebted to Messrs. E. L. Thearle, H. Poritsky, and the late V. Petrovsky, for previous work not published but available within the General Electric Company.

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Discussion

Some Applications of Photoelasticity in Turbine-Generator Design[1]

ELMER K. TIMBY.[2] Progress in stress analysis and design is usually preceded by both analytical and experimental research aimed at the solution of certain hypothetical problems which attract the interest or curiosity of the investigator. Next, someone discovers that some of these ideal solutions are of practical importance and begins to use them in a rational manner. In the paper under discussion we see the happy result of an able research analyst and a practical designer applying methods largely perfected by himself to the satisfactory solution of a difficult problem. The author is to be complimented on the success of his investigations.

It is observed that the model used in finding the so-called "exact" stress-concentration factors for the segment-shaped transverse grooves in the two-pole generator rotor was studied while subjected to a pure bending moment, and consequently no shear. It is doubtful whether this condition exists in the rotor when in service. Undoubtedly the shear stress near the periphery of the rotor is low but is it not possible that shear might have some effect at the bottom of the rather narrow and widely spaced grooves?

Question number 1 then is: Is the author of the opinion that the observed stress-concentration factors would have been the same if the model had been subjected to a combination of transverse shear and bending moment?

With further reference to the rotor, the plane section selected for study was cut from the rotor along the axis of symmetry and at a point which included the deepest portion of the narrow transverse slots. Stress-concentration factors were observed for this location at the bottom of the slots. There is material near the ends of the slots considerably further from the axis than is the material at the bottom of the groove (about 20 per cent further in the rotor shown in Fig. 2 of the paper). It seems reasonable to suspect that failure could not occur at the bottom of the slot (13 in. from the axis) until the material at one end of a slot (15.5 in. from the axis, measured parallel to the 13-inch dimension, and not radially) had suffered considerable deformation or possibly failure.

Question number 2 may be stated as: Was a diametrical section including the ends of the slots studied? If so, what was the strength when compared to the one which was studied; and if it was not studied, why was it disregarded?

The second test describes the experimental determination of centrifugal stresses in the rotor.

It should be noted that K as computed for the core stresses by use of Equation [1] of the paper should be 1.31 instead of 1.13. This may be a misprint since the stresses appear to be correctly plotted in Fig. 5.

Turning now to the subject of stress concentrations in T-heads, it is noted that the author considered it inadvisable to load the specimen at any point closer to the shaft than about twice the radius of the fillet at the junction of head and shaft; and further that it is in this vicinity that the value of the stress-concentration factor is increasing most rapidly. The reason given for not applying the load at a point closer to the shaft is that the material underwent considerable distortion in the neighborhood of the concentrated supports which would probably lead to a false determination of the maximum stress. This reason appears sound, but it also appears to avoid the problem rather than to solve it. Although not recommended, it would have been possible to extrapolate the experimental curve of Fig. 7 to a value of infinity for k as easily as to a value of about 5.45. Furthermore, it is probable that in actual construction the point of support may be very close to the shaft unless the supporting material is purposely cut away to avoid such a condition.

The author states that tests have revealed that stress concentration at such fillets is usually much greater than what one could expect from a comparison with known results obtained with tension and bending specimens of similar fillet ratios.

Question number 3 follows: How reliable does the author consider the dotted portion of the curve for k as plotted in Fig. 7? And question number 4: Is it convenient for the author to explain how the stress-concentration factor is affected by the presence of a loaded boundary at the point at which the stress is to be determined?

While further discussing the matter of the location of the load on the flange the author expresses the belief that the case of simple axial tension corresponds to the location of the loads on the flanges at infinite distances on either side of the shaft. It is difficult for the writer to visualize the similarity between these two conditions of loading. In the case of simple axial tension there must be a plane, normal to the direction of the load, at the mid-depth of the head along which there is no shear and on which the normal stresses vary from some large value at the center line to lower values on either side. Also, from symmetry, there must be no resultant shear on planes parallel to the length of the shaft and located on either side of it. It is not evident that either of these conditions would be produced by loads placed at an infinite distance on either side of the shaft.

As the final question, number 5: What was the line of reasoning which led to the conclusion that the two loading conditions just mentioned were equivalent?

An attempt has been made in this discussion to ask the author to expand certain items which were treated in a brief manner in his original presentation. There has been no intention of questioning the reliability of the work. On the contrary, the author is to be congratulated for having made such distinct progress in the field of stress analysis.

M. M. FROCHT.[3] In 1934 the writer investigated factors of stress concentration at fillets subjected to the combined action of pure compression and bending using a scheme of loading as shown in Fig. 1 of this discussion. The results were incorporated in the paper "Some New Aspects of Stress Concentrations," presented before the American Association for the Advancement of Science at its meeting held in Pittsburgh, in February, 1934.

Fig. 2 of this discussion shows a typical stress pattern, while the writer's Fig. 3 represents a curve giving the factors of stress concentration for $R/d = 0.075$, the case investigated by Dr. Hetényi. It was found convenient to plot k against the ratio σ/σ_b in which

[1] By M. Hetényi. Published in the December, 1939, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 61, 1939, p. A-151.

[2] Princeton University, Princeton, N. J.

[3] Associate Professor of Mechanics, Carnegie Institute of Technology, Pittsburgh, Pa. Mem. A.S.M.E.

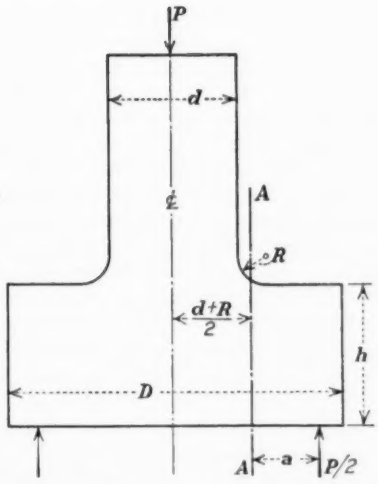


FIG. 1 FILLET SUBJECTED TO COMBINED PURE COMPRESSION AND BENDING

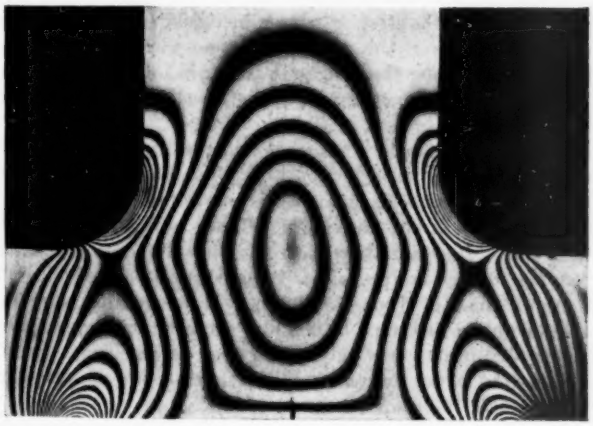


FIG. 2 TYPICAL STRESS PATTERN OF FILLET SHOWN IN FIG. 1

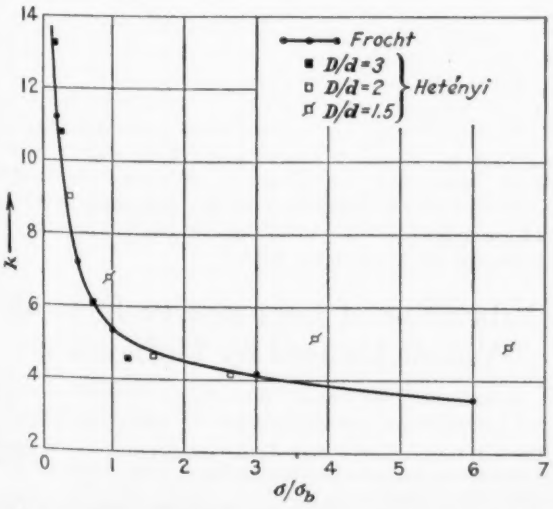


FIG. 3 STRESS-CONCENTRATION FACTORS FOR $R/d = 0.075$

σ and σ_b denote the compressive and bending stresses respectively. These two nominal stresses were calculated from the elementary equations

$$\sigma = \frac{P}{A} \quad \text{and} \quad \sigma_b = \frac{Mc}{I}$$

the bending stress σ_b being computed at section A-A distant $(d + R)/2$ from the axis, as shown in Fig. 1 of this discussion.

It is interesting to point out that all the data in the fifteen curves of Fig. 9 of the paper included in the range between $D/d = 1.8$ and $D/d = 3$ is contained in the single curve of Fig. 3 of this discussion. It would therefore seem that for T-heads in which D/d is not much less than 2, the local effect of the distributed loads can be neglected and factors of stress concentrations can be calculated on the basis of the ratio of direct to bending stresses.

Curves similar to that shown in Fig. 3 of this discussion are now available for a range of $R/d = 0.0572$ to $R/d = 1.00$. The writer hopes to find an opportunity to publish all of these curves in the near future.

A. L. KIMBALL.[4] The question has been raised as to whether the assumption of plane stress is adequate in solving the elastic problem of stress concentration produced by centrifugal body stress at a central hole in a revolving cylinder.

In the case of a thin disk, this assumption is evidently adequate where axial contraction is not restrained, but for a long cylinder revolving about its axis, considerable axial shear will arise because the cylinder is sufficiently long so that its central portions cannot contract axially relatively more than its peripheral portions do. Consequently, a more accurate solution can be obtained by assuming that no differential *strain* can exist along the cylinder axis, but that it be confined to the plane of the cylinder section only. In other words, whereas the assumption of plane *stress* gives a more accurate result in the rotating-disk problem, the assumption of plane *strain* gives a more accurate result in the case of the cylinder problem. These two cases are treated by Love.[5]

A comparison of these two theories shows that, other things being equal, for a small central hole the tangential stress comes out about 5 per cent higher for plane strain than for plane stress, whereas at the edge of the disk the former theory gives a tangential stress 25 per cent lower than the latter.

R. E. NEWTON.[6] The author not only made a judicious choice of problems, but also he has been decidedly realistic in his application of the photoelastic techniques. For example, in studying the problem of the stress concentration in transverse grooves of a rotor he has treated this problem as a two-dimensional one. The use of this simplification is highly desirable from an experimental point of view and the reference to Neuber's theoretical results leaves the reader with confidence in the validity of the approximation. The replacement of a distributed gravitational loading by a pair of externally applied couples seems also justifiable, but the author has neglected to point out the nature of this approximation.

The simplicity and effectiveness of the mounting used to study the centrifugal stresses around the bore are notable. Concerning the quantitative results of this analysis, it is interesting to note that the substitution of $\alpha = 1$ in Equation [1] gives a stress that

[4] Engineering General Department, General Electric Co., Schenectady, N. Y. Mem. A.S.M.E.

[5] "A Treatise on the Mathematical Theory of Elasticity," by A. E. H. Love, University Press, Cambridge, England, third edition, 1920, section 102.

[6] Instructor in Applied Mathematics, Washington University, St. Louis, Mo. Jun. A.S.M.E.

is almost exactly equal to the experimentally determined value of the maximum stress at the edge of the hole. This, of course, would correspond to the case of a winding having the same specific gravity as the rotor body.

The manner in which the variables of the T-head problem have been separated is highly commendable. Altogether too many sets of exhaustive data have been gathered in connection with similar problems, only to be presented in such a manner that they are almost useless to the practical designer.

W. M. MURRAY.[7] Although the paper[1] deals primarily with the problems of rotor design, it brings out two very significant points which are usually either taken for granted or neglected. The first of these involves the effect of adjacent stress concentrations. The effect of a single discontinuity is usually considered rather serious, and rightly so, but, as the author's Table 1 shows, several discontinuities adjacent to one another tend to give a lower value of the stress-concentration factor than the single discontinuity. This is borne out in practice in the cases of threaded connections which stand up in spite of their form where evidently the adjacent threads tend to break down the concentrations.

The problem of the stress concentration at the fillet in the head of a bolt is exceedingly interesting and suggests the possibility of the use of an internal fillet to relieve the stress condition. In the cases of the concentrated loads being applied to the T-heads, it is evidently desirable that the point of application of the load should be at about $d/2$ from the shank. In this case the concentration factor for the author's problem would be about 3.25. Provided that the load is concentrated, the area required to support it will be small so that there would be a good opportunity to use an internal fillet of even larger radius than that considered in the present example. Provided that the head of the bolt is sufficiently thick to make the effect of bending negligible, the internal fillet would not produce a weakening effect and at the same time it would insure that the load could not be applied nearer to the shank than a distance of $2R$, where R is the fillet radius. In the case of the distributed load, about the same conditions would prevail as for the concentrated load with the exception that the presence of the internal fillet would necessarily reduce the bearing area to some extent. The writer feels that entirely too little attention is paid to the use of the internal fillet and that stresses in bolt heads might be reduced through its use, although the cost of manufacture possibly would be increased.

AUTHOR'S CLOSURE

The five questions raised by Professor Timby will be answered in the same sequence as they were given.

The shearing force due to the distributed weight of the rotor will be in any case small around the middle of the span of the rotor where the maximum normal stress from bending occurs. For this reason, it seemed sufficient to test the models in simple bending.

It is true that the nominal bending stress will be higher in the extreme fibers of vertical sections to the right or left of the symmetry section which has been investigated. However, such an increase of the nominal stress will be small in comparison to the decrease of the concentration factor, due to the lesser depth of the grooves in these sections. Taking the extreme case, which is a longitudinal section through one end of the transverse grooves, we will have a nominal stress only about 20 per cent larger than in the symmetry section while, at the same time, the k factor, because of the vanishing grooves, will approach a value of unity. This shows that we must look for the maximum stress value in the axial symmetry section at the deepest point of the grooves.

[7] Instructor in Charge of Photoelasticity, Massachusetts Institute of Technology, Cambridge, Mass. Mem. A.S.M.E.

The author is obliged to Professor Timby for pointing out that the correct value of the k factor in the centrifugal-stress problem is 1.31 instead of 1.13, being a ratio of 85.4 to 65.2.

The dotted portion of the curve in Fig. 7 of the paper aims merely to show the subsequent increase of the concentration factor as the load approaches the fillet. Professor Timby is right in saying that one could just as well assume a singular point with an infinite k value at the point where the straight horizontal boundary joins the fillet. In practical cases, however, the load is always more or less distributed and a singular value would not alter the area along the loaded portion under the influence line, like the one in Fig. 7. At this point, it might be well to call attention to the fact that the mean value of the k ordinates in Fig. 7 is 3.5, the same as the one obtained by a uniformly distributed loading along the flanges of a T-head of the same proportions as shown in Fig. 9.

The subject of the fourth question, namely, the determination of the principal stresses at a loaded boundary, constitutes a separate problem which also can be analyzed photoelastically. Such a problem, however, did not come up in the present T-head tests where the maximum stress is always in the curved part of the fillet, the boundary of which naturally is free from external loading in any case.

The fifth question is probably due to a misunderstanding. The statement, that the stress condition in T-heads subjected to simple axial tension is the same as if the same sample were to be supported along the flanges at points in infinity, referred to T-heads of very large depth (e.g., the one in Fig. 7), where the bending of the flanges is negligible.

Due to the particular scheme of loading which Professor Frocht used, his concentration factors were obtained merely from the bending action in the flanges which is only one of the three sources of the increase in fillet stresses at T-heads as was pointed out in the paper. It was gratifying to find that, for cases where the bending stresses are prevalent, the author's results check with those obtained by Professor Frocht. However, this comprises only one phase of the problem and a complete solution could be represented in any case only by a set of curves, irrespective of whether the factors are referred to a nominal σ stress value or to σ/σ_b ratios of Professor Frocht.

Mr. Kimball's discussion gave further evidence in support of the fact that the core stresses due to the inertia forces in a revolving cylinder can be determined with satisfactory accuracy from plane-model tests.

The fact, pointed out by Mr. Newton, that the substitution $\alpha = 1$ in Equation [1] gives almost exactly the value of the maximum stress at the core, is undoubtedly only a curious coincidence and could not serve as a general rule.

The author agrees with Dr. Murray's statement that in some cases the application of internal fillets might reduce substantially the peak stress value. In order to be able to suggest the best design for a particular case, however, one must know the effect of all the variable proportions in T-heads. This could be achieved only by a systematic study of all the variables in a manner similar to the one advanced in the paper.

Vibration of Locomotive Driving Wheels Caused by Unbalance[1]

R. EKSERGIAN.[2] This paper[1] represents a clear-cut condensation of a complicated research problem in counterbalancing that

[1] By O. J. Horger and C. W. Nelson. Published in the December, 1939, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E. vol. 61, 1939, p. A-177.

[2] Consulting Engineer, Edward G. Budd Mfg. Company, Philadelphia, Pa. Fellow A.S.M.E.

has confronted the railways in the high-speed operation of steam locomotives.

The actual problem is complicated with a system having several degrees of freedom, while the disturbing forces include, in addition to the unbalance in the driving wheels, the effect of a mutual vertical reaction and torque reaction between the drivers and spring-borne parts and a varying tractive force at the drivers and correlated drawbar pull resulting in additional couples on the system. In a first approximation, the dynamic system consists of a two-mass system, the spring-borne parts, and the driving wheels separated by the spring suspension, while the drivers in turn are supported by the elastic rail spring. If all the drivers had the same mass and the unbalance force per driver was the same, the system can be considered as an inverted double pendulum with two degrees of freedom. Actually a better approximation is a system consisting of the spring-borne parts, and a parallel system with individual driving masses and trucks, so that, from a coplanar aspect, we have the number of degrees of freedom equal to the displacements of these masses, with an additional degree of freedom for pitching of the locomotive chassis. But the crosswise motion on roll of the chassis and individual driving wheels adds further degrees of freedom. Finally, to further complicate matters the spring suspension is tied in by equalization of the running gear. This reduces the degrees of freedom and throws us back in a first approximation to a double pendulum for coplanar motion.

On the basis of a double-spring pendulum the displacements of the driving wheel and spring-borne mass (per driver), for the steady state, are as follows. For the driver

$$x_1 = \frac{P_D\,(k - \omega^2 m_2)}{(k_R + k - \omega^2 m_1)\,(k - \omega^2 m_2) - k^2} \quad \ldots\ldots [1]$$

For the spring-borne mass

$$x_2 = \frac{P_D k}{(k_R + k - \omega^2 m_1)\,(k - \omega^2 m_2) - k^2} \quad \ldots\ldots [2]$$

so that the ratio of the motion of the spring-borne parts to that of the driver is

$$\frac{x_2}{x_1} = \frac{k}{k - \omega^2 m_2} = \frac{1}{-(\omega^2/p^2) + 1} \quad \ldots\ldots [3]$$

Now p, the natural frequency of the spring-borne parts with rigid rail, depends only on the static deflection of the driving springs. With a spring constant 10,500 lb per in. and a spring weight of 24,000 lb per wheel, then δ = (spring deflection) = 2.3 in., approximate. Hence, $p/2\pi = 2.06$ vibrations per second or roughly 120 per minute. With operating speeds around 360 rpm this means $x_2/x_1 = -1/8$ and becomes less with increasing speeds. Therefore, the authors' assumption of neglecting the motion of the spring-borne parts is well-founded. Thus, on the basis of coplanar motion, the motion of the driving wheel practically operates as a system with one degree of freedom.

In locomotive-driving-axle assembly the unbalanced applied forces, the spring reactions, and rail reactions are considerably out of plane, though symmetrical to the center line. While the authors have considered the problem as a vibration in a single plane the writer suggests a little greater refinement by including the entire axle assembly. Proceeding with the assumption made by the authors that the motion of the spring-borne parts, compared with the motion of the axle assembly, is small, then for the axle assembly in vibrating in the pedestal guides, we have a system with two degrees of freedom, defined by the vertical coordinate y_g and the angular coordinate θ in the transverse plane. Let G = lateral distance between the elastic rail supports with spring constants k_R and a = lateral distance between spring supports, with spring constant k. Let m = total mass of axle assembly and ρ = the radius of gyration in rotating about a horizontal axis at the center in a transverse plane.

The spring reactions are $k[y_g + (a/2)\theta]$ and $k\,[y_g - (a/2)\theta]$ for either side. The elastic rail reactions are $k_R[y_g + (G/2)\theta]$ and $k_R[y_g - (G/2)\theta]$. Assume the unbalance to be concentrated in the wheel planes or at any lateral distance L apart. Let P_D = the dynamic unbalance = $W_0/g\ \omega^2 R$. Since the cranks are at quadrature and the unbalance is usually spread at an angle greater than $\pi/2$, the applied forces are $P_D \sin \omega t$. For the right wheel and $P_D \sin(\omega t - \beta)$ for the left wheel, where $\beta = (\pi/2) + \alpha$.

Then, for the vertical translation

$$m\ddot{y}_g = -2(k + k_R)y_g + P_D\,[(\sin \omega t + \sin(\omega t - \beta)]\,. \quad [4]$$

and for the angular motion

$$m\rho^2\ddot{\theta} = -2\left[\frac{ka^2}{4} + \frac{k_R G^2}{4}\right]\theta + P_D\left[\frac{L}{2}\sin \omega t - \frac{L}{2}\sin(\omega t - \beta)\right] \quad \ldots\ldots [5]$$

where it is to be observed no coupling between the two motions occurs due to symmetry and equality of the spring constants for either side.

Then, for steady-state condition

$$y_g = \frac{P_D[\sin \omega t + \sin(\omega t - \beta)]}{(p_1^2 - \omega^2)\ W/g} \quad \ldots\ldots [6]$$

$$\theta = \frac{P_D(L/2)[\sin \omega t - \sin(\omega t - \beta)]}{(p_2^2 - \omega^2)\ W\rho^2/g} \quad \ldots\ldots [7]$$

where $m = W/g$.

On the basis of the authors' criterion on lifting from the rail, we must equate the deflection $y_R = y_g + (G/2)\theta$ to $(S + W)/(2k_R)$ where W and S now refer to the entire axle assembly and spring weight per driver.

On combining Equations [6] and [7], we have

$$y_R = \frac{RW_0}{W}\left[\frac{\omega^2}{(p_1^2 - \omega^2)} + \frac{GL}{4\rho^2}\,\frac{\omega^2}{(p_2^2 - \omega^2)}\right]\sin \omega t \quad \ldots\ldots [8]$$

$$+ \frac{RW_0}{W}\left[\frac{\omega^2}{(p_1^2 - \omega^2)} - \frac{GL}{4\rho^2}\,\frac{\omega^2}{(p_2^2 - \omega^2)}\right]\sin(\omega t - \beta) \quad \ldots\ldots [9]$$

where

$$p_1^2 = \frac{2(k + k_R)g}{W}, \qquad p_2^2 = \frac{(ka^2 + k_R G^2)g}{2W\rho^2}$$

While this analysis does not disclose any further characteristics than given by the authors, it does permit a simpler analysis of the effects of the unbalance since in place of $P_D \sin \omega t$ and $P_D \sin(\omega t - \beta)$ we can consider all the unbalance centrifugal forces for the vertical motion by simply summing up their vertical components disregarding their planes of action, and for the angular motion summing up their moments about the center of axle to their respective planes of action. Moreover, the planes of action of the spring forces and rail reactions are properly taken into consideration.

By considering the general solution which includes the free motion, we have the four constants for determining the motion in the second phase when either or both wheels are off the rail and the corresponding rotational angle for this period.

Perhaps the most serious error in both the authors' and the writer's analysis is the disregard of the very large friction damping in the plate springs. The initial friction in locomotive plate

springs may be in the order of 10,000 lb. Due to the large spring constant of the rail, the equivalent friction of the rail spring may not enter materially in the problem.

Since the spring systems of locomotives with inside journals are not likely to be modified, nor the proportions of driving wheel and spring-borne parts and rail conditions, the solution of eliminating serious vibration of locomotive driving wheels appears in reducing the unbalance to a minimum. The Timken Company, through Dr. Horger and Mr. Buckwalter should be congratulated in making this possible through excellent technical design of lightweight rods and reciprocating parts.

ALEXANDER W. LUCE.[3] "This paper is an extremely interesting one, and presents some matters which have been but very little understood, and which appear to be of very great importance in the operation of railroads. The fact that a locomotive driving wheel in ordinary service actually rises entirely clear of the track at high speeds is a matter of so much importance that it certainly should receive great attention from The American Society of Mechanical Engineers." Thus opened a discussion of two papers[4] on counterbalance and rail pressures at the December, 1894, meeting of the Society. Present readers will find them most interesting reading because of the revelation in the present paper of progress in means for investigation, both physical and mathematical. Ample evidence of the seriousness of the problem is presented throughout the former papers as well as in the present one. Comparison of the papers will disclose how the worst reciprocating weights of yesterday were not as bad as the best of today.

The authors are to be congratulated on having adapted recent progress in photography to a problem that Professor Goss undertook to solve by calipering wire rods passed under the drivers of a locomotive on the test stand at Purdue University, and on their use of the advances made in recent years in the mathematical treatment of vibrations. The authors of forty-five years ago spoke of vibrations, oscillations, track stiffness, and spring stiffness, but it has remained for the present authors to handle these factors with confidence.

A further interesting fact is that the Burlington road is prominent in both the former and present papers.

AUTHORS' CLOSURE

Dr. Eksergian's use of two coordinates to express the motion of the axle assembly indicates one of the refinements which can be introduced into the authors' analysis without unduly complicating the problem. During the preparation of their paper, the authors considered this refinement but did not use it because it would not be expected to affect their numerical results greatly, particularly for vibrations such that the wheel did not leave the rail, and because calculation of ρ, the radius of gyration of the axle assembly about an axis through its center of gravity and parallel to the track center line, would be laborious for an actual locomotive.

The authors agree with Dr. Eksergian that the neglect of friction constitutes one of the most serious approximations in their analysis. The need for data concerning the friction forces was emphasized in the authors' paper. However, this and other assumptions in the authors' analysis, such as the neglect of the effect of one driver on vibrations of the other drivers and the neglect of the effect of angularity of the main rod, were necessary if the problem was to be kept simple enough for numerical calculation to permit a better appreciation of the fundamental aspects of the problem. It should be mentioned that operation of a locomotive with too short a cut-off will cause the main drivers to lift from the rail at speeds lower than those reported in the paper due entirely to the vertical component of the steam force transmitted to a main driver by the main rod. This possibility should not be ignored. However, the authors have tried for the most part to focus attention on the effect of overbalance which may be considered of fundamental importance in connection with rail damage observed in service.

The authors agree that the two papers[4] referred to by Professor Luce make most interesting reading. The experiments described in the paper by Professor Goss demonstrated the possibility of a driving wheel lifting and indicated that the maximum lift of a driving wheel lags behind the maximum upward force. The tests by Professor Goss do not detract from the value of the test results given in the authors' paper since his tests were made on a test stand which probably represented a nearly rigid foundation instead of comparatively flexible track. If the foundation under the test stand did have a flexibility comparable to that of a railroad track, then the test conditions were still complicated by the presence, between this foundation and the locomotive drivers, of a large mass consisting of an axle, two supporting wheels, brakes, and other attachments. The advantages of the photographic method described by the authors are principally that (1) it can be used in tests made on actual track either with or without slipping, (2) it is possible to observe the lifting of the wheel during a number of consecutive revolutions, and (3) it is possible to observe not only the motion of the wheels but also the approximate deformations of the rail.

The authors have a few comments to add and some acknowledgments to make concerning the theoretical work in the paper[4] by D. L. Barnes. Mr. Barnes referred to a still earlier paper[5] by R. A. Parke. In his paper, Mr. Parke arrived at the differential equations[6] of motion for a driving wheel on or off the rail. For steady-state motion on the rail, Mr. Parke obtained Equation [2] and the three preceding equations.[7] For motion of sufficient amplitude so that the wheel was off the rail during part of a revolution, Mr. Parke did not consider the constants of integration, specifically A and B in the authors' Equation [1] and two other constants in the equation for x_2. For this reason, Mr. Barnes stated in his paper[8] that "later Mr. Parke found that his analysis did not take into account all of the conditions . . . . . ." Mr. Parke did not examine carefully the significance of his equivalent of the authors' Equation [2]. Due to the form in which he left this equation, he arrived at the erroneous conclusion, "it is evident that a wheel is more easily lifted from a rigid than from an elastic track," meaning that less overbalance would be required to produce lifting at a given speed for rigid track than for comparatively flexible track. Actually Equation [2] shows the opposite to be true except at speeds so low that $\omega < q$, using the authors' notations. At such speeds ridiculous values of overbalance are required to produce lifting. When $\omega = q$, the overbalance required to produce lifting is $\frac{(S+W)g}{R\omega^2} =$ 2680 lb for the locomotive constants given under Fig. 6 of the paper, and $U = 1500$ lb per sq in. Mr. Barnes showed in conclusion (g),[9] that he realized the error in Mr. Parke's conclusion.

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[4] "Counterbalance in Locomotive Drive Wheels," by W. F. M. Goss, and "Rail Pressures of Locomotive Driving Wheels," by D. L. Barnes, Trans. A.S.M.E., vol. 16, 1895, p. 305 and p. 249.

[5] "The Vertical Influence of the Counterbalance," by R. A. Parke, New York Railroad Club, Feb. 15, 1894. Reprinted in *Railroad Gazette*, Feb. 23, 1894, p. 136.

[6] Ref. 1, equation immediately preceding Equation [1], p. A-183; and first equation, p. A-184.

[7] Ibid., p. A-183.

[8] Ref. 4, p. 276.

[9] Ibid., p. 289.

In the paper[4] by Mr. Barnes, the theory of vibrations of a driving wheel was developed in an appendix by Professor J. B. Webb. The latter obtained equations which the authors have identified with Equation [1] for x_1 and the corresponding equation[10] for x_2. At this point Professor Webb and Mr. Barnes found that their mathematical difficulties were just beginning. Using the equations for x_1 and x_2, an attempt was made to determine whether or not the maximum lift should occur when the unbalance was directly up. They did not, however, assume a steady-state vibration but calculated the vertical motion of the wheel through 9 revolutions with "stupendous" labor, beginning with $x_1 = 0$, $\frac{dx_1}{dt} = 0$, at $\omega t = 90$ deg. Mr. Barnes concluded that "the maximum lift of the drivers does not commonly occur when the counterbalance is directly up, neither does the maximum rail pressure occur when the counterbalance is directly down. These maximums may take place either before or after the upper and lower points according to the conditions under which the wheel is revolving at the instant." Professor Webb, however, wrote that "if the power supplied is used up by the friction, there is a chance of getting a regular path," and "friction delays the phase, is a rough statement." The authors feel that their Fig. 8, obtained by neglecting friction while still recognizing the effect of friction in producing steady-state vibrations, is very useful in obtaining an over-all viewpoint of the problem considered. This type of general picture of the problem should precede any detailed calculations for a particular case such as the calculations by Professor Webb and Mr. Barnes. The wheel lift for various points on the curves of Fig. 8 increases from zero, where the curve leaves the horizontal axis, to infinity at the point where the various curves intersect. The infinite amplitude at the latter point results because friction has been neglected.

The Thick-Film Lubrication of Full Journal Bearings of Finite Width[1]

EVERETT O. WATERS.[2] The writer has been interested in solving Reynolds' equation for various boundary conditions, and is glad to have an opportunity to compare the methods of others with those which he has found useful in his own work. After reading the paper, he would like to offer the following comments by way of query or interpretation.

1 The various figures in the paper indicate that computations were carried to eccentricity ratios of the order of 0.6. Is the convergence of the successive-approximation method used by the authors sufficiently rapid to make this method feasible for higher eccentricity ratios—say 0.8 or 0.9—and if so, how many approximations would be necessary for reasonably accurate results?

2 The effect of a circumferential groove may be easily shown by considering Reynolds' equation directly. The corresponding homogeneous equation, using the notation of Howarth and Needs, is

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 0$$

and all the solutions of this equation, when superposed upon the solution for infinite width, give the net values of p vs. x and z, provided the boundary conditions are satisfied. Ordinarily, this requires that the solutions of the homogeneous equation be complicated functions of x and z. However, there is one solution, usually trivial, that exactly fits the boundary conditions for a circumferential groove, namely

$$p = c_1 z + c_2$$

This shows at once that the circumferential groove merely adds a component of pressure varying linearly with z to the pressure already fixed by the other conditions.

A further advantage of the circumferential groove, not mentioned by the authors, is the possibility of maintaining a constant carrying capacity for the bearing regardless of the angular position of the load line, which obviously cannot be done when the oil is fed at a point source or a line source parallel to the journal axis.

3 The results presented by the authors for simple 360-deg bearings without circumferential groove or point source are obviously predicated upon the existence of positive absolute pressure throughout the oil film. But since the pressure at the sides of the bearing is the mean of the highest and lowest value, one concludes that the maximum pressure cannot be greater than twice atmospheric, unless oil is fed to the sides of the bearing at more than atmospheric pressure—an unconventional and probably difficult procedure, to say the least. This limits the validity of the results to a fairly narrow pressure range—a limitation which must be strictly observed in making practical use of the authors' numerical and graphical data.

The foregoing remarks are submitted in no spirit of argument or criticism, but rather with the thought of pointing out one or two directions in which the authors will have to carry their research a few steps further before it can be considered as finished. On the contrary, the pioneer work that they have done on the 360-deg bearing is well deserving of high commendation, and doubtless they have already considered the matter of filling in these gaps in order to make the picture complete.

H. REISSNER.[3] The writer having published a paper[4] about the full journal bearing of finite width, would like to remark about the difference in the starting point, the mathematical method used by the authors and himself, and the future research. The authors started from the Reynolds differential equation of lubrication, that is, Equation [1] of the paper under discussion. This equation is a very plausible approximation, neglecting the inertia forces, the curvature of the streamlines, the transverse velocities, and the transverse pressure gradient. It has been applied until now for the treatment of the bearing of infinite width (Sommerfeld) and for the plane bearing of finite width (Michell).

For the journal bearing of finite width Reynolds' equation brings the inconvenience that its coefficients are functions of the circumference angle so that particular solutions of a trigonometrical development are not possible. Therefore, the authors have right from the beginning put up a development in ascending powers of the relative eccentricity of the two bearing cylinders and have split the original equation into a sequence of ordinary two-dimensional potential equations with quasi mass sources each depending on the solution of the preceding equation. In solving these equations they have also been able to take account of a pressure oil feed assuming a radial linear source going from journal to bearing.

The writer's method resembles in one point the work of the

[10] Ibid., p. 301.

[1] By M. Muskat and F. Morgan. Published in the September, 1939, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 61, 1939, p. A-117.

[2] Associate Professor in Mechanical Engineering, Yale University, New Haven, Conn. Mem. A.S.M.E.

[3] Armour Institute of Technology, Chicago, Ill.

[4] "Ebene und räumliche Strömung zäher, inkompressibler, trägheitsfreier Flüssigkeiten zwischen exzentrischen, relativ zueinander rotierenden Zylinderflächen. Beitrag zur Theorie der Schmierung—II," *Zeitschrift für angewandte Mathematik und Mechanik*, vol. 16, no. 5, October, 1936, pp. 275–286.

authors in so far as it was the first to use the development in powers of the relative eccentricity for the bearing of finite width as the last step of the analysis.

In other respects it differs fundamentally. Instead of the approximative Reynolds equation it uses the exact differential equation taking into account the curvature of the streamlines, the transverse pressure gradient, and the transverse velocities.

The writer does not want to assert that they are not negligible except perhaps in the vicinity of an oil-fed source where there are transverse velocities decidedly of the same order of magnitude as the tangential velocities and in the case of the partly filled journal bearing.

But the advantage of the exact starting equation

$$\nabla^2(u, v, w) = \text{grad } p$$

lies in the fact that in cylindrical coordinates r, θ, z it has constant coefficients in the directions of θ and z and so can be integrated, fulfilling the corresponding boundary conditions by Fourier series in both directions and in each term. Only in the radial direction Bessel functions arise for which, considering the narrow angular space, very few terms of a series are sufficient.

It is only in order to satisfy the boundary conditions appearing in the form of difference equations that a development of the series coefficients in powers of the eccentricity seemed to be the best way for my method.

The process has been executed so far as to lead to final formulas for the pressure and velocity distribution in circumferential, transverse, and axial direction. The average streamlines (taken over the thickness of the film) going axially outward on the pressure side and inward on the suction side have been calculated and, using as a bearing a glass cylinder, photographically confirmed.

Yet the whole analysis was thought only as a program for future detail and extension work.

Now a few words about some future problems in lubrication which ought to be prepared theoretically.

There is first the question of the partly filled journal bearing which has less friction than the full bearing because the friction on the suction side is avoided. For this case the boundary conditions at the ends of the arc for velocity and pressure had until the writer's papers never been given exactly. As the velocity is not zero at the bounding radii, there must be a free surface running along the nearly empty side of the rotating cylinder. Only in this way can the fundamental difficulties of this question be settled. There will then be obtained a certain relation between embracing arc, load, eccentricity, and feed pressure. It will be difficult for the Reynolds method, neglecting transverse velocities and transverse pressure gradient, to formulate this condition.

We all know further that two important difficult problems are still waiting for a theoretical rational treatment. These concern the change of viscosity by pressure and by temperature. Viscosity changes enormously with pressure and temperature and will be very different in different points of the bearing.[5]

The mathematical difficulties have not been surmounted yet. Still a theory approximative perhaps but giving the essential behavior must be found.

S. Kyropoulos.[6] The investigation constitutes, as an application of the hydrodynamic theory to bearings of the type used in practice, a valuable contribution to the understanding of their working mechanism. The extension of the analysis to bearings fed with lubricant at point sources, and the pertaining experiments with small loads and adequate oil pressures permit an illustrative approximate realization of the limiting conditions visualized by Petroff. The discussions of the bearing with a circumferential groove as a source of oil feed emphasize the nonhydrodynamic factor of better cooling which tends to offset the hydrodynamic disadvantages of such bearings in practice. This explanatory discussion in connection with the theoretical investigations of the paper is particularly fortunate in that it may serve to increase the confidence of the practical designer in the scientific development of the fundamentals of bearing design and operation, the importance of which is not yet sufficiently realized, as practice shows.

With regard to the case of bearings fed with lubricant at point sources in the pressure region, it might not be out of place to sound a warning regarding practical applications. It happened not infrequently that in cases where the oil feed was shifted into the pressure zone the feed pump was unable to pump against the pressure of the oil film and that the bearing even pumped oil into the oil pump.[7]

With regard to the subject of thin film lubrication, mentioned in the introductory remarks of the paper, the writer is inclined to believe that under the conditions of practice, where nonviscous lubrication comes into play, such friction is always connected with wear due to the irregularities of real surfaces. The high local pressures involved result in high local temperatures and chemical high-temperature reactions between lubricated surfaces and lubricants or in the disappearing of the lubricant and local exposure of the "metal" surfaces. Prior to this stage there will occur at the highest spots true friction between adsorption films and, perhaps, such friction will occur also on any adsorption layers on local peaks, exposed by high local temperatures in which case the adsorbed films will scarcely be identical with the original lubricant. Thus the physical investigation of the phenomena will ultimately have to aim at a molecular physical understanding of the law of Amontons-Coulomb.[8]

Authors' Closure

In reply to the question of Professor Waters, concerning the convergence of the approximation method used by the authors, we must admit that, beyond eccentricity ratios of 0.6, the number of approximations which have been used would not suffice to give quantitatively accurate results. Since the method is essentially equivalent to an expression of the pressure distribution as a power series in the eccentricity, its convergence naturally becomes increasingly less satisfactory as the eccentricity increases. For eccentricities close to unity, a different method of approximation would be necessary to obtain significant results.

The detailed treatment of the problem of a circumferential groove is given by the authors in a previous paper.[9] While it is true that the linear-pressure-distribution term, required to give the grooving effect, represents but a trivial solution of the Reynolds equation, the authors were surprised in their failure to find any mention of this solution or of its applicability in the lubrication literature.

The authors fully agree with the warning frequently pointed out regarding the limitation of the use of the Reynolds equation solution in the regions of negative pressure. Therefore, their solutions for the flooded bearing were not intended as ultimate representations of the actual situation which will obtain in flooded bearings. Rather, they were derived and developed

[5] "Theory of Lubrication," by M. D. Hersey, John Wiley and Sons, Inc., New York, N. Y.

[6] California Institute of Technology, Pasadena, Calif.

[7] "Die Reibung in Gleitlagern," by M. Ten Bosch, *Schweizerische Bauzeitung*, vol. 99, 1932, p. 321.

[8] "Non-Fluid Lubrication," by S. Kyropoulos, *Refiner and Natural Gasoline Manufacturer*, vol. 18, 1939, pp. 320, 399.

[9] "The Theory of Thick-Film Lubrication of Flooded Journal Bearings and Bearings With Circumferential Grooves," by M. Muskat and F. Morgan, *Journal of Applied Physics*, vol. 10, 1939, pp. 398–407.

mainly as a necessary preliminary for the study of bearings with lubricant sources such as the circumferential groove or the point source. If the feed pressures in these types of bearings are sufficiently high, the negative pressures will be overcome and the contributions due to the flooded-bearing solutions will possess physical validity.

While it is true that the exact hydrodynamic equation used by Dr. Reissner has constant coefficients in the direction of θ, the difficulty of satisfying exactly the boundary conditions in the radial direction would seem to be about as great as the difficulties of treating the Reynolds equation. Moreover the authors cannot agree that the approximations of the Reynolds equation will be of any consequence in treating practical journal-bearing systems, where the radial clearance is but a small fraction of the over-all dimensions. In fact, as Duffing has pointed out, in the case of the infinitely wide bearing the exact hydrodynamic treatment gives identically the same pressure distribution as found by Sommerfeld when using the Reynolds equation. It is difficult to see how the mere finiteness in width would lead to appreciable errors if the radial clearance is kept small. Even where a source is present the approximations of the Reynolds equation should still remain valid except possibly in the immediate vicinity of the source.

The authors do agree with Dr. Reissner's last remark concerning future problems. The question of negative pressures does indeed still seem to be bothersome, and a study of the boundary conditions from the point of view of the rigorous hydrodynamic equations would be the most promising line of attack.

The difficulty expressed by Professor Kyropoulos of applying the lubricant source in the high-pressure zone of the film is well taken. The theory developed by the authors permits a calculation of the pressure which would be required to maintain any rate of lubricant inflow at the point source. Of course, the rate of inflow will decrease with the applied pressure and, if the external sources cannot maintain a pressure at the inflow point equal to the natural pressure which would be developed there in a flooded bearing, the bearing will even pump lubricant back into the oil pump.

The problem of thin-film lubrication is indeed pressing from a practical point of view. The authors agree with Professor Kyropoulos' interpretation regarding this problem. In fact, it is their belief that the hydrodynamic theory will have to be abandoned completely in this region of lubrication, and that the problem will be solved only by direct experimental investigation.

Chain Links Under Cross Forces[1]

M. V. Barton.[2] In this paper the condition of loading assumed by the author was one in which the forces were exerted by the connecting links at right angles to the supported link. This condition of loading probably would not occur in a chain installation because of the necessity for the connecting links to pass around the supporting chain sheave or drum; also, this condition may not necessarily be the most severe encountered in chain service. For these reasons it may be necessary to consider the stresses in the links adjacent to the one analyzed by the author. The adjacent connecting link would be rotated in position 90 deg with respect to the one analyzed by the author and may be assumed to be supported in its vertical position by the sheave groove or by the restraining action of the connecting links.

For purposes of comparison with the author's results, the circular link as illustrated in Fig. 1 of this discussion will be analyzed. This indeterminate structure may be easily analyzed by means of Castigliano's theorem. The structure with redundants M_0 and H_0 acting is shown in Fig. 2 of this discussion. The moment at any point is

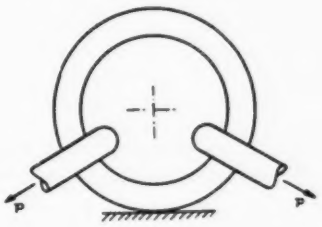


Fig. 1

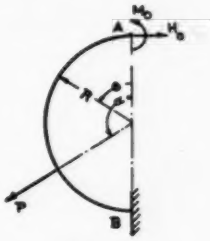


Fig. 2

$$M_1 = M_0 - H_0R(1 - \cos\theta) \quad [1]$$

for $0 \leq \theta \leq \alpha$, and

$$M_2 = M_0 - H_0R(1 - \cos\theta) + PR\sin(\theta - \alpha) \quad [2]$$

for $\alpha \leq \theta \leq \pi$. For continuity, deflection at A in the direction of H_0 equals zero, or

$$\delta_{H_0} = 0 = \frac{\partial V}{\partial H_0} = \int M\frac{\partial M}{\partial H_0}\,d\theta \quad [3]$$

$$= \int_0^{\pi} [M_0 - H_0R(1 - \cos\theta)](1 - \cos\theta)\,d\theta + \int_{\alpha}^{\pi} PR\sin(\theta - \alpha)(1 - \cos\theta)\,d\theta \quad [4]$$

Also change in slope at A equals zero, or

$$\Delta = 0 = \frac{\partial V}{\partial M_0} = \int M\frac{\partial M}{\partial M_0}\,d\theta = \int_0^{\pi} [M_0 - H_0R(1 - \cos\theta)]\,d\theta + \int_{\alpha}^{\pi} PR\sin(\theta - \alpha)\,d\theta \quad [5]$$

Solving these two equations for M_0 and H_0 it is found that

$$M_1 = PR\left[\frac{\pi - \alpha}{\pi}\sin\alpha\cos\theta - \left(\frac{\cos\alpha + 1}{\pi}\right)\right] \quad [6]$$

for $0 \leq \theta \leq \alpha$, and

$$M_2 = PR\left[\frac{\pi - \alpha}{\pi}\sin\alpha\cos\theta - \left(\frac{\cos\alpha + 1}{\pi}\right) + \sin(\theta - \alpha)\right] \quad [7]$$

for $\alpha \leq \theta \leq \pi$.

Investigation shows that for practical angles of the force, the maximum moment occurs where $\theta = \pi$, i.e., at the point of support, then

$$M = \frac{PR}{\pi}[\alpha\sin\alpha - \cos\alpha - 1] \quad [8]$$

and the condition for the maximum value of M is given by

$$2\tan\alpha + \alpha = 0 \quad [9]$$

[1] By Jos. B. Reynolds. Published in the December, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-171.

[2] Instructor in Machine Design, College of Engineering, Cornell University, Ithaca, N. Y.

or for α about equal to 131 deg. For the case $\alpha = 131$ deg

$$M_{max} = 0.44PR \quad \ldots\ldots\ldots\ldots\ldots\ldots [10]$$

This is about 12 per cent less than that found as a maximum by the author. The stress produced however may be higher in this case because the effect of thrust must be added.

Hence

$$\sigma = \frac{Mc}{I} + \frac{H}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots [11]$$

where $H = P \sin \alpha - H_0 = 0.552P$. This stress will of course be increased by the shift of the neutral axis which has not been considered here.

It is evident therefore that it is possible to obtain stresses which are higher than those determined by the author at practical angles of application of the forces.

It might also be pointed out that the deflections found from geometrical considerations by the author may be very easily verified by the use of Castigliano's theorem, that is

$$\delta = \frac{\partial V}{\partial P} = \int_0^{\frac{\pi}{2}} \left[\frac{M_n}{EI_n} \frac{\partial M_n}{\partial P} + \frac{M_t}{GI_t} \frac{\partial M_t}{\partial P} + \frac{KQ}{A_c G} \frac{\partial Q}{\partial P} \right] Rd\theta \quad \ldots\ldots [12]$$

$$= PR^3 \left[\frac{1}{2EI_n} (\pi - 2) + \frac{1}{GI_t} (\pi - 3) \right] + \frac{KPR}{A_c G} \frac{\pi}{2} \quad \ldots\ldots [13]$$

where K is a shear deflection factor having a value of about unity for circular sections and 6/5 for square sections.

Author's Closure

The theory of the paper deals only with the conditions under which tests for verification were run. Many other circumstances are not only possible but in accord with actual conditions. In general, when the link lies flat on the sheave, the forces applied at the ends of the link will not be perpendicular to the plane of the neutral axis of the link, but will have components tending to stretch the link as well as components tending to bend it.

Mr. Barton has made a significant contribution to the general theory. An analogous method might be employed to analyze the case of an oblong link under corresponding circumstances. It can hardly be assumed in these cases, however, that P has the same value as in the case discussed in the paper.

Analysis of Spherical Shells of Variable Wall Thickness[1]

Stefan Bergmann.[2] The author has published a contribution to the theory of elasticity of the spherical shell for the case where certain terms in the differential equations can be neglected, provided the thickness of the shell varies according to the law[3]

$$\delta = d(1 + \alpha\varphi)^2$$

Since the author's (approximate) solution appears in a form very convenient for numerical calculations, his result is to be considered as a valuable contribution to the theory of thin shells.

Nevertheless, it appears to be of practical importance to insert a few remarks on the exact theory of spherical shells as affected by external forces and of the wall thickness which varies according to another law. In this connection the attention of the reader should be called to a method for the solution of related problems, indicated by the writer and used and worked out in detail by Berentzen.[4]

The latter considers the problem of a spherical ring shell the thickness of which varies according to $\delta = 0.16a\varphi/t$, the external forces being represented by its own weight and by hydrostatic pressure; t is a parameter upon which the thickness of the ring shell depends.

The publication[4] was intended mainly as a contribution to the theory of dams. Introducing spherical coordinates a, φ, θ, the neutral surfaces of such a "shell" dam are given by $\varphi_0 \leqq \varphi \leqq \varphi_u$, $\theta_0 \leqq \theta \leqq \theta_u$. Berentzen considers a shell $\varphi_0 \leqq \varphi \leqq \varphi_u$, $0 \leqq \theta \leqq 2\pi$, as shown in Fig. 1 of this discussion. The method employed in his paper aims to give—with the aid of certain functions

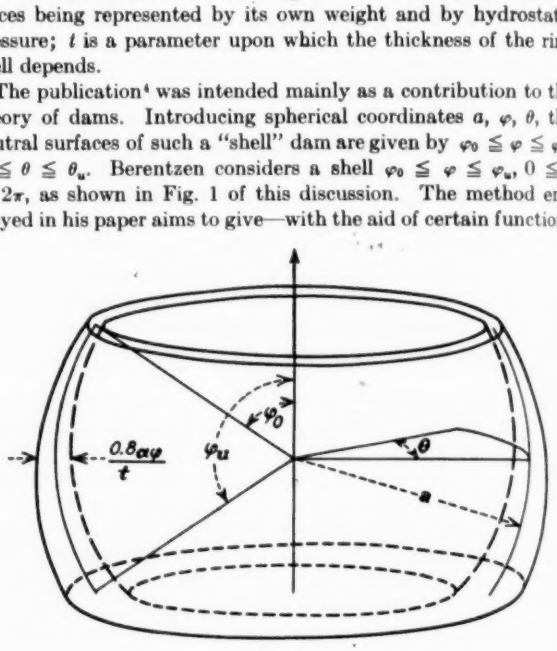


Fig. 1

tabulated once for all—a comparatively concise solution for every value of t, for arbitrary height of the water surface, for arbitrary φ_0 and φ_u, and for arbitrary boundary conditions.

As auxiliary quantities are introduced X and the quantity $M = t^3 \frac{3 \times 10^3 Q}{a^2 E}$ instead of Q. Starting then from Reissner's equations, he obtains (instead of Spotts' Equations [46] and [54]) the following equations

$$\left(\frac{\pi}{2} + \beta\right)^3 X''' + \left(\frac{\pi}{2} + \beta\right)^2 \left(-tg\,\beta + \frac{6}{\pi + 2\beta}\right) X'' + \left(\frac{\pi}{2} + \beta\right)^2 \left(-tg^2\beta - \mu + \frac{6\mu\, tg\,\beta}{\pi + 2\beta}\right) X = -\frac{5(1-\mu^2)}{216} M$$

$$\left(\frac{\pi}{2} + \beta\right) M'' + \left(\frac{\pi}{2} + \beta\right) \left(-tg\,\beta - \frac{2}{\pi + 2\beta}\right) M' + \left(\frac{\pi}{2} + \beta\right)^2 \left(-tg^2\beta + \mu - \frac{2\mu\, tg\,\beta}{\pi + 2\beta}\right) M = \frac{3}{4} 10^3 \left(\frac{\pi}{2} + \beta\right)^2 t^2 X + \frac{3.10E}{8a^2E} t^4 \phi^*(\beta)$$

where $\beta = \varphi - \frac{\pi}{2}$, $\phi^*(\beta) = \phi\left(\beta + \frac{\pi}{2}\right)$, $X(\beta) = \chi(\beta + \pi/2)$

The quantity $\phi^*(\beta)$ depends linearly on parameters α_ν, $\nu = 1, 2, 3, 4$, the values of which depend on the height of the water

[1] By M. F. Spotts. Published in the September, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-97.

[2] Graduate School of Engineering, Harvard University, Cambridge, Mass.

[3] Throughout this discussion, the writer uses the nomenclature adopted by the author.

[4] "Untersuchungen über Spannungen einer unter äusseren Wasserdruck und Eigengewicht stehenden Kugelringschale mit veränderlichen Dicke," by W. Berentzen. Edited Köln, 1936.

surface, the angles φ_0 and φ_u, the specific weight of the material of the shell, the constant t, and Poisson's ratio μ.

The general solution of the system of equations of this discussion appears in the form

$$X(\beta) = \sum_{\nu=1}^{8} \alpha_\nu X_\nu(\beta), \qquad M(\beta) = \sum_{\nu=1}^{8} \alpha_\nu M_\nu(\beta)$$

wherein the $X_\nu(\beta)$ and $M_\nu(\beta)$ are properly chosen particular solutions depending only on t and μ, but independent of all other parameters aforementioned. By choosing in an appropriate manner α_ν we get the sought solution of the considered problem.

It remains to determine the $X_\nu(\beta)$, $M_\nu(\beta)$, $\nu = 1, 2, \ldots 8$ as functions of t and μ. It may be assumed that the variations of μ are small ($\mu_0 - \epsilon \leqq \mu \leqq \mu_0 + \epsilon$, $\epsilon \ll 1$). For $X_\nu(\beta)$ and $M_\nu(\beta)$ certain power series

$$X_\nu(\beta) = \sum_{\vartheta=0}^{\infty} A_{\nu\vartheta}{}^{(1)} \beta^\vartheta, \qquad M_\nu(\beta) = \sum_{\vartheta=0}^{\infty} A_{\nu\vartheta}{}^{(2)} \beta^\vartheta$$

are obtainable whose coefficients $A_{\nu\vartheta}{}^{\delta}$ depend on t and μ. These power series converge for every β in the interval -90 deg $< \beta < 90$ deg. Therefore, if they are considered in a partial interval, e.g., -30 deg $< \beta < 30$ deg, there is rapid convergence and if eight terms are taken the error committed is much less than 1 per cent. The coefficients $A_{\nu\vartheta}{}^{\delta}$ in the power series are polynomials (and not power series of t as it would be in the case of a more general system of equations) in t^2 and μ the degree of which is increasing gradually with ϑ. The coefficients $A_{\nu\vartheta}{}^{\delta}$, $\vartheta \leqq 8$ are not higher than of the second degree in t^2 and, as $\mu - \mu_0$ is very small, we shall have

$$X_\nu = (X_{\nu_1} + X_{\nu_2}t^2 + X_{\nu_3}t^4) + (\mu - \mu_0)(X_{\nu_4} + X_{\nu_5}t^2 + X_{\nu_6}t^4)$$

$$M_\nu = (M_{\nu_1} + M_{\nu_2}t^2 + M_{\nu_3}t^4) + (\mu - \mu_0)(M_{\nu_4} + M_{\nu_5}t^2 + M_{\nu_6}t^4)$$

It is advantageous to tabulate not only $X_{\nu\delta}(\beta)$ and $M_{\nu\delta}(\beta)$ but also $X_{\nu\delta}'(\beta)$, $M_{\nu\delta}'(\beta)$, as well as two other certain quantities occurring in the computations.

Not all of these quantities have been calculated by Berentzen yet. But he establishes all of the equations which are necessary to calculate the series. He calculates furthermore all of the functions $X_\nu(\beta)$, $M_\nu(\beta)$, $X_\nu'(\beta)$, $M_\nu'(\beta)$ mentioned for definite values of t and μ and shows in detail how the constants α_ν are to be determined from the given water height, from the weight, and from the boundary conditions.

M. Hetényi.[5] When a thin shell of revolution is subjected to axially symmetrical edge loading the stresses are of a damped-wave character, rapidly decreasing along the meridian away from the loaded hoop circle. The thinner the shell and the larger the angle of opening, the more concentrated are the stresses in the edge region. For such cases it is satisfactory to use the solution by Geckeler, who takes into account only the highest derivatives in the differential equations of the problem and thereby substitutes the edge region of a shell of revolution by a cylindrical shell. If the shell is somewhat thicker $\left(\frac{a}{\delta} < 40\right)$ or its angle of opening is small ($\varphi < 45$ deg), then Geckeler's solution will involve considerable error and a better approximation is needed, which would take into account also the derivatives next to the highest in the differential equations. The author has accomplished this by a suitable change in the variable and his approximation is of the same order as the one investigated by Blumenthal[6] and by the writer[7] for spherical shells of constant wall thickness.

This type of solution is termed asymptotic and has a point of singularity at the vertex of the shell where it gives infinite values for the unknown quantities. As a rule this singularity affects only the immediate neighborhood of the vertex; however, in Fig. 4 of the paper, even at 5 deg away from the vertex, the angular deflection χ shows no tendency of approaching zero, which value it should assume at the top. It would be interesting to know whether that divergence is due to the singularity at the vertex or to some other reason.

The accuracy of the author's solution is probably high, though it would be of interest to try to estimate the involved percentage error in the same manner as has been done by the writer for shells of constant thickness. The variable wall thickness assumed in the paper is apt to follow the actual stress distribution in shell constructions and the author is to be congratulated for this valuable contribution to the theory of thin shells.

[5] Research Laboratories, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

[6] "Über die Asymptotische Integration," etc., by O. Blumenthal, *Zeitschrift für Mathematik und Physik*, vol. 62, 1914, p. 343.

[7] "Spherical Shells Subjected to Axial Symmetrical Bending," by M. Hetényi, Publication of International Association for Bridge and Structural Engineering, Eidgen. Techn. Hochschule, Zurich, Switzerland, vol. 5, 1938.

The Spring Clutch[1]

A. M. Wahl.[2] The author has presented an interesting analysis of a device which should probably find in the future a wider application than it has had in the past. For example, a possible additional use might be in torque-limiting mechanisms, since, as the author shows, by using a relatively few turns the unwinding torque may be kept practically constant irrespective of the actual magnitude of the coefficient of friction. Such torque-limiting devices could be made to operate in both directions by using two springs, wound in opposite directions, in series on coaxial arbors.

The author derives the expression for limiting torque T by assuming initially a constant force f_0 per unit length produced by assembly of the spring. This force f_0 is determined from energy considerations. The differential relations of an element of the wire are set up and by integration the torque is determined. Near the end of the spring there will be a tendency of the spring to lift from the arbor as indicated in Fig. 1*b* of this discussion, while a concentrated load P' also acts at this end.

It will be of interest to apply curved-bar theory to this problem and to compare the results with those obtained by the author. If r is the mean radius of curvature of the circular ring and u the radial deflection (taken positive toward the center of the ring) at any point at an angle ϕ, as shown in Fig. 1*b*, it may be shown that the following differential equation holds.

$$\frac{d^2u}{d\phi^2} + u = -\frac{Mr^2}{EI} \qquad [1]$$

where M is the bending moment at the angle ϕ.[3]

The equations of equilibrium of an element of the bar may be obtained from Fig. 1*a* (by neglecting small quantities of higher order). These are (for the unwinding condition where the friction force $\mu prd\phi$ is in the direction shown)

$$\frac{dM}{d\phi} = Qr - \frac{\mu prh}{2} \qquad [2]$$

[1] By C. F. Wiebusch. Published in the September, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-103.

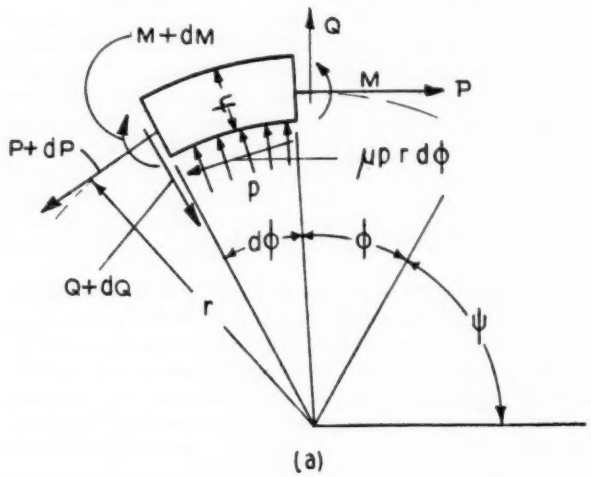
[2] Research Engineer, Research Laboratories, Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

[3] See, for example, "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., New York, N. Y., vol. 2, p. 458.

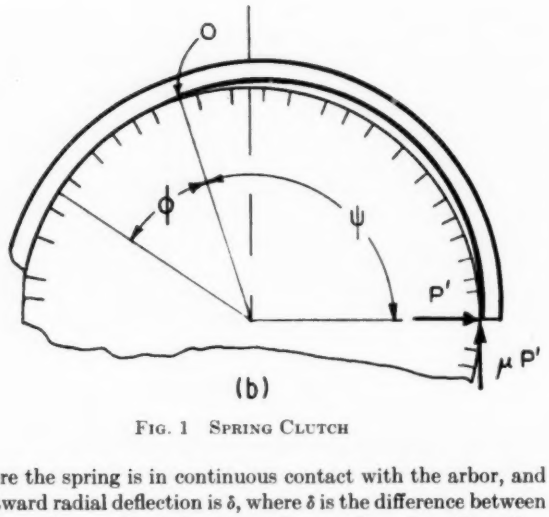
$$\frac{dQ}{d\phi} = pr - P \quad \ldots\ldots [3]$$

$$\frac{dP}{d\phi} = Q - \mu pr \quad \ldots\ldots [4]$$

where Q = shearing force, lb; μ = coefficient of friction; p = radial pressure in pounds per inch acting at angle ϕ; r = mean radius of spring; and h = depth of cross section, in.



(a)



(b)

FIG. 1 SPRING CLUTCH

Where the spring is in continuous contact with the arbor, and the outward radial deflection is δ, where δ is the difference between the arbor radius and initial spring radius, we have from Equation [1] (since $d^2u/d\phi^2 = 0$ for u = const)

$$\delta = \frac{M_0 r^2}{EI} \quad \ldots\ldots [5]$$

where M_0 is the constant moment.

Also where the spring is in continuous contact with the arbor, from Equation [2]

$$\frac{dM}{d\phi} = 0 = Qr - \frac{\mu prh}{2} \quad \ldots\ldots [6]$$

or

$$Q = \mu ph/2 \quad \ldots\ldots [7]$$

From Equation [4], using Equation [7]

$$\frac{dP}{d\phi} = Q - \mu pr = -\mu pr\left(1 - \frac{h}{2r}\right) \quad \ldots\ldots [8]$$

Since in most practical clutch springs h is small compared to r

$$\frac{dP}{d\phi} \approx -\mu pr \quad \ldots\ldots [9]$$

By differentiating Equation [1] and using Equations [2], [3], and [9] we have

$$\frac{dp}{d\phi} + \mu p - \frac{\mu h}{2r}\frac{d^2p}{d\phi^2} = 0 \quad \ldots\ldots [10]$$

The solution of this equation is (for small h/r)

$$p = Ae^{2r\phi/\mu h} + Be^{-\mu\phi} \quad \ldots\ldots [11]$$

For the unwinding condition, it may be expected that $A = 0$, since p cannot continue to increase with increasing ϕ; hence

$$p = Be^{-\mu\phi} \quad \ldots\ldots [12]$$

The angle ψ from the end of the spring at which contact is established with the arbor may be determined as follows:

At $\phi = 0$, the moment, shearing force, and direct load are respectively

$$M_0 = P'r\,[\sin\psi + \mu(1 - \cos\psi)] \quad \ldots\ldots [13]$$

$$Q_0 = P'(\cos\psi + \mu\sin\psi) + X_0 \quad \ldots\ldots [14]$$

$$P_0 = P'(\sin\psi - \mu\cos\psi) - \mu X_0 \quad \ldots\ldots [15]$$

These equations are obtained from conditions of static equilibrium, the force X_0 being an assumed concentrated load acting radially outward at $\phi = 0$; it is not shown in Fig. 1 of this discussion. This force is necessary to satisfy conditions near the end of the spring. In addition, since the end of the spring must deflect outward by an amount $\delta(1 - \cos\psi)$ relative to point 0, shown in Fig. 1 of this discussion (due to forces P' and $\mu P'$), we have from curved-bar theory

$$\delta = \frac{P'r^3}{EI}\left\{\frac{\psi}{2} - \frac{\sin\psi\cos\psi}{2} + \mu(1 - \cos\psi) - \frac{\mu\sin^2\psi}{2}\right\}\left\{\frac{1}{1 - \cos\psi}\right\} \quad \ldots\ldots [16]$$

Also from Equations [2], [3], and [7] for small h/r

$$\left.\begin{aligned} P_0 &= p_0 r = Br \\ Q_0 &= \frac{\mu p_0 h}{2} \end{aligned}\right\} \quad \ldots\ldots [17]$$

where p_0 is the unit radial load at $\phi = 0$.

From Equations [5], [13], and [16], we find if μ is not too large that an approximate expression in radians for ψ is

$$\psi = 2.14 + 1.42\mu \quad \ldots\ldots [18]$$

These equations enable us to determine p_0 by using approximate expressions for $\sin\psi$ and $\cos\psi$. Thus

$$p = \frac{\delta EI}{r^4}\frac{(1 - 0.92\mu)}{(1 + 0.92\mu)}e^{-\mu\phi} \quad \ldots\ldots [19]$$

The free torque is given by

$$T = \int_0^{2\pi N'} \mu p r^2 d\phi + \mu(P' + X_0)r$$

where N' = number of turns in contact with arbor measured from point $\phi = 0$ (note that this is not the total number of turns). Using the values of P' and X_0 found from the foregoing equations, the expression for free torque becomes

$$T = \frac{\delta EI}{r^2}\left[\frac{(1 - \epsilon^{-2\pi N'\mu})\ (1 - 0.92\mu) + 1.84\mu}{(1 + 0.92\mu)}\right] \quad [20]$$

For $N'\mu$ greater than about 1, the exponential term may be neglected and the equation reduces to

$$T = \frac{\delta EI}{r^2} \quad [21]$$

This value of torque coincides with the value given by Equation [5] of the paper[1] if h/r is small and the value of f_0 given by Equation [19] of the paper is used.

For the slipping torque it is merely necessary to change the sign of μ in the preceding equations. In this case the first term on the right side of Equation [11] may be neglected as far as any contribution to the torque is concerned since r/h is assumed very large and with μ negative the exponential term dies out very rapidly. Hence Equation [20] may still be used if the sign of μ is changed.

This gives for the slipping torque

$$T' = -\frac{\delta EI}{r^2}\left[\frac{(\epsilon^{2\pi N'\mu} - 1)\ (1 + 0.92\mu) - 1.84\mu}{1 - 0.92\mu}\right] \quad [22]$$

where the negative sign signifies a torque in an opposite direction to the free torque.

It should be noted that Equation [22] of this discussion differs somewhat from Equation [8] of the paper, the difference being accounted for by the assumptions made as to conditions near the end of the spring. Also the value N' for number of turns in contact with the arbor is used rather than total turns N as used by the author. Since the angle ψ at which contact is established, shown in Fig. 1, is approximately $(2.14 - 1.4\mu)$ radians for the slipping condition (obtained by changing the sign of μ in Equation [18]) it may be expected that N' will be around $^1/_4$ to $^1/_2$ turn less than N. For the larger values of N, if this difference between N and N' is taken into account, the results calculated from Equation [22] of this discussion will not differ much from those calculated from Equation [8] of the paper. In actual springs, however, considerable deviation from the ideal conditions assumed in the analysis may be expected in many cases and this may result in considerable difference between calculated and test values of the slipping torque. Such deviations may also account for the ± 20 per cent variations in slipping torque found experimentally by the author.

It is interesting to note that if we assume a zero coefficient of friction, then from Equation [19] of this discussion the radial pressure becomes constant and equal to

$$\frac{\delta EI}{r^4}$$

For small values of h/r this equation coincides with the expression for f_0 obtained by the author from Equation [19] of the paper for the radial load on the spring when no torque is applied. This expression for f_0 thus involves the assumption that frictional effects are negligible during assembly of the spring on the arbor.

Author's Closure

The curved-beam theory has the advantage of yielding an expression for the end effect, that is, for the portion of the turn which does not contact the arbor. From Equations [18] and [16] of the discussion, it is possible to calculate the inward force P' at the end of the spring. One can then decide whether or not it is necessary to grind or otherwise treat the end of the spring in order to reduce the bearing pressure to a safe value. On small spring clutches no special end treatment is generally required but on larger sizes, intended to give high values of free torque, end treatment is necessary to prevent excessive wear.

The curved-beam theory has the disadvantage of resulting in a more complex derivation and a more complex expression for the torque and in addition it requires, on the part of the reader, a more detailed knowledge of elastic theory than does the energy method. Both methods introduce an approximation regarding the smallness of the ratio of h/R_1 but it is not obvious which derivation results in the better solution in cases where h/R_1 is not very small.

The most sensitive test of the equations are data such as are shown by the torque curve of Fig. 4 of the author's paper. For the case shown in Fig. 4, Equation [21] of the discussion would give about 3.5 per cent lower value of torque at the 0.193-in. arbor diameter and a 3 per cent higher value at the 0.182-in. arbor diameter. It is believed that the data would have shown up such differences. This cannot be taken as conclusive evidence for choosing between the two equations and more data would be required to settle the point. The percentage difference between the two equations is given by $100\ (\delta - h/2)/R_1$ and hence for values of h/R_1 of about 0.2 one might expect a 10 per cent difference between the two methods of calculation. This difference is large enough to permit easy experimental differentiation between the two relations.

The Forces Required for Rolling Steel Strip Under Tension[1]

C. W. MacGregor.[2] Dr. Nádai is to be congratulated upon this noteworthy contribution to our knowledge of the stresses set up during the rolling of steel. The extension of the solution to the cases of front and back tension is very interesting and will prove of considerable value in that it clearly brings out the effects of certain variables, such as friction on the shape of the pressure distribution between the rolls and the bar.

It appears that during rolling under well-lubricated conditions the assumption that p is a principal stress represents the true case very closely. When dry friction is assumed, or if metal to metal contact is present, p will then probably differ somewhat from the principal stress at the point of contact. The writer would therefore like to ask Dr. Nádai if any calculations have been made in order to show the magnitude of this effect.

Recently cold rolling tests were made at the Massachusetts Institute of Technology during which motion pictures were made of the flow layers obtained on the highly polished surface of the rolled bar as it passes between the rolls in the rolling mill.[3] The bars were of Bessemer screw stock, 1 in. square in cross section, and the rolls about 6 in. in diameter. The reductions used were from 1 to $2^1/_2$ per cent. A special apparatus using reflected light was designed in order to photograph the flow layers in motion.

It was found that the flow layers received were simple cycloids checking very closely those predicted by Prandtl's solution for plastic flow between two rough parallel plates.

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[1] By A. Nádai. Published in the June, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-54.

[2] Associate Professor of Applied Mechanics, Massachusetts Institute of Technology, Cambridge, Mass. Jun. A.S.M.E.

[3] "The Plastic Flow of Metals," presented at the A.I.M.E. Meeting during the National Metals Congress in Detroit, Oct. 17, 1938.

Since the rolling problem is indeed very complicated with such a large number of variables entering the problem, the method adopted by Dr. Nádai of inquiring into the effect of changing one variable at a time appears to be perhaps the best procedure to give us the fundamental information so needed to clarify this problem. It is hoped that he will continue the work along these lines.

Author's Closure

The surface friction between rolls and the piece to be rolled mentioned by Dr. MacGregor deeply influences the structure of the rolled piece and the distribution of the stresses in the contact areas. Conditions in this respect may be widely different in the case of hot rolling or when a strip is cold-rolled and at the same time one of the high-pressure lubricants is applied. Axial tension must also have a deep effect on the stress distribution and probably makes it more uniform along a line which is perpendicular to the plane of the sheet. It is known that a few cases have been treated, in which the nonuniform stress distributions were determined in a layer of a plastic material squeezed out under pressure between rigid plates. The case of parallel plates and that of two slightly inclined plates have been treated. It appears, therefore, that it may be possible perhaps to improve the assumptions which were made in the paper for describing the states of stress in the compressed portion of a sheet, so that account also of the tangential forces is taken retarding the flow of the plastic material when it is compressed between the rolls. The experiments by Dr. MacGregor may throw additional light on the influence of the surface friction and should be of assistance in the future, when such improved computations should be carried out.

Mathematical-Machine Determination of the Vibration of Accelerated Unbalanced Rotor[1]

F. M. Lewis.[2] A solution of the problem of the vibration of an accelerated unbalanced rotor can be obtained by the same procedure, using complex functions, which was utilized in the writer's paper[3] "Vibration During Acceleration Through a Critical Speed," which was referred to by the author.[1]

If the eccentricity of the mass M is e then the magnitude of the exciting force becomes

$$M \frac{\omega^2}{\omega_n^2} e\omega_n^2 = \frac{r^2}{q^2} ek \quad \ldots\ldots [1]$$

and if the resonance factor R is now equal to x/e, then the dimensionless solution corresponding to Equation [15] of the 1932 paper[3] becomes

$$R = -\frac{i\pi e^{i\sigma}}{q^2\delta} e^{2\pi i u_3 r}\left[\int_0^r r^2 e^{i\pi[(r^2/q)-2ru_1]}\,dr - e^{2\pi i u_3 r}\int_0^r r^2 e^{i\pi[(r^2/q)-2ru_3]}\,dr\right] \quad \ldots\ldots [2]$$

Here R is the same quantity as the x/e or u of Baker's paper,[1] but the other symbols are identical with those of the 1932 paper.[3]

We will indicate the method of solution without carrying out all the work in detail.

With the complex substitution

$$r = \sqrt{\frac{iz_1 q}{\pi}} + qu_1 \quad \ldots\ldots [3]$$

the equation obtained from Equation [2] of this discussion contains three integrals

$$\int_0^r e^{-z_1}\,dz_1 \quad \ldots\ldots [4]$$

$$\int_0^r \frac{1}{\sqrt{z_1}} e^{-z_1}\,dz_1 \quad \ldots\ldots [5]$$

$$\int_0^r \sqrt{z_1}\, e^{-z_1}\,dz_1 \quad \ldots\ldots [6]$$

The solution of Equation [4] is obvious, while Equation [5] appears in the solution for an exciting force of constant amplitude.

Integrating by parts, Equation [6] of this discussion can be written

$$\int_{z_1}^{\infty r} \sqrt{z}\, e^{-z}\,dz = \sqrt{\pi}\,K + \sqrt{z_1}\, e^{-z_1} + \frac{1}{2}\int_{z_1}^{\infty r} \frac{1}{\sqrt{z}} e^{-z}\,dz \quad \ldots\ldots [7]$$

After these substitutions and reduction there is obtained the solution

$$R = -\frac{1+i}{\sqrt{2}} + \frac{\sqrt{\pi q}}{2\delta}\left[\frac{i}{2\pi q}\left\{\varphi_1 - \varphi_2 + 2\sqrt{\pi}\, e^{z_1} K\right\} + u_1^2(\varphi_1 + 2\sqrt{\pi}\, e^{z_1} K) - u_2^2\varphi_2\right] \quad \ldots\ldots [8]$$

The φ_1 and φ_2 integrals were evaluated for the 1932 paper[3] so that results for the present case can be easily obtained, the only additional operations needed being multiplication and division, provided that the same $r^2\pi q$ values are used for which φ_1 and φ_2 were previously computed.

The writer has evaluated the solution for $q = 40.8$, $\gamma = 0.051$ accelerated motion, which is very close to Baker's curve for $q = 40.8$, $\gamma = 0.050$ and by interpolating between Baker's curves for $\gamma = 0.05$ and $\gamma = 0.1$ a close check can be obtained in this case.

A comparison between points on the curve $q = 40.8$, $\gamma = 0.051$ by the machine and complex integral calculation is given in Table 1 of this discussion.

TABLE 1

β	u_3 or R — Machine	u_3 or R — Lewis' Equation [8]
0.924	4.0	3.98
1.000	7.6	7.58
1.076	12.6	12.42
1.107	13.7	13.45
	Peak	
1.122	13.7	13.30
1.150	11.5	11.34
1.198	3.7	3.68

The curves are almost identical in shape, having their maximum at the same β value. The peak value of the differential-analyzer curve appears to be high by $2^1/_2$ or 3 per cent.

The peak value of the deceleration curve was computed and the machine curve appears to be high by $3^1/_2$ per cent.

In view of the difficult problem given the machine, virtually amounting to summing a considerable number of positive and

[1] By J. G. Baker. Published in the December, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-145.

[2] Professor of Marine Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. A.S.M.E.

[3] "Vibration During Acceleration Through a Critical Speed," by F. M. Lewis, Trans. A.S.M.E., vol. 54, 1932, paper APM-54-24, p. 253.

negative loops, such an agreement can be considered remarkably close. It is to be expected that for lower q values and higher γ values a still closer check could be obtained.

Author's Closure

In the oral discussion attention was called to a paper, "The Transient Vibrations of Machines," by W. E. Johnson, published in the *General Electric Review*. The author was not aware of this paper, which should have been included in the references noted. The present paper differs mainly by including the effect of damping.

Professor Lewis' discussion is especially valuable in that it not only demonstrates an entirely mathematical method of obtaining the results of the paper, but also substantiates the accuracy of the mathematical machine used. The author is grateful for Professor Lewis' contribution.

Book Reviews

The Timoshenko Commemorative Volume

THE MECHANICS OF SOLIDS. A Collective Work. The Macmillan Company, New York, N. Y., 1938. Cloth, 6 × 9 in., 277 pp., illustrated, $5.

REVIEWED BY S. C. HOLLISTER[1]

ON the evening of December 6, 1938, at the time of the Annual Meeting of The American Society of Mechanical Engineers in New York, a notable group met at dinner to do honor to Professor Stephen Timoshenko. On that occasion his many friends presented him with a volume of collected writings to mark his approaching sixtieth birthday. This volume is made up of papers contributed by twenty nine of his friends, associates, and former students, each writing in his special field of interest. It is the first commemorative volume of the kind to appear in this country.

It is only natural, and indeed very fitting, that the group of authors reflect Professor Timoshenko's relation to the International Congresses for Applied Mechanics; his years at Westinghouse; his many friendships and associations in the Applied Mechanics Division of this Society, of which he was a leading sponsor; and his decade of teaching at the University of Michigan. It characterizes the quality of his leadership, both as an authority in the field of mechanics to which he has devoted his life, and as an organizer in strengthening the research and teaching in that field.

During the century or more that engineering training has been given in America, applied mechanics has rested chiefly on the older tradition of the English-speaking countries abroad. In the early part of the nineteenth century, the prevalent works here were those of Navier and Poisson. By the middle of the century the English translation of Weisbach was available, and about that time an American edition of Moseley, and the work of the great Scottish engineer Rankine, came into common use. The latter work especially had a profound influence on the scope and methods of instruction in mechanics in this country. The French tradition had disappeared, and the Russian tradition, founded by Euler, was almost unknown. Such a development was inevitable, chiefly owing to the remoteness of frontiers where other languages were spoken, and the consequent scanty knowledge among engineering students of other tongues than English.

Late in the century American works on mechanics began to appear. The books by Merriman and Church came into general use. They carried on the same tradition as before, with only fragments drawn from French and German schools.

The first two decades of the present century witnessed an attempt to "simplify" the writings of earlier authors. Mechanics in this period did not move forward here as it did abroad. The disparity between the status of the subject here and abroad became increasingly marked. It was into such a situation that Timoshenko came at the end of the World War.

We have witnessed during the last two decades a tremendous stimulation in the study of mechanics in this country. In this development Timoshenko has played a leading part. The books "Applied Elasticity" (1923) by Timoshenko and Lessells, and "Strength of Materials" (1930) by Timoshenko, have exerted a profound influence upon the teaching of mechanics in America. The organization of the A.S.M.E. Applied Mechanics Division and the initiation of that Division's publication, JOURNAL OF APPLIED MECHANICS, to both of which Timoshenko gave impetus, have further strengthened the field by furnishing a forum for the discussion of practical problems in dynamics, elasticity, and plasticity.

It is no wonder, then, that a spontaneous desire arose to do honor to Timoshenko on his sixtieth birthday anniversary. He has personally left a profound impression upon the advancement of mechanics in this country, both on its teaching and upon its practical application to problems in industry. As a teacher he has that rare gift of developing men to a new level of competence, as contrasted with the frequent practice of merely presenting courses.

Handsome as the present volume is as a tribute to the genius of Timoshenko, it is a notable book in its own right. The authors include distinguished European writers as well as many well-known American teachers and engineers. The contributions deal with the whole gamut of mechanics of solids, both in fundamental aspects and in practical applications. As a reference work it will be sought after by many workers in dynamics, elasticity, elastic buckling, plasticity, and soil mechanics.

[1] Dean, College of Engineering, Cornell University. Mem. A.S.M.E.

Bending of Plates

MOMENTENTAFELN UND EINFLUSSFLÄCHEN FÜR KREUZWEISE BEWEHRTE EISENBETONPLATTEN. By Ernst Bittner. Julius Springer, Vienna, 1938. Paper, 7 × 10 in., 86 pp., 16 figs., and 81 tables, 9.60 rm.

REVIEWED BY S. TIMOSHENKO[2]

THE problem of bending of a simply supported rectangular plate by a uniformly distributed load is completely solved not only in the case when the load is distributed over the entire surface of the plate but also in the case when the load is distributed over a rectangular area representing only a portion of the surface of the plate. In the design of plates used as a floor of a highway bridge we have a more complicated problem of bending of plates under the action of concentrated loads. The practical

[2] Professor of Theoretical and Applied Mechanics, Stanford University, Stanford University, California. Fellow A.S.M.E.

solution of this problem is discussed in the Bittner book. To find, in each particular case, the most unfavorable loading condition, influence surfaces are introduced. By using these the maximum bending moment for any combination of loads can be readily obtained. The concentrated forces in this calculation are replaced by loads distributed over small rectangular areas.

The first part of the book contains the general explanation of the use of influence surfaces and gives a numerical example in which the method is illustrated with all necessary detail. The second part of the book contains numerical tables of influence numbers calculated for various ratios of the sides of the plate and for various magnitudes of the loaded rectangle. The third part of the book is of a more theoretical character. It includes the discussion of the general theory of rectangular plates which was used in the calculation of the influence numbers and also a discussion of some approximate solutions which were used in the case of plates with built-in edges.

The book is of considerable importance for engineers who are designing concrete slabs submitted to the action of systems of traveling loads as for case of highway bridges. Most of the numerical tables appear for the first time and will be useful to designers.

Theoretical Mechanics

THEORETICAL MECHANICS. By Carl Jenness Coe, Ph.D., Assistant Professor of Mathematics, The University of Michigan. The Macmillan Company, New York, N. Y., 1938. Cloth $5^1/_2 \times 8^1/_2$ in., xii and 555 pp., 102 figs., $5.

REVIEWED BY J. N. GOODIER[3]

THIS book covers courses given by the author in the department of mathematics, University of Michigan, one introductory and one advanced, throughout the year. A year's work in calculus is suggested as adequate mathematical preparation.

The preface discusses the present position of Newtonian mechanics in relation to relativity mechanics. A very readable introduction justifies the study of the former on the grounds that the extent to which it explains the physical world is much more surprising than the fact that there are cases in which it does not. The point of view throughout the book is correspondingly philosophical, with virtually no regard to practical application. Such regard is of course not pertinent in a "Theoretical Mechanics," except incidentally where the author chooses to illustrate principles by means of actual mechanisms. Then the student may be led to believe that a physical problem has been solved, when, in fact, tacit idealization has destroyed its reality.

The analytical treatment begins with a concise discussion of the rectilinear motion of a particle. The next chapter is an introduction to vectors; vector analysis is then employed consistently throughout the book. The chapter on curvilinear motion proceeds at once to space curves with subsequent specialization to plane curves. The special topics include finite pendulum motion and planetary motion. Rigid-body mechanics begins with the general kinematical theorems for the fixed point and the fixed axis, and includes "twists" or "screws" as general motions. Then comes a chapter on sliding vectors—vectors localized in a line, as forces in statics. This covers the reduction of vector systems, or the composition of forces in the older specialized language. In the chapter on statics of particles and of a rigid body, simple frames and centroids are included; the following chapter deals with the flexible cord and funicular polygon.

The discussion of virtual work is preceded by a section on constraints, where it is emphasized as a principle that motions of a set of particles satisfying the imposed conditions are as a whole neither aided nor opposed by the constraints. It has proved difficult to realize any clear meaning in this as it stands, in spite of physical illustrations provided. It is only later that the author's intentions become apparent, when he lays down postulates as to the work of the constraints, whereupon the "principle" in question seems to become redundant.

The chapter on kinetics of particles introduces the momentum and energy theorems, and, in brief, the problem of n bodies. The treatment of moments of inertia in the chapter on kinetics of a rigid body is in terms of vector analysis, but is very much more cumbersome than the usual coordinate treatment. The mechanical subjects of the chapter are free motion in a plane, motion about a fixed point, including tops, and about a fixed axis. There is then a chapter on general principles of mechanics. Throughout the book are admirable short historical sections.

A final chapter on potential theory is preceded by a treatment of scalar and vector fields, with the theorems of Green and Stokes. The potential theory, as may be expected, is put on a rigorous basis. There is an excellent set of problems, including potentials of ellipsoids of revolution for axial points.

The book contains in all 696 problems for the student.

[3] Professor of Engineering Mechanics, Cornell University, Ithaca, N. Y. Mem. A.S.M.E.

Elements of the Topology of Plane Sets of Points

ELEMENTS OF THE TOPOLOGY OF PLANE SETS OF POINTS. By M. H. A. Newman. The University Press, Cambridge; The Macmillan Company, New York, N. Y., 1939. Cloth $5^1/_2 \times 8^3/_4$ in., viii and 222 pp., 93 figs., $3.50.

REVIEWED BY W. L. AYRES[4]

ROUGHLY topology may be defined as "rubber sheet geometry." While this is not accurate it gives a rather clear conception of the nature of the subject. Each branch of geometry is the study of those properties invariant under a particular transformation. For example, Euclidean geometry is the study of properties which remain unchanged under rigid motions. In topology the transformation in question is any single-valued continuous one which has a single-valued continuous inverse. Such a transformation is called a homeomorphism and topology is the study of those properties of spaces which are unchanged by homeomorphisms. For example, a circle cuts its plane into two regions and one of the oldest and most famous theorems of topology, the Jordan curve theorem, states that every homeomorphic transform of a circle cuts its plane into exactly two regions.

While topology is of prime importance in the various branches of mathematics, for the present it finds little application in any of the engineering sciences. Distance is not an invariant of topology since any figure may be "stretched" to any size by a homeomorphism. For this reason it does not lend itself easily to those sciences where measurement and rigid bodies are of first importance as engineering. At the present time it finds its only application to engineering in the analysis of networks in electrical circuit theory.

The present book gives an account of the topology of plane sets and forms a carefully written introduction to the subject which may be read with no previous knowledge on the part of the reader. The first four chapters, which comprise roughly half the book, give an elegant exposition of the general theory of sets of points. The last four chapters develop the plane topology using rectangular gratings. Among other results a simple and readable proof of the Jordan curve theorem is given.

This book will serve as an excellent introduction to this im-

[4] Associate Professor of Mathematics, University of Michigan, Ann Arbor, Michigan.

portant and fascinating subject. It is to be regretted that the author seems to be unaware of the important American work in plane topology with the exception of the Alexander Lemma.

Hydrodynamics

THEORETICAL HYDRODYNAMICS. By L. M. Milne-Thomson. The Macmillan Company, New York, N. Y., 1938. Cloth, 6¼ × 9¾ in., xxv and 552 pp., 330 figs., 4 plates, and 508 exercises, $11.25.

REVIEWED BY HUGH L. DRYDEN[5]

THIS book, which is an outgrowth of the author's lectures to junior naval constructors at the Royal Naval College, is a clear account of the mathematical theory of the motion of an inviscid or perfect fluid based on vector methods and notation. A single chapter of 37 pages is devoted to viscosity. The book accordingly makes a good textbook for courses in classical fluid mechanics. Over 500 examples are provided, some easy, but many quite difficult. As indicated by the title, there is no attempt to introduce comparisons with experiment or to indicate fields of practical application.

The book contains 19 chapters. The first three deal respectively with Bernoulli's equation, properties of vectors, and general aspects of the equations of motion. Chapter 4 describes those properties of two-dimensional motion which can be treated without the complex variable. Chapter 5 gives an account of the theorems with respect to the complex variable, defined as a vector operator, required in the following treatment. The next nine chapters give a detailed discussion of two-dimensional motion in terms of the complex variable; the chapter headings are streaming motions, airfoils, sources and sinks, moving cylinders, theorem of Schwarz and Christoffel, jets and currents, the wake, rectilinear vortices, and waves. The next four chapters treat three-dimensional problems, first the application of conformal mapping to problems with axial symmetry, then, spheres and ellipsoids, solids moving through a liquid, and vortex motion in three dimensions. The final chapter gives the application of vector methods to viscous liquids.

The experience and skill of the author as a teacher have enabled him to present the subject with the detail essential to ease of comprehension of the mathematical development, yet without prolixity. While the language is mainly mathematical, the text moves along smoothly, and the physical interpretations are given, though often in very brief space. Thus the physical concept, virtual mass, is described in a single sentence on each of two widely separated pages. The author mentions one new method specifically, a method of partial differentiation with respect to a vector applied to obtain Kirchhoff's equations, and there are other original contributions and original methods of presentation throughout the text.

Strength of Materials

FORMULAS FOR STRESS AND STRAIN. By Raymond J. Roark, Professor of Mechanics, University of Wisconsin. Product Development Series, First Edition, McGraw-Hill Book Company, Inc., New York, N. Y., 1938. Cloth, 6 × 9 in., ix and 326 pp., 19 tables, 36 figs., $3.

REVIEWED BY R. E. PETERSON[6]

THIS book combines the sequence and general style of a strength-of-materials textbook and the direct usefulness of a conventional handbook. It differs from the strength-of-materials textbook in not giving the derivation of formulas and also in presenting much information not found in such a textbook. It differs from the handbooks in stating general principles, in making available the results of advanced work, in including literature references for all data, and in giving supplementary explanations and notes. It is obvious in looking over the book that it has been designed primarily to be of ready use as a reference; the tables are excellently arranged and all data are clearly and carefully presented.

In part one definitions and symbols are given.

In part two general statements are made concerning the behavior of engineering materials under various types of loading, the principles used in mathematical analysis, and the main experimental methods used for the solution of stress-distribution and strength problems.

In part three 18 tables are given together with text explanations. Tables of stresses and deflections are not usually found for such cases as curved beams, circular rings, flat plates, pressure vessels, and bodies in contact. The tables of elastic stability of a wide variety of members are also of unusual value. The table of stress-concentration factors will be especially useful to the designer. Two minor suggestions occur to the reviewer in this connection. The term "stress-concentration factor for repeated loading" can be replaced by a more accurate term which has recently come into use, namely "fatigue strength reduction factor." The data under 8 in Table 17 for fillets in tension are now obsolete and represent a considerable error. These data were obtained by Timoshenko and Dietz about fifteen years ago (among the early photoelastic tests in this country); celluloid models and white light were used at that time. The recent results of advanced technique using bakelite models and monochromatic light may be considered highly accurate. Data by Frocht (reference 6) should be used for the foregoing case. The remainder of the table contains the latest data available.

The points just mentioned are indeed quite minor when one considers the large amount of data presented. The reviewer and his associates have found the book to be useful on a number of occasions and can recommend it highly to engineers who need a ready source of information for design problems.

Diesel Engines

DIESEL ENGINES. By B. J. von Bongart. D. Van Nostrand Company, Inc., New York, N. Y., 1938. Cloth, 6 × 9 in., xiii and 335 pp., 346 figs., $5.50.

REVIEWED BY C. FAYETTE TAYLOR[7]

THIS book comprises a varied assortment of information on the modern Diesel engine. Of its 335 pages approximately 230 are devoted to rather uncritical descriptive matter of various types of Diesel engines, fuel pumps, and injection nozzles.

The theoretical portion of the book is contained in four chapters, the first of which is called "Thermodynamics," largely confined to the air cycle and to some very brief data on fuel consumption. This is followed by a brief chapter, "Combustion," giving meager and scattered information on the pressure-time diagram and the delay period. "Fuel Oils" are treated in fifteen pages, mostly devoted to laboratory characteristics, with an extremely brief consideration of ignition lag. The chapter on "Fuel Atomization" appears to be compilations of some of the results obtained by the National Advisory Committee for Aeronautics and by one or two German investigators.

At the end of the book is a chapter called "Miscellaneous," which treats of governors, cooling systems, dynamic balance, and

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[6] Manager, Mechanics Division, Westinghouse Research laboratories, East Pittsburgh, Pa. Mem. A.S.M.E.

[7] Professor of Automotive Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

vibration, all in thirteen pages. The introductory chapter on the history of the Diesel engine, however, is unusually complete.

The sources of the data presented, most of which are in the form of curves, are in many cases not given and in other cases difficult to determine. It is evident, however, that these data have been taken from the literature and are not the result of original work on the part of Mr. von Bongart. That he is somewhat biased in favor of the Diesel engine is indicated by two altitude-performance curves, Figs. 233 and 238, both of which show an advantage of the Diesel engine at altitude, which is not confirmed in practice or by any properly conducted tests (see N.A.C.A. Technical Note No. 619). These curves are extremely misleading and tend to shed doubt on the accuracy of other data in the book. Many of the other curves, unaccompanied by adequate critical analysis, tend to be difficult to understand or even misleading.

It is with regret that this reviewer must state that this book adds nothing of significant value to existing literature on the subject of the Diesel engine.

The Theory of Machines

THE THEORY OF MACHINES. By Thomas Bevan. First edition, Longmans, Green and Company, New York, N. Y., 1939. Cloth, 5½ × 9½ in., vii and 549 pp., 357 figs., $5.40.

REVIEWED BY G. B. KARELITZ[8]

BOOKS on the elementary theory of machines appear frequently. This can only mean that the profession is not satisfied with the existing textbooks. The probable reason is that the subject is practically unlimited in scope and no textbook could cover all requirements. Even a comparatively simple machine presents many problems when its performance, the forces on its members, its vibrations, etc., are analyzed. The information collected by any author must be presented in a limited number of pages; his book may therefore seem inadequate. Moreover, writers often have their pet subjects which overshadow other topics, such as gear-tooth development, or unbalanced forces in reciprocating engines. In addition, the art has moved so fast during recent years that all textbooks on machine design appear somewhat obsolete as soon as they are printed. Any new textbook on the theory of machines is therefore received with considerable interest.

The book of Bevan may be called to the attention of the profession. It was written primarily for "the student who is preparing for a university degree or who intends to sit for the membership examination of one of the engineering institutions." Its contents are therefore chosen and treated to give the student the necessary equipment to answer the problems given during these examinations. The author did a thorough job. He collected carefully the information necessary to satisfy the examiners and has given in the book numerous problems for exercise, with answers appended at the end of the volume. A large proportion of the problems are from the "examination papers" of various institutions. The problems indicate that the examiners endeavored to compile fair questions which would soundly test the familiarity of the aspirants with the theory of machines.

This approach has produced a volume which is no more advanced or modern than several of our current textbooks, but which covers the field very completely. The derivation of expressions is clear and precise while the material is presented in a concise manner. The 550 pages contain a vast store of information, and teachers of mechanics of machinery will be delighted to use this volume.

The table of contents reads as follows: Simple mechanisms; motion, inertia; velocity and acceleration; mechanisms with lower pairs; valve diagrams and valve gears; friction; belt, rope, and chain drives; brakes and dynamometers; cams; toothed gearing; gear trains; the equilibrium of machines, turning moment, the flywheel; governors; balancing; vibrations. Two thirds of the work is given to kinematics, and one third only to dynamics of machinery. However, dynamic balancing machines are briefly described and their action explained. The brief chapter on vibration covers the oscillation of a pendulum, the critical speed of a shaft and the torsional vibration of two-rotor, and multirotor shafts. As usual, the chapter on governors deals mostly with the mechanics of the flyball type.

The text is well illustrated with numerical examples which are carried out in detail. Numerous problems at the end of each chapter add greatly to the value of the book. But this volume does not obviate the need of a good modern textbook on the theory of machines, a book which would reflect the advances of recent years in machine design.

[8] Professor of Mechanical Engineering, Columbia University, New York City. Mem. A.S.M.E.

The Normal Modes of Vibrations of Beams Having Noncollinear Elastic and Mass Axes

By CLYNE F. GARLAND,[1] BERKELEY, CALIF.

This analysis deals with vibration characteristics of cantilever beams in which the longitudinal axis, passing through the mass centers of the elementary sections, is not collinear with the longitudinal axis about which the beam tends to twist under the influence of an applied torsional couple. Expressions are derived from which the natural frequencies and normal modes of vibration of such a beam can be determined. The Rayleigh-Ritz method is employed to determine the frequencies and amplitude ratios. Following the development of the general expressions, more specific equations are derived which express the natural frequencies and relative amplitudes of motion in each of two normal modes of vibration.

The theoretical relationships of the several physical properties of the beam to the natural frequencies of vibration are shown graphically. Finally a numerical example is presented for a particular beam, and the computed natural frequencies and normal modes are compared with those determined experimentally.

DURING recent years, with the development of machines for heavy-duty and high-speed service, there has come a wide interest in the many vibration problems which have arisen. Mechanical vibrations, with their accompaniment of increased stresses, energy losses, and noises, are apt to appear not only in the parts of the machine itself, but also in the structural members of its supporting framework.

A thorough understanding of the relationships among the several factors involved in vibration phenomena has become of great importance to those concerned with the design of machines. To this end expressions for the normal modes of vibration and the corresponding natural frequencies have been developed for a large number of machine parts of simple form (1, 2).[2]

The analysis here presented deals with the vibration characteristics of cantilever beams in which the longitudinal axis G, shown in Fig. 1, which passes through the mass centers of the elementary sections Δy, is not collinear with the longitudinal axis C about which the beam tends to twist under the influence of an applied torsional couple. The axis of twist C, sometimes known as the "elastic axis," may also be defined by the property that it is the only axis along which a transverse load applied to the beam will produce flexure without twisting. The condition of noncollinear axes just described is known to exist in most prismatic bars having nonsymmetrical sections (3, 4, 18), as well as in built-up beams whose structural members are not symmetrically placed (5, 6).[3]

When such a beam is subjected to a transverse vibratory motion, say in the Y-Z plane shown in Fig. 1, torsional oscillations of the beam about the longitudinal axis are caused to be superimposed upon the lateral motion. This torsional motion of any elementary length of the beam is due to the fact that the resultant of the transverse shearing forces on the section acts at C, and therefore tends to rotate the section about its mass center G. Thus, the beam is seen to behave as a coupled elastic system with normal modes of vibration involving simultaneous displacements in flexure and torsion. Accordingly, the natural frequencies of vibration in the several normal modes differ from the frequencies computed for vibration in pure flexure and pure torsion, respectively.

In the present paper general expressions are developed from which the natural frequencies and normal modes of vibration of such a beam can be determined. These expressions are derived from energy considerations which, in turn, are based upon assumed normal elastic deflection curves in bending and torsion. The Rayleigh-Ritz method is employed to determine the frequencies and amplitude ratios. Following the development of

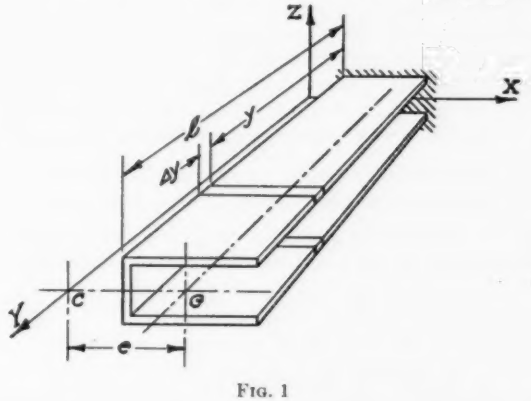


FIG. 1

the general expressions, more specific equations are derived which express the natural frequencies and relative amplitudes of motion in each of two normal modes of vibration. These equations involve dimensionless groups, the values of which depend upon the various physical characteristics of the beam. The theoretical relationships of the several physical properties of the beam to the natural frequencies of vibration are also shown graphically in Fig. 3. Finally, a numerical example is presented for a particular beam, and the computed natural frequencies and normal modes are compared with those determined experimentally.

A first approximation to the solution of this problem was made by Greene and Younger (7, 8)[4] in a study of the flutter of monoplane wings. In their solution the wing (i.e., beam) is treated as a rigid body constrained to rotate about a transverse axis at the built-in end and about a longitudinal axis—corresponding to

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[1] Assistant Professor, Mechanical Engineering, University of California.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Presented at the Annual Meeting, Philadelphia, Pa., December 4-8, 1939, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] The relative positions of these axes in the airplane wing is discussed in these references.

[4] A summary of the theoretical work appears in reference (8).

the X- and Y-axes, respectively, in Fig. 1. The elastic restoring moments are taken to be proportional to the angular motion about these respective axes. Expressions are derived for the natural frequencies of vibration of this idealized wing in each of its two normal modes. A similar treatment of the same problem is presented by Kassner and Fingado (9) in which they treat the wing as a rigid body supported on springs and left free to execute up-and-down vibrations without torsion, or to undergo torsional oscillations about any longitudinal axis. Here again the system has been very highly idealized in that no attempt has been made to take into account the elastic deformations in flexure and torsion. Many other writers (11, 12, 13, 14, 15, 16)[5] who have investigated the problem of flutter in the wings and control surfaces of airplanes have been primarily interested in determining the relationship of the structural characteristics of the wing to the critical air speed at which the phenomenon of flutter is induced, and devote little or no attention to the aspect of the problem to be considered in this paper.

It may be well to note at this point that the problem considered in the present paper is related to the problem of wing flutter in that (*a*) in most cases the monoplane wing is essentially a beam of type under consideration, and (*b*) when such a wing is excited by an air stream moving at the proper (i.e., critical) velocity it tends to vibrate in one of its normal modes and at the corresponding natural frequency (7, 10, 13, 14, 16).[6] The problem of flutter is mentioned here because it has received a great amount of study due to the many spectacular and tragic aircraft accidents which have been traced to this cause. It is generally considered as an example of self-induced vibration (17),[7] a special type of vibration which is not to be confused with the more common forced-vibration phenomenon.

While the principles discussed in this paper are related to the problem of flutter to the extent outlined, it should be clearly understood that their applications are by no means limited to that particular problem. Indeed determination of the natural frequencies and normal modes is essential to the solution of any vibration problem in which such a structural member is involved.

Derivation of Expressions for the Natural Frequencies and Normal Modes—General

The free vibrations of a beam of the type shown in Fig. 1 may be considered to be the resultant of the three components

$$\left.\begin{aligned} z &= [a_1Y_1 + a_2Y_2 + a_3Y_3 + \dots.] \sin \omega_n t \\ \theta &= [\varphi_1 Y_1' + \varphi_2 Y_2' + \varphi_3 Y_3' + \dots.] \sin \omega_n t \\ x &= [b_1 Y_1'' + b_2 Y_2'' + b_3 Y_3'' + \dots.] \sin \omega_n t \end{aligned}\right\} \dots [1]$$

where z denotes the vertical component of displacement at any section; θ denotes the angular displacement at any section; x denotes the horizontal component of displacement at any section; $Y_1, Y_2, \dots Y_1', Y_2', \dots Y_1'', Y_2'', \dots$ are functions of y which satisfy the end conditions for any particular beam; and $a_1, a_2, a_3, \dots \varphi_1, \varphi_2, \varphi_3 \dots b_1, b_2, b_3, \dots$ are the amplitudes of the respective functions Y_n.

In order to reduce the complexity of the equations it will be assumed that the flexural rigidity of the beam in the X-Y plane is very much greater than that in the Y-Z plane. Thus, the x component of the motion may be neglected and the total motion considered to be composed of the z and θ components. This assumption is a reasonable one since most beams of unsymmetrical section are much stiffer in one plane than in the other.

The potential energy of the beam may then be expressed as the sum of the energies stored in the beam due to the displacements z and θ, respectively, or

$$V = \frac{EI_{x-x}}{2} \int_0^l \left(\frac{\partial^2 z}{\partial y^2}\right)^2 dy + \frac{C}{2} \int_0^l \left(\frac{\partial \theta}{\partial y}\right)^2 dy \dots [2]$$

where V is the total potential energy stored in the beam; E is Young's modulus; I_{x-x} is the rectangular moment of inertia of the beam section relative to the x-axis; C is the torsional rigidity of the beam (3, 18);[8] and l is the length of the beam.

If the expressions for z and θ from Equation [1] are substituted into Equation [2], the expression for the potential energy becomes

$$V = \frac{1}{2}\left\{EI_{x-x}\left[a_1^2 \int_0^l \left(\frac{d^2Y_1}{dy^2}\right)^2 dy + a_2^2 \int_0^l \left(\frac{d^2Y_2}{dy^2}\right)^2 dy + \dots\right] + C\left[\varphi_1^2 \int_0^l \left(\frac{dY_1'}{dy}\right)^2 dy + \varphi_2^2 \int_0^l \left(\frac{dY_2'}{dy}\right)^2 dy + \dots\right]\right\} \sin^2 \omega_n t \dots [2a]$$

or

$$V = \frac{1}{2}\left\{K_x[a_1^2Q_1 + a_2^2Q_2 + \dots] + C[\varphi_1^2 R_1 + \varphi_2^2 R_2 + \dots]\right\} \sin^2 \omega_n t \dots [2b]$$

where K_x denotes the flexural rigidity EI_{x-x}; $Q_1, Q_2, \dots$ denote the integrals $\int_0^l \left(\frac{d^2Y_1}{dy^2}\right)^2 dy$, $\int_0^l \left(\frac{d^2Y_2}{dy^2}\right)^2 dy$, ... respectively; and $R_1, R_2, \dots$ denote the integrals $\int_0^l \left(\frac{dY_1'}{dy}\right)^2 dy$, $\int_0^l \left(\frac{dY_2'}{dy}\right)^2 dy$, ... respectively. Or still more briefly

$$V = \frac{1}{2}\{\beta\} \sin^2 \omega_n t \dots [2c]$$

where β denotes the quantity in braces, Equations [2a] and [2b].

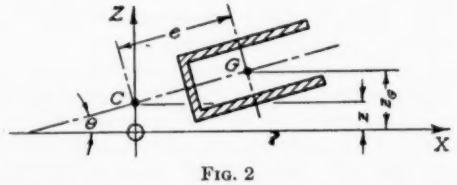


Fig. 2

Applying the principle of motion of the mass center to the elementary transverse section of the beam shown in Fig. 2, the kinetic energy of the element is expressed as the sum of the kinetic energy due to the translation of the mass center and that due to the rotation about the mass center. Thus, integrating over the length of the beam, the expression for the total kinetic energy becomes

$$T = \frac{1}{2}\rho A \int_0^l \dot{z}_G^2 dy + \frac{1}{2}\rho J_G \int_0^l \dot{\theta}^2 dy \dots [3$$

where T is the total kinetic energy at the displacement z, θ; ρ is the mass density of the material of which the beam is made; A is the cross-sectional area of the beam; z_G is the vertical displacement of the mass-center G; $\dot{z}_G$ is the z component of the

[5] For examples see references (11, 12, 13, 14, 15, 16) inclusive. The work of many others is summarized in reference (14).

[6] Discussions of experiments designed to determine the relationship of the natural frequencies of a wing to the critical air speeds are given in these references.

[7] This reference gives a discussion of the general theory of self-induced vibrations, together with a number of examples encountered in the operation of machinery.

[8] Discussions of the torsional rigidity of bars having nonsymmetrical sections are given in these references.

velocity of the mass center; $\dot{\theta}$ is the instantaneous angular velocity of rotation of the section; and J_G is the polar moment of inertia of the section with respect to the gravity axis.

From the geometry of Fig. 2, for small values of θ

$$z_G = z + e\theta \quad \text{(approximately)} \ldots\ldots\ldots\ldots [4]$$

Hence, by differentiating Equation [4] with respect to time

$$\dot{z}_G = \dot{z} + e\dot{\theta} \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

Substituting this value of $\dot{z}_G$ into Equation [3] the expression for the kinetic energy becomes

$$T = \frac{1}{2}\rho A\left[\int_0^l (\dot{z}^2 + 2e\dot{z}\dot{\theta} + e^2\dot{\theta}^2)\,dy + \frac{J_G}{A}\int_0^l \dot{\theta}^2 dy\right] \ldots [3a]$$

From Equation [1], by differentiation

$$\left.\begin{aligned} \dot{z} &= (a_1Y_1 + a_2Y_2 + a_3Y_3 + \ldots.)\ \omega_n \cos \omega_n t \\ \dot{\theta} &= (\varphi_1Y_1' + \varphi_2Y_2' + \varphi_3Y_3' + \ldots.)\,\omega_n \cos \omega_n t \end{aligned}\right\} \ldots [6]$$

Substituting these values into Equation [3a]

$$\begin{aligned} T = \frac{1}{2}\rho A \Bigg\{\Bigg[& a_1^2\int_0^l Y_1^2 dy + a_2^2\int_0^l Y_2^2\,dy \\ & \ldots. 2a_1a_2\int_0^l Y_1Y_2\,dy + \ldots\Bigg] + 2e\Bigg[a_1\varphi_1\int_0^l Y_1Y_1'dy \\ & + a_1\varphi_2\int_0^l Y_1Y_2'dy + \ldots + a_2\varphi_1\int_0^l Y_2Y_1'dy \\ & + a_2\varphi_2\int_0^l Y_2Y_2'dy + \ldots + \ldots\ldots\Bigg] \\ & + \left(e^2 + \frac{J_G}{A}\right)\Bigg[\varphi_1^2\int_0^l Y_1'^2dy + \varphi_2^2\int_0^l Y_2'^2dy \\ & + \ldots 2\varphi_1\varphi_2\int_0^l Y_1'Y_2'dy + \ldots\ldots\ldots\Bigg]\Bigg\}\omega_n^2\cos^2\omega_n t \ldots [3b] \end{aligned}$$

Collecting terms, and denoting $\int_0^l Y_1^2dy$ by S_1, $\int_0^l Y_1'^2$ by $S_{1'}$, $\ldots \int_0^l Y_2^2dy$ by S_2, $\int_0^l Y_2'^2dy$ by $S_{2'}$, $\ldots$, Equation [3b] becomes

$$\begin{aligned} T = \frac{1}{2}\Bigg\{\rho A\Bigg[& (a_1^2S_1 + a_2^2S_2 + \ldots.. 2a_1a_2S_{1-2} + \ldots) \\ & + 2e(a_1\varphi_1S_{1-1'} + a_1\varphi_2S_{1-2'} + \ldots.. + a_2\varphi_1S_{2-1'} + a_2\varphi_2S_{2-2'} \\ & + \ldots.. + \ldots) + \left(e^2 + \frac{J_G}{A}\right)(\varphi_1^2S_{1'} + \varphi_2^2S_{2'} \\ & + \ldots 2\varphi_1\varphi_2S_{1'-2'} + \ldots)\Bigg]\Bigg\}\omega_n^2\cos^2\omega_n t \ldots [3c] \end{aligned}$$

or, more briefly

$$T = \frac{1}{2}\{\alpha\}\,\omega_n^2\cos^2\omega_n t \ldots\ldots\ldots\ldots [3d]$$

where α represents the quantity within the braces in Equation [3c].

Now that the expressions for the kinetic and potential energies of the beam have been determined, the natural frequencies of vibration may be evaluated by applying the Rayleigh-Ritz method. Equating the maximum values of the potential and kinetic energies, as obtained from Equations [2c] and [3d], respectively, and solving for the frequency

$$\omega_n^2 = \frac{\beta}{\alpha} \ldots\ldots\ldots\ldots\ldots\ldots [7]$$

The expressions for both β and α involve the coefficients a_1, $a_2, \ldots$, and $\varphi_1, \varphi_2, \ldots$, which define the assumed elastic deflection curves. Therefore, the values of ω_n obtained from Equation [7] depend upon the assumed curves. By Rayleigh's principle, if the assumed elastic curve is not the exact one, the lowest computed value of natural frequency will be higher than the true fundamental frequency of the beam. Or, to state it a little differently, if several elastic curves are assumed in succession, the one yielding the lowest value of frequency is nearest correct. In order to obtain the closest approximation possible, the coefficients $a_1, a_2, \ldots$, and $\varphi_1, \varphi_2, \ldots$ must be so chosen that the fundamental frequency computed from Equation [7] will be a minimum. This value of ω_n^2 may be found by equating to zero the partial derivative of β/α with respect to each of the coefficients $a_1, a_2, \ldots$, and $\varphi_1, \varphi_2, \ldots$, respectively. Thus

$$\left.\begin{aligned} \alpha\frac{\partial\beta}{\partial a_1} - \beta\frac{\partial\alpha}{\partial a_1} &= 0 \\ \alpha\frac{\partial\beta}{\partial a_2} - \beta\frac{\partial\alpha}{\partial a_2} &= 0 \\ \ldots\ldots\ldots\ldots & \\ \alpha\frac{\partial\beta}{\partial\varphi_1} - \beta\frac{\partial\alpha}{\partial\varphi_1} &= 0 \\ \alpha\frac{\partial\beta}{\partial\varphi_2} - \beta\frac{\partial\alpha}{\partial\varphi_2} &= 0 \\ \ldots\ldots\ldots\ldots & \end{aligned}\right\} \ldots\ldots\ldots\ldots [8]$$

From Equation [7] it is noted that $\beta = \omega_n^2\alpha$. Hence Equation [8] may be written in the form

$$\left.\begin{aligned} \frac{\partial\beta}{\partial a_1} - \omega_n^2\frac{\partial\alpha}{\partial a_1} &= 0 \\ \frac{\partial\beta}{\partial a_2} - \omega_n^2\frac{\partial\alpha}{\partial a_2} &= 0 \\ \ldots\ldots\ldots\ldots & \\ \frac{\partial\beta}{\partial\varphi_1} - \omega_n^2\frac{\partial\alpha}{\partial\varphi_1} &= 0 \\ \frac{\partial\beta}{\partial\varphi_2} - \omega_n^2\frac{\partial\alpha}{\partial\varphi_2} &= 0 \\ \ldots\ldots\ldots\ldots & \end{aligned}\right\} \ldots\ldots\ldots\ldots [9]$$

These equations are seen to be homogeneous and linear in the coefficients $a_1, a_2, \ldots \varphi_1, \varphi_2, \ldots$, and are equal in number to the number of coefficients. By setting the determinant of these equations equal to zero, eliminating the coefficients, and expanding, the frequency equation may be obtained. The frequency equation yields the values of the natural frequencies in the several normal modes.

The amplitude ratios $a_2/a_1, \ldots \varphi_1/a_1, \varphi_2/a_1, \ldots$, which determine the configuration of the beam in each of the normal modes, can then be evaluated by substituting the respective values of ω_n into Equations [9].

Derivation of Frequency Equation for Two Normal Modes

To illustrate the process outlined, a frequency equation yielding two natural frequencies will now be derived. As a first ap-

proximation the instantaneous displacement of the beam may be expressed by the components

$$\left.\begin{aligned} z &= a_1 Y_1 \sin \omega_n t \\ \theta &= \varphi_1 Y_1' \sin \omega_n t \\ x &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots [10]$$

where Y_1' is a function of y which satisfies the end conditions for flexure and Y_1' satisfies the end conditions for torsion.

From Equation [2b]

$$\beta = K_x a_1^2 Q_1 + C\varphi_1^2 R_1 \ldots\ldots\ldots\ldots\ldots [11]$$

and from Equation [3c]

$$\alpha = \rho A\left[a_1^2 S_1 + 2ea_1\varphi_1 S_{1-1'} + \left(e^2 + \frac{J_G}{A}\right)\varphi_1^2 S_1'\right] \ldots [12]$$

Taking partial derivatives with respect to a_1 and φ_1, respectively, and denoting $S_{1-1'}/S_1$ by η, and S_1'/S_1 by γ, the following expressions are obtained

$$\left.\begin{aligned} \frac{\partial \beta}{\partial a_1} &= 2K_x Q_1 a_1 \\ \frac{\partial \beta}{\partial \varphi_1} &= 2CR_1\varphi_1 \\ \frac{\partial \alpha}{\partial a_1} &= 2\rho A S_1[a_1 + \eta e\varphi_1] \\ \frac{\partial \alpha}{\partial \varphi_1} &= 2\rho A S_1\left[\left(e^2 + \frac{J_G}{A}\right)\gamma\varphi_1 + \eta e a_1\right] \end{aligned}\right\} \quad \ldots\ldots [13]$$

Substituting these terms into Equation [9] and rearranging into determinant form

$$\begin{vmatrix} 2[K_x Q_1 - \rho A S_1 \omega_n^2]a_1 & -2[\rho A S_1 \eta e \omega_n^2]\varphi_1 \\ -2[\rho A S_1 \eta e \omega_n^2]a_1 & 2\left[CR_1 - \rho A S_1 \gamma\left(e^2 + \frac{J_G}{A}\right)\omega_n^2\right]\varphi_1 \end{vmatrix} = 0 \ldots [14]$$

Dividing the first column by $2\rho A S_1 a_1$ and the second column by $2\rho A S_1 \varphi_1$, the determinant is reduced to the form

$$\begin{vmatrix} \frac{K_x Q_1}{\rho A S_1} - \omega_n^2 & -\eta e \omega_n^2 \\ -\eta e \omega_n^2 & \frac{CR_1}{\rho A S_1} - \gamma\left(e_2 + \frac{J_G}{A}\right)\omega_n^2 \end{vmatrix} = 0 \ldots [15]$$

At this point it may be noted that the quantity $\left[\frac{K_x Q_1}{\rho A S_1}\right]^{1/2}$ is the familiar expression for the natural frequency of flexural vibration for a uniform beam. This quantity is therefore the natural frequency of the beam under consideration for the particular case in which e is zero. For convenience the natural frequency for this particular case ($e = 0$) may be denoted by the symbol ω_0. Then, with the view to express the natural frequencies as dimensionless ratios, the relationship

$$\frac{K_x Q_1}{\rho A S_1} = \omega_0^2 \ldots\ldots\ldots\ldots\ldots [16]$$

may be substituted into Equation [15] and each term of the equation divided by ω_0^2. After this operation is performed and the determinant is expanded into linear form, simplified, and rearranged, the following quadratic equation is obtained

$$\left[\gamma\left(e^2 + \frac{J_G}{A}\right) - \eta^2 e^2\right]\left(\frac{\omega_n^2}{\omega_0^2}\right)^2 - \left[\frac{CR_1}{K_x Q_1} + \gamma\left(e^2 + \frac{J_G}{A}\right)\right]\frac{\omega_n^2}{\omega_0^2} + \frac{CR_1}{K_x Q_1} = 0 \ldots [17]$$

Introducing the additional dimensionless quantities

$$\lambda = \frac{CR_1 A}{K_x Q_1 \gamma J_G} \ldots\ldots\ldots\ldots [18]$$

$$\text{and } \epsilon = \frac{e^2 A}{J_G} \ldots\ldots\ldots\ldots [19]$$

and dividing by $\frac{\gamma J_G}{A}$, Equation [17] is further simplified to the form

$$\left[\epsilon\left(1 - \frac{\eta^2}{\gamma}\right) + 1\right]\left(\frac{\omega_n^2}{\omega_0^2}\right)^2 - (\lambda + \epsilon + 1)\frac{\omega_n^2}{\omega_0^2} + \lambda = 0 \quad [20]$$

This is the *frequency equation* and its two roots are

$$\frac{\omega_1^2}{\omega_0^2} = \frac{(\lambda + \epsilon + 1) - \sqrt{(\lambda + \epsilon + 1)^2 - 4\lambda\left[\epsilon\left(1 - \frac{\eta^2}{\gamma}\right) + 1\right]}}{2\left[\epsilon\left(1 - \frac{\eta^2}{\gamma}\right) + 1\right]} \quad \ldots\ldots [21]$$

$$\text{and } \frac{\omega_2^2}{\omega_0^2} = \frac{(\lambda + \epsilon + 1) + \sqrt{(\lambda + \epsilon + 1)^2 - 4\lambda\left[\epsilon\left(1 - \frac{\eta^2}{\gamma}\right) + 1\right]}}{2\left[\epsilon\left(1 - \frac{\eta^2}{\gamma}\right) + 1\right]} \quad \ldots\ldots [22]$$

The physical significance of the terms λ and ϵ may now be considered. Referring to Equation [18] it is seen that, of the terms on the right-hand side, the quantities C, A, and J_G are independent of the length of the beam. From the definitions stated following Equations [2b] and [3b], Q_1 is seen to vary inversely with l^3, R_1 inversely with l, and S_1' (i.e., γS_1) directly with l. Hence λ varies with l^2, and may therefore be appropriately called the *length modulus*. Referring to Equation [19], the term ϵ, on the other hand, is seen to be entirely independent of the length and to vary with the square of e, the distance between the gravity axis and the elastic axis of the beam. Since ϵ serves as a measure of the relative positions of the two axes, for lack of a more appropriate term, it will be referred to as the *eccentricity modulus*.

Discussion of the Frequency Diagram

Fig. 3 shows the relationship of the moduli ω_n/ω_0, ϵ, and λ for a limited range of ϵ, six selected values of λ, and the particular value of $\eta^2/\gamma = 0.925$.[9] Each of the curves illustrates the variation of the natural frequencies with ϵ for a particular value of λ, as defined by Equations [21] and [22]. For positive values of ϵ any given ordinate is seen to intersect each of the λ curves in two points. The values of ω_n/ω_0 at these two points of intersection determine the natural frequencies of the two normal modes.

[9] The value of η^2/γ depends upon the functions Y_1 and Y_1' which are chosen to express the elastic curves of the beam in flexure and torsion. The value used here (0.925) is taken from the numerical example presented later. A consideration of functions other than those used in the example indicates that the value of η^2/γ is quite close to 0.925, regardless of the functions chosen.

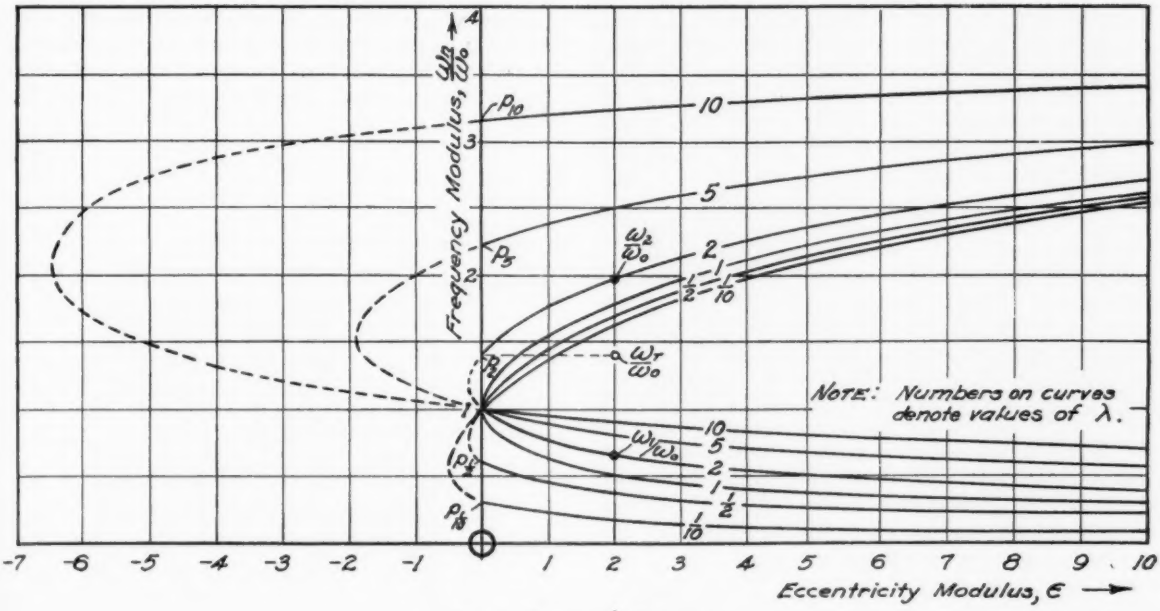


FIG. 3 FREQUENCY DIAGRAM

The dashed portions of the curves, to the left of the origin, have no physical significance. This may be concluded from the fact that, in any actual beam, the quantities e^2, A, and J_G must be real and positive. Hence a value of ϵ, as computed from Equation [19], can never be negative.

Inspection of the diagram also indicates that, for any given length modulus λ, the two natural frequencies become farther and farther apart as the eccentricity increases. Also, for a given amount of eccentricity, the difference between the two frequencies increases slightly with increasing values of λ.

It will be noted also that each of the curves passes through the point $\omega_n/\omega_0 = 1.0$, $\epsilon = 0$. This is explained by the fact that in the actual beam, if $e = 0$, one of the normal modes of vibration is that of pure flexure, and the corresponding natural frequency is ω_0. Therefore, this common point of intersection of the λ curves represents the frequency of the purely flexural vibrations which prevail in one of the normal modes when $e = 0$.

The other intersections (marked p_{10}, p_5, etc.) of the λ curves with the frequency axis have an interesting significance. Our knowledge of the fact that when $e = 0$ the normal modes of vibration of the beam are those of pure flexure and pure torsion, coupled with the fact that $\omega_n/\omega_0 = 1.0$, $\epsilon = 0$, represents the flexural mode, suggests that these points p may represent the frequency of the purely torsional mode of vibration for the case $e = 0$. Indeed such may be demonstrated to be the case.

First it will be noted that when $\epsilon = 0$ is substituted into Equations [21] and [22], the two values of ω_n/ω_0 become 1.0 and $\lambda^{1/2}$, respectively. Next, the expression for the natural frequency of the purely torsional mode of vibration may be determined by equating the maximum values of potential energy and kinetic energy for the θ component of motion. From Equation [2b] the maximum potential energy in the twisted beam is found to be

$$V_{max} = \frac{C}{2}\varphi_1^2 R_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots [23]$$

From Equation [3c] the maximum kinetic energy of rotation is

$$T_{max} = \frac{1}{2}\rho S_1' J_G \varphi_1^2 \omega_n^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [24]$$

Equating V_{max} to T_{max}, substituting γS_1 for S_1', and solving for the frequency, there results the expression

$$\omega_T^2 = \frac{CR_1}{\rho J_G \gamma S_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots [25]$$

where the subscript T is used to denote the torsional mode.

To put Equation [25] into dimensionless form, the left-hand side may be divided by ω_0^2, and the right-hand side by its equivalent, as defined in Equation [16]. Thus

$$\frac{\omega_T^2}{\omega_0^2} = \frac{CR_1}{\rho J_G \gamma S_1} \Big/ \frac{K_x Q_1}{\rho A \gamma S_1} = \frac{CR_1 A}{K_x Q_1 \gamma J_G} \quad \ldots\ldots\ldots [26]$$

But, referring to Equation [18], it is seen that the last quantity in Equation [26] is equal to λ. Hence, the torsional frequency is given by the expression

$$\frac{\omega_T}{\omega_0} = \lambda^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [27]$$

It has been shown already that the values of the ordinates at the points p shown in Fig. 3 are equal to $\lambda^{1/2}$. Therefore, it is concluded that at these points the frequencies are those of the torsional mode of vibration of the beam when $e = 0$.

Thus the diagram indicates the ratios of the true natural frequencies (for $e \neq 0$) to the frequencies in both flexure and torsion when $e = 0$. From these ratios an estimate can be made of the accuracy to be expected of frequency computations in which the effect of e is neglected. Consider, for example, a beam in which $\lambda = 2$ and $\epsilon = 2$. From Fig. 3 it is seen that the true natural frequency ω_1 is about two thirds the value computed for purely flexural vibrations, and the second natural frequency, ω_2, is about one and one half times that computed for the case of pure torsional vibrations. Thus, it is seen that computations of critical frequencies of such a beam, in which the effect of the eccentricity e is neglected, are apt to lead to quite erroneous results.

Still another interesting characteristic of the system is indicated by the relative positions of the points p for the several values of λ. For values of λ greater than unity the points p lie above the value $\omega_n/\omega_0 = 1.0$, whereas for value of λ less than unity the corresponding points lie below the value $\omega_n/\omega_0 = 1.0$. The

interpretation of this is simply that when λ is greater than unity the critical frequency for the torsional mode of vibration is greater than that for the flexural mode, and conversely, when λ is less than unity the critical frequency in torsion is less than that in flexure. It is interesting to note also that, for the borderline case (i.e., for $\lambda = 1.0$), the natural frequencies in the flexural and torsional modes are equal.

Remarks on the Accuracy of the Method

Before terminating the discussion of the frequency curves in Fig. 3 it would seem desirable to say a word about the accuracy of the equation which they represent. It will be recalled by referring to Equation [10] that each of the assumed elastic curves was expressed as a single function of y. It is recognized that by using additional terms in the expressions for z and θ, it is possible to determine the lower frequencies with greater accuracy, and at the same time obtain approximate values of frequency for modes of higher order. For example, if two terms are used in each of the expressions for the elastic curves, the resulting frequency equation is a biquadratic from which four values of frequency can be found.

An estimate of the accuracy of the method may be gained by comparing the results which it gives with those obtained by exact solutions, for cases in which the latter are known. In applying the method to the determination of the natural frequencies of a vibrating string, Timoshenko (1) demonstrates that when only one term is used to express the assumed elastic curve, the computed value for the fundamental frequency differs from the exact value by 0.66 per cent. By using two terms the error in the computed fundamental frequency is reduced to 0.001 per cent. Again, by using three terms, the frequency of the third mode of vibration is computed with an error of less than 0.5 per cent. In another example—the case of a vibrating wedge of constant width, with the thick end built in and the other end free—Timoshenko (1) demonstrates that the error in the computed value of the fundamental frequency is about 3 per cent when only one term is used to express the elastic curve, and less than 0.1 per cent when two terms are used.

Thus it is seen that the method gives very satisfactory results, even when only a few terms are used. This statement is substantiated further by the agreement between the computed and measured frequencies found in the numerical example to follow. Since it is usually the fundamental or, in some instances, the second mode of vibration which is apt to give rise to trouble in the practical case, it is seldom necessary to use more than one term or, at most, two terms in each of the expressions for the elastic curves. Moreover, it should be recognized that when a larger number of terms is used, the extra labor of computation may not be worth the increased accuracy to be gained.

The Normal Modes of Vibration

The natural frequencies of vibration of the beam having been determined, the corresponding normal modes may now be considered. By a "normal mode" (19)[10] is meant a motion of the beam in which every particle executes simple harmonic motion and moves in phase with every other particle. Thus, all particles reach their extreme positions simultaneously and attain their maximum velocities (at their respective positions of rest) simultaneously. Such modes of motion are expressed in Equation [1] and are determined by the relative values of the amplitudes $a_1, a_2, \ldots \varphi_1, \varphi_2, \ldots$ The absolute amplitudes are of course arbitrary since they depend upon the initial displacement of the beam.

[10] A discussion of normal modes of vibration is given in reference (19), pp. 153 and 328.

As stated previously, the relative amplitudes of the assumed elastic curves can be determined from Equation [14] by first substituting for ω_n the values of natural frequency obtained from the frequency equation. Thus the amplitude ratios are found to be

$$\frac{\varphi_1}{a_1} = \frac{K_z Q_1 - \rho A S_1 \omega_n^2}{\eta \rho A S_1 e \omega_n^2} \quad [28]$$

Dividing both the numerator and the denominator of the right-hand side by $\rho A S_1$, and substituting Equation [16], Equation [28] becomes

$$\frac{\varphi_1}{a_1} = \frac{\omega_0^2 - \omega_n^2}{\eta e \omega_n^2} \quad [29]$$

When both sides of Equation [29] are multiplied by ηe, the following dimensionless equation is obtained

$$\frac{\eta e \varphi_1}{a_1} = \frac{1 - (\omega_n^2/\omega_0^2)}{(\omega_n^2/\omega_0^2)} \quad [30]$$

Equation [30] yields two values of the amplitude ratio—one for each of the values of ω_n/ω_0. It will be recalled by referring to Fig. 3 that the lower value of ω_n/ω_0 lies between zero and 1.0, while the higher value is greater than 1.0. Thus, for the first mode, $\omega_n^2/\omega_0^2 < 1.0$, and from Equation [30], $\eta e \varphi_1/a_1 > 0$. Similarly, for the second mode, $\omega_n^2/\omega_0^2 > 1.0$, and from Equation [30], $-1.0 < \eta e \varphi_1/a_1 < 0$.

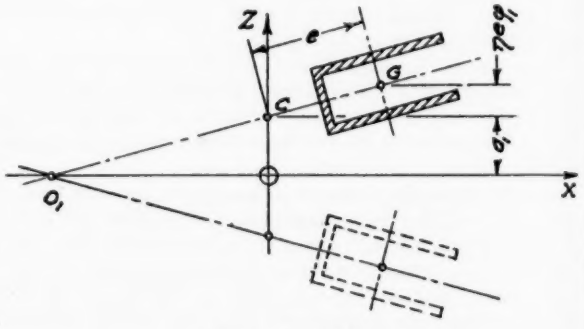


Fig. 4 First Mode

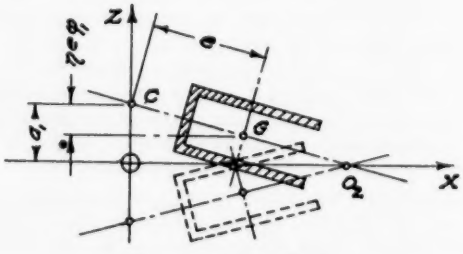


Fig. 5 Second Mode

The relative positions of the elastic axis C, and the mass axis G, at any given section of the beam, are illustrated in Figs. 4 and 5 for the first and second modes of vibration, respectively. In the first mode the nodal point at a given section lies at O_1, in Fig. 4, to the left of C. The motion of the beam at this section is therefore a harmonic oscillation about O_1. Similarly, in the second mode any given section of the beam oscillates about O_2 in Fig. 5. Since z and θ increase from zero at $y = 0$, to a_1 and φ_1, respectively, at $y = l$, according to the shapes of the assumed elastic curves in flexure and torsion, the lines passing through all the points

0_1 and all of the points 0_2, respectively, for the consecutive beam sections, form curves in the X, Y plane. Each of these curves intersects the elastic axis of the beam at the fixed end.

Experimental Work

A beam of the type shown in Fig. 1 was made by forming a strip of 24-gage cold-rolled steel into a channel with $^1/_8$-in. web and 1-in. flanges. One end of this channel was then cast into a base and mounted upon a vertical sliding head as shown in Fig. 6.

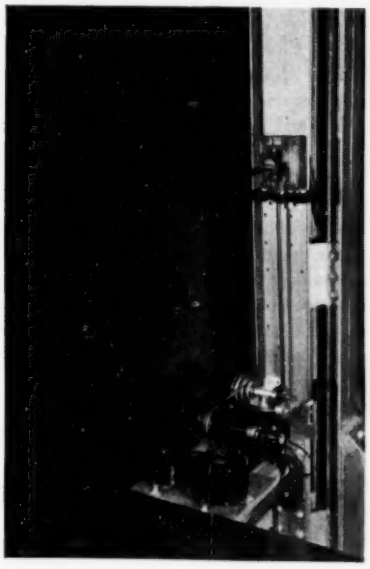


Fig. 6 Experimental Apparatus

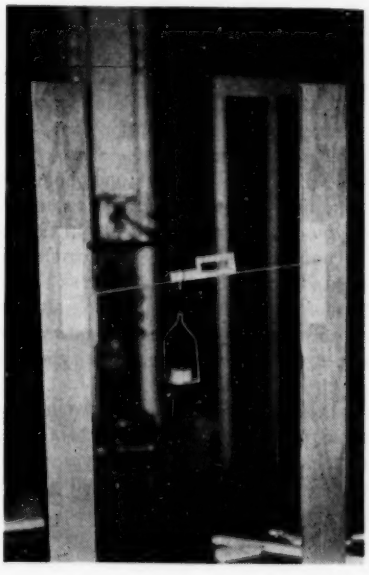


Fig. 7 Loading Frame

At the free end of the beam known loads were applied at various distances from the longitudinal axis by means of the loading device shown in Fig. 7. Lateral and torsional deflections of the beam were measured for various loads placed at several positions and from these data were determined the flexural rigidity

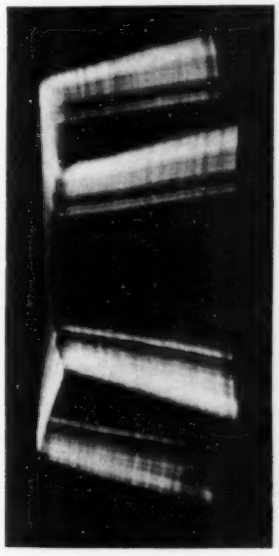


Fig. 8 First Mode, 710 Vibrations per Min

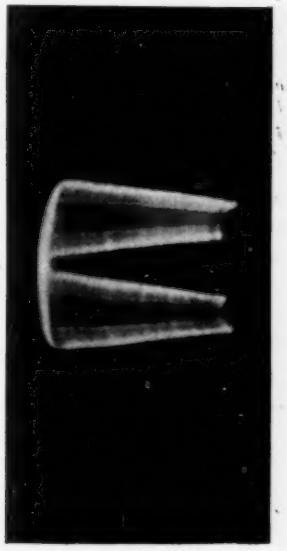


Fig. 9 Second Mode, 2600 Vibrations per Min

EI_{z-z}, the torsional rigidity C, and the location of the axis of twist. These values are used in the numerical solution which follows.

The base (i.e., the fixed end) of the beam was then caused to vibrate with small amplitude (0.018 in.) along the vertical axis by means of the driving mechanism shown in Fig. 6. The frequency of vibration was varied over a wide range and the resonant frequencies noted. A stroboscope was used to measure the frequency of vibration and also to study the nature of the motion. Marked resonant vibrations were found to occur at frequencies of 710 and 2600 vibrations per minute.

Figs. 8 and 9 are the results of time exposures which were made while the beam was vibrating in the steady state at the first and second resonant speeds, respectively. These illustrations show the configurations of the free end of the beam in the two modes.

A comparison of Figs. 8 and 9 with Figs. 4 and 5, respectively, indicates excellent agreement of predicted and observed normal modes. Furthermore, as will be shown by the numerical computations to follow, the computed natural frequencies agree quite satisfactorily with the resonant frequencies obtained by experiment.

Numerical Example

The results of the numerical solution for the two natural frequencies of the beam just described are presented in the following summary. The frequencies are computed from Equations [21] and [22]. The physical constants of the beam (a cross section is shown in Fig. 10) are

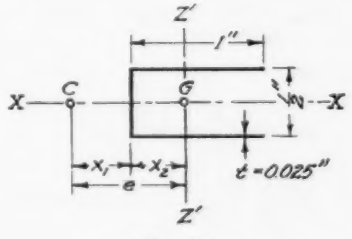


Fig. 10

l	= 40 in.	x_1	= 0.45 in. (measured)
t	= 0.025 in.	x_2	= 0.41 in. (computed)
A	= 0.0625 in.^{2}	e	= 0.86 in.
ρ	= 0.000818 lb sec^2/in.^{4}	ϵ	= 4.49
I_{x-x}	= 0.00379 in.^{4}	C	= 302 in-lb/radian/in.
I_{s-s}	= 0.00651 in.^{4}	K_x	= 116,300 lb-in.^{2}
J_θ	= 0.0103 in.^{4}		

As a first approximation the shape of the elastic curve of the beam in flexure will be expressed by

$$Y_1 = \frac{28}{27} - \cos\frac{\pi y}{2l} - \frac{1}{27}\cos\frac{3\pi y}{2l} \quad \text{[31]}$$

This function satisfies the end conditions of the beam since, at $y = 0$, $Y_1 = 0$ and $dY_1/dy = 0$; and at $y = l$, $d^2Y_1/dy^2 = 0$ and $d^3Y_1/dy^3 = 0$.

The elastic curve in torsion will be expressed by

$$Y_1' = \sin\frac{\pi y}{2l} \quad \text{[32]}$$

This function is seen to satisfy the end conditions since at $y = 0$, $y_1' = 0$ and dY_1'/dy is finite; and at $y = l$, $dY_1'/dy = 0$.

These values of Y_1 and Y_1' yield the following terms for use in the frequency equation

$$S_1 = \int_0^l Y_1^2 dy = 0.271l$$

$$Q_1 = \int_0^l \left(\frac{d^2Y_1}{dy^2}\right)^2 dy = \frac{3.376}{l^3}$$

$$S_{1-1'} = \int_0^l Y_1 Y_1' dy = 0.3538l$$

$$S_{1'} = \int_0^l Y_1'^2 dy = \frac{l}{2}$$

$$R_1 = \int_0^l \left(\frac{dY_1'}{dy}\right)^2 dy = \frac{1.234}{l}$$

$$\eta = \frac{S_{1-1'}}{S_1} = 1.305$$

$$\gamma = \frac{S_{1'}}{S_1} = 1.845$$

$$\frac{\eta^2}{\gamma} = 0.925$$

By Equation [18]

$$\lambda = \frac{CR_1A}{K_xQ_1\gamma J_G} = 3.12\,l^2 = 5.0$$

and by Equation [16]

$$\omega_0^2 = \frac{K_xQ_1}{\rho AS_1} = \frac{2.82 \times 10^{10}}{l^4}$$

$$\omega_0 = \frac{1.68 \times 10^5}{l^2} = 105 \text{ radians per sec}$$

$$N_0 = \omega_0 \times \frac{60}{2\pi} = 1003 \text{ vibrations per min}$$

Substituting these values into Equations [21] and [22], the frequency ratios are found to be

$$\left(\frac{\omega_1}{\omega_0}\right)^2 = 0.51 \qquad \frac{\omega_1}{\omega_0} = 0.71$$

$$\left(\frac{\omega_2}{\omega_0}\right)^2 = 7.36 \qquad \frac{\omega_2}{\omega_0} = 2.71$$

Multiplying these values by the value of N_0, the computed natural frequencies of the beam are

$$N_1 = 0.71 \times 1003 = 712 \text{ vibrations per min}$$

and $$N_2 = 2.71 \times 1003 = 2720 \text{ vibrations per min}$$

A comparison of these computed values with the resonant frequencies (710 and 2600 vibrations per min) indicates that the derived frequency equation gives quite satisfactory results. The computed and observed values agree almost exactly for the first mode, and differ by less than 5 per cent for the second mode.

Summary and Conclusions

The normal modes of vibration of the particular type of beam under consideration have been shown to consist of simultaneous vibrations in flexure and torsion. Each of these normal modes is characterized by a particular natural frequency and by a definite configuration of the beam. These normal modes are expressed in general form by Equation [1].

Equations [21], [22], and [30] express the natural frequencies and the corresponding amplitude ratios in terms of suitable physical constants of the beam, for the first and second normal modes of vibration. Although it is recognized that it is most frequently one or the other of these two modes which underlies vibration problems, the method of extending the solution to include the higher modes of vibration has been indicated.

The effect of the eccentricity, i.e., the displacement of the mass axis from the elastic axis, upon the natural frequencies of the beam is illustrated in Fig. 3. From this figure it is seen that the fundamental frequency becomes greater and the frequency in the second mode becomes less as the amount of the eccentricity is increased.

From a study of Fig. 3 it is also concluded that if values for the frequencies in the purely flexural and purely torsional modes of vibration are computed on the assumption that the effect of the eccentricity is negligible, the lower of these two computed values will be somewhat higher than the true fundamental frequency of the beam, and the higher of the computed values will be somewhat lower than the true natural frequency in the second mode. The error in the computed values which is introduced by such an assumption is apt to be quite large—as much as 50 per cent in the case cited. Therefore, it would seem advisable always to take the eccentricity into account in computing the natural frequencies of machine parts or structural members of the type treated in this analysis.

The excellent agreement of the natural frequencies and normal modes of vibration, as computed from the theoretical equations, with the values determined experimentally indicates that the frequency equation can be expected to yield quite satisfactory results for the two modes considered.

The frequency diagram in Fig. 3 may be used to evaluate the first two natural frequencies of a beam for which the values of ϵ, λ, and ω_0 have been computed.

The method used in this analysis takes into account the elastic deformations of the beam in flexure and in torsion, and therefore yields closer approximations to the correct values of the natural frequencies and amplitude ratios than those obtained by treating the beam as a rigid body. Any desired degree of accuracy can be attained by using a sufficient number of terms in the expressions

for the elastic deformation curves. However, for most practical problems, satisfactory values may be obtained by using only one or two terms to express each of the elastic curves.

By choosing the functions $Y_1, Y_2, \ldots Y_1', Y_2', \ldots$ in Equation [1] to satisfy the end conditions, the method can be applied to a beam with any type of support or any load distribution.

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Strengthening Cylindrical Tanks of Variable Thickness Under External Pressure by Circular Stiffening Rings

By C. B. BIEZENO[1] and J. J. KOCH,[2] DELFT, NETHERLANDS

In a previous paper[3] the authors discussed the buckling of a cylindrical tank of variable thickness under external pressure, and they developed a method by which the factor of safety against buckling was derived in numerical terms.

This present paper deals with the strengthening of a tank whose factor of safety is insufficient. Obviously, great help may be expected from one or two stiffening circular rings, adjusted in the upper part of the tank, where the thickness of the shell is small. The authors show how to calculate such rings.

THE strengthening against buckling of a cylindrical tank of variable thickness under external pressure by circular stiffening rings cannot be understood unless one has at his disposal the method previously developed by the authors.[3] The method is also discussed at great length in "Technische Dynamik."[4]

It is outlined briefly under the following subhead.

1—The Differential Equations of the Buckling Problem of a Cylinder of Constant Thickness h, Subjected to the Radial Pressure q

If a be the radius of the middle surface of the cylinder, l its height, φ and z the azimuthal and axial coordinates of one of its points, u, v, w the radial, tangential, and axial displacements of this point, and q represents the radial pressure per unit surface, then with the abbreviations

$$\left.\begin{aligned} &a\frac{\partial u}{\partial z}=u';\ a\frac{\partial v}{\partial z}=v';\ a\frac{\partial w}{\partial z}=w';\ \frac{\partial u}{\partial\varphi}=u^{\cdot};\ \frac{\partial v}{\partial\varphi}=v^{\cdot};\\ &\frac{\partial w}{\partial\varphi}=w^{\cdot};\ \frac{1}{m}=\nu;\ A=\frac{Eh^3}{12(1-\nu^2)};\ B=\frac{Eh}{1-\nu^2};\\ &\frac{A}{a^2B}=\frac{h^2}{12a^2}=k;\ \frac{qa}{B}=q_1 \end{aligned}\right\}\quad[1]$$

the differential equations of the problem may be written as follows

$$\left.\begin{aligned} &u+v^{\cdot}+\nu w'+k\left[u^{\cdot\cdot\cdot\cdot}+2u^{\prime\prime\cdot\cdot}+u^{\prime\prime\prime\prime}+2u^{\cdot\cdot}+u\right.\\ &\left.-\frac{3-\nu}{2}v^{\prime\prime\cdot}+\frac{1-\nu}{2}w^{\prime\cdot\cdot}-w^{\prime\prime\prime}\right]+q_1[u+u^{\cdot\cdot}+w']=0\\ &u^{\cdot}+v^{\cdot\cdot}+\frac{1-\nu}{2}v''+\frac{1+\nu}{2}w^{\prime\cdot}+k\left[-\frac{3-\nu}{2}u^{\prime\prime\cdot}\right.\\ &\left.+\frac{3}{2}(1-\nu)v''\right]=0\\ &\nu u'+\frac{1+\nu}{2}v^{\prime\cdot}+\frac{1-\nu}{2}w^{\cdot\cdot}+w''+k\left[\frac{1-\nu}{2}u^{\prime\cdot\cdot}-u'''\right.\\ &\left.+\frac{1-\nu}{2}w^{\cdot\cdot}\right]-q_1w^{\cdot\cdot}=0 \end{aligned}\right\}\quad[2]$$

Restricting ourselves to the special case for which the radial and tangential displacements at the edges ($z=0$ and $z=l$) of the cylinder are equal to zero, we may put

$$\left.\begin{aligned} u&=U\cos p\varphi\sin\lambda(z/a)\\ v&=V\sin p\varphi\sin\lambda(z/a)\\ w&=W\cos p\varphi\cos\lambda(z/a) \end{aligned}\right\}\quad[3]$$

where p is a positive integer, $\lambda=n\pi(a/l)$; n is a positive integer, and U, V, and W are constants. Under the condition of loading prevailing in actual practice, and considering that a is very great with respect to h and of the same order as l, it may be expected that, at the moment of buckling, the number of circumferential waves is great with respect to the number of longitudinal waves, so that $\lambda^2/p^2\ll 1$.

By this remark it is possible to compare the order of magnitude of the different terms in Equation [2], and it can be shown that these equations reduce themselves, as far as u is concerned, to

$$u^{::::}+\frac{1-\nu^2}{k}u^{\prime\prime\prime\prime}+\frac{q_1}{k}u_0^{:::}=0\quad[4]$$

2—The Differential Equations of the Buckling Problem of a Tank Built Up of a Number of Cylinders Each of Constant Thickness h

If a tank is built up of a number of cylinders each of constant thickness, Equations [3] can no longer be used because they do not satisfy the boundary conditions laid upon the different cylinders. Therefore we put now

$$\left.\begin{aligned} u&=U\cos p\varphi\\ v&=V\sin p\varphi\\ w&=W\cos p\varphi \end{aligned}\right\}\quad[5]$$

where p is a positive integer, and U, V, W represent unknown functions of z, which for different rings are different and which replace the expressions $U\sin\lambda(z/a)$, $V\sin\lambda(z/a)$, $W\cos\lambda(z/a)$ (U, V, W are constant in Equations [3]). Reasoning, similar to that from section 1, now leads to

$$U''''+\frac{p^6(kp^2-q_1)}{1-\nu^2}U=0\quad[6]$$

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[1] Professor of Applied Mechanics, University of Delft.

[2] Lector in Applied Mechanics, University of Delft.

[3] "The Buckling of a Cylindrical Tank of Variable Thickness Under External Pressure," by C. B. Biezeno and J. J. Koch, Proceedings of the Fifth International Congress of Applied Mechanics, John Wiley & Sons, Inc., New York, N. Y., 1939, pp. 34–49.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

As a matter of fact it is impossible to satisfy all boundary conditions imposed on the tank problem, now that the system given by Equations [2] has been reduced to a system of n differential equations of the type [6]. Therefore it has to be investigated which of these conditions can be maintained, which of them must be neglected, and eventually which of them must be softened.

Without proof we state that our buckling problem is as follows:

Calculate the minimum value of q (or q_1) which allows a solution different from $U = 0$ of the differential equations

$$U_i'''' + \frac{p^6(k_i p^2 - q_{1,i})}{1 - \nu^2} U_i = 0 \quad \ldots\ldots\ldots\ldots [7]$$

where $i = 1, 2 \ldots\ldots n$, and n represents the number of cylinders with constant thickness under the following conditions:

$$\left.\begin{array}{c} U_1 = 0;\ U_1'' = 0 \\ \text{at the upper rim of the top cylinder} \\ U_n = 0;\ U_n'' = 0 \\ \text{at the lower rim of the bottom cylinder} \\ U_i = U_{i+1};\ U_i' = U'_{i+1};\ S_{ii}U_i'' = S_{i,i+1}U''_{i+1}; \\ S_{ii}U_i''' = S_{i,i+1}U'''_{i+1};\ (i = 1, 2 \ldots n-1) \\ \text{at every joint of two consecutive rings, with} \end{array}\right\} [8]$$

$$S_{i,i} = Eh_i \quad \ldots\ldots\ldots\ldots\ldots [9]$$

3—The Substitute Bar

The foregoing formulation of our problem suggests its identification with the problem of an elastically supported bar composed of n segments. If the flexural rigidity of such a segment per unit of width be denoted by S_i $(i = 1, 2 \ldots n)$, its coefficient of elastical support per unit of width by κ_i $(i = 1, 2 \ldots n)$, its load per unit of length by r_i $(i = 1, 2 \ldots n)$, and its deflection by y_i $(i = 1, 2 \ldots n)$ then its differential equation runs

$$S_i \frac{d^4 y_i}{dz^4} + \kappa_i y_i = r_i \quad \ldots\ldots\ldots\ldots\ldots [10]$$

so that, by putting $r_i = 0$ and noticing that $a(dU/dz) = U'$, Equations [7] and [10] can be identified by requiring $S_i = s_i a^4$ and $\kappa_i = s_i \dfrac{p^6(k_i p^2 - q_{1,i})}{1 - \nu^2}$.

It can be proved that the value of the factor of proportionality s_i amounts to

$$s_i = \frac{Eh_i}{a^2 p^4}$$

so that

$$\left.\begin{array}{l} S_i = \dfrac{Eh_i a^2}{p^4} \\ \kappa_i = \dfrac{Eh_i^3 p^4}{12(1-\nu^2)a^4} - \dfrac{p^2}{a} q \end{array}\right\} \quad \ldots\ldots\ldots\ldots [11]$$

The boundary conditions of Equations [8] for the quantities U_i translate themselves into homogeneous conditions for the quantities y_i, if only S_{ii} and U_i be replaced by S_i and y_i. Therefore the buckling problem of the tank can finally be identified with the following bar problem:

A bar of unit width fixedly supported (but not clamped) at its ends consists of n segments of the length l_i $(i = 1, 2 \ldots n)$ each of which has its own flexural rigidity and its own coefficient of elastic support κ_i given by Equations [11]. If p be restricted to positive integer numbers, the minimum value of q is required, for which an equilibrium deflection, different from zero, is possible under the sole action of the elastic support.

4—The Buckling Criterion

From now we restrict ourselves to the case that the tank consists of four cylinders of constant thickness. Consequently the substitute bar consists of four parts I, II, III, IV, such as represented in Fig. 1, each having its own coefficient of flexural rigidity,

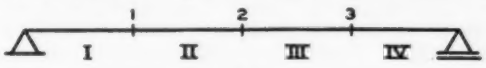


Fig. 1

and its own coefficient κ of elastic support. (The segment I of the bar represents the top cylinder of the tank, while the segment IV represents the bottom cylinder.) As the thickness of the tank increases from top to bottom, it is clear that the coefficient κ of elastic support of the different bar segments increases from left to right if p and q have fixed (but for the rest arbitrary) values, and it is to be expected that κ changes from a negative value for bar I into a positive one for bar IV. It cannot be predicted how many of the left-hand bars will have a negative κ; therefore a plausible assumption (which has to be checked a posteriori) must be made in every special case. It will be supposed that κ_{I} and κ_{II} are negative and κ_{III} and κ_{IV} positive. Instead of tackling the buckling problem in a direct way, an indirect method is followed by calculating the forces P_1, P_2, P_3, which have to be applied at the joints 1, 2, 3 of the bars I and II, II and III, III and IV if in those same points prescribed deflections y_1, y_2, y_3 have to be produced.

Formally the equations relating P_1, P_2, P_3 to y_1, y_2, y_3 may be written in the form

$$\left.\begin{array}{l} P_1 = a_{11}y_1 + a_{12}y_2 + a_{13}y_3 \\ P_2 = a_{21}y_1 + a_{22}y_2 + a_{23}y_3 \\ P_3 = a_{31}y_1 + a_{32}y_2 + a_{33}y_3 \end{array}\right\} \quad \ldots\ldots\ldots\ldots [12]$$

where the a_{ij} represent the so-called "inverse Maxwell numbers."

The mechanical significance of these numbers follows from Equations [12] themselves. Put $y_2 = 1$, $y_1 = y_3 = 0$; then

$$P_1 = a_{12} \quad P_2 = a_{22} \quad P_3 = a_{32}$$

Therefore a_{12}, a_{22}, a_{32} represent those forces which have to be applied in the points i $(i = 1, 2, 3)$ if a forced deflection y_j of unit magnitude at the point $j = 2$ and a deflection $y_j = 0$ at all other points $j \neq 2$ is prescribed.

It can be easily proved that the "inverse" Maxwell numbers are "reciprocal" like the Maxwell numbers themselves, i.e., $a_{ij} = a_{ji}$. Several problems can be treated in a natural way by the use of such numbers.[5]

As buckling of the tank requires bending of the bar not subjected to external forces, buckling only occurs if the right-hand sides of Equations [12] prove to be zero, i.e., if

$$\left.\begin{array}{l} a_{11}y_1 + a_{12}y_2 + a_{13}y_3 = 0 \\ a_{21}y_1 + a_{22}y_2 + a_{23}y_3 = 0 \\ a_{31}y_1 + a_{32}y_2 + a_{33}y_3 = 0 \end{array}\right\} \quad \ldots\ldots\ldots\ldots [13]$$

Therefore the condition of buckling is expressed by

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0 \quad \ldots\ldots\ldots\ldots [14]$$

For any chosen value of p this transcendental equation in q has an infinity of roots $q^{(p)}$, the smallest of which may be designated

[5] "Technische Dynamik," by C. B. Biezeno and R. Grammel, Julius Springer, Berlin, Germany, 1939, p. 95.

by $q^{(p)}_{\min}$. The smallest value of all possible $q^{(p)}_{\min}$ ($p = 1, 2, \ldots$) represents the smallest buckling pressure of the tank.

How the coefficients a_{ij} in Equations [13] can be calculated in a practical way, and how the smallest root $q^{(p)}_{\min}$ of Equation [14] can be determined, has been described in the previous article.[3]

5—The Strengthening Effect of Circular Rings

A new question arises when for a prescribed value of the external pressure q the coefficient of safety s against buckling proves to be insufficient. Then it may be asked whether this coefficient can be improved by providing the tank with one or two stiffening circular rings fitted in the upper (thinnest) cylinder of the tank, for instance, at its lower rim and at half of its height.

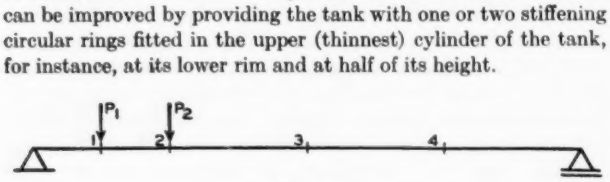


Fig. 2

It is easily seen from Fig. 2 that in our bar model such rings are translated by two external forces P_1 and P_2 at the points 1 and 2, which are proportional to the local deflections y_1 and y_2, so that

$$P_1 = -\mu_1 y_1 \text{ and } P_2 = -\mu_2 y_2 \ldots\ldots\ldots\ldots [15]$$

where μ_1 and μ_2 are essentially positive quantities. Defining inverse Maxwell numbers b_{ij} for the points 1, 2, 3, 4 as described in the preceding paragraph, and supposing y_1, y_2, y_3, y_4 to be the local deflections at the points 1, 2, 3, 4 of the deflected bar, then it is clear that the following equations hold

$$\left.\begin{aligned} P_1 = -\mu_1 y_1 &= b_{11}y_1 + b_{12}y_2 + b_{13}y_3 + b_{14}y_4 \\ P_2 = -\mu_2 y_2 &= b_{21}y_1 + b_{22}y_2 + b_{23}y_3 + b_{24}y_4 \\ 0 &= b_{31}y_1 + b_{32}y_2 + b_{33}y_3 + b_{34}y_4 \\ 0 &= b_{41}y_1 + b_{42}y_2 + b_{43}y_3 + b_{44}y_4 \end{aligned}\right\} \ldots\ldots [16]$$

having a solution (different from $y_1 = y_2 = y_3 = y_4 = 0$) only if

$$\begin{vmatrix} b_{11}+\mu_1 & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22}+\mu_2 & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{vmatrix} = 0 \ldots\ldots [17]$$

Remembering that (different from the problem treated in the sections 1 to 4, inclusive) the buckling pressure q now has a prescribed value, Equation [17] defines a relation between μ_1 and μ_2 for every set of values p and q.

If we wish that the tank be strengthened only by one ring we put μ_1 or $\mu_2 = 0$; then Equation [17] represents (for every p and q) a linear equation. If, on the other hand, we wish to strengthen it by two rings with equal coefficients of elastical support, we put $\mu_1 = \mu_2 = \mu$ and find a quadratic equation for μ, which must supply at least one positive root; only then will its solution have any practical significance for our problem. If two positive roots exist, the greater of them has to be used for the calculation of the rings. As a matter of fact Equation [17] has to be solved for a series of values of p. Every solution gives rise to a value μ_p; with every μ_p corresponds a value $I_{(p)}$ of the required moment of inertia I of the rings; the greatest of all is determinative.

Finally to derive $I_{(p)}$ from $\mu_{(p)}$ we use the well-known fact that a radial load system

$$q = q_0 \cos p\varphi \ldots\ldots\ldots\ldots [18]$$

acting on a ring produces a similar radial displacement y given by

$$y = \frac{q_0 a^4}{(p^2-1)^2 EI_{(p)}} \cos p\varphi \ldots\ldots\ldots [19]$$

From Equations [18] and [19] it follows

$$q = \frac{(p^2-1)^2 EI_{(p)}}{a^4} y \ldots\ldots\ldots\ldots [20]$$

Consequently

$$\mu_p = \frac{(p^2-1)^2 EI_{(p)}}{a^4} \quad \text{or} \quad I_p = \frac{a^4 \mu_p}{E(p^2-1)^2} \ldots\ldots [21]$$

Flow Through Granular Media

By L. P. HATCH,[1] WILSON DAM, ALA.

The author points out that the equation which appears to define flow through granular media is essentially identical to the equation for flow through pipes, but the expression is not useful to determine resistance to flow through granular media unless some of the variables are modified. The author discusses these variables and sets up formulas to determine the resistance to flow through granular media. He gives experimental results which show close agreement between calculated and observed values.

THE equation which appears to define flow through granular media is essentially identical to the equation for flow through pipes, although the flow phenomena are not similar in all respects. The equation in convenient form is

$$\frac{h}{l} = \frac{k}{g}\left(\frac{\mu}{\zeta}\right)^{2-n} U^n D^{n-3} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where h = resistance head
l = length
k = a coefficient
g = acceleration due to gravity
μ = viscosity
ζ = density of fluid (mass per unit volume)
U = average velocity; rate of discharge per unit of cross-sectional area
D = characteristic diameter of flow channel
n = an exponent ranging in value from 1.0 for laminar flow to 2.0 for turbulent flow

In its present form Equation [1] cannot be considered a useful expression of the resistance to flow through granular media, and certain modifications in the form of some of the variables are necessary. In particular the usual concept of diameter and velocity, with respect to flow in pipes, is inadequate to represent the conditions of flow through granular media.

A brief analysis of the open structure within granular media, which forms the complex network of flow channels, will readily disclose the fact that no useful direct physical measurement of individual channels can be made, and that such measurement must be obtained from a consideration of the flow system as a whole. Furthermore, it appears evident that the measurement which characterizes the size of the flow channels will depend, largely, upon the size of the adjacent solid particles, and upon the degree to which they are packed. It is essential, therefore, that we carefully consider these factors, as well as the channel velocity, for in them lies the crux of the whole problem of flow through granular media. To permit a lucid discussion of the three important variables, certain familiar terms are introduced as follows:

α = porosity, or volume of voids divided by total or bulk volume
a = gross or bulk cross-sectional area
a_p = cross-sectional area of pores in granular media
v = average bulk velocity or rate of discharge per unit of gross or bulk cross-sectional area; this corresponds to the velocity of approach in an open portion of a tube containing granular media
v_p = average velocity within the pores or voids in the direction of generalized travel
d = representative linear dimension of solid particles
d_p = characteristic size of pores or flow channels

Velocity

In the past, critical investigations of the permeability of granular beds have, in general, brought forth identical conclusions on at least one important point, namely, the relation between void velocity v_p and the bulk or approach velocity v. The relation is given by the expression $v_p = \frac{v}{\alpha}$. Following an apparently new approach to the problem, Bakhmeteff and Feodoroff (1)[2] do not support this general opinion on the grounds that the foregoing expression is dimensionally unsound. They submit in its place the exponential form $v_p = v/\alpha^{2/3}$. This expression is derived from their relation $a_p = a\alpha^{2/3}$, which is founded upon the assumption that porosity is a volumetric parameter and that its normal exponent 1.0 must be dimensionally consistent with the rest of the equation. For a homogeneous medium, however, the ratio of open area to gross area is the same for every cross section, or

$$\alpha = \frac{a_p}{a} \times \frac{\text{length}}{\text{length}}$$

Therefore, porosity is not necessarily a volumetric parameter, and the author believes that the relation given by Bakhmeteff and Feodoroff (1) is incorrect as indicated in a discussion (2) of their paper.

Diameter of Flow Channels

At the outset, it is well recognized that the diameter of the flow channels must be some linear dimension characteristic of the flow system as a whole, with respect to size of individual channels and the resistance to flow. It appears that such a measure can be derived only in terms of the size of the adjacent particles and the degree of packing as represented, for example, by the porosity. Beyond these requirements no useful specification of the diameter is known to the author.

Numerous theories have been advanced for the derivation of a characteristic channel diameter, most of which are equally dependent upon particle size but relatively few reflect serious consideration of the all-important porosity factor.

Kozeny (3) has derived a permeability equation in which a term equivalent to the characteristic channel diameter is expressed as the surface area of particles per unit of solid volume. This, in turn, was developed from a mathematical treatment of the transverse section and marginal boundary of the unit void, together with the length of flow line. By analogy, Kozeny viewed the problem in the light of flow through tubes of uniform cross section, and he obtained an equation which, with suitable modification, has since been verified experimentally over a wide range

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

of conditions. He states, moreover, that the same equation was previously developed empirically by E. Kruger (4). The equation, given for viscous flow, essentially is

$$v = C\frac{h}{l}\left(\frac{V}{A}\right)^2 \frac{\zeta}{\mu}\frac{\alpha^3}{(1-\alpha)^2} \quad [2]$$

where C = a coefficient, V = volume of particles, and A = surface area of particles.

As part of an investigation of flow through granular media at Harvard University, in which the author was associated with Gordon M. Fair, an expression for the channel diameter was derived in accordance with the concept of the hydraulic radius, a familiar term frequently employed as a size characteristic to define flow through irregular conduits. This approach appears to have paralleled the one followed previously by Kozeny, and the resulting expressions, in so far as porosity is concerned, are identical. What has previously been referred to as the Fair-Hatch expression has been discussed in other articles (5, 6), but the derivation is here reviewed as follows:

$$\text{Hydraulic radius} = \frac{\text{cross-sectional area}}{\text{wetted perimeter}} \times \frac{\text{length}}{\text{length}} =$$

$$\frac{\text{volume of voids}}{\text{surface area of solids}} = \frac{\text{volume of solids}}{\text{surface area of solids}} \times \frac{\alpha}{1-\alpha}$$

Bakhmeteff and Feodoroff (1) have introduced a diameter relationship in which porosity again is said to have suitable dimensional characteristics, i.e., $d_p = d\alpha^{1/3}$. This, also, the author believes to be incorrect on the grounds that the porosity term should take the form $\alpha/(1-\alpha)$, and, furthermore, that the foregoing expression is founded upon the characteristics of an isolated unit void rather than of the unit channel or the flow system as a whole.

Particle Size and Shape

The size of irregular particles is frequently expressed in terms of the size of opening or the "mesh" of sampling screens. The size of individual random particles retained between two screens of slightly different mesh will not be uniform but will vary in accordance with the screen openings. As is commonly true of random variations, it will be found that the measured sizes of these particles will usually follow a geometrically normal distribution. Therefore, with respect to screen sizes, there is reason to assume the geometric mean of the screen openings to most truly represent the size of the particles as a whole.

For careful analysis, however, sampling screens should not be regarded as size-measuring devices, and an independent size determination of each fraction should be made. This may be done by direct measurement of a representative number of particles and expressing the result as an average value calculated in accordance with the size distribution. Because of the multitude of "diameters" which may be used to define size of irregular particles, the results of independent investigations oftentimes are not comparable. As is frequently overlooked, the real objective in many cases is merely to obtain a representative size measure by some convenient method which will give consistent results. There is need for uniformity along these lines, and with this in mind the writer used as a measure of size the maximum projected distance between extreme edges of particles in one consistent direction; the particles lay at random in the measuring field. This method, described elsewhere, has no special merit, but it is convenient and gives consistent results. In the present study the geometric mean of individual measures within each fraction was considered a representative value.

While particle shape should be included in our concept of particle size, it appears that, at present, a separate evaluation of these two factors is necessary. In the Kozeny expression, particle shape is included in the diameter term, V/A, but for practical use this term is better represented by d/S. Here S is the shape factor and d, as before, is the representative particle diameter. Obviously, for spheres, S takes a value of 6.0, but for nonspherical particles, S becomes larger as the degree of irregularity increases. Here again, it is important that some uniform method of measuring particle diameter be adopted in order that numerical shape factors, essential to a full definition of particle size, will be directly comparable. Shape factors[3] ranging from 6.1 for well-rounded sand grains to 7.7 for angular grains were suggested by Fair and Hatch (5). The values are based upon the terminal settling velocity of the particles in water as compared with the settling rate of spheres and should be considered as approximations. Shape factors for the sands used in the present study were evaluated in the same manner, but in this case the particle shape was substantially uniform.

Uniformity of Particle Size

Except for ideal assemblages, granular media are made up of particles of unequal size. Thus, a useful general equation for flow through such material must be similarly unrestricted in its application. If the individual size portions are known, from the familiar screen analysis for example, the size characteristic of any homogeneous assemblage, conveniently represented by the area-volume ratio A/V may be calculated readily. As shown by Fair and Hatch (5), if P = weight percentage of material of uniform size d, and S = shape factor, then

$$\frac{A}{V} = \frac{S}{100}\sum\frac{P}{d} \quad [3]$$

If the assemblage is not homogeneous with respect to particle-size distribution, only one systematic arrangement of particles appears to lend itself to simplified analysis, namely, that of stratified packing. In this case the sizes increase consistently from the finest at one end to the coarsest at the other, while, with uniform particle shape and like conditions of packing, the porosity remains substantially constant. Since the resistance gradient h/l varies in accordance with the particle size, no single unit assembly is representative and the entire depth of packing L must be considered. For this purpose Fair and Hatch (5) suggest an average value of A/V, weighted with respect to flow resistance, and based upon the fact that for individual layers the percentage of total weight P is also percentage of total depth $(l/L)100$. Thus

$$\left(\frac{A}{V}\right)^2 = \frac{S^2}{100}\sum\frac{P}{d^2} \quad [4]$$

If the size distribution of random material is geometrically normal, as is frequently the case, the area-volume ratio A/V may be expressed in terms of the geometric-mean size M_g and the standard geometric deviation σ_g. Suitable equations are given by T. F. Hatch (7) which are derived from a consideration of the size distribution according to the number of particles in relation to the distribution according to weight; the latter is commonly measured by a screen analysis. In this case, for homogeneous assemblages or nonstratified beds

$$\log\frac{A}{V} = \log S - [\log M_g - 1.1513\log^2\sigma_g] \quad [5]$$

[3] Shape factors based upon particle diameters determined from size of opening of sampling screens (manufacturer's rating); these sizes differ considerably from measured diameters.

TABLE 1 ANALYSIS OF EXPERIMENTAL RESULTS

Sand no.	Screen mesh	Measured size, mm	Test run	Sand depth, in.	Porosity α	Porosity factor $\frac{(1-\alpha)^2}{\alpha^3}$	Approach velocity, cm per sec	Resistance head, in. (t = 20 C)	Coefficient k^a
1	20–28	1.00	1	17.54	0.415	4.79	0.0331	1.062	372
							0.0534	1.680	365
							0.1280	3.980	360
			2	17.10	0.403	5.43	0.0312	1.065	358
							0.0851	3.020	372
			3	16.60	0.388	6.42	0.0334	1.340	366
							0.0683	2.800	375
							0.0917	3.750	374
			4	16.10	0.373	7.57	0.0275	1.340	389
							0.0600	2.800	373
							0.0904	4.310	381
2	28–35	0.685	1	17.17	0.427	4.21	0.0735	4.410	381
							0.1470	8.340	360
							0.2800	16.290	369
							0.4700	27.140	366
			2	16.80	0.415	4.77	0.1130	7.150	361
							0.2400	15.160	360
							0.4200	26.450	360
			3	16.20	0.397	5.81	0.0816	6.140	366
							0.2220	16.590	363
							0.379	28.750	369
			4	15.79	0.386	6.56	0.0725	5.920	363
							0.0839	6.840	362
							0.2100	16.840	356
							0.326	26.900	365
							0.346	28.630	367
3	35–48	0.417	1	18.45	0.438	3.76	0.0600	8.320	339
							0.0890	12.800	349
							0.178	25.300	346
							0.226	32.450	352
			2	18.00	0.426	4.25	0.0578	9.000	346
							0.109	17.030	347
							0.151	24.000	353
							0.202	32.000	351
			3	17.45	0.412	4.95	0.0377	6.730	352
							0.0646	11.600	354
							0.101	17.850	348
							0.134	24.400	358
							0.178	33.200	368
							0.196	35.600	358
			4	16.87	0.396	5.88	0.0354	7.420	359
							0.0587	12.700	371
							0.0890	19.300	371
							0.122	26.600	374
							0.151	32.200	366
4	48–60	0.359	1	22.00	0.443	3.57	0.0466	10.300	353
							0.0816	17.790	347
							0.133	28.800	346
			2	21.40	0.429	4.12	0.0432	10.860	358
							0.0705	17.810	360
							0.1120	28.050	357
			3	20.86	0.413	4.90	0.0367	10.540	352
							0.0734	21.100	353
							0.0993	28.680	355
			4	20.58	0.406	5.28	0.0261	8.050	357
							0.0532	16.300	354
							0.0668	20.800	360
							0.0905	28.300	361
5	60–100	0.215	1	18.50	0.439	3.72	0.0350	17.900	335
							0.0499	25.300	331
							0.0654	33.200	331
							0.0684	34.800	332
			2	17.87	0.424	4.36	0.0177	10.210	335
							0.0363	21.040	336
							0.0535	31.000	335
							0.0606	35.900	343
			3	17.45	0.412	4.95	0.0218	14.230	341
							0.0382	24.850	339
							0.0535	35.300	344
			4	16.70	0.391	6.19	0.0171	13.790	351
							0.0316	25.400	351
							0.0452	36.500	353
6[b]	20–100	0.406	1	16.10	0.354	9.39	0.0188	7.000	397
							0.0363	13.400	393
							0.0599	22.050	394
							0.0795	28.600	384
							0.0963	35.800	396
			2	15.80	0.345	10.42	0.0208	8.100	380
							0.0270	10.900	395
							0.0334	13.200	386
							0.0397	15.900	391
							0.0611	24.600	393
							0.0851	34.400	394
			3	15.20	0.324	13.44	0.0285	13.900	384
							0.0459	22.500	386
							0.0717	35.200	386
7[c]	20–60	0.455	1	21.78	0.428	4.18	0.0625	9.850	350
							0.1151	18.090	349
							0.1780	27.980	348
			2	21.00	0.408	5.16	0.0534	10.670	371
							0.0980	18.770	355
							0.1546	30.050	361
							0.1589	31.250	365
			3	20.20	0.385	6.63	0.0459	11.220	370
							0.0773	19.000	371
							0.1200	29.550	372
8[d]	20–60	0.486	1	16.32	0.373	7.59	0.0565	11.200	368
							0.0979	19.820	377
							0.1330	26.600	372
			2	15.90	0.359	8.85	0.0498	11.310	372
							0.0864	19.920	377
							0.1200	27.450	374
			3	15.31	0.340	11.10	0.0397	11.200	382
							0.0414	11.530	378
							0.0744	21.200	386
							0.0980	28.220	390

[a] Coefficient does not include value of shape S^2.

[b] Equal portions of (20–28) (28–35) (35–48) (48–60) (60–100) unstratified packing; size calulated from $\frac{1}{100}\sum\frac{P}{d}$.

[c] Equal portions of (20–28) (35–48) (48–60) stratified packing; size calculated from $\left[\frac{1}{100}\sum\frac{P}{d^3}\right]^{1/3}$.

[d] Equal portions of (20–28) (35–48) (48–60) unstratified packing.

while for stratified beds

$$\log \left(\frac{A}{V}\right)^2 = \log S^2 - [\log M_g^2 - 4.6052 \log^2 \sigma_g] \ldots . [6]$$

Area-Volume Ratio

So far as the author is aware, no satisfactory useful method of determining surface area of irregularly shaped particles has yet been presented. Attempts have been made to correlate surface area with solution rate, and with coating films of molecular thickness, but the procedures are involved and yield results which apparently are subject to gross error.

For particles somewhat symmetrical in outline, however, it would seem that an approximately uniform relationship may exist between the area-volume ratio of the particles and their terminal settling rates in suitable fluids. Certain factors may enter into such a relationship which do not yield to simplified analysis but, for practical purposes, it is conceivable that they would play a minor role within a desired range of motion.

In the present study, each size fraction of sand was made up of grains having practically uniform shape characteristics, in which case the area-volume ratio for each size would be expected to bear a substantially constant relation to a representative particle diameter. Hence, the numerical effect of shape is absorbed in the equation coefficient.

The Generalized Equation

Evaluation of the channel velocity and the channel diameter, including also the particle-size distribution factor, permits a restatement of Equation [1], possibly extending its useful application to the general problem of flow through granular media. Thus

$$\frac{h}{l} = \frac{k}{g}\left(\frac{\mu}{\zeta}\right)^{2-n} \left(\frac{v}{\alpha}\right)^n \left(\frac{A}{V}\,\frac{1-\alpha}{\alpha}\right)^{3-n} \ldots\ldots\ldots [7]$$

Experimental Procedure

There are a number of practical considerations, oftentimes overlooked, with respect to the measurement of flow through granular media, which influence the accuracy of the results. Some of these are as follows: (*a*) The apparatus should be designed for simplicity of operation and to permit close observation of the conditions of packing and flow; (*b*) the packing should follow some specified orderly arrangement; (*c*) all disturbing factors, particularly trapped air bubbles with liquid flow, should be eliminated; (*d*) static equilibrium of pressures must be reached before resistances are recorded; (*e*) substantially constant temperatures must be maintained.

In the present study the medium (sand) was packed in a glass tube about 2 in. in diameter and 5 ft long, having suitable side taps for maintaining a constant liquid level above the medium and permitting a measurement of static pressures. The tube was supported vertically to facilitate the uniform packing of particles and to simplify the measurement of resistance. With the glass tube irregularities in the packing and other disturbing factors could be readily detected.

With uniform, and vertically stratified nonuniform media, the individual particles were first suspended in an upward-flowing stream of distilled water until all air bubbles were removed. The particles were then allowed to settle into place gradually and the direction of flow reversed; deaerated distilled water was used in all tests.

With unstratified nonuniform media, on the other hand, the foregoing procedure obviously could not be used, but good results were obtained by first evacuating the voids in a dry medium and then allowing the deaerated water to enter slowly from the bottom. It was necessary to adjust the rate of flow carefully in order to prevent air from being further released from solution, and also to secure the medium in place while the water was rising.

Experimental Results

To simplify the experimental considerations, Equation [7] may be rearranged in the form

$$\frac{h}{l} = \frac{k}{g}\left(\frac{\mu}{\zeta}\right)^{2-n} v^n \left(\frac{A}{V}\right)^{3-n} \left(\frac{(1-\alpha)^{3-n}}{\alpha^3}\right) \ldots\ldots [8]$$

Briefly, the author's tests covered a range of viscous flow ($n = 1$) of water through sands of similar particle shape, but differing considerably in particle size and size distribution. The last three factors in Equation [8] were investigated in their separate influence upon the resistance gradient and good agreement with the theoretical relationship was found in each case. To bring about a variation of porosity within a single test, each individual sand was placed in the tube with relatively loose packing, and with each successive test run the medium was consolidated by tapping the tube uniformly on the outside.

To simplify the analysis of the data with respect to Equation [8], the combined influence of all variables (except shape factors which were assumed to be constant) is reflected in the value of the coefficient k given in Table 1. The approximate constancy of the coefficient appears to verify the general correctness of Equation [8].

Carman (8), in a study of specific surface of particles, gives extensive experimental data to verify the Kozeny porosity factor in Equation [8], covering a range of porosities of 0.485 to 0.855. The corresponding change in the value of $(1 - \alpha)^2/\alpha^3$ is from 2.32 to 0.034, and the deviation between calculated and observed values is said to be ±5 per cent. The author has never been able to demonstrate such close agreement between theory and experiment, probably for reasons outlined as follows: (*a*) Difficulties of determining accurately the particle size and shape; (*b*) irregularities in packing, especially when mixed sizes were used; (*c*) experimental errors which, even with careful technique, appeared to be unavoidable. For normally accurate work over a substantial range of particle assemblage, the margin of error may well exceed ±10 per cent.

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Effects of a Change of Poisson's Ratio Analyzed by Twinned Gradients

Illustrated by Simple Solutions of Boussinesq's Problem of a Normal Force and Cerruti's Problem of a Tangential Force Acting on the Surface of a Large Solid, and by a Simple Derivation of the Stresses in a Rotating Thick Disk

By H. M. WESTERGAARD,[1] CAMBRIDGE, MASS.

Some problems of elasticity have a simple solution for a particular value of Poisson's ratio. For example, Boussinesq's problem of a normal force and Cerruti's problem of a tangential force, acting on the plane surface of a semi-infinite solid, are solved when Poisson's ratio is $^1/_2$ by referring to Kelvin's problem of a force at a point in the interior of an infinite solid. For, when Poisson's ratio is $^1/_2$, the solution of Kelvin's problem can be stated in terms of one principal stress at each point, acting along the radial line from the point of the load; the other principal stresses are zero; and one half of the total force may be assigned to one half of the infinite solid. For other values of Poisson's ratio terms must be added in the formulas for the displacements and stresses.[2, 3, 4, 5, 6] The derivations that have been available are somewhat lengthy, especially for Cerruti's problem. The difficulties are reduced by a simple analytical device, here called "the twinned gradient." The displacement to be added by the change of Poisson's ratio is stated as the gradient of a potential except that one of the components is replaced by its twin, an identical component in reversed direction. This device also lends itself to a simplification of the analysis of stresses in a rotating thick disk.[7]

Notation

x, y, z = rectangular coordinates
r, θ, z = cylindrical coordinates
R = length of radius vector from the origin to the point x, y, z
$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ = unit vectors in the directions of x, y, and z
$\boldsymbol{\rho}$ = displacement as a vector
ξ, η, ζ = components of $\boldsymbol{\rho}$ in the directions of x, y, and z
$\operatorname{div} \boldsymbol{\rho} = \dfrac{\partial \xi}{\partial x} + \dfrac{\partial \eta}{\partial y} + \dfrac{\partial \zeta}{\partial z}$ = divergence of $\boldsymbol{\rho}$ = increase of volume per unit of volume
$\nabla = \boldsymbol{i}\dfrac{\partial}{\partial x} + \boldsymbol{j}\dfrac{\partial}{\partial y} + \boldsymbol{k}\dfrac{\partial}{\partial z}$ = the operator nabla
$\Delta = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ = Laplace's operator
$\sigma_x, \sigma_y, \sigma_z$ = normal stresses in the directions of x, y, and z, positive in tension
$\tau_{xy}, \tau_{yz}, \tau_{zx}$ = shearing stresses in the directions of x, y, and z
$\sigma_r, \sigma_\theta, \sigma_z, \tau_{r\theta}, \tau_{\theta z}, \tau_{zr}$ = corresponding stresses in the directions of the cylindrical coordinates
$\Theta = \sigma_x + \sigma_y + \sigma_z = \sigma_r + \sigma_\theta + \sigma_z$ = bulk stress
G = modulus of elasticity in shear
m, μ = values of Poisson's ratio
$\boldsymbol{K}$ = body force per unit of volume, a vector

The Twinned Gradient

Assume that a set of displacements

$$\boldsymbol{\rho}' = \boldsymbol{i}\xi' + \boldsymbol{j}\eta' + \boldsymbol{k}\zeta' \quad \ldots\ldots\ldots\ldots\ldots [1]$$

and stresses σ', τ' have been found to apply when Poisson's ratio has the particular value m and when the body forces per unit of volume are $\boldsymbol{K}'$. Then $\boldsymbol{\rho}'$ is a vector function satisfying the equation of elasticity

$$G\left(\Delta + \frac{1}{1-2m}\nabla \operatorname{div}\right)\boldsymbol{\rho}' + \boldsymbol{K}' = 0 \quad \ldots\ldots [2]$$

The corresponding stresses are defined by equations of the form

$$\sigma_x' = 2G\left(\frac{\partial \xi'}{\partial x} + \frac{m}{1-2m}\operatorname{div}\boldsymbol{\rho}'\right) \quad \ldots\ldots [3]$$

$$\tau_{xy}' = G\left(\frac{\partial \xi'}{\partial y} + \frac{\partial \eta'}{\partial x}\right) \quad \ldots\ldots\ldots\ldots [4]$$

$$\Theta' = \sigma_x' + \sigma_y' + \sigma_z' = \frac{2(1+m)G}{1-2m}\operatorname{div}\boldsymbol{\rho}' \quad \ldots\ldots [5]$$

Assume now that Poisson's ratio is changed from m to μ, while the modulus of elasticity in shear G remains unchanged. Assume for the time being that the displacements $\boldsymbol{\rho}'$ are held unchanged. The new set of body forces required by these assumptions need not be expressed. The shearing stresses, according to Equation [4], will remain unchanged. The normal stresses, according to Equation [3], will increase in all directions by the amount

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[2] Kelvin, 1848; Boussinesq, 1878 to 1885; Cerruti, 1882 and 1888.

[3] See "Mathematical Theory of Elasticity," by A. E. H. Love, University Press, Cambridge, England, third edition, 1920, pp. 16, 181–190, and 241.

[4] "Über die Berechnung der Fundamentdeformation," by Fredrik Vogt, Det Norske Videnskaps-Akademi i Oslo, *I. Math.-Naturv. Klasse*, 1925, no. 2.

[5] Or "Force at a Point in the Interior of a Semi-Infinite Solid," by R. D. Mindlin, *Physics*, vol. 7, May, 1936, p. 195.

[6] Kelvin's and Boussinesq's problems are treated in "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, pp. 321 and 328.

[7] Compare S. Timoshenko, loc. cit., pp. 319–320.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

$$\bar{\sigma} = 2G\left[\frac{\mu}{1-2\mu} - \frac{m}{1-2m}\right]\operatorname{div}\boldsymbol{\rho}' = \frac{2(\mu-m)G}{(1-2\mu)(1-2m)}\operatorname{div}\boldsymbol{\rho}' \quad [6]$$

which may be restated by use of Equation [5] as

$$\bar{\sigma} = \frac{\mu-m}{(1+m)(1-2\mu)}\Theta' \quad [7]$$

The change is completed by adding the displacements $\boldsymbol{\rho}''$ to $\boldsymbol{\rho}'$, thereby creating the final displacements

$$\boldsymbol{\rho} = \boldsymbol{\rho}' + \boldsymbol{\rho}'' \quad [8]$$

and the final stresses

$$\sigma = \sigma' + \sigma'', \qquad \tau = \tau' + \tau'' \quad [9]$$

which require the final body forces

$$\boldsymbol{K} = \boldsymbol{K}' + \boldsymbol{K}'' \quad [10]$$

The choice is made to investigate the possibility of expressing $\boldsymbol{\rho}''$ by a twinned gradient, defined by reversal of sign of the z-component of the gradient $-\nabla\varphi$ of a function $-\varphi$. The proposed formula is

$$2G\boldsymbol{\rho}'' = -\boldsymbol{i}\frac{\partial\varphi}{\partial x} - \boldsymbol{j}\frac{\partial\varphi}{\partial y} + \boldsymbol{k}\frac{\partial\varphi}{\partial z} = -\nabla\varphi + \boldsymbol{k}\cdot 2\frac{\partial\varphi}{\partial z} \quad [11]$$

By referring to Equation [4] it is seen that

$$\tau_{zz}'' = \tau_{zy}'' = 0, \quad \tau_{xy}'' = -\frac{\partial^2\varphi}{\partial x\,\partial y} \quad [12]$$

Also

$$2G\operatorname{div}\boldsymbol{\rho}'' = -\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = -\Delta\varphi + 2\frac{\partial^2\varphi}{\partial z^2} \quad [13]$$

By referring to Equation [3] and noting that the normal stresses were previously increased by $\bar{\sigma}$ one finds

$$\sigma_z'' = \bar{\sigma} + 2G\left(\frac{\partial\zeta''}{\partial z} + \frac{\mu}{1-2\mu}\operatorname{div}\boldsymbol{\rho}''\right) \quad [14]$$

or, by Equations [7] and [13]

$$\sigma_z'' = \frac{\mu-m}{(1+m)(1-2\mu)}\Theta' + \frac{1}{1-2\mu}\left(\frac{\partial^2\varphi}{\partial z^2} - \mu\Delta\varphi\right) \quad [15]$$

The further choice is made to impose the requirement

$$\sigma_z'' = 0 \quad [16]$$

This requirement is satisfied when

$$\frac{\partial^2\varphi}{\partial z^2} - \mu\Delta\varphi = \frac{m-\mu}{1+m}\Theta' \quad [17]$$

Equations [3] and [16] permit a computation of σ_x'':

$$\sigma_x'' = \sigma_x'' - \sigma_z'' = 2G\left(\frac{\partial\xi''}{\partial x} - \frac{\partial\zeta''}{\partial z}\right) = -\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial z^2} = \frac{\partial^2\varphi}{\partial y^2} - \Delta\varphi \quad [18]$$

The complete set of stresses produced by the changes may now be recorded:

$$\sigma_z'' = \tau_{zx}'' = \tau_{zy}'' = 0 \quad [19]$$

$$\left.\begin{aligned}\sigma_x'' &= \frac{\partial^2\varphi}{\partial y^2} - \Delta\varphi\\ \sigma_y'' &= \frac{\partial^2\varphi}{\partial x^2} - \Delta\varphi\\ \tau_{xy}'' &= -\frac{\partial^2\varphi}{\partial x\,\partial y}\end{aligned}\right\} \quad [20]$$

It remains to express the body forces that are required to maintain the stresses in Equation [20]. By use of the two equations of equilibrium of the form

$$\frac{\partial\sigma_x''}{\partial x} + \frac{\partial\tau_{xy}''}{\partial y} + K_x'' = 0 \quad [21]$$

one finds

$$\boldsymbol{K}'' = \boldsymbol{i}\frac{\partial\Delta\varphi}{\partial x} + \boldsymbol{j}\frac{\partial\Delta\varphi}{\partial y} \quad [22]$$

The function φ must satisfy Equations [17] and [22].

In the majority of significant cases both the original and the final body forces are zero. That is,

$$\boldsymbol{K}' = \boldsymbol{K}'' = \boldsymbol{K} = 0 \quad [23]$$

Then Equation [22] is satisfied if

$$\Delta\varphi = 0 \quad [24]$$

that is, if φ is a harmonic function. Equation [17] is satisfied if in addition

$$\frac{\partial^2\varphi}{\partial z^2} = \frac{m-\mu}{1+m}\Theta' \quad [25]$$

It is possible to satisfy Equations [24] and [25] at the same time because $\Delta\Theta' = 0$; Θ' is a harmonic function. With $\Delta\varphi$ removed in Equations [20], φ becomes an Airy function for the stresses in the directions of x and y.

Some of the equations that have been stated become indefinite when $m = 1/2$; but considerations of convergence show that the remaining equations remain applicable in this limiting case. This conclusion is reached also by a direct study of the limiting case of $m = 1/2$.

Statement of the Solution of Kelvin's, Boussinesq's, and Cerruti's Problems When Poisson's Ratio Is 1/2

In Kelvin's problem a solid is considered which extends infinitely far in all directions. At the origin of the coordinates a concentrated force $2P$ is applied in the direction of z, which is assumed to be vertical downward. In Boussinesq's problem the semi-infinite solid $z \geqq 0$ is considered; the concentrated normal pressure P acts at the origin of the coordinates in the direction of z. In Cerruti's problem, for the time being the semi-infinite solid $y \geqq 0$ is considered, with the tangential force P acting at the origin of the coordinates in the direction of z.

When Poisson's ratio is 1/2,

$$m = 1/2 \quad [26]$$

the solution of all three problems can be stated by the same formulas. In doing so it is expedient to use, besides the rectangular coordinates x, y, z, also the cylindrical coordinates r, θ, z, and the polar coordinate R; r and R being defined by the equations

$$R^2 = r^2 + z^2 = x^2 + y^2 + z^2 \quad [27]$$

The formulas follow:

Displacement in the direction of the horizontal radial coordinate r:

$$\rho_r' = \frac{P}{4\pi G}\frac{rz}{R^3} \quad \ldots\ldots [28]$$

Resultant displacement:

$$\boldsymbol{\rho}' = \frac{P}{4\pi GR^3}\left[\boldsymbol{i}xz + \boldsymbol{j}yz + \boldsymbol{k}(R^2 + z^2)\right] \quad \ldots\ldots [29]$$

Principal stress, in the direction of the radius vector R, equal to the bulk stress Θ', the other two principal stresses being zero:

$$\sigma_R' = \Theta' = -\frac{3P}{2\pi}\frac{z}{R^3} \quad \ldots\ldots [30]$$

Corresponding components of stress:

$$\sigma_r' = -\frac{3P}{2\pi}\frac{r^2z}{R^5}, \quad \sigma_\theta' = 0, \quad \sigma_z' = -\frac{3P}{2\pi}\frac{z^3}{R^5} \quad \ldots [31]$$

$$\tau_{r\theta}' = 0, \quad \tau_{\theta z}' = 0, \quad \tau_{rz}' = -\frac{3P}{2\pi}\frac{rz^2}{R^5} \quad \ldots\ldots [32]$$

$$\left.\begin{aligned}\sigma_x' &= -\frac{3P}{2\pi}\frac{x^2z}{R^5}\\ \sigma_y' &= -\frac{3P}{2\pi}\frac{y^2z}{R^5}\\ \sigma_z' &= -\frac{3P}{2\pi}\frac{z^3}{R^5}\end{aligned}\right\} \quad \ldots\ldots [33]$$

$$\left.\begin{aligned}\tau_{xy}' &= -\frac{3P}{2\pi}\frac{xyz}{R^5}\\ \tau_{yz}' &= -\frac{3P}{2\pi}\frac{yz^2}{R^5}\\ \tau_{zx}' &= -\frac{3P}{2\pi}\frac{xz^2}{R^5}\end{aligned}\right\} \quad \ldots\ldots [34]$$

BOUSSINESQ'S PROBLEM FOR ANY VALUE OF POISSON'S RATIO

With $m = 1/2$ and Θ' as in Equation [30], Equation [25] gives

$$\frac{\partial^2\varphi}{\partial z^2} = \frac{1-2\mu}{3}\Theta' = -\frac{(1-2\mu)P}{2\pi}\frac{z}{R^3} \quad \ldots\ldots [35]$$

$$\frac{\partial\varphi}{\partial z} = \frac{(1-2\mu)P}{2\pi R}, \quad \varphi = \frac{(1-2\mu)P}{2\pi}\log(R+z) \quad \ldots [36]$$

This φ is a harmonic function. By referring to Equation [11] one finds the displacements to be added to those in Equations [28] and [29]:

$$\left.\begin{aligned}\rho_r'' &= -\frac{1}{2G}\frac{\partial\varphi}{\partial r} = -\frac{(1-2\mu)P}{4\pi G}\frac{r}{R(R+z)}\\ \zeta'' &= \frac{1}{2G}\frac{\partial\varphi}{\partial z} = \frac{(1-2\mu)P}{4\pi GR}\end{aligned}\right\} \quad \ldots [37]$$

By referring to Equations [20] and [24], one finds the two and only two stresses that will be superposed on those in Equations [31] and [32]:

$$\sigma_r'' = \frac{1}{r}\frac{\partial\varphi}{\partial r} = \frac{(1-2\mu)P}{2\pi}\frac{1}{R(R+z)} \quad \ldots\ldots [38]$$

$$\sigma_\theta'' = \frac{\partial^2\varphi}{\partial r^2} = -\frac{1}{r}\frac{\partial\varphi}{\partial r} - \frac{\partial^2\varphi}{\partial z^2}$$

$$= \frac{(1-2\mu)P}{2\pi}\left(-\frac{1}{R(R+z)} + \frac{z}{R^3}\right) \quad \ldots\ldots [39]$$

CERRUTI'S PROBLEM

In dealing with Cerruti's problem it is expedient now to change the directions so that the force P at the origin of coordinates will act in the direction of x on the semi-infinite solid $z \geq 0$. Accordingly, Equations [27] to [34] are reinterpreted with changes of x to y, y to z, z to x, $\boldsymbol{i}$ to $\boldsymbol{j}$, $\boldsymbol{j}$ to $\boldsymbol{k}$, and $\boldsymbol{k}$ to $\boldsymbol{i}$.

Then Equations [25] and [30] give

$$\frac{\partial^2\varphi}{\partial z^2} = \frac{1-2\mu}{3}\Theta' = -\frac{(1-2\mu)P}{2\pi}\frac{x}{R^3} \quad \ldots\ldots [40]$$

By two integrations one finds the two successive harmonic functions

$$\frac{\partial\varphi}{\partial z} = \frac{(1-2\mu)P}{2\pi}\frac{x}{R(R+z)}, \quad \varphi = -\frac{(1-2\mu)P}{2\pi}\frac{x}{R+z} \quad \ldots [41]$$

By substituting this function φ in Equations [11] and [20] and adding the values to those derived from Equations [29], [33], and [34] by changing directions, one obtains the following formulas, which are the solution of the problem:

$$\xi = \frac{P}{4\pi GR}\left[1 + \frac{x^2}{R^2} + (1-2\mu)\left(\frac{R}{R+z} - \frac{x^2}{(R+z)^2}\right)\right] \quad \ldots [42]$$

$$\eta = \frac{P}{4\pi GR}\left[\frac{xy}{R^2} - \frac{(1-2\mu)xy}{(R+z)^2}\right] \quad \ldots\ldots [43]$$

$$\zeta = \frac{P}{4\pi GR}\left[\frac{xz}{R^2} + \frac{(1-2\mu)x}{R+z}\right] \quad \ldots\ldots [44]$$

$$\sigma_x = \frac{Px}{2\pi R^3}\left[-\frac{3x^2}{R^2} + \frac{1-2\mu}{(R+z)^2}\left(R^2 - y^2 - \frac{2Ry^2}{R+z}\right)\right] \quad \ldots [45]$$

$$\sigma_y = \frac{Px}{2\pi R^3}\left[-\frac{3y^2}{R^2} + \frac{1-2\mu}{(R+z)^2}\left(3R^2 - x^2 - \frac{2Rx^2}{R+z}\right)\right] \quad \ldots [46]$$

$$\sigma_z = -\frac{3Pxz^2}{2\pi R^5}, \quad \Theta = -\frac{(1+\mu)Px}{\pi R^3} \quad \ldots\ldots [47]$$

$$\tau_{xy} = \frac{Py}{2\pi R^3}\left[-\frac{3x^2}{R^2} + \frac{1-2\mu}{(R+z)^2}\left(-R^2 + x^2 + \frac{2Rx^2}{R+z}\right)\right] \quad \ldots\ldots [48]$$

$$\tau_{yz} = -\frac{3Pxyz}{2\pi R^5}, \quad \tau_{zx} = -\frac{3Px^2z}{2\pi R^5} \quad \ldots\ldots [49]$$

ROTATING THICK DISK

It is desired to determine the stresses in a disk, with the sides $z = \pm c$ and the edge $r = a$, rotating about the axis of z with a constant angular velocity ω. The body force is the centrifugal force per unit of volume, $w\omega^2 r/g$, in which w/g is the mass per unit of volume. Timoshenko[3] has shown a solution of this problem. The method of the twinned gradient permits a simple derivation.

In the initial state the body forces and the stresses are taken as zero. Then φ may be assumed to account for the total body force and stresses, except that a simple state of stress may be superposed at the end of the computations. Accordingly

$$K = K'', \quad \sigma = \sigma'', \quad \text{and} \quad \tau = \tau'' \quad \ldots\ldots [50]$$

Then Equations [17] and [22] will be satisfied if

[3] Compare S. Timoshenko, loc. cit., pp. 319–320.

$$\frac{\partial^2 \varphi}{\partial z^2} = \mu \Delta\varphi \quad \text{and} \quad \Delta\varphi = \frac{w\omega^2}{2g}(r^2 + f(z)) \ldots\ldots [51]$$

The polynomial

$$\varphi = (w\omega^2/g)(Ar^4 + Br^2z^2 + Cz^4) \ldots\ldots [52]$$

is chosen for investigation. Substitution from Equation [52] in Equations [51] gives three linear equations for the constants A, B, and C. Thereby one finds

$$\varphi = \frac{w\omega^2}{g}\left(\frac{1-\mu}{32} r^4 + \frac{\mu}{4} r^2z^2 + \frac{\mu^2}{12(1-\mu)} z^4\right) \ldots\ldots [53]$$

and

$$\Delta\varphi = \frac{w\omega^2}{g}\left(\tfrac{1}{2} r^2 + \frac{\mu}{1-\mu} z^2\right) \ldots\ldots [54]$$

which satisfy Equations [51].

The two stresses that do not vanish may now be written in accordance with Equations [20], but with a constant tension σ_0 superposed to make the load on the edge vanish as nearly as possible:

$$\sigma_r = \frac{1}{r}\frac{\partial \varphi}{\partial r} - \Delta\varphi + \sigma_0$$

$$= \frac{w\omega^2}{g}\left[\frac{3+\mu}{8}(a^2 - r^2) + \frac{\mu(1+\mu)}{6(1-\mu)}(c^2 - 3z^2)\right] \ldots\ldots [55]$$

$$\sigma_\theta = \frac{\partial^2 \varphi}{\partial r^2} - \Delta\varphi + \sigma_0$$

$$= \frac{w\omega^2}{g}\left[\frac{3+\mu}{8} a^2 - \frac{1+3\mu}{8} r^2 + \frac{\mu(1+\mu)}{6(1-\mu)}(c^2 - 3z^2)\right] \ldots [56]$$

which are the values given by Timoshenko. When c is not too great, the radial stresses which remain at the edge according to Equation [55] constitute a load that balances itself locally. Its removal, by superposing an equal and opposite load, would cause only local effects near the edge.

Torsional Vibration in Geared-Turbine Propulsion Equipment

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Two alternative methods are presented by the authors for the determination of torques in the propelling machinery of ships when friction is taken into account. One is called the "direct method," while the other is called the "method of normal coordinates." The latter is described at length, showing the great saving of numerical work possible by its use when compared to the direct method.

The paper points out that at critical propeller speeds oscillatory torques are limited only by damping. Two types of damping are taken into account: One is damping of the propeller in water; the other is friction in the driving mechanism, introduced as solid friction in the shafts similar to damping in the metal.

IN THE propelling machinery of ships, several modes of torsional vibration may occur at certain critical speeds within the running range. If the amplitudes of these vibrational torques become greater than the respective driving torques in any one of the shafts, chattering of the gear teeth or lifting of the journal affected might result.

The critical frequencies are determined by the inertias, stiffnesses, and gear ratios of the propulsion equipment. The exciting stimulus for the torsional vibrations is provided largely by the blades of the propeller as they approach and recede from the hull or sternpost of the boat. At critical speeds the oscillatory torques are limited only by damping. Two types of damping are taken into account: One is damping of the propeller in water; the other is friction in the driving mechanism, introduced as solid friction in the shafts similar to damping in the metal.

Two alternative methods are presented herein for the determination of torques when friction is taken into account. One of them is referred to as the "direct method" and is explained in section 7. The other one is called the "Method of Normal Coordinates;" it is described at length in sections 8 and 9 and is summarized in section 10. While the method of normal coordinates appears at first sight unduly long and involved, the fact remains that this method, once mastered, will result in a great saving of numerical work as compared with the direct method.

A schematic diagram of the propulsion drive considered is shown in Fig. 1. Most of the discussion applies to other arrangements as well.

An equivalent system, to which the one shown in Fig. 1 is reduced, is shown in Fig. 2.

1—Nomenclature

The nomenclature used in the paper is as follows:

n = number of propeller blades

N = propeller speed, rpm

$\omega_p = N2\pi/60 = 0.1047\,N$ = propeller speed, radians per sec

f = excitation frequency, cycles per sec $= Nn/60$

$\omega = n\omega_p$ = excitation frequency, radians per sec

$I_p, I_1, \ldots = WR^2/g$ lb-in. sec^2 = moment of inertia in mass rather than weight units

$K_0, K_1, \ldots$ = stiffness constants of various shafts, in-lb per radian

$\theta_p, \theta_0, \ldots$ = angular displacement of various inertias or the amplitude[3] of such displacements, radians

$T_0, T_1, \ldots$ = torque in shaft $K_0, K_1 \ldots$, in-lb per radian or the amplitude of this torque

T_e = exciting torque applied at the propeller, or its amplitude, in-lb per radian

φ = solid-friction damping factor = fraction of maximum elastic energy dissipated per amplitude cycle

$\epsilon = \varphi/2\pi$

D = damping constant for a shaft, in-lb sec

D_p = damping constant for propeller, in-lb sec

$C = D_p/\omega$. The introduction of C leads to algebraic simplification

$\omega_0, \omega_1, \ldots \omega_s$ = critical frequencies, in rad per sec; $\omega_0 = 0$ corresponds to the "degenerate" critical frequency $\omega = 0$; ω_1 corresponds to the first true critical frequency, etc.

E = kinetic energy of system

V = potential energy of system

Referring to the normal coordinates in sections 8 and 9:

q_i = normal coordinate of any displacement of system. The coordinate q_i specifies the amount of the ith critical shape of vibration present in an actual displacement. Its dimensions are the same as those of θ.

A_i = constant factor having the dimensions of stiffness constants K, in-lb per radian

B_i = constant factor having the dimensions of inertia constants I, in-lb sec^2

Q_i = "generalized force" defined by the equation: $Q_i dq_i$ = work done in displacement dq_i by the forces or torques whose effect is not included in V

The I's, K's, θ's, T's are for the equivalent system, possessing no gears as discussed in section 2, and the θ's, T's, V, E are reckoned relative to a steady state determined by the average load and speed.

2—Inertia and Stiffness Constants

The calculation of the natural frequencies does not depend on the magnitude of the exciting torque and on the damping but only on the inertia and stiffness constants.

It is convenient to use an "equivalent system" as shown in Fig. 2, which has no gears (or has gears that effect no change of speed) and whose natural frequencies are the same as the frequencies of the original system. This is possible, because, while the double-reduction gears in turn effect a change of rotary speed of ratio R_1:1, R_2:1, it will be noted that an oscillatory vibration will be transmitted through the gears without change of frequency

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[3] In the sense used here the quantity $A \cos(\omega t + \varphi)$ possesses the amplitude A and not $2A$.

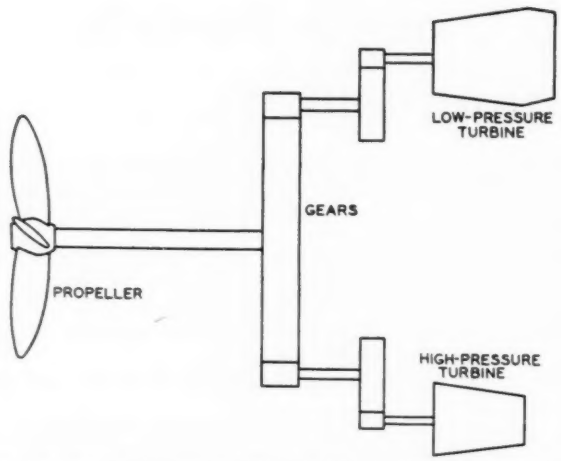


FIG. 1 ARRANGEMENT OF A TYPICAL MARINE-PROPULSION UNIT

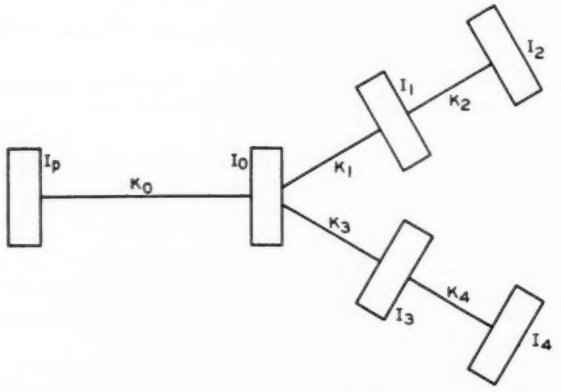


FIG. 2 AN ARRANGEMENT OF A MARINE-PROPULSION UNIT WHICH IS EQUIVALENT TO THE ONE SHOWN IN FIG. 1

but with a proper change of amplitude. The inertias and stiffnesses for the equivalent inertia are obtained as follows.

Starting at the propeller end, I_p and K_0, shown in Fig. 2, are taken equal to the moment of inertia of the propeller and the stiffness of the propeller shaft shown in Fig. 1. Progressing away from the propeller, after one passes through a gear of step-up ratio R:1, the actual I's and K's occurring beyond are multiplied by R^2. If several gears are passed, then the subsequent I's and K's are multiplied by the squares of the step-up ratios of each gear. Thus the inertia I_0 for Fig. 2 is obtained by adding to the moment of inertia of the big gear the moments of inertia of the two pinions multiplied by R_1^2, while I_2, K_2 for Fig. 2 are obtained by multiplying $R_1^2R_2^2$ the I_2, K_2 of Fig. 1. It may be shown that an equivalent system so obtained has the same natural frequencies as the actual system, as well as the same position of nodes (whether in free or forced oscillation). The amplitudes and torques of the two systems are also definitely related, as shown by

$$\text{Multiply the actual } \begin{Bmatrix} I \\ K \\ T \\ \theta \end{Bmatrix} \text{ by } \begin{Bmatrix} R^2 \\ R^2 \\ R \\ 1/R \end{Bmatrix} \text{ to obtain the equivalent value}$$

As explained previously, the step-up ratios R used here are those of the gears encountered in passing from the propeller to the shaft or inertia under consideration.

If we suppose that the equivalent system with gears of speed ratio 1:1 is used to deliver power to the propeller and that the generation of power is properly divided between the two branches, it will be seen that the steady and oscillatory torques are changed in the same ratio in each branch. Thus the equivalent system can be used directly to examine questions of chattering at the gears.

The calculation of inertias and stiffnesses of each member of the drive can be made with good accuracy from the blueprints. The following items must be kept in mind:

1 The inertia of the propeller is increased to allow for entrained water. By various tests, the increase of inertia is estimated to be around 25 per cent of the inertia of the propeller. One authority mentions 23 to 32 per cent (1).[4] The higher figure applies when the propeller blades have broad tips. The shipbuilder commonly supplies a figure for this increase in inertia.

2 The moment of inertia of the shaft which joins two large members is assumed to be divided equally between the two larger inertias. This is not strictly correct. It may be shown that if a node exists at one end of the shaft, then it is fairly correct to add a third of its moment of inertia to the inertia of the member at the other end. If a node exists in the middle, then a third of the inertia of the portion of the shaft to each side of the node should be added onto the respective inertia of the member at that end. Since the position of the node cannot be calculated beforehand, this procedure is not recommended and the straightforward addition of half of the inertia to the members on each side has been adopted. However, it must be added that the effect of the inertia of the shaft is generally quite small.

3 Gear and pinion teeth are assumed to be rigid. This is confirmed by estimates of gear-teeth deflections converted into angular displacements. It is also true of coupling teeth.

3—EXCITATION FACTORS

There is a once-per-revolution excitation due to unbalance or misalignment in any rotating part. The frequency of this may be N (the propeller frequency) or NR_1, or NR_1R_2 (the turbine rotation frequency). However, in every case it tends to produce a *transverse* vibration and cannot be considered as a cause of torsional oscillations.

Defects of gears may serve as a cause of high-frequency torsional oscillations, though with a double-helical type and the high perfection of present gear cutting, this effect is negligible.

Asymmetry of the propeller blades may cause a torsional stimulus of once-per-revolution frequency. The chief cause of torsional vibration, however, lies in the variation of the torque on the propeller due to the variation of the water forces on the blades during their rotation. The frequency of this variation is the blade frequency, that is, the propeller frequency multiplied by the number of blades.

The cause of such torque irregularities is known as circumferential wake variation (2). A partial list of factors which may make the hydrodynamic field around the propeller nonuniform is as follows:

1 The sternpost and the rudder. This is important in single-screw ships, where, with an even number of propeller blades, the effect is doubled. Hence single-screw ships tend to have large torsional excitation factors.

2 The distortion of the hydrodynamic field near the hull. This is an important factor in ships with two or four screws. As each blade tip passes near the hull there is a torque impulse.

3 The propeller operating near or at the surface of the water. This may be very important in the case of a ship lightly loaded. Also it often accounts for the accentuation of the vibration in rough water.

[4] The numbers in parentheses refer to the Bibliography at the end of the paper.

4 Protuberances in the ship's hull, such as propeller shaft bossings.

The amplitude of this exciting torque is generally given as a fraction of the propelling torque. This is possible because the wake around the propeller remains similar at all operating speeds.

One of the first articles on the subject is by A. T. Thorne and J. Calderwood (3). They measured the variation in the propeller resisting torque in a number of ships and, allowing for resonance effects, found that the excitation factor was not likely to exceed 10 per cent for smooth water. A complete study of a twin-screw steamer, the *President Hoover*, was made by F. M. Lewis (4). He found from a careful analysis of model experiments an excitation factor of 3.2 per cent and torsiograph records taken on the actual ship showed 3.3 per cent.

From the foregoing discussion it would seem that a conservative figure for the case where there is more than one screw is 7.5 per cent. A figure as low as $3^1/_2$ per cent has been recommended in some cases; this is based on measurements.

There are also some indirect references to blade frequency excitation. J. L. Kent (5) has mentioned it as being noticeable in one particular case when the stern was rising. J. H. Gibson, in commenting on an article by H. E. Saunders (6), speaks of blade pulsations in the propeller thrust.

It is to be kept in mind that the driving power which is transmitted through the propeller as a torque reacts on the water largely as a thrust. Any fluctuation in propeller torque, therefore, shows up also as a variation in thrust at the thrust bearing. Furthermore, the water forces react directly on the hull or sternpost. This causes hull vibrations.

Again, in transmitting a load through gearing, there is a resultant normal reaction at the gear contact teeth and gear bearings which may again excite transverse vibrations in the ship. These effects are not taken into account here because it is believed they do not affect the natural frequency appreciably; while they may contribute a certain amount of damping, they serve to make the vibrations prominent. It is worth noting that in some cases in which these vibrations were, no doubt, excited torsionally from the propeller, all kinds of remedies were tried before changing the propeller exciting frequency from the natural frequency effected a solution (7).

4—Determination of Natural Frequencies of the Propulsion Drive

Critical frequencies are those frequencies at which, in the absence of friction, the system oscillates with no external or applied torques.

The equations of motion are obtained by equating the rate of change of angular momentum of each inertia to the net torque acting on it, then relating the torque of each shaft to the relative angular displacement of its two ends. Referring to Fig. 2, let all the angular displacements, θ_p, θ_0, ... be measured positive in the same direction, and let the torques be positive when they tend to displace the right-hand end of the shaft in the positive direction relative to its left-hand end. Then there result the equations

$$I_2\ddot{\theta}_2 = -T_2 \quad [1]$$

$$I_1\ddot{\theta}_1 = T_2 - T_1 \quad [2]$$

$$I_4\ddot{\theta}_4 = -T_4 \quad [3]$$

$$I_3\ddot{\theta}_3 = T_4 - T_3 \quad [4]$$

$$I_0\ddot{\theta}_0 = T_1 + T_3 - T_0 \quad [5]$$

$$I_p\ddot{\theta}_p = T_0 \quad [6]$$

$$T_2 = K_2(\theta_2 - \theta_1) \quad [7]$$

$$T_1 = K_1(\theta_1 - \theta_0) \quad [8]$$

$$T_4 = K_4(\theta_4 - \theta_3) \quad [9]$$

$$T_3 = K_3(\theta_3 - \theta_0) \quad [10]$$

$$T_0 = K_0(\theta_0 - \theta_p) \quad [11]$$

To solve for the natural frequencies let us seek solutions of Equations [1] to [11], inclusive, in all of which θ_i and T_i vary as $\cos \omega t$. Assume

$$\theta_i = \theta_i \cos \omega t,\ T_i = T_i \cos \omega t \quad [12]$$

This notation, while using θ_i both for the displacement itself as well as the amplitude of the oscillatory displacement, and similarly for T_i, will not lead to any confusion. Physically, the assumption in Equation [12] amounts to finding the steady-state solution only. Variations in speed, if any, are assumed to be slow enough to allow steady state to obtain. Double time differentiation in Equations [1] to [6], inclusive, may now be replaced by multiplication by $-\omega^2$; upon division by $\cos \omega t$ there results a system of algebraic linear homogeneous equations in the unknown amplitudes. In practice one works along the upper branch from the free end toward θ_0 by means of the equations

$$T_2 = I_2\omega^2\theta_2 \quad [13]$$

$$\theta_1 = \theta_2 - T_2/K_2 \quad [14]$$

$$T_1 = T_2 + I_1\omega^2\theta_1 \quad [15]$$

$$\theta_0 = \theta_1 - T_1/K_1 \quad [16]$$

These equations, known as the Holzer equations, follow from Equations [1], [2], [5], [7], and [8] under the assumption in Equation [12], and determine the torques and displacements in terms of θ_2. Similarly, along the lower branch the equations

$$T_4 = I_4\omega^2\theta_4 \quad [17]$$

$$\theta_3 = \theta_4 - T_4/K_4 \quad [18]$$

$$T_3 = T_4 + I_3\omega^2\theta_3 \quad [19]$$

$$\theta_0 = \theta_3 - T_3/K_3 \quad [20]$$

serve to determine the amplitudes in terms of θ_4. From Equations [16] and [20], θ_4 may be determined in terms of θ_2. Proceeding toward the propeller, one obtains from Equations [5] and [11]

$$T_0 = T_1 + T_3 + I_0\omega^2\theta_0 \quad [21]$$

$$\theta_p = -T_0/K_0 + \theta_0 \quad [22]$$

and modifying Equation [6] by the addition of an applied torque T_a, we obtain

$$-T_a = T_0 + I_p\omega^2\theta_p \quad [23]$$

This T_a may be considered as the torque which, applied at the propeller branch, will produce the vibration with θ_i, T_i. With the sign as in Equation [23], a positive T_a tends to displace the propeller in the sense of positive θ_p. At the critical speeds T_a is zero without all the amplitudes vanishing.

The usual method of solution is to assume values of ω^2 and to solve for T_a/θ_2. A graph is made, and where $T_a/\theta_2 = 0$, there $\omega^2 = \omega_c^2$. All calculations may be made on a tabular form sheet. They should be preserved for use in the torque calculations at the criticals which will be discussed later.

5—Damping in the System

Damping in the system is introduced as solid-friction damping, which we imagine to be distributed throughout the shafts. Actually there is also damping at the shrink fits, at the tooth contacts,

in the gears due to the changing stresses there; there is viscous damping in the oil; also windage losses. Since not enough is known about these various types of damping, it does not seem to be worth-while to take account of them individually, and for this reason damping has been introduced as of the solid-friction type, especially in view of the fact that for sinusoidal oscillations there is a comparatively easy way of introducing damping of this kind.

By solid friction is meant the loss of energy in an elastic material that is undergoing a cyclic change of stress. Experimentally it has been found that in such cases there is energy loss per cycle similar to energy loss in material due to magnetic hysteresis; in fact, it is connected with elastic hysteresis because the strain lags slightly behind the stress. When the stress-strain curve for solid material is plotted, there results a narrow loop whose area gives the loss of energy per cycle. This loss of energy in the case of steel, is likely to be a minor fraction φ (<1 per cent) of the maximum elastic energy per amplitude cycle (8). Tests conducted by the General Electric Company on assembled steel structures composed of elements connected by shrinkage fits, bolts, or riveted joints show energy losses from 2 to 10 per cent of the maximum elastic energy per amplitude cycle. It thus appears that the greater part of the damping loss in such an assembly occurs in the fits rather than in the material itself. As a middle value 5 per cent may be considered to be a conservative estimate of what might be expected with coupled shafting and gear-teeth contacts. This value is much higher than the solid-friction energy losses of the solid materials which are used for the shafts and gears, the excess being thought of as due primarily to the energy losses at the coupling fits and tooth contacts where rubbing contact and viscous oil squeezing during oscillations are doubtless responsible for dissipation of energy.

For the case of sinusoidal vibrations, it will now be shown that solid friction can be taken account of by multiplying the stiffness constants K by the factor

$$1 + j\frac{\varphi}{2\pi} = 1 + j\epsilon \qquad [24]$$

In Equation [24] $j = \sqrt{-1}$ and the use of complex numbers is in the sense in which vectors have been employed representing sinusoidal currents or voltages. This is done by assuming that time enters as a factor

$$e^{j\omega t} \qquad [25]$$

rather than as $\cos \omega t$ as in Equation [12], and using complex amplitudes. As known, the passage to the real sinusoids is made by taking the real (or imaginary) part of the complex expression. Since

$$\begin{aligned}(a + jb)e^{j\omega t} &= (a + jb)(\cos \omega t + j \sin \omega t)\\ &= (a \cos \omega t - b \sin \omega t) + j(b \cos \omega t + a \sin \omega t)\end{aligned}$$

it is seen that if the real part is used, a coefficient $(a + jb)$ of $e^{j\omega t}$ leads to the sinusoid $(a \cos \omega t - b \sin \omega t)$. Geometrically, the complex number $a + jb$ can be plotted as a vector, and it may be shown that its length or absolute value

$$|a + jb| = \sqrt{(a^2 + b^2)}$$

represents the amplitude or maximum value of the sinusoid, and $\tan^{-1} b/a$, the angle that the vector makes with the axis of reals, is the phase of the sinusoid.

To justify the foregoing statement concerning Equation [24], imagine a relatively massless shaft with one end fixed and a sinusoid torque applied at the other end. If there is energy lost in friction, then the displacement of the free end will not be in phase with the torque. The 90-deg out-of-phase displacement component will give a velocity term which will be in phase once more with the applied torque, resulting in an energy loss which is thus identified with friction loss.

A way of recognizing this energy loss is by replacing the ordinary torque relation $T = K\theta$ where θ is the total angle of twist in a shaft, by

$$T = K\theta + D\dot{\theta} \qquad [26]$$

where D is a damping constant. If we assume a solution

$$\theta = c \cos \omega t, \dot{\theta} = -c\omega \sin \omega t$$

then

$$T = Kc \cos \omega t - Kc\omega \sin \omega t$$

The work performed by T is

$$\begin{aligned}\int T d\theta &= K\int \theta d\theta + D\int \dot{\theta} d\theta = K(\theta^2/2) + D\int \dot{\theta}^2 dt\\ &= Kc^2 \cos^2 (\omega t/2) + Dc^2\omega^2 \int \sin^2 \omega t dt\end{aligned}$$

The term involving K represents the elastic energy stored in the shaft, an amount $Kc^2/2$ flowing into it as it deflects, and returning as it approaches the equilibrium position. The term involving D leads to a consumption of energy at the average rate $Dc^2\omega^2/2$, hence of amount $Dc^2\omega\pi$ per cycle $\Delta t = 2\pi/\omega$. By definition then

$$\varphi = \frac{Dc^2\omega\pi}{Kc^2/2} = \frac{2D\omega\pi}{K}$$

Hence

$$D = \frac{\varphi K}{2\pi\omega}$$

If imaginary exponentials are used to represent sinusoidal quantities, letting θ and T vary as $e^{j\omega t}$ as in Equation [25], then from Equation [26]

$$Te^{j\omega t} = Kce^{j\omega t} + Dj\omega ce^{j\omega t}$$

or omitting the factor $e^{j\omega t}$

$$T = \left(K + K\frac{\varphi j}{2\pi}\right)\theta = K\left(1 + j\frac{\varphi}{2\pi}\right)\theta = K(1 + j\epsilon)\theta$$

This shows that complex numbers may be used to account for internal damping by replacing K by $K(1 + j\epsilon)$. It may be shown that this results in elliptical hysteresis curves.

It is customary to represent viscous friction by a term proportional to the velocity and thus similar to $D\dot{\theta}$ in Equation [26] with a constant D. Resistance due to such friction is sometimes called "ohmic resistance" due to its similarity to electrical resistance. If D were constant, then the energy loss per unit time would be constant and independent of ω. By letting D vary as $1/\omega$, the constant energy loss per cycle has been obtained.

6—Propeller Damping

It seems curious at first thought that the propeller which causes the vibration should also contribute to its damping. Yet this is the case and it may be seen as follows:

From the propeller torque-speed curve for equilibrium conditions, shown in Fig. 3, it will be noted that if, as a result of a torsional vibration the propeller speed ω_p oscillates about a mean value at P_1, the driving torque also oscillates and in phase with the propeller speed. The variable component of torque is

$$D_p\omega_p \qquad [27]$$

where the damping coefficient D_p is the slope of the torque-speed curve.

The foregoing argument supposes that for a rapid variation of propeller speed the torque of Fig. 3 is followed. Actually this curve represents the load on the propeller when it is held at various constant speeds, while the ship velocity and the propeller

slip have adjusted themselves to their respective constant values corresponding to this constant velocity of the propeller. If the variation in propeller speed is rapid, the ship velocity will not have time to change and only a variation in the propeller slip will result. For this reason the coefficient D_p should be the slope of the propeller torque-speed curve when the ship's velocity with respect to the water or the speed of advance remains constant. This curve is steeper than the equilibrium curve and is indicated by the dashed line in Fig. 3.

Three methods have been suggested for estimating the value of D_p.

1 *Model Experiments.* F. M. Lewis (9) has derived the following expression for the damping factor

$$D_p = \frac{60}{2\pi} \frac{Q(1 - s)}{N} \frac{\partial Q_m/\partial s}{Q_m}$$

where Q is torque, N is propeller speed in rpm, and s is the slip. The subscript m refers to a model propeller which simulates the actual one; s is the same for both the model and the ship. The quantity $(\partial Q_m/\partial s)/Q_m$ can be evaluated from the proper diagram included in Figs. 185–208 of D. W. Taylor's "Speed and Power of Ships" (first edition) (10).

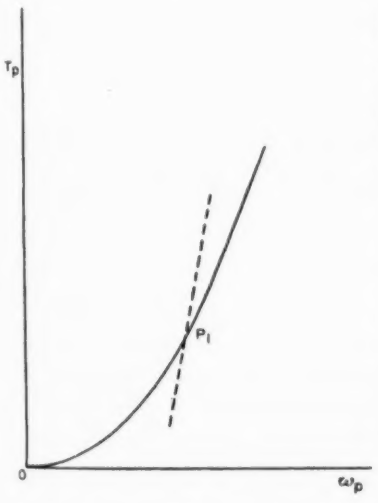


FIG. 3 PROPELLER TORQUE-SPEED CURVE

This method requires a knowledge of propellers and a certain skill is involved in adjusting model experimental data to the propeller on the ship. Furthermore, the data are not available for many propellers of recent design.

2 *Method of Tingey.* The actual torque-speed curve for constant speed of advance can be obtained from a Troost diagram. This has been suggested and developed by R. H. Tingey, chief engineer of the Shipbuilding Division of the Bethlehem Steel Company. On a Troost diagram there are curves of constant values of δ plotted with pitch ratio as the ordinate and basic variable B_p as the abscissa, where

d = propeller diameter, ft
H_0 = pitch at blade tip, ft
P = shaft horsepower
N = propeller speed, rpm
V = ship speed, knots
V_a = speed of advance of propeller in relation to water at stern in which it is working
b = block coefficient of hull

For single-screw ship $V_a = V[1 - (0.5b - 0.10)]$
For twin-screw ship $V_a = V[1 - (0.5b - 0.16)]$

and

$$B_p = \frac{NP^{0.5}}{V_a^{2.5}}, \qquad \delta = \frac{Nd}{V_a}$$

$$\text{Pitch ratio} = \frac{\text{Propeller pitch}}{\text{Propeller diameter}} = \frac{H_0}{d}$$

It is necessary to select the diagram that most nearly applies.

From a knowledge of the equilibrium conditions, if B_p and δ are known, the pitch ratio can be found; or if any one factor in these three quantities is unknown, it can be determined. The pitch ratio is, of course, constant for a given propeller. With constant V_a, δ and B_p can be determined as functions of N. From B_p the shaft horsepower and consequently the propeller torque can be estimated. The damping constant is the slope of propeller torque for constant speed of advance plotted against propeller speed in radians per second.

Shipbuilders, generally speaking, have access to Troost's diagrams for most propellers of recent design, including some with airfoil sections. Similar diagrams are given in D. W. Taylor's "Speed and Power of Ships" (10), 1933 edition, Fig. 204 for three blades and Fig. 211 for four blades. It can be seen that this method also involves a knowledge of the propeller details.

3 *Double-the-Slope Method.* An approximate value of the damping constant can be taken as twice the slope of the equilibrium torque-speed curve. Incidentally, if only one or two points are known on this curve, it may be assumed that the torque varies as the square of the propeller speed. Double-the-slope method gives damping factors which are conservative at low speeds, where propeller damping is significant, and which may be too large at high speeds, where, however, propeller damping is usually negligible compared with that due to internal solid friction.

Because it is based on the best information that is obtainable, and because it is most direct and involves the fewest assumptions, the Tingey method is unquestionably the most accurate. Often, however, the propeller characteristics are not available and then it is necessary to use the third method.

With the assumption Equation [25] that time enters as an imaginary exponential, Equation [23] is now replaced by

$$-T_a = T_0 + \theta_p(I_p\omega^2 + jD_p\omega) \quad \ldots\ldots [28]$$

7—Direct Determination of Amplitude of Oscillating Torques at the Critical Speeds

By introducing solid-friction and propeller damping in the manner explained in the two preceding sections, the Holzer Equations [13] to [23], inclusive, are modified by having K_i replaced by $K_i(i + j\epsilon)$ thus leading, say, from Equation [14] to

$$\theta_1 = \theta_2 - \frac{T_2}{(1 + j\epsilon)K_2} \quad \ldots\ldots [14a]$$

furthermore Equation [23] is replaced by

$$-T_a = T_0 + \theta_p(I_p\omega^2 + jD_p\omega) \quad \ldots\ldots [23a]$$

To solve for the torques at a critical speed, say $\omega^2 = \omega_i^2$, where this critical had been determined by means of Equations [13] to [23], inclusive, we simply set $\omega^2 = \omega_i^2$ and proceed to determine the various torques and amplitudes in terms, say, of θ_2. After the last equation has yielded T_a in terms of θ_2, the assumed value of T_a is inserted, whereupon all the amplitudes are determined absolutely, rather than relatively to θ_2. In solving Equations [14a] and [23a], use is made of the following approximations

$$\frac{1}{1 + j\epsilon} = 1 - j\epsilon \quad \ldots\ldots [29]$$

which holds since ϵ is a very small quantity, though, of course, more exactly

$$\frac{1}{1 + j\epsilon} = \frac{1 - j\epsilon}{1 + \epsilon^2} \quad \ldots\ldots [29a]$$

This method, while straightforward, is quite laborious due to the introduction of imaginary quantities. At certain critical speeds

it is necessary to use a calculating machine in order to maintain proper accuracy, especially at the junction point of the low-pressure and high-pressure turbine branches. It often happens that these branches involve large (equivalent) inertias and also large torques, but that they oscillate one against the other so that in calculating the torque in the propeller shaft from the previously found torques T_1, T_2, higher accuracy is needed than can be obtained by means of a slide rule. In such cases the approximation in Equation [29] may itself lead to error.

The method of normal coordinates, which will now be taken up, simplifies the calculations considerably.

8—Introduction of Normal Coordinates (11)

Consider linear systems in which the forces or torques are linear in the different coordinates which specify the displacements from equilibrium. Then the potential energy V is quadratic in the displacements. The kinetic energy E is, of course, quadratic in the velocities. By "normal coordinates" of a linear system is meant a set of coordinates for which E and V both reduce to a sum of squares (without product terms). These normal coordinates are proper linear combinations of the original variables, with constant coefficients. Their existence follows from a well-known theorem in algebra which states that two quadratics may be simultaneously reduced to algebraic sums of squares by a proper choice of axes.

The present system is linear in the six variables θ_0, θ_p, θ_1, θ_2, θ_3, and θ_4, as noted from the Equations [1] to [11], inclusive. These θ's, however, are not normal coordinates because product terms occur in the potential energy

$$V = \frac{1}{2}[K_0(\theta_p - \theta_0)^2 + K_1(\theta_0 - \theta_1)^2 + \ldots] + K \ldots \quad [30]$$

even though the kinetic energy

$$E = \frac{1}{2}[I_p\dot{\theta}_p^2 + \ldots + I_4\dot{\theta}_4^2] \quad [31]$$

does involve only squares. If $q_0, q_1, \ldots q_5$ are normal coordinates, then each q is a linear combination of the θ's and when V, E are expressed in terms of the q's, there results

$$V = \frac{1}{2}(A_0q_0^2 + A_1q_1^2 + \ldots) \quad [32]$$

$$E = \frac{1}{2}(B_0\dot{q}_0^2 + B_1\dot{q}_1^2 + \ldots) \quad [33]$$

where $A_0, A_1, \ldots$ and $B_0, B_1, \ldots$ are the proper constants.

Applying Lagrange's equations for motion under no external torques, there results the system of equations

$$\left.\begin{aligned} B_0\ddot{q}_0 + A_0q_0 &= 0 \\ B_1\ddot{q}_1 + A_1q_1 &= 0 \\ &\cdots \end{aligned}\right\} \quad [34]$$

It will be noted that as a result of the nonoccurrence of product terms in both V and E, the variables *separate* completely in the equations of motion. Therefore, the solution of these equations is very simple, being given for each q by

$$q_i = C_i \sin(\omega_i t + \varphi_i),\ \omega_i^2 = A_i/B_i \quad [35]$$

where the constants C_i, φ_i are completely arbitrary. Choosing all C_i but one equal to zero, there results an oscillation of frequency ω_i. Now the q's are given by six linear equations

$$\left.\begin{aligned} q_0 &= L_{0p}\theta_p + L_{00}\theta_0 + L_{01}\theta_1 + \ldots \\ q_1 &= L_{1p}\theta_p + L_{10}\theta_0 + L_{11}\varphi_1 + \ldots \\ &\cdots \end{aligned}\right\} \quad [36]$$

whose solution yields for the θ's linear combinations of the q's

$$\left.\begin{aligned} \theta_p &= M_{p0}q_0 + M_{p1}q_1 + M_{p2}q_2 + \ldots \\ \theta_0 &= M_{00}q_0 + M_{01}q_1 + M_{02}q_2 + \ldots \\ &\cdots \end{aligned}\right\} \quad [37]$$

Hence if each q vanishes except q_i, for which Equation [35] holds, there results a motion which can be described as a free oscillation of frequency ω_i, in which all the inertias oscillate sinusoidally and in phase with the same frequency ω_i.

This shows the physical meaning of the normal coordinates, namely, that each normal coordinate q_i is associated with one of the critical frequencies. Each critical frequency carries with it a certain "shape of oscillation," that is, a definite ratio of amplitudes or a definite position of nodes; and each normal coordinate measures, so to speak, the component of that shape in the actual displacement. The subscripts of the q's can conveniently increase in the order of increasing frequencies.

The exact nature of the q's is not completely determined by the foregoing requirement, since a multiplication of each q by a constant is still possible. To avoid this ambiguity, each q will be chosen as equal to unity corresponding to a displacement in which the propeller displacement θ_p is equal to unity. This is equivalent to putting

$$M_{p0} = M_{p1} = \ldots = M_{p5} = 1 \quad [38]$$

in the first Equation [37] which becomes

$$\theta_p = q_0 + q_1 + \ldots + q_5 \quad [39]$$

Now q_1 carries with it a displacement in which the propeller displacement θ_p is equal to q_1, while all the other θ's have the same ratio to θ_p as the amplitudes in the oscillation of frequency ω_1. However, if all q's but q_1 vanish, then the Equations [37] yield for the displacement $M_{01}q_1, M_{11}q_1, \ldots M_{41}q_1$. Hence the coefficients of q_1 in Equation [37] represent precisely the first "shape of vibration," that is, the amplitudes of the various inertias during the critical frequency ω_1. Similarly for the coefficients of $q_2, \ldots, q_5$. As regards q_0, it corresponds to the degenerate frequency $\omega = 0$ corresponding to a constant displacement of all the inertias or to their uniform rotation, and the coefficients of q_0 in Equation [37] are all equal to unity.

To determine A_i, B_i note first that the ratio A_i/B_i is given by Equation [35]; thus only the A's or the B's need be found. The latter are given by

$$B_i = \left[I_p + I_0\left(\frac{\theta_0}{\theta_p}\right)^2 + I_1\left(\frac{\theta_1}{\theta_p}\right)^2 + \ldots\right] \quad [40]$$

where the ratios are of the amplitudes at the ith critical, thus being equal to proper M's of Equations [37]. To prove this, note from Equation [33] that $B_iq_i^2/2$ is the kinetic energy which is possessed by the system when all q's but q_i vanish. Since this happens during the ith critical oscillation, Equation [40] follows readily. Similarly

$$B_0 = I_p + I_0 + \ldots + I_4 \quad [41]$$

9—Application of Normal Coordinates

The applied torque and the propeller damping will first be taken into account. This is done by applying Lagrange's equations of motion in the presence of external forces, that is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i,\ L = E - V \quad [42]$$

Here, as in Equations [30], [31], [32], and [33], E is the kinetic energy and V the potential energy, and Q_i the "generalized" forces

(or torques) whose effect is not included in V, defined by the fact that

$$\Sigma Q_i dq_i \quad [43]$$

is the work done by these forces during the displacement dq_1, dq_2 .... Since the effect of the applied torque and the friction torque on the propeller is not contained in V, they must both be introduced as an external torque Q. Due to an external torque Q on the propeller the work done during a small displacement $d\theta_0$ ..., $d\theta_p$ is $Qd\theta_p$ or, taking Equation [39] into account $Q(dq_0 + dq_1 + \ldots + dq_5)$. Comparing with Equation [43] it follows that $Q_0 = Q_1 = \ldots = Q_5 = Q$.

That is, each "generalized" external force Q_i is equal to the external propeller torque Q. Now the torque acting on the propeller is given by

$$Q = T_a - D_p\dot{\theta}_p = T_a - D_p(\dot{q}_0 + \dot{q}_1 + \ldots + \dot{q}_5) \quad [44]$$

in view of Equation [39].

Substituting in Equation [42], there result the differential equations

$$\left.\begin{aligned} B_0\ddot{q}_0 + A_0q_0 + D_p(\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \ldots) &= T_a \\ B_1\ddot{q}_1 + A_1q_1 + D_p(\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \ldots) &= T_a \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot & \end{aligned}\right\} \quad [45]$$

Now suppose that time enters as $e^{j\omega t}$ as in Equation [25]. Putting

$$D_p = C\omega \quad [46]$$

replacing A_i by $B_i\omega_i^2$ there results[5]

$$\left.\begin{aligned} q_0B_0(\omega_0^2 - \omega^2) + j\omega^2C(q_0 + q_1 + q_2 + \ldots) &= T_a \\ q_1B_1(\omega_1^2 - \omega^2) + j\omega^2C(q_0 + q_1 + q_2 + \ldots) &= T_a \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot & \end{aligned}\right\} \quad [47]$$

If it is desired to take solid friction into account, then it will be recalled from Equation [24] that the stiffness constants K_i had to be replaced by $K_i(1 + j\epsilon)$. This will have the effect of replacing A_i by $A_i(1 + j\epsilon)$ and ω_i^2 by $\omega_i^2(1 + j\epsilon)$, thus modifying Equations [47] to

$$\left.\begin{aligned} q_0B_0[\omega_0^2(1 + j\epsilon) - \omega^2] + j\omega^2C(q_0 + q_1 + q_2 + \ldots) &= T_a \\ q_1B_1[\omega_1^2(1 + j\epsilon) - \omega^2] + j\omega^2C(q_0 + q_1 + q_2 + \ldots) &= T_a \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot & \end{aligned}\right\} \quad [48]$$

By subtracting one equation from another, it will be noted that the ratio of any two q's is uniquely determined. For example

$$\left.\begin{aligned} q_iB_i[\omega_i^2(1 + j\epsilon) - \omega^2] &= q_0B_0[\omega_0^2(1 + j\epsilon) - \omega^2] \\ q_iB_i[\omega_i^2(1 + j\epsilon) - \omega^2] &= q_1B_1[\omega_1^2(1 + j\epsilon) - \omega^2] \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot & \end{aligned}\right\} \quad [49]$$

These ratios are independent of propeller damping and of the applied torque, depending only on the frequency and on the solid friction.

Equations [48] suffice to solve for q_i and since each q carries with it a set of θ's and T's, this solution enables one to determine any displacement and any torque amplitude. The chief advantage of these equations, however, lies in the ease with which they can be applied to calculations at the criticals. Indeed, say at $\omega = \omega_i$, Equations [49] yield

$$\left.\begin{aligned} q_iB_i\omega_i^2(j\epsilon) &= q_0B_0[\omega_0^2 - \omega_i^2 + \omega_0^2(j\epsilon)] \\ q_iB_i\omega_i^2(j\epsilon) &= q_1B_1[\omega_1^2 - \omega_i^2 + \omega_1^2(j\epsilon)] \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot & \end{aligned}\right\} \quad [50]$$

Since ϵ is very small compared with

$$\frac{\omega_0^2}{\omega_i^2} - 1, \quad \frac{\omega_1^2}{\omega_i^2} - 1, \quad \ldots\ldots \left(\text{excluding } \frac{\omega_i^2}{\omega_i^2} - 1\right)$$

it follows that at $\omega = \omega_i$, q_i will be very much larger than the other q's. Therefore, at $\omega = \omega_i$ the other q's may in general be neglected in comparison with q_i and Equations [48] reduce practically to

$$q_ij\omega_i^2[C + B_i\epsilon] = T_a \quad [51]$$

Physically, the last result amounts to a statement that at $\omega = \omega_i$ only the corresponding ith critical mode is excited to any appreciable extent. It will be further noted that q_i is limited by the joint friction C and ϵ, due to propeller damping and solid friction, and that the phase of q_i is 90 deg relative to T_a. The mode q_i is limited by frictional "resistance," whereas the other modes are limited by "dynamic impedance," almost entirely.

In absence of friction the simplified Equations [43] or [47] and [41] yield

$$\theta_p = T_a\left[\frac{1}{B_0(\omega_0^2 - \omega^2)} + \frac{1}{B_1(\omega_1^2 - \omega^2)} + \ldots\right] \quad [52]$$

Note that at $\omega^2 = \omega_i^2$ the term involving $\omega_i^2 - \omega^2$ becomes infinite. Hence if

$$\frac{T_a}{\theta_p} = \frac{1}{X} \quad [53]$$

(where X is the quantity in the bracket in Equation [52]) be plotted against ω^2 it will cross the ω^2 axis at $\omega^2 = \omega_i^2$. It is readily shown that the slope of Equation [53] at that point is B_i. This way of calculating B_i is sometimes more convenient than Equation [40], since the curve of Equation [53] is available from the determination of ω_i.

Note from Equation [52] that the term $1/B_i^2(\omega_i^2 - \omega^2)$ of θ_p/T_a, which is responsible for the infinity at ω^2, if plotted by itself, leads to a hyperbola. If the mean value of θ_p/T_a at two points which are equidistant from the asymptote $\omega^2 = \omega_i^2$ be taken, then the result will be practically equal to the sum of the remaining terms of the bracket in Equation [52]. In this way one may obtain the contribution of the remaining modes to, say, θ_p/T_a, provided frictional terms occurring in these nonresonant terms be neglected in comparison with the dynamic impedance.

Similarly, after the contribution to any torque T_n at the critical ω_i^2 due to q_i has been found by multiplying the known ratio of that torque to θ_p at ω_i^2 by q_i as determined from Equation [51], the contribution to T_n due to the other modes, that is, the nonresonant components, may be determined by taking half the sum of the values of T_n/T_a at two points close to and equidistant from the asymptote $\omega^2 = \omega_i^2$.

These nonresonant components are usually small so that high accuracy is not essential. The omission of damping in their computation is further justified by the fact that the torques thus obtained are too high.

These components are either in phase with the exciting torque or else 180 deg out of phase with it; in other words, they do not contain any j. Therefore, in combining the nonresonant components and the resonant components, the absolute magnitude is determined as the square root of the sum of the squares.

10—Tuned Branches

If both the high-pressure and low-pressure branches are tuned so that they have a common frequency (when oscillating against I_0 held fast), then this frequency is also a natural frequency for the system as a whole. This follows from the fact that the criticals of the system are separated by the branch frequencies. In this oscillation the low-pressure branch can oscillate against the high-pressure branch with a node at I_0 and without the propeller par-

[5] The critical ω_0 actually vanishes and has been retained to give Equations [47] a more uniform appearance.

ticipating at all. When this is the case, an applied propeller torque cannot excite the resonant and usually large mode of oscillation and the critical is not felt at all. When the branches are nearly tuned, the resonant mode is excited only to a slight extent.

To prove this, consider Equation [40]. Since in the K_1, K_3 shafts there are nodes very close to I_0, it follows that the amplitudes θ_1, θ_2, θ_3, and θ_4 are very large compared with θ_0. On the other hand, if the propeller branch frequency is not close to the tuned critical, then if there is a node in it, it is not close to I_0; therefore the θ_p/θ_0 is not very large. Hence θ_1/θ_p, $\theta_2/\theta_p \ldots$ are large and the inertia constant B for this critical will be very large, and as exact tuning is approached B will become infinite. The resulting q will therefore be very small or zero, and only the nonresonant modes of vibration will be excited. Thus the torque curve will possess no peak or but a small one at the critical.

If all three branches are tuned to a common frequency, then this frequency can be considered as the equivalent of two natural frequencies of the system in the sense that two frequencies are trapped between the frequencies of the three branches. The algebraic equation for ω^2 for the whole system now has the critical as a double root.

In this case the low-pressure branch can oscillate against the high-pressure branch without involving the propeller or the propeller can oscillate against one of the branches without involving the other branch. There is always a node at the big gear, but the resonant component *is* excited by a propeller torque. The chief characteristic of this mode of tuning is that one has all the criticals over with in a single running range. On the other hand, if there is a slight mistuning of the two branches, then it often happens that it is more advantageous to mistune the propeller branch even further, as this leads to smaller maxima torques.

Systems with tuned branches are often referred to as nodal systems or *nodal drives*. The term was introduced by J. H. Smith (12) who pointed out the advantages of such drives.

11—Summary

After the equivalent inertia and stiffness constants have been found, the critical frequencies of the whole system are located. This is done by assuming values for ω^2 in Equations [13] to [23], inclusive, and calculating progressively the various θ's and T's say, in terms of θ_2. The curve T_a/θ_2 is plotted and the points where it cuts the axis correspond to the critical speeds.

After the critical frequencies have been determined the damping constants D_p, C, and ϵ are found and the torques at the criticals are proceeded with. This is carried out either by means of the modified Equations [13] to [23], inclusive (see Equations [14a] and [23a]) or preferably by the method of normal coordinates.

For definiteness consider the first critical ω_1, and suppose that it is not too close to ω_2. From the preceding calculations the ratio of the θ's and T's say to θ_p, at ω_1, is known. It is then possible to find B_1 from Equation [40]; B_1 may also be found from the slope of T_a/θ_p at ω_1^2 (see Equations [53], [52], and the following discussion). Using Equation [51], q_1 is now determined. By multiplying T_1/θ_p, $T_2/\theta_p \ldots$. (as determined from Equations [13]-[22]) by this value of q_1, the "resonant components" of the torques, T_1, T_2, .. at ω_1, are obtained.

Unless the high-pressure and low-pressure branches are nearly tuned to ω_1, these resonant components represent the torques sufficiently accurately. However, if the high-pressure and the low-pressure branches are nearly tuned to ω_1, then B_1 will be large, q_1 small, and the resonant component of torque small likewise. Under these conditions a calculation of the nonresonant components is desirable. One may, of course, wish to obtain the value of the nonresonant components even when they are small relative to the resonant values. These nonresonant components, say of the torque T_2, neglecting friction, may be obtained by taking the mean of T_2 as found from Equations [13] to [23], inclusive, at two values of ω^2 close to and equidistant from ω_1^2.

In case the critical in question, say ω_1, is quite close to the next critical ω_2 (this will happen if the lowest frequencies of both turbine branches are close to the propeller branch frequency), then Equation [51] may not be sufficiently accurate and Equations [48] and [49] should be used directly. Now it is proper, however, to neglect all the normal coordinates other than q_1 and q_2, resulting in the solution of only two simultaneous equations for q_1, q_2.

At frequencies removed from the critical ones the term "resonant" is no longer relevant. In general, the effect of friction is negligible at such frequencies, the amplitude being limited chiefly by dynamic impedance, and the values derived from Equations [13] to [23], inclusive, can be used.

While it is felt that at the present state of knowledge of damping in the system the introduction of friction as solid friction is entirely satisfactory, it is to be pointed out that the introduction of velocity friction located at the inertias is possible by multiplying the constants B_i by a proper complex number.

In summarizing, we believe that the application of normal coordinates to this problem is convenient from several points of view. Normal coordinates do not represent a universal panacea for all difficulties encountered in this field, nor do they replace in any way the location of the criticals by the ordinary methods. They can be used, however, to eliminate lengthy calculations involving complex quantities, for instance, those that result when friction terms are introduced directly in the equation of motion. By resolving any quantity into a resonant and nonresonant component, normal coordinates afford a clearer picture of what happens at resonance. Such questions as how nearly the shape of vibration at a critical (including friction) agrees with the shape of free vibration in absence of friction are easily answered by examination of the nonresonant components.

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11 For an interesting application of normal coordinates to engineering vibration systems, see R. Eksergian, *Journal of The Franklin Institute*, vol. 227, May, 1939, pp. 673-690.

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Discussion

Measurement of Impact Strains by a Carbon-Strip Extensometer[1]

R. K. Bernhard.[2] It might be of certain interest to supplement the excellent paper of the authors by describing briefly similar tests which the writer made and published[3] in part in 1928. The purpose of the investigation was the determination of the propagation velocity of longitudinal waves in a complete structure and not in a single bar.

The instruments for measuring these waves were essentially the same. However, instead of a carbon-strip extensometer, a special type of carbon-stack extensometer (pickup units) weighing 0.12 lb each and having a gage length of 1 in. were used. The recording instrument was a standard two-mirror oscillograph with a maximum speed of the light-sensitive paper of 16 fps and equipped with an electric-arc light source. The natural frequency of the pickup unit was 2000 cps of the receiver (galvanometer elements) 20,000 cps, both working with half-aperiodic damping. Hence no distortion of the records with respect to amplitude and phase, up to exciting frequencies of $2000 \div 4 = 500$ cps, was to be expected.

The procedure of the investigation was as follows: The extensometers were fixed on the lower chord of a simple beam railroad truss of 115 ft span, one telemeter being attached to the first chord near the left support and the second telemeter to the last chord close to the right support. The distance between the two pickups was 98 ft. The waves were excited by a locomotive approaching the left support via the adjacent dam and stopping suddenly over the left support by applying the brakes as strongly as possible. Thus, a relatively large impulse could be excited on the superstructure. Both pickups recorded simultaneously on the light-sensitive paper the tension-time diagram excited by this impulse. By comparing the corresponding waves, their phase difference indicated the time of the longitudinal stress wave in propagation from the left to the right support. The result revealed a propagation velocity of 14,500 ft-sec^{-1}. The theoretical velocity in steel is $v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{30 \times 10^6 \times 12^2}{\frac{485}{32.1}}}$ $\sim$ 17,000 ft-sec^{-1}, where E = modulus of elasticity of steel and ρ = density of steel. Hence, the measured value was 17 per cent lower than the theoretical value. A tentative explanation of this time difference might be the retarding effect of parts not contributing to the propagation, i.e., gusset plates, loose rivet connections, etc.

The advantages of the measuring method applied in this manner are:

1 With respect to carbon-stack pickups: No amplification units (electro tubes) for the output of the telemeter are required.

2 With respect to mirror oscillographs: Both records, including the time signal, are obtained on the same light-sensitive paper.

3 Furthermore, no supersensitive film or fast lenses are required, thus simplifying the photographic procedure substantially. Hence, difficulties in timing the release of a limited length of two separate supersensitive films traveling at high speed can be avoided.

Kalman J. DeJuhasz.[4] The investigation of the authors has a considerable value not solely for the information concerning the impact phenomena as such, but beyond that, also for the data covering the entire field of elastic transient phenomena which obey the differential Equation [3] of the paper. Besides elastic rods and springs these also embrace "water hammer" in hydraulic conduits, injection phenomena in Diesel engines, and surges in electric circuits.

The experimental confirmation of the theoretical predictions for impact will strengthen the confidence in applying the theoretical method to cases in which experimental methods are not readily applicable.

At The Pennsylvania State College our interest centers mainly on injection phenomena in Diesel engines. For this research, a graphical method has been developed which is easier to apply and gives a clearer picture than the mathematical analysis. Subsequently this method has been applied also to surges in elastic columns and springs.

For those working in this field, a study of the treatise[5] describing the method and of the several papers given in the accompanying bibliography may be of interest. At this time, only the diagrams of the cases treated by the authors will be given as follows:

1 For two bars of equal length and cross section (Figs. 1, 2, and 3 of the paper) but having flat ends contacting simultaneously over the entire area, the velocity-strain diagram is given in Fig. 1 of this discussion, and the stereograms of velocity and strain, as a function of location and time, in Figs. 2 and 3. Cross sections of these stereograms parallel with the time axis will give the history of velocity, with respect to strain, at any given cross section of the bar. It is readily seen that, after the elapse of $2\,L/a$ interval, the two bars separate, the impacting bar comes to a standstill and the impacted bar will assume the original velocity of the impacting bar.

2 For two bars of equal length and cross section, but with flat ends contacting simultaneously over their entire area, the other end of the impacted bar being fixed (Fig. 5 of the paper), the velocity-strain diagram is given in Fig. 4 of this discussion, and the stereograms of velocity and of strain in Figs. 5 and 6. It shows, in conformity with the experimental results, that the strain of impact doubles on reaching the fixed end, and that the two bars separate after the elapse of $4\,L/a$ interval after the beginning of the impact. The impacting bar will then assume an equal and opposite velocity to that of its original velocity. The negative (tensile) strain shown in Fig. 5 of the paper is not explained by the theory, and on this further explanation is requested.

3 For a stationary, free, long bar impacted by a moving short

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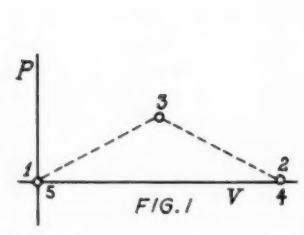


FIG. 1

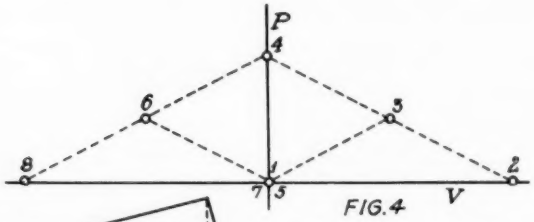


FIG. 4

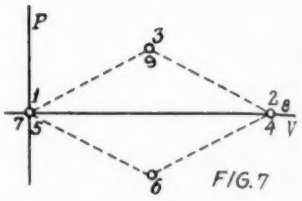


FIG. 7

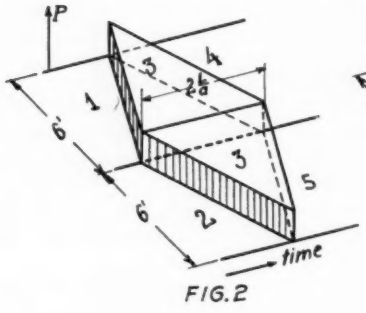


FIG. 2

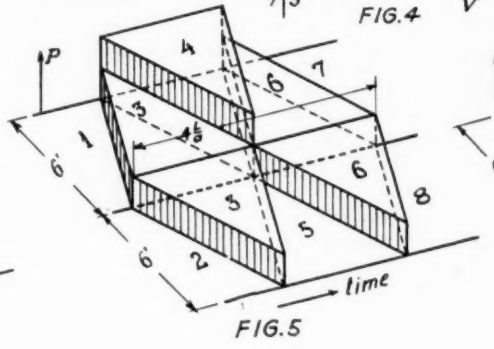


FIG. 5

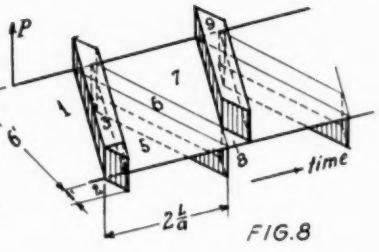


FIG. 8

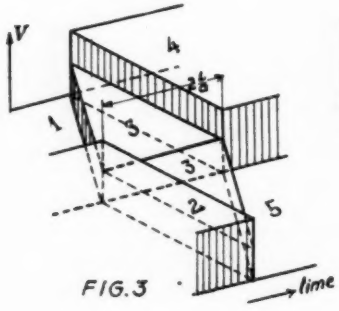


FIG. 3

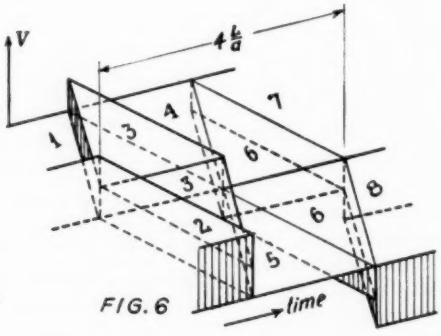


FIG. 6

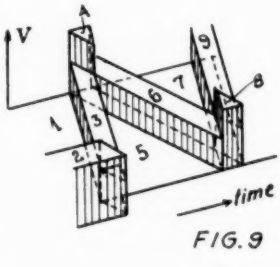


FIG. 9

bar of equal cross section (Fig. 4 of the paper), the velocity-strain diagram is given in Fig. 7 of this discussion, and the stereograms of velocity and strain in Figs. 8 and 9. It is shown that, in the long bar, compressional and tensional strains alternate with strainless periods.

Allowing for the unavoidable contamination always present in experiments as compared with the pure assumptions of theory, the experimental results confirm the theoretical method. It is suggested, however, that making the impacting ends of the bars spherical introduces a seemingly unnecessary complication into the experiments, which would be avoided by using flat ends, taking all possible care that the impact takes place simultaneously over the entire area.

It may be of interest to state that the case of impacting bars of different cross sections is also readily susceptible of graphical treatment. Such cases have been treated by Boussinesq analytically, and subsequently by L. Bergeron[6] graphically.

J. N. Goodier.[7] The development of strain gages suitable for registering the sudden strains of impact raises the question of what is a suitable means of calibration? The authors have naturally chosen what appears to be the simplest type of impact for which analysis is possible. Nevertheless, even this case, the longitudinal impact of rods, presents difficulties. First, the accepted theory, as used by the authors, involves the assumption of a plane wave, i.e., a wave in which, at a given instant, there are no variations over a cross section of the bar. The wave is supposed to travel at constant velocity without change of form, but this is a consequence of the planeness. Then too, Saint Venant's principle is introduced to justify the transition from the local condition at the rounded end of the bar to the plane wave further on.

As to the first point, wave propagation in rods can be investigated without restriction to plane waves.[8] It is found that only sinusoidal waves, with a wave length long compared with the diameter, are propagated as plane waves with velocity $\sqrt{E/\rho}$. The shorter sinusoidal waves involve variation over the cross section and have velocities varying with the wave length. A group of these waves, if generated, would constitute a nonplane wave which changes in form as it is propagated. Such effects would be pronounced in a wave with a steep front. Possibly, however, the wave forms in the authors tests were not steep on the scale of the diameter.

As to the second point, the question is whether there is any evidence to support the extension of Saint Venant's principle from statical to dynamical stress systems. In view of the properties of short sinusoidal waves, it appears likely that the steepness of the wave front, or the suddenness of the impact, will have some importance. In some photographs of impact waves in a bakelite bar, taken by T. R. Cuykendall at Cornell University, plane waves appeared to be the exception rather than the rule.

Authors' Closure

As Professor De Juhasz suggests, the phenomena would be simpler if it were possible to use flat-ended bars which come into

[6] "Methode Graphique de Resolutions des Problems de Propagation d'ondes Planes," by L. Bergeron, Fifth International Congress for Applied Mechanics, Cambridge, Mass., 1938, pp. 694–699.

[7] Professor of Mechanics, Department of Mechanics of Engineering, Cornell University, Ithaca, N. Y. Mem. A.S.M.E.

[8] "Mathematical Theory of Elasticity," by A. E. H. Love, 4th edition, Cambridge University Press, London, England, 1927, p. 289.

contact simultaneously all over their ends. However, the early investigators of impact, notably Saint Venant, attempted to secure this condition, without success and, as is pointed out by Sears in his paper, there are difficulties both practical and theoretical in the way of attainment of this idealized condition. In tests on tension impact we have observed a rise of strain from zero to a maximum within about 20 microseconds, indicating that in this case contact was established fairly rapidly over the impacting surfaces.

As Professor Goodier remarks, the question of calibration under dynamic conditions is a real problem for strain gages. We have recently changed at M.I.T. from carbon strips to gages employing fine wires as the strain-sensitive resistance element, and with these new gages feel confident on the bases of tests made to the present time that the response of these gages is not dependent on the rate of straining, thus allowing the use of the static calibration of the gage, and an electrical calibration of the rest of the equipment. This question is now being actively studied.

With regard to the question of change in wave form as the wave propagates, we may remark that for the only moderately steep wave fronts considered in this paper (several feet in length) any such effect is unimportant even in repeated passages through a 6-ft bar. However, in other experiments in which the wave front was only a few inches long, some evidence of such distortion has been observed, though it is still hardly appreciable in a single passage through a 6-ft bar of this diameter, $1^1/_{16}$ in.

Book Reviews

Mathematical Methods in Engineering

MATHEMATICAL METHODS IN ENGINEERING. By Th. von Kármán and Maurice A. Biot. First edition. McGraw-Hill Book Company, Inc., New York, N. Y., 1940. Cloth, 6 × 9 in., xii and 505 pp., $4.

REVIEWED BY ALBERT E. HEINS[1]

IN recent years there have been many attempts on the part of the pure mathematician to write a textbook on applied mathematics which would fit the requirements of the engineer and physicist. These texts have usually been so written that certain mathematical topics beyond the elements of the calculus are discussed and only superficial applications are indicated. On the whole this procedure has been quite unsatisfactory since there are very few pure mathematicians who appreciate the mathematical requirements of the applied scientist.

The textbook by von Kármán and Biot is unique from several points of view. Both authors are applied mathematicians in their own right. Their work in mechanics, pure and applied, and in similarly related topics has given them a much deeper insight into the mathematical requirements of the engineer. Furthermore instead of forcing mathematical methods to fit physical applications, they build their discussion about one central physical topic, mechanics, and then discuss the various mathematical methods used in this and similarly related fields.

The material which the authors have chosen to discuss forms an excellent intermediate text on mathematics for engineers or physicists. On the mathematical side, we find a discussion of linear and quadratic forms, matrices, ordinary differential equations, certain simple partial-differential equations, boundary value problems, harmonic analysis, transient phenomena, operational calculus, and equations of finite differences. Due attention is given to approximate and graphical methods. These mathematical methods are in turn applied to many interesting problems in dynamics, structures, and related engineering problems. The text is essentially a discussion of mechanics of rigid bodies and continuous media, and some electric-circuit theory. The problems, with the exception of a few mathematical exercises, are drawn mainly from mechanics and to a lesser extent from the electric-circuit theory.

The discussion of mechanics should make a mark for itself with the engineers and the physicists in the same way that the classical text of Paul Appel "Traité de Mécanique Rationelle" has made with the applied mathematicians for the last four decades. The present authors, however, have taken pains to add a more practical flavor and modern dress to their discussion of mechanics than is to be found in Appel's treatise. The authors are to be commended for bringing so many important mathematical methods to the attention of the applied scientist. The book is clearly written and careful attention is given to definitions of mathematical and physical character. The problems are well chosen and graded. Here the student will occasionally meet his Waterloo. Von Kármán and Biot have written a textbook which is a contribution to mathematics as well as to the applied sciences.

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Thermodynamics

SPEZIFISCHE WÄRME, ENTHALPIE, ENTROPIE UND DISSOZIATION TECHNISCHER GASE. By E. Justi. Julius Springer, Berlin, 1938. Paper, $10^1/_4$ × 7 in., vi and 157 pp., 43 figures and 116 tables, 19.80 rm.

REVIEWED BY JOHN A. GOFF[2]

FOR many years since the suggestion as to the quantum nature of radiant energy was put forward as a hypothesis by Planck in 1900, the development of the quantum theory has been regarded as the plaything of the physicist. Little of real practical value was expected from this development even by the physicist himself and the attitude of the engineer has been one of indifference and aloofness. But along with the development of the quantum theory have come notable improvements and refinements in the mathematical apparatus of statistical mechanics. The result has been that now the physicist is in a position to predict, in many cases with greater accuracy than that with which the engineer can measure by calorimetric methods, the properties of substances entering into the processes with which the engineer is directly concerned, and to answer questions whose solution by empirical methods would require much time and elaborate experimental apparatus. As one of the first attempts to explain to the engineer the nature of the physicist's predictions, to collect and comprehensively review for him the data already available in the literature, and to suggest what further contributions may be expected in the future, the book under review is indeed timely and valuable.

Of first importance, perhaps, are the predictions of quantum-theory and statistical mechanics regarding the thermodynamic

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properties of gases at low pressures or, more precisely, at zero pressure. The principal source of the numerical values obtained is the analysis of band spectra. According to the quantum theory, the frequency ν of each observed spectral line corresponds to a transition from one energy level to another such that $\Delta\epsilon = h\nu$, where h denotes Planck's constant. When the various modes of internal motion of which the molecules of a gas are capable have been enumerated and analyzed, it is usually possible to "integrate" the spectrum in such a way as to obtain for each individual mode a series of energy levels ϵ_i which, beginning with an undeterminable "ground-level" value ϵ_0, increase by integral multiples of the energy quantum appropriate to that particular mode. The author briefly describes the various methods of determining these energy levels from spectroscopic measurements.

Having determined the whole series of energy levels for each mode of internal motion, the problem is to estimate the fraction of all molecules occupying each level at any given temperature. Statistical mechanics solves the problem by showing how to construct for each mode a "partition function" Z such that the contribution to the free energy, $F = U - TS$, of an aggregate of N molecules in statistical equilibrium within a volume V is

$$\Delta F = -kNT \ln Z$$

where k denotes Boltzmann's constant. The form of the partition function, except in the case of translational motion, is

$$Z = e^{-\epsilon_0/kT} \sum_{i = 0,1,2 \ldots} e^{-\frac{\epsilon_i - \epsilon_0}{kT}}$$

Applying the identical relations of thermodynamics in order to determine the corresponding contributions to the energy U and entropy S and examining the results, one obtains deductions of practical as well as theoretical importance.

In the first place it appears in accordance with the reasoning of classical thermodynamics that the contribution to the energy can only be evaluated to within an additive constant quantity $N\epsilon_0$ which is the value of the energy contribution at absolute zero ($T = 0$). In the second place, however, the corresponding entropy contribution is entirely independent of the value assigned to $N\epsilon_0$ and indeed its value at absolute zero is absolutely zero.

The last deduction evidently contains the essence of the so-called Third Law of thermodynamics, but in attempting to correlate it with direct experimental data complications often arise. Thus, there may be energy separations satisfying the inequality $\epsilon_i - \epsilon_0 \ll kT$ at the lowest experimentally accessible temperatures and, if so, the contribution to the entropy will still have a finite value at these temperatures. Such "degeneracy" or "multiplicity of the ground level" actually occurs in the rotational levels (except in the case of hydrogen), also in the vibrational, electronic, and nuclear-spin levels. And there are other ways in which observations at accessible temperatures on actual gases must be corrected. To mention only one, even at the lowest accessible temperatures, ordinary hydrogen remains a mixture of two allotropic modifications though if held at still lower temperatures for a sufficiently long time or properly catalyzed the modification with nuclear spins symmetrical would presumably disappear and the entropy contribution would tend to vanish in accordance with the Third Law. The author mentions the more important of these corrections, but his discussion leaves much to be desired.

The final step leading to absolute values of entropy for a gas is the analysis of the contribution from translatory motion. In this case also statistical mechanics predicts "degeneracy" so that even though the entropy contribution remains finite down to extremely low temperatures, possibly of the order of 0.001 deg K, nevertheless at still lower temperatures it tends to vanish in accordance with the Third Law. At temperatures above this range of degeneracy the entropy contribution from translational motion is correctly given by the Sackur-Tetrode equation. It is the only contribution which depends upon the volume.

In the second part of chapter 3 the author assembles and discusses the available data on something like forty different gases. Wherever comparison with direct calorimetric determinations of specific heat is of value or special interest, such comparison is made. The Third Law as first formulated by Nernst and later reformulated by Lewis and Randall was intended to apply only to the crystalline solid state. Again statistical mechanics has shown the necessity of applying corrections in many cases for the existence of energy separations $\epsilon_i - \epsilon_0 \ll kT$ at the lowest experimentally accessible temperatures, but when such corrections have been estimated and applied the entropy of the solid at absolute zero may be taken to be absolutely zero as in the case of gases. This provides the basis of a most interesting comparison between the value of the entropy for a gas obtained from analysis of its band spectrum, and that obtained by extrapolating low-temperature specific-heat data on the solid phase to absolute zero for a starting point, adding entropies calculated from observed specific heats and latent heats through vaporization, and finally adding a correction for the effect of intermolecular forces which may be appreciable if the resulting vapor pressure is not sufficiently low. In all cases where the requisite calorimetric data are available, the author makes this comparison and thereby increases confidence in the accuracy of the predictions of statistical mechanics.

Yet another comparison is obtained from vapor-pressure measurements which the author discusses though not with too much clarity. Unfortunately, also, in the tables many errors have been carried over from the original sources and a number have been added so that the reader must exercise considerable discretion in the use of the data listed. Thus the values listed for monatomic hydrogen in Table 86 are not "Reduced Free Enthalpy" as stated, for they omit the contribution from electron spin and part of the Sackur-Tetrode constant. Again the data in Table 51b for CO are all in error in view of subsequent recalculations.

The collection and discussion of the data from spectroscopic measurements is doubtless the most valuable part of the book, though scarcely less valuable is the matter of chapter 2. Here one finds an excellent discussion of the empirical equations of state of van der Waals and of Berthelot, particular attention being given to the latter by way of comparison with direct experimental data. There is also a discussion of the semiempirical equations of Callendar, Wohl, and Beattie-Bridgeman. The evident purpose of these discussions is to provide means of correcting the spectroscopic data, which are strictly applicable at zero pressure only, to allow for the effect of intermolecular force which enters at moderate to high pressures.

Finally, the author makes a real contribution to the thermodynamics of gas mixtures by clearly stating the problem and then briefly summarizing what has been accomplished to date in the way of verifying the various proposed combination rules for evaluating the so-called interaction constants.

The book is well documented and, as stated, presents a timely and valuable treatment of a subject which is destined to find increasing application in engineering.

Errata

In Table 1 on page A-39 of the March, 1940, issue of the Journal of Applied Mechanics the figures for specific damping capacity should in every case be twice the corresponding figures for logarithmic decrement, instead of one half, so that the correct figures for damping capacity are four times those which appear in this column.

The Impact of a Mass Striking a Beam

By E. H. LEE,[1] STANFORD UNIVERSITY, CALIF.

The integral equation for the horizontal impact of a mass on a beam was developed by Timoshenko in 1912. He showed that any such impact problem can be solved provided sufficient patience and tenacity are applied in using the tedious numerical method of solution by dividing the time into small increments during which the contact force can be considered constant. This method is too lengthy to be applied in practice, and moreover, a solution is required giving characteristics of the impact as a whole, rather than slowly progressing through its duration with no indication of the future development of the impact.

In the first part of this paper a concise approximate solution is developed, making the assumptions that the duration of contact is small in comparison with the period of the fundamental mode of vibration of the beam, and that only the fundamental mode of oscillation of the beam need be considered. Results obtained by this method are checked against those determined by the lengthier numerical method. Good agreement is reached in some examples, but other results show large errors, both in the contact compression force and in the resulting motion.

Light was brought to bear on this difficulty by using a solution based on energy and momentum considerations, giving a curve which supplies a rapid determination of the distribution of energy among the various modes of vibration of the beam. This curve gives directly the condition for the vibrational energy developed in the beam, due to the impact, to be confined mainly to the fundamental mode. This is shown by examples to be the required condition for the validity of the previous approximate method.

The solution using energy and momentum considerations is developed to give the motions of the beam and mass resulting from the impact (free vibrations in the beam and uniform velocity of the mass) so that the subsequent development of the impact can be ascertained and repeated impacts dealt with. The investigation also gives the deflection curves of the beam produced by the impact; these show a wide deviation from static-deflection curves.

NUMEROUS attempts have been made to determine the stresses and deflections produced when a mass strikes a beam. The most popular approach has been from energy considerations, using the transformation of the kinetic energy of the striking mass into potential energy of bending in the beam, with some estimate of energy loss due to impact. Saint Venant (1)[2] carried out a more thorough investigation in which he assumed that, immediately on contact, the mass became part of the beam and moved with it. He considered the general problem of the modes of vibration of the beam and attached mass, from which he obtained the special solution satisfying the initial condition of impact, the mass moving with velocity v_0 and the beam at rest.

A completely satisfactory equation of motion was evolved by Timoshenko (2) using only justifiable assumptions. This equation includes the effect of local elastic deformation at the point of contact using the relationship $\alpha = kF^{2/3}$ developed by Hertz (3), where α is the relative approach of the two bodies in contact, F is the compression force between them, and k is a constant depending on the radii of curvature at contact and on the elastic properties of the bodies. This relationship is strictly true for static loading only. However, since the time for stress waves to traverse the region of contact is extremely small compared with the duration of contact in an impulse, sensibly equilibrium conditions obtain in this region and the Hertz relation holds good. A further simplification is that the vibration of the striking mass can be neglected. Rayleigh (4) showed that very little vibration is induced in an oscillating system under the influence of forces of duration long in comparison with its natural periods. This condition is obeyed by the striking mass since the natural frequencies of a solid body are extremely high, thus eliminating the necessity of considering its vibration.

The theory of horizontal[3] impact on beams can be developed in a general manner for impact at any point along the length of a beam and for various conditions of support. To simplify the theory, the problem of central impact on a uniform beam simply supported at its ends, as shown in Fig. 1, is considered in the body of this paper, the general case being dealt with briefly in the appendix.

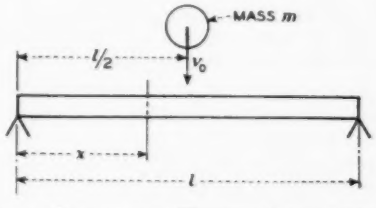


FIG. 1 CENTRAL IMPACT ON A UNIFORM BEAM SIMPLY SUPPORTED AT ITS ENDS

In order to simplify subsequent elaborations, the fundamental integral equation is developed. It represents mathematically the equality between the approach of the mass and the beam in terms of their relative motion, and the approach as deduced from Hertz's law, $\alpha = kF^{2/3}$.

Measuring displacements from the positions of the mass and the beam when contact commences, s denotes the displacement of the striking mass and η that of the center of the beam. Then the relative approach α, as shown in Fig. 2, is

$$\alpha = s - \eta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [1]$$

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[1] Commonwealth Fund Fellow from Cambridge University working at Stanford University.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

This paper is an abstract of a thesis having the same title and submitted to Stanford University, June, 1940, in partial fulfillment for a Ph.D. degree.

Presented at the National Meeting of the Applied Mechanics Division, Ann Arbor, Mich., June 20–21, 1940, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1941, for publication at a later date. Discussion received after closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Horizontal impact is discussed to eliminate gravity considerations. Actually, the impact duration is so brief that gravity would have no appreciable effect on the motion during impact so that the theory developed can also be applied to problems of vertical impact.

The term $F(\tau)$ represents the contact compression force as a function of the time, in terms of which the displacements of the mass and the beam can be expressed. Denoting the mass of the striking body by m, the diminution of its velocity during the interval $d\tau$ is $[F(\tau)/m]d\tau$, which contributes a decrease in the displacement at the time t equal to $[F(\tau)/m]d\tau(t - \tau)$. Thus the displacement at the time t is given by

$$s = v_0 t - \frac{1}{m}\int_0^t F(\tau)(t-\tau)d\tau \quad [2]$$

where v_0 is the initial velocity of the striking body.

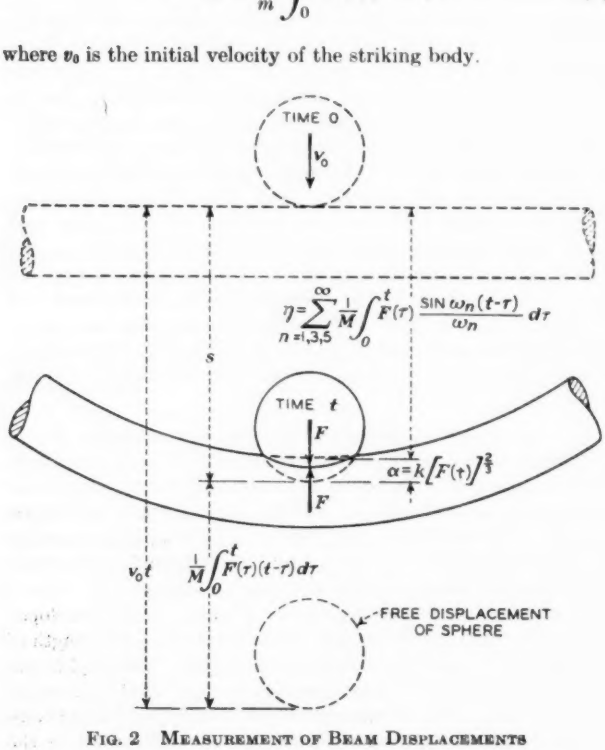


FIG. 2 MEASUREMENT OF BEAM DISPLACEMENTS

The forced vibrations produced in the beam by the varying force $F(\tau)$ at the point of contact are expressed in terms of the normal modes of vibration. In the case of symmetrical vibration of a simply supported beam, the normalized characteristic deflection functions[4] are $\sqrt{(2/l)}\sin(n\pi x/l)$ for $n = 1, 3, 5$, etc., where x is the coordinate of position along the beam of length l. The corresponding angular frequencies of the natural modes of vibration are ω_n, related by the equation $\omega_n = n^2\omega_1$.

The central deflection η due to the forced vibrations is given by (5)

$$\eta = \sum_{n=1,3,5}^{\infty} \frac{2}{ql}\int_0^t F(\tau)\frac{\sin \omega_n(t-\tau)}{\omega_n}d\tau \quad [3]$$

where q is the constant mass per unit length of the beam. $ql/2$, denoted by M, is called the reduced mass and in this particular case is equal to half the mass of the beam.

Substituting Equations [2] and [3] in Equation [1] gives the integral equation for the contact force variation from which the motion can be determined, that is

$$k[F(t)]^{\frac{2}{3}} = v_0 t - \frac{1}{m}\int_0^t F(\tau)(t-\tau)d\tau - \sum_{n=1,3,5}^{\infty}\frac{1}{M}\int_0^t F(\tau)\frac{\sin \omega_n(t-\tau)}{\omega_n}d\tau \quad [4]$$

Due to the nonlinear term, $kF^{\frac{2}{3}}$, direct solution is possible only by a tedious numerical method. The duration of the impact is subdivided into small increments of time during which F can be considered constant for the purpose of integration. Knowing that F is always positive and initially zero, the solution can be developed using a step-by-step process in which the value of F at the end of an interval is obtained in terms of the integrals on the right-hand side of Equation [4] determined by numerical methods.

Although this method is very lengthy, it does supply a possible solution of any impact problem, sufficiently small time intervals providing any desired degree of accuracy. Characteristics of various types of impact are shown in the solution of different problems, but this method does not indicate the general factors leading to these special characteristics. For example, it is found that in certain circumstances the complete impact consists of a succession of separate rapid impacts; the whole occurs in a very short time and appears as one contact to an observer. This process can be easily visualized in the case of a large mass striking a beam. At the first impact, the relatively light beam bounces off the oncoming mass, only to have its motion reversed by its own elasticity and thus encounter repeated impacts, until the velocity of the mass is reversed and it leaves the region of vibration of the beam. That such repeated impacts occur has been shown experimentally by Mason (6), Arnold (7), and Tuzi and Nisida (8). This type of impact is indicated in the step-by-step solution of certain problems; but it would be quite impossible to solve a sufficient number of cases with different masses, natural frequencies, and contact surfaces, in order to arrive at general conditions for its occurrence. In order to study such characteristics, recourse must be made to solutions in terms of functions of time, even if they can only be obtained under certain restrictions.

APPROXIMATE SOLUTION ASSUMING SHORT IMPACT DURATION

J. Lennertz (9) developed a concise method of solution by considering each impact as a whole rather than as a succession of steps, but the method is restricted by additional assumptions. He assumed that the duration of contact is small compared with the period of the fundamental mode of vibration of the beam and also that the fundamental mode only is stimulated into activity. The first assumption is justified in most impact problems and, moreover, is easily checked, but the decision whether or not it is permissible to neglect the effect of the higher modes of vibration of the beam is more difficult. The latter assumption is equivalent to replacing the beam by the one-

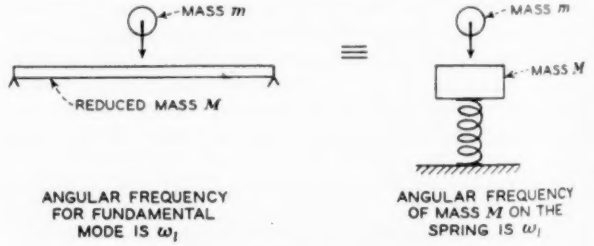


FIG. 3 ASSUMPTION FOR THE APPROXIMATE SOLUTION

[4] The characteristic function $u(x)$ of a mode of vibration is normalized by multiplying it by a constant so that $\int_0^l [u(x)]^2 dx = 1$. In vibration theory, normalized characteristic functions are used to simplify the algebra.

degree-of-freedom system of a mass on a spring as shown in Fig. 3.

The method developed in this paper is an improvement and further development of the one used by Lennertz, since his solution did not utilize the best approximations, and in addition contained some errors. His basic assumptions are retained.

The method of solution is to assume a sinusoidal contact-force variation $F = K \sin \lambda\tau$ which enables the integrals in Equation [4] to be evaluated, thus transforming it into an algebraic equation. Values of K and λ are required which satisfy this equation for all values of the time t during the impact. Having made some simplifying approximations, two simultaneous equations for K and λ are obtained by equating coefficients of $\sin \lambda t$ and t, so that the equation is satisfied for all t. The approximate value of F so obtained can be improved by a method of successive approximation.

Since $F = K \sin \lambda\tau$ cannot become negative, contact occurs for $0 < \lambda\tau < \pi$, so that the duration of contact is $T_D = \pi/\lambda$ sec.

Substituting the sinusoidal variation for F and evaluating the integrals, Equation [4] becomes

$$k(K \sin \lambda t)^{3/2} = v_0 t - \frac{K}{m}\left(\frac{t}{\lambda} - \frac{1}{\lambda^2}\sin \lambda t\right) - \frac{K}{M}\frac{\lambda \sin \omega_1 t - \omega_1 \sin \lambda t}{\omega_1(\lambda^2 - \omega_1^2)} \quad [5]$$

bearing in mind the initial assumption that only the fundamental mode of vibration is to be considered.

Equation [5] is still too complicated for direct solution. Some simplification can be made by writing $\omega_1 t$ for $\sin \omega_1 t$, since $\omega_1 t$ remains small. This results from the assumption that the impact duration π/λ is short compared with the period $2\pi/\omega_1$ of the fundamental mode. In addition the $\sin^{3/2} \lambda t$ in $k(K \sin \lambda t)^{3/2}$ can be approximated by $A \sin \lambda t$; where A is chosen so that the new contact-force variation, which makes $\alpha = kK^{3/2}A \sin \lambda t$, produces the same total change of momentum in the striking mass as did $K \sin \lambda t$. This condition expressed mathematically becomes $\int_0^\pi \sin \theta d\theta = A \int_0^\pi \sin^{3/2} \theta d\theta$ which gives $A = 1.093$.

Inserting these simplifications, Equation [5] becomes

$$kK^{3/2}A \sin \lambda t = v_0 t - \frac{K}{m}\left(\frac{t}{\lambda} - \frac{1}{\lambda^2}\sin \lambda t\right) - \frac{K}{M}\frac{1}{\lambda^2 - \omega_1^2}(\lambda t - \sin \lambda t) \quad [6]$$

Equating the coefficients of $\sin \lambda t$ and t on each side gives two simultaneous equations for the determination of K and λ, that is

$$\left.\begin{aligned} 0 &= v_0 - \frac{K}{m\lambda} - \frac{K}{M}\frac{\lambda}{\lambda^2 - \omega_1^2} \\ kK^{3/2}A &= \frac{K}{m\lambda^2} + \frac{K}{M}\frac{1}{\lambda^2 - \omega_1^2} \end{aligned}\right\} \quad [7]$$

Eliminating K

$$\lambda^5 = v_0 \frac{[(m + M)\lambda^2 - M\omega_1^2]^2}{A^3k^3m^2M^2(\lambda^2 - \omega_1^2)^2} \quad [8]$$

A good approximation for λ can be obtained from this equation by first obtaining λ_1 when ω_1^2 is neglected in comparison with λ^2

$$\lambda_1^5 = v_0 \frac{(m + M)^2}{A^3k^3m^2M^2} \quad [9]$$

Now λ_1 is substituted in Equation [8] to obtain λ_2 which will be a good approximation to the correct value of λ.

Then K is determined from the first of the simultaneous Equations [7] which transforms into

$$K = v_0 \frac{mM\lambda(\lambda^2 - \omega_1^2)}{(m + M)\lambda^2 - M\omega_1^2} \quad [10]$$

The value thus obtained for contact pressure $K \sin \lambda\tau$ can be improved by using a method of successive approximation derived directly from Equation [4]. If $F_\mu(\tau)$ is an approximate solution of the integral equation, then a better[5] approximation $F_{\mu+1}(\tau)$ can be obtained from the equation

$$k[F_{\mu+1}(t)]^{3/2} = v_0 t - \frac{1}{m}\int_0^t F_\mu(\tau)(t - \tau)d\tau - \sum_{n=1,3,5}^{\infty}\frac{1}{M}\int_0^t F_\mu(\tau)\frac{\sin \omega_n(t - \tau)}{\omega_n}d\tau \quad [11]$$

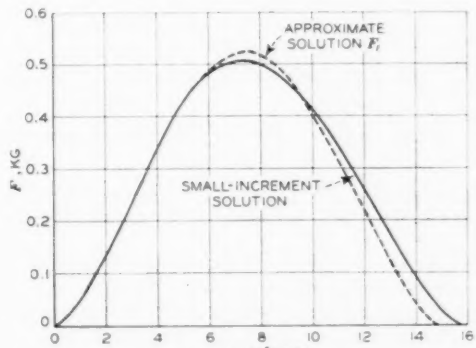


FIG. 4 COMPARISON OF CONTACT-FORCE VARIATIONS USING TIMOSHENKO'S SMALL-INCREMENT METHOD AND THE AUTHOR'S APPROXIMATE SOLUTION

In adopting this process of successive approximation, the first step is to find F_1 when $F_0 = K \sin \lambda t$. In the theory developed in the foregoing, the right-hand side of Equation [11] for $F_0(\tau) = K \sin \lambda\tau$, was equated to $kK^{3/2}A \sin \lambda t$, as shown by Equation [6]. Thus Equation [11] reduces to

$$kF_1^{3/2}(t) = kK^{3/2}A \sin \lambda t$$

or

$$F_1(t) = KA^{2/3} \sin^{3/2} \lambda t \quad [12]$$

But $F_2(\tau)$, a better approximation, can be obtained by expressing $\sin^{3/2} \lambda t$ in a Fourier series for evaluation of the integrals on the right-hand side of Equation [11] when $\mu = 1$. In most cases the assumptions on which the method is based do not justify the expenditure of the time required to calculate $F_2(\tau)$.

Results, obtained by applying this approximate method to the problem solved by Timoshenko (2) using the lengthy small-increment procedure, agree well as shown in Fig. 4. In this problem $m = 32.24$ grams, $M = 61.1$ grams, $v_0 = 1$ cm per sec, and $\omega_1 = 2\pi\ 10^3\ \text{sec}^{-1}$.

Fig. 5 shows the contact-force variation obtained in three different ways for the case of a 50-lb weight striking a beam of total weight 298.5 lb. The angular frequency of the fundamental mode of vibration is 531 sec^{-1} and the initial velocity of the

[5] For a nonlinear equation the convergence of this process is somewhat indefinite, but it appears to be convergent if $F_0(\tau)$, the first approximation, is reasonably close to the correct force variation. For a linear equation this procedure is convergent whatever expression is chosen for $F_0(\tau)$.

striking mass is 11 fps. This problem was solved by the small-increment method taking 360 intervals during the period of the fundamental mode. The solution was obtained in this manner both for the actual conditions when all the modes of vibration can be excited, and for the related problem dealing with a system of one degree of freedom corresponding to the fundamental mode of the beam. Fig. 5 shows that the approximate method of solution developed in this section gives a force variation $F_1(\tau)$ lying

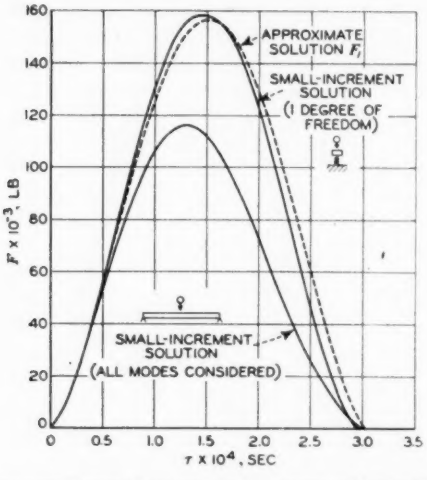


FIG. 5 CONTACT-FORCE VARIATIONS OBTAINED IN THREE DIFFERENT WAYS FOR THE CASE OF A 50-LB WEIGHT STRIKING A 298.5-LB BEAM

very close to that obtained using the small-increment solution for the one-degree-of-freedom system. These curves, however, show much larger contact forces than are developed during impact on a beam when all the modes of vibration can be stimulated. This indicates that for the particular problem considered, the error in the solution lies not in the mathematical approximations adopted but in the physical assumption that only the fundamental mode of the beam affects the impact process.

From this example it is clear that if this approximate method of solution is to prove useful, some criterion must be found which will give conditions justifying the assumption of the equivalence of a one-degree-of-freedom system. Such a criterion is evolved from the energy considerations of the following section.

SOLUTION BY ENERGY AND MOMENTUM CONSIDERATIONS

An important contribution to the understanding of impact characteristics was made by Zener and Feshbach (10) in their considerations of energy transfer. By combining the principle of conservation of energy[6] during the impact, and momentum considerations for the striking mass, the resulting velocity of the mass was determined and also the vibrational energy developed in the beam. Solution of the nonlinear integral Equation [4] was replaced by assuming a function for the time variation of the contact force normalized in such a way that the final motion was insensitive with respect to the choice of this function.

In the present paper this manner of handling the problem is further developed to probe more deeply into the impact process, leading to a simple determination of the energy spectrum of the beam (i.e., the distribution of energy between the normal modes of vibration) and also giving the resulting motion of the beam and striking mass. The former consideration has an important bearing on the validity of the previous approximate solution, leading to conclusions which are confirmed by the few specific impact problems which have been solved by the reliable but lengthy small-increment method.

The procedure in the application of the principle of conservation of energy is to assume a contact-force variation in terms of which the energy imparted to the beam can be represented. An equation is obtained by writing this energy expression equal to the kinetic energy lost by the mass. Using momentum considerations for the striking mass, the force variation is transformed into a normalized function which largely eliminates errors due to the arbitrary choice of the force function. The energy equation then gives directly the rebound velocity of the mass and the energy absorbed by the beam.

Any symmetrical vibration of a simply supported beam can be expressed (11) as a series of characteristic functions in the form

$$u(x,t) = \sum_{n=1,3,5}^{\infty} C_n(t) \sqrt{\frac{2}{l}} \sin \frac{n\pi x}{l} \quad \ldots\ldots [13]$$

where $u(x,t)$ represents the displacement of the section at the distance x from the end of the beam, and at the time t; $C_n(t)$ is the generalized coordinate of the nth mode of vibration.

If a force variation $F(t)$ is applied at the center of the beam, normal to its length, the generalized force stimulating the nth mode is given by $F(t)\sqrt{(2/l)}\sin n\pi/2$ which reduces to $\sqrt{(2/l)}(-1)^{(n-1)/2}F(t)$, and the corresponding differential equation determining the motion is

$$\frac{d^2C_n}{dt^2} + \omega_n^2 C_n = \frac{1}{q}\sqrt{\frac{2}{l}}(-1)^{\frac{n-1}{2}} F(t) \quad \ldots\ldots [14]$$

where q is the mass per unit length of the beam.

Assuming the beam to be initially at rest at the time t_0, when the force commenced to act, the solution of this equation giving the vibration produced in the nth mode is

$$C_n(t) = \sqrt{\frac{2}{l}} \frac{(-1)^{\frac{n-1}{2}}}{\omega_n q} \int_{t_0}^{t} \sin \omega_n(t-\tau) F(\tau) d\tau \quad \ldots [15]$$

The development of this solution is similar to the development of Equation [3].

The vibrational energy absorbed by each mode is composed of the sum of kinetic energy and strain energy of bending at a particular time. The kinetic energy for the nth mode is equal to $(q/2)(dC_n/dt)^2$, the strain energy equals $(q/2)\omega_n^2 C_n^2$ making the total energy, ΔE_n, equal to

$$\Delta E_n = \frac{q}{2}\left[\left(\frac{dC_n}{dt}\right)^2 + \omega_n^2 C_n^2\right] \quad \ldots\ldots [16]$$

Substituting the value of C_n from Equation [15], the energy absorbed by each mode during impact is

$$\Delta E_n = \frac{1}{ql}\left|\int_{t_0}^{t_1} F(\tau) e^{i\omega_n \tau}\, d\tau\right|^2 \quad \ldots\ldots [17]$$

where t_1 is the time when contact ceases.

Errors resulting from ignorance of the contact-force variation $F(\tau)$ can be largely eliminated by expressing $F(\tau)$ in the form of a normalized force. To this end e, a restitution coefficient, is defined by expressing the rebound velocity of the striking mass in the form ev_0, which is positive when its direction is opposite to that of the initial velocity v_0. The total change in the momentum of the mass produced by the contact force is then

$$mv_0(1+e) = \int_{t_0}^{t_1} F(\tau) d\tau \quad \ldots\ldots [18]$$

[6] Throughout this paper elastic impact is considered, assuming that the stress in the contact region does not exceed yield-point stress. Although energy is conserved, the usual concept of loss of kinetic energy by the striking mass holds good, the "lost" energy going into vibration of the beam.

Now, defining the normalized contact force $\mathfrak{F}(\tau)$ by the relation $F(\tau) = mv_0(1 + e)\mathfrak{F}(\tau)$, the momentum Equation [18] becomes

$$\int_{t_0}^{t_1} \mathfrak{F}(\tau)d\tau = 1 \quad \ldots\ldots [19]$$

The advantage of this transformation is that only the shape of the $\mathfrak{F}(\tau)$ curve is arbitrary, the scale being restricted by the condition $\int_{t_0}^{t_1} \mathfrak{F}(\tau)d\tau = 1$, thus minimizing errors in assuming an expression for the contact-force variation. Substituting for $F(\tau)$, Equation [17] becomes

$$\Delta E_n = \frac{m^2v_0^2(1 + e)^2}{ql}\left|\int_{t_0}^{t_1} \mathfrak{F}(\tau)e^{i\omega_n\tau}d\tau\right|^2 \quad \ldots\ldots [20]$$

The total vibrational energy of the beam is equal to the sum of the energies in the separate modes of vibration, i.e., the total vibration energy $\Delta E = \sum_{n=1,3,5}^{\infty} \Delta E_n$.

In order to simplify the development, the function R is introduced, and defined by the equation

$$R = \frac{m}{M}\sum_{n=1,3,5,}^{\infty}\left|\int_{t_0}^{t_1} \mathfrak{F}(\tau)e^{i\omega_n\tau}d\tau\right|^2 \quad \ldots\ldots [21]$$

where $M = ql/2$ is the reduced mass of the beam. The total vibrational energy ΔE now reduces to

$$\Delta E = \sum_{n=1,3,5}^{\infty} \Delta E_n = \frac{1}{2}mv^2_0(1 + e)^2R = E(1 + e)^2R \ldots [22]$$

where E represents the initial kinetic energy of the striking mass.

Since energy is conserved ΔE is also the loss of energy of the striking mass, giving the equation

$$\Delta E = E(1 - e^2) \quad \ldots\ldots [23]$$

Now $\Delta E/E$ can be eliminated from Equations [22] and [23] giving the coefficient e in terms of the function R, or

$$e = \frac{1 - R}{1 + R} \quad \ldots\ldots [24]$$

Using the theory developed in this paper, and assuming an expression for $\mathfrak{F}(\tau)$ satisfying the condition $\int_{t_0}^{t_1} \mathfrak{F}(\tau)d\tau = 1$, R can be determined. This gives directly the coefficient e, the rebound velocity ev_0, and the energy absorbed by the beam.

The remaining requirement is some indication for a suitable choice of $\mathfrak{F}(\tau)$, which must approximate to the actual normalized contact force. Information in this direction can be obtained from the known Hertz (3) solution for impact of the striking mass on the same beam, constrained so that it cannot vibrate. The corresponding normalized force variation $\mathfrak{F}_H(\tau)$ can be calculated directly. Although the vibration of the beam does produce some change in the normalized contact force, the Hertz solution would afford a reasonable approximation for application in the energy solution. It is not convenient to use $\mathfrak{F}_H(\tau)$ directly, however, since its form is such that the integral for R in Equation [21] could not be obtained. To overcome this difficulty, a simple force function is used which is chosen to conform approximately

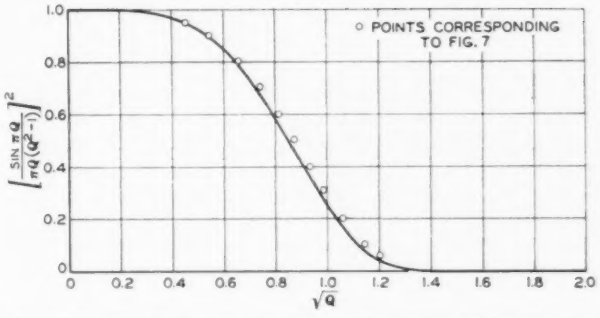


FIG. 6 ENERGY-DISTRIBUTION CURVE COMPARED WITH RESULTS FROM FIG. 7

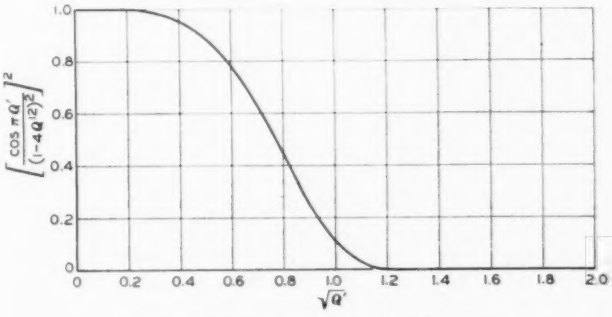


FIG. 7 ENERGY-DISTRIBUTION CURVE

to the Hertz variation $\mathfrak{F}_H(\tau)$. The function adopted by Zener and Feshbach(10) is

$$\mathfrak{F}_1(\tau) = \left(1 + \cos\frac{2\pi\tau}{T}\right)\Big/T$$

for $-T/2 \leq \tau \leq T/2$, and

$$\mathfrak{F}_1(\tau) = 0$$

for $|\tau| > T/2$.

The parameter T is chosen so that $\mathfrak{F}_1(\tau)$ has the same maximum ordinate as $\mathfrak{F}_H(\tau)$. This condition gives $T = 1.0873T_H$, where T_H is the time of contact of the Hertz impact. Substituting $\mathfrak{F}_1(\tau)$ in Equation [21] leads to a comparatively simple expression

$$R = \frac{m}{M}\sum_{n=1,3,5,}^{\infty}\left[\frac{\sin(\pi Q_n)}{\pi Q_n(Q_n^2 - 1)}\right]^2 \quad \ldots\ldots [25]$$

in which Q_n is a nondimensional quantity equal to $(\omega_n/2\pi)T$. The term Q_n has a simple physical significance; it is the number of oscillations of the nth mode of vibration in the time T. In Fig. 6 $\left[\frac{\sin(\pi Q_n)}{\pi Q_n(Q_n^2 - 1)}\right]^2$ is plotted against $\sqrt{Q_n}$ for use in solving specific impact problems.

For a beam simply supported at its ends, $\omega_n = n^2\omega_1$; therefore, the corresponding $Q_n = n^2Q_1$. Equation [25] becomes

$$R = \frac{m}{M}\sum_{n=1,3,5,}^{\infty}\left[\frac{\sin(\pi n^2Q_1)}{\pi n^2Q_1(n^4Q_1^2 - 1)}\right]^2 \quad \ldots\ldots [26]$$

It can be shown that in the limit as $Q_1 \to 0$, $R \to \infty$ as $Q_1^{-1/2}$, so that it is convenient to write

$$R = \frac{m}{MQ_1^{1/2}}G(Q_1) \quad \ldots\ldots [27]$$

TABLE 1 REBOUND VELOCITIES AFTER THE FIRST IMPACT, CALCULATED BY DIFFERENT METHODS

	Rebound velocity			Positive—reversed velocity; Negative—same direction as v_0		
	I	II	III	IV	V	VI
	Small increment method	$F_1(\tau)$ $T = 1.087 T_H$	$F_1(\tau)$ $T = 1.087 T_{MH}$	$F_2(\tau)$ $T' = 0.855 T_H$	$F_2(\tau)$ $T' = 0.855 T_{MH}$	Mean of columns (III) and (IV)
m = 50 lb, M = 149.3 lb v_0 = 11 fps, ω_1 = 531 sec^{-1}	0.932	1.128	0.817	1.068	0.745	0.943
m = 470 lb, M = 149.3 lb v_0 = 11 fps, ω_1 = 531 sec^{-1}	−7.740	−7.270	−8.060	−7.320	−8.120	−7.690
m = 32.24 grams, M = 61.1 grams v_0 = 1 cm per sec, $\omega_1 = 2\pi\ 10^3$ sec^{-1}	0.305	0.312	0.286	0.310	0.2878	0.298
m = 258 grams, M = 122.2 grams v_0 = 1 cm per sec, $\omega_1 = (\pi/2)\ 10^3$ sec^{-1}	−0.550	−0.4956	−0.580	−0.5086	−0.584	−0.544
m = 1.21 lb, M = 0.96 lb v_0 = 5.16 in. per sec, ω_1 = 1,153 sec^{-1}	−3.460	−3.320	−3.670	−3.340	−3.686	−3.500

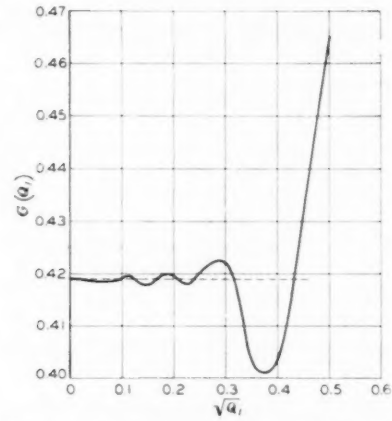


FIG. 8 VARIATION OF THE FUNCTION $G(Q_1)$

Now $G(Q_1)$ remains finite for all values of Q_1 and its variation is plotted against $\sqrt{Q_1}$ in Fig. 8.

The determination of rebound velocity and energy absorbed by the beam for a specific problem is thus made very simple; T_H is first obtained (12) giving immediately T and Q_1. The value of $G(Q_1)$ is ascertained using Fig. 8 giving R using Equation [27] and e from Equation [24].

In order to have some check on the validity of the assumed normalized force, an identical development has been evolved using the function

$$\mathfrak{F}_2(\tau) = \frac{\pi}{2T'} \cos \frac{\pi\tau}{T'}$$

for $-T'/2 \leq \tau \leq T'/2$, and

$$\mathfrak{F}_2(\tau) = 0$$

for $|\tau| > T'/2$.

Again T' is chosen so that the maximum ordinate of $\mathfrak{F}_2(\tau)$ equals that of $\mathfrak{F}_H(\tau)$. This condition gives $T' = 0.855\ T_H$. The integration for R is again quite simple giving

$$R = \frac{m}{M} \sum_{n=1,3,5}^{\infty} \left[\frac{\cos \pi Q_n'}{1 - 4Q_n'^2} \right]^2 \quad \ldots\ldots\ldots [28]$$

where $Q_n' = (\omega_n/2\pi)T'$, and is the number of oscillations of the nth mode in the time T'. Then $\left[\frac{\cos \pi Q_n'}{1 - 4Q_n'^2} \right]^2$ is plotted against $\sqrt{Q_n'}$ in Fig. 7. The difference between the curves in Figs. 6 and 7 is more apparent than actual when it is realized that the abscissas are different expressions in terms of T_H, the fundamental quantity defining this property of the impact. Transferring points from the curve in Fig. 7 to Fig. 6, using abscissas $\sqrt{Q}$ corresponding to the same value of T_H (the ordinate for $\sqrt{Q'}$ in Fig. 7 is plotted against $\sqrt{Q}$ in Fig. 6 using the relation $Q'/0.855 = T_H = Q/1.0873$), the individual points plotted in Fig. 6 are obtained which lie close to the curve. This procedure shows that the marked difference between the two force variations chosen has little effect on the energy absorbed by the beam.

As for the first force variation, the second one leads to the expressions

$$R = \frac{m}{M} \sum_{n=1,3,5}^{\infty} \left[\frac{\cos \pi n^2 Q_1'}{1 - 4n^4 Q_1'^2} \right]^2 \quad \ldots\ldots\ldots [29]$$

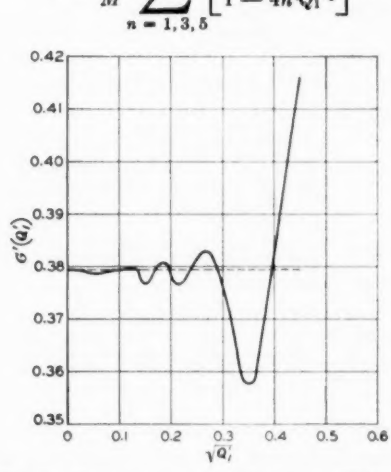


FIG. 9 VARIATION OF THE FUNCTION $G'(Q_1')$

For convenience, R is expressed in the form

$$R = \frac{m}{M(Q_1')^{1/2}} G'(Q_1') \quad \ldots\ldots\ldots [30]$$

where $G'(Q_1')$ is plotted against $\sqrt{Q_1'}$ in Fig. 9. The method of solving a particular problem using this curve is the same as that explained using the function $G(Q)$.

Table 1 gives values of the rebound velocity after the first impact, calculated by different methods for five specific problems enumerated in the first column.

Column I contains values of rebound velocity obtained using the reliable but tedious small-increment method.

Column II gives values obtained using the energy method described in this paper, assuming $\mathfrak{F}_1(\tau)$.

Column IV assumes the second form of force variation $\mathfrak{F}_2(\tau)$.

Columns III and V assume load variations similar to $\mathfrak{F}_1(\tau)$ and $\mathfrak{F}_2(\tau)$, respectively, based on a solution similar to Hertz's solution, but containing some allowance for the vibration of the beam. The development of this modified Hertz solution is given in Appendix 2. The only change which is thus introduced in the method described herein is that T and T' are calculated from T_{MH} (the contact duration for the modified Hertz solution) instead of using T_H, i.e., $T = 1.0873\ T_{MH}$, and $T' = 0.855\ T_{MH}$.

Column VI gives the average of the values in columns III and IV, and in all the cases considered shows excellent agreement

with the velocities in column I. This mean rebound velocity seems to give the most reliable approximate value yet obtained.

The errors shown by the differences between values of the rebound velocity obtained in different ways should be considered in terms of the change of velocity rather than the absolute value. For example, in the first problem tabulated, the sphere is brought almost to rest, and errors introduced are errors in the total change in momentum, i.e., on a basis of 12 fps, and not on a basis of the absolute value of the rebound velocity.

It has been shown that a reliable value for the rebound velocity (and so the coefficient e) is given by the energy method. This leads directly to a determination of the energy absorbed by the modes of vibration of the beam which is considered in the following section of this paper.

The Energy Spectrum and Its Significance

Equation [20] shows the amount of energy absorbed in each mode of vibration of a beam after impact. This expression leads to a direct determination of the energy spectrum of the beam, giving the distribution of the total vibrational energy among the separate modes. It further produces a criterion for impacts in which the vibrational energy is stimulated mainly in the fundamental mode of vibration. As was shown previously in this paper, such impacts can be dealt with satisfactorily by the approximate method described, although the method itself gives no indication whether this assumption is valid for a particular problem. Thus, using the energy method developed just previously, the group of problems to which the approximate solution is applicable can be determined. For this restricted group of impact problems the approximate solution is reliable for the contact duration and contact force. The impact force derived directly from the energy method, $F(\tau) = mv_0(1 + e)\mathfrak{F}_1(\tau)$ or $mv_0(1 + e)\mathfrak{F}_1(\tau)$ cannot be used as an approximation since it may differ appreciably from the correct value, although it gives excellent results for the subsequent motion.

Substituting $\mathfrak{F}_1(\tau)$ for $\mathfrak{F}(\tau)$ in Equation [20]

$$\Delta E_n = E\frac{m}{M}(1 + e)^2\left[\frac{\sin(\pi Q_n)}{\pi Q_n(Q_n^2 - 1)}\right]^2 \quad \ldots\ldots [31]$$

where $E(m/M)(1 + e)^2$ is independent of n, so that the distribution of energy among the modes is proportional to the value of $\left[\frac{\sin(\pi Q_n)}{\pi Q_n(Q_n^2 - 1)}\right]^2$. This can be taken directly from Fig. 6 by measuring the ordinates corresponding to the values of $\sqrt{Q_n}$, i.e., $n\sqrt{Q_1}$, $n = 1, 3, 5$, etc. Fig. 6 is therefore the energy spectrum for the beam, the ratio of the energies in the modes being proportional to the ordinates for $\sqrt{Q}$ equal to $\sqrt{Q_1}$, $3\sqrt{Q_1}$, $5\sqrt{Q_1}$, etc.

This investigation explains the apparent inconsistencies in the application of the approximate solution. The value of $\sqrt{Q_1}$ for the example corresponding to Fig. 4 is 0.434. Fig. 6 shows that the energy is distributed in the proportion 0.956 : 0.012 : 0.00 . . . , between the first, third, etc., modes. Thus, the energy is absorbed almost entirely by the fundamental mode of vibration, and replacing the beam by a system with one degree of freedom is justified. In this case, as shown in Fig. 4, the approximate solution shows good agreement.

The example corresponding to Fig. 5 has $\sqrt{Q_1} = 0.1729$. Fig. 6 gives the distribution of energy between the first few odd modes to be proportional to 1.00 : 0.908 : 0.470 : 0.040 : 0.00, so that to replace the beam by a system of one degree of freedom would be expected to produce appreciably different conditions for the impact. This is shown to be the case in Fig. 5, in which the force variations for the beam and corresponding one-degree-of-freedom system are different, both being calculated by the small-increment method. The approximate solution shows excellent agreement with the force variation for the one-degree-of-freedom spring system which is, however, useless as an approximation to the beam-impact problem.

It appears that the approximate method supplies a simple reliable solution for the group of problems in which energy is absorbed mainly in the fundamental mode of vibration of the beam. This is to be expected, for when this is true, the beam behaves like a one-degree-of-freedom system. A limited amount of energy in the third mode does not affect the force variation appreciably, since central deflection and not energy is the determining factor ($kF^{2/3} = \alpha = \delta - \eta$). The minimum value of $\sqrt{Q_1}$ to give reasonable results is about 0.3.

The assumption that the impact duration is short compared with the period of the fundamental mode of the beam also restricts the range of Q_1 for which the approximate method gives reliable results. From this point of view small values of Q_1 give the best approximation. The limit for which a good approximation can be expected is $Q_1 = 0.25$, $\sqrt{Q_1} = 0.5$. Though toward the end of the impact, $Q_1 = 0.25$ will lead to quite large values of $\omega_1 t$, this is not important since toward the end of impact the contact force is small, producing little effect on the final motion. Therefore, the approximate method is useful for values of $\sqrt{Q_1}$ between 0.3 and 0.5, and it is in this range that many practical problems lie. Values of $\sqrt{Q_1} > 0.5$ are unusual, so that the most frequent obstacle to the application of this approximate method is that the vibrational energy is not sufficiently concentrated in the fundamental mode. This leads to a very important criticism of Lennertz' paper (9), since he considered the validity of the short contact assumption only, and merely stated, without investigation, that the equivalence of a one-degree-of-freedom system was satisfactory.

Motion of the Striking Mass and of the Beam After Impact

The solution of impact problems based on energy considerations leads to an expression for the free vibration of the beam after contact has ceased. The displacement-time curves for the center of the beam and for the mass can be drawn, from which the existence of a repeated impact can be determined.

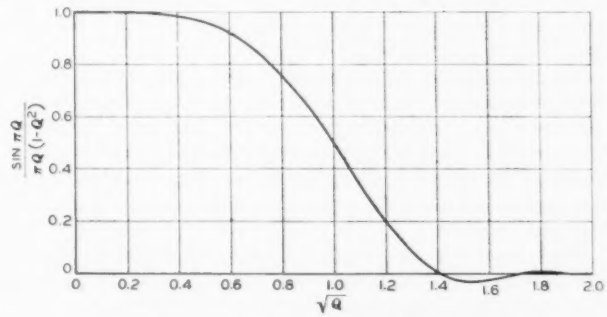


Fig. 10 Graph of Displacement Function

Equation [15] is directly integrable assuming the normalized force variation $\mathfrak{F}_n(\tau)$, giving

$$C_n(t) = \sqrt{\frac{2}{l}}\frac{(-1)^{\frac{n-1}{2}}}{\omega_n q} mv_0(1 + e)\frac{\sin(\pi Q_n)}{\pi Q_n(1 - Q_n^2)}\sin(\omega_n t). \quad [32]$$

$$n = 1, 3, 5 \ldots$$

Using Equation [13], the central deflection due to the nth mode is

$$C_n(t)\sqrt{\frac{2}{l}}\sin\left(\frac{n\pi}{2}\right) = \frac{m}{M}\frac{v_0(1 + e)}{\omega_n}\frac{\sin(\pi Q_n)}{\pi Q_n(1 - Q_n^2)}\sin(\omega_n t) \ldots [33]$$

where $n = 1, 3, 5 \ldots\ldots$, $\omega_n = n^2\omega_1$, and $Q_n = n^2 Q_1$.

The motion of the center of the beam is thus composed of the

superposition of sinusoidal oscillations given by Equation [33]. Their amplitudes decrease rapidly with increasing n, so that normally all but the first two or three modes of vibration remain quiescent. To simplify the determination of the deflection, $\sin(\pi Q/\pi)Q(1-Q^2)$ is plotted against $\sqrt{Q}$ in Fig. 10.

In solving a specific problem, $\sqrt{Q_1}$ and e are calculated as shown previously, the value of $\sin(\pi Q_n)/\pi Q_n(1-Q_n^2)$ being then given as the ordinate for $\sqrt{Q} = n\sqrt{Q_1}$ in Fig. 10. The vibration, both amplitude and phase, is given by substitution in Equation [33]. The sinusoidal displacement-time curves are plotted for each mode, and their superposition gives the motion of the center of the beam. The time origin is at the middle of the impact duration and Equation [33] is only valid after contact has ceased, since integrations were carried out for $t > T/2$. This

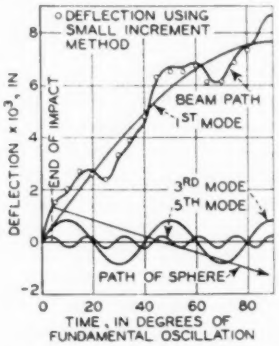


FIG. 11 COMPARISON OF DISPLACEMENTS OBTAINED BY DIFFERENT METHODS, AND REBOUND OF THE STRIKING MASS

explains the apparent inconsistency of zero deflection at the middle of the impact.

This procedure is applied to the problem corresponding to Fig. 5 ($m = 50$ lb, $M = 298.5$ lb, $\omega_1 = 531$ sec^{-1}, and $\sqrt{Q_1} = 0.1729$). The displacement so obtained is compared in Fig. 11 with values given by the small-increment method of solution. Excellent agreement is demonstrated.

The advantage of the displacement curve obtained from energy considerations (in addition to the comparison between a mere half hour's work, and two or three days' tedious ploughing through the small increment method) results, since the motion is already harmonically analyzed, which is more useful than a knowledge of the displacement at isolated time intervals as given by the small increment method.

The rebound of the striking mass is also shown in Fig. 11. The mass moves with constant velocity ev_0 from the position the beam had when contact ceased.

Intersection of the two displacement curves indicates a repeated impact. The statement made by Zener and Feshbach (10), "multiple collisions will occur whenever the calculated e is negative," is true but not necessary. Repeated impacts also occur when e is positive and the rebound velocity is not sufficient to carry the mass out of the range of the vibrating beam before a second impact occurs. Experimental evidence of this state of affairs appears in Tuzi and Nisida's (8) illustrations.

Using a method analogous to that adopted for the first impact, the second impact can be dealt with. Again, a normalized contact-force variation is assumed. The Hertz (3) force variation, calculated for a velocity of approach equal to the relative velocity between the beam and the mass, is used as a basis for the assumed variation $\mathfrak{F}_1(\tau)$, exactly as $\mathfrak{F}_1(\tau)$ was defined for the first impact in the energy method. The free vibration already present in the beam due to the first impact complicates the problem somewhat. We know $C_n(t) = A_n \sin \omega_n t$ are the generalized coordinates of the vibration already disturbing the beam due to the first impact. Defining e_1, a coefficient of restitution for the second impact, by writing $e_1(-ev_0)$ for the velocity of rebound,[7] energy and momentum considerations determine e_1 by the equation

$$(1-e_1) = (1+e_1)R + \frac{2}{(-ev_0)} \sum_{n=1,3,5}^{\infty} A_n\omega_n \sqrt{\frac{2}{l}} (-1)^{\frac{n-1}{2}} \frac{\sin(\pi Q_n)}{\pi Q_n(1-Q_n^2)} \cos(\omega_n t_m) \quad [34]$$

where t_m is the displacement in time between the two impacts, and R is defined in Equation [21].

It is interesting to note that in applying Equation [34] to the first impact $A_n = 0$, and the equation $e_1 = (1-R)/(1+R)$ is obtained which is the same as Equation [24].

When e_1 has been determined from Equation [34], the vibration superimposed on the beam due to the second impact can be calculated from Equation [32], replacing v_0 by $(-ev_0)$, and e by e_1. Adding the result to the vibration already present in the beam, the total motion is obtained.

Investigating the motion after the second impact, the existence of another impact can be decided. Repeated impacts can be studied using Equation [34] until the mass rebounds out of the region covered by the vibrating beam and the whole impact process is complete.

THE FORM OF THE DEFLECTION CURVE OF THE BEAM

The knowledge of the generalized coordinates $C_n(t)$ for the

[7] Here $-ev_0$ is the velocity of the mass between the first and second impacts; it is positive when in the same direction as v_0.

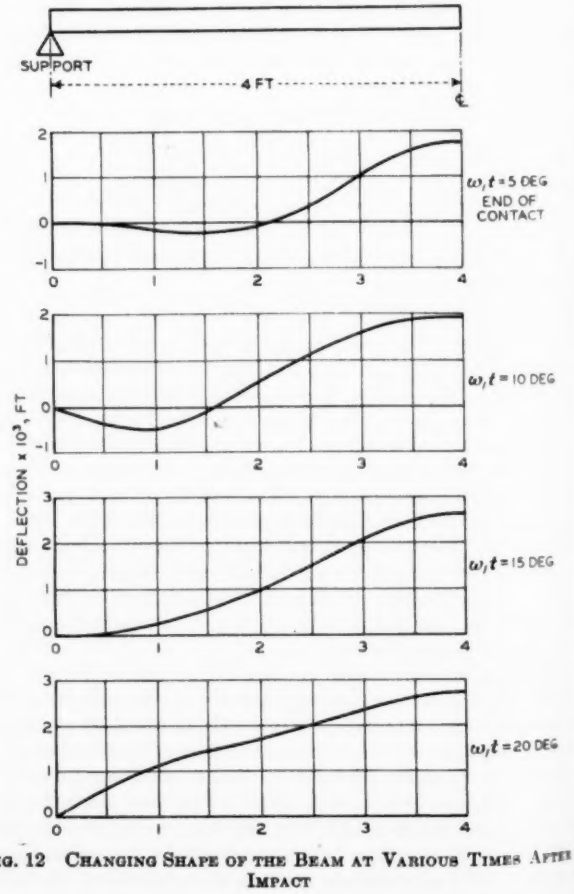


FIG. 12 CHANGING SHAPE OF THE BEAM AT VARIOUS TIMES AFTER IMPACT

vibration of the beam after impact leads directly to the changing deflection curve of the beam.

Equation [13] gives the motion of all points along the beam, and substituting for $C_n(t)$ from Equation [32], becomes

$$u(x,t) = \frac{m}{M} v_0(1+e) \sum_{n=1,3,5} \frac{(-1)^{\frac{n-1}{2}}}{\omega_n} \frac{\sin(\pi Q_n)}{\pi Q_n(1-Q_n^2)} \sin(\omega_n t) \sin\left(\frac{n\pi x}{l}\right) \quad [35]$$

This equation only holds after contact has ceased. The deflection curve of the beam is composed of the superposition of sine terms, the amplitudes of these Fourier terms changing with time. For any time t these amplitudes are given in Equation [33], and can be readily evaluated using Fig. 10 or from the central deflections considered in the last section.

Fig. 12 shows the changing shape of the beam due to the impact considered previously ($m = 50$ lb, $M = 298.5$ lb, $\omega_1 = 531$ sec^{-1}, and $\sqrt{Q_1} = 0.1729$). Immediately after contact has ceased the deflection is limited mainly to the central portion of the beam; near the supports the deflection is small and in the opposite direction. Later, the first mode of vibration dominates. The curves of Fig. 9 show a marked difference from the static-deflection curve for central loading, indicating that Cox's (13) assumption of static-deflection form is unjustified.

Beam-deflection curves, obtained for the impact of a 470-lb mass on the beam just considered, agree well with the experimental investigation carried out by Arnold (7) as interpreted by Smith in the discussion of Arnold's paper. He also obtains the initial negative deflection near the supports due to the third harmonic.

Conclusion

It has been shown that energy considerations lead to many characteristics of the impact process due to a mass striking a beam. For simplicity, this paper has dealt solely with central impact on a beam simply supported at its ends. No difficulties are encountered in generalizing the method for impact at any point along a beam which is supported in any way. A development, analogous to that already evolved, leads to similar conclusions, the relations being expressed in terms of the normalized characteristic functions for the modes of vibration. This general theory is developed briefly in Appendix 1, the procedure being identical with that adopted for the specialized problem.

Acknowledgments

The author wishes to express his appreciation to Professor S. Timoshenko for suggesting the subject and for valuable help offered by him in its development. The author is also grateful to the Commonwealth Fund whose Fellowship made the work possible.

Appendix 1

THE GENERALIZED IMPACT PROBLEM

The development carried out in the body of this paper for central impact on a uniform beam simply supported at its ends is briefly repeated here for a uniform beam with general supporting conditions, the impact occurring at any position along its length. Since the development is parallel to that for the special case already considered, little verbal explanation is given; the corresponding equations are denoted by the same reference number or letter which was used in the simpler development, but carry the suffix a for discussers' reference.

The vibration of the beam is considered in terms of the normalized characteristic functions $u_n(x)$ and the generalized coordinates $C_n(t)$ of the normal modes of vibration. The deflection of any point of the beam is given by

$$u(x,t) = \sum_{n=1,2,3,\ldots} u_n(x) C_n(t) \quad [13a]$$

The general integral equation for impact becomes

$$k[F(t)]^{3/2} = v_0 t - \frac{1}{m}\int_0^t F(\tau)(t-\tau)d\tau - \sum_{n=1,2,3} \frac{1}{M_n}\int_0^t F(\tau)\frac{\sin[\omega_n(t-\tau)]}{\omega_n} d\tau \quad [4a]$$

where the reduced mass of the beam for the nth mode $M_n = q/[u_n(c)]^2$, $x = c$ being the point of impact.

The approximate method remains unchanged, although the range of validity of the assumptions used differs. The introduction of the second mode of vibration, which was eliminated due to symmetry in the special problem dealt with before, makes the consideration of only the first mode a more restrictive assumption.

The concept of the normalized contact force is applicable to this generalized impact problem.

The vibrations produced in the beam due to a force variation $F(\tau)$, acting at the point $x = c$, are given by the differential equation

$$\frac{d^2C_n}{dt^2} + \omega_n^2 C_n^2 = \frac{1}{q} F(t) u_n(c) \quad [14a]$$

Assuming no motion at the time t_0 when the force commences to act

$$C_n(t) = \frac{u_n(c)}{\omega_n q}\int_{t_0}^t \sin[\omega_n(t-\tau)]F(\tau)d\tau \quad [15a]$$

giving the energy in each mode after contact has ceased

$$\Delta E_n = \frac{[u_n(c)]^2}{2q}\left|\int_{t_0}^{t_1} F(\tau)e^{i\omega_n\tau}d\tau\right|^2 \quad [17a]$$

Transferring to the normalized force variation $\mathfrak{F}(\tau) = F(\tau)/mv_0(1+e)$ Equation [17a] becomes

$$\Delta E_n = \frac{m^2v_0^2(1+e)^2}{2q}[u_n(c)]^2\left|\int_{t_0}^{t_1}\mathfrak{F}(\tau)e^{i\omega_n\tau}d\tau\right|^2 \quad [20a]$$

The coefficient of restitution, e, is given by

$$e = \frac{1-R}{1+R} \quad [24a]$$

where

$$R = \frac{m}{q}\sum_{n=1,2,3,\ldots}[u_n(c)]^2\left|\int_{t_0}^{t_1}\mathfrak{F}(\tau)e^{i\omega_n\tau}d\tau\right|^2 \quad [21a]$$

Assuming the normalized force variation $\mathfrak{F}_1(\tau)$

$$R = \frac{m}{q}\sum_{n=1,2,3,\ldots}[u_n(c)]^2\left[\frac{\sin(\pi Q_n)}{\pi Q_n(Q_n^2-1)}\right]^2 \quad [25a]$$

For any particular problem R can be calculated using Fig. 6.

This gives e from Equation [24a], in terms of which the energy distribution after contact is expressed as

$$\Delta E_n = E\frac{m}{q}(1+e)^2[u_n(c)]^2\left[\frac{\sin(\pi Q_n)}{\pi Q_n(Q_n^2-1)}\right]^2 \ldots [31a]$$

Using Fig. 6, Equation [31a] gives the energy distribution between the modes of vibration of the beam.

The motion after contact is given by

$$u(x,t) = \sum_{n=1,2,3,\ldots}^{\infty} C_n(t)u_n(x) =$$

$$\frac{mv_0(1+e)}{q}\sum_{n=1,2,3,\ldots}^{\infty}\frac{u_n(c)}{\omega_n}\frac{\sin(\pi Q_n)}{\pi Q_n(1-Q_n^2)}\sin(\omega_n t)\cdot u_n(x) \ldots [35a]$$

This equation includes the motion of the point of contact of the beam, leading to a study of the subsequent impact procedure. Equation [35a] also gives the deflection form of the beam when contact has ceased.

The energy-distribution condition for the validity of the approximate method can be studied using Equation [31a], which gives the energy spectrum for the beam. The appearance of the second harmonic will make the restriction that the energy is absorbed mainly in the fundamental mode more stringent, and special care is needed to insure that the assumptions are valid when this approximate method is used.

This theory developed for impact on a beam can be extended to impact on plates, the plate deflections being expressed in terms of the characteristic functions for the normal modes of vibration. Since, in the case of plate vibrations, the natural frequencies of the normal modes are closer to the fundamental than for beams, special care must be taken in considering the validity of the approximations used for the approximate method.

Appendix 2

A MODIFIED HERTZ SOLUTION

This approximate solution has been developed as a basis from which an approximation for the normalized contact force variation can be derived. It gives a better approximation to the correct contact force than the Hertz (3) solution, since it contains some allowance for the effect of the vibrating beam, whereas the Hertz solution applies to impact of a mass on a beam constrained so that it cannot vibrate.

The fundamental mode of vibration only is considered and the duration of contact is assumed small compared with the period of the fundamental. The second assumption enables $\sin\omega_1(t-\tau)$ to be replaced by $\omega_1(t-\tau)$ and Equation [4] becomes

$$k[F(t)]^{3/2} = v_0 t - \frac{1}{m}\int_0^t F(\tau)(t-\tau)d\tau - \frac{1}{M}\int_0^t F(\tau)(t-\tau)d\tau \ldots [37]$$

Combining the last two terms and writing $\frac{1}{M_e} = \frac{1}{m} + \frac{1}{M}$, this equation reduces to the Hertz equation for a mass M_e striking the constrained beam with a velocity v_0. The corresponding force variation is given by Hertz's formula and is termed the "modified Hertz solution."

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Analysis of Clamped Rectangular Plates

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This paper attempts to solve the problem of the bending action of rectangular plates clamped at all four edges and subjected to lateral loading. Analytical in nature, the author's investigation is based upon the ordinary theory of bending of thin plates as treated in Lagrange's equation of the middle surface. The superposition method is used and applied to a number of loadings not hitherto studied.

THE bending of rectangular plates which are clamped at all four edges and subjected to the action of lateral loads is a problem which has received considerable attention because of its technical importance. Two main methods of attack have been used for obtaining solutions, namely, (*a*) Ritz's energy method (1),[2] a recent discussion of which has been given by G. Pickett (2), and (*b*) the superposition method proposed by S. P. Timoshenko (3) as a generalization of Hencky's solution (4). These methods are approximate in the rigorous mathematical sense but, practically, the results may be obtained to any desired degree of accuracy by taking sufficient terms in the infinite series of simultaneous equations which occur in both these methods. To date, complete analyses are available only for the case of a uniform load and for the case of a central concentrated load. In this paper, Timoshenko's method is applied to a number of loadings which have not been treated before. Consideration is given first to symmetrically placed loads and to antisymmetrical loadings. Then unsymmetrically distributed loads are obtained by superposition of the preceding cases. The numerical calculations have been carried out for several loadings and values for the deflections, moments, and shears at critical points are given.

This is an analytical investigation based upon the ordinary theory of bending of thin plates, as governed by Lagrange's equation of the middle surface. Discussion of this theory, derivation of the differential equation, and the relations for moments and shears are available in references such as (5) and (6), and will not be given here. For the plates treated in this paper it is assumed that all four edges are perfectly fixed so there is no deflection or change in slope at the edges under the action of the lateral loads.

Nomenclature

The following nomenclature is used in the paper:

- a = length of plate in x direction
- b = length of plate in y direction
- h = thickness of plate (assumed constant in this investigation)
- E = modulus of elasticity of plate material
- ν = Poisson's ratio (assumed equal to 0.3 in numerical calculations)
- D = $Eh^3/12(1 - \nu^2)$ flexural rigidity of plate
- M_x, M_y = bending moments per unit length of plate on sections perpendicular to x and y axes, respectively, taken as positive when producing compression at top of plate
- $(M)_{ss}$ = bending moment for a simply supported plate carrying the load under consideration
- Q_x, Q_y = vertical shearing forces per unit length of plate on sections perpendicular to the x and y axes, respectively, taken as positive on a rectangular element of plate when acting upward on the side of the element having the smaller value of x or y, respectively.
- w = deflection of plate at any point, assumed positive downward
- P = concentrated load, assumed positive when acting downward
- q = intensity of a distributed lateral load, assumed positive when acting downward
- p = intensity of a triangularly distributed load at unit distance from line of zero load
- x, y = rectangular coordinates in plane of middle surface of plate. For convenience in discussion, the plane of the plate will be assumed horizontal and loads will then be acting vertically. The origin of coordinates is taken at center of plate
- k, m = integers, with values as indicated on the summation sign where they occur
- A, B, G, H = coefficients which are defined where used

$$\alpha_m = \frac{m\pi b}{2a} \qquad \delta_m = \frac{m\pi a}{b}$$

Symmetrical Loadings

A symmetrical load is one which is symmetrically distributed about each of the coordinate axes through the center of the plate. The general method for analyzing clamped rectangular plates under such loadings has been given by Timoshenko (3). Detailed results for a uniformly loaded plate have been worked out by I. A. Wojtaszak (7) and T. H. Evans (8). Calculations for a central concentrated load have also been given (9). Deflections, moments, and shears for uniformly loaded plates of several proportions are summarized in Table 1 for later use. These results were taken from Wojtaszak's paper with some additional values computed by the author.

Antisymmetrical Loads

The term "antisymmetrical" is used to designate load distributions which have equal but opposite intensities at corresponding positive and negative values of the coordinates of one axis. The loading is assumed symmetrically distributed parallel to the other axis. In the following work the loads are taken as antisymmetrically distributed about the x axis and symmetrically distributed about the y axis, so that $q(x, y_1) = -q(x, -y_1)$ and $q(x_1, y) = q(-x_1, y)$. These loadings are not of practical interest by themselves, but are for use in combination with symmetrical loads to obtain cases of unsymmetrical loadings. Since Timoshenko's method has not been previously carried through in detail for this type of loading, it appears worth-while to develop here the necessary equations.

The problem is to find a solution of the differential equation of the middle surface of a bent plate

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

[1] University of Connecticut.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1941, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

that will satisfy the boundary conditions for clamped edges

$$(w)_{x=\pm a/2} = (w)_{y=\pm b/2} = 0\,, \quad \left(\frac{\partial w}{\partial x}\right)_{x=\pm a/2} = \left(\frac{\partial w}{\partial y}\right)_{y=\pm b/2} = 0$$

This may be done by taking w as the sum of three solutions

$$w = w_1 + w_2 + w_3 \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

For the function w_1 we take the solution for a simply supported plate carrying the given antisymmetrical lateral load. In most cases this is readily derived using known methods. For w_2 we take the solution for a simply supported plate with symmetrically distributed edge moments applied at $y = {}^1/_2 b$, and equal but opposite moments applied at $y = -{}^1/_2 b$. It is convenient to take these moments in the form of a cosine series

$$(M_y)_{y=b/2} = -(M_y)_{y=-b/2} = \sum_{m=1,3,5,\ldots} A_m(-1)^{(m-1)/2} \cos\frac{m\pi x}{a} \ldots [2]$$

and then the function w_2 is given by

$$w_2 = \frac{a^2}{2\pi^2 D}\sum_{m=1,3,5\ldots}\frac{A_m(-1)^{(m-1)/2}}{m^2\sinh\alpha_m}\left(\alpha_m\coth\alpha_m\sinh\frac{m\pi y}{a} - \frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right)\cos\frac{m\pi x}{a} \ldots [3]$$

For w_3 we take the solution for a simply supported plate with antisymmetrically distributed edge moments of the form

$$(M_x)_{x=\pm a/2} = \sum_{m=1}^{\infty} B_m \sin\frac{2m\pi y}{b} \ldots\ldots\ldots [4]$$

applied along the edges $x = \pm {}^1/_2 a$. The deflection function for this case is given by

$$w_3 = \frac{b^2}{8\pi^2 D}\sum_{m=1}^{\infty}\frac{B_m}{m^2\cosh\delta_m}\left(\delta_m\tanh\delta_m\cosh\frac{2m\pi x}{b} - \frac{2m\pi x}{b}\sinh\frac{2m\pi x}{b}\right)\sin\frac{2m\pi y}{b} \ldots [5]$$

The coefficients A_m and B_m in the preceding equations are to be determined from the condition that the slope at the boundaries is zero. Thus, taking $\left(\frac{\partial w}{\partial y}\right)_{y=b/2} = 0$ and expressing the terms containing $\cosh\frac{2m\pi x}{b}$ and $\frac{2m\pi x}{b}\sinh\frac{2m\pi x}{b}$ by a cosine series, we obtain

$$\left(\frac{\partial w}{\partial y}\right)_{y=b/2} = \left(\frac{\partial w_1}{\partial y}\right)_{y=b/2} + \frac{a}{2\pi D}\sum_{m=1,3,5,\ldots} A_m\frac{(-1)^{(m-1)/2}}{m}\left(\frac{\alpha_m}{\sinh^2\alpha_m} - \coth\alpha_m\right)\cos\frac{m\pi x}{a} + \frac{8a^2}{\pi^2 bD}\sum_{m=1}^{\infty}\frac{B_m(-1)^m}{m^3} - \sum_{k=1,3,5,\ldots}\frac{k(-1)^{(k-1)/2}}{\left(\frac{4a^2}{b^2}+\frac{k^2}{m^2}\right)^2}\cos\frac{k\pi x}{a} = 0 \ldots [6]$$

Since this relation holds for any cosine term, say $\cos\frac{k\pi x}{a}$, the summation signs may be dropped and the cosine term canceled, thus giving

$$A_k\left[\coth\alpha_k - \frac{\alpha_k}{\sinh^2\alpha_k}\right] - \frac{k^2b^2}{\pi a^2}\sum_{m=1}^{\infty}\frac{m(-1)^m B_m}{\left(m^2+\frac{k^2b^2}{4a^2}\right)^2} = \frac{2\pi Dk(-1)^{(k-1)/2}}{a}G_k \ldots [7]$$

where G_k is found by expressing the slope at $y = {}^1/_2 b$ due to w_1 in a cosine series of the form

$$\left(\frac{\partial w_1}{\partial y}\right)_{y=b/2} = \sum_{k=1,3,5,\ldots} G_k\cos\frac{k\pi x}{a} \ldots\ldots [8]$$

One equation in the unknowns A_k and B_m is obtained for each value of $k = 1, 3, 5, \ldots$, substituted in Equation [7].

Similarly, for zero slope at $x = {}^1/_2 a$ we find the expression

$$-B_m(-1)^m\left[\tanh\delta_m + \frac{\delta_m}{\cosh^2\delta_m}\right] + \frac{32m^2a^3}{\pi b^3}\sum_{k=1,3,5\ldots}\frac{kA_k}{\left(k^2+\frac{4m^2a^2}{b^2}\right)^2} = -\frac{4\pi m(-1)^m D}{b}H_m \ldots [9]$$

where H_m is obtained by expressing the slope at $x = {}^1/_2 a$ due to w_1 in a sine series of the form

$$\left(\frac{\partial w_1}{\partial x}\right)_{x=a/2} = \sum_{m=1}^{\infty} H_m\sin\frac{2m\pi y}{b} \ldots\ldots [10]$$

From Equation [9] we obtain one equation in the unknown coefficients A_k and B_m for each value of $m = 1, 2, 3, 4 \ldots\ldots$

Equations [7] and [9] lead to a double-infinite series of simultaneous equations in the unknowns A_k and B_m, which may be solved approximately by assuming that terms higher than a given order may be neglected. In general, it was found most expeditious to solve this system of equations by successive approximations as illustrated in (3, 7, 8). The left-hand members of these equations are independent of the load and depend only upon the ratio b/a of the length of the sides, so that once the numerical values of these terms are computed for a plate of given proportion, they may be used for any loading.

The numerical calculation of the coefficients A_k and B_m completes the solution of Equation [1]. Values of the deflection, moments, and shears at any point may then be determined from the known relations in the theory of plates. Detailed solutions for two particular antisymmetrical loadings will now be given.

Triangular Load Distribution.

The load intensity in this case varies linearly from the x axis to the edges $y = \pm {}^1/_2 b$ as shown in Fig. 1. The intensity at any point is given by the expression $q = py$, where p is the intensity at unit distance from the x axis. It is easily verified that the deflection of a simply supported plate carrying this loading is given by

$$w_1 = \frac{pa^4}{\pi^5 D}\sum_{m=1,3,5\ldots}\frac{(-1)^{(m-1)/2}}{m^5}\left[4y - \frac{b(2+\alpha_m\coth\alpha_m)}{\sinh\alpha_m}\sinh\frac{m\pi y}{a} + \frac{b}{\sinh\alpha_m}\times\frac{m\pi y}{a}\times\cosh\frac{m\pi y}{a}\right]\cos\frac{m\pi x}{a} \ldots [11]$$

An alternate expression for w_1 which is useful for determining the load coefficient H_m, is as follows

$$w_1 = \frac{pb^5}{16\pi^5 D}\sum_{m=1}^{\infty}\frac{(-1)^m}{m^5}\left[-1 + \frac{{}^1/_2\delta_m\tanh\delta_m + 1}{\cosh\delta_m}\cosh\frac{2m\pi x}{b} - \frac{1}{2\cosh\delta_m}\times\frac{2m\pi x}{b}\times\sinh\frac{2m\pi x}{b}\right]\sin\frac{2m\pi y}{b} \ldots [12]$$

Table 2. Antisymmetrical triangular loading (see Fig.1).

	$\frac{b}{a}=2$	$\frac{b}{a}=1.5$	$\frac{b}{a}=1$	$\frac{b}{a}=\frac{1}{2}$	$\frac{b}{a}=0$
Coefficients A_m and B_m					
A_1	$-0.128\,28\ \frac{qa^2}{\pi^2}$	$-0.183\,29\ \frac{qa^2}{\pi^2}$	$-0.331\,96\ \frac{qa^2}{\pi^2}$	$-0.191\,89\ \frac{qa^2}{\pi^2}$	
A_3	0.009 81	0.00878	0.00883	−0.001 37	
A_5	0.003 79	0.00730	0.00703	−0.000 58	
A_7	0.002 97	0.00417	0.00333	0.00384	
A_9	0.001 76	0.00291	0.00275	0.00418	
A_{11}	0.001 13	0.00181	0.00233	0.00339	
A_{13}	0.000 78	0.00110	0.00179	0.00259	
A_{15}	0.000 57	0.000 79	0.001 30	0.001 98	
B_1	$-0.108\,43\ \frac{qa^2}{\pi^2}$	$-0.128\,85\ \frac{qa^2}{\pi^2}$	$-0.137\,48\ \frac{qa^2}{\pi^2}$	$-0.137\,61\ \frac{qa^2}{\pi^2}$	
B_3	0.005 28	−0.000 72	−0.00409	−0.00491	
B_5	0.004 03	0.005 96	0.006 76	0.007 65	
B_7	−0.003 68	−0.00691	−0.00471	−0.00500	
B_9	0.002 67	0.003 80	0.003 17	0.003 44	
B_{11}	−0.001 90	−0.002 64	−0.002 19	−0.002 48	
B_{13}	0.001 36	0.001 86	0.001 73	0.001 87	
B_{15}	−0.001 00	−0.001 06	−0.001 16	−0.001 43	
Deflections, moments and shears					
w at $x=0, y=\frac{b}{2}$	$0.000\,0330\ \frac{qa^4}{D}$	$0.000\,0411\ \frac{qa^4}{D}$	$0.000\,0880\ \frac{qa^4}{D}$	$0.000\,0723\ \frac{qa^4}{D}$	$0.000\,0725\ \frac{qa^4}{D}$
M_y at $x=0, y=\frac{b}{2}$	$-0.004\,36\ qa^2$	$-0.006\,05\ qa^2$	$-0.007\,79\ qa^2$	$-0.008\,51\ qa^2$	$-0.008\,33\ qa^2$
M_x at $x=\frac{a}{2}, y=\frac{b}{2}$	$-0.003\,79\ qa^2$	$-0.004\,58\ qa^2$	$-0.004\,60\ qa^2$	$-0.004\,61\ qa^2$	
Q_y at $x=0, y=\frac{b}{2}$	$-0.02\ qa$	$-0.04\ qa$	$-0.10\ qa$	$-0.10\ qa$	$-0.100\ qa$
Q_x at $x=\frac{a}{2}, y=\frac{b}{2}$	$-0.030\ qa$	$-0.034\ qa$	$-0.034\ qa$	$-0.034\ qa$	

Table 3. Uniform load in opposite direction on each half of plate (see Fig 2).

	$\frac{b}{a}=2$	$\frac{b}{a}=1.5$	$\frac{b}{a}=1$	$\frac{b}{a}=\frac{1}{2}$	$\frac{b}{a}=0$
Coefficients A_m and B_m					
A_1	$-0.397\,00\ \frac{qa^2}{\pi^2}$	$-0.614\,14\ \frac{qa^2}{\pi^2}$	$-0.901\,00\ \frac{qa^2}{\pi^2}$	$-1.014\,40\ \frac{qa^2}{\pi^2}$	
A_3	0.044 71	0.053 47	0.060 79	−0.104 36	
A_5	0.019 90	0.029 06	0.037 32	0.017 72	
A_7	0.008 92	0.014 31	0.033 16	0.032 20	
A_9	0.004 83	0.007 78	0.013 85	0.019 59	
A_{11}	0.002 93	0.004 64	0.008 69	0.013 79	
A_{13}	0.001 93	0.002 99	0.005 71	0.009 70	
A_{15}	0.001 36	0.002 04	0.004 02	0.006 94	
B_1	$-0.446\,18\ \frac{qa^2}{\pi^2}$	$-0.559\,13\ \frac{qa^2}{\pi^2}$	$-0.598\,90\ \frac{qa^2}{\pi^2}$	$-0.599\,78\ \frac{qa^2}{\pi^2}$	
B_3	−0.074 33	−0.102 71	−0.118 14	−0.130 13	
B_5	0.021 39	0.030 46	0.036 88	0.041 11	
B_7	−0.011 88	−0.016 99	−0.020 20	−0.022 37	
B_9	0.007 14	0.010 32	0.012 10	0.013 34	
B_{11}	−0.004 96	−0.007 06	−0.008 43	−0.010 32	
B_{13}	0.003 68	0.005 03	0.005 84	0.006 34	
B_{15}	−0.002 85	−0.004 09	−0.004 85	−0.005 28	
Deflections and moments					
w at $x=0, y=\frac{b}{2}$	$0.000\,0982\ \frac{qa^4}{D}$	$0.000\,1190\ \frac{qa^4}{D}$	$0.000\,2805\ \frac{qa^4}{D}$	$0.000\,3210\ \frac{qa^4}{D}$	$0.000\,3273\ \frac{qa^4}{D}$
M_y at $x=0, y=\frac{b}{2}$	$-0.0128\ qa^2$	$-0.0169\ qa^2$	$-0.0187\ qa^2$	$-0.0211\ qa^2$	$-0.02125\ qa^2$
M_x at $x=\frac{a}{2}, y=\frac{b}{2}$	$-0.0170\ qa^2$	$-0.0183\ qa^2$	$-0.0196\ qa^2$	$-0.0197\ qa^2$	

Table 6. Square clamped plate with concentrated load at quarter points of y-axis ($a=\frac{b}{2}$)

	Two sym. concentrated loads (see Fig 6)	Two antisym. concentrated loads (see Fig 7)	Non-central concentrated load (see Fig 8)
w at $x=0, y=0$	$0.00490\ \frac{Pa^2}{D}$	0	$0.00249\ \frac{Pa^2}{D}$
w at $x=0, y=\frac{b}{2}$	$0.00403\ \frac{Pa^2}{D}$	$0.00249\ \frac{Pa^2}{D}$	$0.00326\ \frac{Pa^2}{D}$
M_y at $x=0, y=\frac{b}{2}$	$-0.2821\ P$	$-0.2115\ P$	$-0.2468\ P$
M_y at $x=0, y=\frac{b}{4}$	$-0.2821\ P$	$0.2115\ P$	$-0.0353\ P$
M_y at $x=0, y=0$	$0.026\ P$	0	$0.013\ P$
M_x at $x=0, y=0$	$0.091\ P$	0	$0.046\ P$
M_x at $x=\frac{a}{2}, y=0$	$-0.1731\ P$	0	$-0.0706\ P$
M_x at $x=\frac{a}{2}, y=\frac{b}{2}$	$-0.0737\ P$	$-0.0389\ P$	$-0.0463\ P$
M_x at $x=\frac{a}{2}, y=\frac{b}{4}$	$-0.0337\ P$	$0.0389\ P$	$-0.0074\ P$
M_y at $x=0, y=\frac{b}{2}$	$(M_y)_{\nu=0}=-0.0994\ P$	$(M_y)_{\nu=0}=-0.0507\ P$	$(M_y)_{\nu=0}=-0.0752\ P$
M_x at $x=0, y=\frac{b}{2}$	$(M_x)_{\nu=0}=-0.0920\ P$	$(M_x)_{\nu=0}=-0.0322\ P$	$(M_x)_{\nu=0}=-0.0622\ P$

Table 1. Deflections, moments and shears for clamped rectangular plate with uniform load.

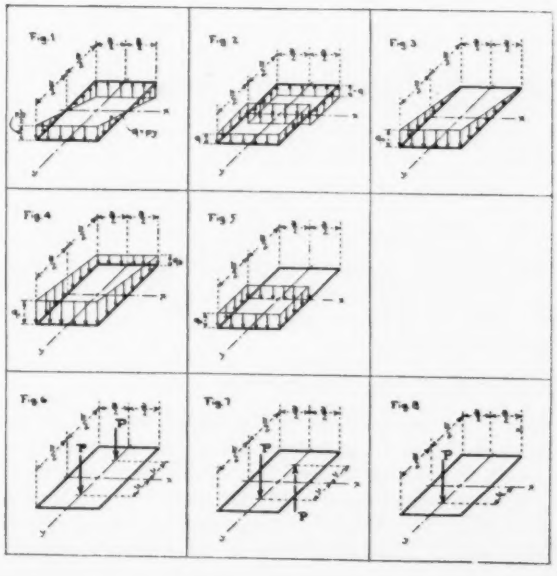
	$\frac{b}{a}=2$	$\frac{b}{a}=1.5$	$\frac{b}{a}=1$	$\frac{b}{a}=\frac{1}{2}$	$\frac{b}{a}=0$
w at $x=0, y=0$	$0.000\,1159\ \frac{qa^4}{D}$	$0.000\,1434\ \frac{qa^4}{D}$	$0.000\,1266\ \frac{qa^4}{D}$	$0.000\,2170\ \frac{qa^4}{D}$	$0.000\,2460\ \frac{qa^4}{D}$
M_y at $x=0, y=\frac{b}{2}$	$-0.0148\ qa^2$	$-0.0353\ qa^2$	$-0.0713\ qa^2$	$-0.0757\ qa^2$	$-0.0833\ qa^2$
M_y at $x=0, y=\frac{b}{4}$	$0.00446\ qa^2$	$0.00719\ qa^2$	$0.01093\ qa^2$	$0.01196\ qa^2$	$0.01042\ qa^2$
M_y at $x=0, y=0$	$0.00396\ qa^2$	$0.00901\ qa^2$	$0.0228\ qa^2$	$0.0368\ qa^2$	$0.0417\ qa^2$
M_x at $x=0, y=0$	$0.01039\ qa^2$	$0.01834\ qa^2$	$0.0229\ qa^2$	$0.0203\ qa^2$	
M_x at $x=0, y=\frac{b}{4}$	$0.00886\ qa^2$	$0.01094\ qa^2$	$0.01254\ qa^2$	$0.00844\ qa^2$	
M_x at $x=\frac{a}{2}, y=0$	$-0.0307\ qa^2$	$-0.0336\ qa^2$	$-0.0713\ qa^2$	$-0.0570\ qa^2$	
M_x at $x=\frac{a}{2}, y=\frac{b}{4}$	$-0.0167\ qa^2$	$-0.0339\ qa^2$	$-0.0324\ qa^2$	$-0.0346\ qa^2$	
Q_y at $x=0, y=\frac{b}{4}$	$-0.33\ qb$	$-0.31\ qb$	$-0.44\ qb$	$-0.52\ qb$	$-0.50\ qb$
Q_x at $x=\frac{a}{2}, y=0$	$-0.36\ qb$	$-0.35\ qb$	$-0.44\ qb$	$-0.47\ qb$	

Table 4. Deflections, moments and shears for triangular loading (see Fig.3)

	$\frac{b}{a}=2$	$\frac{b}{a}=1.5$	$\frac{b}{a}=1$	$\frac{b}{a}=\frac{1}{2}$	$\frac{b}{a}=0$
w at $x=0, y=0$	$0.000\,0880\ \frac{qa^4}{D}$	$0.000\,2171\ \frac{qa^4}{D}$	$0.000\,6301\ \frac{qa^4}{D}$	$0.001\,10\ \frac{qa^4}{D}$	$0.001\,302\ \frac{qa^4}{D}$
M_y at $x=0, y=\frac{b}{2}$	$-0.0113\ qa^2$	$-0.0187\ qa^2$	$-0.0334\ qa^2$	$-0.0462\ qa^2$	$-0.0500\ qa^2$
M_y at $x=0, y=0$	$0.00198\ qa^2$	$0.00451\ qa^2$	$0.0115\ qa^2$	$0.0184\ qa^2$	$0.0208\ qa^2$
M_x at $x=0, y=0$	$0.00515\ qa^2$	$0.00817\ qa^2$	$0.0115\ qa^2$	$0.0102\ qa^2$	
M_x at $x=\frac{a}{2}, y=0$	$-0.0104\ qa^2$	$-0.0168\ qa^2$	$-0.0257\ qa^2$	$-0.0285\ qa^2$	
M_x at $x=\frac{a}{2}, y=\frac{b}{2}$	$-0.0119\ qa^2$	$-0.0163\ qa^2$	$-0.0208\ qa^2$	$-0.0219\ qa^2$	
Q_y at $x=0, y=\frac{b}{2}$	$-0.30\ qb$	$-0.29\ qb$	$-0.32\ qb$	$-0.34\ qb$	$-0.35\ qb$
Q_x at $x=\frac{a}{2}, y=0$	$-0.18\ qb$	$-0.23\ qb$	$-0.27\ qb$	$-0.29\ qb$	

Table 5. Deflections and moments for uniform load over half the plate (see Fig.5)

	$\frac{b}{a}=2$	$\frac{b}{a}=1.5$	$\frac{b}{a}=1$	$\frac{b}{a}=\frac{1}{2}$	$\frac{b}{a}=0$
w at $x=0, y=0$	$0.000\,0880\ \frac{qa^4}{D}$	$0.000\,2171\ \frac{qa^4}{D}$	$0.000\,6301\ \frac{qa^4}{D}$	$0.001\,10\ \frac{qa^4}{D}$	$0.001\,302\ \frac{qa^4}{D}$
M_y at $x=0, y=\frac{b}{2}$	$-0.0140\ qa^2$	$-0.0231\ qa^2$	$-0.0400\ qa^2$	$-0.0534\ qa^2$	$-0.0573\ qa^2$
M_y at $x=0, y=0$	$0.00198\ qa^2$	$0.00451\ qa^2$	$0.0115\ qa^2$	$0.0184\ qa^2$	$0.0208\ qa^2$
M_x at $x=0, y=0$	$0.00515\ qa^2$	$0.00817\ qa^2$	$0.0115\ qa^2$	$0.0102\ qa^2$	
M_x at $x=\frac{a}{2}, y=0$	$-0.0104\ qa^2$	$-0.0168\ qa^2$	$-0.0257\ qa^2$	$-0.0285\ qa^2$	
M_x at $x=\frac{a}{2}, y=\frac{b}{2}$	$-0.0159\ qa^2$	$-0.0211\ qa^2$	$-0.0260\ qa^2$	$-0.0272\ qa^2$	



From these two equations we find

$$G_k = \frac{pa^4(-1)^{(k-1)/2}}{\pi^4 k^4 D}\left[\frac{4}{\pi k} - \frac{b}{a}\left(\frac{\alpha_k}{\sinh^2 \alpha_k} + \coth \alpha_k\right)\right] \quad [13]$$

$$H_m = \frac{pb^4(-1)^m}{16\pi^4 m^4 D}\left[\tanh \delta_m - \frac{\delta_m}{\cosh^2 \delta_m}\right] \quad [14]$$

Substituting these expressions for G_k and H_m into Equations [7] and [9], we obtain the system of equations from which the coefficients A_k and B_m may be calculated. This has been done for several different ratios b/a of the length of the sides and the results are given in Table 2. Using these values, deflections, moments, and shears have been calculated for various critical points and these are given in the same table.

Uniform Load in Opposite Directions on Each Half of Plate.

This loading is shown in Fig. 2. There is a uniform load of intensity q in the region $y > 0$, and a uniform load of intensity $-q$ in the region $y < 0$. The deflection for a simply supported plate carrying this load may be expressed as

$$w_1 = \frac{2pa^4}{\pi^5 D}\sum_{m=1,3,5\ldots}\frac{(-1)^{(m-1)/2}}{m^5}\left[2 - 2\cosh\frac{m\pi y}{a} + \frac{m\pi y}{a}\sinh\frac{m\pi y}{a} + \frac{1}{\sinh^2 \alpha_m}(\alpha_m - \alpha_m \cosh \alpha_m - 2\sinh \alpha_m + 2\sinh \alpha_m \cosh \alpha_m)\sinh\frac{m\pi y}{a} + \frac{1 - \cosh \alpha_m}{\sinh \alpha_m} \times \frac{m\pi y}{a} \times \cosh\frac{m\pi y}{a}\right]\cos\frac{m\pi x}{a} \quad [15]$$

which is valid only in the region $y > 0$. An alternate expression for w_1 which is useful in the determination of H_m is

$$w_1 = \frac{pb^4}{8\pi^5 D}\sum_{m=1,3,5\ldots}\frac{1}{m^5}\left[2 - \frac{2 + \delta_m \tanh \delta_m}{\cosh \delta_m}\cosh\frac{2m\pi x}{b} + \frac{2m\pi x}{b\cosh \delta_m}\sinh\frac{2m\pi x}{b}\right]\sin\frac{2m\pi y}{b} \quad [16]$$

From these two equations the load coefficients are found to be

$$G_k = -\frac{2pa^3(-1)^{(k-1)/2}}{\pi^4 k^4 D}\left[\tanh\frac{\alpha_k}{2} - \frac{\frac{1}{2}\alpha_k}{\cosh^2\frac{\alpha_k}{2}}\right] \quad [17]$$

$$H_m = -\frac{pb^3}{4\pi^4 m^4 D}\left[\tanh \delta_m - \frac{\delta_m}{\cosh^2 \delta_m}\right] \text{ for } m = 1, 3, 5 \ldots . \quad [18]$$

$$H_m = 0 \text{ for } m = 2, 4, 6 \ldots .$$

The results calculated for this case are given in Table 3.

Unsymmetrical Loadings

Solutions for loads which are unsymmetrically distributed about one of the coordinate axes (here taken as the x axis) may be readily obtained by superposing solutions for symmetrical and antisymmetrical loadings.

Triangular (Hydrostatic) Loading.

Let q_0 be the maximum intensity of loading for this case which is shown in Fig. 3. The solution for this load is obtained by superposition of the solutions for a uniform load of intensity $\frac{1}{2}q_0$ and for an antisymmetrical triangular loading of intensity $q = (q_0/b)y$. Results for these two component loadings are given in Tables 1 and 2. Combining these in the manner specified, we obtain the deflections, moments, and shears for the hydrostatic loadings. The values for this case are summarized in Table 4.

Trapezoidal Load Distribution.

For this case which is shown in Fig. 4, the load intensity varies linearly from a minimum value q_2 at $y = -\frac{1}{2}b$, to a maximum q_1 at $y = \frac{1}{2}b$. The average intensity is $q_a = \frac{1}{2}(q_1 + q_2)$ and the increase in load intensity per unit length is $p = q_b/b$ where $q_b = q_1 - q_2$. The results for this case are then found by superposition of the results for a uniform load of intensity q_a and the results for an antisymmetrical triangular load of intensity $q = (q_b/b)y$.

Uniform Load Over Half the Plate.

For this loading which is shown in Fig. 5, there is a uniform load of intensity q_0 over the region $y > 0$, but no load whatsoever in the region $y < 0$. This loading is obtained by superposition of a uniform load of $\frac{1}{2}q_0$ over the entire plate and of an antisymmetrical uniform load of intensity $\frac{1}{2}q_0$ in opposite directions on each half of the plate. Results for the two component loadings may be found in Tables 1 and 3. The deflections and moments for the unsymmetrical load are then as given in Table 5.

Noncentral Concentrated Load.

A plate with a concentrated load P at a point $x = u$, $y = 0$, as shown in Fig. 8, can be handled by superposition of the symmetrical case shown in Fig. 6, and the antisymmetrical case shown in Fig. 7. Solutions for these later two loadings may be carried through by the methods described. As an example, the particular case of a square plate with load P at the quarter point, i.e., $u = \frac{1}{4}b$, has been worked out and the results given in Table 6.

As is well known, in the ordinary theory of bending of plates, the moments under a concentrated load become infinite unless consideration is given to the area over which the load is distributed. This has been discussed by Timoshenko (10) for a simply supported rectangular plate and he has given expressions for this moment in terms of the area over which the load is distributed. Call this moment $(M)_{ss}$. Then for a clamped plate, the moment at the concentrated load may be written as $(M)_{ss} - M_{2,3}$, where $M_{2,3}$ is the moment at the load due the functions w_2 and w_3. In Table 6 the moments at the concentrated load are thus expressed.

Acknowledgment

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The Orthogonally Stiffened Plate Under Uniform Lateral Load

By HENRY A. SCHADE,[1] WASHINGTON, D. C.

The problem of designing structures with orthogonally stiffened plates under lateral loading has been solved approximately by the infinite-strip method and has been applied, for example, to the double-bottom structure of a ship. Under certain conditions where the side ratio, that is, the ratio of the longitudinal dimension to the transverse dimension, is small, the approximate solution is inadequate. The author gives herein an exact solution applicable to all side ratios.

THE problem here discussed may be described as follows: A rectangular grid of orthogonally intersecting beams rests on rigid supports on all four edges, as shown in Fig. 1. The grid may be plated on one or both of its surfaces and supports a uniformly distributed lateral load. On two parallel edges, termed the "transverse edges," the beams are fixed. On the other two edges, termed the "longitudinal edges," the beams are freely supported. The beams in the longitudinal system are all identical and equally spaced, except the central longitudinal beam, which may be stiffer than the others in the system. The beams in the transverse system are all identical and equally spaced but their size and spacing are not necessarily the same as in the other direction.

The nomenclature is as follows:

- p = uniform lateral load
- E = modulus of elasticity
- μ = Poisson's ratio
- w = deflection
- I_x = moment of inertia of longitudinal stiffener (with effective width of plating)
- I_y = moment of inertia of transverse stiffener (with effective width of plating)
- I_m = moment of inertia of central longitudinal stiffener (with effective width of plating)
- i_x = equivalent unit moment of inertia of longitudinal stiffener $= \left(\frac{I_x}{s_x} + 2\,\frac{(I_m - I_x)}{b}\right)$
- i_y = unit moment of inertia of transverse stiffener $= \left(\frac{I_y}{s_y}\right)$
- i_f = average unit moment of inertia of flanges (plating) alone $= \left(\frac{i_{fx} + i_{fy}}{2}\right)$
- η = torsion coefficient $= \left(\frac{i_f}{\sqrt{i_x i_y}}\right)$
- σ_x = bending stress in flanges (plating) in the longitudinal direction
- τ_x = vertical shear stress in longitudinal stiffeners (webs)
- σ_y = bending stress in flanges (plating) in the transverse direction
- τ_y = vertical shear stress in transverse stiffeners (webs)
- A_x = sectional area of web of longitudinal stiffener
- A_m = sectional area of web of central longitudinal stiffener
- A_y = sectional area of web of transverse stiffener
- r_x = distance from neutral axis of longitudinal stiffener to most strained fiber
- r_y = distance from neutral axis of transverse stiffener to most strained fiber

This problem was solved in previous papers (1, 2)[2] approximately by means of the infinite-strip method, whereby the solution for a strip of such stiffened plating, infinitely long, freely supported on the edges, and with a single transverse support was obtained. The solution for the single transverse support was then simply superimposed upon the solution for another such

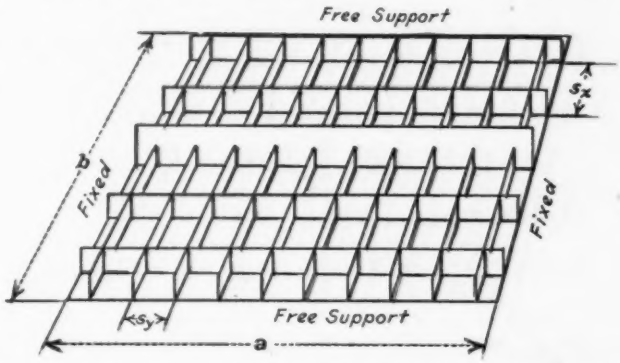


FIG. 1 A RECTANGULAR GRID OF ORTHOGONALLY INTERSECTING BEAMS

transverse support, which gave an approximation to the solution for the plate included between two such transverse supports. In the previous papers (1, 2), this method of solution was applied to the double-bottom structure of a ship. It is equally applicable to deck structure and many other elements of a ship, and also to many airplane structural problems. Under certain conditions, where the side ratio, that is, the ratio of the longitudinal dimension to the transverse dimension, is small, the approximate solution is inadequate; consequently, the following exact solution, applicable to all side ratios, has been developed. It follows, in general, the method used by Huber (3) for the orthotropic plate.

From Equation 30 in one of the author's previous papers (1), the deflection of the infinite strip, with no transverse support, under uniform lateral load, where $n = 1, 3, 5 \ldots$, is

$$w_p = \frac{4pb^4}{\pi^5 E i_y} \sum \frac{1}{n^5} \sin \frac{n\pi y}{b} \quad \ldots\ldots\ldots\ldots [1]$$

and the deflection due to the introduction of a negative transverse lineal load (of which rigid transverse support is a special case) is

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1941, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to the Bibliography at end of paper.

$$-w_b = \sum Ce^{-\alpha x}\left(\cos\beta x + \frac{\alpha}{\beta}\sin\beta x\right)\sin\frac{n\pi y}{b} = \sum CF(x)\sin\frac{n\pi y}{b} \quad [2]$$

where C is a constant depending on the lineal load, and the parameters α and β are

$$\left.\begin{aligned}\alpha &= \frac{n}{b}\frac{\pi}{\sqrt{2}}\sqrt[4]{\frac{i_y}{i_x}}\sqrt{(1+\eta)}\\ \beta &= \frac{\sqrt{(1-\eta)}}{\sqrt{(1+\eta)}}\alpha\end{aligned}\right\} \quad [3]$$

If it be imagined that there exists an infinite number of negative transverse lineal loads equally spaced across the infinite strip, then a panel between any two of these lineal loads will represent the rectangular element of stiffened plating under discussion, provided the boundary conditions at the locations of the lineal loads are suitably determined. If the spacing of these lineal loads is a then the deflection due to an infinite number of such loads may be written

$$-w_b = \sum_{m=0,1,2\ldots}\sum_n CF(ma + x)\sin\frac{n\pi y}{b} + \sum_{m=1,2,3\ldots}\sum_n CF(ma - x)\sin\frac{n\pi y}{b} \quad [4]$$

The first term represents the effect of all lineal loads to the left of a point whose x coordinate from any one station is x and the second term represents the effect of all such loads to the right of that point. The equation may also be written

$$-w_b = \sum_n CF'(x)\sin\frac{n\pi y}{b} \quad [5]$$

where the F' function is

$$F'(x) = \sum_{m=0,1,2} e^{-\alpha(ma+x)}\left[\cos\beta(ma+x) + \frac{\alpha}{\beta}\sin\beta(ma+x)\right] + \sum_{m=1,2,3} e^{-\alpha(ma-x)}\left[\cos\beta(ma-x) + \frac{\alpha}{\beta}\sin\beta(ma-x)\right] \quad [6]$$

The function $F'(x)$, after the trigonometric functions have been expanded and the terms collected, is

$$\begin{aligned}F'(x) = {} & e^{-\alpha x}\left(\cos\beta x + \frac{\alpha}{\beta}\sin\beta x\right)\sum_{m=0,1,2}(e^{-\alpha a})^m\cos\beta ma\\ & + e^{-\alpha x}\left(\frac{\alpha}{\beta}\cos\beta x - \sin\beta x\right)\sum_{m=0,1,2}(e^{-\alpha a})^m\sin\beta ma\\ & + e^{\alpha x}\left(\cos\beta x - \frac{\alpha}{\beta}\sin\beta x\right)\sum_{m=1,2,3,\ldots}(e^{-\alpha a})^m\cos\beta ma\\ & + e^{\alpha x}\left(\frac{\alpha}{\beta}\cos\beta x + \sin\beta x\right)\sum_{m=1,2,3\ldots}(e^{-\alpha a})^m\sin\beta ma \quad [7]\end{aligned}$$

The summations indicated in Equation [7] have finite values and, after evaluating them, rearranging, and eliminating the exponential functions in favor of hyperbolic functions, the equation becomes

$$F'(x) = \frac{1}{\cosh\alpha a - \cos\beta a}\left[\sinh\alpha(a-x)\cos\beta x + \frac{\alpha}{\beta}\cosh\alpha(a-x)\sin\beta x + \sinh\alpha x\cos\beta(a-x) + \frac{\alpha}{\beta}\cosh\alpha x\sin\beta(a-x)\right] \quad [8]$$

The boundary condition for determining the constant C in Equation [5] is that the deflection due to the uniform lateral load alone, according to Equation [1], must be equal and opposite to the deflection due to the infinite number of lineal loads, according to Equation [5], at the transverse supports, so that the two components of deflection cancel each other at the supports, where $x = a, 2a, 3a$, etc. Accordingly, for this condition to be satisfied

$$C = \frac{4pb^4}{\pi^5 Ei_y}\frac{1}{n^5}\frac{\cosh\alpha a - \cos\beta a}{\sinh\alpha a + \frac{\alpha}{\beta}\sin\beta a} \quad [9]$$

The total deflection between supports is, of course, the sum of that due to the uniform lateral load w_p and that due to the lineal loads w_b. These are given by Equations [1] and [5] with the value of C from Equation [9] inserted, so that the expression for total deflection is

$$w = \frac{4pb^4}{\pi^5 Ei_y}\sum\frac{1}{n^5}\sin\frac{n\pi y}{b}\left\{1 - \frac{1}{\sinh\alpha a + \frac{\alpha}{\beta}\sin\beta a}\times\left[\sinh\alpha(a-x)\cos\beta x + \frac{\alpha}{\beta}\cosh\alpha(a-x)\sin\beta a + \sinh\alpha x\cos\beta(a-x) + \frac{\alpha}{\beta}\cosh\alpha x\sin\beta(a-x)\right]\right\} \quad [10]$$

This is the fundamental equation for deflection of the orthogonally stiffened plate panel, clamped on two parallel edges and freely supported on the other two. Undoubtedly, the same result could be obtained by other means, i.e., by considering directly a panel of finite length, and imposing suitable boundary conditions on the two transverse boundaries. Such a solution would probably be more cumbersome to derive than that given here, particularly since the solution for the infinite strip is already available.

When dealing with rectangular panels of unstiffened plating, it is usual to frame the equations in terms of the ratio of the sides of the panel. A similar procedure can be adopted here, using instead of the actual side ratio a "virtual side ratio" ρ defined by

$$\rho = \frac{a}{b}\sqrt[4]{\frac{i_y}{i_x}} \quad [11]$$

Then in lieu of the parameters α and β, defined in Equation [3], it is more convenient to use dimensionless parameters, defined by

$$\left.\begin{aligned}\alpha' &= \frac{\alpha}{n}a = \frac{\pi}{\sqrt{2}}\rho\sqrt{(1+\eta)}\\ \beta' &= \frac{\beta}{n}a = \frac{\pi}{\sqrt{2}}\rho\sqrt{(1-\eta)}\end{aligned}\right\} \quad [12]$$

Then the deflection at the center of the panel, where $x = a/2$ and $y = b/2$, may be written

$$w_{a/2,\,b/2} = \frac{pb^4}{Ei_y}\left(\frac{5}{384} - \frac{8}{\pi^5}\sum\right) = K_1\frac{pb^4}{Ei_y} \quad [13]$$

where

$$\sum = \sum(-1)^{\frac{n-1}{2}}\frac{1}{n^5}\frac{\sinh n\frac{\alpha'}{2}\cos n\frac{\beta'}{2} + \frac{\alpha'}{\beta'}\cosh n\frac{\alpha'}{2}\sin n\frac{\beta'}{2}}{\sinh n\alpha' + \frac{\alpha'}{\beta'}\sin n\beta'}$$

For the special case that $\eta = 1$, and $i_x = i_y$, the summation becomes

$$\sum = \sum (-1)^{\frac{n-1}{2}} \frac{1}{n^5} \frac{\sinh n\frac{\pi}{2}\frac{a}{b} + n\frac{\pi}{2}\frac{a}{b}\cosh n\frac{\pi}{2}\frac{a}{b}}{\sinh n\pi\frac{a}{b} + n\pi\frac{a}{b}} \quad [14]$$

which is the form for the isotropic, unstiffened plate (4). The isotropy of the plate is indicated by the condition that $i_x = i_y$, and the fact that the structure is a solid plate, without stiffening, is indicated by the condition that the torsion coefficient η is equal to unity. This is clear from the physical meaning of η. It is, from its definition, $\eta = i_f/\sqrt{(i_x i_y)}$, the ratio of the unit stiffness of the material able to take horizontal shear stress to the geometric mean of the unit stiffnesses of the material able to take bending stresses in the two coordinate directions. In other words, η is a measure of the effect of horizontal shear or torsion. In a solid plate all material is equally able to take horizontal shear and bending stresses and $\eta = 1$. In a structure made up of horizontal plate and vertical stiffening webs, the webs are unable to take horizontal shear stresses (at least, to any significant degree) and here $0 < \eta < 1$. In a structure made up of a grid of intersecting webs, without horizontal plating, there is no material to take horizontal shear stresses and $\eta = 0$. The theory therefore comprehends unstiffened plates, stiffened plates (so long as the stiffening is orthotropic) and grids, and the equations can be used for any of these cases by choosing the proper value of η.

With the deflection function established, deriving the equations for bending stresses in the flanges (horizontal plating), and vertical shear stresses in the webs, is simply a matter of differentiating the function and applying Equations [1] and [2] from the author's previous paper (1) in the same manner as was done for the approximate solution. The maximum value of bending stress in the longitudinal direction occurs at the center of a transverse (fixed) boundary, with a secondary maximum at the center of the panel. The maximum value of bending stress in the transverse direction occurs at the center of the panel, since the longitudinal edges are freely supported. The maximums of shear in the vertical webs occur at the mid-points of the edges in each case.

The bending stress in the longitudinal direction at the center of a transverse boundary is

$$\sigma_{x(0,b/2)} = \frac{E}{1-\mu^2} r_x \frac{\partial^2 w}{\partial x^2}\Bigg]_{\substack{x=0\\y=b/2}}$$

$$= \frac{pb^2 r_x}{\sqrt{(i_x i_y)}}\left(\frac{1}{7.28} - \frac{8}{0.91\pi^3}\sum\right) = K_2 \frac{pb^2 r_x}{\sqrt{(i_x i_y)}} \quad [15]$$

with the value of 0.3 assigned to μ, and with

$$\sum = \sum (-1)^{\frac{n-1}{2}} \frac{1}{n^3} \frac{\frac{\alpha'}{\beta'}\sin n\beta'}{\sinh n\alpha' + \frac{\alpha'}{\beta'}\sin n\beta'}$$

The bending stress in the longitudinal direction at the center of the panel is

$$\sigma_{x(a/2,b/2)} = \frac{E}{1-\mu^2}\left(r_x\frac{\partial^2 w}{\partial x^2} + \mu r_y \frac{\partial^2 w}{\partial y^2}\right)\Bigg]_{\substack{x=a/2\\y=b/2}}$$

$$= \frac{pb^2 r_x}{\sqrt{(i_x i_y)}}\left[\frac{1}{1-\mu^2}\left(\frac{\mu'}{8} - \frac{8}{\pi^3}\sum\right)\right] = K_3 \frac{pb^2 r_x}{\sqrt{i_x i_y}} \quad [16]$$

where

$$\sum = \sum (-1)^{\frac{n-1}{2}} \frac{1}{n^3} \frac{(1+\mu')\sinh n\frac{\alpha'}{2}\cos n\frac{\beta'}{2} - (1-\mu')\frac{\alpha'}{\beta'}\cosh n\frac{\alpha'}{2}\sin n\frac{\beta'}{2}}{\sinh n\alpha' + \frac{\alpha'}{\beta'}\sin n\beta'}$$

and

$$\mu' = \mu\frac{r_y}{r_x}\sqrt{\frac{i_x}{i_y}}$$

To a close degree of approximation μ' may be assumed equal to μ, and with the value of 0.3 assigned to μ', the equation becomes

$$\sigma_{x(a/2,b/2)} = \frac{pb^2 r_x}{\sqrt{(i_x i_y)}}\left(\frac{0.3}{7.28} - \frac{8}{0.91\pi^3}\sum\right) = K_3\frac{pb^2 r_x}{\sqrt{(i_x i_y)}} \quad [17]$$

where

$$\sum = \sum (-1)^{\frac{n-1}{2}} \frac{1}{n^3} \frac{1.3\sinh n\frac{\alpha'}{2}\cos n\frac{\beta'}{2} - 0.7\frac{\alpha'}{\beta'}\cosh n\frac{\alpha'}{2}\sin n\frac{\beta'}{2}}{\sinh n\alpha' + \frac{\alpha'}{\beta'}\sin n\beta'}$$

The bending stress in the transverse direction at the center of the panel is

$$\sigma_{y(a/2,b/2)} = \frac{E}{1-\mu^2}\left(r_y\frac{\partial^2 w}{\partial y^2} + \mu r_x\frac{\partial^2 w}{\partial x^2}\right)\Bigg]_{\substack{x=a/2\\y=b/2}}$$

$$= \frac{pb^2 r_y}{i_y}\left(\frac{1}{7.28} - \frac{8}{0.91\pi^3}\sum\right) = K_4\frac{pb^2 r_y}{i_y} \quad [18]$$

where

$$\sum = \sum (-1)^{\frac{n-1}{2}} \frac{1}{n^3} \frac{1.3\sinh n\frac{\alpha'}{2}\cos n\frac{\beta'}{2} + 0.7\frac{\alpha'}{\beta'}\cosh n\frac{\alpha'}{2}\sin n\frac{\beta'}{2}}{\sinh n\alpha' + \frac{\alpha'}{\beta'}\sin n\beta'}$$

with the value of 0.3 assigned to μ, and using again the close approximation that

$$\frac{r_x}{r_y}\sqrt{\frac{i_y}{i_x}} = 1 \quad [19]$$

The shear stress in the longitudinal webs at a transverse boundary is

$$\tau_{x(0,y)} = -E\frac{i_x s_x}{A_x}\frac{\partial^2 w}{\partial x^2}\Bigg]_{x=0} = \frac{pbi_x s_x}{A_x\sqrt[4]{(i_x^3 i_y)}}$$

$$\left[\frac{4\sqrt{2}}{\pi^2}\sqrt{(1+\eta)}\sum\frac{1}{n^2}\frac{\cosh n\alpha' - \cos n\beta'}{\sinh n\alpha' + \frac{\alpha'}{\beta'}\sin n\beta'}\sin\frac{n\pi y}{b}\right] \quad [20]$$

At the center of a transverse boundary this becomes

$$\tau_{x(0,\,b/2)} = \frac{pbI_m}{A_m\sqrt[4]{(i_x{}^2 i_y)}}\left[\frac{4\sqrt{2}}{\pi^2}\sqrt{(1+\eta)}\sum\right] = K_5\frac{pbI_m}{A_m\sqrt[4]{(i_x{}^2 i_y)}} \quad \ldots [21]$$

where

$$\sum = \sum (-1)^{\frac{n-1}{2}}\frac{1}{n^2}\frac{\cosh n\alpha' - \cos n\beta'}{\sinh n\alpha' + \frac{\alpha'}{\beta'}\sin n\beta'}$$

and this is the maximum value unless the cross-sectional area of the central longitudinal stiffener A_m is considerably greater than the area of the other longitudinal stiffeners A_s, in which case the maximum value of vertical shear stress will occur in the first web adjacent to the central longitudinal web. Equation [20] would then be used for the maximum. In case the central longitudinal web is identical with the other longitudinal webs, Equation [21] becomes

$$\tau_{x(0,\,b/2)} = K_5\frac{pbs_x}{A_x}\sqrt[4]{\frac{i_x}{i_y}} \quad \ldots [22]$$

The vertical shear stress in the transverse web at the center of a longitudinal boundary is

$$\tau_{y(a/2,\,0)} = -\frac{Es_y}{A_y}\left(i_y\frac{\partial^3 w}{\partial y^3} + i_f\frac{\partial^3 w}{\partial x^2\partial y}\right)\Bigg]_{\substack{x=a/2\\ y=0}}$$

$$= \frac{pbs_y}{A_y}\left[\frac{1}{2} - \frac{8}{\pi^2}\sum\right] = K_6\frac{pbs_y}{A_y} \quad \ldots [23]$$

where

$$\sum = \sum\frac{1}{n^2}\,\frac{(1+\eta)\sinh n\frac{\alpha'}{2}\cos n\frac{\beta'}{2} + (1-\eta)\frac{\alpha'}{\beta'}\cosh n\frac{\alpha'}{2}\sin n\frac{\beta'}{2}}{\sinh n\alpha' + \frac{\alpha'}{\beta'}\sin n\beta'}$$

It will be noted from Equations [13], [15], [17], [18], [21], and [23] that the maximum deflection, bending, and shear stresses are all expressed in terms of the loading, the dimensions of the structure, and the modulus of elasticity, multiplied by coefficients (the K's) which are functions only of the virtual side ratio ρ and the torsion coefficient η. Thus it is possible to represent the numerical values of these coefficients (K's) by contour curves for various values of η with the values of the virtual side ratio as abscissa. Such curves for the maximum bending stress in the longitudinal direction, calculated from Equation [15], are shown in Fig. 2.

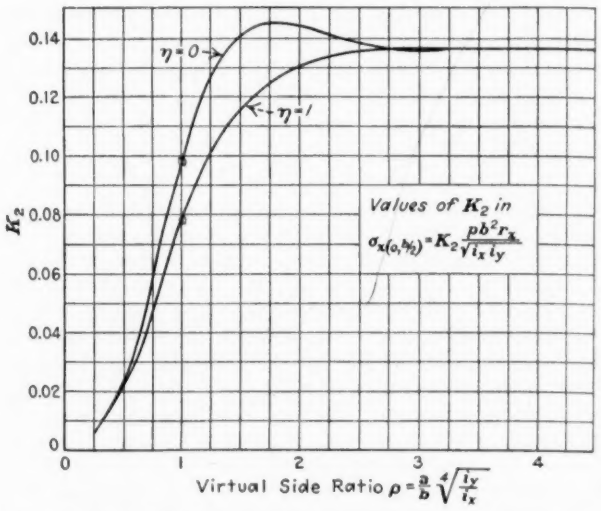


FIG. 2 CURVES FOR MAXIMUM BENDING STRESS IN THE LONGITUDINAL DIRECTION CALCULATED FROM EQUATION [15]

Since for the unstiffened plate η is equal to unity, the curve for that value of η will be identical with the curve for the unstiffened plate with the same loading and boundary conditions, as given, for example, in the curves for case 5 in the Wojtaszak Design Data (5). Also, if the stiffening on a stiffened plate is such that η may be assumed equal to unity as an approximation, then curves for unstiffened plating may be used for stiffened plating, provided the "virtual side ratio" is taken into account. Curves for other values of η can be used for design purposes for stiffened plating exactly as the Wojtaszak curves (5) are used for unstiffened plating. The calculation of maximum deflection and stresses in orthogonally stiffened plating, with the assumed loading and boundary conditions, becomes no more difficult than the calculation of equivalent values in unstiffened plating.

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Some Observations on the Theory of Contact Pressures

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The stress system in a semi-infinite elastic solid with given normal surface loading is of importance in many problems of machine and structural design and strength of materials. It often happens in such cases that the stresses in which one is primarily interested are those in the surface of the body. This paper presents some observations that can be made regarding the stresses and displacements in the surface.

SOME of the observations that can be made regarding stresses and displacements in a semi-infinite elastic solid with given normal surface loading are briefly as follows:

(*a*) There are a number of cases in which an infinite number of different load distributions inside a region R of the surface produce the same system of stresses and horizontal displacements in the surface outside R.

(*b*) The horizontal displacements in the surface are shown to stand in a simple proportionality relation to the slopes in the x and y directions of a stretched membrane loaded in the same way as the elastic solid. The stresses in the surface are related to the second derivatives of the membrane deflection.

(*c*) An analogy can be set up between the stresses in the surface of the semi-infinite solid, when it carries a loading $q = f(x)$ along a line segment from $x = -c$, $y = 0$ to $x = c$, $y = 0$, and the shearing stresses on certain planes in a semi-infinite solid in a plane state of strain loaded by the pressure distribution $p = f(x)$ along the band between $x = -c$ and $x = +c$. The same analogy also holds when the latter body is a thin sheet in a state of plane stress.

The Semi-Infinite Body With Vertical Loading on the Surface

The first serious attempt to deal with this problem was made by Lamé and Clapeyron in 1828(1).[2] However, they used a series method which led to quadruple integral expressions and was hence of slight practical value. It was not until 1878 that Boussinesq (2), using the method of singularities developed earlier by Green in his potential theory, obtained a satisfactory solution.

Let the plane $z = 0$ be the bounding surface and let the body fill all space for $z > 0$. Suppose a concentrated load P is applied at the point $(x', y', 0)$ as shown in Fig. 1. We want to know the displacements and stresses at a general point (x, y, z). Let

$$\psi = P/\rho \quad [1]$$

$$\phi = P \log (\rho + z) \quad [2]$$

$$\rho = \sqrt{[(x-x')^2 + (y-y')^2 + z^2]} \quad [3]$$

Boussinesq showed that the displacements and stresses at the point x, y, z could be expressed in terms of ψ and ϕ as follows

$$\left.\begin{aligned} u &= -\frac{z}{4\pi G}\frac{\partial\psi}{\partial x} - \frac{1-2\mu}{4\pi G}\frac{\partial\phi}{\partial x} \\ v &= -\frac{z}{4\pi G}\frac{\partial\psi}{\partial y} - \frac{1-2\mu}{4\pi G}\frac{\partial\phi}{\partial y} \\ w &= -\frac{z}{4\pi G}\frac{\partial\psi}{\partial z} + \frac{1-\mu}{2\pi G}\psi \end{aligned}\right\} \quad [4]$$

$$\left.\begin{aligned} \sigma_x &= \frac{\mu}{\pi}\frac{\partial\psi}{\partial z} - \frac{z}{2\pi}\frac{\partial^2\psi}{\partial x^2} - \frac{1-2\mu}{2\pi}\frac{\partial^2\phi}{\partial x^2} & \tau_{xy} &= -\frac{z}{2\pi}\frac{\partial^2\psi}{\partial x\partial y} - \frac{1-2\mu}{2\pi}\frac{\partial^2\phi}{\partial x\partial y} \\ \sigma_y &= \frac{\mu}{\pi}\frac{\partial\psi}{\partial z} - \frac{z}{2\pi}\frac{\partial^2\psi}{\partial y^2} - \frac{1-2\mu}{2\pi}\frac{\partial^2\phi}{\partial y^2} & \tau_{yz} &= -\frac{z}{2\pi}\frac{\partial^2\psi}{\partial y\partial z} \\ \sigma_z &= \frac{1}{2\pi}\frac{\partial\psi}{\partial z} - \frac{z}{2\pi}\frac{\partial^2\psi}{\partial z^2} & \tau_{zx} &= -\frac{z}{2\pi}\frac{\partial^2\psi}{\partial z\partial x} \end{aligned}\right\} \quad [5]$$

For any normal surface loading over a region R, Equations [4] and [5] remain valid if we simply form the integrals, as did Boussinesq, of the right-hand members of Equations [1] and [2], replacing P by $p\,dx'dy'$. For any distributed load over R, therefore, ψ and ϕ become

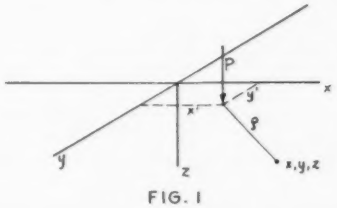


FIG. 1

$$\psi = \int_R\int \frac{p}{\rho}\,dx'dy' \quad [6]$$

$$\phi = \int_R\int p \log (\rho + z)\,dx'dy' \quad [7]$$

Equations [1] and [2] may now be regarded merely as special cases of Equations [6] and [7]. Coordinates with primes will always refer to a point where a load is acting.

One may easily verify by differentiation that ψ and ϕ satisfy Laplace's equation everywhere inside the body

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = 0; \quad \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad [8]$$

It should be noted, however, that ψ does not satisfy Laplace's equation in the loaded area itself, for $\partial\psi/\partial z$ is discontinuous at $z = 0$ for points inside R, as shown in the following:

In case the load is distributed along a curve C with intensity ω per unit length, the integrals for ψ and ϕ become

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[1] Research Engineer, Westinghouse Electric & Mfg. Co.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Presented at the National Meeting of the Applied Mechanics Division, Ann Arbor, Mich., June 20–21, 1940, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1941, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

$$\psi = \int_C \frac{\omega}{\rho}\, ds \quad [9]$$

$$\phi = \int_C \omega \log (\rho + z) ds \quad [10]$$

where ds is an element of arc of C.

We also observe that in all cases

$$\psi = \partial\phi/\partial z \quad [11]$$

We see that ψ, as defined by Equation [6], is the Newtonian potential due to a distribution of matter of surface density p in the plane $z = 0$. Suppose we have a volume V whose surface is S

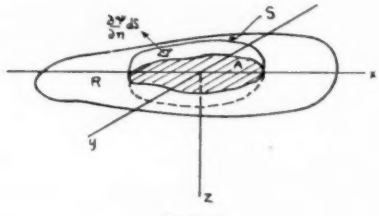


FIG. 2

and which intersects the plane $z = 0$ in an area A, as shown in Fig. 2. The theorem of Gauss then takes the form

$$\int_S \int \frac{\partial\psi}{\partial n}\, dS = -4\pi \int_A \int p dA \quad [12]$$

If we take for the volume V a short right cylinder whose ends are in the planes $z = \delta z$, $z = -\delta z$ and whose section is dS, we will have $dS = dA$, and by letting δz approach zero we will obtain

$$\left(\frac{\partial\psi}{\partial z}\right)_+ - \left(\frac{\partial\psi}{\partial z}\right)_- = -4\pi p \quad [13]$$

There is thus a discontinuity in $\partial\psi/\partial z$ in the plane $z = 0$ for points inside R of amount $-4\pi p$. Outside R, since $p = 0$, there is no discontinuity in $\partial\psi/\partial z$. Since ψ is symmetrical about the plane $z = 0$, we have

$$\left(\frac{\partial\psi}{\partial z}\right)_+ = -2\pi p \quad [14]$$

Hence, for $z = 0$, the equation for σ_z in Equations [5] gives $\sigma_z = -p$ inside R and $\sigma_z = 0$ outside R, since p is then equal to zero.

For the displacements and stresses in the surface $z = 0$ we will introduce the subscript zero. From Equations [4] and [5] these quantities become

$$u_0 = -\frac{1-2\mu}{4\pi G}\frac{\partial\phi_0}{\partial x};\quad v_0 = -\frac{1-2\mu}{4\pi G}\frac{\partial\phi_0}{\partial y};\quad w_0 = \frac{1-\mu}{2\pi G}\psi_0 \quad [15]$$

$$\left.\begin{aligned} &\sigma_{x0} = -2\mu p - \frac{1-2\mu}{2\pi}\frac{\partial^2\phi_0}{\partial x^2};\quad \tau_{xy0} = -\frac{1-2\mu}{2\pi}\frac{\partial^2\phi_0}{\partial x\partial y}\\ &\sigma_{y0} = -2\mu p - \frac{1-2\mu}{2\pi}\frac{\partial^2\phi_0}{\partial y^2};\quad \tau_{yz0} = 0\\ &\sigma_{z0} = -p;\qquad\qquad \tau_{zx0} = 0 \end{aligned}\right\} \quad [16]$$

in which ψ_0 and ϕ_0 are defined by the integrals

$$\psi_0 = \int_R \int \frac{p}{\rho_0}\, dx'dy' \quad [17]$$

$$\phi_0 = \int_R \int p \log \rho_0 dx'dy' \quad [18]$$

where

$$\rho_0 = \sqrt{[(x-x')^2 + (y-y')^2]} \quad [19]$$

If we use the symbol Δ for the Laplacian operator in two dimensions

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad [20]$$

the function ϕ_0 will satisfy Poisson's equation in its two-dimensional form

$$\Delta\phi_0 = 2\pi p \quad [21]$$

Everywhere outside R where $p = 0$, we will have

$$\Delta\phi_0 = 0 \quad [22]$$

The sum of the stresses σ_{x0} and σ_{y0} becomes

$$\sigma_{x0} + \sigma_{y0} = -(1 + 2\mu)p \quad [23]$$

which reduces to zero everywhere outside R.

If A is a region in the plane $z = 0$ which contains part or all of R,

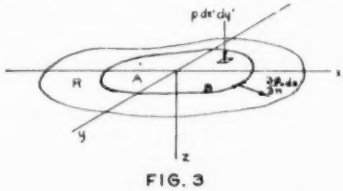


FIG. 3

and if B is the boundary of A, as shown in Fig. 3, Gauss's theorem in two dimensions tells us that

$$\oint_B \left(\frac{\partial\phi_0}{\partial n}\right) ds = 2\pi \int_A \int p dx'dy' \quad [24]$$

or the total outward flux of the gradient of a logarithmic potential ϕ_0 due to sources p per unit area in a region A is equal to 2π times the total of the sources in A.

If the loading is of intensity ω along a curve C, we may apply Gauss's theorem, Equation [24], to a small rectangle enclosing an element ds of C and derive the value of the discontinuity in $\partial\phi_0/\partial n$ at C, or

$$\left(\frac{\partial\phi_0}{\partial n}\right)_1 - \left(\frac{\partial\phi_0}{\partial n}\right)_2 = 2\pi\omega \quad [25]$$

We may call attention to the fact that the theory of elasticity breaks down near curves carrying loading along their length and near concentrated loads, as the displacements become discontinuous or excessively large at such places.

In terms of polar coordinates, the displacements and stresses in the surface are given by

$$\delta_{r0} = -\frac{1-2\mu}{4\pi G}\frac{\partial\phi_0}{\partial r};\quad \delta_{\theta 0} = -\frac{1-2\mu}{4\pi G}\frac{1}{r}\frac{\partial\phi_0}{\partial\theta};\quad w_0 = \frac{1-\mu}{2\pi G}\psi_0 \quad [26]$$

$$\left.\begin{aligned} &\sigma_{r0} = -2\mu p - \frac{1-2\mu}{2\pi}\frac{\partial^2\phi_0}{\partial r^2};\qquad \sigma_{z0} = -p\\ &\sigma_{\theta 0} = -2\mu p - \frac{1-2\mu}{2\pi}\left(\frac{1}{r}\frac{\partial\phi_0}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi_0}{\partial\theta^2}\right);\\ &\tau_{r\theta 0} = -\frac{1-2\mu}{2\pi}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi_0}{\partial\theta}\right) \end{aligned}\right\} \quad [27]$$

Different Loadings Producing the Same Surface Stresses

There are a number of cases in which the logarithmic potential ϕ_0 defined by Equation [18] will be of a certain form outside a

region R for an infinite number of different load distributions inside R. It follows that in such cases, an infinite number of different load distributions inside R will give rise to the same stresses and horizontal displacements in the surface outside R. We here list some of these cases. Each one is discussed more fully in the Appendix.

Case I. Let R be a circle of radius a with center at the origin. Any radially symmetrical pressure distribution $p = f(r')$ within R as shown in Fig. 4*a* will produce the same logarithmic potential ϕ_0

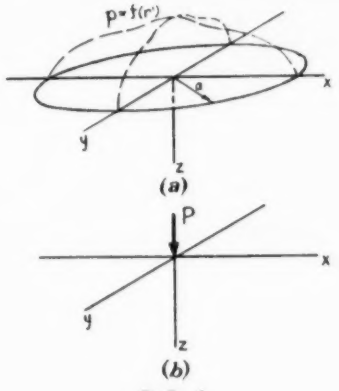


FIG. 4

and the same horizontal displacements and stresses in the surface outside R as a concentrated force P at the origin as shown in Fig. 4*b*, providing

$$P = \int_0^a 2\pi r' f(r') dr' \quad \ldots\ldots\ldots\ldots\ldots [28]$$

that is, if P is the total of the pressures p.

For both cases of loading, if $r > a$, we will have

$$\phi_0 = P \log r; \quad \delta_{r0} = -\frac{1-2\mu}{4\pi G}\cdot\frac{P}{r}; \quad \delta_{\theta 0} = 0 \ldots\ldots [29]$$

$$\left.\begin{aligned} \sigma_{r0} &= \frac{1-2\mu}{2\pi}\cdot\frac{P}{r^2}; \quad \sigma_{z0} = 0 \\ \sigma_{\theta 0} &= -\frac{1-2\mu}{2\pi}\cdot\frac{P}{r^2}; \quad \tau_{r\theta 0} = 0 \end{aligned}\right\} \ldots\ldots\ldots [30]$$

Any self-equilibrating vertical loading, radially symmetrical, distributed over R will produce no stresses or horizontal displacements in the surface outside R.

Case II. Suppose we have a loading ω per unit length of arc around a circle of radius a which varies according to the law

$$\omega = \frac{m}{\pi a^2}\cos\theta' \ldots\ldots\ldots\ldots\ldots [31]$$

as shown in Fig. 5*a*. This loading will produce the same logarithmic potential and the same stresses and horizontal displacements in the surface outside the circle as a concentrated couple m at the origin, with axis along OY, as shown in Fig. 5*b*. The logarithmic potential and the horizontal displacements and stresses will be, for $r > a$

$$\phi_0 = -\frac{m}{r}\cos\theta; \quad \delta_{r0} = -\frac{1-2\mu}{4\pi G}\cdot\frac{m}{r^2}\cos\theta;$$

$$\delta_{\theta 0} = -\frac{1-2\mu}{4\pi G}\cdot\frac{m}{r^2}\sin\theta \ldots\ldots [32]$$

$$\left.\begin{aligned} \sigma_{r0} &= \frac{1-2\mu}{\pi}\cdot\frac{m}{r^3}\cos\theta; \quad \sigma_{z0} = 0 \\ \sigma_{\theta 0} &= -\frac{1-2\mu}{\pi}\cdot\frac{m}{r^3}\cos\theta; \quad \tau_{r\theta 0} = \frac{1-2\mu}{\pi}\cdot\frac{m}{r^3}\sin\theta \end{aligned}\right\} \ldots [33]$$

These expressions are, of course, also valid for $r < a$ for the concentrated moment loading.

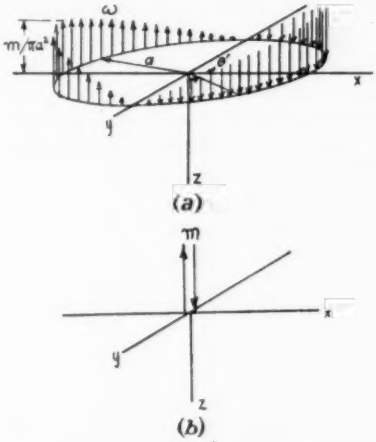


FIG. 5

The moment of the ω loading about OY is equal to m.

Cases of loading of the form

$$p = f(r')\cos\theta' \ldots\ldots\ldots\ldots\ldots [34]$$

over a circle of radius a can be investigated by superposition of ring loadings of the type shown in Fig. 5*a*. The equivalent couple M at the origin with axis along OY would be the moment of the p loading about OY.

Case III. Let $p(x', y')$ be any load distribution over a region R' and let C be a closed curve outside of and enclosing R', as shown in Fig. 6*a*. Let R be the region bounded by the curve C

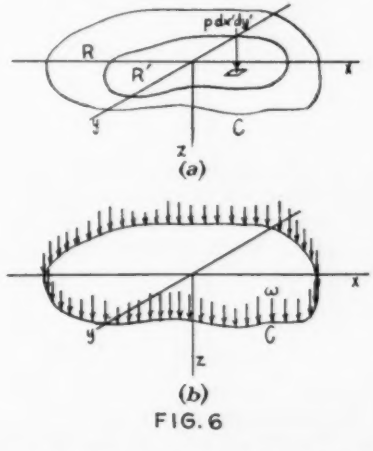


FIG. 6

and let ϕ_0 be the logarithmic potential due to loading p. The logarithmic potential, the stresses, and horizontal displacements outside R due to loading $p(x', y')$ will be the same as those due to a loading ω distributed along C as shown in Fig. 6*b* where ω is defined by the relation

$$\omega = \frac{1}{2\pi}\left(\frac{\partial\phi'}{\partial n_1} - \frac{\partial\phi'}{\partial n_2}\right) \ldots\ldots\ldots\ldots [35]$$

Here, ϕ' is a logarithmic potential function defined by the requirements that $\Delta\phi' = 0$ inside C and $\phi' = \phi_0$ outside C; the outward normal component of the gradient of ϕ' takes the value $\partial\phi'/\partial n_1$ just outside C and the value $\partial\phi'/\partial n_2$ just inside C. The potential ϕ' is that due to the loading ω alone. The total of the ω loading is equal to the total amount of the p loading on R'.

An important subcase of III is that for which ϕ_0 is constant on C. Then ϕ' is constant in R and ω is simply given by

$$\omega = \frac{1}{2\pi}\left(\frac{\partial\phi_0}{\partial n}\right)_C \quad \ldots\ldots [36]$$

An interesting example of this subcase is the following. Let the loaded region be a line segment extending from $x = -c$, $y = 0$ to $x = c$, $y = 0$, and let the intensity of loading per unit length of this segment be given by

$$q = \frac{Q}{\pi c\sqrt{\left[1 - \left(\frac{x'^2}{c^2}\right)\right]}} \quad \ldots\ldots [37]$$

where Q is the total amount of the load. This loading is represented in Fig. 7*a*. The logarithmic potential ϕ_0 due to this loading, the horizontal displacements, and the stresses in the surface will be the same as those due to a loading ω distributed around an ellipse C, whose foci are the end points of the line segment, where ω varies according to the law

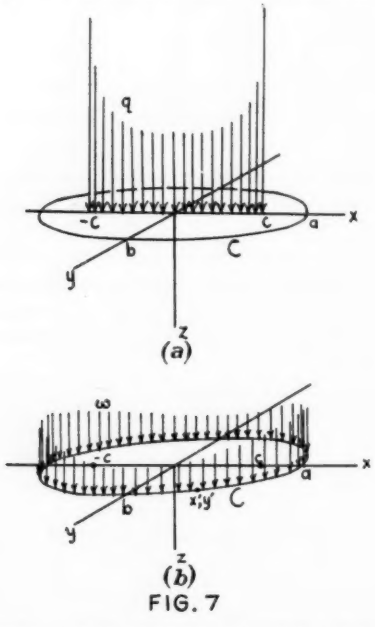


FIG. 7

$$\omega = \frac{Q}{2\pi ab\sqrt{\left(\frac{x'^2}{a^4} + \frac{y'^2}{b^4}\right)}} \quad \ldots\ldots [38]$$

Here a and b are the semiaxes of the ellipse, and (x', y') is a point on the ellipse. The total of the ω loading is Q. This loading is shown in Fig. 7*b*.

We may go one step further and replace the ellipse C by a narrow ring between the geometrically similar ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots\ldots [39]$$

$$\frac{x^2}{(a - ak)^2} + \frac{y^2}{(b - bk)^2} = 1 \quad \ldots\ldots [40]$$

and apply on this ring the uniform pressure

$$p_0 = \frac{Q}{2\pi abk} \quad \ldots\ldots [41]$$

The load per unit length on the ring will be everywhere equal to ω, and the total load is Q. The outer boundary of the ring, given by Equation [39], is the ellipse C. If the ring is very narrow the logarithmic potential due to the loading upon it, the horizontal diplacements, and stresses outside C will be the same as for the ω loading on C or the q loading on the line segment.

To describe this stress system it is convenient to use elliptic coordinates λ, ν defined by

$$\left.\begin{aligned} \frac{x^2}{c^2\cosh^2\lambda} + \frac{y^2}{c^2\sinh^2\lambda} &= 1 \\ \frac{x^2}{c^2\cos^2\nu} - \frac{y^2}{c^2\sin^2\nu} &= 1 \end{aligned}\right\} \quad \ldots\ldots [42]$$

Outside C, the logarithmic potential, the horizontal displacements, and stresses will be

$$\phi_0 = Q\left(\lambda - \log\frac{2}{c}\right);\ \delta_\lambda = -\frac{1-2\mu}{4\pi G}Qh;\ \delta_\nu = 0 \quad \ldots [43]$$

$$\left.\begin{aligned} \sigma_\lambda &= \frac{1-2\mu}{2\pi}Qc^2h^4\sinh\lambda\cosh\lambda \\ \sigma_\nu &= -\frac{1-2\mu}{2\pi}Qc^2h^4\sinh\lambda\cosh\lambda \\ \tau_{\lambda\nu} &= \frac{1-2\mu}{2\pi}Qc^2h^4\sin\nu\cos\nu \end{aligned}\right\} \quad \ldots\ldots [44]$$

in which

$$h = \frac{1}{c\sqrt{(\cos^2\nu\sinh^2\lambda + \sin^2\nu\cosh^2\lambda)}} \quad \ldots\ldots [45]$$

The value of λ on the ellipse C is $\lambda_0 = \cosh^{-1} a/c$. The quantity h is the magnitude of the gradient of λ or the gradient of ν. Inside C, for the case of the loaded ellipse, the stresses and horizontal displacements are zero, since ϕ_0 is constant in that region.

Case IV. Any two uniformly loaded confocal elliptical areas give rise to the same potential ϕ_0 in the region external to both if the total loads are the same. Consequently, the stresses and horizontal displacements due to two such loadings are alike in the region external to both.

More specifically, let C and C_1 be ellipses whose equations are

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots\ldots [46]$$

$$\frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1 \quad \ldots\ldots [47]$$

and let

$$a^2 - b^2 = a_1^2 - b_1^2 = c^2 \quad \ldots\ldots [48]$$

so that the ellipses are confocal. We assume $a > a_1$, $b > b_1$. Let p_0 be the pressure on the area within C and p_1 be the pressure on the area within C_1. (It is not meant that we are superimposing these loadings; we are comparing them, assuming each exists separately.) The total loads on the two ellipses will be alike if

$$p_1a_1b_1 = p_0ab = Q/\pi \quad \ldots\ldots [49]$$

Both loaded areas will then give rise to the same logarithmic potential outside C.

Now, let b_1 approach zero while a_1 approaches c and $p_1a_1b_1$ re-

mains constant. The inner ellipse C_1 approaches the line segment from $(-c, 0)$ to $(+c, 0)$ and the loading becomes a force per unit length on this segment of amount

$$q = \frac{2Q}{\pi c}\sqrt{\left[1 - \left(\frac{x'^2}{c^2}\right)\right]} \quad \ldots\ldots [50]$$

The stresses and horizontal displacements in the surface outside C due to the q loading, as shown in Fig. 8*a*, will be the same as

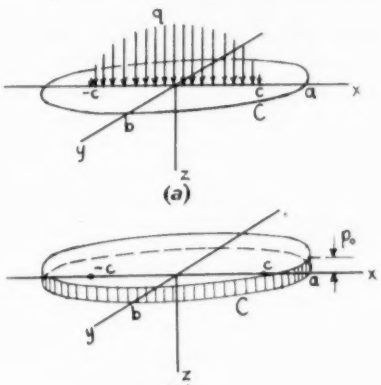


FIG. 8

those due to the uniformly loaded ellipse shown in Fig. 8*b*. These quantities, for the region outside C in both cases, will be

$$\left.\begin{aligned} u_0 &= -\frac{1-2\mu}{2\pi G}\left(\frac{Qx}{\sqrt{c^2+\gamma}\,(\sqrt{c^2+\gamma}+\sqrt{\gamma})}\right); \\ v_0 &= -\frac{1-2\mu}{2\pi G}\left(\frac{Qy}{\sqrt{\gamma}\,(\sqrt{c^2+\gamma}+\sqrt{\gamma})}\right) \end{aligned}\right\} \quad \ldots [51]$$

$$\left.\begin{aligned} \sigma_{x0} &= \frac{(1-2\mu)Q}{\pi c^2}\left[\sqrt{\frac{\gamma+c^2}{\gamma}}\left(\frac{\gamma^2}{\gamma^2+y^2c^2}\right)-1\right] \\ \sigma_{y0} &= -\sigma_{x0} \\ \tau_{xy0} &= \frac{(1-2\mu)}{\pi}\cdot\frac{Qxy\sqrt{\gamma}}{\sqrt{c^2+\gamma}\,(\gamma^2+y^2c^2)} \end{aligned}\right\} \quad \ldots [52]$$

where

$$\gamma = \frac{1}{2}\left\{x^2+y^2-c^2+\sqrt{[(x^2+y^2-c^2)^2+4y^2c^2]}\right\} \quad \ldots [53]$$

These expressions are also valid inside C for the case of the loaded line segment shown in Fig. 8*a*.

Case V. Compare the effects of a semiellipsoidal loading which varies according to the relation

$$p = p_0\sqrt{\left(1-\frac{x'^2}{a^2}-\frac{y'^2}{b^2}\right)} \quad \ldots\ldots [54]$$

over an ellipse C of semiaxes a and b, with a parabolic loading along a line segment from $(-c, 0)$ to $(+c, 0)$ which is given by

$$q = \frac{\pi abp_0}{2c}\left(1-\frac{x'^2}{c^2}\right) \quad \ldots\ldots [55]$$

where

$$c = \sqrt{(a^2-b^2)}$$

The two loadings are shown in Figs. 9*a* and 9*b*. The total amounts of the two loadings are alike, being

$$Q = \frac{2}{3}\pi abp_0 \quad \ldots\ldots [56]$$

The horizontal displacements and stresses outside C will be the same for both cases of loading. They are found to be

$$\left.\begin{aligned} u_0 &= -\frac{(1-2\mu)p_0ab}{8Gc}\left[\frac{2xy}{c^2}\left(\tan^{-1}\frac{x-c}{y}-\tan^{-1}\frac{x+c}{y}\right)\right. \\ &\left.+\frac{1}{2}\left(\frac{x^2}{c^2}-\frac{y^2}{c^2}-1\right)\log\frac{(x-c)^2+y^2}{(x+c)^2+y^2}+\frac{2x}{c}\right] \\ v_0 &= -\frac{(1-2\mu)p_0ab}{8Gc}\left[\left(\frac{x^2}{c^2}-\frac{y^2}{c^2}-1\right)\left(\tan^{-1}\frac{x-c}{y}\right.\right. \\ &\left.\left.-\tan^{-1}\frac{x+c}{y}\right)-\frac{2y}{c}-\frac{xy}{c^2}\log\frac{(x-c)^2+y^2}{(x+c)^2+y^2}\right] \end{aligned}\right\} \quad [57]$$

$$\left.\begin{aligned} \sigma_{x0} &= -\frac{(1-2\mu)}{2c^2}p_0ab\left[2+\frac{x}{2c}\log\frac{(x-c)^2+y^2}{(x+c)^2+y^2}\right. \\ &\left.+\frac{y}{}\left(\tan^{-1}\frac{x-c}{y}-\tan^{-1}\frac{x+c}{y}\right)\right] \\ \sigma_{y0} &= -\sigma_{x0} \\ \tau_{xy0} &= -\frac{(1-2\mu)}{2c^2}p_0ab\left[\frac{x}{c}\left(\tan^{-1}\frac{x-c}{y}\right.\right. \\ &\left.\left.-\tan^{-1}\frac{x+c}{y}\right)-\frac{y}{2c}\log\frac{(x-c)^2+y^2}{(x+c)^2+y^2}\right] \end{aligned}\right\} \quad [58]$$

These expressions are also valid inside C for the case of the q loading.

At the ends of the major and minor axes the stresses in the surface become

$$\left.\begin{aligned} (\sigma_{x0})_a &= -(\sigma_{y0})_a = -\frac{(1-2\mu)}{2c^2}p_0ab\left(2+\frac{a}{c}\log\frac{a-c}{a+c}\right) \\ (\sigma_{x0})_b &= -(\sigma_{y0})_b = -\frac{(1-2\mu)}{2c^2}p_0ab\left(2-\frac{2b}{c}\tan^{-1}\frac{c}{b}\right) \\ (\tau_{xy0})_a &= (\tau_{xy0})_b = 0 \end{aligned}\right\} \quad [59]$$

These values agree with those obtained by Belajef, referred to by Timoshenko (3).

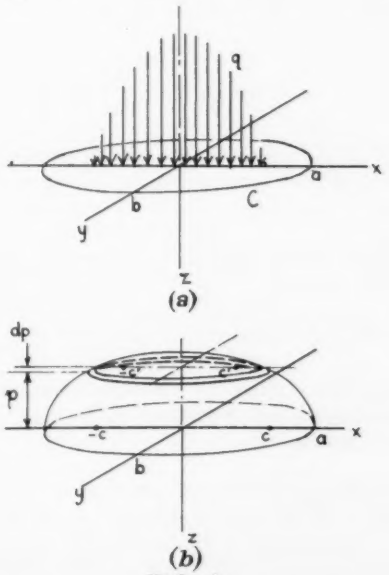


FIG. 9

The loading of Fig. 9*b* is that which results when two elastic solids with slightly curved surfaces are brought into contact (4, 5, 6).

Case VI. Compare the effects of a uniform loading of intensity $Q/2c$ per unit length along a line segment from $(-c, 0)$ to $(+c, 0)$, as shown in Fig. 10*a*, with those of the loading due to a perfectly

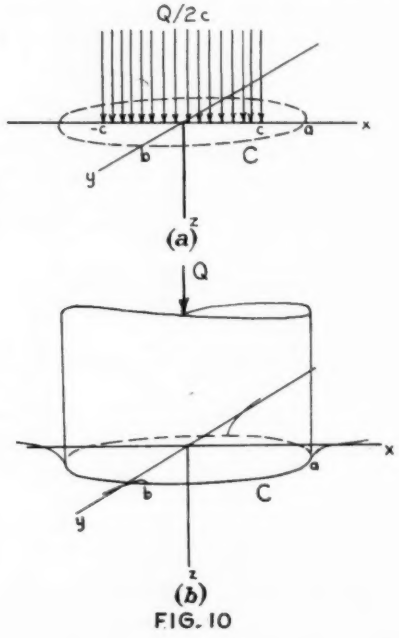


FIG. 10

rigid punch in the form of an elliptical cylinder with a flat end pressed against the surface with a force Q, as shown in Fig. 10*b*. Call the periphery of the loaded area C. The elliptical section of the punch and its position on the surface are assumed to be such that $(-c, 0)$ and $(+c, 0)$ are the foci of the ellipse C.

J. Boussinesq (7) called attention to the fact that the problem of calculating the pressure distribution under any rigid flat-ended punch is essentially the same as that of calculating the electric-charge distribution in a thin conducting disk of the same shape as the cross section of the punch, and concluded that the pressure distribution beneath the elliptical punch is given by

$$p = \frac{Q}{2\pi ab\sqrt{\left(1 - \frac{x'^2}{a^2} - \frac{y'^2}{b^2}\right)}} \quad \ldots\ldots [60]$$

The potential ϕ_0 due to either the p loading under the punch or the loading $Q/2c$ along the segment, for the region outside C will be

$$\phi_0 = -\frac{Q}{4c}\left[(x-c)\log\{(x-c)^2 + y^2\} - (x+c)\log\{(x+c)^2 + y^2\} + 4c + 2y\left(\tan^{-1}\frac{x-c}{y} - \tan^{-1}\frac{x+c}{y}\right)\right] \quad [61]$$

which is also valid within C for the loaded segment. The horizontal displacements and the stresses in the surface, outside C, for both cases become

$$\left.\begin{aligned} u_0 &= \frac{(1-2\mu)Q}{16\pi Gc}\log\frac{(x-c)^2 + y^2}{(x+c)^2 + y^2}; \\ v_0 &= \frac{(1-2\mu)Q}{8\pi Gc}\left(\tan^{-1}\frac{x-c}{y} - \tan^{-1}\frac{x+c}{y}\right) \end{aligned}\right\} \quad [62]$$

$$\left.\begin{aligned} \sigma_{x0} &= \frac{(1-2\mu)Q}{2\pi}\left\{\frac{x^2 - y^2 - c^2}{(x^2 + y^2 + c^2)^2 - 4x^2c^2}\right\} \\ \sigma_{y0} &= -\sigma_{x0} \\ \tau_{xy0} &= \frac{(1-2\mu)Q}{\pi}\left\{\frac{xy}{(x^2 + y^2 + c^2)^2 - 4x^2c^2}\right\} \end{aligned}\right\} \quad [63]$$

and Equations [62] and [63] are also valid inside C for the loading on the line segment.

The Logarithmic Potential ϕ_0 as the Deflection of a Membrane

The function ϕ_0 as defined in Equation [18] may be looked upon as the deflection of a uniformly stretched membrane of circular form with a radius very large compared to the dimensions of the loaded area R.

Let A be the radius of the membrane, and assume it can be taken indefinitely large. Let ζ be the deflection of the membrane relative to its elevation at $r = A$ which is assumed constant.

If a concentrated force P is applied to the membrane at point x', y' situated a distance r' from the center, and if r'/A is very small, the slope of the membrane surface in the direction of lines projecting radially from P will be

$$\frac{\partial\zeta}{\partial\rho_0} = -\frac{P}{2\pi N\rho_0} \quad \ldots\ldots [64]$$

where ρ_0 is the distance from x', y' to the point x, y where $\partial\zeta/\partial\rho_0$ is evaluated, and N is the tension in the membrane.

From vertical-equilibrium Equation [64] we have

$$\zeta = -\frac{P}{2\pi N}\log\rho_0 + C \quad \ldots\ldots [65]$$

where C is a constant. If the deflection is zero on the boundary, and if A is indefinitely large, we will have

$$\zeta = -\frac{P}{2\pi N}\log\rho_0 + \frac{P}{2\pi N}\log A \quad \ldots\ldots [66]$$

For any distribution of pressure over a region R, every point of which is within a distance ϵ of the origin which is infinitesimal compared to A, ζ will be

$$\zeta = -\frac{1}{2\pi N}\int_R\int p\log\rho_0 dx'dy' + \frac{P}{2\pi N}\log A \quad [67]$$

in which P is the total load on R.

This expression will represent the deflection within an error that can be made smaller than any preassigned quantity by taking A sufficiently large compared to ϵ.

Introducing ϕ_0 as defined by Equation [18] we have the relations

$$\left.\begin{aligned} \zeta &= -\frac{1}{2\pi N}\phi_0 + \frac{P}{2\pi N}\log A \\ \phi_0 &= -2\pi N\zeta + P\log A \end{aligned}\right\} \quad \ldots\ldots [68]$$

$$\left.\begin{aligned} \text{grad}\,\phi_0 &= -2\pi N\,\text{grad}\,\zeta \\ \Delta\phi_0 &= -2\pi N\Delta\zeta = 2\pi p \end{aligned}\right\} \quad \ldots\ldots [69]$$

The slope of the membrane in a given direction when multiplied by $(1-2\mu)(N/2G)$ will therefore be analogous to the horizontal displacement in the surface of the elastic solid in that direction. The stresses can be expressed in terms of the second derivatives of ζ.

An Analogy Between Two- and Three-Dimensional Stress Systems for Semi-Infinite Bodies

Let the load on the semi-infinite elastic body be distributed with an intensity $q = f(x')$ along the line segment from $(-c, 0)$ to $(+c, 0)$ as shown in Fig. 11*a*.

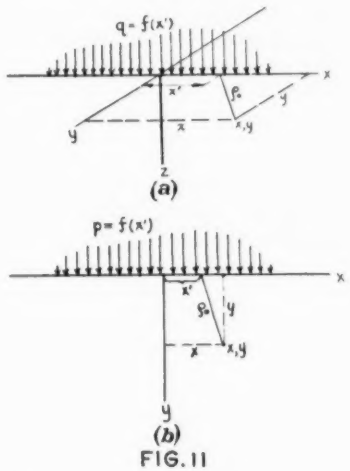


FIG. 11

The stresses in the surface will then be given by

$$\left.\begin{aligned} \sigma_{x0} &= -\frac{1-2\mu}{2\pi}\frac{\partial^2\phi_0}{\partial x^2}; \quad \sigma_{y0} = -\frac{1-2\mu}{2\pi}\frac{\partial^2\phi_0}{\partial y^2}; \\ \tau_{xy0} &= -\frac{1-2\mu}{2\pi}\frac{\partial^2\phi_0}{\partial x\partial y} \end{aligned}\right\} \quad [70]$$

$$\phi_0 = \int_{-c}^{c} f(x') \log \rho_0 \, dx' \quad [71]$$

If we have a semi-infinite solid in a state of plane strain loaded by pressure distributed along the band between $x = -c$ and $x = c$ with an intensity $p = f(x')$ as shown in Fig. 11*b* and if we take the x, z plane for the surface of the solid, the stresses in the interior will be

$$\left.\begin{aligned} \sigma_x &= \frac{1}{2\pi}\frac{\partial V}{\partial y} - \frac{y}{2\pi}\frac{\partial^2 V}{\partial x^2} \\ \sigma_y &= \frac{1}{2\pi}\frac{\partial V}{\partial y} - \frac{y}{2\pi}\frac{\partial^2 V}{\partial y^2} \\ \tau_{xy} &= -\frac{y}{2\pi}\frac{\partial^2 V}{\partial x\partial y} \end{aligned}\right\} \quad [72]$$

where

$$V = -2\int_{-c}^{c} f(x') \log \rho_0 \, dx' \quad [73]$$

The function V is simply the value taken by ψ, as defined in Equation [6], when the new orientation of the axes is used, one integration is carried out, and an infinite constant is dropped. The constant contributes nothing to the stresses.

Consider the shearing stress on 45-deg planes obtained by forming half the difference of σ_x and σ_y. Call this τ_{45}. This shearing stress, since $\Delta V = 0$, will be

$$\tau_{45} = -\frac{y}{2\pi}\frac{\partial^2 V}{\partial x^2} \quad [74]$$

We may now put down the following relations between the quantities in the plane-strain problem and those in the loaded-segment problem

$$V = -2\phi_0 \quad [75]$$

$$\left.\begin{aligned} \tau_{45} &= \frac{-2y}{1-2\mu}\sigma_{x0} = \frac{2y}{1-2\mu}\sigma_{y0} \\ \tau_{xy} &= \frac{-2y}{1-2\mu}\tau_{xy0} \end{aligned}\right\} \quad [76]$$

Therefore, if we know the shear stresses on 45-deg planes and on planes perpendicular to the x and y axes in the plane-strain problem, we will be able to calculate at once the stresses σ_{x0}, σ_{y0}, τ_{xy0} in the surface of a semi-infinite solid loaded along a line segment.

To illustrate this analogy, suppose we have a uniform load p_0 over the band between $x = -c$ and $x = c$ in the surface $y = 0$. The stresses will then be (8)

$$\left.\begin{aligned} \sigma_x &= -\frac{p_0}{2\pi}\left[2\left(\tan^{-1}\frac{x+c}{y} - \tan^{-1}\frac{x-c}{y}\right) + \frac{y(x-c)}{(x-c)^2+y^2} - \frac{y(x+c)}{(x+c)^2+y^2}\right] \\ \sigma_y &= -\frac{p_0}{2\pi}\left[2\left(\tan^{-1}\frac{x+c}{y} - \tan^{-1}\frac{x-c}{y}\right) - \frac{y(x-c)}{(x-c)^2+y^2} + \frac{y(x+c)}{(x+c)^2+y^2}\right] \\ \tau_{xy} &= \frac{p_0}{2\pi}\left[\frac{(x-c)^2-y^2}{(x-c)^2+y^2} - \frac{(x+c)^2-y^2}{(x+c)^2+y^2}\right] \end{aligned}\right\} \quad [77]$$

and we have

$$\tau_{45} = -\frac{p_0}{\pi}\left[\frac{y(x-c)}{(x-c)^2+y^2} - \frac{y(x+c)}{(x+c)^2+y^2}\right] \quad [78]$$

We therefore conclude that for the case of a uniform load q_0 per unit length along a line segment on the surface of a semi-infinite body, extending from $(-c, 0)$ to $(+c, 0)$

$$\left.\begin{aligned} \sigma_{x0} = -\sigma_{y0} &= \frac{(1-2\mu)}{2\pi} q_0 \left[\frac{x-c}{(x-c)^2+y^2} - \frac{x+c}{(x+c)^2+y^2}\right] \\ \tau_{xy0} &= -\frac{(1-2\mu)}{4\pi y} q_0 \left[\frac{(x-c)^2-y^2}{(x-c)^2+y^2} - \frac{(x+c)^2-y^2}{(x+c)^2+y^2}\right] \end{aligned}\right\} \quad [79]$$

An algebraic simplification shows that these expressions are identical with those given by Equations [63] when we put $q_0 = Q/2c$. Since the stresses in the surface due to the loaded line segment are the same as those for a rigid elliptical punch loading outside the loaded region, we can calculate the latter stresses at once if we know the stresses for the uniform load over a portion of the boundary for the case of plane strain.

This analogy holds equally well for a state of plane stress instead of plane strain. The loading on the edge of the plate will then be $pt = tf(x')$ per unit length, where t is the thickness of the plate.

Since photoelastic tests can be made to determine the shear stresses τ_{xy} and τ_{45} (it is only necessary to observe the principal stress difference and the orientation) the way is open to the evaluation of the stresses in the surface for loading along a line segment by means of plane stress photoelastic tests.

SUMMARY

I There are a number of cases in which an infinite variety of loadings inside a region R give rise to the same stresses and horizontal displacements in the surface outside R. Cases investigated are shown in Fig. 12.

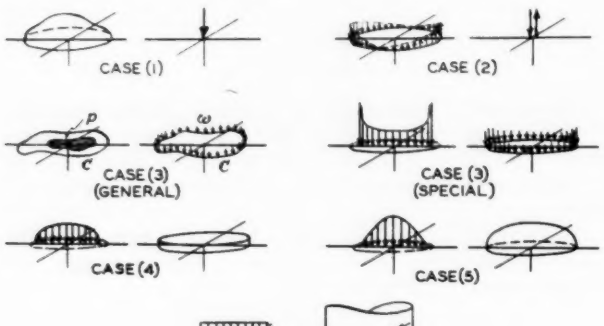


FIG. 12

II The analogy between the logarithmic potential ϕ_0 due to the loading on the solid and the deflection ζ of a large circular membrane with the same loading states that

$$\phi_0 = -2\pi N\zeta + P \log A$$

III Shear stresses τ_{xy} and τ_{45} in the plane strain problem of the semi-infinite body with loading $p = f(x')$ on the surface $y = 0$ are analogous to stresses τ_{xy0} and σ_{x0} (or $-\sigma_{y0}$) in the surface $z = 0$ of a semi-infinite solid with loading $q = f(x')$ along the x-axis. The latter quantities are multiplied by the factor $-2y/(1 - 2\mu)$.

Appendix

If R is a region in the finite part of the plane $z = 0$, and if two different source distributions $p = f(x', y')$ and $p_1 = f_1(x', y')$ in R give rise to logarithmic potentials

$$\phi_0 = \int_R\int p \log \rho_0 dx'dy' \quad [80]$$

$$\phi_{01} = \int_R\int p_1 \log \rho_0 dx'dy' \quad [81]$$

such that outside R

$$\text{grad } \phi_0 = \text{grad } \phi_{01} \quad [82]$$

then we can say that $\phi_0 = \phi_{01}$ everywhere outside R. For, by Gauss's theorem, if B be any closed curve outside of and surrounding R

$$\oint_B \text{grad}_n \phi_0 ds = \oint_B \text{grad}_n \phi_{01} ds = 2\pi \int_R\int p dx'dy' = 2\pi \int_R\int p_1 dx'dy' \quad [83]$$

and the total loads in R in both cases will be alike and equal to P. If there is any difference between ϕ_0 and ϕ_{01} outside R, they will differ by an additive constant, by virtue of hypothesis of Equation [82]. But since the loadings in both cases are of the same total amount we have

$$\left.\begin{aligned} \lim_{r\to\infty} (\phi_0 - P \log r) &= 0 \\ \lim_{r\to\infty} (\phi_{01} - P \log r) &= 0 \end{aligned}\right\} \quad [84]$$

which is impossible if ϕ_0 differs from ϕ_{01} by a constant. Hence ϕ_0 must equal ϕ_{01} everywhere outside R. This proposition is useful because it reduces the problem of proving two logarithmic potential functions, Equations [80] and [81], equal to that of proving that their gradients are equal.

We will now amplify the treatment of the different cases of loading producing the same stresses and horizontal displacements in the surface outside the loaded area.

Case I. Consider the loading $p = f(r')$ on the circle of radius a. Applying Gauss's theorem in two-dimensional form of Equation [24] to a circle outside $r = a$ and of radius r we get

$$2\pi r \frac{\partial \phi_0}{\partial r} = 2\pi P \quad [85]$$

$$\frac{\partial \phi_0}{\partial r} = \frac{P}{r} \quad [86]$$

But this is the same as the radial component of the gradient due to the concentrated force P at the origin. The tangential component of the gradient is zero in both cases by radial symmetry. Hence, for $r > a$ in both cases the gradients of the logarithmic potentials are alike and the potentials themselves are alike and equal to

$$\phi_0 = P \log r \quad [87]$$

and the stresses and horizontal displacements are easily calculated by Equations [26] and [27] and shown to be the expressions given by Equations [29] and [30].

The stresses inside the loaded area can also easily be found for the case of the loaded circle. Again, by Gauss's theorem, for $r<a$

$$2\pi r \frac{\partial \phi_0}{\partial r} = 2\pi \int_0^r 2\pi r' f(r') dr' = 2\pi P_r \quad [88]$$

$$\frac{\partial \phi_0}{\partial r} = \frac{P_r}{r} \quad [89]$$

$$\frac{\partial^2 \phi_0}{\partial r^2} = 2\pi p - \frac{P_r}{r^2} \quad [90]$$

so that the horizontal displacements and stresses will be

$$\delta_{r0} = -\frac{1-2\mu}{4\pi G} \cdot \frac{P_r}{r}; \quad \delta_{\theta 0} = 0 \quad [91]$$

$$\left.\begin{aligned} \sigma_{r0} &= -p + \frac{1-2\mu}{2\pi} \frac{P_r}{r^2}; \quad \sigma_{z0} = -p \\ \sigma_{\theta 0} &= -2\mu p - \frac{1-2\mu}{2\pi} \frac{P_r}{r^2}; \quad \tau_{r\theta 0} = 0 \end{aligned}\right\} \quad [92]$$

Outside the circle $r = a$ we have $P_r = P$ and $p = 0$ so that Equations [29] and [30] are particular cases of Equations [91] and [92].

Comparing the stresses just inside the circle of radius a with those just outside, we see that there is a discontinuity in σ_{r0} at $r = a$ of amount p_a, the pressure at $r = a$, and a discontinuity in $\sigma_{\theta 0}$ of amount $2\mu p_a$.

The vertical displacements of the surface will not be alike for the two cases of loading. For the loaded circle

$$w_0 = \frac{1-\mu}{2\pi G} \int_0^a \int_0^{2\pi} \frac{f(r')r'dr'd\theta'}{\rho_0} \quad [93]$$

while for the concentrated load

$$w_0 = \frac{1-\mu}{2\pi G}\frac{P}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots [94]$$

If the pressure distribution in the circle is such that $p = 0$ for $r < b$ and $p = f(r')$ for $b < r < a$, the stresses and horizontal displacements would be zero inside the ring.

Case II. The equivalence of the logarithmic potentials outside the circle of radius a for the loadings shown in Figs. 5*a* and 5*b* can be demonstrated as follows.

Let P be a downward force at $x = c$, $y = 0$ and $-P$ be an upward force at $x = -c$, $y = 0$. Let $P = m/2c$. Now if we allow c to approach zero, keeping m constant we obtain the concentrated couple of Fig. 5*b*. The potential due to force P will be equal to the potential due to a uniform pressure $p_0 = P/\pi a^2$ over a circle of radius a and center at $(+c, 0)$. The potential due to force $-P$ will be equal to that due to pressure $-p_0$ over a circle of radius a with center at $(-c, 0)$. As c is allowed to approach zero the circles overlap and their loads cancel except for semicircular crescent-shaped regions on either side of the y-axis. The loading on the crescent on the side $x > 0$ is p_0 and the loading on the other crescent is $-p_0$ per unit area. When c becomes very small, the width of the crescent region becomes

$$t = 2c\cos\theta' \quad \ldots\ldots\ldots\ldots\ldots\ldots [95]$$

and the load per unit length around the circle $r = a$, becomes in the limit as $c \to 0$

$$\omega = \lim_{c\to 0} p_0 t = \lim_{c\to 0} \frac{m}{2c}\frac{1}{\pi a^2} 2c\cos\theta' \quad \ldots\ldots\ldots [96]$$

$$\omega = \frac{m}{\pi a^2}\cos\theta'$$

Since the potential due to the loaded circles was at all times equal to the potential due to the forces P and $-P$ during this process, in the limit the potential due to the ω loading becomes equal to the potential due to the concentrated couple m, for the region outside the circle $r = a$. The logarithmic potential of a plane doublet of moment m is well known and is given by the first of Equations [32]. The horizontal displacements and stresses are easily obtained from ϕ_0.

Inside the circle of radius a for the loading of Fig. 5*a* the potential gradient may be calculated by a similar type of reasoning and found to be

$$\frac{\partial\phi_0}{\partial r} = -\frac{m\cos\theta}{a^2};\quad \frac{\partial\phi_0}{r\partial\theta} = \frac{m\sin\theta}{a^2} \quad \ldots\ldots\ldots [97]$$

The stresses in the surface are zero inside the circle $r = a$. There is a discontinuity in $\partial\phi_0/\partial r$ at $r = a$ of amount $2\pi\omega$ as would be anticipated by Equation [25].

The vertical displacements are not alike for the two cases of loading, being

$$w_0 = \frac{1-\mu}{2\pi G}\frac{m\cos\theta}{r^2} \quad \ldots\ldots\ldots\ldots [98]$$

for the concentrated couple, and

$$w_0 = \frac{1-\mu}{2\pi G} m\cos\theta\left[\frac{2(r+a)}{\pi r a^2}\left\{\frac{r^2+a^2}{(r+a)^2}F'(n) - E'(n)\right\}\right] \quad [99]$$

for the ω loading on the circle, where n is the number

$$n = \frac{2\sqrt{ra}}{r+a}$$

and F' and E' are the complete elliptic integrals of first and second type; symbols K and E are sometimes used, as by Peirce (9). Since w_0 for the ω loading varies as the product of $\cos\theta$ with a function of r, points on a circle $r =$ constant will remain in a plane after the vertical displacements have taken place.

Case III. Here we had $p(x',y')$ as a source distribution in a region R' and C a closed curve around R' and outside it as shown in Fig. 6*a*. Let the value of ϕ_0 due to the sources p be ϕ_{0c} on C and not necessarily constant along this curve. Now it is possible to find one and only one source distribution ω along C such that the logarithmic potential due to sources ω is equal to ϕ_{0c} on C and equal to ϕ_0 outside C, and this source distribution is the one defined by Equation [35]. This constitutes the two-dimensional statement of a proposition in potential theory due to Green (10). The proof that the potential due to sources ω on C is actually ϕ_0 outside C may be outlined as follows.

Let the potential due to sources ω be V. Then, by definition of ω and V

$$V = \frac{1}{2\pi}\oint_C \log\rho_0\left(\frac{\partial\phi'}{\partial n_1} - \frac{\partial\phi'}{\partial n_2}\right)ds \quad \ldots\ldots [100]$$

where ρ_0, as usual, is the distance from a point x',y' on C to the point x,y where V is to be evaluated. Noting that $\partial\phi'/\partial n_1 = \partial\phi_0/\partial n_1$ since $\phi' = \phi_0$ outside C, we have, by Green's transformation (11)

$$V = \frac{1}{2\pi}\left[\oint_C\left(\phi_0\frac{\partial}{\partial n_1}\log\rho_0 - \phi'\frac{\partial}{\partial n_2}\log\rho_0\right)ds - \int_R\int\{\phi_0\,\Delta\log\rho_0 - \phi'\Delta\log\rho_0 - \log\rho_0(\Delta\phi_0 - \Delta\phi')\}dA\right] \quad [101]$$

The line integral vanishes since $\phi_0 = \phi' = \phi_{0c}$ on C, and the normal derivative of $\log\rho_0$ is continuous from one side of C to the other. In the integral over R all terms except $\Delta\phi_0\log\rho_0$ are zero. Therefore

$$V = \frac{1}{2\pi}\int\int_R \Delta\phi_0\log\rho_0 dA \quad \ldots\ldots\ldots [102]$$

But $\Delta\phi_0$ is $2\pi p$ where p is the source intensity in R' giving rise to ϕ_0, and consequently

$$V = \int_{R'}\int p\log\rho_0 dA \quad \ldots\ldots\ldots\ldots [103]$$

But this is precisely ϕ_0. Therefore, the sources ω on C give rise to potential ϕ_0 outside C. Inside C, for the source distribution ω, we will have $\Delta V = 0$ and V takes the value ϕ_{0c} on C. Hence $V = \phi'$ inside C, since Laplace's equation has a unique solution in a region when the value around the boundary is given.

The loading ω defined by Equation [38] is the one which makes the ellipse C on which it is applied an equipotential curve. We are guided in the treatment of this problem by the well-known theory of the electric potential due to a charged ellipsoidal conductor (12). When the ellipsoid is made infinitely long it becomes an elliptical cylinder. The electric charge distributes itself on the surface so as to make this surface one of constant potential. When the elliptic coordinates λ, ν are used to describe the potential field it is at once evident that the potential is linearly related to λ, since λ is a harmonic function of x and y and therefore fulfills the requirement for the potential outside the charged cylinder and since λ is constant on the charged surface.

In the first of Equations [43], for ϕ_0, the term $-\log 2/c$ is introduced so as to make the potential approach $Q\log r$ at an infinite

distance. The source distribution ω on C is obtained by applying Equation [25], noting that $\partial\phi_0/\partial n_2$ is zero since ϕ_0 is constant within the ellipse. This gives us

$$\omega = \frac{1}{2\pi} Q \operatorname{grad} \lambda = \frac{Qh}{2\pi} \quad [104]$$

Expressing h in terms of rectangular coordinates x', y' of a point on the ellipse we obtain the distribution given by Equation [38].

Since λ is a function of x, y and the focal distance c, the potential given by the first of Equations [43] will be the one corresponding to a source distribution ω around any ellipse having foci $(-c, 0)$, $(+c, 0)$. We can state that all confocal ellipses carrying a source distribution given by Equation [38] will produce the same potential in the region external to all, when the total source strength is the same in all cases. The smallest one of such confocal ellipses is an infinitely narrow one which coincides with the line segment between the common foci. The loading per unit length on this line segment will be

$$q = \lim_{\substack{b \to 0 \\ a \to c}} 2\omega = \frac{Q}{\pi c \sqrt{[1 - (x'^2/c^2)]}} \quad [105]$$

The q loading on the line segment given by Equation [37] thus produces the same potential outside C as the ω loading on C given by Equation [38].

In connection with the discussion of the elliptical ring having C as its outer boundary, the width of this ring may be shown to be

$$t = \frac{k}{\sqrt{\left(\frac{x'^2}{a^4} + \frac{y'^2}{b^4}\right)}} \quad [106]$$

so that the load per unit length, $p_0 t$, is

$$p_0 t = \frac{Qt}{2\pi abk} = \frac{Q}{2\pi ab\sqrt{\left(\frac{x'^2}{a^4} + \frac{y'^2}{b^4}\right)}} = \omega \quad [107]$$

The width of the ring t at point x',y' is the product of k and the distance from the origin to the tangent line to C at x',y', as may be seen by geometrical reasoning. The distance to the tangent line is

$$\frac{1}{\sqrt{\left(\frac{x'^2}{a^4} + \frac{y'^2}{b^4}\right)}} \quad [108]$$

so that we see the loading ω is distributed around C proportionately to this distance.

Case IV. The proposition here stated, that any two confocal ellipses having uniform source distributions over their areas give rise to the same logarithmic potential in the region external to both if the totals of the sources on them are alike, is the two-dimensional form of Maclaurin's theorem (13). This theorem states that two homogeneous, solid, confocal ellipsoids of the same total mass produce the same gravitational attraction (and potential) in the region external to both. One may prove the two-dimensional form of this theorem either by first establishing it for ellipsoids and then choosing the ellipsoids infinitely long, or else by starting at the beginning and reconstructing the proof for the two-dimensional case. We will not go into the proof here, though it is of theoretical interest and not difficult.

The two-dimensional problem of calculating the logarithmic potential due to a uniform source distribution over an elliptical disk is the same as that of calculating the Newtonian potential due to an infinitely long elliptical cylinder. This problem is most easily handled by first obtaining the expression for the potential of a solid homogeneous ellipsoid (13) and then allowing one axis of the ellipsoid to increase indefinitely. In this way we find that the logarithmic potential of the uniformly loaded ellipse is

$$\phi_0 = -\frac{1}{2}(V - V_A - 2p_0 \pi ab \log A) \quad [109]$$

where A is a distance r that increases without limit and

$$V = \pi abp_0 \int_\gamma^\infty \frac{\left(1 - \frac{x^2}{c^2 + u} - \frac{y^2}{u}\right) du}{\sqrt{[u(c^2 + u)]}} \quad [110]$$

and γ, given in Equation [53], is the positive root of the equation

$$\frac{x^2}{c^2 + \gamma} + \frac{y^2}{\gamma} = 1 \quad [111]$$

The function V is the Newtonian potential of an infinitely long elliptical cylinder of density p_0. The integral in Equation [110] can be evaluated without difficulty and, for external points, we find

$$\phi_0 = \frac{\pi ab}{2} p_0 \left[\log \frac{\sqrt{\gamma(c^2 + \gamma)} + \gamma + (c^2/2)}{2} + \frac{2x^2}{c^2}\left(1 - \sqrt{\frac{\gamma}{c^2 + \gamma}}\right) - \frac{2y^2}{c^2}\left(1 - \sqrt{\frac{c^2 + \gamma}{\gamma}}\right)\right] \quad [112]$$

For large values of $x^2 + y^2$ we find that ϕ_0 approaches $\pi abp_0 \log \sqrt{(x^2 + y^2)}$ as it should. The constants in Equation [109] were chosen for this purpose. For points inside the ellipse we take $\gamma = b^2$ in evaluating the integral Equation [110] and obtain

$$\phi_0 = \frac{\pi ab}{2} p_0 \left[\log \frac{(a + b)^2}{4} + \frac{2x^2}{c^2}\left(1 - \frac{b}{a}\right) - \frac{2y^2}{c^2}\left(1 - \frac{a}{b}\right)\right] \quad [113]$$

From Equation [112] the gradient of ϕ_0 can be easily calculated and Equations [51] and [52] are derived. For points inside the loaded ellipse the gradient is calculated from Equation [113] and the horizontal displacements and stresses become

$$u_0 = -\frac{1 - 2\mu}{2\pi G}\left(\frac{Qx}{a(a + b)}\right); \quad v_0 = -\frac{1 - 2\mu}{2\pi G}\left(\frac{Qy}{b(a + b)}\right) \quad [114]$$

$$\left.\begin{aligned} \sigma_{x0} &= -2\mu p_0 - \frac{1 - 2\mu}{\pi a}\frac{Q}{(a + b)} \\ \sigma_{y0} &= -2\mu p_0 - \frac{1 - 2\mu}{\pi b}\frac{Q}{(a + b)} \\ \tau_{xy0} &= 0 \\ \sigma_{z0} &= -p_0 \end{aligned}\right\} \quad [115]$$

Inside the ellipse, therefore, σ_{x0}, σ_{y0}, σ_{z0} are the principal stresses and they are constant everywhere over the elliptical area. The resultant horizontal displacement $\sqrt{(u_0^2 + v_0^2)}$ is constant around ellipses geometrically similar to C.

The stresses are discontinuous at the edge of the loaded ellipse. For, as the ellipse is approached from the outside the stresses become, by Equation [52]

$$\sigma_{x0} = \frac{1 - 2\mu}{\pi c^2} Q\left(\frac{ab^3}{b^4 + y^2c^2} - 1\right); \quad \sigma_{y0} = -\sigma_{x0};$$

$$\tau_{xy0} = \frac{1 - 2\mu}{\pi} Q\left(\frac{xyb}{a(b^4 + y^2c^2)}\right) \quad [116]$$

Case V. In this case we compared the semiellipsoidal loading of Fig. 9*b* with the parabolic loading on a line segment between the foci of the loaded ellipse, as shown in Fig. 9*a*.

That these two load distributions produce the same logarithmic potential outside C may be shown by dividing the semiellipsoidal loading into a number of uniformly loaded elliptical areas, as indicated in Fig. 9*b*, each carrying a pressure dp. A typical one of these elements would be the ellipse of semiaxes $a\sqrt{[1-(p^2/p_0^2)]}$ and $b\sqrt{[1-(p^2/p_0^2)]}$ with foci at $x = \pm c', y = 0$ where $c' = c\sqrt{[1-(p^2/p_0^2)]}$. By the proposition of case IV, the potential due to this element will be equal, outside it and hence outside C, to the potential due to loading dq per unit length along the line segment from $(-c', 0)$ to $(+c', 0)$ which varies as

$$dq = \frac{2dQ}{\pi c'}\sqrt{\left(1-\frac{x'^2}{c'^2}\right)} \quad \ldots\ldots\ldots\ldots [117]$$

where dQ is the total load on the element, given by

$$dQ = \pi abdp[1-(p^2/p_0^2)] \quad \ldots\ldots\ldots\ldots [118]$$

When the elemental ellipse with load dp has a focal distance equal to x', the magnitude dq of the semielliptical load corresponding to it at x' is zero. All of the elliptical elements smaller than this one will have corresponding semielliptical loadings which do not extend as far as x'. Consequently, the load on the line segment corresponding to the full semiellipsoidal loading will have magnitude at x' given by

$$q(x') = \frac{2}{\pi}\int_0^{\bar{p}} \frac{dQ}{c'}\sqrt{\left(1-\frac{x'^2}{c'^2}\right)} = \frac{\pi abp_0}{2c}\left(1-\frac{x'^2}{c^2}\right). [119]$$

where

$$\bar{p} = p_0\sqrt{[1-(x'^2/c^2)]}$$

Here $\bar{p}$ is the value of p for which the corresponding elliptical element has focal distance $c' = x'$.

The logarithmic potential due to both cases of loading is found by direct integration, using the parabolic loading on the line segment rather than the semiellipsoidal loading, to simplify the integration.

The vertical displacements in the surface are different in the two loadings. For the semiellipsoidal loading we have inside C

$$w_0 = \frac{3(1-\mu)}{4\pi G}\frac{Q}{a}\left[F'\left(\frac{c}{a}\right) - \frac{x^2}{c^2}\left\{F'\left(\frac{c}{a}\right) - E'\left(\frac{c}{a}\right)\right\} - \frac{y^2}{c^2}\left\{\frac{a^2}{b^2}E'\left(\frac{c}{a}\right) - F'\left(\frac{c}{a}\right)\right\}\right] \ldots [120]$$

and outside C

$$w_0 = \frac{3(1-\mu)}{4\pi G}\frac{Q}{a}\left[F\left(\alpha,\frac{c}{a}\right) - \frac{x^2}{c^2}\left\{F\left(\alpha,\frac{c}{a}\right) - E\left(\alpha,\frac{c}{a}\right)\right\} - \frac{y^2}{c^2}\left\{\frac{a^2}{b^2}E\left(\alpha,\frac{c}{a}\right) - F\left(\alpha,\frac{c}{a}\right) - \frac{a}{\sqrt{(\gamma-b^2)}}\left(\frac{a^2}{b^2}\sqrt{\frac{\gamma}{c_2+\gamma}} - \sqrt{\frac{c_2+\gamma}{\gamma}}\right)\right\}\right] .. [121]$$

where $\alpha = \cot^{-1}\sqrt{[(\gamma-b^2)/a^2]}$ and γ is as defined in Equations [53] and [111]. We see that inside C the deformed surface is an elliptical paraboloid. The evaluation of the integrals involved outside C is due to V. A. Hoersch (14).

The displacement w_0 for the parabolic loading is

$$w_0 = -\frac{(1-\mu)}{4G}\cdot\frac{p_0ab}{c^2}\left\{\left(c^2-x^2+\frac{y^2}{2}\right)\right.$$

$$\times \log\frac{x-c+\sqrt{[(x-c)^2+y^2]}}{x+c+\sqrt{[(x+c)^2+y^2]}} + \frac{3x+c}{2}\sqrt{[(x-c)^2+y^2]} - \frac{3x-c}{2}\sqrt{[(x+c)^2+y^2]}\left.\right\} .. [122]$$

Case VI. The validity of the statement that both types of loading shown in Figs. 10*a* and 10*b* in this case produce the same logarithmic potential in the region outside C can be shown by dividing the loaded elliptical area into elliptical rings bounded by geometrically similar ellipses. The pressure on any one of these rings will be uniform around its periphery and will produce the same logarithmic potential outside C as a loading on a line segment of the type discussed in case III. Superimposing all such segment loadings corresponding to all the loaded ring elements we obtain the uniformly loaded line segment of Fig. 10*a*. The logarithmic potential due to this loaded segment may be easily calculated. The vertical displacements w_0 are different for the two types of loading. For the line-segment loading

$$w_0 = \frac{1-\mu}{2\pi G}\frac{Q}{c}\coth^{-1}\frac{\sqrt{(c^2+\gamma)}}{c} \quad \ldots\ldots\ldots [123]$$

while for loading applied by the rigid punch we obtain, outside C

$$w_0 = \frac{1-\mu}{2\pi G}\frac{Q}{a}F\left(\alpha,\frac{c}{a}\right) \quad \ldots\ldots\ldots\ldots [124]$$

and within C $\quad w_0 = \frac{1-\mu}{2\pi G}\cdot\frac{Q}{a}F'\left(\frac{c}{a}\right) = \text{const.} \quad \ldots\ldots\ldots [125]$

At a great distance from the origin Equations [123] and [124] both approach $[(1-\mu)/2\pi G]\cdot(Q/r)$ asymptotically. Since α depends on γ and since the curves $\gamma = \text{constant}$ are confocal ellipses we see that for both types of loading the surface deflection is constant around ellipses confocal with C.

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Forty-Eight Ordinate Harmonic Analysis and the Harmonic Spectrum of Two-Cycle Diesel Torque

By NANCY KLOCK,[1] EAST HARTFORD, CONN.

For the harmonic analysis of periodic curves, machines as well as numerical methods are available. The machines are expensive and not generally available, and time is required to learn their operation. Therefore, an occasional analysis of a curve consumes even less time by the numerical method than by the machine. The best known numerical method is that of Runge (1)[2] described in many places; a good account is found in Scarborough's book (2). The principle is clear, but writers usually give the method for 6, 12, or 24 ordinates for the sake of simplicity, which is not sufficiently accurate for most applications. In this paper the 48-ordinate scheme is fully given in Tables 1, 2, and 3.

For the analysis of torsional vibrations the harmonic content of the torque curve of the disturbing engine is required. Such analyses have been published for the four-cycle Diesel engine (3) and for the four-cycle gasoline engine for aviation purposes (4). But to the author's knowledge none have been published for the two-cycle Diesel engine. Such a spectrum is given herein.

OCCASIONALLY it is found desirable to determine the high-order Fourier harmonics present in a repeating curve. A simple numerical method to obtain orders as high as desired has been worked out by Runge (1). Scarborough (2) gives an excellent description of the general method together with a complete scheme for 24 ordinates which allows the determination of the first 12 harmonics. For most cases the accuracy obtainable is insufficient. Other numerical methods for special purposes have been proposed by Lewis (3) and Zipperer (5).

When higher harmonics, or better accuracy in the lower ones, are wanted, more than 24 ordinates from the original curve must be chosen. If the ordinates at 48 equally spaced abscissas are taken, the constant term and 23 sine and 24 cosine components can be obtained. This article is primarily intended to make available a simple procedure by which from 15 to 20 harmonics can be computed quite accurately, the remaining high harmonics becoming less accurate. Since Runge's method has been well described in several articles and books (2), his theory will not be repeated in this paper, but only the procedure itself is described.

To simplify the description of the procedure, it is applied to a specific case. An indicator card, supplied by the Busch-Sulzer Bros. Diesel Engine Co., has been converted to a curve of torque versus crank angle, as shown in Fig. 1. The torque magnitude is given in arbitrary units and the harmonics are later converted to per cent of mean torque which is represented by the dotted line in the figure. From this curve the values of the torque harmonics with respect to engine speed are computed as follows:

1 The 360-deg cycle is divided into 48 equal parts, that is, into 7.5-deg intervals. The ordinates at these divisions are called $y_0, y_1, y_2, \ldots y_{47}$, corresponding to $\omega t = 0, 7.5, 15, \ldots 352.5$ deg, respectively. The y values, including signs, must then be placed in the second and third rows of Table 1 in the order indicated by the printed numbers of the first row in this table. (At this point it should be noted that y_{24}, at the mid-point of the cycle, is placed at the beginning of the third row and y_{25} is then placed under y_{23} at the end of the row, etc.)

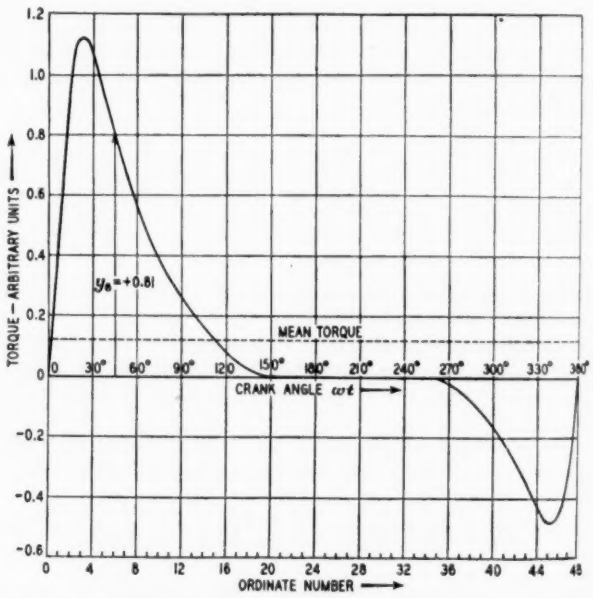


FIG. 1 TORQUE CURVE OF A TWO-CYCLE DIESEL ENGINE
(The torque values have been obtained from an indicator card supplied by Busch-Sulzer Bros. Diesel Engine Company. The values are for full-load conditions.)

2 Adding algebraically $(y_0 + y_{24})$, $(y_1 + y_{47})$, $(y_2 + y_{46})$, $\ldots$ $(y_{23} + y_{25})$, the values for the sums $c_0, c_1, c_2, \ldots c_{23}$ are then placed in the fourth row of this table. Subtracting algebraically, $(y_0 - y_{24})$, $(y_1 - y_{47})$, $(y_2 - y_{46})$, $\ldots$ $(y_{23} - y_{25})$, similar values for the difference terms d are found and placed in the fifth row. This completes the first part of Table 1.

3 The values of c_0 through c_{23} are then copied in the second rectangle on the left, those for d_0 through d_{23} in the second rectangle on the right. The addition and subtraction procedure is now repeated, resulting in e and f terms from the c's, and g and h terms from the d's.

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

4 The remaining computations in Table 1 are similarly filled in as directed.

5 The values A_1 (= sin 7½ deg), A_2 (= sin 15 deg), ... A_{11} (= sin 82½ deg), are included in Table 1, making the sheet complete with all the information necessary for further work in the determination of the desired harmonics.

6 The remaining procedure is equally simple. Tables 2 and 3 give the values for the sine and cosine components of each frequency and by substituting values from Table 1 into these equations, the sine terms (a_1 through a_{23}) and cosine terms (b_0 through b_{24}) can be found. These are the coefficients which go into the Fourier series

$$f(\omega t) = b_0 + b_1 \cos \omega t + b_2 \cos (2\,\omega t + \phi_2) + \ldots\ldots + b_{24} \cos(24\,\omega t + \phi_{24}) + a_1 \sin \omega t + a_2 \sin (2\,\omega t + \phi_2) + \ldots\ldots + a_{23} \sin (23\,\omega t + \phi_{23})$$

As an example take the ninth harmonic

$$24\,a_9 = -(1.49 - 0.280) - 0.383\,(+ 2.325) + 0.707\,(+ 1.945) + 0.924\,(+ 1.045) = + 0.240$$

$$24\,b_9 = + (0 - 0.340) + 0.383\,(- 0.475) - 0.707\,(- 0.155) - 0.924\,(- 0.005) = - 0.408$$

so that $a_9 = + 0.0100$ and $b_9 = - 0.0170$, both in the arbitrary units chosen. The value found for b_0 in these units

$$\frac{1}{48}\,(p_0 + p_1 + p_2) = \frac{1}{48}\,(+ 1.895 + 1.785 + 2.095) = + 0.1203$$

represents the mean torque, and the sine and cosine functions, in per cent of this mean value, are respectively

a_9 = 8.3 per cent of mean torque
b_9 = 14.1 per cent of mean torque

TABLE 1 48-ORDINATE FOURIER ANALYSIS

y	0 (24)	1 (47)	2 (46)	3 (45)	4 (44)	5 (43)	6 (42)	7 (41)	8 (40)	9 (39)	10 (38)	11 (37)	12 (36)	13 (35)	14 (34)	15 (33)	16 (32)	17 (31)	18 (30)	19 (29)	20 (28)	21 (27)	22 (26)	23 (25)
	0	.500	.980	1.120	1.060	.940	.810	.675	.575	.450	.375	.310	.260	.210	.160	.115	.080	.045	.020	.005	0	0	0	0
	0	-.315	-.495	-.475	-.430	-.350	-.270	-.190	-.155	-.115	-.075	-.040	-.020	-.005	0	0	0	0	0	0	0	0	0	0
sum: C	0	.185	.505	.645	.630	.590	.540	.485	.420	.335	.300	.270	.240	.205	.160	.115	.080	.045	.020	.005	0	0	0	0
diff: d	0	.815	1.455	1.595	1.490	1.290	1.080	.865	.730	.565	.450	.350	.280	.215	.160	.115	.080	.045	.020	.005	0	0	0	0

c	0 (12)	1 (23)	2 (22)	3 (21)	4 (20)	5 (19)	6 (18)	7 (17)	8 (16)	9 (15)	10 (14)	11 (13)
	0	.185	.505	.645	.630	.590	.540	.485	.420	.335	.300	.270
	.240	0	0	0	0	.005	.020	.045	.080	.115	.160	.205
sum: e	.240	.185	.505	.645	.630	.595	.560	.530	.500	.450	.460	.475
diff: f	-.240	.185	.505	.645	.630	.585	.520	.440	.340	.220	.140	.065

d	0 (12)	1 (23)	2 (22)	3 (21)	4 (20)	5 (19)	6 (18)	7 (17)	8 (16)	9 (15)	10 (14)	11 (13)
	0	.815	1.455	1.595	1.490	1.290	1.080	.865	.730	.565	.450	.350
	.280	0	0	0	0	.005	.020	.045	.080	.115	.160	.215
sum: g	.280	.815	1.455	1.595	1.490	1.295	1.100	.910	.810	.680	.610	.565
diff: h	-.280	.815	1.455	1.595	1.490	1.285	1.060	.820	.650	.450	.290	.135

e	0 (6)	1 (11)	2 (10)	3 (9)	4 (8)	5 (7)
	.240	.185	.505	.645	.630	.595
	.560	.475	.460	.450	.500	.530
sum: k	.800	.660	.965	1.095	1.130	1.125
diff: l	-.320	-.290	.045	.195	.130	.065

$f_1 - f_7 - f_9 = i_1 = -.475$
$f_2 - f_6 - f_{10} = i_2 = -.155$
$f_3 - f_5 - f_{11} = i_3 = -.005$

$g_1 + g_7 - g_9 = j_1 = +1.045$
$g_2 + g_6 - g_{10} = j_2 = +1.945$
$g_3 + g_5 - g_{11} = j_3 = +2.325$

h	0 (6)	1 (11)	2 (10)	3 (9)	4 (8)	5 (7)
	.280	.815	1.455	1.595	1.490	1.285
	1.060	.135	.290	.450	.650	.820
sum: m	1.340	.950	1.745	2.045	2.140	2.105
diff: n	-.780	.680	1.165	1.145	.840	.465

k	0 (3)	1 (5)	2 (4)
	.800	.660	.965
	1.095	1.125	1.130
sum: p	1.895	1.785	2.095
diff: q	-.295	-.465	-.165

l	0 (3)	1 (5)	2 (4)
	-.320	-.290	.045
	.195	.065	.130
sum: r	-.125	-.225	.175
diff: s	-.575	-.355	-.085

$A_1 = .131$
$A_2 = .259$
$A_3 = .383$
$A_4 = .500$
$A_5 = .609$
$A_6 = .707$
$A_7 = .793$
$A_8 = .866$
$A_9 = .924$
$A_{10} = .966$
$A_{11} = .991$

m	0 (3)	1 (5)	2 (4)
	.780	.950	1.745
	2.045	2.105	2.140
sum: t	2.825	3.055	3.885
diff: u	-1.265	-1.155	-.395

n	0 (3)	1 (5)	2 (4)
	-1.340	.680	1.165
	1.145	.465	.840
sum: V	-.195	1.145	2.005
diff: W	-2.485	.215	.325

TABLE 2 SINE TERMS FOR FOURIER ANALYSIS WITH 48 ORDINATES

$$24\,a_1 = + d_{12} + A_1g_{11} + A_2g_2 + A_3g_3 + A_4g_4 + A_5g_5 + A_6g_6 + A_7g_7 + A_8g_8 + A_9g_9 + A_{10}g_{10} + A_{11}g_{11}$$

$$24\,a_2 = + h_6 + A_2m_1 + A_4m_2 + A_6m_3 + A_8m_4 + A_{10}m_5$$

$$24\,a_3 = + (g_4 - d_{12}) + A_3j_1 + A_6j_2 + A_9j_3$$

$$24\,a_4 = + n_2 + A_4v_1 + A_8v_2$$

$$24\,a_5 = + d_{12} - A_1g_7 + A_2g_{10} - A_3g_9 + A_4g_4 + A_5g_1 - A_6g_6 + A_7g_{11} - A_8g_8 + A_9g_3 + A_{10}g_2 - A_{11}g_7$$

$$24\,a_6 = + (m_2 - h_6) + A_6\,(u_1 + m_3)$$

$$24\,a_7 = - d_{12} + A_1g_7 + A_2g_{10} + A_3g_9 - A_4g_4 - A_5g_{11} - A_6g_6 + A_7g_1 + A_8g_8 + A_9g_3 + A_{10}g_2 - A_{11}g_9$$

$$24\,a_8 = + A_8\,(w_1 + w_2)$$

$$24\,a_9 = - (g_4 - d_{12}) - A_3j_3 + A_6j_2 + A_9j_1$$

$$24\,a_{10} = + h_6 + A_2m_6 + A_4m_5 - A_6m_4 - A_8m_3 + A_{10}m_1$$

$$24\,a_{11} = - d_{12} - A_1g_{11} + A_2g_2 + A_3g_3 - A_4g_4 - A_5g_7 + A_6g_6 + A_7g_5 - A_8g_8 - A_9g_9 + A_{10}g_{10} + A_{11}g_1$$

$$24\,a_{12} = + v_1 - n_3$$

$$24\,a_{13} = + d_{12} - A_1g_{11} - A_2g_2 + A_3g_3 + A_4g_4 - A_5g_7 - A_6g_6 + A_7g_5 + A_8g_8 - A_9g_9 - A_{10}g_{10} + A_{11}g_1$$

$$24\,a_{14} = - h_6 + A_2m_5 - A_4m_2 - A_6m_3 + A_8m_4 + A_{10}m_1$$

$$24\,a_{15} = + (g_4 - d_{12}) - A_3j_3 - A_6j_2 + A_9j_1$$

$$24\,a_{16} = + A_8\,(w_1 - w_2)$$

$$24\,a_{17} = + d_{12} + A_1g_7 - A_2g_{10} + A_3g_9 + A_4g_4 - A_5g_{11} + A_6g_6 + A_7g_1 - A_8g_8 + A_9g_3 - A_{10}g_2 - A_{11}g_9$$

$$24\,a_{18} = - (m_2 - h_6) + A_6\,(u_1 + m_3)$$

$$24\,a_{19} = - d_{12} - A_1g_5 - A_2g_{10} - A_3g_9 - A_4g_4 + A_5g_1 + A_6g_6 + A_7g_{11} + A_8g_8 + A_9g_3 - A_{10}g_2 - A_{11}g_7$$

$$24\,a_{20} = + n_2 + A_4v_1 - A_8v_2$$

$$24\,a_{21} = - (g_4 - d_{12}) + A_3j_1 - A_6j_2 + A_9j_3$$

$$24\,a_{22} = - h_6 + A_2m_1 - A_4m_2 + A_6m_3 - A_8m_4 + A_{10}m_5$$

$$24\,a_{23} = - d_{12} + A_1g_1 - A_2g_2 + A_3g_3 - A_4g_4 + A_5g_5 - A_6g_6 + A_7g_7 - A_8g_8 + A_9g_9 - A_{10}g_{10} + A_{11}g_{11}$$

TABLE 3 COSINE TERMS FOR FOURIER ANALYSIS WITH 48 ORDINATES

$$48\,b_0 = + p_0 + p_1 + p_2$$

$$24\,b_1 = + d_0 + A_1f_{11} + A_2f_{10} + A_3f_9 + A_4f_8 + A_5f_7 + A_6f_6 + A_7f_5 + A_8f_4 + A_9f_3 + A_{10}f_2 + A_{11}f_1$$

$$24\,b_2 = + f_0 + A_2l_3 + A_4l_4 + A_6l_3 + A_8l_2 + A_{10}l_1$$

$$24\,b_3 = + (d_0 - f_8) + A_3i_3 + A_6i_2 + A_9i_1$$

$$24\,b_4 = + l_0 + A_4q_2 + A_8q_1$$

$$24\,b_5 = + d_0 - A_1f_7 + A_2f_9 - A_3f_8 + A_4f_8 + A_5f_{11} - A_6f_6 + A_7f_1 - A_8f_4 + A_9f_2 + A_{10}f_{10} - A_{11}f_3$$

$$24\,b_6 = + (f_0 - l_4) + A_6\,(i_1 - l_2)$$

$$24\,b_7 = + d_0 - A_1f_5 - A_2f_2 - A_3f_9 + A_4f_8 + A_5f_1 + A_6f_6 - A_7f_{11} - A_8f_4 - A_9f_3 - A_{10}f_{10} + A_{11}f_7$$

$$24\,b_8 = + q_0 + A_4\,(p_1 - p_2)$$

$$24\,b_9 = + (d_0 - f_6) + A_3i_1 - A_6i_2 - A_9i_3$$

$$24\,b_{10} = + f_0 + A_2l_1 + A_4l_4 - A_6l_3 - A_8l_2 + A_{10}l_5$$

$$24\,b_{11} = + d_0 + A_1f_1 - A_2f_{10} - A_3f_9 + A_4f_8 + A_5f_5 - A_6f_6 - A_7f_7 + A_8f_4 + A_9f_3 - A_{10}f_2 - A_{11}f_{11}$$

$$24\,b_{12} = + l_0 - q_2$$

$$24\,b_{13} = + d_0 - A_1f_1 - A_2f_{10} + A_3f_9 + A_4f_8 - A_5f_5 - A_6f_6 + A_7f_7 + A_8f_4 - A_9f_3 - A_{10}f_2 + A_{11}f_{11}$$

$$24\,b_{14} = + f_0 - A_2l_1 + A_4l_4 + A_6l_3 - A_8l_2 - A_{10}l_5$$

$$24\,b_{15} = + (d_0 - f_8) - A_3i_1 - A_6i_2 + A_9i_3$$

$$24\,b_{16} = + p_0 - A_4\,(p_1 + p_2)$$

$$24\,b_{17} = + d_0 + A_1f_5 - A_2f_2 + A_3f_9 + A_4f_8 - A_5f_1 + A_6f_6 + A_7f_{11} - A_8f_4 + A_9f_3 - A_{10}f_{10} - A_{11}f_7$$

$$24\,b_{18} = + (f_0 - l_4) - A_6\,(i_1 - l_2)$$

$$24\,b_{19} = + d_0 + A_1f_7 + A_2f_9 + A_3f_8 + A_4f_8 - A_5f_{11} - A_6f_6 - A_7f_1 - A_8f_4 - A_9f_2 + A_{10}f_{10} + A_{11}f_3$$

$$24\,b_{20} = + l_0 + A_4q_2 - A_8q_1$$

$$24\,b_{21} = + (d_0 - f_8) - A_3i_3 + A_6i_2 - A_9i_1$$

$$24\,b_{22} = + f_0 - A_2l_3 + A_4l_4 - A_6l_3 + A_8l_2 - A_{10}l_1$$

$$24\,b_{23} = + d_0 - A_1f_{11} + A_2\,f_{10} - A_3f_9 + A_4f_8 - A_5f_7 + A_6f_6 - A_7f_5 + A_8f_4 - A_9f_3 + A_{10}f_2 - A_{11}f_1$$

$$24\,b_{24} = + q_0 - p_1 + p_2$$

7 The absolute magnitude of each frequency in this series is then obtained by adding the squares of the sine and cosine components for this 9th order torque:

$$T_9 = \sqrt{(8.3^2 + 14.1^2)} = 16 \text{ per cent of mean torque}$$

In this manner the other torque harmonics present in the two-cycle Diesel engine have been computed and these values are represented by the solid curve of Fig. 2. The values are given

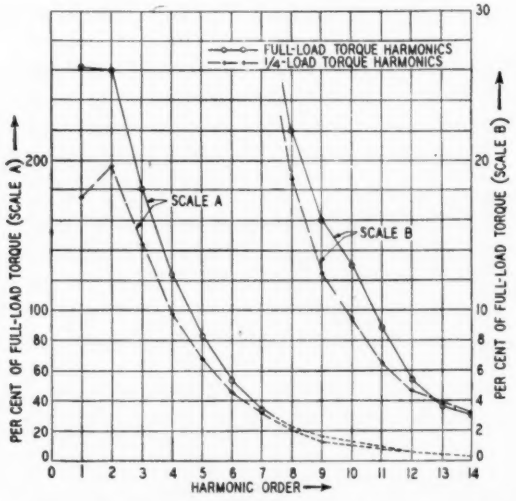


FIG. 2 TORQUE HARMONICS PRESENT IN A TWO-CYCLE BUSCH-SULZER DIESEL ENGINE

up through the 14th order, in terms of the full-load mean torque. As mentioned previously, the procedure herein described gives very good accuracy for the harmonics up to about the twentieth, while the still higher ones become unreliable.

The indicator card from which these harmonic torques were determined was for one of six cylinders under full-load conditions. The dashed line of Fig. 2 is the result of a similar analysis for half indicated power; because of internal friction, the brake horsepower for this condition is about one fourth the full-load brake horsepower. The ordinates for this harmonic curve are, as before, in per cent of full-load torque, not of mean quarter-load torque.

It is interesting to note that the peak pressure at half indicated power is about 10 per cent lower than the peak pressure at full load and that the harmonic disturbances are less by a similar order of magnitude in spite of the fact that the mean torque has been halved.

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Turbine-Blade Vibration Due to Partial Admission

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This paper deals with stresses in steam-turbine blades set up during partial admission, that is, when the steam loading on the blades is intermittent. By making simplifying assumptions about the character of the loading, it is possible to develop simple relations showing how the blade stress depends on frequency, running speed, and damping. Apparatus to determine experimentally the damping of blade steels and optical equipment to investigate the exact nature of the steam forces acting on the blades in actual operation are described.

THE term "partial admission" is used whenever steam is admitted to the rotating blades of a turbine over only a portion of the periphery. Fig. 1 illustrates this type of steam flow. It shows a single wheel propelled by a steam jet emerging from a series of nozzles, covering but a small part of the circumference.

Partial admission has been applied since the earliest days of the steam turbine. Because it produces high turbine efficiency at low loads, it has been used on units of all sizes. However, the use of partial admission on large high-pressure high-temperature machines has created an industry-wide problem of shroud and blade failures.

The blades, passing through the steam jet every revolution, are loaded intermittently. The general character of the force acting on the blades is represented in Fig. 2. In this figure the force is plotted as a function of time, or as a function of distance traveled by the blade, during one revolution of the rotor spindle.

First occurs a period in which the blade, entering the nozzle jet, is subjected to variable steam forces before steady flow through the blade passages on either side of the blade is established.

During the following or second period, the force on the blade is substantially constant, except for some slight periodic variations caused by wakes in the steam flow behind the nozzle vanes.

The third period corresponds to the blade leaving the steam jet.

As they rotate, the blades are subjected to a variable loading such as shown in Fig. 2 once every revolution. If the conditions are such that the steam forces in the first and third periods are for a major part in phase with the blade velocity, energy is fed into the blade system continuously and large vibratory motions with high stresses result.

Vibratory Conditions of Blades

It will be advantageous to examine first the vibratory conditions of blades with the simplest possible loading diagram, shown in Fig. 3.

In design, it is of special interest to know the maximum amplitudes of vibration which can exist for a given steam force P_0. Considering the type of loading shown in Fig. 3 to be a series of successive instantaneous loadings and unloadings, it is obvious from energy considerations that maximum motion will result as the load is applied when the blade is in its extreme backward position, and removed when the blade is in its extreme forward position. Actually the timing of the steam-jet loading with respect to the blade motion is hard to predict accurately because there are a large number of vibration cycles, say, between 20 and 60 for every revolution.

Fig. 4 shows the motions of blade with a single steam jet and with the worst possible timing of the load. In the following, the blades are considered as having a single degree of freedom so that at all times amplitude and stress are proportional. Let y_{st} be the static deflection of the blade under the force P_0 and let there be n cycles of vibration per revolution, n being an integer. Let there be n_2 cycles in the loaded period and n_1 cycles in the unloaded portion of the revolution. For maximum amplitudes n_2 should be equal to any integer $+1/2$.

A sudden application of the load is equivalent to a shift of the neutral point about which the vibration takes place. With δ being the logarithmic decrement, the various amplitudes, shown in Fig. 4, are related as follows

$$b = a + y_{st}$$
$$c = b e^{-n_2\delta}$$
$$d = c + y_{st}$$
$$a = d e^{-n_1\delta}$$

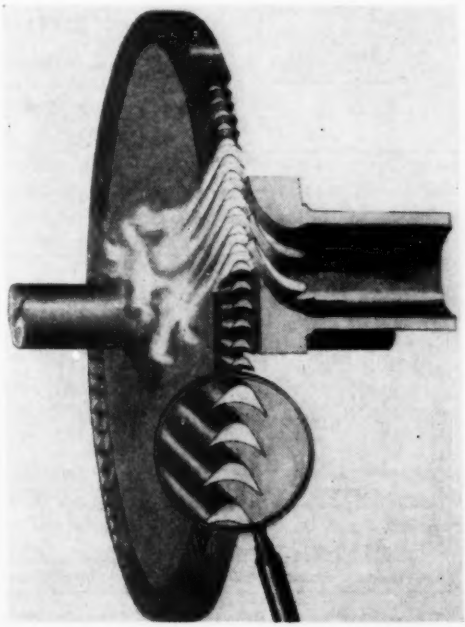


Fig. 1 Turbine Wheel Operating With Partial Admission

[1] Manager Experimental Division, Westinghouse Electric & Manufacturing Company, South Philadelphia Works. Jun. A.S.M.E.

Presented at the National Meeting of the Applied Mechanics Division, Ann Arbor, Mich., June 20–21, 1940, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1941, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

These equations are sufficient to solve for the unknown amplitudes a, b, c, and d. For example

$$b = y_{st}\frac{1 + e^{-n_1\delta}}{1 - e^{-n\delta}}$$

and

$$d = y_{st}\frac{1 + e^{-n_2\delta}}{1 - e^{-n\delta}}$$

Of particular interest is the amplitude range which will be denoted as $2\,y_{osc} = b' + y_{st} + d'$. Since $b' = be^{-\delta/2}$ and $d' = de^{-\delta/2}$, we have

$$y_{osc} = \frac{y_{st}}{2} + \frac{y_{st}\,(2 + e^{-n_1\delta} + e^{-n_2\delta})e^{-\delta/2}}{2(1 - e^{-n\delta})}$$

$$y_{osc} = y_{st}\left[\frac{1}{2} + \frac{2 + e^{-n_1\delta} + e^{-n_2\delta}}{2(1 - e^{-n\delta})e^{\delta/2}}\right] \quad \ldots\ldots\ldots\ldots [1]$$

The logarithmic decrement δ is a very small quantity, and even the product $n\delta$ is usually less than unity. Under those circumstances an approximation of Equation [1] by means of series expansion is indicated, especially since it allows a better insight into some of the factors involved.

With

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} \ldots\ldots$$

it follows that

$$y_{osc} = y_{st}\left[\frac{1}{2} + \frac{2 + \left(1 - n_1\delta + \frac{n_1^2\delta^2}{2} - \frac{n_1^3\delta^3}{6} \ldots\ldots\right) + \left(1 - n_2\delta + \frac{n_2\delta^2}{2} - \frac{n_2^3\delta^3}{6} \ldots\ldots\right)}{2\left(n\delta - \frac{n^2\delta^2}{2} + \frac{n^3\delta^3}{6} \ldots\ldots\right)\left(1 + \frac{\delta}{2} + \frac{\delta^2}{8} \ldots\ldots\right)}\right]$$

Since $n_1 + n_2 = n$, this can be written

$$y_{osc} = y_{st}\left[\frac{1}{2} + \frac{4 - n\delta + \frac{n^2\delta^2}{2} - n_1n_2\delta^2 - \frac{n_1^3\delta^3}{6} - \frac{n_2^3\delta^3}{6} \ldots\ldots}{n\delta\left(2 - \frac{n\delta}{2} + \frac{n^2\delta^2}{6} \ldots\right)\left(1 + \frac{\delta}{2} + \frac{\delta^2}{8} \ldots\right)}\right]$$

Neglecting higher orders, this reduces to

$$y_{osc} = y_{st}\left[\frac{1}{2} + \frac{2 + \frac{n\delta}{2}}{n\delta\left(1 + \frac{\delta}{2}\right)}\right]$$

Neglecting further $\delta/2$ compared with 1, the following simple formula is derived

$$y_{osc} = y_{st}\left(1 + \frac{2}{n\delta}\right) \quad \ldots\ldots\ldots\ldots\ldots [2]$$

While Equation [2] is approximate, it shows the essentials clearly. As would be expected, with zero damping, amplitude and stress become infinite.

With a given steam force P_s on a given blade, the length of the arc of admission has a negligible effect on the blade stress. It is represented by n_2, which does not enter into the approximate Equation [2].

In a typical case (n = 30 cycles per revolution and logarithmic decrement δ = 0.01) Equation [2] would give

$$y_{osc} = y_{st}\left(1 + \frac{2}{0.3}\right) = 7.67\,y_{st}$$

The exact Equation [1] gives, for $n_1 = 14^1/_2$ cycles, $n_2 = 15^1/_2$ cycles

$$y_{osc} = y_{st}\left[\frac{1}{2} + \frac{2 + 0.865 + 0.856}{2(1 - 0.7408)(1.005)}\right] = 7.65\,y_{st}$$

and for $n_1 = 2^1/_2$ cycles, $n_2 = 27^1/_2$ cycles

$$y_{osc} = y_{st}\left[\frac{1}{2} + \frac{2 + 0.975 + 0.769}{2(1 - 0.7408)\,1.005}\right] = 7.70\,y_{st}$$

It is seen that oscillating stresses equal to many times the static steam stresses can be set up. It is this magnification of stress which endangers the life of impulse blades.

For the type of loading assumed, these stresses are practically inversely proportional to the damping. This emphasizes the great importance of damping as a quality with which blade constructions must be endowed.

On low-pressure blades, the remedy against large vibratory stresses consists of tuning the blade carefully so that it does not operate at resonance. This, of course, is possible only in machines with constant speed. On impulse blades it is not possible to tune the blades so they operate out of resonance, i.e., the resonant frequencies are too close together. If there is resonance

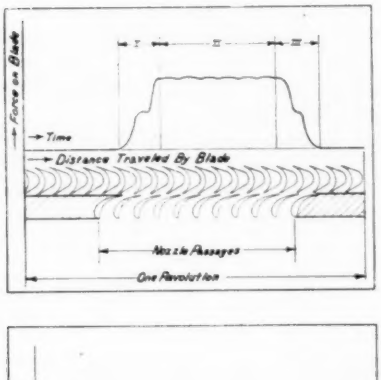


FIG. 2 GENERAL DIAGRAM OF INTERMITTENT STEAM LOAD

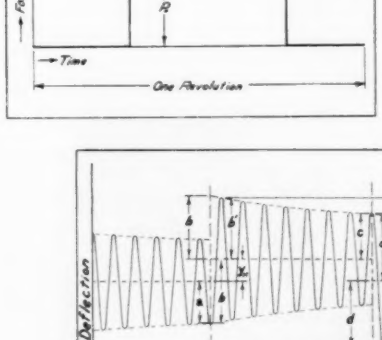


FIG. 3 SIMPLIFIED BLADE-LOAD DIAGRAM

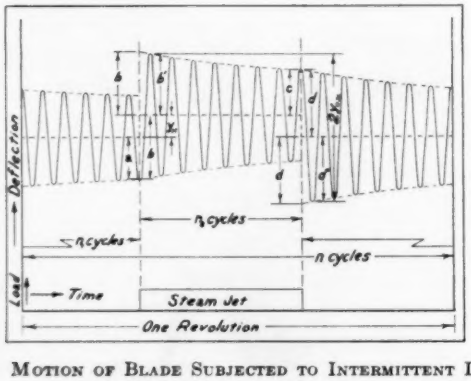


FIG. 4 MOTION OF BLADE SUBJECTED TO INTERMITTENT LOADING FORCES SHOWN IN FIG. 3

with 40 cycles per revolution at 3549 rpm, the next higher resonant speed will be 3640 rpm with 39 cycles per revolution. Thus, in this typical case the resonant speeds are $2^1/_2$ per cent apart. To make certain that the blades would operate between critical speeds, they would have to be tuned to within 1 per cent or less. This is practically impossible because of temperature variations (which influence the modulus of elasticity of the material and, hence, the frequency) and because of variations in the fit between rotor and blade.

Effect of Turbine Speed on Blade Stress

It is of particular interest to review the influence which the increase of turbine speed has had on impulse-blade stress. For

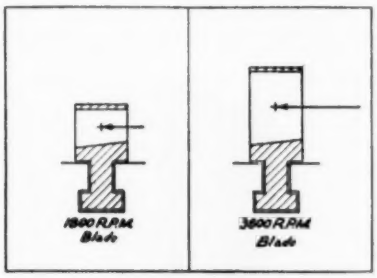


Fig. 5 Comparison Between Blades for 1800 Rpm and 3600 Rpm at Same Turbine Output

simplicity, let us compare two blades of the same cross section on turbines of the same kilowatt output, one machine operating at 1800 rpm, the other at 3600 rpm. Let us assume that the blade velocities, steam conditions, and arc of admission are identical on both turbines. The blades on the 3600-rpm machine will then be located on a mean diameter one half that of the 1800-turn machine.

To pass the same quantity of steam the blade port, or that part of the blade by which the steam is guided, must be twice as high for 3600 rpm as for 1800 rpm. Generally, the weakest section of the blades will be found where they are supported in the rotor. This section is a certain distance below the bottom of the blade port. In a typical case, shown in Fig. 5, the steady stresses in the weakest section would be in the ratio 8:3.

The blade frequencies vary approximately inversely with the square of the length. Therefore, the blade for 3600 rpm has roughly $^4/_9$ the frequency of the 1800-rpm blade, and n, the number of vibration cycles per revolution, is for the 3600-rpm turbine only $^2/_9$ of what it is for the 1800-rpm. This means that for very small damping values the oscillatory stresses in the 3600-rpm turbine are $\frac{9}{2} \times \frac{8}{3} = 12$ times as high as those of the 1800-rpm machine. With higher damping values this ratio decreases. This example illustrates the tremendous effect that increase in speed from 1800 to 3600 rpm has on impulse-blade stresses.

So far we have limited the discussion to one steam jet over the periphery. Often, for constructional reasons, it appears to be advantageous to have one series of nozzle passages in the base of the cylinder and another group in the cylinder cover. From Equation [2] it is clear, however, that if these passages would be located so each time the application and removal of the steam load would take place at the most unfavorable moment, the blade stress would be almost double that for the single series of nozzle passages. This means that this type of construction should be used only on blades with inherently low stress, or in such cases where the spacing of the nozzles is such that the additive effect of the "shocks" is precluded with certainty.

From low-pressure blades, operating in a slightly nonuniform steam flow (full admission), we are accustomed to having mechanical resonance when the blade frequency is a multiple of the running speed, e.g., when there is a whole number of free vibrations per revolution.

With the diagram assumed in Fig. 3 this is yet the case but, by taking diagrams of other shapes, it may easily be proved that the largest motions are often obtained when the number of vibrations per revolution is not an integer.

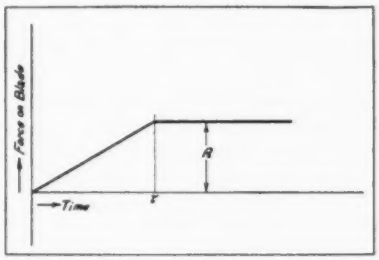


Fig. 6 Diagram of Load Applied Gradually

By applying the load more gradually, the oscillatory stresses can be reduced materially. A loading diagram such as shown in Fig. 6 is especially of interest.

If the load P_0 is applied gradually between $t = 0$ and $t = \tau$, the differential equation of the motion in this period is

$$m\frac{d^2y}{dt^2} + ky = P_0\frac{t}{\tau} \quad \ldots\ldots\ldots\ldots\ldots [3]$$

Here m and k are the equivalent mass and spring constant of the vibratory system. Damping forces during the loading period are neglected.

The solution of Equation [3] is, with $\omega = \sqrt{\frac{k}{m}}$

$$y = A\cos\omega t + B\sin\omega t + \frac{P_0 t}{k\tau} = A\cos\omega t + B\sin\omega t + y_{st}\frac{t}{\tau} \quad \ldots\ldots [3a]$$

At $t = \tau$ the amplitude is

$$y_\tau = A\cos\omega\tau + B\sin\omega\tau + y_{st} \quad \ldots\ldots\ldots\ldots [4]$$

and the velocity

$$v_\tau = -A\omega\sin\omega\tau + B\omega\cos\omega\tau + \frac{y_{st}}{\tau} \quad \ldots\ldots [5]$$

If the maximum amplitude reached after $t = \tau$ is a', we have the energy relation

$$\frac{1}{2}k(a'^2 - y_\tau^2) = P_0(a' - y_\tau) + \frac{1}{2}mv_\tau^2$$

Introducing Equations [4] and [5], this yields

$$(a' - y_{st})^2 = A^2 + B^2 + \frac{y_{st}}{(\omega\tau)^2} - \frac{2Ay_{st}}{\omega\tau}\sin\omega\tau + \frac{2By_{st}}{\omega\tau}\cos\omega\tau \quad \ldots [6]$$

If the amplitude before $t = 0$ was a, and if φ represents a phase angle between the time of maximum amplitude and the moment $t = 0$ when the load begins to be applied, we get from Equation [3a] the following initial boundary conditions at $t = 0$ (same displacement, same velocity as the free vibration existing for $t < 0$)

$$A = -a\cos\varphi$$

$$B = a \sin \varphi - \frac{y_{st}}{\omega\tau}$$

Inserting these values in Equation [6], we obtain with $\lambda = \omega\tau$

$$(a' - y_{st})^2 = a^2 + \frac{2y_{st}^2}{\lambda^2} - \frac{2y_{st}^2}{\lambda^2}\cos\lambda + \frac{2y_{st}}{\lambda}a\,(-\sin\varphi + \cos\varphi\sin\lambda + \sin\varphi\cos\lambda)$$

or

$$(a' - y_{st})^2 = a^2 + \frac{2y_{st}^2}{\lambda^2}(1 - \cos\lambda) + \frac{4y_{st}a}{\lambda}\left(\cos\frac{2\varphi+\lambda}{2}\sin\frac{\lambda}{2}\right)$$

So far, we have not specified at what phase of the existing vibration of amplitude a the load shall be applied. We will now choose the phase angle φ to give us the worst timing of the load, resulting in the largest increase in amplitude $(a' - a)$.

The maximum positive value of $\cos\frac{2\varphi+\lambda}{2}\sin\frac{\lambda}{2}$ is $\pm\sin\frac{\lambda}{2}$ for $\cos\frac{2\varphi+\lambda}{2} = \pm 1$, depending upon whether $\sin\frac{\lambda}{2}$ is positive or negative.

Thus we get for the largest increase in amplitude

$$(a' - y_{st})^2 = a^2 + \frac{4y_{st}^2}{\lambda^2}\sin^2\frac{\lambda}{2} \pm \frac{4y_{st}}{\lambda}a\sin\frac{\lambda}{2}$$

or

$$(a' - a) - y_{st} = \pm\frac{2y_{st}}{\lambda}\sin\frac{\lambda}{2} \quad \ldots\ldots\ldots\ldots [7]$$

Equation [7] gives a relation between the maximum increase in amplitude and the time in which the load is gradually increased from 0 to its final value. Since the natural period T of the free undamped vibration is $2\pi/\omega$, λ can be written as $\frac{2\pi\tau}{T}$, and Equation [7] becomes

$$\eta = \frac{(a' - a) - y_{st}}{y_{st}} = \frac{T}{\pi\tau}\sin\frac{\pi\tau}{T} \quad \ldots\ldots\ldots\ldots [8]$$

The factor η, which is a measure of the severity of the load changes, is thus seen to depend upon the length of loading period compared with the natural period of the blades.

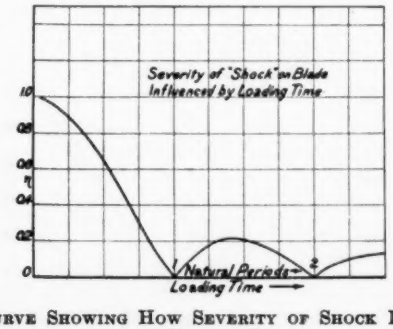


FIG. 7 CURVE SHOWING HOW SEVERITY OF SHOCK DEPENDS ON LOADING TIME OF BLADES

A plot of η against T/τ is given in Fig. 7.

It is at once clear that, where the loading is completed in one, two, or more natural periods ($\sin\pi\tau/T = 0$) the factor η will become zero. This means that, in this case, no matter what the phase relation is between the instant the load begins to be applied and the existing vibration, the only effect of the load will be that the oscillatory motion is shifted an amount equal to the static deflection.

If the load is both applied and removed in this fashion, the stress varies simply between zero and the steady steam stress, just as if the steam load were applied at an infinitely slow rate. This corresponds to the ideal minimum duty of the blade required to carry a certain steam force with partial admission.

The foregoing analysis indicates one way of reducing blade stresses under partial admission by means of a more gradual application of the steam load.

While blade frequencies can be precalculated with a reasonable degree of accuracy, the two other factors involved, namely, the exact nature of the steam shock and the damping of the blades, can be reliably determined by experiment only.

DETERMINING THE DAMPING OF BLADE STEELS

To determine the damping of blade steels at the temperature and the frequency pertaining to actual operation, we resorted to testing specimens in the shape of large tuning forks. This form of specimen had two obvious advantages. The vibratory forces in the fork are so nearly balanced that the exact way in which the fork is held is immaterial so far as the vibration of the prongs is concerned. The stress distribution in the prong is similar to that of the blade, so that any doubtful transfer from one state of stress to another is avoided.

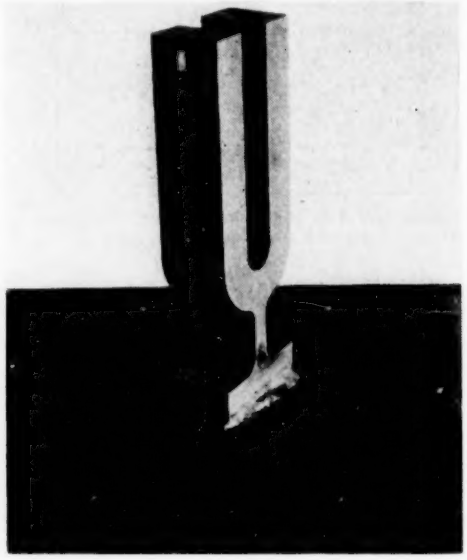


FIG. 8 TUNING-FORK SETUP FOR MEASURING DAMPING CAPACITY

Fig. 8 shows one of the test tuning forks mounted on a heavy block. The vibration of the tuning fork is recorded optically by means of a mirror clamped to the tip of one of the prongs, so that no energy is dissipated in the recording system. Suspicions about air friction around the forks were removed by making some of the tests in vacuum.

Even in recent literature the damping values of steels are innocently given as constant. We now know that the decrement depends vitally on stress and also on frequency and, as would be expected, on temperature.

For example, Dr. S. Siegel, physicist at the Westinghouse Research Laboratories, testing small bars in longitudinal vibration at 35,000 cycles per sec with stresses of ± 30 psi, finds damping values only $^1/_{30}$ of those found on large tuning forks (bending) at 750 cycles per sec and at a maximum stress of 20,000 psi.

Some typical damping curves obtained on various blade materials at room temperature are shown in Fig. 9 in the working-stress range. It is seen that there is considerable difference

between these materials. The 12 per cent Cr steel has, in the working-stress range, about 10 times as much damping as the Cr-Ni steels.

Some other characteristics, damping as function of temperature and frequency, are shown in Figs. 10 and 11. A great deal more fundamental work seems to be necessary before a clear understanding of what constitutes damping can be reached.

Besides internal damping in the materials, there is a certain amount of damping caused by microscopic rubbing and impacting along the fits between rotor and blades and in the shroud connections. However, unless there has been established a very definite lower limit of this external damping, it appears to be good practice to take for design purposes only the available internal damping of the blade materials. Our experience with *T*-roots indicates that the damping contributed by small motions between rotor and blade is negligible in this case.

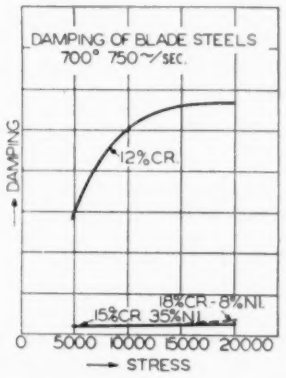


FIG. 9 DAMPING OF BLADE MATERIALS AS A FUNCTION OF STRESS

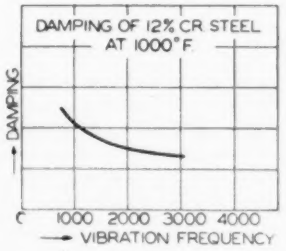


FIG. 10 DAMPING OF BLADE MATERIALS AS A FUNCTION OF FREQUENCY

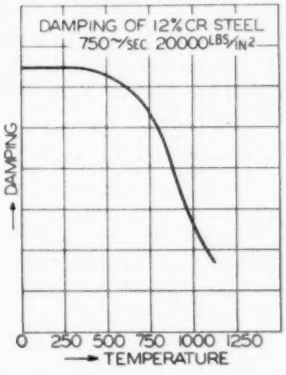


FIG. 11 DAMPING OF BLADE MATERIALS AS A FUNCTION OF TEMPERATURE

NATURE OF STEAM FORCES ACTING ON BLADE

To determine the nature of the steam forces acting on the blades while in operation, an optical system is used. This system and the experimental turbines on which it was applied have been described[2] recently and will only be mentioned here.

[2] "Superposed Turbine Blade Research," by F. T. Hague, *Mechanical Engineering*, vol. 61, April, 1940, pp. 275–277.

Light from an external source, Fig. 12, is deflected by a stationary slanted mirror into the hollow shaft. After passing a quartz window, which is needed to prevent the steam from leaking out, the light beam strikes a whirling mirror which projects it through a hole in the rotor and the blade toward a small mirror plug held at the tip of the blade. This small mirror reflects the light back by way of the inclined rotating mirror out through the shaft to the screen or film. The light beam is focused on the screen eccentrically so that, when the spindle rotates without blade vibration, the light spot describes a circle. When the blades vibrate, the light beam is deflected away from this circular path and waves appear. The stresses in the blades are deduced directly from the magnitude of the waves.

Turbine-blade vibrations occur both in the circumferential direction (in the plane of rotation) and in the lateral direction. Since we wish to record both motions separately, we need some sort of filtering device. For this purpose we have in the optical system a cylindrical lens of short focal length. The light beams striking this lens are so nearly parallel that the lens focuses them all practically on one focal line in the plane of the screen. While motion of the light beam along the longitudinal axis of the lens is transmitted undistorted along the focal line, motion in the perpendicular direction is cut out almost entirely. As shown in Fig. 12, the setup will record lateral blade motions; to record circumferential motion the cylindrical-lens assembly should be rotated 90 deg.

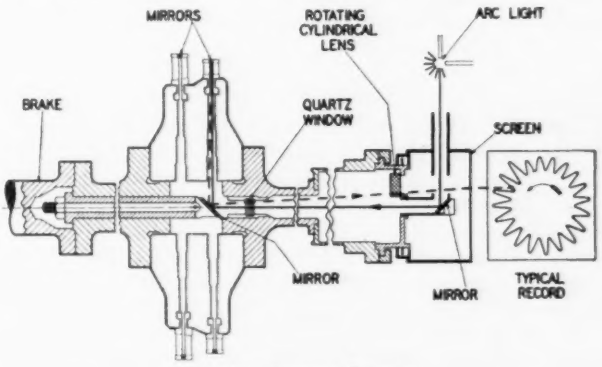


FIG. 12 OPTICAL SYSTEM

To date more than 30,000 individual records of blade vibration have been obtained for a wide range of operating conditions. After this test work is completed, it will be the subject of another paper.

Once the fundamentals are clearly understood, the nature of the various means of reducing blade stress under partial admission is clearly indicated:

1 The designer should, in general, strive to obtain short blades and light steam loads on the blades as long as this does not result in poor turbine efficiency. Shorter blades also mean higher natural frequencies so that the vibratory motion can die out more rapidly between successive loading periods.

2 Arranging the nozzles in a continuous sequence so that there is never more than one loading and unloading of the blades during one revolution is advantageous.

3 By applying effectively the steam load gradually at a certain rate, the stresses can be reduced materially.

4 More damping, internal or external, decreases blade stress.

ACKNOWLEDGMENT

The author wishes to express his indebtedness to his collaborators, especially to Messrs. C. C. Davenport, O. E. Rodgers, T. J. Putz, and W. A. Williams for their invaluable help in several phases of this work.

Convergence of Hardy Cross's Balancing Process

By RUFUS OLDENBURGER,[1] CHICAGO, ILL.

It can be shown easily that the Cross method of structural analysis may be applied to a given structure in such a manner that the process does not converge. In this paper the author gives necessary and sufficient conditions for the convergence of the Cross method, and exhibits a convergent process of balancing any given structure. In particular he shows that a balancing process can be described by real linear transformation, that is, by a matrix of real numbers, and that the process converges in the sense of this paper if and only if the infinite power of this matrix exists and is zero. The study is restricted to the case of a continuous beam.

1—Introduction

HARDY CROSS has introduced a process[2] of structural analysis by balancing moments against each other in successive steps, this process being part of an infinite procedure. It has been used by many engineers, as for example by Grinter.[3] Southwell[4] has given an equivalent method, the convergence of which was considered in a paper by Temple.[5] Using energy considerations, Temple gave two convergent methods of applying the Southwell process to a given structure.

One can easily show that the Cross method may be applied to a given structure in such a manner that the process does not converge.[6] In the present paper the author gives necessary and sufficient conditions for the convergence of the Cross method, and exhibits a convergent process of balancing any given structure. In particular, he shows that a balancing process P can be described by real linear transformation, that is, by a matrix composed of real numbers, and that the process P converges in the sense of this paper if and only if the infinite power of this matrix exists and is zero. The existence and vanishing of the infinite power of a real matrix has been treated elsewhere.[7] For a discussion of matrices and their applications, the reader is referred to a book by Frazer, Duncan, and Collar.[8]

The study in this paper is restricted to a continuous beam. The argument for a network of beams is similar to that given here.

2—Description of the Hardy Cross Process for a Continuous Beam

We suppose that we have a beam B supported at both ends, these ends being free or fixed. Let S be an intermediate support. We associate with the left and right sides of S real numbers, so that with S is associated a pair of real numbers (a, b). Although the meaning of these numbers is immaterial to the following mathematical deductions it may be mentioned here that they are certain bending moments which would exist if the system were changed to a system of independent beams each fixed at all of the given supports. In the present paper we shall follow a convention used by Grinter[3] about positive and negative signs attributed to moments so that the final equilibrium is reached if the sum of the moments at each support is zero.

When the support S is balanced by the Hardy Cross method, the pair of numbers (a, b) is replaced in a certain way by a pair of real numbers $(\alpha, -\alpha)$ whose sum vanishes. With an end support E of B we associated one real number a, and when E is balanced we replace a by 0. A fixed end E is not balanced. The purpose of the Hardy Cross balancing process is through balancing to replace the numbers at the supports of B by a set of numbers where the sum of the numbers at each support is arbitrarily small.

For further descriptive details the reader is recommended to read the papers[2,3,4,5,7] referred to previously. A precise mathematical formulation of the Hardy Cross process will follow immediately.

3—The Balancing Process in Terms of Matrices

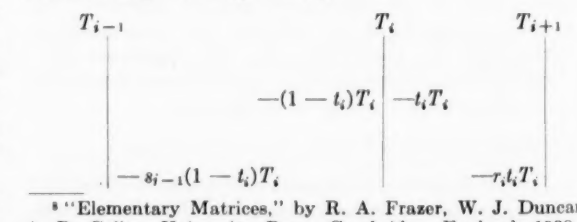
At any stage in the balancing process, the numbers at the i-th support add up to a number which we shall denote by T_i. If we now balance this support the theory of beams on unyielding supports prescribes that only the numbers at the $(i-1)$-st, i-th, and $(i+1)$-st supports are affected. Explicitly, we have the pattern

T_{i-1}	T_i	T_{i+1}

and after balancing we have the pattern

$T_{i-1} - s_{i-1}(1-t_i)T_i$	0	$T_{i+1} - r_i t_i T_i$

obtained by performing the steps



[1] Associate Professor of Mathematics, Armour Institute of Technology.

[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, Transactions of the American Society of Civil Engineers, vol. 96, 1932, pp. 1–15.

[3] "Wind Stress Analysis Simplified," by L. E. Grinter, Transactions of the American Society of Civil Engineers, vol. 99, 1934, pp. 610–634. The author is indebted to Mr. Grinter for calling his attention to the existence of the problem of convergence of the Hardy Cross process.

[4] "Stress Calculation in Frameworks by the Method of 'Systematic Relaxation of Constraints,' " by R. V. Southwell, Proceedings of the Royal Society of London, series A, vol. 151, 1935, pp. 56–95.

[5] "The General Theory of Relaxation Methods Applied to Linear Systems," by G. Temple, Proceedings of the Royal Society of London, series A, vol. 169, 1938–1939, pp. 476–500.

[6] For example, one could "balance" one support a finite number of times only, and the remaining supports an unlimited number of times.

[7] "Infinite Powers of Matrices and Characteristic Roots," by R. Oldenburger, *Duke Mathematical Journal*, vol. 8, June, 1940.

Presented at the National Meeting of the Applied Mechanics Division, Ann Arbor, Mich., June 20–21, 1940, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1941, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[8] "Elementary Matrices," by R. A. Frazer, W. J. Duncan, and A. R. Collar, University Press, Cambridge, England, 1938. The authors give an extensive treatment of matrix powers and discuss in special cases how high powers of a real matrix a are related to the latent (characteristic) roots of a, and how the existence of the infinite power of a depends on the latent roots.

The r's, s's, and t's are given constants which depend on the elastic properties of the beams right and left of the support considered.

Let there be n supports so that i ranges over 1, 2, ..., n. If the first support is a free end, $t_1 = 1$; similarly, if the n-th end is a free end, $t_n = 0$. If one or both of the end supports is a fixed end, we balance only the remaining supports.

The sets of constants $T_1, \ldots, T_n$ can be arranged in a column, called a "column vector," namely

$$v = \begin{Vmatrix} T_1 \\ T_2 \\ \cdot \\ \cdot \\ \cdot \\ T_n \end{Vmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

The corresponding vector after balancing the i-th support is, in the sense of matrix multiplication,[9] the product $a_i v$, where

$$a_i = \begin{Vmatrix} 1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ & \cdot & & & & & \\ & & \cdot & & & & \\ & & & \cdot & & & \\ 0 & \ldots & 01 & 0 & 0 & 0 \ldots & 0 \\ 0 & \ldots & 0 & 1 & [-s_{i-1}(1-t_i)] & 0\ 0 \ldots & 0 \\ 0 & \ldots & 0 & 0 & 0 & 0\ 0 \ldots & 0 \\ 0 & \ldots & 0 & -r_i t_i & 1 & 0 \ldots & 0 \\ 0 & \ldots & 0 & 0 & 0 & 1\ 0 \ldots & 0 \\ & & & & & & \cdot \\ & & & & & & \cdot \\ & & & & & & \cdot \\ 0 & & & \cdot & \cdot & \cdot & 01 \end{Vmatrix} \quad \ldots [2]$$

which can be written as

$$a_i = \begin{Vmatrix} I_1 & 0 & 0 \\ 0 & B_i & 0 \\ 0 & 0 & I_2 \end{Vmatrix} \quad \ldots\ldots\ldots\ldots\ldots [3]$$

where I_1, I_2 are "minors" with 1's on the diagonal and zeros elsewhere, and B_i is the minor

$$\begin{Vmatrix} 1 & -s_{i-1}(1-t_i) & 0 \\ 0 & 0 & \\ 0 & -r_i t_i & 1 \end{Vmatrix}$$

If we now balance the j-th column the numbers at the supports form a column vector given by $a_j a_i v$. Thus after a sequence of balancing the numbers at the supports are given by a product

$$a_\alpha a_\beta \ \ldots\ a_\lambda a_\xi a_\delta v \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

which can be written more simply as av, where a is the matrix $a_\alpha a_\beta \ldots a_\delta$.

Since we do not balance fixed ends, we may restrict v to be a vector composed of the numbers at the remaining supports.

We term the element in the i-th row and j-th column of a matrix the (i, j)-th element of this matrix.

Let $c_1, c_2, \ldots$ be an infinite sequence of n by m matrices.[10] We say that this sequence has a limit c if for each pair of values i, j the (i, j)-th elements of $c_1, c_2, \ldots$ have the (i, j)-th element of c as a limit in the usual sense. We write this as

$$\lim_{\alpha \to \infty} c_i = c \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

The balancing process, if performed indefinitely, converges in the sense of this paper if and only if the successive products v, $a_\delta v$, $a_\xi a_\delta v$, $a_\lambda a_\xi a_\delta v$, ... of the type [4] approach the vector

$$0 = \begin{Vmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{Vmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

as a limit.

Example. We consider a case of four supports with the right end fixed. We take the constants as indicated in the pattern

$$\begin{array}{cccc} T_1 = 5 & T_2 = 2 & T_3 = 3 & T_4 = 1 \\ \Big| & \Big| & \Big| & \Big| \\ r_1 = 1/2 & r_2 = 1/3 & r_3 = 1/6 & \\ s_1 = 1/4 & s_2 = 1/5 & s_3 = 1/2 & \\ \Big| & \Big| & \Big| & \\ t_1 = 1 & t_2 = 1/3 & t_3 = 1/2 & \end{array}$$

We take

$$v = \begin{Vmatrix} T_1 \\ T_2 \\ T_3 \end{Vmatrix} = \begin{Vmatrix} 5 \\ 2 \\ 3 \end{Vmatrix} \quad \ldots\ldots\ldots\ldots\ldots [7]$$

The matrices a_1, a_2, a_3 which correspond to balancing the first, second, and third supports are respectively

$$\begin{Vmatrix} 0 & 0 & 0 \\ -r_1 t_1 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} = \begin{Vmatrix} 0 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} \quad \ldots\ldots\ldots [8]$$

$$\begin{Vmatrix} 1 & -s_1(1-t_2) & 0 \\ 0 & 0 & 0 \\ 0 & -r_2 t_2 & 1 \end{Vmatrix} = \begin{Vmatrix} 1 & -1/8 & 0 \\ 0 & 0 & 0 \\ 0 & -1/6 & 1 \end{Vmatrix} \quad \ldots\ldots\ldots [9]$$

$$\begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & -s_2(1-t_3) \\ 0 & 0 & 0 \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/10 \\ 0 & 0 & 0 \end{Vmatrix} \quad \ldots\ldots [10]$$

We are disregarding the fourth support since it is a fixed end.

If we balance the second, third, and first supports in this order we obtain $v_1 = a_1 a_3 a_2 v$ where specifically

$$v_1 = \begin{Vmatrix} 0 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/10 \\ 0 & 0 & 0 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & -1/8 & 0 \\ 0 & 0 & 0 \\ 0 & -1/6 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 5 \\ 2 \\ 3 \end{Vmatrix} \quad \ldots [11]$$

which upon multiplying the matrices successively yields

$$v_1 = \begin{Vmatrix} 0 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/10 \\ 0 & 0 & 0 \end{Vmatrix} \cdot \begin{Vmatrix} \frac{14}{3} \\ 0 \\ \frac{25}{9} \end{Vmatrix} =$$

$$\begin{Vmatrix} 0 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} \frac{14}{3} \\ -5/18 \\ 0 \end{Vmatrix} = \begin{Vmatrix} 0 \\ -47/18 \\ 0 \end{Vmatrix} \quad \ldots\ldots [12]$$

Now if we write $a = a_1 a_3 a_2$, then $v_1 = av$ and

$$a = \begin{Vmatrix} 0 & 0 & 0 \\ -1/2 & \frac{17}{180} & -\frac{1}{10} \\ 0 & 0 & 0 \end{Vmatrix} \quad \ldots\ldots\ldots [13]$$

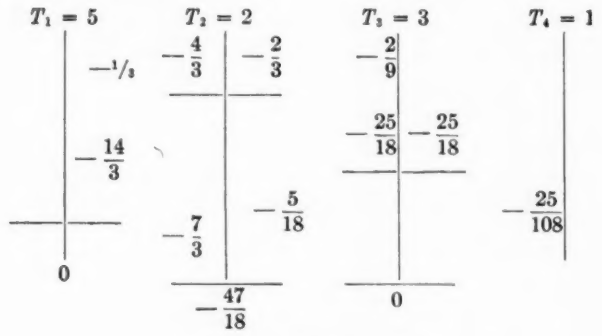
Thus the numbers at the supports are now

0	$-\frac{47}{18}$	0	*

[9] For the matrix theory used here the reader is referred to "Theory of Matrices," by C. C. MacDuffee, *Ergebnisse der Mathematik*, Julius Springer & Sons, Berlin.

[10] An n by m matrix is by definition a matrix with n rows and m columns.

Direct balancing yields the same result as is clear from the scheme which follows



4—Repeated Balancing

It is obvious that restrictions must be put on the balancing process to insure its convergence in the sense of this paper. Since a balancing performed by an individual can involve only a finite set F of steps, it is no restriction to assume that if the balancing were continued indefinitely, we would do this by repeating the set F of steps over and over. Let v be the original vector, whose elements are the numbers $T_1, T_2, \ldots, T_n$ at the supports to be balanced. The vector v_1 obtained by the steps F from v, is $v_1 = av$, where a is a product of matrices of the type a_i defined in Section 3. If we perform the steps F again we obtain from v_1 a vector v_2 related to v_1 by the formula

$$v_2 = av_1 \quad [14]$$

whence

$$v_2 = a(av) = a^2v \quad [15]$$

Thus after performing the set F of steps m times we arrive at a vector v_m related to v by the formula

$$v_m = a^m v \quad [16]$$

By Section 3 the balancing process converges if and only if

$$\lim_{m \to \infty} v_m = 0 \quad [17]$$

For arbitrary v, $\lim v_m = 0$ as $m \to \infty$ if and only if $\lim a^m = 0$ as $m \to \infty$. We shall use the notation a^∞ for

$$\lim_{m \to \infty} a^m$$

It has been proved[7] that $a^\infty = 0$ if and only if the characteristic roots of a are in absolute value less than 1. We term a the matrix of the balancing process. We have therefore arrived at the following result: (See next column.)

Theorem: Balancings with matrix a converge if and only if the characteristic roots of a are less than 1 in absolute value.

Example. We consider the matrix a in the example of Section 3. Subtracting a variable λ from the diagonal elements of a we obtain the matrix $a - \lambda I$ given by

$$\begin{Vmatrix} -\lambda & 0 & 0 \\ -\frac{1}{2} & \frac{17}{180} - \lambda & -\frac{1}{10} \\ 0 & 0 & -\lambda \end{Vmatrix}$$

Here I denotes the identity matrix with 3 rows and 3 columns; I is a matrix with 1's on the diagonal and zeros elsewhere. The characteristic determinant of a is the determinant $|a - \lambda I|$ given by

$$\begin{vmatrix} -\lambda & 0 & 0 \\ -\frac{1}{2} & \frac{17}{180} - \lambda & -\frac{1}{10} \\ 0 & 0 & -\lambda \end{vmatrix}$$

The characteristic equation of a is the equation $|a - \lambda I| = 0$, whose roots are 0, $\frac{17}{180}$, and 0. Since these roots are in absolute value less than 1, the balancing process with matrix a converges if repeated indefinitely.

5—A Converging Process

In Section 4 we have given an answer to the question: When does the balancing process converge? In the present section we shall give a converging balancing process that may be applied to any given set of supports.

Let there be n given supports. It is no restriction to suppose that $0 \leqq t_i \leqq 1$ and $0 \leqq r_i < 1, 0 \leqq s_i < 1$. We can obviously balance the 1st, 3rd, 5th, ... supports so that the vector of the numbers at these supports is

$$v = \begin{Vmatrix} 0 \\ T_2 \\ 0 \\ T_4 \\ 0 \\ \vdots \\ T_n \end{Vmatrix} \quad [18]$$

The last element T_n is zero if n is odd. We now balance the even-numbered supports, and then the odd-numbered supports. In place of the vector v we now have a vector $v' = av$, where (See Equation 19 below.)

$$a = \begin{Vmatrix} 0 & 0 & 0 & 0 & 0 & 0 \ldots 0 \\ 0 & \left[\begin{matrix} r_1 t_1 s_1 (1 - t_2) \\ + r_2 t_2 s_2 (1 - t_3) \end{matrix}\right] & 0 & s_2 s_3 (1 - t_3)(1 - t_4) & 0 & 0 \ldots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \ldots 0 \\ 0 & r_2 r_3 t_2 t_3 & 0 & \left[\begin{matrix} r_3 t_3 s_3 (1 - t_4) \\ + r_4 t_4 s_4 (1 - t_5) \end{matrix}\right] & 0 & s_4 s_5 (1 - t_5)(1 - t_6) \quad 0 \quad 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \ldots 0 \\ 0 & 0 & 0 & r_4 r_5 t_4 t_5 & 0 & \left[\begin{matrix} r_5 t_5 s_5 (1 - t_6) \\ + r_6 t_6 s_6 (1 - t_7) \end{matrix}\right] \quad 0 \quad 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \ldots 0 \quad 0 \quad 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \qquad \cdot \quad \cdot \end{Vmatrix} \quad [19]$$

If a is of order n, n even, the last bracket quantity in a is simply $[r_{n-1}t_{n-1}s_{n-1}(1-t_n)]$. As noted in Section 4, if the first end is free, $t_1 = 1$, if the last is free, $t_n = 0$. Except for a power of λ, the characteristic determinant $|a - \lambda I|$ (I here is again the identity matrix with 1's on the diagonal and zeros elsewhere) of a is the characteristic determinant of the matrix a', where

$$a' = \begin{Vmatrix} \begin{bmatrix} r_1s_1t_1(1-t_2) \\ + r_2s_2t_2(1-t_3) \end{bmatrix} & s_2s_3(1-t_3)(1-t_4) & 0 & 0 \cdot \cdot \cdot \cdot 0 \\ r_3r_3t_2t_3 & \begin{bmatrix} r_3s_3t_3(1-t_4) \\ + r_4s_4t_4(1-t_5) \end{bmatrix} & s_4s_5(1-t_5)(1-t_6) & 0 \cdot \cdot \cdot \cdot 0 \\ 0 & r_4r_5t_4t_5 & \begin{bmatrix} r_5s_5t_5(1-t_6) \\ + r_6s_6t_6(1-t_7) \end{bmatrix} & s_6s_7(1-t_7)(1-t_8) \; 0 \; 0 \\ 0 & 0 & r_6r_7t_6t_7 & \begin{bmatrix} r_7s_7t_7(1-t_8) \\ + r_8s_8t_8(1-t_9) \end{bmatrix} \; \cdot \; \cdot \\ \cdot & \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot \cdot \end{Vmatrix} \quad \cdots\cdots\cdots [20]$$

The characteristic roots of a are 0 and the roots of $|a' - \lambda I| = 0$. We note that a' has the shape

$$a = \begin{Vmatrix} * & * & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ * & * & * & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & * & * & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & & \cdot & & & & \vdots \\ \cdot & & & & \cdot & & \cdot & 0 \\ \cdot & & \cdot & & & & * & * \\ 0 & \cdot & \cdot & 0 & & & * & * \end{Vmatrix} \quad \cdots\cdots\cdots [21]$$

where the asterisks stand for numbers that may be different from zero.

We observe that the elements of a' are positive or zero. It has been proved[7] that an increase in some of these numbers will either leave the maximum absolute value of the characteristic roots of a' unchanged, or it will increase this value. Since for each i we have $r_i < 1$ and $s_i < 1$, there is a positive number r less than 1 such that the r_i and s_i are less than r for each i. Replace each r_i and each s_i in a' by r to obtain a matrix

$$a'' = \begin{Vmatrix} r^2[t_1(1-t_2) + t_2(1-t_3)] & r^2(1-t_3)(1-t_4) & 0 & 0 \; 0 \\ r^2t_2t_3 & r^2[t_3(1-t_4) + t_4(1-t_5)] & r^2(1-t_5)(1-t_6) & 0 \; 0 \\ 0 & r^2t_4t_5 & r^2[t_5(1-t_6) + t_6(1-t_7)] & \cdot \cdot \cdot \\ \cdot & \cdot & \cdot & \cdot \cdot \cdot \end{Vmatrix} \quad \cdots\cdots [22]$$

We let p denote the order of a''. We let B denote the matrix obtained from a'' by replacing r^2 by 1 wherever r^2 occurs in a''. We can write

$$B = \begin{Vmatrix} [1-a_1-b_1] & a_2 & 0 & 0 & \cdot \cdot \cdot & 0 & 0 \\ b_1 & [1-a_2-b_2] & a_3 & 0 & \cdot \cdot \cdot & 0 & 0 \\ 0 & b_2 & [1-a_3-b_3] & a_4 & \cdot \cdot \cdot & 0 & 0 \\ 0 & 0 & b_3 & \cdot & & 0 & 0 \\ & & & \cdot & \cdot & a_{p-1} & 0 \\ & & & & \cdot & [1-a_{p-1}-b_{p-1}] & a_p \\ 0 & 0 & 0 \cdot \cdot \cdot \cdot 0 & & & b_{p-1} & [1-a_p-b_p] \end{Vmatrix} \quad \cdots\cdots [23]$$

where $a_i = (1 - t_{2i-1})(1 - t_{2i})$ for $i = 1, 2, \ldots, p$, and $b_i = t_{2i}t_{2i+1}$ for $i = 1, 2, \ldots, p$. Dividing each of the rows of $|a'' - \lambda I|$ by r^2 we find that the characteristic roots of a'' are the roots of $|B - (\lambda/r^2)I| = 0$. We write $\mu = \lambda/r^2$. If $\mu = 1$, then $\lambda = r^2 < 1$. Thus the maximum absolute value of the characteristic roots λ of a'' is less than 1 if the maximum absolute value of the characteristic roots μ of B is less than or equal to 1.

Frobenius[11] proved almost completely that a matrix a all of whose elements are positive or zero has a real positive characteristic root not less in absolute value than any other characteristic root of a. Since the t's are between 0 and 1, it is easy to see that the elements of B are between 0 and 1. To prove that the roots of $|B - \mu I| = 0$ are in absolute value not greater than 1, it is necessary to prove only that the maximum positive root μ of $|B - \mu I| = 0$ is not greater than 1. We thus wish to show that $|B - \mu I| = 0$ has no root $\mu = 1 + \epsilon$ where $\epsilon > 0$. Substituting $1 + \epsilon$ for μ in $|B - \mu I|$, and changing the sign of each row we obtain

$$D(\epsilon) = \begin{vmatrix} a_1 + b_1 + \epsilon & -a_2 & & & \bigcirc \\ -b_1 & a_2 + b_2 + \epsilon & \cdot & & \\ & \cdot & \cdot & & \\ & & & \cdot & -a_p \\ \bigcirc & & & -b_{p-1} & a_p + b_p + \epsilon \end{vmatrix} \quad \cdots\cdots [24]$$

[11] "Über Matrizen aus positiven Elementen," by S. B. Frobenius, Kgl. Preuss. Akad. d. Wiss., 1908, pp. 471–476.

Expanding $D(\epsilon)$ we get

$$D(\epsilon) = \epsilon^p + \alpha_1\epsilon^{p-1} + \alpha_2\epsilon^{p-2} + \alpha_{p-1}\epsilon + \alpha_p \quad [25]$$

By determinant theory the coefficient α_i of ϵ_i in Equation [25] is the sum of the principal determinant minors of $D(0)$ of order i. It is recalled that a principal determinant minor of $D(0)$ is a minor obtained by striking out the same rows as columns from $D(0)$.

We let m_i denote the leading principal determinant minor of $D(0)$ of order i. It is

$$m_i = \begin{vmatrix} (a_1 + b_1) & -a_2 & & \bigcirc \\ -b_1 & (a_2 + b_2) & & \\ & & & \\ \bigcirc & & -b_{i-1} & (a_i + b_i) \end{vmatrix} \quad [26]$$

We write $m_i[a_1, a_2, \ldots, a_i;\ b_1, b_2, \ldots, b_i]$ since m_i is a function of these constants. If we write $m_0 = 1$ we can prove that

$$m_i = a_1 m_{i-1}[a_2, \ldots, a_i;\ b_2, \ldots, b_i] + b_1 b_2 \ldots b_i \quad [27]$$

Here $m_{i-1}(a_2, \ldots, a_j;\ b_2, \ldots, b_i)$ is obtained from $m_{i-1}[a_1, a_2, \ldots, a_{i-1};\ b_1, \ldots, b_{i-1}]$ by replacing $a_1, \ldots, a_{i-1}$ by $a_2, \ldots, a_i$ respectively, and making a similar substitution on the b's. Since $a_1 \geqq 0$, $b_1 \geqq 0$, $\ldots$, $b_i \geqq 0$, it follows that if $m_{i-1} \geqq 0$, then $m_i \geqq 0$. Since $m_0 > 0$, it follows by mathematical induction that $m_i \geqq 0$ for each i. It can be easily proved that each principal minor is either of the same shape as m_i, or a product of such minors. Thus each principal determinant minor of $D(0)$ is $\geqq 0$, whence the coefficients $\alpha_1, \ldots, \alpha_p$ in Equation [25] are $\geqq 0$. Directly, or by Descartes rule of signs, one can see that there is no positive root of the equation $D(\epsilon) = 0$. Thus there is no root μ of $|B - \mu I| = 0$ greater than 1. It follows by retracing the foregoing argument that the characteristic roots of a are in absolute value less than 1 so that $a^\infty = 0$. Thus we have proved the following result:

Theorem: The process of balancing alternately odd and even supports converges.

Example. We take a case of four supports with free ends. We also take all the r's, s's and t's except t_1 and t_4 to be $\frac{1}{2}$. Evidently, $t_1 = 1$, $t_4 = 0$. Let the given T's be 1, —1, —2, and 2 respectively. The balancing of the odd-numbered supports yields

| 1 | —1 | —2 | 2 | ⟶ | 0 | —1 | 0 | 5/2 |

Thus

$$v = \begin{Vmatrix} 0 \\ -1 \\ 0 \\ 5/2 \end{Vmatrix} \quad [28]$$

If we balance the even-numbered supports, we obtain

| 0 | —1 | 0 | 5/2 | ⟶ | 1/4 | 0 | —1 | 0 |

If now we balance the odd-numbered supports, we get

| 1/4 | 0 | —1 | 0 | ⟶ | 0 | 1/8 | 0 | 1/4 |

The vector

$$v' = \begin{Vmatrix} 0 \\ 1/8 \\ 0 \\ 1/4 \end{Vmatrix} \quad [29]$$

is related to v by the equation $v' = av$ with matrix

$$a = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 3/16 & 0 & 1/8 \\ 0 & 0 & 0 & 0 \\ 0 & 1/16 & 0 & 1/8 \end{Vmatrix} \quad [30]$$

Now

$$|a - \lambda I| = \begin{vmatrix} -\lambda & 0 & 0 & 0 \\ 0 & 3/16 - \lambda & 0 & 1/8 \\ 0 & 0 & -\lambda & 0 \\ 0 & 1/16 & 0 & 1/8 - \lambda \end{vmatrix} = \lambda^4 - 5/16\lambda^3 + 1/64\lambda^2 \quad [31]$$

The characteristic roots of a are 0, 0, 1/16, 1/4 which are all in absolute value less than 1. Hence $a^\infty = 0$, and $\lim_{n \to \infty} a^n v = 0$.

Discussion

Boundary Friction in Bearings at Low Loads[1]

J. L. Fletcher.[2] It appears that the authors have confirmed what might be expected, that is, for boundary lubrication the rougher the surface the higher the coefficient of friction. And further, that for a given grinding process the softer the metal the rougher the surface.

A study of this paper has led to the following questions:

1 As finishing processes approach producing journal surfaces of zero roughness, or a roughness comparable with an oil-molecule thickness, would not the friction coefficient become practically equal for all types of metals?

2 If it were possible to get equal surfaces (rather than equal surfacing processes) would not the friction coefficient be equal for all metals, assuming no metal scoring?

3 This then leads to the question: Should not softer metals be finished by different processes from those for harder metals when boundary friction is to be encountered to an appreciable extent?

4 As the stainless steel showed considerable wear, it would be interesting to know whether or not the coefficient of friction for stainless was approaching that of nickel steel as the surface of the stainless approached the condition of the surface of the nickel steel. Because of this wear, can data taken at start of test be reliably compared with data taken at end of test?

The writer would welcome the authors' opinion on these questions. It is the writer's belief that as the metal surfaces approach being equal the coefficients of friction for boundary lubrication would become nearly equal. Further it is probable that it would pay to use a process for finishing the softer metals that would give a surface equal to that of a ground hard surface when there is much difference in material and hardening costs of the metals being considered.

J. R. Connelly.[3] The method of oil feed employed by the authors seems susceptible to uncontrolled variations in the test conditions. Some years ago the writer experimented with various rates of oil feed in connection with such work on "thin-film lubrication." From the results obtained it was concluded that if one desired results that would be reproducible by different experimenters some method of applying lubricant should be used that supplied the lubricant in unlimited quantities and that the viscosity should be varied by other means than starving the bearing.

To say that there was provided "enough oil for boundary lubrication but not enough to form a fluid film or to provide cooling" seems to the writer to be unsupported by the evidence. Any fluid or plastic (such as grease) third material inserted between journal and bearing is bound to take some energy from localized hot spots and distribute the energy through the third material. Is not this cooling? The amount of cooling then is affected by the amount of third material and the cooling will never be absent unless the third material is absent.

Whether a fluid film is formed depends on what is meant by a fluid film. The transition from "thick-film" to "thin-film" is not likely to be as definitely noticeable as the transition from thin-film to conditions of contact between the journal and bearing. The question remains then as to where in the transition zone between thick-film and thin-film, or even in the contact range, these tests were run.

Paul G. Exline.[4] It is well known that the roughness of bearing surfaces influences the behavior of a journal bearing to a considerable degree, particularly when operating in the boundary region. The recent development of instruments capable of measuring surface conditions enables investigators to add materially to the value of their papers, as the authors have done, by reporting the roughness data together with the performance data. As it seems evident that this commendable practice will increase in the future, the writer feels that an early discussion of the relative importance of different measurements is necessary. The authors report the maximum depths of the grinding scratches as measured on a line parallel to the axis of the journal. The writer in his own work uses an instrument which indicates the root-mean-square value of the heights of the irregularities but, due to the difficulty of circumferential measurements, the value is also taken on a line parallel to the axis. It is quite certain that under some conditions measurements made in the two directions would show widely varying results. A well-run-in journal quite often carries fairly deep grooves running completely around it which apparently do not interfere with its operation.

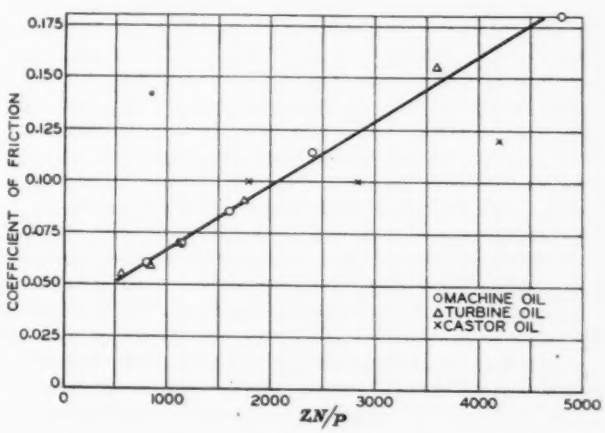


Fig. 1

The conditions under which a bearing operates in the boundary lubrication region are generally understood to be those which are in close proximity to the minimum point of the curve of coefficient of friction plotted against ZN/P. The method, used by the authors, in which the boundary region is reached by regulating the rate of oil flow to the bearing appears to offer a new technique for investigation but one which makes it exceedingly difficult to determine when the desired conditions are reached. Calculation

[1] By L. M. Tichvinsky and E. G. Fischer. Published in the September, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-109.

[2] Refrigeration Engineering Department, Sunbeam Electric Manufacturing Co., Evansville, Ind.

[3] Associate Professor of Industrial Engineering, Lehigh University, Bethlehem, Pa.

[4] Gulf Research & Development Company, Pittsburgh, Pa.

based on the data presented in Figs. 10 and 16 of the paper indicate that the bearings were operating at very high values of ZN/P; and Fig. 1 of this discussion, plotted from such calculated data, gives no evidence that conditions were other than those of thick-film lubrication. Consequently, it would be desirable to know what criterion was used by the authors to indicate the correct rate of oil feed to the bearing to bring about the conditions of boundary lubrication.

F. P. Bowden.[5] The writer is in full agreement with the authors that the friction and wear between boundary lubricated metals was very dependent upon the nature of the metals. Experiments carried out in Cambridge[6] have shown that the sliding between metal surfaces might not be a continuous process. There is evidence that an intermittent clutching and breaking away of the surfaces occurs and that the friction, surface temperature, and area of contact all show violent fluctuations during sliding.

Even if the surfaces are ground flat and carefully polished, hills and valleys are still present on the surface, and where the metals are placed in contact they are supported on the summits of these irregularities so that the real area of contact is very small. This means that even with lightly loaded surfaces the local pressure is very great.[7] Experiment suggests that the pressure is great enough to cause the plastic flow of steel. If the sliding speed is high enough the heat liberated at the localized points of contact is sufficient to raise them to a very high temperature.

The results suggest that the friction is due to the formation and subsequent breaking of metallic junctions between the surfaces at these points of contact; although the junctions are small their dimensions are large compared with the dimensions of a molecule, so that when motion occurs and the junctions are broken, the metals are torn and distorted to a considerable degree beneath the surface.

Even with the best lubricant films the writer has always observed metallic contact and damage of the underlying metal. The picture reproduced by the authors of the adsorbed lubricant films sliding over one another might represent the ideal case, but the writer does not think it occurs in practice in kinetic friction. The intense local pressure causes a breakdown of the film so that local contact of the metals occurs. During sliding, this breakdown is aided by the local high temperature. The writer considers that friction cannot be regarded as a surface phenomenon. The strength of the junctions and the magnitude of the friction is influenced by the bulk properties of the metals. The effects of sliding extend to a great depth beneath the metal surface.

Experiments have suggested that the metallic junctions might be of three main types. The exact type which predominated is influenced by the *relative* physical properties, particularly by the relative melting point and relative hardness of the two metals. The worst type of junction which causes the highest friction and wear is that formed between similar metals and between metals of similar melting points. Because of this it is an advantage to choose metals of very different melting points, and the less important surface, e.g., the bearing, should have the lower melting point. Melting and flow of the surface will then occur before the local temperature can reach a sufficiently high value to cause a softening and damage of the journal. It is interesting that the authors found the lowest friction with babbitt metal.

[5] Laboratory of Physical Chemistry, Cambridge University, Cambridge, England.

[6] "The Nature of Sliding and the Analysis of Friction," by F. P. Bowden and L. Leben, Proceedings of the Royal Society of London, series A, vol. 169, Feb., 1939, p. 371.

[7] "The Area of Contact Between Stationary and Between Moving Surfaces," by F. P. Bowden and D. Tabor, Proceedings of the Royal Society of London, series A, vol. 169, Feb., 1939, p. 391.

The writer has also found, in agreement with the authors, that the exact value of the friction and wear is influenced to some extent by the method of preparation and surface finish of the metals. This effect might, however, be of much less importance than the bulk composition of the metals themselves. Some of the values of kinetic friction given in the paper were below 0.05. The writer has not observed values of boundary lubrication quite as low as this. Did the authors consider it possible that some fluid lubrication was occurring in their experiment?

There is also no doubt that the boundary kinetic friction and wear are dependent upon the shape and size of the lubricant molecule and on the method of attachment to the surface. In general, mineral oils are bad lubricants but recent experiments have shown that the presence on the surface of certain long-chain polar molecules is able to change the nature of the sliding so that intermittent stick-slip is prevented and the sliding proceeds continuously. At the same time the wear of the surfaces is greatly reduced. Substances which cause this change are in general good lubricants.

S. Kyropoulos.[8] The paper under discussion is a typical investigation from an engineering point of view. It differs from some other papers of these last years with similar objects by a more rigorous consideration of the factors involved, of which only the attention paid to surface conditions may be mentioned. For this reason the investigation is undoubtedly bound to meet with the appreciation it deserves in engineering circles. The rather well-defined conditions of experiment and the clarity with which conclusions are drawn must as well appeal to those studying the phenomena of boundary lubrication from a physicochemical viewpoint.

In view of the experience that the clear and well-supported conclusions of such investigations are apt to be expanded into general conclusions by later workers and in view of the introductory physicochemical remarks of the investigators which invite physicochemical comment, a few pertinent remarks might not be out of place.

In view of the introduction one would logically expect that the authors would attempt a correlation of data of static or kinetic friction of the oils used with the results obtained. Apart from the very important viscosity data of the oils no such properties are listed in the paper, an omission which, incidentally, the writer does not consider a serious deficiency. Nevertheless one has to draw conclusions from data on a by-effect (viscous lubrication) in order to estimate the effect appearing in the title of the paper. With regard to the very illustrative photomicrographs of the journal surfaces it would have appeared desirable to have pictures of the surfaces before testing and not only before running in. Furthermore, it should be emphasized that in all the experiments described in the paper we are dealing with mixed fluid and nonfluid (i.e., "boundary") lubrication. This follows from the investigations of Beare and Bowden[9] quoted by the authors, as well as Kyropoulos and Shobert.[10] This also follows from Fig. 16 of the paper[1] to which we shall refer later.

Bearing this in mind is helpful in the interpretation, e.g., of Figs. 11 and 12, representing starting and running friction, of which the latter must necessarily show a greater influence of viscosity effects. Furthermore, it would be of interest to know to which oil Figs. 11 to 14 refer or whether different oils showed

[8] California Institute of Technology, Pasadena, Calif.

[9] "Physical Properties of Surfaces. I—Kinetic Friction," by W. G. Beare and F. P. Bowden, Philosophical Transactions of the Royal Society of London, series A, vol. 234, 1935, p. 329.

[10] "A Simple Method for Measuring the Coefficient of Nonviscous Friction of Thin Lubricating Layers," by S. Kyropoulos and E. I. Shobert, Review of Scientific Instruments, vol. 8, 1937, p. 151.

any difference, which probably was not the case. This once more tends to show that (1) viscosity effects are rather strong in all these experiments and (2) that the behavior of the bearing materials is largely determined by properties of these materials which are not taken into account by the concepts at which the introduction hints.

The writer would be inclined to interpret the running properties as they appear in the experiments, as determined by interlocking of asperities,[6] cold work, and Coulomb friction between the metal surfaces, contaminated in some not necessarily predictable way. Chromium and stainless steel will certainly be contaminated by a surface layer of oxygen or oxide (passivity).

The well-known properties of pure silver may perhaps throw some light on the interpretation of Fig. 13 showing the outstanding low starting friction of silver. Silver is the only strictly homogeneous and ductile bearing metal tested. All others contain hard components. Moreover silver is certainly the least reactive at local high temperatures. In running friction tests under the conditions of experiment one would qualitatively expect a larger contribution by fluid friction with the less homogeneous and less ductile metals. Such a metal should actually bear at fewer points and thus permit more easily the formation of a bearing wedge of the lubricant. As a result, the bearing friction would fall in comparison with silver. Attempting and further testing such interpretations is of vital importance for the practical application of the paper. Possibly the order of the curves of Fig. 11, i.e., strictly reverse order of starting friction and hardness, may be largely determined by the same phenomenon. Unfortunately the surfaces of the bearings have not apparently been examined microscopically. In view of the wear of the journals there must be surface changes.

Regarding Fig. 16, the curves of running friction go with the oil viscosities, which shows once more that fluid lubrication predominates under these conditions and that the method of lubrication used still amply feeds lubricant. The same order of the oils appears in starting friction up to 75 lb per sq in. from which it follows that up to this pressure fluid lubrication definitely predominates—at least in this bearing combination. At higher pressures castor oil shows the behavior one would expect in nonfluid lubrication.

Those who attribute particular significance to the wetting power of oils might be reminded of the good performance of chromium-plated and stainless-steel journals which are certainly only poorly wetted.

R. SCHNURMANN.[11] The authors' tests were made with loads of from 25 to 140 lb and at sliding speeds of 203.2 cm per sec. A thermocouple was inserted in the middle of the bottom half of the bearing "as close as possible to the bearing surface." The highest temperature thus measured was 64 C, when a monel-metal journal was running at a load of 140 lb (i.e., 63,560 gram weight) in a tin-base babbitt bearing, and castor oil was used as lubricant. It would be interesting to have a more detailed description of the location of the thermocouple and to know as closely as possible its actual distance from the bearing surface.

The melting point of the tin-base babbitt is about 232 C, and it does not seem likely that a temperature gradient of about 168 C can be maintained in the mass of the bearing metal over a very short distance from its surface. The assumption of "high localized temperatures" on a metal surface with asperities of a height of certainly less than 5×10^{-4} cm is not plausible whatever the geometrical shape, cylindrical or pyramidal, of the surface asperities may be through which the heat of friction is conducted away. It appears that if any "wiping" of the bearing metal occurred it could only be effected by the normal rise of bearing temperature, which the authors have measured, but not by imaginary considerably higher "localized temperatures," the occurrence of which the high values of the heat conductivity of metals exclude.

[11] London, Midland and Scottish Railway Company, Research Laboratory, Derby, England.

AUTHORS' CLOSURE

In connection with J. L. Fletcher's questions, the authors are of the definite opinion that the value of the coefficient of friction for the same "zero-roughness" surface will not be the same for all types of metals. This is due to such physical properties of an oil as its affinity to various surfaces corresponding to different degrees of adsorption. Experimentally this was demonstrated[12] by placing a drop of oil directly on the line of contact of equally well polished surfaces of different materials M_1 and M_2, Fig. 2 of this closure. As may be seen from the sketch, the drop T assumes an unsymmetrical position between M_1 and M_2 due to higher affinity to M_1 than to M_2.

The questions raised by both J. R. Connelly and Paul G. Exline are very important. Did the bearings perform in the region of boundary friction? The testing machine used was built for boundary conditions of friction. We notice the low L/D value, which is equal to 0.736. The short bearing is not provided with any circumferential sealing grooves and, therefore, the side

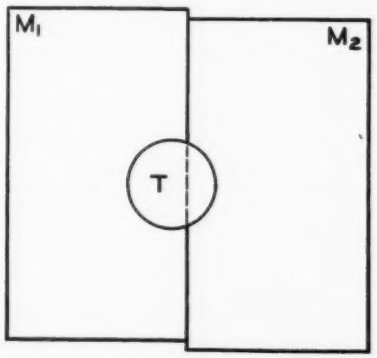


FIG. 2 AFFINITY OF OIL TO DIFFERENT METAL SURFACES

leakage is very large. The oil supply was never maintained higher than 10 drops per min. Finally, the fact that the roughness diminished during tests serves to indicate that the rubbing surfaces were never separated by a continuous film.

Fig. 1 of Mr. Exline's discussion shows the calculated values of ZN/P which suggests that the bearing performs in the region of perfect fluid lubrication. Referring to such curves, we might deduce somewhat confusing conclusions. For example, consider oil-lubricated steel-mill bearings. Some of these operate with values of ZN/P less than 2 (Z = 250 cp, N = 40 rpm, and P = 2500 lb per sq in.). A mirror-finish journal runs in such a bearing with a continuous film of oil, without any wear. However, if one refers to the value of ZN/P equal to 2 it might appear that such a bearing could not perform at all because it is too close to the region of dry friction.

In view of this fact, it is possible that fluid-film lubrication was also taking place during the experiments, but the bearings were performing predominantly in the region of boundary or semi-fluid friction.

It is interesting to note from F. P. Bowden's remarks the fact that sliding between metal surfaces might not be a continuous process. This was found to be the case during laboratory experiments with the use of flat contact surfaces. Undoubtedly

[12] "Über die Messung der Schmierfähigkeit von Schmierölen," by Richard von Dallwitz-Wegener, *Zeit. technische Physik*, vol. 5, 1924, pp. 378–384.

similar performance might occur in cylindrically shaped surfaces, i.e., standard circular bearings.

Dr. S. Kyropoulos indicates the possibility that the experiments were carried out under conditions of mixed nonfluid (boundary) and fluid lubrication. His explanation of the low starting friction of silver bearings is interesting.

The authors believe that their work will give an impetus for similar correlation of friction and surface-finish data. Developed surface-finish apparatus provide complete facilities for such accurate measurements. Friction versus finish data will undoubtedly constitute a field for further development in laboratories and in industry. In this connection, the collaboration of physicists and engineers will prove very beneficial in elucidating the complex phenomena of friction which some authors consider close to the "terra incognita."

A Photoelastic Study of Stresses in Rotating Disks[1]

C. W. MacGregor.[2] The author has presented an interesting study of the stresses in rotating disks containing holes by applying the newer three-dimensional stress-freezing technique to what is essentially a two-dimensional problem.

A question which occurred to the writer, however, concerns the ratio of the diameter of the holes used to the thickness of the plate. In model tests A and B the diameter-thickness ratios are 0.5 and 0.75, respectively, according to the data given in the paper. This means that the problem falls somewhere between the cases of plane strain and plane stress.

It was shown some time ago by R. W. Vose[3] that a considerable reduction in the stress-concentration factor determined photoelastically was found for the case of a hole in a plate subjected to tension in one direction, as compared to the theoretical value when the ratio of hole diameter to the thickness of the plate was less than 2 to 1. This would seem to indicate that the stresses found photoelastically in the case of the rotating disks described in the present paper are lower than the actual values. Although a direct numerical correction based upon Vose's values, as applied to the case under discussion, may not give the true picture, it is indicative of the order of magnitude of the effect. The factors by which the photoelastic-stress value should be multiplied to obtain the theoretical value in the case of the plate with a single hole subjected to tension in one direction only would be 1.305 and 1.167, respectively, for the diameter-to-thickness ratios used in model tests A and B of the paper.

The writer would like to ask the author if any experiments were made to determine whether or not such effects were of appreciable magnitude in the present tests.

A. Yorgiadis.[4] Referring to Equation [4a] of the paper and the preceding paragraph, the ratio of the cross-sectional area, along the circle of centers before drilling the holes, to the area after drilling is not

$$\frac{2\pi a}{2\pi a - 6d} \quad \ldots\ldots [5] \quad \text{but} \quad \frac{2\pi}{2\pi - 24 \arcsin (d/4a)} \quad \ldots\ldots [6]$$

Even this corrected expression would be unsuitable for this purpose because the maximum reduction in area does not occur in the line of centers, but in the circle, the radius of which is $\sqrt{a^2 - 0.25\,d^2}$ and at that section the reduction in area due to drilling is

$$\frac{2\pi}{2\pi - 12 \arcsin \dfrac{d}{2a}} \quad \ldots\ldots [7]$$

When d is small relative to a, the author's equation introduces only a slight error. However, this error becomes appreciable as this ratio increases. For example when $a = 1$ in. and $d = {}^3/_8$ in., Equation [7] of this discussion gives a value 0.3 per cent more than Equation [5]. When $a = 1$ in. and $d = {}^7/_8$ in., Equation [7] gives a value 23 per cent more than Equation [5]. At the limiting value, $d = a$, when the drilled holes are tangent to each other, the author's equation yields $\dfrac{\pi}{\pi - 3}$, whereas this value should be $\dfrac{\pi}{\pi - \pi}$, or ∞ as calculated from Equation [7].

Author's Closure

Referring to the corrections suggested by Dr. MacGregor, the author wishes to state that no such corrections were made because none was believed necessary. It seems to the author that Vose's experimental results have been seriously misinterpreted by a number of photoelasticians. Although it is not defined in Vose's article, the quantity which he has called the "stress-concentration ratio" seems to be the ratio of the change of thickness at the point of highest stress to the change of thickness at a point in the uniform stress field. Customary usage would define the stress-concentration factor in the problem of the hole in a tension field as the ratio of the maximum principal stress to the tension in the uniform field. The two definitions will yield identical values as long as a condition of plane stress exists in the plate. However, when the ratio of the diameter of the hole to the thickness of the plate is small the stress system in the plate is three-dimensional and the strain gage does not measure the sum of the two principal stresses in the plane of the plate. Theory of elasticity shows that the stress-concentration factor is equal to 3 for the limiting cases of plane stress and plane strain and there is no apparent reason to anticipate an appreciable change in the intermediate range. Some unpublished experimental results (obtained optically, by use of a polariscope) of M. M. Frocht confirm this opinion.

In the problem of the rotating disk the variation of the body forces causes a difference in the theoretical solutions under the assumptions of plane stress and plane strain. It is well known that the drilling of a small hole in the center of a solid rotating disk of uniform thickness doubles the maximum principal stress in the disk. Although this stress-concentration factor is usually derived under the assumption of plane stress, it is equally correct for the case of plane strain, despite the difference in the magnitudes of the stresses in the two cases. There is no available theoretical solution for the problem dealt with experimentally in the author's paper, but, in the light of the foregoing considerations, it seems reasonable to assume that the correction suggested by Dr. MacGregor is unwarranted.

The author readily concedes that Mr. Yorgiadis' mathematics is correct. At the time of writing of the paper the approximate nature of the correction factor, Equation [5], was recognized, but this form was used for the sake of simplicity. The 0.3 per cent error involved is considerably smaller than the probable experimental error. In the extreme hypothetical cases cited by Mr. Yorgiadis not even his "corrected" correction factors would be suitable, because the assumption stated in the paper, upon which the use of the correction factor is based, is grossly violated.

[1] By R. E. Newton. Published in the June, 1940, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 62, p. A-57.

[2] Associate Professor of Applied Mechanics, Massachusetts Institute of Technology, Cambridge, Mass. Mem. A.S.M.E.

[3] "An Application of the Interferometer Strain Gage in Photoelasticity," by R. W. Vose, Journal of Applied Mechanics, Trans. A.S.M.E., vol. 57, 1935, p. A-99.

[4] East Rockaway, Long Island, N. Y.

Book Reviews

Strength of Marine-Engine Shafting

STRENGTH OF MARINE-ENGINE SHAFTING. By S. F. Dorey, D.Sc., Chief Engineer Surveyor, Lloyd's Register of Shipping. A paper with copious discussion published by the North-East Coast Institution of Engineers and Shipbuilders, Newcastle-on-Tyne, England, 1939. Cloth, 6 × 9 in., 150 pp., figs., 5 s.

REVIEWED BY J. P. DEN HARTOG[1]

THE paper gives a very elaborate and complete account of the cause and frequency of failures of marine shafting during the last decade. It can be divided roughly into three parts: The first deals exhaustively with the various strength properties of materials used in crankshafts, the second part is concerned with torsional vibration in the drive system, and the third part discusses axial vibration.

It has been found, in the author's experience, that a number of failures in crankshafts could not be explained by torsional excitation, but would have to be traced down to a longitudinal or axial vibration of the entire drive structure. Methods of analysis for calculating these effects, very similar to those employed for torsional vibrations, are given. The method used throughout the paper is the usual one originated by Holzer. An innovation consists in writing the hysteresis loss in the shaft as being proportional to the square of the stress rather than to the 2.3 power, which has been common practice heretofore. The history of this number 2.3 is rather interesting. It appears that tests prior to 1920 by an English investigator on mild steel gave a hysteresis loss proportional to the 2.3 power of the stress. Modern crankshaft steels have hysteresis losses several times smaller than those found by the original investigator and, moreover, the total loss in the engine is about ten times as high as the hysteresis loss in the crankshaft. Nevertheless, it has become customary to calculate the hysteresis loss in the crankshaft on the basis of the power 2.3 and then to multiply by an empirical constant of roughly 10. This procedure surprisingly leads to results that are in decent agreement with practice, but an analysis on the basis of the simple power 2 would do just as well and is very much simpler to handle. It is gratifying therefore that an authority on Lloyd's Register of Shipping has taken the initiative toward this simplification.

The paper is illustrated with a large number of records and test setups and the discussion contains contributions from almost every authority in the field of vibrations in marine engines.

Biomechanics

RESEARCHES IN THE BIODYNAMICS OF LOCOMOTION. N. Bernstein, editor. Volume 1, V.I.E.M. (Institute of Experimental Medicine), U.S.S.R., 1935. 243 pp., 107 figs., 88 tables. $2.00 (Rbls. 7.75) (Russian).

FEARFULLY AND WONDERFULLY MADE. By Renée von Eulenburg-Wiener. The Macmillan Company, New York, 1938. Cloth, 6 × 9½ in., xii and 472 pp., $3.50.

REVIEWED BY GEORGE B. KARELITZ[2]

THE word "biomechanics" is now rather fashionable. It denotes the active interest which medical people have recently shown in studying the mechanics of the human body, in addition to the chemistry of the thousands of involved reactions in our organisms.

The analysis of the mechanics of the human body should be basically easier than the study of the physiological reactions, nevertheless exact information in biomechanics is scarce. This may be attributed to the fact that mechanical models and adequate instruments are expensive when compared with the equipment necessary for biological and chemical investigations.

The mechanics of the body can be studied from various angles and the term biomechanics is really too all-inclusive as used at present. In the first place, the body can be considered as a machine designed to perform certain mechanical motions and produce mechanical work. The kinematics of the various linkages; both static and dynamic forces acting on the various members and joints of the body; the stresses and strength of the structural bones, muscles, and tendons would fall under this classification. Then the heat balance of the body may be analyzed. This would involve the conversion of energy of combustion and of all the complicated reactions into heat and work, i.e., consideration of the body as a power generator. Again, the control systems of the body can be studied: the thermostatic function of the skin, the optical, acoustical, and muscular mechanisms, and the flow of blood through the vessels.

The first book, although somewhat old for a book review, is hardly known here. It is the first treatise in a series devoted to human locomotion, and deals with the walking of an adult. The locomotion of a child, the military step, the dynamics of running, and the motion on protheses are promised in the next volume. The reports published in this volume are the results of ten years of experimentation by Bernstein and his group; the technique used was a substantial improvement upon the methods employed by Braune and Fischer.[3] A number of small lamps were attached to various joints of the investigated men, and photographs at the rate of 98 frames per sec were taken of the lamps' positions while the men walked. After careful time plotting of the coordinates so determined, the velocities and accelerations of the points were determined and, therefore, the dynamic forces acting on the various joints could be ascertained.

For instance, the velocity of the shoulder in a normal man was found to vary from 4.7 fps to 5.9 fps during each step, while the pressure of his foot on the ground varied from 90 to 213 lb. The weight of the man was 152 lb and his speed was 122 paces per min.

The first two chapters of the book describe the experimental technique. Chapter 3 treats the free walking of an adult; chapter 4 gives the information for a laborer carrying loads; the last chapter discusses fatigue after long walking, both free and with load.

The book of Eulenburg-Wiener is of interest to those who would consider the human body as a power generator. In a short volume "the author has succeeded in presenting and explaining the most important facts now known about the human body." The book consists of 32 chapters. The first several chapters deal with the digestion of food, its chemical transformations, and absorption while it travels through the body.

Four chapters discuss the blood-circulation system. The pressure drop of from 30 to 40 mm of mercury in the capillary system is discussed; the mechanics of blood vessels as described

[1] Associate Professor of Applied Mechanics, Harvard University, Cambridge, Mass. Mem. A.S.M.E.

[2] Professor of Mechanical Engineering, Columbia University, New York. Mem. A.S.M.E.

[3] "Der Gang des Manschen," by O. Fischer, Trans. Society of Saxony, Teubner, Leipzig 1895; 1899–1901, 1904.

is somewhat confused; it would offer a challenging problem for one interested in variable flow in elastic tubes.

The energetics of the body is treated in several chapters. It is rather remarkable that the animal body produces the same amount of heat during the digestion of a given food, such as carbohydrates or true fat, as the food would yield if burned in a bomb calorimeter. Yet the temperatures at which all the reactions are taking place are so much lower than any which are used in laboratories.

The importance of the skin as an instrument for regulation of the body temperature is pointed out in chapter 28. Minute areas respond to either hot or cold sensations. These spots are sensitive to infrared radiation lying within the range of from 8000 to 1,000,000 A. The last chapter gives an integrated picture of the human body as a whole, surveying the various sensations and activities in their interconnections.

The book leaves one with the impression that enormous strides were made toward understanding the fine physiological reactions and interrelations in the many complicated organisms comprising the human body. In comparison, the lack of information on the simple mechanics of the body is striking. Thirty years passed between the work of Braune and Fischer and that of the Bernstein group. Some published work on testing tendon tissues for strength is of very recent origin.[4,5] An indication that the wings of birds, a duck, and a hawk are constructed with the factor of safety of two was obtained in 1938 in the State Aeronautical Research Laboratory at Moscow. Other scattered bits of information can be found here and there. It seems that it is time for such organizations as the powerful insurance companies to take up the subject of biomechanics, find a number of able engineers and physicians genuinely interested in the mechanics of the human body, and let them work on a long-time program. The result of such an investigation, properly endowed and planned, would no doubt repay manyfold the investment of funds and energy involved in the investigation of this neglected field.

Vapor Charts and Special Tables for Turbine Calculations

VAPOR CHARTS AND SPECIAL TABLES FOR TURBINE CALCULATIONS. By Frank O. Ellenwood and Charles O. Mackey, Professors of Heat Power Engineering, Cornell University, John Wiley & Sons, Inc., New York, N. Y., 1939. Cloth, 8½ × 11 in., 43 pages, charts, tables. $2.50.

REVIEWED BY C. HAROLD BERRY[6]

THE original Ellenwood steam charts, based upon the Marks and Davis tables, appeared in 1914, and rapidly gained favor among engineers having occasion to make frequent computations for changes of state of steam. This replotting uses the Keenan and Keyes tables and therefore covers a wider range with superior accuracy. The plotting and reproduction are splendidly done.

The chart giving the properties of saturated and superheated steam covers a wide range to a large scale. If it were on one sheet, it would be 40 × 47 in. For convenience, it is divided into twenty-one sections, each occupying one page of the volume. Once the user is familiar with the arrangement (a matter of a few minutes' examination) he should find this volume much more convenient and durable than a large "blanket" chart. A small-scale skeleton chart serves as an index which will be used at first and soon outgrown.

The ordinate scale of enthalpy is divided to single Btu per pound; fractions can be estimated. The abscissa is a logarithmic scale of specific volume comprising five 9-inch cycles with divisions not as close as those of the 10-inch slide rule; readings can be estimated to three digits throughout the range from the critical volume (0.05) to 3500 cu ft per lb, which is a trifle larger than the specific volume of saturated vapor at 32 F.

Lines of constant pressure, constant entropy, constant per cent moisture (in the wet region), and constant temperature (in the superheated region) are provided at satisfactory intervals, wide enough to make the chart easy to read, and close enough to permit estimating between consecutive curves to a degree of precision that agrees well with the probable accuracy of observed data.

The choice of enthalpy and volume as coordinates provides what is probably the most widely useful chart of vapor properties. Since specific volumes can be read directly, in addition to all the quantities given by the widely used Mollier enthalpy-entropy chart, the Ellenwood chart makes possible a direct solution in terms of ratio of expansion which demands a tedious cut-and-try solution by other means. The chart may appear at first sight to be somewhat more complex, but this impression is soon dispelled.

One could wish that the authors had found it feasible to provide a generous overlap on all four edges of each section of the chart. Apart from this minor item, one can find no fault with the chart, which should promptly resume its place as the most convenient means for carrying out steam computations with a degree of accuracy sufficient for most applications.

The chart giving the properties of compressed liquid water is the best that the writer has seen. It provides for direct solution of all types of problems that arise, and permits reading to a degree of accuracy that is as good as the readings of thermometers and pressure gages. It is much more convenient and more accurate than the skeleton table which is the only other convenient source of data in this field. This chart, covering two pages, is a valuable contribution to the engineer's equipment.

Supplementary tables and charts greatly increase the usefulness of the volume. For the turbine designer there are tables of jet velocities for enthalpy drops up to 80 Btu by tenths, and up to 599 by units, and a table of squares of numbers from 200 to 2000, expressed in millions. For the air conditioner there are three pages of psychrometric charts for dry-bulb temperatures from —40 to +350 F, plotted in convenient form so as to yield easy readings of relative and specific humidity, volume per pound of air, and the sigma function (enthalpy — enthalpy of liquid). For the refrigerator there are charts of the properties of ammonia and freon-12, with enthalpy plotted against the logarithm of pressure, covering the entire two-phase liquid-vapor region and an adequate portion of the superheated-vapor region. For everybody there are tables for correcting mercurial barometer readings for gravity, temperature, and capillarity.

This replotting of a chart that has always been one of the most useful should meet with a hearty welcome from engineers and others who have occasion to make thermodynamic computations. The authors are to be congratulated for having produced so good a job.

In a publication involving so much plotting, it is astonishing to find so few errors. There are seven errors in marking numerical values on the lines—these will be immediately evident to the user of the chart—five of them are noted on an errata sheet.

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[4] "Biomechanical Studies of Fibrous Tissues Applied to Fascial Surgery," by C. M. Gratz, *Archives of Surgery*, vol. 34, 1937, p. 461.

[5] "Engineering Methods in Medical Research," by C. M. Gratz and S. N. Blackberg, *Mechanical Engineering*, vol. 57, April, 1935, pp. 217–220.

[6] Gordon McKay Professor of Mechanical Engineering, The Graduate School of Engineering, Harvard University, Cambridge, Mass. Mem. A.S.M.E.

Transactions of the A.S.M.E.

IN TWO SECTIONS—SECTION TWO

SOCIETY RECORDS—PART 1

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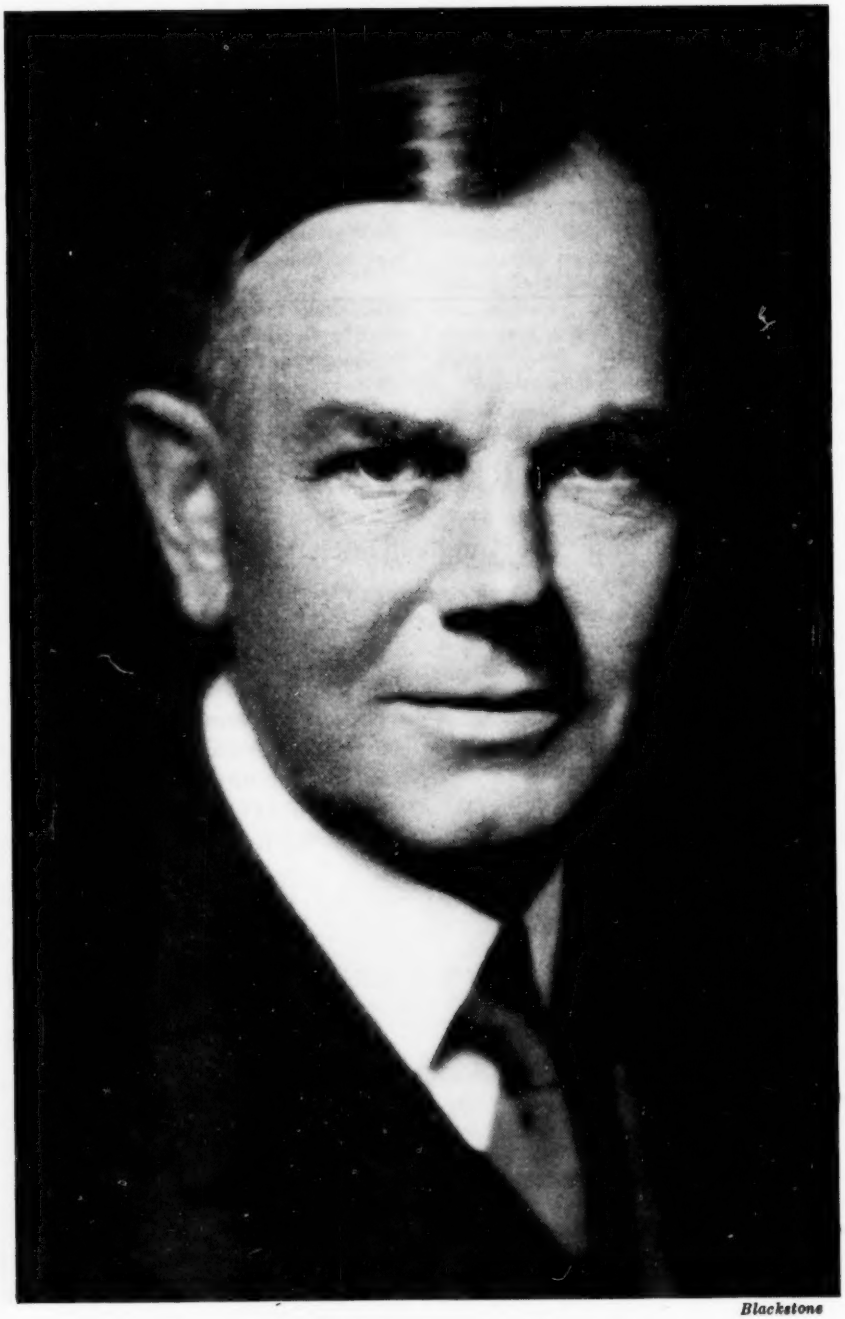
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FEBRUARY, 1940

VOL. 62, NO. 2

PRESIDENT



Blackstone

WARREN H. McBRYDE
1939-1940

Warren Horton McBryde

Warren Horton McBryde, consulting engineer, industrialist, and world traveler, of San Francisco, California, President of The American Society of Mechanical Engineers for the year 1939-1940, was born at Mobile, Ala., in 1876. At the age of 17, he entered the Alabama Polytechnic Institute, Auburn, where he specialized in electrical and mechanical engineering. He was graduated in 1897 with the degree of B.S.

Immediately after graduation, he was employed by the Electric Lighting Co. of Mobile, following which he did architectural designing and drafting for George F. Barber & Co., Knoxville, Tenn. During the Spanish-American War, he served for a few months in the U.S. Lighthouse Department and U.S. Engineers Corps and had charge of the submarine mines, searchlights, and electrical equipment guarding the entrance to Mobile Bay. Following this he was made chief electrician of the U.S. Army Transport *Sheridan*, on which he made a voyage from New York, N.Y., via the Suez Canal to the Philippines; thence via Nagasaki, Japan, to San Francisco, where he was honorably discharged. From 1899 to 1903, Mr. McBryde was employed by the Yuba Electric Power Company as assistant resident engineer in charge of construction on the Colgate hydroelectric powerhouse, in northern California, by the Peyton Chemical Company, San Francisco, as assistant superintendent of construction and by the Bay Counties Power Company, also of San Francisco, as chief draftsman and assistant to the chief electrical and mechanical engineer.

He was the first resident engineer in charge of construction at Gibbstown, N.J., for the E. I. du Pont de Nemours Powder Company from 1903 to 1905, and was transferred during the year 1905 to Hercules, Calif., where he constructed large sulphuric acid plants. In 1906, he was appointed the first chief of the West Coast engineering and construction department of the du Pont organization and, subsequently, became assistant superintendent of its Hercules plant and retained this position until 1919. During this period, Mr. McBryde was in charge of all maintenance and repairs, and handled many important engineering problems in connection with the hurried construction of 12 large TNT plants, the necessary nitric- and sulphuric-acid factories, and many other industrial buildings during the World War. He was also in charge of the labor- and industrial-relations department of the company and secured positions for more than three quarters of the 4000 men under his supervision when demobilization came.

In 1919, he became secretary of the California & Hawaiian Sugar Refining Co., San Francisco, and assistant to the general manager, and handled many important engineering problems for the company until 1927, when he engaged on his own account in his present business as a consulting engineer.

Mr. McBryde was elected a member of the Society in 1921. He was chairman of the San Francisco Section in 1925 and 1926, when the Spring Meeting was held in San Francisco in the latter year. As a delegate from the Section, he attended the 1926 Annual Meeting of the Society in New York, N.Y., and the celebration at Washington, D.C., in 1930, of the fiftieth anniversary of the formation of the A.S.M.E.

In company with Calvin W. Rice, Mr. McBryde represented the Society at the 75th anniversary of the Verein deutscher Ingenieure at Cologne, Germany, in June, 1931; he also attended the 1931 and 1935 spring meetings in England of The Institution of Mechanical Engineers at Cambridge and Bath, respectively. He was one of the founders of the Engineering Societies Employment Service office in San Francisco. In 1937, Mr. McBryde was elected vice-president of the A.S.M.E. and in his official capacity attended the 1937 and 1938 Annual Meetings in New York. He has taken a great deal of interest in the activities of A.S.M.E. student members and has attended their meetings at several schools on the West Coast as well as the Student Group Meetings, in 1938 and 1939, at Los Angeles and San Francisco, respectively.

In more recent years Mr. McBryde made his second and third circumnavigations of the globe, including voyages which took him through the Suez and Panama Canals, the Strait of Magellan and around the Cape of Good Hope, and visited and studied some eighty of the most prominent countries of the world, some two hundred and fifty of the most important world ports, and the capitals and principal cities of most of these countries.

These extended trips were made to study first-hand the engineering progress and industrial developments, the harbor improvements and port facilities, the transportation systems—rail, highways, and inland waterways—also the industrial, mineral, and agricultural products of these countries, as well as their domestic commerce and foreign trade, their economic conditions, and their political problems. During these trips he met many of the world's leading engineers and the directors of the most important museums of science and industry.

Other activities of Mr. McBryde include election as mayor of Hercules, Calif., and supervisor of Contra Costa County, while a resident of that city; the San Francisco Engineers' Club; the San Francisco Electrical Development League; and the presidency of the Industrial Association of San Francisco, in 1921, in recognition of his experience as an industrial employer and his interest in workingmen.

The American Society of Mechanical Engineers

HEADQUARTERS: 29 West 39th St., New York, N. Y.
MID-WEST OFFICE: Room 1617, 205 West Wacker Drive, Chicago, Ill.
MID-CONTINENT OFFICE: 211 Midco Building, Tulsa, Okla.

The members of the Council and of its standing and special committees given on the following pages are those in office on January 1, 1940, serving for the official year 1939–1940. The terms of office of members of other committees are not fixed by the official calendar.

OFFICERS AND COUNCIL

PRESIDENT

Warren H. McBryde

PAST-PRESIDENTS

Terms expire December

Ralph E. Flanders (1940)
William L. Batt (1941)
James H. Herron (1942)
Harvey N. Davis (1943)
Alexander G. Christie (1944)

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Terms expire December, 1940

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Alfred Iddles
James W. Parker

Terms expire December, 1941

Kenneth H. Condit
Francis Hodgkinson
Jerome C. Hunsaker
Kilshaw M. Irwin

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Terms expire December, 1940

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Samuel B. Earle
Frank H. Prouty

Terms expire December, 1941

Clarke Freeman
William H. Winterrowd
Willis R. Woolrich

Terms expire December, 1942

Joseph W. Eshelman
Linn Helander
Guy T. Shoemaker

TREASURER

W. D. Ennis

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C. E. Davies

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Representatives on Council without vote

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Meetings and Program, Erik Oberg
Publications, C. B. Peck
Admissions, R. L. Sackett
Professional Divisions, Harte Cooke
Local Sections, A. J. Kerr
Constitution and By-Laws, G. E. Hulse
Honors and Awards, J. W. Roe
Relations with Colleges, H. O. Croft
Education and Training for the Industries, Warner Seely
Library, A. R. Mumford
Research, L. W. Wallace
Standardization, W. C. Mueller
Power Test Codes, Francis Hodgkinson
Safety, T. F. Hatch
Professional Conduct, B. F. Wood

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James W. Parker, *Vice-Chairman*
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William H. Winterrowd

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C. B. LePage, *Assistant Secretary* (Technical Committees)
George A. Stetson, *Editor*
Frederick Lask, *Advertising Manager*
D. C. A. Bosworth, *Comptroller*

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J. L. Kopf, *Vice-Chairman* (1941)
K. W. Jappe (1942)
G. L. Knight (1943)
W. I. Slichter (1944)

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K. M. Irwin (1941)

Junior Adviser

John S. Leslie (1940)

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W. J. Wohlenberg (1941)
A. L. Kimball (1942)
N. E. Funk (1943)
L. K. Sillcox (1944)

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D. E. Zeliff (1941)

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Coleman Sellers, 3d (1941)
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A. R. Stevenson, Jr. (1944)

Advisory Members (1940)

W. L. Dudley
N. C. Ebaugh
A. N. Goddard
E. J. Kates
L. S. Marks
J. M. Todd

Junior Advisers

H. B. Fernald, Jr. (1940)
C. C. Kirby (1941)

(Personnel of Special Committee, p. RI-3)

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H. E. Molé (1941)
T. M. Knoop (1942)
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L. R. Ford (1944)

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F. C. Spencer (1940)

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G. B. Karelitz (1942)
W. A. Carter (1943)
C. T. Ripley (1944)

Junior Advisers

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A. E. Blirer (1941)

(Personnel of Professional Divisions' Executive Committees, p. RI-6)

*Representative on the Council.

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J. P. Ferris (1943)
F. W. Marquis (1944)

Junior Advisers

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S. Davidson (1941)

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S. R. Beitler (1941)
A. T. Dupont (1942)
L. H. Kenney (1943)
F. B. Orr (1944)

Junior Adviser

H. M. Black (1940)

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J. W. Roe, *Chairman** (1941)
W. H. Carrier (1940)
R. V. Wright (1942)
D. C. Jackson (1943)
C. L. Bausch (1944)

(Personnel of Special Committee, p. RI-3)

RELATIONS WITH COLLEGES

H. O. Croft, *Chairman** (1940)
E. W. O'Brien (1941)
A. C. Chick (1942)
J. D. Cunningham (1943)
P. B. Eaton (1944)

Advisory Members (1940)

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S. F. Duncan
E. O. Eastwood
F. H. Prouty
G. L. Sullivan

Junior Adviser

J. I. Yellott (1940)

(Student Branches and Officers, p. RI-18)

EDUCATION AND TRAINING FOR THE INDUSTRIES

Warner Seely, *Chairman** (1940)
J. H. Ploehn (1941)
A. R. Stevenson, Jr. (1942)
M. R. Bowerman (1943)
A. C. Harper (1944)

Advisory Member (1940)

F. O. Clements

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W. H. Kenerson (1942)
C. E. Waddell (1943)
V. E. Alden (1944)

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J. E. Lovely (1942)
J. R. Weaver (1943)
T. E. French (1944)

POWER TEST CODES †

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A. G. Christie, *Vice-Chairman* (1941)
W. W. Lawrence, *Junior Observer* (194

Term expires 1940

A. T. Brown
P. H. Hardie
B. V. E. Nordberg
R. J. S. Pigott
M. C. Stuart

Term expires 1941

A. G. Christie
Paul Diserens
G. A. Orrok
L. A. Quayle
W. M. White

Term expires 1942

W. A. Carter
Harte Cooke
E. R. Fish
H. B. Oatley
W. J. Wohlenberg

Term expires 1943

Louis Elliott
G. A. Horne
H. B. Reynolds
P. W. Swain
E. N. Trump

Term expires 1944

C. H. Berry
D. S. Jacobus
Francis Hodgkinson
L. F. Moody
E. B. Ricketts

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D. L. Royer (1940)
A. W. Luce (1942)
A. E. Windle (1943)
H. C. Houghton (1944)

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A. R. Mumford, *Chairman** (1940)
John Blizard (1941)
E. F. Church, Jr. (1942)
The Secretary, C. E. Davies, *Ex-Officio*

† Personnel of all Technical Committe
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G. A. Orrok
J. W. Roe
W. H. Winterrowd

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T. E. Durban
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M. F. Moore
H. H. Vaughan
H. LeRoy Whitney

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MEDALS

(Special Committee of Board of Honors and Awards)

Term expires 1940

W. H. Carrier
E. R. Fish
H. C. Meyer, Jr.
L. W. Wallace

Term expires 1941

H. A. Everett
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G. A. Orrok
J. W. Roe

Term expires 1942

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E. S. Pearce
R. V. Wright

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(Continued)

Term expires 1943

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W. A. Hanley
D. C. Jackson

Term expires 1944

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Francis Hodgkinson
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(Continued)

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St. Louis
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Utah
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(Dates in parentheses denote expiration of terms)

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L. W. Wallace (Research)
Harte Cooke (Professional Divisions)
C. B. Peck (Publications)
Erik Oberg (Meetings and Program)

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P. T. Norton, Jr.

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S. H. Libby
W. R. Webster

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H. L. Whittemore
W. E. Wickenden

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E. C. Hutchinson
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J. L. Walsh
H. K. Rutherford (War Dept. Rep.)
A. B. Anderson (Navy Dept. Rep.)

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J. B. Wright
R. V. Wright

A.S.M.E. REPRESENTATIVES ON OTHER ACTIVITIES

See also A.S.M.E. Representatives on Other Research Committees, etc., pages RI-21, 27, 28, 31, 33
(Dates in parentheses denote expiration of terms)

AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

SECTION M, ENGINEERING

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R. L. Sackett

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Alternate

C. B. LePage (1940)

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A. A. Potter (1943)

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J. W. Roe

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S. R. Dows, San Francisco Board
Ernest Hartford, *Chairman*, Metropolitan Board

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A. G. Christie (1943)

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E. H. Barry
F. M. Gibson
H. Drake Harkins
J. C. Hobbs
A. L. Penniman, Jr.
E. B. Ricketts
E. H. Tenney

GANTT MEDAL BOARD OF AWARD

W. D. Ennis (1940)
L. C. Morrow (1941)
Lillian M. Gilbreth (1942)

DANIEL GUGGENHEIM MEDAL FUND, INC.

W. B. Mayo (1940)
E. A. Sperry, Jr. (1941)
Alex. Klemin (1942)

JOSEPH A. HOLMES SAFETY ASSOCIATION

J. F. Barkley

HOOVER MEDAL BOARD OF AWARD

W. H. Kenerson (1941)
S. F. Voorhees (1943)
W. L. Batt (1945)

INTERNATIONAL ELECTROTECHNICAL COMMISSION

U. S. NATIONAL COMMITTEE

H. N. Davis
Francis Hodgkinson
Paul Diserens

Alternate

C. Harold Berry

MARSTON AWARD

G. A. Orrok

NATIONAL BUREAU OF ENGINEERING REGISTRATION

V. M. Palmer

NATIONAL FIRE WASTE COUNCIL

John A. Neale

NATIONAL MANAGEMENT COUNCIL

J. A. Piacitelli (1941)—F. L. Eidmann, *Alternate*
L. C. Morrow (1942)—W. H. Kushnick, *Alternate*
L. P. Alford (1943)—C. W. Lytle, *Alternate*

NATIONAL RESEARCH COUNCIL

DIVISION OF ENGINEERING AND INDUSTRIAL RESEARCH

W. L. Batt (1942)

ALFRED NOBLE PRIZE

A. M. Greene, Jr.

UNITED ENGINEERING TRUSTEES, INC.

K. H. Condit (1940)
H. A. Lardner (1942)
To be appointed (1943)

ENGINEERING SOCIETIES MONOGRAPHS COMMITTEE

C. B. Peck
G. B. Karelitz

VERMILYE MEDAL ADVISORY COMMITTEE

R. A. Wentworth

WASHINGTON AWARD COMMISSION

C. B. Nolte (1940)
W. L. Abbott (1941)

PROFESSIONAL DIVISIONS

ARTICLE B6A, PAR. 16: The Standing Committee on Professional Divisions shall, under the direction of the Council, have supervision of the Professional Divisions of the Society.

STANDING COMMITTEE

HARTE COOKE, *Chairman* (1940)
VICTOR WICHUM (1941)
G. B. KARELITZ (1942)
W. A. CARTER (1943)
C. T. RIPLEY (1944)

Junior Advisers

SAM SHOOR (1940)
A. E. BLIRER (1941)

Aeronautic

Organized, 1920

ALEXANDER KLEMIN, *Chairman*

EXECUTIVE COMMITTEE

C. H. DOLAN, II, *Chairman*
B. M. WOODS, *Vice-Chairman*
JEROME LEDERER, *Secretary*
J. M. CLARK
R. F. GAGG
ALEXANDER KLEMIN

Dayton Representative

P. H. KEMMER

GENERAL COMMITTEE

E. A. SPERRY, *Chairman*

Applied Mechanics

Organized, 1937

J. P. DEN HARTOG, *Chairman*

EXECUTIVE COMMITTEE

J. P. DEN HARTOG, *Chairman*
H. L. DRYDEN, *Secretary*
J. H. KEENAN
J. ORMONDROYD
B. M. WOODS

Associates

R. EKSERGIAN
J. C. HUNSAKER
A. L. KIMBALL
F. M. LEWIS
G. B. PEGRAM
C. R. SODERBERG
E. O. WATERS
H. M. WESTERGAARD

SPONSORS

H. L. DRYDEN, Fluid Mechanics
J. A. GOFF, Thermodynamics
G. B. KARELITZ, Lubrication
A. NADAI, Plasticity
R. E. PETERSON, Strength of Materials
E. L. THEARLE, Dynamics
S. TIMOSHENKO, Elasticity

JOURNAL OF APPLIED MECHANICS

J. M. LESSELLS, *Editor*

Fuels

Organized, 1920

H. O. CROFT, *Chairman*

EXECUTIVE COMMITTEE

H. O. CROFT, *Chairman* (1940)
A. R. MUMFORD, *Secretary* (1942)
W. G. CHRISTY (1941)
H. F. HEBLEY (1943)
A. W. THORSON (1944)

Associates

J. B. BENNETT, 3RD
L. C. BOSLER
O. F. CAMPBELL
B. J. CROSS
M. D. ENGLE
E. R. KAISER
T. A. MARSH
J. E. TOBEY

COOPERATION WITH A.I.M.E.

A. W. THORSON, *Chairman*
H. F. HEBLEY
J. E. TOBEY

MODEL SMOKE LAW

J. F. BARKLEY, *Chairman*
O. F. CAMPBELL
A. G. CHRISTIE
W. G. CHRISTY
T. A. MARSH
T. E. PURCELL
R. A. SHERMAN
R. R. TUCKER

PROGRAM

A. W. THORSON, *Chairman*

REVIEW OF PAPERS

H. O. CROFT, *Chairman*
A. R. MUMFORD
A. W. THORSON

Graphic Arts

Organized, 1922

B. D. STEVENS, *Chairman*

EXECUTIVE COMMITTEE

B. D. STEVENS, *Chairman*
T. E. DALTON, *Secretary*
A. E. GIEGENGACK
F. W. HOCH
W. B. LAUGHTON
R. G. MACDONALD
W. M. PASSANO
B. L. SITES
B. L. WEHMHOFF

Associates

G. H. CARTER
EDWARD EPSTEAN
W. C. GLASS
W. C. HUEBNER
W. R. MAULL

Graphic Arts

(*Continued*)

GRAPHIC ARTS FORUM

F. W. HOCH, *Chairman*
T. E. DALTON
R. G. MACDONALD

Heat Transfer

(Professional Group)

Organized, 1938

T. B. DREW, *Chairman*
W. S. PATTERSON, *Group Secretary*

EXECUTIVE COMMITTEE

T. B. DREW, *Chairman* (1942)
W. S. PATTERSON, *Group Secretary*
A. K. SCOTT (1940)
E. D. GRIMISON (1941)
H. C. HOTTEL (1943)
R. A. BOWMAN (1944)
J. L. MENSON, *Junior Representative*

Advisory Associates

C. E. LUCKE
J. H. SENGSTAKEN

COORDINATION

W. S. PATTERSON, *Chairman*

Divisional Representatives

Fuels, B. J. CROSS
Iron and Steel, W. TRINKS
Oil and Gas Power, F. G. HECHLER
Petroleum, T. D. TIFFT
Power, G. A. GAFFERT
Process Industries, A. WEISSELBERG

Members at Large

L. M. K. BOELTER
R. H. HEILMAN
G. L. TUVE

Liaison Officer for Local Sections

A. K. SCOTT

PAPERS

E. D. GRIMISON, *Chairman*
W. L. DEBAUFRE (1941)
R. A. SHERMAN (1942)
R. A. BOWMAN (1943)
C. F. KAYAN (1944)

TESTING TECHNIQUE

H. C. HOTTEL, *Chairman*
B. J. CROSS
J. H. RUSHTON
A. K. SCOTT

THERMO-PHYSICAL PROPERTIES OF MATERIALS

R. H. NORRIS, *Chairman*
T. H. CHILTON
J. F. DOWNIE-SMITH
MAX JAKOB

Hydraulic

Organized, 1926

F. G. SWITZER, *Chairman*

EXECUTIVE COMMITTEE

F. G. SWITZER, *Chairman*
M. P. O'BRIEN, *Vice-Chairman*
L. J. HOOPER, *Secretary*
FORREST NAGLER
E. B. STROWGER
J. D. SCOVILLE

CAVITATION

E. B. STROWGER, *Sponsor*
L. F. MOODY, *Chairman*

GENERAL HYDRAULICS

M. P. O'BRIEN, *Sponsor*

HYDRAULIC PRIME MOVERS

FORREST NAGLER, *Sponsor*
J. FRANK ROBERTS, *Chairman*

PUMPING MACHINERY

F. G. SWITZER, *Sponsor*
R. L. DAUGHERTY, *Chairman*

WATER HAMMER

Honorary Chairman, LORENZO ALLIEVI, Rome, Italy

S. LOGAN KERR, *Sponsor and Chairman*
N. R. GIBSON
EUGENE HALMOS
L. F. MOODY
R. S. QUICK
J. D. SCOVILLE
E. B. STROWGER

Affiliated Societies and Their Representatives:

American Society of Civil Engineers, N. R. GIBSON and FORD KURTZ
American Water Works Association, F. M. DAWSON and L. H. KESSLER

Associate Members, Representing:

Australia, GEORGE HIGGINS
Brazil, A. W. K. BILLINGS and F. KNAPP
Engineering Institute of Canada, R. W. ANGUS and F. M. WOOD
France, LOUIS BERGERON and CHARLES CAMICHEL
Germany and Verein deutscher Ingenieure, D. THOMA
Great Britain and Institution of Mechanical Engineers, E. BRUCE BALL and A. H. GIBSON
Italy, GAUDENZIO FANTOLI and ALBINO PASINI
Switzerland, CHARLES JAEGER and O. SCHNYDER

Iron and Steel

Organized, 1927

W. TRINKS, *Chairman*

EXECUTIVE COMMITTEE

W. TRINKS, *Chairman*
M. D. STONE, *Secretary*
J. A. CLAUSS
G. L. FISK
S. M. WECKSTEIN
T. H. WICKENDEN

Associates

A. J. BOYNTON
J. H. HITCHCOCK
S. M. MARSHALL
B. C. MCFADDEN
J. H. ROMANN
R. J. WEAN
W. R. WEBSTER

Machine Shop Practice

Organized, 1921

A. M. JOHNSON, *Chairman*

EXECUTIVE COMMITTEE

A. M. JOHNSON, *Chairman*
WARNER SEELY, *Secretary*
O. W. BUENTING
SOL EINSTEIN
ERIK OBERG
B. G. TANG

CUTTING METALS

HANS ERNST, *Chairman*

(See also Research Technical Committee on Cutting of Metals, page RI-20)

FOUNDRY PRACTICE

JAMES THOMSON, *Chairman*
R. E. KENNEDY, *Secretary*

LUBRICATION ENGINEERING

B. G. TANG, *Chairman*
C. M. LARSON
H. J. MASSON

MACHINE DESIGN

E. S. AULT, *Chairman*
A. E. R. DE JONGE
J. MARIN

WELDING

C. W. OBERT, *Chairman*

Management

Organized, 1920

W. H. KUSHNICK, *Chairman*

EXECUTIVE COMMITTEE

W. H. KUSHNICK, *Chairman*
G. VARGA, *Secretary*
J. R. BANGS
H. B. BERGEN
L. C. MORROW

GENERAL COMMITTEE

L. P. ALFORD
L. A. APPLEY
R. M. BARNES
W. L. BATT
C. W. BEESE
F. B. BELL
H. V. COES
K. H. CONDIT
N. E. ELSAS
R. E. FLANDERS
W. D. FULLER
LEE GALLOWAY
L. M. GILBRETH
P. E. HOLDEN
W. F. HOSFORD
D. S. KIMBALL
T. S. MCEWAN
D. B. PORTER
WALTER RAUTENSTRAUCH
F. E. RAYMOND
J. W. ROE
D. L. ROYER
E. H. SCHELL
E. D. SMITH
L. W. WALLACE
JOHN YOUNGER

LOCAL SECTIONS REPRESENTATIVES

Kansas City, A. H. SLUSS
Metropolitan, W. F. CARHART
Milwaukee, B. V. E. NORDBERG
Philadelphia, C. S. GOTWALS
Western Washington, H. J. MCINTYRE

Materials Handling

Organized, 1920

G. E. HAGEMANN, *Chairman*

EXECUTIVE COMMITTEE

G. E. HAGEMANN, *Chairman*
F. J. SHEPARD, JR., *Secretary*
C. F. DIETZ
J. A. JACKSON
M. C. MAXWELL
R. B. RENNER

Associates

NIXON ELMER
R. H. MCLAIN
F. E. MOORE
E. D. SMITH
J. B. WEBB

Oil and Gas Power

Organized, 1921

F. G. HECHLER, *Chairman*

EXECUTIVE COMMITTEE

F. G. HECHLER, *Chairman*
W. J. HARGEST, *Secretary*
E. S. DENNISON
W. L. H. DOYLE
C. W. GOOD
L. H. MORRISON

Associates

R. D. CAMPBELL
H. E. DEGLER
L. R. FORD
E. J. KATES
BRUNO NORDBERG
M. J. REED
LEE SCHNEITTER
A. R. WALKER

Petroleum

Organized, 1925

Reorganized, 1937

T. D. TIFFT, *Chairman*

EXECUTIVE COMMITTEE

T. D. TIFFT, *Chairman*
W. J. OVERTON, *Staff Secretary*
W. F. HERBERT, *First Vice-Chairman*
H. L. EGGLESTON, *Second Vice-Chairman*
E. H. BARLOW, *Third Vice-Chairman*
W. H. CARSON, *Fourth Vice-Chairman*

Power

Organized, 1920

E. L. HOPPING, *Chairman*

EXECUTIVE COMMITTEE

E. L. HOPPING, *Chairman*
THEODORE BAUMEISTER, JR., *Secretary*
O. F. CAMPBELL
G. C. EATON
G. A. GAFFERT

Process Industries

Organized, 1934

J. W. HUNTER, *Chairman*

EXECUTIVE COMMITTEE

J. W. HUNTER, *Chairman*
T. R. OLIVE, *Secretary*
THEODORE BAUMEISTER, JR.
C. E. HARRINGTON
R. F. O'MARA
WM. RAISCH
J. H. SENGSTAKEN
A. F. SPITZGLASS
ARNOLD WEISSELBERG
W. R. WOOLRICH
J. I. YELLOTT
F. L. YERZLEY

AIR CONDITIONING

C. F. KAYAN, *Chairman*

DRYING

ARNOLD WEISSELBERG, *Chairman*

FOOD PROCESSING

G. L. MONTGOMERY, *Chairman*

INDUSTRIAL INSTRUMENTS AND REGULATORS

EXECUTIVE COMMITTEE

E. S. SMITH, *Chairman*
E. D. HAIGLER, *Vice-Chairman*
A. F. SPITZGLASS, *Secretary*
GEORGE PERAZICH, *Chairman,* Bibliography Committee
P. G. EXLINE, *Chairman,* Pressure and Temperature Committee
J. J. GREBE, *Chairman,* Program Committee
A. F. SPITZGLASS, *Chairman,* Quantitative Analysis Committee
J. I. YELLOTT, *Chairman,* Terminology Committee
M. J. ZUCROW, *Chairman,* Translation Committee

Process Industries

(*Continued*)

MECHANICAL SEPARATION

R. F. O'MARA, *Chairman*

PAPER AWARDS AND HONORS

C. E. LUCKE, *Chairman*

PROGRAM

J. W. HUNTER, *Chairman*

PULP AND PAPER

J. A. MCPHERSON, *Chairman*

SANITATION

WILLIAM RAISCH, *Chairman*

SUGAR

F. M. GIBSON, *Chairman*

SULPHUR

B. E. SHORT, *Chairman*

VEGETABLE OILS

ROSCOE MORTON, *Chairman*

Sub-Division on Rubber and Plastics

EXECUTIVE COMMITTEE

F. L. YERZLEY, *Chairman*
J. F. D. SMITH, *Secretary*
L. E. JERMY
G. M. KLINE

Railroad

Organized, 1920

L. H. FRY, *Chairman*

EXECUTIVE COMMITTEE

L. H. FRY, *Chairman* (1940)
C. L. COMBES, *Secretary*
A. I. LIPETZ (1941)
D. S. ELLIS (1942)
J. R. JACKSON (1943)
J. G. ADAIR (1944)

(*Continued on following page*)

Railroad

(Continued)

GENERAL COMMITTEE (RR2)

L. H. Fry, *Chairman*
R. S. Binkerd
B. S. Cain
O. C. Cromwell
W. E. Dunham
L. B. Jones
F. G. Lister
F. E. Lyford
K. F. Nystrom
John Roberts
R. W. Salisbury
W. C. Sanders
W. M. Sheehan
H. C. Wilcox
Dennistoun Wood
E. G. Young
G. A. Young

PAST-CHAIRMEN (RR3)

James Partington (1923–1924)
H. B. Oatley (1926–1927)
Wm. Elmer (1928)
R. S. McConnell (1929)
A. F. Stuebing (1930)
Eliot Sumner (1931)
T. C. McBride (1932)
L. K. Sillcox (1933)
C. B. Peck (1934)
C. E. Barba (1935)
Geo. W. Rink (1936)
W. H. Winterrowd (1937)
E. C. Schmidt (1938)
C. T. Ripley (1939)

Textile

Organized, 1921

R. DeVere Hope, *Chairman*

EXECUTIVE COMMITTEE

R. DeVere Hope, *Chairman*
A. D. Asbury, *Vice-Chairman*
W. B. Heinz, *Secretary*
H. H. Iler
Albert Palmer
J. D. Robertson
E. R. Stall
W. W. Starke, *Metropolitan Representative*
S. B. Earle, *Southern Representative*

Associates

A. W. Benoit
W. S. Brown
Winn Chase
M. A. Golbick, Jr.

Wood Industries

Organized, 1921

C. B. Norris, *Chairman*

EXECUTIVE COMMITTEE

C. B. Norris, *Chairman*
M. J. MacDonald
Sern Madsen
T. D. Perry
R. H. McCarthy, *Metropolitan Representative*

Associates

C. L. Babcock
P. H. Bilhuber
H. B. Carpenter
F. P. Cartwright
G. E. French
A. W. Keuffel
A. S. Kurkjian
J. S. Mathewson
E. D. May
A. D. Smith, Jr.
H. M. Sutton
Chas. White

SUBCOMMITTEE CHAIRMEN

Dimensional Limits and Allowances, Sern Madsen
Machining of Wood, R. H. McCarthy
Use of Plywood as an Engineering Material, T. D. Perry
Wood Finishing, M. J. MacDonald

LOCAL SECTIONS

ARTICLE B6A, PAR. 17: The Standing Committee on Local Sections shall, under the direction of the Council, have supervision of the Local Sections of the Society.

STANDING COMMITTEE ON LOCAL SECTIONS

A. J. KERR, *Chairman* (1940)
H. L. EGGLESTON (1941) J. P. FERRIS (1943)
J. N. LANDIS (1942) F. W. MARQUIS (1944)

Junior Advisers

J. R. CAVE, JR. (1940) S. DAVIDSON (1941)

REGIONAL GROUP DELEGATES TO ANNUAL CONFERENCES

Terms expire October, 1940

S. F. DUNCAN, *Speaker for 1939 Conference*, Group VII
R. A. SPENCE, *Secretary*, Group I
C. A. HESCHELES, Group II N. C. EBAUGH, Group IV
J. T. REA, Group III C. J. FREUND, Group V
R. E. TURNER, Group VI

Terms expire October, 1941

A. R. ACHESON, *Speaker for 1940 Conference*, Group III
G. R. TRUEDSSON, *Secretary*, Group I
O. B. SCHIER, II, Group II H. M. GANO, Group V
A. D. ASBURY, Group IV FRED H. DORNER, Group VI
W. D. TURPIN, Group VII

AKRON-CANTON

Organized: 1920
Territory: Counties of Richland, Ashland, Medina, Summit, Portage, Wayne, Stark, Holmes, Tuscarawas, Carroll, and Coshocton in Ohio
Place of Meeting: As selected monthly
Number of Members: 126

EXECUTIVE COMMITTEE

A. D. MACLACHLAN, *Chairman*
A. W. SEEKINS, *Vice-Chairman*
A. G. WALKER, *Secretary-Treasurer*
OSCAR BIESHELT
M. R. BOWERMAN
J. W. BOYD
V. R. CAMP
JAMES FORREST
E. D. GEORGE
C. W. NELSON
G. J. SCHOESSOW
A. E. SHETLER

ANTHRACITE-LEHIGH VALLEY

Organized: 1920, as Lehigh Valley; reorganized, 1928, as Anthracite-Lehigh Valley
Territory: Counties of Bradford, Susquehanna, Wayne, Sullivan, Wyoming, Lackawanna, Columbia, Luzerne, Monroe, Pike, Schuylkill, Carbon, Berks, Lehigh, Northampton in Pennsylvania, and Warren in New Jersey
Place of Meeting: One meeting annually at Allentown, Bethlehem, Easton, Hazleton, Pottsville, Reading, Scranton, and Wilkes-Barre
Local Organization: The Engineers' Club of Lehigh Valley
Number of Members: 191

EXECUTIVE COMMITTEE

J. W. STEINMEYER, *Chairman*
A. W. LUCE, *Vice-Chairman*
F. C. PETERS, *Vice-Chairman*
W. P. SAUNIER, *Vice-Chairman*
C. M. MERRICK, 3RD, *Secretary*
M. C. STUART, *Treasurer*
G. W. FARNHAM
W. W. HAGERTY
R. E. MOYER, JR.
E. A. SCHMIDT
W. T. SOULIS
W. TALLGREN
R. L. WILLIS
C. R. CHISHOLM
R. C. DANNETTEL
A. M. GOMPF
K. P. HANSON
W. P. HILL
G. W. KEEN
S. M. WHITELEY

ATLANTA

Organized: 1913
Territory: Radius of sixty miles from Atlanta, Ga.
Place of Meeting: Atlanta Athletic Club
Luncheon meeting every Monday at 12:30 p.m. at Atlanta Athletic Club
Number of Members: 85

EXECUTIVE COMMITTEE

M. F. MERL, *Chairman*
F. C. SMITH, *Vice-Chairman*
J. M. RITTELMEYER, *Secretary-Treasurer*
R. N. BENJAMIN
E. A. HARPER
R. S. HOWELL
A. H. KOCH
R. J. WODDALL, *Student Chairman*

JUNIOR GROUP

W. D. BOYNTON, *Chairman*
W. P. HILL, *Vice-Chairman*
F. S. KNABE, *Secretary*
F. C. COOK
C. F. HILDENBRAND
P. T. MANGER
T. W. MELE

BALTIMORE

Organized: 1916
Territory: Radius of thirty miles from Baltimore, Md.
Place of Meeting: Engineers Club of Baltimore
Luncheon meeting every Wednesday at 12:00 noon at Engineers Club
Number of Members: 189

EXECUTIVE COMMITTEE

L. F. COFFIN, *Chairman*
S. B. SEXTON, 3RD, *Secretary-Treasurer*
W. D. BOYNTON

BIRMINGHAM

Organized: 1915
Territory: Radius of sixty miles from Birmingham, Ala.
Place of Meeting: Tutwiler Hotel
Number of Members: 90

EXECUTIVE COMMITTEE

R. A. POLGLAZE, *Chairman*
H. S. KENT, *Vice-Chairman*
C. F. VON HERRMAN, JR., *Secretary-Treasurer*
J. E. GETZEN
W. JOE MOORE

BOSTON

Organized: 1909
Territory: Radius of thirty miles from Boston, Mass.
Place of Meeting: Mass. Inst. of Technol
Local Organization: Engineering Societ of New England
Number of Members: 559

BOSTON

(*Continued*)

EXECUTIVE COMMITTEE

EDGAR MACNAUGHTON, *Chairman*
KERR ATKINSON, *Vice-Chairman*
R. A. SPENCE, *Secretary-Treasurer*
H. J. BROWN
G. K. SAURWEIN
J. W. ZELLER

JUNIOR GROUP

E. I. BOWER, *Chairman*
R. N. GILBERT, *Vice-Chairman*
R. A. SPENCE, *Secretary-Treasurer*
F. ROBERT HARTIN
F. M. MAGEE
ANTON SALECKER
A. A. SHAMES

BRIDGEPORT

Organized: 1917, as a Branch of Connecticut Section; reorganized as a Section, 1923
Territory: Fairfield County, Conn.
Place of Meeting: Stratfield Hotel
Local Organization: Engineers' Club of Bridgeport
Number of Members: 121

EXECUTIVE COMMITTEE

A. H. BEEDE, *Chairman*
C. N. HOAGLAND, *Vice-Chairman*
W. H. SNIFFEN, *Secretary*
A. W. HAGAN, *Treasurer*
RUDOLF BECK
C. A. BUSS
H. P. HARRIS
R. C. MOODY
O. J. RICHMOND
J. D. SKINNER
D. E. STEER
J. H. VAN YORX, JR.

JUNIOR GROUP

C. A. BUSS, *Chairman*
W. J. CHELGREN, *Vice-Chairman*
D. E. STEER, *Secretary-Treasurer*

BUFFALO

Organized: 1915
Territory: Radius of thirty miles from Buffalo, N.Y.
Place of Meeting: University Club, Delaware Ave.
Local Organization: Engineering Society of Buffalo
Number of Members: 191

EXECUTIVE COMMITTEE

W. M. KAUFFMANN, *Acting Chairman*
E. J. WEITZMAN, *Acting Vice-Chairman*
W. A. MILLER, *Secretary*
C. E. HARRINGTON, *Treasurer*
PAUL DUBOSCLARD
C. G. ROSE
J. L. YATES, *Adviser for Juniors*

CENTRAL ILLINOIS

Organized: 1937
Territory: All the territory in Central Illinois between the following counties on the northern boundary: Bureau, LaSalle, Knox, Stark, Putnam, Marshall, Livingston, Peoria; counties on the southern boundary: Pike, Scott, Morgan, Sangamon, Macon, Piatt, Douglas, and Edgar
Place of Meeting: Hotel Pere Marquette or Caterpillar Show Room
Number of Members: 94

EXECUTIVE COMMITTEE

E. D. EDWARDS, *Chairman*
O. A. LEUTWILER, *Vice-Chairman*
F. L. MEYER, *Vice-Chairman*
C. O. SMITH, *Vice-Chairman*
R. T. MEES, *Secretary-Treasurer*
L. E. JOHNSON, *Assistant Secretary*

JUNIOR GROUP

L. E. JOHNSON, *Chairman*

CENTRAL INDIANA

Organized: 1916
Territory: Radius of eighty miles from Indianapolis, within Indiana
Place of Meeting: Place varies
Local Organization: Indiana Engineering Society
Number of Members: 123

EXECUTIVE COMMITTEE

HANS SKABO, *Chairman*
J. A. DROGUE, *Vice-Chairman*
W. J. COPE, *Secretary-Treasurer*
E. A. GRIMMER
C. M. GROSS
RONALD UPDIKE
CARL WISCHMEYER

CENTRAL PENNSYLVANIA

Organized: 1921
Territory: Radius of approximately sixty miles from State College, Pa.
Place of Meeting: State College and Altoona, Pa.
Number of Members: 85

EXECUTIVE COMMITTEE

F. C. STEWART, *Chairman*
J. O. P. HUMMEL, *Secretary-Treasurer*
C. L. ALLEN
W. D. GARMAN
G. L. GUILLET

CHARLOTTE

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of seventy-five miles from Charlotte, N.C.
Luncheon Meeting every Monday at 1:00 p.m. at Efirds Department Store Dining Room
Local Organization: Charlotte Engineers Club
Number of Members: 37

EXECUTIVE COMMITTEE

W. W. LEROY, *Chairman*
R. P. REECE, *1st Vice-Chairman*
M. D. THOMASON, *2nd Vice-Chairman*
A. W. DUNBAR, *3rd Vice-Chairman*
J. S. BASS, JR., *Secretary-Treasurer*
A. W. BOLLENBACK
T. C. HEYWARD
ASA HOSMER
W. E. MCDOWELL
T. O. SILLS

CHICAGO

Organized: 1913
Territory: Radius of fifty miles from Chicago, Ill.
Headquarters: Mid-West A.S.M.E. Office, Room 1617, 205 West Wacker Drive, Chicago, Ill.
Place of Meeting: Civic Opera Bldg., 20 N. Wacker Dr.
Luncheon Meeting every Tuesday at 12:15 p.m. at Chicago Engineers' Club
Local Organization: Western Society of Engineers
Number of Members: 784

EXECUTIVE COMMITTEE

L. M. ELLISON, *Chairman*
H. M. BLACK, *Vice-Chairman*
F. C. EVERITT, *Vice-Chairman*
H. F. HEBLEY, *Vice-Chairman*
D. ROESCH, *Vice-Chairman*
F. B. ORR, *Secretary-Treasurer*
C. C. AUSTIN
A. D. BAILEY
C. L. CHARLES
W. E. DUNHAM
J. A. FOLSE
A. H. JENS
J. S. KOZACKA
R. KRAUSE
F. H. LANE
J. P. MAGOS
J. C. MARSHALL
T. S. MCEWAN
J. R. MICHEL
H. L. NACHMAN
C. W. PARSONS
H. S. PHILBRICK
J. G. REID
C. ROBERTS
M. W. SHERWOOD
B. D. STEVENS
R. E. TURNER
C. L. WACHS

JUNIOR GROUP

R. KRAUSE, *Co-Chairman*
J. C. MARSHALL, *Co-Chairman*

CINCINNATI

Organized: 1912
Territory: Radius of thirty miles from Cincinnati, Ohio
Place of Meeting: Engineers' Club Rooms, Ninth & Race Sts.
Local Organization: Engineers' Club of Cincinnati
Number of Members: 185

EXECUTIVE COMMITTEE

E. H. SCHUBERT, *Chairman*
C. E. SOMOGYI, *Vice-Chairman*
J. W. BUNTING, *Secretary-Treasurer*
H. B. BRANDT
D. S. BROWN

CINCINNATI

(Continued)

A. G. Bruck
E. J. Martin
L. E. Nenninger
E. S. Sauerbrunn
C. H. Spiehler
H. P. Thompson

CLEVELAND

Organized: 1918
Territory: Counties of Lorain, Cuyahoga Lake, Geauga, and Ashtabula in Ohio
Place of Meeting: Case Club or Cleveland Engineering Society Rooms
Local Organization: Cleveland Engineering Society
Number of Members: 248

Executive Committee

G. B. Carson, *Chairman*
E. L. Lindseth, *Vice-Chairman*
E. R. McCarthy, *Secretary*
F. A. Barnes, *Treasurer*
A. B. Einig
E. B. Heavilon
J. M. Main
H. A. Schwartz
D. K. Wright

Junior Group

R. E. Erickson, *Chairman*
W. A. Lynam, *Secretary-Treasurer*
J. O. Geissbuhler

COLORADO

Organized: 1919
Territory: Entire State of Colorado
Place of Meeting: Parisienne Rotisserie Inn, Denver, Colo.
Local Organization: Colorado Engineering Council (Colorado Society of Engineers)
Number of Members: 77

Executive Committee

A. L. Hill, *Chairman*
R. F. Throne, *Secretary-Treasurer*
C. E. Beatty
L. D. Crain
F. H. Prouty
G. A. Richter
S. L. Simmering
J. Tayler Strate

COLUMBUS

Organized: 1920
Territory: Counties of Union, Delaware, Licking, Madison, Franklin, Fayette, Pickaway, and Ross in Ohio
Place of Meeting: Battelle Memorial Institute and The Ohio State University
Local Organization: Engineers' Club of Columbus
Luncheon Meeting third Friday of each month at 12:00 noon at Engineers' Club, Columbus
Number of Members: 76

Executive Committee

R. N. Tucker, *Chairman*
E. M. Sampson, *Vice-Chairman*
R. C. Cross, *Secretary-Treasurer*
H. N. Booker
A. I. Brown
H. R. Limbacher
C. P. Roberts
W. A. Welcker, Jr.

DAYTON

Organized: 1926
Territory: Counties of Drake, Miami, Champaign, Preble, Montgomery, Greene, and northern part of Butler and Warren in Ohio
Place of Meeting: Engineers' Club of Dayton
Local Organization: Engineers' Club of Dayton
Number of Members: 82

Executive Committee

P. H. Kemmer, *Chairman*
A. R. Weber, *Vice-Chairman*
(Miss) M. Elsa Gardner, *Secretary*
B. W. Treadwell, *Treasurer*
C. L. Bauer
H. M. Gano
R. W. Martin
T. F. Rataiczak
W. E. Sooy

DETROIT

Organized: 1916
Territory: Radius of thirty miles from Detroit, Mich.
Place of Meeting: Place varies
Local Organization: Engineering Society of Detroit
Number of Members: 456

Executive Committee

B. W. Beyer, Jr., *Chairman*
Tom Jeffords, *Secretary-Treasurer*
E. J. Abbott
J. A. Carlin
M. L. Fox
J. F. Jarnagin
J. B. Macauley
D. E. McGuire
Jesse Ormondroyd
J. P. Schechter
A. M. Selvey
M. W. Benjamin, *Ex-Officio*

Junior Group

George Scranton, *Chairman*
Clarence Green, *Secretary*
B. W. Lane

EAST TENNESSEE

Organized: 1922
Territory: All counties in Tennessee east of the west boundary of Scott, Morgan, Cumberland, White, Warren, Coffee, Moore, Franklin; Belle County in Kentucky; and Rossville, Dade, Walker, Cattasa, Whitfield, Murray, Gordon, Chattooga in Georgia
Place of Meeting: Place varies
Local Organization: Chattanooga Engineers Club and Knoxville Technical Club
Luncheon Meeting every Monday noon at Chattanooga Engineers Club
Number of Members: 94

Executive Committee

W. R. Chambers, *Chairman*
T. C. Ervin, *1st Vice-Chairman*
David Shearer, *2nd Vice-Chairman*
R. B. Taylor, *3rd Vice-Chairman*
J. M. Tucker, *Secretary*
G. E. Campbell
C. E. Ferris
J. M. Johnson
Elmer Torok
A. S. Wells

ERIE

Organized: 1917
Territory: Radius of thirty miles from Erie, Pa.
Place of Meeting: Auditorium of Pennsylvania Telephone Company
Number of Members: 75

Executive Committee

C. T. Oergel, *Chairman*
G. F. Linde, *Vice-Chairman*
R. R. Blunt, *Secretary*
E. E. Ims, *Treasurer*
G. W. Bach
E. H. Horstkotte
W. L. Hunter
McD. S. Reed
R. W. Rogers

FLORIDA

Organized: 1925
Territory: State of Florida
Place of Meeting: Various Cities in State
Local Organization: Florida Engineering Society, Gainesville, Fla.
Number of Members: 74

Executive Committee

N. C. Ebaugh, *Chairman*
J. H. Clouse, *1st Vice-Chairman*
M. H. C. Brombacher, *2nd Vice-Chairman*
V. C. Couchman, *Secretary-Treasurer*
W. E. Drew
A. B. Hale
W. A. Stansfield
C. H. Stanton, *Student Branch Chairman*

FORT WAYNE

Organized: 1939
Territory: Counties of LaGrange, Steuben, Noble, DeKalb, Whitley, Allen, Wabash, Huntington, Wells, Adams, Miami, Blackford and Jay in Indiana; Counties of Williams, Defiance, Paulding, Van Wert and Mercer in Ohio
Local Organization: Fort Wayne Engineers' Society
Number of Members: 27

Executive Committee

N. T. Bourke, *Chairman*
W. L. Knaus, *Vice-Chairman*
F. T. McInerney, *Secretary*
F. L. Ruoff, *Treasurer*
J. H. Powers
W. S. Richhart

GREEN MOUNTAIN

Organized: 1923
Territory: Entire State of Vermont and neighboring and closely related communities of Claremont and Hanover, N.H.
Place of Meeting: Springfield, Windsor, Vt., and Claremont, N.H.
Local Organization: Vermont Engineering Society
Number of Members: 39

Executive Committee

M. H. Arms, *Chairman*
Leif Fersing, *Secretary-Treasurer*
P. H. Adams
C. J. Dewell
Douglas Hamilton
J. B. Johnson
E. L. Sussdorff

Junior Group

E. L. Sussdorff, *Chairman*

GREENVILLE

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of sixty miles from Greenville, S.C.
Place of Meeting: Meetings held at Greenville, Clemson College, S.C., Canton, Asheville, and Enka, N.C.
Number of Members: 40

Executive Committee

A. D. Asbury, *Chairman*
R. B. Fuller, *Vice-Chairman*
J. H. Sams, *Secretary-Treasurer*
B. E. Fernow
J. R. Gill
H. H. Iler
R. S. Pruitt
J. E. Waldrep

HARTFORD

Organized: 1917, as Branch of Conn. Section; reorganized, 1923
Territory: Hartford County except that portion served by Meriden and New Britain Sections
Place of Meeting: Hartford Electric Light Company
Number of Members: 115

Executive Committee

D. K. Morgan, *Chairman*
H. Burdick, *Vice-Chairman*
R. D. Keller, *Secretary*
S. A. Brandenburg
R. F. Dow
E. P. Herrick
F. O. Hoagland
W. S. Paine
R. S. Shaw
S. J. Teller
G. R. Truedsson
H. B. Van Zelm
J. C. Ward, Jr.

INLAND EMPIRE

Organized: 1921
Territory: East of Columbia River in State of Washington, and Counties of Okanogan and Benton, and part of Northern Idaho
Place of Meeting: Davenport Hotel, Spokane
Luncheons Wednesdays at 12:00 noon, Davenport Hotel, Spokane
Local Organization: Associated Engineers of Spokane
Number of Members: 26

Executive Committee

E. B. Parker, *Chairman*
Alex. Lindsay, *Vice-Chairman*
A. R. Karlsten, *Secretary-Treasurer*
H. F. Gauss
D. R. Gray
H. H. Langdon

ITHACA

Organized: 1936
Territory: Radius of thirty miles from Ithaca plus following cities: Binghamton, Corning, Endicott, Geneva, Painted Post
Place of Meeting: Willard Straight Hall, Cornell Campus, Ithaca, N.Y.
Number of Members: 85

Executive Committee

W. M. Sawdon, *Chairman*
L. C. Wilder, *Vice-Chairman*
F. S. Erdman, *Secretary-Treasurer*
R. E. Kinsman
F. G. Switzer
M. P. Whitney
N. R. Wickersham

KANSAS CITY

Organized: 1921
Territory: Radius of sixty miles from Kansas City, Mo.
Place of Meeting: University Club
Local Organization: Engineers' Club of Kansas City
Number of Members: 135

Executive Committee

E. D. Hay, *Chairman*
H. L. Crain, *Vice-Chairman*
E. M. Bruzelius, *Secretary*
J. R. Stone, *Treasurer*
C. E. Brown
Chester Cotter
M. A. Durland
H. E. Manuel
W. B. Rollins
H. A. Smith
R. V. Sutherland

Junior Group

H. E. Manual, *Chairman*
G. G. Brauninger, *Secretary-Treasurer*
E. M. Bruzelius
R. F. Glore
R. P. Hahn

LOS ANGELES

Organized: 1915
Territory: South of southern boundaries of following counties: Monterey, Kings, Tulares, and Inyo, Calif.
Place of Meeting: Barker Bros. Store
Local Organization: Technical Societies of Los Angeles
Luncheon Meetings Thursdays at 12:00 noon at Engineers' Club
Number of Members: 454

Executive Committee

J. Roy Hoffman, *Chairman*
C. H. Smith, *Vice-Chairman*
E. K. Springer, *Secretary-Treasurer*
P. L. Armstrong
J. C. Brown
D. S. Clark
B. E. Del Mar
S. F. Duncan
A. Hollander
Scott Jensen
J. A. Whitaker

Junior Group

B. E. Del Mar, *Chairman*
O. R. Broberg
R. W. Jones

LOUISVILLE

Organized: 1922
Territory: Radius of thirty miles from Louisville, Ky.
Place of Meeting: Engineers and Architects Club of Louisville
Local Organization: Engineers and Architects Club
Number of Members: 50

Executive Committee

Ray Haselberger, *Chairman*
M. Sack, *Vice-Chairman*
L. L. Amidon, *Secretary*
W. L. Lucas, *Treasurer*
H. H. Fenwick
J. K. Meyer
J. A. Rabuck
J. H. Romann

MEMPHIS

Organized: 1923
Territory: Radius of sixty miles from Memphis, Tenn., and eastern half of Arkansas including all the territory east of a line drawn north and south through the western boundary of the city of Little Rock
Number of Members: 20

Executive Committee

M. D. Rust, *Chairman*
W. J. Fransioli, Jr., *Vice-Chairman*
R. B. H. Shepherd, *Secretary-Treasurer*
T. H. Allen
C. M. McCord
W. H. Roberts

METROPOLITAN

Organized: 1910
Territory: Metropolitan District, New York and New Jersey
Place of Meeting: Engineering Societies Building, 29 West 39th Street, New York, N.Y.
Number of Members: 3316

Executive Committee

T. B. Allardice, *Chairman*
E. H. Fezandie, *Secretary*
E. J. Billings, *Treasurer*

METROPOLITAN

(Continued)

F. D. CARVIN, *Chairman of Program Committee*
W. G. BLAKE
P. E. FRANK
W. McC. McKEE
A. R. MUMFORD
O. B. SCHIER, II
J. M. DRISCOLL, *Ex-Officio*

JUNIOR GROUP

SAM SHOOR, *Chairman*
C. K. HOLLAND, *Vice-Chairman*
A. E. BLIRER, *Secretary*
SIDNEY DAVIDSON
H. B. FERNALD, JR.
W. W. LAWRENCE
H. G. OLIVER, JR.
R. F. WARNER, JR.

MID-CONTINENT

Organized: 1919
Territory: Entire state of Oklahoma; territory in Arkansas not included in Memphis Section; part of Louisiana; and territory in Texas north of the southern boundaries of the counties of Gaines, Dawson, Bordon, Scurry, Fisher, Jones, and Shackelford
Place of Meeting: Usually Mayo Hotel, Tulsa, Okla.
Luncheon Meetings with Engineers Club of Tulsa, Mondays at 12:00 noon
Local Organization: Engineers Club of Tulsa
Number of Members: 142

EXECUTIVE COMMITTEE

H. R. AUERSWALD, *Chairman*
A. G. BLANCHARD, *Vice-Chairman*
L. C. PRICE, *Vice-Chairman*
G. RAYMOND, *Vice-Chairman*
HENRY SCHAEFER, *Vice-Chairman*
MAX WISE, *Secretary*
HERMAN WIENECKE, *Treasurer*
E. E. AMBROSIUS
R. G. AYERS
E. C. BAKER
W. L. DUCKER
W. G. HELTZEL
D. K. HUTCHCRAFT
A. J. KERR
H. T. SEARS
FRANK STIVERS

MILWAUKEE

Organized: 1909
Territory: Radius of fifty miles from Milwaukee, Wis.
Place of Meeting: Wisconsin Club
Local Organization: Engineers' Society of Milwaukee
Luncheon Meetings once each month, 3rd Wednesday at Wisconsin Club
Number of Members: 208

EXECUTIVE COMMITTEE

B. V. E. NORDBERG, *Chairman*
A. W. LINDSTROM, *Secretary-Treasurer*
W. D. BLISS
H. DAHLSTRAND
F. H. DORNER
A. H. LUEDICKE
W. C. LINDEMANN
R. C. NEWHOUSE
E. C. SCHUM
A. SIMON

JUNIOR GROUP

G. V. MINIBERGER, *Chairman*
R. H. MILLER, *Secretary*
GEORGE FRATCHER
RUSSELL HAHN
C. A. POFAHL, JR.
W. T. SAVELAND, JR.
ROBERT STRASSMAN

MINNESOTA

Organized: Minneapolis, 1913; St. Paul, 1913; the two Sections merged, 1934
Territory: Entire State of Minnesota
Place of Meeting: Minnesota Union, Univ. of Minnesota
Local Organization: Minneapolis Engineers' Club, Minnesota Federation of Architectural and Engineering Societies
Number of Members: 118

EXECUTIVE COMMITTEE

K. A. POWELL, *Chairman*
C. F. SHOOP, *Vice-Chairman*
L. G. STRAUB, *Secretary-Treasurer*
N. J. STERNAL
J. C. VANSELOW

NEBRASKA

Organized: 1922
Territory: State of Nebraska, and Council Bluffs, Iowa
Place of Meeting: Lincoln and Omaha
Local Organization: Engineers' Club of Lincoln and Omaha
Luncheon Meeting every Wednesday noon at the Omaha Engineers' Club—4th Monday Evening at Lincoln
Number of Members: 38

EXECUTIVE COMMITTEE

N. H. BERNARD, *Chairman*
A. A. LUEBS, *Vice-Chairman*
G. A. ROGERS, *Secretary-Treasurer*
J. W. BURNETT
J. H. COLSON
J. L. WHITE

NEW BRITAIN

Organized: 1921, as a Branch of Connecticut Section; reorganized as a Section, 1923
Territory: New Britain, Plainville, Forestville, Bristol, Kensington, and Berlin, Conn.
Place of Meeting: Auditorium of the State Trade School
Number of Members: 39

EXECUTIVE COMMITTEE

B. S. LEWIS, *Chairman*
C. N. FLAGG, *Vice-Chairman*
H. MICHELSEN, *Secretary-Treasurer*
P. W. BAUER
H. C. BARNES
H. RAMM
C. C. STEVENS
S. H. STONER

NEW HAVEN

Organized: 1912; reorganized, 1923
Territory: Portions of New Haven and Middlesex Counties, Conn.
Place of Meeting: Mason Laboratory, Yale University
Number of Members: 88

EXECUTIVE COMMITTEE

R. A. NORTH, *Chairman*
L. H. VON OHLSEN, *Secretary-Treasurer*
W. F. THOMPSON
A. L. BRECKENRIDGE
I. T. HOOK
W. L. TANN
F. J. COCHRAN

NEW ORLEANS

Organized: 1916
Territory: All of Louisiana except the northern part allotted to Mid-Continent Section
Place of Meeting: Room 422, St. Charles Hotel
Local Organization: Louisiana Engineering Society
Number of Members: 98

EXECUTIVE COMMITTEE

K. P. KAMMER, *Chairman*
L. J. LASSALLE, *Vice-Chairman*
G. R. HAMMETT, *Secretary-Treasurer*
C. A. BENDER
L. J. CUCULLU
J. S. HUEY
J. K. MAYER

JUNIOR GROUP

J. K. MAYER, *Chairman*
WALDEMAR NELSON, *Vice-Chairman*
R. B. JAMES, *Secretary*
E. A. MCLELLAN, *Treasurer*
J. R. ROMBACH, JR.
HENRY VOELKEL

NORTH TEXAS

Organized: 1922
Territory: Radius of one hundred and twenty-five miles from Dallas, in Texas
Place of Meeting: Dallas Power & Light Co. Bldg. Auditorium
Local Organization: Technical Club of Dallas
Number of Members: 93

EXECUTIVE COMMITTEE

N. G. HARDY, *Chairman*
C. A. COWLES, JR., *Vice-Chairman*
F. C. JUSTICE, *Secretary-Treasurer*
LEONARD COLE
C. L. KRIBS, JR.
R. M. MATSON
W. B. GREGORY, *Ex-Officio*

JUNIOR GROUP

J. K. CHATTEY, *Chairman*

NORWICH

Organized: 1930
Territory: Counties of Tolland, Windham, and New London in Connecticut, and Westerly District in Rhode Island

NORWICH

(Continued)

Place of Meeting: Arcanum Club, 150 Main St., Norwich
Number of Members: 39

EXECUTIVE COMMITTEE

E. S. DENNISON, *Chairman*
A. D. ANDRIOLA, *Secretary-Treasurer*
W. E. EDEL
F. S. ENGLISH
H. LEUHRS
A. E. WELCH
R. WOSAK

ONTARIO

Organized: 1917
Territory: Province of Ontario, Canada
Place of Meeting: Hart House, University of Toronto
Number of Members: 161

EXECUTIVE COMMITTEE

S. G. CLARKE, *Chairman*
G. E. ELLSWORTH, *Secretary-Treasurer*
O. H. ANDERSON
W. S. BALL
A. C. BLUE
D. F. CORNISH
C. R. DAVIS
F. G. EAST
R. B. ELLIOTT
W. G. MCINTOSH
W. E. MICKLETHWAITE
R. L. RUDE

JUNIOR GROUP

F. G. EAST, *Chairman*

OREGON

Organized: 1919
Territory: State of Oregon and that territory in Washington within a radius of thirty miles from Portland, Ore.
Place of Meeting: Usually Public Service Bldg., Portland, Ore.
Local Organization: Oregon Society of Engineers
Number of Members: 56

EXECUTIVE COMMITTEE

E. N. BATES, *Chairman*
R. B. ROWAN, *Vice-Chairman*
A. A. OSIPOVICH, *Secretary*
P. L. HESLOP
J. C. OTHUS
TOM PERRY

JUNIOR GROUP

H. JOHNSON, *Chairman*
W. E. CARSON, *Vice-Chairman*
R. R. STEINKE, *Secretary-Treasurer*

PENINSULA

Organized: 1923
Territory: West of the east boundaries of the following counties: Emmet, Charlevoix, Antrim, Kalkaska, Missaukee, Clare, Isabella, Gratiot, Clinton, Eaton, Calhoun, and Branch, Mich.
Place of Meeting: Grand Rapids, Mich.
Luncheon Meeting Fifth Thursday noon each month
Local Organization: Engineers' Club of Grand Rapids
Number of Members: 55

EXECUTIVE COMMITTEE

C. A. HAMILTON, *Chairman*
R. E. KLISE, *Secretary-Treasurer*
C. G. LOHMANN
E. E. NORMAN
B. E. PORTER

PHILADELPHIA

Organized: 1912
Territory: Counties of Bucks, Montgomery, Chester, Philadelphia, Delaware, Pa., and the State of Delaware
Place of Meeting: Philadelphia Engineers' Club, 1317 Spruce Street, Philadelphia, Pa.
Local Organization: Philadelphia Engineers' Club
Luncheon Meeting every Thursday noon at 12:30 p.m. at Philadelphia Engineers' Club
Number of Members: 849

EXECUTIVE COMMITTEE

E. L. HOPPING, *Chairman*
L. P. HYNES, *Vice-Chairman*
J. S. MOREHOUSE, *Secretary-Treasurer*
C. S. GOTWALS
L. N. GULICK
C. C. JONES

JUNIOR GROUP

J. P. CLARK, *Chairman*
T. M. POMEROY, JR., *Vice-Chairman*
J. D. PETERSON, *Secretary*
G. K. FISHER, *Treasurer*
E. W. GRISCOM
R. K. KNIPE
W. B. PEGRAM
Z. T. WOBENSMITH, 2ND

PITTSBURGH

Organized: 1920
Territory: Counties bounded by and including Beaver, Butler, Venango, Forest, Jefferson, Indiana, Somerset, Fayette, Greene, and Washington, Pa.
Place of Meeting: Engineers' Society of Western Pennsylvania, William Penn Hotel
Local Organization: Engineers' Society of Western Pennsylvania
Number of Members: 424

EXECUTIVE COMMITTEE

W. D. CANAN, *Chairman*
M. R. MCCONNEL, *Secretary*
K. F. TRESCHOW, *Treasurer*
A. BUTCHER
B. C. MCFADDEN
T. E. PURCELL

PLAINFIELD

Organized: 1921
Territory: Plainfield and territory included between Elizabeth, Bound Brook, Metuchen, and Watchung, N.J.
Place of Meeting: Elizabeth Carteret Hotel, Elizabeth, and Plainfield Masonic Temple, Plainfield
Local Organization: Plainfield Engineers Club, Singer Engineering Society
Number of Members: 192

EXECUTIVE COMMITTEE

D. V. WATERS, *Chairman*
G. E. LEAVITT, JR., *Vice-Chairman*
F. C. SPENCER, JR., *Secretary*
C. G. HOLMBERG, JR., *Treasurer*
R. S. AUSTIN
F. C. FROLANDER
JOHN HAYDOCK
JOSEPH KERR

PROVIDENCE

Organized: 1920
Territory: Radius of thirty miles from Providence, R.I.
Place of Meeting: Providence Engineering Society Building, 195 Angell St., Providence, R.I.
Local Organization: Providence Engineering Society
Number of Members: 156

EXECUTIVE COMMITTEE

H. S. SIZER, *Chairman*
A. W. CALDER, JR., *Vice-Chairman*
R. M. SCOTT, *Secretary-Treasurer*
F. S. BLACKALL, JR.
J. D. ELDERT
J. E. ENGLUND
E. W. FREEMAN
N. D. MACLEOD
S. A. VAULE
R. L. WALES

RALEIGH

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of sixty miles from Raleigh, N.C.
Place of Meeting: N.C. State College, Raleigh, N.C.
Local Organization: N.C. Engineering Council, Raleigh Engineers Club
Number of Members: 36

EXECUTIVE COMMITTEE

R. S. WILBUR, *Chairman*
C. E. KERCHNER, *Vice-Chairman*
C. N. SANFORD, *Secretary-Treasurer*
F. F. GROSECLOSE
E. G. HOEFER
F. J. REED

ROCHESTER

Organized: 1919
Territory: Radius of thirty miles from Rochester, N.Y.
Place of Meeting: Rochester Engineering Society Rooms, Sagamore Hotel
Local Organization: Rochester Engineering Society, Sagamore Hotel
Luncheon Meeting every Tuesday at 12:15 p.m. at Sagamore Hotel
Number of Members: 108

ROCHESTER

(Continued)

EXECUTIVE COMMITTEE

J. H. SNYDER, *Chairman*
I. S. BRADLEY, *Vice-Chairman*
I. G. MCCHESNEY, *Secretary-Treasurer*
J. W. GAVETT
K. H. HUBBARD
H. F. KURTZ
P. B. WESSON

ROCK RIVER VALLEY

Organized: 1926
Territory: Radius of thirty miles from Rockford, Ill., plus members in Madison, Wis.
Meeting Place: Place varies
Local Organization: Rockford Engineering Society
Number of Members: 61

EXECUTIVE COMMITTEE

G. L. LARSON, *Chairman*
C. L. AVERY, *Vice-Chairman*
L. A. WILSON, *Secretary*
B. G. ELLIOTT
R. C. GLAZEBROOK
W. C. VON FISCHER
C. KRAUS, *Junior Member*

ST. JOSEPH VALLEY

Organized: 1929
Territory: Counties of La Porte, Starke, Pulaski, St. Joseph, Marshall, Fulton, Elkhart, and Kosciusko in Indiana, and Cass and Berrien Counties in Michigan
Place of Meeting: Morningside Hotel, South Bend, Ind.
Local Organization: St. Joseph Valley Engineers' Club
Number of Members: 36

EXECUTIVE COMMITTEE

C. C. WILCOX, *Chairman*
C. R. ADAMS, *Vice-Chairman*
K. W. KNORR, *Secretary-Treasurer*

ST. LOUIS

Organized: 1909
Territory: Radius of thirty miles from St. Louis, Mo.
Place of Meeting: Place varies
Local Organization: Engineers' Club of St. Louis
Number of Members: 208

EXECUTIVE COMMITTEE

D. E. DICKEY, *Chairman*
ALBERT VIGNE, *Vice-Chairman*
C. B. BRISCOE, *Secretary-Treasurer*
C. J. COLLEY
A. L. HEINTZE
R. W. MERKLE

SAN FRANCISCO

Organized: 1910
Territory: All territory north of the northern boundaries of the counties of San Luis Obispo, Kern, and San Bernardino
Place of Meeting: Engineers' Club, 206 Sansome St.
Luncheon Meetings, Tuesdays, California Hotel, Oakland; Thursdays, Engineers' Club, San Francisco
Local Organization: San Francisco Engineers' Club
Number of Members: 385

EXECUTIVE COMMITTEE

G. H. RAITT, *Chairman*
V. F. ESTCOURT, *Vice-Chairman*
H. B. LANGILLE, *Secretary-Treasurer*
WARREN DOBLE, *Corresponding Secretary*
K. B. ANDERSON
W. HARRY ARCHER
H. T. AVERY
R. B. PLASS
G. N. SOMERVILLE

JUNIOR GROUP

G. GALE SULLIVAN, *Chairman*
W. M. BRASHEAR, *Secretary*
BERT HIRSCH
J. W. R. LEMERY
BERT TRUETT
J. E. ZORN

SAVANNAH

Organized: 1923
Territory: Radius of 125 miles from Savannah in Georgia
Place of Meeting: Savannah Hotel
Local Organization: Engineers' Council of Savannah Chamber of Commerce
Number of Members: 21

EXECUTIVE COMMITTEE

A. P. KEISKER, *Chairman*
W. R. BROWN, *Secretary*
ALEX. M. ORMOND
LOUIS ROESEL
BRUCE SAMS

SCHENECTADY

Organized: As a Branch, 1919; as a Section, 1927
Territory: Radius of thirty miles from Schenectady, N.Y.
Place of Meeting: Rice Hall
Number of Members: 172

EXECUTIVE COMMITTEE

A. L. RUIZ, *Chairman*
S. L. JAMESON, *Vice-Chairman*
E. E. PARKER, *Vice-Chairman*
A. R. STEVENSON, JR., *Vice-Chairman*
R. H. NORRIS, *Secretary*
CARL SCHABTACH, *Treasurer*
J. L. FLETCHER
H. M. OTTO
O. L. WOOD, JR.

SOUTH TEXAS

Organized: 1919
Territory: South Texas and the northern part of the State not included in the North Texas Section territory
Place of Meeting: Electric Bldg., Houston, Tex.
Number of Members: 171

EXECUTIVE COMMITTEE

J. J. KING, *Chairman*
C. W. CRAWFORD, *Vice-Chairman*
H. F. MOLLER, *Secretary-Treasurer*
HERBERT ALLEN
D. D. ALTON
J. W. BERETTA
A. H. BURR
H. E. DEGLER
JOHN DOGGETT, JR.
H. G. HIEBELER
F. M. LEVERETT
J. S. NETHERWOOD
C. L. ORR

SUSQUEHANNA

Organized: 1927
Territory: Counties of Cumberland, Dauphine, Lebanon, Adams, York, and Lancaster
Place of Meeting: Engineering Society of York, and at Lancaster Twice a Year
Local Organization: Engineering Society of York and Engineers' Society of Pennsylvania, Harrisburg, Pa.
Number of Members: 79

EXECUTIVE COMMITTEE

W. E. BELINE, *Chairman*
O. E. WEBER, *Vice-Chairman*
E. T. P. NEUBAUER, *Secretary*
E. E. AUGHENBAUCH
T. K. BREDA
M. G. LEESON
H. B. MARTIN
W. NOYES
GERALD SMITH
S. P. SOLING

SYRACUSE

Organized: 1920
Territory: Radius of thirty miles from Syracuse, N.Y.
Place of meeting: Ball Room of the Onondaga Hotel
Local Organization: The Technology Club of Syracuse
Number of Members: 81

EXECUTIVE COMMITTEE

MORRIS F. WILLIAMS, *Chairman*
R. C. PAUL, *Vice-Chairman*
E. KEITH RHODES, *Secretary-Treasurer*
H. B. CRAIG
C. F. DIETZ
E. A. FAILMEZGER
W. F. JONES

TOLEDO

Organized: 1920
Territory: Radius of thirty miles from Toledo, Ohio
Place of Meeting: University Club, Toledo, Ohio
Local Organization: Affiliated Technical Societies of Toledo
Number of Members: 62

EXECUTIVE COMMITTEE

R. F. HILL, *Chairman*
H. O. KRANICH, *Vice-Chairman*

TOLEDO

(Continued)

N. R. Ohler, *Secretary-Treasurer*
C. A. Alexander
J. W. Dean
Carl Frautschi
John Gillett
H. E. Happel
R. J. Mugfor
D. M. Palmer
W. Schroeder
H. R. Schultz
H. H. Vogel
E. W. Weaver
I. F. Zarobsky

TRI-CITIES

Organized: 1920
Territory: Radius of thirty miles from Moline, Ill.
Place of Meeting: Rock Island, Ill., Moline, Ill., and Davenport, Iowa
Luncheon Meeting every Wednesday, Davenport Hotel, 12:00 noon
Number of Members: 61

Executive Committee

R. A. Cross, *Chairman*
H. A. Kleinman, *Vice-Chairman*
C. A. Carlson, *Secretary-Treasurer*
P. E. Anderson
R. M. Barnes
J. M. Hartman

UTAH

Organized: 1923
Territory: State of Utah
Place of Meeting: University Club, Salt Lake City
Local Organization: Utah Society of Engineers
Number of Members: 40

Executive Committee

H. M. Ferguson, *Chairman*
W. D. Turpin, *Vice-Chairman*
G. M. Jones, *Secretary*
R. D. Baker
W. H. Trask, Jr.

VIRGINIA

Organized: 1919
Territory: State of Virginia
Place of Meeting: Richmond, Norfolk, Charlottesville, Roanoke, University, Petersburg
Local Organization: Central Virginia Engineers Club
Number of Members: 142

Executive Committee

G. L. Bascome, *Chairman*
G. C. Molleson, *Vice-Chairman*
J. B. Jones, *Secretary*
F. S. Roop, Jr., *Treasurer*
E. B. Boynton
D. F. Evans
L. R. Gardner
D. G. Moorhead
W. E. Segl

WASHINGTON, D.C.

Organized: 1919
Territory: District of Columbia
Place of Meeting: Auditorium, Potomac Electric Power Co., 10th & E Sts., Washington, D.C.
Number of Members: 234

Executive Committee

H. G. Thielscher, *Chairman*
G. F. Jenks, *Vice-Chairman*
C. E. Miller, *Secretary-Treasurer*
A. T. Dupont
W. B. Ensinger
A. F. Johnson
Paul Weeks

WATERBURY

Organized: 1917, as a Branch; reorganized as a Section, 1923
Territory: Litchfield County and a portion of New Haven County
Place of Meeting: Elton Hotel
Number of Members: 73

Executive Committee

W. C. Schneider, *Chairman*
L. J. Niekirk, *Vice-Chairman*
R. B. Bass, *Secretary-Treasurer*
C. W. Childs
A. J. German
R. C. Perry

Junior Group

W. B. McKenzie, Jr., *Chairman*
R. W. Simpson, Jr., *Secretary-Treasurer*
H. C. Ashley
G. N. Tyack

WESTERN MASSACHUSETTS

Organized: 1922
Territory: Includes counties of Berkshire, Franklin, Hampden, and Hampshire
Place of Meeting: Highland Hotel, Springfield, Mass.
Local Organization: Engineering Society of Western Massachusetts
Number of Members: 87

Executive Committee

J. L. Scherner, *Chairman*
A. H. Lewis, *Vice-Chairman*
L. G. Carlton, *Secretary-Treasurer*
A. E. Benson
Lester Campbell
F. R. Shoemaker

WESTERN WASHINGTON

Organized: 1919
Territory: State of Washington west of Columbia River with exception of territory included in 30-mile radius of Portland, Ore.
Place of Meeting: Engineers' Club, Seattle, Wash.
Local Organization: Seattle Engineers' Club
Luncheon Meetings daily at noon at Engineers' Club, Seattle
Number of Members: 101

Executive Committee

H. J. McIntyre, *Chairman*
Eddy Floodeen, *Vice-Chairman*
R. E. Walter, *Secretary-Treasurer*
H. L. Adams
H. P. Ford
B. T. McMinn
C. J. Yost

WEST VIRGINIA

Organized: 1925
Territory: State of West Virginia, South of Parallel 39
Place of Meeting: Charleston, W.Va.
Number of Members: 60

Executive Committee

C. L. Johnson, *Chairman*
M. S. Bloomsburg, *Vice-Chairman*
H. B. Hickman, *Secretary-Treasurer*
B. B. Betty
H. L. Carspecken, Jr.
E. L. Hudson
F. LeRoy Schaefer
H. C. Skaggs, Jr.

WORCESTER

Organized: 1915
Territory: Radius of thirty miles from Worcester, Mass.
Place of Meeting: Sanford Riley Hall, Worcester Poly. Inst.
Local Organization: Worcester Engineering Society
Number of Members: 125

Executive Committee

F. R. Jones, *Chairman*
R. P. Kolb, *Vice-Chairman*
R. H. Wood, *Secretary-Treasurer*
E. K. Allen, Jr.
E. N. Armstrong
H. P. Crane
R. F. Gow
C. M. McMahon
Otto Muller
E. H. Smith

Junior Group

W. M. Wilcox, *Chairman*
Wilfred Ogg, *Vice-Chairman*
W. H. Davenport, *Secretary-Treasurer*
W. F. Beth
J. E. Guild
Thos. Hill
Geo. Palmenter
Irwin Peterson
W. S. Snow
N. G. Stradtherr

YOUNGSTOWN

Organized: 1928
Territory: Counties of Trumbull, Mahoning, and Columbiana in Ohio, and Mercer and Lawrence in Pennsylvania
Place of Meeting: Republic Rubber Co. Club Rooms, Albert St., Youngstown, Ohio
Number of Members: 58

Executive Committee

H. W. Smith, *Chairman*
L. A. Kline, *Vice-Chairman*
C. W. Foard, *Secretary-Treasurer*
F. J. Bowers
W. B. Jenkins
H. E. Melin
E. O. Oyen

STUDENT BRANCHES

ARTICLE B6A, PAR. 20: The Standing Committee on Relations With Colleges shall, under the direction of the Council, have supervision of the Student Branches of the Society and of such work of the Society as aims to further the education of engineers through the colleges and schools of accepted standing.

STANDING COMMITTEE, RELATIONS WITH COLLEGES

H. O. CROFT, *Chairman* (1940)
E. W. O'BRIEN (1941)
A. C. CHICK (1942)
J. D. CUNNINGHAM (1943)
P. B. EATON (1944)

H. E. DEGLER
S. F. DUNCAN
E. O. EASTWOOD
F. H. PROUTY
G. L. SULLIVAN
Advisory Members (1940)

J. I. YELLOTT, *Junior Adviser* (1940)

Communicate with Student Branch through Honorary Chairman

Name and Location	Year Authorized	No. of Members †	Chairman	Secretary	Honorary Chairman
Akron, Univ. of, Akron, Ohio	1924	34	HERBERT DEIST	CHARLES CEHRS	F. S. GRIFFIN
Alabama Polytechnic Inst., Auburn, Ala.	1920	49	F. W. CAYCE	T. R. LODER, JR.	C. R. HIXON
Alabama, Univ. of, University, Ala.	1931	31	J. A. LOMBARD	LEONARD MANDELL	J. M. GALLALEE
Arizona, Univ. of, Tucson, Ariz.	1937	21	ANGUS MCVICAR	JOHN KERR	M. L. THORNBURG
Arkansas, Univ. of, Fayetteville, Ark.	1910	18	J. P. JOHNSON	JOSEPH MURRAY	L. C. PRICE
Armour Inst. of Tech., Chicago, Ill.	1909	167	R. W. SCOTT	JOHN CATLIN	DANIEL ROESCH
British Columbia, Univ. of, Vancouver, B.C., Can.	1938	34	R. T. BOGLE	H. J. MORRIS	F. W. VERNON
Brown Univ., Providence, R.I.	1923	17	G. R. KELLER	R. H. STARRETT	S. J. BERARD
Bucknell Univ., Lewisburg, Pa.	1916	12	JOSEPH LINK	SAMUEL FISHER	G. M. KUNKEL
California Inst. of Tech., Pasadena, Calif.	1914	60	G. W. HOFELLER	NEWELL PARTCH	R. L. DAUGHERTY
California, Univ. of, Berkeley, Calif.	1912	129	R. A. SLIPPER	W. M. GAYMAN	G. F. GARLAND
Carnegie Inst. of Tech., Pittsburgh, Pa.	1913	52	W. L. STOVER	G. A. CONNOR	D. C. SAYLOR
Case School of Applied Science, Cleveland, Ohio	1913	89	J. F. LAWLER	W. T. GLOOR	F. H. VOSE
Catholic Univ. of America, Washington, D.C.	1922	30	C. E. CHANEY	R. J. DECESARE	M. W. WESCHLER
Cincinnati, Univ. of, Cincinnati, Ohio	1909	59	GUY ODOM	JAMES TARKINGTON	R. L. SMITH
Clarkson College of Tech., Potsdam, N.Y.	1930	37	R. D. WALSER	W. C. NELLIS	F. A. CLARY
Clemson A.&M. College, Clemson College, S.C.	1921	39	F. C. MILLS	M. W. ACKERMAN	J. H. SAMS
Colorado State College of A.&M. Arts, Fort Collins, Colo.	1914	26	LAWRENCE WILKENS	WILLIAM SUTHERLAND	J. T. STRATE
Colorado, Univ. of, Boulder, Colo.	1914	31	LAWRENCE BURT	ROBERT LAUTH	W. S. BEATTIE
Colorado School of Mines Division, Golden		21	LINCOLN ELKINS	F. R. FISHER	J. C. REED
Columbia University, New York, N.Y.					
Management Division	1909	12			FRED DUTCHER *WALTER RAUTENSTRAUCH
Mechanical Division	1909	24	R. M. TAYLOR	G. W. PLOSSL	FRED DUTCHER
Cooper Union, New York, N.Y.	1920				
Day Division		38	WILTON BREITMAN	ARTHUR SVENSON	E. A. SALMA
Evening Division		80	P. E. HAGERTY	E. G. LUCKEN	E. A. SALMA
Cornell Univ., Ithaca, N.Y.	1908	79	BEACH BARRETT	J. E. BILLINGS	P. H. BLACK
Delaware, Univ. of, Newark, Del.	1929	44	EDWARD SCHWARTZ	W. H. DAWSON	W. F. LINDELL
Detroit, Univ. of, Detroit, Mich.	1930	63	A. J. HOFWEBER, JR.	V. D. POLHLMEYER	F. J. LINSENMEYER
Drexel Inst. of Tech., Philadelphia, Pa.	1920	44	W. W. CATHCART	D. E. WIGFIELD	W. J. STEVENS
Duke Univ., Durham, N.C.	1935	23	H. O. SCHMIDT	D. E. WOOLLARD	F. J. REED
Florida, Univ. of, Gainesville, Fla.	1926	23	C. H. STANTON	A. D. JONES	R. A. THOMPSON
George Washington Univ., Washington, D.C.	1924	37	J. A. HARTMAN	J. C. GOFF	A. F. JOHNSON
Georgia School of Tech., Atlanta, Ga.	1915	48	R. J. WOODDALL	W. B. ASHBY	R. S. HOWELL
Idaho, Univ. of, Moscow, Idaho	1925	38	BARRY MERRILL	EARL JELTER	H. F. GAUSS
Illinois, Univ. of, Urbana, Ill.	1909	166	R. W. ROOSE	K. L. CARTER	D. G. RYAN
Iowa State College, Ames, Iowa	1919	44	EDWARD OTT	JOSEPH BROGAN	R. E. ROUDEBUSH
Iowa, State Univ. of, Iowa City, Iowa	1913	37	J. P. MAY	H. L. YAKISH	H. L. GODEKE
Johns Hopkins Univ., Baltimore, Md.	1917	46	CHARLES FLAGLE	C. D. MONTGOMERY	K. P. HANSON
Kansas State College, Manhattan, Kan.	1914	60	ROBERT PLYE	HARDEN TUBBS	B. B. BRAINARD
Kansas, Univ. of, Lawrence, Kan.	1909	20	C. H. MCCABE	J. W. FOGWELL	R. S. TAIT
Kentucky, Univ. of, Lexington, Ky.	1911	24	T. W. COZINE	THOMAS JACKSON	C. C. JETT
Lafayette College, Easton, Pa.	1919	35	ROBERT CORBY	J. E. HAMMERSTONE	P. B. EATON
Lehigh Univ., Bethlehem, Pa.	1911	27	J. F. MCQUILLIN	E. R. SNOVEL	T. E. JACKSON
Lewis Inst., Chicago, Ill.	1933	52	E. J. BURROWES	HENRY DEA	J. S. KOZACKA
Louisiana State Univ., University, La.	1916	56	S. A. BROWNING	W. J. SPERIER	G. F. MATTHES
Louisville, Univ. of, Louisville, Ky.	1928	20	C. A. EVANS	B. E. MCKERCHER	L. L. AMIDON
Maine, Univ. of, Orono, Maine	1910	48	D. H. CARR	F. S. MARTIN	I. H. PRAGEMAN
Marquette Univ., Milwaukee, Wis.	1923	26	EUGENE THOMA	SEBASTIAN JUDD	R. J. SMITH
Maryland, Univ. of, College Park, Md.	1937	48	F. C. MORRIS	T. G. TIMBERLANE	J. W. HUCKERT
Massachusetts Inst. of Tech., Cambridge, Mass.	1909	114	JOHN MCMULLIN, III	W. R. SCHULER	ALVIN SLOANE
Michigan College of Min. & Tech., Houghton	1930	65	PAUL MARTIN	CLYDE TURNER	A. P. YOUNG
Michigan State College, E. Lansing, Mich.	1917	64	G. R. GRENZKE	R. J. MARTIN	C. N. RIX
Michigan, Univ. of, Ann Arbor, Mich.	1914	81	R. S. JONES	R. H. JEFFERS	E. T. VINCENT
Minnesota, Univ. of, Minneapolis, Minn.	1913	142	B. L. ERTSGAARD	L. L. MILLER	C. A. KOEPKE

* Faculty Adviser. † As of January 1, 1940.

Name and Location	Year Authorized	No. of Members	Chairman	Secretary	Honorary Chairman
Mississippi State College, State College, Miss.	1926	61	STANLEY LIVINGSTON	HERMAN BOPP	H. P. NEAL
Missouri School of Mines & Metallurgy, Rolla, Mo.	1930	52	VERNON RIEKE	LYNN RIEGE	A. J. MILES
Missouri, Univ. of, Columbia, Mo.	1909	21	IRWIN TROWBRIDGE	H. E. KETTER	R. L. SCORAH
Montana State College, Bozeman, Mont.	1920	33	D. H. MIKKELSON	JAMES COPENHAVER	R. T. CHALLENDER
Nebraska, Univ. of, Lincoln, Neb.	1909	62	GAIUS CADWELL	B. R. SCHLUCKEBIER	N. H. BARNARD
Nevada, Univ. of, Reno, Nev.	1923	19	M. M. TILLEY	HERMAN KONNERTH	W. H. DAVIDSON
Newark College of Engineering, Newark, N.J.	1924	110	G. L. BREUR	R. P. JOHNSON	F. J. BURNS
New Hampshire, Univ. of, Durham, N.H.	1926	34	A. L. HARDY	H. S. BUTTERWORTH	E. T. DONOVAN
New Mexico State College of A.&M. Arts, State College, New Mex.	1938	16	JOHN VALKENAAR	E. B. PACK	M. T. LEWELLEN
New Mexico, Univ. of, Albuquerque, New Mex.	1935	17	C. W. MERRITT	PHILLIP HOOD	A. D. FORD
New York, College of the City of, New York, N.Y.	1922	53	WILLIAM SCHEINGOLD	CARL MARQUIT	D. A. FISHER
New York University, New York, N.Y.					
Aeronautic Division	1917	26	B. S. MEAD, JR.	HENRY BYCH	J. M. LABBERTON *F. K. TEICHMANN
Mechanical Division		48	EDWIN BLUESTONE	J. J. FOGEL	J. M. LABBERTON
New York Univ. Evening School, New York, N. Y.	1933	39	W. A. SHAW	F. W. LYNCH, JR.	J. M. LABBERTON
North Carolina State College, Raleigh, N.C.	1920	64	M. M. DALTON	J. H. KENNEDY	F. F. GROSECLOSE
North Dakota Agricultural College, Fargo, N.D.	1929	18	ARTHUR HEWITT	NORMAN GEHLHAR	A. W. ANDERSON
North Dakota, Univ. of, Grand Forks, N.D.	1923	15	F. L. SVORE	STANLEY VOAK	A. J. DIAKOFF
Northeastern Univ., Boston, Mass.	1922	95	T. C. MCDONOUGH	R. L. COREY	A. J. FERRETTI
Northwestern Univ., Evanston, Ill.	1935	16	F. P. LOCKARD	FLOYD JACOBSEN	EDWARD OBERT
Notre Dame, Univ. of, Notre Dame, Ind.	1929	37	J. J. LINNEHAN	R. W. WITCHGER	C. C. WILCOX
Ohio Northern Univ., Ada, Ohio	1922	10	R. W. ROTH	ROBERT BOWDEN	J. A. NEEDY
Ohio State Univ., Columbus, Ohio	1911	53	J. M. HELDACK	S. F. WHITEHEAD	PAUL BUCHER
Oklahoma A.&M. College, Stillwater, Okla.	1921	12	JOHN WILLIAMSON, JR.	GERTRUDE HILL	V. W. YOUNG
Oklahoma, Univ. of, Norman, Okla.	1917	83	W. F. FORD	JACK LUCAS	E. M. SIMS
Oregon State Agricultural College, Corvallis, Ore.	1909	33	R. P. LANKOW	C. E. ELSTON	J. C. OTHUS
Pennsylvania State College, State College, Pa.	1909	67	NORMAN YOUNG	ROBERT GUNDEL	C. M. ALLEN
Pennsylvania, Univ. of, Philadelphia, Pa.	1925	46	SAMUEL GRIFFIN	J. R. LYNCH	L. N. GULICK
Pittsburgh, Univ. of, Pittsburgh, Pa.	1917	44	P. F. GIBSON	FRANK KRISTL	I. A. GRADISAR
Polytechnic Inst. of Brooklyn, Brooklyn, N.Y.					
Day Division	1909	50	WILLIAM ABOKAIR	JACK BABAKIAN	F. W. MING
Evening Division		47	M. R. NICASTRO	W. A. CUSHIE	F. W. MING
Pratt Inst., Brooklyn, N.Y.	1923	72	LEE MAHER	VERNON CLARK	J. W. HUNTER
Princeton Univ., Princeton, N.J.	1926	39	R. S. LOVELACE	F. I. WALSH	A. M. GREENE, JR.
Puerto Rico, Univ. of, Mayaguez, P.R.	1923	31	J. D. MARTINEZ	PEDRO HILERA	GUILLERMO DEYA
Purdue, Univ. of, W. Lafayette, Ind.	1909	152	J. K. BRYAN	T. P. PEPPLER	W. J. COPE
Rensselaer Polytechnic Inst., Troy, N.Y.	1910	75	E. E. HENRY	R. C. JONES	G. K. PALSGROVE
Rhode Island State College, Kingston, R.I.	1930	52	E. A. JOHNSON	W. C. SMITH	E. L. CARPENTER
Rice Inst., Houston, Tex.	1926	24	S. E. HAGGARD	HARDY JACKSON	C. A. HALL
Rose Polytechnic Inst., Terre Haute, Ind.	1926	32	N. G. EDER	FREDERICK THODAL	CARL WISCHMEYER
Rutgers Univ., New Brunswick, N.J.	1920	33	R. L. LARRABURE	A. M. LIPSKY	N. P. BAILEY
Santa Clara, Univ. of, Santa Clara, Calif.	1925	17	W. T. BOX	F. M. BOOTH	R. A. SEBAN
South Dakota State College, Brookings, S.D.	1935	23	EARL BOWAR	ROGER ST. JOHN	R. E. GIBBS
Southern California, Univ. of, Los Angeles, Calif.	1929	33	R. MCS. BRIDGES	J. O. SUFFRON	T. T. EYRE
Southern Methodist Univ., Dallas, Tex.	1933	19	RAYMOND HOLLAND	CHARLES GRIMES	R. M. MATSON
Stanford Univ., Stanford University, Calif.	1909	44	H. P. MOOIJ	F. T. COLLINS	M. S. HUGO
Stevens Inst. of Tech., Hoboken, N.J.	1908	72	M. G. BELINE	STEVEN DANATOS	WILLIAM HALLIDAY
Swarthmore College, Swarthmore, Pa.	1921	22	A. D. COX, JR.	M. G. SMITH	C. G. THATCHER
Syracuse Univ., Syracuse, N.Y.	1912	28	W. N. COLE	WILLIAM NELSON	A. R. ACHESON
Tennessee, Univ. of, Knoxville, Tenn.	1923	52	J. T. EDWARDS	V. A. HINES	R. W. MORTON
Texas, A.&M. College of, College Station, Tex.	1921	160	B. H. BURNS	J. J. WALKER	C. W. CRAWFORD
Texas Technological College, Lubbock, Tex.	1930	34	R. W. HURN	T. C. NOWELL	H. F. GODEKE
Texas, Univ. of, Austin, Tex.	1921	58	J. E. KAINER	W. R. WOOLRICH	V. L. DOUGHTIE
Toronto, Univ. of, Toronto, Ont., Can.	1933	49	J. M. VANWINCKLE	W. F. WOODLEY	R. C. WIREN
Tufts College, Tufts College, Mass.	1917	36	R. W. EDWARDS	G. H. SAWTELL	EDGAR MACNAUGHTON
Tulane Univ. of Louisiana, New Orleans, La.	1933	28	E. L. MURPHY	R. T. BASNETT	E. R. STEPHAN
U. S. Naval Academy, Post-Graduate School, Annapolis, Md.	1925				P. J. KIEFER
Utah, Univ. of, Salt Lake City, Utah	1923	30	GRANT BAGLEY	A. C. MCMASTER	R. D. BAKER
Vanderbilt Univ., Nashville, Tenn.	1928	29	J. M. WRIGHT	C. D. FULTON	J. E. BOYNTON
Vermont, Univ. of, Burlington, Vt.	1922	15	F. A. JOHNSON	F. S. HEATH	E. L. SUSSDORFF
Villanova College, Villanova, Pa.	1925	15	J. A. HECK	CASIMIR KUREK	W. J. BARBER
Virginia Polytechnic Inst., Blacksburg, Va.	1915	72	P. D. DALE	G. G. VIA	F. S. ROOP
Virginia, Univ. of, University, Va.	1923	20	T. G. HOBBS, JR.	C. E. BATCHELDER, JR.	H. C. HESSE
Washington, State College of, Pullman, Wash.	1930	28	J. S. MILLER	D. R. ROSS	F. W. CANDEE
Washington, Univ. of, Seattle, Wash.	1917	62	OTIS LAMSON	R. S. LAING	R. W. CRAIN
Washington Univ., St. Louis, Mo.	1911	34	EDWIN MATHAE	DOUGLAS OWENS	HERBERT KUENZEL
West Virginia Univ., Morgantown, W. Va.	1922	25	R. L. QUEEN	C. J. SNYDER, JR.	H. M. CATHER
Wisconsin, Univ. of, Madison, Wis.	1909	76	R. H. BENNEWITZ	R. E. WRIGHT	B. G. ELLIOTT
Worcester Polytechnic Inst., Worcester, Mass.	1914	41	K. R. BLAISDELL	E. M. BATES, JR.	E. W. ARMSTRONG
Wyoming, Univ. of, Laramie, Wyo.	1925	28	L. C. CLAUSEN	M. H. STEFFENSEN	R. S. SINK
Yale Univ., New Haven, Conn.	1910	34	C. T. ROELKE	DARRAH CORBET, JR.	S. W. DUDLEY

* Faculty Adviser.

RESEARCH TECHNICAL COMMITTEES

ARTICLE B6A, PAR. 24: The Standing Committee on Research shall advise the Council on the research work of the Society.

The first Standing Committee on Research was organized in 1909.

STANDING COMMITTEE

L. W. WALLACE, *Chairman* (1940)
E. G. BAILEY (1941)
W. TRINKS (1942)
M. D. HERSEY (1943)
J. H. WALKER (1944)

LUBRICATION

Appointed October, 1915, to investigate the fundamental problems of lubrication, to formulate results of investigations previously made, and to keep in touch with contemporary research in this field

(Reorganized May, 1936)

G. B. KARELITZ, *Chairman*
S. J. NEEDS, *Secretary*
A. L. BEALL
OSCAR BRIDGEMAN
W. E. CAMPBELL
H. P. DAHLSTRAND
H. A. EVERETT
A. E. FLOWERS
J. C. GENIESSE
RAYMOND HASKELL
M. D. HERSEY
B. F. HUNTER
C. M. LARSON
F. C. LINN
G. L. NEELY
B. L. NEWKIRK
A. E. NORTON
E. S. PEARCE
ERNEST WOOLER

FLUID METERS

Appointed 1916 to develop the theory of fluid meters of all kinds and to report on the best methods for their installation and use

(Reorganized July, 1926)

R. J. S. PIGOTT, *Chairman*
J. R. CARLTON, *Secretary*
H. S. BEAN
S. R. BEITLER
E. O. BENNETT
R. K. BLANCHARD
B. O. BUCKLAND
LOUIS GESS
A. J. KERR
T. H. KERR
M. P. O'BRIEN
W. S. PARDOE
L. K. SPINK
R. E. SPRENKLE
E. C. M. STAHL
T. R. WEYMOUTH
M. J. ZUCROW

THERMAL PROPERTIES OF STEAM

Appointed in December, 1921, to direct research on the thermal properties of water-vapor and steam from 0 C to the upper limits of temperature and pressure

(Reorganized April, 1929)

W. L. ABBOTT, *Vice-Chairman*
H. N. DAVIS
H. C. DICKINSON
A. M. GREENE, JR.
R. C. H. HECK
D. S. JACOBUS
MAX JAKOB
J. H. KEENAN
F. G. KEYES
L. S. MARKS
G. A. ORROK
R. J. S. PIGOTT
H. V. RASMUSSEN
E. L. ROBINSON

STRENGTH OF GEAR TEETH

Appointed in December, 1921. Is investigating factors affecting the strength and life of gear teeth

R. E. FLANDERS, *Chairman*
C. H. LOGUE, *Secretary*
C. G. BARTH
EARLE BUCKINGHAM
A. M. GREENE, JR.
C. W. HAM
F. E. MCMULLEN
E. W. MILLER
ERNEST WILDHABER

CUTTING OF METALS

Appointed in September, 1923. Is studying the problems of metal cutting, including tool materials, tool design, lubrication, cooling, and speeds and fuels

M. F. JUDKINS, *Chairman*
L. N. GULICK, *Secretary*
L. P. ALFORD
O. W. BOSTON
R. C. DEALE
A. L. DELEEUW
C. M. THOMPSON, JR.

MECHANICAL SPRINGS

Appointed May, 1924, to determine the status of the mechanical-spring art, to promote and conduct necessary and adequate research, and to develop the art to the point of standardization

J. R. TOWNSEND, *Chairman*
C. T. EDGERTON, *Secretary*
C. E. BARBA
W. G. BROMBACHER
R. W. COOK
W. T. DONKIN
RUPEN EKSERGIAN
G. E. HANSEN
BENJAMIN LIEBOWITZ
DAVID LOFTS
(R. D. BRIZZOLARA, *Alternate*)
D. J. MCADAM, JR.
R. E. PETERSON
J. B. REYNOLDS
J. W. ROCKEFELLER, JR.
B. W. ST. CLAIR
M. F. SAYRE
T. R. WEBER
KEITH WILLIAMS
J. K. WOOD
F. P. ZIMMERLI
O. B. ZIMMERMAN

ELEVATORS

Appointed June, 1924, as a subcommittee of the Sectional Committee on Safety Code for Elevators, to study the function and operation of elevator safeties and buffers and their associated mechanisms and to develop methods of test for the approval of elevator safety devices

D. J. PURINTON, *Chairman*
O. P. CUMMINGS, *Vice-Chairman*
J. A. DICKINSON, *Secretary*
BASSETT JONES
D. L. LINDQUIST
M. G. LLOYD
J. J. MATSON
M. B. MCLAUTHLIN
W. S. PAINE

EFFECT OF TEMPERATURE ON THE PROPERTIES OF METALS

Appointed December, 1924, as a joint research committee of the A.S.T.M. and the A.S.M.E. to encourage the investigation and accumulation of data on the properties of metals used in the mechanic arts at extremely high and low temperatures

N. L. MOCHEL, *Chairman*
H. J. KERR, *Vice-Chairman*
J. W. BOLTON, *Secretary*
R. H. ABORN
W. H. ARMACOST
A. B. BAGSAR
A. D. BAILEY
F. E. BASH
E. S. DIXON
F. B. FOLEY
J. R. FREEMAN, JR.
H. J. FRENCH
H. W. GILLETT
A. J. HERZIG
G. F. JENKS
J. J. KANTER
C. E. MACQUIGG
P. E. MCKINNEY
E. L. ROBINSON
A. E. WHITE
Director, National Bureau of Standards, U.S. Department of Commerce
Representative of Bureau of Engineering, U.S. Navy Department

BOILER FEEDWATER STUDIES

Appointed March, 1925, as a Joint Research Committee of the American Boiler Manufacturers Association, American Railway Engineering Association, American Water Works Association, Edison Electric Institute, the American Society for Testing Materials, and the A.S.M.E. to study methods of analysis and treatment of boiler feedwater for stationary and railroad practice

BOILER FEEDWATER STUDIES

(Continued)

EXECUTIVE COMMITTEE (Total personnel 41)

C. H. FELLOWS, *Chairman*
R. C. BARDWELL, *Vice-Chairman*
J. B. ROMER, *Secretary*
A. G. CHRISTIE *
R. E. COUGHLIN
B. W. DE GEER
MAX HECHT
H. E. JORDAN
W. F. KEENAN, JR.
P. B. PLACE
S. T. POWELL
F. N. SPELLER
M. F. STACK
E. H. TENNEY
A. E. WHITE *

CONDENSER TUBES

Appointed May, 1925, to investigate and report on the causes of failure of tubes used in steam condensers and similar heat interchange apparatus

A. E. WHITE, *Chairman*
D. C. WEEKS, *Vice-Chairman*
P. A. BANCELL
D. K. CRAMPTON
H. M. CUSHING
R. E. DILLON
O. B. J. FRASER
J. R. FREEMAN, JR.
V. M. FROST
C. F. HARWOOD
G. C. HOLDER
O. Z. KLOPSCH
H. W. LEITCH
E. F. MILLER
W. B. PRICE
M. F. STACK
H. A. STAPLES
W. R. WEBSTER
Director, Bureau of Engineering, U.S. Navy Department

BOILER FURNACE REFRACTORIES

Appointed June, 1925, to determine the principal factors governing the failure of refractories in various types of installations, to subject these factors to detailed experimental analysis, to undertake the formulation of suitability tests and, if necessary, to attempt to develop a suitable refractory to meet the needs of severe service

W. A. CARTER, *Chairman*
S. H. BARNUM
G. A. BOLE
M. C. BOOZE
W. H. FULWEILER
C. B. GRADY
R. A. HEINDL
R. K. HURSH
N. E. LEWIS
J. S. McDOWELL
(F. A. HARVEY, *Alternate*)
S. J. McDOWELL
PERCY NICHOLLS
S. M. PHELPS
E. B. POWELL
R. A. SHERMAN
R. B. SOSMAN
L. J. TROSTEL
G. B. WILKES

* Official A.S.M.E. representatives serving on this committee.

WORM GEARS

Appointed May, 1927, to investigate certain problems in connection with the action of worm gear drives and to recommend improvements in their design, manufacture, and use

EARLE BUCKINGHAM, *Chairman*
G. H. ACKER
L. R. BUCKENDALE
W. H. HIMES
D. L. LINDQUIST
A. A. ROSS
B. F. WATERMAN
Representative of Bureau of Engineering, U.S. Navy Department

MEASURES OF MANAGEMENT

Appointed March, 1928, to attempt the reconciliation of certain economic laws affecting production, to develop formulas for management, and to collect and report information on management research

W. E. FREELAND, *Chairman*
F. E. RAYMOND, *Secretary*
J. H. BARBER
T. H. BROWN
R. C. DAVIS
G. E. HAGEMANN

ABSORPTION OF RADIANT HEAT IN BOILER FURNACES

Appointed April, 1928, to make a study of the absorption of radiant heat in boiler furnaces with the purpose of developing recommendations on improved furnace design

W. J. WOHLENBERG, *Chairman*
E. G. BAILEY
R. M. GATES
C. W. GORDON
E. L. LINDSETH
G. A. ORROK
R. J. S. PIGOTT
JOHN VAN BRUNT

STRENGTH OF VESSELS UNDER EXTERNAL PRESSURE

Appointed June, 1929, to develop reliable design data on the strength of cylindrical and spherical surfaces under external pressure, particularly with reference to jacketed vessels

W. D. HALSEY, *Chairman*
F. V. HARTMAN
M. B. HIGGINS
A. W. LIMONT, JR.
H. E. SAUNDERS
E. E. SHANOR
D. B. WESSTROM
F. S. G. WILLIAMS
D. F. WINDENBURG

WIRE ROPE

Appointed April, 1930, to investigate existing rope so that it may be better understood and more effectively used

W. H. FULWEILER, *Chairman*
H. LeR. BRINK
D. L. LINDQUIST
G. W. MARTIN
C. A. McCUNE
A. H. McDOUGALL
B. V. E. NORDBERG, JR.
W. S. PAINE
W. J. RYAN
GEORGE SIMPSON
L. E. YOUNG

CRITICAL PRESSURE STEAM BOILERS

Appointed June, 1931, to study the characteristics of high-pressure forced-circulation steam-generating units

H. L. SOLBERG, *Chairman*
W. H. ARMACOST
A. D. BAILEY
E. G. BAILEY
F. S. CLARK
C. H. FELLOWS
H. J. KERR
M. W. LINK
G. A. ORROK
E. L. ROBINSON
P. W. THOMPSON

COTTONSEED PROCESSING

Appointed December, 1932, to study the mechanical problems involved in storing, conditioning, and cooking cottonseed meats

W. R. WOOLRICH, *Chairman*
HOMER BARNES
E. L. CARPENTER
C. E. GARNER
B. J. SAMS
R. B. TAYLOR

ROLLING OF STEEL (PLASTICITY)

Appointed October, 1938, to study plasticity in the particular field of rolling of steel

W. TRINKS, *Chairman*
E. C. BAIN
C. L. EKSERGIAN
J. H. HITCHCOCK
G. B. KARELITZ
C. W. MacGREGOR
A. NADAI
MORRIS STONE

A.S.M.E. Representatives on Other Research Committees

See also A.S.M.E. Representatives on Other Activities, page RI-5

AMERICAN COORDINATING COMMITTEE ON CORROSION

American Society for Testing Materials

S. L. KERR
(C. H. FELLOWS, *Alternate*)

CORROSION COMMITTEE

American Society of Refrigerating Engineers

(To be appointed)

FATIGUE PHENOMENA OF METALS

American Society for Testing Materials

C. T. EDGERTON

HEAT-TREATMENT OF ROCK DRILL STEELS

Advisory Board of the National Bureau of Standards and Bureau of Mines

(To be appointed)

METALLURGICAL RESEARCH

Advisory Committee to the National Bureau of Standards

C. H. BIERBAUM

PROPERTIES OF REFRACTORY MATERIALS

Advisory Committee to the National Bureau of Standards

E. B. POWELL

WATER FOR INDUSTRIAL USES

American Society for Testing Materials

J. H. WALKER

STANDARDIZATION TECHNICAL COMMITTEES

ARTICLE B6A, PAR. 23: The Standing Committee on Standardization shall advise the Council on the dimensional standardization work of the Society, including relations with the American Standards Association.

The first Standing Committee on Standardization was organized in April, 1911.

STANDING COMMITTEE

W. C. MUELLER, *Chairman* (1940)
A. L. BAKER (1941)
J. E. LOVELY (1942)
J. R. WEAVER (1943)
T. E. FRENCH (1944)

STANDARDIZATION AND UNIFICATION OF SCREW THREADS (B1)

* *Joint sponsorship with the Society of Automotive Engineers. Sectional Committee originally organized in June, 1921. Reorganized in February, 1929*

A.S.M.E. Members (Total personnel, 35)

R. E. FLANDERS, *Chairman* †
EARLE BUCKINGHAM, *Secretary*
E. J. BRYANT
G. S. CASE
T. G. CRAWFORD
A. M. HOUSER †
H. C. E. MEYER
P. V. MILLER †
W. C. MUELLER
R. H. PERRY
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMEN

No. 1 on Scope, Arrangement, and Editing of American National Standard, R. E. FLANDERS
No. 2 on Terminology and Thread Specifications, Except Gages, C. W. BETTCHER
No. 3 on Special Threads and Twelve Pitch Series, Except Gages
No. 4 on Acme and Other Similar Threads, Except Gages, EARLE BUCKINGHAM
No. 5 on Screw Thread Gages and Inspection, G. S. CASE
No. 7 on Wood Screws, ARTHUR BOOR

Special Subcommittee on Revision of American Standard, P. V. MILLER

* Note: All of these standards committees for which the Society is sponsor or joint sponsor, or on which it has representation, are organized under the procedure of the American Standards Association.

† Official A.S.M.E. representative serving on this committee.

PIPE THREADS (B2)

* *Joint sponsorship with the American Gas Association. Sectional Committee originally organized in 1913. Reorganized May, 1927*

A.S.M.E. Members (Total personnel, 46)

A. S. MILLER, *Chairman*
C. B. LEPAGE, *Acting Secretary*
A. F. BREITENSTEIN
E. J. BRYANT
C. S. COLE
E. S. CORNELL, JR.
J. J. CROTTY
A. P. DENTON
J. J. HARMAN
A. M. HOUSER †
A. H. JARECKI
P. V. MILLER
F. H. MOREHEAD
W. C. MORRIS
S. F. NEWMAN
L. N. SHANNON
W. D. SIZER
J. H. WILLIAMS

SUBCOMMITTEE CHAIRMEN

No. 1 on Editing and Gaging, A. M. HOUSER
No. 2 on Taper Pipe Threads, S. B. TERRY
No. 3 on Straight Pipe Threads, A. S. MILLER
No. 4 on Plumbers' Threads, A. F. BREITENSTEIN
No. 5 on Screw Threads for Rigid Steel Conduit, JAMES BARTON
No. 6 on Special Threads for Thin Tubes, C. C. WINTER

Special Subcommittee on Truncation, E. J. BRYANT

BALL AND ROLLER BEARINGS (B3)

* *Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized December, 1920*

A.S.M.E. Members (Total personnel, 14)

W. P. KENNEDY, *Vice-Chairman* †
D. E. BATESOLE †
L. A. CUMMINGS
F. G. HUGHES
G. E. HULSE †
L. F. NENNINGER
ERNEST WOOLER

ALLOWANCES AND TOLERANCES FOR CYLINDRICAL PARTS AND LIMIT GAGES (B4)

* *Sole sponsorship. Sectional Committee originally organized in June, 1920. Reorganized in September, 1930*

A.S.M.E. Members (Total personnel, 43)

J. E. LOVELY, *Chairman* †
F. E. BANFIELD, JR.
F. S. BLACKALL, JR.
E. J. BRYANT
EARLE BUCKINGHAM †
F. H. COLVIN †
T. G. CRAWFORD
R. E. W. HARRISON
F. O. HOAGLAND
N. E. JACOBI
H. C. E. MEYER
P. V. MILLER
W. C. MUELLER
E. C. PECK †
R. H. PERRY
W. C. SCHOENFELDT
C. C. STEVENS
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMAN

No. 1 on Tolerance Systems, R. E. W. HARRISON

SMALL TOOLS AND MACHINE TOOL ELEMENTS (B5)

* *Joint sponsorship with the National Machine Tool Builders Association and the Society of Automotive Engineers. Sectional Committee organized September, 1922*

A.S.M.E. Members (Total personnel, 25)

W. C. MUELLER, *Chairman* †
F. O. HOAGLAND, *Vice-Chairman*
J. B. ARMITAGE
O. W. BOSTON
E. J. BRYANT
EARLE BUCKINGHAM
F. H. COLVIN †
S. A. EINSTEIN
H. E. HARRIS †
JOHN HAYDOCK
J. P. LAUX †
J. E. LOVELY
ERIK OBERG

SMALL TOOLS AND MACHINE TOOL ELEMENTS (*Continued*)

TECHNICAL COMMITTEES

No. 1 ON T-SLOTS

A.S.M.E. Members (Total personnel, 7)

ERIK OBERG, *Chairman* †
J. B. ARMITAGE
HARRY CADWALLADER, JR.†
S. A. EINSTEIN
F. O. HOAGLAND †

No. 2 ON TOOL POSTS AND TOOL SHANKS

A.S.M.E. Members (Total personnel, 8)

O. W. BOSTON, *Chairman*
F. S. BLACKALL, JR.
GRANGER DAVENPORT †
M. E. LANGE

No. 3 ON MACHINE TAPERS

A.S.M.E. Members (Total personnel, 20)

E. J. BRYANT, *Chairman* †
C. B. LEPAGE, *Acting Secretary*
J. B. ARMITAGE
F. S. BLACKALL, JR.
EARLE BUCKINGHAM †
F. H. COLVIN
J. B. DILLARD
(T. F. GITHENS, *Alternate*)
B. P. GRAVES
H. E. HARRIS
F. O. HOAGLAND †
J. H. HORIGAN
A. H. LYON
L. F. NENNINGER

SUBGROUP CHAIRMEN

Steep Tapers Series, S. MCMULLAN
Revision of Slow Taper Standard, E. J. BRYANT

No. 4 ON SPINDLE NOSES AND COLLETS FOR MACHINE TOOLS

A.S.M.E. Members (Total personnel, 26)

J. E. LOVELY, *Chairman* †
L. F. NENNINGER, *Secretary*
J. B. ARMITAGE
B. P. GRAVES
F. O. HOAGLAND †
A. M. JOHNSON
M. E. LANGE
J. H. MANSFIELD
L. D. SPENCE

SUBGROUP CHAIRMEN

No. 1 on Milling Machines, Small and Medium, J. B. ARMITAGE
No. 2 on Large Milling Machines, F. B. KAMPMEIER
No. 3 on Grinding Machine Spindles, H. J. GRIFFING
No. 5 on Drilling Machines and Horizontal Boring Machines, S. MCMULLAN
No. 6 on Turning Machines, Including Automatic Screw Machines, Lathes, Automatic Lathes, Turret Lathes, and Automatic Chucking Machines, J. E. LOVELY
No. 8 on Correlation of Counter Proposals for Spindle Noses, J. E. LOVELY

No. 5 ON MILLING CUTTERS

A.S.M.E. Members (Total personnel, 20)

J. B. ARMITAGE
A. N. GODDARD †
J. H. HORIGAN
G. L. MARKLAND, JR.
E. K. MORGAN
ERIK OBERG †
E. D. VANCIL

SUBGROUP CHAIRMEN

No. 1 on Profile Cutters, E. D. VANCIL
No. 2 on Keyways, J. B. ARMITAGE
No. 3 on Nomenclature, A. C. DANEKIND
No. 4 on Limits, J. H. HORIGAN
No. 5 on Formed Cutters, H. C. HUNGERFORD
No. 6 on Hobs, G. L. MARKLAND, JR.
No. 7 on Inserted Tooth Cutters, J. B. ARMITAGE

No. 6 ON DESIGNATIONS AND WORKING RANGES OF MACHINE TOOLS

A.S.M.E. Members (Total personnel, 19)

JOHN HAYDOCK, *Chairman*
EARLE BUCKINGHAM †
T. H. DOAN, JR.
B. P. GRAVES †
J. J. MCBRIDE
E. R. SMITH

No. 7 ON TWIST DRILL SIZES

A.S.M.E. Members (Total personnel, 7)

W. C. MUELLER, *Chairman* †
J. H. HORIGAN †

No. 8 ON JIG BUSHINGS

A.S.M.E. Member (Total personnel, 8)

J. H. HORIGAN †

No. 9 ON PUNCH PRESS TOOLS

A.S.M.E. Members (Total personnel, 15)

D. H. CHASON
N. W. DORMAN
H. E. HARRIS †
D. M. PALMER

No. 10 ON FORMING TOOLS AND HOLDERS

A.S.M.E. Members (Total personnel, 9)

W. C. MUELLER, *Chairman* †
WILLIAM HARTMAN †
L. D. SPENCE

No. 11 ON CHUCKS AND CHUCK JAWS

A.S.M.E. Member (Total personnel, 10)

J. E. LOVELY, *Chairman* †

SUBGROUP CHAIRMAN

No. 1 on Master Chuck Jaws, J. E. LOVELY
No. 2 on Adapters for Air Cylinders, J. E. LOVELY

No. 12 ON CUT AND GROUND THREAD TAPS

(Total personnel, 7)

No. 13 ON SPLINES AND SPLINED SHAFTS

A.S.M.E. Members (Total personnel, 14)

J. B. ARMITAGE †
R. E. W. HARRISON
F. O. HOAGLAND
J. E. LOVELY †
B. F. WATERMAN

SUBGROUP CHAIRMEN

Machine Tool Splines, R. E. W. HARRISON
Automotive Splines, W. L. BARTH
Involute Splines, C. K. LENNING

No. 17 ON NOMENCLATURE FOR SMALL TOOLS AND MACHINE TOOL ELEMENTS

A.S.M.E. Members (Total personnel, 14)

O. W. BOSTON, *Chairman and Secretary*
F. S. BLACKALL, JR.
F. H. COLVIN †
H. E. HARRIS
F. O. HOAGLAND

Ex-Officio Members

A. N. GODDARD
B. P. GRAVES
W. C. MUELLER †

No. 19 ON SINGLE POINT CUTTING TOOLS

A.S.M.E. Members (Total personnel, 2)

F. H. COLVIN, *Chairman* †
O. W. BOSTON, *Secretary*

No. 20 ON REAMERS

A.S.M.E. Members (Total personnel, 15)

F. H. COLVIN
T. F. GITHENS †
J. H. HORIGAN †

No. 21 ON TOOL-LIFE TESTS FOR SINGLE-POINT TOOLS

(Being organized)

O. W. BOSTON, *Chairman*

GEARS (B6)

** Joint sponsorship with the American Gear Manufacturers Association. Sectional Committee organized June, 1921*

A.S.M.E. Members (Total personnel, 27)

EARLE BUCKINGHAM, *Vice-Chairman* †
C. B. LEPAGE, *Acting Secretary*
G. H. ACKER
U. S. EBERHARDT
L. H. FRY
C. B. HAMILTON, JR.
D. T. HAMILTON
M. R. HANNA
O. A. LEUTWILER †
G. L. MARKLAND, JR.
CARLETON REYNELL

SUBCOMMITTEE CHAIRMEN

No. 1 on Program
No. 2 on Editing Reports
No. 3 on Nomenclature, D. T. HAMILTON
No. 4 on Tooth Form (Spur Gears), U. S. EBERHARDT
No. 5 on Helical Gears, W. P. SCHMITTER
No. 6 on Worm Gears, T. R. RIDEOUT
No. 7 on Bevel Gears, F. L. KNOWLES

GEARS (*Continued*)

No. 8 on Materials, C. B. HAMILTON, JR.
No. 9 on Inspection, J. P. BREUER
No. 10 on Horsepower Rating, EARLE BUCKINGHAM

PIPE FLANGES AND FITTINGS (B16)

** Joint sponsorship with the Heating, Piping, and Air Conditioning Contractors National Association and the Manufacturers Standardization Society of the Valve and Fittings Industry. Sectional Committee organized October, 1921*

A.S.M.E. Members (Total personnel, 49)

C. P. BLISS, *Chairman* †
J. J. HARMAN, *Secretary*
L. W. BENOIT †
A. L. BROWN
SABIN CROCKER
FERDINAND FINK
H. E. HALLER
J. S. HESS
H. A. HOFFER †
E. L. HOPPING
A. M. HOUSER
D. S. JACOBUS
C. A. KELTING †
J. R. KRUSE (JOHN BLIZARD, *Alternate*)
M. B. MACNEILLE
F. H. MOREHEAD
L. S. MORSE
LUDWIG SKOG
J. R. TANNER †
J. H. TAYLOR
H. L. UNDERHILL
G. W. WATTS
J. H. WILLIAMS

SUBCOMMITTEE CHAIRMEN

No. 1 on Cast Iron Flanges and Flanged Fittings, A. M. HOUSER
No. 2 on Screwed Fittings, F. H. MOREHEAD
No. 3 on Steel Flanges and Flanged Fittings, C. P. BLISS
No. 4 on Materials and Stresses, A. M. HOUSER
No. 5 on Face to Face Dimensions of Ferrous Flanged Valves, J. R. TANNER
No. 6 on Malleable Iron or Steel Brass Seat Unions
No. 7 on Rating of Pipe Fittings
No. 8 on Marking of Pipe Fittings
No. 9 on Port Openings, W. W. HUBBARD

SHAFTING (B17)

** Sole sponsorship. Organized October, 1918*

A.S.M.E. Members (Total personnel, 11)

C. M. CHAPMAN *Chairman* †
H. C. E. MEYER
L. C. MORROW
J. M. SHIMER
G. N. VAN DERHOEF †
L. W. WILLIAMS †

BOLT, NUT, AND RIVET PROPORTIONS (B18)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized March, 1922*

A.S.M.E. Members (Total personnel, 57)

A. E. NORTON, *Chairman* †
H. E. ALDRICH
F. C. BILLINGS
B. G. BRAINE
G. S. CASE
T. G. CRAWFORD
H. P. FREAR
HERMAN KOESTER
S. F. NEWMAN
R. J. WHELAN
E. M. WHITING
V. R. WILLOUGHBY
(J. J. MCBRIDE, *Alternate*)
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMEN

No. 1 on Large and Small Rivets
No. 2 on Wrench Head Bolts and Nuts, W. K. MENDENHALL, JR.
No. 3 on Slotted Head Proportions
No. 4 on Track Bolts and Nuts
No. 5 on Round Unslotted Head Bolts (Carriage Bolts), M. C. HORINE
No. 6 on Plow Bolts, O. B. ZIMMERMAN
No. 7 on Body Dimensions and Materials
No. 8 on Nomenclature, G. S. CASE
No. 9 on Socket Head Cap and Set Screws, HERMAN KOESTER

PLAIN AND LOCK WASHERS (B27)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized August, 1925*

A.S.M.E. Members (Total personnel, 37)

EUGENE CALDWELL
T. G. CRAWFORD
B. S. LEWIS †
C. H. LOUTREL
J. J. MCBRIDE
H. C. E. MEYER
W. C. MUELLER †
E. M. WHITING †
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMEN

No. 1 on Plain Washers, W. L. BARTH
No. 2 on Lock Washers, A. W. MELLOWES

TRANSMISSION CHAINS AND SPROCKETS (B29)

** Joint sponsorship with the Society of Automotive Engineers and the American Gear Manufacturers Association. Sectional Committee organized September, 1917. Reorganized December, 1926*

A.S.M.E. Members (Total personnel, 15)

W. J. BELCHER
C. B. JAHNKE †
JOSEPH JOY
L. V. LUDY †
D. B. PERRY
G. A. YOUNG
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMEN

No. 1 on Roller Chain Standardization
No. 2 on Silent Chain Standardization, G. A. YOUNG

CODE FOR PRESSURE PIPING (B31)

** Sole sponsorship. Sectional Committee organized March, 1926. Reorganized December, 1937*

A.S.M.E. Members (Total personnel, 101)

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R. E. BRYANT
C. S. COLE
H. C. COOPER
D. H. COREY
SABIN CROCKER
H. D. EDWARDS
E. R. FISH
CHARLES FITZGERALD
V. M. FROST
T. W. GREENE
H. E. HALLER
W. D. HALSEY
J. S. HAUG
(E. B. SEVERS, *Alternate*)
H. A. HOFFER
G. G. HOLLINS
E. L. HOPPING
A. M. HOUSER †
ALFRED IDDLES †
D. S. JACOBUS
T. M. JASPER
C. A. KELTING
G. S. LARSEN
M. B. MACNEILLE
G. W. MARTIN
H. C. E. MEYER
J. W. MOORE
(J. D. CAPRON, *Alternate*)
F. H. MOREHEAD
(W. W. CRAWFORD, *Alternate*)
(J. J. HARMAN, *Alternate*)
H. H. MORGAN
L. S. MORSE
A. W. MOULDER
E. W. NORRIS
C. W. OBERT
G. A. ORROK
A. L. PENNIMAN, JR.
C. S. ROBINSON †
J. H. ROMANN
D. B. ROSSHEIM
G. W. SAATHOFF
G. K. SAURWEIN
LUDWIG SKOG
H. S. SMITH
(H. H. MOSS, *Alternate*)
F. N. SPELLER
J. R. TANNER
J. H. TAYLOR
J. H. VANCE
H. L. WHITTEMORE
J. H. WILLIAMS
T. F. WOLFE

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan, Scope, and Editing, SABIN CROCKER
No. 2 on Power Piping, ALFRED IDDLES
No. 4 on Gas and Air Piping, J. S. HAUG
No. 5 on Refrigeration Piping, A. H. BAER
No. 6 on Oil Piping, A. D. SANDERSON
No. 7 on Piping Materials and Identification, F. H. MOREHEAD
No. 8 on Fabrication Details, LUDWIG SKOG
No. 9 on Code for District Heating Piping, G. K. SAURWEIN

WIRE AND SHEET METAL GAGES (B32)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized November, 1927. Reorganized November, 1939*

A.S.M.E. Members (Total personnel, 33)

C. S. COLE
A. P. COTTLE
J. F. HOWE †
F. G. WILSON †

WIRE AND SHEET METAL GAGES (*Continued*)

SUBCOMMITTEE CHAIRMAN

Wire and Sheet Metal Gages, H. W. TENNEY

SCREW THREADS FOR HOSE COUPLINGS (B33)

** Sole sponsorship. Sectional Committee organized August, 1928*

A.S.M.E. Members (Total personnel, 27)

A. L. BROWN, *Secretary*
A. F. BREITENSTEIN †
J. J. CROTTY
W. L. CURTISS
W. E. DUNHAM †
J. J. HARMAN
(F. C. ERNST, *Alternate*)
A. M. HOUSER
H. C. E. MEYER
J. H. WILLIAMS

SUBCOMMITTEE CHAIRMEN

No. 1 to Draft Recommended Specifications, J. H. HOWLAND
No. 2 on Basic Thread Dimensions, D. R. MILLER

WROUGHT IRON AND WROUGHT STEEL PIPE AND TUBING (B36)

** Joint sponsorship with the American Society for Testing Materials. Sectional Committee organized April, 1928*

A.S.M.E. Members (Total personnel, 44)

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SABIN CROCKER, *Secretary*
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H. E. ALDRICH
E. L. HOPPING
(A. B. MORGAN, *Alternate*)
A. M. HOUSER †
D. S. JACOBUS †
(F. S. CLARK, *Alternate*) †
J. J. KANTER
H. C. E. MEYER
F. H. MOREHEAD
H. B. OATLEY †
LUDWIG SKOG †
F. N. SPELLER
J. R. TANNER
A. E. WHITE

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan, Scope, and Editing, H. H. MORGAN
No. 2 on Pipe and Tubing for Low Temperature Service, J. J. SHUMAN
No. 3 on Pipe and Tubing for High Temperature Service, J. R. TANNER
No. 4 on Materials, F. H. MOREHEAD

PRESSURE AND VACUUM GAGES (B40)

** Sole sponsorship. Sectional Committee organized July, 1930*

A.S.M.E. Members (Total personnel, 44)

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A. W. LENDEROTH, *Secretary* †
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J. P. CAVANAUGH †
PAUL DISERENS
C. H. GRAESSER
W. F. JONES
R. J. KEHL
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W. C. SCHOENFELDT

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No. 1 on Plan and Scope, M. D. ENGLE
No. 2 on Definitions, C. F. SCHWEP
No. 3 on Gage Sizes and Mounting Dimensions, H. B. REYNOLDS
No. 4 on Accuracy and Test Methods, O. J. HODGE

STOCK SIZES, SHAPES AND LENGTHS FOR HOT AND COLD FINISHED IRON AND STEEL BARS (B41)

** Sole sponsorship. Sectional Committee organized April, 1929*

A.S.M.E. Members (Total personnel, 27)

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L. W. WILLIAMS †
G. H. WOODROFFE
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMEN

No. 1 on Hot Rolled Steel, HENRY WYSOR
No. 2 on Cold Finished Steels, L. E. CREIGHTON
No. 3 on Hot Rolled Iron

SPECIFICATIONS FOR LEATHER BELTING (B42)

** Sole sponsorship. Sectional Committee organized February, 1931*

A.S.M.E. Members (Total personnel, 28)

H. T. COATES
R. W. DRAKE †
KING HATHAWAY
J. E. RHOADS
G. A. SCHIEREN
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMEN

No. 1 on Standard Specifications, R. C. BOWKER
No. 2 on Recommendations for Selection, Care, and Installation, G. A. SCHIEREN

MACHINE PINS (B43)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized March, 1926*

A.S.M.E. Members (Total personnel, 14)

E. J. BRYANT †
J. J. MCBRIDE
H. C. E. MEYER
O. B. ZIMMERMAN

SUBCOMMITTEES

No. 1 on Straight, Taper, and Dowel Pins
No. 2 on Split Pins

CLASSIFICATION AND DESIGNATION OF SURFACE QUALITIES (B46)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized May, 1932*

A.S.M.E. Members (Total personnel, 60)

J. R. WEAVER, *Chairman*
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E. J. BRYANT
C. S. COLE
T. G. CRAWFORD
R. C. DEALE †
U. S. EBERHARDT
S. A. EINSTEIN
R. F. GAGG
W. W. GILBERT
J. J. HARMAN
R. E. W. HARRISON †
F. V. HARTMAN
F. O. HOAGLAND
H. J. HOLTZCLAW
R. T. KENT
H. F. KURTZ
A. H. LYON
M. J. REED
F. C. SPENCER
C. C. STEVENS
J. S. TAWRESEY
C. H. WHITAKER
ERNEST WOOLER

SUBCOMMITTEE CHAIRMEN

No. 1 on Surfaces Produced by Tools and Abrasives, JOHN CETRULE
No. 2 on Surfaces Produced by Molds, Dies, Rolls, or Any Other Means of Deforming Materials
No. 3 on Coated Surfaces, G. B. HOGABOOM
No. 4 on Symbols for Indicating Surface Quality on Drawings, T. G. CRAWFORD
No. 5 on Ways, Means, and Apparatus for Measuring Quality of Surface, J. R. WEAVER
No. 6. on Finance, R. E. W. HARRISON

SCHEME FOR IDENTIFICATION OF PIPING SYSTEMS (A13)

** Joint sponsorship with the National Safety Council. Sectional Committee organized June, 1922*

A.S.M.E. Members (Total personnel, 33)

E. E. ASHLEY
W. L. BUNKER
CROSBY FIELD
E. L. HOPPING
H. L. MINER
H. S. SMITH
FRANK THORNTON

SUBCOMMITTEE CHAIRMEN

Identification by Colors
Classification, CROSBY FIELD
Identification Markings Other Than Color

COMBUSTION SPACE FOR SOLID FUELS (B50)

** Sole sponsorship. Sectional Committee organized June, 1933*

A.S.M.E. Members (Total personnel, 21)

C. E. BRONSON, *Chairman*
W. G. CHRISTY
JOHN HUNTER
A. J. JOHNSON
V. G. LEACH †

COMBUSTION SPACE FOR SOLID FUELS (*Continued*)

J. P. Magos
J. F. McIntire
F. L. Meyer
C. A. Reed
John Van Brunt †

SUBCOMMITTEE CHAIRMEN

No. 1 on Purpose and Scope, C. E. Bronson
No. 2 on Combustion and Design, B. M. Guthrie
No. 3 on Warm Air Furnaces, J. H. Manny
No. 4 on Steel Heating Boilers, W. B. Russell
No. 5. on Cast Iron Boilers, J. F. McIntire

MINIMUM REQUIREMENTS FOR PLUMBING AND STANDARDIZATION OF PLUMBING EQUIPMENT (A40)

** Joint sponsorship with the American Society of Sanitary Engineering. Sectional Committee organized August, 1928*

A.S.M.E. Members (Total personnel, 55)

C. B. LePage, *Acting Secretary*
J. F. Carney
C. S. Cole
J. J. Crotty
A. M. Houser
G. W. Martin
(A. H. Morgan, *Alternate*)
W. K. McAfee
W. R. Webster †

SUBCOMMITTEE CHAIRMEN

No. 1 on Minimum Requirements for Plumbing, W. C. Groeniger
No. 2 on Staple Vitreous China Plumbing Fixtures, H. R. Van Sciver
No. 3 on Staple Porcelain (All Clay) Plumbing Equipment, H. R. Van Sciver
No. 4 on Enameled Sanitary Ware, J. J. Hall
No. 5 on Traps, A. R. McGonegal
No. 6 on Brass Plumbing Products, J. L. Murphy
No. 7 on Brass Fittings for Flared Copper Tubes, F. L. Riggin
No. 8 on Cast Iron Soil Pipe and Fittings, J. J. Crotty
No. 9 on Gasoline, Oil, and Grease Separators, J. J. Crotty
No. 11 on Soldered Fittings for Tubing, A. M. Houser
No. 12 on Minimum Air Gaps in Plumbing Systems, W. K. McAfee

Joint Committee on Threaded Cast Iron Pipe, F. H. Morehead

ELECTRIC MOTOR FRAME DIMENSIONS (C28)

** Joint Sponsorship with the National Electrical Manufacturers Association. Sectional Committee organized November, 1927*

A.S.M.E. Members (Total personnel, 29)

C. A. Adams
S. A. Einstein
E. W. Ely
F. S. English
W. F. Jones
A. G. Trumbull †
Rudolph Michel
F. W. Ming
W. C. Mueller
E. B. Neil
J. W. Owens
F. C. Panuska
E. S. Smith †

ROLLED THREADS FOR SCREW SHELLS OF ELECTRIC SOCKETS AND LAMP BASES (C44)

** Joint sponsorship with the National Electrical Manufacturers Association. Sectional Committee organized March, 1929*

A.S.M.E. Members (Total personnel, 16)

E. J. Bryant †
Earle Buckingham †
A. B. Morgan
E. S. Sanderson †

LETTER SYMBOLS AND ABBREVIATIONS FOR SCIENCE AND ENGINEERING (Z10)

** Joint sponsorship with the American Association for the Advancement of Science, American Institute of Electrical Engineers, American Society of Civil Engineers, and the Society for the Promotion of Engineering Education. Sectional Committee organized January, 1926. Reorganized October, 1935*

A.S.M.E. Members (Total personnel, 37)

S. A. Moss, *Vice-Chairman* †
R. M. Anderson
(E. P. Warner, *Alternate*)
K. H. Condit
R. J. S. Pigott †
(S. R. Beitler, *Alternate*) †
Frank Thornton, Jr.

SUBCOMMITTEE CHAIRMEN

No. 1 on Letter Symbols and Signs for Mathematics, A. A. Bennett
No. 2 on Symbols for Hydraulics, J. C. Stevens
No. 3 on Symbols for Mechanics, R. E. Peterson
No. 4 on Symbols for Structural Analysis, Albert Haertlein
No. 5 on Symbols for Heat and Thermodynamics, S. A. Moss
No. 6 on Symbols for Photometry, E. C. Crittenden
No. 7 on Aeronautical Symbols, G. W. Lewis
No. 8 on Symbols for Electric and Magnetic Quantities, J. F. Meyer
No. 9 on Symbols for Radio, H. M. Turner
No. 10 on Symbols for Physics, H. K. Hughes
No. 11 on Abbreviations for Engineering and Scientific Terms, G. A. Stetson

DRAWINGS AND DRAFTING ROOM PRACTICE (Z14)

** Joint sponsorship with the Society for the Promotion of Engineering Education. Sectional Committee organized July, 1926*

A.S.M.E. Members (Total personnel, 53)

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H. P. Frear
T. E. French
A. C. Harper
E. R. Hill
A. M. Houser
Alfred Iddles
Samuel Ketchum †
F. R. Laney
H. B. Langille

GRAPHIC PRESENTATION (Z15)

** Sole sponsorship. Sectional Committee organized November, 1926*

A.S.M.E. Members (Total personnel, 31)

G. E. Hagemann, *Secretary* †
C. M. Bigelow
Wallace Clark
T. E. French
D. B. Porter †

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan and Scope
No. 2 on Terminology
No. 3 on Preferred Practice for Time Series Charts, A. H. Richardson
No. 4 on Engineering and Scientific Graphs, W. A. Shewhart

SPEEDS OF MACHINERY (Z18)

** Sole sponsorship. Sectional Committee organized May, 1928*

A.S.M.E. Members (Total personnel, 29)

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J. F. Daggett
R. C. Deale †
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F. S. English
D. C. Jackson
John Reid
P. G. Rhoads
F. C. Spencer
O. B. Zimmerman

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan and Scope, A. E. Hall
No. 2 on Questionnaire and Canvass to Industry, F. S. English
No. 3—Special Reviewing Committee

GRAPHICAL SYMBOLS AND ABBREVIATIONS FOR USE IN DRAWINGS (Z32)

** Joint sponsorship with American Institute of Electrical Engineers. Sectional Committee organized April, 1936*

A.S.M.E. Members (Total personnel, 51)

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J. M. Barnes
T. E. French †
G. F. Habach
D. T. Hamilton
A. M. Houser
(J. J. Harman, *Alternate*)
W. C. Mueller
L. L. Munier
J. W. Owens
F. C. Panuska †
M. J. Reed
T. R. Thomas

SUBCOMMITTEE CHAIRMEN

No. 1 on Symbols for Use in Mechanical Engineering, T. E. French
No. 2 on Symbols for Use in Electrical Engineering, H. W. Samson

DEVELOPMENT OF APPLICATIONS OF STATISTICS IN ENGINEERING AND MANUFACTURING

Joint Sponsorship with the American Society for Testing Materials. Appointed in December, 1929

A.S.M.E. Members (Total personnel, 8)

A. G. Ashcroft
W. H. Fulweiler
L. K. Sillcox †
J. S. Tawresey †

A. S. M. E. Representatives on Miscellaneous Standardization Committees

See also A.S.M.E. Representatives on Other Activities, page RI-5

ACOUSTICAL MEASUREMENTS AND TERMINOLOGY

* *Sponsor body: Acoustical Society of America*

P. H. Bilhuber
W. B. White
(R. V. Parsons, *Alternate*)
(J. S. Parkinson, *Alternate*)

AERONAUTICS

* *Sponsor body: Society of Automotive Engineers*

E. A. Sperry, Jr.

APPROVAL AND INSTALLATION REQUIREMENTS FOR GAS BURNING APPLIANCES

* *Sponsor body: American Gas Association*

O. F. Campbell

BUILDING CODE REQUIREMENTS FOR LIGHT AND VENTILATION

* *Sponsor bodies: Federal Housing Administration and U.S. Public Health Service*

F. R. Scherer

CLASSIFICATION OF COALS

* *Sponsor body: American Society for Testing Materials*

E. G. Bailey

DEFINITIONS OF ELECTRICAL TERMS

* *Sponsor body: American Institute of Electrical Engineers*

C. H. Berry

DRAINAGE OF COAL MINES

* *Sponsor body: American Mining Congress*

O. M. Pruitt

ELECTRIC WELDING APPARATUS

* *Sponsor bodies: American Institute of Electrical Engineers and the National Electrical Manufacturers Association*

R. E. Kinkead

FOREST FIRE PROTECTION

Committee of National Fire Protection Association

C. B. White

GEAR LUBRICANTS

Committee of American Gear Manufacturers Association

G. B. Karelitz

LOADING PLATFORMS AT FREIGHT TERMINALS AND WAREHOUSES

* *Sponsor body: American Trucking Association*

M. C. Maxwell

MANHOLE FRAMES AND COVERS

* *Sponsor bodies: A.S.A. Telephone Group and American Society of Civil Engineers*

Anton Hansen
Homer Rupard

METHODS OF TESTING WOOD

* *Sponsor bodies: U.S. Forest Service and the American Society for Testing Materials*

C. M. Bigelow

MISCELLANEOUS OUTSIDE COAL-HANDLING EQUIPMENT

* *Sponsor body: American Mining Congress*

(To be appointed)

PETROLEUM PRODUCTS AND LUBRICANTS

* *Sponsor body: American Society for Testing Materials*

G. B. Karelitz
(H. J. Masson, *Alternate*)
(S. J. Needs, *Alternate*)

PREFERRED NUMBERS

* *Special Committee of A.S.A.*

K. H. Condit

RATING OF RIVERS

* *Sponsor body: U.S. Geological Survey*

D. W. Mead

ROTATING ELECTRICAL MACHINERY

* *Sponsor bodies: American Institute of Electrical Engineers and National Electrical Manufacturers Association*

C. A. Booth, *Alternate*

SPECIFICATIONS FOR CAST IRON PIPE AND SPECIAL CASTINGS

* *Sponsor bodies: American Gas Association, American Society for Testing Materials, American Water Works Association, and the New England Water Works Association*

J. E. Gibson
L. R. Howson

SPECIFICATIONS FOR CLEAN BITUMINOUS COAL

* *Sponsor body: American Institute of Mining and Metallurgical Engineers*

R. A. Sherman
(E. L. Lindseth, *Alternate*)

SPECIFICATIONS FOR FIRE TESTS OF BUILDING CONSTRUCTION AND MATERIALS

* *Sponsor bodies: A.S.A. Fire Protection Group, National Bureau of Standards, and the American Society for Testing Materials*

R. C. Parlett

SPECIFICATIONS FOR SIEVES FOR TESTING PURPOSES

* *Sponsor bodies: American Society for Testing Materials and National Bureau of Standards*

R. M. Hardgrove

THERMAL INSULATING MATERIALS

Committee of American Society for Testing Materials

R. H. Heilman

VOLUME WATER HEATING

Committee of American Gas Association

Mark Resek

WIRE ROPE FOR MINES

* *Sponsor body: American Mining Congress*

J. L. Harrington

SAFETY TECHNICAL COMMITTEES

ARTICLE B6A, PAR. 25: The Standing Committee on Safety shall advise the Council on the activities of the Society having to do with engineering and industrial safety, except the activities of the Boiler Code Committee, for which special provision is made.

The first Standing Committee on Safety was appointed in October, 1921.

STANDING COMMITTEE

T. F. HATCH, *Chairman* (1941)
D. L. ROYER (1940)
A. W. LUCE (1942)
A. E. WINDLE (1943)
H. C. HOUGHTON (1944)

SAFETY CODE FOR ELEVATORS (A17)

** Joint Sponsorship with the American Institute of Architects and the National Bureau of Standards. Sectional Committee organized November, 1922*

A.S.M.E. Members (Total personnel, 41)

O. P. CUMMINGS, *Vice-Chairman*
C. R. CALLAWAY
D. L. HOLBROOK †
D. L. LINDQUIST
N. O. LINDSTROM †
M. B. MCLAUTHLIN
W. S. PAINE

SUBCOMMITTEE CHAIRMEN

Research, Recommendations, and Interpretations, D. J. PURINTON
Inspectors' Handbook, K. A. COLAHAN
Elevator Systems Safe Against Fire, W. S. PAINE
Interlocks and Hoistway Enclosures, B. JONES

SAFETY CODE FOR MECHANICAL POWER-TRANSMISSION APPARATUS (B15)

** Joint sponsorship with the International Association of Industrial Accident Boards and Commissions and the National Conservation Bureau. Sectional Committee organized February, 1921*

A.S.M.E. Members (Total personnel, 31)

G. M. NAYLOR, *Chairman* †
P. G. RHOADS, *Secretary*
C. C. FOSTER
W. W. NICHOLS
W. S. PAINE
D. C. WRIGHT †
(G. N. VAN DERHOEF, *Alternate*) †

SUBCOMMITTEE CHAIRMEN

No. 1 on Detail Classification of Belts
No. 2 on Modification of Rule 223 for Cone Pulley Belts
No. 3 on Mechanical Power Control, W. S. PAINE
No. 4 on Use of ASA Code Versus State Codes
No. 5 on Statistics on Place of Occurrence of Accidents
No. 6 on V-Belt Drives, D. M. JOHNSON

* Note: All of the safety committees for which the Society is sponsor or joint sponsor, or on which it has representation, are organized under the procedure of the American Standards Association.

† Official A.S.M.E. representative serving on this committee.

SAFETY CODE ON COMPRESSED AIR MACHINERY AND EQUIPMENT (B19)

** Joint sponsorship with the American Society of Safety Engineers—Engineering Section, National Safety Council. Sectional Committee organized May, 1923*

A.S.M.E. Members (Total personnel, 24)

D. L. ROYER, *Chairman*
H. D. EDWARDS
W. J. GRAVES

SAFETY CODE FOR CONVEYORS AND CONVEYING MACHINERY (B20)

** Joint Sponsorship with the National Conservation Bureau. Sectional Committee organized November, 1925, Reorganized, April 1937*

A.S.M.E. Members (Total personnel, 53)

D. L. ROYER, *Chairman*
C. T. COLLEY
W. J. GRAVES
M. A. KENDALL †
(N. W. ELMER, *Alternate*) †
P. T. ONDERDONK
C. G. PFEIFFER
R. B. RENNER
F. J. SHEPARD, JR.
J. G. WHEATLEY

SUBCOMMITTEE CHAIRMEN

No. 1 on All Types of Chain Conveyors, Belt Conveyors, Belt Elevators Including Steel Belt, and Screw, Track or Scraper Conveyors, C. G. PFEIFFER
No. 2 on Gravity Conveyors and Chutes, Live Roll Conveyors, H. G. DALTON
No. 3 on Cable-Operated and Cable Flight Conveyors and Cableways, R. MCA. KEOWN
No. 4 on Air, Steam, or Liquid Conveyors, J. J. MCNULTA
No. 5 on Tiering, Piling, and Stacking Conveyors, J. G. WHEATLEY

SAFETY CODE FOR CRANES, DERRICKS, AND HOISTS (B30)

** Joint sponsorship with U. S. Navy Department, Bureau of Yards and Docks. Sectional Committee organized November, 1926*

A.S.M.E. Members (Total personnel, 56)

LEWIS PRICE †
F. H. SCHWERIN
R. H. WHITE †
H. L. WHITTEMORE

SUBCOMMITTEE CHAIRMEN

No. 1 on Overhead and Gantry Cranes, R. H. WHITE
No. 2 on Locomotive and Tractor Cranes, H. H. VERNON
No. 3 on Derricks and Hoists, LEWIS PRICE
No. 4 on Miscellaneous Equipment for Cranes and Hoists, L. W. HOPKINS
No. 5 on Jacks, E. W. CARUTHERS

A.S.M.E. Representatives on Other Safety Committees

See also A.S.M.E. Representatives on Other Activities, page RI-5

SAFETY CODE FOR ABRASIVE WHEELS

** Sponsor bodies: Grinding Wheel Manufacturers Association of United States and Canada, and International Association of Industrial Accident Boards and Commissions*

J. B. CHALMERS

SAFETY CODE FOR CONSTRUCTION WORK

** Sponsor bodies: American Institute of Architects and National Safety Council*

C. H. O'NEIL

COOPERATION WITH OTHER ENGINEERING SOCIETIES

Committee of American Society of Safety Engineers—Engineering Section, National Safety Council

H. L. MINER

ASA SAFETY CODE CORRELATING COMMITTEE

D. L. ROYER
(J. B. CHALMERS, *Alternate*)

SAFETY CODE FOR EXHAUST SYSTEMS

** Sponsor body: International Association of Industrial Accident Boards and Commissions*

T. F. HATCH

SAFETY CODE FOR FLOOR AND WALL OPENINGS, RAILINGS, AND TOE BOARDS

** Sponsor body: National Safety Council*

A. E. WINDLE

SAFETY CODE FOR FORGING AND HOT METAL STAMPING

** Sponsor bodies: American Drop Forging Institute and National Safety Council*

C. F. PARK

SAFETY CODE ON COLORS FOR IDENTIFICATION OF GAS MASK CANISTERS

** Sponsor body: National Safety Council*

L. C. LICHTY

SAFETY CODE FOR LADDERS

** Sponsor body: American Society of Safety Engineers—Engineering Section, National Safety Council*

H. C. Houghton

SAFETY CODE FOR LAUNDRY MACHINERY AND OPERATION

** Sponsor bodies: American Institute of Laundering, International Association of Governmental Labor Officials, and National Association of Mutual Casualty Companies*

E. J. Carroll

SAFETY CODE FOR LIGHTING FACTORIES, MILLS, AND OTHER WORK PLACES

** Sponsor body: Illuminating Engineering Society*

A. W. Luce

LOW VOLTAGE ELECTRICAL HAZARDS

Special Committee of the American Society of Safety Engineers—Engineering Section, National Safety Council

J. P. Jackson

SAFETY CODE FOR MECHANICAL REFRIGERATION

** Sponsor body: American Society of Refrigerating Engineers*

O. A. Anderson
Crosby Field
E. W. Gallenkamp
W. F. Jones
(A. W. Oakley, *Alternate to all A.S.M.E. Representatives*)

SAFETY CODE FOR PAPER AND PULP MILLS

** Sponsor body: National Safety Council*

R. L. Weldon

SAFETY CODE FOR POWER PRESSES, AND FOOT AND HAND PRESSES

** Sponsor body: National Safety Council*

J. B. Chalmers

SAFETY CODE FOR PREVENTION OF DUST EXPLOSIONS

** Sponsor bodies: National Fire Protection Association and U.S. Department of Agriculture*

J. H. Morrow
(R. M. Ferry, *Alternate*)

SAFETY CODE FOR PROTECTION OF HEADS, EYES, AND RESPIRATORY ORGANS OF INDUSTRIAL WORKERS

** Sponsor body: National Bureau of Standards*

T. A. Walsh, Jr.
(T. F. Hatch, *Alternate*)

SAFETY CODE FOR PROTECTION OF INDUSTRIAL WORKERS IN FOUNDRIES

** Sponsor bodies: American Foundrymen's Association and National Founders Association*

H. M. Lane

SAFETY CODE FOR RUBBER MACHINERY

** Sponsor bodies: National Safety Council and International Association of Industrial Accident Boards and Commissions*

E. S. Ault

SPECIFICATIONS AND METHOD OF TEST FOR SAFETY GLASS

** Sponsor bodies: National Conservation Bureau and National Bureau of Standards*

T. A. Walsh, Jr.

SAFETY CODE FOR TEXTILES

** Sponsor body: National Safety Council*

M. A. Golrick, Jr.

SAFETY CODE FOR VENTILATION

** Sponsor body: American Society of Heating and Ventilating Engineers*

T. F. Hatch

SAFETY CODE FOR WALKWAY SURFACES

** Sponsor bodies: American Institute of Architects and American Society of Safety Engineers—Engineering Section, National Safety Council*

G. K. Palsgrove

SAFETY CODE FOR WORK IN COMPRESSED AIR

** Sponsor body: International Association of Industrial Accident Boards and Commissions*

L. J. Eibsen

POWER TEST CODES TECHNICAL COMMITTEES

ARTICLE B6A, PAR. 27: The Standing Committee on Power Test Codes shall, under the direction of the Council, have supervision of all the activities of the Society in connection with the A.S.M.E. Power Test Codes, including the interpretation of such codes.

The first Standing Committee on Power Test Codes was organized in December, 1918, to revise and extend the Power Test Codes which had been formulated by various technical committees appointed to develop particular codes. This work began in 1884.

STANDING COMMITTEE

FRANCIS HODGKINSON, *Chairman* (1944)
A. G. CHRISTIE, *Vice-Chairman* (1941)
W. W. LAWRENCE, *Junior Observer* (1940)

Term expires 1940

A. T. BROWN
P. H. HARDIE
B. V. E. NORDBERG
R. J. S. PIGOTT
M. C. STUART

Term expires 1941

A. G. CHRISTIE
PAUL DISERENS
G. A. ORROK
L. A. QUAYLE
W. M. WHITE

Term expires 1942

W. A. CARTER
HARTE COOKE
E. R. FISH
H. B. OATLEY
W. J. WOHLENBERG

Term expires 1943

LOUIS ELLIOTT
G. A. HORNE
H. B. REYNOLDS
P. W. SWAIN
E. N. TRUMP

Term expires 1944

C. H. BERRY
D. S. JACOBUS
FRANCIS HODGKINSON
L. F. MOODY
E. B. RICKETTS

---

(1) GENERAL INSTRUCTIONS

Appointed December, 1918
Reorganized, 1939

THEODORE BAUMEISTER, JR., *Chairman*
PAUL DISERENS
HENRY KREISINGER
A. R. MUMFORD
C. R. SODERBERG
M. C. STUART
P. W. SWAIN

(2) DEFINITIONS AND VALUES

Appointed December, 1918
Reorganized, 1936

R. J. S. PIGOTT, *Chairman*
L. J. BRIGGS
W. F. DAVIDSON
A. L. KIMBALL
L. S. MARKS
F. G. PHILO
J. C. SMALLWOOD
P. W. SWAIN
A. C. WOOD

(3) FUELS

Appointed December, 1918

W. J. WOHLENBERG, *Chairman*
E. G. BAILEY
B. L. BOYE
L. P. BRECKENRIDGE
H. W. BROOKS
S. B. FLAGG
D. M. MYERS
F. G. PHILO
G. S. POPE
E. B. RICKETTS
F. M. ROGERS
E. X. SCHMIDT
NICHOLAS STAHL
E. N. TRUMP

(4) STATIONARY STEAM-GENERATING UNITS

Appointed December, 1918

E. R. FISH, *Chairman*
A. D. BAILEY
M. W. BENJAMIN
B. J. CROSS
MARTIN FRISCH
P. H. HARDIE
R. M. HARDGROVE
ALFRED IDDLES
E. L. LINDSETH
E. L. MCDONALD
E. B. POWELL
R. SHELLENBERGER
R. L. SPENCER

(5) RECIPROCATING STEAM ENGINES

Appointed December, 1918
Reorganized, 1931

A. G. CHRISTIE, *Chairman*
HARTE COOKE
K. S. M. DAVIDSON
HENRIK GREGER
J. A. HUNTER
H. G. MUELLER
B. V. E. NORDBERG
A. V. SAHAROFF
A. G. WITTING

(6) STEAM TURBINES

Appointed December, 1918

C. H. BERRY, *Chairman*
I. E. MOULTROP, *Secretary*
O. D. H. BENTLEY
W. E. CALDWELL
C. B. CAMPBELL
A. G. CHRISTIE
H. P. DAHLSTRAND
V. M. FROST
A. E. GRUNERT
FRANCIS HODGKINSON
S. A. MOSS
R. O. MULLER
T. E. PURCELL
G. B. WARREN

(7) RECIPROCATING STEAM-DRIVEN DISPLACEMENT PUMPS

Appointed December, 1918

R. D. HALL, *Chairman*
E. H. BROWN
J. N. CHESTER
J. E. GIBSON
G. L. KOLLBERG
M. B. MACNEILLE
D. W. MEAD
L. A. QUAYLE

(8) CENTRIFUGAL AND ROTARY PUMPS

Appointed December, 1918
Reorganized, 1936

M. B. MACNEILLE, *Chairman*
H. E. BECKWITH
R. L. DAUGHERTY
R. G. FOLSOM
R. C. GLAZEBROOK
W. B. GREGORY
R. T. KNAPP
J. B. LINCOLN
L. F. MOODY
ARVID PETERSON
F. H. ROGERS
W. C. RUDD
MAX SPILLMAN
F. G. SWITZER
W. M. WHITE
I. A. WINTER

(9) DISPLACEMENT COMPRESSORS AND BLOWERS

Appointed December, 1918
Reorganized, 1935

PAUL DISERENS, *Chairman*
G. T. FELBECK
C. R. HOUGHTON
J. F. HUVANE
R. M. JOHNSON
J. F. D. SMITH

(10) CENTRIFUGAL AND TURBO-COMPRESSORS AND BLOWERS

Appointed December, 1918
Reorganized, 1929

A. T. BROWN, *Chairman*
E. L. ANDERSON
C. A. BOOTH
W. H. CARRIER
THOMAS CHESTER
L. E. DAY
E. S. DEAN
Z. G. DEUTSCH
S. H. DOWNS
P. E. GOOD
J. J. GROB
H. F. HAGEN
PAUL HOFFMAN

H. D. Kelsey
A. L. Kimball
R. D. Madison
L. S. Marks
Arvid Peterson
H. F. Schmidt
M. C. Stuart

(11) COMPLETE STEAM POWER PLANTS

Appointed December, 1918
Reorganized, 1933

F. M. Van Deventer, *Chairman*
D. S. Wegg, *Secretary*
W. F. Davidson
C. H. Fellows
R. A. Foresman
V. M. Frost
W. W. Johnson
E. W. Norris
H. S. Whiton

(12) CONDENSERS, WATER HEATING, AND COOLING EQUIPMENT

Appointed December, 1918

G. A. Orrok, *Chairman*
P. H. Hardie, *Secretary*
C. H. Baker, Jr.
R. N. Ehrhart
J. F. Grace
D. W. R. Morgan
H. B. Reynolds
P. E. Reynolds

(13) REFRIGERATING SYSTEMS

Appointed December, 1918
Reorganized May, 1939

B. H. Jennings, *Chairman* †
A. C. Buensod
(R. W. Waterfill, *Alternate*)
J. C. Consley
(H. B. Pownall, *Alternate*)
H. A. Feldbush †
Walter Jones †
A. W. Oakley
C. L. Svenson
Frank Zumbro †

(14) EVAPORATING APPARATUS

Appointed December, 1918

E. N. Trump, *Chairman*
B. N. Bump
E. A. Newhall
H. L. Parr
L. C. Rogers

(15) STEAM LOCOMOTIVES

Appointed December, 1918

E. C. Schmidt, *Chairman*
W. F. Kiesel, Jr.

† Official A.S.M.E. representatives serving on this committee.

H. B. Oatley
G. E. Rhoads
L. K. Sillcox
W. E. Woodard

(16) GAS PRODUCERS

Appointed December, 1918

C. D. Smith
H. F. Smith

(17) INTERNAL-COMBUSTION ENGINES

Appointed December, 1918
Reorganized, 1939

Lee Schneitter, *Chairman*
F. H. Dutcher, *Secretary*
J. C. Barnaby
Harte Cooke
H. E. Degler
W. L. H. Doyle
L. R. Ford
L. B. Jackson
E. J. Kates
E. C. Magdeburger
B. V. E. Nordberg
M. J. Reed
Max Rotter
O. D. Treiber

(18) HYDRAULIC PRIME MOVERS

Appointed December, 1918
Reorganized, 1931

S. L. Kerr, *Chairman*
C. M. Allen
L. M. Davis
H. L. Doolittle
W. F. Durand
H. N. Eaton
N. R. Gibson
J. P. Growdon
T. H. Hogg
L. J. Hooper
C. W. Hubbard
E. C. Hutchinson
D. J. McCormack
L. F. Moody
W. J. Rheingans
E. B. Strowger
R. V. Terry
W. M. White

(19) INSTRUMENTS AND APPARATUS

Appointed, December, 1918

W. A. Carter, *Chairman*
C. M. Allen
W. C. Andrae
E. G. Bailey
H. S. Bean
L. J. Briggs
J. D. Davis
K. J. De Juhasz
R. E. Dillon
F. M. Farmer
J. B. Grumbein
W. W. Johnson
W. H. Kenerson
E. S. Lee
E. L. Lindseth
Osborn Monnett
S. A. Moss
R. J. S. Pigott
E. B. Ricketts
W. A. Sloan
R. B. Smith
I. M. Stein

(20) SPEED-RESPONSIVE GOVERNORS

Appointed December, 1921

Francis Hodgkinson, *Chairman*
Harte Cooke
F. H. Rogers

(21) DUST SEPARATING APPARATUS

Appointed October, 1934

M. D. Engle, *Chairman*
Ollison Craig, *Secretary*
E. L. Anderson
A. D. Bailey
H. H. Bubar
W. G. Christy
H. O. Croft
J. M. DallaValle
H. C. Dohrmann
Philip Drinker
J. W. Fehnel
H. F. Hagen
P. H. Hardie
C. W. Hedberg
J. H. Leech
H. E. Macomber
H. B. Meller
H. C. Murphy
B. F. Tillson

A.S.M.E. Representatives on Other Technical Committees

See also A.S.M.E. Representatives on Other Activities, page RI-5

DEVELOPMENT OF DEFINITIONS FOR THE NET CALORIFIC VALUE AND GROSS CALORIFIC VALUE OF FUELS

Sponsor body: American Society for Testing Materials

W. J. Wohlenberg

COMMITTEE ON REDEFINING SO-CALLED STANDARD TON OF REFRIGERATION

Sponsor body: American Society of Refrigerating Engineers

G. B. Bright

BOILER CODE COMMITTEES

ARTICLE B6A, PAR. 26: The Special Committee on Boiler Code shall, under the direction of the Council, have supervision of all the activities of the Society in connection with the A.S.M.E. Codes for Pressure Vessels, including the interpretations of these codes.

The first Special Committee on Boiler Code was organized in September, 1911.

SPECIAL COMMITTEE

D. S. JACOBUS, *Chairman*
E. R. FISH, *Vice-Chairman*
C. W. OBERT, *Honorary Secretary*
M. JURIST, *Acting Secretary*
C. A. ADAMS
H. E. ALDRICH
H. C. BOARDMAN
PERRY CASSIDY
R. E. CECIL
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C. E. GORTON
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J. O. LEECH
I. E. MOULTROP
C. O. MYERS
H. B. OATLEY
JAMES PARTINGTON
WALTER SAMANS
S. K. VARNES
A. C. WEIGEL

Honorary Members

W. H. BOEHM
F. W. DEAN
W. F. DURAND
T. E. DURBAN
C. L. HUSTON
W. F. KIESEL, JR.
M. F. MOORE
H. H. VAUGHAN
H. LEROY WHITNEY

CONFERENCE COMMITTEE

W. E. ALLEN, St. Louis, Mo.
T. R. ARCHER, Delaware
L. M. BARRINGER, Seattle, Wash.
J. G. BOLLOCK, St. Joseph, Mo.
B. M. BOOK, Pennsylvania
H. S. BRUNSON, Minnesota
CITY BOILER INSPECTOR, Parkersburg, W. Va.
E. S. CARPENTER, Rhode Island
L. M. CAVE, Maryland
A. J. CONWAY, Indiana
M. A. EDGAR, Wisconsin
C. W. FOSTER, Omaha, Neb.
E. M. FITZGERALD, Louisiana
M. R. FRANCIS, West Virginia
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GERALD GEARON, Chicago, Ill.
J. M. GILLESPIE, Tulsa, Okla.
C. H. GRAM, Oregon
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F. A. HECKINGER, Memphis, Tenn.
H. K. KUGEL, District of Columbia
JOE KUNSCHIK, Texas
G. A. LUCK, Massachusetts
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H. H. MILLS, Detroit, Mich.
C. O. MYERS, Ohio
J. D. NEWCOMB, JR., Arkansas
W. L. NEWTON, Oklahoma
F. A. PAGE, California
L. C. PEAL, Nashville, Tenn.
E. K. SAWYER, Maine
A. H. SCHLEMAN, Tampa, Fla.
J. F. SCOTT, New Jersey
J. N. Seiger, Evanston, Ill.
C. I. SMITH, Utah
H. B. SMITH, North Carolina
WM. E. SMITH, Hawaiian Islands
JOHN H. THORPE, Michigan

EXECUTIVE COMMITTEE

D. S. JACOBUS, *Chairman*
H. E. ALDRICH, *Vice-Chairman*
E. R. FISH
V. M. FROST
C. E. GORTON
C. W. OBERT
JAMES PARTINGTON

SUBCOMMITTEES

BOILERS OF LOCOMOTIVES

JAMES PARTINGTON, *Chairman*
F. H. CLARK
J. M. HALL
H. B. OATLEY

CARE OF STEAM BOILERS AND OTHER PRESSURE VESSELS IN SERVICE

F. M. GIBSON, *Chairman*
D. C. CARMICHAEL
V. M. FROST
J. R. GILL
FRANK HENRY
J. A. HUNTER
H. J. KERR
P. B. PLACE
S. T. POWELL
C. W. RICE
J. B. ROMER
W. C. SCHROEDER
NICHOLAS STAHL
F. G. STRAUB

FERROUS MATERIALS

D. B. ROSSHEIM, *Chairman*
A. B. BAGSAR
E. C. CHAPMAN
A. J. ELY
H. J. FRENCH
W. R. GRUNOW
M. B. HIGGINS
W. G. HUMPTON
A. HURTGEN
T. MCLEAN JASPER
J. J. KANTER
E. F. KENNEY
H. J. KERR
A. B. KINZEL
L. J. MASON
N. L. MOCHEL
E. L. ROBINSON
S. K. VARNES
A. E. WHITE
R. WILSON

HEATING BOILERS

C. E. GORTON, *Acting Chairman*
C. E. BRONSON
J. A. DARTS
WM. FERGUSON
W. E. STARK
J. W. TURNER

MATERIAL SPECIFICATIONS

PERRY CASSIDY, *Chairman*
A. M. GREENE, JR.
W. G. HUMPTON
J. O. LEECH
P. J. SMITH
A. C. WEIGEL

MINIATURE BOILERS

C. E. GORTON, *Chairman*
W. H. FURMAN
G. A. LUCK
C. W. OBERT

NONFERROUS MATERIALS

H. B. OATLEY, *Chairman*
J. J. AULL
D. K. CRAMPTON
J. R. FREEMAN, JR.
A. M. HOUSER
F. P. HUSTON
E. F. MILLER
JOSEPH PRICE
R. L. TEMPLIN

POWER BOILERS

H. E. ALDRICH, *Chairman*
PERRY CASSIDY
E. R. FISH
V. M. FROST
D. L. ROYER
A. C. WEIGEL

RULES FOR INSPECTION

(This subcommittee is being reorganized)

SPECIAL DESIGN

D. B. WESSTROM, *Chairman*
H. C. BOARDMAN
R. E. CECIL
T. W. GREENE
D. B. ROSSHEIM
W. H. ROWAND
E. O. WATERS
F. S. G. WILLIAMS

UNFIRED PRESSURE VESSELS

E. R. FISH, *Chairman*
C. A. ADAMS
C. E. BRONSON
R. E. CECIL
PAUL DISERENS
H. S. SMITH
D. B. WESSTROM

WELDING

Members of A.S.M.E. Boiler Code Committee

JAMES PARTINGTON, *Chairman*
E. C. CHAPMAN
J. H. DEPPELER
W. D. HALSEY
J. C. HODGE
R. K. HOPKINS
T. J. PHILLIPS
L. A. SHELDON

Members of Conference Committee of American Welding Society

C. W. OBERT, *Chairman*
C. A. ADAMS
H. C. BOARDMAN
WALTER SAMANS
A. C. WEIGEL

SPECIAL COMMITTEES

APPROVAL OF NEW MATERIALS

C. A. ADAMS, *Chairman*

CLAD VESSELS

S. K. VARNES, *Chairman*

SPECIAL COMMITTEE ON COORDINATION

V. M. FROST, *Chairman*

EXTENSION OF FUSION WELDING REQUIREMENTS

H. E. ALDRICH, *Chairman*

FEEDWATER

C. W. RICE, *Chairman*

ISSUANCE OF CODE SYMBOL STAMPS

C. O. MYERS, *Chairman*

RADIOGRAPHIC EXAMINATION OF WELDED JOINTS

C. A. ADAMS, *Chairman*

REVISION OF SECTION VIII OF THE A.S.M.E. BOILER CODE

E. R. FISH, *Chairman*

RULES FOR BOLTED FLANGED CONNECTIONS

D. B. WESSTROM, *Chairman*

RULES FOR OPENINGS

T. D. TIFFT, *Chairman*

SAFETY VALVE REQUIREMENTS

H. B. OATLEY, *Chairman*

API-ASME COMMITTEE ON UNFIRED PRESSURE VESSELS

WALTER SAMANS, *Chairman*

A.S.M.E. Representatives

R. E. CECIL
E. R. FISH
D. S. JACOBUS
T. MCLEAN JASPER
JAMES PARTINGTON

A.P.I. Representatives

A. J. ELY
K. V. KING
(P. D. MCELFISH, *Alternate*)
R. C. POWELL
WALTER SAMANS
T. D. TIFFT

THE WOMAN'S AUXILIARY TO THE A.S.M.E.

The Woman's Auxiliary to the A.S.M.E. was organized on May 10, 1923, and its Constitution and By-Laws was approved by the Council of the A.S.M.E. on October 27, 1924. The objects of the Auxiliary are to render service to all that pertains to the interest of the profession of mechanical engineering; to cooperate with any committees of the A.S.M.E.; and to assist the sons and daughters of the members of the Society or worthy students of mechanical engineering in obtaining scholarships; and to promote any other objects consistent with the aims or objects of the A.S.M.E.

OFFICERS

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First Vice-President, MRS. COLLINS P. BLISS
Second Vice-President, MRS. CALVIN W. RICE
Third Vice-President, MRS. CROSBY FIELD
Fourth Vice-President, MRS. J. PAGE HARBESON
Fifth Vice-President, MRS. HARRY A. SCHWARTZ
Recording Secretary, MRS. CHARLES H. FAY
Corresponding Secretary, MRS. F. M. GIBSON
Treasurer, MRS. A. H. MORGAN

STANDING COMMITTEE CHAIRMEN

Education, MRS. ROY V. WRIGHT
Membership, MRS. GEORGE E. HAGEMANN
Publicity and Custodian, MISS BURTIE HAAR

COUNCIL REPRESENTATIVES

To be appointed

OFFICERS OF LOCAL SECTIONS

BALTIMORE

Chairman, MRS. D. E. DONOVAN
Vice-Chairman, MRS. A. G. CHRISTIE
Secretary, MRS. J. H. BERRYMAN
Treasurer, MRS. LOUIS F. COFFIN

CLEVELAND

Chairman, MRS. HARRY A. SCHWARTZ
Vice-Chairman, MRS. T. F. GITHENS
Secretary, MRS. E. R. MCCARTHY
Treasurer, MRS. H. L. SPENCE

LOS ANGELES

Chairman, MRS. BERNARD TOBEN
Secretary, MRS. WILBUR W. REASER
Treasurer, MRS. WILLIAM T. POWELL

METROPOLITAN

Chairman, MRS. E. C. M. STAHL
First Vice-President, MRS. J. NOBLE LANDIS
Second Vice-President, MRS. R. B. PURDY
Third Vice-President, MRS. W. TALLMADGE
Recording Secretary, MRS. C. H. YOUNG
Corresponding Secretary, MRS. A. C. COONRADT
Treasurer, MRS. CHARLES GUS

PHILADELPHIA

Chairman, MRS. J. PAGE HARBESON, JR.
Vice-Chairman, MRS. E. F. ZEINER
Recording Secretary, MRS. JUSTIN MCCARTHY
Corresponding Secretary, MRS. F. E. WASHBURN
Treasurer, MRS. WILLIAM F. GLIMM

AWARDS

The following paragraphs deal with the medals, awards, scholarships, and loan funds which come within the jurisdiction of the A.S.M.E. Other awards available to Student Members are listed in *Mechanical Engineering*, February, 1938, page 183. The Society also participates with other engineering societies in a number of joint awards. Further details concerning all the awards will be found in a series of articles beginning in the October, 1938, issue of *Mechanical Engineering*.

Honorary Membership, to which persons of acknowledged professional eminence are elected by unanimous vote of Council under the provisions of the By-Laws and Rules. A list of honorary members is given on page RI-37.

Life Membership, which may be conferred by the Council for distinguished service to the Society; or secured by a member by payment for an annuity in accordance with the provisions of the By-Laws.

A.S.M.E. Medal, established by the Society in 1920 to be presented, together with an engraved certificate, for distinguished service in engineering and science. May be awarded for general service in science having possible application in engineering.

Holley Medal, instituted and endowed in 1924 by George I. Rockwood, Past Vice-President of the Society, to be bestowed, together with an engraved certificate, for some great and unique act of genius of engineering nature that has accomplished a great and timely public benefit.

Worcester Reed Warner Medal, provision for which was made in the will of Worcester Reed Warner, Honorary Member of the Society, is a gold medal to be bestowed, together with an engraved certificate, on the author of the most worthy paper received, dealing with progressive ideas in mechanical engineering or efficiency in management.

Melville Medal, established in 1914 by the bequest of Rear-Admiral George W. Melville, Honorary Member and Past-President of the Society, to be presented, together with an engraved certificate, for an original paper or thesis of exceptional merit, presented to the Society for discussion and publication, to encourage excellence in papers. The medal may be presented annually.

Spirit of St. Louis Medal, established by an endowment fund created in 1929 by citizens of St. Louis, Mo., to be awarded for meritorious service in the advancement of aeronautics. This medal will be awarded at the discretion of the Council of the Society at approximately three-year periods upon the recommendation of its Board of Honors and Awards.

Junior Award, annual cash award of $50, established in 1914, from a fund created by Henry Hess, Past Vice-President of the Society, to be presented, together with an engraved certificate, for the best paper or thesis submitted by a Junior Member.

Pi Tau Sigma Medal Award, established in 1938, endowed by Pi Tau Sigma, the national honorary mechanical engineering fraternity, to be presented annually, together with an engraved certificate, to the young mechanical engineer for outstanding achievement in his profession within the ten years after graduation from a regular four-year mechanical engineering course of a recognized American college or university. Any mechanical engineering graduate, not more than thirty-five years of age, whose achievement has been all or in part in any field including industrial, educational, political, research, civic, etc., is eligible.

Charles T. Main Award, annual cash award of $150, established in 1919 from a fund created by Charles T. Main, Past-President of the Society, to be awarded, together with an engraved certificate, to a Student Member of the Society, for the best paper within the general subject of the influence of the profession upon public life. The exact subject is assigned by the Board of Honors and Awards, subject to the approval of the Council, and is announced each year through the Honorary Chairman of the Student Branches.

Student Awards, two annual cash awards of $25 each, established in 1914, from a fund created by Henry Hess, Past Vice-President of the Society, to be presented, together with engraved certificates, for the best papers or theses submitted by Student Members. The awards for 1932 and subsequent years have been given, one for undergraduate and the other for postgraduate work.

SCHOLARSHIPS AND LOAN FUNDS

Max Toltz: Loan Fund of $15,000 established by Major Max Toltz, former member of the Council of the Society, the income to be used for assistance to Student Members.

John R. Freeman: Fund of $25,000 established in 1926 by John R. Freeman, Past-President of the Society, the income to be used for travel scholarships and research.

Woman's Auxiliary: Scholarship or Fellowship offered by the Woman's Auxiliary to the Society to assist sons and daughters of members or worthy students of mechanical engineering.

RECIPIENTS OF AWARDS

The names of the recipients of the different awards to date are given in the following lists, together with the dates of presentation, and the services or papers for which the awards were made. There were no awards for the years not listed.

A.S.M.E. Medal

1921 Hjalmar Gotfried Carlson, in recognition of the services rendered the Government because of his invention and part in the production of 20,000,000 Mark III drawn steel booster casings used principally as a component of 75-mm high-explosive shells, but also used extensively in gas shells and bombs

1922 Frederick Arthur Halsey, for his paper describing the premium system of wage payments presented before the Society at the Providence Meeting in 1891, as the adoption of the methods there proposed has had a profound effect toward harmonizing the relations of worker and employer

1923 John Ripley Freeman, for his eminent service in engineering and manufacturing by his meritorious work in fire prevention and the preservation of property

1926 R. A. Millikan, in recognition of his contributions to science and engineering

1927 Wilfred Lewis, for his contributions to the design and construction of gear teeth

1928 Julian Kennedy, for his services and contributions to the iron and steel industry

1929 William LeRoy Emmet, for his contributions in the development of the steam turbine, electric propulsion of ships, and other power-generating apparatus

1931 Albert Kingsbury, for his research and development work in the field of lubrication

1933 Ambrose Swasey, for his contributions to the advancement of the engineering profession and for his part in the development of the turret lathe and the astronomical telescope

1934 Willis H. Carrier, in recognition of his research and development work in air-conditioning

1935 Charles T. Main, for distinguished achievements in the textile and other industries, in engineering education, and for eminent service to the engineering profession

1936 Edward Bausch, for meritorious mechanical developments in the field of optics

1937 Edward P. Bullard, for outstanding leadership in the development of station-type machine tools

1938 Stephen J. Pigott, for outstanding leadership in marine propulsion and construction

1939 James E. Gleason, for service to the cause of safer and better transportation.

Holley Medal

1924 Hjalmar Gotfried Carlson, for his inventions and processes which made possible the timely production of drawn steel booster casings for artillery ammunition, thereby aiding victory in the World War (diploma in recognition of achievements presented in 1921)

1927 Elmer Ambrose Sperry, for achievements and inventions that have advanced the naval arts, including the gyroscope that has freed navigation from the dangers of the fluctuating magnetic compass

1929 Baron Chuzaburo Shiba, for his contributions to knowledge through fundamental research, including the field of aerodynamics, by the development of ultra-rapid kinematographic methods.

1934 Irving Langmuir, for his contributions to science and engineering, including the development of gas-filled incandescent lamps, thoriated filament for thermionic emission, atomic hydrogen welding, phase control operation of the thyratron tube, and fundamental research in oil films
1936 Henry Ford, for revolutionary influence through invention and practice on transportation and on mass production methods in manufacturing
1937 Frederick G. Cottrell, for preeminent public service—the invention of electric precipitation—advancement of the science of gas liquefaction—gifts for engineering research
1938 Francis Hodgkinson, for meritorious services in the development of the steam turbine
1939 Carl E. Johansson, in recognition of his pioneer work in the development of basic measuring gages.

Worcester Reed Warner Medal

1933 Dexter S. Kimball, for his contributions to efficiency in management as exemplified by his recently revised "Principles of Industrial Organization" and by his many articles, engineering society papers, and public addresses
1934 Ralph E. Flanders, for his contributions to a better understanding of the relationship of the engineer to economic problems and social trends as exemplified by the many papers which he has presented
1935 Stephen Timoshenko, for his contributions to the theory of the design of elastic structures and the treatment of dynamics of moving machinery
1936 Charles M. Allen, for his early and continued hydraulic laboratory work and for the permanent value of the papers on his development of methods of testing large hydraulic turbine installations
1937 Clarence F. Hirshfeld, for his research and contributions to the theory and practice of heat-power engineering as exemplified by books and papers
1938 Lawford H. Fry, for contributions relating to improved locomotive boiler design and utilization of better materials in railway equipment
1939 Rupen Eksergian, for influential papers of permanent value in A.S.M.E. Transactions.

Melville Medal

1927 Leon P. Alford, "Laws of Manufacturing Management"
1929 Joseph W. Roe, "Principles of Jig and Fixture Practice"
1930 Herman Diederichs and William D. Pomeroy, "The Occurrence and Elimination of Surge or Oscillating Pressure in Discharge Lines From Reciprocating Pumps"
1931 Arthur E. Grunert, "Comparative Performance of a Pulverized-Coal-Fired Boiler Using Bin System and Unit System of Firing"
1932 Alexey J. Stepanoff, "Leakage Loss and Axial Thrust in Centrifugal Pumps"
1933 William E. Caldwell, "Characteristics of Large Hell Gate Direct-Fired Boiler Units"
1935 Oscar R. Wikander, "Draft-Gear Action in Long Trains"
1936 H. A. Stevens Howarth, "The Loading and Friction of Thrust and Journal Bearings With Perfect Lubrication"
1937 Alfred J. Büchi, "Supercharging of Internal-Combustion Engines With Blowers Driven by Exhaust-Gas Turbines"
1938 Alphonse I. Lipetz, "Air Resistance of Railroad Equipment"
1939 Lester M. Goldsmith, for his paper, "High-Pressure High-Temperature Turbine-Electric Steamship *J. W. Van Dyke*."

Spirit of Saint Louis Medal

1929 Daniel Guggenheim, founder of the Guggenheim Fund for the Promotion of Aeronautics
1932 Paul Litchfield, for his work in encouraging and sponsoring airship design and construction in this country
1935 Will Rogers, for his splendid, constructive, and unselfish work in the achievement of aviation, and the building up of public confidence in aviation through his articles in the press, over the radio, and from the speaker's platform
1938 James H. Doolittle, for meritorious service in the advancement of aeronautics.

Junior Award

1915 Ernest O. Hickstein, "Flow of Air Through Thin Plate Orifices"
1916 L. M. McMillan, "The Heat Insulating Properties of Commercial Steam-Pipe Coverings"
1919 E. D. Whalen, "Properties of Airplane Fabrics"
1921 S. Logan Kerr, "Moody Ejector Turbine"
1922 R. H. Heilman, "Heat Losses From Bare and Covered Wrought-Iron Pipe at Temperatures up to 800 Degrees Fahrenheit"
F. L. Kallam, "Preliminary Report on the Investigation of the Thermal Conductivity of Liquids"
1923 S. S. Sanford and S. Crocker, "The Elasticity of Pipe Bends"
1924 R. H. Heilman, "Heat Losses Through Insulating Material"
1925 Gilbert S. Schaller, "An Investigation of Seattle as a Location for a Synthetic Foundry Industry"
1927 William M. Frame, "Stresses Occurring in the Walls of an Elliptical Tank Subjected to Low Internal Pressure"
1928 M. D. Aisenstein, "A New Method of Separating the Hydraulic Losses in a Centrifugal Pump"
1929 Arthur M. Wahl, "Stresses in Heavy, Closely Coiled Helical Springs"
1930 Ed Sinclair Smith, "Quantity-Rate Fluid Meters"
1931 M. K. Drewry, "Radiant-Superheater Developments"
1932 Edmond M. Wagner, "Frictional Resistance of a Cylinder Rotating in a Viscous Fluid Within a Coaxial Cylinder"
1933 Townsend Tinker, "Surface Condenser Design and Operating Characteristics"
1934 John I. Yellott, Jr., "Supersaturated Steam"
1935 Stanley J. Mikina, "Effect of Skewing and Pole Spacing on Magnetic Noise in Electrical Machinery"
1936 Harwood F. Mullikan, Jr., "Evaluation of Effective Radiant Heating Surface and Application of the Stefan-Boltzman Law to Heat Absorption in Boiler Furnaces"
1937 Leslie J. Hooper, "American Hydraulic-Laboratory Practice"
1938 Arthur C. Stern, "Separation and Emission of Cinders and Fly Ash."

Pi Tau Sigma Medal

1938 Wilfrid E. Johnson, for his development work in the field of refrigeration
1939 John I. Yellott, Jr., in recognition of significant achievements in steam-flow research and engineering education; also contributions on "Supersaturated Steam" and "Condensation of Flowing Steam in Diverging Nozzles."

Charles T. Main Award

1925 Clement R. Brown, Catholic University of America. Subject: "The Influence of the Locomotive on the Unity of the United States"
1926 W. C. Saylor, Johns Hopkins University. Subject: "The Effect of the Cotton Gin Upon the History of the United States During Its First Seventy Years"
1927 No Award. Subject: "Scientific Management and Its Effect Upon the Industries"
1928 Robert M. Meyer, Newark College of Engineering. Subject: "Scientific Management and Its Effect Upon Manufacturing"
1929 No award. Subject: "The Influence of Engineering on Farm Production"
1930 Jules Podnossoff, Polytechnic Institute of Brooklyn. Subject: "The Value of the Safety Movement in the Industries"
1931 Robert E. Klise, University of Michigan. Subject: "Interchangeability—Its Development and Significance in Industry"
1932 Marshall Anderson, University of Michigan. Subject: "Apprenticeship and Vocational Training"
1933 George D. Wilkinson, Jr., Newark College of Engineering. Subject: "Progress in the Prevention of Smoke and Atmospheric Pollution"
1934 Philip P. Self, Colorado State College, Subject: "Air Conditioning—Its Practicability and Relation to Public Welfare"
1935 G. Lowell Williams, Lafayette College. Subject: "Coordinated Transportation—An Economic Comparison of Railroad, Bus, Truck, Water, and Air Transportation for Long and Short Haul"

1936 No award. Subject: "Development in the Generation and Distribution of Power and Their Effect Upon the Consumer"

1937 ALLAN P. STERN, Case School of Applied Science. Subject: "The Influence of the Introduction of Labor Saving Machinery Upon Employment in the United States"

1938 EDWARD W. CONNOLLY, University of Detroit. Subject: "Economic Limitations in Engineering Design, With Concrete Examples"

1939 JAMES R. BRIGHT, Lehigh University. Subject: "The Economics of Investment in New Manufacturing Equipment—With Concrete Cases."

STUDENT AWARD

1916 BOYNTON M. GREEN, Stanford University, "Bearing Lubrication"
HOWARD E. STEVENS, Rensselaer Polytechnic Institute, "An Investigation of the Dynamic Pressure on Submerged Flat Plates"
M. ADAM, Louisiana State University, "The Adaptability of the Internal Combustion Engine to Sugar Factories and Estates"

1917 H. R. HAMMOND and C. W. HOLMBERG, The Pennsylvania State College, "Study of Surface Resistance With Glass as the Transmission Medium"

1919 C. F. LEH and F. G. HAMPTON, Stanford University, "An Experimental Investigation of Steel Belting"
W. E. HELMICK, Stanford University, "An Experimental Investigation of Steel Belting"

1920 HOWARD G. ALLEN, Cornell University, "Wire Stitching Through Paper"

1921 KARL H. WHITE, University of Kansas, "Forces in Rotary Motors"
RICHARD H. MORRIS and ALBERT J. R. HOUSTON, University of California, "A Report Upon an Investigation of the Herschel Type of Improved Weir"

1923 CHARLES F. OLMSTEAD, University of Minnesota, "Oil Burning for Domestic Heating"
H. E. DOOLITTLE, University of California, "The Integrating Gate: A. Device for Gaging in Open Channels"

1924 GEORGE STUART CLARK, Stanford University, "Two Methods Used for the Determination of the Gasoline Content of Absorption Oils in Absorption Plants"
L. J. FRANKLIN and CHARLES H. SMITH, Stanford University, "The Effect of Inaccuracy of Spacing on the Strength of Gear Teeth"

1925 HARRY PEASE COX, JR., Rensselaer Polytechnic Institute, "A Study of the Effect of End Shape on the Towing Resistance of a Barge Model"
W. S. MONTGOMERY, JR., and E. RAY ENDERS, JR., Pennsylvania State College, "Some Attempts to Measure the Drawing Properties of Metals"

1926 R. E. PETERSON, University of Illinois, "An Investigation of Stress Concentration by Means of Plaster of Paris Specimens"
CECIL G. HEARD, University of Toronto, "Pressure Distribution Over U.S.A. 27 Aerofoil With Square Wing Tips—Model Tests"

1927 ALFRED H. MARSHALL, Princeton University, "Evaporative Cooling"
ROGER IRWIN EBY, University of Washington, "Measurement of the Angular Displacement of Flywheels"

1928 CLARENCE C. FRANCK, Johns Hopkins University, "Condition Curves and Reheat Factors for Steam Turbines"

1929 FRANK VERNON BISTROM, University of Washington, "An Investigation of a Rotary Pump"
WILLIAM WALLACE WHITE, University of Washington, "An Investigation of a Rotary Pump"

1930 GERARD EDEN CLAUSSEN, Polytechnic Institute of Brooklyn, "High-Temperature Oxidation of Steel"
HAROLD L. ADAMS and RICHARD L. STITH, University of Washington, "A Wind Tunnel for Undergraduate Laboratory Experiments"

1931 JULES PODNOSSOFF, Polytechnic Institute of Brooklyn, "Pressure and Energy Distribution in Multi-Stage Steam Turbines Operating Under Varying Conditions"

1932 H. E. FOSTER, JR., University of Tennessee, "Factors Affecting Spray Pond Design" (Undergraduate Award)
WILLIAM A. MASON, Stanford University, "An Experimental Investigation of the Flame Propagation in Internal-Combustion Engines" (Postgraduate Award)

1933 HUGO V. CORDIANO, Polytechnic Institute of Brooklyn, "Thermal Analysis of Lithium-Magnesium System of Alloys" (Undergraduate Award)
JAMES A. OSTRAND, JR., Princeton University, "Sudden Enlargement in the Open Channel" (Postgraduate Award)

1934 H. REYNOLDS HUDSON, Georgia School of Technology, "Dynamic Balance and Functional Utility Applied to Automotive Design" (Undergraduate Award)

1935 CHARLES P. BACHA, Rutgers University, "The Behavior of Metals Subjected to Combined Stress" (Postgraduate Award)
ROBERT W. BEAL, Oregon State College, "Do Lubricating Oils Wear Out?" (Undergraduate Award)

1936 LEON B. STINSON, Oklahoma Agricultural and Mechanical College, "Polymerized Motor Fuels; Their Economic Significance" (Undergraduate Award)
DEWITT D. BARLOW, JR., Princeton University, "The Critical Speeds of Lateral Vibrations of Shafts With Gyroscopic Effects" (Postgraduate Award)

1937 GINO J. MARINELLI, Rensselaer Polytechnic Institute, "Investigation of the Towing Resistance of a Model Submarine Hull" (Undergraduate Award)

1938 MARSHALL C. LONG, Princeton University, "An Investigation Into the Angular Characteristics of an Adjustable Blade Current Meter" (Postgraduate Award)
DONALD C. MCSORLEY, Michigan State College, "Humidity Insulation" (Undergraduate Award)

1939 DAVID T. JAMES, Michigan State College, "Bells—Concerning Their Tones" (Undergraduate Award).

FREEMAN TRAVEL SCHOLARSHIP

1927 HERBERT N. EATON
1928 BLAKE R. VAN LEER
1929 ROBERT T. KNAPP
1931 REGINALD WHITAKER
1932 G. ROSS LORD
1933, 1934 H. J. CASEY
1935, 1936 VICTOR L. STREETER

HONORARY MEMBERS

HONORARY MEMBERS IN PERPETUITY

ALEXANDER LYMAN HOLLEY, Founder of the Society. Died 1882.

JOHN EDSON SWEET, Founder of the Society. Died 1916.

HENRY ROSSITER WORTHINGTON, Founder of the Society. Died 1880.

DECEASED HONORARY MEMBERS

	ELECTED	DIED
HORATIO ALLEN	1880	1889
SIR WILLIAM ARROL	1905	1913
SIR JOHN AUDLEY FREDERICK ASPINALL	1911	1937
WILLIAM WALLACE ATTERBURY	1925	1935
SIR BENJAMIN BAKER	1886	1907
JOHANN BAUSCHINGER	1884	1893
SIR HENRY BESSEMER	1891	1898
SIR FREDERICK JOSEPH BRAMWELL	1884	1903
JOHN ALFRED BRASHEAR	1908	1920
GUSTAVE CANET	1900	1908
ANDREW CARNEGIE	1907	1919
DANIEL KINNEAR CLARK	1882	1896
RUDOLPH JULIUS EMMANUEL CLAUSIUS	1882	1888
SIR JOHN GOODE	1889	1892
PETER COOPER	1882	1883
CHARLES DE FRÉMINVILLE	1919	1936
CARL GUSTAF PATRICK DE LAVAL	1912	1913
RUDOLPH DIESEL	1912	1913
JAMES DREDGE	1886	1906
VICTOR DWELSHAUVERS-DERY	1886	1913
THOMAS ALVA EDISON	1904	1931
ALEXANDRE GUSTAVE EIFFEL	1889	1923
MARSHAL FERDINAND FOCH	1921	1929
SIR CHARLES DOUGLAS FOX	1900	1921
JOHN RIPLEY FREEMAN	1932	1932
JOHN FRITZ	1900	1913
MAJOR-GENERAL GEORGE WASHINGTON GOETHALS	1917	1928
FRANZ GRASHOF	1884	1893
REAR-ADMIRAL ROBERT STANISLAU GRIFFIN	1920	1933
OTTO HALLAUER	1882	1883
CHARLES HAYNES HASWELL	1905	1907
NATHANAEL GREENE HERRESHOFF	1921	1938
FRIEDRICH GUSTAV HERRMANN	1884	1907
GUSTAV ADOLPH HIRN	1882	1890
JOSEPH HIRSCH	1889	1901
IRA N. HOLLIS	1928	1930
ROBERT WOOLSTON HUNT	1920	1923
BENJAMIN FRANKLIN ISHERWOOD	1894	1915
HENRI LÉAUTÉ	1891	1916
ERASMUS DARWIN LEAVITT	1915	1916
HENRI LE CHATELIER	1927	1936
ANATOLE MALLET	1912	1919
CHARLES H. MANNING	1913	1919
REAR-ADMIRAL GEORGE WALLACE MELVILLE	1910	1912
THE HONORABLE SIR CHARLES ALGERNON PARSONS	1920	1931
CHARLES TALBOT PORTER	1890	1910
AUGUSTE C. E. RATEAU	1919	1930
SIR EDWARD J. REED	1882	1906
FRANZ REULEAUX	1882	1905
CALVIN WINSOR RICE	1931	1934
PALMER C. RICKETTS	1931	1934
HENRI ADOLPHE-EUGENE SCHNEIDER	1882	1898
CHARLES M. SCHWAB	1918	1939
C. WILLIAM SIEMANS	1882	1883
VISCOUNT EI-ICHI SHIBUSAWA	1929	1931
AMBROSE SWASEY	1916	1937
ELIHU THOMSON	1930	1937
HENRY ROBINSON TOWNE	1921	1924
HENRI TRESCA	1882	1885
WILLIAM CAWTHORNE UNWIN	1898	1933
OSKAR VON MILLER	1912	1934
FRANCIS A. WALKER	1886	1897
WORCESTER REED WARNER	1925	1929
GEORGE WESTINGHOUSE	1897	1914
SIR WILLIAM HENRY WHITE	1900	1913
SIR ALFRED FERNANDEZ YARROW	1914	1932

LIVING HONORARY MEMBERS

	ELECTED
LORENZO ALLIEVI	1937
EDMUND BRUCE BALL	1939
HUTCHINSON I. CONE	1936
MORTIMER ELWYN COOLEY	1928
ALEX DOW	1936
WILLIAM FREDERICK DURAND	1934
HERBERT CLARK HOOVER	1925
DAVID SCHENCK JACOBUS	1934
MASAWO KAMO	1929
DEXTER SIMPSON KIMBALL	1939
CHARLES THOMAS MAIN	1939
GEORGE A. ORROK	1936
GRANDE UFFICIALE ING. PIO PERRONE	1920
EDWIN JAY PRINDLE	1939
WILLIAM H. TSCHAPPAT	1938
SAMUEL MATTHEWS VAUCLAIN	1920
HENRY HAGUE VAUGHAN	1939
RIGHT HONORABLE LORD WEIR	1920
ORVILLE WRIGHT	1918

PAST-PRESIDENTS

A list of past vice-presidents, managers, treasurers, and secretaries will be found in the 1930 Record and Index, pages 10-12. Dates in parentheses denote year of death.

ALEXANDER LYMAN HOLLEY, *Chairman of the Preliminary Meeting for Organization of The American Society of Mechanical Engineers* (1882)

1880–1882 ROBERT HENRY THURSTON (1903)
1883 ERASMUS DARWIN LEAVITT (1916)
1884 JOHN EDSON SWEET (1916)
1885 JOSEPHUS FLAVIUS HOLLOWAY (1896)
1886 COLEMAN SELLERS (1907)
1887 GEORGE H. BABCOCK (1893)
1888 HORACE SEE (1909)
1889 HENRY ROBINSON TOWNE (1924)
1890 OBERLIN SMITH (1926)
1891 ROBERT WOOLSTON HUNT (1923)
1892 CHARLES HARDING LORING (1907)
1893–1894 ECKLEY BRINTON COXE (1895)
1895 EDWARD F. C. DAVIS (1895)
1895 CHARLES ETHAN BILLINGS (1920)
1896 JOHN FRITZ (1913)
1897 WORCESTER REED WARNER (1929)
1898 CHARLES WALLACE HUNT (1911)
1899 GEORGE WALLACE MELVILLE (1912)
1900 CHARLES HILL MORGAN (1911)
1901 SAMUEL T. WELLMAN (1919)
1902 EDWIN REYNOLDS (1909)
1903 JAMES MAPES DODGE (1915)
1904 AMBROSE SWASEY (1937)
1905 JOHN RIPLEY FREEMAN (1932)
1906 FREDERICK WINSLOW TAYLOR (1915)
1907 FREDERICK REMSEN HUTTON (1918)
1908 MINARD LAFEVER HOLMAN (1925)
1909 JESSE MERRICK SMITH (1927)
1910 GEORGE WESTINGHOUSE (1914)
1911 EDWARD DANIEL MEIER (1914)
1912 ALEXANDER CROMBIE HUMPHREYS (1927)
1913 WILLIAM FREEMAN MYRICK GOSS (1928)
1914 JAMES HARTNESS (1934)
1915 JOHN ALFRED BRASHEAR (1920)
1916 DAVID SCHENCK JACOBUS
1917 IRA NELSON HOLLIS (1930)
1918 CHARLES THOMAS MAIN
1919 MORTIMER ELWYN COOLEY
1920 FRED J. MILLER (1939)
1921 EDWIN S. CARMAN
1922 DEXTER SIMPSON KIMBALL
1923 JOHN LYLE HARRINGTON
1924 FREDERICK ROLLINS LOW (1936)
1925 WILLIAM FREDERICK DURAND
1926 WILLIAM LAMONT ABBOTT
1927 CHARLES M. SCHWAB (1939)
1928 ALEX DOW
1929 ELMER AMBROSE SPERRY (1930)
1930 CHARLES PIEZ (1933)
1931 ROY V. WRIGHT
1932 CONRAD N. LAUER
1933 A. A. POTTER
1934 PAUL DOTY (1938)
1935 RALPH E. FLANDERS
1936 WILLIAM L. BATT
1937 JAMES H. HERRON
1938 HARVEY N. DAVIS
1939 ALEXANDER G. CHRISTIE

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Memorial Biographies

SOCIETY RECORDS—Part II

Part I of Society Records for the year 1940, containing Council and Committee personnel and other general information, was issued as Section Two of the Transactions for February, 1940. Part II is published for inclusion with all bound copies of Volume 62 of the Transactions and distributed to members of the Society upon request.

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Memorial Biographies

THE purpose of Memorial Biographies is to place on permanent record the biographical and professional data relating to deceased members of The American Society of Mechanical Engineers. Hence every effort is expended to insure accuracy, and to make the memorials as inclusive as is reasonably possible.

The first source of information upon which these notices are based is the Society's file of membership applications and transfers. In the case of the more recent member, these application records are fairly complete. The applications of those who became members many years ago, however, contain less detailed data, and in many cases the sponsors are no longer alive, so that it is difficult to obtain assistance from this source. If the member has been retired for several years prior to his death, his business associates are frequently hard to locate, and, in some cases, members of his family cannot be found. While all these factors add to the difficulty of obtaining accurate and fairly complete data, every possible source of information is explored, with the result that publication of the notice is sometimes delayed. A number of the memorials in this issue have been prepared under these difficulties.

It is the practice of the Committee on Publications in the case of some deceased members to ask former friends and associates to prepare the obituary. The object is to secure a final record that will be more valuable for having been prepared by men who knew the deceased and are competent to evaluate his work. Memorials prepared in accordance with this policy are signed by those who wrote them or who collaborated in their preparation. To all persons who have thus cooperated, the Committee acknowledges its gratitude.

The Committee also appreciates and acknowledges the assistance that has been given by relatives, business associates, and friends in the preparation of all other memorials. It also acknowledges its debt to such sources as Who's Who in Engineering, Who's Who in America, and similar publications; the Encyclopedia of American Biography, and the National Cyclopedia of American Biography; the technical and daily press; colleges and universities and their alumni associations; and engineering and other societies which have supplied information from their records.

Relatives, business associates, and Local Section and Student Branch officers are urged to notify the Society promptly of the deaths of members. Newspaper clippings or obituaries in any other form should be sent whenever available and the names and addresses of those who can supply further information should be furnished. A special form for supplying complete details will be forwarded by the Office of the Society upon request.

THE COMMITTEE ON PUBLICATIONS

Memorial Biographies

GEORGE PENNELL ABORN (1863–1938)

George Pennell Aborn, in charge of the Industrial Department of the Harvard Trust Company, Cambridge, Mass., died on July 7, 1938, of coronary thrombosis, at Waban, Mass.

Mr. Aborn was born at Wakefield, Mass., on June 8, 1863, the son of George W. and Mary Frances (Pennell) Aborn. He prepared for college in the Wakefield High School, followed by one year at the Phillips Academy, Andover, Mass., and was graduated in mechanical engineering from the Massachusetts Institute of Technology in 1886, with the B.S. degree. During his college years he was employed in the vacation periods in the office of Percy M. Blake, waterworks consulting engineer of Boston, and there gained early practical experience which was a valuable asset in his later work.

In October of 1886, Mr. Aborn went to work as draftsman at the Knowles Steam Pump Works, Warren, Mass., advancing to assistant constructing engineer in 1890.

The following account of his work from then until the end of 1926, when his fortieth anniversary with the company was celebrated, is taken from "Simplexities," the Blake and Knowles house organ, December, 1926.

"It was at about this time that the compound duplex direct-acting pump was being looked upon with general favor as the most suitable type of engine for municipal water works pumping stations and for large mining service, and Mr. Aborn was responsible for the design and successful operation of the many units, some of them the largest ever made in the history of direct-acting pumping engine manufacture, for which the Knowles Works was justly well known in those days. He developed and patented a high-duty pumping engine at this period which showed much higher economy than the direct-acting pumps as regularly built, but the gradual introduction of flywheel pumping engines for large water pumping duty prevented further development along this line.

"Mr. Aborn also invented and patented many improvements in pumping machinery, air compressors, and power-plant auxiliaries which became adopted practice in the Knowles product. Of a generally inventive mind, his activities naturally overflowed to outside lines and several patents stand to his credit covering devices of a widely varied character.

"In 1896, Mr. Aborn was made chief engineer and assistant to the manager. When the Knowles Steam Pump Works was consolidated with the George F. Blake Manufacturing Company, at East Cambridge in 1897, the title of the company then becoming Blake and Knowles Steam Pump Works, Mr. Aborn was made assistant manager and chief engineer of the combined plants, and in 1905, he became manager of the Blake and Knowles Steam Pump Works (now the Blake and Knowles Works of the Worthington Pump and Machinery Corporation).

"During this East Cambridge period, Mr. Aborn's inventive genius has from time to time manifested itself in valuable improvements applied to the various lines of product built by the company, covering sinking pumps, air compressors, attachments and unloaders, exhaust relief valves, engine valves, a turbinated pump valve, novel arrangements of compound steam ends, etc. He was one of the first to investigate the possibilities of a combined wet and dry vacuum pump.

"Mr. Aborn has always been deeply interested in the matter of steam valve gearing, and has given much attention to the subject, particularly with reference to the operation of single pumps, and has devised many improvements in the means of accomplishing satisfactory valve movements, among them being the well-known simplex style 'A' gear which for twelve years has been adopted as the standard for all Blake-Knowles single pumps, and entirely supersedes both the original Knowles and the Blake types. All the thousands of simplex pumps built for the Naval program during the past war were equipped with this gear, and its great popularity with naval engineers and with pump operators generally is a sufficient testimonial of its excellence.

"Mr. Aborn has also developed a simplex cut-off valve gear, a radical departure in direct-acting pump operation which has shown an increase of efficiency and decrease in steam consumption amounting to from 15 to 20 per cent."

In February, 1928, after the East Cambridge plant closed, Mr. Aborn joined the staff of the Harvard Trust Company, Cambridge, Mass., in its Industrial Department, and subsequently was appointed manager, which position he was holding at the time of his death.

Mr. Aborn joined the A.S.M.E. as a junior in 1889, and was promoted to the member grade in 1892. He was also a member of the Society of Naval Architects and Marine Engineers and of the American Society of Naval Engineers. He was a past-president of the Boston Branch of the National Metal Trades Association, of the New England Foundrymen's Association, and of the American Association of Foundrymen. In 1931–1933 he was treasurer of the Cambridge Industrial Association and he served as its president in 1933–1934. He also belonged to the Masonic fraternity and to a number of Boston clubs, including the Engineers and University.

In 1890, Mr. Aborn married Maude Nutting Powers, of Warren, who survives him, together with their two children, Rachel (Aborn) Andersen and Pennell N. Aborn.

HARALD AHLQVIST (1876–1938)

Harald Ahlqvist, consulting engineer, New York, N.Y., died of coronary thrombosis, on September 6, 1938, at Kingston, Jamaica, while on a West Indian cruise. Mr. Ahlqvist was born on October 16, 1876, in Fredrikshamn, Finland, the son of Carl Henrik and Anna Luisa Ahlqvist. He attended the Real Lyceum, Fredrikshamn, and a school in Hamburg, Germany, and in 1901 was graduated in mechanical engineering from the Royal Polytechnicum, Dresden, Germany.

From March, 1896, to March, 1898, he served an apprenticeship at machine building in the works of Sulzer Brothers, Winterthur, Switzerland, and during 1899 was in military training in the Finnish Dragoons. For three months in 1900 he was employed in the drafting room of Berliner Maschinenbau, Berlin, Germany. In 1901, Mr. Ahlqvist came to the United States and was employed by the Southwark Foundry & Machine Co., Philadelphia, Pa., and by the Sharon Steel Co., Sharon, Pa., designing rod and wire machines, especially those used for producing barbed wire and wire netting. This was followed by a term as engineer for the Compania Industria de Turba in Mexico (November, 1903, to May, 1904), erecting a plant and installing machinery for peat briquetting. From then until August, 1905, he was engineer for Oficina Technica Pablo Bergner S. en C., Mexico, D.F., testing and designing mining and milling machinery, milling plants, and plants for the manufacture of rubber from guayule.

In 1906, he returned to the United States and joined the staff of Semet-Solvay Company, and the Solvay Process Company, Syracuse, N.Y., as designing engineer. He remained with this firm until 1922, becoming superintendent of operations in 1908 and assistant to the general manager in 1918. In 1919 he made a survey for the company in the Southwest which led to the establishment of three large alkali plants in Louisiana and Texas.

In 1922, Mr. Ahlqvist opened his own office as consulting engineer, specializing in the design, construction, and operation of soda ash, caustic soda, sodium bicarbonate, calcium chloride, and potassium chloride plants, lime kilns, and the manufacture of alkaline products in general. He designed and built plants in the United States and many other parts of the world, including Mexico, Venezuela, Germany, and Japan. He also remodelled and modernized the equipment of many plants, among which was the Mathieson Alkali Works, at Saltville, Va. He had taken out patents on an ammonia still, a sulphur recovery process, and a clarifier, designed primarily for ammonia soda liquors but usable for filtering other liquids.

Mr. Ahlqvist joined the A.S.M.E. as a member in 1909. He was also a member of the American Institute of Mining and Metallurgical Engineers, American Chemical Society, and American Institute of Chemical Engineers. His clubs included the Chemists' Club, New York, and the Apawamis, Manursing, and American Yacht clubs of Rye, N.Y., where he had for many years made his home. He was a fluent linguist, speaking not only his native tongue and English, but Swedish, French, and German as well. He became an American citizen in June, 1912, at Syracuse, N.Y.

He married, in 1903, Eleanor St. Barbe Pattee, of Brookline, Mass., now residing in San Diego, Calif.

SAMUEL AITKEN (1882–1938)

Samuel Aitken, vice-president of Moore & McCormack Co., Inc., (now Moore-McCormack Lines, Inc.), steamship agents, New York, N.Y., died on August 12, 1938.

Mr. Aitken was born on July 21, 1882, in Belfast, Ireland, the son of Robert and Annie (Kane) Aitken. He attended Belfast schools, including the Academical Institute, and then served as apprentice engineer in the famous shipyards of Harland & Wolff, Belfast, covering the years 1897–1902. At the completion of this training, Mr. Aitken went to sea as junior engineer, and continued in service with the White Star Line and the Pacific Mail Steamship Company until 1916, advancing to the position of chief engineer. In 1911, he was chief engineer of the S.S. *Asia* when it was wrecked on Finger Rocks in the

China Sea. Mr. Aitken, together with the captain and chief officer, spent the whole night fighting off Chinese pirates, during which they fired some five-hundred rounds of ammunition. The vessel had on board more than 200 passengers, all of whom were landed safely, and the cargo was kept intact.

In 1916–1917 Mr. Aitken was assistant superintendent of repairs for the Moore & Scott Iron Works, San Francisco, Calif. From then until 1919, he was employed by the U.S. Shipping Board, first as assistant to its representative in the Far East, stationed at Manila, where he was in charge of repairing interned German ships, and later as engineer-representative of the Board of Inspection and port engineer for the North Atlantic district. He organized the Engineering Department of the Board of Inspection and as its senior engineer had charge of the supervision of maintenance and personnel of U.S. Shipping Board vessels in the department. The Inspection Board, which investigated all accidents, strandings, etc., covered, during the latter part of 1918, New York, Philadelphia, Baltimore, Norfolk, and Newport News. Mr. Aitken's duties as post engineer were similar to those for the Board of Inspection.

In 1919, Mr. Aitken was appointed superintendent-engineer of Moore & McCormack Co., Inc., steamship agents, in charge of the Engine Department and of repairs and new construction in all departments of vessels controlled by the company. He was elected vice-president of the company in 1923. During the time he served the company in that office he made alterations to the Hog Island type of vessel to raise its speed from the 11 knots for which it was designed to maintain a sea speed of $13^1/_2$ knots. In this connection, Mr. Aitken took out several patents, foremost among which was the "drag eliminator," which was a form of streamlining around the stern post and rudder of the vessel.

Mr. Aitken joined the A.S.M.E. as an associate-member in 1921, and in 1924 was promoted to the member grade. Besides his membership in the A.S.M.E., he was also a civil member of the Society of Naval Engineers, and a member of the Society of American Military Engineers, the Society of Naval Architects and Marine Engineers, and the Institution of Marine Engineers of Great Britain. In 1936, he was appointed by the U.S. Secretary of Labor as the delegate to the International Maritime Labor Conference at Geneva, representing American steamship owners. He was also a member of the Board of Visitors of the New York State Merchant Marine Academy. He belonged to the Shackmaxon Country Club and Downtown Athletic Club. He held licenses as chief engineer from both the British and American governments. Though born in Ireland, he was a citizen of the United States.

In 1910, Mr. Aitken married Margaret Eugenia Scott, a native of Philadelphia, and is survived by her and by their son, George Robert Aitken.

WILLIAM ALEXANDER, II (1910–1938)

William Alexander, II, production foreman, Scott Paper Company, Chester, Pa., died on February 6, 1938, at Hackensack, N.J.

Mr. Alexander was born in Edinburgh, Scotland, on November 24, 1910, the son of William and Annisabel (Cox) Alexander. As a child, he emigrated with his parents to the United States, and he obtained his preparatory education in the Hasbrouck Heights, N.J., high school. He later attended the Rensselaer Polytechnic Institute for two years and the Massachusetts Institute of Technology, where he was graduated with the degree of B.S. in mechanical engineering, in 1932. He later did postgraduate work in accounting at the University of Pennsylvania.

Immediately after the completion of his technical education, Mr. Alexander went to work for the Scott Paper Company, Chester. From August, 1932, to December of that year he operated a paper converting machine; from then until July, 1934, he worked in the cost department; and in July, 1934, he was promoted to production foreman, the position he held at the time of his death.

Mr. Alexander joined the A.S.M.E. as a junior in 1932. He was an American citizen due to his father's being naturalized while he, the son, was a minor. He was unmarried.

GEORGE FRANCIS ANDERSON (1884–1938)

George Francis Anderson, chief engineer of England-Walton & Co., Hazelwood, N.C., died in Philadelphia, Pa., on June 23, 1938.

Mr. Anderson was born in Oakland, Calif., on October 20, 1884, the son of John Francis and Maria Agnes Anderson. His education was obtained in the public high school, followed by classes at the Humbolt Evening Technical School, and he served a four-years' apprenticeship as machinist, 1901–1904, with the Risdon Iron Works, San Francisco. He then spent about twelve years at sea, first as assistant engineer with the American-Hawaiian and Pacific Mail Steamship Line, 1905–1912, and during the next four years as chief engineer for the Eschen & Minor Steamship Co. During the year 1916–1917, Mr. Anderson was employed as lubrication and sales engineer by the Texas Oil Company, and Standard Oil Company of California. In 1918, he was a first lieutenant in the Air Service of the U.S. Army. From 1919 to 1925 he served at Philadelphia as senior inspector of hulls and machinery for the U.S. Shipping Board, Emergency Fleet Corporation.

When applying for membership in the A.S.M.E. in 1925, Mr. Anderson stated that although he had not taken out any patents he had done considerable design work for changes in vessels under his charge. He mentioned tie bolts for holding brickwork in Foster boilers; improvements in baffles; a time-saving tool for back-facing holes in headers for header plugs; oil burners; oil alarm devices to indicate low lubricating oil in turbine units; and engineroom appliances.

Following his service with the Shipping Board, Mr. Anderson joined the staff of the A. C. Lawrence Leather Co., being attached to its subsidiary, England-Walton & Co., of Hazelwood, N.C., as chief engineer, in charge of six separate plants, and he was holding this position at the time of his death.

Mr. Anderson joined the A.S.M.E. as a member in 1925. He was also a member of the Society of American Military Engineers. In 1921, he married Blanche Oliver, of Parkersburg, W.Va., who survives him.

JAMES DHU ANDREW (1874–1937)

James Dhu Andrew, manager of the American Boiler Manufacturers Association, New York, N.Y., died of pneumonia at his home in Englewood, N.J., on March 22, 1937. Mr. Andrew was born in Brooklyn, N.Y., on October 29, 1874, the son of James D. and Olive (Stevens) Andrew. He attended grammar and high school in Brooklyn, and subsequently spent two years in the School of Mines of Columbia University.

From 1896 to 1899, Mr. Andrew worked as erecting and testing engineer for the Edward P. Allis Company, Milwaukee, Wis. From 1899 to 1902, during the period of its electrification, he served as mechanical engineer for the Metropolitan Street Railway Company, New York, and for the next three years was chief engineer of the New York Edison Company.

In 1907, Mr. Andrew began a term of six years as superintendent of power for the Boston (Mass.) Elevated Railway system, and then spent four years as superintendent of station engineering for the Edison Electric Illuminating Company of Boston. From 1917 to 1920, he was general superintendent of plant construction and manager of hull construction of the American International Shipbuilding Corporation, Hog Island, Pa. During the next three years he was president, first of the American Balsa & Balsa Refrigerator Company, New York, and then of the Standard Tank Car Company, Sharon, Pa. From 1923 to 1926 he was vice-president of Stevens & Wood, Inc., engineers and constructors, New York, in charge of the design, construction, and operation of some power plants in Ohio and Pennsylvania, and also engaged in engineering surveys in South America.

Mr. Andrew acted as general consulting engineer for Armour & Co., of Chicago, Ill., from 1926 to 1929, and then became chief engineer of the Niagara & Hudson Power Corp. In 1933, he was appointed manager of the American Boiler Manufacturers Association and Affiliated Industries, and was thus employed at the time of his death.

In 1904, Mr. Andrew became a member of the A.S.M.E. He was also a member of the American Institute of Electrical Engineers, Society of Naval Architects and Marine Engineers, the Engineers Club, New York, the American Trade Association Executives, Winslow Lewis Lodge, F. & A. M., and Aleppo Shrine in Boston. He served as a member of the Board of Governors of the Englewood Hospital Association.

Mr. Andrew married Laura Ward, of Napanee, Ontario, in 1902, and is survived by her and by their son, James D., Jr., of Essex Fells, N.J.

JESSE OLIVER ARKEBAUER (1871–1937)

Jesse Oliver Arkebauer, marine surveyor and consulting engineer, New York, N.Y., died at the Marine Hospital, on Staten Island, on April 13, 1937, after nearly a year's illness. He is survived by his widow, Ada (Wheeler) Arkebauer, whom he married in 1900, and by a daughter, Gladys.

Mr. Arkebauer was well known in steamship and shipbuilding circles throughout the United States. He held an unlimited license as chief engineer on ocean steamships; had received a certificate of proficiency from the Brooklyn Polytechnic Institute for two years' special training in the design, construction, and operation of Diesel engines;

and was a graduate from the five-year course in general science of Cooper Union, receiving a mechanical engineering degree in 1919. He was commissioned an Engineer Officer in the 2nd Battalion of the New York State Naval Militia in May, 1915, and was retired in September, 1935, with the rank of Lieutenant-Commander in the U.S. Naval Reserve. He had been a member of the A.S.M.E. since 1921, and was also a member of the Society of Naval Architects and Marine Engineers, Propeller Club of the U.S. Port of New York, Marine Square Club of New York, and Port Tampa Lodge, F. & A. M.

Mr. Arkebauer was born at Nokomis, Ill., on September 1, 1871, son of Hye G. and Martha (Peyton) Arkebauer. He went to sea at the age of twelve, working his way up, rank by rank, to the berth of chief engineer, first serving in that capacity on the army transport S.S. *Torpon* in 1898. Later during the Spanish-American War he was assigned to the S.S. *Margarette* and S.S. *Florida*. Subsequent to the war he continued at sea until June, 1905, being first assistant engineer on the S.S. *Havana*, S.S. *Esperanza*, and S.S. *Morro Castle*. Then, after a few months' engineering work in New York, he was made chief engineer of the ferryboat, *Brooklyn*, in 1905. He remained in that service until 1907, when he was appointed assistant inspector of boilers for the United States Steamboat Inspection Service at the Port of New York. He was engaged continuously in this work until April, 1917, with the exception of a few weeks' active service in the New York State Naval Militia earlier that year. From then until November, 1919, he served in the United States Navy as machinery inspector at the New York Navy Yard, Brooklyn. During this period he supervised the fitting out of a large number of submarine chasers, overseas troop ships, and other classes of vessels. For the next nine years he was superintending engineer for Phelps Bros. & Co., New York, the American and Canadian agents for the Cosulich Steamship Line, of Trieste, Italy. Since 1928 he had been engaged in private practice as marine surveyor and consulting engineer.

CARL BENNETT AUEL (1870–1937)

Carl Bennett Auel, manager of the Employees Service Department of the Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa., died on April 4, 1937, at his home in Irwin, Pa.

Mr. Auel was born in Brooklyn, N.Y., on March 21, 1870, son of Charles and Annie (Lambert) Auel. He attended the Polytechnic Institute of Brooklyn and the School of Engineering of Columbia University, then entered the School of Mechanical Engineering of Cornell University, from which he received the degree of mechanical engineer in 1892. He spent a part of the following year in graduate work, then entered the employ of the Westinghouse company, Pittsburgh, as assistant general foreman. He continued with the company until 1897, being in charge of the Production and Specification Departments during the latter part of the time.

For the next two years Mr. Auel was, first, general manager of the Clark Brake Company, Pittsburgh, then engineer of the Westinghouse Airbrake Company, Wilmerding, Pa. In 1901 he went to Manchester, England, as assistant to the general manager of the British Westinghouse Electric & Manufacturing Co., later becoming assistant general manager. He returned to the United States in 1905, resuming his association with the Westinghouse company here as assistant to the manager of works. Subsequently he became manager of the Railway and Control Departments, then assistant manager of works, director of standards, processes, and materials, and in 1920, manager of the Employees Service and Works Standards Departments. He established the company's building and loan association and tuberculosis sanatorium and developed a medical service for the employees. He was also in charge of the pension system, company restaurants, and Materials Disposition Department.

Mr. Auel had been a member of the A.S.M.E. since 1914. He was chairman of the Sectional Committee on a Safety Code for Mechanical Power Transmission Apparatus from the time of its organization in 1921 until his death, serving as one of the representatives of the A.S.M.E. He was appointed a member of Subcommittee No. 3 on Mechanical Power Control when it was formed in February, 1929, and was elected its chairman, in which capacity he served until the committee was discharged in December, 1935. He was also active in the work of the Management Division of the Society, and particularly in its Management Week campaigns and waste elimination activities; he had been chairman of the Subcommittee on the Elimination of Waste since the time of its organization in 1930, and was instrumental in the compilation and publication of the Waste Materials Dictionary. He was the author of many articles and addresses on industrial relations. Mr. Auel was also a past-president of the National Safety Council (1924–1925), and a member of the Western Pennsylvania Safety Council. He served as president of the Allegheny Public Health Association in 1925–1926 and belonged to the Pennsylvania State and the Pittsburgh Chambers of Commerce, Pennsylvania State Council for the Blind, Pennsylvania State Board of Health, and Philadelphia Board of Trade. He was a member of the Methodist church and Delta Upsilon fraternity, and was a Republican in politics. He enjoyed reading, especially historical works, gardening, camping, and historical and genealogical writing.

Surviving him are his widow, Louisa (Clark) Auel, whom he married in 1901, a son, Carl Clark Auel, and three sisters.

TOM BARKER (1884–1938)

Tom Barker, industrial engineer for the United Parcel Service, New York, N.Y., died on March 14, 1938.

Mr. Barker was born at Baraboo, Wis., on September 26, 1884, the son of John and Alice (Druse) Barker. He prepared for college in the Englewood (Ill.) High School, and then attended Dartmouth College, where he was graduated in 1906 with an A.B. degree.

His first employment, 1906–1908, was with the Western Electric Company in Chicago, Ill., where he took a general shop training course and did considerable clerical work. From 1908 to 1910, he worked for the Rex Tool & Manufacturing Co., Chicago, doing shop work in gasoline engine production, tools and dies. In 1910–1914, he was employed by the Rogers Addresser Company, Chicago, as manager of manufacturing addressing machines. In 1917–1919, Mr. Barker served as First Lieutenant of Infantry, 314th Machine Gun Battalion, 80th Division, A.E.F., in France.

On his return to civilian life, he worked in 1919–1920 in the Hawthorne Plant of the Western Electric Company in the establishment of piece rates, and followed this by a year's employment by Arison Goodwin & Associates, Chicago, consulting industrial engineers. Following this position, Mr. Barker became associated with the United Parcel Service and remained in their employ up to the time of his death.

He joined the A.S.M.E. as an associate-member in 1922, and in 1935 was automatically transferred to the member grade. Mr. Barker married Marie Therese Polet in 1937, and is survived by her and also by two brothers, Ralph and Hugh Barker, also engineers.

BERNARD CLEVELAND BECKER (1887–1938)

Bernard Cleveland Becker, development engineer with The Hoover Company, North Canton, Ohio, died suddenly of a heart attack on March 19, 1938.

Mr. Becker was born on November 16, 1887, in York, Pa., the son of Henry Franklin and Rebecca Jane (Zeigler) Becker. He was educated in public schools and afterwards took the complete course in mechanical engineering as given by the International Correspondence Schools. He began work in 1903 and had four years of practice in general machine shop work and toolmaking. In 1908 he entered the employ of the Heany Laboratories, Inc., New Haven, Conn., where he worked successively for ten years as machinist, draftsman, factory engineer, and chief engineer, occupying the last position for eight years.

For the year 1919, Mr. Becker was employed by the Tabulating Machine Company, Endicott, N.Y., as machine designer. In 1920 he began private practice as patent attorney at York, Pa., and carried on this work until 1926, though in 1924 he became associated with the York Manufacturing Company in the development and design of refrigerating machinery.

In 1927, Mr. Becker was engaged by The Hoover Company of North Canton, Ohio, "to create, improve, and develop mechanical and electrical devices." The company wrote that his ten years of service was marked by many successful achievements in the creative-inventive field and his work was outstanding. His attack on a problem was thorough and from many angles, which usually resulted in a number of designs from which a choice could be made. His layout work was clear, concise, and well planned, making it easy to interpret in the model construction department." Mr. Becker also invented a suction cleaner and several mechanical lead pencils.

Mr. Becker married Bernice E. Deardorff in 1926, and is survived by her and by two sons, Charles H., of Orange, Texas, and Ned G., of Lawton, Okla., as well as by his mother.

He became a member of the A.S.M.E. in 1932. He was also a member of Federal Lodge No. 1, F. & A. M., Capital Chapter, R.A.M. and Washington Commandery No. 1, Knights Templar, Washington, D.C., and of the Elks Club, of Flint, Mich.

HENRY LÉON BANCEL BINSSE (1852–1938)

Henry Léon Bancel Binsse, of Washington, D.C., died at his summer home at Pointe au Pic, County Charlevoix, Que., Can., on July 2, 1938. He was born in New York, N.Y., on November 14, 1852, son of Louis Bancel and Mary Delia (Carpenter) Binsse, and was de-

scended from the French families of Binsse de Saint Victor and Bancel, who came to New York at the time of the French Revolution. His early education included a period of engineering study at Düsseldorf, Germany.

When Mr. Binsse applied for membership in the A.S.M.E. in 1882 he submitted the following record: "Following my graduation with the degree of C.E. from Rensselaer Polytechnic Institute, Troy, N.Y., in 1875, I took a position at the North Chicago Rolling Mill, Chicago, Ill. I found that my taste led me to mechanical engineering and I, consequently, entered the works of C. H. Delamater & Co., New York, and familiarized myself with the trade. In 1879, I entered the firm of George A. Ohl & Co., engineers and machinists, of East Newark, N.J., of which firm I am still an active member." He made improvements in small turning lathes and horizontal boring machines while with this company.

In 1887 he established his own business, the Newark Machine Tool Works, in Newark, and he continued the manufacture of machine tools under that name until 1902 and as president of the Binsse Machine Company, Newark, from then until his retirement in 1914. He took out a patent on a screw-jack in 1883, one on a slotting machine in 1889, and one on gearing in 1899.

Mr. Binsse married Elizabeth Hewlett Scudder on November 10, 1903, and is survived by her and a son, Harry Lorin Binsse, residents of Washington, D.C., and Glen Cove, L.I., N.Y., respectively; also by a sister, Alice (Mrs. Schuyler N. Warren), of New York, N.Y.

He resigned his membership in the A.S.M.E. in 1911 but was reinstated as of 1882 by action of the Council in 1917, with permanent remission of dues.

WOODROW W. BLACK (1912–1938)

Woodrow W. Black, an employee of the Reid Diesel Engine Company, of Oil City, Pa., died on August 2, 1938, of colitis, in the General Hospital, Buffalo, N.Y.

Mr. Black was born on January 21, 1912, at Cyril, Okla., the son of Warner and Annie Caroline (Williams) Black. He was graduated from the University of Oklahoma in June, 1937, with a B.S. degree, and had been in the employ of the Reid Diesel Engine Company since that time.

He became a junior member of the A.S.M.E. in 1938, and was unmarried.

EDMUND E. BLAKE (1870–1938)

Edmund E. Blake, chief engineer of Saco-Lowell Shops, Biddeford, Maine, died in Portland, Maine, on June 21, 1938, following a major operation.

Mr. Blake was born in Boston, Mass., on November 13, 1870, the son of James M. and Mary H. (Brown) Blake. He prepared for college in the Newton High School, Newton, Mass., and was graduated in 1893 from the Massachusetts Institute of Technology with an B.S. degree in mechanical engineering.

Mr. Blake's entire business career was devoted to Saco-Lowell Shops and its predecessors. Following his graduation, he served a year's apprenticeship in the shops of the Pettee Machine Works, Newton Upper Falls, Mass., and continued with that company until 1897, when he was in charge of machinery installations. He was associated with H. S. Chadwick, textile and mill engineer, in 1897–1899. He then took the position of salesman and engineer for the Saco and Pettee Machine Shops and in 1904 became manager of the Saco-Pettee Company at Biddeford. He was appointed chief engineer of Saco-Lowell Shops in 1923, and was responsible for many valuable improvements and inventions in cotton-mill machinery, the most notable being the commercial development of long-draft processing which has revolutionized the cotton-spinning industry in the last decade. He lived across the river in Saco, Maine, and was a valued member of the community, serving it in many capacities for 34 years.

Mr. Blake became a member of the A.S.M.E. in 1913. In 1896, he married Clara S. Sheppard, of Newton, Mass., and they had two sons, Malcolm S. and Edmund G., and a daughter, Mary, all married, and all of whom, together with six grandchildren, survive him.

HARVEY BOLTWOOD (1875–1938)

Harvey Boltwood, director of the Bureau of Service of the Interstate Commerce Commission, Washington, D.C., died suddenly of a coronary occlusion on September 2, 1938, while on the way to his office.

Mr. Boltwood was born at Albany, N.Y., on July 6, 1875, the son of Ransom Haight and Sarah Effie (Clarkson) Boltwood. He moved to Denver with his parents in 1888 and completed his preliminary education in the public schools there. He then worked for two years with the Union Pacific Railroad in Denver, after which he spent a year at Colorado College, Colorado Springs. His studies there were supplemented later by correspondence courses through the International Correspondence Schools.

In 1899, Mr. Boltwood began four years' service as machinist in railroad, mine, and mill work in Colorado, Idaho, and Washington. From 1903 to 1907, he was supervising locomotive construction and maintenance for the Denver & Rio Grande R.R. in New Mexico and Colorado. The next three years Mr. Boltwood spent in cyanide mill construction and operation in Colorado, as master mechanic. He gave this up in 1910 and was employed for the following year in charge of enginehouse and locomotive maintenance for the Union Pacific Railroad at Denver.

Mr. Boltwood's first service with the Interstate Commerce Commission began in Washington in 1911 when the Bureau of Locomotive Inspection was organized. He was one of the original fifty district inspectors. In 1918, he was transferred to the Division of Operation of the U.S. Railroad Administration as supervisor of equipment. In this connection he was sent to many points to make surveys of plant layouts and equipment with a view to initiating changes to increase output. He had some part in the design of standardized equipment at that time and later made a study of criticisms of it for the Department of Equipment, Division of Liquidation Claims. He was appointed mechanical engineer for this department in 1920, when the railroads were returned to private operation, his duties relating chiefly to handling engineering features in connection with the settlement of claims against the Railroad Administration. In 1922–1923 he was engaged in commercial education work in Colorado, and then he returned to the Interstate Commerce Commission, and was stationed at Shreveport, La., in the Bureau of Locomotive Inspection.

In April, 1925, Mr. Boltwood was appointed assistant director, Bureau of Service, for the Interstate Commerce Commission, in charge of the Section on Efficiency and Economy of Operation. He also was consultant in matters relating to the design of containers for shipping explosives and other dangerous items, particularly tank cars. In March, 1937, he was made director of the Bureau of Service, the position he was filling at the time of his death.

Mr. Boltwood joined the A.S.M.E. as an associate-member in 1921, and was promoted to member in 1930. He was also a member of the Mechanical Division of the American Railway Association, the International Association of Machinists, the Sons of the American Revolution, and the Masonic order. He was a member of the Galilee Baptist Church in Denver for many years and belonged to the National Baptist Memorial Church in Washington. In the A.S.M.E., Mr. Boltwood served on the Executive Committee of the Washington, D.C., Section from 1929 to 1932, as its chairman the last year. He was a member of the Section's Subcommittee on the Economic Status of the Engineer in 1928. He was also appointed a member of the General Committee (RR 2) of the Railroad Division in 1937, for the term 1937–1939. During 1937, he was also a member of a committee appointed by the District of Columbia Council of Engineering and Architectural Societies to study and report upon proposals looking toward the enactment of a law covering the practice of these professions in the District.

In 1917, Mr. Boltwood married Maude Margaret Morgan, who survived him, and they had two children, both living: Ransom Harvey and Charlotte Boltwood.

HENRY EDWARD BRETT (1858–1937)

Henry Edward Brett, consulting engineer, of Los Angeles, Calif., died at his home in that city on April 3, 1937. Mr. Brett was born in British Guiana, South America, on November 13, 1858, the son of Rev. William H. Brett and Caroline Brett. He was educated in the Grammar School of Loughborough, England, and by private tuition. He then spent some six years as apprentice in engineering works, covering patternmaking, machine shop, and millwright work, and as student draftsman with W. H. Thornbery, consulting engineer, of Birmingham, England.

He arrived in the United States in September, 1880, and spent four years as draftsman with E. D. Leavitt, Jr., Cambridgeport, Mass.; I. P. Morris Co., Philadelphia, Pa.; Robert Wetherill & Co., Chester, Pa.; and the Scott Foundry, Reading, Pa. He then went to the West Coast, where he served with the Pacific Rolling Mill Company of San Francisco, as draftsman, engineer in charge of the construction of cable railways in San Francisco and Oakland, and assistant to superintendent in charge of steel works. In 1887 he returned to the I. P. Morris Co., in Philadelphia, as draftsman, and subsequently was with Wm. Wharton Jr. & Co., Inc., of that city, as draftsman and designer of special machinery. Then, in 1890, he went back to the West Coast and joined the staff of the Pacific Rolling Mill Com-

pany, serving as constructing engineer for the Los Angeles Consolidated Electric Railway, of Los Angeles, where he was in full charge of track work, design and construction of powerhouse, and installation of steam and electric equipment and rolling stock. From 1892 to 1893, he held a similar position with the San Diego Electric Railway, of San Diego, Calif., and next served for one year with the Baker Iron Works, of Los Angeles, as mechanical engineer.

From 1894 up to his death, Mr. Brett practiced as consulting mechanical engineer in Los Angeles. His work covered general practice: machine design, development of inventions, structural work, and investigations and reports. Among other things he designed machinery for can making and canning equipment, machines for flax treatment, pressure filters, excavators, and shipyard machinery.

Mr. Brett obtained a number of patents, including oil burners, crematory furnaces, a continuous pressure cooker, and various machines for special purposes. In 1905 he served as a member of the Commission to Investigate the Construction of an outfall sewer for the City of Los Angeles. He was a registered civil engineer, State of California. He joined the A.S.M.E. as a member in 1902 and served on the Executive Committee of the Los Angeles Section in 1921–1922 and 1928. He was at one time a member of the Engineers' and Architects' Association for Southern California and had taught mechanical drawing in the Y.M.C.A. of Los Angeles.

On November 25, 1885, Mr. Brett married Miss Emilie T. Bridle, of Philadelphia, and they had three sons: H. Austin, born December 29, 1887; Edward C. N., born June 9, 1892; and Arthur T., born February 10, 1894. He became a citizen of the United States by naturalization in Los Angeles, 1896.

BENNETT MATTINGLY BRIGMAN (1881–1938)

Bennett Mattingly Brigman, best known as organizer and first dean of the Speed Scientific School of the University of Louisville—Ben to a host of personal friends—was a true Kentuckian. Born in Louisville, on February 25, 1881, the son of William Frederick and Margaret (Lehman) Brigman, he was long active in the professional and educational activities of state and city and died in Louisville, February 8, 1938. Mrs. Brigman, the former Alice Jessel, of Louisville, whom he married in 1904, survived him.

He attended the Universities of Kentucky and Louisville, receiving the degrees of B.S. and M.S. from the latter, in 1908 and 1912, respectively, and did special graduate work at the University of Wisconsin in the summer of 1915. He engaged in practical engineering for a number of years, principally in the shops, engineering offices, and test plants of the Louisville & Nashville Railroad. Long interested in boys, he left this work to organize and direct their pre-college and vocational training, first in the Manual Training Department of the University School, a one-time private preparatory school in Louisville, and then in the DuPont Manual Training High School of Louisville. From there he went to the University of Louisville as instructor in physics, later becoming professor of engineering and drawing.

Thus by broad training and by natural interest, he was well qualified to undertake the planning, building, and initial operation in 1924 of the Speed Scientific School, for which he served as inspiration as well as administrative head until his death. It was on the basis of his prospectus that gifts were made by William S. Speed and Mrs. Olive (Speed) Sackett to establish the James Breckenridge Speed Foundation. He served as guiding architect and dean of this, the first southern school to be accredited in all its branches of engineering, until his death.

He contributed much to his native city and to his native state. But it is as a nationally known engineer and personality, well acquainted throughout this country, that most of us knew him—as a leader in the Council of National Defense and in the Bureau of War Savings Stamps during the World War—as an authority on zoning and planning, municipal and community, and on smoke abatement—as a member and participant in many national projects of educational and professional nature—as a member and officer of national educational and engineering associations, particularly as vice-president of The American Society of Mechanical Engineers and of the Society for the Promotion of Engineering Education.

He became a member of the A.S.M.E. in 1923 and one of its Fellows in 1936. He served as chairman of its Louisville Local Section, 1925–1934; as a member of the Special Committee on Policies and Budget, 1933–1936; as a delegate to the American Engineering Council, 1933–1935; and as a manager of the Society, 1934–1937, and vice-president from then until his death.

These are the formal activities and accomplishments for which Dean Brigman will long be remembered—but those who knew him well personally, who sat on Council or Committee with him, in conversation with him in his home, or beside him during long train rides to conferences, or who joined his ever-present group of friends at conventions, will prefer to remember certain distinctive personal traits. There was his ever-impelling interest in his fellowmen, particularly his fellow students, whose ranks he never left—his never-failing infectious sense of humor, which made him take his job, but not himself, seriously—his unbounded enthusiasm and industry, despite a relatively weak physique—his sheer likableness as a man, as a teacher, and as an engineer.

These are the characteristics that made Ben Brigman stand well above his fellows.—[Memorial prepared by EUGENE W. O'BRIEN, Atlanta, Ga., Past Vice-President, A.S.M.E.]

MORTIMER COWPERTHWAITE BROWN (1887–1938)

Mortimer Cowperthwaite Brown, whose death occurred at his home in Mount Vernon, N.Y., on August 20, 1938, was born there on November 5, 1887, son of Edgar K. and Emily (Cowperthwaite) Brown. He prepared for college in the Mount Vernon schools and was graduated from Stevens Institute of Technology as a mechanical engineer in 1910. During the next eighteen months he was employed by the Russell, Burdsall & Ward Bolt & Nut Co., Port Chester, N.Y., as assistant to the foreman of the machine shop, and by the Max Ams Machine Company, Bridgeport, Conn., in drafting work on automatic machinery. He then became draftsman for the Hodgman Rubber Company, Tuckahoe, N.Y., with which he remained until the fall of 1914, being promoted during that time to chief draftsman and then second assistant chief engineer.

Beginning in January, 1915, Mr. Brown was connected for about four years with the Aetna Explosives Company, serving at first as draftsman and inspector at New York, and later as chief engineer at the Fayville, Ill., plant of the company. He returned to New York in 1919 as mechanical engineer for the Pratt Engineering & Machine Co., with which he continued until April, 1921.

His next employment of note was with the Western Electric Company, New York, for three years beginning in May, 1925. During the following year he was an estimator for Coverdale & Colpitts, consulting engineers, New York, in connection with a project covering the cost and valuation of the Interborough Rapid Transit Company. Subsequently he was engaged in layout and drafting for the American Cyanamid Company, General Chemical Company, and John Johnson Sales Corporation, in New York. He was senior engineer and valuation checker for Maurice R. Scharff, consulting engineer, New York, September, 1934, through February, 1935, and from April to October of that year was engaged in making an inventory of gas plants and pumping stations for the Consolidated Gas Company, New York. He then was employed by the Law Department of the City of New York as valuation engineer on the appraisement for tax purposes of public utility power plant and substation machinery and equipment. He was transferred to the Tax Department at the beginning of 1936, and employed there until the end of the year and again from March, 1937, to February, 1938, when all engineers engaged on this work were released, later to be replaced by civil service engineers. During the spring of 1938 he was a provisional assistant electrical engineer for the New York Board of Transportation, in connection with listing and pricing equipment on elevated lines.

Mr. Brown became an associate-member of the A.S.M.E. in 1915 and was automatically transferred to full membership in 1925. His wife, Ethel (Stickney) Brown, whom he married in 1917, survives him.

ARTHUR BRUCKNER (1872–1937)

Arthur Bruckner, professor and head of the department of mechanical engineering in the School of Technology of the College of the City of New York, died of a heart attack on August 29, 1937, at his home in Hastings-on-Hudson, N.Y. He had been on leave of absence from the college since February because of ill health.

Professor Bruckner was born in New York on April 22, 1872, the son of Herman and Susan (Hoffman) Bruckner. His preparatory education was obtained in the New York public schools. He was graduated with a B.S. degree from the College of the City of New York in 1892, and in 1898 from the Sibley College of Cornell University, with an M.E. degree. He later took one year of graduate work in Cornell University, and three years of evening work in the New York Trade Schools.

From 1892 to 1897, Mr. Bruckner taught mechanic arts in the College of the City of New York. The next two years he spent at Cornell for his M.E. degree and graduate work. From 1899 to 1904, he was instructor in physics and the mechanic arts at the College of the City of New York, and organized the work of laboratory instruction in physics. In the years 1904–1907, he was busy in the planning, construction, and equipment of the Mechanic Arts Building of the College of the City of New York. He personally designed much of

the equipment for the laboratories and supervised its installation, and served as instructor in the mechanic arts and in the mechanical laboratory.

In 1907, Mr. Bruckner was made director of the Mechanic Arts Laboratories and instructor in various mechanical engineering subjects. In 1917 he was appointed assistant professor of mechanical engineering. He advanced to the rank of associate professor at the beginning of 1929, and was appointed full professor of mechanical engineering, in charge of the department, on January 1, 1936.

Professor Bruckner became a member of the A.S.M.E. in 1919. He was also a member of the Society for the Promotion of Engineering Education and the American Association of University Professors. He was much interested in education in general, and at the time of his death was serving his second term as trustee of the Union Free School District No. 4, Town of Greenburgh, Westchester County, N.Y. He had long made his home in Hastings-on-Hudson, N.Y., where he had served as a member of the Hastings Board of Education since 1935, and a member of the Hastings Taxpayers Association, and was both a member and secretary of the original Riverview Manor Association.

In September, 1898, Professor Bruckner married Mabel C. Earll, of Ithaca. He married a second time, in December, 1906, Ella May Ford, of Washington, D.C., who survives him. Surviving also are a son, Robert Earll, of Vineland, N.J., and three daughters, Mrs. Mabel Amelia Chomsland, of Ridgefield Park, N.J., and Mrs. Helen Sue Fagans and Mrs. Virginia Ford von Isecke, of Basking Ridge, N.J. He also left a sister, Mrs. Annie Hillenbrand, of New York.

EDWIN BURHORN (1866–1937)

Edwin Burhorn, president of the Edwin Burhorn Company, of Hoboken, N.J., died in Margaretville, N.Y., on September 26, 1937, after a brief illness. A sister, Miss Etta Burhorn, and a son, Alfred Burhorn, survived him. He had been a member of the A.S.M.E. since 1914.

Mr. Burhorn was graduated from the Stevens Institute of Technology in 1885, with a degree in mechanical engineering. The Morton Memorial volume published by the Institute in 1905 contains the following biography of Mr. Burhorn.

"Edwin Burhorn was born in New York, June 21, 1866, the son of August and Henrietta W. Bickel Burhorn. Both parents were born in Germany and came to the United States about 1849, their parents being revolutionary exiles. He was educated in the public schools of Hoboken, N.J., graduating from Hoboken High School and receiving the Stevens scholarship. He served as draughtsman with Henry Warden, Germantown Junction, Philadelphia, Pa., manufacturer of boilers and special wrought-iron work, etc., 1885; and as assistant to the general manager of the Franklin Sugar Refinery, Philadelphia, 1885. While in the latter position he took entire charge of testing boilers, engines, evaporating apparatus, etc., designed and installed a special system of draining bag filters by vacuum process, filling char filters by automatic spreading-machines, etc. He became interested with Mr. B. H. Coffey, '85 (Stevens Institute), in the Cycle Water Filter, having obtained a patent on a special controlling valve mechanism, and joined with Mr. Coffey at the shops of Henry Warden, improving the filter and placing it successfully on the market. He was with the Link-Belt Engineering Co., of Philadelphia and New York, 1890–1893, planning many systems of handling material by machinery of special design, and afterward took entire charge of the designing department of the New York office of the same company.

"In 1893 he started in business as an engineer and contractor under the firm name of Warren & Burhorn, changed the following year to Burhorn & Granger,* of New York, the firm acting as manufacturers' agents in addition to professional work. This partnership lasted for eight years, during which time it installed many complete plants for power, heat, and electric light; made several improvements in the Woodbury high-speed automatic engine, built by the Stearns Manufacturing Co., and placed the engine in successful operation in many places, notably in the *World* Building, New York. In 1901 the firm of Burhorn & Granger was dissolved, Mr. Burhorn opening an office for himself in New York, as engineer and contractor, which business he is now carrying on. Some of the installations made under Mr. Burhorn's supervision are as follows: Boiler plant at the rope-walks of the Boston Navy Yard (750 horsepower); electrical transmission plant for the factory of John Mehl & Co., Jersey City Heights; a motor-driven swing-bridge over the Passaic River, at Fourth Avenue, Newark, N.J.; char-drying equipment and incinerator for sugar-baskets at the Arbuckle sugar-refinery, Brooklyn, N.Y., and an Acme water-cooling tower at Baltimore, Md. . . . showing a class of work of which Mr. Burhorn is making a specialty,—namely, recooling water from condensers in steam plants, or, in ice plants, for recooling water from ammonia condensers. He is a member of The Franklin Institute of Philadelphia."

About the year 1905, cooling towers were designed with perforated pans for the cooling surface and with wire mesh placed on the outside of the tower to act as louvers to prevent the water from being blown away by the wind. Up to the time of his death in 1937, Mr. Burhorn had been granted more than fifty patents on cooling towers and auxiliary apparatus. Some of these developments include all-steel and all-wood construction, the narrow-type tower, indoor-type installation, forced-draft tower, louver fence, and many others for use with refrigerating machines, Diesel engines, air compressors, oil-refining equipment, and air-conditioning apparatus. Burhorn cooling towers are found in all parts of the world and in almost every state in this country, in use by manufacturers, food industries, breweries, meat packers, dairies, and ice plants.

In June, 1910, Mr. Burhorn incorporated his company and was president and chairman of the board until his death. Because of the individual requirements of each cooling-tower installation, the Edwin Burhorn Company, engineers, contractors, and manufacturers, carries through a design from the beginning to the final erection. Other equipment manufactured by the company includes spray-pond apparatus.

* For biography of Abbott D. Granger, see page RI-55.

JOHN ALLEN CAPP (1870–1938)

John Allen Capp died on January 6, 1938, at the age of 68 years. He was born in Philadelphia, Pa., on January 14, 1870, and received his education at the Central High School (where Prof. Elihu Thomson was, at that period, associated with the science courses), and at the University of Pennsylvania. In 1892 Capp was engaged by the Thomson Houston Electric Company in the mechanical testing laboratory at the company's West Lynn Works. That laboratory and two other laboratories, dedicated respectively to chemical and electrical testing and investigations, were located in a very large basement, below the office building. The equipment of the three laboratories was remarkably complete, considering the state of advancement in science and engineering at that early period.

Today such an assemblage of apparatus, machinery, and personnel would be termed a research laboratory. It is probable that, at that time, no finer testing facilities in the field of general engineering and scientific research had ever been provided by any manufacturing corporation in this country. E. W. Rice, Elihu Thomson, and A. L. Rohrer, who had collaborated in the design of this laboratory, were justly very proud of it and they brought down from the offices many visitors from all over the world. Everything was shown to the visitors without any reservation whatsoever. Incidentally it served as a fine inspiration to the personnel to meet so many distinguished pioneering scientists and engineers.

Capp's assignments related chiefly to materials, especially to metals and insulation. In those early days, relatively little was known concerning the properties and characteristics of materials used in the manufacture of electrical equipment and products. Much remained to be learned. In the course of his professional career of 45 years, Capp contributed a very noteworthy share of the knowledge of these materials, which have since been developed, and he aided greatly in improving them to their present high quality. In 1892, cast steel was just emerging as a rival to cast iron and wrought iron for generators, railway motors, and other electrical apparatus. (Outside of railway motors, there was at that time no quantity production of electric motors for other than extremely small powers.) The kind of sheet iron available for armature cores and for transformers not only had a high hysteresis loss, but this loss increased rapidly with time in service. However, in conformance with common experience, since we had no knowledge that core material several times better ever would be developed, most people concerned were fairly contented. Few of the faults in insulating materials had yet been overcome; indeed they scarcely were realized. The quality of the oil used in transformers left much to be desired, and the nature of its faults already was being vigorously investigated. The investigations are continuing today, almost half a century later.

In May, 1894, after the Thomson Houston Company and the Edison Machine Company had been consolidated as the General Electric Company, Capp was transferred to the Schenectady Works. In 1896 he was placed in charge of the Materials Testing Laboratory. In this capacity Capp had the responsibility for the preparation of specifications for all materials purchased by the company and for their inspection and testing. In parallel with these duties Capp was constantly carrying on special experimental work and researches on materials and apparatus. Along with other materials, coal was an important item in the list of Capp's responsibilities and early received his active attention. At first, the coal people resented the in-

trusion of specifications concerning their "natural" product, but soon they became impressed by Capp's fairness and they accepted the methods he proposed for sampling and for testing the ash and sulphur content and the calorific value. The major burden of these expanding activities was borne by Capp up to 1927, when his laboratory was merged in the new Schenectady Works Laboratory, in which Capp became engineer of materials and continued in that position for the remaining 11 years of his life.

Throughout his professional career, Capp was a very active member of several engineering societies, notably The American Society of Mechanical Engineers, of which he became a member in 1901, and the American Society for Testing Materials, which he served as president for the 1919–1920 term. Honorary membership in that society was conferred on him in 1937. Many technical papers were contributed by Capp in the course of his professional career, not only to these societies but also to the American Institute of Mining and Metallurgical Engineers, the British Iron and Steel Institute, and the Institute of Metals.

In 1919 Capp was appointed one of a group of three representatives of the A.S.T.M. to develop, in collaboration with representatives from other societies, the plans for organizing an American Engineering Standards Committee. Capp entered upon this task with high enthusiasm. The early steps were beset with tremendous difficulties and it was due in no small measure to Capp's sound judgment and organizing experience and ability that finally there emerged from those early efforts the highly successful American Standards Association of today. Dr. Agnew, who has been its secretary from almost the very beginning, has spoken in the highest terms of Capp's valuable contributions to its successful evolution at various critical stages.

Cloyd M. Chapman, who also had much to do with the ASA, writes: "My impressions of Capp concerned chiefly his clear, distinct, precise thinking and the exactness he displayed in expressing his ideas. He hewed to a line and was definite in his ideas. He decided quickly and was slow to change or modify a decision once made. He seldom needed to. I have enjoyed some lovely arguments with Capp even though I often came off second best."

While the ASA achievement probably is the one of greatest consequence in the committee field of Capp's activities, mention certainly should also be made of much other fine committee work contributed by him. Altogether, as a member of the A.S.T.M. since its founding in 1898, Capp served on 18 different committees. Beginning with the establishment of the Committee on Copper and Copper Alloy Wires in 1909, Capp was its chairman, and his expert knowledge of testing methods for determining the properties of nonferrous wire products contributed to the development of specifications and practices which were for years the standards of the copper industry. Also as an active member of the Committee on Steel, Capp headed up various subcommittees and served as chairman of the committee in 1918. From 1920 to 1927 he gave much effort to the reorganization of the Committee on Methods of Testing and served as its chairman until 1927. As a member of the A.S.M.E., he served on the Research Committee Subcommittee on Bearing Metals (later called the Special Research Committee on Bearing Metals), from the time of its appointment in August, 1916, for the study of microstructure and hardness of bearing crystals, until its discharge in December, 1927.

A. L. Rohrer calls attention to the high opinion of Capp's abilities held by Dr. Chas. B. Dudley in the early years of this century. Dr. Dudley was a great pioneer and authority in the field of materials specifications and testing, particularly as relating to steel rails and similar materials. He acted as Capp's mentor on many occasions and visits were frequently exchanged between himself and Capp at Altoona and Schenectady.

G. H. Wright (who was Capp's close associate for many years, including the period of the Schenectady activities) and A. L. Rohrer, recall that, around 1920, Capp did some early testing which led to the substitution of stainless steel for the monel metal buckets in the first stages of his company's steam turbines. With time and experience, this practice was extended through the entire stages. Since the last few stages require strength beyond the range of monel metal, stainless steel buckets replaced 3½ per cent nickel steel. Capp is believed to have been the first to suggest this substitution and he worked very hard for its adoption.

In addition to his engineering interests, Capp took great delight in music. He was one of the founders of the Schenectady Civic Music Association and served as its president from 1932 to 1934. Dr. Felix Kleeberg (who was Capp's chief for some time, in the laboratory work in the 1894–1895 period) in reminiscencing about those early times, speaks of a trio which frequently met and played the classical Haydn trios, Capp playing the piano, a mutual friend (Frank Holden) the cello, and Kleeberg the violin.

There are but few now living who were associated professionally with Capp when he went to work at Lynn at 21 years of age. J. M. Darke, of the Lynn Works of the General Electric Company, referring to that early time, states that they all considered Capp as "a very efficient man, very methodical in his work, and very original in such work." A review of his accomplishments in later life more than justified these impressions of his early associates.

A fitting conclusion to this memorial has been furnished by F. M. Farmer, long associated with him in committee work, who writes: "John Allen Capp leaves with a wide circle of friends and acquaintances the memory of a man who could be depended upon to conscientiously carry out any assignment he accepted, a man whose integrity was never questioned and who would strenuously contend for what he believed to be the right decision, a man whose sound advice was widely sought and was always freely given."—[Memorial prepared by HENRY M. HOBART, Schenectady, N.Y., Mem. A.S.M.E.]

JAMES ADAMS CHARTER (1865–1937)

James Adams Charter, a member of the A.S.M.E. since 1899, died on April 21, 1937. He was born in Sterling, Ill., on July 24, 1865, the son of John and Rachel (Adams) Charter. His public school education was supplemented by studies under a private tutor and an apprenticeship in designing and mechanical work with the Williams & Orton Manufacturing Co., of Sterling. Following the completion of his apprenticeship in 1883 he worked as a draftsman for this company and for H. W. Caldwell & Co., of Chicago, for which he later became general superintendent. Other early connections were with the Sterling Gas Company and the Sterling Gas & Electric Light Co.

Mr. Charter was associated with his father in the design of gas engines, the first patents on which were taken out in 1882. He later described the invention as "the first internal-combustion engine to use liquid fuel (gasoline) injected directly into the suction intake of the engine (now called the carburetor)," and stated that their designs "made it possible to adopt gasoline for powering both automobiles and aeroplanes using internal-combustion engines." From 1894 to 1908 he was connected with Fairbanks, Morse & Co., Beloit, Wis., in charge of the manufacture of these engines, also representing this company in Europe for some time, and during the next four years he was with the Austin Manufacturing Company, Harvey, Ill., as designing engineer in connection with the manufacture of gasoline-driven road rollers.

In 1910 Mr. Charter became president of the Charter Single Sleeve Motor Company, Chicago, developing the rotary-sleeve type of engine for automobiles and aeroplanes. In 1923 he invented a new type engine starter drive, and incorporated a company for its manufacture, Charter Drive, Inc., Chicago, of which he served as president until his death.

Mr. Charter married Miss Julia Wheeler, of Chicago, in 1913, and is survived by her.

LAWRENCE RIPLEY CLAPP (1886–1937)

Lawrence Ripley Clapp, vice-president, The New Haven Copper Company, Seymour, Conn., died on July 9, 1937. He was born on February 25, 1886, in Brooklyn, N.Y., the son of William Gamwell and Sarah Louise (Ripley) Clapp. He prepared for college in the Morristown High School, Morristown, N.J., and was professionally educated at Columbia University, where he was graduated with an M.E. degree in 1910. As a student at Columbia he was awarded the Illig Medal for his outstanding work in metallurgy and was made a member of the honorary societies of Tau Beta Pi and Sigma Xi. Later, besides becoming a member of the A.S.M.E. in 1921, he also joined the American Institute of Mining and Metallurgical Engineers and the Canadian Institute of Mining and Metallurgy, and was a licensed and registered engineer in British Columbia.

From 1905 to 1907, prior to his graduation from Columbia, Mr. Clapp served as assayer, surveyor, miner, and millman at various mines in the United States and Mexico, for the Cananea Copper Company, Federal Lead Company, and Ms. Tecolotes y Anexas. From the time of his graduation in 1910 until the World War in 1917, he served as superintendent, Loreto Mill (silver), Cia de Real del Monte, Pachuca, Mexico; Silver Lake Mines (silver, lead, gold, copper), Silverton, Colo.; Sunnyside Mill (silver, lead, zinc), Eureka, Colo.

When the United States entered the World War, Mr. Clapp became associated with the U.S. Fuel Administration, Washington, D.C., as chief of States Conservation Section; U.S. Army, New York District Ordnance Claims Board, New York, N.Y., general assistant; and until 1922 he practiced as a consulting engineer in plant management and valuation in New York, N.Y.

Returning to the mining business in 1922 he served as assistant general manager, Granby Consolidated Mining, Smelting & Power Co., Anyox and Copper Mountain, B.C.; and in 1924 he returned to this country as vice-president and general manager, The New Haven

Copper Company, Seymour, Conn., the position he held at the time of his death.

Mr. Clapp stated that he had contributed "occasional papers in the technical press;" considered himself "best qualified for the mining and metallurgy of copper, including rolling-mill practice;" and had taken out "several patents on the rolling of copper and other ductile metals."

On April 17, 1915, Mr. Clapp married Miss Helen Tilge, of Philadelphia, Pa., and they had two children: Eleanor Lawrence and Marjorie Ripley.

ERIC HERBERT COSTER (1867–1937)

Several of the more important railroad power plants in this country, together with some private plants, bear testimony to the skill of Eric Herbert Coster, engineer of mechanical design for the Interborough Rapid Transit Company, New York, N.Y., from 1924 until his death in 1937.

Mr. Coster was born in London, England, on June 9, 1867, the son of Fredrick Leonard and Elizabeth (Bradley) Coster, but went to Sweden at an early age. He received his preparatory education at the grammar school in Uddevalla, Sweden, and the high school in Gothenburg, Sweden, and then attended the Chalmers College of Engineering, Gothenburg, receiving his diploma on his twenty-first birthday. Beginning in March, 1889, he worked for eighteen months as a fitter and erecting engineer on steam fire engines and pumping machinery for the Ludwigsberg Engineering Company, Stockholm. From then until May, 1896, when he came to this country, Mr. Coster was superintendent of his father's woodworking plant and planing and sawmills at Uddevalla.

His first employment after coming to the United States was with Sheaff & Jaastad, consulting engineers, Boston, Mass., as a draftsman on general plans and details for powerhouses and equipment. In April, 1897, he went with the Boston Elevated Railway Company, where he was engaged until December, 1898, in drafting and field work on powerhouse equipment, especially piping, and in the operation of two powerhouses. A brief engagement as a draftsman on piping with the New England Gas & Coke Co., Boston, followed, and, in March, 1899, Mr. Coster began an association with Westinghouse, Church, Kerr & Co., New York, N.Y., which lasted, with a single brief interruption, for fifteen years.

Drafting on building and equipment plans and details for the Kingsbridge powerhouse at New York and other powerhouses was Mr. Coster's first work with Westinghouse, Church, Kerr & Co. and was interrupted early in 1901, when he went to the London office of the British Westinghouse Electric & Manufacturing Co. to design a power plant for the Mersey Railway Company, at Birkenhead, and the Neasden power plant for the Metropolitan Railway, London. He returned to the United States late in 1902 and resumed work for Westinghouse, Church, Kerr & Co. as an assistant engineer, being subsequently promoted to the position of engineer-in-charge. Some of the more important work with which he was associated for the company included the design of five electrical substations for the Long Island Railroad and inspection of field work in this connection; general design of the power plant for the Congressional Office Buildings at Washington, D.C.; general design of the Cos Cob, Conn., power plant of the New York, New Haven & Hartford R.R., and arrangement of equipment; design and equipment of the Waterbury power plant for the Connecticut Company; design of producer-gas and gas-engine plants for the Harriman Estate at Harriman, N.Y., and purchase of all equipment, and supervision of field work in connection with this plant; similar work in connection with a mechanical-stoker and forced-draft installation at the Chicago plant of the N. K. Fairbank Company; and design and construction of an extension to the plant of the Fall River (Mass.) Electric Light Company. During these years Mr. Coster also did some appraisal work in connection with power plants.

From 1915 to 1917 he was engaged in the design and construction of new power plants and extensions to existing power plants for E. I. du Pont de Nemours & Co., Wilmington, Del. This employment was followed by work with Gibbs & Hill, consulting engineers, New York, in connection with the proposed improvement of the Pennsylvania Railroad Station at Pittsburgh, from June, 1917, to July, 1918. From then to February, 1920, he was engineer-in-charge for Henry R. Kent & Co., engineers and constructors, New York. The following April he took a position as assistant mechanical engineer with The Foundation Company, New York, with which he remained until May, 1921. About a year later he was appointed assistant appraisal engineer with the Board of Public Utility Commissioners, State of New Jersey, where he continued until May, 1923. He then went to Reading, Pa., where he was associated with the W. S. Barstow Management Association, until September, 1924, as mechanical engineer in charge of design and construction of a power plant at Middletown, Pa., for the Metropolitan Edison Company. From that time until his death, which occurred at New York on January 29, 1937, Mr. Coster was engineer of mechanical design for the Interborough Rapid Transit Company.

His widow, Mary (Seward) Coster, whom he married in 1903, a daughter, Elizabeth, of Holyoke, Mass., and a son, Herbert S. Coster, of Orange, N.J., survive him.

Mr. Coster became a naturalized citizen of the United States in 1906 and a member of the A.S.M.E. in 1910.

DUDLEY PEAK CRAIG (1890–1937)

Dudley Peak Craig, head of the Mechanical Engineering Department of the Colorado State College, Fort Collins, Colo., died on March 26, 1937, of pneumonia.

Mr. Craig was born in Ghent, Ky., on November 8, 1890, the son of Ulysses Peak and Mollie (Davis) Craig. His early education was obtained in the Manual Training High School of Indianapolis, Ind., and his engineering course was taken in Purdue University, where he was graduated with a B.S. degree in mechanical engineering in 1912, and in 1924 was granted an M.E. degree.

On graduating in 1912, Mr. Craig went to work with the Western Electric Company, Chicago, Ill., where he remained five years. His work was devoted particularly to manufacturing methods and time study, and the development of lead-covered cable. From 1917 to 1918, Mr. Craig was employed by Landers, Frary & Clark, New Britain, Conn., on time studies and factory costs. In 1918, he was in the U.S. Army in a field artillery officers training camp, but soon afterwards took a position with the Firestone Tire & Rubber Co., Akron, Ohio, where he stayed until 1921. He was attached to the Industrial Engineering Department in charge of the factory suggestion system.

In 1921, Mr. Craig returned to Purdue University as instructor and graduate student, obtaining his M.E. degree in 1924. He was appointed assistant professor in 1925, and associate professor in 1931. He remained in this position until September, 1936, when he went to Fort Collins, to take over the chair of mechanical engineering and head of the department at Colorado State College, following the retirement of Prof. L. D. Crain.

Professor Craig joined the A.S.M.E. as a member in 1925. He was honorary chairman of the A.S.M.E. Student Branch at Colorado State College at the time of his death. He was also a member of the Society for the Promotion of Engineering Education and corresponding secretary of its Rocky Mountain Section, belonged to the Tau Beta Pi and Pi Tau Sigma fraternities, and held the rank of major in the U.S.A. Field Artillery Reserves. In collaboration with Hubert J. Anderson, he was author of a book, "Steam Power and Internal Combustion Engines," which was brought out by the McGraw-Hill Book Company in 1931 (revised edition in 1937). Like many other young engineering students and instructors, Professor Craig engaged in occasional outside work; thus he worked as machinist apprentice for the Big Four Railroad, Beech Grove, Ind., during the summers from 1908 to 1911, and also put in another summer (1924) with the Cleveland Electric Illuminating Company, on acceptance tests of powdered coal boilers, and contributed papers to the *Purdue Engineering Review* and to *Power*.

In 1914, Professor Craig married Bessie Sheffer, of Chicago. He is survived by her and their four children, Gordon Davis, Mary Ruth, Elizabeth Ann, and Donald John, as well as by his mother and three brothers, all engineers.

CALVIN HENRY CROUCH (1870–1937)

Calvin Henry Crouch, a resident of Cliffside Park, N.J., died suddenly in Oswego, N.Y., on July 13, 1937, shortly after attending a reunion of his Class of 1892 at Cornell University.

Mr. Crouch was born on April 25, 1870, near Oswego, N.Y., the son of Henry Theodore and Ruth Lydia (Kenyon) Crouch. He prepared for college in the Oswego public schools and then attended Cornell University, where he was graduated with an M.E. degree.

Immediately after his graduation, Mr. Crouch spent a year as a student apprentice with the Ames Iron Works, Oswego. He then worked for a year with the Rome, Watertown & Ogdensburg R.R., Oswego, as a machinist, and followed this position by working for three years, from 1894 to 1897, as erecting engineer and in experimental and test work with the Deane Steam Pump Company of Holyoke, Mass. He next spent three years as instructor in machine shop practice at the Williamson Free School of Mechanical Trades, Delaware County, Pa. This was followed by one year erecting locomotives in Europe as traveling engineer for the Baldwin Locomotive Works, of Philadelphia, Pa.

In 1901, Mr. Crouch went to the University of North Dakota, Grand Forks, N.D., as dean of the College of Mechanical and Electrical Engineering and professor of mechanical engineering. He held this appointment until 1916, when all the engineering interests in the university were combined into the College of Technology. Under the reorganization, Mr. Crouch became professor of mechanical engineering and director of shops.

In 1919 Professor Crouch took the position of acting dean of the College of Technology and professor of mechanical engineering at the University of New Hampshire, Durham, N.H. He became dean the following year and remained at the college until 1925. From then until 1932, he was employed by E. L. Phillips & Co., New York, N.Y., as mechanical engineer.

In 1898, Mr. Crouch married Della L. Newman of Oswego, N.Y., who survives him.

He joined the A.S.M.E. as an associate in 1898, and became a member in 1908. He was also a member of the Society for the Promotion of Engineering Education. Mr. Crouch was a Knight Templar in the Masonic order, a Baptist in religion, and a Republican in politics. His paper, "Experiments With North Dakota Lignite in a Steam Power Plant and a Gas Producer," was published in the A.S.M.E. Transactions (vol. 34, 1912, page 795), and the University of North Dakota published a Bulletin by him, "Vocational Training." He also wrote several papers that were published in the Proceedings of the S.P.E.E.

ARCHER DAVIDSON (1881–1938)

Archer Davidson, sales engineer at Boston, Mass., for the Westinghouse Electric & Manufacturing Co., died on July 7, 1938. Mr. Davidson was born on February 18, 1881, at Farmville, Va., the son of William and Julia Flippen (Wiltsie) Davidson. His education consisted of six years at public schools, two years at Farmville High School, and three years at the Virginia Polytechnic Institute, Blacksburg, Va., where he received a B.S. degree in June, 1902.

During a portion of the time Mr. Davidson spent at the Virginia Polytechnic Institute, he worked at steam fitting in Lynchburg, Va., and also on the power plant and steam-heating system for the Virginia State Normal School for Women. Upon his graduation from college, he went to work as an apprentice machinist with the Westinghouse Machine Company, Pittsburgh, Pa., and was soon promoted to assistant foreman in the steam turbine Erecting and Testing Department.

In 1903, he was made an assistant in the Turbine Test Department and was employed as erecting engineer, under the direction of Westinghouse, Church, Kerr & Co., in connection with installations at two plants in Maine. In 1904 he returned to the Westinghouse Machine Company as erecting engineer, and during the following year he was connected with the installation of some twenty turbine units for plants furnishing power for cotton mills, electric railway and lighting service, and general industrial purposes.

Mr. Davidson was appointed district engineer for the Atlanta District of the company in 1905, and remained there until 1915, when he was transferred to the Boston Office as sales engineer, in which work he continued until his death. He was highly regarded in engineering circles for his ability and experience in the design, operation, and application of steam equipment, and he was responsible for many of the large turbine installations throughout New England.

During his long service with the Westinghouse organization, Mr. Davidson took out a number of patents. Probably the most outstanding was one covering the combination impulse and reaction turbine exploited by the Westinghouse Machine Company, which materially shortened the length of the turbine as compared with one of equal power having all reaction blading. This patent was sufficiently broad in scope to cover any combination type turbine.

A second patent of importance covered a power plant arrangement wherein with an existing non-extraction type turbine, an extraction type is installed with feedwater heaters supplied from the extraction type unit, serving to progressively increase the temperature of condensate passing from the condensers of both turbines to separate boilers, the boiler supplying steam to the extraction turbine operating at higher pressure and temperature with the object of improving the overall performance.

Mr. Davidson married Florence Meadows in 1922 and is survived by her and by two sisters. He joined the A.S.M.E. in 1907 as an associate.

HENRY S. DEMAREST (1867–1937)

Henry S. Demarest died on July 11, 1937, at his home in Hempstead, L.I., N.Y., after a long illness. Mr. Demarest was president and treasurer of Greene, Tweed & Co., New York, N.Y., manufacturers of mill supplies. He was the son of Albert Alonso and Constance Marie (Eakin) Demarest and was born on February 1, 1867, in Brooklyn, N.Y., in the public schools of which city he obtained his education.

Mr. Demarest began his business career with the Worthington Pump & Machinery Co., then known as Henry R. Worthington. While with that firm his attention was drawn to a little-known brand of asbestos packing. The qualities of this packing so impressed him that he gave up the lucrative position he held and persuaded Greene, Tweed & Co., the manufacturers of the packing, to permit him to attempt to place their product on the market. His untiring efforts, judgment, and faith in the product were eventually rewarded, the packing becoming well and favorably known throughout the world. Later Mr. Demarest acquired the controlling interest in the company.

Besides his own business, Mr. Demarest was a director of the Union Ferry Company of New York and Brooklyn. He was a vice-president of the American-Polish Chamber of Commerce and was decorated by the Polish government with the medal of the Order of Polonia Restituta. During the first World War, he was a member of the Executive Committee of the Hardware Metals and Allied Trades Committee.

Mr. Demarest became an associate of the A.S.M.E. in 1914. He was also a member of the Union League Club, the Merchants Club, the Export Managers Club, the British Chamber of Commerce, the Huguenot Society, and the Holland Society of New York.

Mr. Demarest's chief hobby was his business; but he was also keenly interested in his library and garden.

He is survived by his widow, Mrs. Frieda J. (Nicholas) Demarest, whom he married in 1931, in London, England.

HERMAN DIEDERICHS (1874–1937)

Herman Diederichs was elected a junior member of the A.S.M.E. in 1904 and promoted to full membership in 1913. In 1930 the Melville Medal of the Society was awarded jointly to him and to William D. Pomeroy for their paper entitled "The Occurrence and Elimination of Surge or Oscillating Pressure in Discharge Lines From Reciprocating Pumps."

He was appointed a member of the Committee on Awards of the Society in 1933 to fill an unexpired term and served as chairman of the committee during 1936. He was also a member of the Nominating Committee for 1936. He served as a member of the Power Test Codes Committee on Reciprocating Steam Engines from 1918 until his death, and as a member of the Special Research Committee on Bearing Metals from its appointment in 1916 until its discharge in 1927. He was a representative of the Society on the Division of Engineering and Industrial Research of the National Research Council from 1930 to 1933.

Dean Diederichs had been on the faculty of the College of Engineering of Cornell University since his graduation there in 1897. The following memorial is adapted from an obituary by Dexter S. Kimball (whom he succeeded as dean of the College of Engineering) which was published in *The Cornell Engineer*, October, 1937. The same issue contained tributes by others who had been associated with him, including Livingston Farrand, S. C. Hollister, P. M. Lincoln, W. N. Barnard, and Albert W. Smith.

Dean Herman Diederichs passed away on August 31, 1937, at Clifton Springs (N.Y.) Sanitarium of an obscure anemic disease and after several months of illness. His death removes another of the outstanding figures that have not only made famous the College of Engineering of Cornell University, but who, by reason of their personal characteristics, gave luster to the entire institution.

Dean Diederichs was born in Muenchen-Gladbach, Germany, August 12, 1874, and received his elementary education there. He was the son of Johann Peter and Anna Marie (Kamps) Diederichs, and was the oldest of a family of six, all of the remainder of whom survive him. These are Mrs. Katherine Shafer, of Brooklyn, N.Y.; Mrs. Louise Weber, of Waterbury, Conn.; P. Paul, of Dolgeville, N.Y.; Hugo N., of Owens, W.Va.; and William J., of Upper Darby, Pa.

In 1888 his parents migrated to this country and settled in Dolgeville, N.Y. The family was one of modest circumstances, yet means were found to send him and others of the family to high school at Dolgeville, where he won a state scholarship. He walked to Herkimer, more than twenty miles distant, to take the competitive examinations. With this and a promise of financial assistance from friends (which did not materialize), he entered Cornell University in 1893. Here, largely through his own exertions, he not only completed his course in engineering in the prescribed four years, with such a good record as to be elected to the honorary society of Sigma Xi, but he also found time to engage in student activities and was

prominent as a shot-putter. He graduated with the degree of M.E. in 1897, which, for a person who had to learn English at fourteen years of age, must be considered as an accomplishment.

He won the first Sibley prize in his senior year and the Sibley Fellowship for graduate study. After a year on this fellowship he was appointed an instructor in experimental engineering under Professor Rolla C. Carpenter. In 1902 he was promoted to an assistant professorship, to a professorship in 1907, and succeeded Professor Carpenter as head of the Department of Experimental Engineering in 1920. In 1928 he was appointed the first incumbent of the John Edson Sweet Professorship, founded in honor of the distinguished engineer who at one time was a professor at Cornell. In 1921 he became director of the Sibley School of Mechanical Engineering and in 1936 he was appointed to the deanship of the College of Engineering—a well-deserved honor, which he was fated to enjoy only for a little over one year.

His entire manhood was, therefore, devoted to the service of his alma mater, not only in his chosen field, but in many of the broader aspects of university life. No man of the faculty was called upon more frequently to serve on important committees, and the confidence of his colleagues in his sound judgment and honesty is attested by his election by the university faculty in 1929 to serve as one of the faculty representatives on the Board of Trustees, which office he filled for three years. Space does not permit even an enumeration of the many services of this sort that he rendered to the university community. His service to the College of Engineering was even greater. For many years no important question whether of academic or administrative character has been decided without seeking his advice and judgment. The curriculum of mechanical engineering, in particular, has been shaped under his guiding hand. He was indeed a wise adviser.

He was no less active and interested in the problem of student life. Interested naturally in athletics, he was for many years a member of the Athletic Association and for some years served as president of that body. Here, again, his sterling character impressed itself upon the difficult problems of athletics, and that this was amply appreciated by the student body is shown by the following dedication of *The Cornellian* of 1935:

"To Herman Diederichs, who for forty-two years has served his alma mater as student, teacher, and administrator, and, who, through his intense interest and untiring efforts, has succeeded in inaugurating a new era in Cornell athletics. For his active participation in campus affairs, he will long be remembered, and as a stern teacher and a sympathetic, honest friend, the Class of 1935 will revere him—Cornell's Man of the Year."

Surely student praise can attain no higher level of appreciation.

He was vice-president of the second district of the National Collegiate Athletic Association and chairman of the Board of Athletic Control of Cornell University.

Dean Diederichs was essentially a man of scholarly mind and his outlook on life was naturally colored by this fine quality. He was an authority in the field of experimental engineering. In 1905 he and Professor Carpenter collaborated in writing a treatise entitled "Internal Combustion Engines" and in 1911 they produced "Experimental Engineering," which is a standard work in that field. In 1930 he and Professor W. C. Andrae collaborated in a monumental treatise entitled "Experimental Mechanical Engineering" (Volume I, Engineering Instruments) and Volume II, on testing of power plant apparatus, was in preparation. He also translated Güldner's "Internal Combustion Engines" from the German. He contributed a number of articles to the *Sibley Journal* and other publications, beginning as early as 1900. Among these were "Analysis of Coals," "Comparative Tests of Plain and Para-Pneumatic Pulleys," and several reports on gas-engine tests. He was the co-author of three bulletins of the Engineering Experiment Station at Cornell, dealing with "The Purification of Salt Made From Central New York Brines," "The Heat Transfer From Steam to Heavy Fuel Oil," and "Flame Propagation in Closed Cylinders." He also made many tests and investigations as a consultant, among which may be mentioned a power test on the steamer *Morro Castle*, of the Ward Line, a report on a plant for the production of oxygen, for the American Oxygen Company, and, particularly, a voluminous report on the power plant of the Hall of Records in New York, N.Y. This test consumed a year and the report constitutes a book of 673 pages.

Dean Diederichs was also a member of the following honorary and scientific societies: Quill and Dagger, senior honorary society; Phi Sigma Kappa fraternity; Sigma Xi; Tau Beta Pi; Phi Kappa Phi; Society of Automotive Engineers; American Society for Metals; Verein deutscher Ingenieur; and Society for the Promotion of Engineering Education.

His teaching naturally reflected his scholarly habits and Germanic thoroughness. He had little sympathy with lazy or indifferent students, but he would go to any length to make a point clear and no student went to him for help and came away empty-handed. Perhaps his best teaching work was his lectures on materials of engineering given to many generations of electrical and mechanical engineers and which, in the opinion of the writer, were classics. He was an excellent lecturer and could interest the student in this highly technical subject. Of lasting value also is his work in helping to develop the present course in experimental engineering than which there is none better, if as good, in this country. It should be remembered that the engineering experimental laboratory was conceived by Dr. Robert H. Thurston at Stevens Institute and brought by him to Cornell in 1885. The background of the present course was developed under his guidance by Professor Rolla C. Carpenter and has been widely copied elsewhere. Professor Diederichs, therefore, drank from the original source and carried with him to his death the inspiration of these two great pioneers. It has been his labor to modify and adapt the course to an ever changing industrial world and this he has done in a masterly manner, keeping the course in advance of the times, though almost always handicapped for lack of money and equipment. It was this same inspiration that enabled him to keep the spirit of research alive in mechanical engineering, for, in spite of many handicaps, too many to enumerate, the long list of scientific publications issued under his guidance has continued to grow. His influence and knowledge in this important side of the work of the college will be one of its heaviest losses, and one that will be impossible to compensate for. Men of his thorough scholarly and scientific background and long experience in experimental work are rare indeed, and difficult to replace. One of the reasons for his wide and deep knowledge lay in his omnivorous reading, not only of scientific and engineering literature, but also of a broad general character, and this wide reading habit embraced German literature as well as American and English. He was a very well-informed man.

I have always held him in high regard as an administrator. He was my close friend for nearly forty years, and my most valued adviser during my deanship. Not that we have always agreed on all matters, but because of his openmindedness it was always possible to come to a conclusion that seemed best and which was mutually satisfactory. He never allowed his personal feelings or opinions to affect in any way the welfare of any member of the faculty, and his interest in the life of the college was deep and fair-minded. He had certain definite policies for the future of the college, which I had hoped he would have an opportunity to develop, but fate ordained otherwise. That these good qualities were fully appreciated by his colleagues, was attested by the almost unanimous vote by which his nomination was approved.

His relations to his students were quite unique. Despite the fact that he was a bachelor, he had a real fondness for young people and sympathy with their problems. He was far from being a demonstrative person, and his rather severe countenance and somewhat brusque manner were often forbidding to the new student. It usually took some experience with him to discover that this rough exterior hid a warm and understanding heart. Dr. C. F. Hirshfeld, in an appraisal of him on the occasion of his appointment to the deanship, has very aptly expressed this quality thus—"In some respects the poorer scholars and breakers of rules and regulations among the older classes were more fortunate than those who traveled the straight and narrow path of scholarship and good conduct. Many of the weak, the lame, and mischievous who appeared before Professor Diederichs and his associates, by special though not sought after invitation, discovered that back of the severe manner of the strict disciplinarian there dwelt a kindly, understanding, and human living soul." He had a fine sense of humor through which he could always view student manners, and he was fond of relating humorous situations that arose in his contacts with underclassmen. Only a short time ago he related to me with glee, two absolutely new and equally spurious reasons advanced by freshmen, why it was imperative for them to get away several days before the beginning of the spring recess. He said that because the reasons were at least new and ingenious, though of doubtful validity, he granted the requests and laughed heartily.

No student ever went to him for advice or help on any matter without receiving assistance, and as freshmen grew into seniors, they acquired respect, admiration, and often real affection for this rugged and unique personality. To hundreds of returning alumni, the campus will not be quite the same, because "Died" will not be there.

One of Dean Diederichs' most outstanding virtues was his essential honesty, not only in matters pertaining to his professional work and his teaching, but in all his personal contacts with people. I have never known a man on whose word I could more thoroughly rely, or one of greater integrity, in all the many transactions that transpired between us. He was modest to an extreme degree, which accounted, no doubt, for some of his apparent great reserve. He was a sociable man, fond of company, and a first-class companion. His wide range of reading enabled him to converse intelligently on almost any topic. Frailties he no doubt had, as have all of us, but they were over-

shadowed by his rugged, upstanding personality, and by the many helpful services he rendered to all around him. Like Joseph of Arimathæa, "he was a good man, and a just," and Cornell University and this entire community are the better because he lived and worked among us.

AMELIA EARHART (1898–1937)

Amelia Earhart was born in Atchison, Kan., on July 24, 1898, the daughter of Edwin S. and Amy Otis Earhart. She was educated in the Hyde Park High School of Chicago, at Columbia University, and at various other colleges. From 1926 to 1928, she was in charge of the girls' work at Denison House in Boston, and also taught an extension course for the Commonwealth of Massachusetts. On June 17, 1928, she was the first woman to cross the Atlantic Ocean in an airplane, going from Trespassey Bay, Newfoundland, to Burryport, Wales. From 1928 to 1930, she was aviation editor of the *Cosmopolitan Magazine*, and began her career as a pioneer. She has been described as the first woman to fly the Atlantic alone, the first person to fly the Atlantic alone twice, the first woman to fly an autogiro, the first person to cross the United States in an autogiro, the first woman to receive the Distinguished Flying Cross, the first woman to fly nonstop across the United States, and the first woman to fly from Hawaii to the United States.

In 1931, she married George Palmer Putnam, who cooperated in every way to make her flying possible and to encourage her in her pioneer work. The best account of her achievements and her interests is contained in her two books, "20 Hours, 40 Minutes," published in 1928, and "The Fun of It," published in 1931. They show her two chief interests to have been flying and furthering the progress of aviation, and increasing opportunities for women to select and go forward into careers and at the same time, enjoy home and family life.

Purdue University is proud that Amelia Earhart became a member of its faculty and made outstanding contributions, both in the field of aeronautics and in that of careers for women. Her "Flying Laboratory" in which she made her last flight was financed by men affiliated with Purdue, and the tragic ending in July, 1937, of her exploratory trip around the world, which shocked and saddened the entire world, took from Purdue not only a valued and inspiring leader, but a beloved friend. Her portrait hangs in one of the residence halls for women and each of the three residence halls has a memorial library maintained by the students in affectionate recognition of her contributions. President Edward C. Elliott has characterized her perfectly in a speech made on the Purdue campus at a Conference on Women's Work and Opportunities, October 31, 1937, when he said, "Her primary interest in life was not in this career of adventure upon which she had embarked, but rather in an effort to find and make some addition to the solution of the problem of careers for women."

Miss Earhart had many honors and medals. She was a member of many technical groups, including the A.S.M.E., which she joined as an associate in 1931, and of many nontechnical groups and clubs. Her contributions can be measured by her achievements, but also include stimulating and encouraging youth everywhere to adventure and to achievement.—[Memorial prepared by LILLIAN M. GILBRETH, Montclair, N.J., Mem. A.S.M.E.]

RALPH EARLE (1874–1939)

Ralph Earle, Rear-Admiral, U.S. Navy, Retired, president of Worcester Polytechnic Institute, died at Worcester, Mass., on February 13, 1939. He had been president of the Institute since 1925 and was made a member of the A.S.M.E. in 1926.

President Earle was descended from the early settlers of Leicester, Mass. He was born in Worcester, May 3, 1874, son of Stephen Carpenter and Mary Eaton (Brown) Earle, and after his early education in the public schools of Worcester, was admitted to the Worcester Polytechnic Institute in January, 1892. A few months later, he took the competitive examination for entrance to the United States Naval Academy at Annapolis, won the appointment, and was graduated there in 1896.

He was promoted through the various commissioned grades of the Navy, serving in the war with Spain, the World War, and in the Philippine and Mexican Campaigns. He soon became conspicuous in matters pertaining to naval ordnance, and in 1916, shortly after attaining the rank of captain, he was made the Chief of the Bureau of Ordnance, with the rank of Rear-Admiral during the exacting World War period. He was responsible for the conception of two of the outstanding activities in which the American Navy was engaged during the World War. The first had much to do with the defeat of the enemy's submarine warfare and consisted of the design and construction of a mine barrage across the North Sea. This barrage consisted of a submerged mine field 230 miles in length and 25 miles wide and was the culminating step in combatting the submarine.

The second accomplishment was that of the design and the construction of the 14-inch railway batteries which saw active service in France. These naval guns were the largest at the front and resulted in cutting the enemy's main line of communication and unquestionably paved the way for the Armistice.

In 1925, he retired from the Navy to accept the call to become the president of the Worcester Polytechnic Institute in his native city. His record there was one of marked progress for the school, both professionally and materially, and, at the time of his sudden death in 1939, his comprehensive plan to keep that well-established school abreast the field of engineering was well under way and is now being pushed to completion. As sixth president of the Institute, his administration will be long remembered for the success which he achieved in training youth for leadership in the world of science and engineering.

Admiral Earle was a Bachelor of Science, United States Naval Academy, and received the following honorary degrees: D.Sc., Worcester Polytechnic Institute, 1925; D.Eng., Rensselaer Polytechnic Institute, 1926; LL.D., Amherst College, 1929; and LL.D., Clark University, 1930. During his naval career, he was honored with many decorations both by his own and foreign countries. He was a member of the Society of Naval Architects and Marine Engineers and of the Society for the Promotion of Engineering Education, as well as of a number of other societies and clubs, and was the author of several books.

President Earle is survived by his widow, Janet Turner (Schenck) Earle, whom he married in 1898 and who is the daughter of a naval officer; a son, Ralph, Jr., who is a naval officer; and a daughter, Mary Janet Hines, who is the wife of a naval officer.

President Earle's whole career was stamped with the loyalty engendered in his early naval life and he had to an unusual degree the gift of winning the loyalty of his subordinates. Added to this gift were his human qualities which made him beloved by all with whom he came in contact.—[Memorial prepared by ADMIRAL WAT TYLER CLUVERIUS, President, Worcester Polytechnic Institute.]

CHARLES ISAAC EARLL (1863–1938)

Charles Isaac Earll, of York, Pa., died in New York, N.Y., on June 14, 1938, while on a business trip. He was severely injured in an automobile accident near Norwalk, Ohio, on October 28, 1937; and although he apparently recovered from the effects of the accident, he was never well for long afterward, and his death, eight months later, was probably an indirect result of the injuries sustained at that time.

Mr. Earll was born at Van Buren, N.Y., on April 11, 1863, the son of Myron H. and Asenath (Styles) Earll. He obtained his preparatory education at the State Normal School, Whitewater, Wis., and was graduated with a B.S. degree in mechanical engineering from the University of Wisconsin in 1885. He later took one and one-half years' work in the New York Law School.

Immediately on his graduation from the University of Wisconsin, Mr. Earll went to work for the Filer & Stowell Co., Milwaukee, Wis., as machinist and draftsman in the building of steam engines and sawmill machinery. From 1886 through 1890, he worked as draftsman and designer on the Paige typesetter, an enterprise conducted by James W. Paige, Hartford, Conn., and financed by Samuel L. Clemens (Mark Twain). In January, 1891, Mr. Earll went to New York and was employed as assistant engineer on the several cable-operated street railways then in use on Broadway, Columbus, Seventh, Ninth, and Lexington Avenues, and Twenty-Third Street.

From 1894 to 1898, Mr. Earll was superintendent of shops for the De Laval Separator Company, Poughkeepsie, N.Y., and from 1898 to 1915, he practiced in New York as patent solicitor and consulting engineer. In 1915, he established his own business in York, Pa., manufacturing trolley catchers and trolley retrievers, and continued in that work until his death.

During his long professional career, Mr. Earll took out a number of patents relating to cable railways and grips for these railways; also on a geared hoist and on a reel for clotheslines, and several on springs, working mechanisms, and other details for trolley catchers and retrievers.

Mr. Earll became a member of the A.S.M.E. in 1892. He served on the Publications Committee of the Society from 1911 to 1917, being chairman in 1914 and 1915; and on the Executive Committee of the Susquehanna Section in 1928–1930. He was also a member of the Engineering Society of York, and of the Manufacturers' Association, the Chamber of Commerce, the Lafayette Club, the Country Club, and the Rotary Club, all of York, and of the Engineers' Club, of New York.

In 1905, Mr. Earll married Mary H. Flynn, of New York, who survives him.

SAMUEL SUMNER EDMANDS (1877–1938)

Samuel Sumner Edmands was born in Kalamazoo, Mich., on April 30, 1877, his father being John Edmands, and his mother Maria (Goodwin) Edmands. His formal technical education was received at Worcester Polytechnic Institute, Worcester, Mass., where he was graduated in 1899. During the following year he was employed by the American Telephone & Telegraph Co., Providence, R.I. In 1900 he returned to the Middle West, serving as instructor in electrical engineering at Ohio State University for one year. In 1901 he began his service at Pratt Institute, Brooklyn, N.Y., where he was destined to continue for thirty-seven years, and to leave his imprint on the development of technical education of the technical institute type in this country.

He served Pratt the first year as instructor, and became head of the electrical department the following year. In 1910 Arthur L. Williston retired from his position as director of the School of Science and Technology at Pratt to become principal of Wentworth Institute, at Boston, and the young head of the electrical department was elevated to fill his position.

The period which followed 1910 was an important one for technical institutes. They were to be called upon, with other educational institutions, to give preliminary trade training to United States soldiers in 1918, and later, from 1919 to 1925, to train many demobilized soldiers who had been wounded in service, or whose physical weaknesses had been aggravated by service. For this work the technical institute type of institution proved very useful, for it was accustomed to devise courses to meet specific needs, and was free of the inertia of large educational systems. Those interested in such courses found much to learn at Pratt Institute, because of the prominent position of this institute in its field. During all this period Director Edmands was quietly effective. During more recent years he expanded the Institute's two-year course to three years, with an optional fourth year for a degree.

Director Edmands joined the A.S.M.E. in 1923 and was a member of the Committee on Education and Training for the Industries from 1924 to 1933. He declined the chairmanship of this committee because of the pressure of other duties. His service on the committee was wholehearted. He helped select many of the papers that were presented at the meetings, and his acquaintance with educators and industrial leaders in the East brought the committee into contact with effective speakers in the field of education for industry. He was also a member of the Brooklyn Engineers' Club, and of the Society for the Promotion of Engineering Education.

In 1930 Director Edmands received the honorary degree of Doctor of Engineering from Worcester Polytechnic Institute. Efforts to attract him to positions of greater apparent prominence in education failed to move him from Pratt Institute. He was primarily interested in the development of the technical institute type of course with which Pratt Institute had long been identified, and in which his own influence had been great.

Dr. Edmands' death on May 24, 1938, was the result of injuries suffered in the crash of an automobile in which he was a passenger. He was at the height of his powers and would have made a considerable additional contribution to the field in which he had already done much, had he been spared. He was married in 1905 to Althea Florence Miller, of Brooklyn, N.Y., and is survived by his widow and a daughter, Miss Patricia Edmands.—[Memorial prepared by JOHN T. FAIG, Cincinnati, Ohio, Mem. A.S.M.E.]

LOUIS G. ENGEL (1859–1938)

Louis G. Engel, who retired from active professional work about 1905, died on April 19, 1938, at his summer home, New Canaan, Conn. Mr. Engel was born on November 29, 1859, in Brooklyn, N.Y. He was the son of Frederick Eberhard and Caroline Leonora (Gunzenhauser) Engel. He attended Brooklyn public schools, and then entered the School of Mines of Columbia University, where he was graduated with the Class of 1880 with the degree of Mining Engineer (E.M.). He then served a partial apprenticeship in shop work, and became familiar with ordinary machine tools.

This was followed by three years' service as mining engineer for the Tilly Foster Iron Mines, Tilly Foster, N.Y., where he had direct charge of all underground work. He was then employed for about three years as draftsman by the Brooklyn Sugar Refining Company and for over a year further as superintendent of construction, with full responsibility for all new work. About 1890, Mr. Engel associated himself with the New York Sugar Refining Company, under the presidency of Claus Doscher. This company later became the National Sugar Refining Company, and Mr. Engel remained with it until his retirement in 1905.

Mr. Engel married Miss Gesine Doscher, who survives him. He became a member of the A.S.M.E. in 1887.

ROBERT HEYWOOD FERNALD (1871–1937)

Robert Heywood Fernald, director of the Mechanical Engineering Department and dean of the Towne Scientific School of the University of Pennsylvania, died on April 24, 1937, in the University of Pennsylvania Hospital, Philadelphia, Pa., of a heart ailment, following a long illness.

Dean Fernald was the son of Dr. Merritt Caldwell Fernald, a former president of the Maine State College of Agriculture and the Mechanic Arts (now the University of Maine), and Mary Lovejoy (Heywood) Fernald. He was born in Orono, Maine, on December 19, 1871, prepared for college in the Orono High School, and was graduated from Maine State College with a B.S. degree in mechanical engineering in 1892. He did graduate work at the Massachusetts Institute of Technology, 1892–1893, and in 1898 was granted an M.E. degree by the Case School of Applied Science. He was a Fellow in Columbia University in 1900–1902, obtaining the A.M. degree there in 1901, and the Ph.D. in 1902.

It was not at all Dr. Fernald's ambition as a young man to take up educational work—he had architecture in mind; but following his graduate work at M.I.T., he ran into the depression of 1893, and being offered an instructorship in mathematics and mechanical engineering at the Case School of Applied Science, he accepted it, and decided to teach until economic conditions improved. He liked his work so well that he never subsequently dropped it, although he found time to engage in a wide variety of consulting and professional work.

In 1900, Dr. Fernald left the Case School, where he had been assistant professor of mechanical engineering since 1896, to take a fellowship at Columbia University. Upon the completion of his work there in 1902 he became professor of mechanical engineering at Washington University, St. Louis, Mo., and remained there until 1907, when he returned to the Case School to head its Mechanical Engineering Department. In 1912, he was called to the University of Pennsylvania as Whitney professor of dynamical engineering and director of the Mechanical Engineering Department. In February of 1930, Dr. Fernald was made dean of the Towne Scientific School, which appointment was terminated only by his death.

In addition to his several earned degrees, the University of Pennsylvania in 1924 conferred upon Dr. Fernald the honorary degree of Doctor of Science. He was also a member of the honorary fraternities of Sigma Xi, Phi Kappa Phi, Tau Beta Pi, and Phi Beta Kappa. He became an associate of the A.S.M.E. in 1900, member in 1903, and Fellow in 1936. He was an active member of the Society for the Promotion of Engineering Education. He was a member of the Joint Committee of the National Industrial Conference Board and Society for the Promotion of Engineering Education on relations between technical education and the industries, 1922–1928; and served as official representative of the Towne Scientific School in the Society for the Promotion of Engineering Education. He belonged to The Franklin Institute, and served as president of the Cleveland Engineering Society in 1912. He was a secretary of the Engineers' Club of Philadelphia in 1916, its president in 1922–1924, and a member of its Board of Directors, 1924–1927. His club memberships included the Engineers, University, Manufacturers, and Lenape clubs, of Philadelphia.

Dr. Fernald directed and conducted extensive investigations in the United States and Europe for the U.S. Geological Survey and Bureau of Mines and prepared exhaustive reports for these bureaus. He was engineer-in-charge of the Technologic Branch, U.S. Geological Survey, from September 1, 1904, to July 1, 1910, and consulting engineer, Fuel Division, Bureau of Mines, July 1, 1910–1920. As consulting engineer for the Public Service Commission of Pennsylvania, 1913–1915, he formulated rules and regulations for gas, heating, and water utilities of Pennsylvania. He was a member of the Conservation State Board, Pennsylvania, U.S. Fuel Administration, 1918–1919, engineer-member, Giant Power Survey Board of Pennsylvania, 1923–1926; member, Executive Committee, Third World Power Conference, 1935–1936; member, Science Advisory Committee, Mechanical Engineering Division, Chicago Century of Progress Exposition, held in 1933; member, Board of Directors, Philadelphia Sesquicentennial Exposition, held in 1926; member, Executive Committee, Traffic Commission, City of Philadelphia, 1930–1933; member, Greater Pennsylvania Council, 1931; chairman, Committee on Development of Pymatuning Area, Mercer County, Pa.; honorary vice-chairman, Engineers' National Hoover Committee, Pennsylvania Section, 1932; engineer-member, Advisory Committee, Philadelphia Agency of the Reconstruction Finance Corporation, 1932–1933; and chairman, Executive Committee, Technical Advisory Council affiliated with Philadelphia Chamber of Commerce, 1936–1937. He also carried on a general consulting practice in mechanical engineering, especially in fuels and power,

and made commercial investigations for numerous firms and individuals.

An easy and prolific writer, Dean Fernald contributed many papers, not only to purely technical journals, but to many non-technical periodicals. He was also the co-author, with G. A. Orrok, of a 600-page university textbook, "Engineering of Power Plants," which passed through three editions, 1916, 1921, and 1927.

As a member of the A.S.M.E., Dr. Fernald served with competence and loyalty. He was a manager of the Society, 1916–1919, and vice-president, 1920–1921. He was one of the Society's delegates to the American Engineering Council in 1921. His committee service began in 1911, when he became chairman of the Gas Power Section. He was chairman of the Executive Committee of the Philadelphia Section in 1915–1916, and a member of the Meetings and Program Committee, 1913–1917, serving as chairman the last year. He was a member of the Power Test Codes Committee from 1918 until his death, and was appointed its acting chairman in 1933 and permanent chairman in 1936. He was temporary chairman of Power Test Codes Committee No. 16 on Gas Producers during its first year, 1918–1919, and continued as a member of the committee until his death.

In June, 1905, Dr. Fernald married Catherine Mason Coupland, of Boone, Iowa. They had three children: Merritt Caldwell, Frances Mason, and Mason. His widow and two sons survive him. He also had three brothers, Merritt Lyndon Fernald, Fisher Professor of Natural History, Harvard University, George Bancroft Fernald, Master of English at St. Marks School, Southborough, Mass., and Reginald Lovejoy Fernald, proprietor of the Pratt Teachers Agency, New York, N.Y.; and a sister, Harriet Converse (Fernald) Pierce.

WESLEY MONTEITH GRAFF (1890–1937)

Wesley Monteith Graff, director of the Safety Engineering Division of the National Bureau of Casualty and Surety Underwriters, New York, N.Y., died on June 5, 1937. Mr. Graff was born on May 11, 1890, in Brooklyn, N.Y., the son of Charles Edward and Margaret (Barclay) Graff. He was educated in the Polytechnic Preparatory School, Brooklyn, and in the Sheffield Scientific School of Yale University, where he obtained a Ph.B. degree in 1911 and M.E. degree in 1913.

Mr. Graff's first practical experience was secured during his college vacations: In 1910, with the Pennsylvania Tunnel & Terminal R.R. Co., as draftsman in connection with tunnels and terminals in New York, and as assistant engineer on acceptance tests of electric locomotives; in 1911, with the Hartford Electric Light Company, in charge of an investigation and cost-comparisons of steam and water power for central-station use; in 1912 and 1913, with the Edison Electrical Illuminating Company of Brooklyn as assistant power engineer on sales of industrial electric power, and laying out and supervising the installation of industrial power, heating, and lighting equipment.

After his graduation, from September, 1913, to April, 1918, Mr. Graff was employed by the Graves Engineering Co., Inc., New York, consulting engineers, first as chief mechanical engineer, and, from October, 1917, as vice-president. His work chiefly concerned the design, construction, and operation of electric public utilities; design, specifications, investigations, and reports pertaining to industrial power, heat, and illumination; studies of special mechanical manufacturing processes; and the design of special machinery.

Mr. Graff served from April, 1928, to December of the same year with the Engineering Division of the Ordnance Department, Washington, D.C., as director of the preparation of specifications for the Motor Equipment Section, covering trucks, tractors, tanks, mobile machine and repair shops, and allied matters. In December, 1918, he became vice-president of the Graves, Graff & Dresser Co., New York, and in August of the following year, he formed his own company, the Graff Engineering Corporation, New York, consulting engineers, specializing in design, investigations, and reports pertaining to industrial power, heat and illumination, and to the elimination of industrial waste by the improvement of manufacturing processes and equipment. He was president of the company until it was discontinued in 1931 when he became director of the Safety Engineering Division of the National Bureau of Casualty and Surety Underwriters.

Mr. Graff was elected a member of the A.S.M.E. in 1923. He served on the Society's Safety Committee from 1931 until his death, being a member the first three years, chairman two years, and advisory member the remainder of the time. He was the Society's representative on the Sectional Committee on a Safety Code for Walkway Surfaces from October, 1932, until his death. He had served since 1931 for the National Bureau of Casualty and Surety Underwriters on several other safety code committees. He was also a member of the Illuminating Engineering Society, Alpha Chi Rho fraternity, and the honorary society of Sigma Xi, and the Union Club of New Jersey. He had served as chairman of the Republican Committee in Westfield, N.J., where he made his home.

In 1913, Mr. Graff married Marjorie Higgins, and three children were born to them: Marian Barclay, Wesley Monteith, Jr., and Barbara Preston Graff.

ABBOTT DEAN GRANGER (1870–1938)

Abbott Dean Granger, president of the Granger Machinery Corporation, New York, N.Y., died at his home in Oakland, N.J., on September 27, 1938. Surviving him are Mrs. James S. (Peters) Granger, whom he married in 1895, and three daughters, Althea (Granger) Ross and Barbara S. Granger, of Ridgewood, N.J., and Olive (Granger) Oliver, of White Plains, N.Y.

Mr. Granger was born in New York on December 3, 1870, son of John Calvin and Althea (Dean) Granger. He attended high school in Brooklyn and secured a C.E. degree at the School of Mines, Columbia University, in 1892. For about eight years thereafter he was associated with Edwin Burhorn* in the firm of Burhorn & Granger, engaged in the installation and equipment of plants for power, heat, and electric light, and other contracting and engineering work. He then established the A. D. Granger Co., New York, and served as president of it and of its successor, in 1935, the Granger Machinery Corporation, until his death. The company designed, sold, and installed power plant machinery and other equipment.

Mr. Granger was elected a member of the A.S.M.E. in 1915 but accepted a loss in seniority in 1936 and is recorded as a member of that year. He belonged to the Columbia University Club and was at one time a member of the Brooklyn Engineers Club.

JOHN WILSON WISHART HALLOCK (1888–1937)

John Wilson Wishart Hallock, secretary of the General Alumni Association of the University of Pittsburgh, died on July 17, 1937, in the West Penn Hospital, Pittsburgh, Pa., after a long illness.

He was born at Pittsburgh on December 27, 1888, the son of Harvey T. and Ellen (Wishart) Hallock. He secured his preliminary education in the public schools of his native city, and then entered the University of Pittsburgh, where he was graduated with the B.S. degree in mechanical engineering in 1912, and in 1915 was granted the M.E. degree. In 1936 the degree of doctor of philosophy in economics was conferred upon him by the university.

From July, 1912, to October, 1913, he worked as assistant engineer for the Harris Pump & Supply Co., Pittsburgh, on the design, construction, and erection of hydraulic machinery. He was then employed for seven months as draftsman and junior engineer by the War Department, in the Engineer Depot, Washington, D.C. In June, 1914, he returned to the Harris firm in Pittsburgh, as assistant to the vice-president in executive and sales work. A year later he was engaged as instructor in cooperative work, by the School of Engineering, University of Pittsburgh, and subsequently became director of this work. An article by him entitled, "The Cooperative Plan of Engineering Education," published in the Bulletin of the Society for the Promotion of Engineering Education (vol. VIII, no. 1, September, 1917, pp. 12–24), describes the plan in some detail. In May, 1917, he began a term of active service with the U.S. Army, Corps of Engineers, entering as second lieutenant and being discharged, February 10, 1919, with the rank of captain in that corps. During the greater part of the time he was in charge of production districts for the General Engineer Depot. At the time of his death, he was colonel in the Corps of Engineer Reserves, and chief of the Pittsburgh Procurement District.

After leaving active service in 1919, Dr. Hallock resumed his former occupation as director of cooperative work in the University of Pittsburgh, but in October, 1919, he returned to the Harris Pump & Supply Co., as sales manager. However, about a year later he went back to the University of Pittsburgh as professor and head of the newly established Department of Industrial Engineering, in addition to resuming the directorship of the cooperative work; he also continued to act for the Harris Company in a consulting and sales organizing capacity. After eighteen years of active faculty service, Dr. Hallock was asked to take charge of the reorganization of the University's General Alumni Association. He became its secretary on August 1, 1933, and soon had it functioning. He took over this task when interest was at a low ebb, and built up its files of names and addresses from about 7,000 to more than 25,000.

He joined the A.S.M.E. as an associate-member in 1919 and was promoted to member in 1922. He was also a member of the Society for the Promotion of Engineering Education and the American Economics Association, of the national fraternity of Phi Gamma Delta, the Rotary and University clubs of Pittsburgh, the honorary

* For biography of Edwin Burhorn, see page RI-48.

fraternities, Sigma Tau and Omicron Delta Kappa, and of Scabbard and Blade, honorary military fraternity. He was a ruling elder in the Shadyside Presbyterian Church of Pittsburgh and general superintendent of the church school.

Dr. Hallock is survived by his widow, Mrs. Eva (White) Hallock, whom he married in 1914, and by two daughters, Janet Thomson and Eleanor Wishart Hallock.

He was the author of a number of published articles, principally on industrial safety, and of a widely known text, "Production Planning, Its Engineering Elements," published by the Ronald Press in 1929. In 1931 the outline of his work in industrial safety engineering was published by the National Safety Council and adopted as a standard text.

RUDOLPH K. F. HARTIG (1892–1937)

Rudolph K. F. Hartig, an engineer in the service of the Brooklyn Edison Company, Inc., died on June 9, 1937. Mr. Hartig was born at Waukesha, Wis., on March 24, 1892, the son of William and Fredrika (Dittman) Hartig, and obtained his preparatory education in the Waukesha public schools. He served in the U.S. Navy as a fireman, 1917–1919, and after the close of the World War entered Montana State College, from which he was graduated in 1925 with a B.S. degree in mechanical engineering.

From October, 1925, to July, 1927, Mr. Hartig was employed by the Brooklyn Edison Co., Inc., as junior engineer in power plant work, supervising the installation and maintenance of central station instruments. He also assisted in testing various equipment units, such as turbogenerators, condensers, boilers, stokers, and auxiliaries. In July, 1927, he was transferred to the main office of the Brooklyn Edison Co., where he was given charge of the preparation of reports and articles for A.S.M.E. and N.E.L.A. publications. He cooperated in obtaining and preparing data for a number of progress reports of the A.S.M.E. Special Research Committee on Condenser Tubes, 1927–1933, and prepared the N.E.L.A. Prime Movers Committee Serial Reports on Stoker Equipment and Furnaces, 1927–1928, and the Foreign Developments Report, 1929–1930. He also made plant betterment studies for the company.

Mr. Hartig joined the A.S.M.E. as an associate-member in 1930, and was automatically transferred to the grade of member in 1935. He served on the Annual Meeting Excursions Committee in 1929, 1930, and 1931, as chairman the last year.

In 1926, Mr. Hartig married Anna O'Hare of Brooklyn, N.Y., who, together with their two children, William J. and John G. Hartig, survives him.

PHILIP MOULTON HATHEWAY (1885–1937)

Philip Moulton Hatheway, of Brooklyn, N.Y., died at "Mowglis, School of the Open," East Hebron, N.H., on July 21, 1937. Mr. Hatheway was born in Boston, Mass., on October 19, 1885, the son of Amos L. and Cora L. (Moulton) Hatheway. He prepared for college at the Phillips Exeter Academy, from which he entered the Sheffield Scientific School of Yale University, being graduated from there in 1907 with a Ph.B. degree.

Following his graduation, Mr. Hatheway entered the employ of the Stanley Works, New Britain, Conn., where he worked in the office and cold rolled steel mill for about two years. He then served successively as field clerk, draftsman, and assistant engineer with Westinghouse, Church, Kerr & Co. He engaged in both inside and outside work in connection with various construction jobs and was connected with organization work at the main office of the company in New York, N.Y. In 1913, he went with the American Felt Company, Boston, Mass., as plant manager, remaining there until 1915, when he took a position with the Associated Factory Mutual Fire Insurance Company, Boston, making fire insurance surveys, inspections, and reports. In 1916–1917 he was connected with the International Steel & Ordnance Corp., Lowell, Mass., putting a detonator plant into production. The following year he was employed by the International Register Company, of Chicago, on war work. He acted as assistant to the president, directly connected with the manufacture and inspection of navy, ordnance, and signal corps matériel.

From 1918 to 1932, Mr. Hatheway was employed by the National Lead Company, of New York, N.Y. He was chief engineer at the Atlantic Works, Brooklyn, during the early part of the time, later becoming assistant superintendent of the plant, which position he held until he resigned in 1932.

Mr. Hatheway married Clitheroe James, of Brookline, Mass., in 1918, and is survived by her and their son Dean Moulton, and daughter, Clitheroe Dean Hatheway. Mr. Hatheway listed his recreations as canoeing and camping, and his hobby as photography; his religion as Congregational and his politics as Republican. He was a member of the Yale Club of New York, Appalachian Mountain Club of Boston, and the Masonic fraternity. He joined the A.S.M.E. as a junior in 1913, and was promoted to the member grade in 1921.

CHARLES GILBERT HAWLEY (1868–1939)

Charles Gilbert Hawley, lawyer, scientist, and internationally known inventor, son of the inventor, Benjamin Ruggles Hawley, and Rosamond (Hall) Hawley, was born in Normal, Ill., on June 20, 1868. His parents were early Illinois pioneers.

Mr. Hawley's early schooling was in Chicago, Ill. He attended Cornell University for his academic work and took his graduate work at Northwestern University, from whose Law School he received the Bachelor of Laws degree on June 11, 1890. He immediately entered the active practice of the law, specializing in all branches of patent law. Because of his keen inventive analytical mind he rose rapidly and was in great demand by prominent corporations who recognized his great ability as both technical adviser and trial expert. In the beginning of the century he moved his offices from Minneapolis, Minn., where he started his practice, to Chicago, where he had his headquarters, but divided his time between his offices there and in New York and Washington. He kept his legal residence in Chicago until January, 1938, when he changed it to Cleveland, Ohio.

One of Mr. Hawley's early associations was as counsel and large shareholder of the Universal Rim Company, who owned the patents on the Baker demountable rim, the first demountable rim on the market. He was also one of the first inventors to conceive the idea of the front-wheel drive for automobiles, but did not push it because of his belief that it was too far in advance of the industry to be at that time profitably marketable.

Mr. Hawley was an officer and large stockholder of the Locomotive Fire Box Company, who manufactured and marketed, under patents assigned to it by Mr. Hawley, the Nicholson thermic syphon, used by more than one hundred railroad companies in this and foreign countries. He severed his connections with this company a few years ago, after which he took out domestic and foreign patents on an improved downflow thermic syphon, which patents he assigned to the Downflow Syphon Company, of which he became an officer, large stockholder, and technical adviser. This new syphon is now being marketed by this company.

In 1924 Mr. Hawley organized and became president of Centrifix Corporation, with headquarters in Cleveland. This company manufactures and markets outstanding high-efficiency centrifugal separators and purifiers for the removal of solids and other objectionable entrainment from steam, gas, air, and vapor. Through Mr. Hawley's ingenious inventions this company also developed high-speed tar extractors, fractionating units, and gas scrubbers of a standard of efficiency equal to that of his purifiers. During the time Mr. Hawley was directing head of Centrifix Corporation, a position he held until his death on April 16, 1939, he invented, perfected, and assigned to this company eighty-six United States patents and a great number of foreign patents under which the company now operates. The basis and most remarkable part of these inventions is that none of the equipment has any moving parts, which makes them of lifelong duration.

During the last fifteen years of his life Mr. Hawley spent much of his time in the development of a new type of sewage treatment plant. His main aim in this was to be able to give to the public at a very low cost a complete sewage treatment apparatus that required little space, is easily installed, operates at a minimum operating cost, and is easily added to if necessary. These plants were designed especially for small communities which in the past have been denied the benefits of sewage treatment because of the costs involved. The Hawley Engineering Company has taken up where Mr. Hawley left off and is carrying forward his ideas for the further development and marketing of these sewage disposal plants. The future development of these plants is based upon patents granted or applied for by Mr. Hawley during his life.

Mr. Hawley had been a member of the A.S.M.E. since 1926. He also belonged to the Masonic fraternity.

Mr. Hawley was an individualist of the old school. He was a man of force and indomitable will, but endowed with a big heart and a loyalty to his friends which was unshakable. His friends and those who knew him loved and respected him and have deeply mourned their loss. His work and his family occupied all of a full, successful, and praiseworthy life. His death brought to him a well deserved rest—a rest he did not have time to take during his life.

Mr. Hawley is survived by his wife, Virginia (Taylor) Hawley, and three children, Hope (Hawley) Degenhardt, John Stuart Hawley, and Charlotte (Hawley) Laier.—[Memorial prepared by RANDALL T. ELLIOTT and HUBERT H. RICHARDSON, Cleveland, Ohio, nephews of Mr. Hawley.]

ROBERT THOMPSON HAZELTON (1883–1937)

Robert Thompson Hazelton, secretary-treasurer of The Cincinnati Shaper Company, died suddenly of thrombosis on March 3, 1937, in Cincinnati, Ohio. Mr. Hazelton was born in Petoskey, Mich., on July 17, 1883, the son of Wilbur and Margaret (Patterson) Hazelton. He obtained his early education in the public schools of Petoskey, and Medina and Rochester, N.Y. He also attended the Rochester Business Institute for one year; took a four-years' course in the Mechanics' Institute of Rochester, covering such subjects as are usual in a course in mechanical engineering; and served an apprenticeship as a machinist at the A. L. Sweet Iron Works, Medina, working on general manufacturing, tools, and jigs. During the summer of 1901 he worked in the drafting room of the A. L. Sweet Electric Light & Power Co., manufacturers of motors and other electrical apparatus.

In October, 1901, Mr. Hazelton entered the employ of Charles Bridgeford, Rochester, N.Y., manufacturer of machine tools and special machinery, and spent about six months there on shop work. Later he designed and patented the line of machine tools built by the Bridgeford Machine Tool Works, of Rochester, which company was successor to Charles Bridgeford. Here he had charge of drafting and design, and specialized in designs for jigs and fixtures used with the machine tools manufactured by the company.

Mr. Hazelton did special work at the University of Rochester in 1909, and then returned to the Bridgeford Machine Tool Works as superintendent. In March, 1911, he was engaged by the Cincinnati Milling Machine Company, on supervision of tools; in 1914, he was made head of the Engineering Department; in 1915, works manager and chief engineer. In June, 1920, Mr. Hazelton became works manager of The Cincinnati Shaper Company and was made treasurer in 1927. While at The Cincinnati Shaper Company he developed a line of machine tools and sheet metal working equipment; these machines today are recognized leaders in their particular fields.

He joined the A.S.M.E. as a junior in 1909, and was promoted to member in 1916. He served from September, 1929, to October, 1936, on Subgroup No. 7 on Gear Cutting and Hobbing Machines of Technical Committee No. 4 on the Standardization of Spindle Noses and Collets for Machine Tools of the Sectional Committee on the Standardization of Small Tools and Machine Tool Elements. He was also a member of Technical Committee No. 1 on T-Slots of the Sectional Committee on the Standardization of Small Tools and Machine Tool Elements from September, 1924, to the date of his death, representing the National Machine Tool Builders' Association. Mr. Hazelton was an active member of the committee appointed in 1931 by the Industrial Commission of Ohio to study and to develop better means for the safeguarding of metal-working machinery.

Mr. Hazelton was director of the Engineers' Club of Cincinnati for several years. He also belonged to the Cincinnati Club, the Hyde Park Country Club, and the Masonic order. He was twice married; his first wife, Miss Rose Westcott, predeceased him. His second wife was Mrs. Lydia Jenkins, by whom he is survived, as well as by three children: Mrs. Rupert Benson of Camp Hill, Pa.; Burton Hazelton, of St. Louis, Mo.; and Merrill Hazelton, of Cincinnati, Ohio.

NATHANAEL GREENE HERRESHOFF (1848–1938)

Nathanael Greene Herreshoff, retired president of the Herreshoff Manufacturing Company, died at his home in Bristol, R.I., on June 2, 1938, after a year's illness. At the time of his death Mr. Herreshoff was ninety years of age.

While famous primarily for his achievements in the field of yacht design, which gained for him among yachtsmen the name "Wizard of Bristol," Mr. Herreshoff had to his credit many noteworthy accomplishments in mechanical engineering, which led to his election to honorary membership in the A.S.M.E. in 1921.

Mr. Herreshoff was born in Bristol, R.I., on March 18, 1848, the son of Charles Frederic and Julia Ann (Lewis) Herreshoff. He received his early education in the public schools of Bristol and his advanced technical training at Massachusetts Institute of Technology, which he attended from October, 1866, to May, 1869. He was awarded the honorary degree of Master of Science in 1896 by Brown University, a recognition peculiarly fitting, in consideration of the fact that his great-grandfather, John Brown, was the founder of that institution.

Upon completion of his education, Mr. Herreshoff joined the Corliss Steam Engine Company of Providence, spending nine years with that organization, first as draftsman, later engaged in engine adjusting, and finally in charge of steam and hydraulic engineering, particularly in connection with the development of large pumping engines. While with the Corliss company one of his most important responsibilities was the conduct of experiments and the testing of material for the construction of the famous engine which furnished the motive power for the Centennial Exposition in Philadelphia in 1876. Mr. Herreshoff was also in charge of the installation of this engine, starting it, and conducting power tests. During this period in his career as one of the pioneers in the design of compound steam engines, he was awarded two patents on steam engine governors.

The Herreshoff Manufacturing Company of Bristol was founded in 1863 by Mr. Herreshoff's older brother, John Brown Herreshoff, who remained at the head of the concern until his death in 1915. Nathanael Herreshoff's interest in boats dated from the early years of his youth on the shores of Narragansett Bay, for it is reported that at the age of ten he designed and built a twenty-foot catboat, the forerunner of the series of fast sailing yachts which made his name known throughout the yachting world. While still connected with the Corliss company he did a certain amount of designing for his brother John, hence it is not surprising that in 1878 he joined the Herreshoff company as designing engineer and superintendent of construction. After the death of his brother, Mr. Herreshoff became president of the company, serving in this capacity until 1925 when the concern was reorganized and he retired.

During its early years the Herreshoff Company was engaged in the construction of sailing vessels, but in the 1870's its interest turned to the production of fast steam-driven craft, Nathanael Herreshoff's knowledge of steam power playing a large part in its success. In 1876 he designed the first torpedo boat for the United States Navy, the boiler being designed by his brother James. The success of this craft led to the receipt of orders for many similar vessels from several foreign governments. During this period Mr. Herreshoff was awarded several patents on details of construction of steam engines, and perfected the design of coil boilers for use on steam yachts and other fast steam-powered vessels. His ingenuity in the reduction of unnecessary weight in power plants became evident at this time, and his engineer's craving for efficiency in design carried over into his later work with fast sailing yachts. The fundamental soundness and advanced character of his designs may be appreciated when it is realized that many winches and other devices designed by him were still in use on *Ranger*, the last America's Cup Defender, sailed in 1937.

In the late 1870's Mr. Herreshoff began his experimenting in the field of fast sailing yachts, the field in which he was preeminent until his retirement. Probably the first of his creations to cause excitement in yachting circles was *Amaryllis*, a double-hulled sailing catamaran built in 1877, which showed remarkable speed and led to arguments among racing committees of that day. This craft was followed in 1891 by *Gloriana*, a boat which may be said to have truly revolutionized the design of racing yachts, since she was the first to embody such features as the spoon bow and long overhangs which gave the yacht a greatly increased sailing length when heeled. *Gloriana's* phenomenal speed and racing success, together with that of *Wasp*, built in the following year, brought the designer to the attention of the yachtsmen of this country and led to the production of a long line of yachts which were to make international yacht-racing history.

In 1893, when another challenge for the America's Cup was received from England, Nathanael Herreshoff was in the front rank of American yacht designers, and it was inevitable that he should be called upon to design a defender. He produced *Vigilant*, which defeated Lord Dunraven's *Valkyrie II* in three straight races. This success was followed by *Defender*, which defeated *Valkyrie III* in 1895 and by *Columbia* which defeated the first of Sir Thomas Lipton's *Shamrocks* in 1899 and repeated against *Shamrock II* in 1901. The long series of Herreshoff defenders was completed by *Reliance* in 1903 and *Resolute* in 1920, these two defeating Lipton's *Shamrocks III* and *IV*.

In 1925 the Herreshoff Manufacturing Company was reorganized and Nathanael Herreshoff retired from active participation in the business. The company, however, continued its connection with the defense of the America's Cup, for *Enterprise* and *Rainbow*, the successful defenders of 1930 and 1934, were built at its Bristol plant from designs by W. Starling Burgess. Mr. Herreshoff continued his active interest in yachting and yacht designing as long as his health would permit, his last design being the yawl *Belisarius*, which he designed for Commodore Carl B. Rockwell of Bristol in 1935.

On March 18, 1938, the Town Council of Bristol took note of Mr. Herreshoff's ninetieth birthday by the passage of a testimonial resolution which evidenced the high esteem in which he was held by his fellow citizens. This resolution testified in part that "by his extraordinary talents and genius for marine architecture—by his sterling character and integrity, by his interest in Bristol both as a just and fair employer and as a worthy citizen, extending over an unusual period of years, he has endeared himself to our people and brought renown upon his native town, placing our citizens under lasting obligations to him."

While Mr. Herreshoff's fame depended chiefly upon his achieve-

ments as a designer of racing yachts, his accomplishments as an engineer in steam power plant development would in themselves have made his career of importance. His genius lay in the combination of accurate science with the less definite characteristics of art which has marked the great designers in all fields.

In addition to his honorary membership in the A.S.M.E., Mr. Herreshoff was also a member of The Society of Naval Architects and Marine Engineers, The Franklin Institute, and the Institution of Naval Architects of London.

Mr. Herreshoff was married in 1883 to Clara A. DeWolf, who died in 1905. In 1915 he married Ann R. Roebuck, who survives him. Surviving him also are four sons: A. Sidney of Bristol, A. Griswold of Grosse Pointe, Mich., L. Francis of Marblehead, Mass., and Clarence DeWolf of Bristol; also one daughter, Miss Agnes M. Herreshoff of Bristol. Of the sons, A. Sidney and L. Francis are following their father's career of naval architecture.—[Memorial prepared by Z. R. Bliss, Providence, R.I., Mem. A.S.M.E.]

ED HILL (1876–1937)

Ed Hill, superintendent of the City Water and Light Plant, Jonesboro, Ark., died on July 17, 1937.

Mr. Hill was born at Olney, Ill., on May 4, 1876, the son of Kizziah and Edward F. Hill. He attended the Olney schools and in 1890 entered the service of the Olney Edison Electric Light Company, as apprentice operating engineer. He remained in Olney until 1895, when he went to Flora, Ill., as assistant foreman on the construction of a power plant, of which he subsequently became operating engineer.

He was employed from 1900 to 1904 by the St. Louis-San Francisco Railway Company, at Springfield, Mo., on field construction work. He then became lineman for the City Water and Light Plant of Jonesboro, for which he later served as chief engineer, and subsequently superintendent in charge of construction, operation, and maintenance of plants, which position he was holding at the time of his death. Mr. Hill was largely responsible for the design, construction, and equipment of the present Jonesboro power plant.

Mr. Hill became a member of the A.S.M.E. in 1932. He married Miss Kate Galbraith, of Flora, Ill., in 1899, and is survived by her and one daughter, Helen (Hill) Roth.

GEORGE N. HINCHMAN (1866–1937)

George N. Hinchman, of Webster Groves, Mo., died on May 25, 1937. At the time of his death, Mr. Hinchman was engineer in charge of development and design for Otto Eick, Inc., of St. Louis, Mo.

He was born on March 17, 1866, at St. Louis, the son of George Nichol and Elizabeth (McFarland) Hinchman. He obtained his elementary education in the grade schools of St. Louis, and was graduated in 1884 from Washington University, in mechanical engineering. He served as an apprentice from 1884 to 1886 in the Standard Foundry Company of St. Louis, and from 1886 to 1890, under Charles K. Pickles, of St. Louis, learning drafting and machine design.

From 1890 to 1899, Mr. Hinchman was in charge of drafting and design for the Falls Rivet & Machinery Co., Cuyahoga Falls, Ohio, and as their engineer was responsible for all plant layouts and designs made by that firm. In 1899, he went to the Pittsburgh Meter Company, East Pittsburgh, Pa., and was in charge of design, special inspection, and erections. He returned to St. Louis in 1902, as designer and experimental engineer for the Standard Adding Machine Company. He was superintendent of the Compressed Air & Vacuum Machinery Co., St. Louis, in 1910–1911, and then until 1918 was employed by Otto Eick, a brewery engineer, designing and erecting bottling machinery. Subsequently he was connected with the Alvey Manufacturing Company, St. Louis, designing elevators and conveying machinery, and with the Wagner Electric Company, St. Louis, designing airplane motors for the United States government.

From 1922 to 1928, Mr. Hinchman was in charge of the Maintenance Department of the Commonwealth Steel Company, Granite City, Ill., where he was responsible for all construction designs. From 1928 to 1932, he was employed to handle all special design work for the W. W. Sly Manufacturing Co., Cleveland, Ohio, and in 1932, he returned to Otto Eick, Inc., as engineer in charge of all development and design, which position he held up to the time of his death.

Mr. Hinchman became a member of the A.S.M.E. in 1919, and was also active in the Masonic order in Webster Groves and Cleveland. He married Elizabeth Scown in 1890 and they had one daughter, Laura, now Mrs. R. R. Johnson, of Dallas, Texas.

CLARENCE FLOYD HIRSHFELD (1881–1939)

Clarence Floyd Hirshfeld was born in San Francisco, Calif., on January 30, 1881, and received his elementary education in the public schools of that city. It was his early ambition to be a mining engineer but he grew away from this and in 1898 entered the University of California to study electrical engineering. Graduating in 1902 with the degree of bachelor of science in electrical engineering, he entered Cornell University in 1903 as a graduate student in mechanical engineering and received the degree of M.E. in 1905.

During his first year at Cornell, Dr. Hirshfeld became an instructor in experimental engineering and advanced in the following years to the positions of assistant professor and finally professor of mechanical engineering, teaching successively experimental engineering, heat power engineering, gas engine design, gas manufacture, and gas distribution. He remained a member of the faculty until 1914 and was author and co-author of several textbooks, including:

Engineering Thermodynamics, 1913
Farm Gas Engines (with T. C. Ulbricht), 1913
Steam Power (with T. C. Ulbricht), 1913
Elements of Heat Power Engineering (with W. N. Barnard), 1915
Economic Operation of Steam-Turbo-Electric Stations (with C. L. Karr), 1918
Elements of Heat Power Engineering (with W. N. Barnard and F. O. Ellenwood), Latest Edition, 1933.

Early in his teaching career, Dr. Hirshfeld undertook a private consulting practice which grew to wide proportions and broad scope. In 1913 he was engaged by Alex Dow, president of The Detroit Edison Company, for consulting services in connection with that company's power system. He became active in many phases of the company's business, notably in the field of industrial electric heating, and in 1914 resigned from Cornell to become chief of research for The Detroit Edison Company under an arrangement which provided for the continuation of his private consulting work.

In addition to his duties as research adviser to the management, Dr. Hirshfeld organized and headed the company's Research Department, credited as being one of the first of its kind in an electricity supply company, and one now widely known in this country and abroad for its pioneering in many fields, the high quality of its work, and its many contributions to the art. As an essential part of the department, Dr. Hirshfeld organized and developed the company's library, which is outstanding of its kind and which has served as a model for many other organizations. Continuing with the company until his death, Dr. Hirshfeld has left many marks of his work, his influence, and his inspiration. He had an active part in the design of power plants, the development of scientific purchasing methods, the institution of an inspection division, the development of standardization practices, the establishment of patent policies, and the general development of the research attitude throughout the organization.

He was called into the service of the War Department late in 1917, and was first commissioned as major in the U.S. Army Ordnance Department and later as lieutenant-colonel. His previous engineering training made him immediately available for the task in hand, that of general supervision of the inspection of artillery and artillery ammunition purchased for the army. He forthwith devoted himself with characteristic energy to the effective administration of inspection forces quickly organized for the war effort and drawn largely from civilian life. Shortly after his entrance upon this work, the inspection section of this departmental division was decentralized by the establishment of district ordnance offices and it became Dr. Hirshfeld's duty to act as liaison officer between the several districts and the headquarters in Washington. He served in this capacity with marked intelligence and undoubted distinction until he was honorably discharged in May of 1919. He returned then to his place in the Detroit Edison organization, from which he had been given temporary leave, and resumed the activities interrupted by the war service.

As in many other cases where men had similar experience in war materials procurement, Dr. Hirshfeld continued his interest in ordnance work through membership in the Army Ordnance Association. During the last four years of his life he was assistant chief of the Detroit Ordnance District, his knowledge of the subject, based upon experience in time of actual war, making his counsel invaluable.

In various associations of the electrical operating industry Dr. Hirshfeld was a prominent figure. He served as member, and more often as chairman, of many committees of the National Electric Light Association and in 1926 he was made chairman of the National Technical Section, one of the four major divisions of the organization. Likewise he was active on many committees of the Association of Edison Illuminating Companies, notably the Committee on Power Generation and the Committee on Electric Switching and Switchgear, which he headed in 1928 at the time of its organization. He was representative for several years of the electric light and power group on the American Engineering Standards Committee and its successor, the American Standards Association, and he served as a member of the United States National Committee of the International Electrotechnical Commission.

Under the auspices of the Association of Edison Illuminating Companies, a conference of electric utility executives and research men was held in 1933 to discuss the feasibility of a joint research organization of electric operating companies. The result of the conference was the formation of the Edison Research Coordinating Council, of which Dr. Hirshfeld was made vice-chairman and later chairman. He served also as chairman of the council's research committee on metallurgical applications of electricity. When the council was later incorporated under the name, Utilities Coordinated Research, Inc., he was elected president.

Although an active member of many technical and professional organizations, Dr. Hirshfeld was most prominent in The American Society of Mechanical Engineers, of which he became a junior member in 1905, member in 1919, and Fellow in 1936; to which he contributed many outstanding technical papers; and of which he was an officer and member of the Council as a manager from 1929 to 1932 and as vice-president from 1932 to 1934.

His early work in the Society dealt largely with standardization, test codes, and special research projects. For many years he served as a member of the main Power Test Codes Committee, counseling it on the broad aspects of acceptance tests, and as chairman of the individual committee dealing with instruments and apparatus he rendered both the committee and the Society extremely valuable and efficient service.

More recently he was actively concerned with the professional training and development of the engineer. In 1928 he became a member of the Committee on the Economic Status of the Engineer, which sponsored the study on earnings of engineers for the year 1930 and later recommended the steps which led to the formation of the Engineers' Council for Professional Development. Dr. Hirshfeld served as chairman of the preliminary organization committee of E.C.P.D. and for two years as first chairman of that body. He exerted a valuable influence in its formative years by his clear concept of the ideals of the new organization and by the selection of men of vision and understanding to carry on its work. After his retirement as chairman he remained an active member of the E.C.P.D. until his death.

Dr. Hirshfeld's full appreciation of the need for providing orderly access to technical literature led to his active support of The Engineering Index, on the Advisory Board of which he represented The American Society of Mechanical Engineers, and to his devoted service as a director when it became a separate undertaking as Engineering Index, Incorporated.

Other organizations of which Dr. Hirshfeld was a member included the American Institute of Electrical Engineers, American Association for the Advancement of Science, American Chemical Society, American Institute of Mining and Metallurgical Engineers, American Society for Testing Materials, Econometric Society, The Newcomen Society, Sigma Xi, and the Princeton Engineering Society.

An able lecturer and finished public speaker, Dr. Hirshfeld gave generously of his time in filling demands for his appearance on programs of technical, professional, and industrial organizations. He addressed sales conferences, educators, purchasing agents, operating men, and executives with the same skill that made him a great teacher, for he possessed an outstanding ability to convey new and complex subject matter in such unique and interesting fashion that his listeners absorbed it without conscious effort. In 1932 he delivered the commencement address at Rensselaer Polytechnic Institute which at that time honored him with the degree of Doctor of Engineering; in 1935, under the auspices of Science Service, he gave a radio broadcast on the subject of Science in Transportation, and in 1936, the Brackett Lecture on Industrial Research at Princeton University. A prophet not without honor in his own country, he received the degree of Doctor of Engineering from the University of Detroit in June of 1938.

Dr. Hirshfeld was well known in America and Europe as the author of scientific and engineering papers and articles. He wrote extensively on the many phases of electricity supply business and his writings on engineering education, professional training, and the place of the engineer in society won nation-wide comment and praise. In 1934 his paper, "Engineers of the Next Generation," won the American Institute of Electrical Engineers' national prize for the best paper on public relations and education, and he was the recipient of the Worcester Reed Warner Medal at the Annual Meeting of The American Society of Mechanical Engineers in December, 1937. He was also author of the paper, "Preparation and Combustion of Fuel," read before the First World Power Conference in London, 1924, and "Research Relating to Power Development," before the Second World Power Conference in Berlin. In 1926 his paper, "What Is the Demand for Overload in the U.S.A.?" was presented before the International Electrotechnical Congress. He was author of the chapter, "The Spirit of Power," in the composite book, "Toward Civilization," edited by Dr. Charles A. Beard and published in 1930.

Gaining recognition first as an educator, Dr. Hirshfeld later became known as one of the world's leading figures in the field of industrial research. Early in life he adopted what he termed "the research method of thought," and his naturally inquisitive mind, his remarkable powers of observation, and an enviable memory, contributed to the ultimate development of that type of mental approach to a high degree of perfection. He applied it not only to consideration of phenomena in the physical world but to the behavior of human beings as well. He cautioned against living in the present without knowledge of the past and appreciation of the possibility of the future, and he taught the intelligent use of known and procurable facts in the solution of problems of all types, whether scientific, industrial, civic, economic, or social.

One of his greatest achievements was the ability to assemble, rationalize, and coordinate facts and knowledge of all kinds into a single pattern which he referred to as "the whole scheme of things." Even the obscure and apparently unimportant observations found their place in that pattern. To him, that which divides knowledge into different classes was merely a matter of terminology; in all he saw the same elements, the same forces, and the same behavior under comparable influences, so that he interpreted all experiences by a single set of laws.

He often referred to research as merely a fact-finding process, but those who knew him realize it was his ability to interpret the significance of new and often apparently unimportant facts in terms of already existing knowledge that made him outstanding as a research engineer. He possessed a fertile imagination and the necessary courage to venture on new lines of thought even though they might lead along untraditional paths and run contrary to conventional belief and practice. It was probably this attitude of mind, together with his connections with many research undertakings, that led to his service on the Advisory Board of the Purdue Research Foundation; the National Board of Councillors of the United States Bureau of Mines; the Ohio State University Research Foundation; the National Research Council; and the American Institute of Physics' Advisory Council on Applied Physics.

In 1930 a committee of street railway executives, known as the Electric Railway Presidents' Conference Committee, was organized to deal with an acute situation which had arisen in their industry. Increasing competition of the rubber-tired, automotive type of vehicle had reached such proportions that fear was expressed for the future existence of the electric street car, and the huge investment in street railway properties throughout the nation appeared seriously jeopardized. Despite certain advantages of the rail-type vehicle in mass transportation and despite miscellaneous efforts to modernize it, the street car lacked the public appeal offered by its competitors. Recognizing that the electric car had developed largely along traditional lines by rule-of-thumb methods, whereas its competitors had grown out of the great mass of scientific research that forms the background of the automotive industry, the committee was aware that a street car capable of recapturing public favor must be more than simply the old mechanism in a new dress; it must be a new product of scientific effort under the most able direction available. The choice of Dr. Hirshfeld for the task stands as a great tribute and a fine expression of the confidence in this man who had had no experience in the field of transportation and had not been identified with this branch of engineering.

He was appointed chief engineer of the committee in the fall of 1930 and immediately started an analysis of the problem. In this he conferred not only with street railway executives, engineers, and operators, but in characteristic fashion with factory hands, office workers, and housewives who made use of public means of transportation. He diagnosed the shortcomings and found the need for many improvements, but there was available little information as to the specifications to be met, or the inherent possibilities and limitations of a rail vehicle.

Accordingly Dr. Hirshfeld set up a program of research and experimentation, established a laboratory, and employed a group of engineers and technicians to work under his direction in a fundamental study of the whole problem of street car design and performance. After four years of intensive activity, an initial car was produced containing many revolutionary developments in truck design, wheel construction, control equipment, lighting, ventilation, and body. Steel springs had given way to rubber springs, the wheels contained rubber disks insulating the steel rim from the rest of the structure, and other novel features had been successfully developed and applied. A rail vehicle had been created which could equal or outperform the modern automobile in accelerating or braking rates. The smoothness of ride by actual test excelled that of any other street vehicle, and the noise level had been reduced to a point where the "sing" of the trolley wire became the distinguishing evidence of the car's approach. A new type of welded body with modern lines and appearance, heated, lighted, and ventilated by improved methods,

constituted the new dress in which this revolutionary street car made its appearance.

Exhibited at the Cleveland Convention of the American Transit Association in September, 1934, the new car met with enthusiastic approval of the street railway industry. It was called the Presidents' Conference Committee Street Car and is now commonly known as the P.C.C. car. When commercial production was undertaken, the committee's interests were transferred to the newly incorporated Transit Research Corporation. Dr. Hirshfeld continued as chief engineer of the new organization until the time of his death.

That the P.C.C. car has met with public favor is evidenced by the fact that at the end of 1939 there were approximately 1130 cars of this type in operation or under construction for thirteen cities in the United States and Canada.

Teacher, scientist, engineer, lecturer, and always a scholar; it would seem that such achievements and accomplishments must have completely filled a comparatively brief life span of 58 years. Yet, Dr. Hirshfeld took time to listen to those who brought their individual problems to him; to encourage them in the development of worthwhile ideas; and to divert their misdirected efforts to more logical ends. His instant recognition of the best in those with whom he came in contact, his deep understanding and broad tolerance of humanity's shortcomings, and his desire, without a trace of the overzeal of the reformer, to make the world a better place for his having lived in it, won respect, admiration, and genuine affection from his great circle of friends.

His casual observation and his more considered advice were sought by his neighbors and friends who valued the honesty and integrity of his opinion, while those privileged to know him more intimately not only appreciated his good humor and innate modesty but found in him the love, devotion, sincerity, and hospitality which added so much to the realization of an unusually happy home life.

In addition to the recognition which was his, living, it is fitting that this final honor, the award of the John Fritz Medal, should have come to him. There could be no finer memorial than the words of the citation, "for notable leadership through research and development in power generation and electric traction and for being a great teacher and friend of man, young and old."

Dr. Hirshfeld's death occurred on April 19, 1939, at the Grace Hospital in Detroit, after an illness of several months. Surviving him are his widow, the former Elizabeth Bishop Winslow, of Ithaca, N.Y., and two sons, John W. and James F. Hirshfeld.—[Biography, slightly expanded, prepared by F. DOUGLAS CAMPBELL, personal assistant to Dr. Hirshfeld at The Detroit Edison Company, for the John Fritz Medal Board of Award.]

JAMES DAVID HOFFMAN (1868–1938)

James David Hoffman, affectionately known as "Jimmie" by his many friends, a member of the engineering staff of Purdue University from 1890 to 1911 and from 1917 to 1938, died at midnight on Saturday, August 13, 1938, at his home in West Lafayette, Ind. He was born in Auburn, Ind., on January 23, 1868, son of Daniel Zinn and Rachael C. (Goetschius) Hoffman. He is survived by his widow, Zoelah M. (Burroughs) Hoffman, whom he married in 1913, and by one son, James David Hoffman. His first wife, Kate B. (Peterson) Hoffman, whom he married in 1890, died in 1902.

Professor Hoffman served as apprentice machinist with the Auburn Foundry and Machine Works between the time he was graduated from the Auburn High School in 1883 and his entrance to Purdue University in 1886. He was graduated from Purdue University with the degree of B.M.E. in 1890, and the degree of M.E. was conferred upon him in 1893. During the summer of 1890 he was draftsman and designer for the Buckeye Engine Company at Salem, Ohio, but was recalled the same fall to Purdue University to the post of assistant in practical mechanics, working under the direction of the late Dean W. F. M. Goss. He was promoted to the rank of instructor in 1891 and to assistant professor in the same department in 1901. In 1902 he was appointed assistant professor of machine design at Purdue University and was promoted to associate professor in 1903 and to professor in 1908. His connection with Purdue University was interrupted from 1911 to 1917, when he held the position of professor of mechanical engineering and practical mechanics at the University of Nebraska. He was recalled to Purdue University in 1917 to the position of professor and head of the department of practical mechanics, as successor to Michael Joseph Golden. Since 1929 he had also been in charge of the curriculum of industrial education. Professor Hoffman was retired from active service at Purdue University with the rank of professor-emeritus, effective September 1, 1938.

He was author or co-author of a number of books on engine and boiler design, machine design, and heating and ventilating; also of five pamphlets on house construction and heating of homes. His contributions to the technical press included numerous articles and papers, published in technical magazines and in the transactions of engineering societies.

In addition to the courses which he taught at Purdue University and at the University of Nebraska, Professor Hoffman delivered frequent nontechnical lectures and radio talks on residence construction, heating and ventilating, and shop practice.

His society affiliations included the following: The American Society of Mechanical Engineers (junior 1894, member 1903), American Society of Heating and Ventilating Engineers (life member and past-president), National District Heating Association (honorary member), National Warm Air Heating and Air Conditioning Association (honorary member), Warm Air Furnace Code Committee (first and only chairman from its organization in 1921 until his death), Society for the Promotion of Engineering Education (life member), Executive Committee, Central Indiana Section, A.S.M.E., Indiana Engineering Society, Advisory Committee for the Indiana Federal Housing Administration, Tau Beta Pi, Sigma Xi, Sigma Tau, Kappa Delta Rho, Rotary International (past-president, Lafayette Rotary Club). He was also an active member of the Presbyterian Church at Lafayette, Ind., and at Lincoln, Neb., having served on all of its official boards. He was a public-spirited citizen and as such devoted himself unselfishly to his community and state.

An editorial appearing in the Lafayette *Journal Courier* of August 14, 1938, reviewed his life and work in the following manner:

"With the passing of Professor James D. Hoffman, another of the sturdy, mature dependables who helped make Old Purdue a power in education, disappears from the campus he graced for so long. Professor Hoffman had been retired at seventy but he was to continue to have an office on campus, and had been awarded emeritus honors.

"Activity, hard work and constant association with youth had kept Professor Hoffman young in heart and spirit. He was a jolly companion, a good and cheery neighbor, a splendid citizen, a successful and greatly beloved teacher. Perhaps he was best described as 'a Christian gentleman.'

"His practical helpfulness in his relations with the young, over and above the functions of a teacher, have been attested many times. He knew how to be helpful when help was really needed, and there was no ostentation in his well-doing. His influence on behalf of clean living and integrity was far beyond that of a mere professor. His simplicity and sincerity, his friendliness and unassuming guidance by example rather than preachment, made him a real power in college, community, club and church, among young and old alike. 'Jimmie' Hoffman leaves a vacancy in the ranks of sound civic life and better education which it will be hard to fill."—[Memorial prepared by L. V. LUDY, Lafayette, Ind., Mem. A.S.M.E.]

JOHN A. HUNTER (1868–1939)

John A. Hunter entered this world at Stormstown, Pa., on May 15, 1868, son of John A. and Elizabeth (Elder) Hunter. By way of contrast, the word "storm" had no place in the tranquil life which he was destined to live. His bland sense of humor, his ability to appreciate the other fellow's point of view, and the basic spirit of fair play which dominated his life, left him unusually free of enemies and from the normal strife of life. He did not avoid making adverse criticisms, but such criticisms were usually accompanied by helpful suggestions and with a crack of humor which defied resentment. He seemed to anticipate friction between human beings and his efforts to maintain peace and goodwill were successful to an amazing degree.

After graduating from Pennsylvania State College in 1890 with the degree of B.S. in mechanical engineering he spent three years teaching manual arts in Georgia. He then took graduate work at Cornell University and Pennsylvania State College, where he gained an M.E. degree in 1896. He was an instructor and assistant professor of mechanical engineering there from 1894 to 1904 and then went to the University of Colorado as an assistant professor. Two years later he was made a full professor and placed at the head of the department of mechanical and chemical engineering, a position he continued to hold until reaching the retirement age.

Professor Hunter's humor and friendly smile were appreciated as fully by the women students in the university as by the men. One day Professor Hunter entered a dining room during a rather furious "boarding house" debate when one of the girls flew to his side in an appeal for support and called him "Uncle John." This girl is now the wife of Alvin Forbess, chief engineer for the Amalgamated Sugar Company of Utah. There was a significance to this little episode which became a characteristic part of the remaining twenty-five years of Professor Hunter's life. Not only did he become "Uncle John" from then on to every acquaintance from the governor of the state down to the boy in the streets, but through the use of that simple and affectionate salutation, "Uncle John," all barriers were

down and future hundreds were destined to do as Mrs. Forbess had done—fly to his side for advice, comfort, and protection.

John Hunter had a keen political mind and this ability was soon recognized by the president of the university. For many years the president sought his advice on political affairs affecting the university. Professor Hunter also persuaded many of the best regents ever to serve on the board that it was their duty to make the necessary sacrifice and seek election to this non-salaried position. Professor Hunter also contributed his own efforts to the cause of public government and civic interests. He was a member of Boulder City Council from 1912 to 1916. He served the Chamber of Commerce as president during 1923, 1924, and 1925. He was president of the Boulder Club over a long period of years. He headed many campaigns, such as community chest, etc., and he was rightfully considered one of the leading citizens of Boulder.

From 1896 until 1918 Professor Hunter spent his vacations in the employ of such firms as Westinghouse Machine Company, Pittsburgh Locomotive Works, Baldwin Locomotive Works, and the Pennsylvania Railroad, and his practical knowledge was one of his greatest assets.

He searched continuously for employment opportunities for his graduated students and many outstanding engineers today are grateful to "Uncle John" for his assistance in getting a good start. *The Colorado Engineer*, published by the student engineering body, nominated him for the "Hall of Fame." In May, 1936, the engineering faculty held a banquet in his honor and used the occasion to present him with a gold watch. When he retired he was presented with a bronze plaque on which was cast the actual signature of every member of the faculty of the university from the president to the youngest instructor.

For a period of twenty years he served as weather observer for the City of Boulder, but probably the extra-curricular activity which gave him the greatest pleasure was his position as a deputy state oil inspector. During this period every dollar he received from the oil inspector's salary was used to help needy and worthy boys through college by employing them to do extra testing work in the oil laboratory.

Professor Hunter was a member of the Colorado State Board of Examiners for Engineers and Surveyors from 1921 until his death. He rewrote the first Colorado registration law and personally steered the revised law through the Colorado legislature. Because of the intense study which he gave to this work, this new Colorado law is almost a verbatim duplicate of the present "Model Registration Law" of the American Society of Civil Engineers, although it appeared several years in advance of the "Model Law."

Professor Hunter helped develop and helped finance the first mechanically operated vertical automobile storage elevator. The rights to this elevator were purchased by Westinghouse and one operates today in the heart of downtown Chicago. He was also interested in the development of a road "file" and a rotary type engine.

In 1919 romance entered the life of "Uncle John" when he married a member of the faculty, Miss Alice Downing. His marital happiness ran strong but ended abruptly when his bride was suddenly stricken and died in January, 1920. The grief attending this tragedy caused "Uncle John" to be absent from the only Colorado Section meeting he missed from the time he personally organized the Section in 1918 until his own health failed nineteen years later.

Professor Hunter became a member of A.S.M.E. in 1914 and he was one of the first chairmen of the Colorado Section. From 1932 through 1935, Professor Hunter served the Society as manager and during the next two years he was a vice-president. He was elected to the Fellow grade in 1936. From the time he joined the Society until his death he served its interests with an ardor analogous to religious zeal.

Following the death of his wife, a sister, Miss Anna Hunter, presided over his home. The courteous and thoughtful consideration with which he unfailingly treated this sister prompted many appreciative comments. At his death, which occurred on December 17, 1939, Anna Hunter went to live with the other remaining sister, Mrs. A. Woodward Smith of Blairsville, Pa.

Professor Hunter was an active Mason, and a member of Acacia, Tau Beta Pi, and Alpha Chi Sigma. From 1905 until his retirement he was a member of the Society for the Promotion of Engineering Education.

His physical stamina astounded physicians when he recovered from a condition where he seemed completely paralyzed, with death but a matter of hours. His recovery was so s .fficient that he again attended A.S.M.E. meetings, but within the year he succumbed to a combination of uremic poisoning and heart trouble.

On December 19, 1939, he was laid to rest beside his bride of a few months, in Fairmont Cemetery in Denver.—[Memorial prepared by FRANK H. PROUTY, Denver, Colo., Mem. A.S.M.E.]

MELVILLE WILLIAM KANTROW (1880–1937)

Melville William Kantrow, mechanical engineer in charge of inspection for the American Safety Razor Corporation, Brooklyn, N.Y., died on October 15, 1937.

Mr. Kantrow, who simplified his name from Kantrovitz in 1923, was born in Virginia City, Nev., on December 8, 1880, the son of Charles Marks and Henrietta (Mayer) Kantrovitz. His early education was obtained in public schools in New York, N.Y., and at Cooper Union in that city, from whose Art Department he was graduated in 1898.

Following his graduation, Mr. Kantrow worked at various jobs such as assistant draftsman in both mechanical and architectural drafting rooms, and mechanic's helper in machine shops. Then, from 1903 to 1908, he was with the Yale & Towne Manufacturing Co., in Stamford, Conn., first as apprentice and later as assistant in their toolroom, in charge of die work in connection with Keller automatic die sinking machines. From 1909 to 1915, he was employed by Joseph Mayer & Bros., Seattle, Wash., in charge of the manufacture and installation of tower and street clocks and electric time-clock systems. From 1915 to 1919, he worked for the Remington Arms & Ammunition Co., Bridgeport, Conn., first as a foreman in the die room, later being made general foreman in charge of the sub-assembly of the Browning machine gun. He also installed and put into operation a battery of Keller automatic die-cutting machines, and served as assistant in the Drop Forge Department. He designed many jigs, fixtures, and machines to speed up production during the World War.

In 1919, Mr. Kantrow went with the Keller Mechanical Engraving Company, Brooklyn, N.Y., as superintendent of their Automatic Die-Sinking Machine Department, and also directed the setting up and putting into production of these machines for purchasers.

Mr. Kantrow had been employed by the American Safety Razor Corporation since 1926. At first he was engaged in checking production control in the Frame Department. Later he organized an inspection system for the entire plant, also organized and started production in their Double-Edged Blade Department, was in charge of the Pile Wire and Plating Departments, and acted as general "trouble-shooter" in locating and eliminating "choke points" throughout the entire plant.

In 1921, Mr. Kantrow joined the A.S.M.E. as an associate-member and he was automatically transferred to the member grade in 1935. He married Rose Klipper, of New York, in 1928, and is survived by her and by a son, Harry E., of New York, and twins, Lillian and Leon, of Seattle, Wash., as well as by his mother.

MOSES BATES KAVEN (1864–1937)

Moses Bates Kaven, vice-president and consulting engineer of the United Shoe Machinery Corporation, Boston, Mass., died on July 1, 1937.

Mr. Kaven was born on September 28, 1864, at Plymouth, Mass., the son of James M. and Harriet (Barnfield) Kaven. His early education was obtained in the public schools of East Bridgewater; his professional schooling at the Worcester Polytechnic Institute, Worcester, Mass., from which he was graduated in 1885 with the degree of B.S. in mechanical engineering. He was awarded the D. Eng. degree by his alma mater in 1928.

In February, 1886, Mr. Kaven was employed at the Atlantic Works, East Boston, as draftsman on marine engines. In November of that same year, he transferred to the Simonds Rolling Machine Company, Fitchburg, Mass., where he was engaged in drafting work for dies and special machinery for rolled forgings. In October, 1887, he was sent to England and was there employed as works manager of the Simonds Steel & Iron Forging Co., Ltd., of London and Birmingham, until June of 1890.

Mr. Kaven returned to this country in 1890, and became assistant mechanical engineer on marine engines and general machinery with the Lockwood Manufacturing Company, East Boston, Mass. He left there in 1892, and until 1898 acted as assistant to the manager of the Portland Company, Portland, Maine, building marine engines and general machinery and boilers.

In 1898, Mr. Kaven became associated with predecessors of the United Shoe Machinery Corporation, serving successively as superintendent, general superintendent, vice-president in charge of manufacturing, and vice-president and consulting engineer; also as director in sixteen affiliated and subsidiary companies. Mr. Kaven continued in this work until his death.

He became a member of the A.S.M.E. in 1891. He was also a member of the University Club and the Engineers Club, both of Boston; of the Worcester Club of Worcester, Mass., in which city he had resided since 1925; and of the Tedesco Country Club, of Swampscott, Mass. He was a trustee of the Worcester Polytechnic Institute. He was a Republican in politics and a Unitarian in religion.

In 1890, Mr. Kaven married Helen M. Kidder, of West Boylston, Mass., who died on November 30, 1935.

ROBERT RUSSELL KEITH (1879–1937)

Robert Russell Keith, works manager of the Tractor Works of the J. I. Case Co., Racine, Wis., died suddenly on June 12, 1937, while attending the 35th reunion of his class at Iowa State College.

Mr. Keith was born on February 13, 1879, at West Liberty, Iowa, the son of Albert F. and Rachel E. Keith. He obtained his preparatory education in the public high school of West Des Moines, Iowa, where he was later instructor in manual training for a year, and his professional education at the Iowa State College, where he was graduated with a bachelor degree in mechanical engineering in 1902. He then worked as an apprentice machinist in the roundhouse of the Chicago Great Western R.R. Co., South Des Moines, from July to October, 1902. This was followed by service as draftsman in Des Moines with the Eagle Iron Works; one year in the foundry of the Keith Furnace Company, West Liberty; and two years, 1903–1905, with the Louisiana Purchase Exposition at St. Louis. He had charge of the foundation work for all power units for the exposition, some 22,000 hp; was assistant to the superintendent of maintenance and operation; and then directed dismantling and shipping.

Next, Mr. Keith spent five years with the Sight Feed Oil Pump Company, Milwaukee, whose name was later changed to Richardson-Phoenix Company. He acted as shop superintendent and designed the entire line of product. This was followed by six months as assistant superintendent at the A. O. Smith Co., Milwaukee, and eighteen months as general manager, Sterling Machine Company, Norwich, Conn.

From 1912 to 1917, Mr. Keith served as superintendent of the Sheffield Car Company, Three Rivers, Mich., a plant of Fairbanks, Morse & Co.; 1917 to 1919, plant manager of the Holt Caterpillar Company, Peoria, Ill.; 1919 to 1928, as superintendent of the Tractor Works of the Moline Plow Company, Rock Island, Ill., for a short period, and then with International Harvester Company, first as superintendent of Tractor Works at Chicago, and later chief engineer of the Motor Trucks and Bus Division for that company. He took out several automotive patents which he assigned to the company. In January, 1928, he became works manager of the Tractor Works for J. I. Case Co., Racine, Wis., where he was employed at the time of his death.

Mr. Keith joined the A.S.M.E. as a junior in 1904, was promoted to associate in 1911, and to member in 1914. He was also a member of the Society of Automotive Engineers, and at the time of his death was vice-president of that society, representing the Production Engineering Activity. He had also served on the society's National Meetings Committee and the Production, Truck, and Engine Divisions of the Standards Committee.

Mr. Keith married Helen Marks, of Davenport, Iowa, in 1920 and is survived by her and by four children: Dorothy, Herbert, Gordon, and Mary.

JOHN ALFRED ALEXANDER LINDSTROM (1866–1937)

John A. A. Lindstrom, research engineer, National Lead Company, Brooklyn, N.Y., died at the Post Graduate Hospital, New York, N.Y., on September 28, 1937, of uremic poisoning.

Mr. Lindstrom was born on August 20, 1866, in Abo, Finland, the son of Alfred Emanuel and Sophia Gustava (Lindquist) Lindstrom. He obtained both his elementary and professional education in Finland, where he was graduated from the Institute of Technology of Abo in 1884. His first professional employment was as assistant engineer in the office of the city engineer and architect of Abo, from 1884 to 1893. He then emigrated to the United States, and was employed from 1895 to 1899 on the design and construction of new works for Howard & Bullough, American Machine Company, Pawtucket, R.I. In 1903–1904, he worked on the design of the Kingsbridge Power Station, New York, N.Y., and then was employed until 1906 by the Atlas Portland Cement Company at Northampton, Pa., on the design and construction of their plants Nos. 4, 5, 6, and 7. In 1908–1909, he was superintendent of erection and inspector under W. T. Donnelly, chief engineer, for an 8000-ton floating drydock, Erie Basin, Brooklyn.

From 1909 until his resignation in 1929 Mr. Lindstrom was employed by the General Chemical Company, New York, as engineer, estimator, and designer of chemical plants. These were situated in Bay Point, Calif.; Calumet and East St. Louis, Ill.; Newell, Pa.; Pulaski, Va.; Marcus Hook, Pa.; and elsewhere. This work included also wharves and tramways for the Marcus Hook and the Hudson River plants.

In 1934–1936, he was chief engineer, U.S.A., at Fort Hamilton; and in 1937, began his service with the National Lead Company, Brooklyn, and was there employed up to his death.

Mr. Lindstrom joined the A.S.M.E. as a member in 1911. He was also for thirty years a member of the Bergen Beach Yacht Club. He became a naturalized citizen of the United States in 1900, and supplemented his early education in Finland by taking correspondence courses in this country.

He is survived by his widow, Fanny (Spencer) Lindstrom, whom he married in 1920.

ALAN NELSON LUKENS (1867–1937)

Alan Nelson Lukens, chief engineer for the Railway Steel-Spring Division of the American Locomotive Company, New York, N.Y., died at his home in Elizabeth, N.J., on October 20, 1937.

Mr. Lukens was born on March 29, 1867, at McKeesport, Pa. He prepared for college at the Pingry School, and then entered the Stevens Institute of Technology, Hoboken, N.J., where he was a student in mechanical engineering for two years. He left in 1886 to work with Cyrus Currier and Sons, Newark, N.J., as a special apprentice. After two years of this training, he was employed as superintendent by the Conshohocken (Pa.) Tube Company, in charge of their wrought iron pipe mill.

In 1892, Mr. Lukens left this company and took a position as designing engineer for the Artificial Gas Producer Company, Pittsburgh, Pa. He remained there until 1896, when he was engaged as superintendent by the Charles Scott Spring Company, Philadelphia, Pa. Here he had entire charge of the factory, and supervised the erection of new buildings and the reinstallation of all machinery after a destructive fire. In 1902 this company became one of the properties of the newly incorporated Railway Steel Spring Company, and Mr. Lukens was given entire charge of all engineering of five plants, later reduced to three and greatly enlarged. He was responsible for much research and development work on modern steel springs, and numerous inventions and patents on springs, miscellaneous car parts, and special machinery used in their manufacture. After the company became the Railway Steel-Spring Division of the American Locomotive Company in 1926 Mr. Lukens continued active in its affairs up to the time of the short illness which ended his life.

Mr. Lukens became a member of the A.S.M.E. in 1931, and was greatly interested in its work. He was a member of the Special Research Committee on Mechanical Springs from its organization in 1924 up to his death. He also served on the Subcommittees of Heavy Helical Springs and Elliptical Springs. He was a member of the American Society for Testing Materials, and served as Clerk of Sessions for the Westminster Presbyterian Church of Elizabeth.

Surviving are two brothers, the Rev. Victor H. Lukens, rector emeritus of the First Presbyterian and Trinity Church of South Orange, and the Rev. Frank Lukens, rector of the Burlington, N.J., Presbyterian Church; and two daughters, Mrs. Samuel R. MacPhee, of Williamsburg, Pa., and Miss Dorothy Lukens, of Elizabeth.

WILLIS McKEE (1873–1937)

Willis McKee, consulting engineer and contractor, of Cleveland, Ohio, died of pneumonia on December 4, 1937. Mr. McKee was born at State College, Pa., on June 24, 1873, the son of Professor James Y. McKee, then vice-president of The Pennsylvania State College, and Margaret G. McKee.

Mr. McKee spent his entire childhood on the campus of The Pennsylvania State College, and obtained his preliminary education in the A and B preparatory departments there. In 1888 he entered the college in the course in civil engineering, and was graduated with honors with a B.S. degree in 1892. In 1902, he received the degree of M.E. from the college. He was appointed vice-president of the Alumni Association of the college in 1935 and continued in that office until his death. He was also a member of several committees of the association.

After his graduation in 1892, he entered the employ of the Union Switch & Signal Co. and was engaged in erecting block signals along the lines of the N.Y. Central & Hudson River R.R. From 1895 to 1897, he taught in the high school at Mt. Carmel, Pa., and in 1897, he went to work with the Carnegie Steel Company, at Duquesne, Pa., as draftsman. He transferred to the Engineering Department of the National Steel Company, Youngstown, Ohio, in 1899, and after a few months was put in charge of the engineering of their Mill Department. In the early part of 1901, when the National Steel Company became the Ohio Works of the Carnegie Steel Company, he was made chief engineer there. He resigned this position to take the same title with the Republic Iron & Steel Co., in 1902, and in 1904 left that company to become general superintendent of the Elyria Iron & Steel

Co. He remained with this company and its successor, Steel & Tubes, Inc., for about thirteen years, serving as treasurer during the last year.

In 1918 Mr. McKee joined the organization of Arthur G. McKee & Co., which had previously been established by his brother, Arthur G. McKee. For several years he served this company as general superintendent of construction and later as manager of the Mill Department. In 1923 he opened offices as a consulting engineer in Cleveland and was engaged in this business and in marketing rolling mill equipment, on which he had about twenty patents, until the time of his death.

In 1900, Mr. McKee married Sara Pascoe of Carnegie, Pa., and they had two daughters, Margaret R. (McKee) McFarlane, and Helen P. (McKee) Hunzicker, who survive him. He is also survived by two brothers, George C., of Greenville, Mich., and Arthur G., and one sister, Mary (Mrs. H. E. Stitt), both of Cleveland.

Mr. McKee became a member of the A.S.M.E. in 1920. He was also a member of the Cleveland Engineering Society. In the A.S.M.E., he served on the Executive Committee of the Iron and Steel Division as associate in 1933 and 1934, and as member from then until his death.

ALBERT CLEMENT MIDDLETON (1870–1939)

Albert Clement Middleton, son of Frank Barclay and Sally (Albertson) Middleton, was born in Camden, N.J., on November 4, 1870. After attending Friends, a private school in Camden, he entered the Friends' Central High School, Philadelphia, Pa. When he had completed his studies there, he served an apprenticeship under his uncle, Howard W. Middleton, of the firm of C. W. and H. W. Middleton, of Philadelphia, learning the iron and steel trade, and continued in that work until his uncle's death, in 1893, terminated the business.

During the next two years Mr. Middleton engaged in the iron and steel business with H. McCalla, of Philadelphia, with whom his brother, E. M. Middleton, and Walter Ivins were also associated, and from 1895 to 1897 he was connected with the Cole Machine Company, Camden, manufacturers of engines for steam launches and small boats.

In 1897 Mr. Middleton joined Eldridge R. Johnson, Camden, who had a machine shop there and contracted for the making of all motors for the Berliner Gramophone Company, of Philadelphia. Mr. Middleton served as secretary of the business until 1901, when it was incorporated as the Victor Talking Machine Company, and from then until he retired in 1913 he was this company's secretary and assistant treasurer.

Mr. Middleton took an active part in civic and political affairs in the State of New Jersey. He was elected treasurer of the state on January 31, 1928, and served until April 1, 1934. He had been chairman of the South Jersey Port Commission since its organization in April, 1926. This commission constructed port facilities at Camden, where it operates the Camden Marine Terminals, and has aided greatly in developing transportation to serve the industrial and commercial interests of South Jersey.

Mr. Middleton had also served as a director of the West Jersey & Seashore R.R. Co., and of the First Camden National Bank & Trust Co., the Camden Safe Deposit & Trust Co., and the Camden Fire Insurance Association. He became an associate of the A.S.M.E. in 1911, and was a member of The Franklin Institute, the Camden County Historical Society, and several clubs in New York and Philadelphia. Gunning, yachting, and fishing were his chief recreations.

Mr. Middleton's death occurred on March 4, 1939. He is survived by Mrs. Middleton, the former Rena E. Lord, of Haddonfield, N.J., whom he married in 1898, and by a daughter, Rena.

ALBERT ELIPHALET MITCHELL (1855–1937)

Albert Eliphalet Mitchell, president and treasurer of the Wyckoff Pipe & Creosoting Co., Inc., New York, N.Y., died on December 26, 1937. He was born in Madison, Maine, on February 3, 1855, son of Thomas Gilmore and Laura Ann (Packard) Mitchell.

Mr. Mitchell's preliminary education was obtained in the common schools, and he later attended the University of Maine, which in his student days was known as the Maine State College of Agriculture and the Mechanic Arts. He was graduated as a mechanical engineer in 1875, of which class he is believed to have been the last surviving member.

Mr. Mitchell served as apprentice machinist with the Baldwin Locomotive Works in 1876–1877 and then entered the employ of the Pennsylvania Railroad at Altoona, Pa., where he worked successively in the machine shop, the Test Department, and the Signal Department. He then spent a year, 1881–1882, in the Yale & Towne Manufacturing Co., Stamford, Conn., as a designer of hoisting machinery, followed by two years as chief draftsman with the old New York & New England R.R., now a part of the New York, New Haven & Hartford R.R. He next spent about two years in Cleveland, Ohio, where he was employed as mechanical engineer and plant superintendent by the French Furnace Company and the Arctic Ice Machine Manufacturing Company. In 1886 and 1887, he was employed by the New York, Lake Erie & Western R.R. Co. (now the Erie Railroad) on special work in the Motive Power Department; from 1887 to 1889, he was engineer of signals; in 1889 and 1890, engineer of tests; from 1890 to 1892, mechanical engineer; and from 1890 to 1902, superintendent of motive power.

In 1901–1902, he was assistant superintendent of motive power for the Chicago, Milwaukee & St. Paul R.R., and in 1902–1904, superintendent of motive power for the Northern Pacific Railroad. In 1904–1906, he held a similar position with the Lehigh Valley Railroad, and in 1906–1908, was employed by the N.Y., N.H. & H. Railroad, on appraisal of rolling stock and as manager of purchases and supplies, fuel agent, etc. In 1908, Mr. Mitchell became vice-president of the Wyckoff Pipe & Creosoting Co., later becoming its president and treasurer, and continuing in this position to the time of his death. He held several patents, chiefly in the railway field.

Mr. Mitchell became a member of the A.S.M.E. in 1891. He was also a member of the N.Y. Railroad Club and served as its president from 1896 to 1898. He belonged to the Cleveland Engineering Society, the Mayflower Society, and the Phi Gamma Delta fraternity.

He was twice married. His first wife was Nellie Vernon Knapp, whom he married in Stamford, Conn., in 1884; his second wife was Mrs. Eliza Anna Wyckoff, nee Corel, whom he married in 1908, and who died in 1933. He had one daughter, Hazel Vernon, who also predeceased him.

Mr. Mitchell was a member of the Jury of Awards of the Paris, France, Exposition of 1892. He was for many years a member of both the Master Car Builders' and the Master Mechanics' Associations, and during that time, wrote various reports for these associations.

GEORGE LUTON MORTON (1858–1937)

George Luton Morton, former primary examiner in the U.S. Patent Office, who retired on June 30, 1932, after forty-six years of service in the Patent Office, died on May 13, 1937, in Washington, D.C., of pneumonia. He was born on August 19, 1858, at Parkman, Ohio, the son of Perry and Laura (Chapman) Morton.

Mr. Morton took a six-years' course in mechanical engineering in the Ohio State University, graduating in 1884, during which time he worked at intervals in various shop processes outside the university. He later took a three-years' course in law at the National Law School, Washington, D.C., and was admitted to the bar in the spring of 1889. He was also admitted to practice before the Supreme Court of the United States.

After his graduation in 1884, he was variously engaged: for one year as teacher of mechanical drawing in the School of Design, Cleveland, Ohio; as draftsman for Professor Robinson at Columbus, Ohio; and as a student of patent law in the office of Ex-Commissioner Gen. M. D. Leggett at Cleveland; also on the mathematical work connected with an article, "Cushioning in Engines," published in Van Nostrand's *Engineering Magazine*, June, 1885. Following these activities, Mr. Morton worked for nine months as machinist and draftsman for the Cleveland Rubber Company, and then went into the office of the superintendent of motive power, New York, Lake Erie & Western Railway Company, as draftsman. He remained here until 1886, when as the result of a successful competitive examination he was appointed an assistant examiner in the U.S. Patent Office, and assigned to Division 13.

Mr. Morton rose to be chief clerk in the Patent Office, in 1895, and three years later was appointed primary examiner of Division 36, which was organized under his supervision. He held this position until his retirement in 1932. He became a member of the A.S.M.E. in 1890; he was also a member of the Masonic order and of the University Club in Washington.

He married, in 1905, Alice M. B. Simmons, of Washington, D.C., who survives him.

ANDREW HENRY NEUREUTHER (1877–1937)

Andrew Henry Neureuther, consulting engineer and patent attorney for the Westclox Division of the General Time Instruments Corporation, of La Salle and Peru, Ill., was killed in an automobile accident on October 16, 1937, near Princeton, Ill.

Mr. Neureuther was born on April 3, 1877, at Peru, the son of Charles F. and Louise (Oesterle) Neureuther. He attended the Peru schools and entered the University of Illinois in the Mechanical Engi-

neering Department with the Class of 1898. After being granted his B.S. degree in mechanical engineering he spent a year in postgraduate work at the University, securing his M.E. in 1899. He then joined the staff of the Western Clock Company, La Salle, a small local plant, as engineer. Mr. Neureuther grew up with the company, soon becoming its chief engineer, a position he held until 1932, when he was made consulting engineer and patent attorney. As chief engineer, he supervised every addition to the plant from 1899 on, one of his first jobs being to change the illuminating system over from gas to electricity. The modernization of the heating and power plants, the establishment of physical and chemical testing departments and an experimental machine shop, the conduct of research, and the design and manufacture of special machinery all came under his supervision. He held some twenty patents in the United States and Great Britain relating to clocks and clock parts, and particularly to automatic casting machines and other special machinery used in the manufacture of clocks.

In addition to his engineering course, Mr. Neureuther took up the study of law in his spare time, specializing in patent law, and secured his LL.B. degree from the Chicago Correspondence School of Law in 1903 and shortly thereafter was admitted to the Illinois Bar Association. In 1930 the Western Clock Company was acquired by the General Time Instruments Corporation, of which it became the Westclox Division, and since 1932 Mr. Neureuther had devoted practically all of his time to the patent work of the corporation. He attained a wide reputation in patent work and had served as an expert witness in important cases.

Mr. Neureuther was a contributor to technical publications including the *Street Railway Review* and *Machinery*. He was serving as president of the National Sheet Metal Company and of the Citizens' Hotel Company, both of Peru, at the time of his death and had served as director of The Illinois Zinc Company and The Central Life Insurance Company of Illinois, Chicago, and of two banks in Peru.

Mr. Neureuther became a junior in the A.S.M.E. in 1899, and was promoted to the member grade in 1928. He was a member of Tau Beta Pi, honorary engineering fraternity, and of the Masonic order and was active in many organizations for community betterment. He had served as a trustee of the Peru Congregational Church, president of the public library board, vice-president of the Business Men's Association, and member of the school board and hospital association, and was actively interested in Boy Scout work. He was also secretary of the executive board of the Civic Federation of La Salle, Peru, and vicinity, and president and director of the Ottawa (Ill.) Chautauqua Association.

In 1902 he married Nettie Jane Robinson, of La Salle, who died in 1922. Three children, Anita Nettie Connellee, Andrew Wesley Neureuther, and Mabel Elaine Small, survive him. In 1926 Mr. Neureuther married Ada M. Baysoar, who also survives him.

CARL VICTOR NORDBERG (1872–1937)

Carl Victor Nordberg, of the Nordberg-Rowe Engineering Company, Butte, Mont., died on June 3, 1937, of angina pectoris.

Mr. Nordberg was born in Björneborg, Finland, on July 10, 1872, the son of Carl Victor and Dores (Heinze) Nordberg. His education and early engineering experience were acquired in Finland, where he was graduated with an M.E. degree from the Polytechnic Institute of Finland in 1900. Prior to this, he was employed from June to September, 1896, with Abo J.M.A.B., manufacturers of steam engines; from September, 1896, to June, 1897, with Nobel Bros., St. Petersburg, builders of mining machinery and steam engines; from June to September, 1897, and in 1898, with Björneborg Mek. Verkstad, Finland, builders of steamships and marine machinery; from June, 1899, to August, 1900, with A. G. Electricitäts Werke Vorm. Kummer & Co., Dresden, Germany, as assistant to the chief engineer, building an electric railway and power plant for the city of Helsingfors, Finland.

Following his graduation from the Polytechnic Institute Mr. Nordberg came to the United States and entered the employ of the Nordberg Manufacturing Company, Milwaukee, Wis., which had been founded by his older brother, Bruno V. Nordberg (Mem. A.S.M.E.; for obituary see Trans. A.S.M.E., vol. 46, 1924, page 1318). For about five years he worked on designs for air compressors, blowing engines, hoisting, pumping and stationary steam engines and stamps.

From September, 1905, to August, 1916, Mr. Nordberg served as representative of the Nordberg Manufacturing Company in the western states, selling machinery and supervising the installation of power plants. In August, 1916, he formed the corporation known as C. V. Nordberg Machinery Company, at Butte, Mont. On November 1, 1919, he assumed charge of the Missoula Iron Works, which he managed in addition to the C. V. Nordberg Machinery Company until October, 1924. In November, 1926, he became associated with the Nordberg-Rowe Engineering Company as consultant in research and engineering. He terminated this connection in 1932 to devote his time to designing and building portable compressors, which line he followed up to the time of his death. He supervised the installation of compressor valve systems for a number of mining companies, and a central compressor plant for the Anaconda Copper Mining Company, Butte, Mont., one of the largest of such plants in the world. He took out several patents, including one on cooling towers, and one on clutches.

Mr. Nordberg became a member of the A.S.M.E. in 1908. He was a member of the Navy Consulting Board of the United States, appointed by Secretary Josephus Daniels, in September, 1916. He became a naturalized citizen of the United States in 1912.

In 1902, Mr. Nordberg married Ida Viola Stenstrom in New York, N.Y. She died on February 27, 1924. He is survived by a daughter, Greta (Nordberg) Quayle, and a son, Carl Victor Nordberg, Jr., both of Butte, Montana; also by two grandchildren, Dolores Ann Quayle and Carl Victor Nordberg.

JULIAN DAY PAGE (1873–1936)

Julian Day Page was born at La Salle, Ill., on April 6, 1873, son of Thomas Mason and Kata (Brown) Page. He was graduated from Purdue University in 1894 with a bachelor's degree in mechanical engineering and took his master's degree three years later. During that period he worked on crane design for the Whiting Foundry Equipment Company, Harvey, Ill. From 1897 to 1901 he worked in Cleveland, Ohio, first for the Brown Hoisting Machinery Company, then for the McMyler Manufacturing Company, on crane design. He went to Youngstown, Ohio, in 1901, and for about three years was chief engineer for the Youngstown Engineering Company, also in connection with crane manufacture.

In 1904 he turned to plate work as chief engineer of the Enterprise Boiler Company, of Youngstown. He continued in this field for some years, working for the Wm. B. Pollock Co., Youngstown, from 1905 to 1907 and again, as assistant engineer, in 1910–1911; at the Ohio Works of the Carnegie-Illinois Steel Corporation, Youngstown, in a similar capacity, from 1907 to 1910; and for the Struthers Furnace Company, at Struthers, Ohio, as chief engineer, from 1911 to 1915. He received a safety medal in connection with his work in this position.

Subsequently Mr. Page spent about a year each with the Wm. Tod Co., Youngstown, and Mark Manufacturing Company, Chicago, Ill., and then went to New York, where he was chief draftsman with Perin & Marshall for several years. From May, 1918, until March, 1920, and again from July, 1923, to June, 1932, he was connected with the Wilputte Coke Oven Corporation, New York, as chief draftsman during the most of the time. He was with the International Coal Products Corporation in Newark, N.J., in 1920–1922.

Since 1934 he had been designing engineer for the Lummus Company, New York. He died in Plainfield, N.J., on February 3, 1936, after an illness of almost a year.

Mr. Page became a member of the A.S.M.E. in 1918. He was a Knight Templar and 32nd degree Mason. He was much interested in horticulture and in the promotion of welfare for crippled children.

Surviving Mr. Page are his widow, Helen Grinnell (Sill) Page, whom he married in 1903, and two children, Julian Day Page, Jr., and Helen Jane Page, who reside with their mother in Westfield, N.J.

EDMUND J. MASON PARRY (1876–1937)

Edmund J. Mason Parry, for more than thirty years associated with the Hartford Steam Boiler Inspection & Insurance Co., died suddenly at his desk in the home office of that company in Hartford, Conn., on October 4, 1937.

Mr. Parry was born on October 10, 1876, at Menai Bridge, Isle of Anglesey, North Wales. He was educated in Menai Bridge and Beaumarais schools and then served apprenticeships with De Winton, Carnarvon, Wales, and with Harland and Wolff, shipbuilders and engineers, Belfast, Ireland. At the completion of this course of training in 1896, Mr. Parry went to sea as assistant engineer in the Dominion Line Steamship Company, Liverpool, England, and in December, 1900, he was granted the certificate of the British Board of Trade as first-class marine chief engineer. He continued at sea, after transferring to the White Star Line in 1900, until April, 1906, with the rank of Engineer Officer, assistant to the chief engineer.

In June, 1906, having emigrated to the United States, Mr. Parry joined the Hartford Steam Boiler Inspection & Insurance Co. as inspector of steam boilers and flywheels, in its Boston office. He occupied this position until March, 1913, when he was transferred to Hartford as directing inspector for the company, in charge of the inspection of details relative to the design, construction, and installa-

tion of boilers and other pressure vessels, with authority over the work of other inspectors.

Following the death of Frank S. Allen in 1918, Mr. Parry succeeded to the position of chief inspector. For the next five years he served as head of the home department's inspection staff and in 1923 was transferred to the New York department as chief inspector. In the spring of 1933 he was granted leave of absence for rest and recuperation from a long illness and spent the next several months in his native England. On his return Mr. Parry went to Hartford as a member of the engineering staff of the boiler division and served in this capacity until the time of his death.

Mr. Parry became a member of the A.S.M.E. in 1918. He belonged to the Masonic fraternity and was a Shriner and a member of the Order of de Molay. For many years he was associated with the Hartford Hospital as a member of the board of directors and at the time of his death was its secretary. He was also a member of the Municipal Incinerator Commission, and of the Trinity Episcopal Church of Hartford. His clubs included the St. David Society of the State of New York, and the Professional Engineers Club of Connecticut.

He is survived by his wife, Elizabeth (Olson) Parry; three sisters in England; and a brother, Osmond Parry, in South America.

JOHANN FRIEDRICH MAX PATITZ (1866–1937)

Johann Friedrich Max Patitz—generally known as Max Patitz—chief consulting engineer for the Allis-Chalmers Manufacturing Company, Milwaukee, Wis., died suddenly of a heart attack on January 3, 1937.

Mr. Patitz was born on May 21, 1866, in Muegeln, near Leipzig, in Saxony. The following account of his life is based largely upon an obituary in *Milwaukee Engineering*, Vol. XVII, No. 4, of January, 1937.

He was educated in the Citizens' School and Royal High School or Gymnasium, of Dresden. With his parents, he emigrated to the United States, and found his first employment in Pittsburgh, Pa., where he spent four years working in various factories and machine shops. Then, in June, 1885, he entered the employ of the Edward P. Allis Company at the Reliance Works in Milwaukee, where he held a number of successive minor positions during the first years. His ability, however, soon found recognition, both by Mr. Allis and the late Edwin Reynolds, and he was transferred to the engineering department. He became one of the company's designing engineers; and in 1901, when the Edward P. Allis Company became part of the Allis-Chalmers Company, he was put in charge of the building of steam engines and compressors. As early as 1899, Mr. Patitz saw the possibilities of the steam turbine, and in 1902 he was sent abroad to investigate the subject, and on his return did much to further the building of steam turbines by the company. He later made other trips to Europe for the company, investigating the various designs of gas engines, diesel engines, compressors, tillers, and tractors, and was intimately connected with the early development of Allis-Chalmers tractors and farm machinery.

Mr. Patitz was granted many patents. His studies of the parallel operation of alternating-current generators began as early as 1900. He became an outstanding authority on critical speeds and vibrations in high-speed machinery, and on the intricate mathematical calculations of modern machine design.

Mr. Patitz became a junior member of the A.S.M.E. in 1891 and a member in 1900. He was appointed a member of the Power Test Codes Committee No. 5 on Reciprocating Steam Engines in January, 1918; and of the P.T.C. Committee No. 20, on Speed Responsive Governors, in April, 1922, and was still serving on these committees at the time of his death. He was also a member of the Subcommittee on Steam of the Research Committee from the time of its appointment in 1911 until the completion of its work in 1915. In addition, Mr. Patitz was a member of the Society of Automotive Engineers, Verein deutscher Ingenieure, Engineers' Society of Milwaukee, American Society of Agricultural Engineers, American Association for the Advancement of Science, and Deutsche Landwirtschafts Gesellschaft.

FRANCIS GLADHEIM PEASE (1881–1938)

Francis Gladheim Pease was born at Cambridge, Mass., on January 14, 1881, and received his early education at Highland Park, Ill. He studied science and engineering at Armour Institute of Technology, Chicago, and received his B.S. degree in 1901. His alma mater conferred the degree of M.S. in 1924 and Sc.D. in 1927. While attending Armour, Pease worked evenings and weekends in the optical shop of Petitdidier, where he mastered many practical details of instrument making.

After graduating from Armour, Pease was recommended by Petitdidier to Professor G. W. Ritchie of the Yerkes Observatory. Here he first took part in the routine work of grinding lenses and mirrors and later began work in instrument design under Professor Ritchie and also took part in the astronomical work as an observer under Dr. George Ellery Hale. This was a splendid apprenticeship for a man who combined great mechanical talent with good training in engineering theory. To these Pease also brought imagination and great enthusiasm.

When the work at Mt. Wilson Observatory was inaugurated in 1904 under a grant from Carnegie Institution, Pease and others accompanied Dr. Hale to Pasadena, Calif. Much of the development of this fine observatory must be credited to Dr. Pease. He designed in large part the Snow telescope, the 60- and the 150-ft tower sun telescopes, the 60- and 100-in. reflecting telescopes, the 50-ft interferometer, besides a great host of auxiliary instruments. He also supervised much of the work of grinding and testing the 60- and 100-in. reflectors. After the erection of the 60-in. telescope, Dr. Pease carried on the regular work of an astronomer. His astronomical observations cover a wide range, including fine photographs of star clusters and faint nebulae which are used for comparison with previous and later observations. He made many observations of the spectra of faint nebulae. His moon photographs are said to be the finest in existence.

In 1919 Dr. Pease assisted Dr. A. A. Michaelson in designing and constructing a giant interferometer and in measuring the diameter of Betelgeuse. Later, Pease made other measurements of the diameters of many of the nearer stars. He also assisted Dr. Michaelson in two determinations of the velocity of light, the last of which was completed by him after Dr. Michaelson's death. In this later experiment he designed and built a pipe line a mile long which was evacuated to one one-thousandth of an atmosphere. In 1929 he repeated the famous Michaelson-Morley experiment on ether drift, using an 85-ft interferometer, and confirmed the previously found negative results.

Following the design of the 100-in. telescope, Pease became greatly interested in the design of still larger telescopes. In 1926 he made a careful study of the problems involved in the design of a 300-in. reflector. He advocated the fork type mounting and proposed the use of pyrex glass for the mirror.

When the work on the 200-in. reflector was inaugurated in 1930 at the California Institute of Technology, Dr. Pease devoted half of his time to assisting in the design. He was engaged in this work at the time of his very sudden illness and death, on February 7, 1938.

Dr. Pease had been a member of the A.S.M.E. since 1915 and served the Society as chairman of the Los Angeles Section and in many other capacities. He is survived by his widow whom, as Miss Caroline T. Furness, he met at Yerkes Observatory and married in 1905. Dr. Pease was a large man, a viking physically, and he radiated health and friendliness. He will be sorely missed.—[Memorial prepared by Wm. Howard Clapp, Pasadena, Calif., Mem. A.S.M.E.]

HARRY ADAM PICKEL (1874–1937)

Harry Adam Pickel, superintendent of generation for the Pennsylvania Power & Light Co., at Hazleton, Pa., died on January 18, 1937, in the Good Samaritan Hospital at Lebanon, Pa., the result of an automobile accident two days previously.

Mr. Pickel was born at Montclair, Pa., on August 11, 1874, the son of Uriah Leonard Pickel and Elizabeth (Van Artsdalen) Pickel. He had no formal education beyond grammar school and home study work. From 1893 to 1896, he served an apprenticeship at woodworking with Samuel Gourley of Philadelphia, from which he entered the employ of M. R. Mucklé, Jr., & Co., electrical and mechanical engineers and contractors. Here he served as electrical mechanic and foreman on several important jobs, the largest being the United States Gas Improvement Building at Broad and Arch Streets, Philadelphia, when he was in charge of all electrical work. In 1899, Mr. Pickel went to the Philadelphia Bourse Building as electrician, and had become assistant chief engineer when he left, in 1902, to serve as superintendent and chief engineer of the Colonial Trust Building, Reading, Pa., a new building, ten stories high, and the first modern office building in that city.

In 1906, Mr. Pickel joined the staff of the Engineer Company of New York, specialists in boiler efficiency and owners of the balanced draft system. After serving for some time in New York as salesman and superintendent of construction, he was sent to Philadelphia as district manager, and while there he designed and built the first device to operate the damper of a steam boiler by the variation of the furnace gases.

From 1908 to 1913, Mr. Pickel was building superintendent and chief engineer of the Central Branch Y.M.C.A. in Philadelphia, one of the largest Y.M.C.A. buildings in the country, and he was also its house manager. He was also consulting engineer for the General

Board of the Y.M.C.A. in Philadelphia and in that capacity designed the plumbing, heating, and electrical work for the Elkins Memorial addition to the Central Branch, and also power plants for other branches in the city.

In 1913, he went into business with two partners as the Quaker City Engineering Company, of Philadelphia. This business was terminated in 1916, because of lack of capital, and Mr. Pickel became superintendent of power for the Hercules Powder Company, Kenvil, N.J. Here he remained until August, 1919, when he took the position of superintendent of the Hauto Steam Electric Station of the Pennsylvania Power & Light Co. He became superintendent of generation for the company on April 1, 1925.

Mr. Pickel joined the A.S.M.E. as an associate-member in 1917, and was automatically transferred to the grade of member in 1935. He was active in the establishment of the Lehigh Valley Section in 1920, and was one of the promoters in the expansion of the territory to include the anthracite area, which occurred in 1928. He was chairman of the Executive Committee of the Anthracite-Lehigh Valley Section in 1935–1936, and vice-chairman in 1928–1929, 1931–1934, and 1936–1937.

In 1901, Mr. Pickel married Bertha May Hubbs, who survived him though injured in the same accident that was the cause of her husband's death. He was also survived by two sons, Harry A., Jr., living in Ringtown, Pa., and Robert, who was a student at Gettysburg College at the time of his father's death.

In addition to his membership in the A.S.M.E., Mr. Pickel had been a member of the Engineers Club of the Lehigh Valley since its organization in 1921, and served on important committees at various times. He was active in the work of the Boy Scouts of America, and of the Rotary Club. He was a Royal Arch Mason and Knight Templar and a member of the First Presbyterian Church of Hazleton, Pa., in which he served as president of the Men's League.

EDWARD RIVETT (1851–1937)

Edward Rivett, of Allerton, Mass., died on December 16, 1937, of thrombosis of the brain, after a nine weeks' illness.

Mr. Rivett was born on January 5, 1851, in L'Assomption, Que., Can., the son of Antoine and Elise (Marsolais) Rivett. He had only a common school education, and went to work in 1865 in a cotton mill, at Manville, R.I. In the following year he also worked in a cotton mill, in Holyoke, Mass. He then put in two years in shoe shops in Worcester and Lynn, Mass.; then two years as night overseer in a textile factory in Baton Rouge, La. In 1871, he was employed by the N.Y. Watch Co. in its factory in Springfield, Mass., and this started him on what he made his lifework. He went to Boston in 1872, and got work with C. A. W. Crosby, watchmaker and jeweler. He invented a number of watchmakers' tools, and in 1884 started manufacturing tools and lathes with Mr. Crosby as silent partner, under the firm name of the Faneuil Watch Tool Company, at Faneuil, Mass. The first "factory" was a wooden extension of his own home. After Mr. Crosby's death, Mr. Rivett bought out his interest and incorporated the business as the Rivett Lathe Manufacturing Company, Brighton, Mass.

In 1912, Mr. Rivett sold out this business on account of failing health. From then on up to his death, he considered himself as "retired," but he continued to experiment and improve watchmaking tools and machinery, often remarking to friends that he was "too busy to take a vacation."

During his early days with Mr. Crosby, he had his first experience in the buying of diamonds, in which he became an expert.

Concerning his work the *American Machinist*, in its issue of December 29, 1937, said, in part:

"Mr. Rivett believed himself to be the originator of precision in bench lathes and small internal grinders. Unusual mechanical skill, an inquiring mind, and initiative were qualities that aided Mr. Rivett in building precision into his machines. His designs were worked out at the bench and his men built early models from sample pieces. In the search for greater precision he visited many shops in this country and abroad to pick up ideas or materials. From Switzerland he obtained gages, in Turkey he bought a lifetime supply of lapping powder. If he wanted special gib stock, he went directly to the mill and showed them what he needed. In his opinion many modern shops have an impersonal attitude toward their products, and it was his delight to find one where no amount of time or trouble was too much to give in the search for perfection."

Mr. Rivett became a member of the A.S.M.E. in 1904. He was a Knight Templar and a Shriner. He was a skilful fisherman, good shot, and excellent woodsman, and was very fond of such recreation. His wife, Alma (Pinceloup) Rivett, whom he married in 1882, died in 1895. A daughter, Ida F. (Rivett) Lockwood, of Newton, Mass., survives him.

MARTIN LUTHER SIBERT (1908–1937)

Martin Luther Sibert, a junior member of the A.S.M.E., died on March 31, 1937. He was born on December 1, 1908, in Gadsden, Ala., the son of Samuel Houston and Emma Lula (Penny) Sibert, and was a nephew of Major-General William L. Sibert, Retired, of the U.S. Army Corps of Engineers.

He prepared for college in the Gadsden High School and was graduated, with a B.S. degree in mechanical engineering, from the Georgia School of Technology in 1931, later studying electro-acoustical engineering through correspondence courses. During part of 1932–1933 he was engaged in business for himself as a merchant and subsequently was employed by the U.S. Geological Survey as an engineering draftsman and computer, assigned, since August 15, 1934, to the Tennessee Valley Authority, in Chattanooga, Tenn.

Mr. Sibert became a member of the Student Branch of the A.S.M.E. at the Georgia School of Technology in 1928, and transferred to junior membership in the Society in 1932. He was commissioned as second lieutenant in the 345th Infantry Reserve in 1930. He was unmarried.

EDMUND RAY STEVENSON (1859–1935)

Edmund Ray Stevenson was born at Burgettstown, Pa., on April 14, 1859, and received his early education at the public school there, which was in session only for a six months' winter term. Nevertheless, at the age of eighteen he passed an examination qualifying him to teach in the public schools of his native county of Washington. Because of his youthful appearance, however, the superintendent of schools thought it best not to make the certificate valid.

Mr. Stevenson therefore entered upon an apprenticeship in the steamboat shop of James Rees in Pittsburgh, upon the completion of which he secured employment with the Yale & Towne Manufacturing Co., Stamford, Conn. Here he studied steam engineering under James McBride (Mem. A.S.M.E.) and Thomas Shaw, who later founded the Shaw Crane Company. He continued his studies under H. P. Morgan, chief engineer of the Norwalk Iron Works Company, South Norwalk, Conn., where he spent 15 years as machinist, toolmaker, toolroom foreman, supervisor of machine repairs, and designer of jigs, tools, and fixtures. He was also one of the early pupils of the International Correspondence Schools, enrolling in the mechanical course in January, 1901. This course enabled him to teach evening classes in mechanical drawing and mathematics.

He left the Norwalk Iron Works Company to work as tool designer under Joseph E. Aue (Mem. A.S.M.E.) for the De La Vergne Machine Company, New York. During a year there he helped to design, among other things, an attachment for use on planers to plane cylindrical valve stems, a cam grinder, a cylinder boring mill, and a large floor milling machine. For the next five years he was in charge of power and transmission at the plant of the Union Hardware Company, Torrington, Conn., and performed all the millright work. He supervised the installation of considerable new equipment and the construction of a new grinding room.

The remainder of Mr. Stevenson's work was in Detroit, Mich. He was employed by The Timken-Detroit Axle Company in 1915 as tool trouble man and four years later was transferred to the Tool Design Department. He taught in the vestibule or shop training school throughout the World War, and improved the company's practice in the welding of tools by substituting a butt-welding machine for the spot-welding machine that was formerly used to join the high-speed steel and Stellite tips to cold-rolled stock, thereby effecting savings in time and current.

He next went to the Kelsey Wheel Company to design tools for the manufacture of Ford front tractor wheels. This led to his connection, in 1924, with the Ford Motor Company, to design and build fixtures for body assembly; he also installed safety attachments on cranes, and improved the tools used in the plant and the fixtures for repairing them. He was appointed an instructor at the Henry Ford Trade School in 1928 and remained there until 1933.

Mr. Stevenson was especially interested in increasing production by the standardization of parts, and the use of conveying systems instead of men and trucks to move material from one part of the plant to another. In addition to the work already mentioned, he invented an attachment for use in connection with universal cutter and tool grinders, and developed a multiple stamp mill for the Stamford Bronze Company, and a line of machinery for the Boas Thread Company which would print labels, stamp and label spools, and wind thread.

He joined the A.S.M.E. in 1921 as an associate-member but resigned in 1927. He was reinstated as an associate-member in 1933. His death occurred on June 29, 1935. Mrs. Ethel R. Stevenson, of Detroit, survives him.

GEORGE GODFREY STEWART (1888–1937)

George Godfrey Stewart, engineer at the Donora Steel Works of the American Steel & Wire Co., Donora, Pa., died on January 14, 1937, of an embolism following an operation for appendicitis.

Mr. Stewart was born at Merchantville, N.J., on December 23, 1888, the son of Wickliff Baldwin and Justina (Godfrey) Stewart. He was graduated from Temple University in 1907, and from Drexel Institute in 1916. He served a special apprenticeship from 1904 to 1907 as a machinist with the Pennsylvania Railroad ferries at Camden, N.J., and two further years, 1908–1910, in the P.R.R. shops at Hoboken, N.J. During the latter period, he secured considerable engineroom experience on P.R.R. ferry- and tugboats in New York harbor, and before and during his course at Drexel Institute he piloted ferryboats of the P.R.R. between Camden and Philadelphia.

In 1916–1918, Mr. Stewart was inspector of tests in the Test Department of the P.R.R., with headquarters at Altoona, Pa. In 1918–1919, he served as assistant superintendent of shop and field machinery at the Gloucester, N.J., and Bristol, Pa., plants of the U.S. Shipping Board, Emergency Fleet Corporation. He began work for the American Steel & Wire Co. in 1921 as machine operator in the shops at Donora, and in 1922 was made foreman of the Engines and Pumps Department. Retrenchments by the company necessitated his transfer to work as engine repairman in May, 1932, but the following year he was placed in charge of the boiler house, with the title of head engineer, and continued in that position until his death. He held patents on a safety valve and on a key ring.

Mr. Stewart joined the A.S.M.E. as an associate-member in 1928, and was automatically transferred to the member grade in 1935. He was a member of the Engineers Club of Philadelphia from 1914 to 1938. He was also a member of the Masonic order and a Shriner.

In 1920, Mr. Stewart married Margaret M. Pipes of Donora, Pa., who survives him.

WILLIAM FRASER STEWART, JR. (1897–1937)

William Fraser Stewart, Jr., chief engineer in connection with fire prevention and safety engineering work with W. H. Markham & Co., St. Louis, Mo., died at the Central Hospital, in that city, on September 9, 1937, of a duodenal ulcer.

Mr. Stewart was born in St. Louis on January 1, 1897, the son of William Fraser and Daisy Beatrice (Lamb) Stewart. His formal education ended with the completion of three years in the Central High School, St. Louis, after which he served an apprenticeship with David Ranken, St. Louis, and with Hispano Suiza, Paris, France.

In July, 1917, he enrolled in the U.S. Naval Reserve Force at St. Louis, as chief machinist's mate, and was sent to France, where he was assigned to aviation, in charge of bomb sight design and machine shops at St. Trojone. On his return to the United States in 1919 he became superintendent of the American Heater Corporation, St. Louis, designers and builders of automatic water heaters. He held this position until 1922, and then served until 1925 with the Western Electric Company as division inspector, in charge of inspection and tests of automatic telephone exchange apparatus. In 1925, Mr. Stewart was appointed engineer for the Ocean Accident & Guarantee Corp., his duties relating to safety engineering, general consulting work, and the inspection of powerhouse equipment. His association with W. H. Markham & Co., began in 1930. In addition to his responsibilities as chief engineer he served the company in connection with building and equipment values, on which he was well informed.

Mr. Stewart joined the A.S.M.E. as an associate-member in 1929, and was automatically transferred to the member grade in 1935. He was also a member of the Safety Council, St. Louis, Army Ordnance Association, and American Legion, and a former member (1924) of the Flying Club of St. Louis, and belonged to the Masonic Lodge in Clayton, Mo. He founded and was chief of the West Overland (Mo.) Fire Department (volunteer) from 1929 to 1934, when it was disbanded. He continued in the Naval Reserve Force for a number of years after the War, finally accepting his discharge in 1925. He was an excellent shot and made a hobby of collecting old firearms.

He married Marion Clare Williams in 1927, and is survived by her and their three children, William Robert, John Fraser, and Beverley Ann.

ARCHIE MILTON STILES (1878–1937)

Archie Milton Stiles, who had been connected with the Water Division of the Department of Public Utilities of the City of St. Louis, Mo., since 1901, died in that city on March 5, 1937. Mr. Stiles was born in St. Louis on June 9, 1878, the son of Andrew and Catherine (Cathbert) Stiles, and secured his preliminary education in the public schools there; his professional education he obtained through the International Correspondence Schools.

From 1892 to 1896 he worked as machinist-apprentice with the Rankin-Fritsch Foundry & Machine Co. of St. Louis and during the next five years he was machinist with several different companies, making dies, castings, and tools. In 1901, he began his long service with the Water Division of the City of St. Louis. He served until 1905 as machinist and from then to 1912 as foreman of machine, blacksmith, tin, and pattern shops. In 1912, he became inspector for steam boiler and pumping machinery, and held this position until 1920, being resident inspector at various plants throughout the United States, during the manufacture of pumps, boilers, compressors, valves, piping, and general supplies and machinery for the City of St. Louis. In 1930, Mr. Stiles was appointed superintendent of construction of mechanical equipment by the City Water Division, and was so employed up to his death.

He joined the A.S.M.E. as a member in 1930, and was also a life member of the Masonic fraternity and of the Low Twelve Club of Chief Engineers, of St. Louis. He contributed occasional papers to *Power* and *American Machinist*. He was especially fond of opera and vocal music and enjoyed water sports and fishing.

Mr. Stiles is survived by his widow, Marcelina F. (Ryan) Stiles, whom he married in 1930.

GEORGE EDWIN TAYLOR (1886–1936)

George Edwin Taylor, district manager at San Francisco, Calif., for the Chain Belt Company of Milwaukee, died on July 23, 1936. He had been associated with the company since 1923.

Mr. Taylor was born in Chicago, Ill., on January 22, 1886, son of Edwin and Lilla (Knight) Taylor. He took a three-year technical course at the Crane Technical High School and Junior College in Chicago (1901–1903) and an evening course in mechanical drawing at the Chicago Technical College in 1905–1906. After leaving high school he was employed successively in the shops of the Western Electric Company and by the Arthur Frantzen Company, electrical contractors, Chicago, for several years, then became a junior draftsman with the H. W. Caldwell & Son Co., Chicago, where he remained until April, 1912. He then took the position of chief draftsman for the Meese & Gottfried Co., of San Francisco. Subsequently he was put in charge of all work in the engineering department. He helped to develop a full line of power transmitting machinery, embodying friction clutch design, chain, gear, and belt drives. In a letter written in 1921 he mentioned several examples of his work, as follows:

"I have had the pleasure of seeing the successful operation of truck conveyors installed at the Bay piers for unloading ships for the California Navigation and Improvement Company, also large conveying systems for handling sugar for the California and Hawaiian Sugar Refining Company at Crockett, Calif. I have made complete layouts of timber structure and large rock screens for gravel plants such as were installed for the Grant Rock and Gravel Company at Fresno and the Grant Gravel Company at Pleasanton, Calif. We have just completed and shipped five self-propelled automatic trippers for belt conveyors to be used at the Shasta Zinc and Copper Company's plant."

During his years with this company Mr. Taylor also gave evening instruction in mathematics at the Healds Engineering School in San Francisco. In 1921 he joined the R. & J. Dick Co., Inc., of Passaic, N.J., as sales engineer, and he continued in that work until he entered the employ of the Chain Belt Company in 1923.

Mr. Taylor married Anita Leith, of San Francisco, and is survived by her. He enjoyed music, was a baseball, basketball, and football fan, and liked to fish and hunt. He was a charter member of the Cleveland Lodge No. 211, F. & A.M., Chicago, Ill., held the 32d degree in the Scottish Rite, San Francisco Consistory, and was a member of Islam Temple, A.A.O.N.M.S., San Francisco. He became a junior member of the A.S.M.E. in 1919, was advanced to the grade of associate-member in 1921, and automatically transferred to full membership in 1935.

ROGER TAYLOR (1881–1937)

Roger Taylor, superintendent of the plant of the Susquehanna Electric Company at Conowingo, Md., died on April 27, 1937. Mr. Taylor was born on October 7, 1881, at Ashton-under-Lyne, England, the son of Jesse and Mary Schofield Taylor. He received his professional education at the technical school in Ashton-under-Lyne, and later took two years' work in the Drexel Institute of Philadelphia, Pa.

Mr. Taylor's first employment after coming to the United States began in October, 1898, in the Motive Power Department of the Philadelphia (Pa.) Rapid Transit Company, where he worked on boiler and turbine tests and indicating steam engines until August, 1906, when he was appointed engineer-in-charge and assistant for several of the company's reciprocating and turbine plants.

In October, 1912, he joined the Philadelphia Suburban Gas & Electric Co. as superintendent of plants in Pottstown and Phoenixville, Pa. This assignment covered the parallel operation of turbine, hydroelectric, and reciprocating engine plants, considerable new construction, and work on gas plant operation.

In October, 1916, Mr. Taylor joined the Operating Department of the Philadelphia Electric Company. He assisted in the design and construction of the company's Delaware station and served as its superintendent. He also was closely connected with the engineering design and development of the Conowingo hydroelectric project. He remained with this company for nearly ten years, when he became associated with the Susquehanna Electric Company, a subsidiary of the Philadelphia Electric Company, as plant superintendent, which position he held for the remainder of his life.

Mr. Taylor became an associate-member of the A.S.M.E. in 1917 and was promoted to member in 1927. He also belonged to the American Institute of Electrical Engineers. He served on the National Hydraulic Committee of the National Electric Light Association and on the Prime Movers and Hydraulic Committees of the Pennsylvania Electric Association, which represented the eastern geographic division of the N.E.L.A. prior to 1933.

Although born in England, he came a citizen of the United States by his father's naturalization. He married Sarah McIlhenny, of Philadelphia, in 1906, and is survived by her and by their two children, Roger S., an electrical engineer, and S. Eleanor Taylor. Mr. Taylor's brother, Jesse Taylor, Jr., is a member of the A.S.M.E.

CARL CLAPP THOMAS (1872–1938)

Dr. Carl Clapp Thomas died at his residence at Pasadena, Calif., on June 5, 1938. He was born in Detroit, Mich., on July 14, 1872, the son of George Roscoe and Caroline Melissa (Clapp) Thomas. His preparatory education was obtained in the public schools of Detroit and later in those of Pasadena, where his father became an early resident. Carl Thomas entered in 1891 the first class at Leland Stanford University, along with the Hon. Herbert Hoover and others who have since been outstanding personalities in American life. In 1894 he transferred to Cornell University, from which he received the degree of Mechanical Engineer in 1895. In 1937, The Johns Hopkins University honored him by conferring the degree of Doctor of Engineering.

Dr. Thomas was employed for four years after graduation with the Globe Iron Works Company, shipbuilders, Cleveland, Ohio, in the shops, as draftsman, as chief draftsman, and finally as chief engineer. From 1899 to 1901 he served as chief draftsman in the marine department of the Maryland Steel Company, Sparrows Point, Md. He was next professor of marine engineering and naval architecture at New York University from 1901 to 1903, and at the same time maintained a consulting practice as a marine engineer. He was assistant professor of mechanical engineering at the University of California, Berkeley, Calif., during the college year 1903–1904, at which time he served as consultant on the design and building of the U.S.S. *Nebraska*, of dredges for the North American Dredging Company, and of other ships of various types.

In the spring of 1904, the late George Westinghouse engaged Dr. Thomas to carry out some special work for him. Later in the summer Dr. Thomas became interested in the Steam Turbine Department and spent some time on the test floor before going to Cornell University that fall as professor of marine engineering. This interest in steam turbines resulted in the offering by Professor Thomas during the following year of a course of lectures on steam turbines at Cornell University, probably one of the first courses on steam turbines in America, and, later, in the writing of a textbook on steam turbines (John Wiley & Sons, Inc.) which first appeared in 1906 and which ran through four editions. At the time it was written this book formed an outstanding contribution to engineering literature.

The summer of 1905 was spent on experimental work at the General Electric Company's laboratories at Schenectady on the specific heat of superheated steam. This was followed by extensive research on this subject at Cornell which resulted in the contribution of a paper to the A.S.M.E. on "The Specific Heat of Superheated Steam" (Trans. A.S.M.E., vol. 29, 1907, p. 1021). A by-product of this research was the invention and development of the Thomas electric calorimeter for the exact measurement of the quality of wet steam. Professor Thomas spent the summer of 1906 in the steam turbine department of the Fore River Shipbuilding Company, Quincy, Mass. During 1907–1908 he served as president and manager of Thomas and Grant, engineers and boat builders, Ithaca, N.Y.

Following the death of Prof. Storm Bull, Dr. Thomas was called to the University of Wisconsin as professor of mechanical engineering in 1908 and remained until 1913, with the exception of one year spent in research in Europe. At Wisconsin he directed much research, from some of which resulted the invention of the Thomas electric gas meter that has since been used widely throughout the world. Another invention less widely known was a method of supercharging internal-combustion engines. A third contribution during this period was the design of a form of bent tube boiler that has since been widely used. Other investigations covered steam flow, steam nozzles, air measurement, properties of bearings, boiler circulation, and properties of heat insulation materials. Many technical papers were written during this period, two of which appeared in the A.S.M.E. Transactions.

In 1913 he was called to The Johns Hopkins University, Baltimore, Md., to assist in organizing the new School of Engineering and to become its first professor of mechanical engineering, which position he filled until 1920, with the exception of the war years, 1917–1919, when he was on leave of absence to serve as manager of machinery design and fabrication, American International Shipbuilding Corporation, at Hog Island, Pa. This was the largest shipyard ever constructed, and it was here that freighters and transports were built in great numbers to aid America and her Allies in World War I.

While at The Johns Hopkins University he carried on extensive experimental work, and developed devices for the cooling of water in spray ponds, for the recovery of used oil, for the low-temperature carbonization of coal, flotation methods for minerals, etc. He also invented the electric recording gas calorimeter.

In 1920, Professor Thomas moved to Pasadena and became western representative and vice-president of Dwight P. Robinson & Co., Inc., which positions he held until his death. He had a large part in the design and construction of the Seal Beach Station of the former Los Angeles Gas & Electric Co. and of the expansion of the American Potash & Chemical Co.'s plant at Trona, Calif.

Professor Thomas became associate in engineering research at the California Institute of Technology in 1925, and continued in this position until his death. One of the results of his research work at the Institute was the development of a new apparatus to separate liquids from mixtures with vapors.

The contributions of Dr. Thomas to the civic affairs of Pasadena were of no small order. He was one of the Board of City Directors of Pasadena from 1921 to 1924. He was a director and vice-president of the Pasadena Hospital. He was a director of the Pasadena Art Institute and also of the Pasadena Civic Orchestra Association.

Dr. Thomas was a member of the American Society of Naval Architects and Marine Engineers, of The Franklin Institute (from which he received the Edward Longstreth Medal in 1912 for his paper on "The Measurement of Gases," published in the November, 1911, issue of the *Journal* of The Franklin Institute), the American Gas Institute, the American Association for the Advancement of Science, and the Inventors Guild.

He joined the A.S.M.E. in 1908 and served on its Nominating Committee in 1913. From 1913 to 1917 he was a member of the Society's Special Committee on Changes in Patent Laws of the United States. He was chairman of the Committee on Air Machinery from 1916 to 1920; a member of the Main Committee on Research, 1916–1920; and a member of the Power Test Codes Committee on Steam Turbines since 1920. He was a manager of the Society from 1920 to 1923, and a delegate to the American Engineering Council during 1924–1926.

Dr. Thomas was married on July 14, 1899, to Katherine L. Nash. Mrs. Thomas and two sons, Alfred Randall and Roscoe Carl, all of Pasadena, survive him.

Dr. Thomas possessed a keen analytical mind as evidenced by his prolific research and many inventions. He was a capable engineer and an able administrator. In university work, he was an inspiring teacher and was loved by students and associates. He was devoted to good music and throughout his life played the violin with skill and great enjoyment.

Only great dynamic energy and boundless enthusiasm could have enabled Dr. Thomas, in his relatively short life, to accomplish so much in so many diverse fields. Yet, while he was highly respected as an engineer, a scientist, a teacher, and an inventor, it is Carl Thomas the man who is remembered by his friends. He had a genius for friendship. His warm interest in people, his eagerness to help without thought of self, his uncompromising devotion to the highest ideals, combined with tender sympathy and an understanding of life's problems—these are characteristics that will make all who knew him remember Carl C. Thomas as a beloved friend.—[Memorial prepared by A. G. Christie, Baltimore, Md., Fellow and Past-President, A.S.M.E.]

WILLIAM FYFE TURNBULL (1881–1937)

William Fyfe Turnbull, engineering examiner of the New York City Municipal Civil Service Commission, died on May 14, 1937.

Mr. Turnbull was born in Belleville, Ont., Can., on August 9, 1881, the son of Alexander and Mary Esther Turnbull. He prepared for college at the high school in Orange, N.J., and then attended Columbia University, where he was graduated with the A.B. degree in 1903, and took a year of graduate work, 1903–1904. He also took two years' work in mechanical engineering at the Massachusetts Institute of Technology, 1904–1905 and 1906–1907. During his college years, he had some shop experience with the H. K. Porter Co., builders of locomotives, Pittsburgh, Pa. In 1905–1906, he worked as draftsman for the Erie Railroad, and in the same capacity for the N.Y. Central R.R. in 1907.

In 1908 and 1909, Mr. Turnbull served as assistant to the treasurer of the Hampton Institute, Virginia, his work there covering also inspection of the steam and electric light plant and the sewer system. From 1909 to 1915, he was estimator and technical correspondent with the American Locomotive Company, New York, also lecturer in strength of materials for evening classes at Teachers College, Columbia University. From 1915 to 1918, he served as instructor in machine design at the University of Pennsylvania, and in 1918–1919 was attached to the U.S. Department of Labor on war service. Following this assignment, Mr. Turnbull resumed teaching, serving on the faculty of Yale University, 1919–1921, and New York University from then until September, 1927, when he was employed by the Third Avenue Railway System of the City of New York. Here he worked on shop drawings, road tests, mechanical improvements, traffic surveys, cost of operation of buses, and proposed trolley buses. In 1934 he was appointed to the New York City Municipal Civil Service Commission as engineering examiner.

Like many engineers who divided their professional labors between teaching and commercial lines, Mr. Turnbull did considerable independent and consulting work. He was licensed as a professional engineer in the State of New York in 1924. This work included surveys for the West Shore Railroad, Erie Railroad, and Central Railroad of New Jersey, for which he also checked all steel-work drawings for the Newark Bay Viaduct.

In 1916, he married Clara Kramer, of Tuckahoe, N.Y., who survives him. They had no children. Mr. Turnbull was an American citizen, having been naturalized in 1902, at Newark, N.J. He became an associate of the A.S.M.E. in 1913, but resigned in 1917. However, he was reinstated in 1923 with the grade of member. Several brief contributions by him, dealing with engineering economics, were published in *Mechanical Engineering* and he served on the Subcommittee on Economics of the Management Division of the Society in 1933–1935. He was also a member of the Society for the Promotion of Engineering Education and the New York Railroad Club.

ARTHUR WILLIAMS (1868–1937)

Arthur Williams, former vice-president of the New York Edison Company, died on April 14, 1937. Mr. Williams was born on August 14, 1868, at Norfolk, Va., the son of Rev. Christopher Stephen and Hannah Sanford (Rogers) Williams, and was educated in public and private schools in Hartford, Conn., and in New York, N.Y.

He began work in August, 1884, with the firm of Rennie & Smith, electrical contractors, New York. In February, 1885, he entered the service of the Edison Electric Illuminating Company of New York as assistant in the Chemical Meter Department, becoming superintendent of interior construction in 1887; electrician of the company later in 1887; superintendent of the third district, 1888, and of the Underground Department, 1889; general inspector, 1890, and general agent, 1893. From then on, until 1915, he exercised the function of these two positions in this company and in the New York Edison Company, into which it was merged in 1901. He was then appointed general commercial manager, and in 1924 he was made vice-president in charge of commercial relations, from which position he retired in 1928.

Besides his lifework in the electrical industry, Mr. Williams was active in many other fields. In 1917, he was appointed to the post of Federal food administrator for New York, N.Y., and served in that capacity until his resignation in 1920. During the Spanish-American War, he served as commanding officer of the New York Volunteer Defense Forces, mining New York Harbor. In 1912, he was decorated by the King of Spain, receiving the rank of Knight in the Royal Order of Isabel the Catholic, in recognition of his work in the interests of safety; and in 1926 he was made a Chevalier of the French Legion of Honor.

Mr. Williams joined the A.S.M.E. as an associate in 1915, and was a Fellow of the American Institute of Electrical Engineers. He was a past-president of the American Association for the Advancement of Science, Association of Edison Illuminating Companies, National Electric Light Association, New York Electrical Society, and American Museum of Safety, and was chairman of the Greater New York Safety Council, 1936–1937. He had served as president of the Electrical Board of Trade and of the Electrical and Industrial Exhibition, held annually at Grand Central Palace, New York, for a number of years. He was also a member of the American Academy of Political and Social Sciences, American Electrochemical Society, American Association for Labor Legislation, International Industrial Relations Association, Society of American Military Engineers, and other technical and civic organizations, as well as of many clubs. He was the author of numerous papers on municipal ownership, electrical subjects, relations with employees, and allied topics.

Mr. Williams was deeply interested in social and civic welfare. He was unmarried, and made his home in Roslyn, Long Island, of which village he was a trustee.

RUDOLF WINTZER (1873–1937)

Rudolf Wintzer was born on May 14, 1873, at Iserlohn, Westphalia, Germany. He was one of four children of Marie Koenig and Wilhelm Wintzer. During his early life, his father acquired a factory at Guetersloh, Germany, at which wagon wheels were made. Upon the death of his father, the family established a home in Berlin. Mr. Wintzer completed his education at the Technische Hochschule, Charlottenburg, and for several years worked in various machine shops and foundries in Germany. From 1896 to 1900 he worked under Professor Riedler, consulting engineer, Charlottenburg, and later became his assistant. This work related particularly to the design of pumping and blowing engines, compressions, and all kinds of machinery for mines and iron and steel works. He was also assistant at the technical high school. In 1901–1902 he had full charge of the drafting room and shop of the Stahl und Eisen Aktiengesellschaft at Hörde, Germany, manufacturing steam engines and boilers.

His work with Professor Riedler prepared him for his next position, which was with the Westinghouse Works, at Manchester, England, where he went in 1903. From there he was called to the United States by the Guggenheim interests for consultation, and after completion of his work with them, again returned to Manchester, where he was active for several years, designing and testing large gas engines. He was finally again called to the United States by the Guggenheim organization and engaged by them for carrying on work then being done at the Snow Steam Pump Works of the International Steam Pump Company, at Buffalo. In 1906 or 1907 he became chief engineer of the Power & Mining Machinery Co., at Cudahy, Wis., which in 1906 had become part of the International Steam Pump Company. During the next few years he was kept busy traveling for the company and in work at the Buffalo and the Cudahy works.

In 1911 or 1912, while seeking other employment, he contacted the Nordberg Manufacturing Company, Milwaukee, and was engaged by the Peck Concentrator Company to design a special separator, which work he carried on in the shops of the Nordberg Manufacturing Company. He was finally employed by the Nordberg Manufacturing Company, toward the end of 1912, and remained with it until his death. Between 1922 and 1924 he acted as assistant to Dr. Bruno V. Nordberg, at that time chief engineer, and upon the latter's death in 1924, Mr. Wintzer became chief engineer of the company. Having led a very active life, he was finally appointed as consulting engineer in 1935 and his assistant, Emil Grieshaber, thereupon took over his duties as chief engineer.

During his years with the Nordberg Manufacturing Company, Mr. Wintzer developed a number of interesting units. While associated with Dr. Nordberg, he detailed and carried out special designs on a very successful hydrogen compressor. With Dr. Nordberg, he also developed large rolling mill engines, and later on, when he assumed the position of chief engineer, this experience stood him in good stead. He thereafter developed the largest hydrogen compressors so far built, as well as some very heavy rolling mill engines, river boat engines, compressors, pumps, and the like. During his lifetime, he invented a number of things and held twelve patents, either in his own name or in combination with others. He was continually striving to improve any device which came to his notice and his intense interest in all mechanical means kept him active along these lines until the last.

Mr. Wintzer was of a very retiring nature, but traveled a great deal and formed many acquaintances and friends. He often spoke of the particularly happy years spent in association with Dr. Nordberg. It seems both men recognized in each other those engineering attributes which sometimes lead to lasting friendships. His death, from heart failure, occurred on December 16, 1937. He is survived by his widow, Emmy Wintzer, whom he married in 1910, and by two boys, Herman and Fred, both of whom are engineers in the employ of chemical industries in this country, and a sister living in Hamburg, Germany. He had been a member of the A.S.M.E. since 1908.

IN TWO SECTIONS—SECTION TWO

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of the

A.S.M.E.

SOCIETY RECORDS—Part III

(Including Indexes to Publications)

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Library of the Escuela de Ingenieros Mecanicos y Electricistas

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Portugal

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Svenska Teknologföreninger
Gothenburg.....Chalmers Tekniska Institut

Switzerland

Zurich.....Bibliothek der Eidg. Technischen Hochschule

Turkey
Istanbul.....Robert College

U.S.S.R.
Kharkov.....Supreme Economic Council of Ukraine
Leningrad.....Leningrad Polytechnic Institute
Moscow.....Supreme Council of National Economy
Tomsk.....Tomsk Polytechnic Institute

Wales
Cardiff.....Cardiff Public Library

Indexes to A.S.M.E. Papers and Publications

THIS and the following pages will serve as a guide to the current publications of the A.S.M.E. during the calendar year 1940, and also to publications developed by the technical committees.

Regular Society Publications, 1940

Mechanical Engineering, monthly (see index on page RI-79)
A.S.M.E. Transactions, monthly (see index on page RI-89)
Mechanical Catalog and Directory, 1941 edition

Special Publications Issued in 1940

1939 Oil Engine Power Cost Report
1940 Proceedings of the Oil and Gas Power Division
Autobiography of an Engineer by William LeRoy Emmet
Autobiography of John Fritz

Boiler Construction Code, 1940 Edition

Locomotive Boiler Code
Low-Pressure Heating Boiler Code
Miniature Boiler Code
Power Boiler Code Including Rules for Inspection
Unfired Pressure Vessel Code
Specifications for Materials
Suggested Rules for Care of Power Boilers

Power Test Codes

Evaporating Apparatus
Steam Locomotives

Auxiliary Sections

Part 3—Temperature Measurement, Chapter 8
Part 5—Chapter 4, Flow Measurement by Means of Standardized Nozzles and Orifice Plates
Part 11—Determination of Quality of Steam

How to Find Papers Presented at 1940 A.S.M.E. Meetings

THE technical programs of the meetings of the Society and of its Professional Divisions have been published in *Mechanical Engineering* and may be located by consulting the index on pages RI-79–88. A majority of these papers were published, or will be published, in *Mechanical Engineering* or the Transactions (including the *Journal of Applied Mechanics*) and may be located by reference to the indexes of these publications. Several additional papers and reports included in these 1940 programs were not published during the year in either Transactions or *Mechanical Engineering*, but were issued in mimeographed or photo-offset form.

Complete sets of these are on file for reference purposes at the office of the Society and the Engineering Societies Library, under the title of "Miscellaneous Papers Presented at A.S.M.E. Meetings, 1940." Photostat copies of any of the papers may be secured from the Library at twenty-five cents a page to members.

Publications Developed by the Technical Committees

THE Society's technical committees, the first of which was organized many years ago and all of which have been continuously at work on codes, standards, research, and other special reports, have developed a series of publications of permanent value to the membership. The following list is presented here for record and for ready reference. This list covers the entire group of publications of these committees completed to date which are now available.

To assist the members in securing copies of these publications the sale price is also given. A discount of 10 per cent is allowed to A.S.M.E. members on standards and a 20 per cent discount on all other publications except where otherwise noted.

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Bolt, Nut, and Rivet Proportions

Large Rivets (B18.4—1937), $0.65
Plow Bolts (B18f—1928), $0.35
Round Unslotted-Head Bolts (B18.5—1939), $0.50
Slotted-Head Proportions: Machine Screws, Cap Screws, and Wood Screws (B18c—1930), $0.45
Small Rivets (B18a—1927), $0.30
Socket Set Screws and Socket-Head Cap Screws (B18.3—1936), $0.40
Tinners', Coopers', and Belt Rivets (B18g—1929), $0.35
Track Bolts and Nuts (B18d—1930), $0.40
Wrench-Head Bolts and Nuts and Wrench Openings (B18.2—1933), $0.50
Price of Set of Standards on Bolt, Nut, and Rivet Proportions, including binder, $4.55

Piping and Pipe Fittings

Brass Fittings for Flared Copper Tubes (A40.2—1936), $0.35
Cast-Iron Pipe Flanges and Flanged Fittings for 25 Lb Maximum Saturated Steam Pressure (B16b2—1931), $0.40
Cast-Iron Pipe Flanges and Flanged Fittings for 125 Lb Maximum Saturated Steam Pressure (B16a—1939), $0.60
Cast-Iron Pipe Flanges and Flanged Fittings for 250 Lb Maximum Saturated Steam Pressure (B16b—1928), $0.50
Cast-Iron Pipe Flanges and Flanged Fittings for 800 Lb Maximum Hydraulic Pressure (B16b1—1931), $0.35
Cast-Iron Soil Pipe and Fittings (A40.1—1935), $0.65
Cast-Iron Long Turn Sprinkler Fittings for 150 and 250 Lb Maximum Saturated Steam Pressure (B16g—1929), and Addendum (B16g1—1937), $0.50
Cast-Iron Screwed Fittings for 125 and 250 Lb Maximum Saturated Steam Pressure (B16d—1927), $0.35
Code for Pressure Piping (no discount) (B31.1—1935), $1.00
Face-to-Face Dimensions of Ferrous Flanged and Welding End Valves (B16.10—1939), $0.55
Malleable-Iron Screwed Fittings for 150 Lb Maximum Saturated Steam Pressure (B16c—1939), $0.50
Pipe Plugs (B16e2—1936), $0.35
Scheme for the Identification of Piping Systems (A13—1928), $0.50
Steel Pipe Flanges and Flanged Fittings for 150 to 2500 Lb Maximum Steam Service Pressure (B16e—1939), $1.25
Wrought-Iron and Wrought-Steel Pipe (B36.10—1939), $0.50

Letter and Graphical Symbols and Charts

Aeronautical Symbols (Z10e—1929), $0.35
Drawings and Drafting-Room Practice (Z14.1—1935), $0.50
Engineering and Scientific Charts for Lantern Slides (Z15.1—1932), $0.50
Graphical Symbols for Electric Power and Wiring (Z10g2—1933), $0.20

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